Volatility Analysis of Bitcoin and Stockmarket Prices

Master Thesis

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1 Introduction

Cryptocurrencies are a growing phenomenon being frequently discussed by media, financial, and governmental instutions (Glaser *et al.*, 2014). Most strikingly, the Chair of the Board of Governors of the U.S. Federal Reserve highlighted the need to study financial innovations such as distributed ledger technologies on which blockchain technologies and cryptocurrencies, in particular, are based (Chan *et al.*, 2017). This motivates to start unraveling cryptocurrencies as a critical element of new financial technology.

In sharp contrast to fiat currencies that rely on a central bank with discretionary decision making, cryptocurrencies are decentralized in terms of money creation and transaction processing. While there is a large variety of cryptocurrencies by now, Bitcoin was the first cryptocurrency introduced and certainly attracts the most attention. It was proposed by the pseudonym Nakamoto (2008).

Figure 1 illustrates the percentage market capitalization of the most popular cryptocurrencies between 2013 and 2018 (Coinmarketcap, 2018). It is evident that Bitcoin plays a dominant role throughout the period in the cryptocurrency market with a percentage market cap ranging from 33 to 96%. At present, Bitcoin represents about 48% of the total estimated cryptocurrency capitalization.[1] Moreover, Bitcoin was the first cryptocurrency - it was introduced in July 2010. This makes it the most appropriate choice for the underlying analysis because it provides the longest time series compared to other cryptocurrencies and is most representative for the crypto market.

Within only eight years after Bitcoin entered the market, its total value increased to a market capitalization of $333.4 billion as of 17th December 2017. Interestingly, the dollar value of $790 per Bitcoin in December 2016 skyrocketed through $19.900 in December 2017 (+2519%), and eventually declined sharply to $5.900 in June 2018 ($-70\%$).[2]

These fluctuations in Bitcoin prices have led to periods of high volatility. The latter can be seen as a first indicator that Bitcoin is not primarily used as a transaction system, but also as a speculative (financial) asset. In fact, empirical studies have shown that Bitcoin is rather used as an asset instead of as a currency (Glaser *et al.*, 2014; Dyhrberg, 2016a;

[1]coinmarketcap.com accessed on July 30th 2018

[2]In comparison, the S&P 500 stock index increased from $2.2 to $2.72 (+23%) and decreased to $2.7 ($-1.2\%$) in the same period.

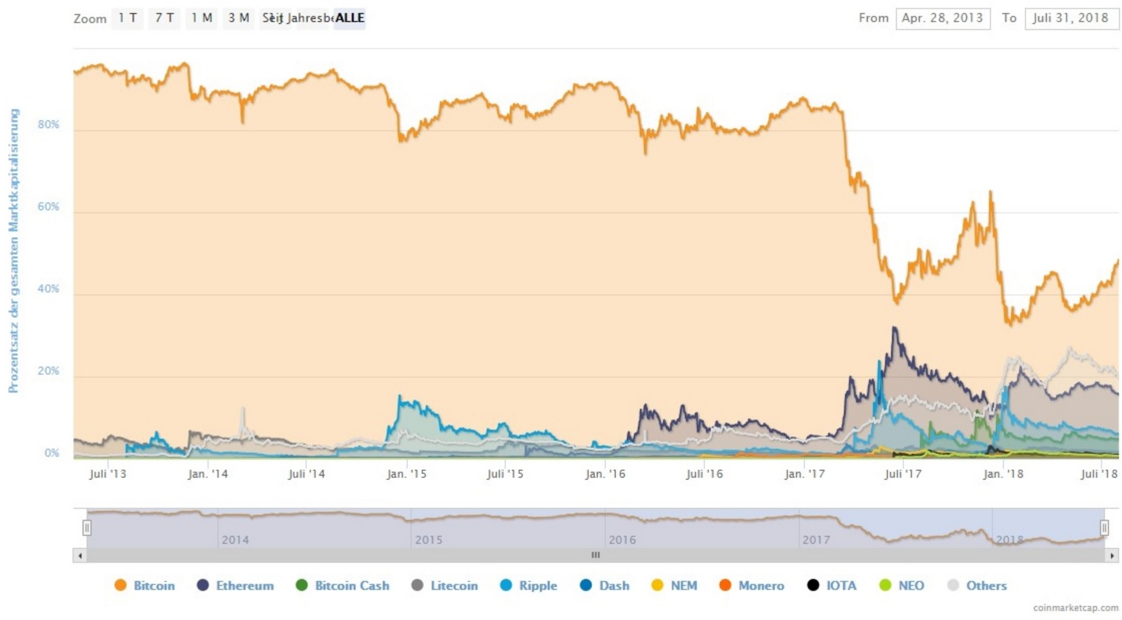


Figure 1
Cryptocurrency Market Capitalization

The chart displays the percentage market capitalizations (y-axis) of cryptocurrencies over time (x-axis).

Source: Coinmarketcap (2018)

Burniske and White, 2017; Baur *et al.*, 2018; Eom *et al.*, 2019). In other words, the average users' interest in Bitcoin is more driven by its appeal as an asset than as a currency (Glaser *et al.*, 2014). However, Bitcoin is also used for the trade of illegal products and services (Rosenberger, 2018) and the U.S. Drug Enforcement Administration found evidence that it is used as a means for money laundering (DEA, 2018).

According to Baur *et al.* (2018) and Burniske and White (2017), Bitcoin returns are not correlated with major asset classes - neither in „normal“ nor in volatile market periods. This offers diversification potential for investors. Bariviera *et al.* (2017) find the presence of persistent volatility in Bitcoin returns, enabling the application of (Generalized) Autoregressive Conditional Heteroskedasticity ((G)ARCH) models. Using asymmetric GARCH models, Dyhrberg (2016b) showed that it can be used as a hedge against stocks in the Financial Stock Exchange Index and the US Dollar in the short run. This shows that Bitcoin has its place in the financial markets and is worth considering for portfolio management. Hence, studying its volatility is crucial.

Persistent volatility has also been observed for other asset return series including stock returns. That is to say, an occurring high/low volatility persists for some time which is also referred to volatility clustering (Mandelbrot, 1963). ARCH models that were proposed by Engle (1982) describe the dynamic of volatility and thereby capture volatility clustering. Notably, the deviations from the return mean are modeled as dependent and this dependence can be captured by a function of lagged mean deviations to describe the variance of the return. Generalized ARCH (GARCH) models by Bollerslev (1986) additionally describe the variance as a function of its own lagged values. In this way, the model defines the evolution of volatility by an exact function.

As being part of the extreme value theory, I further consider the Hill estimator, introduced by Hill (1975), in my analysis. Corresponding Hill plots illustrate the distribution of the *tail index*. The latter can be seen as a statistic for the distribution of extreme observations or a as a measure of the maximum finite non-integer moment of a distribution (Hols and de Vries, 1991). Thereby, it provides useful information about the frequency of both large positive and negative outcomes (Jansen and de Vries, 1991). Most importantly, the *tail index* is not immaterial to my thesis as it is based on stable distributions that are nested with ARCH distributions with regard to their tail shapes (Ghose and Kroner, 1995; Groenendijk *et al.*, 1995). This allows for an empirical comparison between the two concepts.

Taking into account these considerations as well as recent developments, I pose the following central research question: What are the properties and applications of Bitcoin and how are they reflected in its return distribution in comparison to stocks? In particular, this thesis aims at giving theoretical and empirical insights on whether the properties and stylized facts of Bitcoin are confirmed by the results of (G)ARCH models and Hill estimates. Based on my evaluation, I find indications that Bitcoin has unique properties that are reflected in the first four moments of its return distribution.

The thesis is organised as follows. Section 2 provides some background information on cryptocurrencies and Bitcoin in particular. Taking into account recent developments, I elaborate on whether Bitcoin fulfills the properties of a currency or an asset. In Section 3, I carry out a descriptive time series analysis of Bitcoin, the Standard & Poor's 500 (S&P 500) stock index, and six publicly quoted stocks. In Section 4, I explain and apply (G)ARCH models and briefly discuss their results. Section 5 gives the Hill plots and a comparison with the GARCH results. Finally, Section 6 summarizes the main findings and provides concluding remarks.

2 Properties of Bitcoin

2.1 What are cryptocurrencies?

In 2008, an unknown author under the pseudonym Satoshi Nakamoto invented the digital currency Bitcoin by publishing the paper „Bitcoin: A Peer-to-Peer Electronic Cash System“. Nakamoto (2008) introduced a peer-to-peer payment system which enables the transfer of digital assets without a trusted intermediary.

The network of users records Bitcoin transactions by hashing them into a publicly available ledger (blockchain). The hashing process requires solving a mathematical problem (mining) that consumes computing (CPU) power of voluntary participants from the network. Users are incentivized to engage in mining by a monetary reward in form of Bitcoins in case they solve the mathematical problem before other participating users. This generates the supply of Bitcoins.

To avoid double spending of a Bitcoin, all transactions are publicly recorded in the blockchain and all users must agree on the history of the blockchain order. In particular, users accept valid blocks by working on them with their CPU power. They reject invalid blocks by not working on them and thus not extending them. In this way, the longest blockchain can be seen as a proof both for the validity of the blockchain history and for coming from the largest CPU power in the network. Nakamoto (2008) refers to this as „proof-of-work“.

This system is today known as cryptocurrency. The definition by Nakamoto suggests that Bitcoin serves as an alternative currency, but it is also used as a speculative investment. For instance, according to a report of Coinbase and ARK Invest, around 57% of Coinbase users facilitate the Bitcoin strictly as an investment between 2013 and 2016 (Burniske and White, 2017).[3] This raises the question whether Bitcoin rather fulfills the properties of an asset than of a currency. For stakeholders including policy makers, entrepreneurs, and investors, the classification of Bitcoin is of particular interest (Eom *et al.*, 2019). It is directly linked to regulatory processes and investment strategies that depend on the classification of the respective asset.

2.2 Is Bitcoin a currency or an asset?

2.2.1. Bitcoin as a currency

According to Law (2018), a currency is defined as any kind of money which circulates in an economy or a particular country. Apart from the fact that Bitcoin is intended to be a medium of exchange in the first place, there is an increasing number of online merchants accepting Bitcoin as a form of payment (Hileman and Rauchs, 2017). In other words, Bitcoin is circulating in today's economy and hence satisfies the definition of a currency.

Money, however, is defined as a medium of exchange with little intrinsic value (1) (Law, 2018). It is primarily used for the trade of goods or services (2). Besides, it serves as a unit of account (3), a means for deferred payment (4), and a store of value (5).

First, Bitcoin is a medium of exchange by design. It is held in digital user accounts (wallets) with obviously no intrinsic value.

Second, according to Hileman and Rauchs (2017), most of the Bitcoin transactions are not daily purchases by consumers but cross-border payments by companies that are denominated in national currencies. In this context, the authors refer to the Bitcoin system as being a fast and cost-efficient „payment rail". In addition to that, the majority of users does not use it as currency at all (Burniske and White, 2017) and it can be concluded that Bitcoin is not primarily used for the purchase of goods and services - as far as legal purchases are concerned.

Rosenberger (2018) states that Bitcoin, among other cryptocurrencies, is used for the trade of illegal products and services in the so-called Darknet. For instance, the illegal online market place Silk Road was the first organisation that processed payments exclusively with Bitcoins. This demonstrated the applicability of Bitcoin as a means of payment. According to a report of the U.S. Drug Enforcement Administration (DEA, 2018), cryptocurrencies provide a money laundering method for criminals that is more secure compared to other methods such as bulk cash smuggling. The seizing of the latter

[3] Coinbase Inc. was the largest trading platform for Bitcoins at that time (see Figures 16 and 17 in Appendix).

has been steadily decreasing over the past years which may point to increasing usage of cryptocurrencies as an alternative method. However, the extent to what Bitcoin is used for illegal trade and money laundering cannot be exactly quantified and hence does not play a role in my analysis.

Third, in order to serve as a unit of account (numéraire), a currency must provide a reference point in terms of prices for goods and services (Yermack, 2015). For example, the price of a cup of coffee is €1 at Spar and €2.75 at Starbucks. Potential customers can conclude that the latter is almost three times as expensive as the former. However, since Bitcoin prices are extremely volatile, its value in US dollars changes drastically compared to the Euro (see Figure 18 in Appendix). If coffee sellers offered Bitcoin prices, they would have to recalculate the price constantly which is costly for them and confusing for the customers. Obviously, this problem would be solved with Bitcoin as a principal currency.

Moreover, the current Bitcoin market prices differ greatly. For instance, while I am writing this paragraph, the Bitcoin prices range from \$6436 to \$6470. This diversity in Bitcoin market prices represents a violation of the law of one price and would not persist in developed currency markets due to „the ease of arbitrage“ (Yermack, 2015). Even if someone would take the average of all current Bitcoin prices, it would not adequately reflect the cost of buying or selling the Bitcoin at that time. This further aggravates the problem of high volatility in Bitcoin prices, because a valid reference point for consumer prices cannot be established. Taken together, Bitcoin performs poorly as a unit of account.

Fourth, there is no evidence that Bitcoin serves as a means for deferred payment. This is arguably related to the instability of Bitcoin prices compared to conventional currencies. It is not rational to use Bitcoin as a means for deferred payment, because its value changes constantly and thereby imposing a risk on both the payer and payee of deferred payment.

Fifth, to serve as a store of value, the economic value of a currency must be stable over a given period. This includes protection against theft. Bitcoins are held in wallets that are facing the threat of hackers stealing it which counteracts the stability of Bitcoin's economic value. Most importantly, the prevailing extreme volatility in Bitcoin prices does not make it an appropriate store of value.

According to a report of the European Central Bank (ECB, 2015), Bitcoin does not only match the economic definition of money due to its high volatility but also due to its low level of acceptance. The authors refer to Bitcoin as a decentralized „digital representation of value“ that serves as a substitute for money under certain circumstances.

As shown above, Bitcoin cannot be classified as money in a narrow sense of the word. In the broader sense, one can distinguish between two different forms of money, namely commodity money and fiat money. Commodity money is naturally scarce and has other use than a medium of exchange, such as gold. Fiat money is naturally not scarce (issued by a central bank) and serves primarily as a medium of exchange, such as the US dollar.

Selgin (2013) points to the fact that Bitcoin shares properties of both commodity money and fiat money while not fitting the definition of either. On the one hand, Bitcoin is scarce since it can only be increased by competitive mining (i.e. it cannot be issued at near-zero marginal costs) and it's total supply is finite by design; it will not increase beyond the year 2040[5]. In addition, it does not depend on a central institution. Bitcoin shares these properties with commodity money like gold. On the other hand, Bitcoin has no intrinsic value just like fiat money. Consequently, Selgin (2013) classifies it as „synthetic commodity money“.

Taken together, Bitcoin only meets the properties of money partially. In particular, its high volatility in prices compared to other currencies is a major reason as to why it performs poorly as a substitute for fiat money. In addition, it imposes a high short-term risk on users.[4] However, Bitcoin is a currency by design and already found its markets - but only to a limited extent.

2.2.2. Bitcoin as an asset

An asset is broadly defined as any tangible or intangible object that is valued by its owner, and that can be converted into money (Law, 2018). First of all, in contrast to money, an asset does not possess the feature of a medium of exchange and a unit of account but only of a store of value (Baur *et al.*, 2018).

[5] Theoretically, someone could enforce a supply beyond the year 2040. This would require 51% of the CPU power of the network.

[4] Assuming that users do not make use of any swaps, forwards, or other derivative markets to mitigate the risk.

Greer (1997) distinguishes between three asset classes, namely capital assets (1), consumable/transformable assets (2), and store of value assets (3). First, capital assets represent a continuous income source such as equities, bonds, and income-producing real estate. Second, consumable/transformable assets can be both consumed and transformed into another asset but do not generate continuous income, e.g. physical commodities (grains) or precious metals (gold). Third, store of value assets neither can be consumed nor generate continuous income such as precious metals (gold), currencies, and fine art. Gold both matches the definition of consumable/transformable assets and store of value assets showing that the lines between the classes are blurred.

As a matter of fact, Bitcoin does not match any of the asset classes from above. Someone may assign it to store of value assets, but as shown in Section 2.2.1. its extreme volatility in prices does not make it an appropriate store of value. This raises the question of whether Bitcoin is an asset at all, and if so, to which asset class it belongs.

By naming their paper „Bitcoin: Ringing the Bell for a New Asset Class“, Burniske and White (2017) already make a case for classifying Bitcoin as an asset. The authors find that Bitcoin has properties of a unique asset class contrasting with the three traditional asset classes from above. For instance, the fact that Bitcoin has a predictable supply and a fixed total amount already makes it unique. Furthermore, it shows a low correlation (smaller than $|0.4|$) to other assets such as stocks, bonds, gold, and oil. Thus, Bitcoin is not related to their market swings. According to the authors, this is a crucial condition for declaring Bitcoin to a new asset class.

Most importantly, the chart in Figure 2 shows that the global Bitcoin trading volume increased at a higher rate than the global Bitcoin transaction volume between 2012 and 2016. According to the graph, the trading volume constantly outnumbers the transacting volume since the end of 2015. This points to the fact that the Bitcoin is more and more used as a speculative investment (asset) than a currency in the given period. Since Bitcoin cannot be assigned to the currency asset class (store of value asset), Burniske and White (2017) question the naming cryptocurrency and suggest to use other names such as „blockchain asset“ or „digital asset“.

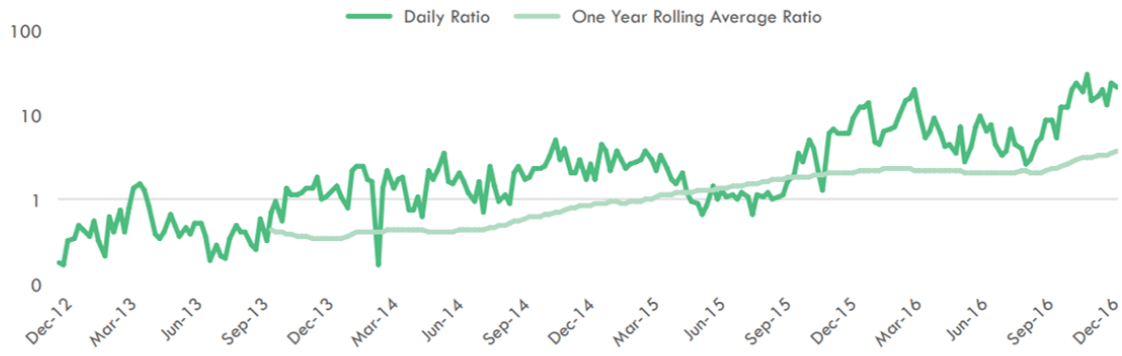


Figure 2
Global Daily Bitcoin Volumes: Trading Relative to Transacting
Source: Burniske and White (2016)

According to an empirical analysis of Glaser *et al.* (2014), Bitcoin is mostly used as an asset instead of as a currency. This makes the Bitcoin market more speculative and volatile than the market for common currencies. Moreover, users appear to be biased towards good news. Bad news on prices tends to be neglected, whereas good news on prices attract new users while previous users stay invested. This indicates Bitcoin users' limitations concerning their objectivity and professionality.[6]

Consequently, Bitcoin's volatility might be asymmetrically affected by good and bad news. This is generally referred to as the leverage effect[7] (Zivot, 2009) and implies the need for models capturing this behavior such as the exponential GARCH (EGARCH) model by Nelson (1991), the asymmetric power ARCH (APARCH) model by Ding *et al.* (1993), and the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model by Glosten *et al.* (1993). Estimating these models, however, results in insignificant model paramaters for the Bitcoin series and hence they are not considered in my analysis.

I conclude that Bitcoin can rather be seen as an asset than as a currency. It can be best described as a „digital asset" (Burniske and White, 2017) or „digital representation of value" (ECB, 2015). Since it is mainly used as an investment, it competes with traditional assets including stocks suggesting a comparison in terms of volatility to be interesting. My analysis will show that Bitcoin's unique properties as an asset are reflected in its distribution that clearly separates it from the stock/index distributions.

[6] Detzel *et al.* (2018) point to the fact that Bitcoin lacks common value indicators such as dividends or accounting statements in contrast to stocks. As a consequence, users are dependent on other indicators such as price and volume; making it harder to conclude about Bitcoin's economic value.

[7] The sample correlation between the lagged return r_{t-1} and the squared return r_t^2 equals -0.085 for the Bitcoin series providing evidence for possible leverage effects.

Given the finite supply of Bitcoins in the future, the demand might become larger than the supply which in turn would imply deflationary effects (Baur *et al.*, 2018). This makes it even more likely for the Bitcoin to be used as an investment instead of a medium of exchange in the long run, because it would not be rational to spend a share x of a Bitcoin in period t if the expected value of x is greater in period $t+1$.

However, Bitcoin is an open-source software and evolves constantly, thereby affecting its applications and properties. Latest innovations like smart contracts or side chains, for instance, could facilitate completely new financial services such as liquid private markets or peer-to-peer loan issuance. Hence, my conclusion of Bitcoin being primarily a financial asset must be regarded as the most recent snapshot that may change in the near future.

3 Descriptive Time Series Analysis

3.1 Stylized Facts

In the following part, I will compare Bitcoin closing prices with the S&P 500 index and stock adjusted[8] closing prices on a daily basis. The data includes stock prices of MSCI Inc. (stock market indexes and portfolio analytics), Nestlé S.A. (food processing), The Procter & Gamble Company (consumer goods), BP p.l.c. (oil and gas), Goldman Sachs Group, Inc. (financial services), and Meritor (commercial vehicles systems and components), thus covering a wide range of industries. All price data were obtained from YahooFinance (2018) and range from 17th July 2010 to 19th October 2018. This corresponds to a total of 3017 observations for the Bitcoin series and 2079 observations for the stock/index series.

The Bitcoin data contains prices of the most popular trading platforms, and there are closing prices available from any day since Bitcoin can be traded around the clock. In contrast, stocks are not traded on weekends and stock exchange holidays which explains the difference in the number of observations. To ensure comparability, I synchronize the underlying time series with respect to the available dates, i.e. I drop 938 Bitcoin prices.[9] This results in 2079 observations for each time series ranging from 19th July 2010 to 18th October 2018. I performed the data analysis using the open source software R.

