Hierarchy in Anarchy

Incentives to Discourage Mutiny in Criminal Organisations

Simon van Tartwijk

Student number: 430341

Supervisor: Dr. Jurjen Kamphorst

Second Assessor: Dr. Dana Sisak

Bachelor Thesis Economics & Business Economics

ERASMUS SCHOOL OF ECONOMICS
Erasmus University Rotterdam
The Netherlands

15 July 2019

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Simon van Tartwijk*

Abstract

In a criminal organisation, a leader is not protected by laws: the agents in the organisation can depose him or her by starting a mutiny. In this paper, I study, by means of a game-theoretical model, how the distribution of the organisation's profits is affected by the threat of mutiny. Leaders who give higher wages are better able to incentivise agents to not mutiny. Furthermore, in larger organisations, leaders will offer lower wages even though agents are more eager to mutiny, which results in mutinies occurring more often. Additionally, leaders who make higher profits will offer higher wages and therefore face fewer mutinies.

1 Introduction

Economists often view the government as a disturbing force, meddling with the smooth working of economic processes that result in efficient outcomes through the mechanism of the invisible hand of Adam Smith (1838). However, even the father of laissez-faire thinking believed the government did have an important role to play in society: most importantly, the government needs to administer justice and enforce property rights.

Some organisations, however, exist outside the reach of the government. These extralegal organisations have the same need for order as any organisation operating within the confines of the law. To satisfy this need, even in the absence of government, forms of law and order emerge in a wide range of environments (Skarbek, 2012).

Amongst criminals, organised crime arises as a consequence of the unmet need for protection when the government can or will not provide this (Skarbek, 2012). The criminal organisation can become, in this sense, the governing power within the criminal environment. As these criminal organisations grow, coordinating activities between members gets more difficult, as does monitoring behaviour and enforcing cooperation (Piano, 2017). Because of these difficulties, the organisation develops a hierarchy, often with —to some extent— autocratic rule.

*Erasmus School of Economics, Erasmus University Rotterdam

Since the autocratic leaders of criminal organisations are not protected by laws, the agents can depose him or her by starting a mutiny. The purpose of this paper is to explain how the distribution of the organisation's profits is affected by this threat of mutiny.

When one thinks of mutiny, pirates will often come to mind. The organisation aboard a pirate ship provides a prime example of an extralegal organisation. Once aboard the ship, the crew could not get off, and they found themselves inside a miniature society, in which abiding by the rules was of paramount importance. Inside this society, order had to be maintained, payments had to be distributed, and discipline had to be administered (Leeson, 2007). The establishment of the office of captain was the solution, and was essential for the undertaking's success. On some ships, the captain's authority was even formalised by a code resembling a constitution (Leeson, 2009). However, a captain who abused his privilege was sure to die with his boots on, as the crew appreciated fairness above all.

Another example of a type of organisation that operates by its own laws is that of the one-percent motorcycle clubs. Originally a group of men hungering for a life free of law (not unlike pirates, but more closely resembling outlaws in the Wild West), these motorcycle enthusiasts created a community for individuals with violent predispositions (Quinn & Koch, 2003). These clubs evolved into criminal organisations with tens of thousands of members, engaging in grievous criminal activity, such as drug-dealing and prostitution (Piano, 2017). At the head of the organisation is the leader of the mother chapter, the subgroup that enforces smooth workings between the other chapters.

Such hierarchical structures as aboard pirate ships and in one-percent motorcycle clubs, are found in nearly every criminal organisation. Other examples of organisations operating in a highly hierarchical manner are the Sicilian maffia (Gambetta, 1996) and large drug-dealing organisations (Kostelnik & Skarbek, 2013). That said, not every leader of a criminal organisation will be loved by his or her subordinates and successful in the execution of his or her tasks. More than anywhere else, the stability of the leader's position is unsound in the criminal organisation and rebellions are likely.

To examine the position of the criminal organisation's leader more closely, let us look back to the example of the pirate ship. Because of the closed character of the pirate organisation, it gives a clear glimpse into the mechanisms that regulate criminal institutions (Leeson, 2007). On the pirate ship, a captain who was cruel or cowardly or demonstrated poor judgement, got deposed by his or her crew (Leeson, 2007). However, mutiny — where one pirate would try to appropriate the position of captain for him or herself— did not occur as often as one might think. This was largely because captains often did not manage to appropriate a large share of the booty, since the captain had to incentivise the crew to abide by his or her rule, leaving little residual for the captain to claim. In many instances the captain did not even get a cabin, and would sleep with the crew (Leeson, 2007). That does not hold for every captain though. Famous pirate captains, such as Henry Avery —who became captain through mutiny himself (Baer, 1994)— and Edward Teach, also known as Blackbeard, would not have made the history books if they could not even manage to keep their own crew in check, and they were exceedingly wealthy, so they had a high residual claim.

A leader of an organisation, or a captain of a pirate ship, can have many ways of incentivising his agents. Clark and Wilson (1961) distinguish three such incentives. First, a leader can give material incentives, which are any form of material reward, i.e., paying a higher wage. Furthermore, solidary incentives can be used. These are incentives unrelated to the organisation's ends, such as good working conditions. Finally, agents can receive purposive incentives, which come as a result of the utility the agents obtain by fulfilling the goals of the organisation.

On most pirate ships, the rules of conduct as laid out in the pirate code were adopted (Leeson, 2014). This means solidary incentives were not the captain's to give. He or she had to treat the crew in a 'lawful' manner. Purposive incentives are usually associated with working for charities. Those who work there feel good about themselves by accomplishing their tasks. These incentives might have played a role for pirates. Pirates lived aboard their ships with the same crew every day. If their miniature society didn't fare well, the mood on the ship would turn sour. So, a skilful leader, who is able to steer the crew towards great treasures, incentivises the crew not only by providing them fortune (material incentives), but also by giving them a sense of purpose (purposive incentives). A skilful captain might then not have had to share the booty as equally as he or she would have had to do otherwise.

In this paper, I study how the threat of mutiny affects wages in criminal organisations. I model the criminal organisation as a structure with one principal who is the residual claimant of the profits, and multiple agents who are employed by him. I study why some leaders are more likely to face mutinies than others, and analyse how a leader can incentivise his or her agents to not engage in mutiny.

For the mutiny decision in my model, it is crucial that the agent who instigates the mutiny is the one who becomes leader if the mutiny is successful. The mutineer is motivated to revolt, because he or she wants to obtain the high residual claim of the leader, or because he or she is dissatisfied with the leader's performance, and believes he or she can do better. If the mutineer manages to sway the other members of the organisation, he or she can dispose of the leader. However, swaying the other members, will be more difficult if the leader has kept the agents satisfied, either by giving them high wages or by making the organisation successful. A mutineer who starts the mutiny, but fails to inspire his or her fellow agents, will be 'made to walk the plank'.

I consider a static model in which one random member gets the opportunity to start a revolt. This allows me to focus on the most interesting aspect of the model —how the principal can use material incentives to dissuade his or her agents to mutiny— without overly complicating the decision of the mutineer. In addition, this is realistic, because if a mutineer managed to 'steal' the loyalty of the other agents to become leader, these agents are not likely to change allegiance immediately after. And, in case of an unsuccessful mutiny, the leader will be wary of other mutinies or will be actively working to ensure the agents are more complacent.

The principal can improve the odds of staying in charge by giving a higher wage, but the effectiveness of this measure depends on how the agents —other than the mutineer— choose to allocate their loyalty. In fact, the nature of the competition for the loyalty of the other agents between the incumbent and the challenger constitutes most of the dynamics surrounding the stability of the leader's position. I study four different competitions. First, I consider a competition in which the allocation of loyalty cannot be influenced by either the leader or the mutineer, and in which the likelihood of winning is therefore exogenous. Relatedly, I examine the situation in which a principal with better leadership skills has a higher chance of winning; both incumbent and challenger cannot affect the competition here either. Then, I consider a competition in which the principal can buy the loyalty of the other agents through higher wages. And, finally, I consider a competition in which the leader can buy loyalty, but in which the mutineer can try to outbid the leader.

In my model the focus is on how the leader of a criminal organisation can remain authoritative towards his or her agents, whereas most theoretical models on organised crime are concerned with how contact with the law can be avoided. For example, Jennings (1984) models how criminal organisations tend to operate in those crimes where the possibility of collaboration with the police by the agents is the smallest, and Baccara and Bar-Isaac (2008) model how higher cooperation within a criminal organisation increases detectability by police. In these studies the criminal organisation was not modelled as hierarchy but as a joint effort of agents.

Other studies have modelled organised crime from the perspective of the leader of a criminal organisation. Garoupa (2000) modelled a criminal organisation, where the principal extracts rents from the agents through extortion, in line with Konrad and Skaperdas (1997, 1998). Garoupa (2000) did, however, not consider how criminals move up in the organisation and go from being the extorted to the extortionist, but only how transfers from individual criminals to the criminal organisation affect social welfare and expenditures on law enforcement.

Furthermore, criminal organisations in which a principal faces disloyalty by agents who cooperate with police, are studied in a number of papers. Garoupa (2007) models a principal who decides on the optimal size of the criminal organisation, and models how this decision is affected by the choice of expenditures on detection of crimes by the government. Similar to me, he models the criminal organisation as having one principal, who decides on the labor contract, and several agents who expose him to danger. Unlike me, he assumes that danger is that the principal has to pay a sanction if he is detected by the police as a result of the disloyalty of an agent, but after the sanction continues to run the enterprise. This means that the leadership position of the principal is perfectly stable, in contrast to my model. Gamba, Immordino, and Piccolo (2018) studied how criminal bosses remain in power through subverting the law by bribing public officials, and how leniency programmes help to avoid this. In this model, the agent can also exhibit disloyalty through cooperating with the police. The person the leader has to incentivise is in this game not the agent, but a third person —the public official— who can neutralise the effect of the disloyalty of the agent. No attention is given to who will lead the criminal organisation in case a boss failed to protect his position.

