Information, delegation and communication

Optimal decision-making process and cheap talk with an endogenously informed principal

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ABSTRACT

This paper examines the optimal decision-making procedure in a cheap-talk model with an exogenously informed agent and endogenous information acquisition by the principal. I analyze two kinds of equilibria: the equilibrium with the most informative communication and the partial pooling equilibrium in which the principal has acquired the maximum amount of information. I show that there is no difference in terms of outcome between delegation and retainment of the decision-making authority for either player in the most informative equilibria. Conversely, in the partial pooling equilibria, it is advantageous for the principal to retain her decision-making authority. Additionally, my results show information acquisition and communication to be supplements. In the most informative equilibrium, the principal acquires relatively little information, which allows for greater information transmission between the players, whereas information transmission in the partial pooling equilibrium is much more limited. This causes the latter equilibrium to be significantly less attractive for both players.

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1. Introduction

Recently, Dutch Minister for Legal Protection Sander Dekker was under pressure for the events surrounding the intended early release of criminal Michael P. During an unsupervised leave, which was in preparation of this early release, P. committed a murder on a young woman. In recent times, similar cases in the Netherlands have occurred, in which a criminal psychiatric patient was released and thereafter relapsed into crime, including the murder of Els Borst and a fatal stabbing in the metro of Amsterdam (Boersema, 2019; Meijer, 2019).The decision to release a criminal from a psychiatric clinic is obviously complicated, but will entail a cost-benefit analysis nonetheless. On one side, there is the, rather easy to observe, reduction in costs after a patient is released, such as the cost of an occupied cell and the cost of continued therapy. On the other side, the potential risks (or costs) of releasing the patient – i.e. will the patient recidivate – is much harder to observe, even for specialists. This problem is intensified by the fact that budget cuts by the government have led to understaffing in the mental health care sector (Meijer, 2019; Nieuwenhuis, 2019). Now suppose that the decision to release a patient is made by a psychiatrist in consultation with the director of the patient's psychiatric clinic (hereafter: "the director") by comparing the cost reductions with the probability of recidivating. By spending time on this specific patient, the psychiatrist is able to obtain an estimate of the probability of the patient recidivating. The accuracy of this estimate increases with the time invested by the psychiatrist. The director, on the other hand, is able to observe the exact cost reduction resulting from the release of the patient, with virtually zero effort. Additionally, suppose that due to private motives, for example a very tight clinic-wide budget or wanting to achieve certain performance goals, the director is just somewhat biased towards releasing the patient in comparison to the psychiatrist. In order to arrive at the best possible decision for society as a whole, who should be given the authority to make the final decision? The (initially) less informed psychiatrist or the biased director? Additionally, given the difference in preferences, how much information can they credibly communicate to one another? And how much time is the psychiatrist going to invest in the patient to obtain information?

This example, albeit possibly a somewhat simplified version of the actual procedures, illustrates the intricacy of information acquisition, communication and decision making when faced with incongruent preferences. The communication in this paper takes the form of cheap talk. Decision making in conjunction with cheap-talk communication was first researched by Crawford & Sobel (1982) (hereafter: CS).[1] One of the implicit characteristics of the CS-model is that it would be better for the receiver, who is responsible

[1] More specifically, when referring to CS, I'm referring to the well-known uniform-quadratic example of section 4 of Crawford & Sobel (1982).

for making the final decision, to let the sender make the final decision and essentially cut himself out of the process altogether. This is due to the receiver holding no information, except for what he is told by the sender. In reality, a person charged with the responsibility to make executive decisions often has the possibility to collect at least some information regarding the subject at hand. When the amount or accuracy of this information is less than that of the other person involved, this introduces a tradeoff. One could keep the decision-making authority to himself, knowing that he has less information than the other, or the other person could be asked to make the decision, even though that person has different preferences. Of course, the person that is not making the final decision can be asked to disclose his information to the other, but without being able to confirm the correctness of the shared information, this leaves room for strategic communication.

The main objective of this paper is to examine the influence of the possibility of information acquisition for the principal on the communication and optimal decision-making process in the presence of incongruent preferences between the principal and the agent. To this end, I develop a model where two players, a principal and an agent, face a decision regarding the implementation of a project with an uncertain payoff. This payoff is the resultant of the revenues and expenditures associated with the project, which are independent of each other. Both players care equally about the payoff of the project, except for the agent being somewhat biased towards its implementation. Before anything else, the principal has to decide on the allocation of the decision-making authority. She can either decide to delegate the authority to the agent or keep it herself. In both scenarios, the principal is able to acquire an estimate of the revenues of the project by spending costly effort. The more effort spent, the more accurate the estimate. The agent, on the other hand, can simply observe the expenditures associated with the project without any cost. In the delegation scenario, the principal will send a cheap talk message regarding her information on the revenues of the project to the agent. In the nondelegation scenario, these roles are reversed; the principal will receive a cheap talk message from the agent prior to making the implementation decision.

The main result of this paper is that delegation of the decision-making authority yields identical results as retainment of this authority for both the principal and the agent in terms of payoff, given that the most informative equilibrium is reached.[2] Conversely, in this most informative equilibrium, the principal has acquired relatively little information regarding the revenues of the project. In both the delegation and the nondelegation scenario, the coarseness of the principal's information allows for more informative communication between the players and thus, for better decision making, compared to the case where the principal has acquired full information regarding the revenues of the project. This even applies when the acquisition of the principal's information is assumed to be costless. As the principal is ex ante less informed than the agent, this result contrasts with the original cheap talk model of CS, where it would always be better for both players to let the ex ante best informed player, i.e. the sender, make the final decision.

Intuitively, the results of the most informative equilibria can be explained through the information acquisition mechanism. Through this mechanism, the principal observes an estimate of the revenues of the project, which is deliberately left relatively coarse in the most informative equilibria. In order to make the implementation decision, the decision maker (which of the two players this is, depends on whether or not the decision-making authority was delegated) compares the expected revenues to the expected costs. In the delegation scenario, the principal is able to communicate all her information to the agent, due to the deliberate coarseness of this information. Thus, at the moment of the implementation decision, the agent will have obtained all the information available in the game, leading to an optimal decision for the agent. In the nondelegation scenario, the agent does not communicate all his information to the principal. Instead, he constructs message intervals such that the principal is able to infer whether the *expected* costs are lower or higher than the expected revenues for all possible values of the expected revenues. However, the agent constructs the messages in such a way that the principal will also implement the project in those cases where this is profitable for the agent, but detrimental to the principal. The principal is not able to remedy this, again due to the coarseness of her information. The decision made by the principal is therefore again

[2] Cheap talk models generally face the issue of having multiple equilibria. The maximum amount of informative communication is limited by the degree of incongruence between the players' preferences, but equilibria with less communication than this maximum usually also exist. In this paper I focus on two types of equilibria: the equilibrium with the most informative communication and the equilibrium where the principal has acquired the maximum amount of information prior to communication. The first type is the main focus of the paper, since this is expected to lead to the best decision making. The latter type serves to show what happens when the information acquisition mechanism is essentially replaced with the assumption that the principal simply observes the value of the revenues. I abstain from discussing the problem of equilibrium refinement. For a suitable application of this to my model, see Section 4 of Bijkerk, Delfgaauw, Karamychev, & Swank (2018).

optimal for the agent, given the available information. Hence, the outcome is equal between the two scenarios.

Even though these outcomes are optimal for the agent in both cases, the principal benefits from the relatively high level of communication as well. In the equilibria where the principal acquires full information regarding the revenues of the project, thereby being able to perfectly observe its value, the decision making is inferior. This is due to the limited communication that is possible in these equilibria, which recedes to the form of the communication in CS.

The foundations of the cheap talk literature, and therefore of this paper, lie in the paper of CS. One of the implicit characteristics of CS is that in all cases where informative communication could have taken place, it is better for both players if the sender[3] would simply make the final decision, meaning that the receiver would take no part in the game. This result was made more explicit by Dessein (2002), who considered the allocation of the decision rights to be endogenous. Le Quement (2009) extends this model to include information acquisition by the agent. In line with Dessein, he finds that both players prefer delegation of the decision rights whenever the agent's information is sufficiently high. However, for lower levels of information acquisition, the principal is better off making the final decision himself. Similar to Dessein and Le Quement, in my model, the right to allocate the decision authority is held by the principal. However, my results do not show either of the two players to be making better decisions for both players compared to the other player.

Both CS and Dessein (2002) assume that the receiver relies solely on the sender for information, meaning that there can be no communication in the case of delegation of the decision authority to the sender. The main feature of my model is the possibility of information acquisition for the principal, resulting in both players (potentially) observing information. Therefore, informative communication can exist, regardless of who holds the decision-making authority. Two-sided information is not new to cheap talk models, however. Even so, most papers have assumed this information to be exogenously given. Both Chen (2009) and Moreno de Barreda (2010) discuss a framework in which the decision maker observes a signal regarding the state of the world. Chen finds that the decision maker cannot improve communication by trying to

[3] Throughout the literature, the two players have received different names, depending on their exact role. What I call 'the agent' is in other papers often referred to as 'the expert' or 'the sender'. Similarly, 'the principal' is often referred to as 'the decision maker' or 'the receiver'. However, in my paper the agent and principal switch between sending information and making the implementation decision. Therefore, throughout this paper, the terms 'the sender' and 'the decision maker' refer to whichever player is fulfilling these roles, regardless of the delegation decision.

reveal her private information to the sender, prior to communication. Moreno de Barreda finds that private information of the decision maker hampers communication, since it makes exaggeration more attractive. Similarly, Lai (2014) assumes the decision maker observes whether the state of the world is low or high. Again, this information hampers communication, but contrary to Moreno de Barreda, Lai finds that the private information is often enough to offset the loss in communication, meaning that the decision maker benefits from the private information. Interestingly, Ishida & Shimizu (2018) show that the availability of private information for a decision maker can actually improve communication, when the decision maker is uncertain whether his information is correct. They argue that information acquisition by the decision maker and communication can either be complements or substitutes, depending on the degree of uncertainty concerning the correctness of the principal's information. These papers all assumed the private information of the decision maker to concern the same variable as the sender's information. Watson (1996) introduced the notion of the decision maker observing a second variable, where both variables are relevant for the decision maker's optimal decision, which is similar to my model. He finds that the presence of two-sided information makes it possible for the sender to fully reveal his information. This is in line with my results in the delegation scenario, in which case the principal is the sender of the communication.

The information of the principal is not exogenously given in my model. Instead, the information is assumed to be endogenously acquired by the principal. In the literature different mechanisms of endogenous information acquisition have been incorporated into the CS model.[4] The mechanism of information acquisition of my model is most related to the models of Ivanov (2010), Pei (2015), Argenziano, Severinov & Squintani (2016) and Bijkerk, Delfgaauw, Karamychev, & Swank (2018). Ivanov considers the case where a principal is able to limit the information observed by the agent, by dividing the interval on which the state of the world is distributed into subintervals of any size, after which the agent only observes in which subinterval the true state of the world falls. This informational control is costless. Ivanov compares this informational control to delegation of the decision authority as two tools that the principal has in order to elicit a higher payoff. He finds that informational control is superior to delegation whenever informational communication is possible. Pei incorporates the same information acquisition mechanism as Ivanov, except in Pei's model, the sender is able to determine the information acquisition himself. Additionally, in Pei's model, finer subintervals are costlier. Pei finds that the agent will always communicate all his information,

[4] Austen-Smith (1994) and Hidir (2017) consider a sender that can buy a perfectly revealing signal of the state of the world for a given price. Other papers, such as Che & Kartik (2009), Dewatripont and Tirole (1999), Le Quement (2009) and Dur & Swank (2005) assume that the sender is able to invest in information, where a higher investment means a higher probability that the observed signal is fully informative. See Sobel (2013) for a more extensive summary of some of the different information acquisition mechanisms used in the one-sided information models.

since if he is planning to withhold information, he can always make the intervals coarser and incur less costs in the process. Both models assume that the size of the subintervals can be of any size. Conversely, my model assumes the size of the subintervals to be uniform in size. Argenziano, Severinov & Squintani model the information acquisition by letting the agent decide on a number of binary trials. The probability that the trial is a success is equal to the true value of the state of the world. Thus, more successes mean a higher expected state of the world.

However, none of the previously discussed papers analyze the presence of two-sided information on two separate variables in combination with information acquisition. Bijkerk et al. (2018) do exactly that. Their model contains a company's executive and insider contemplating the implementation of a project. The insider is able to observe one of the two performance indicators of the project, whereas the executive is able to invest costly effort in order to acquire information regarding the second indicator. She does this by partitioning the total interval in equally sized subintervals and subsequently observing in which subinterval the indicator falls. The executive communicates to the insider and two external parties. The executive wants to accurately inform the insider, but at the same time wants the value of the performance indicator perceived by the external parties to be as high as possible. They find that transparent communication, where all parties can observe the executive's message, is key for informative communication to the external parties, which in turn constrains communication to the insider. Additionally, they find that less accurate information acquisition allows for more informative communication. My model can be perceived as a stripped-down model of Bijkerk et al., where the assumption of the agent's bias replaces the need to impress the external parties. My main contribution is the consideration of the possibility of delegation in this setting. This is, to the best of my knowledge, the first time that the option to delegate is considered in a setting where the principal has acquired his own (imperfect) information.

The next section describes the model. In order to gain some intuition behind the model, a relatively simple version of the model is discussed in Section 3. In this version, the principal can maximally observe the revenues of the project to be high or low, if she chooses to acquire any information. Section 4 discusses the full model, which entails the principal being able to acquire more accurate information if she so desires. In this section the focus will be on two types of equilibria, the most informative equilibria and the equilibria where the principal has acquired full information. Section 5 compares these equilibria in terms of dynamics and outcome. Section 5 will also contain a comparison between the equilibria of my model and CS. I conclude in Section 6.

2. The model

The model includes two players, the principal P (she) and the agent A (he). Together, they have to decide on the implementation of a project. For ease of exposition, a principal-agent setting is used throughout the model of this paper. However, it should be noted that the model is also applicable to situations where there is no authority relationship between the players, such as the introductory example regarding the potential release of the patient from the psychiatric clinic. In this light, the project can also be thought of as a public good, such as a new highway, that could potentially be provided to the public, where two independent governmental institutions must decide on its merits for society. Additionally, an example which fits the traditional principal-agent setting better would be an investment opportunity faced by a firm, where a CEO together with a lower-level manager must judge the potential investment on its profitability.

