Pricing the term structure with linear regressions: three different looks at the term premium

Bachelor Thesis Double Degree Econometrics & Economics

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Date final version: July 2019

Abstract

This paper reviews the Adrian, Crump, and Moench (2013) 3-step term structure estimation approach. We use a more up-to-date data set and extend the methodology by tackling the high persistence of yields using bootstrap bias correction and a coefficient restriction, in the first step of the Adrian et al. (2013) procedure. We find the two considered extensions fit the yield data to a similar degree as the authors' original model. In addition, a comparison of the forecasting performance of the two additional models reveals the original, non-altered model produces the most accurate 12-step ahead forecasts. Finally, we find that the term premium produced by the coefficient restricted model is the most economically justified term premium, as compared to the original and bias-corrected premia. The coefficient restricted model is therefore deemed more desirable to decompose the term premium. The unaltered model is preferred for forecasting purposes.

*The views stated in this thesis are those of the author and not necessarily those of Erasmus School of Economics or Erasmus University Rotterdam.

Contents

1 Introduction

The relationship between zero-coupon bond yields and the respective maturities of these bonds is what is known as the term structure of interest rates. When this relationship is graphed, the yield curve is observed. Market participants' expectations about future interest rates are reflected in the yield curve. The shape of the yield curve then gives an indication of the expectations in economic activity. The term structure, therefore, plays a central role in the workings of the economy. When longer-term yields fall below short term yields, the yield curve inverts. An inverted yield curve is generally considered an accurate predictor of a looming recession (Chinn & Kucko, 2015), however, changes in the term premium can distort the recession signal given by an inverted yield curve (Rosenberg & Maurer, 2008). If the term premium contains valuable information regarding the accuracy of an inverted yield curve as a predictor of recessions, it seems of crucial importance to model this phenomenon correctly. Is the current literature accurately representing the US term premium?

Given its importance and the fact that modelling the term structure represents one of the most challenging topics in the recent financial economics literature, it has been theoretically and empirically widely explored (Gibson, Lhabitant, & Talay, 2010). The affine term structure literature most commonly uses the Maximum Likelihood approach to estimate the term structure. By using these kinds of models, practitioners incorporate distributional assumptions and are able to provide a complete characterisation of the joint distribution of yields. Analytical solutions, to the maximisation problems maximum likelihood estimation lead to, do however not exist. As a consequence, researchers must resort to numerical optimisation techniques. These methods are prone to diverse issues such as (1) a large number of parameters to be estimated, (2) a highly non-linear likelihood function and (3) the existence of local optima that can never guarantee the solutions found are global solutions. (Duffee & Stanton, 2012)

Alternative approaches, that circumvent the need to estimate all parameters using numerical techniques, are however available. Joslin, Singleton, and Zhu (2011) use a maximum likelihood approach but reduce the numerical challenges faced in the estimation by splitting the parameters to be estimated into two subsets, the first of which can be estimated via Ordinary Least Squares (OLS). De los Rios (2015) and Adrian et al. (2013) both purely rely on linear regressions to estimate the term structure. These relatively recent lines of research set the stage for less computationally intensive methods to take a foothold in the term structure literature.

This paper firstly replicates part of the methodology in Adrian et al. (2013). Using the Gurkaynak et al. (2007) data-set that ranges from 1987 until 2018, we use five principal components of yields as pricing factors, and apply the authors three-step OLS approach, to produce an OLS estimator. This estimator is subsequently used in recursive pricing formulas, to produce a term structure plot. The term premium is calculated from these parameters by differencing the risk-neutral yield and the model-implied fitted yield. The five-factor model is employed as it is the preferred specification due to superior out-of-sample performance (Adrian et al., 2013).

Thereafter, however, we turn to the more interesting contributions of this paper. In particular, we refine the original methodology by mitigating the issues relating to the time series dynamics of the pricing factors. We use methods to alleviate the small-sample problem. Researchers are faced with highly persistent risk factors and small available data samples to infer the dynamic properties of the model, which means that the uncertainty around the VAR parameters remains high. This problem is addressed by employing bootstrap bias correction, to eliminate the bias in the VAR(1) model that is estimated in the first step of the Adrian et al. (2013) three-step procedure. Furthermore, a larger, more up-to-date, data-set is used, that incorporates information up to and including the year 2018. We thus include more observations at times of unprecedented central bank interventions, that kept short term rates near the zero lower bound for years (Bauer & Rudebusch, 2016). The level principal component in bond yields is sometimes considered to be a non-stationary process, Kozicki and Tinsley (1997) find that the long-horizon expectation of the level of the term structure may need to be revised period-by-period to the current value. As such, we model the level principal component in bond yields as a random walk, such that the possible non-stationarity in the yields process is examined.