Additionally, a principal who is confronted with agents displaying disloyalty through stealing is considered as well. Polo (1995), like me, analyses the internal rather than the market relations of a criminal organisation. He models the ability of a principal of a criminal organisation to incentivise his or her agents to be loyal or to punish opportunistic behaviour by agents. He assumes punishment is not costly, and serves solely as threat for the agents. The principal's position is not compromised if he fails to punish the agent. Similar to Polo (1995), Bravard, Durieu, Kamphorst, Roche, and Sémirat (2018) studied how a boss of a criminal organisation can incentivise his or her workers to not steal from the organisation, by either offering a high wage or threatening to punish the worker. In this model punishment is assumed to be costly. Furthermore, a leader who fails to discourage the workers to steal, loses his or her position. However, unlike in my model, here it is assumed that a new leader is found outside the organisation.

As far as I am aware, the current paper is the first in which disloyalty by agents in the form of mutiny is analysed.

This paper is structured in the following manner. I will firstly discuss the model in section 2. In section 3, I will analyse the four different variants of the mutiny game. Then, in section 4, I will discuss an extension to the model. Lastly, I will summarise and discuss my findings in section 5.

2 Model

This static game represents the decision of an agent in an extralegal organisation to mutiny, and how the leader of that organisation can discourage the agent to start the mutiny. Two types of players exist, both risk-neutral. First, there is one principal who is the head of the organisation, referred to as she. In the model, she is denoted by subscript P. The principal decides on a non-negotiable distribution of the organisation's revenue. Second, there is a finite number of agents, who work for the organisation, $A = \{1, \ldots, n\}$. One of these agents gets the opportunity to mutiny against the leader.

The game starts with an organisation with one principal with a given, randomly chosen, leadership skill level $S_P \sim [0, 1]$, and n agents. Every agent also has a skill level S_i, which is uniformly distributed on the interval $[0, 1]$. I assume the agent's skill level is private information. The revenue that the organisation will make is determined by the skill level of the principal and the number of agents, such that the revenue function is $R(S_P, n)$. For simplicity, it will be set to: $R(S_P, n) = S_P \cdot n$. I assume every player is perfectly informed about this revenue function, and therefore about the skill level of the principal. Given the revenue that the organisation will make, the principal sets for all agents an equal wage w. As leader of the organisation she is the residual claimant of the revenue. Therefore, the organisation's profit is her payoff: $\pi(S_P, w)$.

After the principal has made her proposition, one agents i, further referred to as he, gets the choice to accept the distribution, or, to start a rebellion. Should agent i decide to engage in mutiny, the principal and he enter a competition for the loyalty of the other agents.

The probability of winning for the agent would in reality depend on a number of factors. For that reason, I consider different probabilities in this model to try and capture the essential elements. First, I start with a basic competition: I assume the probability of winning is determined by an exogenous factor, $\rho \sim U[0,1]$, which is known by everyone. With this probability the model is easiest and gives elementary insights. Then, I consider the situation where the outcome of the competition depends only on the leadership skill of the principal, such that the probability of winning for the agent becomes: $\rho = 1 - S_P$. This probability captures the fact that a stronger leader will be better able to sway the agents to her side, in addition to the fact that her agents will be more satisfied, because they receive purposive incentives. Then, I study the situation in which the principal can buy loyalty, because the probability of winning depends on the wage the agents get. This probability captures the fact that agents are more likely to mutiny if they are unhappy with the way they are treated. Agents who receive higher material rewards are more likely to stay loyal to the leader. If the principal offers a low wage even though she expects to make high profits, more agents will join the cause and the chances of the mutiny being successful increase. The probability of winning the competition for the mutineer is therefore extended to the following: $\rho - w$. Finally, and most interestingly, I will allow for the mutineer to offer a new wage w' to the other agents in order to persuade them to join his side. The probability of winning the competition for the challenger then depends on the following: $\rho + \frac{w'-w}{w}$.

If the odds do not favour the agent, and he fails to inspire his fellow agents to join him in the rebellion, and therefore, loses the competition against the principal, he will die and leave the game having received no payoff whatsoever. I assume no further mutinies can be started. Then, a new agent is hired to replace the mutineer, wages are paid, and the leader claims her residual.

However, if the agent succeeds in usurping the principal's position, the principal will die and leave the game. The agent then becomes the new leader himself. I assume that after a successful mutiny takes place, no further mutinies can be started. Consequently, the agent's skill level will now determine the profits. I also assume the agent cannot lower the wage that was already offered to the agents by the previous leader, since that would lose him the loyalty of the agents that just helped him become the leader. Furthermore, with the first three probabilities he has no reason to increase the wage, since no other agents will mutiny. Only when the competition depends on the wage increase, w', he promises, will he give a higher wage to the agents. If he does not uphold his promise, he will be deposed. Next, a new agent is hired at the same wage as the other agents to replace the now open position of the mutineer. Then all wages are paid, and agent i claims the residual profits.

To conclude, I will briefly summarise all the steps involved in this game.

1. The game starts with a leader with skill $S_P \sim U[0,1]$, who generates revenue, $R(S_P, n)$.
2. The leader decides on a wage w and announces this to the agents.
3. Agent i gets the opportunity to mutiny.
4. If i chooses not to mutiny, the distribution is accepted, and payoffs are paid out.
5. If i chooses to mutiny, the leader and the agent enter a competition. The mutiny is successful with probability:
   (a) $\Pr(success) = \rho$
   (b) $\Pr(success) = 1 - S_P$
   (c) $\Pr(success) = \rho - w$
   (d) $\Pr(success) = \rho + \frac{w' - w}{w}$
6. Outcome competition:
   (a) If the principal wins, i leaves the game, a new agent is hired, and payoffs are paid out.
   (b) If agent i wins, the leader leaves the game, agent i becomes the new leader, a new agent is hired, and payoffs are paid out.

3 Analysis

3.1 Exogenous probability of winning the mutiny

In order to gather the elementary lessons of the model, I will start with the most basic competition for the loyalty of the agents (from now on referred to as 'the competition'), i.e., a probability of winning the mutiny that does not depend on any endogenous factors, but on some random probability ρ that is known to all. This allows me to illustrate the underlying dynamics affecting a mutiny within a criminal organisation.

The mutiny decision

Consider the last decision that is made in the mutiny game. For any agent i, it holds that he is willing to mutiny against his principal if the chance of winning the competition, ρ, times the profits he will make as leader is larger than the wage he would earn as an agent under the current principal. This decision therefore, depends strongly on the wage the principal sets. Firstly, because the wage is the agent's payoff if he does not mutiny, and, secondly, because he will not be able to lower this wage if he were to become the leader himself. The profits the mutineer makes as leader are the revenue he is able to generate with his skill level minus the wage costs, such that his profits are $n(S_i - w)$. Agent i will decide to engage in mutiny if the following condition holds ($w \geq 0$):

$$\rho n(S_i - w) \geq w \tag{1}$$

The principal anticipates that agents will follow this rule, and from that she can deduce how the percentage of agents willing to mutiny changes depending on the wage she offers. The leader recognises that the agent who is indifferent to mutinying, has the following skill level S_A^* [1].

$$S_A^* = \frac{\rho n + 1}{\rho n} w \tag{2}$$

Every agent with a skill level above S_A^* is willing to mutiny. The fraction of agents willing to mutiny $(1 - S_A^*)$ is affected by ρ, n and w in the following ways. Firstly, a higher probability of winning the competition leads to a larger fraction of agents willing to mutiny and a higher wage leads to a lower fraction of agents willing to mutiny. Furthermore, a higher number of agents, n, leads to a larger fraction of agents being willing to mutiny. This stems from the fact that becoming a principal becomes more attractive the more agents there are in the organisation, because more revenue is made, and a potentially higher residual can be claimed. These observations are summarised in the following Lemmas.

Lemma 1 *Consider the mutiny game with an exogenous probability of winning the competition. The threat of mutiny is smaller when wages are higher.*

Lemma 2 *Consider the mutiny game with an exogenous probability of winning the competition. Mutinying is more attractive in larger organisations.*

The wage-setting stage

The agents' responses to the wage the principal sets, enables the principal to determine her expected profits. Her profits depend on the revenue she is able to generate and the wages she has to pay, $\pi = n(S_P - w)$. Next, her expected profits also depend on the probability that she stays in charge, which in turn depends on the wage she sets. The probability that she stays in charge is equal to the sum of the probability that no-one mutinies ($\Pr(S_i \leq S_A^*) = S_A^*$) and the probability that if someone mutinies, the mutiny fails ($\Pr(fail|S_i \geq S_A^*) = (1 - S_A^*)(1 - \rho)$). This leads to the following expected profits:

$$E[\pi] = n(S_P - w)\left(S_A^* + (1 - S_A^*)(1 - \rho)\right) \tag{3}$$

[1] $\rho n(S_A^* - w) = w \Leftrightarrow S_A^* - w = \frac{w}{\rho n} \Leftrightarrow S_A^* = \frac{w}{\rho n} + w = \frac{\rho n+1}{\rho n} w$

By substituting equation 2 for S_A^* in equation 3, the objective function of the principal is found[2].

$$E[\pi] = n(S_P - w)\left(1 - \rho + \frac{\rho n + 1}{n} w\right) \tag{4}$$

Taking the derivative of this expression with right to w gives the wage-setting rule that the principal will use[3].

$$w = \frac{1}{2}\left(S_P - \frac{1-\rho}{\rho + \frac{1}{n}}\right) \tag{5}$$

The wage-setting rule leads to the following insights[4]. A more highly-skilled leader will set a higher wage. However, she will also claim a higher residual, since the principal pays a maximum w of $\frac{S_P}{2}$ to the agents. That means, that the residual she gets is at least $\frac{S_P}{2} \cdot n$. A more highly-skilled leader thus pays a higher wage, because she has more to lose. From equation 2 it can be observed that a more highly-skilled leader will —because of the higher wage— also face fewer mutinies. Furthermore, a higher chance of a mutiny being successful increases the optimal wage. This is a result of higher expected costs of mutiny. The principal want to decrease the likelihood of a mutiny occurring when she is more likely to lose the mutiny. Finally, a higher number of agents leads to a lower wage. This results from a higher cost of discouraging the mutiny, since the leader has to pay all n agents the higher wage. The basic results are summarised in the following lemmas.