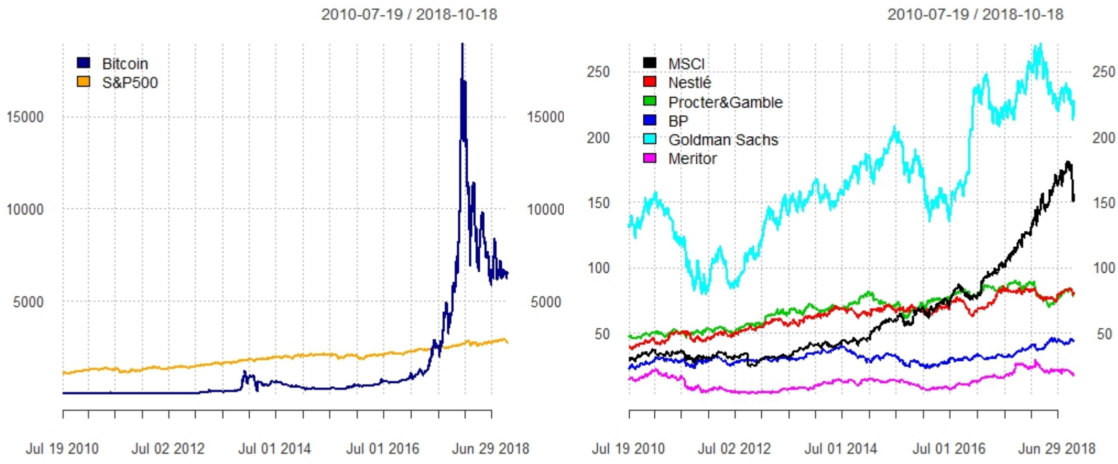


Figure 3
Closing Prices in US Dollar (2010-2018)

[8]The stock prices are adjusted for stock splits and dividend payments.
[9]This does not affect the conclusions drawn from my analysis.

The displayed time series in Figure 3 seem to be non-stationary processes (or integrated of order one: I(1)) because the mean levels of the prices show a clear upward trend thereby suggesting random walk behavior. This is validated by a unit root test below. In addition, there appears to be volatility variability especially for the Bitcoin and Goldman Sachs time series.

I perform an Augmented Dickey-Fuller (ADF) test for the underlying time series to validate the visual assumption of their non-stationarity. According to Wooldridge (2015), a stochastic process $\{y_t : t = 1,2,...\}$ is said to be stationary if the joint distribution of $(y_{t_1}, y_{t_2}, ..., y_{t_m})$ is equal to the joint distribution of $(y_{t_1+h}, y_{t_2+h}, ..., y_{t_m+h})$ for all integers $h \geq 1$ and $1 \leq t_1 < t_2 < ... < t_m$.

Given the following AR(1) model

$$y_t = \alpha + \rho y_{t-1} + e_t, \quad t = 1,2,... \tag{1}$$

where y_t are the observed prices, e_t the error terms which are assumed to be independent from previous prices, and α is the intercept.

$$E(e_t | y_{t-1}, y_{t-2}, ..., y_0) = 0 \tag{2}$$

In case the sequence $\{y_t\}$ follows the model in equation (1), it has a unit root if, and only if, $\rho = 1$. To make the t statistic an appropriate choice for a unit root test, I subtract y_{t-1} from both sides of equation (1).

$$\begin{aligned} y_t - y_{t-1} &= \alpha + (\rho - 1) y_{t-1} + e_t \\ \Leftrightarrow \Delta y_t &= \alpha + \theta y_{t-1} + e_t, \end{aligned} \tag{3}$$

If $\{y_t\}$ follows equation (3) with $\rho = 1$, then Δy_t is serially uncorrelated and follows a random walk. The equation can be augmented by adding further lags of Δy_t to it.

$$\Delta y_t = \alpha + \theta y_{t-1} + \gamma_1 \Delta y_{t-1} + e_t, \tag{4}$$

where $\gamma_1 < 1$ to make sure that under $H_0 : \theta = 0$, $\{y_t\}$ follows a stable AR(1) model and under $H_1 : \theta < 0$, $\{y_t\}$ follows a stable AR(2) model. More lags of Δy_t can be added to the model to take the dynamics of the time series process into account. To sum up, I test the null hypothesis $H_0 : \theta = 0$, implying a unit root is present in the time series sample (i.e. $\{y_t\}$ follows a random walk and is I(1)) against the alternative hypothesis $H_1 : \theta < 0$, implying stationarity (i.e. $\{y_t\}$ does not follow a random walk and is I(0)).

I fail to reject the H_0 for the underlying time series at any reasonable significance level and irrespective of the number of lags, implying no substantial evidence against the H_0 given the data. It can be concluded that a unit root is present in the series and $\{y_t\}$ follows a random walk in each case. This validates the assumption of the visual examination from Figure 3 and the time series processes can be described as non-stationary (or integrated of order 0: I(0)).

The underlying time series $\{y_t\}$ display unit-root behavior and thus cannot be modeled as stationary. Consequently, I calculate the log-returns (hereafter referred to as the returns) of all time series to generate stationarity and remove potential trends in the prices.

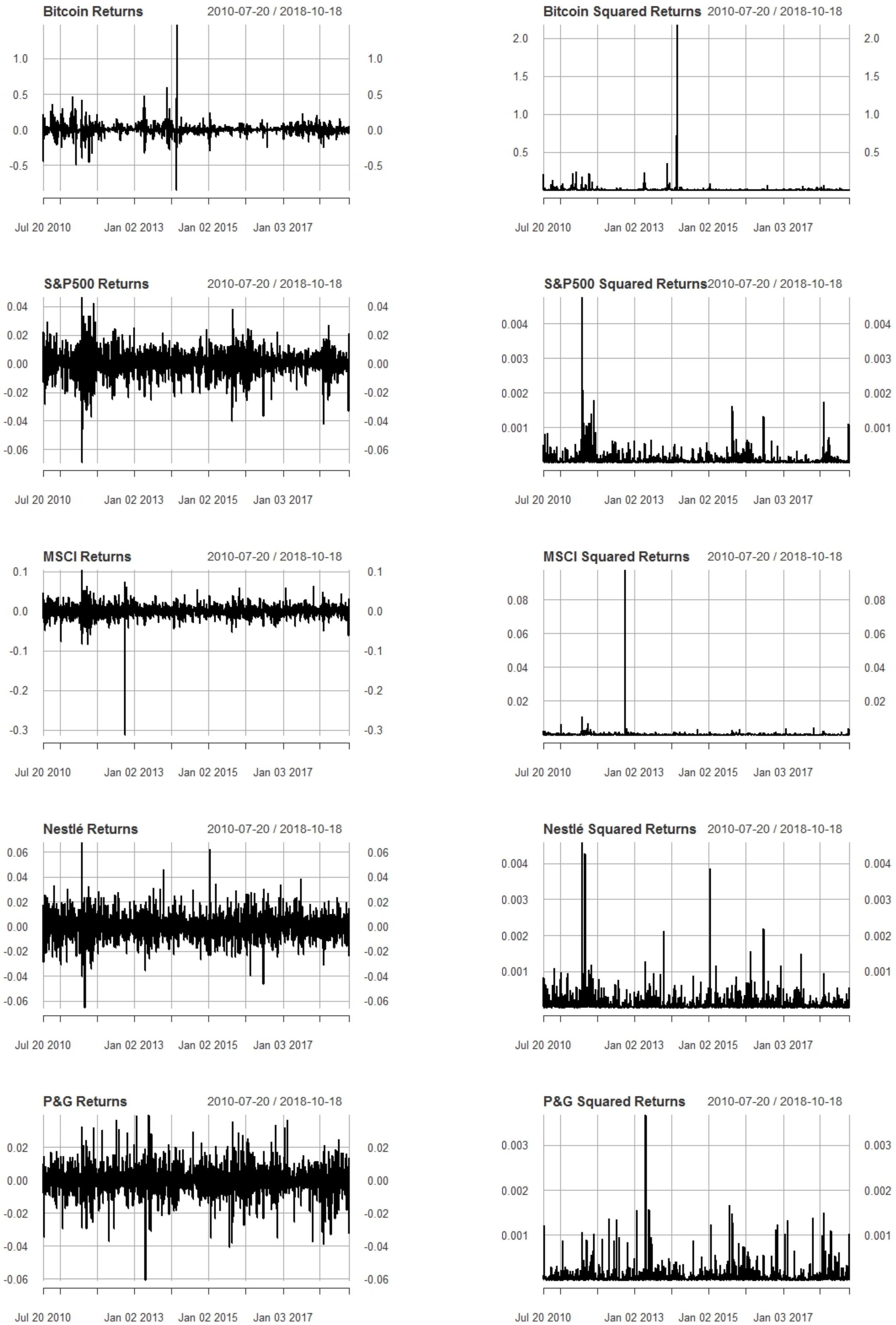
$$(5) \quad r_t = log(y_t) - log(y_{t-1}) = log\left(\frac{y_t}{y_{t-1}}\right),$$

where y denotes the daily closing price of the respective asset. The return r_t can be regarded as the logarithmized profit of buying yesterday and selling today assuming the absence of any transaction costs.

Figure 4 shows the return and squared return charts of the underlying series. The most striking abnormal returns are found in the Bitcoin charts in the beginning of 2014. This can be explained by the bankruptcy of the most popular trading platform for Bitcoins at that time (Mt. Gox) causing the disappearance of Bitcoins worth hundreds of millions of US Dollars (Yermack, 2015). On 20$^{\text{th}}$ February 2014, the daily return was -84.9%, whereas only six days later it increased to 147.4%.

Another striking abnormal return is found in the MSCI charts in 2012. It can be traced back to the 2$^{\text{nd}}$ October 2012 where the largest U.S. mutual fund manager (Vanguard Group) declared the cancellation of 22 index funds provided by MSCI Inc. (Pressman, 2012). This caused the MSCI return series to drop by -31.2% on the same day.

As can be seen from the graphs in Figure 4, the scale of the Bitcoin return volatility exceeds the scale of the stock/index returns volatility by far. This is reflected in the standard deviation, being 0.079 for the Bitcoin return and 0.015 for the average stock/index return (see Table 1).


Bitcoin Returns
2010-07-20 / 2018-10-18
Bitcoin Squared Returns
2010-07-20 / 2018-10-18
S&P500 Returns
2010-07-20 / 2018-10-18
S&P500 Squared Returns
2010-07-20 / 2018-10-18
MSCI Returns
2010-07-20 / 2018-10-18
MSCI Squared Returns
2010-07-20 / 2018-10-18
Nestlé Returns
2010-07-20 / 2018-10-18
Nestlé Squared Returns
2010-07-20 / 2018-10-18
P&G Returns
2010-07-20 / 2018-10-18
P&G Squared Returns
2010-07-20 / 2018-10-18
Jul 20 2010
Jan 02 2013
Jan 02 2015
Jan 03 2017

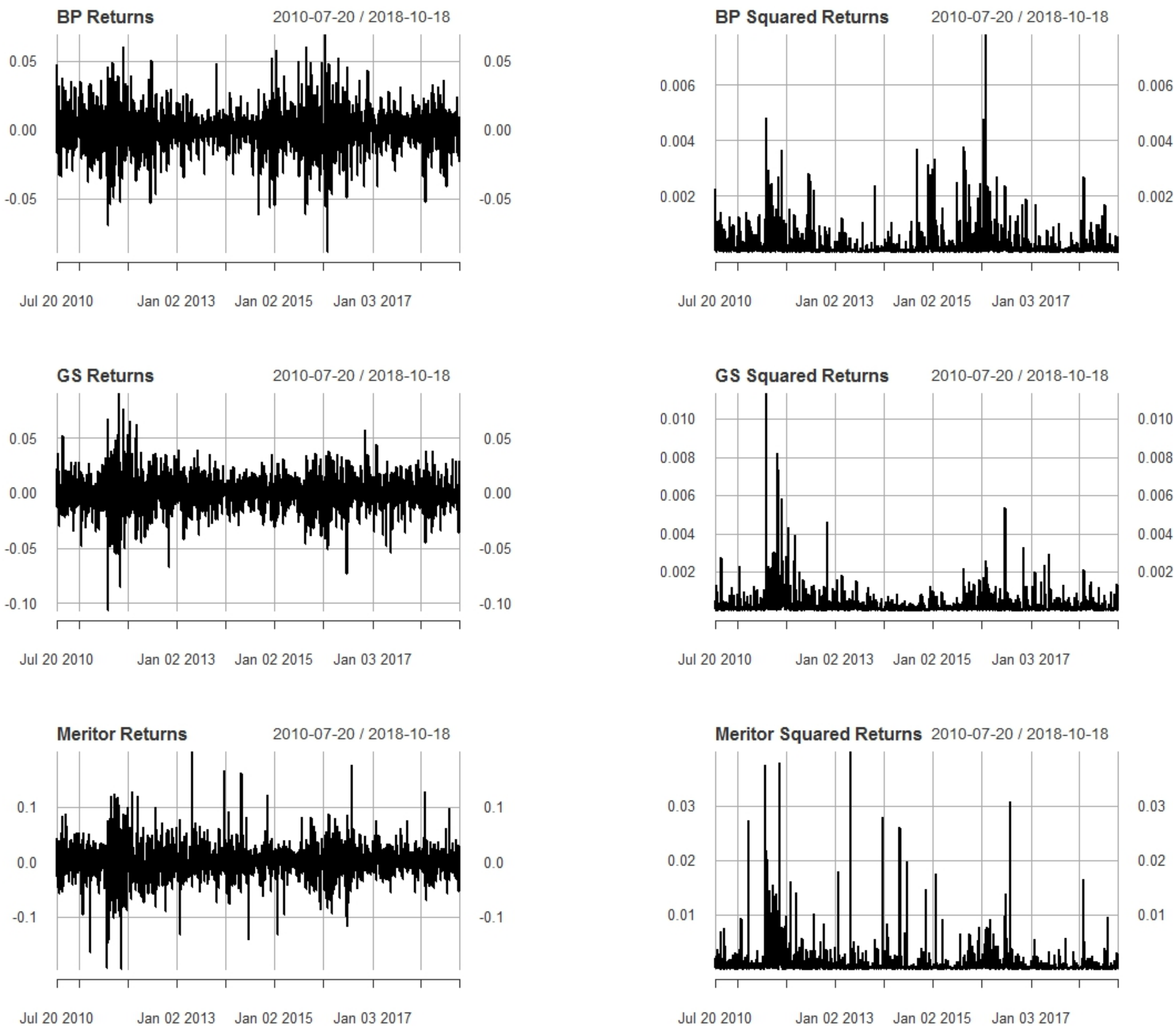


FIGURE 4
RETURNS AND SQUARED RETURNS

Furthermore, the squared returns of all series show some persistence in volatility. More precisely, the return series seem to exhibit clusters of high and low volatility which is a common feature among financial assets. This pattern is referred to volatility clustering, i.e. large (small) changes in returns are followed by large (small) changes in returns, implying that the volatility is conditionally autoregressive and persistent (Cuthbertson and Nitzsche, 2004). Therefore, the underlying volatilities seem follow an autoregressive conditional heteroscedasticity (ARCH) process. However, this assumption will be further checked below. Corresponding ARCH and GARCH models will be explained and estimated in Section 4.

Besides having higher volatility, the Bitcoin return mean is more than 5 times higher than the average stock/index return mean (see Table 1). The daily returns of the latter have a mean close to 0. This remarkable difference can be explained by an extreme price increase of Bitcoin during the underlying period (see Figure 3).

The empirical density function of the return series are displayed in Figure 5 by means of histograms. All histograms appear to be unimodal with modus 0 in each case.

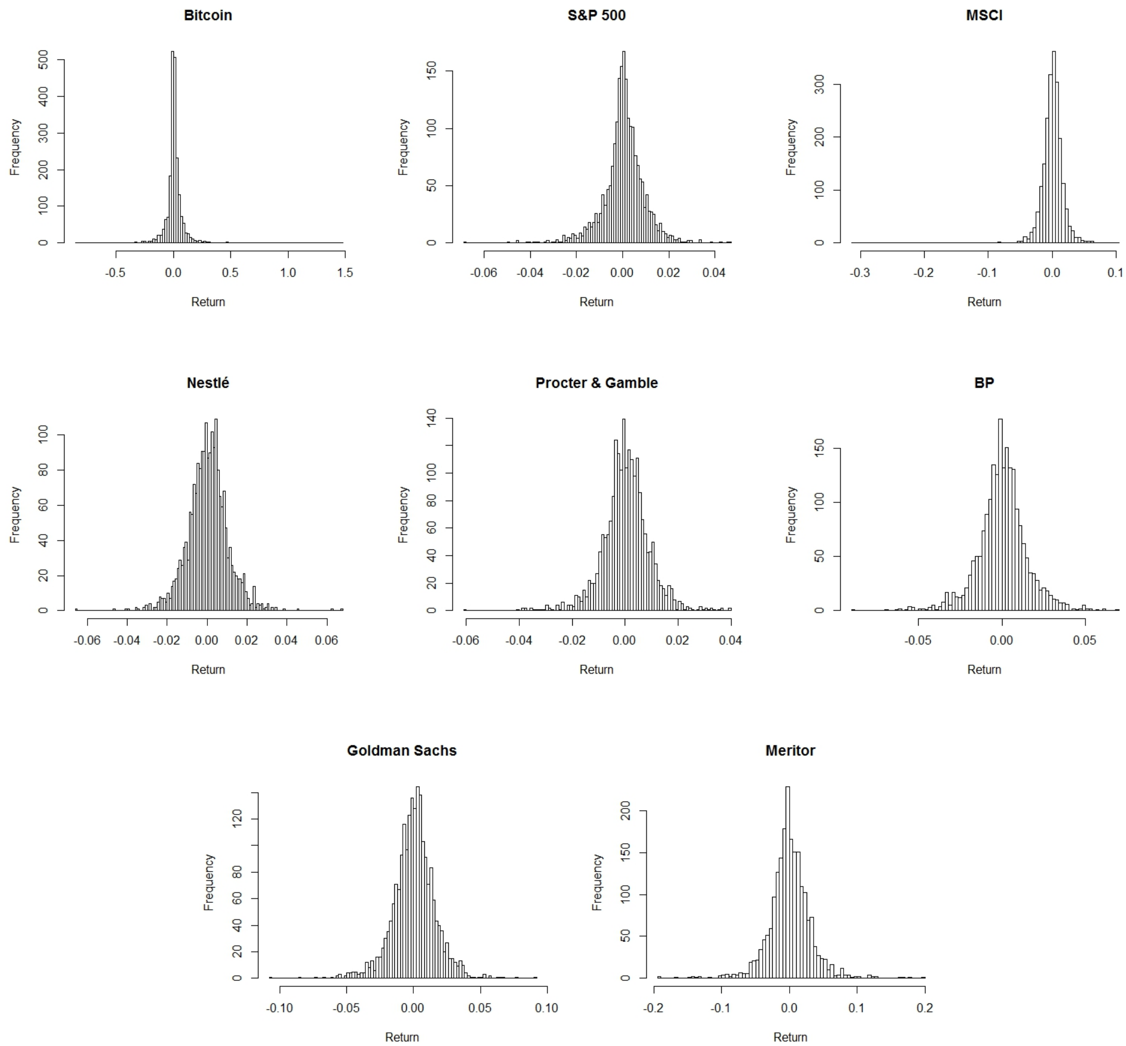


Figure 5
Return Histograms

It is apparent from Figure 5 that the Bitcoin return density has bigger tails compared to the other densities, implying a wider return-spread of the former. According to the coefficient of skewness from Table 1, the Bitcoin returns are right-skewed whereas the stock returns are, except Nestlé, left-skewed. Thus, the densities of the Bitcoin and Nestlé returns have asymmetric tails extending more towards positive values, whereas the densities of the other returns have asymmetric tails extending more towards negative values. The skewness of the MSCI and Bitcoin distributions (MSCI: -3.217, Bitcoin: 2.591) are extreme compared to the relatively low skewness of the other distributions that are ranging between -0.556 and 0.069. Hence, the latter are more symmetric compared to the Bitcoin and MSCI densities which is also reflected in the histograms from Figure 5.[10]

Referring to Table 1, both the Bitcoin and the MSCI return distributions are characterized by a high kurtosis (Bitcoin: 69.039, MSCI: 63.256), indicating higher frequencies of outcomes at both the extreme negative and positive ends of the distribution, i.e. heavy tails. The kurtosises of the other distributions range between 2.776 and 5.469, implying fewer outliers compared to the Bitcoin and MSCI return distributions. This becomes obvious in Figure 5 and is in line with higher minimum/maximum values of especially the Bitcoin return. The boundaries of the stock/index returns are between -0.312 and 0.2 which is low in comparison to the Bitcoin boundaries of -0.849 and 1.474.

In summary, the Bitcoin and MSCI distributions are characterized by higher values of skewness and kurtosis compared to the other distributions. The latter are closer to a normal distribution in terms of skewness and kurtosis (skewness: 0, kurtosis: 3). However, all underlying return distributions can be described as heavy-tailed and to further verify their non-normality, I perform a Jarque-Bera test.

Table 1
Descriptive Statistics of Returns

	Bitcoin	S&P500	MSCI	Néstle	P&G	BP	GS	Meritor
Minimum	-0.849	-0.069	-0.312	-0.065	-0.061	-0.088	-0.106	-0.195
Maximum	1.474	0.046	0.104	0.068	0.040	0.069	0.090	0.200
1. Quartile	-0.015	-0.003	-0.007	-0.006	-0.004	-0.007	-0.008	-0.015
3. Quartile	0.025	0.005	0.009	0.006	0.005	0.008	0.009	0.016
Mean	0.005	0.000	0.001	0.000	0.000	0.000	0.000	0.000
Median	0.002	0.001	0.001	0.000	0.000	0.000	0.001	0.000
Variance	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.001
Stdev	0.079	0.009	0.017	0.010	0.009	0.015	0.016	0.031
Skewness	2.591	-0.556	-3.217	0.069	-0.288	-0.200	-0.234	-0.048
Kurtosis	69.039	5.109	63.256	3.264	3.479	2.776	3.373	5.469

[10] It is noteworthy that MSCI's return skewness and kurtosis is much lower when excluding the minimum value of -0.312. The resulting new minimum value, skewness, and kurtosis are -0.0825, 0.023, and 3.790, respectively. Hence, the density function is more symmetric and the excess kurtosis significantly lower.

Suppose that $\{r_1, ...r_T\}$ is a random sample of returns with T observations. The sample mean (first moment) measures the central location of a distribution and is defined as $\hat{\mu}_r = \frac{1}{T}\sum_{t=1}^{T} r_t$ (Tsay, 2010). The sample variance (second moment) quantifies the variability of the return and is defined as $\hat{\sigma}_r^2 = \frac{1}{(T-1)}\sum_{t=1}^{T}(r_t - \hat{\mu}_r)^2$. Further, the sample skewness (third moment) $\hat{S}(r) = \frac{1}{(T-1)\hat{\sigma}_r^3}\sum_{t=1}^{T}(r_t - \hat{\mu}_r)^3$ provides information about the symmetry of the return around the mean. Lastly, the sample kurtosis (fourth moment) $\hat{K}(r) = \frac{1}{(T-1)\hat{\sigma}_r^4}\sum_{t=1}^{T}(r_t - \hat{\mu}_r)^4$ quantifies the thickness of tail.