Given the small-sample and persistence problems yield time series dynamics give rise to, we investigate whether the three-step linear regression approach of Adrian et al. (2013) can be improved upon; in terms of forecasting performance. We compare the different models by investigating their ability to predict future interest rates. Given that we employ two different methods to tackle the persistence of yields, we specify two research questions:

1. *To what extent does bootstrap bias-correction, applied to the 5-factor model of Adrian et al. (2013), improve the model's ability to predict future interest rates, as measured by the root mean square prediction error*

2. *To what extent does imposing a random walk on the first principal component of yields in the 5-factor model of Adrian et al. (2013), improve the model's ability to predict future interest rates, as measured by the root mean square prediction error*

We are also interested in how the term premium is affected. In theory, if yield dynamics are in fact more persistent than is previously thought, the term premia produced will be too volatile. The magnitude of the premium may also vary depending on how the term structure is estimated. Given that three different term premias are constructed from three different specifications; the standard Adrian et al. (2013) model, the random walk model and the bias-corrected model, we also investigate which term premium could be considered the most accurate. To proxy for an "accurate" term premium we explain these with macro-economic variables and judge which are best explained by these variables. We thus propose a final research question.

3. *To what extent do the term premiums obtained vary, in terms of how well they are explained by macro-economic variables?*

Studying the yield curve is, as mentioned, of crucial importance in the fields of finance and macro-economics. For financial market practitioners, the availability of accurate interest rate forecasts can be the key to a successful trading strategy. In the academic domain, models describing the term structure have been proliferating since the seminal works of Vasicek (1977) and Cox, Ingersoll, and Ross (1985). With good reason, since this topic lies at the core of the most basic and elaborate valuations problems encountered in finance. Certainly, all financial assets can be valued by discounting the asset's expected future cash flows, using an appropriate discount rate function that lodges an underlying rationale about risk premia and the term structure. Furthermore, policymakers widely rely on the yield curve to predict future movements in the economy, as the current yield curve contains information about the future path of the economy. Indeed, yield spreads have been useful in forecasting not only real activity (Harvey, 1988) but also inflation (Fama, 1990). In addition, debt policy sees itself largely affected. When issuing new debt, governments must decide on the maturity of the bonds that fund budget deficits, the term structure will undoubtedly play a role in what decisions are ultimately made by the Treasury Department.

The term premium is the difference between a bond's yield and the expectation of the risk-free rate over the life of the bond. It is thus the compensation investors require for holding a long-term bond compared to rolling over a series of short-term bonds with lower maturity. It is of economic interest because it allows for inferences to be made as to what risk-adjusted expectations of the path of future short-term interest rates are. Especially since the introduction of large-scale asset purchase programs in the United States, Japan, and Europe, the term premium is of crucial importance. The size of term premia enables investors to assess the risks and rewards of investing across different fixed income markets. Furthermore, issuers of bonds can, using the term premium, analyse and break down the components of their funding cost and hence enforce the management of their primary market activities (James, Leister, & Rieger, 2016). Lastly, as mentioned, an inverted yield curve might signal a downturn in the economy, however, term premia effects may be the cause of this inversion and as such, the yield curve inversion might lose its reliability in signalling an imminent recession (Rosenberg & Maurer, 2008).

This research shows the two considered extensions fit the yield data to a similar degree as the original Adrian et al. (2013) three-step procedure. The two extensions produce future yield forecasts that are less accurate than the original model. The term premium obtained by the coefficient restricted model does, however, gain economic plausibility, with respect to the original premium and as such we consider it a more accurate representation of the "true" term premium.

The remaining sections of this paper are structured as follows. We provide an overview of the term structure literature in Section 2, after which the methodology used in this study is presented - in Section 3. An overview of the Data follows in Section 4. Section 5 presents a discussion of the results in this paper and Section 6 concludes. The Appendix at the end presents a description of the programs in the attached Zip file.

2 Literature Review

As mentioned, the majority of the term structure literature relies on numerical optimisation. A step towards circumventing the need for numerical procedures is undertaken by Joslin et al. (2011).

The authors use linear combinations of yields as observable pricing factors and split the model parameters into two subsets, the first of which are estimated via OLS. Then, in the second step, the authors estimate the remaining model parameters which govern the evolution of pricing factors under the risk-neutral measure, via a numerical solution of the likelihood function using the OLS estimates of the first step as constants in the estimation. Hamilton and Wu (2012) also combine linear regressions with numerical optimisation, the methodology employed relies on minimising a Wald test statistic with the null hypothesis that the restrictions on the model implied by the no-arbitrage model, are consistent with the data used. In other words, a quadratic objective function, that measures the deviations of the estimated reduced-form parameters and the reduced-form coefficients implied by the no-arbitrage model, is minimised. However, both Hamilton and Wu (2012) and Joslin et al. (2011) rely on on the assumption that exactly K linear combinations of yields are observed and priced without error while the remaining yields are observed with error. The two also require that fitted yields' errors are not serially correlated, and Adrian et al. (2013) derive that uncorrelated yield errors give rise to auto-correlation in the bonds' return pricing errors, this then generates excess return predictability that can not be captured by the principal components of the yields (the pricing factors). The empirical results of Adrian et al. (2013) suggest that there is indeed serial correlation in yield fitting errors while there is little to no auto-correlation in return pricing errors, suggesting that the initial assumptions made may be questionable. Furthermore, Joslin et al. (2011) and Hamilton and Wu (2012) of course still partly rely on undesirable numerical optimisation methods.