Lemma 3 *Consider the mutiny game with an exogenous probability of winning the competition. In organisations with more agents wages are lower.*

Lemma 4 *Consider the mutiny game with an exogenous probability of winning the competition. A highly-skilled leader will offer higher wages.*

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[2] $E(\pi) = n\left(S_P - w\right)\left(\frac{\rho n+1}{\rho n} w + \left(1 - \frac{\rho n+1}{\rho n} w\right)(1-\rho)\right) = n(S_P - w)\left(1 - \rho + \frac{\rho n+1}{n} w\right)$

[3] $\frac{\partial E[\pi]}{\partial w} = n\left(-(1-\rho) + \frac{\rho n+1}{n}(S_P - 2w)\right) = 0 \Leftrightarrow S_P - 2w = \frac{n(1-\rho)}{\rho n+1} \Leftrightarrow w = \frac{1}{2}\left(S_P - \frac{n(1-\rho)}{\rho n+1}\right) = \frac{1}{2}\left(S_P - \frac{1-\rho}{\rho+\frac{1}{n}}\right)$

[4] Potentially, for some values of ρ, n and S_P, the wage-setting rule leads to an optimal wage such that S_A^* would become bigger than 1 (or the probability of no mutiny occurring would be larger than 1), which is impossible. Such a wage strategy is not optimal, since the principal gives an incentive wage higher than necessary to discourage all agents from mutinying. Therefore, I need to check if at some skill level S_P of the leader, given ρ and n, the principal sets the wage too high and should not follow the wage-setting rule anymore. For this purpose, equation 6 is set equal to 1. This gives $\overline{S_P}$ that signifies the highest skill level of the leader that should still follow the wage-setting rule.

$S_A^* = \frac{\rho n+1}{\rho n}\left(\frac{1}{2}\left(S_P - \frac{1-\rho}{\rho+\frac{1}{n}}\right)\right) = \frac{\rho n+1}{2\rho n}\frac{\left(\rho+\frac{1}{n}\right)S_P-(1-\rho)}{\rho+\frac{1}{n}} = \frac{(\rho n+1)S_P - n(1-\rho)}{2\rho n} = 1 \Leftrightarrow (\rho n+1)\overline{S_P} - n(1-\rho) = 2\rho n \Leftrightarrow (\rho n+1)\overline{S_P} = \rho n + n \Leftrightarrow \overline{S_P} = \frac{\rho+1}{\rho+\frac{1}{n}}$

By observing this condition, the following becomes clear. If n is equal to 1, then $\overline{S_P}$ will be equal to 1 too. Further, given an n bigger than 1, $\overline{S_L}$ will be bigger than 1 as well. So, there are no leaders given any ρ and n such that the wage-setting rule is not optimal given an exogenous probability of winning the competition.

Considering that I found in Lemmas 1 and 2 that for agents mutinying is more attractive when wages are lower and if the organisation is larger, and considering that I found in Lemma 3 that in larger organisations wages are lower, the following result holds.

Proposition 5 *Consider the mutiny game with an exogenous probability of winning the competition. In larger organisations mutinies will occur more frequently.*

Furthermore considering that I found in Lemma 1 that the threat of mutiny is smaller when wages are higher, and in Lemma 4 that highly-skilled leaders offer higher wages, the following result holds.

Proposition 6 *Consider the mutiny game with an exogenous probability of winning the competition. A highly-skilled leader is less likely to face a mutiny.*

Numerical simulations

To clearly demonstrate how the threat of mutiny affects the wages depending on ρ, the size of the organisation, and the skills of the principal, I have included Table 1 below with numerical values of w and of S_A^* in parantheses.

TABLE 1 Numerical values of w and (S_A^*) given certain n and S_P

		S_P									
n	ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.25	0	0	0	0	0	0	0.050	0.100	0.150	0.200
		(0)	(0)	(0)	(0)	(0)	(0)	(0.25)	(0.50)	(0.75)	(1)
	0.5	0	0	0	0.033	0.083	0.133	0.183	0.233	0.283	0.333
		(0)	(0)	(0)	(0.10)	(0.25)	(0.40)	(0.55)	(0.70)	(0.85)	(1)
	0.75	0	0.029	0.079	0.129	0.179	0.229	0.279	0.329	0.379	0.429
		(0)	(0.07)	(0.18)	(0.30)	(0.42)	(0.53)	(0.65)	(0.77)	(0.88)	(1)
5	0.25	0	0	0	0	0	0	0	0	0	0
		(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	0.5	0	0	0	0	0	0	0	0.043	0.093	0.143
		(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0.06)	(0.13)	(0.20)
	0.75	0	0	0.018	0.069	0.118	0.168	0.218	0.268	0.318	0.368
		(0)	(0)	(0.02)	(0.09)	(0.15)	(0.21)	(0.28)	(0.34)	(0.40)	(0.47)
100	0.25	0	0	0	0	0	0	0	0	0	0
		(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
	0.5	0	0	0	0	0	0	0	0	0	0.010
		(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0.01)
	0.75	0	0	0	0.036	0.086	0.136	0.186	0.236	0.286	0.336
		(0)	(0)	(0)	(0.04)	(0.09)	(0.14)	(0.19)	(0.24)	(0.29)	(0.34)

The same effects that were outlined in Lemmas 3 and 4 can be seen in this table. Now, the strength of the effects becomes clearer. Noticeably, the probability of the mutiny being successful influences the wage strongly. Especially a weaker principal sets wages that are extremely low when the probability of the mutiny being successful is low. In larger organisations (already when $n = 5$), the principal will stop to offer wages when there is a low threat of a successful mutiny, regardless of her skill level. The principal stops offering wages, because she is risk neutral. In larger organisations dissuading agents is very costly (but affordable because the residual claim is higher as well). Consequently, the principal accepts high risks in order to obtain extreme wealth.

3.2 Strong leaders exact the loyalty of the agents

In reality, the probability of winning the competition for the loyalty of the agents will not be exogenous, but will depend on a number of endogenous factors. Imaginably, the leadership skill level of the principal is one of these. A stronger leader will be able to make the organisation successful, because of this the agents receive purposive incentives: they are happier to work for a successful than a non-successful organisation. Furthermore, a strong principal might more easily command loyalty, since she is a more skilled leader. Therefore, in this section, the influence of the principal's skill level on the competition is analysed, i.e., this is a special case of section 3.1, in which $\rho = 1 - S_P$.

The mutiny decision

With probability $1 - S_P$, equation 2 from section 3.1 can be used to determine the skill level of the indifferent agent. By substituting $1 - S_P$ for ρ in equation 2, the following S_A^* is found.

$$S_A^* = \frac{n(1 - S_P) + 1}{n(1 - S_P)} w \tag{6}$$

As for the situation with equation 2, there will be more agents that would mutiny if n is lower and if w is lower, if given the chance. Therefore, Lemmas 1 and 2 also hold in this section. Differently, now a stronger principal with a high skill level S_P will face fewer mutinies independently of the wage she proposes. This result is summarised in the following lemma.

Lemma 7 *Consider the mutiny game with a probability of winning the competition that depends on the leader's skill level. Ceteris paribus, a highly-skilled leader will face fewer mutinies.*

The wage-setting stage

Because ρ and $1 - S_P$ are both constants for all intents and purposes of this model, ρ can be substituted for $1 - S_P$ in equation 5, to get the new wage-setting rule[5].

$$w = \frac{\left(\frac{1}{n} - S_P\right) S_P}{2(1 - S_P + \frac{1}{n})} \tag{7}$$

As with an exogenous probability of winning the mutiny, the more agents there are in the organisation, the lower the wage. Lemma 3, therefore, holds for an exogenous probability and for a probability that depends on the principal's skill level, and therefore, Proposition 5 holds as well. The relation between w and S_P, on the other hand, has become ambiguous, and largely depends on n. Lemma 4 does not hold up for the special case that $\rho = 1 - S_P$. This corresponds with the two mechanisms that work in theory. Firstly, as S_P increases, the profits of the principal increase. Therefore, she has more to lose, and, can afford higher incentive wages. Secondly, as S_P increases, the likelihood that the principal wins the mutiny (should it occur) becomes larger. Therefore, the need for deterring a mutiny becomes smaller. The main result is summarised in the following Lemma.

Lemma 8 *Consider the mutiny game with a probability of winning the competition that depends on the leader's skill level. Wages in an organisation have a parabolic relation with the height of the principal's skill level.*

Considering Lemma 7, the threat of mutiny is lower for stronger principals, and considering Lemma 8, this leads to lower wages when the principal is highly-skilled. So, there is one skill level of the leader for which the wages are highest, denoted by $\widehat{S_P}$, which can be found by maximising w with respect to S_P[6]. After that point the threat of mutiny does not result in higher wages anymore. The result of this section is summarised in the following proposition.

Proposition 9 *Consider the mutiny game with a probability of winning the competition that depends on the leader's skill level. Agents obtain the highest wage with increasingly lower-skilled leaders in larger organisations. Wages are highest when the principal has skill level:*

$$\widehat{S_P} = 1 + \frac{1}{n} - \sqrt{1 + \frac{1}{n}} \tag{8}$$

[5] $w = \frac{1}{2}\left(S_P - \frac{1-\rho}{\rho+\frac{1}{n}}\right) = \frac{1}{2}\left(S_P - \frac{1-1+S_P}{1-S_P-\frac{1}{n}}\right) = \frac{1}{2}\left(1 - \frac{1}{1-S_P+\frac{1}{n}}\right) S_L = \frac{\left(\frac{1}{n}-S_P\right)S_P}{2(1-S_P+\frac{1}{n})}$

[6] $w = \frac{\left(S_P-\frac{1}{n}\right)S_P}{2(1-S_P+\frac{1}{n})} = \frac{1}{2}\left(1 - \frac{1}{1-S_P+\frac{1}{n}}\right) S_P \Leftrightarrow \frac{\partial w}{\partial S_P} = \frac{1}{2}\left(1 - \frac{1}{1-S_P+\frac{1}{n}} - \frac{S_P}{(1-S_P+\frac{1}{n})^2}\right) = 0 \Leftrightarrow \frac{1}{2}\left(1 - \frac{1-S_P+\frac{1}{n}}{(1-S_P+\frac{1}{n})^2} - \frac{S_P}{(1-S_P+\frac{1}{n})^2}\right) = 0 \Leftrightarrow \frac{1}{2}\left(1 - \frac{1+\frac{1}{n}}{(1-S_P+\frac{1}{n})^2}\right) = 0 \Leftrightarrow (1-S_P+\frac{1}{n})^2 = 1+\frac{1}{n} \Leftrightarrow 1-S_P+\frac{1}{n} = \sqrt{1+\frac{1}{n}} \Leftrightarrow S_P = 1+\frac{1}{n} - \sqrt{1+\frac{1}{n}}$

Numerical simulations

To illustrate the relation between w and S_P more clearly, I have included Table 2 below, which has numerical values of w, and S_A^* in parentheses, given various levels of n and S_P.