The skewness $S(r)$ of a normally distributed variable is 0. Generally, excess kurtosis is defined as $K(r) - 3$. The excess kurtosis of a normally distributed random variable is 0, because its kurtosis equals 3. A positive excess kurtosis implies heavy tails of the underlying distribution, i.e. the distribution exhibits more extreme values compared to a normal distribution. As shown above, the underlying series exhibit excess kurtosis and non-zero skewness implying non-normality.

Assuming that a sample is normally distributed, $\hat{S}(r)$ and $\hat{K}(r) - 3$ are asymptotically normally distributed with 0 mean and corresponding variances $6/T$ and $24/T$. To test for asymmetry, consider the test statistics

$$(6) \quad t = \frac{\hat{S}(r)}{\sqrt{6/T}},$$

with the corresponding null hypothesis $H_0 : S(r) = 0$ against the alternative hypothesis $H_1 : S(r) \neq 0$. The null hypothesis is rejected if $|t| > Z_{\alpha/2}$, where α is the significance level and $Z_{\alpha/2}$ stands for the upper $100(\alpha/2)^{\text{th}}$ quantile of the standard normal distribution.

To test for excess kurtosis, consider the test statistics

$$(7) \quad t = \frac{\hat{K}(r)}{\sqrt{24/T}},$$

with the null hypothesis $H_0 : K(r) - 3 = 0$ against the alternative hypothesis $H_1 : K(r) - 3 \neq 0$. The null hypothesis is rejected if the p-value of the test statistic is smaller than α. The Jarque-Bera test statistic brings both test statisitcs together and is defined as (Tsay, 2010)

$$(8) \quad JB = \frac{\hat{S}^2(r)}{6/T} + \frac{[\hat{K}(r) - 3]^2}{24/T}.$$

It is asymptotically χ^2 distributed with 2 degrees of freedom and the H_0 : the sample of returns comes from an iid normally distributed population is tested against the H_1: the sample of returns does not come from an iid normally distributed population. The H_0 is rejected if the p-value of the test statistic is smaller than the significance level α. For the underlying series, the H_0 of the Jarque-Bera test can be rejected at the significance level of 1% with p-values being close to 0.[11] In other words, the test result indicates that normality of underlying time series is very unlikely.

While the Jarque-Bera test statistic is based on the skewness and kurtosis, the test result does not provide any further information on how the underlying distributions differ from the normal distribution. In contrast, the quantile-quantile (Q-Q) plots in Figure 6 and the Kernel density plots in Figure 8 allow us to see how the underlying distributions appear to be different from the normal distribution.

Figure 6, shows the Q-Q plots of the underlying series against a normal distribution. The sample quantiles are plotted on the y-axis against the theoretical (normal) quantiles on the x-axis. A resulting black dotted curve displays the intersection points from the sample quantiles and normal quantiles, and a straight colored line shows the Q-Q plots of a normal distribution against a normal distribution. Hence, most of the black dotted intersection points shall lie on the colored line if the underlying return distribution is close to a normal distribution.

However, this is only partially the case since the outer ends deviate from the straight colored line in each case. In particular, the „S“ shape of all black curves indicate that heavy tails are present in the distributions. This is especially the case for the Bitcoin series since the magnitude of the deviation from the straight colored line is relatively high. Taken together, the normal Q-Q plots do not indicate a good fit to a normal distribution.

[11] This is confirmed by a Shapiro-Wilk test.

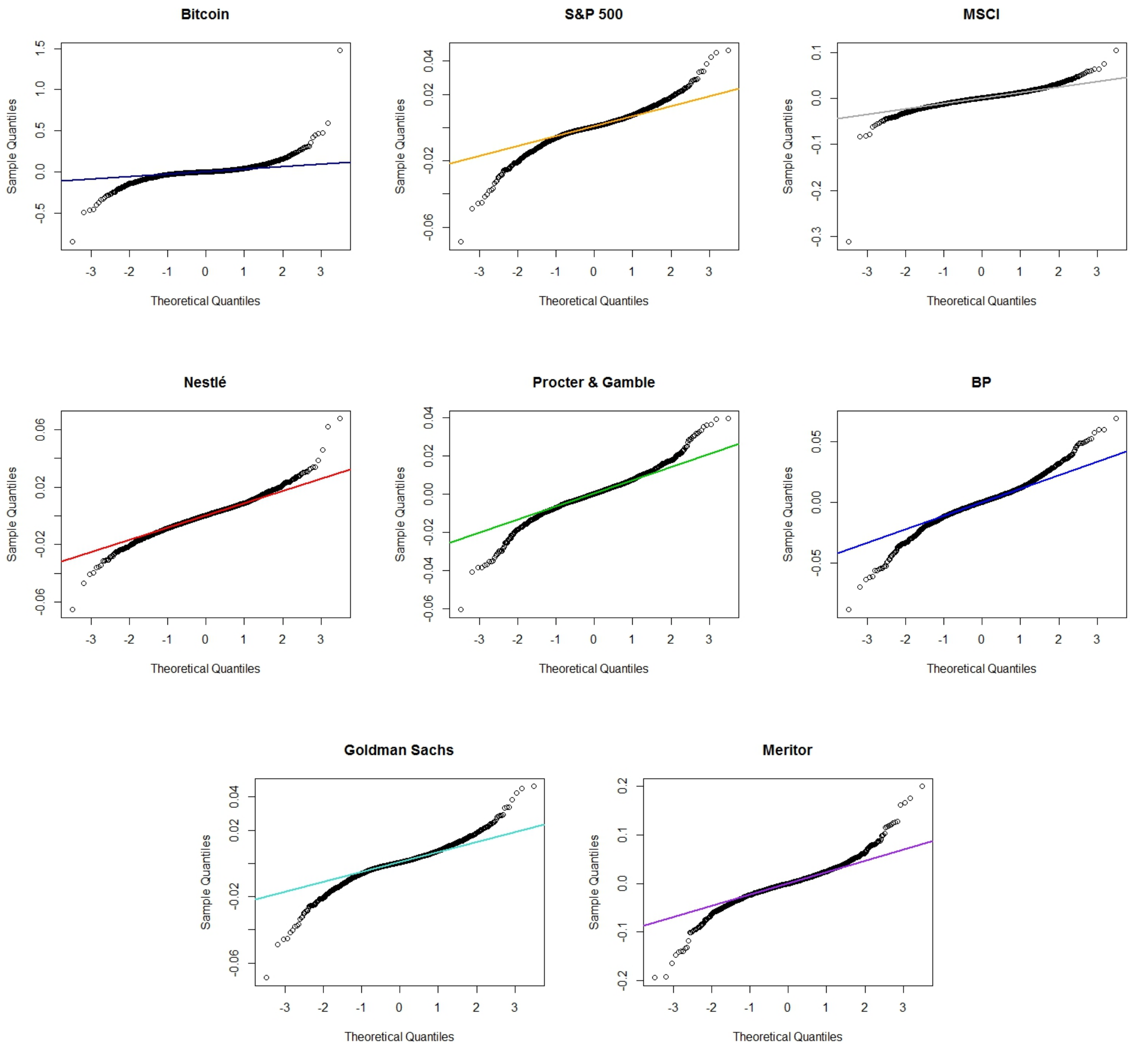


FIGURE 6
NORMAL Q-Q PLOTS

Figure 7 shows the Q-Q plots of the underlying series against a Student-t distribution.[12] In contrast to the normal Q-Q plots, the Student-t Q-Q plots clearly display a better fit of the straight colored lines and the black dotted curves. The black dotted curves are slightly „S“ shaped implying the presence of heavier tails compared to a Student-t distribution. However, all in all, the Student-t distribution seems to capture the higher than normal kurtosis and indicates a good fit to the data.

Figure 8 displays Kernel Density plots. Within each plot, the red line represents a Kernel density estimation[13] of the return series and the blue line represents a random

[12]The degrees of freedom are chosen according to the best fit. Bitcoin: 2, S&P 500: 3.5, MSCI: 5, Nestlé: 7, Procter & Gamble: 4.5, BP: 4, Goldman Sachs: 5, and Meritor: 4.

[13]The following bandwidth estimator is used: $bw = 0.9n^{-1/5}\min\left[sd(x), \frac{IQR(x)}{1.34}\right]$, where n denotes the sample size, $sd()$ the standard deviation, and $IQR()$ the interquartile range function of the respective asset. The $IQR()$ calculates the difference between the upper and lower quartile, i.e.$q_{0.75} - q_{0.25}$.

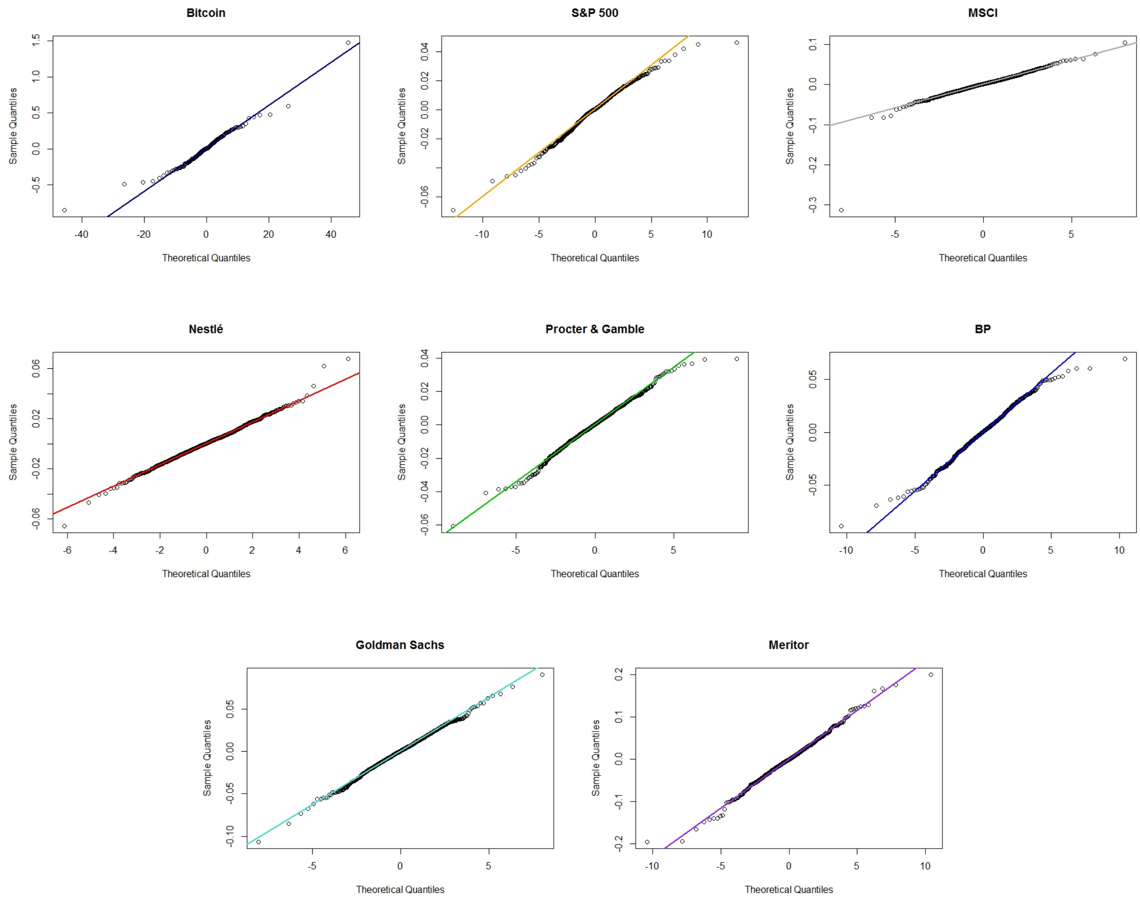


FIGURE 7
STUDENT-T Q-Q PLOTS

sample of a normal distribution with the mean and the variance of the respective series. The Figure illustrates the presence of heavy tails in all underlying distributions indicated by the longer tails of the Kernel density (red) in comparison to the tails of the normal densities (blue).

In particular, the extreme skewness of the Bitcoin (2.591) and the MSCI (-3.217) distribution compared to the moderate skewnesses of the other stock distributions that are ranging between -0.556 and 0.069 becomes evident in the plots. Moreover, the positive excess kurtosises of all underlying distributions can be clearly identified in the Figure. The extreme excess kurtosis of Bitcoin is by far the most obvious.

Taken together, the descriptive analysis showed that Bitcoin prices are characterized by extreme price movements that become obvious in comparison to stocks. The scale of Bitcoin's return volatility clearly exceeds the scale of the stock/index return volatilities by far. This is reflected in Bitcoin's standard deviation being more than 5 times higher

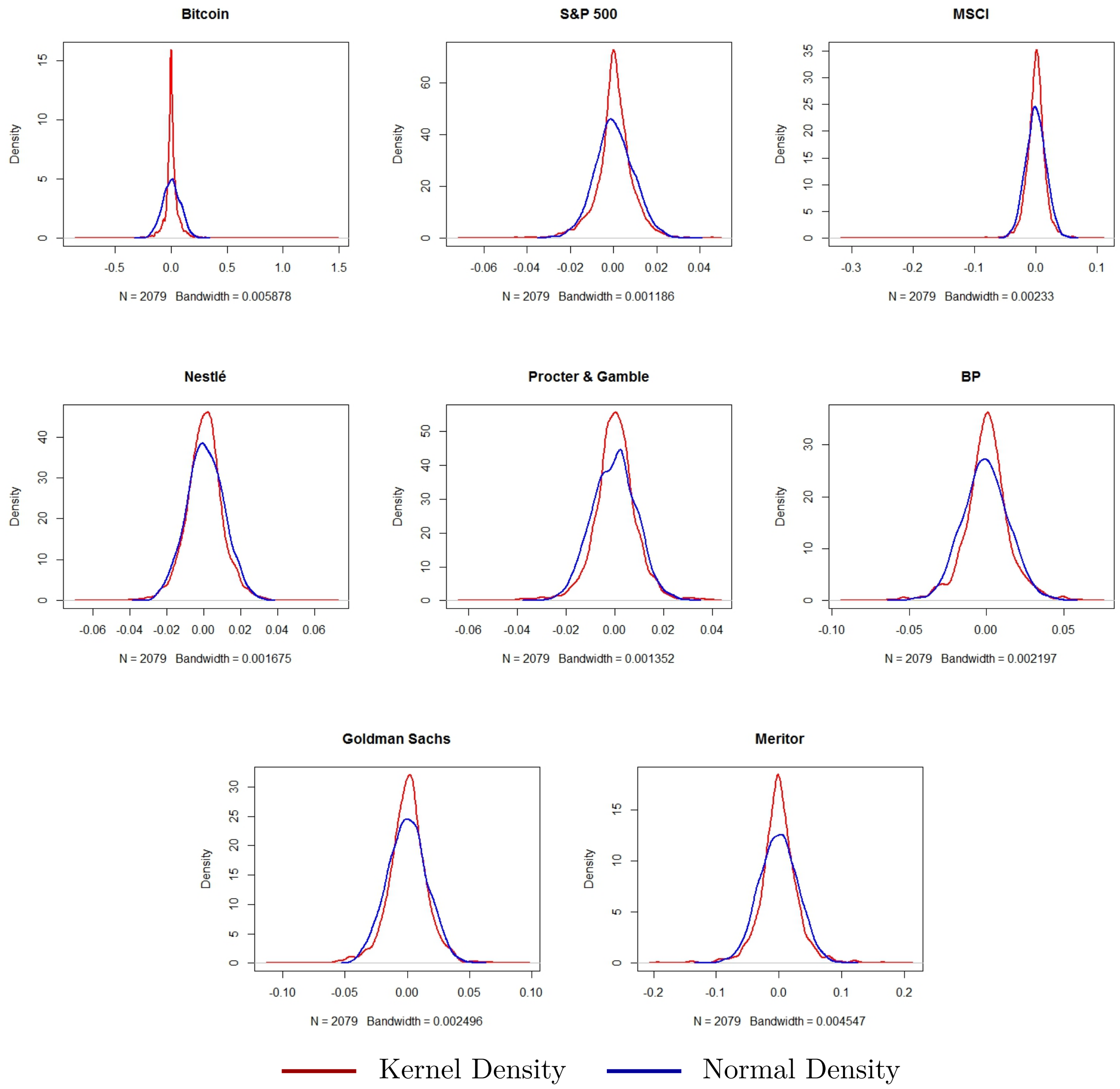


FIGURE 8
KERNEL DENSITY PLOTS

compared to stock/index return standard deviations on average. Besides, Bitcoin's return mean is more than 5 times higher than the stock/index return means. This can be explained by Bitcoin's remarkable price increase in the given period.

Neither the descriptive statistics of returns nor the statistical tests of normality suggest that the underlying return distributions are normally distributed. This is manifested in excess skewness and kurtosis compared to a normal distribution. In particular, Bitcoin's skewness of 2.591 indicates a relatively strong asymmetry of the distribution. Bitcoin's kurtosis indicates higher frequencies of outcomes at both the extreme negative and positive ends of the distribution. In other words, Bitcoin's return distribution has heavier tails compared to the stock/index return distributions implying a wider return spread of the former. Both it's higher skewness and kurtosis become apparent in the Kernel density plots. Finally, the normal Q-Q plots indicate non-normality, whereas the Student-t Q-Q plots indicate a good fit to all underlying series.

3.2 Correlation Analysis

3.2.1. Autocorrelation

It is well known that financial time series are characterized by autocorrelation (Alvarez-Ramirez and Rodriguez, 2018). That is to say, series are closely related to its previous values. To check for autocorrelation among the underlying return series, I calculate autocorrelation functions and test for serial correlation. I further create and interpret corresponding (partial) autocorrelation plots. This is important because (G)ARCH-type models are based on autoregressive behavior since past lagged values are included as independent variables in order to estimate return and volatility.

Under the assumption that the underlying series are at least weakly stationary, the correlation coefficient between the returns r_t and r_{t-l} is named the lag-l autocorrelation of r_t and is denoted by ρ_l in equation (9) (Tsay, 2010). Recall that stationarity was generated by taking the log return of the underlying series.

$$(9) \quad \rho_l = \frac{Cov(r_t, r_{t-l})}{\sqrt{Var(r_t)Var(r_{t-l})}} = \frac{Cov(r_t, r_{t-l})}{Var(r_t)},$$

where $Var(r_t) = Var(r_{t-l})$ due to the fact that the series are weakly stationary. Note that by definition $\rho_0 = 1$, $\rho_l = \rho_{-l}$, and $-1 \leq \rho_l \leq 1$.

The lag-l *sample* autocorrelation is denoted by $\hat{\rho}_l$ in equation (10).

$$(10) \quad \hat{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t - \bar{r})(r_{t-l} - \bar{r})}{\sum_{t=1}^{T}(r_t - \bar{r})^2}, \quad 0 \leq l < T - 1,$$

where the sample of returns is defined as $\{r_t\}_{t=1}^{T}$ and the sample mean is defined as $\bar{r} = \sum_{t=1}^{T} r_t / T$. Assuming that the sequence of $\{r_t\}$ is independent and identically distributed (iid) and that $E[r_t^2] < \infty$, $\hat{\rho}_l$ is asymptotically normally distributed with zero mean and variance $1/T$, given a positive integer l. In this case, $\hat{\rho}_l$ is a consistent estimate of ρ_l.

These conditions allow for hypothesis testing. To examine potential autocorrelation among the series, I use a Ljung-Box test with the null hypothesis $H_0 : \rho_1 = ... = \rho_m = 0$

against the alternative hypothesis $H_1 : \rho_l \neq 0$ for $l \in \{1,...,m\}$. In other words, I test H_0: the return series are independently distributed (i.e. the correlations in the population are 0) against the alternative hypothesis H_1: the return series are not independently distributed and exhibit serial correlation. The corresponding test statistic is (Tsay, 2010)

$$(11) \quad Q(m) = T(T+2)\sum_{l=1}^{m} \frac{\hat{\rho_l}^2}{T-l}.$$

Assuming that $\{r_t\}$ is an idd sequence, $Q(m)$ is asymptotically χ^2 distributed with m degrees of freedom. Hence, H_0 is rejected if $Q(m) > \chi^2_\alpha$. The latter stands for the $100(1-\alpha)^{\text{th}}$ percentile of a χ^2 distribution with m degrees of freedom.

The Ljung-Box test statistic results are displayed in the charts of Figure 9. Within each chart, the number of lags m are found on the x-axis and the corresponding p-value on the y-axis. The dashed horizontal line displays the 0.05 level of the p-value. Therefore, I fail to reject the H_0 at the significance level of $\alpha = 0.05$ if the $p-$value is above that line for any given lag.

The test result exhibits significant correlation for (the population of) the Bitcoin, S&P 500, and MSCI return series. The corresponding p-values indicate that the H_0 can be rejected at the $\alpha = 0.05$ level (except for the first lag of the Bitcoin and the second lag of the S&P 500 series). Similarly, the p-values of Nestlé are found to be close to 0.05 throughout lag $1-35$ indicating significant correlation of the series for the most part. In contrast, the Procter & Gamble chart clearly shows no significant correlation of the returns for any given lag indicating that this series is independently distributed.

The test results of BP are best described as ambiguous since the p-values in the corresponding chart seem to move in a wave-like pattern with only a few lags indicating significant correlation. The p-values of the Goldman Sachs returns are decreasing with higher lags exhibiting significant correlation from the 16^{th} lag onward, whereas the p-values of the Meritor returns are increasing with higher lags exhibiting significant correlation for only a few smaller lags.