De los Rios (2015) introduces an asymptotic least-square estimator defined by the no-arbitrage conditions upon which term structure models are constructed. The method employed differs from Adrian et al. (2013) in the distributional assumption of the pricing errors. Adrian et al. (2013) assume uncorrelated pricing errors of excess returns, instead of uncorrelated pricing errors of yields. De los Rios (2015) states that his OLS estimates of the reduced-form parameters remain consistent and asymptotically normal under the assumption of uncorrelated yield errors. De los Rios (2015) mentions that his estimator, when compared to the estimator in Adrian et al. (2013), provides asymptotic efficiency gains, given that Adrian et al. (2013) use an identity weighting matrix and do not impose self-consistency of the model. Joslin, Le, and Singleton (2013) also point out this internal consistency problem, the researchers state that the regression models are under-identified in the sense that they contain redundant parameters.

Bauer, Rudebusch, and Wu (2012) highlight another issue affecting term structure models. It seems that estimates of the time series dynamics of the pricing factors used in the estimation of term structure models may suffer from bias, due to the high persistence of interest rates. The authors find that in general, the estimates are biased towards making the system less persistent, that is, the data generating process actually reverts to the unconditional mean more slowly than what the estimates suggest. This results in the component of long-term yields reflecting expected future short rates being too stable and as such the term premia produced are too volatile. On the other hand, Kim and Orphanides (2012) improve the identification of the persistence of bond yields by including additional information, in the form of survey expectations of future short-term interest rates. The authors suggest that including such information completely mitigates the small-sample problem. Joyce, Lildholdt, and Sorensen (2010) and Guimaraes (2014) also aim to alleviate this problem, they use survey expectations of inflation and survey expectations of both short-term interest rates and inflation data, respectively, to improve the identification of parameters. It thus seems noteworthy to study potential non-stationarity further.

A general finding in the literature is that term premia are counter-cyclical, which has lead to the inclusion of macro-economic variables as additional variables within term structure models. This follows from the work of Ang and Piazzesi (2003). The authors investigate how macro variables affect bond prices and the dynamics of the yield curve and find that the forecasting performance of the VAR model improves when macro-economic variables are included. In more recent work, Joslin, Priebisch, and Singleton (2014) find that macro-economic variables are unspanned, thus imperfectly correlated with, the yield curve, yet these variables contain information about future excess returns on bonds beyond the information that the bond yields contain. Macro-economic variables may, therefore, be important in explaining the time series dynamics of yield curve factors. The researchers also find that a large portion of the variation in forward terms premiums are captured by these macro-economic risks. Kopp and Williams (2018), in a very recent study, also point out these results. Including macro-economic variables may thus provide a more accurate term structure and consequently also more accurate term premiums. Term premiums obtained from models that include macro-economic information are further found to be counter-cyclical (G. H. Bauer & de los Rios, 2012), this is then in line with standard asset pricing theory and thus reinforces the argument for the inclusion of such variables. These general findings are however questioned by Cochrane (2015), in an unpublished set of notes, Cochrane casts doubt on the usefulness of macro-economic variables such as growth and inflation as additional forecasters of yield returns. The author detects that including a simple linear trend drives out the additional forecasting performance of macro-economic variables. Indeed, Adrian et al. (2013), in an extension of their own work, mention unspanned macro factors provide a somewhat poorer fit to the cross-section of Treasury yields than the 5-factor specification.

Turning to the term premium, term premia are driven by a wide variety of variables. Callaghan (2019) justifies the term premium obtained for 10-year New Zealand bond by regressing the term premium on the volatility of inflation, unemployment, and the MOVE index. A significant amount of the variation in the term premium is found to be explained by these three variables, with an R^2 of the simple OLS regression being close to 75%. These are, of course, not the only possible explanatory variables, Bonis, Ihrig, and Wei (2017) estimate the term premium effect on the 10-year Treasury yield large-scale asset purchases and the maturity extension program of the Federal Reserve after the Federal Funds rate was at its effective lower bound following the Great Recession. The authors find quantitative easing has a negative effect on the term premium. Furthermore, Chadha, Turner, and Zampolli (2013) find government debt structure has a significant effect on the 10-year Treasury bond term premium. It is found that an increase in the average maturity of debt outstanding held outside the US Central Bank is accompanied by a rise in the 10-year term premium. Finally, Cochrane (2009) also states that the covariance of payoffs with future marginal consumption growth is a factor that influences the term premium.

3 Methodology

3.1 The term structure models of Adrian et al. (2013)

We outline the key equations of a standard Gaussian affine term structure model (ATSM). This derivation follows Adrian et al. (2013), to which the reader is referred to for further details.

Assuming that the dynamics of a (K x 1) vector of pricing factors, X_t, evolve according to a Gaussian Vector Auto Regression (VAR) with one lag, we have:

$$X_{t+1} = \mu + \Phi X_t + v_{t+1}, \tag{1}$$

where the shocks $v_{t+1} \sim N(0, \Sigma)$ are conditionally Gaussian, homoscedastic, and independent across time.