The value of the project is dependent on two independent variables. The first variable represents the benefits or revenues resulting from the project, denoted by $r \sim U[0,1]$. The second variable $e \sim U[0,1]$ represent the costs or expenses associated with the project. The total value of the project thus amounts to $r - e$. The two players are able to obtain information regarding the values of r and e respectively. At the start of the game, P must decide on the accuracy of the information he receives on the value of r. This accuracy is determined through $s \in \mathbb{N}$. The interval $[0,1]$ on which r is distributed is divided into s subintervals of equal length. P then observes in which subinterval r falls. This particular subinterval $[\bar{r}_{t-1}, \bar{r}_t]$ is denoted by t, which is drawn from the set of feasible types $T = \{1, \dots, s\}$, with $\bar{r}_t = \frac{t}{s}$. Throughout this paper t is referred to as P's type. When $s = 1$, there is only one interval and therefore P will have no additional information. More accurate information is costlier; the cost of the information is c per extra subinterval, meaning that the total cost of the information acquisition amounts to $(s-1)c$. After learning her type, P's expectation of r, denoted by r_t, will be:

$$E[r|t,s] = r_t = \frac{2t-1}{2s} \tag{1}$$

On the other hand, A simply observes the value of e perfectly and without cost. e will hereafter be referred to as A's type. While P's choice of s is assumed to be common knowledge, both players' types remain private information. Additionally, no proof can be provided regarding the values of these variables. Figure 1 provides a visual representation of the information acquisition for both players. As an example, it is assumed that $r = \frac{9}{16}$ and $e = \frac{1}{3}$. The top line depicts the interval of r, where P has set $s = 4$. P does not observe the value of r, but instead observes $t = 3$. The bottom line depicts the interval on which e is distributed. The value of e is simply observed by A.

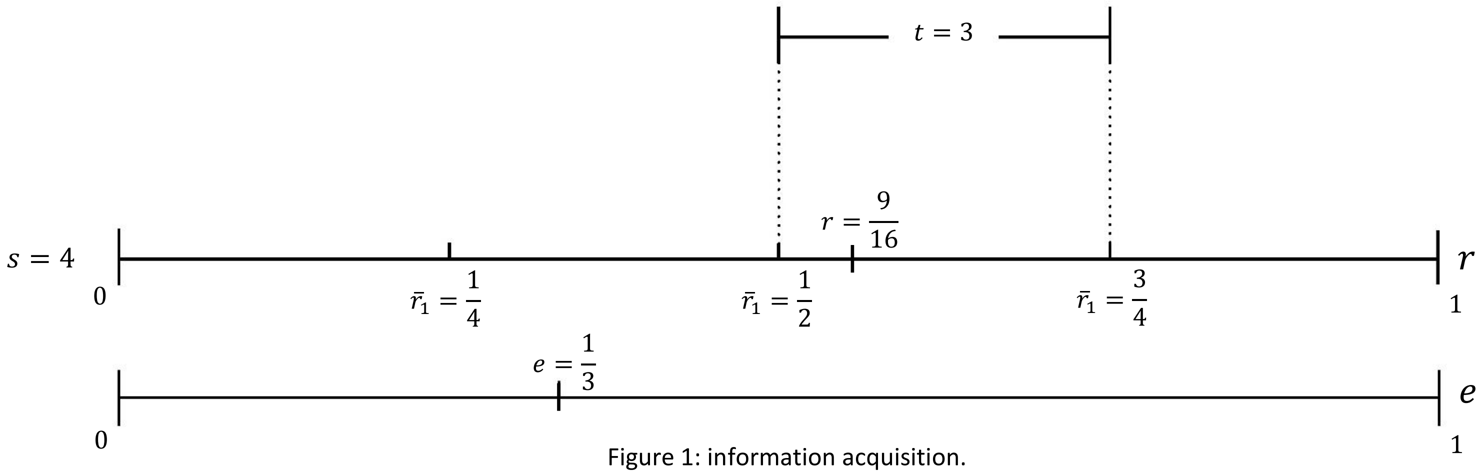

Figure 1: information acquisition.

The decision-making process can proceed in two different manners, hereafter referred to as the delegation and nondelegation scenario. Before the game starts, it is P's responsibility to decide which of the two scenarios will be effectuated. In the delegation scenario, P will send a cheap talk message m^P to A in order to convey information regarding the value of r. In a cheap talk model, the actual form of the messages is irrelevant for the players' payoffs; the information conveyed in these messages is only able to affect the equilibrium outcome through the implementation decision of the decision maker. This means that in equilibrium, the communication can essentially take any form, as long as the content of the message is clear to the other player. In order to avoid working with an infinitely big message space, it is assumed that the set of possible messages M^P is just big enough for P to be able to disclose all information she possesses to A, meaning that $M^P = T$. Adding any additional messages to M^P does not lead to any different outcomes of this scenario. Figure 2 depicts this for $s = 4$, where the number of messages is equal to the number of possible types. After receiving the message, A will make the final implementation decision $d^A \in \{0,1\}$ regarding the project, where $d^A = 0$ denotes A deciding against implementation of the project, and $d^A = 1$ denotes the decision to implement the project. Rejection of the project means that both players receive a payoff of 0, minus possible costs for the information acquisition regarding the value of r. Implementation of the project means that the project value is realized for both players.

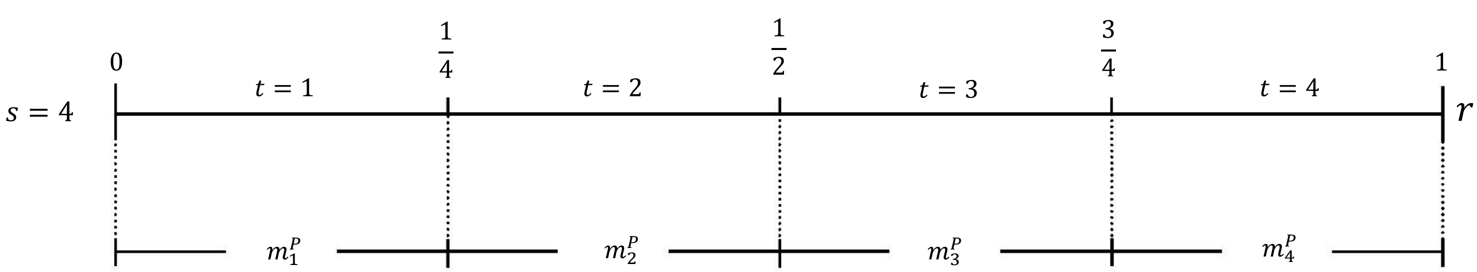

Figure 2: message space in the delegation scenario for $s = 4$.

In the nondelegation scenario, A will send a cheap talk message m^A to P regarding the value of e, after which P will make the implementation decision $d^P \in \{0,1\}$. Again, it is assumed that the message space of A is just big enough to be able to convey all useful information to P. Figure 3 depicts an example for $s = 4$. When $s = 4$, there exist four potential values of r_t: one for each different type. Since the implementation decision is binary, the maximum amount of relevant information P can receive from A is knowing whether e is smaller or bigger than r_t. However, since A does not observe t, he cannot simply communicate whether e is higher than r_t. Instead, when A wants to communicate the maximum amount of useful information, he has to use four messages to communicate that $e < r_t$ for $t \epsilon \{1,2,3,4\}$, plus one message communicating that e is even bigger than r_4. For example, message m_2^A tells P that the costs of the project are smaller than the expected revenues for $t = 2$ (and thus, for $t > 2$), but that the costs are higher than the expected revenues for $t = 1$. In general, the message space that is needed for the exchange of the maximum amount of relevant information in the nondelegation scenario is $M^A = T + 1$. Again, adding any additional messages does not lead to any different outcomes. Note that both M^P and M^A consist of the messages that the players *can* use to communicate in the delegation and nondelegation scenarios respectively, but whether this will actually be an equilibrium strategy depends on the degree of incongruence between the players' preferences.

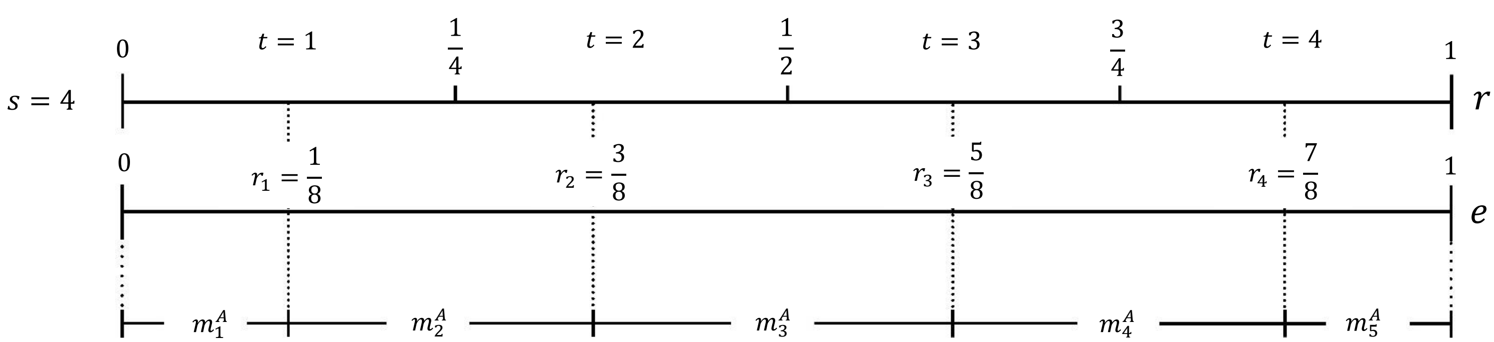

Figure 3: message space in the nondelegation scenario for $s = 4$.

With respect to P's implementation decision, $d^P = 0$ denotes no implementation of the project, whereas $d^P = 1$ means that P decides to implement the project. Whilst either sending the message or making the implementation decision, both players apply a strategy aimed at maximizing their own payoffs. It is assumed that P's payoff simply consists of the project value contingent on implementation, minus the cost of information acquisition. For $i \in \{P, A\}$, the payoff of P is equal to:

$$U^P = (r - e)d^i - (s - 1)c \qquad (2)$$

Regarding $A's$ payoff, it is assumed that his payoff is the same as P's payoff, apart from a bias towards wanting the project to be implemented. This bias can be interpreted as an additional motive of A to want to see the project implemented, not shared by others. In the example of the decision to release the patient from the psychiatric clinic, this bias might represent the director being under pressure for his clinic to perform well, where one of the performance indicators is the number of patients cured. In this case, signing the patient off as cured will look good on paper, and will additionally free up valuable resources towards other or new patients. This bias is represented by the constant b. A's payoff is therefore:

$$U^A = (r + b - e)d^i - (s - 1)c \tag{3}$$

Thus, the project will still be profitable for A when $r + b \geq e$, whilst P will only want to see the project implemented when $r \geq e$. The value of b is common knowledge. This creates a tradeoff. Either P grants decision-making rights to A, who is better informed than her but at the same time has a bias towards implementation, or she will make the implementation decision herself. Of course, this all depends on the communication between the players and how much information they are willing and able to share with one another.

The two scenarios are solved for Perfect Bayesian Equilibria (PBE). In the delegation scenario, such an equilibrium consists of a collection $(\sigma, \mu^P(s,t))$ of strategies for P and an approval strategy $\delta^A(e,s,m^P)$ and belief $G(t|s,m^P)$ regarding $P's$ type for A, such that:

1. for any e, s and m^P, decision $d^A = \delta^A(e,s,m^P)$ maximizes $A's$ expected value of (3), given belief $G(t|s,m^P)$;
2. for any s and t, sending message $m^P = \mu^P(s,t)$ maximizes $P's$ expected value of (2).
3. the accuracy of information $s = \sigma$ maximizes $P's$ expected value of (2).
4. belief $G(t|s,m^P)$ follows Bayes' rule on all information sets.

In the nondelegation scenario, a PBE consists of a collection $(\sigma, \delta^P(s,t,m^P))$ of strategies and belief $H(e|m^A)$ regarding the value of e for P and a messaging strategy $\mu^A(e,s)$ for A, such that:

1. for any s, t and m^A, decision $d^P = \delta^P(s,t,m^A)$ maximizes $P's$ expected value of (2), given belief $H(e|m^A)$;
2. for any e and s, sending message $m^A = \mu^A(e,s)$ maximizes $A's$ expected value of (3).
3. the accuracy of information $s = \sigma$ maximizes $P's$ expected value of (2).
4. belief $H(e|m^A)$ follows Bayes' rule on all information sets.

3. Restricted possibility for information acquisition

The two scenarios will first be analyzed for the relatively simple case where the maximum number of subintervals that can be chosen by P is exogenously restricted to two, essentially creating a model where P observes the revenues of the project to be either low or high. The four aspects that constitute a PBE, as discussed in the previous section, will hereafter be discussed consecutively per scenario. The sections below offer an intuitive and relatively informal analysis of the model. A more formal approach can be found in Appendix 1 and 2 for the delegation and nondelegation scenario respectively.

3.1 Delegation

In the delegation scenario, the implementation decision is made by A after receiving a cheap talk message from P. Using backward induction, the analysis starts at the last aspect of the players' strategies, which is the implementation decision. After that, the communication strategy of P is discussed, together with A's beliefs after receiving the message. Finally, P's decision regarding the accuracy of the information acquisition is analyzed.

3.1.1 A's decision strategy

After the information acquisition and the communication have taken place, A will implement the project if he expects the project to yield him a positive payoff. From (3) it can be seen that this will only be the case if the costs of the project are less than the expected revenues plus the value of A's bias, meaning that:

$d^A = 1$ if $e < E[r|m^P, s] + b$

$d^A = 0$ otherwise

3.1.2 Communication from P to A

Next, the communication between P and A is analyzed. It is assumed for now that P has set $s = 2$, such that there actually is some information to be communicated. In the next section it will be discussed when P has an incentive to deviate from $s = 2$.

The decision strategy described in the previous section is anticipated by P. A will implement the project if the costs are lower than the expected revenues plus A's bias, meaning that for P, conditional upon implementation of the project, the expected value of e equals:

$$E[e|d^A = 1] = \frac{1}{2}E[r|m^P, s] + \frac{1}{2}b$$

Furthermore, P knows that the probability that A implements the project is equal to the probability that the costs are smaller than the expected revenues plus A's bias. Since e is uniformly distributed on the interval [0,1], this yields:

$$P(d^A = 1) = P(e < E[r|m^P, s] + b) = E[r|m^P, s] + b$$

These last two expressions are dependent on the expected value of r from A's perspective, which is in turn dependent on the message received. From P's perspective, the expected value of r for the low type will be $r_1 = \frac{1}{4}$ and for the high type $r_2 = \frac{3}{4}$. Since $s = 2$, if P wants to convey any information to A, her only option is that both types will send messages to A wherein they truthfully reveal their types. Let these messages be denoted by m_t^A, where $t \in \{1,2\}$. In Appendix 1 it is shown that this equilibrium only holds for $b \leq \frac{1}{4}$. A visual representation of this communication strategy by P is provided in Figure 4.