Referring to equation (10), the statistics $\hat{\rho_l} = \hat{\rho_1}, \hat{\rho_2}, ..., \hat{\rho_m}$ is called the *sample autocorrelation function* (ACF). Within an ACF plot, the lag m is displayed on the x-axis and the corresponding autocorrelation $\hat{\rho_l}$ is displayed on the y-axis. The dashed horizontal

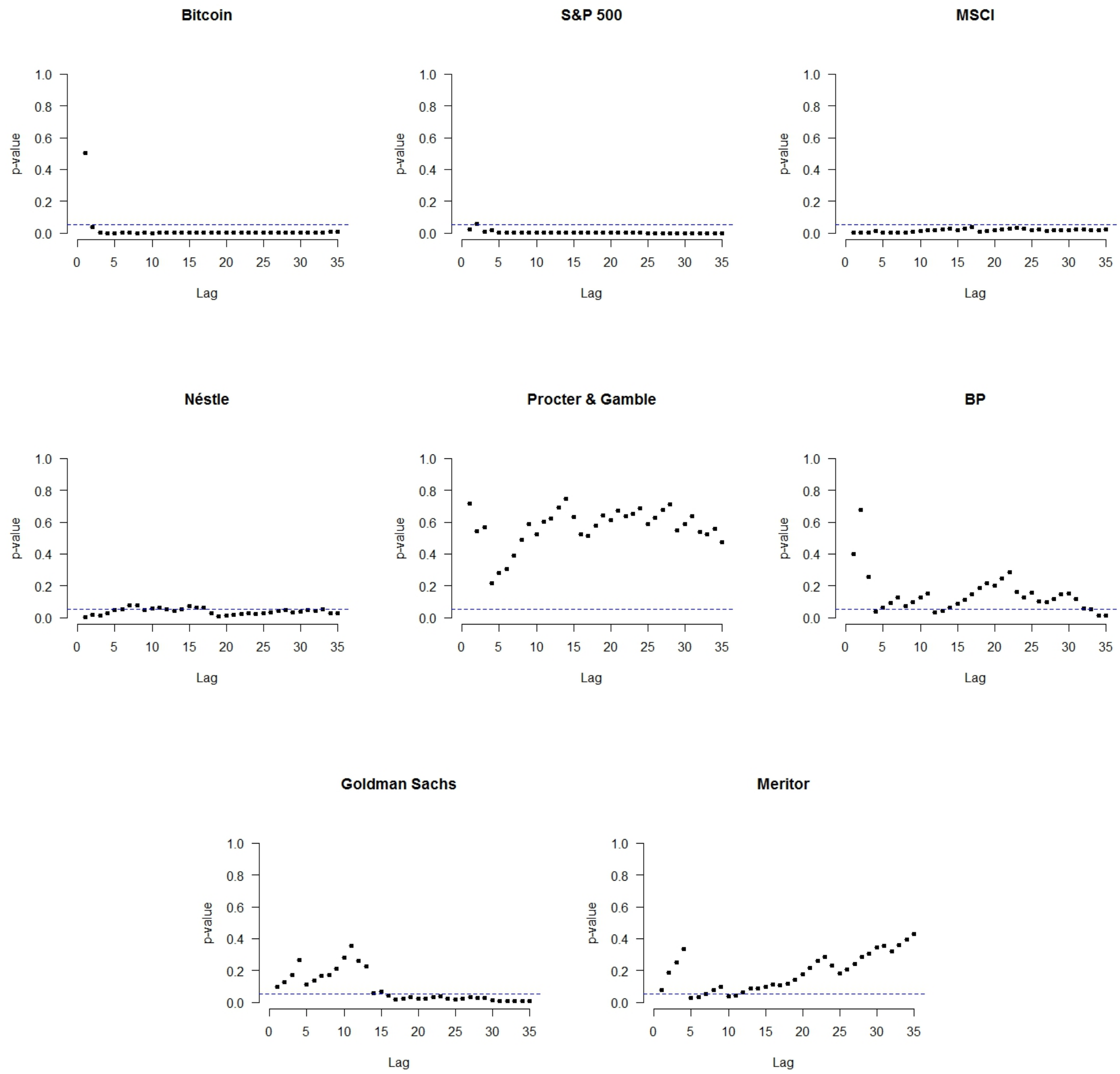


Figure 9
P-Values of Ljung-Box Test Statistic for the ACF

lines are bounds equal to $\pm 2\sqrt{T}$, where T denotes the length of the time series. Any autocorrelation greater than these bounds is significantly different from 0 at the $\alpha = 0.05$ significance level. According to Shumway and Stoffer (2017) this can be explained by the fact that $\hat{\rho}_l$, $l = 1, 2, ..., m$, follow a normal distribution in approximation with zero mean and standard deviation $\frac{1}{\sqrt{T}}$. Note that m has to be fixed and T sufficiently large. The peaks in $\hat{\rho}_l$ are said to be insignificant if they lay between the bounds $\pm 2\sqrt{T}$ (plus/minus twice the standard errors). 95% of the sample ACF spikes are expected to be within these bounds for a white noise series.

As illustrated in the ACF plots on the left in Figure 10, the Bitcoin return autocorrelation is significantly different from 0 for a time lag of 2, 3, 4, and 8 days. In addition, there are several larger spikes that are, however, not significant at the $\alpha = 0.05$ level. On the whole, this corresponds with the Ljung-Box test results that exhibited serial correlation for the series.

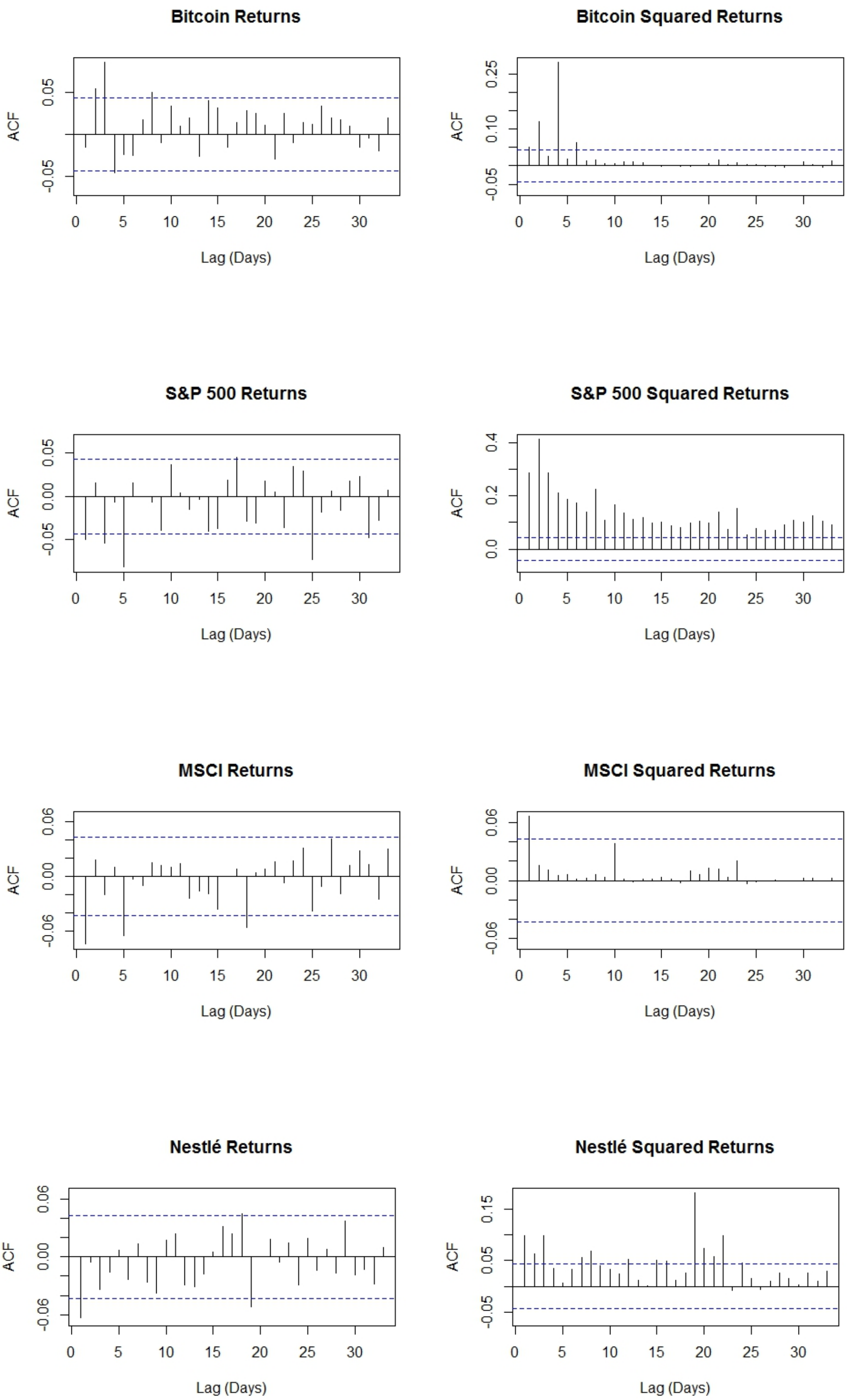
In contrast, the autocorrelation of Procter & Gamble is only significantly different from 0 for a time lag of 29 days. All other autocorrelations in the ACF plot are close to 0 which makes it virtually a white noise series. This is in line with the fact that the Ljung-Box test did not exhibit any significant autocorrelation for the series (see Figure 9).

The example of Goldman Sachs also illustrates the link between the Ljung-Box test results and the ACF plots. As can be seen in Figure 9, the Ljung-Box test indicates 0 correlation for the first 15 lags and significant serial correlation from the 16$^{\text{th}}$ lag onwards. This is reflected in the corresponding ACF plot from Figure 10 with larger and partially significant spikes occurring from lag 14 onwards.

Apart from Procter & Gamble, all ACF plots clearly suggest that the underlying return series are not serially independent. In fact, they are dependent (although the serial correlations are weak) and this dependence can be captured by volatility models which are covered in Section 4.

More importantly, the significant autocorrelation in all *squared* returns from Figure 10 imply the presence of conditional heteroscedasticity, i.e. autoregressive conditional heteroscedastic (ARCH) effects, for all series. In other words, the series are autoregressive in the second moment and hence ARCH models are an appropriate choice to model their volatilities.

This can also be seen in Figure 20 in the Appendix that shows the partial ACF (PACF) plots of the squared returns. In contrast to the ACF, the PACF considers the correlation between two single returns exclusively. For instance, the lag-3 PACF spike illustrates the added effect of the lag-3 return r_{t-3} to r_t, all else equal (Tsay, 2010). The effect can be estimated using the least-square method in a linear regression. The above explains why the PACF is more informative about the order of dependence for AR models compared to the ACF (Shumway and Stoffer, 2017). The big spikes in the PACF suggest that the returns are not serially independent and have ARCH effects.


Bitcoin Returns
Bitcoin Squared Returns
S&P 500 Returns
S&P 500 Squared Returns
MSCI Returns
MSCI Squared Returns
Nestlé Returns
Nestlé Squared Returns
ACF
Lag (Days)

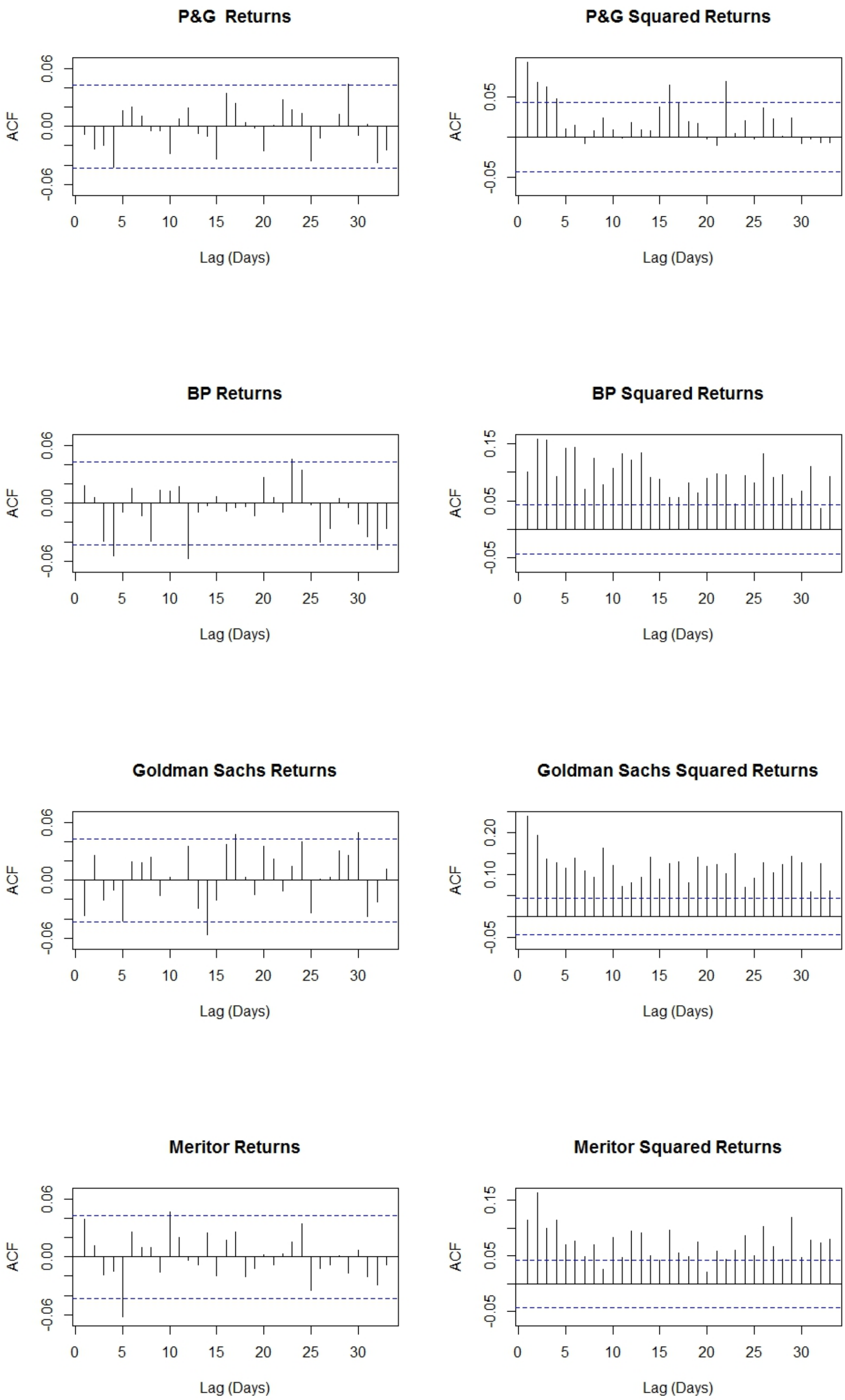


FIGURE 10
ACF PLOTS

Interestingly, the dependencies in the PACF of the squared Bitcoin and MSCI returns are not as long as the dependencies in the PACF of the other squared returns. This is also the case for the ACF of the squared returns. In fact, the (P)ACF of Bitcoin and MSCI fall off after 4 and 1 days, respectively. In comparison, the other series exhibit significant spikes in the (P)ACF up until 33 days, except Procter & Gamble. The latter exhibits relatively weak dependencies in the (P)ACF. GARCH estimates reflect this stylized fact and I will come back to this matter in Section 4.2.1..

The short dependencies in the squared returns of Bitcoin are in line with the results of Eom *et al.* (2019) who find negative evidence of volatility persistence in the Bitcoin return series between 2011 and 2017. The authors argue that this is caused by extreme price jumps.

Taken together, the significant autocorrelation of all squared returns suggests predictability of the return's volatility indicating the adequacy of the underlying series for (G)ARCH models.

3.2.2. ARCH effects

The ACF and PACF of the squared returns in Figure 10 and Figure 20 illustrate the presence of conditional heteroscedasticity and this is at least partially (i.e. for some lags) validated by a Ljung-Box test of the return series (except from Procter & Gamble). To further check for conditional heteroskedasticity, I use an *ARCH test*. The test result points to potential serial correlation among the innovations a_t.

A Ljung-Box test statistic $Q(m)$ can be applied to the series $\{a_t^2\}$ with the corresponding null hypthesis H_0 : the first m ACF lags of the series are equal to 0 and the alternative hypothesis H_1 : the first m ACF lags of the series are unequal to 0.

The Ljung-Box test statistics of the Bitcoin a_t^2 series exhibits strong ARCH effects with $Q(12) = 625.151$ and a p-value equal to 0. Consequently, I reject the null hypothesis and conclude that there is significant autocorrelation among the first 12 lags of the squared innovations a_t^2, i.e. the latter are conditionally heteroskedastic. This conclusion holds for different numbers of lags $(1 - 50)$.

Likewise, the Ljung-Box test statistics of the stock/index a_t^2 series exhibit ARCH effects for lag $1 - 50$, except the MSCI series. This can be seen in Figure 11 where the Ljung-Box test results for the squared innovations a_t^2 of the Bitcoin and MSCI return series are displayed. The lags (in days) are on the x-axis and the corresponding p-values on the y-axis.

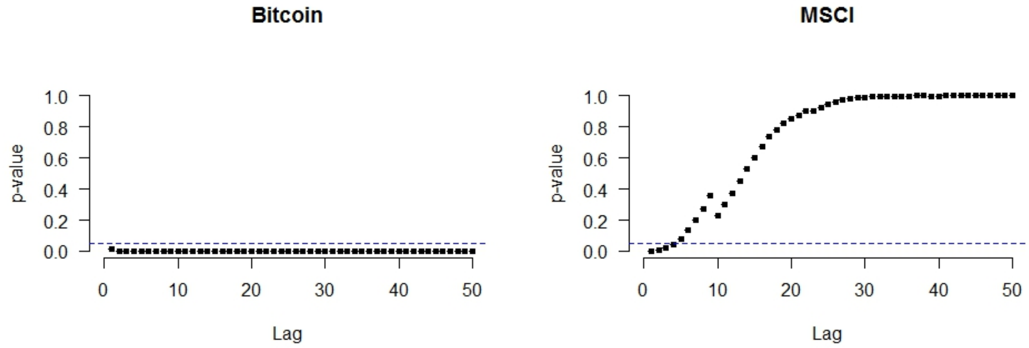


FIGURE 11
P-VALUES OF LJUNG-BOX TEST STATISTIC FOR THE SQUARED INNOVATIONS a_t^2

The Bitcoin chart on the left of the figure is representative for the test results of the other underlying time series. In fact, they exhibit the same charts with p-values close to 0, implying that the first $1-50$ lags of the squared innovations are conditionally heteroskedastic, i.e. there are strong ARCH effects. In contrast, the MSCI chart on the right of the figure implies conditionally heteroskedasticity only among the first $1-4$ lags and from the 5^{th} lag onwards the null hypothesis is rejected indicating no significant autocorrelation anymore - for unknown reasons and a more detailed analysis would go beyond the scope of this thesis. However, the MSCI series exhibits at least some ARCH effects.

3.2.3. Drivers of Autocorrelation in Returns

In terms of arbitrage, ACF with spikes outside the boundaries should not happen, because someone could make use of the fact that a return series is autocorrelated by trading on it. Next to volatility clustering, the momentum effect can explain the presence of autocorrelated returns. The effect states that assets, that outperform the market at present, also tend to have excess returns in the near future (Hens and Rieger, 2016). It is empirically proven, e.g. by Rouwenhorst (1997).

Investors can be divided into two groups, namely news watchers and chartists, i.e. momentum traders. Information from the news is assumed to move slowly (underreaction of prices) and this results in a particular price trend towards the „truth“ (Hong and Stein, 1997). The chartists traders identify the price trend and participate (buy/sell the asset), thereby boosting the current trend (overreaction of prices). After some time correction takes place and a reversal occurs.

Bitcoin traders both rely mostly on price and volume as indicators for Bitcoin's economic value (Detzel *et al.*, 2018) and are biased towards good news (Glaser *et al.*, 2014) implying limitations concerning their objectivity. This suggests that they can be mostly assigned to the group of momentum traders. Liu and Tsyvinski (2018) find significant momentum for daily Bitcoin returns between 2011 and 2018.

In this context, Eom *et al.* (2019) find that investor sentiment, i.e. the general mood of investors concerning a particular asset, can explain changes in Bitcoin's volatility. Interestingly, the Google Trend Index serves as a proxy for investor sentiment in their analysis. That is to say, Google searches can predict the change of Bitcoin's volatility in future days. Liu and Tsyvinski (2018) use the same proxy for investor sentiment and show that it can predict both positive and negative Bitcoin future returns. This supports existing empirical evidence that Bitcoin can be seen as an investment asset instead of as a currency since the volatility of investment assets is sensitive to investor's behavior by definition.

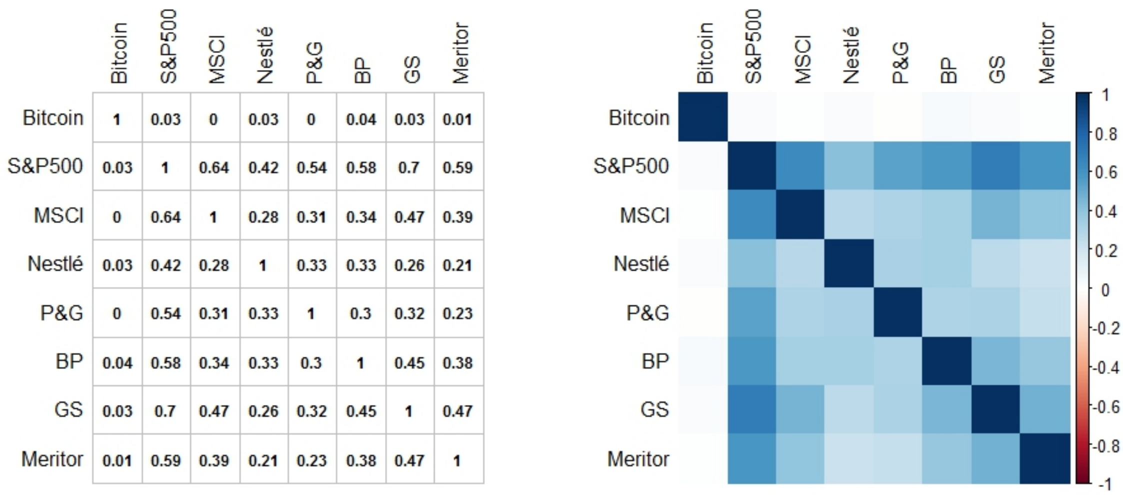
Another explanation for autocorrelated returns can be institutional trading. As a matter of fact, institutional traders do not acquire bigger orders of equity in a single day, but over multiple days - usually at the same time of the day. Murphy and Thirumalai (2017) find evidence that today's stock returns are highly predictive of future day's returns within a particular daytime. Arguably, this could explain why the majority of stock/index returns exhibits longer autocorrelations compared to the Bitcoin return; assuming that institutional investors do not acquire Bitcoin as systematic as the underlying stocks/index.