TABLE 2 Numerical values of w and (S_A^*) given certain n and S_P

	S_P									
n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.024	0.044	0.062	0.075	0.086	0.083	0.081	0.067	0.041	0
	(0.05)	(0.10)	(0.15)	(0.20)	(0.25)	(0.30)	(0.35)	(0.40)	(0.45)	(0.50)
2	0.014	0.023	0.025	0.018	0	0	0	0	0	0
	(0.02)	(0.04)	(0.04)	(0.03)	(0)	(0)	(0)	(0)	(0)	(0)
5	0.005	0	0	0	0	0	0	0	0	0
	(0.01)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)

This table shows that offering any wage, quickly becomes unattractive as n increases. That is, because the competition only depends on S_P. With large n it is expensive for principals to dissuade agents from mutinying. Furthermore, for a highly-skilled leader the probability of staying in charge is already quite good without offering a wage, whereas weaker principals have a poor chance of staying in charge if they do not manage to dissuade agents. In section 3.1, it were only the highly-skilled principals that offered wages in larger organisations. Now, their superiority in the competition outweighs that they can afford higher wages and have more to lose. Therefore, wages are even lower when the competition depends on S_P.

3.3 Buying the loyalty of the agents

In this section, I will consider how the game changes if the probability of winning the competition depends on the wage the principal offers. Agents care about having a strong leader, but this is largely caused by the fact that a strong leader can give them opportunities to earn more as an organisation, and because of that provide them with higher wages. Next to that, agents do not appreciate a strong leader who takes all of the profits for herself. Therefore, I will consider the probability of winning the competition of $\rho - w$.

The mutiny decision

Like before, all agents for whom it holds that the chance of winning the mutiny times the benefit of becoming leader is larger than the wage, are willing to mutiny. This means that for these agents the following condition must hold:

$$n(\rho - w)(S_i - w) \geq w \tag{9}$$

This condition leads to the following skill level S_A^* of the agent indifferent between mutinying and not:

$$S_A^* = \frac{n(\rho - w) + 1}{n(\rho - w)} w \tag{10}$$

Similarly, to the previous probabilities, it is more attractive to mutiny in larger organisations; Lemma 2 holds up with this variation. The percentage of agents willing to mutiny is also still negatively affected by the height of the wage; Lemma 1 holds up as well. This relation, however, is now stronger: not only is this negative relation caused by the fact that being an agent is more attractive, as with previous probabilities, but also by the fact that the probability of winning the mutiny is now negatively affected by the wage.

The wage-setting stage

By substituting the new S_A^*, and $\rho - w$ into equation 3 of section 3.1, the following expected profits of the principal are obtained[7].

$$E[\pi] = n(S_P - w)\left(1 - \rho + \left(1 + \rho + \frac{1}{n}\right) w - w^2\right) \tag{11}$$

After maximising this equation with respect to w, the principal finds his new wage-setting rule[8].

$$w = \frac{1}{3}\left(1 + \rho + \frac{1}{n} + S_P - \sqrt{(S_P)^2 - \left(1 + \rho + \frac{1}{n}\right) S_P + \left(1 + \rho + \frac{1}{n}\right)^2 - 3(1 - \rho)}\right) \tag{12}$$

Note that in small organisations or if the probability of winning the mutiny is already low without incentive wages, it can be easy to dissuade all agents from mutinying. Therefore, I need to determine when S_A^* is larger than 1 with the wage-setting rule. This happens if w is larger than $\widehat{w}$, which gives the maximum feasible value of w: the one for which it holds that $S_A^*(w) = 1$ [9].

$$\widehat{w} = \frac{(1 + \rho)n + 1 - \sqrt{(1 - \rho)^2 n^2 + 2(1 + \rho)n + 1}}{2n} \tag{13}$$

[7] $E[\pi] = n(S_P - w)\left(\frac{n(\rho-w)+1}{n(\rho-w)} w + \left(1 - \frac{n(\rho-w)+1}{n(\rho-w)} w\right)(1 - \rho + w)\right) = n(S_P - w)\left(1 - \rho + w + (\rho - w)\frac{n(\rho-w)+1}{n(\rho-w)} w\right) = n(S_P - w)\left(1 - \rho + w + \rho w - w^2 + \frac{w}{n}\right) = n(S_P - w)\left(1 - \rho + \left(1 + \rho + \frac{1}{n}\right) w - w^2\right)$

[8] $\frac{\partial E[\pi]}{\partial w} = n\left(\left(1 + \rho + \frac{1}{n}\right) S_P - 2S_P w + 1 - \rho - 2(1 + \rho + \frac{1}{n})w + 3w^2\right) = 0 \Leftrightarrow 3w^2 - 2\left(1 + \rho + \frac{1}{n} + S_P\right) w + 1 - \rho + (1 + \rho + \frac{1}{n})S_P = 0 \Leftrightarrow w = \frac{2(1+\rho+\frac{1}{n}+S_P) - \sqrt{4(1+\rho+\frac{1}{n}+S_P)^2 - 12(1-\rho+(1+\rho+\frac{1}{n})S_P)}}{6} = \frac{1}{3}\left(1 + \rho + \frac{1}{n} + S_P - \sqrt{(S_P)^2 - (1 + \rho + \frac{1}{n})S_P + \frac{1}{n^2} + \frac{2(1+\rho)}{n} - 2 + 5\rho + \rho^2}\right)$

[9] $S_A^* = \frac{n(\rho-\widehat{w})+1}{n(\rho-\widehat{w})}\widehat{w} = 1 \Leftrightarrow n(\rho - \widehat{w})\widehat{w} + \widehat{w} = n(\rho - \widehat{w}) \Leftrightarrow n\widehat{w}^2 - ((1 + \rho)n + 1)\widehat{w} + n\rho = 0 \Leftrightarrow \widehat{w} = \frac{(1+\rho)n+1-\sqrt{(1+\rho)^2n^2+2(1+\rho)n+1-4n^2\rho}}{2n} = \frac{(1+\rho)n+1-\sqrt{(\rho^2-2\rho+1)n^2+2(1+\rho)n+1}}{2n} = \frac{(1+\rho)n+1-\sqrt{(1-\rho)^2n^2+2(1+\rho)n+1}}{2n}$

Numerical simulations

To put the wage-setting rule in perspective, I have included Table 3 with numerical values of w and of S_A^* (given this w) in parentheses.

TABLE 3 Numerical values of w and (S_A^*) given certain n and S_P

		S_P									
n	ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.25	0.000	0.000	0.000	0.029	0.071	0.112	0.117†	0.117†	0.117†	0.117†
		(0.00)	(0.00)	(0.00)	(0.16)	(0.47)	(0.92)	(1)	(1)	(1)	(1)
	0.5	0.000	0.000	0.046	0.090	0.134	0.176	0.218	0.219†	0.219†	0.219†
		(0.00)	(0.00)	(0.15)	(0.31)	(0.50)	(0.72)	(0.99)	(1)	(1)	(1)
	0.75	0.004	0.052	0.099	0.145	0.190	0.233	0.276	0.307†	0.307†	0.307†
		(0.01)	(0.13)	(0.25)	(0.38)	(0.53)	(0.69)	(0.86)	(1)	(1)	(1)
5	0.25	0.000	0.000	0.000	0.000	0.000	0.030	0.06	0.097	0.129	0.158
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.06)	(0.13)	(0.23)	(0.34)	(0.50)
	0.5	0.000	0.000	0.003	0.044	0.084	0.123	0.160	0.195	0.228	0.26
		(0.00)	(0.00)	(0.00)	(0.06)	(0.13)	(0.19)	(0.25)	(0.32)	(0.40)	(0.48)
	0.75	0.000	0.033	0.079	0.122	0.165	0.205	0.244	0.281	0.317	0.351
		(0.00)	(0.04)	(0.10)	(0.16)	(0.22)	(0.28)	(0.34)	(0.40)	(0.46)	(0.53)
100	0.25	0.000	0.000	0.000	0.000	0.000	0.002	0.035	0.066	0.095	0.123
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.04)	(0.07)	(0.10)	(0.13)
	0.5	0.000	0.000	0.000	0.028	0.067	0.104	0.139	0.173	0.204	0.234
		(0.00)	(0.00)	(0.00)	(0.03)	(0.07)	(0.11)	(0.14)	(0.18)	(0.21)	(0.24)
	0.75	0	0.027	0.071	0.114	0.155	0.195	0.233	0.269	0.302	0.334
		(0)	(0.03)	(0.07)	(0.12)	(0.16)	(0.20)	(0.24)	(0.27)	(0.31)	(0.34)

†Every agent is dissuaded, $w = \widehat{w}$

Relative to the other sections, the size of the organisation has a smaller impact on the wage-setting rule, but wages are still lower in larger organisations. However, the wages are very similar to that of section 3.1 when the organisation is large and the chance of a successful mutiny is high. This is caused by the fact that in both sections almost every agent who can afford to do so is willing to mutiny. This means that the principal does not dissuade any more agents from mutinying than before. However, her chance of winning the competition is better. Also, highly-skilled principals will as in section 3.1 offer higher wages than weaker leaders. All in all, the same general effects hold as in section 3.1. Consequently, Propositions 5 and 6 hold for this case.

