Master Thesis Financial Economics

"The Diversification Benefits of Emerging and Frontier Equity Markets"

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Abstract

This study aims to analyze the time-varying diversification benefits of emerging and frontier equity markets to a global equity portfolio of developed markets, and a global asset allocation portfolio (including equities, bonds, and commodities). The analysis is based on MSCI index return data. Equities in developed, emerging, and frontier markets are represented by the MSCI World index (developed markets), the MSCI Emerging Markets index, and the MSCI Frontier Markets index respectively. Bonds and commodities, which are incorporated for the diversification over asset classes, are represented by the JPM Global Government Bond index and the S&P GSCI Commodity index. The Dynamic Conditional Correlation (DCC) GARCH model is estimated to examine the time-variation in correlations. Further, this research evaluates diversification strategies in the equity market and over different asset classes to find the best diversification strategy for the investors. We evaluate an equally weighted portfolio, a minimum variance portfolio, a maximum Sharpe ratio portfolio, and portfolios with fixed weights, using numerous portfolio performance measurements. The results reveal that frontier equity markets exhibit lower levels of correlation with developed equity markets compared to emerging equity markets. The inclusion of frontier equity markets in a global asset allocation portfolio can offer significant diversification benefits. In addition, emerging equity markets still offer diversification benefits to a portfolio of developed equity markets. Lastly, investing in emerging and frontier equity markets offer diversification benefits and added value to a portfolio of developed equity markets and a global asset allocation portfolio.

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1. Introduction

One of the investors' strategic options to reduce the risk of an equity investment portfolio is to invest their available funds in listed equities of different countries. Emerging equity markets are well-known for their low correlation with developed equity markets, and institutional investors typically invest in these markets to better diversify their global equity portfolios and achieve higher risk-adjusted returns (Harvey, 1995). However, due to today's strongly integrated global economy and financial markets, the correlation between developed and emerging equity markets has increased considerably over time, implying decreasing diversification benefits for investors (Bekaert and Harvey, 2017). As a consequence of decreasing diversification potential of emerging equity markets, investors started to have an interest in the less developed "frontier markets". Frontier markets may offer promising diversification benefits to investors; for example, Speidell and Krohne (2007) find a low correlation between frontier and developed equity markets. Berger, Pukthuanthong, and Yang (2011) find that frontier market equities offer significant diversification benefits, given their low integration with the world market.

This master thesis aims to add to the existing academic literature by examining the time-varying diversification benefits of emerging and frontier equity markets to a global equity portfolio of developed markets, and a global asset allocation portfolio (including equities, bonds, and commodities). Hence, we will examine how the correlations among developed, emerging, and frontier equity markets change over time and which is the appropriate investment proportion of each asset to diversify the investment portfolio. The working title of the thesis is "The diversification benefits of emerging and frontier equity markets," and the research questions that we are trying to answer are:

1) "Do emerging markets still offer strong diversification benefits compared to developed equity markets and to which extent?"

2) "Do less developed "frontier markets" offer better diversification properties to investors?"

3) "Which is the best allocation strategy for investors to diversify their portfolio?"

This study offers three main contributions to the academic literature. First, the results provide empirical evidence about the diversification benefits among developed, emerging, and frontier equity markets, unlike existing studies focusing on developed and emerging markets (e.g., Bekaert and Harvey, 2017) and that do not include frontier equity markets. Second, analyzing the time-varying correlations among developed, emerging, and frontier equity markets using the Dynamic Conditional Correlation (DCC) GARCH model, contributing to the literature on international equity diversification. This method was applied by Piljak and Swinkels (2017) to analyze time-varying dynamic correlations of frontier and emerging government bond markets with the U.S. government and corporate bond market. Third, this study combines international equity diversification with diversification over different asset classes to provide additional evidence on diversification properties of emerging and frontier equity markets in portfolio management. This approach was also applied by Jacobs, Müller, and Weber (2014).

The aim of this thesis is twofold. First, we analyze the time-varying diversification benefits of emerging and frontier equity markets to a global equity portfolio of developed markets. In particular, we estimate the time-varying correlations among emerging, developed, and frontier equity markets, using a moving-window approach based on the previous 60 months to gain an initial idea about how the correlations change through time. This method was also used in previous studies (e.g., Bekaert and Harvey, 2017; Jacobs, Müller, and Weber, 2014). We then estimate the time-varying correlations among emerging, developed, and frontier equity markets, using the Dynamic Conditional Correlation (DCC) GARCH model. This analysis allows the correlations to change over time in a formal econometric model, so it gives a better estimation of the correlation process. Engle proposed this model in 2002, and it has also been used by Jacobs, Müller, and Weber (2014). Second, we examine the added value of emerging and frontier equity markets to a global equity portfolio of developed markets, as well as to a global asset allocation portfolio by comparing an equally weighted portfolio, a minimum variance portfolio, a maximum Sharpe ratio portfolio, and portfolios with fixed weights, using numerous portfolio performance measurements (volatility, mean return, Sharpe ratio, and M2 method).

The results of our analysis reveal that frontier equity markets exhibit lower levels of correlation with developed equity markets compared to emerging equity markets. This finding is in line with the existing literature as Berger, Pukthuanthong, and Yang (2011), demonstrate that frontier equity markets have a lower level of integration with the world equity market, in comparison with emerging and developed markets. We show that the inclusion of frontier equity markets in a global asset allocation portfolio can provide significant diversification benefits, and thus, improve the risk-adjusted returns of the portfolios. This also corroborates the existing findings in the literature (e.g., Girard and Sinha, 2008). We find that emerging equity markets still offer diversification benefits to a portfolio of developed equity markets; nevertheless, the correlation between these markets has increased considerably throughout the years. This finding is also in line with the existing research (e.g., Bekaert and Harvey, 2017). Lastly, our results demonstrate that investing in emerging and frontier equity markets improve the risk-adjusted return, and thus, the diversification benefits of a portfolio of developed equity markets and a global asset allocation portfolio.

The remainder of this paper is organized as follows. In Section 2, we provide a brief review of the related literature. Section 3 presents the data sample and the data description. Section 4 describes the methodology. In Section 5, we analyze the results of this study. Finally, in Section 6, we conclude.

2. Literature Review

This section provides a background discussion of the diversification benefits of emerging and frontier equity markets, citing the most relevant literature referring to the diversification concept, the international diversification, the diversification benefits and the evolution of emerging markets, the emergence of frontier markets and their alternative diversification opportunities, as well as the asset allocation models.

2.1. Diversification

Diversification is commonly associated with the phrase "Do not put all your eggs in one basket" that most people are familiar with. Diversification is a popular method applied in portfolio management. It is defined as a technique that allows investors to minimize the risk of their portfolios by investing in assets that have a low correlation with each other. This can be achieved by selecting either different asset classes or assets operating in different industries or assets traded in different markets/nations. In this thesis, we focus on diversification across foreign equity markets or specifically across developed, emerging, and frontier equity markets. Hence, it is important first to discuss international diversification in equity markets.

2.2. International diversification

International diversification is defined as a method that allows investors to reduce the risk of their portfolios by investing across nations that have relatively low return correlations with each other. The goal of international diversification is to construct a portfolio that includes investments in more than one nation in order to improve the portfolio's risk-return trade-off. Early studies in international diversification indicate that investing internationally is beneficial for investors (Grubel, 1968; Levy and Sarnat, 1970; Solnik, 1974; Grauer and Hakansson, 1987). For instance, Levy and Sarnat (1970) examine the correlations between developed and developing equity markets and reveal that combining these markets in a portfolio can reduce the portfolio's risk considerably. Solnik (1974) demonstrates that the risk of a portfolio, including stocks from U.S. and European countries, is half of the risk of a portfolio including only U.S. stocks. Grauer and Hakansson (1987) investigate that the gains from including non-U.S. asset categories in a universe of international securities were remarkably substantial.

2.3. Emerging markets

Over the last two decades, the diversification benefits offered by emerging markets have attracted considerable attention in the international finance literature (see early study of Errunza, 1977; recent study of Barry, Peavy, and Rodriquez, 1998), while previously the focus was on the diversification benefits offered by developed markets (see Solnik, 1974). This transition occurred due to increasing interdependence among international equity markets in the mid-1990s (see Pukthuanthong and Roll, 2009). Thus, investors have searched for alternative investment opportunities to diversify their portfolios and to achieve higher risk-adjusted returns. The rising interest of investors in emerging markets is also because of their increased opportunities for investment and higher economic growth. In general, emerging markets are commonly associated with higher expected returns, higher volatility, and lower correlation of returns in comparison with developed markets (Harvey, 1995).

An extensive stream of literature has emerged to provide evidence for the features that are commonly associated with emerging markets. For example, Barry, Peavy, and Rodriguez (1998) examine the performance characteristics of emerging markets, and they provide evidence that emerging markets have high return and volatility. Bekaert and Harvey (1997) find that emerging markets exhibit high and predictable returns, as well as low linkages with developed markets by examining whether news and shocks that originate from the rest of the world may influence emerging markets. Li and Majerowska (2008) find that emerging markets are weakly linked to developed markets, by testing the linkages between emerging stock markets in Poland and Hungary and developed stock markets in Germany and the U.S.

However, due to globalization and integration of financial markets, recent studies show that the international diversification benefits are diminishing, as correlations among international financial markets are increasing (see Kearney and Lucey, 2004). Consequently, several studies have emerged to measure financial market integration, such as Bekaert and Harvey (1995), and Carrieri, Errunza, and Hogan (2007).

The increased market liberalizations across emerging markets, which means that these markets have become more accessible for foreign investors, and the further globalization, pose questions for the diversification benefits that these markets offer. Bekaert and Harvey (2017) find that the correlations of emerging markets with the rest of the world have increased, reducing the diversification benefits of investing in emerging markets, although the process of integration of emerging markets into world markets is far from complete. According to Bekaert et al. (2011), emerging markets should still be considered as a separate asset class from developed markets, as emerging markets are not fully integrated within world markets.

Because of the high integration of the global economy and financial markets, the interdependence among developed and emerging equity markets has increased considerably. As a consequence of the decreasing diversification properties of emerging equity markets, frontier equity markets have received increased attention on behalf of international investors, given their low correlation with world markets and their attractive high returns, as reviewed in the next section.

2.4. Frontier markets

Frontier markets are smaller, less accessible, but still "investable" countries in comparison with emerging market counties. The term "frontier markets" originated in the 1990s, when International Finance Corporation (IFC) started to publish data of frontier markets as part of its Emerging Markets Database (EMDB). Standard and Poor's (S&P) acquired IFC's EMDB in 2000 and launched the first investable indexes: Select Frontier Index (including 30 of the largest companies from 11 counties) and Extended Frontier Index (including 150 companies from 27 countries) in 2007. Given the increased interest of investors in these markets, Morgan Stanley Capital International (MSCI) also began a Frontier Markets Index in 2007. In addition, frontier markets have emerged further through the creation of frontier market mutual funds since 2008, and exchange-traded funds (ETFs). Hence, investors can easily access these markets nowadays. For instance, Berger, Pukthuanthong, and Yang (2013) investigate whether frontier markets offer diversification benefits, and their analysis is based on frontier market exchange-traded funds (ETFs).

To be classified as a frontier market depends on the market classification framework that each index company follows. MSCI evaluates a country based on economic development, size and liquidity, and market accessibility to be included in its Frontier Markets index. As of March 2019, MSCI has a list of 29 countries that it classifies in this group. Most of the index stocks are from sectors: financials, consumer staples, and telecommunication services. Kuwait, Vietnam, and Argentina combined account for 55% of the total index market value. Furthermore, out of 116 stocks in the index, the five largest represent about 24% of the total index weight. Therefore, investors should be cautious of the index concentration in terms of sector and country.

The potential diversification benefits of frontier markets have been documented by Berger, Pukthuanthong, and Yang (2011), who demonstrate that frontier markets have a lower level of integration with the world market, in comparison with emerging and developed markets. Furthermore, Speidell and Krohne (2007) report a low correlation between frontier and developed equity markets. The profitability of frontier markets has been evaluated by De Groot, Pang, and Swinkels (2012), who examine the cross-section of returns in 1400 frontier equity markets and find that portfolios based on value and momentum in frontier market derive higher excess returns than that in emerging and developed markets. In addition, Girard and Sinha (2008) study the cross-section of risk premiums of 360 stocks in 19 frontier markets. They find that frontier markets offer higher return potentials than developed and emerging markets. Therefore, the inclusion of frontier markets in a portfolio of developed and emerging markets, provides significant diversification benefits and more efficient portfolios for the investors, as their portfolios' risk-adjusted returns are improved.

2.5. Asset allocation models

Significant progress has been achieved in the literature of asset allocation. The field started with the prominent work of Markowitz (1952), which led to the establishment of Modern Portfolio Theory and derived the mean-variance framework for constructing efficient investment portfolios in order to achieve the highest expected (mean) return with the lowest variance of returns. However, when implemented in practice, optimized mean-variance portfolios often perform poorly out-of-sample and can contain rather extreme weights. Early and recent studies comparing the out-of-sample performance of an equally weighted portfolio with the performance of optimized portfolios, for instance, Frankfurter et al. (1971), and DeMiguel et al. (2009) demonstrate that various optimized portfolios are often unable to outperform equally weighted portfolios. Hence, it is insufficient to limit the thesis to extensions of the Markowitz (1952) model.

The equally weighted portfolio or 1/N portfolio is also called a naïve diversification strategy. The naïve diversification strategy is easy to implement, as it does not rely either on the estimation of expected returns or on optimization compared to traditional optimized mean-variance efficient portfolios. In addition, despite the considerable advances in sophisticated asset allocation models, investors continue to use the simple 1/N portfolio strategy to allocate their wealth across each of N assets (see Benartzi and Thaler, 2001).