Bitcoin autocorrelation might be explained by illegal trades that are already mentioned in Section 2.2.1.. Individuals who are involved in criminal activities are dependent on Bitcoin as a medium of exchange or as a means for money laundering.

Last but not least, with respect to the definition of the autocorrelation function from equation (9), the ACF plots are only valid if the variance of the Bitcoin series is finite. As will be shown, the second moment of the Bitcoin return possibly does not exist. Therefore, interpretations of its ACF should be treated with caution.

3.2.4. Correlation according to Spearman and Kendall

Spearman and Kendall estimates are rank-based measures of correlation. In contrast to other measures of correlation such as the Pearson correlation coefficient, both methods do not rely on the assumption that the data is bivariate normally distributed; making them an appropriate choice for the underlying analysis.



	Bitcoin	S&P500	MSCI	Nestlé	P&G	BP	GS	Meritor
Bitcoin	1	0.03	0	0.03	0	0.04	0.03	0.01
S&P500	0.03	1	0.64	0.42	0.54	0.58	0.7	0.59
MSCI	0	0.64	1	0.28	0.31	0.34	0.47	0.39
Nestlé	0.03	0.42	0.28	1	0.33	0.33	0.26	0.21
P&G	0	0.54	0.31	0.33	1	0.3	0.32	0.23
BP	0.04	0.58	0.34	0.33	0.3	1	0.45	0.38
GS	0.03	0.7	0.47	0.26	0.32	0.45	1	0.47
Meritor	0.01	0.59	0.39	0.21	0.23	0.38	0.47	1

Figure 12
Spearman Correlation Coefficients

The results of the Spearman correlation coefficients are displayed in Figure 12.[14] Both correlation matrices show that the Spearman correlation coefficients between the Bitcoin and the stock/index returns are only marginally different from 0. They are ranging between 0 and 0.04 and this is indicated by the almost white shaded boxes in the right matrix of Figure 12. In comparison, the light blue shaded boxes in the same matrix

[14]They do not differ greatly from the Kendall correlation coefficients. Hence I refrain from displaying both of them.

indicate a higher correlation between the stock returns themselves. They are ranging between 0.21 and 0.48 which is, however, still classified as a low/moderate correlation.

It is not surprising that the S&P 500 exhibits a higher correlation to the stocks; it ranges from 0.42 to 0.7. The index incorporates the market capitalizations of the 500 largest US companies that have common stock listed on the NASDAQ or NYSE stock exchange (including Procter & Gamble, Goldman Sachs, and MSCI). Hence, it is a representation of the stock market which explains the greater magnitude of correlations compared to the correlations between the stock returns themselves.

Most importantly, the virtually non-existent correlation between the underlying Bitcoin and stock/index returns translate into a diversification potential for investors, i.e. potential losses of Bitcoin can be compensated by the stock/index returns, and vice versa because market swings do not affect both returns in the same way.

This is in line with the results of Burniske and White (2017) and Baur *et al.* (2018) who find that Bitcoin returns are uncorrelated with major asset classes such as stocks, bonds, commodities, energy, and fiat currencies (see Figure 19 in Appendix) - both in „normal" and in volatile market periods. My result confirms the conclusion drawn by Burniske and White (2017) who argue that Bitcoin's near 0 correlation to other asset classes is a vital condition for declaring it to a unique asset class.

4 (G)ARCH Modelling

First, let me recall some stylised facts regarding the volatility of daily stock/index returns. They are partially suggested by the descriptive analysis from above and are based on Tsay (2010).

i Volatility clustering: large (small) changes in returns are followed by large (small) changes in returns in either direction.

ii Volatility evolves over time and sudden changes are infrequent.

iii Volatility is stationary, i.e. it varies in a specific/fixed range.

iv Leverage effect, i.e. volatility changes differently according to a price increase or price drop.

It is noteworthy that an AR- or ARMA-process does not adequately model the stylized facts from above. In contrast, ARCH models can explain and capture these facts. This is important, because time-varying variances are vital for the implementation of portfolio allocation and risk measurements such as the Value at Risk (Cuthbertson and Nitzsche, 2004).

4.1 The ARCH Model

Given a return series $\{r_t\}_{t=1}^{T}$, the mean equation of the return is defined as (Tsay, 2010)

(12) $r_t = \mu_t + a_t,$

where μ_t describes the predictable component and a_t the unpredictable component that is assumed to be a heteroscedastic white noise process, i.e. $a_t \sim WN(0, \sigma_t^2)$. From equation (12), it follows that

(13) $a_t = r_t - \mu_t,$

where a_t describe the residuals of the mean equation (hereafter referred to as the innovation of r_t). The conditional variance of the innovation a_t is denoted by

(14) $Var(a_t|F_{t-1}) = \sigma_t^2,$

where F_{t-1} stands for the information available at time $t-1$, e.g. it incorporates all known prices $F_{t-1} = \{y_{t-1}, y_{t-2}, ...\}$. The conditional expected value of r_t is denoted by

$$(15)\quad E(r_t|F_{t-1}) = \mu_t.$$

The above implies

$$(16)\quad Var(r_t|F_{t-1}) = E[(r_t - \mu_t)^2|F_{t-1}] = E[a_t^2|F_{t-1}] = \sigma_t^2.$$

Therefore, the conditional variance of the return equals the conditional variance of the innovation in the model.

That being said, there are two key ideas inherent to the ARCH model. First, the sequence of innovations a_t is serially uncorrelated but dependent. Second, this dependence of a_t can be captured by a function of its own lagged and squared values. This is described by an ARCH(m) model in equation (17).

$$(17)\quad r_t = \mu_t + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2,$$

where the model paramters $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i > 0$, since σ_t^2 is a variance and positive by definition. Further, $\{\epsilon_t\}$ is assumed to be a sequence of iid standard normally distributed random variables, i.e. standard Gaussian white noise. The mean equation $r_t = \mu_t + a_t$ describes an AR(1) process.

It can be seen from the model that large squared innovations from the past $\{a_{t-i}^2\}_{i=1}^m$ indicate a large conditional variance σ_t^2 for today's innovation a_t (and return r_t). Hence, large innovations are more likely to be followed by large innovations than by small ones on expectation, and vice versa. This explains why ARCH models can capture volatility clustering that was suggested in above.

4.1.1. Properties of ARCH Models

I elaborate on the properties of ARCH models using the example of $m = 1$, i.e. an ARCH(1) model (Tsay, 2010).

$$(18)\quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2.$$

Under the assumption that ϵ_t is standard normally distributed with mean 0 and variance 1, the unconditional mean of a_t is equal to 0.

(19) $E(a_t) = E[E(a_t|F_{t-1})] = E[\sigma_t E(\epsilon_t)] = 0.$

This shows that an ARCH process gives a martingale difference sequence and hence arbitrage is not possible. The unconditional variance of a_t is defined as

$$\begin{aligned} Var(a_t) &= E[(a_t - E(a_t))^2] = E(a_t^2) = E[E(a_t^2|F_{t-1})] \\ &= E[\alpha_0 + \alpha_1 a_{t-1}^2] = \alpha_0 + \alpha_1 E(a_{t-1}^2). \end{aligned} \tag{20}$$

Since a_t is assumed to be a stationary process, we have $Var(a_t) = Var(a_{t-1}) = E(a_{t-1}^2)$ and it follows that $Var(a_t) = \alpha_0 + \alpha_1 Var(a_t)$ which implies $Var(a_t) = \alpha_0/(1-\alpha_1)$. To allow higher order moments of a_t to exist, α_1 must satisfy additional constraints. Assuming that ϵ_t is normally distributed, the fourth conditional moment of a_t is equal to

(21) $E(a_t^4|F_{t-1}) = 3[E(a_t^2|F_{t-1})]^2 = 3(\alpha_0 + \alpha_1 a_{t-1}^2)^2.$

It follows that

(22) $E(a_t^4) = E[E(a_t^2|F_{t-1})] = 3E(\alpha_0 + \alpha_1 a_{t-1}^2)^2 = 3E(\alpha_0^2 + 2\alpha_0\alpha_1 a_{t-1}^2 + \alpha_1^2 a_{t-1}^4).$

In case the fourth order of a_t is stationary, say $E(a_t^4) = m_4$, we have

(23) $m_4 = 3[\alpha_0^2 + 2\alpha_0\alpha_1 Var(a_t) + \alpha_1^2 m_4] = 3\alpha_0^2\left(1 + 2\frac{\alpha_1}{1-\alpha_1}\right) + 3\alpha_1^2 m_4.$

And hence

(24) $m_4 = \dfrac{3\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}.$

There are two important suggestions of this result. First, $1 - 3\alpha_1^2 > 0$, since the fourth moment a_t is greater than 0. This implies $0 \leq \alpha_1^2 < 1/3$. Second, as can be seen in equation (25), the unconditional kurtosis of a_t is larger than 3 which implies that a_t has heavier tails than a normally distributed process.

(25) $\dfrac{E(a_t^4)}{[Var(a_t)]^2} = 3\dfrac{\alpha_0^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)} \times \dfrac{(1-\alpha_1)^2}{\alpha_0^2} = 3\dfrac{1-\alpha_1^2}{1-3\alpha_1^2} > 3.$

Therefore, the innovation a_t of a conditional ARCH(1) model produces more outliers than an iid series following a normal distribution. However, it does not capture the excess kurtosis of the underlying series that was demonstrated in the descriptive analysis from Section 3.

4.1.2. Estimation of ARCH Models

Keeping the assumption that ϵ_t from equation (17) is following a normal distribution, the ARCH(m) likelihood function is defined as

$$
\begin{aligned}
f(a_1, ..., a_T|\alpha) &= f(a_T|F_{T-1}) f(a_{T-1}|F_{T-2}) ... f(a_{m+1}|F_m) f(a_1, ..., a_m|\alpha) \\
&= \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{a_t^2}{2\sigma_t^2}\right) \times f(a_1, ..., a_m|\alpha),
\end{aligned} \tag{26}
$$

where α is equal to $(\alpha_0, \alpha_1, ..., \alpha_m)'$ and the joint probability density function of $a_1, ..., a_m$ is denoted by $f(a_1, ..., a_m|\alpha)$. The latter can be dropped when the sample size T is sufficiently large which results in the conditional-likelihood function

$$
f(a_{m+1}, ..., a_T|\alpha, a_1, ..., a_m) = \prod_{t=m+1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{a_t^2}{2\sigma_t^2}\right), \tag{27}
$$

where σ_t^2 can be estimated by the maximum-likelihood method. I take the logarithm of the function to simplify the maximization which results in the conditional log-likelihood function

$$
l(a_{m+1}, ..., a_T|\alpha, a_1, ..., a_m) = \sum_{t=m+1}^{T} \left[-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{a_t^2}{\sigma_t^2}\right]. \tag{28}
$$

Since there are no parameters in $\ln(2\pi)$, the function can be further simplified to

$$
l(a_{m+1}, ..., a_T|\alpha, a_1, ..., a_m) = -\sum_{t=m+1}^{T} \left[\frac{1}{2}\ln(\sigma_t^2) + \frac{1}{2}\frac{a_t^2}{\sigma_t^2}\right], \tag{29}
$$

The maximization process described above differs according to the assumptions about the sequence $\{\epsilon_t\}$. For instance, assuming that it follows a standardized Student-t distribution with heavy tails. The latter effectively results in both the best ARCH and GARCH model fits for the underlying series. It is noteworthy that this assumption also exhibits a superior model fit compared to a skewed Student-t assumption.

4.1.3. Model Checking

I consider two different information criteria that are informative about the goodness of fit of an estimated ARCH model, namely the Akaike Information Criterion (AIC) and the

Bayesian Infomation Criterion (BIC). The AIC is defined as (Shumway and Stoffer, 2017)

$$(30)\quad \text{AIC} = \log\, \hat{\sigma}_k^2 + \frac{T+2k}{T},$$

where T denotes the sample size and k the number regression coefficients in the model. Further, $\hat{\sigma}_k^2 = \frac{SSE(k)}{n}$ is the maximum likelihood estimator for the variance with $SSE(k)$ as the sum of squared residuals for a model with k regression coefficients. The BIC is defined as (Shumway and Stoffer, 2017)

$$(31)\quad \text{BIC} = \log\, \hat{\sigma}_k^2 + \frac{k\, \log\, T}{T}.$$

The minimum AIC/BIC specifies the best model. It is apparent that the two criteria only differ with respect to the second expression, i.e. the „penalty term". This name derives from the fact that it would be rational to minimize log $\hat{\sigma}_k^2$ by choosing a larger k ($\hat{\sigma}_k^2$ decreases monotonically in k). In sharp contrast, the penalty term increases in k, i.e. it penalizes the error variance proportionally to the number of parameters k. This introduces a trade-off to the choice of the latter.

The penalty term of the BIC penalizes model complexity more heavily compared to the penalty term of the AIC. This explains the tendency of the BIC to pick models with a smaller number of parameters k. The BIC is the better reference point for large samples, whereas the AIC is the better reference point for small samples with a relatively large number of parameters k (Shumway and Stoffer, 2017). For more discussion on the AIC/BIC, see Burnham and Anderson (2004).

The standardized innovations

$$(32)\quad \tilde{a}_t = \frac{a_t}{\sigma_t}$$

represent a sequence of iid random variables with mean 0 and variance 1 if the model is correctly specified. In other words, the sequence of the innovations should not be serially correlated, conditional heteroskedastic, or nonlinearly dependent (Zivot, 2009). To check for the latter, I perform Ljung-Box tests of the sequences $\{\tilde{a}_t\}$ and $\{\tilde{a}_t{}^2\}$. The results give information about the adequacy of the mean equation $r_t = \mu_t + a_t$ and the volatility equation $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_m a_{t-m}^2$, respectively. The series are independently distributed under the H_0, whereas the series are not independently distributed and exhibit serial correlation under the H_1. Hence, the corresponding p-values of a Ljung-Box test shall be sufficiently large to ensure the validity of the estimated model.

4.1.4. Application of ARCH Models

Table 2 displays the results of the estimated ARCH models. Since the empirical distributions of the underlying returns exhibit heavy tails and the Student-t distribution gives a better fit compared to a normal distribution (see Q-Q plots from Figure 6 and Figure 7), ϵ is assumed to follow a standardized Student-t distribution in each case. This effectively results in a better model fit, i.e. lower AIC/BIC, compared to the normality assumption.

The shape of the model is estimated together with the other model parameters. In the given case of the Students-t distribution, it corresponds to the (optimal) number of degrees of freedom of the ARCH process. Notably, since the Students-t distribution's shape is determined by its degrees of freedom, the shape of the model also corresponds to the *tail index* of the distribution (Horváth and Šopov, 2016). In Section 5, I will compare the implicit tail behavior of the (G)ARCH processes with Hill estimates.

The estimated drift parameter $\hat{\alpha_0}$ of the volatility equation is statistically significant for all series. Likewise, the estimated ARCH parameters $\{\hat{\alpha_i}\}_{i=1}^{m}$, measuring the effect of the squared innovations on the variance, are significant in all volatility equations. According to the Ljung-Box tests of the (squared) standardized innovations, almost all volatility and mean equations are adequate. Exceptions are the mean equations of Bitcoin and Nestlé (Q(20)) and the volatility equations of BP, Goldman Sachs, and Meritor (Q(20)).

Since there are significant ARCH effects in the series, the PACF of a_t^2 is an appropriate tool to determine the order m of the model (Tsay, 2010).

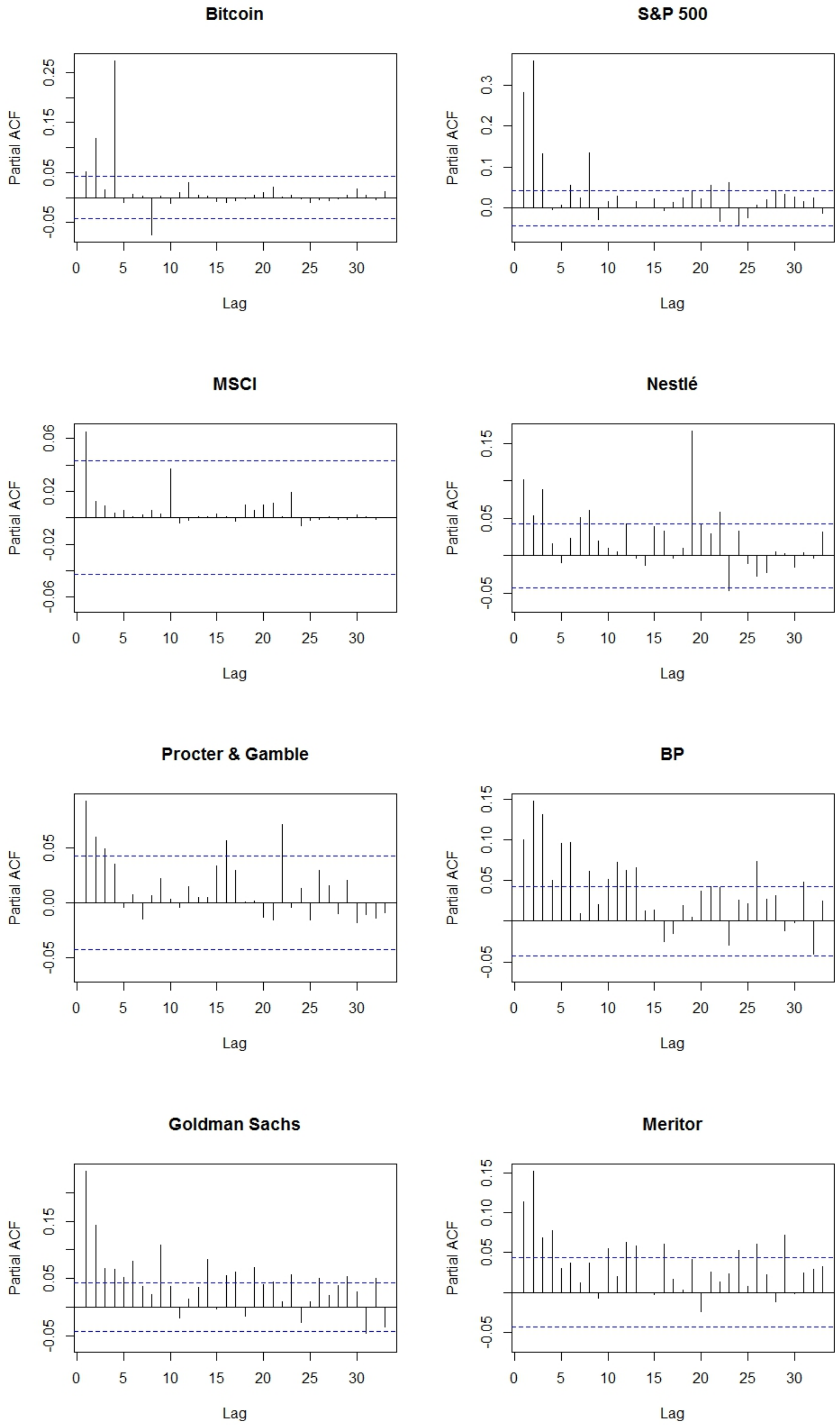


Figure 13
PACF of the Squared Innovations a_t^2

Referring to Figure 13, the PACF of the a_t^2 Bitcoin series is falling off a few times within the first 10 lags. Therefore, I consider several model specifications. It turns out that an ARCH(7) model generates the lowest information criterion statistics, particularly AIC$= -3.343$ and BIC$= -3.315$. Besides, all estimates including $\hat{\mu}$ and $\hat{\alpha_0}, ..., \hat{\alpha_7}$ are at least significant at the 10% level. I specify the model accordingly.

$$(33) \quad r_t = \mu + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + ... + \alpha_7 a_{t-7}^2.$$

The resulting fitted model is

$$(34) \quad \begin{aligned} r_t = 0.002 + a_t, \quad \sigma_t^2 = 0.001 + 1a_{t-1}^2 + 0.845a_{t-2}^2 - 0.535a_{t-3}^2 - 0.216a_{t-4}^2 + 0.508 \\ a_{t-5}^2 + 0.210a_{t-6} + 0.306a_{t-7}. \end{aligned}$$

The Ljung-Box test statistic (see equation (11)) of the standardized innovations gives $Q(20) = 61.824$, i.e. lag 20, with a p-value close to 0 indicating that the mean equation is not adequate at the 0.1% significance level.[15] The Ljung-Box test statistic of the squared standardized innovations gives $Q(20) = 9.049$ with a p-value of 0.982 indicating that the volatility equation is adequate. The restriction $0 \leq \alpha_1 < 1$ is violated since $\hat{\alpha_1} = 1$ and this is causing the unconditional variance $0.001/1 - (\hat{\alpha_1} + \hat{\alpha_2} + ... + \hat{\alpha_7})$ to be negative, i.e. non-existent. Table 7 in the Appendix shows that this is also the case for the Bitcoin series with all available observations.

Similar to the Bitcoin series, the PACF of the a_t^2 S&P 500 series falls off a few times within the first 10 lags (see Figure 13), therefore I consider several model specifications. Under the condition that all parameter estimates are significant, an ARCH(6) model generates the lowest AIC/BIC ($-6.937/-6.912$). Moreover, both the mean and volatility equation is adequate according to the Ljung-Box test statistics of the standardized innovations. Therefore, the ARCH(6) model is sufficient for describing the conditional heteroscedasticity of the S&P 500 return data. The expected daily return for the S&P 500 is about 0.1%. The unconditional variance of r_t is $0.00002/(1 - 0.145 - 0.225 - 0.167 - 0.135 - 0.075 - 0.113) = 0.00002/0.14 \approx 0.0001$.