It is noticeable that principals do not offer wages even near to what they could afford. Especially, in larger organisations, they could afford to make their position significantly more secure. The principal sacrifices certainty to make higher profits. She does that because obtaining the same amount of certainty is very costly in large organisations, and because she is risk-neutral.

Overall, being able to buy the loyalty of the agents, does not change the mutiny game significantly, except for the fact that wages are slightly higher, and more constant with regards to n and ρ.

3.4 Bidding for the loyalty of the agents

Thus far, I assumed that the mutineer would not alter the wage that the principal had set. That is, because he would always be worse off by doing so: either he lowers the wage and is deposed, or he raises the wage and makes lower profits. However, in this section, I assume that the likelihood of swaying the other agents to his side depends on a wage w' he offers them, so he does have an incentive to offer a higher wage. To model this, I extend the probability of winning the competition to the following: $\rho + \frac{w'-w}{w}$. This probability has some attractive features.

First, the outcome of the competition depends on actions by both players to persuade the other agents. Just as in every other election, both candidates need to persuade voters to vote for them. An election with only the incumbent being able to sway voters, or none of the candidates being able to do so, seems less realistic.

Second, the fact that the wage can change after the mutiny means that organisations can change, and become fairer. Previously, the wage the leader offered determined the wage structure in the organisation with no possibility of ever changing. As the biggest reason to join a mutiny is dissatisfaction with the current state of affairs, it is more realistic if the mutiny can change the current situation. Therefore, this variation constitutes the most important and realistic version of the model.

The persuasion stage

Now, before we enter the mutiny stage, there is a new final stage in the game. After the principal has proposed a wage to the agents, and after the agent in question decided to start a mutiny, he has to promise a certain wage w' to the other agents in order to gain their loyalty. The idea is that the other agents would mutiny immediately if the usurper does not uphold his pledge, after he becomes the new leader, and therefore his promise is binding. He chooses the w' that maximises his expected profits, which are as follows.

$$E[\pi] = n\left(\rho + \frac{w'-w}{w}\right)(S_i - w') \tag{14}$$

Taking the derivative with respect to w' gives us the following rule for the wage the mutineer should promise[10].

$$w' = \frac{S_i + (1-\rho)w}{2} \tag{15}$$

[10] $E[\pi] = n(\rho + \frac{w'-w}{w})(S_i - w') \Leftrightarrow \frac{\partial E[\pi]}{\partial w'} = -\rho + \frac{S_i}{w} - \frac{2}{w}w' + 1 = 0 \Leftrightarrow w' = \frac{w}{2}\left(\frac{S_i}{w} + 1 - \rho\right) \Leftrightarrow w' = \frac{S_i+(1-\rho)w}{2}$

The wage the mutineer promises thus depends on his own skill level, the exogenous part of the probability of winning the competition, and on the wage the principal offers. The principal can make it less attractive to mutiny by offering a high wage. To be able to sway the other agents, the mutineer then has to offer an even higher wage, making the mutiny less attractive. For this reason, the wage increase the challenger promises becomes less as the wage the incumbent offers rises, and therefore, the probability that the mutiny is successful, decreases. Furthermore, a mutineer with stronger leadership skills will promise a higher wage, since he will be able to earn more as leader, and therefore has more to win. These findings are summarised in the following Lemmas.

Lemma 10 *Consider the mutiny game with a persuasion stage. A leader that offers higher wages has a higher chance of winning the mutiny, because the mutineer offers relatively smaller wage increases.*

Lemma 11 *Consider the mutiny game with a persuasion stage. A mutineer with better leadership skills will promise higher wages to persuade the other agents to join his mutiny, and will have a higher chance of winning the mutiny.*

However, the wage increase rule for the mutineer has two constraints. First, as in every section before, I assume that the mutineer cannot set a lower wage than the leader, because immediate rebellion would follow. Contrastingly, the expression for w' allows for decreases in wages, i.e., with this wage rule, some agents would offer a wage that is lower than that of the incumbent. In fact, the agent for whom it holds that he would set exactly $w' = w$ has skill level $\overline{S_A}$[11].

$$\overline{S_A} = (1 + \rho)w \tag{16}$$

The lower constraint that is set is therefore: every agent for whose skill level it holds that $S_i \leq \overline{S_A}$ promises, if faced with a mutiny opportunity, a wage such that $w' = w$.

Second, the wage increase rule also allows for wages w' that are so high that the probability of winning the competition is larger than 1. Of course, this is not possible, and therefore, these high wages are not a feasible option for the mutinying agents. A mutineer maximally sets w' high enough such that the probability of winning the competition is equal to 1. The upper constraint of w' is denoted by $\widehat{w'}$, which is the maximum wage any mutineer would ever set[12].

$$\widehat{w'} = (2 - \rho)w \tag{17}$$

For the agent with skill level $\widehat{S_A}$, it holds that the wage increase rule gives a w' equal to this maximum wage. Every agent with a higher skill level than that would set the wage exactly equal to $\widehat{w'}$ as well. This tipping point occurs at the following skill level[13].

$$\widehat{S_A} = (3 - \rho)w \tag{18}$$

[11] $w' = w = \frac{\overline{S_A}+(1-\rho)w}{2} \Leftrightarrow \frac{\overline{S_A}}{2} = \frac{2w-(1-\rho)w}{2} \Leftrightarrow \overline{S_A} = (1+\rho)w$

[12] $\rho + \frac{\widehat{w'}-w}{w} = 1 \Leftrightarrow \frac{\widehat{w'}}{w} = 1 - \rho + \frac{w}{w} = 2 - \rho \Leftrightarrow \widehat{w'} = (2-\rho)w$

[13] $\frac{\widehat{S_A}+(1-\rho)w}{2} = (2-\rho)w \Leftrightarrow \widehat{S_A} = 2(2-\rho)w - (1-\rho)w = (3-\rho)w$

So, if the mutineer has a skill level that is anywhere above $\widehat{S_A}$, the chance of winning for the principal is equal to zero. However, if $\widehat{S_A} \geq 1$, there will be no skill levels such that the mutineer would definitely win. The principal can influence this by increasing w. If the leader sets w in the following manner, $\widehat{S_i}$ will be larger than 1, and no agent would set $w' \geq \widehat{w'}$ [14].

$$w \geq \frac{1}{3-\rho} \tag{19}$$

The mutiny decision

Different from the previous sections, the mutineer is likely to change the wage if he becomes leader. The profits he will make as leader are therefore not equal to $n(S_i - w)$, but are equal to $n(S_i - w')$. The agent will decide to start a mutiny if his skill level satisfies the following condition.

$$n\left(\rho + \frac{w'-w}{w}\right)(S_i - w') \geq w \tag{20}$$

Anticipating this mutiny decision, and anticipating the wage the mutineer will offer the agents, the leader deduces that the skill level of the indifferent agent will be as follows[15].

$$S_A^* = \left(\frac{2}{\sqrt{n}} + 1 - \rho\right) w \tag{21}$$

This leaves us to the conclusion that Lemmas 1 and 2 still hold up when the persuasion stage is introduced: it is still less attractive to mutiny when wages are higher, and more attractive to mutiny in larger organisations.

However, if it holds that $\overline{S_A} \geq S_A^*$, then S_A^* does not constitute the skill level of the indifferent agent anymore. In that case the skill level of the indifferent agent is the same as with the exogenous probability, namely: $\underline{S_A} = \frac{\rho n+1}{\rho} w$. That is, because if it holds for the usurper that $\overline{S_A} \geq S_i$, then $w' = w$ and therefore $\rho + \frac{w'-w}{w} = \rho$. $\underline{S_A}$ is therefore the skill level of the indifferent agent that should be accounted for, because everyone below that level would only be willing to mutiny with a wage w' lower than w. It holds that $\overline{S_A} \geq S_A^*$, when the following condition is satisfied[16].

$$\rho \geq \frac{1}{\sqrt{n}} \tag{22}$$

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[14] $\widehat{S_i} \geq 1 \Leftrightarrow (3-\rho)w \geq 1 \Leftrightarrow w \geq \frac{1}{3-\rho}$

[15] $n\left(\rho + \frac{w'-w}{w}\right)(S_i - w') \geq w \Leftrightarrow \left(\rho + \frac{S_i}{2w} + \frac{1-\rho}{2} - 1\right)\left(S_i - \frac{S_i}{2} - \frac{(1-\rho)w}{2}\right) \geq \frac{w}{n} \Leftrightarrow \left(\rho + \frac{S_i}{w} - 1\right)(S_i - (1-\rho)w) \geq \frac{4w}{n} \Leftrightarrow \left(\frac{S_i}{w} - (1-\rho)\right)(S_i - (1-\rho)w) \geq \frac{4w}{n} \Leftrightarrow (S_i - (1-\rho)w)^2 \geq \frac{4w^2}{n} \Leftrightarrow S_i - (1-\rho)w \geq \frac{2}{\sqrt{n}} w \Leftrightarrow S_i \geq \left(\frac{2}{\sqrt{n}} + 1 - \rho\right) w$

[16] $\overline{S_A} \geq S_A^* \Leftrightarrow (1+\rho)w \geq \left(\frac{2}{\sqrt{n}} + 1 - \rho\right) w \Leftrightarrow 2\rho \geq \frac{2}{\sqrt{n}} \Leftrightarrow \rho \geq \frac{1}{\sqrt{n}}$

Next, the skill level S_A^* provides a restriction on the maximum wage $\widehat{w}$ that the incumbent would ever set. The principal would never set a wage higher than the wage necessary to dissuade all agents, such that $S_A^* = 1$. Therefore, the maximum wage the principal sets is as follows[17]

$$\widehat{w} = \frac{1}{\frac{2}{\sqrt{n}} + 1 - \rho} \tag{23}$$

The wage-setting stage

The principal now has to consider more factors when deciding upon the optimal wage. First of all, in the previous sections it was often optimal to give no wages to her agents. This is not smart when the mutineer can outbid the principal, because the mutineer can set a very low wage and already be sure to win the mutiny (only relative wages matter). The principal will thus be more inclined to invest at least some of her earnings into securing her position. Furthermore, since only relative wages matter, offering a higher wage will ensure a higher probability of winning the mutiny (the mutineer promises a relatively smaller wage increase when w is higher).