Minimum variance portfolios also do not require mean return forecasts and focus only on volatility. The minimum variance portfolio is the portfolio of risky assets with the lowest risk, usually measured with the variance of the portfolio return. Haugen and Baker (1991) and Chopra and Ziemba. (1993), among others, demonstrate the superior performance of minimum variance portfolios. A study by Clarke, De Silva, and Thorley (2006) investigated that minimum variance portfolio, including the 1,000 largest market capitalization U.S. stocks, delivered lower volatility of about 25 percent than the market portfolio, and at the same time a higher average return. The implementation of minimum variance portfolios needs the estimation of variances and covariances, but not the estimation of expected returns (which are relatively complicated and noisy to estimate; see Chopra and Ziemba, 1993).

The maximum Sharpe ratio portfolio, which is also referred to as the Tangency Portfolio, is the portfolio with the highest Sharpe ratio, or with the highest expected return-to-risk trade-off. Sharpe ratio is a widely used measure for calculating risk-adjusted return and for evaluating the performance of investment portfolios, which first introduced by Sharpe (1966). Maller, Durand, and Jafarpour (2010) argue that a portfolio is optimized using the maximum Sharpe ratio. If a risk-free asset is available, and investors agree on the expected returns and covariance matrix, they would all invest in the tangency portfolio (Bodie, Kane, and Marcus, 10th Edition). However, similar to other mean-variance efficient portfolios, the tangency portfolio suffers from high estimation risk and often performs poorly out-of-sample (DeMiguel et al., 2009).

3. Data

In this section, we first describe the equity market dataset for the analysis of the diversification benefits among equity markets, and the dataset on other asset classes for the examination of the diversification benefits over different asset classes. Second, we explain the choice of our data sample, indicating why the time horizon is suitable for answering our research questions and how we collect the data.

3.1. Data Description

3.1.1. Equity market data

The equity market dataset includes developed, emerging, and frontier equity markets represented by the Morgan Stanley Capital International (MSCI) global equity indexes which are classified in developed, emerging, and frontier markets. In particular, these indexes are the MSCI World (developed markets), the MSCI Emerging Markets and the MSCI Frontier Markets.

These indexes have been widely used in previous research studies (e.g., Bekaert and Harvey, 2017; Jacobs, Muller, and Weber, 2014) to represent the global equity markets and by investors to track the performance of the global stock universe. These indexes collectively measure the performance of 2,871 stocks across 76 countries according to the latest MSCI review. These indexes have as a target to capture approximately 85% of the free float-adjusted market capitalization of the respective equity universe.

MSCI Index methodology

The MSCI global equity indexes are free float-adjusted market cap-weighted indexes, which means that the index’s stocks are weighted according to their free float-adjusted market capitalization, which is calculated by multiplying the stock price with the number of shares available in the market excluding strategic public or private shares, e.g., holders of this shares are governments, banks, corporations. The non-strategic shares are holding from e.g., individuals, investment funds, mutual funds, pension funds. Thus, the largest market capitalized security or country has the highest weighting within the index and the highest impact on the index. In particular, the price of an MSCI index is constructed as follows.

For instance, the price of the MSCI Emerging Markets index is calculated based on the following formula:

$$MSCI_{Emerging\ Markets,t} = \sum_{i=1}^{N} w_i\, P_{it}$$

where *N* = the number of stocks included in the MSCI Emerging Markets index (target to capture 85% of the free float-adjusted market capitalization), P_{it} = the U.S. dollar-denominated price of each stock included in the index, w_i = the weight of each stock included in the index, where

$$w_i = \frac{free\ float{-}adjusted\ mcap_i}{\sum_{i=1}^{N} free\ float{-}adjusted\ mcap_i}$$

The price index tracks the performance of the emerging markets with the total of free float-weighted market capitalization returns of all its stocks on any given day. To find the total return index, we should also incorporate the dividend payments.

MSCI World index

The MSCI World index was launched on March 31st, 1986 to capture the developed equity markets. As of February 28th, 2019, the index consists of 23 countries and tracks the performance of 1,632 stocks. The including countries are from the regions Americas, Europe & Middle East, and Pacific and they are the following: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Most of the countries which have been classified as developed markets from the beginning of index establishment have remained in the same market category. However, some countries are migrating from one index to another, such as Greece which was included in developed markets in 2001 and then downgraded to the emerging market category in 2013, or Israel which was upgraded from emerging to developed market in 2010.

MSCI Emerging Markets index

MSCI launched on January 1st, 2001 the MSCI Emerging Markets index, which initially included ten countries representing less than 1% of world market capitalization, while today it consists of twenty-four countries representing 10% of world market capitalization.

With the attractiveness of the emerging equity markets in the 1990s, the international investors started to search for indexes to provide accurate information for the investment environment in these markets. This index currently measures the performance of 1,125 securities across the regions Americas, Europe, Middle East & Africa and Asia.

The countries included in the MSCI Emerging Markets index are the following: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Qatar, Russia, South Africa, Taiwan, Thailand, Turkey, and United Arab Emirates. Certain countries appear and disappear throughout the examining period, based on the index inclusion rules (see the Appendix). For instance, Argentina (2009), Morocco (2013), Jordan (2008) have been downgraded from emerging to frontier markets, while Portugal (1997), Greece (2001) and Israel (2010) have been upgraded from emerging to developed. Greece re-entered in the emerging markets index in 2013.

MSCI Frontier Markets index

The MSCI Frontier Markets Index was launched on December 18th,2007. It represents a large and mid-cap equity performance of 114 stocks across 29 countries. The term “frontier markets” has recently been developed to distinguish these markets from an earlier wave of developing countries accessing international financial markets in the 1980s and 1990s (i.e., emerging markets).

The countries included in the MSCI Frontier Markets index are from the regions Americas, Europe & CIS (the Commonwealth of Independent States), Middle East and Asia and they are the following: Argentina, Bahrain, Bangladesh, Burkina Faso, Benin, Croatia, Estonia, Guinea-Bissau, Ivory Coast, Jordan, Kenya, Kuwait, Lebanon, Lithuania, Kazakhstan, Mauritius, Mali, Morocco, Niger, Nigeria, Oman, Romania, Serbia, Senegal, Slovenia, Sri Lanka, Togo, Tunisia and Vietnam. Continuous deletions and additions of countries according to the inclusion rules of the index, ensure that it represents the equity frontier markets accurately.

More specifically, Pakistan has been upgraded from frontier to emerging market in 2017, while Bulgaria and Ukraine were downgraded from frontier to standalone index in 2016 and 2015 respectively. Bulgaria had a steady decline in size and liquidity of its equity market, as the securities determined by MSCI as investable fell below the minimum liquidity and size requirement of frontier markets index. Ukraine did not meet the minimum market accessibility requirement of the Frontier Markets index, as the National Bank of Ukraine introduced restrictions on capital flows which had an impact on international institutional investors' equity market accessibility (statements released on MSCI website).

The distinction between frontier and emerging markets

According to MSCI, the distinction between emerging and frontier equity markets is based on the MSCI market classification framework (see in the Appendix: Exhibit I), which consists of three criteria: economic development, size and liquidity, and market accessibility in order to classify the countries as developed, emerging and frontier markets. In order to classify the developed markets, MSCI uses the economic development criterion, which considers the sustainability of economic development using the Gross National Income per capita (GNI per capita) that the World Bank uses to categorize the markets. Even though frontier markets consist mostly of countries that are less economically developed than emerging markets, this criterion is not used by MSCI to distinct the two markets, as it is not considered relevant given their wide range of development levels.

Frontier markets consist of companies that usually have smaller size and liquidity than emerging markets. According to MSCI, a country is classified as a frontier market country when at least two companies should meet minimum-company size of USD 797 mm (full market cap), minimum-security size of USD 71 mm (float market cap), and minimum-security liquidity of 2.5% Annualized Traded Value Ratio (ATVR). When a country is classified as an emerging market at least three companies should meet minimum-company size of USD 1,594 mm, minimum-security size of USD 797 mm, and minimum-security liquidity of 15% ATVR.

Frontier market companies have lower market accessibility than emerging ones, which means that international institutional investors have lower access to this market. To calculate this criterion, MSCI examines several sub-criteria such as openness to foreign ownership, ease of capital inflows/outflows, efficiency of operational framework, competitive landscape, and stability of the institutional framework.

3.1.2. Data on other asset classes

The dataset for the global asset allocation includes equities, bonds, and commodities. Equities are represented by the equity indexes as mentioned above, which are the MSCI World (developed markets), the MSCI Emerging Markets, and the MSCI Frontier Markets.

Bonds are included in the dataset because they provide low correlation with equities (e.g., Chiang and Li, 2009). Bonds are represented by the J.P. Morgan Global Government Bond index, which consists of investment-grade government bonds of different maturities. This index was launched in 1989 and has been frequently employed in the literature (e.g., Jacobs, Muller, and Weber, 2014) as it provides extended historical data availability. Investors have also used it as a global government bond benchmark of developed countries.

Commodities are incorporated in the dataset as they offer diversification properties according to academic research. For instance, Harvey and Erb (2006) demonstrated that commodities have low and even negative correlations with equities and bonds. Becker and Finnerty (2000) exhibited that incorporating commodities into portfolios with equities and bonds can improve the risk-adjusted return of the portfolios. For commodities, we use the total return on the Standard & Poor's Goldman Sachs Commodity Index (S&P GSCI), which tracks the performance of actively traded commodity futures contracts. This index has been used in previous research studies (e.g., Jacobs, Muller, and Weber, 2014; Kurach, 2012) to represent the commodity market and in the investor community as a benchmark to track the commodity market performance.

Goldman Sachs introduced this index (previously known as Goldman Sachs Commodity Index) in 1991, which was later acquired by Standard & Poor's (S&P) in 2007. The S&P GSCI includes 24 commodity futures contracts over the sectors of energy, industrial, agricultural, precious metals, and livestock. In particular, the index provides a broad range across commodity sectors: six energy products, five industrial products, eight agricultural products, two precious metals, and three livestock products. This index is weighted by each commodity's global production, which is calculated by the average production in the last five years. The number of commodity futures contracts included in the index is not limited; however, these contracts should satisfy specific criteria in terms of liquidity and investability in order to be considered. It is worth mentioning that the index is heavily weighted to the energy sector, which provides the highest production in the commodity market, and thus, it accounts for the highest weighting in the index. As of 2018, the energy sector accounts for 58.6 % of the weighting in the S&P GSCI, according to the announcement of S&P Dow Jones Indices for the S&P GSCI composition.

Real estate is not included as an alternative asset in the dataset as individual investors are often already heavily exposed to real estate risk and it might decrease the diversification of the portfolio (e.g., Calvet et al., 2007). Furthermore, hedge funds and private equity funds have not to be taken into account because of their limited diversification benefits (e.g., Amin and Kat, 2003; Phalippou and Gottschalg, 2009).

3.2. Data Sample

Data Sources

For all the indexes, we use U.S. dollar-denominated total return indexes (RI) extracted from Thomson Reuters DataStream. We choose to convert all index levels into U.S. dollars against the local currency since the focus of the research is on U.S investors. DataStream's Time Series Request function can calculate prices in any given currency automatically by multiplying local prices with the daily exchange rate. In addition, we use the total return index, which includes re-invested dividends in addition to price changes, rather than the price index, which includes only price changes. The reason for this is that we cannot ignore dividend payments when determining the returns since it constitutes the standard approach for investment research.

DataStream defines the total return index (RI) as "the theoretical growth in value of a shareholding over a specified period, assuming that dividends are re-invested to purchase additional units of equity at the closing price applicable on the ex-dividend date."

The formula is the following:

$$RI_t = RI_{t-1} * \frac{PI_t}{PI_{t-1}} * \left(1 + \frac{DY_t}{100} * \frac{1}{N}\right)$$

where: RI_t = return index on day t, RI_{t-1} = return index on day t-1, PI_t = price index on day t, PI_{t-1} = price index on day t-1, DY_t = dividend yield % on day t, N = number of working days in the year (assumed 252 days).

Time frame

One of the main concerns in a research study with time series data is the length of the dataset. The more historical data are available, the more reliable the results. This study used monthly return data. An exception is the part of diversification over other asset classes from the portfolio analysis, in which daily return data was used.

For the moving-window correlation analysis among emerging and developed equity markets, the sample period started on January 1st, 1988, reflecting the day where the MSCI Emerging Markets index was launched and ends on March 1st, 2019. For the moving-window correlation analysis among emerging and frontier equity markets, the starting point was on June 1st, 2002, which is the earliest date that monthly historical return data are available and ends on March 1st, 2019. For the moving-window correlation analysis between frontier and developed equity markets, for the dynamic conditional correlation analysis, as well as for the portfolio analysis, the same period was used. For the second part of the portfolio analysis, daily return data was used with a starting point on June 1st, 2002 and ends on March 15th, 2019.

3.3. Reliability

The main reasons for selecting these indexes presented above for our analysis are:

1) Transparency; the index construction and the index methodology are disclosed by the index provider,

2) Representativeness; the indexes cover the equity market of developed, emerging, and frontier markets, the investment-grade government bond market, and the commodity market in order to examine the diversification benefits of these markets,

3) Investment access; international investors can invest directly in these indexes through exchange-trade-funds (ETFs), and they have high liquidity as there is minimum liquidity requirement for the inclusion of securities in these indexes,

4) Data availability; there is extended availability of historical data to conduct accurate and reliable results.

4. Methodology

The methodology consists of two parts. In the first part, we estimate the time-varying correlations among emerging, developed, and frontier equity markets. The first step is to estimate the time-varying correlations, using a moving-window approach based on the previous 60 months to gain an initial idea about how the correlations change through time. This method was used in previous studies (see, Bekaert and Harvey, 2014; Jacobs, Muller, and Weber, 2014).

The next step is to estimate the dynamic conditional correlations among emerging, developed, and frontier equity markets, using the Dynamic Conditional Correlation (DCC) GARCH model. This analysis allows the correlations to change over time in a formal econometric model, so it gives a better estimation of the correlation process. This model was proposed by Engle in 2002, and it has been used by Jacobs, Muller, and Weber (2014), among others.

In the second part, we examine the added value of emerging and frontier equity markets to a global equity portfolio of developed markets, as well as to a global asset allocation (including equities, bonds, and commodities). This occurs by comparing an equally weighted portfolio, a minimum variance portfolio, a maximum Sharpe ratio portfolio and portfolios with fixed weights, using the following portfolio performance measurements: volatility (standard deviation), mean return, Sharpe ratio, and M2 method.