Table 3 summarizes the expected return and unconditional variance of each return series according to the ARCH estimates. Note that the mean equations of Bitcoin and Nestlé,

[15] According to Shumway and Stoffer (2017), the number of lags are chosen arbitrarily. However, they are typically set to $m = 20$

Table 2
ARCH Estimates

	Bitcoin	S&P 500	MSCI	Nestlé	P&G	BP	GS	Meritor
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\hat{\mu}$	0.002***	0.001***	0.001***	0.0003*	0.0004**	0.001*	0.001*	0.0001
	(0.001)	(0.0001)	(0.0003)	(0.0002)	(0.0002)	(0.0003)	(0.0003)	(0.001)
$\hat{\alpha}_0$	0.001***	0.00002***	0.0001***	0.0001***	0.0001***	0.0001***	0.0001***	0.001***
	(0.0003)	(0.00000)	(0.00001)	(0.00001)	(0.00000)	(0.00001)	(0.00001)	(0.0001)
$\hat{\alpha}_1$	1.000***	0.145***	0.255***	0.082**	0.172***	0.041**	0.155***	0.191***
	(0.318)	(0.038)	(0.051)	(0.032)	(0.044)	(0.020)	(0.038)	(0.050)
$\hat{\alpha}_2$	0.845***	0.225***	0.132***	0.091***	0.124***	0.142***	0.165***	0.160***
	(0.314)	(0.045)	(0.042)	(0.032)	(0.040)	(0.033)	(0.043)	(0.045)
$\hat{\alpha}_3$	0.535**	0.167***	0.161***	0.087**	0.058*	0.109***	0.058*	0.134***
	(0.212)	(0.042)	(0.041)	(0.036)	(0.032)	(0.030)	(0.031)	(0.042)
$\hat{\alpha}_4$	0.216*	0.135***				0.051*	0.135***	
	(0.124)	(0.044)				(0.026)	(0.037)	
$\hat{\alpha}_5$	0.508**	0.075**				0.079***		
	(0.207)	(0.032)				(0.026)		
$\hat{\alpha}_6$	0.210*	0.113***				0.135***		
	(0.115)	(0.035)				(0.030)		
$\hat{\alpha}_7$	0.306**							
	(0.145)							
shape	2.232***	4.737***	4.618***	5.644***	4.415***	4.513***	5.692***	3.943***
	(0.083)	(0.533)	(0.472)	(0.686)	(0.439)	(0.473)	(0.720)	(0.353)
Observations	2,079	2,079	2,079	2,079	2,079	2,079	2,079	2,079
Log Likelihood	−3,484.648	−7,219.571	−5,962.236	−6,690.073	−7,024.393	−5,921.623	−5,836.987	−4,512.960
AIC	−3.343	−6.937	−5.730	−6.430	−6.752	−5.689	−5.608	−4.336
BIC	−3.315	−6.912	−5.714	−6.414	−6.735	−5.667	−5.589	−4.319

Note: *p<0.1; **p<0.05; ***p<0.01

as well as the volatility equations of BP, Goldman Sachs, and Meritor, are not adequate according to the Ljung-Box test results. The expected Bitcoin return is more than twice as high as the stock/index expected returns on average. According to the model estimates, Bitcoin's unconditional variance is not defined. The stock/index unconditional variances range between 0.0001 and 0.002.

Table 3
ARCH Results

	Expected Return	Unconditional Variance
Bitcoin	0.2%	/
S&P 500	0.1%	0.0001
MSCI	0.1%	0.0002
Nestle	0.03%	0.0001
P&G	0.04%	0.0001
BP	0.1%	0.0002
GS	0.1%	0.0002
Meritor	0.01%	0.002

It must be considered that the ARCH model does not take the leverage effect into account. For instance, the model also does not distinguish between positive and negative innovations a_t because both affect the volatility in the same way by being squared and this might cause a bad fit of the underlying return series. The leverage effect mentioned in Section 2.2.2. suggests that this does not properly reflect reality. Glaser *et al.* (2014) provide evidence that the Bitcoin return, in particular, is asymmetrically affected by good and bad news. However, (G)ARCH models capturing this asymmetric behavior (EGARCH, APARCH, GJR-GARCH) do not result in significant parameter estimates for the Bitcoin series and hence are not considered in my analysis.

4.2 The GARCH Model

Bollerslev (1986) developed the ARCH model and named it GARCH. In contrast to ARCH models, GARCH models additionally describe the conditional variance of the return as a function of its own lagged values. This results in fewer parameters needed to describe the volatility of returns adequately.

The innovation a_t follows a GARCH(m,s) model if (Tsay, 2010)

$$(35)\quad r_t = \mu_t + a_t, \quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2,$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{max(m,s)}(\alpha_i + \beta_i) < 1$. The last constraint ensures that the conditional variance of a_t/r_t (σ_t^2) evolves over time and the unconditional variance of r_t is finite, i.e. the GARCH model has a stationary solution. The unconditional variance of a_t/r_t is defined as

$$(36)\quad E(a_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{max(m,s)}(\alpha_i + \beta_i)}$$

Just like in the ARCH model, the sequence of $\{\epsilon_t\}$ is iid with mean 0 and variance 1, e.g. a standard normal or a standardized Student-t distribution. The α_i are referred to be the ARCH parameters, whereas the β_j are referred to be the GARCH parameters. Note that if $s = 0$, the model simplifies to an ARCH(m) model.

I specify a GARCH$(1,1)$ model to elaborate on the properties of GARCH models.

$$(37)\quad a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad 0 \leq \alpha_1, \beta_1 \leq 1, (\alpha_1 + \beta_1) < 1.$$

It is apparent from the model that large a_{t-1}^2/σ_{t-1}^2 indicate a large conditional variance σ_t^2, i.e. large innovations are more likely to be followed by large innovations than by small ones on expectation, and vice versa. Hence, GARCH models can capture volatility clustering mentioned in Section 3 just like ARCH models. Second, the GARCH(1,1) distribution also exhibits heavy tails compared to a normal distribution. However, the model also does not distinguish between negative and positive shocks.

4.2.1. Application of GARCH Models

Table 4 displays the results of the estimated GARCH models.[16] The sequences of ϵ_t are assumed to follow a standardized Student-t distribution resulting in a better fit compared to other distributions. The estimated drift parameter $\hat{\alpha_0}$ of the volatility equation is statistically significant for all series, except BP. The estimated ARCH parameter $\hat{\alpha_1}$ and

[16]Note that 0.00000 should not be interpreted as an absolute 0. The first five decimal places are simply equal to 0.

the estimated GARCH parameter $\hat{\beta}_1$ are significant in all volatility equations at least at the 5% significance level.

Note that model checking described in Section 4.1.3. also applies to GARCH models. For daily returns, model selection according to AIC/BIC typically results in large values of m for ARCH(m) models and low values of m/s for GARCH(m,s) models with $m,s \leq 2$ (Zivot, 2009; Tsay, 2010). This is in line with my results because a GARCH$(1,1)$ turns out to be the best model fit in each case. According to the Ljung-Box tests of the (squared) standardized innovations, almost all volatility and mean equations are adequate. Exceptions are the mean equations of Bitcoin and MSCI (Q(20)), and the volatility equation of Goldman Sachs.

According to AIC/BIC $(-3.354/-3.341)$, the GARCH$(1,1)$ model is the most appropriate choice for the Bitcoin return series. I specify the model according to equation (37) and the corresponding fitted model is

$$(38) \quad r_t = 0.002 + a_t, \quad \sigma_t^2 = 0.0002 + 1a_{t-1}^2 + 0.744\sigma_{t-1}^2.$$

The fitted model exhibits significant estimates at the 5% level. But the condition $\alpha_1 + \beta_1 < 1$ is violated since $\hat{\alpha}_1 + \hat{\beta}_1 = 1 + 0.744 = 1.744$ implying an undefined unconditional variance $\alpha_0/1-(\alpha_1+\beta_1)$. This points to a non-stationary and explosive series since the conditional variance is unbounded (Cuthbertson and Nitzsche, 2004). However, according to Nelson (1990) and Groenendijk *et al.* (1995), the stochastic process is still stationary and this will be further discussed in Section 5.1.

The Ljung-Box test statistic (see equation (11)) of the standardized innovations gives $Q(20) = 75.349$ with a p-value close to 0 indicating that the mean equation is not adequate at the 0.01% significance level. The Ljung-Box test statistic of the squared standardized innovations gives $Q(20) = 6.159$ with a p-value close to 1 indicating that the volatility equation is adequate. Table 8 shows that the estimates do not differ greatly from the Bitcoin series covering all observations.

The extended ACF (Tsay and Tiao, 1984) of the Bitcoin return indicates the adequacy of an ARMA$(2,3)$ model, but its combination with a GARCH$(1,1)$ model results in not applicable estimators. This raises the question of whether there are better models for fitting the underlying Bitcoin return data that would imply more reliable results.

Although the GARCH$(1,1)$ is hard to outperform (Hansen and Lunde, 2005; Zivot, 2009), Hansen and Lunde (2005) suggest an APARCH model for return series, because this model captures the leverage effect which distinguishes it from a regular (G)ARCH models.

Chan *et al.* (2017) find that the daily Bitcoin returns are best described by a generalized hyperbolic distribution and hence the authors suggest to use this distribution for the innovation processes in GARCH modeling. Chu *et al.* (2017) find that IGARCH$(1,1)$ model with normally distributed innovations gives the best fit (lowest AIC/BIC) to the Bitcoin data between June 2014 and May 2017. The authors compared the fit of 12 different models to the data including GARCH, APARCH, and EGARCH.

Katsiampa (2017) identifies an AR(1)-CGARCH(1,1) model as the best fit to the Bitcoin return data within a time frame ranging from July 2010 to October 2016. According to the author, this model specification minimizes information criteria AIC/BIC. It shall also exhibit significant parameter estimates and appropriate mean/volatility equations. The goodness of fit of this model specification highlights the importance of including both a short- and long-run part of the conditional variance.

However, model estimation according to the suggestions from above neither results in a significantly lower AIC/BIC compared to a GARCH$(1,1)$ model nor in an adequate mean equation. Moreover, the models partially exhibit insignificant parameter estimates for the Bitcoin series. In contrast to the cited papers, the underlying Bitcoin return data ranges from June 2010 to October 2018 and thus covers a more extended period. This might explain why the suggestions do not improve the model fit in the underlying analysis. Most importantly, the estimated parameters of the alternative models also imply an undefined unconditional variance of the Bitcoin series.

I provide more detailed information about the fitting process of the stock/index series by the examples of S&P 500 and Nestlé. The ACF of the S&P 500 return series in Figure 10 shows significant correlations at lag $1-3$. The PACF of the squared returns in Figure 20 shows strong linear dependence (key feature). The ACF of the standardized innovations $\tilde{a}_t$ and the squared standardized innovations $\tilde{a}_t{}^2$ in Figure 21 do not suggest any serial correlation/conditional heteroscedasticity in the standardized residual series for the most part. This is in line with the Ljung-Box test results, and the model seems to be adequate in describing the linear dependence in the return and volatility series except from $Q(20) = 28.681$ of $\tilde{a_t}$ with a p-value of 0.094. The implied unconditional variance of r_t from the volatility equation is $0.000003/1-(0.175+0.811) = 0.0002$. The expected daily return is about 0.1%.

For the Nestlé return series, a GARCH$(1,2)$ model generated the lowest AIC$=-6.383$, whereas a GARCH$(1,1)$ model generates the lowest BIC$= -6.373$ under the assumption that ϵ_t follows an iid normal distribution. Since the BIC is the superior reference point for larger samples (Shumway and Stoffer, 2017), I choose GARCH$(1,1)$. The model exhibits significant estimators at the 5% significance level. The Ljung-Box test results for the standardized innovations $\tilde{a_t}$ and squared standardized innovations $\tilde{a_t}^2$ indicate the adequacy of the mean and volatility equation, respectively. This can also be seen in Figure 21 where the ACF plots of the (squared) standardized innovations do not suggest any conditional heteroscedasticity.

Under the assumption that ϵ_t a follow Student-t distribution, both the AIC$= -6.441$ and BIC$= -6.427$ are minimized by a GARCH$(1,1)$ model and all estimates are significant at the 5% level. It is noteworthy that other distribution assumptions such as the skewed Student-t distribution do not improve the model fit for the stock/index return series. According to the model estimates, the unconditional variance of r_t is $0.000002/(1-0.025-0.959) = 0.0001$. The expected daily return for the Nestlé stock is around 0.03%.

Recall that the ARCH parameter α_1 measures the effect of the squared innovations on the variance. The estimated effect is relatively similar for the stock/index series and ranges between 0.025 (Nestlé) and 0.175 (MSCI). In sharp contrast, the estimated $\hat{\alpha_1}$ value for the Bitcoin series is equal to 1. Concerning the estimated GARCH parameters $\hat{\beta_1}$, there are no such considerable differences between the series. They range between 0.733 (Procter & Gamble) and 0.959 (Nestlé).

However, the GARCH parameters reflect the relatively weak and short dependencies seen in the (P)ACF plots of Bitcoin, MSCI, and Procter & Gamble. The estimated $\hat{\beta_1}$ values of the latter are 0.744, 0.755, and 0.733, respectively. This points to a lower persistency in volatility of the three series indicating a lower relevance of today's variance

Table 4
GARCH Estimates

	Bitcoin	S&P 500	MSCI	Nestlé	P&G	BP	GS	Meritor
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\hat{\mu}$	0.002***	0.001***	0.001***	0.0003*	0.0004**	0.001**	0.001**	0.0003
	(0.001)	(0.0001)	(0.0003)	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.001)
$\hat{\alpha_0}$	0.0002**	0.00000***	0.00002***	0.00000**	0.00001**	0.00000	0.00000**	0.00001**
	(0.0001)	(0.00000)	(0.00001)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
$\hat{\alpha_1}$	1.000***	0.175***	0.159***	0.025***	0.136***	0.037***	0.064***	0.040***
	(0.386)	(0.028)	(0.036)	(0.008)	(0.038)	(0.007)	(0.016)	(0.010)
$\hat{\beta_1}$	0.744***	0.811***	0.755***	0.959***	0.733***	0.961***	0.918***	0.950***
	(0.024)	(0.027)	(0.053)	(0.014)	(0.082)	(0.007)	(0.022)	(0.014)
shape	2.200***	4.893***	4.935***	5.985***	4.397***	5.175***	6.187***	4.301***
	(0.087)	(0.554)	(0.520)	(0.752)	(0.432)	(0.566)	(0.824)	(0.394)
Observations	2,079	2,079	2,079	2,079	2,079	2,079	2,079	2,079
Log Likelihood	−3,491.718	−7,226.271	−5,971.155	−6,700.291	−7,030.163	−6,035.367	−5,864.020	−4,539.508
AIC	−3.354	−6.947	−5.739	−6.441	−6.758	−5.801	−5.636	−4.362
BIC	−3.341	−6.933	−5.726	−6.427	−6.745	−5.788	−5.623	−4.349

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5
GARCH Results

	Expected Return	Unconditional Variance
Bitcoin	0.2%	/
S&P 500	0.1%	0.0002
MSCI	0.1%	0.0002
Nestle	0.03%	0.0001
P&G	0.04%	0.00008
BP	0.1%	0.0003
GS	0.1%	0.0003
Meritor	0.04%	0.001

to the conditional variance in the future. In contrast, the estimates of the other series imply a higher persistency in volatility which is in line with the P(ACF) plots.

For instance, referring to the estimates of S&P 500, $\hat{\alpha_1} + \hat{\beta_1} = 0.986$ is close to 1 which points to highly persistent shocks ($\hat{\alpha_1} + \hat{\beta_1}$) to the variance and basically make the model an IGARCH$(1,1)$ model (Cuthbertson and Nitzsche, 2004). This is referred to a GARCH process integrated in variance or a GARCH process with persistence (Bollerslev, 1986). In other words, today's information is relevant to the conditional variance of all future periods. This also applies to the coefficients of Nestlé (0.984), BP (0.998), Goldman Sachs (0.982), and Meritor (0.990).

Table 5 shows the expected return and unconditional variance of each return series according to the GARCH estimates. Note that the mean equations of Bitcoin and MSCI(Q(20)), as well as the volatility equation of Goldman Sachs, are not adequate according to the Ljung-Box test results. However, it becomes evident that the expected Bitcoin return is more than twice as high as the stock/index expected returns on average. The model estimates imply that Bitcoin's unconditional variance is not defined, whereas the stock/index unconditional variances range between 0.00008 and 0.001. There are no considerable differences between the expected returns and unconditional variances of the ARCH and GARCH models from Tables 3 and 5, respectively.

5 Extreme Value Theory

The extreme value theory is concerned with modeling the asymptotic characteristics of return distributions, i.e. extreme behavior of returns (Wagner and Marsh, 2003). In this section, I consider the tail behavior of the underlying returns by looking at the *tail index*. The latter is denoted by α (not to confuse with the (G)ARCH parameters) and provides information about the tail's shape of a distribution. Notably, Ghose and Kroner (1995) and Groenendijk *et al.* (1995) showed that so-called stable distributions, on which the *tail index* is based, are nested with ARCH distributions with regard to their tail shapes. This allows for an empirical comparison between the two concepts.

The descriptive analysis from Section 3 showed that extreme returns occur frequently in the underlying series and this is manifested by the presence of heavy tails compared to a normal distribution; especially concerning the Bitcoin return series. Likewise, (G)ARCH parameters estimated in Section 4 implied information about the underlying distribution characteristics including the shape and (non-)existence of the unconditional variance. In comparison to the stock/index estimates, Bitcoin's parameter estimates point to an undefined unconditional variance and the estimated degrees of freedom (2.2) is significantly lower. This suggests an estimation of the *tail index* to be interesting since it indicates how much mass is in the tails of a distribution and thereby revealing useful information about the whole distribution including its variance (Hols and de Vries, 1991).

Recent empirical studies show that *tail indices* of financial returns are located in the interval $\alpha \in (2, 4)$ (Ibragimov *et al.*, 2015); implying finite first and second moments, but infinite fourth moments of the underlying distributions. This is valuable information for financial industries, risk managers, financial regulators, and investors who are interested in analyzing loss exceedance probabilities and in calculating risk measures such as the Value at Risk or Expected Shortfall. Finiteness of second moments for asset returns are critical for the application of regression and least square methods. Finiteness of fourth moments are key for the application of autocorrelation-based methods. The following Hill estimates indicate infiniteness of the fourth moment but finiteness of the second moment for most of the underlying series being in line with recent empirical studies of financial returns.

Hill (1975) introduced an estimator for the *shape parameter* ξ using a nonparametric method that is referred to as the Hill estimator. It is defined as

$$(39) \quad \xi(q) = \frac{1}{q}\sum_{i=1}^{q}\Big[\ln(r_{(T-i+1)} - \ln(r_{(T-q)})\Big],$$

where q denotes the order statistic, and each underlying return sequence is described as $\{r_t\}_{t=1}^{T}$ with $T = 2079$. The minimum return (smallest order statistic) of the sequence is denoted by $r_{(1)}$ and the maximum return (maximum order statistic) is denoted by $r_{(T)}$. Hence, the order statistics of the sequence is $r_{(1)} \leq r_{(2)} \leq ... \leq r_{(T)}$.

Most importantly, the *tail index* of a distribution is defined by the parameter $\alpha = 1/\xi(q)$. It can either be seen as a statistic for the distribution of extreme observations or as a measure of the maximum finite non-integer moment (Hols and de Vries, 1991). For instance, a *tail index* below 2 would imply the infiniteness of the second moment, i.e. non-existence of the variance of the underlying distribution. To explain the presence of an infinite variance in general, consider the stable Pareto distribution with the cumulative distribution function $F(x) = P(X \leq x) = 1 - x^{-\alpha}$ and the corresponding probability density function $f(x) = \alpha x^{-\alpha-1}$. The m^{th} moment of the distribution is defined as

$$(40) \quad \begin{aligned} E[x^m] &= \int_1^\infty x^m f(x)dx = \int_1^\infty x^m \alpha x^{-\alpha-1}dx = \alpha\int_1^\infty x^{m-\alpha-1}dx \\ &= \alpha\left[\frac{1}{m-\alpha}x^{m-\alpha}\right]_1^\infty = \begin{cases} -\frac{\alpha}{m-\alpha}, \text{if } m < \alpha \\ \infty, \text{if } m \geq \alpha \end{cases} \end{aligned}$$

Equation (40) implies that only moments up to α exist. In particular, $E[x^m]$ is bounded if $m < \alpha$ and $E[x^m]$ is unbounded if $m \geq \alpha$. For instance, in the case of $\alpha = 1/2$, not even the mean $(m = 1)$ exists, because the integrals that define it are infinite.

The ARCH paramters of the stock/index returns estimated in Section 4.2.1. imply that the unconditional variances are defined. In contrast, Bitcoin's unconditional variance is undefined and this points to a non-stationary explosive series in Bitcoin's conditional variance (Cuthbertson and Nitzsche, 2004). Hence, the second moment fails implying that the process is not covariance stationary. However, although Bitcoin's ARCH process is not covariance stationary, it is still stationary (Nelson, 1990; Groenendijk *et al.*, 1995).

That is, the joint distribution of one slice of the data is equal to the joint distribution of any other slice of the data (Wooldridge, 2015). This does not require the moments of the distribution - the second moment in particular - to be bounded.

Particularly with respect to an ARCH(1) model, the ARCH process is covariance stationary if $0 < \alpha_1 < 1$, stationary if $1 \leq \alpha_1 < \bar{\alpha}$, and non-stationary if $\alpha_1 > \bar{\alpha}$ (Groenendijk *et al.*, 1995).[17] Most importantly, this result also applies to a GARCH(1,1) process with $\hat{\alpha_1} + \hat{\beta_1} > 1$.

Figure 14 shows the *tail index* on the y-axis as a function of the ARCH parameter α_1 on the x-axis. The curve is based on Monte Carlo simulations and implies that the two concepts are partially nested. In particular, the slope indicates a negative relationship between the ARCH parameter and the *tail index*. For instance, concerning the underlying Bitcoin series, the estimated $\hat{\alpha_1}$ of an ARCH(1) model equals 1 and corresponds to a *tail index* of around 2.1.

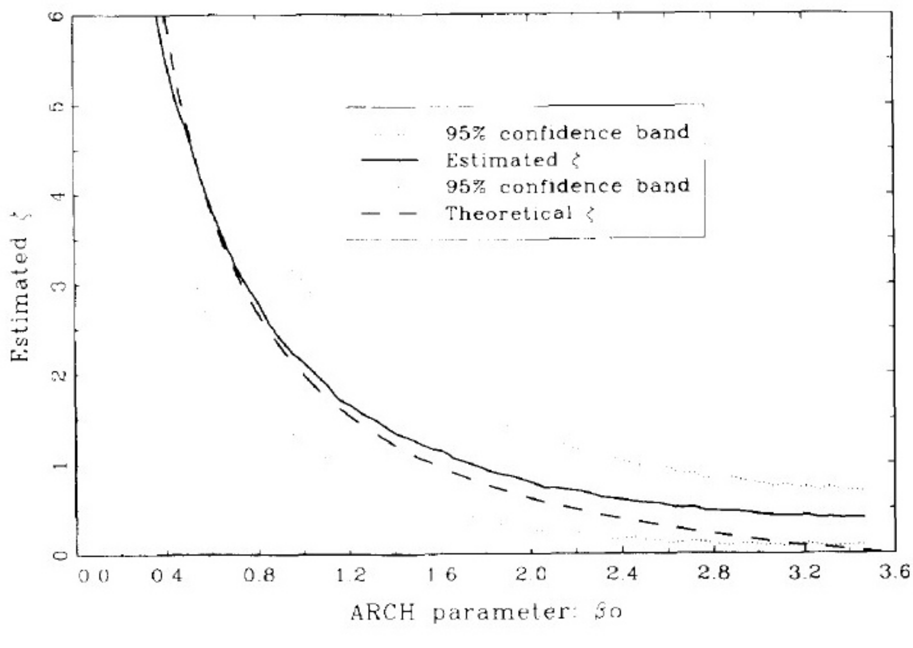


FIGURE 14
TAIL INDEX ESTIMATOR FOR ARCH
The *tail index* (y-axis) is plotted as a function of the ARCH parameter α_1 (x-axis).
Source: Groenendijk *et al.* (1995)

Figure 15 illustrates the scatterplots of the estimated *tail index* and its pointwise 95% confidence interval on the x-axis against the order statistics q on the y-axis. Within each plot, there is quite some variation in α for small numbers of the order statistic since there

[17] For the case $\alpha_1 = \bar{\alpha}$, see Vervaat (1979).

are too few observations. Typically, the α value stabilizes for higher order statistics and a long plateau can be identified that points to the correct α value (Tsay, 2010).