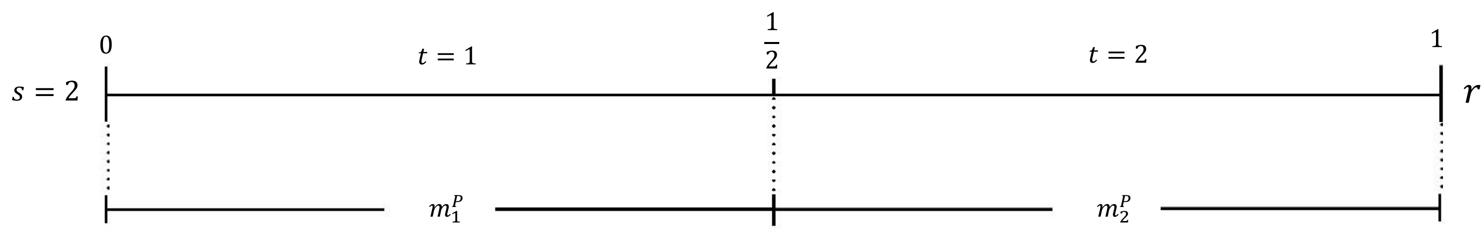

Figure 4: Communication by P for $s = 2$ and $b \leq \frac{1}{4}$

The reason P is not able to exploit a more nuanced communication strategy is because of the limitedness of her information. Since P only knows whether her type is high or low, those will be the only messages she will be able to use. After all, there is no point in stating "my type is medium" when the receiver of the communication is aware that you have no way of knowing this. Both the low and the high type will therefore have to disclose their types honestly if any information is to be conveyed. Logically, when either of the two types has an incentive to deviate and send the other type's message, both types will pool and no information will be conveyed. This happens when the players' preferences run too far apart, which is at $b > \frac{1}{4}$. The high type of P will then want to send the low type's message in order to counteract A's eagerness to implement the project.

Revisiting A's decision strategy, the communication described in this section means that A has the same information as P regarding r, meaning that $E[r|m^P, s] = r_t$. The decision strategy of A is therefore equal to:

$d^A = 1$ if $e < r_t + b$

$d^A = 0$ otherwise

Finally, the ex ante payoff of both players in the equilibrium with $s = 2$ is:

$$E[U^P|s = 2] = \frac{5}{32} - \frac{1}{2}b^2 - c \quad (4)$$

$$E[U^A|s = 2] = \frac{5}{32} + \frac{1}{2}b + \frac{1}{2}b^2 - c \quad (5)$$

Unsurprisingly, P's payoff has a negative relationship with b. The negative effect of b is due to the cases where $r_t < e < r_t + b$, for which $d^A = 1$, representing a distortion in the decision making from P's perspective. Conversely, A's payoff has a positive relationship with b, since the value of b is added to A's payoff whenever the project is implemented. Note that, given the available information, the decision making in this equilibrium is optimal for A. Indeed, even if A would be able to do the information acquisition regarding the revenues himself, meaning that P is cut out of the whole process, the results would not be different.

3.1.3 Information acquisition by P

Up until now it was assumed that $s = 2$. The remaining question is whether P has an incentive to deviate from choosing $s = 2$. When choosing $s = 2$, her expected payoff will be given by (4). When choosing $s = 1$, her expected payoff will be:

$$E[U^P|s = 1] = \left(\frac{1}{2} - \frac{1}{4} - \frac{1}{2}b\right)\left(\frac{1}{2} + b\right) = \frac{1}{8} - \frac{1}{2}b^2$$

Therefore, P will choose $s = 2$ if $c < \frac{1}{32}$ and $s = 1$ otherwise.

To summarize, given $s \in \{1,2\}$, the following equilibrium exists:

- P's decision on the amount of information acquisition:
  - $\sigma = 1$ if $c > \frac{1}{32}$ or $b > \frac{1}{4}$
  - $\sigma = 2$ if $c \leq \frac{1}{32}$ and $b \leq \frac{1}{4}$

- P's messaging strategy:
  - P sends a truthful message about her type to A: $\mu^P(s,t) = m_t^P$

- A's decision strategy:
  - $\delta^A(e,s,m^P) = 1$ If $e < r_t + b$
  - $\delta^A(e,s,m^P) = 0$ otherwise

In the original CS-model, the sender has perfect information. Barring credibility problems, this means the sender is able to essentially send an infinite number of messages regarding the value of her type. At first glance, the sender in the delegation scenario (P) might seem to be at a disadvantage compared to the sender from CS, since in my model the sender is not able to influence the decision maker (A) towards making a more favorable decision for herself by having a more nuanced communication strategy. However, in CS, this temptation to influence the receiver leads to a situation where communication needs to be relatively coarse, since the communication would not be credible otherwise. The results obtained here suggest that the limited possibilities of the sender in terms of communication might not be a disadvantage, since in both my model as the CS-model, two distinct messages are possible in equilibrium for equal levels of incongruency between the players' preferences, namely $b \leq \frac{1}{4}$. In fact, the resulting equilibrium described in my model contains two messages that represent an equal-sized subinterval, whereas in the CS-model the messages are unbalanced by default. This means that the messages used in the equilibrium described in this section contain more information than the messages used in CS. In general, given any number of messages, the use of the messages is more efficient if the subintervals belonging to these messages are more balanced.[5] This notion, and the comparison with the CS model, will be further examined in Section 4, where the model is analyzed without the restriction on the amount of information that P can acquire.

3.2 Nondelegation

The implementation decision in the nondelegation scenario is made by P herself. In order to make a more informed decision, P will ask A to inform her on the value of e, which A does by sending message m^A. The analysis is again based on backward induction, meaning that P's decision strategy is discussed first. It is still assumed, for now, that $s \in \{1,2\}$.

[5] As an example, two messages that are sent by an equal number of types such that the interval $[0,1]$ is divided evenly, convey more information than when the two messages represent $\frac{3}{4}$ and $\frac{1}{4}$ of the interval respectively. The first case has variance $\int_0^{\frac{1}{2}}(v-\frac{1}{4})2\,dv+\int_{\frac{1}{2}}^{1}(v-\frac{1}{4})2\,dv=\frac{1}{24}$, whilst the second case has variance $\int_0^{\frac{3}{4}}(v-\frac{3}{8})\frac{4}{3}\,dv+\int_{\frac{3}{4}}^{1}(v-\frac{7}{8})4\,dv=\frac{5}{96}$. Since $\frac{5}{96}>\frac{1}{24}$, this means that the balanced messages convey more information.

3.2.1 P's decision strategy

P will implement the project if he expects the project to be profitable to her. From (1) and (2) it can be seen that this will be the case if the expected revenues of the project exceed the expected costs. Thus, P's decision strategy will be:

$d^P = 1$ if $E[e|m^A] < r_t$

$d^P = 0$ otherwise

3.2.2 Communication from A to P

At the time P receives the message from A, P will know her own type and will compare the resulting r_t to the expected value of e. The latter is dependent on the message received. It is again assumed, for now, that $s = 2$. This means that the content of A's message can result in one of the following three reactions by P:

1. The expected value of e is low enough for P to implement the project, even if her type is low.
2. The expected value of e is only low enough for P to implement the project when her type is high and thus, she will reject the project when her type is low.
3. The expected value of e is too high for any type of P to implement the project.

Let the messages that prompt reaction 1, 2 and 3 be denoted by m_1^A, m_2^A and m_3^A. In Appendix 2 it is shown that in this equilibrium, A will send m_1^A when $e \in [0, \frac{1}{4} + b]$, m_2^A when $e \in [\frac{1}{4} + b, \frac{3}{4} + b]$, and m_3^A when $e \in [\frac{3}{4} + b, 1]$. This messaging strategy is depicted in Figure 5, where it can be seen that A divides the interval of e in three partitions, each belonging to a separate message. The boundaries of these messages are equal to $r_t + b$ for $t \in \{1,2\}$. Additionally, this equilibrium, just like the equilibrium in the delegation scenario, is only feasible when $b \le \frac{1}{4}$. When $b > \frac{1}{4}$, the expected value of the cost upon observing m_1^A and m_2^A is too high for P to behave as described under reaction 1 and 2 above.

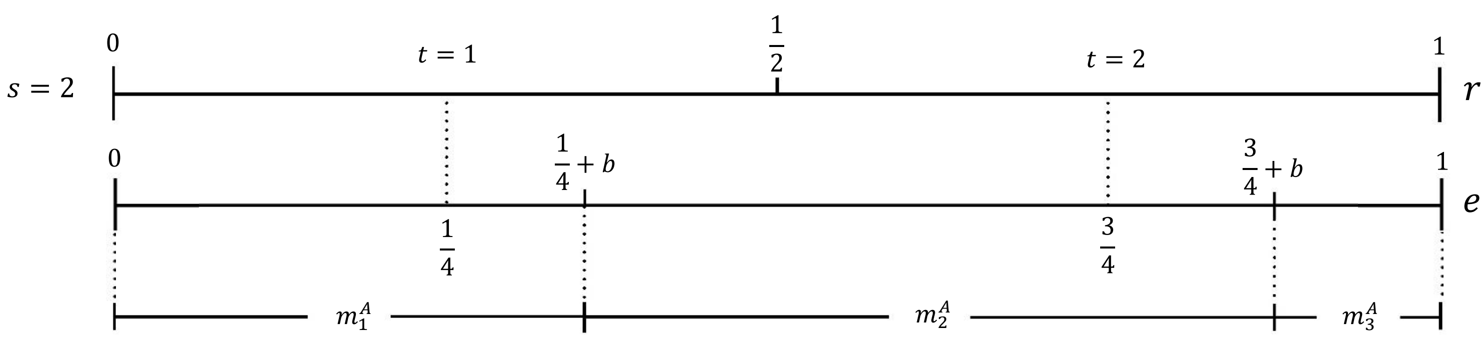

Figure 5: Communication by A for $s = 2$ and $b \le \frac{1}{4}$

Additionally, in Appendix 2 it is shown that other equilibria, where A only sends two messages instead of three, also require $b \leq \frac{1}{4}$. The three-message equilibrium is superior in terms of information transmission and subsequent decision making, making it the more logical equilibrium to focus on.

The dynamics here are different than the delegation scenario. In the delegation scenario, the sender of the information (P) had rather limited information on the value of r, meaning that she was equally limited in the maximum amount of information that she could convey. In the nondelegation scenario, the sender (A) has perfect information regarding the value of e. This means that he could essentially send any value between 0 and 1 to P. However, P knows that A has an incentive to understate the value of the costs, meaning that messages with very specific values of e are not credible. Instead, just as in the CS-model, A will have to send messages stating that e is in a specific partition. These partitions have to be big enough to ensure that it becomes too expensive for A to understate the value of e by sending the message belonging to a lower partition. P compares the expected revenues to the expected costs in order to make the implementation decision. This means that the messages sent by A will have to be tailored to the information that P possesses on the expected revenues of the project. P knows whether his type is low, meaning that the expected revenues are $\frac{1}{4}$, or high, with expected revenues of $\frac{3}{4}$. It is therefore of no use when, for example, A sends two messages, where one states 'the costs are in the interval $[\frac{1}{4}, \frac{1}{2}]$' and the other states 'the costs are in the interval $[\frac{1}{2}, \frac{3}{4}]$', since for both messages P will only implement the project when his type is high, resulting in the exact same outcome. The maximum number of messages that is not outcome-equivalent is three, which is the communication described above.

When comparing the communication described here to the communication in the delegation scenario, it stands out that for an equal bias, more messages can be sent. The partition belonging to the second message in the nondelegation scenario is of equal size as either of the two messages in the delegation scenario. Hence, the communication under nondelegation conveys more information on the value of e than the communication under delegation on the value of r, since the size of the partition of the remaining message in the delegation scenario is divided over two messages in the nondelegation scenario.

The payoffs for both players that follow from this communication are as follows:

$$E[U^P | s = 2] = \frac{5}{32} - \frac{1}{2}b^2 - c \quad (6)$$

$$E[U^A | s = 2] = \frac{5}{32} + \frac{1}{2}b + \frac{1}{2}b^2 - c \quad (7)$$

These payoffs are the same as under the delegation scenario with $s = 2$. The same distorting effect on the decision making from P's perspective can thus be observed in this scenario.

3.2.3 P's decision on the information acquisition

For P, choosing $s = 2$ will yield her the expected payoff given by (6). The alternative to $s = 2$ is the situation where no information is acquired by P, meaning that $E[r] = \frac{1}{2}$. For $b < \frac{1}{2}$, A can still communicate some information to P. Dependent on the value of e, A can send either m_1^A, when $e \in [0, \frac{1}{2} + b]$, or m_2^A, when $e \in [\frac{1}{2} + b, 1]$. In turn, P will only implement the project after receiving m_1^A, which will yield her:

$$E[U^P | m^A, s = 1] = \left(\frac{1}{2} - \frac{1}{4} - \frac{1}{2}b\right)\left(\frac{1}{2} + b\right) = \frac{1}{8} - \frac{1}{2}b^2$$

meaning that, just like in the delegation scenario, P will choose $s = 2$ if $c \leq \frac{1}{32}$ and $s = 1$ otherwise.

To summarize, given $s \in \{1,2\}$, the following equilibrium exists:

- P's decision on the amount of information acquisition:
  - $\sigma = 1$ if $c > \frac{1}{32}$ or $b > \frac{1}{4}$
  - $\sigma = 2$ if $c \leq \frac{1}{32}$ and $b \leq \frac{1}{4}$

- A's messaging strategy, if $\sigma = 2$:
  - $$\mu^A(e, s) = \begin{cases} m_1^A & \text{If } e \in [0, \frac{1}{4} + b] \\ m_2^A & \text{If } e \in [\frac{1}{4} + b, \frac{3}{4} + b] \\ m_3^A & \text{If } e \in [\frac{3}{4} + b, 1] \end{cases}$$

- P's decision strategy:
  - $$\delta^P(s, t, m^A) = 1 \text{ if } \begin{cases} t = 1 \text{ and } m^A = m_1^A \\ t = 2 \text{ and } m^A \epsilon \{m_1^A, m_2^A\} \end{cases}$$
  - $\delta^P(s, t, m^A) = 0$ otherwise

In conclusion, when the principal is limited to $s \in \{1,2\}$, the decision-making processes of the two scenarios yield equal results for both players. Under delegation, A is the one to make the implementation decision. In the equilibrium where $s = 2$ and informational communication has taken place, A knows both the exact value of the costs and P's type. This means that A possesses all the available information regarding the project value before making the implementation decision. However, apart from the cases where $e < r_t$, A will also implement the project for situations where $r_t < e < r_t + b$, which is detrimental for P's payoff. In the nondelegation scenario, P is the one to make the implementation decision. She knows her type but has somewhat limited information on the value of e since she only knows in which of the three partitions e falls. This is also where A's bias comes into play. The three messages sent by A only inform P on whether the costs of the project are smaller or larger than $r_t + b$. If the costs are lower than $r_t + b$, P will implement the project, since the expected value of the costs is then lower than the expected revenues of the project. Thus, due to the design of the messages, P will also inadvertently implement the project for some cases where the costs are higher than the revenues, or $r_t < e < r_t + b$.