The principal anticipates the wage w' any mutineer would set given the mutineer's skill level. However, she does not know in advance the skill level of the agent who gets the opportunity to mutiny, so she cannot foresee which wage the mutineer promises to the other agents. Therefore, different from previously, the probability of winning the mutiny is not known. This means that the leader has to use expectations of this probability to determine her expected profits.

The principal has to account for four different scenarios. The first situation is when there are no constraints on w'. This holds if there are no agents, for whom it holds that they are willing to mutiny if $w' = w$, but would rather promise $w' < w$. Previously it was stated that this holds when $\overline{S_A}$, the skill level of the agent that sets exactly $w' = w$, is smaller than S_A^*, the skill level of the indifferent agent without constraints. Furthermore, in this situation, there are no agents that would promise a wage so high that they would be sure to win the competition ($\forall S_i : \rho + \frac{w'(S_i) - w}{w} < 1$). This only holds when the skill level of the agent who would set w' so high he would be sure to win ($\widehat{S_A}$) lies above 1. Second, the incumbent could face agents that would only be willing to mutiny if they were able to offer a wage $w' \leq w$, which means that $\overline{S_A} \geq S_A^*$. In this scenario it also holds that $\widehat{S_A} > 1$. Third and fourth, the principal should consider both the previous situations in case there actually are agents that would set w' high enough to be sure to win the competition, such that $\widehat{S_A} \leq 1$

Situation 1: no constraints on w'

In this scenario, no constraints are present in the model. It holds that the mutineer (given

[17] $S_A^* = 1 = \left(\frac{2}{\sqrt{n}} + 1 - \rho\right)\widehat{w} \Leftrightarrow \widehat{w} = \frac{1}{\frac{2}{\sqrt{n}}+1-\rho}$

that $S_i \geq S_A^*$) promises $w' = \frac{S_i+(1-\rho)w}{2}$ regardless of his skill level and that the agent indifferent to mutiny has skill level S_A^*. Therefore, the expected profits of the principal are similar to the ones of the previous sections with the exception that the incumbent has to make predictions about the chance of winning the mutiny (should it occur), since w' is not known. The expected probability of staying in charge, which from now on I will refer to as the survival probability, is in this case equal to the probability that no mutiny occurs plus the chance that a mutiny does occur and the mutineer fails to usurp the position of leader.

$$E[\Pr(survive)] = S_A^* + (1 - S_A^*) \cdot E\left[\Pr(fail|S_i \geq S_A^*)\right] \tag{24}$$

The principal's expectations of winning the competition if a mutiny occurs is equal to $1-\rho$ (as in section 3.1) minus the expected wage increase the mutineer promises.

$$E[\Pr(win|S_i \geq S_A^*)] = 1 - \rho - \frac{E[w'|S_i \geq S_A^*] - w}{w} \tag{25}$$

Next, the leader expects that if a mutiny occurs, the wage the mutineer promises will be the average of the wages the agents with skill levels between S_A^* and 1 would promise.

$$E[w'|S_i \geq S_A^*] = \frac{E[S_i|S_i \geq S_A^*] + (1-\rho)w}{2} = \frac{\frac{1+S_A^*}{2} + (1-\rho)w}{2} \tag{26}$$

Given that the skill level of the indifferent agent is $S_A^* = \left(\frac{2}{\sqrt{n}} + 1 - \rho\right)w$, the leader calculates that the objective function, her expected profits, is the following[18].

$$E[\pi|S_A^* \geq \overline{S_A}, \widehat{S_A} \geq 1] = n(S_P - w)\left(\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)w + \frac{3-\rho}{2} - \frac{1}{4w}\right) \tag{27}$$

By taking the derivative of these expected profits with respect to w and then setting this equal to zero, the leader can find the optimal wage to decide upon. However, since the derivative is cubic[19], the wage rule is not a comprehensible formula, which is why it is

[18] $E[w'|S_i \geq S_A^*] = \frac{1+\left(\frac{2}{\sqrt{n}}+3(1-\rho)\right)w}{4} \Leftrightarrow E[\Pr(fail|S_i \geq S_A^*)] = \left(1 - \rho - \frac{\frac{1+\left(\frac{2}{\sqrt{n}}+3(1-\rho)\right)w}{4} - w}{w}\right) = \left(\frac{5}{4} - \frac{\rho}{4} - \frac{1}{2\sqrt{n}} - \frac{1}{4w}\right) \Leftrightarrow E[\Pr(survive|S_i \geq S_A^*)] = S_A^* + (1 - S_A^*) \cdot E[\Pr(fail|S_i \geq S_A^*)] = E[\Pr(fail|S_i \geq S_A^*)] + S_A^*(1 - E[\Pr(fail|S_i \geq S_A^*)]) = E[\Pr(fail|S_i \geq S_A^*)] + S_A^*\left(-\frac{1-\rho}{4} + \frac{1}{2\sqrt{n}} + \frac{1}{4w}\right) = E[\Pr(fail|S_i \geq S_A^*) + \frac{1}{4}\left(\frac{2}{\sqrt{n}} + 1 - \rho\right) + \left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)w = \left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)w + \frac{3-\rho}{2} - \frac{1}{4w} \Leftrightarrow E[\pi] = n(S_P - w)\left(\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)w + \frac{3-\rho}{2} - \frac{1}{4w}\right)$

[19] $\frac{\partial E[\pi}{\partial w} = n\left(\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)S_L - \frac{3-\rho}{2} - 2\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)w + \frac{S_L}{4w^2}\right) = 0 \Leftrightarrow -2\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)w^3 + \left(\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right)S_L - \frac{3-\rho}{2}\right)w^2 + S_L = 0$

not included here. At the end of this section I have included numerical simulations of the wages given ρ, n and S_P for the four situations.

Situation 2: lower constraint on w'

In this situation, which will generally hold for organisations with larger n, there is a lower constraint on w'. This is a result of the fact that some agents with $\underline{S_A} \geq S_i \geq S_A^*$ are only willing to mutiny with a w' that is lower than w. Because this is not allowed in the model, since immediate rebellion would follow, the actual skill level of the indifferent agent is not given by S_A^*, but by $\underline{S_A}$, which is the skill level of the indifferent agent when the competition is only determined by an exogenous probability. There is a significant percentage of agents $(\overline{S_A} - \underline{S_A})$ who would mutiny given the chance, but would promise $w' = w$. The chance of winning for these agents is therefore the exogenous probability ρ. Above $\overline{S_A}$, this situation is similar to situation 1. Furthermore, the chance of no mutiny occurring is $\underline{S_A}$. The survival probability is then as follows.

$$E[\Pr(survive|\overline{S_A} \geq S_A^*, \widehat{S_A} \geq 1)] = \underline{S_A} + (\overline{S_A} - \underline{S_A})(1-\rho) + (1 - \overline{S_A}) \cdot E[\Pr(fail|S_i \geq \overline{S_A})] \tag{28}$$

The probability of winning is similar to that in situation 1, except for that the leader now makes expectations about the wage the mutineer promises given that his skill level is between 1 and $\overline{S_A}$ instead of S_A^*.

$$E[w'|S_i \geq \overline{S_A}] = \frac{w'[S_i = 1] + w'[S_i = \overline{S_A}]}{2} = \frac{\frac{1+(1-\rho)w}{2} + w}{2} = \frac{1 + (3-\rho)w}{4} \tag{29}$$

This survival probability and expected wage give the same expected profits as in situation 1[20]. Compared to having no constraints, there is a smaller fraction of agents willing to mutiny, which leads to a higher probability of staying in charge. The likelihood that no mutiny will be started increased with $\underline{S_A} - S_A^*$ from situation 1 to situation 2. However, the chance of winning the mutiny, if it is started, decreases, which exactly offsets the previous effect. This is a result of that the agents with skill levels between $\underline{S_A}$ and $\overline{S_A}$ have to set $w' = w$, whereas in situation 1, they would have set $w' \leq w$. The fact that these agents are forced to promise a higher wage, means that their chances of winning the mutiny is higher. Consequently, the wage the leader sets is the same in situation 1 and 2, but the percentage of agents willing to mutiny, and the probability of the mutiny being successful are different.

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[20] $E[\Pr(win|S_i \geq \overline{S_i})] = 1 - \rho - \frac{E[w'|S_i \geq \overline{S_i}] - w}{w} = 1 - \rho - \frac{\frac{1+(3-\rho)w}{4} - w}{w} = \frac{5-3\rho}{4} - \frac{1}{4w} \Leftrightarrow E[\Pr(survive|\overline{S_i} \geq S_i^*, \widehat{S_i} \geq 1)] = \left(1 + \frac{1}{\rho n}\right) w + (1-\rho)\left(\rho - \frac{1}{\rho n}\right) w + (1 - (1+\rho)w)\left(\frac{5-3\rho}{4} - \frac{1}{4w}\right) = \left(1 + \rho - \rho^2 + \frac{1}{n} - \frac{(1+\rho)(5-3\rho)}{4}\right) w + \frac{5-3\rho}{4} + \frac{1+\rho}{4} - \frac{1}{4w} = \left(-\frac{1}{4} + \frac{\rho}{2} - \frac{\rho^2}{4} + \frac{1}{n}\right) + \frac{3-\rho}{2} - \frac{1}{4w} = \left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right) w + \frac{3-\rho}{2} - \frac{1}{4w} \Leftrightarrow E[\pi|\overline{S_i} \geq S_i^*, \widehat{S_i} \geq 1] = n(S_P - w)\left(\left(\frac{1}{n} - \frac{(1-\rho)^2}{4}\right) w + \frac{3-\rho}{2} - \frac{1}{4w}\right)$