Corresponding *tail indices* are listed in Table 6. The table displays the sum of the GARCH(1,1) coefficients, i.e. $\hat{\alpha_1} + \hat{\beta_1}$, compared to the approximate *tail index* of each series. The Hill plots point to a *tail index* of around 2.3 for the Bitcoin series.[18] This is close to 2 and indicates that the distribution may have no second moment. In contrast, the stock/index *tail indices* are clearly greater than 2 - they range between 2.7 (Meritor) and 4.1 (BP). This may point, however, to infinite third/fourth moments.[19]

Table 6
Sum of GARCH Coefficients and Tail Index

	GARCH	Tail Index
Bitcoin	1.744	2.3
BP	0.998	4.1
Meritor	0.990	2.8
S&P 500	0.986	3.6
Nestlé	0.984	3.7
GS	0.982	3.3
MSCI	0.914	3.5
P&G	0.869	3.9

Recall that the GARCH(1,1) estimates of the stock/index returns from Section 4.2.1. indicate existing conditional variances with sums of the estimated coefficients $\hat{\alpha_1}$ and $\hat{\beta_1}$ being smaller than 1 in each case. In comparison, the sum of the estimated coefficients is greater than 1 for the Bitcoin return pointing to a non-existent conditional variance. The estimated *tail indeces* from Table 6 indicate existing variances of all return series. Referring to the Bitcoin return, the GARCH estimates point to an infinite variance, whereas the *tail index* of 2.3 effectively points to a finite variance of the Bitcoin distribution; although it is close to 2.

However, the numbers in the table confirm the theory that higher sums of GARCH(1,1) coefficients generally come along with lower *tail indices.* Hence, the GARCH estimates are validated by the Hill plots. That is to say, bigger tails come along with the a greater sum of GARCH(1,1) coefficients and fewer existing moments. Furthermore, both methods

[18]Figure 22 shows that this result holds for the Bitcoin series covering all observations.

[19]Conclusions drawn from statistical methods that require finiteness of the second/third/fourth moment should be hence treated with caution. For example, Bitcoin's correlations to other assets (second moment required) or (G)ARCH estimates (fourth moment required).

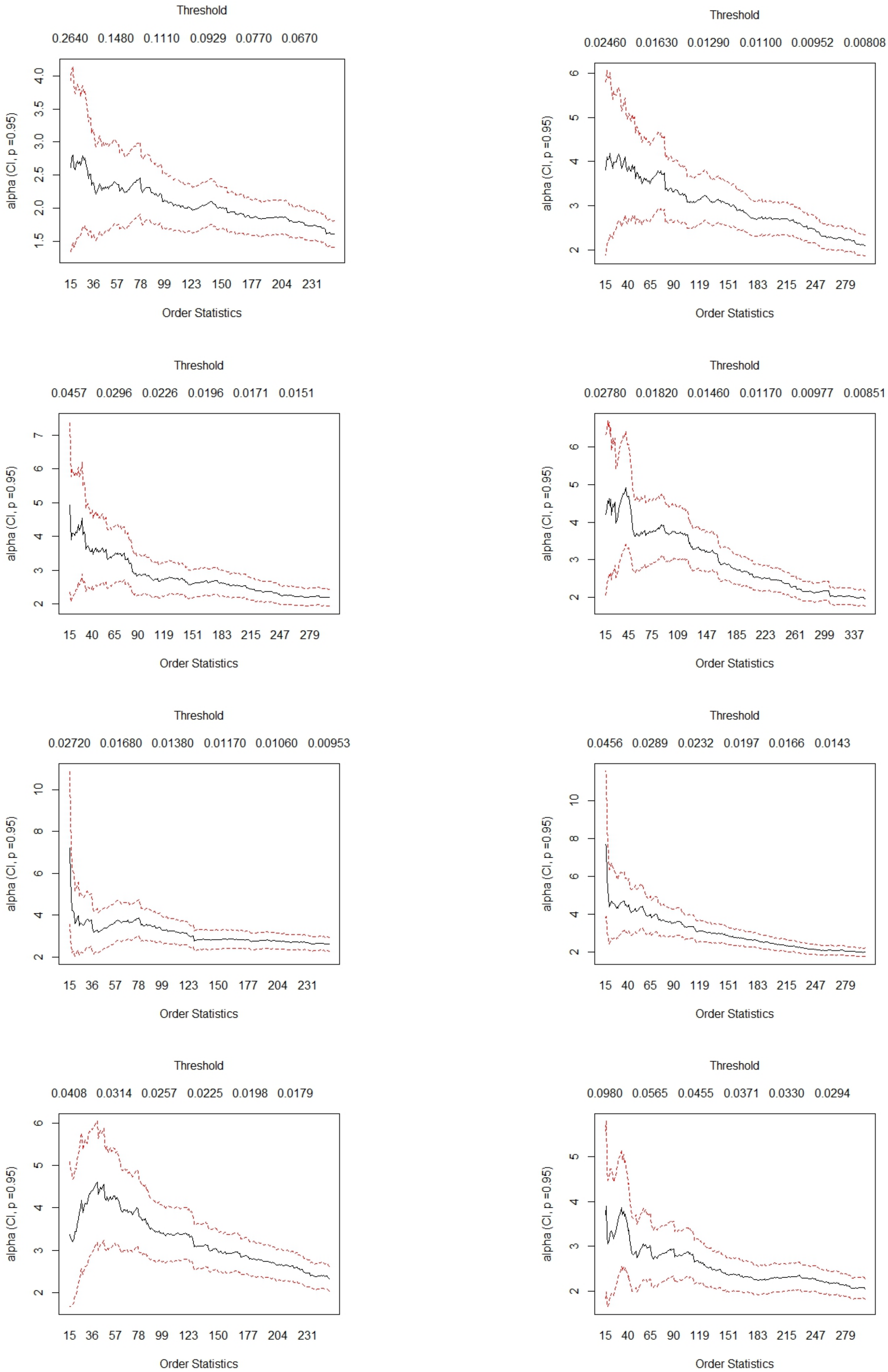


Figure 15
Hill Plots

The charts refer to Bitcoin (first row left), S&P 500 (first row right), MSCI (second row left), Nestlé (second row rigth), Procter & Gamble (third row left), BP (third row right), Goldman Sachs (last row left) and Meritor (last row right).

reflect the presence of heavy tails in particular of the Bitcoin returns, as described in Section 3. It is noteworthy that the *tail index* also coincides with the estimated tail shape of the (G)ARCH models. The latter can be seen as the optimal number of degrees of freedom of the (G)ARCH process that determine the Student-t distribution's tail shape.

Taken together, the sum of the estimated GARCH coefficients ($1.744 > 1$) and the estimated *tail index* (2.3) of the Bitcoin series validate the presence of heavier tails in the Bitcoin series compared to the stock/index series; implying a greater probability mass in the tails of the distribution (Ibragimov *et al.*, 2015), i.e. a greater likelihood of extreme fluctuations in the return.

6 Conclusion

I investigated the applications and properties of Bitcoin as well as its distributional characteristics compared to the S&P 500 stock index and six publicly quoted stocks over the period from July 2010 to October 2018. My thesis yields exciting insights on how Bitcoin's unique properties are reflected in its return distribution and volatility in particular.

There is theoretical and empirical evidence that Bitcoin rather fulfills the properties of an asset instead of a currency (Glaser *et al.*, 2014; Yermack, 2015; Dyhrberg, 2016a; Burniske and White, 2017; Baur *et al.*, 2018; Eom *et al.*, 2019) - mainly because of its high volatility in prices. Consequently, Bitcoin should be considered primarily as a financial asset that is used for the sake of investment - either as a long-term investment in emerging technology or as a short-term investment (Chu *et al.*, 2017).

Moreover, Bitcoin's properties are different from any other traditional asset in the financial market. For instance, Selgin (2013) classifies it as „synthetic commodity money“ sharing properties of commodity money such as gold and fiat money such as the US dollar. In other words, Bitcoin is „ringing the bell for a new asset class“ (Burniske and White, 2017) and one major reason is the fact that its return is not correlated with traditional asset classes - both in „normal“ and volatile market periods (Burniske and White, 2017; Baur *et al.*, 2018). This is in line with my correlation analysis according to Spearman/Kendall showing that the correlation between the underlying Bitcoin and stock/index returns are non-existent, i.e. close to 0; enabling new possibilities for risk management, portfolio analysis, and consumer sentiment analysis (Dyhrberg, 2016a).

The initial descriptive analysis showed that the Bitcoin return is characterized by a mean and standard deviation that is 5 times higher compared to the means and standard deviations of the stock/index returns on average. Furthermore, Bitcoin's return distribution exhibits both higher skewness and excess kurtosis compared to the stock/index return distributions. The relatively high excess kurtosis indicates heavier tails of the Bitcoin return. However, the empirical distributions of the stock/index returns also exhibit heavy tails, and the Q-Q plots clearly show that the Student-t distribution gives a better fit compared to a normal distribution for all series. Interestingly, the autocorrelations of the squared Bitcoin returns are falling off after only a few days in contrast to the vast majority of the stock/index return series. This is reflected in the GARCH estimates and is potentially driven by extreme price jumps Eom *et al.* (2019).

The main focus of my thesis was to explore the volatility properties of the Bitcoin return compared to stock/index returns by using (G)ARCH models and Hill estimates. The results of the (G)ARCH models show that the unconditional variance of the Bitcoin return is non-existent pointing to a non-stationary and explosive series in the conditional variance. In particular, with respect to the GARCH(1,1) model, the sum of the estimated $\hat{\alpha_1}$ and $\hat{\beta_1}$ coefficients equals $1.744 > 1$. However, according to Nelson (1990) and Groenendijk *et al.* (1995), the stochastic process is still stationary. In contrast, referring to the stock/index returns, the sums of the estimated $\hat{\alpha_1}$ and $\hat{\beta_1}$ coefficients of a GARCH(1,1) model are below 1 implying existing unconditional variances.

The Hill plots point to a *tail index* of 2.3 of the Bitcoin series that is low compared to the *tail indeces* of the other series ranging between 2.8 and 4.1. The latter are distinctly greater than 2 indicating finite second moments but potentially infinite fourth moments of the corresponding return distributions. Although Bitcoin's *tail index* points to a finite second moment, it is only marginally greater than 2 implying the possibility of a non-existent variance. The GARCH and Hill estimates are in line with the findings of Ghose and Kroner (1995) and Groenendijk *et al.* (1995) who stated a negative relationship between the sum of GARCH$(1,1)$ coefficients and the *tail index*. Importantly, both methods validate each other and confirm the presence of heavier tails in the Bitcoin return distribution compared to the other distributions.

I draw the conclusion that Bitcoin's unique properties as an asset are reflected in its return distribution. In particular, Bitcoin's return distribution exhibits significantly heavier tails compared to the stock/index return distributions being manifested by GARCH and Hill estimates. Bitcoin is an asset that is characterized by a smaller number of finite moments compared to stock return distributions. Most strikingly, Bitcoin's second moment might not be bounded according to the GARCH and Hill estimates pointing to a potentially non-existent variance. This is an important fact for financial regulators, risk managers, and investors who rely on loss exceedance probabilities and risk measures that require finiteness of the second moment. Finally, I expect to contribute to future research examining the properties and distributional characteristics of Bitcoin and cryptocurrencies in general.

7 Appendix

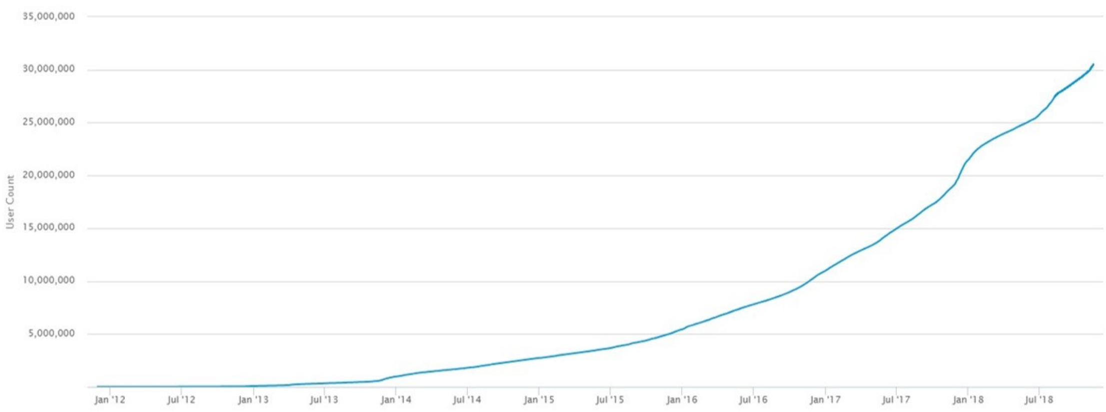


Figure 16
Blockchain Wallet Users

The chart displays the total number of blockchain wallets (y-axis) over time (x-axis).
Source: Blockchain (2018)

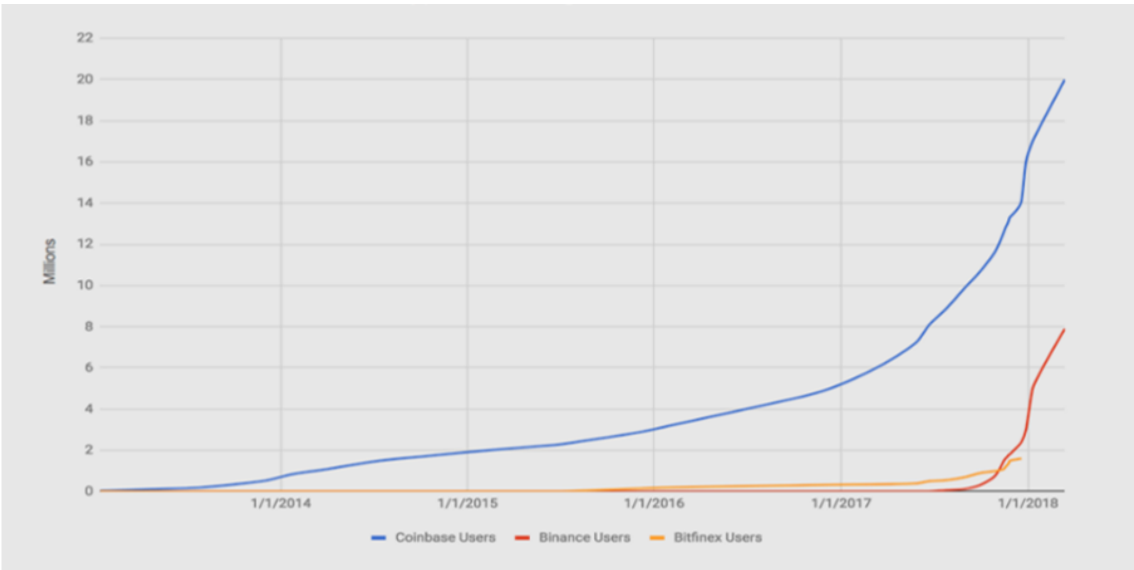


Figure 17
Crypto Exchange User Growth

The chart displays the number of crypto exchange users of Coinbase, Binance, and Bitfinex (y-axis) over time (x-axis).
Source: McCann (2018)

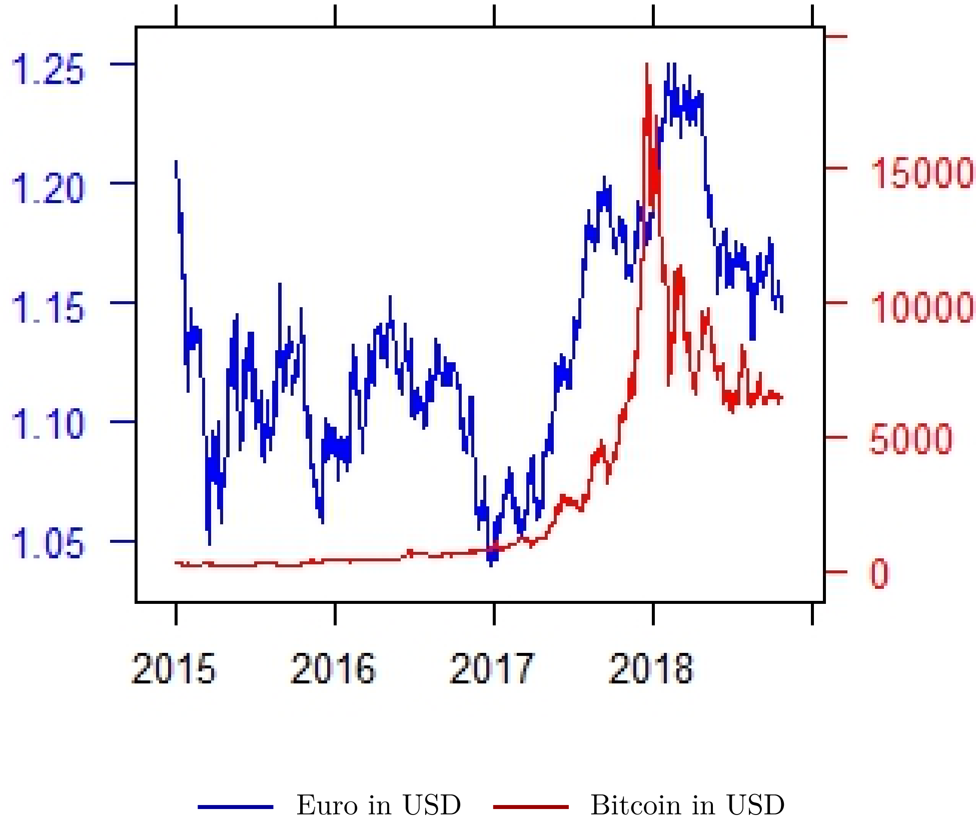


Figure 18
Euro and Bitcoin over Time in USD
Data sourced from YahooFinance (2018)

Correlation Matrix.

Correl	*bitr*	*sp5r*	*sp6r*	*gldr*	*silvr*	*eurr*	*audr*	*jpyr*	*gbpr*	*cnyr*	*hufr*	*twus*	*wtir*	*hhr*	*cbr*	*tbr*	*hbr*
bitr	1.00	0.05	0.05	0.04	0.02	0.01	−0.02	0.01	0.01	0.02	−0.01	−0.01	0.01	0.00	−0.01	−0.03	0.05
sp5r		1.00	0.92	0.05	0.04	0.15	0.18	0.06	0.14	−0.03	−0.20	−0.41	0.36	0.01	−0.06	−0.48	0.32
sp6r			1.00	0.06	0.05	0.12	0.14	0.04	0.10	−0.02	−0.16	−0.37	0.32	0.00	−0.08	−0.45	0.24
gldr				1.00	0.81	0.33	0.38	−0.24	0.34	−0.14	−0.29	−0.29	0.05	0.06	0.04	−0.03	0.06
silvr					1.00	0.33	0.42	−0.12	0.34	−0.15	−0.32	−0.29	0.09	0.07	0.06	−0.04	0.14
eurr						1.00	0.55	−0.22	0.65	−0.24	−0.80	−0.51	0.14	0.05	0.07	−0.12	0.16
audr							1.00	−0.22	0.50	−0.20	−0.59	−0.55	0.18	0.03	0.10	−0.15	0.32
jpyr								1.00	−0.21	0.09	0.09	0.25	−0.04	−0.03	−0.02	−0.05	0.06
gbpr									1.00	−0.21	−0.57	−0.43	0.15	0.05	0.05	−0.13	0.20
cnyr										1.00	0.20	0.21	−0.02	−0.11	0.00	0.05	−0.04
hufr											1.00	0.48	−0.16	−0.03	−0.06	0.15	−0.24
twus												1.00	−0.37	−0.06	−0.19	0.15	−0.31
wtir													1.00	0.12	−0.03	−0.24	0.19
hhr														1.00	−0.01	−0.04	−0.01
cbr															1.00	0.64	0.25
tbr																1.00	−0.12
hbr																	1.00

This table reports the return correlation between 17 assets used in the analysis, including Bitcoin. Daily data between July 2010 and June 2015 is used. Bitcoin to USD data is from the WinkDex website. Prices for all other data are from Bloomberg.

Variables List.

Variable	Explanation	Asset class
bitr	WinkDex (Bitcoin exchange rate index)	Digital Currency
sp5r	S&P500 (US equity index)	Equity
sp6r	S&P600 (US equity index)	Equity
gldr	Gold Spot	Precious Metal
silvr	Silver Spot	Precious Metal
eurr	EUR USD (Euro to US Dollar exchange rate)	Currency
audr	AUD USD (Australian Dollar to US Dollar exchange rate)	Currency
jpyr	JPY USD (Japanese Yen to US Dollar exchange rate)	Currency
gbpr	GBP USD (British Pounds to US Dollar exchange rate)	Currency
cnyr	CNY USD (Chinese Yuen to US Dollar exchange rate)	Currency
hufr	HUF USD (Hungarian Forint to US Dollar exchange rate)	Currency
twus	Trade weighted US dollar index	Currency
wtir	WTI 1 month (Crude oil index)	Energy
hhr	HH 1 month (Natural gas index)	Energy
cbr	Bloomberg US Corporate Bond Index	Bond
tbr	Bloomberg US Treasury Bond Index	Bond
hbr	Bloomberg USD High Yield Corporate Bond Index	Bond

This table reports the list of 17 variables, explanation of the variables and the asset classes of the variables used in this analysis.