The assumption of no-arbitrage implies the existence of a pricing kernel M_{t+1}, that discounts the expected future price of a zero coupon bond, to its price in the current period, such that:

$$P_t^{(n)} = E_t[M_{t+1} P_{t+1}^{(n-1)}]. \tag{2}$$

Now, assuming that the pricing kernel M_{t+1} is exponentially affine:

$$M_{t+1} = exp(-r_t - 0.5\lambda_t^T \lambda_t - \lambda_t^T \Sigma^{-0.5} v_{t+1}), \tag{3}$$

where $r_t = Ln(P_t^{(1)})$ denotes the continuously compounded one-period risk-free rate. Assuming market prices of risk (λ_t) are affine in the factors:

$$\lambda_t = \Sigma^{-0.5}(\lambda_0 + \lambda_1 X_t). \tag{4}$$

Further, the log excess one-period holding return of a zero-coupon bond that matures in n periods is:

$$rx_{t+1}^{(n-1)} = Ln(P_{t+1}^{(n-1)}) - Ln(P_t^{(n)}) - r_t. \tag{5}$$

Combining all these elements, excess holding period returns can be expressed as a function of the expected return to holding bonds, a convexity adjustment, the shocks to the pricing factors, and an orthogonal return pricing innovation. The return generating process is then

$$rx_{t+1}^{(n-1)} = \beta^{(n-1)T}(\lambda_0 + \lambda_1 X_t) - 0.5(\beta^{(n-1)T}\Sigma\beta^{(n-1)} + \sigma^2) + \beta^{(n-1)T} v_{t+1} + e_{t+1}^{(n-1)}. \tag{6}$$

Finally, stacked across maturities and time periods, this can be re-written as:

$$rx = \beta^T(\lambda_0\iota_T^T + \lambda_1 X_-) - 0.5(B^* vec(\Sigma) + \sigma^2\iota_N)\iota_T^T + \beta^T V + E. \tag{7}$$

Before any parameters can be estimated, principal components must be extracted from the yields data provided by Gurkaynak et al. (2007). As mentioned, the first five principal components are retained as pricing factors. We now proceed with the three-step regression, based on equation (7). We estimate the parameters of the model as follows.

Firstly, ordinary least squares is applied to Equation (1). We calibrate $\mu = 0$ as principal components of demeaned yields are taken, such that one can assume the level is in fact zero. This allows for the decomposition of X_t into a predictable component and an estimate of the error $\tilde{v}_{t+1}$. These innovations are stacked into the (K x N) matrix $\tilde{V}$. The state variable variance–covariance matrix estimator is then constructed as $\tilde{\Sigma} = \tilde{V}\tilde{V}/T$

Secondly, excess returns, constructed according to Equation (5) are regressed on a constant, contemporaneous pricing factor innovations and lagged pricing factors, according to:

$$rx = \boldsymbol{a}\iota_T^T + \beta^T\tilde{V} + \boldsymbol{c}X_- + E \tag{8}$$

where, from Equation (7) it is clear that $\boldsymbol{a} = \beta^T\lambda_0 - 0.5(B^* vec(\Sigma) + \sigma^2\iota_N)$ and $\boldsymbol{c} = \beta^T\lambda_1$. The residuals from this regression are collected into the (N x T) matrix $\tilde{E}$. We then are able to estimate $\tilde{\sigma}^2 = tr(\tilde{E}\tilde{E}^T)/NT$. $\tilde{B}^*$ is constructed from $\tilde{\beta}$ by the following formula:

$$\tilde{B}^* = [vec(\beta^1\beta^{1^T})...vec(\beta^N\beta^{N^T})]. \tag{9}$$

In the last step of the three-step procedure, the price of risk estimators λ_0 and λ_1 are estimated via a cross-sectional regression. Using the expressions for $\boldsymbol{a}$ and $\boldsymbol{c}$, the following estimators are obtained.

$$\tilde{\lambda}_0 = (\tilde{\beta}\tilde{\beta}^T)^{-1}\tilde{\beta}(\tilde{\boldsymbol{a}} + 0.5(\tilde{B}^* vec(\tilde{\Sigma}) + \tilde{\sigma}^2\iota_N)) \tag{10}$$

and

$$\tilde{\lambda}_1 = (\tilde{\beta}\tilde{\beta}^T)^{-1}\tilde{\beta}\tilde{c} \tag{11}$$

From these model parameters, a zero-coupon yield curve can be modelled. We first obtain estimates for the short-term interest rate, which is assumed to be measured with error $\epsilon_t \sim iid(0, \sigma_\epsilon^2)$. This is done by regressing the risk-free rate, proxied by the 1-month t-bill (r_t), on a constant and pricing factors. The corresponding equation is then

$$r_t = \delta_0 + \delta_1^T x_t + \epsilon_t. \tag{12}$$

The parameter estimates $\tilde{\delta}_0$ and $\tilde{\delta}_1$ will be used in the linear restrictions for the bond pricing parameters.