4 Unrestricted possibility for information acquisition

The analysis in the previous section was restricted to the situation where P could choose s to be either 1 or 2, meaning that at best P could observe whether the revenues of the project were 'high' or 'low'. This section describes the more general case, where $s \in \mathbb{N}^+$. Thus, in this section P is able to set any number of subintervals and thereby determine the accuracy of the acquired information more freely, bearing in mind that more accurate information is, of course, still more expensive.

Allowing $s \in \mathbb{N}^+$ introduces a problem, however. Like many other cheap talk models, the model of this paper has numerous equilibria that satisfy the requirements of a PBE. These equilibria include separating equilibria, partial pooling equilibria and babbling equilibria. A separating equilibrium in the delegation scenario is an equilibrium in which each type of P sends a unique message to A, thereby always revealing her type. Letting N denote the number of informational messages that is being used in equilibrium, this means that in a separating equilibrium $N = s$. This is different for a partial pooling equilibrium, in which multiple types of P send the same message to A, but still different messages are used by different groups of types. A partial pooling equilibrium can therefore be characterized by $1 < N < s$. Finally, a babbling equilibrium is an equilibrium in which no information is conveyed, because the message that is sent by P is independent of its type. In Section 4.1, the analysis will first focus on the separating equilibria, since this yields the highest number of messages relative to the cost of information and can therefore be expected to yield the highest payoffs. This equilibrium will hereafter be referred to as the 'most informative equilibrium' of the delegation scenario. In Section 4.2 a different equilibrium will be analyzed, namely where P sets $s \to \infty$. The resulting equilibrium will hereafter be referred to as the 'partial pooling equilibrium' of the delegation scenario. Ex ante, this equilibrium seems attractive, since P obtains perfect information regarding the revenues of the project. However, as will be shown, this actually leads to a very limited capacity to communicate information, much in the same way as the CS-model.

In the nondelegation scenario, in which A perfectly observes e and sends a message to P regarding its value, a separating equilibrium is technically not possible for $b > 0$. After all, a separating equilibrium means, by definition, that A would reveal the exact value of e to P, which would not be credible for even the smallest bias. Instead, there is an equilibrium in the nondelegation scenario that is equivalent to the most informative equilibrium in the delegation scenario. This is the equilibrium where the number of messages being used by A is one greater than the number of subintervals set by P, or $N = s + 1$. This equilibrium is equivalent to the most informative equilibrium of the delegation scenario in the sense that it utilizes the maximum number of meaningful messages, given s. Therefore, it will hereafter also be referred to as the 'most informative equilibrium' of the nondelegation scenario and will be discussed in Section 4.3. In Section 4.4, the partial pooling equilibrium with $s \rightarrow \infty$ will again be analyzed, but this time under the nondelegation scenario. This equilibrium will hereafter simply be referred to as the 'partial pooling equilibrium' of the nondelegation scenario. The outcomes of these different equilibria in the delegation and nondelegation scenarios will be compared to each other in Section 5.

4.1 Most informative equilibrium under delegation

4.1.1 A's decision strategy

A's decision strategy is unaffected by the fact that $s \in \mathbb{N}^+$, Thus, he will implement the project if the costs of the project are lower than the expected revenues, given m^P, plus his bias. He will reject the project otherwise. As stated previously, the most informative equilibrium in the delegation scenario is a separating equilibrium, which means that P honestly reveals his type to A. This means that $E[r|m^P, s] = E[r|t, s] = r_t$. A's decision strategy is therefore:

$d^A = 1$ if $e < r_t + b$

$d^A = 0$ otherwise

4.1.2 Communication from P to A

Let a type t send message m_t^P. For this to hold, a type t must prefer sending m_t^P over m_{t-1}^P. The reason for solely considering a deviation to m_{t-1}^P is because A is more eager to implement the project than P, due to A's bias. Therefore, A might implement the project, even though the project might not be profitable for P. This causes P to be tempted to downplay the expected revenues, such that A will only implement the project when it is actually profitable to P. Hence, P has nothing to gain by communicating m_{t+1}^P. Additionally, as b increases, P will want to downplay the expected value of r with small amounts first, meaning that she will first be inclined to deviate to m_{t-1}^P rather than m_{t-2}^P or any other message, at which point the separating equilibrium will already break down. The formal proof of this equilibrium is shown in Appendix 3, considering both the communication and P's choice of s. A more intuitive exposition follows here.

The question is how much subintervals can be set by P whilst still supporting a separating equilibrium. From P's perspective, A's bias causes him to be too eager to implement the project. P's reaction to that would be to downplay the expected revenues of the project. For any given bias, P would optimally like to downplay the expected revenues by the exact value of the bias. In the hypothetical case where A would believe the downplayed value communicated by P, A would then proceed to implement the project only when $r_t - b + b > e$, which is the optimal decision from P's perspective. The problem with this is that A will not believe the value communicated by P. Instead, A will adjust his beliefs because he knows P has an incentive to lie, to which P will react by downplaying the value of the expected revenues even more, to which A will adjust his beliefs even more, etc. This does not lead to an informative equilibrium. For P to commit herself to telling the truth in equilibrium, the costs of lying about her type need to be sufficiently high. This can be achieved by keeping the intervals observed by P intentionally coarse, such that downplaying the expected revenues by sending the message belonging to the type that is one lower, m_{t-1}^P, is too big of a deviation from the truth. This also means that a lower bias will allow for finer subintervals and thus for more accurate information acquisition by P. In Appendix 3 it is shown that the most informative equilibrium is feasible when $b < \frac{1}{2s}$. Put differently, letting $\bar{s}$ denote the maximum number of subintervals that supports the most informative equilibrium, it follows that:

$$\bar{s} = \left\lfloor \frac{1}{2b} \right\rfloor \quad (8)$$ [6]

4.1.3 P's decision on the information acquisition

The value of $\bar{s}$ is the maximum number of subintervals that supports the most informative equilibrium. Whether P will also set $s = \bar{s}$ is dependent on the cost of information acquisition, which is c per extra subinterval. Let thee ex ante payoff of both players in the most informative equilibrium be denoted by $U_{MI}^i(s)$. This is the expected utility of the players as a function of s, before P's type is revealed, yielding:

$$U_{MI}^P(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) - \frac{1}{2}b^2 - (s-1)c$$

$$U_{MI}^A(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) + \frac{1}{2}b + \frac{1}{2}b^2 - (s-1)c$$

b's effect is twofold. As can be seen by (8), b limits the amount of information that can be acquired. Additionally, b distorts the decision making by A. Remarkably, the degree to which the latter effect affects the payoffs is independent of the number of messages used in equilibrium. This distorting effect of b is caused by the implementation of the project for cases where $r_t < e < r_t + b$. Indeed, this will occur with a probability equal to the value of b regardless of P's type and the value of s. As will be shown below, this is different for partial pooling equilibria.

[6] The notation $y = \lfloor x \rfloor$ denotes the so-called *floor function* of x, which means that y takes the value of the greatest integer smaller than or equal to x. For example $y = \lfloor 2.86 \rfloor = 2$. Similarly, as will be used later on in this paper, $y = \lfloor x \rceil$ denotes the *nearest integer function* of x, which means that y takes the value of the integer nearest to x. For example $y = \lfloor 2.86 \rceil = 3$.

P's optimal choice of s is a tradeoff between the costs of the information acquisition and better decision making by A. At the same time, P is limited by $\bar{s}$, since for any $s > \bar{s}$ a separating equilibrium will not be possible. If the costs of the information acquisition are sufficiently low, P will set $s = \bar{s}$. If the costs are too high however, P will set s to the nearest integer of the value where the marginal benefits of an extra subinterval are equal to the costs, which is $s^{opt} = \sqrt[3]{\frac{1}{12c}}$. This yields the following value for s^*, which is the optimal value for s in the most informative equilibrium:

$$s^* = min\left(\left\lfloor \sqrt[3]{\frac{1}{12c}} \right\rceil, \left\lfloor \frac{1}{2b} \right\rfloor\right)$$

Interestingly, since A has all the available information at the time of making the implementation decision, he could not make a better decision even if he could observe the expected revenues himself, given s. After all, in this most informative equilibrium, all types of P honestly reveal their type to A. Moreover, in the cases where $s^{opt} < \bar{s}$, A could not have done better himself, even if he was able to control the information acquisition himself, since in those cases the limitation on the information acquisition are not caused by the discrepancy between the players' preferences. The outcome of the most informative equilibrium is therefore rather favorable towards A.

In short, the most informative equilibrium in the delegation scenario with $s \in \mathbb{N}^+$ is as follows:

- P's decision on the amount of information acquisition:
  - $\sigma = min\left(\left\lfloor \sqrt[3]{\frac{1}{12c}} \right\rceil, \left\lfloor \frac{1}{2b} \right\rfloor\right)$

- P's messaging strategy:
  - P sends a truthful message about her type to A: $\mu^P(s,t) = m_t^P$.

- A's decision strategy:
  - $\delta^A(e,s,m^P) = 1$ if $e < r_t + b$
  - $\delta^A(e,s,m^P) = 0$ otherwise

4.2 Partial pooling equilibrium under delegation

Up until this point the focus was on the most informative equilibrium in the form of a separating equilibrium. A question that remains is whether such equilibria are the most optimal for both players in terms of payoffs, given the value of b. For example, for $\frac{1}{6} < b \leq \frac{1}{4}$, it follows from (8) that $\bar{s} = 2$ in the most informative equilibrium. However, both players might still be better off in an equilibrium where $s > 2$. In such an equilibrium not every type of P will communicate her exact type, meaning this equilibrium can be characterized as a partial pooling equilibrium. One apparent advantage of such an equilibrium is the fact that P acquires more information in comparison to the most informative equilibrium. In this section I will assume that $c = 0$ so that P can acquire any amount of information for free. It will further be assumed that, given this free information acquisition, P will acquire all the information she can, meaning that $s \to \infty$. This allows a direct comparison of the model of this paper with the model of CS. Setting $s \to \infty$ means that P has full information regarding the value of r, which is identical to CS, where the sender has full information regarding the variable on which he wants to communicate as well.

When P decides to set $s \to \infty$, the payoffs that both players receive depends on the number of messages that are used in equilibrium, which in turn depends on b. The number of messages that the players will want to use, denoted by N^*, can be found using a method similar to the method used in CS. The formal proof is shown in Appendix 4.

A's decision strategy has not changed, meaning that he will implement the project only if $e < E[r|m^P, s] + b$. Concerning the communication, if the value of b is not too large, the interval $[0,1]$ will be split up into different partitions, where all the types that fall into a certain partition $[t_{n-1}, t_n]$ send message m_n^P.[7] Types that are exactly on the boundary between two signals, hereinafter called boundary types, should then be indifferent between sending the two adjacent messages. For example, a type t_n should be indifferent between sending m_n^P and m_{n+1}^P. Just as in Section 4.1, any type of P would like A's expectations of the project's revenues to be somewhat lower than is actually the case. This gives P an incentive to downplay her type, where the ideal belief of A from P's perspective would be b lower than the actual revenues of the project. For example, when P has type t_n, the belief of A that would be optimal for P is: $E[r|m^P] = t_n - b$. In order for a boundary type to be indifferent between the two messages, A's belief following these messages should be equally far from this optimal belief. This means that the lower of the two messages must correspond to a bigger partition than the higher message, in order for the lower message to be a

[7] Note that, since $s \to \infty$, P will essentially observe r. However, for the sake of consistency, t is used to denote P's type, and t_n to denote boundary types.

greater deviation from the truth. This offsets the fact that P is already inclined to slightly downplay the expected revenues. The notion that $A's$ beliefs should be equally far from the optimal belief for either message yields:

$$t_n - b - \frac{t_{n-1} + t_n}{2} = \frac{t_n + t_{n+1}}{2} - (t_n - b)$$

$$t_{n+1} - t_n = t_n - t_{n-1} - 4b$$

It follows that that the lower message is sent by types that fall into an interval which is $4b$ larger than the interval belonging to the higher message. This means that the number of intervals that fit on the interval $[0,1]$ increases as b gets smaller. The maximum number of intervals, denoted by $\overline{N}$, equals:

$$\overline{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

Interestingly, this result is the same as in CS. The partial pooling equilibrium of my model thus allows the same number of messages to be credibly used when $s \to \infty$. The difference is that in CS the first partition is the smallest and each subsequent interval is $4b$ larger, whilst in this model the last partition is the smallest and every partition before that is $4b$ larger. This is because in the model of this paper, the sender has an incentive to understate her type, rather than to overexaggerate.

In order to determine whether $N = \overline{N}$ is actually optimal for both players, the payoffs of both players are needed. A player's ex ante expected payoff, denoted by $U^i_{PPE}(N)$, is the average payoff over all possible types as a function of the number of messages used. These payoffs are:

$$U^P_{PPE}(N) = \tfrac{1}{24}(4 - \tfrac{1}{N^2}) - \tfrac{1}{3}b^2 - \tfrac{1}{6}N^2b^2 \tag{9}$$

$$U^A_{PPE}(N) = \tfrac{1}{24}(4 - \tfrac{1}{N^2}) + \tfrac{1}{2}b + \tfrac{2}{3}b^2 - \tfrac{1}{6}N^2b^2 \tag{10}$$

Maximizing (9) and (10) yields the optimal number of messages for P and A respectively, denoted by N^{opt}, if they were totally free to set any number of messages. This yields $N^{opt} = \sqrt[4]{\frac{1}{4b^2}}$ for both players, which is somewhat lower than $\overline{N}$, but the difference is always less than 1. For example, when $b = \frac{1}{12}$, $\overline{N} = 3$ but $N^{opt} \approx 2{,}45$. However, since N needs to be an integer, the players have to choose between $N = \overline{N}$ or $N = \overline{N} - 1$. It turns out that the players are indifferent between using $\overline{N}$ or $\overline{N} - 1$ when b has the exact value that allows for the use of an extra message. As b decreases from this exact value, the players will strictly prefer to use the higher number of messages $\overline{N}$. Thus, setting $N = \overline{N}$ weakly dominates $N = \overline{N} - 1$, essentially meaning that:

$$N^* = \overline{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

This concludes the analysis of the partial pooling equilibrium in the delegation scenario. Section 5.2 will compare the most informative equilibria with the partial pooling equilibria. In the next section I proceed with the analysis of the most informative equilibrium and the partial pooling equilibria of the nondelegation scenario.