FIGURE 19

BITCOIN CORRELATION WITH OTHER ASSETS

Source: Baur *et al.* (2018)

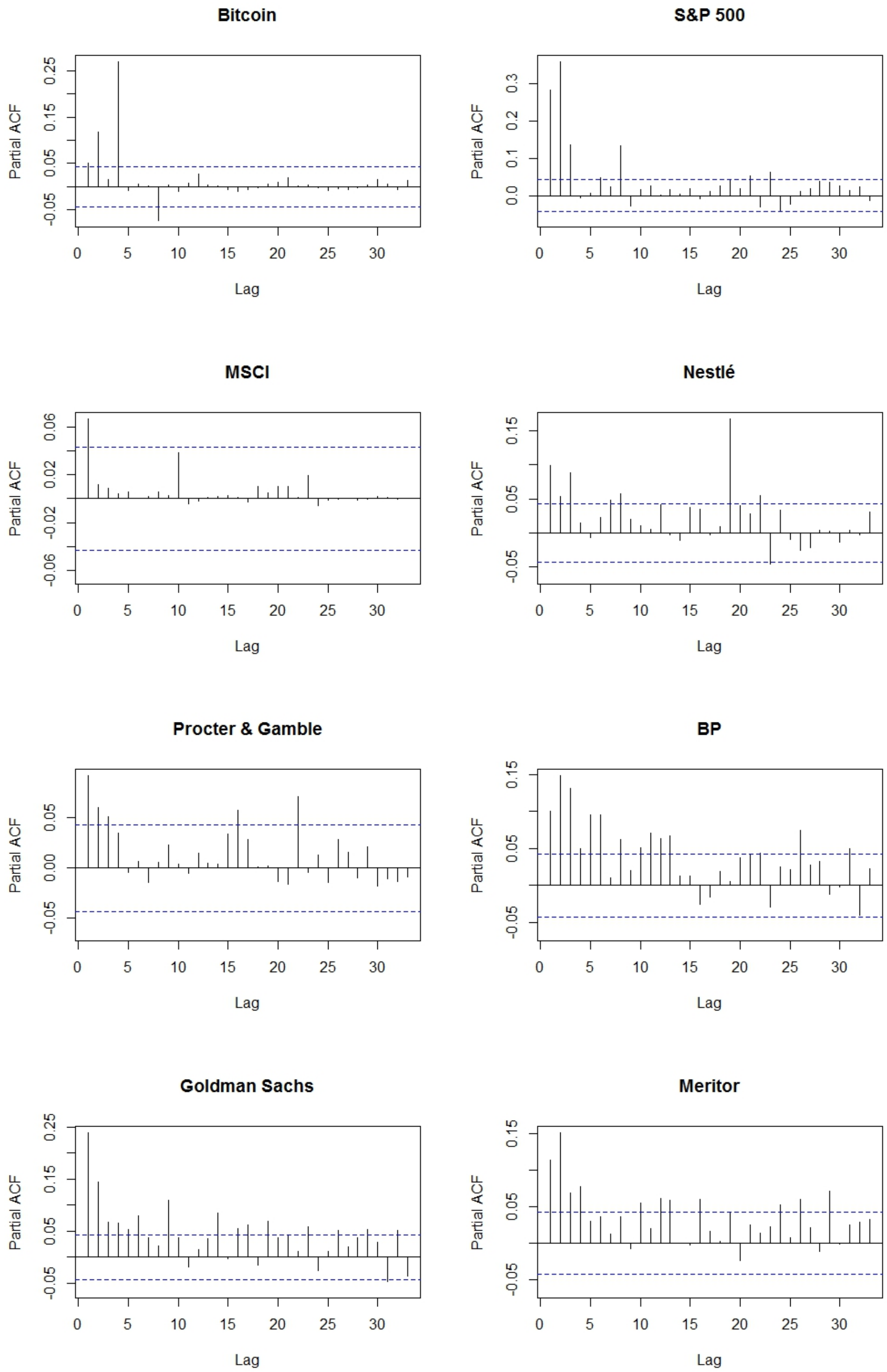
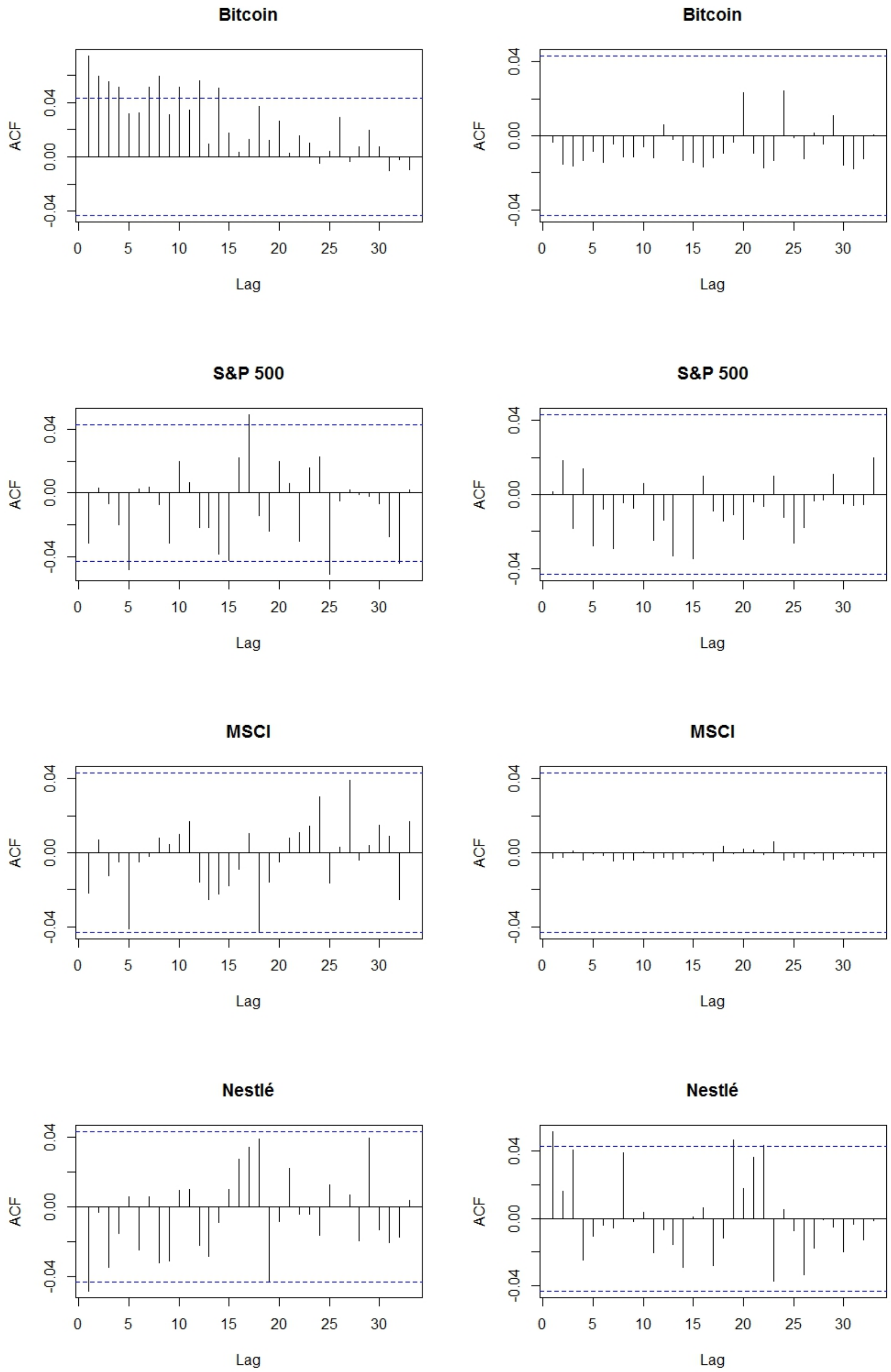


Figure 20
PACF of the Squared Returns


Bitcoin
Bitcoin
S&P 500
S&P 500
MSCI
MSCI
Nestlé
Nestlé
ACF
Lag

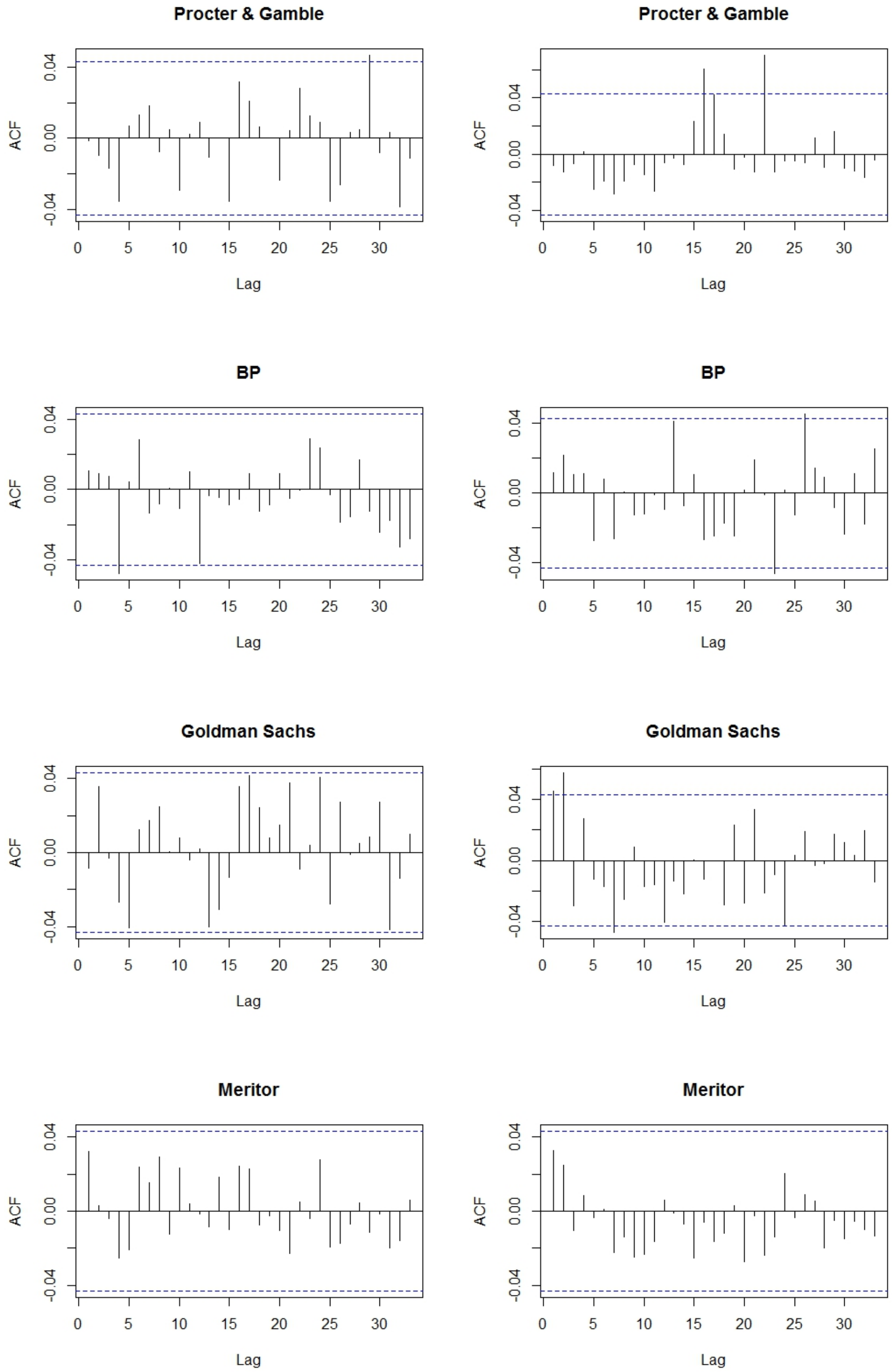


Figure 21
ACF of the Standardized Innovations (Left) and Squared Standardized Innovations (Right)

Table 7
Bitcoin ARCH Estimates

	Dependent variable:	
	Bitcoin (small sample)	Bitcoin (full sample)
	(1)	(2)
$\hat{\mu}$	0.002***	0.002***
	(0.001)	(0.0004)
$\hat{\alpha_0}$	0.001***	0.0004***
	(0.0003)	(0.0001)
$\hat{\alpha_1}$	1.000***	1.000***
	(0.318)	(0.262)
$\hat{\alpha_2}$	0.845***	0.420***
	(0.314)	(0.140)
$\hat{\alpha_3}$	0.535**	0.363***
	(0.212)	(0.128)
$\hat{\alpha_4}$	0.216*	0.413***
	(0.124)	(0.138)
$\hat{\alpha_5}$	0.508**	0.374***
	(0.207)	(0.124)
$\hat{\alpha_6}$	0.210*	0.201**
	(0.115)	(0.087)
$\hat{\alpha_7}$	0.306**	0.189**
	(0.145)	(0.086)
shape	2.232***	2.329***
	(0.083)	(0.100)
Observations	2,079	3,016
Log Likelihood	−3,484.648	−5,596.173
AIC	−3.343	−3.704
BIC	−3.315	−3.684
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 8
Bitcoin GARCH Estimates

	Dependent variable:	
	Bitcoin (small sample)	Bitcoin (full sample)
	(1)	(2)
$\hat{\mu}$	0.002***	0.002***
	(0.001)	(0.0004)
$\hat{\alpha_0}$	0.0002**	0.0001**
	(0.0001)	(0.00003)
$\hat{\alpha_1}$	1.000***	0.709***
	(0.386)	(0.234)
$\hat{\beta_1}$	0.744***	0.758***
	(0.024)	(0.021)
shape	2.200***	2.317***
	(0.087)	(0.123)
Observations	2,079	3,016
Log Likelihood	−3,491.718	−5,608.441
AIC	−3.354	−3.716
BIC	−3.341	−3.706
Note:		*p<0.1; **p<0.05; ***p<0.01

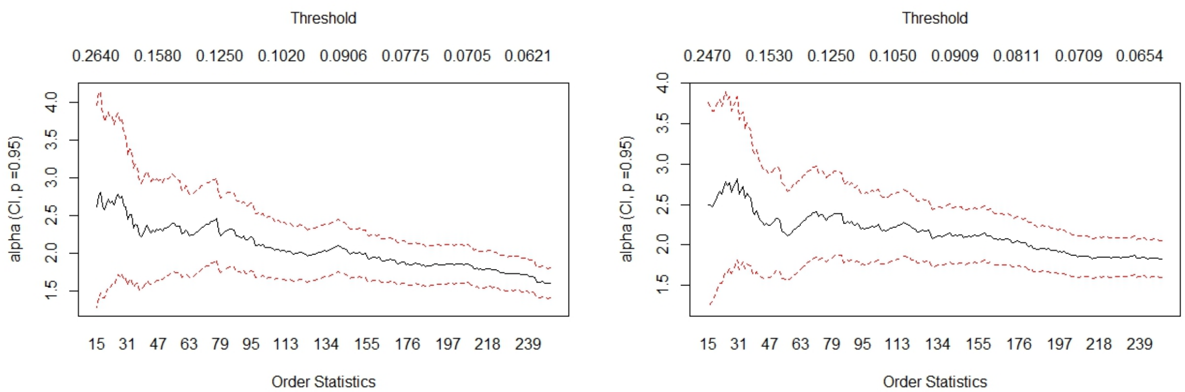


Figure 22
Bitcoin Hill Plots: Small Sample (Left) Versus Full Sample (Right)

REFERENCES

Alvarez-Ramirez, J. and Rodriguez, E. (2018). AR(p)-based detrended fluctuation analysis. *Physica A: Statistical Mechanics and its Applications*, **502**, 49–57.

Bariviera, A. F., Basgall, M. J., Hasperué, W. and Naiouf, M. (2017). Some stylized facts of the bitcoin market. *Physica A: Statistical Mechanics and its Applications*, **484**, 82–90.

Baur, D. G., Hong, K. and Lee, A. D. (2018). Bitcoin: Medium of exchange or speculative assets? *Journal of International Financial Markets, Institutions and Money*, **54**, 177–189.

Blockchain (2018). https://www.blockchain.com/.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, **31** (3), 307–327.

Burnham, K. P. and Anderson, D. R. (2004). Multimodel inference. *Sociological Methods & Research*, **33** (2), 261–304.

Burniske, C. and White, A. (2017). *Bitcoin: Ringing the Bell for a New Asset Class*. Tech. rep., Ark Invest Research.

Chan, S., Chu, J., Nadarajah, S. and Osterrieder, J. (2017). A statistical analysis of cryptocurrencies. *Journal of Risk and Financial Management*, **10** (2), 12.

Chu, J., Chan, S., Nadarajah, S. and Osterrieder, J. (2017). GARCH modelling of cryptocurrencies. *Journal of Risk and Financial Management*, **10** (4), 17.

Coinmarketcap (2018). https://coinmarketcap.com/.

Cuthbertson, K. and Nitzsche, D. (2004). *Quantitative Financial Economics: Stocks, Bonds and Foreign Exchange*. John Wiley & Sons, Ltd.

DEA (2018). National drug threat assessment 2018.

Detzel, A. L., Liu, H., Strauss, J., Zhou, G. and Zhu, Y. (2018). Bitcoin: Predictability and profitability via technical analysis. *SSRN Electronic Journal*.

Ding, Z., Granger, C. W. and Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, **1** (1), 83–106.

Dyhrberg, A. H. (2016a). Bitcoin, gold and the dollar – a GARCH volatility analysis. *Finance Research Letters*, **16**, 85–92.

— (2016b). Hedging capabilities of bitcoin. is it the virtual gold? *Finance Research Letters*, **16**, 139–144.

ECB (2015). *Virtual currency schemes - a further analysis*. Tech. rep., European Central Bank.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica*, **50** (4), 987–1007.

Eom, C., Kaizoji, T., Kang, S. H. and Pichl, L. (2019). Bitcoin and investor sentiment: Statistical characteristics and predictability. *Physica A: Statistical Mechanics and its Applications*, **514**, 511–521.

Ghose, D. and Kroner, K. F. (1995). The relationship between GARCH and symmetric stable processes: Finding the source of fat tails in financial data. *Journal of Empirical Finance*, **2** (3), 225–251.

Glaser, F., Zimmermann, K., Haferkorn, M., Weber, M. C. and Siering, M. (2014). Bitcoin - asset or currency? revealing users' hidden intentions. In *Proceedings of the 22nd European Conference on Information Systems*.

Glosten, L. R., Jagannathan, R. and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, **48** (5), 1779–1801.

Greer, R. J. (1997). What is an asset class, anyway? *The Journal of Portfolio Management*, **23** (2), 86–91.

Groenendijk, P. A., Lucas, A. and de Vries, C. G. (1995). A note on the relationship between GARCH and symmetric stable processes. *Journal of Empirical Finance*, **2** (3), 253–264.

Hansen, P. R. and Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a GARCH(1,1)? *Journal of Applied Econometrics*, **20** (7), 873–889.

Hens, T. and Rieger, M. O. (2016). *Financial Economics*. Springer Berlin Heidelberg.

Hileman, G. and Rauchs, M. (2017). 2017 global cryptocurrency benchmarking study. *SSRN Electronic Journal*.

Hill, B. M. (1975). A simple general approach to inference about the tail of a distribution. *The Annals of Statistics*, **3** (5), 1163–1174.

Hols, M. C. A. B. and de Vries, C. G. (1991). The limiting distribution of extremal exchange rate returns. *Journal of Applied Econometrics*, **6** (3), 287–302.

Hong, H. and Stein, J. (1997). *A Unified Theory of Underreaction, Momentum Trading and Overreaction in Asset Markets*. Tech. rep.

Horváth, R. and Šopov, B. (2016). GARCH models, tail indexes and error distributions: An empirical investigation. *The North American Journal of Economics and Finance*, **37**, 1–15.

Ibragimov, M., Ibragimov, R. and Walden, J. (2015). *Heavy-Tailed Distributions and Robustness in Economics and Finance*. Springer International Publishing.

Jansen, D. W. and de Vries, C. G. (1991). On the frequency of large stock returns: Putting booms and busts into perspective. *The Review of Economics and Statistics*, **73** (1), 18.

Katsiampa, P. (2017). Volatility estimation for bitcoin: A comparison of GARCH models. *Economics Letters*, **158**, 3–6.

Law, J. (2018). *A Dictionary of Finance and Banking*. Oxford University Press.

Liu, Y. and Tsyvinski, A. (2018). Risks and returns of cryptocurrency. *SSRN Electronic Journal*.

Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, **36** (4), 394.

McCann, C. (2018). https://medium.com/@mccannatron/12-graphs-that-show-just-how-early-the-cryptocurrency-market-is-653a4b8b2720.

Murphy, D. P. and Thirumalai, R. S. (2017). Short-term return predictability and repetitive institutional net order activity. *Journal of Financial Research*, **40** (4), 455–477.

Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system.

Nelson, D. B. (1990). Stationarity and persistence in the GARCH(1,1) model. *Econometric Theory*, **6** (03), 318.

— (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, **59** (2), 347.

Pressman, A. (2012). https://uk.reuters.com/article/us-vanguard-indexes-idusbre8910py20121002.

Rosenberger, P. (2018). *Bitcoin und Blockchain.* Springer-Verlag GmbH.

Rouwenhorst, K. G. (1997). International momentum strategies. *SSRN Electronic Journal.*

Selgin, G. (2013). Synthetic commodity money. *Journal of Financial Stability*, **17**, 92–99.

Shumway, R. H. and Stoffer, D. S. (2017). *Time Series Analysis and Its Applications.* Springer-Verlag GmbH.

Tsay, R. S. (2010). *Analysis of Financial Time Series.* John Wiley and Sons Ltd.

— and Tiao, G. C. (1984). Consistent estimates of autoregressive parameters and extended sample autocorrelation function for stationary and nonstationary ARMA models. *Journal of the American Statistical Association*, **79** (385), 84–96.

Vervaat, W. (1979). On a stochastic difference equation and a representation of non–negative infinitely divisible random variables. *Advances in Applied Probability*, **11** (04), 750–783.

Wagner, N. and Marsh, T. A. (2003). Measuring tail thickness under GARCH and an application to extreme exchange rate changes. *SSRN Electronic Journal.*

Wooldridge, J. M. S. U. (2015). *Introductory Econometrics.* Cengage Learning, Inc.

YahooFinance (2018). https://finance.yahoo.com/.

Yermack, D. (2015). *Handbook of Digital Currency.* Academic Press.

Zivot, E. (2009). Practical issues in the analysis of univariate GARCH models. In *Handbook of Financial Time Series*, Springer Berlin Heidelberg, pp. 113–155.