As shown in Adrian et al. (2013), bond prices are exponentially affine in the vector of state variables:

$$Ln(P_t^{(n)}) = A_n + B_n^T X_t + u_t^{(n)}. \tag{13}$$

Excess returns can be written as:

$$rx_{t+1}^{n-1} = A_{n-1} + B_{n-1}^T X_{t+1} + u_{t+1}^{(n-1)} - A_n - B_n^T X_t - u_t^{(n)} + A_1 + B_1^T X_t + u_t^{(1)}. \tag{14}$$

Equating this expression for excess returns with the return generating process equation:

$$\begin{aligned}
&A_{n-1} + B_{n-1}^T(\mu + \Phi X_t + v_{t+1}) + u_{t+1}^{(n-1)} - A_n - B_n^T X_t - u_t^{(n)} + A_1 + B_1^T X_t + u_t^{(1)} \\
&= \\
&\beta^{(n-1)T}(\lambda_0 + \lambda_1 X_t + v_{t+1}) - 0.5(\beta^{(n-1)T}\Sigma\beta^{(n-1)} + \sigma^2) + e_{t+1}^{(n-1)}.
\end{aligned} \tag{15}$$

Fitted excess returns are then computed as:

$$rx_{t+1}^{(n-1)} = \tilde{B}_{(n-1)}^T(\lambda_0 + \lambda_1 X_t) - 0.5(\tilde{B}_{(n-1)}^T \Sigma \tilde{B}_{(n-1)} + \tilde{\sigma}^2) + \tilde{v}_{t+1}. \tag{16}$$

Let $A_1 = \delta_0$ and $B_1 = -\delta_1$, matching terms, the system of recursive linear restrictions for the bond pricing parameters are:

$$A_n = A_{n-1} + B_{N-1}^T(\mu - \lambda_0) + 0.5(B_{N-1}^T \Sigma B_{N-1} + \sigma^2) - \delta_0 \tag{17}$$

$$B_n^T = B_{n-1}^T(\Phi - \lambda_1) - \delta_1^T \tag{18}$$

Here, $A_0 = 0$ and $B_0^T = 0$. Also, $\beta^{(n)} = B_n^T$

The derivation of log bond prices is precise, provided that $\beta^{(n)} = B_n$. The Affine Term Structure Models' parameters are all obtained in the above three-step procedure. The equations are recalculated with the market prices of risk set to zero to produce the risk-neutral yield curve. Specifically, the average short-rate predictions are computed by setting the price of risk parameters λ_0 and λ_1 to zero in the recursions of equations (17) and (18). This generates risk-adjusted bond pricing parameters (A_n^{ra} and B_n^{ra}) and a feature of these is that

$$E_t^{(n)}[asr] = -1/n(A_n^{ra} + B_n^{ra} X_t) \tag{19}$$

where $E_t^{(n)}[asr]$ is the time t expectation of average future short rates over the next n periods. The term-premium is then calculated as the difference between the risk-neutral yield and model-implied fitted yield.

3.2 Extending the Adrian et al. (2013) methodology

3.2.1 Imposing a random walk on the first principal component

In this study, we compare two extreme cases, one in which the VAR process is left unrestricted - this case is identical to that of Adrian et al. (2013). In the other case, the first principal component is restricted to follow a random walk.

In the latter case, extending the Duffee (2011) methodology to the form of 5 pricing factors, we restrict the parameter Φ in equation (1). The modified matrix Φ, in equation (1), then looks as follows.

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} \\ 0 & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} \\ 0 & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} \\ 0 & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} \end{bmatrix} \tag{20}$$

This restriction ensures the level principal component of the term structure follows an I(1) process, this can be observed by writing out the first row of the model. The zeros in the first column of the Φ parameter are necessary for the stationarity of the second, third, fourth and fifth principal components. The coefficient restriction, of course, impacts the estimates of Φ and subsequently the error terms that are used in the second step of the procedure, as well as the parameters in the bond pricing recursions.

3.2.2 Bootstrapping

To address the small-sample bias that might be present, we propose bias-correction in the first stage of the three-step regression procedure of Adrian et al. (2013), when the time series dynamics of the factors are determined. Bootstrap mean bias-correction, that relies on the bootstrap to estimate the bias is employed. Data are simulated using a distribution-less residual bootstrap, taking the VAR parameter estimates as the data-generating parameters. The procedure is, in a similar fashion to Malik and Meldrum (2016), performed as follows.