4.3 Most informative equilibrium under nondelegation

The nondelegation scenario is analyzed in this section for the case where P can set $s \in \mathbb{N}^+$. In this scenario, P makes the implementation decision herself, after receiving a message from A regarding the value of e. First, the focus will be on the most informative equilibrium under nondelegation, after which the partial pooling equilibrium with $c = 0$ and $s \to \infty$ will be discussed in Section 4.4.

4.3.1 P's decision strategy

P's decision strategy remains unchanged from the situation where $s \in \{1,2\}$. She will implement the project if the expected revenues exceed the costs she expects after observing A's message, and reject the project otherwise. More formally:

$d^P = 1$ if $E[e|m^A] < r_t$

$d^A = 0$ otherwise

4.3.2 Communication from A to P

As before, a relatively informal and intuitive analysis follows here. A more formal analysis is shown in Appendix 5.

In the most informative equilibrium of this scenario, the messages sent by A are adapted to the information that P possesses. In doing so, P will be informed by A whether the average costs are larger or smaller than the expected revenues for all $t \in T$. Consequently, the first message, m_1^A, means that e falls into the lowest category, which in turn means that all types of P will implement the project. The second message, m_2^A, means that all types of P will implement the project, except for $t = 1$, and so forth. In general, for every possible type there is a message stating that the expected costs are small enough for that type to implement the project, but not for one type lower. Since there is no point in having two messages that each communicate a different value of e, but for which all types of P react exactly the same, there is exactly one message for each type. Additionally, there is one message stating that the costs are too high for any type to implement the project. The number of messages that is necessary to achieve this is $N = s + 1$, where sending a message m_n^A means that only P with type $t \geq n$ will implement the project.

Each message m_n^A is sent by all A that observe e to be in between certain boundaries, denoted by $[e_{n-1}, e_n]$, where $e_0 = 0$ and $e_N = 1$. These boundaries are determined by A in such a way that P is only able to distinguish whether the costs of the project are smaller or larger than $r_t + b$ for all $t \in T$, meaning that $e_n = \frac{2n-1}{2s} + b$. For a sufficiently small value of b, the *expected* value of the costs, given $t = n$, will be lower than the expected revenues. Hence, when P receives m_n^A, she will implement the project when $t = n$ because on average the costs are lower than the expected revenues, given that message. However, in doing so, she will inadvertently implement the project for the cases where $r_t < e < r_t + b$. This is what makes this messaging strategy attractive for A.

As covered above, it should indeed be the case that all P with type $t \geq n$ will implement the project when receiving message m_n^A. This means that the expected costs of the project, given m_n^A, need to be smaller than the expected revenues when $t = n$. Since the boundaries of the messages are skewed relative to the expected revenues with the value of b, a higher bias means that the messages become more skewed. If the partitions belonging to the messages are too fine, the expected costs of the project will be higher than the expected revenues for $t = n$. By making the partitions sufficiently coarse, the relative impact of b on the expected costs associated with a given message is decreased, ensuring that the average costs of the project are smaller than the expected revenues for $t = n$. In other words, a lower bias allows for finer messages. In turn, this means that the most informative equilibrium can support a higher number of subintervals for a lower bias. Just as before, the value of b for which the most informative equilibrium is feasible is $b < \frac{1}{2s}$. Thus, again letting $\bar{s}$ denote the maximum number of subintervals that can be set by P which will support the most informative equilibrium, it follows that:

$$\bar{s} = \left\lfloor \frac{1}{2b} \right\rfloor$$

In short, as long as $s \leq \bar{s}$, upon observing $e_{n-1} < e < e_n$, where $e_n = \frac{2n-1}{2s} + b$, A will send message m_n^A to P, after which P will decide that $d^P = 1$ if $t \geq n$ and $d^P = 0$ otherwise. The maximum number of subintervals is the same as in the most informative equilibrium under the delegation scenario, meaning that in both scenarios the maximum amount of information that is observed by P is the same.

4.3.3 Information acquisition by P

Just as in the delegation scenario, the most informative equilibrium is possible as long as P sets $s \leq \bar{s} = \left\lfloor \frac{1}{2b} \right\rfloor$. Again, let a player's ex ante expected payoff in the most informative equilibrium be denoted by $U_{MI}^{i}(s)$. These payoffs are:

$$U_{MI}^{P}(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) - \frac{1}{2}b^2 - (s-1)c$$

$$U_{MI}^{A}(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) + \frac{1}{2}b + \frac{1}{2}b^2 - (s-1)c$$

These are the exact same payoffs as the ones resulting from the most informative equilibrium in the delegation scenario. Additionally, since $\bar{s}$ is also the same for the two scenarios, this means that all the results are identical. Thus, letting s^* denote the optimal value for s in the most informative equilibrium, this yields:

$$s^* = min\left(\left\lceil \sqrt[3]{\frac{1}{12c}} \right\rceil, \left\lfloor \frac{1}{2b} \right\rfloor\right)$$

Thus, the results obtained here are equal to the most informative equilibrium in the delegation scenario. A further comparison will follow in Section 5.1. In conclusion, the most informative equilibrium under the nondelegation scenario with $s \in \mathbb{N}^+$ is:

- P's decision on the amount of information acquisition:
  - $\sigma = min\left(\left\lceil \sqrt[3]{\frac{1}{12c}} \right\rceil, \left\lfloor \frac{1}{2b} \right\rfloor\right)$

- A's messaging strategy:
  - $\mu^A(e,s) = m_n^A$ if $e \in [e_{n-1}, e_n]$, where $e_n = \frac{2n-1}{2s} + b$

- P's decision strategy:
  - $\delta^P(t,s,m_n^A) = 1$ if $t \geq n$
  - $\delta^P(t,s,m_n^A) = 0$ otherwise

4.4 Partial pooling equilibrium under nondelegation

In Section 4.2 the partial pooling equilibrium was analyzed for the delegation scenario. This section will do the same for the nondelegation scenario. It is again assumed that $c = 0$ and $s \to \infty$, meaning that P will perfectly observe the value of r. The formal proof of this equilibrium is shown in Appendix 6.

P's decision strategy will be the same as under Section 4.3, meaning that she will implement the project only if $E[e|m^A] < r_t$. Turning towards the communication, the interval $[0,1]$ on which e is distributed will again be divided into different partitions, where a message m_n^A will be sent by all A that observe e to fall into partition $[e_{n-1}, e_n]$. Thus, e_n is the boundary type between sending m_n^A and m_{n+1}^A. Such a boundary type should be indifferent between sending m_n^A and m_{n+1}^A. In general, A will want to understate the value of the costs to P, since he is more eager to implement the project due to his bias. A perceived lower value of the costs will make P implement the project in more cases, even if the actual costs slightly exceed the revenues. Optimally, he wants P to believe that e is b lower than it actually is, because P will then implement the project whenever $e - b \leq r$, which is optimal for A. In order to make A with type e_n indifferent between sending m_n^A and m_{n+1}^A, P's belief upon receiving either of these messages should be equally far from this optimal belief for both messages. Similar to the partial pooling equilibrium in the delegation scenario, this yields:

$$e_{n+1} - e_n = e_n - e_{n-1} - 4b$$

Thus, the lower partition is always $4b$ larger than the higher partition. The maximum number of messages that can be used in this equilibrium is therefore equal to the maximum number in the partial pooling equilibrium of the delegation scenario:

$$\bar{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

which is again identical to the number of messages in CS. The payoffs of the players are not identical to the partial pooling equilibrium in the delegation scenario, however. This is due to P having different preferences than A, whilst having the same amount of information at the moment of making the implementation decision. More on this in Section 5.3. The resulting payoffs are:

$$U_{PPE}^{P}(N) = \frac{1}{24}\left(4 - \frac{1}{N^2}\right) + \frac{1}{6}b^2 - \frac{1}{6}N^2 b^2$$

$$U_{PPE}^{A}(N) = \frac{1}{24}(4 - \frac{1}{N^2}) + \frac{1}{2}b + \frac{1}{6}b^2 - \frac{1}{6}N^2 b^2$$

Maximizing these two payoffs with regard to N yields $N^{opt} = \sqrt[4]{\frac{1}{4b^2}}$ for both players. This means that both N^{opt} and $\overline{N}$ are the same as in the partial pooling equilibrium in the delegation scenario. Thus, given that $N \leq \overline{N}$ and that N is by definition an integer, the optimal number of messages that can be achieved in a partial pooling equilibrium is the same in the delegation scenario, which is:

$$N^* = \overline{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

This concludes the analysis of the partial pooling equilibrium in the nondelegation case. In the next section, I will compare the outcomes of the different equilibria discussed in Section 4.

5 Comparative statics

The previous sections have described a multitude of equilibria in the two scenarios of the model of this paper. I will refrain from touching upon the problem of equilibrium selection. However, even though there are numerous more equilibria in my model, there are some interesting insights to be gained from comparing the equilibria described thus far. In this section, these equilibria will be compared in terms of mechanisms and payoffs. The first three subsections will compare the different equilibria with each other. The last subsection will compare the outcomes of my paper to the paper of Crawford & Sobel (1982).

5.1 Most informative equilibria: delegation versus nondelegation

When comparing the most informative equilibrium of both scenarios, the results show that there is no difference in payoff for either player. In both scenarios, the players' payoffs are:

$$U_{MI}^{P}(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) - \frac{1}{2}b^2 - (s-1)c$$

$$U_{MI}^{A} = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) + \frac{1}{2}b + \frac{1}{2}b^2 - (s-1)c$$

The optimal number of subintervals for which the most informative equilibrium is still possible is equal between the two scenarios as well, being:

$$s^* = min\left(\left\lfloor\sqrt[3]{\frac{1}{12c}}\right\rfloor, \left\lfloor\frac{1}{2b}\right\rfloor\right)$$

It is assumed that the principal is able to determine who holds decision-making authority and who sends the communication. Since there is no difference in P's payoff between the scenarios, she will be indifferent between these equilibria in the two scenarios. Additionally, A is indifferent between the two scenarios as well. The reason for the equivalence in payoffs can be found in the information acquisition. In the most informative equilibrium, the information acquired by the principal is deliberately left relatively coarse. In the delegation scenario, this allows P to honestly reveal her type to A, which is all the information that P possesses. Therefore, A has obtained all the available information regarding the value of the project before making the implementation decision. He is therefore able to make the best possible decision for himself, given the amount of information acquired. In the nondelegation scenario, A does not communicate all the information he possesses to P, meaning that P remains unknowledgeable regarding the exact value of the costs. Instead, he constructs the messages in such a way that P is informed on whether the costs are small enough *on average* for the different possible types of P to implement the project. By doing so, A ensures that P also implements the project for those cases where this is profitable for A, but detrimental to P. This is the case when the expected revenues plus A's bias exceed the costs of the project, while the expected revenues alone are smaller than the costs. However, due to the coarseness of her information, P is not able to remedy this. Thus, the outcome of the most informative equilibrium is ideal for A in both scenarios, given the acquired information.

5.2 Partial pooling equilibrium: delegation versus nondelegation

The payoffs of player P and A respectively, resulting from the partial pooling equilibrium in the delegation scenario, are given by:

$$U_{PPE}^{P}(N) \ = \frac{1}{24}(4 - \frac{1}{N^2}) - \frac{1}{3}b^2 - \frac{1}{6}N^2 b^2$$

$$U_{PPE}^{A}(N) = \frac{1}{24}\left(4 - \frac{1}{N^2}\right) + \frac{1}{2}b + \frac{2}{3}b^2 - \frac{1}{6}N^2 b^2$$

whereas in the nondelegation scenario, the payoffs in the partial pooling equilibrium are:

$$U_{PPE}^{P}(N) = \frac{1}{24}\left(4 - \frac{1}{N^2}\right) + \frac{1}{6}b^2 - \frac{1}{6}N^2 b^2$$

$$U_{PPE}^{A}(N) = \frac{1}{24}(4 - \frac{1}{N^2}) + \frac{1}{2}b + \frac{1}{6}b^2 - \frac{1}{6}N^2 b^2$$

In both scenarios, the same number of messages can be communicated by the sender, being:

$$N^* = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

P's payoff is larger in the partial pooling equilibrium when she retains her decision-making authority. The opposite is true for A. From these results it follows that it is advantageous for a player to be able to make the final implementation decision rather than to be the sender of the communication. It is noteworthy that the total payoff of the two players combined is the same in both scenarios. This is due to the fact that the player that makes the implementation decision is equally informed between the two scenarios. In the delegation scenario, A perfectly observes the value of e and receives N^* messages regarding the value of r. Similarly, in the nondelegation scenario, P perfectly observes r and receives N^* messages regarding the value of e. Additionally, these N^*messages will have an equal distribution. Since r and e have an equal weight in determining the project's profitability, the efficacy of the implementation decision will be the same between the two scenarios, resulting in an equal total payoff. The difference in payoff per player between the two scenarios is thus the result of rent seeking. Given that A under delegation and P under nondelegation have equal information at the time of making the implementation decision, having the right to make the implementation decision means that that player is simply able to make a decision that is more favorable to him-/herself.

5.3 Most informative equilibrium versus partial pooling equilibrium

In the derivation of the partial pooling equilibrium, it was assumed that $c = 0$. In order to provide a fair comparison between the partial pooling equilibria and the most informative equilibria, the payoffs of both players in the most informative equilibria are needed for $c = 0$, which yields:

$$U_{MI}^{P}(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) - \frac{1}{2}b^2$$

$$U_{MI}^{A}(s) = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) + \frac{1}{2}b^2 + \frac{1}{2}b$$

Additionally, from Sections 4.1.3 and 4.3.3 it follows that P's optimal number of subintervals in the most informative equilibria is $s^* = min\left(\left\lceil\sqrt[3]{\frac{1}{12c}}\right\rceil, \left\lfloor\frac{1}{2b}\right\rfloor\right)$. When $c = 0$, it follows that $\sqrt[3]{\frac{1}{12c}} \to \infty$, meaning that $s^* = \left\lfloor\frac{1}{2b}\right\rfloor$. These results will be compared with the results of the partial pooling equilibria.