4.1. Correlations

In this section, we explain the time-varying correlations among developed, emerging, and frontier equity markets using the moving window approach, followed by the dynamic conditional correlations, using the DCC GARCH model.

4.1.1. Moving window

A method that is used to estimate correlations that change over time is the moving window approach. This method is usually used to smooth the day-to-day price fluctuations of the financial markets in order to allow investors to identify trends. In this study, we use 60 months as a moving window of time, for calculating the correlations among developed, emerging, and frontier equity markets. Specifically, in each month t, starting from t = 60+1, we use the data of the previous 60 months to estimate the correlation for each month t. This process is continued by adding the returns of the indexes for the next period in the dataset and dropping the oldest returns. Thus, this method takes into account current information and gives equal weight to all the returns used in the calculation of the moving window correlations. However, a more formal estimation of the time-varying correlations can be executed by using a multivariate GARCH model, which is described in the following section.

4.1.2. DCC GARCH model

We estimate the time-varying correlations among developed, emerging, and frontier equity markets, by employing the Dynamic Conditional Correlation (DCC) GARCH model, which was first introduced by Engle (2002). This model is part of the family of multivariate GARCH models and offers an effective way to capture the time-varying nature of the correlations.

The multivariate GARCH model assumes that returns are conditionally multivariate normal with zero expected value and covariance matrix H_t,

$$r_t|F_{t-1} \sim N\ (0, H_t)$$

where r_t is the vector containing the log-returns of n equities, F_{t-1} is the information set at time $t-1$, and H_t is the conditional covariance matrix.

The multivariate conditional covariance matrix is specified as follows:

$$H_t \equiv D_t R_t D_t$$

where D_t is the (n × n) diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ on the i^{th} diagonal ($D_t = diag\{\sqrt{h_{i,t}}\}$) i=1,2, …., n; R_t is the (n × n) time-varying conditional correlation matrix ($R_t \equiv \{\rho_{i,j}\}_t$).

Engle (2002) proposed a two-stage estimation for the conditional covariance matrix H_t. In the first stage, a univariate GARCH model:

$$h_{i,t} = \omega_i + a_i \varepsilon_{i,t-1}^2 + b_i h_{i,t-1}$$

where i=1,2, …., n and $h_{i,t}$ is the conditional variance of the error term $\varepsilon_{ij,t}$, are fitted for each of the stock return series, and estimates of $\sqrt{h_{i,t}}$ are obtained. These estimates are used as inputs in the second stage to calculate the standardized residuals. The standardized residuals are then utilized to estimate the dynamic conditional correlation parameters. The time-varying dynamic conditional correlation is specified as follows:

$$q_{ij,t} = (1 - \alpha - \beta)\bar{\rho}_{ij} + \alpha\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta q_{ij,t-1}$$

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}}\sqrt{q_{jj,t}}} \qquad i, j = 1,2, \dots, n, and\ i \neq j$$

where $q_{ij,t}$ is the (n × n) time-varying covariance matrix of $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$; $\rho_{ij,t}$ is the conditional correlation between the equity returns of market i and j; $\bar{\rho}_{ij}$ is the (n × n) unconditional covariance between the standardized residuals $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$; α and β are non-negative scalar parameters. Parameters α and β should satisfy α+β< 1, ensuring that $q_{ij,t}$ is positive and mean-reverting. The detailed estimation procedure of the DCC model is described in Engle and Sheppard (2001).

4.2. Asset allocation models

In this section, we discuss the asset allocation models considered for portfolio selection in the case of both global equity diversification and diversification over asset classes.

While academic research focuses on extensions of the Markowitz (1952) model, we concentrate on other well-established methods. Markowitz mean-variance efficient portfolios, while being optimal in theory, often perform poorly out-of-sample and can produce extreme weights in practice. The main problem with portfolio optimization in practice is that expected returns are difficult to estimate (noisy) and the use of simple historical averages can give poor results unless the time-series are really long (see Kritzman et al., 2010). After comparing asset allocation strategies, DeMiguel et al. (2009) have argued that the optimization strategies are not consistently better than a simple equally weighted portfolio in terms of Sharpe ratio, certainly equivalent return, or turnover. Hence, the first model we apply is the equally weighted portfolio (or naïve portfolio), as reviewed in the next section.

4.2.1. Equally weighted portfolio

The equally weighted portfolio or 1/*N* portfolio has weight $w_t = 1/N$ in each of the *N* assets or simply allocates equally across all *N* assets. It is also referred to as a naïve diversification strategy. It has become standard in the literature to consider the 1/*N* portfolio as a benchmark for more sophisticated asset allocation models. First, the 1/*N* portfolio is easy to implement, as it does not rely either on the estimation of expected returns or on optimization compared to the traditional optimized mean-variance efficient portfolio. Second, investors continue to employ this naïve diversification strategy to allocate their wealth across assets, as shown by Benartzi and Thaler (2001). For instance, in the case of global equity diversification, the equally weighted portfolio will be constructed with weights of 33% in developed markets, 33% emerging markets, and 33% frontier markets. The weights of an equally weighted portfolio are specified as follows:

$$w_t^* = \frac{1}{N}\,\mathbf{1}$$

where *w* is the vector of portfolio weights, *N* is the number of different assets, and 1 is a column vector of an appropriate dimension whose entries are equal to one.

4.2.2. Minimum variance portfolio

The second model we implement is the minimum variance portfolio with short-sale constraints, which is the portfolio that minimizes the variance of returns or the risk. A minimum variance portfolio does not require mean return forecasts, which are relatively noisy to forecast (see DeMiguel et al.; 2009) and focus only on estimates of the variance and the covariance. Studies such as Haugen and Baker (1991), and Chopra and Ziemba (1993) have shown the superior performance of minimum variance portfolios. The weights of a minimum variance portfolio are given as:

$$w_t^* = \frac{\sum_t^{-1}\mathbf{1}}{\mathbf{1}'\,\sum_t^{-1}\mathbf{1}}$$

where *w* is the vector of portfolio weights, *N* is the number of different assets, Σ is the *N x N* matrix which contains the return variances and covariances, **1** is a column vector of an appropriate dimension whose entries are equal to one.

4.2.3. Maximum Sharpe ratio portfolio

The third model we employ is the maximum Sharpe ratio portfolio, which is often referred to as the Tangency portfolio. The objective of this model is to maximize the Sharpe ratio of the portfolio, which intends to maximize the risk-adjusted return of the portfolio. On the one hand, Maller, Durand, and Jafarpour (2010) provide evidence that a portfolio is optimized using the maximum Sharpe ratio. On the other hand, DeMiguel et al. (2009) find that the tangency portfolio underperforms compared to the equally weighted portfolio out-of-sample. The same occurs when it is compared to the optimized mean-variance portfolio. The weights of a maximum Sharpe ratio portfolio are provided as follows:

$$w_t^* = \frac{\sum_t^{-1} \mu_t}{\mathbf{1}' \sum_t^{-1} \mu_t}$$

where w is the vector of portfolio weights, N is the number of different assets, μ represents the expected returns of the N assets, Σ is the N x N matrix which contains the return variances and covariances, **1** is a column vector of an appropriate dimension whose entries are equal to one.

The detailed theoretical description of the portfolio weights can be found in Christoffersen et al. (2014).

4.2.4. Portfolios with fixed weights

We construct portfolios with restrictions on the weights in each N assets. In the case of global equity diversification, we select the following weights:

1) 100% in developed markets
2) 85% in developed markets, 10% in emerging markets and 5% in frontier markets
3) 55% in developed markets, 30% in emerging markets and 15% in frontier markets

In the case of diversification over other asset classes, the selected weights are as follows:

1) 33% in developed markets, 33% in emerging markets and 33% in frontier markets
2) 28% in developed markets, 28% in emerging markets, 28% in frontier markets, 10% in bonds, and 5% in commodities
3) 18% in developed markets, 18% in emerging markets, 18% in frontier markets, 30% in bonds, and 15% in commodities

4) 100% in bonds
5) 100% in commodities

4.3. Performance evaluation

In this section, we describe the measurements that will be used to evaluate the performance of the portfolios. These are:

- The mean return, which is calculated as the annualized average return
- The volatility, which is measured as the annualized standard deviation
- The Sharpe ratio (SR), which is defined as the average monthly (or daily) excess return over the risk-free rate, divided by the standard deviation of monthly (or daily) returns and was introduced in 1966 by Sharpe. Despite the considerable development of alternative metrics of investment performance, the Sharpe ratio remains one of the most popular measurements of portfolio performance. It is calculated as follows:

$$SR_i = \frac{r_i - r_f}{\sigma_i}$$

where r_i is the return of the portfolio, r_f is the risk-free rate (the yield of 3 month U.S. Treasury bill), $r_i - r_f$ is the average excess return of the portfolio i, σ_i is the standard deviation of the portfolio returns.

- The M2 method, was developed by Modigliani and Modigliani (1997) and is also known as Modigliani risk-adjusted performance measure. The M2 is equivalent to the return that a portfolio would have achieved if its standard deviation was the same as that of the benchmark. In this study, the benchmark is the MSCI World index. The M2 is calculated as follows:

$$M2 = \frac{r_i - r_f}{\sigma_i} \cdot \sigma_M + r_f$$

where $r_i - r_f$ is the average excess return of portfolio i, σ_i is the standard deviation of the portfolio, σ_M is the standard deviation of the market (i.e., the standard deviation of the benchmark MSCI World index), and r_f is the risk-free rate (the yield of 3 month U.S. Treasury bill).

5. Results

5.1. Correlation analysis

In this section, we examine the time-varying correlations among emerging, developed, and frontier equity markets. This will allow us to understand the relationship that exists among these markets; thus, it will help us to recognize their diversification benefits.

5.1.1. Moving window

In this part, the time-varying correlations are examined using the moving window approach.

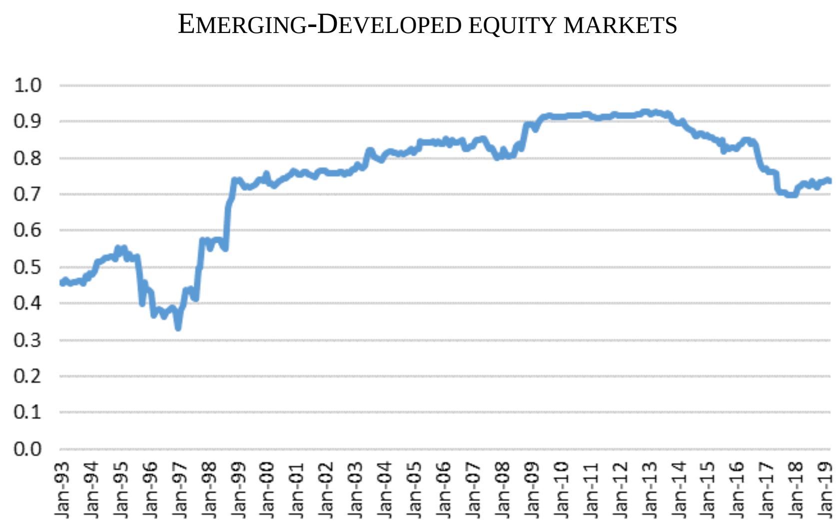

Fig. 1. The change of correlation between emerging and developed equity markets
This figure illustrates the change of correlation between the MSCI World index and the MSCI Emerging Markets index from January 1, 1988 to March 1, 2019. The correlation is calculated using the monthly returns of indexes and then averaged using a moving-window approach based on the previous 60 months.

Fig. 1 reveals that the correlation between emerging and developed equity markets in the early 1990s was between 0.4 and 0.5, which is considered low. During this period, the diversification benefits offered by emerging markets attracted considerable attention from investors. However, this correlation has increased and was between 0.6 and 0.7 in 1998-2003. From 2014 to 2015, it remained high and stable, between 0.8 and 0.9. Then from 2016 onwards, it decreased to approximately 0.7. Overall, this suggests that in recent years the diversification benefits from investing in emerging equity markets are probably not very high to a global equity portfolio of developed markets, although not zero either (as the correlation is not equal to 1).

FRONTIER-EMERGING EQUITY MARKETS

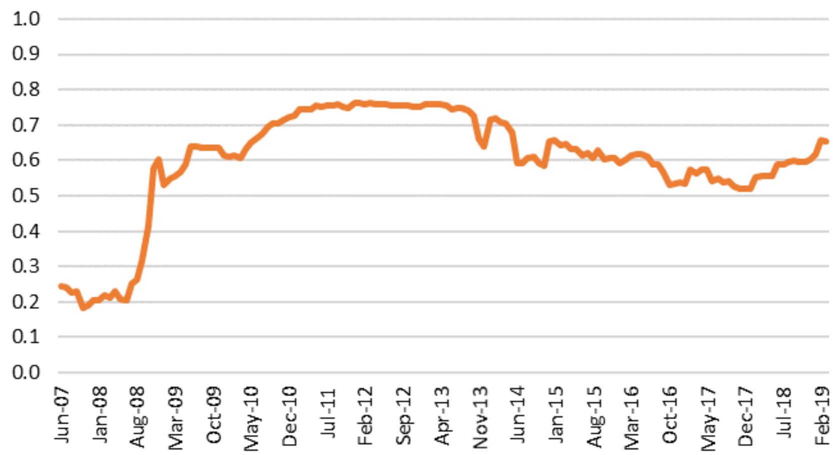

Fig. 2. The change of correlation between frontier and emerging equity markets
This figure reveals the change of correlation between the MSCI Frontier Markets index and the MSCI Emerging Markets index from June 1, 2002 to March 1, 2019. The correlation is calculated using the monthly returns of indexes and then averaged using a moving-window approach based on the previous 60 months.

Fig. 2 illustrates that the correlation between frontier and emerging equity markets increased significantly, from 0.2 to 0.6 approximately, during the period 2008 (one year after the MSCI frontiers market index's launch)-2009. Thus, we observe that the correlations increased through the years to a level ranging from 0.5 to 0.8. This demonstrates that frontier and emerging markets have a tendency to move in the same direction, although frontier markets may still offer some diversification benefits to investors who have invested in emerging markets since the correlation is not perfect.