Situation 3: upper constraint on w', no lower constraint

The last two situations are the scenarios in which there are agents ($S_i \geq \widehat{S_A} = (3-\rho)w$) who promise such a high w' that they always win the competition. These agents promise $w' = (2-\rho)w$, which is the upper constraint on w'. Given that there are such agents, it may hold that there is a lower constraint as well, however, I will show that this does not affect the wage the leader will set. First, I will consider the situation in which there is no lower constraint on w'. The survival probability is in this case the following

$$E[\Pr(survive|S_A^* \geq \overline{S_A}, \widehat{S_A} \leq 1)] = S_A^* + (\widehat{S_A} - S_A^*) \cdot E[\Pr(fail|\widehat{S_A} \geq S_i \geq S_A^*)] \tag{30}$$

The expected probability of winning the mutiny, if it occurs, depends on the expected wage the mutineer promises in the same way as in situation 1 and 2. The principal expects an agent with skill level between $\widehat{S_A}$ and S_A^*, promises the following wage[21].

$$E[w'|\widehat{S_A} \geq S_i \geq S_A^*] = \frac{w'[S_i = \widehat{S_A}] + w'[S_i = S_A^*]}{2} = \frac{\frac{2}{\sqrt{n}} + (6-4\rho)}{4} w \tag{31}$$

After substituting S_A^*, $\widehat{S_A}$, and $E[w'|\widehat{S_A} \geq S_i \geq S_A^*]$, the objective function of the leader becomes[22].

$$E[\pi|S_A^* \geq \overline{S_A}, \widehat{S_A} \leq 1] = n(S_P - w)\left(2 - \rho + \frac{1}{n}\right) w \tag{32}$$

Taking the derivative with respect to w gives the wage-rule in case there are agents that set w' so high they always win, and no agents that would only mutiny if they were allowed to set $w' \leq w$.

$$w[S_P|S_A^* \geq \overline{S_A}, \widehat{S_A} \leq 1] = \frac{S_P}{2} \tag{33}$$

Situation 4: upper and lower constraint on w'

Finally, I will consider the same situation as before, but including a lower constraint on w'. The survival probability is in this case the following.

$$E[\Pr(survive|\overline{S_A} \geq S_A^*, \widehat{S_A} \leq 1)] = \underline{S_A} + (\overline{S_A} - \underline{S_A}) + (\widehat{S_A} - \overline{S_A}) \cdot E[\Pr(fail|\widehat{S_A} \geq S_i \geq \overline{S_A})] \tag{34}$$

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[21] $E[w'|\widehat{S_A} \geq S_i \geq S_A^*] = \frac{w'[S_i = \widehat{S_A}] + w'[S_i = S_A^*]}{2} = \frac{(2-\rho)w + \frac{S_A^* + (1-\rho)w}{2}}{2} = \frac{S_A* + (5-3\rho)w}{4} = \frac{\frac{2}{\sqrt{n}} + (6-4\rho)}{4} w$

[22] $\Pr(win|\widehat{S_A} \geq S_i \geq S_A^*) = 1 - \rho - \frac{\frac{\frac{2}{\sqrt{n}} + (6-4\rho)}{4} w - w}{w} = \frac{1}{2} - \frac{1}{2\sqrt{n}} \Leftrightarrow \Pr(survive|S_A^* \geq \overline{S_A}, \widehat{S_A} \leq 1) = \left(\frac{2}{\sqrt{n}} + 1 - \rho\right) w + \left(2 - \frac{2}{\sqrt{n}}\right)\left(\frac{1}{2} - \frac{1}{2\sqrt{n}}\right) w = \left(\frac{2}{\sqrt{n}} + 1 - \rho + \frac{1}{4}\left(2 - \frac{2}{\sqrt{n}}\right)^2\right) w = \left(2 - \rho + \frac{1}{n}\right) w \Leftrightarrow E[\pi|S_A^* \geq \overline{S_A}, \widehat{S_A} \leq 1] = n(S_P - w)\left(2 - \rho + \frac{1}{n}\right) w$

The principal deduces that the average wage agents, who have a skill level between $\widehat{S_A}$ and $\overline{S_A}$, will promise is as follows.

$$E[w'|\widehat{S_A} \geq S_i \geq \overline{S_A}] = \frac{w'[S_i = \widehat{S_A}] + w'[S_i = \overline{S_A}]}{2} = \frac{(2-\rho)w + w}{2} = \frac{3-\rho}{2}w \tag{35}$$

The expected profits of the leader are in this scenario (just as they were the same in situation 1 and 2) equal to the expected profits in situation 3[23]. Therefore, the wage-setting rule is also the same in situation 4 as it was in situation 3. The wage-setting rule of situation 3 can be generalised to all situations in which it holds that there are agents that would always win the mutiny.

$$w[S_P|\widehat{S_A} \leq 1] = \frac{S_P}{2} \tag{36}$$

Numerical simulations

To illustrate how the wages depend on the size of the organisation, the skill level of the leader, and on ρ, I have included the Table 4 with numerical values of w, and in parentheses the skill level of the agent that is indifferent to mutiny.

[23] $E[\Pr(fail|\widehat{S_A} \geq S_i \geq \overline{S_A})] = 1 - \rho - \frac{\frac{3-\rho}{2}w - w}{w} = 1 - \rho - \frac{1-\rho}{2} = \frac{1-\rho}{2} \Leftrightarrow E[\Pr(survive|\overline{S_A} \geq S_A^*, \widehat{S_A} \leq 1)] = \left(1 + \frac{1}{\rho n}\right) w + (1-\rho)\left(\rho - \frac{1}{\rho n}\right) w + 2(1-\rho)w \cdot \frac{1-\rho}{2} = \left(1 + \rho - \rho^2 + \frac{1}{n} + (1-\rho)^2\right) w = \left(2 - \rho + \frac{1}{n}\right) w \Leftrightarrow E[\pi|\overline{S_A} \geq S_A^*, \widehat{S_A} \leq 1] = n(S_P - w)\left(2 - \rho + \frac{1}{n}\right) w$

TABLE 4		Numerical values of w and (S_A^* or $\underline{S_A}$) given certain n and S_P									
		S_P									
n	ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.25§	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.364†	0.364†	0.364†
		(0.14)	(0.28)	(0.41)	(0.55)	(0.69)	(0.83)	(0.96)	(1)	(1)	(1)
	0.5§	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.4†	0.4†	0.4†
		(0.13)	(0.25)	(0.38)	(0.50)	(0.63)	(0.75)	(0.88)	(1)	(1)	(1)
	0.75§	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.4*	0.444†	0.444†
		(0.11)	(0.23)	(0.34)	(0.45)	(0.56)	(0.68)	(0.79)	(0.90)	(1)	(1)
5	0.25§	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.382	0.405	0.426
		(0.05)	(0.11)	(0.18)	(0.26)	(0.35)	(0.45)	(0.59)	(0.63)	(0.67)	(0.70)
	0.5	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.400	0.425	0.450
		(0.07)	(0.14)	(0.21)	(0.28)	(0.35)	(0.42)	(0.49)	(0.56)	(0.60)	(0.63)
	0.75	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.4*	0.447	0.474
		(0.06)	(0.13)	(0.19)	(0.25)	(0.32)	(0.38)	(0.44)	(0.51)	(0.57)	(0.60)
100	0.25	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.381	0.403	0.423
		(0.05)	(0.10)	(0.16)	(0.21)	(0.26)	(0.31)	(0.36)	(0.40)	(0.42)	(0.44)
	0.5	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.400	0.424	0.446
		(0.05)	(0.10)	(0.15)	(0.20)	(0.26)	(0.31)	(0.36)	(0.41)	(0.43)	(0.46)
	0.75	0.05*	0.1*	0.15*	0.2*	0.25*	0.3*	0.35*	0.4*	0.447	0.471
		(0.05)	(0.10)	(0.15)	(0.20)	(0.25)	(0.30)	(0.35)	(0.41)	(0.45)	(0.48)

§Situation 1 or 3: $S_A^* \geq \overline{S_A}$
*Situation 3 or 4: $\frac{1}{3-\rho} \geq w$: it holds that the wage is $w[S_P|\widehat{S_A} \leq 1] = \frac{S_P}{2}$
†Every agent is dissuaded, $w = \widehat{w} = \frac{1}{\frac{2}{\sqrt{n}}+1-\rho}$

Wages are very similar for small and large organisations, even though in small organisations it is very easy to dissuade (almost) every agent to start a mutiny, whereas in large organisations this is not easy at all. The table shows that for an organisation that employs a hundred agents almost every agent who is financially able to afford to do so, will be willing to start a mutiny. This is a result of the position of leader being so lucrative that it is worth incurring almost any risk. Interestingly, in organisations with strong leaders, this results in a higher percentage of agents being willing to mutiny if the probability of winning for the agent is lower. This is caused by strong leaders offering higher wages when the chances of winning are lower for them.

If we compare these results with the previous sections, the one thing that stands out, is that there are no principals with skill levels such that they offer no wages. The threat of the mutineer successfully persuading the other agents, thus leads to significantly higher wages. Furthermore, even though wages are more similar between small and large organisation than before, the effect of Lemma 3 still holds: wages are lower in larger organisations. Considering that I found that Lemma 2 also still holds —mutinying is more attractive in larger organisations— Proposition 5 will also hold: also with a persuasion stage, mutinies will occur more frequently in larger organisations. However, the effect of the organisation size on the probability of a mutiny occurring is considerably smaller now.

From the table it can be observed that, as in section 3.1, stronger principals offer higher wages (confirming Lemma 4). Since, Lemma 1 —the threat of mutiny is smaller when wages are higher— also holds, it means that Proposition 6 will also hold: a highly-skilled leader is less likely to face a mutiny.

Furthermore, adding the persuasion stage to the mutiny game has benefited the position of highly-skilled principals in case a mutiny does occur: Lemma 10 states that a stronger principal has a higher chance of winning a mutiny. In section 3.1 the skill-level of the principal did not matter for the chances of winning the mutiny. In addition, a highly-skilled leader is also less likely to face a mutiny. This leads to the following result:

Proposition 12 *Consider the mutiny game with a persuasion stage. Highly-skilled principals are more likely to stay in charge.*

Furthermore, since highly-skilled agents are more likely to mutiny, and have a higher chance of winning the mutiny (from Lemma 11; differently from previous sections), the following proposition holds.