First, regarding the number of messages in each equilibrium, it turns out that the number of possible messages is (much) larger in the most informative equilibria. Looking at the delegation scenario, this result means that, even though P is better informed in the partial pooling equilibrium, the amount of information she can convey is considerably less than in the most informative equilibrium. This paradox is caused by the fact that in the most informative equilibrium, P has no way of committing herself to telling the truth. Suppose that the players would try to communicate in similar fashion to the most informative equilibrium, but after P has set $s = \infty$. In other words, they would try to set an amount of equal-sized messages which is equal to $N = \left\lfloor\frac{1}{2b}\right\rfloor$. Since $s = \infty$, P has exact information on the value of the revenues of the project, which is different from the most informative equilibrium. This makes it possible that P observes a value of r that is very close to the lower boundary of the partition belonging to a message. When this is the case, she will have an incentive to deviate to the lower message, which is why this cannot be an equilibrium. In order to be able to credibly send some information to A in a partial pooling equilibrium, the message intervals need to be coarser and each lower message needs to increase in size, resulting in less possible messages. On the contrary, in the most informative equilibrium where $s = N = \left\lfloor \frac{1}{2b} \right\rfloor$, P never knows exactly where in the subinterval the revenues fall, meaning that she will not have the same incentive to deviate. A similar line of reasoning applies when the partial pooling equilibrium and the most informative equilibrium of the nondelegation scenario are compared. In the most informative equilibrium, the messages of A are adapted to the information that P has acquired. Suppose that in the most informative equilibrium, A observes a value of e which falls on the border of the intervals of m_n^A and m_{n-1}^A, meaning that A is indifferent between sending these messages. When A decides to send m_{n-1}^A, all P with type $t \geq n-1$ will implement the project. In the case where $t = n-1$, P will inadvertently implement the project for the lower values of r that still fall in the subinterval of type $t = n-1$. If P had been able to observe the exact value of r, these values of r towards the lower end of the type boundary would not have led to implementation of the project. In the partial pooling equilibrium, this is exactly what happens. P perfectly observes the value of r, meaning that, given message m_{n-1}^A, a smaller and (on average) higher range of r leads to implementation. In turn, this causes A to strictly prefer m_{n-1}^A over m_n^A. Thus, similar to the delegation scenario, player A has no way of committing himself to a finer distribution of messages in the partial pooling equilibrium. This is remedied by a coarser distribution of messages with uneven intervals, where a given message interval is $4b$ larger than the interval of the message that is one higher.

Even if the number of messages in both types of equilibria would be the same, the payoffs for both P and A would still be larger in the most informative equilibria. In these equilibria it can be seen that b has a negative effect on P's payoff due to the inferior decision making by A (delegation scenario) or the biased communication (nondelegation scenario). Also, if $\bar{s} < s^{opt}$, b causes less than optimal information acquisition. In a partial pooling equilibrium, b has an additional effect on the payoffs. In Appendices 4 and 6, it is shown in equation $(A5)$ for the delegation scenario and $(A12)$ for the nondelegation scenario, that b has a negative relationship with d, which is the size of the smallest message interval. This means that, as b increases, the messages become less balanced in size. As described in Section 3.1.3, the use of the messages is more efficient if the partitions belonging to these messages are more balanced. The effect of A's bias in the partial pooling equilibrium is therefore threefold, as it disturbs the decision making in the delegation scenario, decreases the number of informational messages that can be used and skews the partitions of these messages.

5.4 Comparison with Crawford & Sobel (1982)

5.4.1 Most informative equilibria

Lastly, both the most informative equilibria and the partial pooling equilibria will be compared to the results of CS. When comparing the most informative equilibria to the model of CS, the results obtained in the model of this paper are rather surprising. In CS, the sender has full information, whereas the receiver has no information at all. Consequently, if the sender would be able to make the final decision himself, the payoffs would be higher *for both players* for any value of b for which informative communication could have taken place. This means that, due to the information he possesses, the sender would be able to make a decision that is so much better than the receiver's decision that it compensates for the difference in preferences. For higher values of b, at which point no communication is possible, the totally uninformed receiver would be better off taking the final decision, since the sender's decisions would be distorted too much. One could expect that this concept, where the ex ante most well-informed player makes the best decision for both players, should apply to the model of this paper as well. However, it turns out that in the most informative equilibria, there is no difference between delegation and nondelegation in terms of payoff for either player, even though ex ante P always has less information than A when $b > 0$.

Interestingly, even when $s = 1$, such that P obtains no information, there is no difference in outcome between the two scenarios. In the delegation scenario, when $b < \frac{1}{2}$ and $s = 1$, A receives no information from P and will simply implement the project if $e < \frac{1}{2} + b$. In the nondelegation scenario, when $b < \frac{1}{2}$ and $s = 1$, A is still able to send two messages in order to inform P whether the expected costs are lower or higher than the ex ante expected value of r, which is $\frac{1}{2}$. Both cases lead to identical payoffs for both players. When $b > \frac{1}{2}$, A is not able to credibly communicate any information in the delegation scenario. P will then have to make the implementation decision without any information whatsoever, meaning that the expected project value and therewith P's payoff is 0. In the nondelegation scenario with $b > \frac{1}{2}$, A will always implement the project, since e is always smaller than $\frac{1}{2} + b$. This means that the ex ante expected project value is again equal to 0.

The reason there is no difference in the players' payoffs between the most informative equilibria of the two scenarios has been discussed in Section 5.1. In my model, the only relevant information for the decision maker is whether the project is profitable to him or her. To determine this, the decision maker needs to be able to compare the costs of the project with the revenues of the project. By limiting the information on the revenues of the project, the value of the information on the costs of the project is limited as well. This is different in CS. In CS, the final decision is a continuous one, where the goal of the receiver is to match his action to the state of the world, which is observed by the sender. Consequently, the receiver will be able to adjust his decision making to even the most accurate information received from the sender. This means that the accuracy of the information is of higher importance, since the receiver will not be able to make an optimal decision unless he knows the exact value of the state of the world. Of course, the sender is not able to credibly communicate information with this level of accuracy due to his bias and therefore, both players are better off when the sender would be able to make the final decision himself, given the fact that he knows the exact state of the world.

5.4.2 Partial pooling equilibrium

Turning towards the partial pooling equilibria in comparison to CS, the first thing to note is that the same number of messages, with a similar distribution, can be used by the sender in both models. In CS however, the sender is prone to overstating the value of the variable on which he is trying to communicate information, whereas in my model the sender is prone to understating this value. This causes the first message in CS to have the smallest interval, and every next message interval to be $4b$ larger than the last. Conversely, in my model, the last message interval is the smallest and every prior interval is $4b$ larger than the higher interval.

In CS, the receiver has no information prior to any communication. On the contrary, in the partial pooling equilibria of my model, prior to any communication, the decision maker has an equal amount of information as the sender, since both players perfectly observe one of the two variables that determine the project value. In both the partial pooling equilibria and CS, the receiver of the communication will be able to adjust his decision making to even the most accurate information received from the other player.[8] Much in the same way as described in Section 5.3, this leads to equally limited information transmission in CS and the partial pooling equilibria.

Even though the communication is very similar between the partial pooling equilibria and CS, the outcomes are rather different. In CS, The receiver observes no information ex ante and receives limited information from the sender. Therefore, both players would be better off if the sender would simply make the final decision, thereby forgoing all communication. From the receiver's perspective, even though the sender will make suboptimal decisions due to his bias, it is still better if the sender makes the final decision, since the

[8] This is in contrast to, for example, the most informative equilibrium of the nondelegation scenario, where P (the receiver) is only able to adjust his decision making to information up until a certain accuracy; any communication that is more accurate is redundant. This is discussed earlier in Section 5.3

receiver has a lot less information. In the partial pooling equilibria of my model, both players are ex ante equally informed. The decision maker will therefore always be better informed than the sender at the time of the implementation decision. Thus, contrary to CS, the decision maker is never better off when the sender would make the implementation decision. Additionally, for either scenario, the sender is only better off if he/she could make the implementation decision for large values of b, namely $b > \sqrt{\frac{1}{32}} \approx 0.18$, for which $N^* = 2$. Any lower value of b, whether or not combined with additional messages, results in higher payoffs for both players when the implementation decision is made by the receiver of the communication.

6. Concluding remarks

While a principal, responsible for the potential implementation of a project, will often be able to obtain some insight in the project's profitability, she might simultaneously be dependent on an agent to provide her with more specific information. The aim of this paper was to research the optimal decision-making procedure in such a setting, in the presence of incongruent preferences between the principal and the agent. In order to do so, it was assumed that the principal is able to acquire costly information regarding the revenues of the project, whilst the costs of the project are simply observed by the agent. The results show that, whenever the players arrive at the most informative equilibrium, the principal will be indifferent between making the implementation decision herself and delegating the decision authority to the agent. In the case of delegation, the principal communicates all the information she acquired to the agent. This is not possible in the nondelegation case. Instead, the agent communicates to the principal in such a way that the principal will always make the optimal decision for the agent. The bias of the agent in the delegation case is exactly enough to offset the lack of information in the nondelegation case, leading to equal payoffs.

These most informative equilibria are only possible for a relatively limited amount of information acquisition by the principal. This allows for greater information transmission between the players and subsequently, for better decision making, as compared to the equilibria where the principle has acquired the maximum amount of information. This result even holds in the case where the cost of information acquisition is zero, which is in line with Bijkerk et al. (2018). Additionally, in this case where the principal has acquired the maximum amount of information, such that the agent and the principle are ex ante equally informed, it is advantageous for either player be the one to make the implementation decision.

The results of this paper suggest a slight departure from the results of Crawford & Sobel (1982) and Dessein (2002). In these models, only one player observes information prior to communication. In both papers, it would be advantageous for both players if the player with the information would be able to make the final decision, even though his preferences are biased. My model shows there is no difference in terms of outcome between the scenario where the better-informed player makes the implementation decision and the scenario where the less-informed player does, even if the less-informed player observes no information. The key assumption for this result is the design of the information acquisition. Because of this, the principal is able to keep her information relatively coarse, which allows the communication that is needed for the decision maker to compare the expected revenues of the project to its expected costs, irrelevant of which of the two players makes this decision.

My model thus provides a potential explanation for the centralization of decision authority in organizations whilst still assuming that lower level employees are potentially better informed. My results suggest that, in the case of the potential implementation of a project, the decision made by a manager will at least be as good as the decision of a subordinate would be. In cases where the manager is equally informed as the subordinate, the manager's decision would even be better than the subordinate's decision. Additionally, centralization would be even more effective if a manager is to verify the information of the subordinate to some extent, as is often the case in reality. This is in line with the practice in large, hierarchical companies, such as law firms, where the partners decide which cases to pursue, usually after the advice of one or more associates. In this light, a natural extension of my model is to analyze a similar framework with multiple senders, where each sender observes independent information on the value of the project at hand. In such a setting the principal can either make the implementation decision herself after receiving information from the agents or delegate the decision to one of the agents. Additionally, in the case of delegation, the question arises whether the principal should ask the agents to communicate to each other or whether the communication should flow through her.

Finally, the results of my model are based on two strong assumptions, being that the agent's bias is common knowledge and that the agent's information is exogenous. The results show that delegation is not profitable in the presence of endogenous information acquisition by the principal. This raises the question whether this result will hold when the agent's information acquisition is endogenized as well. Additionally, if it is assumed that the principal is able to observe the amount of information acquisition by the agent, another possible extension is to drop the assumption that the agent's bias is common knowledge. Apart from its intrinsic value, information acquisition by the agent will then also serve as a signal regarding the agent's bias.

References

Argenziano, R., Severinov, S., & Squintani, F. (2016). Strategic information acquisition and transmission. *American Economic Journal: Microeconomics, 8*(3), 119-155.

Austen-Smith, D. (1994). Strategic transmission of costly information. *Econometrica, 62*(4), 955-963.

Bijkerk, S. H., Delfgaauw, J., Karamychev, V., & Swank, O. H. (2018). Need to know? On information Systems in Firms (Tinbergen Institute Discussion Paper 2018-091/VII). Retrieved from Tinbergen website: https://www.tinbergen.nl/discussion-paper/2836/18-091-vii-need-to-know-on-information-systems-in-firms

Boersema, W. (2019, april 03). Minister Dekker door de mangel in beladen Faber-debat. *Trouw*. Retrieved from Trouw: http://www.trouw.nl

Che, Y.-K., & Kartik, N. (2009). Opinions as incentives. *Journal of Political Economy, 117*(5), 815-860.

Chen, Y. (2009). Communication with two-sided assymetric information. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1344818&download=yes

Crawford, V. P., & Sobel, J. (1982). Strategic Information Transmission. *Econometrica, 50*(6), 1431-1451.

Dessein, W. (2002). Authority and Communication in Organizations. *Review of Economic Studies, 69*(4), 811-838.

Dewatripont, M., & Tirole, J. (1999). Advocates. *The Journal of Political Economy, 107*(1), 1-39.

Dur, R., & Swank, O. H. (2005). Producing and manipulating information. *Economic Journal, 115*(500), 185-199.

Hidir, S. (2017). Information acquisition and credibility in cheap talk (CRETA discussion paper series No. 36). Retrieved from Centre for Research in Economic Theory and its Applications website: https://warwick.ac.uk/fac/soc/economics/research/centres/creta/papers/manage/36_-_creta_hidir.pdf

Ishida, J., & Shimizu, T. (2018). Cheap talk when the receiver has uncertain information. Advance online publication. *Economic Theory*. Retrieved from https://link.springer.com/journal/199

Ivanov, M. (2010). Informational control and organizational design. *Journal of Economic Theory, 145*(2), 721-751.

Lai, E. K. (2014). Expert advice for Amateurs. *Journal of Economic Behavior & Organization, 103*(C), 1-16.

Le Quement, M. (2009). Cheap talk, information acquisition and conditional delegation. Retrieved from http://ecares.ulb.ac.be/

Meijer, R. (2019, april 3). Kamerleden verzoeken minister Dekker af te treden: 'U beschermt criminelen maar niet de dochters van Nederland'. *De Volkskrant*. Retrieved from https://www.volkskrant.nl

Moreno de Barreda, I. (2010). Cheap talk with two-sided private information. Retrieved from https://www.researchgate.net/publication/228593956_Cheap_Talk_With_Two-Sided_Private_Information_Job_Market_Paper

Nieuwenhuis, M. (2019, april 18). Keihard rapport over GGZ-instellingen: ' Veiligheid patiënt en omgeving in gevaar'. *Algemeen Dagblad*. Retrieved from http://www.ad.nl

Pei, H. D. (2015). Communication with endogenous information acquisition. *Journal of Economic Theory, 160*, 132-149.

Sobel, J. (2013). Giving and receiving advice. In D. Acemoglu, M. Arellano & E. Dekkel (eds.). *Advances in economics and econometrics* (pp. 305-341). Cambridge, England: Cambridge University Press.

Watson, J. (1996). Information transmission when the informed party is confused. *Games and Economic Behavior, 12*(1), 143-161.

Appendices

Appendix 1: Delegation in the limited model

Consider the delegation scenario where $s = 2$ and P truthfully reveals her type by sending message m_t^P to A. For A, this results in the following expectations and decision strategy:

$E[r|m_1^P, s] = \frac{1}{4}$

$d^A(m_1^P) = 1$ if $e < \frac{1}{4} + b$

$d^A(m_1^P) = 0$ otherwise, and

$E[r|m_2^P, s] = \frac{3}{4}$

$d^A(m_2^P) = 1$ if $e < \frac{3}{4} + b$

$d^A(m_2^P) = 0$ otherwise.