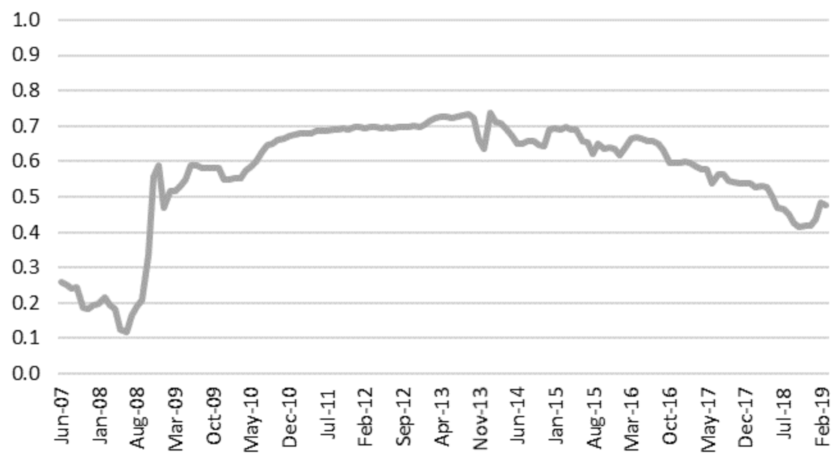

Fig. 3. The change of correlation between frontier and developed equity markets
This figure depicts the change of correlation between the MSCI Frontier Markets index and the MSCI World index from June 1, 2002 to March 1, 2019. The correlation is calculated using the monthly returns of indexes and then averaged using a moving-window approach based on the previous 60 months.

Fig. 3 indicates that the correlation between frontier and developed equity markets increased from low levels close to zero in the year of the MSCI frontiers market index's launch to higher positive levels the next years (from 0.5 to 0.7). We also observe a slightly decreasing correlation between frontier and developed markets in 2014 and 2017, although the correlation level always stayed above 0.4. This shows a slightly lower level of dependence of frontier with developed markets compared to emerging markets. This possibly means that frontier markets offer slightly more diversification benefits than emerging markets to investors whose portfolios include equities from developed markets.

Panel A: The entire period: 1 June 2002 - 1 March 2019			
	Developed markets	*Emerging markets*	*Frontier markets*
Developed markets	1.000		
Emerging markets	0.852	1.000	
Frontier markets	0.573	0.599	1.000
Panel B: The most recent 60-month period: 1 April 2014 – 1 March 2019			
	Developed markets	*Emerging markets*	*Frontier markets*
Developed markets	1.000		
Emerging markets	0.792	1.000	
Frontier markets	0.590	0.592	1.000

Table 1. Average correlations among *Developed markets*, *Emerging markets*, and *Frontier markets* for different time horizons using the moving-window approach
This table demonstrates the average time-varying correlations using the moving-window approach between the MSCI World index, the MSCI Emerging Markets index, and the MSCI Frontier Markets index in the entire period from June 1, 2002 to March 1, 2019 in Panel A and in the most recent 60-month period from April 1, 2014 to March 1, 2019 in Panel B. The correlations are calculated using the monthly returns of indexes and then averaged using a moving-window approach based on the previous 60 months.

Table 1 demonstrates the average correlations for different time horizons using the moving window approach. Panel A shows the average correlations over the entire period, while Panel B presents the average correlations in the most recent 60-month period, which is probably more relevant for the near future.

From this table, we derive that the highest correlation is between emerging and developed markets, with an average correlation of 0.852 over the entire period and 0.792 in the most recent 60-month period. Given these results, the inclusion of emerging markets in a portfolio of developed markets might offer less efficient diversification benefits. On the contrary, frontier markets can offer better diversification benefits compared to emerging markets, as the correlation of frontier with developed markets is lower (0.573 in Panel A and 0.590 in Panel B). The combination of frontier and emerging markets in a portfolio can offer diversification benefits, as their correlation is 0.599 in Panel A and 0.592 in Panel B. Nevertheless, we should keep in mind that both markets are influenced to some extent by the same factors.

5.1.2. DCC GARCH

We previously analyzed how the correlations change over time using a moving window of time. In this part, we analyze the time-varying correlations using the Dynamic Conditional Correlation (DCC) GARCH model, which provides a better estimation of correlations, and thus, a more accurate estimation of time-varying diversification benefits of investing in emerging and frontier equity markets.

The first step is the estimation of the MGARCH (or Multivariate GARCH) model in Stata. The MGARCH implements conditional correlation models, and one of them is the Dynamic Conditional Correlation (DCC) model. The correlations generated from the DCC GARCH model are presented in Table 2.

DYNAMIC CONDITIONAL CORRELATION ESTIMATES BY DCC GARCH MODEL

	Developed markets	*Emerging markets*	*Frontier markets*
Developed markets	1.000		
Emerging markets	0.828*** (0.032)	1.000	
Frontier markets	0.585*** (0.070)	0.631*** (0.064)	1.000

Table 2. Dynamic conditional correlations among *Developed markets*, *Emerging markets*, and *Frontier markets* produced by the DCC GARCH model
This table summarizes the estimated dynamic conditional correlations between the MSCI World index, the MSCI Emerging Markets index, and the MSCI Frontier Markets index over the entire period from June 1, 2002 to March 1, 2019. Values in ( ) are standard errors. *, **, and *** indicate significance at 10%, 5%, and 1%. The correlations are calculated using the monthly returns of indexes and the DCC GARCH model through Stata. All the DCC GARCH model estimates, including a test for the DCC model versus constant correlation, are provided in the Appendix: Exhibit II.

Table 2 shows that the correlations produced by the DCC GARCH model are each positive and statistically significant at 1% level. The highest correlation is between emerging and developed markets (0.828), suggesting that the co-movement between these markets is high. The correlation between emerging and frontier markets is 0.631, followed by that between frontier and developed markets (0.585).

Comparing the correlations using the DCC GARCH model (Table 2) with the correlations using the moving window approach (Table 1, Panel A), we find that the correlation between emerging and developed markets in Table 2 is slightly lower than Table 1. The correlation of emerging and frontier markets and that of frontier and developed markets in

Table 2 are somewhat higher than Table 1. Therefore, the correlation estimates using the DCC GARCH model gives similar findings with the correlation estimates using the moving window approach.

In the next step, we examine how the estimations of diversification benefits among developed, emerging, and frontier equity markets change over time. It occurs by examining the estimates of correlations, and for their calculations, we need the estimates of variances and co-variances. For example, the calculation of the correlation of variable 1 and variable 2 is provided by the co-variance of variable 1 and variable 2, which is divided by the conditional standard deviation of variable 1 and the conditional standard deviation of variable 2. Consequently, we first depict the variance and co-variance time-series, which provide additional evidence to understand how diversification benefits change over time.

DYNAMIC CONDITIONAL VARIANCE ESTIMATES BY DCC GARCH MODEL

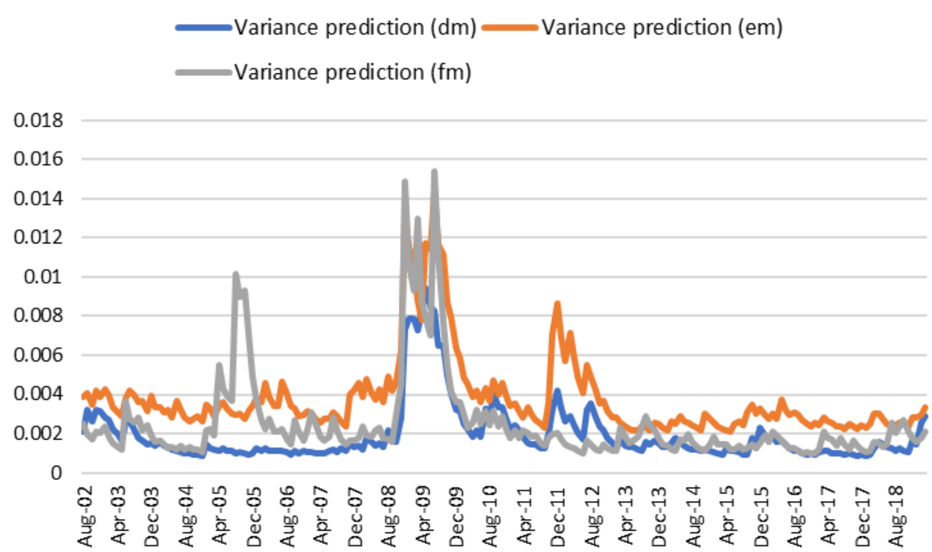

Fig. 4. The evolution of dynamic conditional variances among developed, emerging and frontier equity markets over time.
This figure compares the dynamic conditional variances of the MSCI World index (blue line in graph), the MSCI Emerging Markets (orange line), and the MSCI Frontier Markets index (grey line) in the period from June 1, 2002 to March 1, 2019. The dynamic conditional variances are based on the monthly equity index returns and calculated using the DCC GARCH model through Stata. See in the Appendix: Exhibit III.

In Fig. 6 we see the time-varying conditional variances of developed, emerging, and frontier equity markets based on the DCC GARCH model. The orange line represents the variance estimation of emerging markets, which ranges from 0.002 to 0.004 from 2002 to mid-2008. It reaches a peak (0.014) during the period of end-2008 and 2009, which can be credited to the financial crisis and a second peak (0.008) between the end of 2011 and 2012. From 2013 onwards, it decreased to levels between 0.002 and 0.003. The variance of developed markets is represented by the blue line, which takes values from 0.002 to 0.001 from 2002 to mid-2008 as well as from 2012 and onwards. Exceptions are the peak (0.008) between the end of 2008 and 2009, followed by a second peak (0.004) in 2011-2012. Frontier markets are represented by the grey line, which usually reaches levels of 0.002-0.001, except for the peaks in 2005 and from the end of 2008 to 2009. The last peak (0.015) surpassed the level of emerging markets. Overall, emerging markets have the highest level of volatility compared to the others, developed markets have the lowest one, while during the past few years frontier markets have the same low levels.

DYNAMIC CONDITIONAL CO-VARIANCE ESTIMATES BY DCC GARCH MODEL

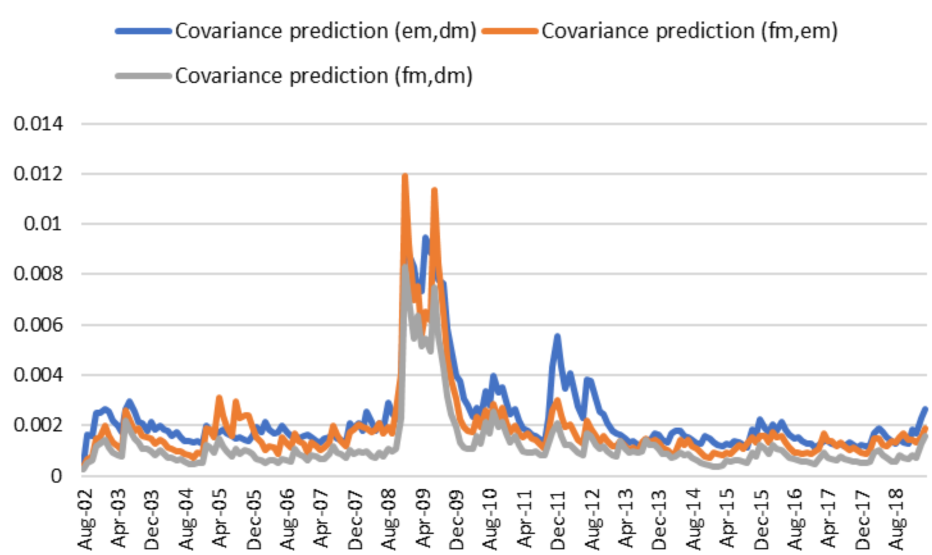

Fig. 5. The evolution of dynamic conditional co-variances among developed, emerging and frontier equity markets over time

This figure compares the dynamic conditional co-variances of the MSCI World index with the MSCI Emerging Markets index (blue line in graph), the MSCI Frontier Markets index with the MSCI Emerging Markets (orange line), and the MSCI Frontier Markets index with the MSCI World index (grey line) in the period from June 1, 2002 to March 1, 2019. The dynamic conditional co-variances are based on the monthly equity index returns and calculated using the DCC GARCH model through Stata. See in the Appendix: Exhibit IV.

As shown in Fig. 5, we also demonstrate the estimates of the dynamic conditional co-variances based on the DCC GARCH model. We observe that most of the highest covariance levels were reached by emerging and developed markets, suggesting that these markets are influenced strongly by the same events. The second highest co-variance is observed between emerging and frontier markets, which reached a peak of 0.012 during the financial crisis (end-2008 and 2009). Lastly, we see that a portfolio that consists of frontier and developed markets has lower co-variance in returns compared to a portfolio that includes emerging and developed markets.

We also examine the conditional correlation time-series data, since variance and co-variance do not provide a clear picture of the diversification benefits.