Proposition 13 *Consider the mutiny game with a persuasion stage. Highly-skilled agents will be more likely to usurp the position of leader.*

Since, it is true that both highly-skilled agents and highly-skilled leaders are more likely to survive mutinies, it must also hold that organisations evolve to have strong leaders. This is summarised in the following proposition.

Proposition 14 *Consider the mutiny game with a persuasion stage. Highly-skilled leaders are more likely to be in charge of a criminal organisations, because the chance of surviving a mutiny is larger for them.*

4 Extension: All agents get the opportunity to mutiny

The model can be extended by adding the possibility that a second agent can mutiny if the first agent to have this opportunity does not take it, and after him the other $n-2$ agents. In this extension, I assume that only one mutiny can be started. After one agent has decided to engage in mutiny, successful or not, no further mutinies occur. However, since only one agent can mutiny in this model, the principal has a very different task. Now, she has to dissuade all agents, or no agents at all. If the principal does not manage to dissuade all agents from mutinying, a mutiny will occur nonetheless, and every bit of wage she put into dissuading the other agents will have been wasted. In this extension, I will only work with a probability of winning the competition for the agents that is equal to ρ.

The wage-setting stage

When all agents sequentially get the opportunity to start a mutiny, instead of one random agent having a monopoly on this opportunity, the game changes. Since there can be only one mutiny, and since I assume as in section 3.1 that the wages will not change regardless of who will eventually be the leader, the mutiny decision is the same as in section 3.1 and the indifferent agent has the same skill level as in equation 2: $S_A^* = \frac{\rho n+1}{\rho n} w$.

The wage-setting stage, on the other hand, is complicated because of the multiple opportunities to mutiny: the likelihood of a mutiny happening changes when multiple agents get the chance to mutiny. Before, the chance of no mutiny occurring was S_A^*. Now, this probability becomes $(S_A^*)^n$. Hence, the probability of winning a mutiny (should it occur) is $(1-(S_A^*)^n)(1-\rho)$. This means that the expected profit of the leader will be as follows:

$$E[\pi] = n(S_L - w)((S_A^*)^n + (1-(S_A^*)^n)(1-\rho)) \tag{37}$$

I substitute S_A^* from equation 2 into this condition to obtain the objective function.

$$E[\pi] = n(S_L - w)\left(\left(\frac{\rho n+1}{\rho n}\right)^n w^n + \left(1-\left(\frac{\rho n+1}{\rho n}\right)^n w^n\right)(1-\rho)\right) \tag{38}$$

By taking the derivative of this function with respect to w, the optimal wage is obtained. Again, this is too complicated to do manually, so I have included the following numerical simulations.

Numerical simulations

In the following table numerical values of w are given, for a number of values of n and S_L.

TABLE 5 Numerical values of w given certain n and S_P

		S_P									
n	ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.25	0.000	0.000	0.000	0.000	0.000	0.000	0.050	0.100	0.150	0.200
	0.5	0.000	0.000	0.000	0.033	0.083	0.133	0.183	0.233	0.283	0.333
	0.75	0.000	0.029	0.079	0.129	0.179	0.229	0.279	0.329	0.379	0.429
5	0.25	0.080	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833
	0.5	0.000	0.165	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833
	0.75	0.000	0.164	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833
25	†	0.096	0.192	0.288	0.385	0.481	0.577	0.673	0.769	0.865	0.962
100	†	0.099	0.198	0.297	0.396	0.495	0.594	0.693	0.792	0.891	0.990
10000	†	0.038	0.076	0.115	0.153	0.191	0.229	0.267	0.306	0.344	0.382

†Outcomes are the same for $\rho = (0.25, 0.5, 0.75)$

The results are now different. Formerly, wages were lower in larger organisations. Now, wages first rise to almost their maximum values, after which they drop again. This results from the fact that giving incentive wages now —up to some point— has a considerably smaller effect on the probability of no mutiny occurring than before. In the other sections, a principal would dissuade a percentage of agents from mutinying, which —because only one agent got the opportunity to mutiny— constituted the likelihood of no mutiny occurring. Now, if the principal dissuades the same percentage of agents, the likelihood that there will be no mutiny is a lot smaller than that percentage, especially in large organisations. The likelihood that no mutiny occurs is now the likelihood that all the agents are dissuaded. So, principals set higher wages to obtain similar levels of certainty. This holds up till one point where the chance of a mutiny occurring is almost a hundred percent and the only reason to give incentive wages is to set a financial barrier for lower skilled agents. With large organisations being a leader is so profitable that almost anyone who can afford to do so, wants to mutiny. Consequently, instead of wages serving as an incentive to keep agents satisfied, it serves more as a financial barrier for lower skilled agents, who cannot afford to give such high wages.

Furthermore, in larger organisations the probability of actually winning the mutiny does not matter greatly for the wage. This is also a result of wages serving solely as a financial barrier for agents wanting to mutiny.

5 Discussion

In this paper, I showed how the threat of mutiny affects the wage in criminal organisations. I analysed different aspects that could determine the successfulness of a mutiny and found the following.

Firstly, in general, more mutinies will occur in large organisations. This is a result of two things: larger organisations are more profitable and therefore are more attractive to lead, and it is more costly for principals of a large organisation to discourage mutinies, which means they will do so to a lesser extent. In large organisations almost every agent that can afford to do so will mutiny. Consequently, the incentive wages serve only as a financial barrier.

Secondly, stronger principals will offer higher wages (except for when being a highly-skilled principal is a great advantage in the competition for the loyalty of the agents). Stronger principals lead organisations that are more profitable. Therefore, they are able to afford higher incentive wages, which they will also do, because they have a higher residual claim to lose in a mutiny. In section 3.2, I showed that this mechanism is offset rapidly when stronger principals have a better chance of staying in charge: in most cases strong leaders will not even bother to pay any wage. However, if they do pay higher wages, they are less likely to face a mutiny.

Thirdly, when the wage the principal offers matters for the competition, such as in section 3.3 and section 3.4, wages will be significantly higher. This results from the benefit of offering incentive wages now being larger. Not only is the likelihood of a mutiny occurring reduced, also the likelihood of winning the mutiny (should it occur) is increased. Therefore, principals are more willing to accept a reduction in residual claim to secure a higher likelihood of staying in charge.

Lastly, if the competition for the loyalty of the agents depends on both the wage the principal promises and on the wage the mutineer promises, as in section 3.4, then both a high-skilled principal and a high-skilled mutineer are more likely to win the mutiny, because they offer relatively higher wages. This means that due to mutinies, the leaders of criminal organisations will get more skilled.

In the process of finding these results, I have made a number of assumptions. Most importantly, I assumed that only one agent is afforded the opportunity to mutiny, and only one mutiny could occur regardless of the success of this mutiny. In section 4, I showed how the mutiny game changes if this assumption is relaxed slightly. In this extension, multiple agents got the opportunity to mutiny sequentially, but similarly only one mutiny could be started. The results remain similar, but wages are higher and the relation between the wages and the size of the organisation changes. Now, wages are significantly higher in larger organisations, up until the point that the wages are almost the maximum of what the leader can afford, which occurs at a size of roughly 100 employees. Then, wages drop again quickly to similar heights of that of an organisation with only one employee. This new relation between the wage and the size of the organisation is a result of an increased chance of a mutiny occurring. At first, the leader tries hard to dissuade all agents, and then at some point there are so many agents that need to be dissuaded, that it is no longer worthwhile. However, apart from the relation between w and n, the results remain similar.

Moreover, the assumption that only one agent gets the opportunity to mutiny could be relaxed entirely, so that multiple successful and unsuccessful mutinies can be started. Likely, the leader will then offer even higher incentive wages, because the probability of winning multiple mutinies is exponentially smaller than the probability of winning one mutiny. However, agents are less inclined to mutiny as well, since they would have to fend off multiple mutineers too if they become leader themselves, which decreases the expected benefits of mutinying. Which effect will dominate is hard to tell. However, for who attempts to journey down this route, I have to say the following: "Pirates, ye be warned". Having multiple mutinies means that the mutinying agent has to assess the likelihood of another agent mutinying after he has become the leader, in addition to the likelihood of winning the competition with the current leader. In turn, this holds for the other second agent as well: his required skill level is linked to both the skill level of the first agent and to the required skill level of some third agent. This loop only ends when there are no agents left to mutiny (i.e. after n repetitions). However, if by the hand of a skilled programmer this model could be created, interesting insights could be gained regarding how criminal organisations evolve over time, and become more or less fair to the agents.

Furthermore, I assumed that all the players in the mutiny game are risk-neutral. This has led to the outcome that principals generally pay lower wages in large organisations, because the cost of discouraging a mutiny becomes larger. However, if the principal was risk-averse and was earning a high residual claim in a larger organisation, she would surely be prepared to invest in order to have more certainty. Imaginably, wages would then be so high that no agent would want to mutiny, especially if they are risk-averse as well. This could lead to a situation where wages are higher in larger organisations, in contrast to what I have found now.

Lastly, I assumed that the number of agents in the organisation was fixed by some exogenous force. This assumption could be relaxed by extending the model with a hiring stage. In real life the wage the leader pays to her agents will strongly determine how many agents she is able to hire. If this is added to the model, the leader cannot give a wage lower than the participation constraint of the agents. Additionally, if the organisation is characterised by decreasing productivity of labour, the organisations in the criminal environment will likely be smaller than without threat of mutiny. This would occur because the principals of the criminal organisations set wages higher than they would if there was no threat of mutiny, which leads them to hire less labour. Then, both in a perfect labour market and in monopsonistic labour market, a higher threat of mutiny within criminal organisations leads to less crime. Furthermore, if a hiring stage is introduced, heterogeneous productivity between agents could also be considered in the model. In this case, if some agents are more productive —because they have higher skill levels— they are also paid more. This can lead to a situation where, automatically, the agents that are more likely to mutiny, are also more disincentivised to do so. Then, mutinies are less frequent.

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