Given the expected value of r, the decision strategy of A and the resulting expected value of e, $P's$ expected payoff, when revealing her type truthfully, is:

$$E[U^P|t = 1, m_1^P, s] = \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{2}b\right)\left(\frac{1}{4} + b\right) - c$$

$$E[U^P|t = 2, m_2^P, s] = \left(\frac{3}{4} - \frac{3}{8} - \frac{1}{2}b\right)\left(\frac{3}{4} + b\right) - c$$

For P to be willing to reveal her type it is necessary that for each of the types, sending the corresponding message will yield the highest payoff. If that is not the case, one of the two types of P will have an incentive to deviate by sending the different message, resulting in an equilibrium where no information is conveyed. Therefore, the separating equilibrium will hold if:

$$E[U^P|t = 1, m_1^P, s] - E[U^P|t = 1, m_2^P, s] \geq 0$$

$$E[U^P|t = 2, m_2^P, s] - E[U^P|t = 2, m_1^P, s] \geq 0$$

Starting with the first constraint, a type $t = 1$ sending message m_2^P yields:

$$E[U^P|t = 1, m_2^P, s] = \left(\frac{1}{4} - \frac{3}{8} - \frac{1}{2}b\right)\left(\frac{3}{4} + b\right) - c$$

which means that:

$$E[U^P|t=1, m_1^P, s] - E[U^P|t=1, m_2^P, s] = \frac{1}{8} + \frac{1}{2}b$$

$\frac{1}{8} + \frac{1}{2}b \geq 0$ is true for every value of b. Therefore, P never has an incentive to lie about her type when $t = 1$. Concerning the second constraint, a type $t = 2$ sending message m_1^P yields:

$$E[U^P|t=2, m_1^P, s] = \left(\frac{3}{4} - \frac{1}{8} - \frac{1}{2}b\right)\left(\frac{1}{4} + b\right) - c$$

which means that:

$$E[U^P|t=2, m_2^P, s] - E[U^P|t=2, m_1^P, s] = \frac{1}{8} - \frac{1}{2}b$$

$\frac{1}{8} - \frac{1}{2}b \geq 0$ yields $b \leq \frac{1}{4}$, meaning that honest communication from P regarding her type is only feasible for $b \leq \frac{1}{4}$, completing the equilibrium with regard to the communication.

The payoffs for both players in this equilibrium, given $s = 2$ will amount to:

$$E[U^P|t, m_t^P, s=2] = \frac{1}{2}E[U^P|t=1, m_1^P, s=2] + \frac{1}{2}E[U^P|t=2, m_2^P, s=2] = \frac{5}{32} - \frac{1}{2}b^2 - c$$

$$E[U^A|t, m_t^P, s=2] = \frac{1}{2}[U^A|t=1, m_1^P, s=2] + \frac{1}{2}E[U^A|t=2, m_2^P, s=2] = \frac{5}{32} + \frac{1}{2}b + \frac{1}{2}b^2 - c$$

Appendix 2: Nondelegation in the limited model

Consider the communication in the nondelegation scenario where $s = 2$. The messages sent by A can prompt one of the following reactions by P:

1. $d^P = 1$ for $t \in \{1,2\}$;
2. $d^P = 1$ for $t = 2$ and $D^P = 0$ for $t = 1$;
3. $d^P = 0$ for $t \in \{1,2\}$.

Let the messages that prompt reaction 1, 2 and 3 be denoted by m_1^A, m_2^A and m_3^A respectively. These three messages will result in the following expected payoffs for A:

1. $E[U^A|m_1^A, s=2] = \frac{1}{2}\left(\frac{1}{4} + b - e\right) + \frac{1}{2}\left(\frac{3}{4} + b - e\right) - c$
2. $E[U^A|m_2^A, s=2] = \frac{1}{2}\left(\frac{3}{4} + b - e\right) - c$
3. $E[U^A|m_3^A, s=2] = -c$

For A to send a certain message it is necessary that his payoff is higher than or equal to his payoff when sending either of the other two messages. Let m_1^A be sent by all A with types $e \in [0, e_1]$, m_2^A when $e \in [e_1, e_2]$ and m_3^A when $e \in [e_2, 1]$. When e is equal to the boundary between two intervals, A should be indifferent between sending the two adjacent messages. For $e = e_1$, this means that $E\left[U^A \middle| m_1^A, e_1, s = 2\right] = E\left[U^A \middle| m_2^A, e_1, s = 2\right]$, yielding $e_1 = b + \frac{1}{4}$. Similarly, when $e = e_2$, it should hold that $E\left[U^A \middle| m_2^A, e_2, s = 2\right] = E\left[U^A \middle| m_3^A, e_2, s = 2\right]$, yielding $e_2 = b + \frac{3}{4}$.

Upon observing m_1^A or m_2^A, P's expected value of e will be $E\left[e \middle| m_1^A\right] = \frac{1}{2}b + \frac{1}{8}$ and $E\left[e \middle| m_2^A\right] = b + \frac{1}{2}$ respectively. For P to implement the project upon receiving m_1^A when her type is $t = 1$, it should be the case that $\frac{1}{2}b + \frac{1}{8} \leq \frac{1}{4}$, meaning that $b \leq \frac{1}{4}$. The same is true – *mutatis mutandis* – for $t = 2$ receiving m_2^A. Thus, the equilibrium where $s = 2$ and A sends the three messages as described above is only feasible for $b \leq \frac{1}{4}$.

In the case where $b \leq \frac{1}{4}$ and $s = 2$, the following payoffs will be realized for P and A respectively:

$$E[U^P | t, m^A, s = 2] =$$

$$\frac{1}{2}E[U^P | t = 1, m_1^A, s = 2] + \frac{1}{2}\left(E[U^P | t = 2, m_1^A, s = 2] + E[U^P | t = 2, m_2^A, s = 2]\right) = \frac{5}{32} - \frac{1}{2}b^2 - c$$

$$E[U^A | t, m^A, s = 2] =$$

$$\frac{1}{2}[U^A | t = 1, m_1^A, s = 2] + \frac{1}{2}(E[U^A | t = 2, m_1^A, s = 2] + E[U^A | t = 2, m_2^A, s = 2]) = \frac{5}{32} + \frac{1}{2}b + \frac{1}{2}b^2 - c$$

I will now analyze possible alternative equilibria for $s = 2$ where only two messages are sent by A. First, I assume that P's reaction to messages m_1^A and m_2^A are the following:

1. $d^P = 1$ for $t \in \{1,2\}$;
2. $d^P = 0$ for $t \in \{1,2\}$.

This results in the following payoffs for A:

1. $E[U^A | m_1^A, s = 2] = \frac{1}{2}\left(\frac{1}{4} + b - e\right) + \frac{1}{2}\left(\frac{3}{4} + b - e\right) - c$
2. $E[U^A | m_2^A, s = 2] = -c$

Let m_1^A be sent by all A with types $e \in [0, e_1]$ and m_2^A with types $e \in [e_1, 1]$. When $e = e_1$, A should then be indifferent between sending m_1^A and m_2^A. $E\left[U^A \middle| m_1^A, e_1\right] = E\left[U^A \middle| m_2^A, e_1\right]$ yields $e_1 = b + \frac{1}{2}$. However, this will mean that a type $t = 1$ will not implement the project for m_1^A, since $r_t = \frac{1}{4} < E[e|m_1^A] = \frac{1}{2}b + \frac{1}{4}$. Therefore, this is not an equilibrium.

Secondly, the reactions to m_1^A and m_2^A are assumed to be the following:

1. $d^P = 1$ for $t \in \{1,2\}$;
2. $d^P = 1$ for $t = 2$ and $d^P = 0$ for $t = 1$.

Resulting in the following payoffs for A:

1. $E[U^A|m_1^A, s = 2] = \frac{1}{2}\left(\frac{1}{4} + b - e\right) + \frac{1}{2}\left(\frac{3}{4} + b - e\right) - c$
2. $E[U^A|m_2^A, s = 2] = \frac{1}{2}\left(\frac{3}{4} + b - e\right) - c$

Again, let e_1 be the border value between the two messages, meaning that $e_1 = b + \frac{1}{4}$. P with type $t = 1$ will only implement the project upon receiving m_1^A if $\frac{1}{4} \geq \frac{1}{2}b + \frac{1}{8}$, yielding $b \leq \frac{1}{4}$, just as in the equilibrium with three messages.

Lastly, the reactions to m_1^A and m_2^A are assumed to be the following:

1. $d^P = 1$ for $t = 2$ and $d^P = 0$ for $t = 1$;
2. $d^P = 0$ for $t \in \{1,2\}$.

resulting in the following payoffs for A:

1. $E[U^A|m_1^A, s = 2] = \frac{1}{2}\left(\frac{3}{4} + b - e\right) - c$
2. $E[U^A|m_2^A, s = 2] = -c$

The analysis here is similar to the previous equilibrium. For this equilibrium to be feasible it needs to hold that $b \leq \frac{1}{4}$.

Appendix 3: Most informative equilibrium under delegation in the full model

Consider the delegation scenario with $s \in \mathbb{N}^+$ where P honestly reveals her type. It follows from (3) that A will implement the project if $e < E[r|m^P] + b = r_t + b$. When a type t sends message m_t^P, it follows that $E[e|d^A = 1, m_t^P] = \frac{1}{2}(\frac{2t-1}{2s} + b)$ and $P(d^A = 1|m_t^P) = \frac{2t-1}{2s} + b$. For P, this means that revealing her type yields:

$$E[U^P|t, m_t^P, s] = \left(\frac{2t-1}{2s} - \frac{2t-1}{4s} - \frac{1}{2}b\right)\left(\frac{2t-1}{2s} + b\right) - (s-1)c$$

whilst deviating to m_{t-1}^P will yield P:

$$E[U^P \big| t, m_{t-1}^P, s] = \left(\frac{2t-1}{2s} - \frac{2t-3}{4s} - \frac{1}{2}b\right)\left(\frac{2t-3}{2s} + b\right) - (s-1)c$$

Thus, for P to honestly reveal her type it must hold that $E[U^P|t, m_t^P] - E[U^P \big| t, m_{t-1}^P] \geq 0$. Substracting the two payoffs yields:

$$E[U^P|t, m_t^P, s] - E[U^P \big| t, m_{t-1}^P, s] = \frac{1-2sb}{2s^2}$$

$\frac{1-2sb}{2s^2} \geq 0$ yields $b \leq \frac{1}{2s}$, meaning that b should be smaller than (or equal to) $\frac{1}{2s}$ for a separating equilibrium to be feasible. In other words, the maximum number of subintervals $\bar{s}$ that supports the most informative equilibrium is given by:

$$\bar{s} = \left\lfloor \frac{1}{2b} \right\rfloor$$

After P has observed her type and has sent a message to A, the payoffs of both players resulting from P revealing her type are given by:

$$E[U^P|t, m_t^P, s] = \left(r_t - \frac{1}{2}r_t - \frac{1}{2}b\right)(r_t + b) - (s-1)c = \frac{1}{2}{r_t}^2 - \frac{1}{2}b^2 - (s-1)c$$

$$E[U^A|t, m_t^P, s] = \left(r_t + b - \frac{1}{2}r_t - \frac{1}{2}b\right)(r_t + b) - (s-1)c = \frac{1}{2}{r_t}^2 + r_t b + \frac{1}{2}b^2 - (s-1)c$$

Substituting for (1) yields:

$$E[U^P|t, m_t^P, s] = \frac{(2t-1)^2}{8s^2} - \frac{1}{2}b^2 - (s-1)c \qquad (A1)$$

$$E[U^A|t, m_t^P, s] = \frac{(2t-1)^2}{8s^2} + \frac{2t-1}{2s}b + \frac{1}{2}b^2 - (s-1)c \qquad (A2)$$

Let the ex-ante payoff of both players in the most informative equilibrium be denoted by $U_{MI}^i(s)$. This is the expected utility of the players as a function of s, before P's type is revealed. In order to derive $U_{MI}^i(s)$, the average of the expected payoffs given in $(A1)$ and $(A2)$ is taken over all types t, yielding:

$$U_{MI}^P(s) = \frac{1}{s}\sum_{t=1}^{s}\left(\frac{(2t-1)^2}{8s^2} - \frac{1}{2}b^2\right) - (s-1)c = \frac{1}{24}\left(4 - \frac{1}{s^2}\right) - \frac{1}{2}b^2 - (s-1)c \qquad (A3)$$

$$U_{MI}^{A}(s) = \frac{1}{s}\sum_{t=1}^{s}\left(\frac{(2t-1)^2}{8s^2} + \frac{2t-1}{2s}b + \frac{1}{2}b^2\right) - (s-1)c$$

$$= \frac{1}{24}\left(4 - \frac{1}{s^2}\right) + \frac{1}{2}b + \frac{1}{2}b^2 - (s-1)c \qquad (A4)$$

Let s^{opt} denote the number of subintervals that P would set in the hypothetical case where $\bar{s}$ does not play a role. s^{opt}can be found by maximizing $(A3)$ and rounding off to the nearest integer, yielding, $s^{opt} = \left\lfloor \sqrt[3]{\frac{1}{12c}} \right\rceil$. Let s^* denote the optimal number of subintervals that can still support the most informative equilibrium. Accordingly, s^* will be equal to the lowest value of s^{opt} or $\bar{s}$, yielding:

$$s^* = min(\left\lfloor \sqrt[3]{\frac{1}{12c}} \right\rceil, \left\lfloor \frac{1}{2b} \right\rfloor)$$

Appendix 4: Partial pooling equilibrium under delegation in the full model

Consider a partial pooling equilibrium in the delegation scenario with $c = 0$ and $s \to \infty$. Let a message m_n^P be sent by all types in the interval $[t_{n-1}, t_n]$, where $t_0 = 0$ and $t_N = 1$. These so-called boundary types t_n should be indifferent between sending m_n^P and m_{n+1}^P. The expected payoff of type t_n when sending m_n^P amounts to:

$$E[U^P | t_n, m_n^P] = \left(t_n - \frac{t_{n-1} + t_n}{4} - \frac{1}{2}b\right)\left(\frac{t_{n-1} + t_n}{2} + b\right)$$

whilst the expected payoff of type t_n when sending m_{n+1}^P yields:

$$E[U^P | t_n, m_{n+1}^P] = \left(t_n - \frac{t_n + t_{n+1}}{4} - \frac{1}{2}b\right)\left(\frac{t_n + t_{n+1}}{2} + b\right)$$