DYNAMIC CONDITIONAL CORRELATION ESTIMATES BY DCC GARCH MODEL

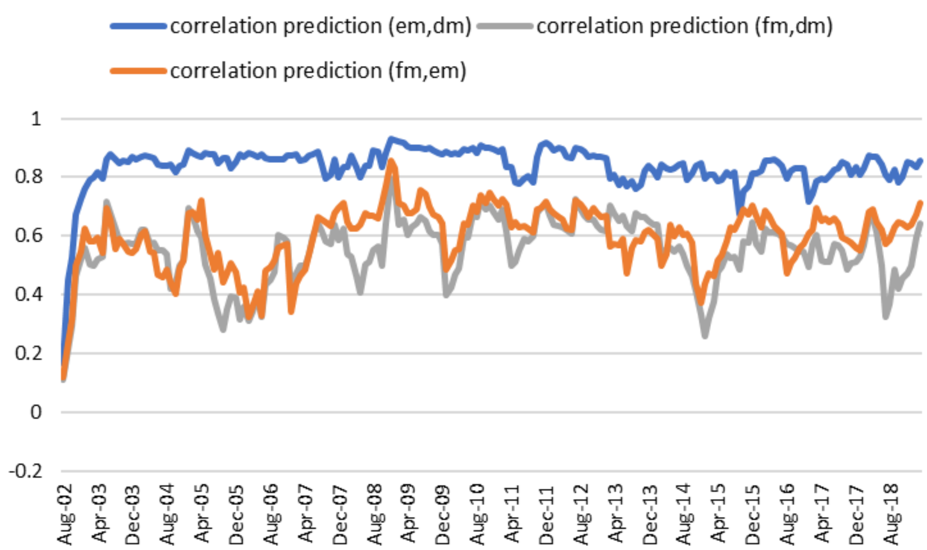

Fig. 6. The evolution of dynamic conditional correlations among developed, emerging, and frontier equity markets over time

This figure compares the dynamic conditional correlations of the MSCI World index with the MSCI Emerging Markets index (blue line in graph), the MSCI Frontier Markets index with the MSCI Emerging Markets (orange line), and the MSCI Frontier Markets index with the MSCI World index (grey line) in the period from June 1, 2002 to March 1, 2019. The dynamic conditional correlations are based on the monthly equity index returns and calculated using the DCC GARCH model through Stata.

Fig. 6 clearly shows that the correlation between emerging and developed markets is consistently higher than the correlation of frontier and emerging markets and that of frontier and developed markets. The correlation between emerging and developed markets remains high and stable between 0.7 and 0.9 over the sample period. Given the fact that the correlation is not equal to 1, emerging markets might offer potential diversification benefits to investors who have portfolios with equities from developed markets. The correlation between frontier and developed markets varies over time from 0.2 to 0.7. This demonstrates that frontier markets can offer additional diversification potential to investors compared to the potential offered by emerging markets. The correlation between frontier and emerging markets ranges from 0.3 to 0.8, suggesting that there might be diversification benefits for investors when investing in both emerging and frontier equity markets.

5.2. Performance analysis

In this section, we examine the volatility and the performance among emerging, developed and frontier equity markets using the moving window approach, in order to understand if the risk level influences the results for the diversification benefits and if they both influence the results for the Sharpe ratios.

THE VOLATILITY OF DEVELOPED, EMERGING AND FRONTIER EQUITY MARKETS

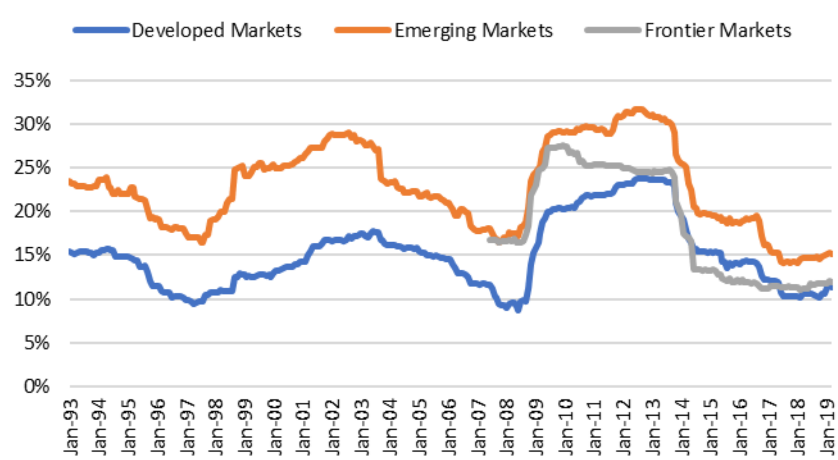

Fig. 7. The volatility among developed, emerging and frontier equity markets
This figure illustrates the change of the volatility among the MSCI World index, the MSCI Emerging Markets index and the MSCI Frontier Markets index from January 1, 1988 to March 1, 2019, with exception the MSCI Frontier Markets index which starts to have available data from June 1, 2002. The volatility is calculated using the annualized standard return deviation and then averaged using a moving-window approach based on the previous 60 months. Exhibit V in the Appendix provides a table of summary statistics.

Fig. 7 compares the volatility of the developed, emerging, and frontier markets, by calculating the annualized standard return deviation. Emerging markets have the highest volatility of all three, ranging from 14% to 32%, followed by the frontier markets, which fluctuate from 11% to 28%. Developed markets have a volatility of 9% to 24%. Overall, the volatility of all three markets tends to decrease throughout the years, while we can observe that the risk of frontier markets has decreased significantly, and it is similar to the risk level of developed markets.

MEAN RETURNS OF DEVELOPED, EMERGING AND FRONTIER EQUITY MARKETS

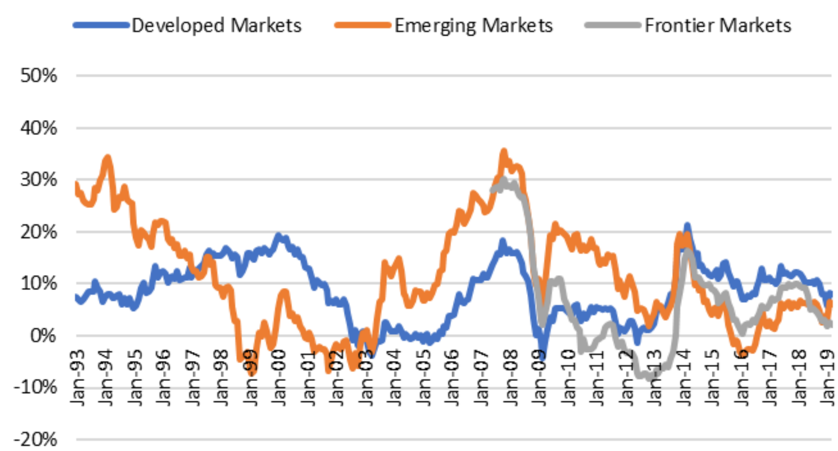

Fig. 8. The mean returns among developed, emerging and frontier equity markets
This figure displays the change of the mean returns among the MSCI World index, the MSCI Emerging Markets and the MSCI Frontier Markets index from January 1, 1988 to March 1, 2019, with exception the MSCI frontier markets index which starts to have available data from June 1, 2002. The mean returns are calculated using the average of annualized returns and then averaged using a moving-window approach based on the previous 60 months. Exhibit V in the Appendix provides a table of summary statistics.

Fig. 8 summarizes the performance of developed, emerging, and frontier markets, by measuring their annualized average returns. The higher fluctuations are observed in emerging markets, ranging from -7% to 36%, as well as in frontier markets taking values from -8% to 30%, while the range of developed markets is from -4% to 21%. The highest performance was reached by emerging markets during 2004-2013, while the last years developed markets have the highest annualized average returns. Frontier markets have also surpassed emerging markets in terms of performance during the last years. Given the low correlation that exists between frontier and developed markets, frontier markets can offer greater diversification benefits than emerging markets, possibly making them the next-generation markets.

5.3. Portfolio analysis

This section consists of two parts. In the first part, we examine the diversification benefits of emerging and frontier equity markets to a global equity portfolio of developed markets. The second part examines the diversification benefits of emerging and frontier equity markets, when expanding the portfolio to other assets, such as bonds and commodities.

5.3.1. Diversification over global equities

In order to examine the diversification benefits over global equities, we compare six different portfolios, which are an equally weighted portfolio, a minimum variance portfolio, a maximum Sharpe ratio portfolio and three portfolios with fixed weights. The first portfolios are constructed using unconditional correlations and measured with the mean return, standard deviation, Sharpe ratio, and M2 measure. These measurements will help us to understand which portfolio performs better. In the Appendix: Exhibit VI, we can also find the mean returns, standard deviations, and correlations for developed, emerging, and frontier equity markets that we used for these calculations.

	Portfolio models					
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
	Portfolio weights					
Developed markets	33%	64%	24%	100%	85%	55%
Emerging markets	33%	0%	38%	0%	10%	30%
Frontier markets	33%	36%	38%	0%	5%	15%
	Portfolio performance measurements					
Mean return	9.79%	8.72%	9.97%	8.53%	8.88%	9.59%
Std. dev.	16.93%	15.23%	17.27%	16.25%	16.35%	16.94%
Sharpe ratio	0.503	0.489	0.504	0.447	0.466	0.491
M2 measure	0.094	0.092	0.094	0.085	0.088	0.092

Table 3. Global equity portfolios using unconditional correlations.
This table reports the portfolio weights in developed, emerging, and frontier equity markets, as well as the portfolio performance measurements: mean annualized returns, annualized standard deviation (std. dev.), annualized Sharpe ratio, and annualized M2 measure. Portfolios are constructed using an equally weighted portfolio model, a minimum variance portfolio model, a maximum Sharpe ratio portfolio model and three

portfolios with fixed weights. The performance measurements are reported from June 1, 2002 to March 1, 2019 and are calculated using monthly returns of indexes. See Section 4 for the description of portfolios' construction. Details on the mean returns, standard deviations, and correlations for developed, emerging, and frontier equity markets that we used for these calculations are provided in the Appendix: Exhibit VI. Summary statistics for the calculation of global equity portfolios, when the correlations are based on the last 60 months are provided in the Appendix: Exhibit VII.

In Table 3, we observe that the maximum Sharpe ratio portfolio has the highest mean return (9.97%), followed by the portfolio with fixed weights (55% developed, 30% emerging, and 15% frontier markets) and the equally weighted portfolio with 9.59% and 9.79% respectively. The minimum variance portfolio has, as expected, the lowest standard deviation (15.23%). The maximum Sharpe ratio portfolio achieves the highest Sharpe ratio (0.504). However, this value is only slightly higher than the Sharpe ratios of the equally weighted portfolio, the portfolio with fixed weights (55% developed, 30% emerging, and 15% frontier markets) and the minimum variance portfolio (0.503, 0.491, and 0.489 respectively). Both the portfolio with 100% allocation in developed markets, and that with fixed weights (85% developed, 10% emerging, 5% frontier markets) perform worse with Sharpe ratios of 0.447 and 0.466 respectively. Both the maximum Sharpe ratio portfolio and the equally weighted portfolio reach the highest value of M2 measure (0.094). Nevertheless, the M2 of the minimum variance portfolio and the portfolio with fixed weights (55% developed, 30% emerging, and 15% frontier markets) is 0.092, which is close to the highest value of M2.

Overall, the maximum Sharpe ratio portfolio and the equally weighted portfolio are the best performing portfolios according to the mean return, the Sharpe ratio, and the M2 measure. These findings suggest that emerging and frontier equity markets offer diversification benefits to a portfolio of developed markets since the max Sharpe ratio portfolio has 38% allocation in emerging and 38% in frontier markets, as well as the equally weighted portfolio which has 33% weight in emerging and 33% in frontier markets. It is worth mentioning that the minimum variance portfolio has 0% allocation in emerging and 36% in frontier markets in order to achieve the lowest volatility. Thus, the combination of frontier and developed markets in a portfolio delivers smaller fluctuations to returns than a portfolio with emerging and developed markets.

We repeat the portfolio construction using this time the dynamic conditional correlations from the DCC GARCH model. The DCC GARCH provides a more accurate estimation of correlations, and thus, the portfolios using dynamic conditional correlations can lead to a better estimation of diversification benefits.

	Portfolio models					
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
	Portfolio weights					
Developed markets	33%	28%	0%	100%	85%	55%
Emerging markets	33%	0%	54%	0%	10%	30%
Frontier markets	33%	72%	46%	0%	5%	15%
	Portfolio performance measurements					
Mean return	9.79%	8.91%	10.53%	8.53%	8.88%	9.59%
Std. dev.	16.57%	15.29%	16.82%	18.59%	18.09%	17.42%
Sharpe ratio	0.514	0.499	0.550	0.391	0.421	0.477
M2 measure	0.096	0.093	0.101	0.076	0.081	0.090

Table 4. Global equity portfolios using DCC GARCH correlations.
This table reports the portfolio weights in developed, emerging, and frontier equity markets, as well as the portfolio performance measurements: mean annualized returns, annualized standard deviation (std. dev.), annualized Sharpe ratio, and annualized M2 measure. Portfolios are constructed using an equally weighted portfolio model, a minimum variance portfolio model, a maximum Sharpe ratio portfolio model and three portfolios with fixed weights. The performance measurements are reported from June 1, 2002 to March 1, 2019 and are calculated using monthly returns of indexes. See Section 4 for the description of portfolios' construction. Details on the mean returns, standard deviations and correlations for developed, emerging, and frontier equity markets that we used for these calculations, are provided in the Appendix: Exhibit VI.

Table 4 shows that the maximum Sharpe ratio portfolio achieves the highest mean return (10.53%). The minimum variance portfolio reaches the lowest standard deviation (15.29%). The highest Sharpe ratio is achieved by the maximum Sharpe ratio portfolio (0.550). This value is somewhat higher than the Sharpe ratios of the equally weighted portfolio and the minimum variance portfolio (0.514, and 0.499 respectively). The maximum Sharpe ratio portfolio achieves the highest M2 value of 0.101, followed by the equally weighted portfolio and the minimum variance portfolio with a M2 value of 0.096 and 0.093, respectively.

All in all, the maximum Sharpe ratio portfolio is the best performing portfolio, as it achieves the highest mean return, Sharpe ratio, and M2 but not the highest standard deviation. Right after the maximum Sharpe ratio portfolio, the equally weighted portfolio and the minimum variance portfolio perform best. These findings suggest that investors can benefit from diversification expanding their portfolios to emerging and frontier markets, since the portfolios which perform better, have allocations in emerging and frontier markets. An exception is the minimum variance portfolio, which has 0% proportion invested in emerging and 72% in frontier markets, suggesting that frontier markets might offer more diversification benefits than emerging markets.