(1) Estimate equation (1) using OLS as before, store the residuals and OLS estimate $\tilde{\Phi}$
(2) Starting with h = 1, generate a bootstrap sample for periods t = 1, 2 ..., T. For t = 1, randomly select $X_1^{(h)}$ from the original sample. For t > 1, randomly select the residuals, with replacement $v_t^{(h)}$ and produce $X_t^{(h)} = \tilde{\Phi} X_{t-1}^{(h)} + v_t^{(h)}$
(3) Calculate the OLS estimate of $\tilde{\Phi}^{(h)}$ using the h^{th} bootstrapped sample.
(4) For each h = 2, 3, ... 10000 perform steps (2) and (3)
(5) Calculate the sample mean of the bootstrapped samples: $\Phi_{bs} = H^{-1} \sum_{h=1}^{H} \tilde{\Phi}^{(h)}$
(6) The bootstrap bias-corrected estimate is now $\Phi_{bc}^{H} = \tilde{\Phi} - [\Phi_{bs} - \tilde{\Phi}]$

Now there are two options, if the bias-corrected estimate Φ_{bc}^{H} has eigenvalues that lie outside the unit circle, such that the corrected estimates have explosive roots, the bias estimate is shrunk toward zero until the stability restriction is satisfied. Specifically, following G. H. Bauer and de los Rios (2012) we apply the adjustment proposed in Kilian (1998). The modified bias-corrected estimate is given by $\Phi_{bc}^{H} = \tilde{\Phi} - \delta_i[\Phi_{bs} - \tilde{\Phi}]$ Starting with $\delta_i = 1$, and iterating with $\delta_i = \delta_{i-1} - 0.01$ until all eigenvalues lie within the unit circle.

If the eigenvalues lie outside the unit-circle we have a stable time series and we can use the estimate as is. The bias-corrected estimate, Φ_{bc}^{H} is then subsequently used in the recursive linear restrictions of the bond pricing parameters in equations (18) and (17).

3.2.3 Evaluating out of sample performance

In order to assess whether the proposed extensions to the work of Adrian et al. (2013) increase the 5-factor models' predictive power, we elaborate on the means of evaluation. Firstly, forecasts must be made for both short and long term interest rates. We use the 1987:01–2017:12 period as a training sample and employ expanding window estimation, re-estimating the model at a monthly frequency. Estimating the next month ahead yields on bonds at all the considered maturities. The decision to forecast one year ahead yields stems from the nature of the VAR(1) process forecasts, which at long horizons simply converge to the unconditional mean.

The forecasts are evaluated by comparing the predictions with the actual, realised data. To evaluate the forecasts, the root mean square prediction error formula is applied, as in Franses and van Dijk (2000):

$$RMSPE = \sqrt{\frac{1}{P} \sum_{i=1}^{P} (r_i - \tilde{r}_i)^2}, \tag{21}$$

where P is the forecast sample size, r is the realised interest rate in the Gurkaynak et al. (2007) data-set and $\tilde{r}$ is the model implied interest rate. We motivate the use of forecasts to evaluate the extensions due to this also being the means of evaluation in Adrian et al. (2013), to evaluate the different factor models the authors produce.

3.2.4 Evaluating the term premiums obtained

To evaluate the three different term premia we employ a simple linear regression and extract the R^2's of these regressions, as a measure of how economically significant they are. The linear regression then looks as follows

$$TP_i = \beta_0 + \beta_1 VI_i + \beta_2 U_i + \beta_3 MOVE_i + \epsilon_i, \tag{22}$$

where TP_i refers to the term premium obtained by the three different models. The explanatory variables are those of Section 4, VI is the volatility of inflation proxied by the standard deviation of inflation, U is the civilian unemployment rate and $MOVE$ refers to the MOVE index. β_0 is the level component of the term premium and ϵ_i is the error term.

Even though, as discussed in Section 2, the term premium is affected by many more variables than those discussed in Section 4, we limit our explanation of the term premia to these variables. The term premia obtained by the three different models refer to the same 10-year bond term premium. The variation between the premia obtained lies in how they are constructed, the degree of explanation by certain macro-variables should, theoretically, be the same. It must be noted that only using the R^2 as a measure of economic significance may be disputed. Callaghan (2019) includes these variables to validate the plausibility of his term structure estimate. The author shows the term premium is correlated with external variables in a way that one would expect. Omitted variable bias will undoubtedly impact the regressions. Other variables, like the government debt structure, the degree of quantitative easing entertained by the central bank, amongst others, determine the term premium. However, the omitted variable bias should impact the three regressions to the same degree, as the underlying dependant variable is theoretically identical. Because of this, the explanatory variables should be explained to the same degree by the variables we introduce.

4 Data

This section firstly explores the yield data that is used to produce the term structure plots. The subsequent sub-section presents macro-economic data that are used to validate the term premia produced by the three considered models; the original Adrian et al. (2013) model, the bootstrap corrected model, and the restricted model.

4.1 The U.S. Treasury yield curve data by Gurkaynak et al. (2007)

The five principal components factor model employed in this study is estimated using zero-coupon yield data provided by Gurkaynak et al. (2007). These data are also used by Adrian et al. (2013), for the construction of pricing factors. The data, a long history of high-frequency yield curve estimates, concerns fitted Nelson-Siegel-Svensson curves (Svensson, 1994). The authors publish the parameters of the fitted curves, along with the estimated zero-coupon curve on a quarterly basis. This allows for a more up-to-date data-set to be used in this study, deviating from Adrian et al. (2013), who are limited to data of the year 2011. The parameters used in this study concern the period: 1987:01 – 2018:12. The data are originally daily data but we take end of month observations, as this is the general practice in the term structure literature. The final sample consists of 372 monthly observations. It must be noted that although we treat these data as given, they concern estimates and as with any kind of modelling, estimation errors are certain to have been made.