Indifference between sending these two signals is achieved when a boundary type receives equal payoffs when sending either of the two messages. $E[U^P | t_n, M_n^P] = E[U^P | t_n, M_{n+1}^P]$ yields:

$$t_{n+1} - t_n = t_n - t_{n-1} - 4b$$

Let the last, and thus smallest, partition on the total interval be of length d. The total interval [0,1] can then be summed up as follows:

$$(d + (N-1)4b) + \cdots + (d + 4b) + d = 1$$

$$Nd + N(N-1)2b = 1 \qquad (A5)$$

Let the maximum number of messages that can be used in this equilibrium be denoted by $\bar{N}$. Since Nd must be positive, but can be infinitely small, the maximum number of informative messages is the value of N that solves N$(N-1)2b < 1$, meaning that:

$$\bar{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor \qquad (A6)$$

From $(A5)$ it can be seen that the size of the last partition, with types that send message m_N^P, is equal to $d = t_N - t_{N-1} = \frac{1}{N} + (2-2N)b$. The partition before that, with types that send m_{N-1}^P, is $4b$ bigger, yielding $t_{N-1} - t_{N-2} = \frac{1}{N} + (6-2N)b$. The partition before that will again be $4b$ bigger, etc. In general, the size of interval $n \epsilon \{1, \ldots, N\}$ is equal to $\frac{1}{N} + (2N - 4n + 2)b = t_n - t_{n-1}$. This means that the first partition is of length $t_1 - t_0 = \frac{1}{N} + (2N-2)b$. Since $t_0 = 0$, the first boundary type is $t_1 = \frac{1}{N} + (2N-2)b$. As $t_2 - t_1 = \frac{1}{N} + (2N-6)b$, this means that $t_2 = \frac{2}{N} + (4N-8)b$. In general, a boundary type t_n in an equilibrium with N messages is equal to:

$$t_n = \frac{n}{N} + (2nN - 2n^2)b \qquad (A7)$$

When P sends a message m_n^P after observing $t \in [t_{n-1}, t_n]$, her expected payoff is:

$$E[U^P | t, m_n^P] = \left(\frac{t_{n-1} + t_n}{2} - \frac{1}{2}\left(\frac{t_{n-1} + t_n}{2} + b\right)\right)\left(\frac{t_{n-1} + t_n}{2} + b\right)$$

Let a player's ex ante expected payoff, meaning before P has observed her type and before A has received any message, in the partial pooling equilibrium be denoted by $U_{PPE}^i(N)$. $U_{PPE}^i(N)$ is the average payoff over sending all messages m_n^P, where $n = \{1, \ldots, N\}$. In order to arrive at this average, the payoff of sending each message is multiplied with the probability that $t_{n-1} \leq t \leq t_n$ which is $t_n - t_{n-1}$, yielding:

$$U_{PPE}^P(N) = \sum_{n=1}^{N} (t_n - t_{n-1}) \left(\frac{t_{n-1} + t_n}{2} - \frac{1}{2}\left(\frac{t_{n-1} + t_n}{2} + b\right)\right)\left(\frac{t_{n-1} + t_n}{2} + b\right) \qquad (A8)$$

Substituting $(A7)$ into $(A8)$ yields:

$$U_{PPE}^P(N) = \frac{1}{24}(4 - \frac{1}{N^2}) - \frac{1}{3}b^2 - \frac{1}{6}N^2 b^2 \qquad (A9)$$

A's payoff is retrieved in a similar fashion, yielding:

$$U_{PPE}^{A}(N) = \frac{1}{24}(4 - \frac{1}{N^2}) + \frac{1}{2}b + \frac{2}{3}b^2 - \frac{1}{6}N^2b^2 \qquad (A10)$$

Maximizing $(A9)$ and $(A10)$ yields the optimal number of messages for P and A respectively, denoted by N^{opt}, if they were totally free to set any number of messages. For both players, maximizing their payoffs with regards to N yields $N^{opt} = \sqrt[4]{\frac{1}{4b^2}}$. However, two restrictions apply to this. Firstly, N cannot be bigger than $\overline{N}$ and secondly, N needs to be an integer. Regarding the first restriction, it turns out that for $b \in [0,1]$, $N^{opt} < \overline{N}$, meaning the players are not hindered by $\overline{N}$ in this sense. Considering the second restriction, when b has the exact value that allows for an extra message to be used, both players would optimally like to use a number of messages that is somewhat lower than the maximum number possible, as is evidenced by $N^{opt} < \overline{N}$. This difference between N^{opt} and $\overline{N}$ is always smaller than 1. Thus, the players have to choose between $N = \overline{N}$ or $N = \overline{N} - 1$. Firstly, it follows from $(A6)$ that the value of b corresponding to a certain maximum number of messages $\overline{N}$ is equal to $b = \frac{1}{2\overline{N}(\overline{N}-1)}$. For $\overline{N} = \{1,2, \dots, \infty\}$, this value of b represents the threshold for allowing the use of an extra message in equilibrium. Secondly, the value of b for which both players are indifferent between $N = \overline{N}$ or $N = \overline{N} - 1$ follows from $U_{PPE}^{i}(\overline{N}) = U_{PPE}^{i}(\overline{N} - 1)$, also yielding $b = \frac{1}{2\overline{N}(\overline{N}-1)}$. This means that, at the exact value of b for which the maximum number of messages increases by one, the players are indifferent between using this maximum number of messages $\overline{N}$ and using $\overline{N} - 1$. As b gets any lower than $\frac{1}{2\overline{N}(\overline{N}-1)}$, they will strictly prefer $\overline{N}$, which essentially means that:

$$N^{*} = \overline{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

Appendix 5 Most informative equilibrium under nondelegation in the full model:

Consider the nondelegation scenario with $s \in \mathbb{N}^{+}$. It follows from (2) that P will implement the project if $r_t > E[e|m^A]$. Let a message m_n^A be sent by all A that observe $e \in [e_{n-1}, e_n]$, where $e_0 = 0$ and $e_N = 1$, such that all P with $t \geq n$ will implement the project upon receiving m_n^A. For this to hold, A should be indifferent between sending m_n^A and m_{n+1}^A when $e = e_n$. When sending m_n^A all P with type $t \geq n$ will implement the project. When $e = e_n$, this yields the following payoff for A:

$$E[U^A|m_n^A, e_n] = \sum\nolimits_{t=n}^{s} \frac{1}{s}\left(\frac{2t-1}{2s} + b - e_n\right) - (s-1)c$$

whilst sending m^A_{n+1} when $e = e_n$ will yield:

$$E[U^A|m^A_{n+1}, e_n] = \sum\nolimits_{t=n+1}^{s} \frac{1}{s}\left(\frac{2t-1}{2s} + b - e_n\right) - (s-1)c$$

For indifference, the difference between these payoffs needs to be equal to 0. $\frac{1}{s}\left(\frac{2n-1}{2s} + b - e_n\right) = 0$ yields:

$$e_n = \frac{2n-1}{2s} + b \qquad (A11)$$

In order for this to be an equilibrium, it should indeed be the case that all P with $t \geq n$ will implement the project when receiving message m^A_n. Given P's decision strategy, this is true when $E[e|m^A_n] < r_t$ for $n = t$. It follows from $(A11)$ that:

$$E[e|m^A_n] = \frac{e_t + e_{n-1}}{2} = \frac{n-1}{s} + b$$

$r_t \geq E[e|m^A_n]$ for $n = t$ yields $s \leq \frac{1}{2b}$. Letting $\bar{s}$ denote the maximum number of subintervals that can support a most informative equilibrium in the nondelegation scenario, this means that

$$\bar{s} = \left\lfloor \frac{1}{2b} \right\rfloor$$

After P observes both her type t and the message m^A_n, she will only implement the project if $n \leq t$. The largest value of e for which A will send a message $n \leq t$ is the upper boundary of the partition belonging to the message $n = t$, which is e_t. It then follows from $(A11)$ that, before P has observed m^A, the probability that she will implement the project is $P(d^P = 1) = P(e \leq e_t) = \frac{2t-1}{2s} + b$. Since $e \leq e_t$ when A approves the project, the expected value of e is $E[e|d^P = 1] = \frac{2t-1}{4s} + \frac{1}{2}b$. This yields the following payoff for P and A, when P's type is a given:

$$E[U^P|t, m^A_n] = \left(\frac{2t-1}{2s} - \frac{2t-1}{4s} - \frac{1}{2}b\right)\left(\frac{2t-1}{2s} + b\right) - (s-1)c = \frac{(2t-1)^2}{8s^2} - \frac{1}{2}b^2 - (s-1)c$$

$$E[U^A|t, m^A_n] = \left(\frac{2t-1}{2s} + b - \frac{2t-1}{4s} - \frac{1}{2}b\right)\left(\frac{2t-1}{2s} + b\right) - (s-1)c$$
$$= \frac{(2t-1)^2}{8s^2} + \frac{2t-1}{2s}b + \frac{1}{2}b^2 - (s-1)c$$

These payoffs and the maximum number of subintervals that can support the most informative equilibrium are equal to the results of the delegation scenario. The rest of the analysis is therefore identical to Appendix 3, resulting in:

$$U_{MI}^{P}(s)=\frac{1}{s}\sum_{t=1}^{s}\left(\frac{(2t-1)^2}{8s^2}-\frac{1}{2}b^2\right)-(s-1)c=\frac{1}{24}\left(4-\frac{1}{s^2}\right)-\frac{1}{2}b^2-(s-1)c$$

$$U_{MI}^{A}(s)=\frac{1}{s}\sum_{t=1}^{s}\left(\frac{(2t-1)^2}{8s^2}+\frac{2t-1}{2s}b+\frac{1}{2}b^2\right)-(s-1)c$$

$$=\frac{1}{24}\left(4-\frac{1}{s^2}\right)+\frac{1}{2}b+\frac{1}{2}b^2-(s-1)c$$

$$s^*=min(\left\lceil\sqrt[3]{\frac{1}{12c}}\right\rceil,\left\lfloor\frac{1}{2b}\right\rfloor)$$

Appendix 6: Partial pooling equilibrium under nondelegation in the full model

Consider a partial pooling equilibrium in the nondelegation scenario with $c=0$ and $s\to\infty$. Let a message m_n^A be sent by all types that observe e to be in the interval $[e_{n-1},e_n]$, where $e_0=0$ and $e_N=1$. e_n is the boundary type between m_n^A and m_{n+1}^A and should therefore be indifferent between sending either of these two messages. When sending m_n^A, P will decide $d^P=1$ if $r>E[e|m_n^A]=\frac{e_{n-1}+e_n}{2}$ and $d^P=0$ otherwise. This means that $P(d^P=1)=1-\frac{e_{n-1}+e_n}{2}$ and $E[r|d^P=1]=\frac{1}{2}(\frac{e_{n-1}+e_n}{2}+1)$. Thus, sending m_n^A when $e=e_n$ yields the following payoff for A:

$$E[U^A|e=e_n,m_n^A]=\left(\frac{e_{n-1}+e_n+2}{4}+b-e_n\right)\left(1-\frac{e_{n-1+}e_n}{2}\right)$$

Similarly, sending m_{n+1}^A when $e=e_n$ yields A:

$$E[U^A\big|e=e_n,m_{n+1}^A]=\left(\frac{e_n+e_{n+1}+2}{4}+b-e_n\right)\left(1-\frac{e_{n+}e_{n+1}}{2}\right)$$

The expected payoff for A of sending either of the two messages should be equal to each other in order for boundary types to be indifferent between sending the two adjacent messages. $E[U^A|e=e_n,m_n^A]=E[U^B\big|e=e_n,m_{n+1}^A]$ yields:

$$e_{n+1}-e_n=e_n-e_{n-1}-4b$$

Just as in the partial pooling equilibrium in delegation scenario, each next interval of types that send one message is $4b$ smaller. In order to find how many messages can be used, let the last, and thus smallest, interval be of length d. The total interval $[0,1]$ can then be summed up as:

$$(d + (N-1)4b) + \cdots + (d + 4b) + d = 1$$

$$Nd + N(N-1)2b = 1 \qquad (A12)$$

Similar to the partial pooling equilibrium in the delegation scenario, this means the maximum number of messages that can be used in this equilibrium, denoted by $\bar{N}$, is equal to:

$$\bar{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$

Since the division of the total interval given in $(A12)$ is the same as under the partial pooling equilibrium in the delegation scenario, the values of the boundary types for A in the nondelegation scenario will be equal to the values of the boundary types for P in the delegation scenario. This means that:

$$e_n = \frac{n}{N} + (2nN - 2n^2)b \qquad (A13)$$

When A sends a message m_n^A after observing that $e \in [e_{n-1}, e_n]$ it follows that $P(d^P = 1) = 1 - \frac{e_{n-1}+e_n}{2}$ and $E[r|d^P = 1] = \frac{1}{2}\left(\frac{e_{n-1}+e_n}{2} + 1\right)$. This means that his expected payoff, after sending m_n^A, becomes:

$$E[U^A|e, m_n^A] = \left(\frac{1}{2}\left(\frac{e_{n-1}+e_n}{2} + 1\right) + b - \left(\frac{e_{n-1}+e_n}{2}\right)\right)\left(1 - \frac{e_{n-1}+e_n}{2}\right)$$

Let $U_{PPE}^i(N)$ denote a player's payoff before any information is acquired and any communication has taken place. $U_{PPE}^i(N)$ is the weighted average payoff over all possible messages, which is the sum of the payoff resulting from each message multiplied with the probability that e falls into the subinterval belonging to that message. This yields the following ex ante payoff for A:

$$U_{PPE}^A(N) = \sum_{n=1}^{N} (e_n - e_{n-1})\left(\frac{1}{2}\left(\frac{e_{n-1}+e_n}{2} + 1\right) + b - \left(\frac{e_{n-1}+e_n}{2}\right)\right)\left(1 - \frac{e_{n-1}+e_n}{2}\right)$$

Substituting for $(A13)$ yields:

$$U_{PPE}^A(N) = \frac{1}{24}(4 - \frac{1}{N^2}) + \frac{1}{2}b + \frac{1}{6}b^2 - \frac{1}{6}N^2b^2$$

P's payoff can be derived similarly, yielding:

$$U_{PPE}^{P}(N) = \frac{1}{24}\left(4 - \frac{1}{N^2}\right) + \frac{1}{6}b^2 - \frac{1}{6}N^2 b^2$$

Maximizing these two payoffs with regard to N yields $N^{opt} = \sqrt[4]{\frac{1}{4b^2}}$. This means that both N^{opt} and $\overline{N}$ are the same as in the partial pooling equilibrium in the delegation scenario. Thus, given that $N \leq \overline{N}$ and that N needs to be an integer, the optimal number of messages in this partial pooling equilibrium is the same as in the delegation scenario which is:

$$N^* = \overline{N} = \left\lfloor \frac{1}{2}(1 + \sqrt{1 + \frac{2}{b}}) \right\rfloor$$