Comparing the results from the global equity portfolios using unconditional correlations with the results from the global equity portfolios using DCC GARCH correlations, we find that the maximum Sharpe ratio portfolio is the best performing portfolio in both cases. The second-best performing portfolio is the equally weighted portfolio. The third-best performing portfolio is the portfolio with fixed weights (55% developed, 30% emerging, and 15% frontier markets) in the first case, while in the second case is the minimum variance portfolio. The aforementioned portfolios have allocations in emerging and frontier markets, with the exception of the minimum variance portfolio which has no allocation in emerging markets. We can conclude that the inclusion of emerging and frontier markets in a portfolio of developed markets offers diversification benefits.

The portfolios that have the worst performance in both cases are the portfolio with 100% allocation in developed markets, and that with fixed weights (85% developed, 10% emerging, 5% frontier markets), suggesting less efficient diversification opportunities.

5.3.2. Diversification over other asset classes

In this part, we also include bonds and commodities in our analysis. We compare the performance of eight portfolios, which are constructed using an equally weighted portfolio model, a minimum variance portfolio model, a maximum Sharpe ratio portfolio model, and five portfolios with fixed weights. Our analysis in this part is based on daily index return data, since we find an estimation error in the calculation of the DCC GARCH model for other asset classes using monthly index return data. Thus, we have global asset allocation portfolios using unconditional correlations (which are provided in the Appendix: Exhibit X) but not using DCC GARCH correlations of monthly return indexes. In order to have both global asset allocation portfolios, we choose to make this analysis based on daily index return data.

	Portfolio models							
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
	Portfolio weights							
Developed markets	20%	10%	9%	33%	28%	18%	0%	0%
Emerging markets	20%	0%	9%	33%	28%	18%	0%	0%
Frontier markets	20%	17%	26%	33%	28%	18%	0%	0%
Bonds	20%	72%	56%	0%	10%	30%	100%	0%
Commodities	20%	1%	0%	0%	5%	15%	0%	100%
	Portfolio performance measurements							
Mean return	6.56%	5.43%	6.29%	8.98%	8.15%	6.49%	4.53%	1.32%
Std. dev.	10.28%	5.53%	6.06%	12.54%	11.16%	9.02%	6.50%	22.65%
Sharpe ratio	0.515	0.754	0.830	0.615	0.617	0.580	0.502	0.002
M2 measure	0.093	0.131	0.143	0.109	0.109	0.104	0.091	0.013

Table 5. Global asset allocation portfolios using unconditional correlations.
This table reports the portfolio weights in equities (developed, emerging and frontier markets), bonds, and commodities, as well as the portfolio performance measurements: mean annualized returns, annualized standard deviation (std. dev.), annualized Sharpe ratio, and annualized M2 measure. Portfolios are constructed using an equally weighted portfolio model, a minimum variance portfolio model, a maximum Sharpe ratio portfolio model and three portfolios with fixed weights. The performance measurements are reported from June 1, 2002 to March 15, 2019 and are calculated using daily returns of indexes. See Section 4 for the description of portfolios' construction. Details on the mean returns, standard deviations and correlations for (developed, emerging, and frontier markets) equities, bonds, and commodities that we used for these calculations are provided in the Appendix: Exhibit IX.

Table 5 demonstrates that the highest mean return is achieved by the portfolio which has fixed weights only in equities (33% developed, 33% emerging, and 33% frontier markets). The minimum variance portfolio exhibits the lowest standard deviation (5.53%). The maximum Sharpe ratio portfolio achieves the highest Sharpe ratio (0.830), followed by the minimum variance portfolio with a Sharpe ratio of 0.754. The highest M2 is achieved by the maximum Sharpe ratio portfolio (0.143), while the M2 of the minimum variance portfolio is slightly lower (0.131).

Overall, the portfolios that perform the worst are first the portfolio, which invests 100% in commodities and second the portfolio, which allocates 100% in bonds. Consequently, these portfolios offer less efficient diversification benefits and the most important is that they have 0% proportion invested in emerging and frontier equity markets. The maximum Sharpe ratio portfolio is the best performing portfolio, while the minimum variance portfolio is the second-best performing portfolio. The maximum Sharpe ratio portfolio has 26% allocation in frontier and 9% in emerging markets, whereas the minimum variance portfolio has 17% proportion invested in frontier and 0% in emerging markets.

We repeat the portfolio construction using DCC GARCH correlations instead of unconditional correlations.

	Portfolio models							
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
	Portfolio weights							
Developed markets	20%	9%	11%	33%	28%	18%	0%	0%
Emerging markets	20%	0%	4%	33%	28%	18%	0%	0%
Frontier markets	20%	11%	19%	33%	28%	18%	0%	0%
Bonds	20%	78%	66%	0%	10%	30%	100%	0%
Commodities	20%	1%	0%	0%	5%	15%	0%	100%
	Portfolio performance measurements							
Mean return	6.56%	5.24%	5.83%	8.98%	8.15%	6.49%	4.53%	1.32%
Std. dev.	6.66%	3.00%	3.22%	8.02%	7.13%	5.76%	3.47%	15.87%
Sharpe ratio	0.794	1.326	1.419	0.962	0.965	0.907	0.940	0.003
M2 measure	0.137	0.221	0.235	0.164	0.164	0.155	0.160	0.013

Table 6. Global asset allocation portfolios using DCC GARCH correlations.
This table reports the portfolio weights in equities (developed, emerging and frontier markets), bonds, and commodities as well as the portfolio performance measurements: mean annualized returns, annualized standard deviation (std. dev.), annualized Sharpe ratio, and annualized M2 measure. Portfolios are constructed using an equally weighted portfolio model, a minimum variance portfolio model, a maximum Sharpe ratio portfolio model and three portfolios with fixed weights. The performance measurements are reported from June 1, 2002 to March 15, 2019 and are calculated using daily returns of indexes. See Section 4 for the description of portfolios' construction. Details on the mean returns, standard deviations and correlations for (developed, emerging, and frontier markets) equities, bonds, and commodities that we used for these calculations are provided in the Appendix: Exhibit IX.

From Table 6, we derive that the portfolio with fixed weights (33% developed, 33% emerging, and 33% frontier equity markets) achieves the highest mean return (8.98%). The minimum variance has, as expected, the lowest standard deviation (3.00%), followed by the maximum Sharpe ratio portfolio with a standard deviation of 3.22%. The highest Sharpe ratio and M2 are achieved by the maximum Sharpe ratio portfolio, while the minimum variance portfolio achieves the second-highest Sharpe ratio and M2.

All in all, the maximum Sharpe ratio portfolio, which has 19% allocation in frontier and 4% in emerging markets, outperforms the other examined portfolios. The minimum variance portfolio is the second-best performing portfolio with 11% proportion invested in frontier and 0% in emerging markets. The third-best performing portfolio is the portfolio with fixed weights (33% developed, 33% emerging, and 33% frontier equity markets). The worst performing portfolios are again the portfolio which invests 100% in commodities and the portfolio which allocates 100% in bonds. It is worth mentioning that the equally weighted portfolio does not perform as well as in global equity portfolios. Therefore, the findings from the global asset allocation portfolios using DCC GARCH correlations are similar to that from the global asset allocation portfolios using unconditional correlations with only small differences in the weights.

To conclude, the maximum Sharpe ratio portfolio is the best performing portfolio with allocations in emerging and frontier equity markets in all cases, suggesting efficient diversification benefits to investors by investing in these markets. The second-best performing portfolio is the equally weighted portfolio when we have global equity portfolios, while in the case of global asset allocation portfolios it is the minimum variance portfolio. These findings support the opinion that emerging, and frontier equity markets offer diversification benefits to a portfolio of developed markets and to global asset allocation, achieving higher risk-adjusted returns.

Summary findings results

This part summarizes the results of our analysis in order to have a clear overview.

Conducting a correlation analysis using the moving-window approach gives us the following results:

- The correlation between emerging and developed equity markets has increased substantially from the early 1990s until now.
- The correlation of emerging with frontier equity markets has increased significantly, although it is at lower levels compared to the correlation between emerging and developed equity markets.
- The correlation between frontier and developed equity markets increased from low levels, nearly zero, to positive levels, however it is lower than the other correlations.

These findings show that even today emerging markets continue to offer diversification potential in a portfolio of developed markets, however in lower levels compared to the early 1990s. The combination of emerging and frontier markets in a portfolio still offers diversification benefits. The slightly lower level of dependence between frontier and developed markets, compared to that of emerging markets, suggests that frontier markets offer slightly more diversification benefits than emerging markets to a portfolio of developed markets.

The correlation analysis using the DCC GARCH model demonstrates that either in the case of the correlations using DCC GARCH or in the case of the correlations using the moving window approach, the results predict the same findings as presented above. The key differences are the following:

- The DCC GARCH model predicted a slightly higher correlation between emerging and developed equity markets than that of the moving window approach.
- Both the correlation of emerging with frontier equity markets, as well as the correlation of frontier and developed equity markets are predicted to be slightly lower than that of the moving window approach.

Next, we find that the results from the variance estimations using the moving window approach as well as using the DCC GARCH model predict the same findings:

- Emerging markets achieve, as expected, the highest variance compared to frontier and developed markets.
- The volatility of frontier markets has decreased considerably during the last years, and it has similar volatility levels with that of developed markets.

In general, the volatility of all three markets tends to decrease throughout the years.

The results from the co-variance estimations using the DCC GARCH model reveal that:

- The highest co-variance is observed between emerging and developed markets.
- It is followed by the co-variance of emerging and frontier markets, and that of frontier and developed markets.

Given the results of the co-variances, we find that a portfolio that consists of frontier and developed markets has lower co-variance in returns compared to a portfolio that combines emerging and developed markets.

In addition, the results from the mean returns estimations using the moving window approach show that:

- Emerging markets perform better than frontier and developed markets on average.
- During the last years, developed markets have achieved the highest mean returns and frontier markets have surpassed emerging markets in terms of performance.

Considering the low correlation and co-variance that exists between frontier and developed markets, their high performance and their decreasing risk during the last years, frontier markets can offer greater diversification benefits than emerging markets.

Now, we move on the portfolio analysis and we find that the results from the global equity portfolio analysis both in the case of using unconditional correlations or in the case of using the DCC GARCH model reveal that:

- The maximum Sharpe ratio portfolio and the equally weighted portfolio are the best performing portfolios

These findings suggest that emerging and frontier equity markets offer diversification benefits to a portfolio of developed markets. Because the max Sharpe ratio portfolio, as well as the equally weighted portfolio, have allocations in emerging and in frontier markets in both cases.

Given the results from the global asset allocation portfolio analysis, we find that either in the case of using unconditional correlations or in the case of using the DCC GARCH model:

- The maximum Sharpe ratio portfolio and the minimum variance portfolio are the best performing portfolios

The first thing that these findings suggest is that emerging, and frontier equity markets offer diversification benefits to a portfolio of developed markets since the maximum Sharpe ratio portfolio has an allocation in frontier and in emerging markets in both cases. Second, the minimum variance portfolio has an allocation in frontier, but 0% proportion invested in emerging markets in both cases. This implies that the combination of frontier and developed markets in a portfolio achieves lower volatility than a portfolio with emerging and developed markets. In addition, it shows that the inclusion of frontier markets in a global asset allocation portfolio can offer better diversification benefits compared to emerging markets.

Therefore, our portfolio analysis predicts that the maximum Sharpe ratio portfolio is the best performing portfolio with allocations in emerging and frontier equity markets in all cases, suggesting efficient diversification benefits to investors from investing in these markets. When we have global equity portfolios, the second-best performing portfolio is the equally weighted portfolio, while the minimum variance portfolio when we have global asset allocation portfolios. These findings predict that emerging, and frontier equity markets offer diversification benefits to a portfolio of developed markets and to global asset allocation, as well as that the inclusion of frontier markets in a global asset allocation portfolio can offer better diversification benefits compared to emerging markets.

6. Conclusions

This study aims to fill the gap in the academic literature regarding frontier equity markets, to further analyze the emerging equity markets and to provide additional evidence in the portfolio management literature regarding the diversification benefits of emerging and frontier equity markets. The research questions that this study strives to answer are:

1) *"Do emerging markets still offer strong diversification benefits compared to developed equity markets, and to which extent?"*
2) *"Do less developed "frontier markets" offer better diversification properties to investors?"*
3) *"Which is the best allocation strategy for investors to diversify their portfolio?"*

In order to answer the research questions, we analyze the time-varying diversification benefits of emerging and frontier equity markets to a global equity portfolio of developed markets, as well as to a global asset allocation portfolio (including equities, bonds, and commodities). The tools for analyzing the time-varying correlations between developed, emerging and frontier equity markets are the moving window approach and the DCC GARCH model. Either in the case of the DCC GARCH or in the case of the moving window approach, our results predict first that even today emerging equity markets continue to offer diversification potential in a portfolio of developed markets, however at lower levels compared to the early 1990s. This finding is in line with the existing research (Bekaert and Harvey, 2017).

The second prediction is that frontier equity markets exhibit lower levels of correlation with developed equity markets compared to emerging equity markets. This finding is in also line with the existing literature as Berger, Pukthuanthong, and Yang (2011), demonstrate that frontier equity markets have a lower level of integration with the world equity markets, in comparison with emerging and developed markets. In addition, the low levels of both correlation and co-variance between frontier and developed equity markets, the high performance and the decreasing risk of frontier equity markets during the last years, compared to emerging markets equity markets, lead to the prediction that frontier markets offer greater diversification benefits than emerging markets to a portfolio of developed markets.

In order to analyze the added value of emerging and frontier equity markets in a portfolio context, we construct global equity portfolios as well as global asset allocation portfolios and compare their performances. The results either in the global equity portfolios or in the global asset allocation portfolios reveal that the maximum Sharpe ratio portfolio is the best performing portfolio in terms of risk-adjusted expected return. The second-best performing portfolio is the equally weighted portfolio when we have global equity portfolios, while the minimum variance portfolio when we have global asset allocation portfolios.

All the above portfolios have allocations in frontier and in emerging equity markets, with an exception the minimum variance portfolio, which has an allocation in frontier, but 0% proportion invested in emerging equity markets. These findings suggest first that investing in emerging, and frontier equity markets improves the risk-adjusted return and the diversification benefits of a portfolio of developed equity markets and a global asset allocation portfolio. Second, the inclusion of frontier equity markets in a global asset allocation portfolio can provide significant diversification benefits, and thus, improve the risk-adjusted returns of the portfolios, in comparison with emerging equity markets.