The published parameters are used to produce a cross-section of yields for maturities n = 3, ..., 120 months. Given these yields, we apply principal component analysis and extract the first 5 factors. Taking the risk-free rate as the n = 1-month yield, excess returns for n = 6, 12, 18, 24, ..., 84, and 120 month zero coupon bonds are computed. The precise formulation can be found in Section 3.

Figure 1 presents a 3-dimensional plot of the yield curve across time, that uses the data provided by Gurkaynak et al. (2007). In the plot, one can see that the level of yields at the start of the sample period is much higher than towards the end of the sample. There is a somewhat downward trend in general. Some yield inversions can also be observed, the most notable of which at the start of the sample in 1989, another in the year 2000 and lastly in 2006. The period following the Great Recession is characterised by extremely low, both long and short term, interest rates, comparatively speaking. The areas in dark blue clearly show the zero lower bound is reached, with the Federal Reserve setting federal funds rate at zero for a prolonged period of time. It can also be seen that rates are finally in an upward trend since around 2014.

4.2 Explaining the term premium - macro-economic variables

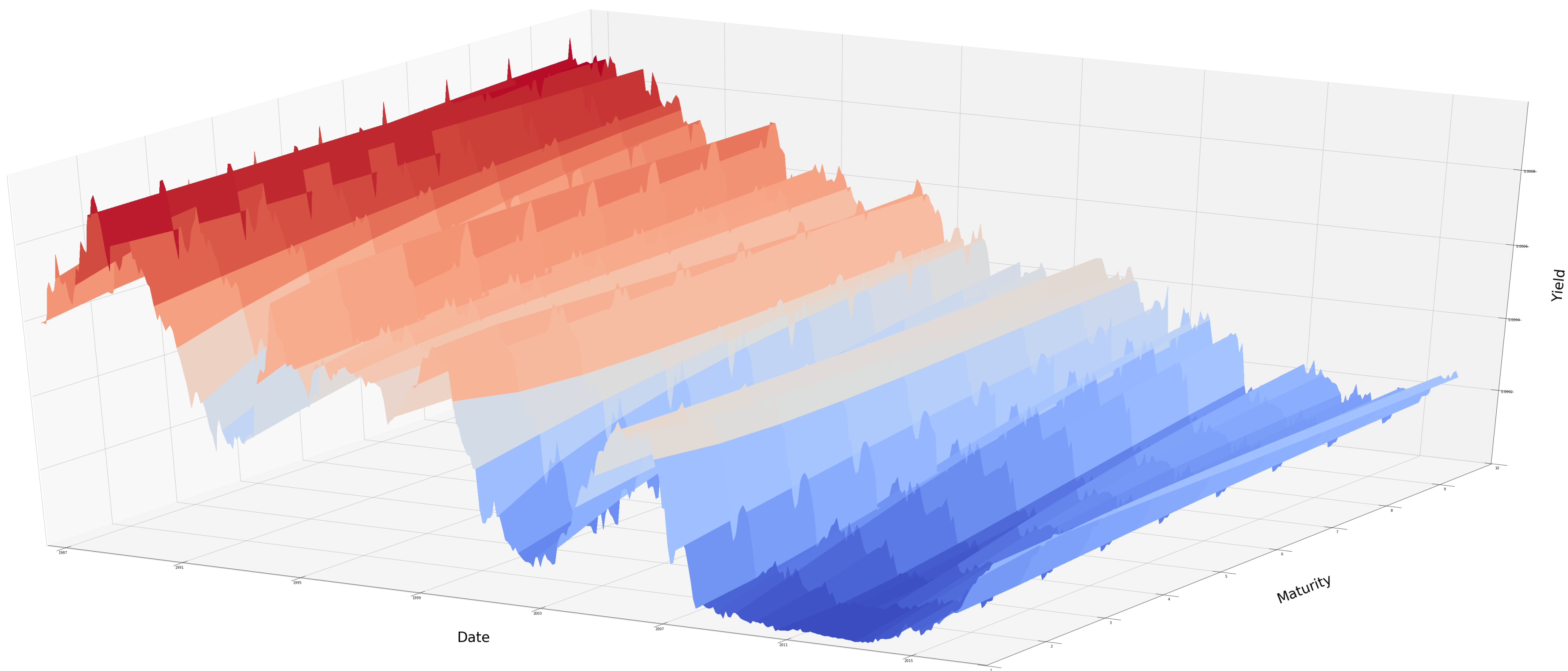
The term premium is the compensation that investors require for bearing the interest rate risk from holding long-term instead of short-term debt. Interest rate risk can be explained by a wide variety of economic variables, however, in this study, we consider only three. The term premia constructed theoretically refer to the same interest rate risk. The premia obtained should thus be relatively equally explained by the variables we consider; inflation volatility, unemployment and the volatility of Treasuries. These are the variables also considered in Callaghan (2019) to economically justify the term premium the author obtains from the term structure model employed.