The findings of this research are useful for the investment community, institutional investors, individual investors, asset allocation management, portfolio management. This study found that investors can continue to benefit by investing in emerging and frontier equity markets, since the correlations of frontier with emerging markets as well as those of frontier and developed markets are considered low, while the correlation between emerging and developed markets is not perfect yet. Thus, investors can improve the risk-adjusted returns of their portfolios by investing in emerging and frontier equity markets.

One limitation concerning this research is that the analysis could be extended to other data if the availability of reliable data was provided. For instance, either more asset classes than bonds and commodities, or other indexes to represent the markets of our analysis, or analysis focus not only on an aggregate level (i.e., emerging markets) but also on the country level (i.e., countries of emerging markets). Another limitation is that the time-varying diversification of these markets could be analyzed with an alternative approach. For example, instead of focusing primarily on the correlation part, it also focuses on other factors that can influence the diversification. Since this study focuses only on some allocation models, it is interesting for future research to include more asset allocation models to have more reliable findings. Therefore, this study can be further extended in the future by considering the limitations which are presented above.

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Appendix

Exhibit I: MSCI Market Classification Framework

The Morgan Stanley Capital International (MSCI) equity indexes are classified in developed, emerging and frontier equity markets based on the MSCI market classification framework. The MSCI uses the following three criteria: economic development, size and liquidity as well as market accessibility for the market classification. For instance, a country should meet the requirements of all three criteria which are mentioned below, to be classified as developed, emerging and frontier market.

Criteria	Frontier	Emerging	Developed
A. Economic Development			
A.1. Sustainability of economic development	No requirement	No requirement	Country GNI per capita 25% above the World Bank high income threshold* for 3 consecutive years
B. Size and Liquidity Requirements			
B.1. Number of companies meeting the following	2	3	5
Standard Index criteria			
Company size (full market cap) **	USD 797 mm	USD 1,594 mm	USD 3,187 mm
Security size (float market cap) **	USD 71 mm	USD 797 mm	USD 1,594 mm
Security liquidity	2.5% ATVR	15% ATVR	20% ATVR
C. Market Accessibility Criteria			
C.1. Openness to foreign ownership	At least some	Significant	Very high
C.2. Ease of capital inflows/outflows	At least partial	Significant	Very high
C.3. Efficiency of operational framework	Modest	Good and tested	Very high
C.4. Competitive landscape	High	High	Unrestricted
C.5. Stability of the institutional framework	Modest	Modest	Very high

* High income threshold for 2017: GNI per capita of USD 12,235 (World Bank, Atlas method), ** Minimum in use for the May 2018 Semi-Annual Index Review (updated on a semi-annual basis), ATVR stands for Annualized Traded Value Ratio, GNI per capita stands for Gross National Income per capita and it is used by the World Bank to classify economies.

Source: MSCI, MSCI Market Classification Framework (December 2018)

Exhibit II: DCC GARCH model estimates, including a test of constant correlation

The table presents all the estimates produced by the DCC GARCH model. We use a three-dimensional dataset consisting of the monthly returns of the MSCI World index (*Developed markets*), the MSCI Emerging Markets index (*Emerging markets*), and the MSCI Frontier Markets index (*Frontier markets*). The estimation period is from June 1, 2002 to March 1, 2019 (201 number of observations). We also use multivariate Gaussian error distribution. The DCC GARCH model is implemented through Stata.

		Panel A	
	Developed markets	***Emerging markets***	***Frontier markets***
arch term	0.156**	0.132**	0.239**
garch term	0.739***	0.735***	0.656***
constant term	0.000*	0.000**	0.000**
		Panel B	
	Developed markets	***Emerging markets***	***Frontier markets***
Developed markets	1.000		
Emerging markets	0.828***	1.000	
Frontier markets	0.585***	0.631***	1.000
		Panel C	
	[Adjustment] lambda1	***[Adjustment] lambda2***	
Coefficient	***0.090****	***0.731******	
Std. error	0.487	0.138	
z -Stat	1.85	5.28	
p-value	0.064	0.000	

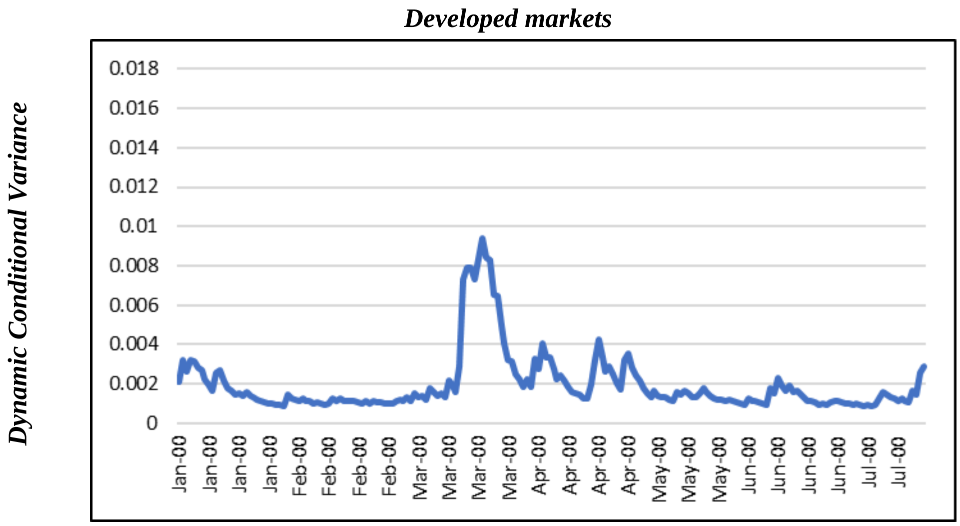
Test for DCC model versus constant correlation

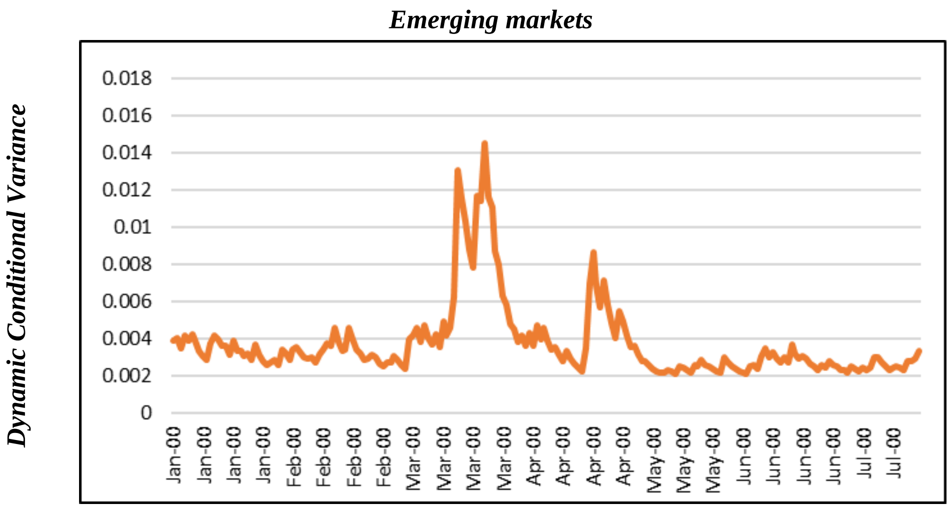
test _b [Adjustment: lambda1] = _b [Adjustment: lambda2] = 0
(1) [Adjustment]lambda1 - [Adjustment]lambda2 = 0
(2) [Adjustment]lambda1 = 0
chi2(2) = 128.53 Prob > chi2 = 0.0000
The p-value is less than 5%, then the null hypothesis is rejected, and the DCC model is better than constant correlation.

*, **, and *** indicate significance at 10%, 5%, and 1%

Exhibit III: Dynamic conditional variances estimated by DCC GARCH

Over the period from June 1, 2002 to March 1, 2019





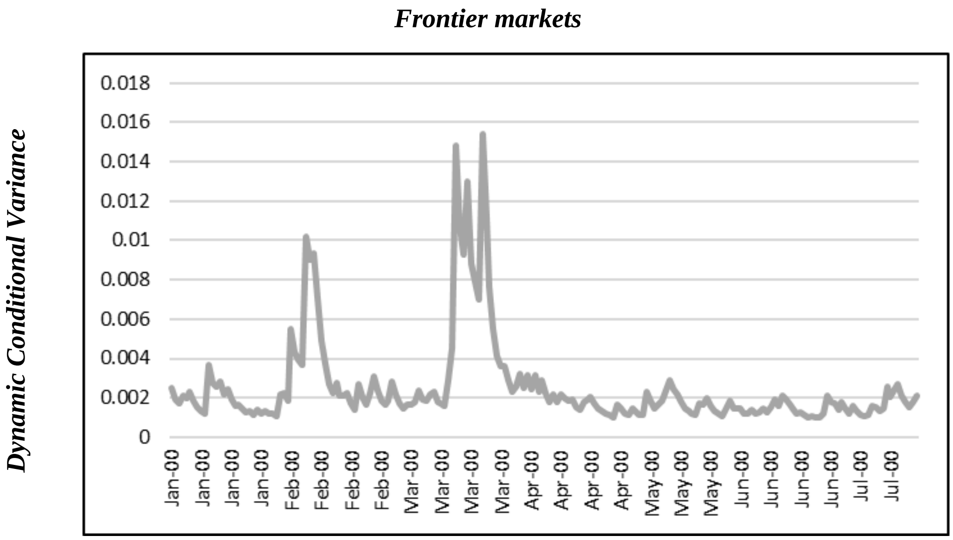
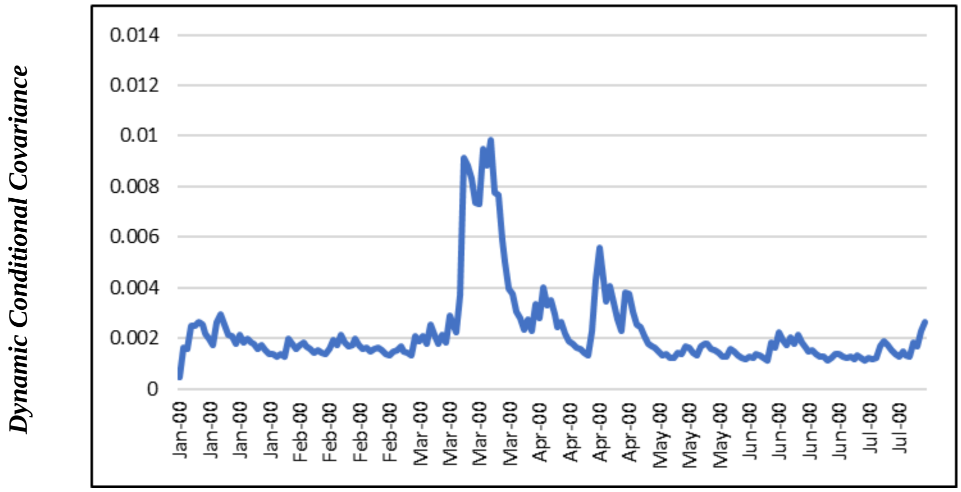


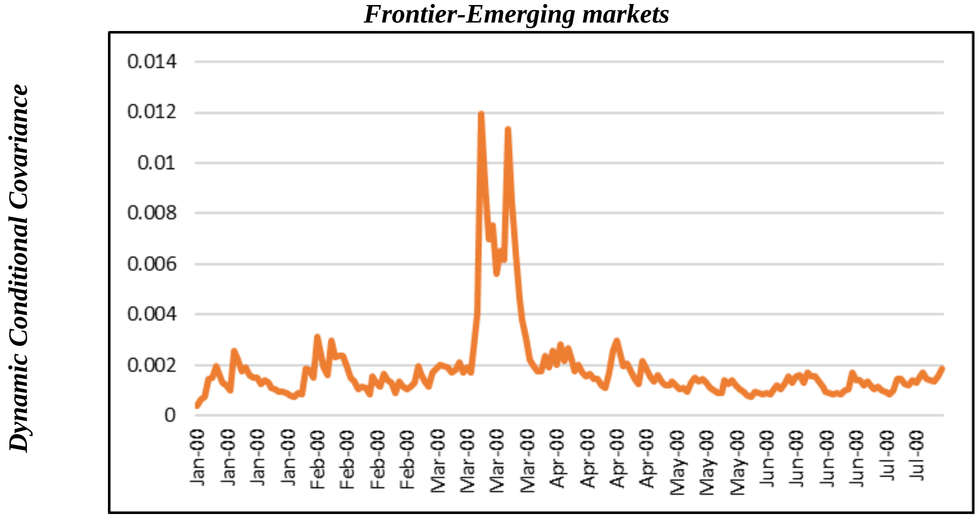
Exhibit IV: Dynamic conditional co-variances estimated by DCC GARCH

Over the period from June 1, 2002 to March 1, 2019

Emerging-Developed markets



Frontier-Emerging markets



Frontier-Developed markets

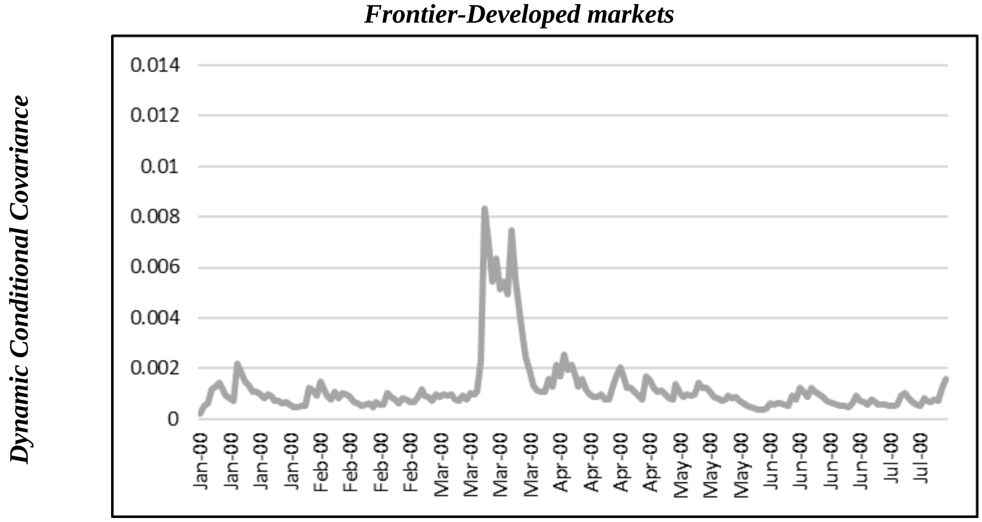


Exhibit V: Summary statistics for developed, emerging and frontier equity markets based on moving-window approach

The table reports the mean return (annualized average return) and annualized standard deviation for developed, emerging, and frontier equity markets based on the moving-window approach. The monthly returns data of the MSCI World index, MSCI Emerging Markets index, and MSCI Frontier Markets index over the period from June 1, 2002 to March 1, 2019 (201 number of observations) are used. The annualized average return and annualized standard deviation are calculated using a moving-window approach based on the previous 60 months.

	Annualized Mean Return	Annualized Std. Dev.	Description
Developed markets	0.0848	0.1644	Average return of MSCI World index (60-M moving window)
Emerging markets	0.1106	0.2277	Average return of MSCI Emerging Markets index (60-M moving window)
Frontier markets	0.0646	0.1852	Average return of MSCI Frontier Markets index (60-M moving window)

Exhibit VI: Summary statistics for the calculation of global equity portfolios

The table reports the mean return (annualized average return), mean excess return (annualized average excess return), and annualized standard deviation in Panel A, the unconditional Variances-Covariances in Panel B, and the conditional Variances-Covariances in Panel C for developed, emerging, and frontier equity markets that we used for the calculation of both global equity portfolios. The dataset consists of the monthly return data of the MSCI World index, MSCI Emerging Markets index, and MSCI Frontier Markets index over the period from June 1, 2002 to March 1, 2019 (201 number of observations).

Panel A: Summary statistics			
	Mean return	**Mean Excess return**	**Std. dev.**
Developed markets	8.53%	7.26%	16.11%
Emerging markets	11.78%	10.51%	22.01 %
Frontier markets	9.06%	7.78%	18.24%
Panel B: Unconditional Variance-Covariance Matrix			
	Developed markets	**Emerging markets**	**Frontier markets**
Developed markets	0.00220	0.00261	0.00145
Emerging markets	0.00261	0.00411	0.00212
Frontier markets	0.00145	0.00212	0.00282
Panel C: Conditional Variance-Covariance Matrix			
	Developed markets	**Emerging markets**	**Frontier markets**
Developed markets	0.00288	0.00267	0.00158
Emerging markets	0.00267	0.00335	0.00188
Frontier markets	0.00158	0.00188	0.00210

Exhibit VII: Summary statistics for the calculation of global equity portfolios & the global equity portfolios when the time horizon is different

The table reports the mean return (annualized average return), mean excess return (annualized average excess return), and annualized standard deviation in Panel A, the unconditional Variances-Covariances in Panel B, and the conditional Variances-Covariances in Panel C for developed, emerging, and frontier equity markets that we used for the calculation of both global equity portfolios. The dataset consists of the monthly return data of the MSCI World index, MSCI Emerging Markets index, and MSCI Frontier Markets index in the most recent 60-month period, from April 1, 2014 to March 1, 2019. In this case, we do not have conditional Variances-Covariance matrix, since we cannot implement DCC GARCH model only for 60 observations. Therefore, we do not also have global equity portfolio based on conditional (monthly return) correlations in the most recent 60-month period.

Panel A: Summary statistics			
	Mean return	**Mean Excess return**	**Std. dev.**
Developed markets	7.89%	7.14%	11.40%
Emerging markets	5.90%	5.15%	15.19%
Frontier markets	2.19%	1.44%	11.99%
Panel B: Unconditional Variance-Covariance Matrix			
	Developed markets	***Emerging markets***	***Frontier markets***
Developed markets	0.00111	0.00108	0.00055
Emerging markets	0.00108	0.00192	0.00099
Frontier markets	0.00055	0.00099	0.00118

The first table presents the global equity portfolios based on unconditional (monthly return) correlations in the most recent 60-month period. The mean return, mean excess return, and standard deviation in Panel A, and the unconditional Variances-Covariances in Panel B, are used for the calculations in the first table.

Table: Global equity portfolios using unconditional (monthly return) correlations in the most recent 60-month period

Portfolio models						
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
Portfolio weights						
Developed markets	33%	53%	100%	100%	85%	55%
Emerging markets	33%	0%	0%	0%	10%	30%
Frontier markets	33%	47%	0%	0%	5%	15%
Portfolio performance measurements						
Mean return	5.33%	5.22%	7.89%	7.89%	7.41%	6.44%
Std. dev.	11.21%	10.07%	11.54%	11.54%	11.29%	11.33%
Sharpe ratio	0.408	0.443	0.619	0.619	0.589	0.502
M2 measure	0.054	0.058	0.078	0.078	0.075	0.065

Exhibit VIII: Summary statistics for the calculation of global equity portfolios & the global equity portfolios when the data frequency is daily

The table reports the mean return (annualized average return), mean excess return (annualized average excess return), and annualized standard deviation in Panel A, the unconditional Variances-Covariances in Panel B, and the conditional Variances-Covariances in Panel C for developed, emerging, and frontier equity markets that we used for the calculation of both global equity portfolios. The dataset consists of the daily return data of the MSCI World index, MSCI Emerging Markets index, and MSCI Frontier Markets index over the period from June 1, 2002 to March 15, 2019 (4,380 number of observations).

Panel A: Summary statistics			
	Mean return	**Mean Excess return**	**Std. dev.**
Developed markets	8.12%	6.86%	15.68%
Emerging markets	10.82%	9.56%	18.87%
Frontier markets	7.99%	6.72%	11.87%
Panel B: Unconditional Variance-Covariance Matrix			
	Developed markets	**Emerging markets**	**Frontier markets**
Developed markets	0.000098	0.000081	0.000019
Emerging markets	0.000081	0.000141	0.000033
Frontier markets	0.000019	0.000033	0.000056
Panel C: Conditional Variance-Covariance Matrix			
	Developed markets	**Emerging markets**	**Frontier markets**
Developed markets	0.000036	0.000029	0.000012
Emerging markets	0.000029	0.000056	0.000016
Frontier markets	0.000012	0.000016	0.000023

The first table presents the global equity portfolios based on unconditional (daily return) correlations. The mean return, mean excess return, and standard deviation in Panel A, and the unconditional Variances-Covariances in Panel B, are used for the calculations in the first table.

The second table shows the global equity portfolios based on conditional (daily return) correlations. The mean return, mean excess return, and standard deviation in Panel A, and the conditional Variances-Covariances in Panel C, are used for the calculations in the second table.

Table: Global equity portfolios using unconditional (daily return) correlations

Portfolio models						
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
Portfolio weights						
Developed markets	33%	32%	18%	100%	85%	55%
Emerging markets	33%	0%	20%	0%	10%	30%
Frontier markets	33%	68%	62%	0%	5%	15%
Portfolio performance measurements						
Mean return	8.98%	8.03%	8.58%	8.12%	8.39%	8.91%
Std. dev.	12.54%	10.54%	11.06%	15.69%	14.88%	13.88%
Sharpe ratio	0.615	0.642	0.661	0.437	0.479	0.551
M2 measure	0.109	0.113	0.116	0.081	0.088	0.099

Table: Global equity portfolios using conditional (daily return) correlations

Portfolio models						
	Equal Weights	**Min Variance**	**Max Sharpe Ratio**	**Fixed Weights**	**Fixed Weights**	**Fixed Weights**
Portfolio weights						
Developed markets	33%	32%	17%	100%	85%	55%
Emerging markets	33%	0%	24%	0%	10%	30%
Frontier markets	33%	68%	59%	0%	5%	15%
Portfolio performance measurements						
Mean return	8.98%	8.03%	8.69%	8.12%	8.39%	8.91%
Std. dev.	7.99%	7.01%	7.37%	9.56%	9.10%	8.60%
Sharpe ratio	0.965	0.966	1.007	0.717	0.783	0.889
M2 measure	0.164	0.164	0.171	0.125	0.135	0.152

Exhibit IX: Summary statistics for the calculation of global asset allocation portfolios

The table reports the mean return (annualized average return), mean excess return (annualized average excess return), and annualized standard deviation in Panel A, the unconditional Variances-Covariances in Panel B, and the conditional Variances-Covariances in Panel C for equities (developed, emerging, and frontier markets), bonds, and commodities that we used for the calculation of both global asset allocation portfolios. The dataset consists of the daily return data of the MSCI World index, MSCI Emerging Markets index, the MSCI Frontier Markets index, the JPM Global Government Bond index, and the S&P GSCI Commodity index over the period from June 1, 2002 to March 15, 2019 (4,380 number of observations).

Panel A: Summary statistics

	Mean return	**Mean Excess return**	**Std. dev.**
Developed markets	8.12%	6.86%	15.68%
Emerging markets	10.82%	9.56%	18.87%
Frontier markets	7.99%	6.72%	11.87%
Bonds	4.53%	3.27%	6.50%
Commodities	1.32%	0.05%	22.65%

Panel B: Unconditional Variance-Covariance Matrix

	Developed markets	**Emerging markets**	**Frontier markets**	**Bonds**	**Commodities**
Developed markets	0.000098	0.000081	0.000019	-0.000002	0.000055
Emerging markets	0.000081	0.000141	0.000033	-0.000001	0.000062
Frontier markets	0.000019	0.000033	0.000056	0.000001	0.000017
Bonds	-0.000002	-0.000001	0.000001	0.000017	0.000002
Commodities	0.000055	0.000062	0.000017	0.000002	0.000204

Panel C: Conditional Variance-Covariance Matrix

	Developed markets	**Emerging markets**	**Frontier markets**	**Bonds**	**Commodities**
Developed markets	0.000036	0.000029	0.000011	-0.000002	0.000024
Emerging markets	0.000029	0.000057	0.000016	0.000000	0.000022
Frontier markets	0.000011	0.000016	0.000023	0.000000	0.000010
Bonds	-0.000002	0.000000	0.000000	0.000005	-0.000001
Commodities	0.000024	0.000022	0.000010	-0.000001	0.000100

Exhibit X: Summary statistics for the calculation of global asset allocation portfolios & the global asset allocation portfolios when the data frequency is monthly

The table reports the mean return (annualized average return), mean excess return (annualized average excess return), and annualized standard deviation in Panel A, and the unconditional Variances-Covariances in Panel B for equities (developed, emerging, and frontier markets), bonds, and commodities that we used for the calculation of both global asset allocation portfolios. The dataset consists of the monthly return data of the MSCI World index, MSCI Emerging Markets index, the MSCI Frontier Markets index, the JPM Global Government Bond index, and the S&P GSCI Commodity index over the period from June 1, 2002 to March 1, 2019 (201 number of observations). In this case, we do not have conditional Variances-Covariance matrix based on monthly returns, since there is an estimation error.

Panel A: Summary statistics

	Mean return	**Mean Excess return**	**Std. dev.**
Developed markets	8.53%	7.26%	16.11%
Emerging markets	11.78%	10.51%	22.01%
Frontier markets	9.06%	7.78%	18.24%
Bonds	4.64%	3.37%	6.60%
Commodities	1.42%	0.15%	23.50%

Panel B: Unconditional Variance-Covariance Matrix

	Developed markets	**Emerging markets**	**Frontier markets**	**Bonds**	**Commodities**
Developed markets	0.002201	0.002613	0.001447	0.000126	0.001519
Emerging markets	0.002613	0.004114	0.002121	0.000252	0.002181
Frontier markets	0.001447	0.002121	0.002815	0.000099	0.001754
Bonds	0.000126	0.000252	0.000099	0.000376	0.000140
Commodities	0.001519	0.002181	0.001754	0.000140	0.004581

The following table presents the global asset allocation portfolios based on unconditional (monthly return) correlations. The mean return, mean excess return, and standard deviation in Panel A, and the unconditional Variances-Covariances in Panel B, are used for the calculations in the this table.

Table: Global asset allocation portfolios using unconditional (monthly return) correlations

	Portfolio models							
	Equal Wt.	**Min Var**	**Max SR**	**Fixed Wt.**	**Fixed Wt.**	**Fixed Wt.**	**Fixed Wt.**	**Fixed Wt.**
	Portfolio weights							
Developed markets	20%	7%	12%	33%	28%	18%	0%	0%
Emerging markets	20%	0%	5%	33%	28%	18%	0%	0%
Frontier markets	20%	5%	13%	33%	28%	18%	0%	0%
Bonds	20%	87%	71%	0%	10%	30%	100%	0%
Commodities	20%	1%	0%	0%	5%	15%	0%	100%
	Portfolio performance measurements							
Mean return	7.09%	5.12%	6.00%	9.79%	8.86%	6.99%	4.64%	1.42%
Std. dev.	13.61%	6.42%	7.16%	16.93%	15.19%	12.12%	6.72%	23.45%
Sharpe ratio	0.427	0.600	0.661	0.503	0.499	0.472	0.502	0.006
M2 measure	0.082	0.109	0.119	0.094	0.093	0.089	0.094	0.014

Exhibit XI: Table with the estimates of correlations, variances and covariances produced by DCC GARCH

This table shows the results when the DCC GARCH correlations, variances, and co-variances are the predicted ones for the last month in our sample.

Panel A: Correlations			
	Developed markets	**Emerging markets**	**Frontier markets**
Developed markets	1		
Emerging markets	0.8586409	1	
Frontier markets	0.6421262	0.711143	1
Panel B: Variances & Co-variances			
	Developed markets	**Emerging markets**	**Frontier markets**
Developed markets	0.0028813		
Emerging markets	0.0026665	0.0033472	
Frontier markets	0.0015782	0.0018838	0.0020965