4.2.1 Volatility of inflation

Volatile inflation can cause investors to require greater compensation for holding fixed-income securities. We thus consider the volatility of inflation as an explanatory variable for the term premium. Edward Hulseman and Alan Detmeister publish the Standard Deviation of Change in 12-month total CPI Inflation, with data from the Bureau of Labor Statistics of a monthly frequency. The authors compute the standard deviation of inflation using a 60-month rolling window. The data are published in the FED Notes section on the Federal Reserve website and proxy for the volatility of inflation. The considered data-set ranges from April 1988 until April 2017 and is thus a set of 349 observations. This slightly restricted data-set is used due to the availability of the variables considered. The MOVE index was only established in 1988, by Merrill Lynch and the data by Hulseman and Detmeister are limited to the year 2017. Figure 2 presents a plot of the considered data. A downward trend can be observed until the year 1996. After that, the trend reverses, and a clear spike occurs during the Great Recession. This is then followed by the initiation of another downward trend.

Figure 1: The US yield curve across time

The colouring of the surface is indicative of the magnitude of the yield. The magnitude of yields, ranging from high to low is as follows. Dark red, orange, grey, light blue and dark blue



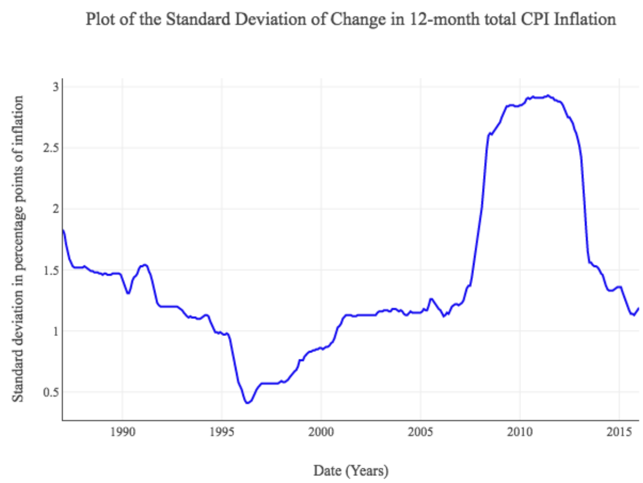


Figure 2: Plot of the Standard Deviation of Change in 12-month CPI Inflation. Monthly data are used.

4.2.2 Unemployment

Cochrane and Piazzesi (2005) show that term premia are positively correlated with the unemployment rate, suggesting that the term premium is counter-cyclical. We use the economic data of the FRED. Specifically, the seasonally adjusted Civilian Unemployment Rate of a monthly frequency is obtained from their website. We consider the sample from April 1988 until April 2017, again 349 observations. Figure 3 presents the corresponding data. It marks clear increases in the unemployment rate following the recessions in the U.S. in the years; 1990, 2001, and 2008.

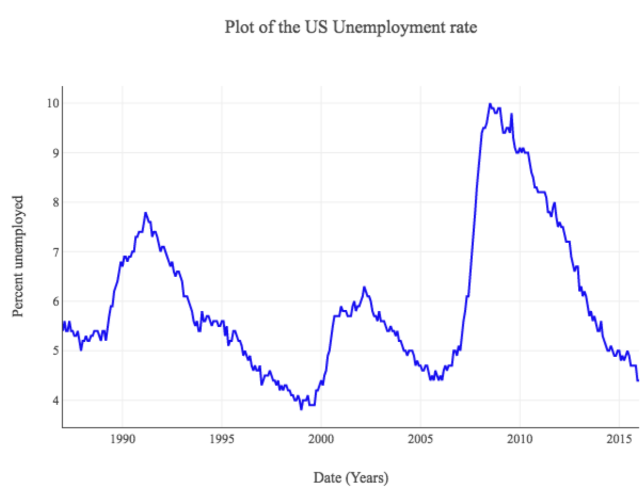


Figure 3: Plot of the seasonally adjusted Civilian unemployment rate of a monthly frequency.

4.2.3 The volatility of Treasuries - MOVE Index

A good proxy for the volatility of US Treasuries is the Merrill Lynch Option Volatility Estimate Index (MOVE), a measure of implied volatilities from options on US Treasury futures. It is the bond market's equivalent of the Chicago Board of Options Exchange Volatility Index (VIX) and helps to gauge the current level of fear in fixed income markets. The original authors of the three-step procedure, Adrian et al. (2013), show that their US term premium estimate is strongly correlated with the MOVE index, which would suggest the term premium reflects the risk of holding US Treasuries. The sample considered is in the same range as the previous two variables and is end-month data retrieved from the Bloomberg Terminal. The data is displayed in Figure 4. A clear spike occurs during the Lehman Brothers collapse in September 2008, followed by a roughly downward trend following a few months after the collapse.

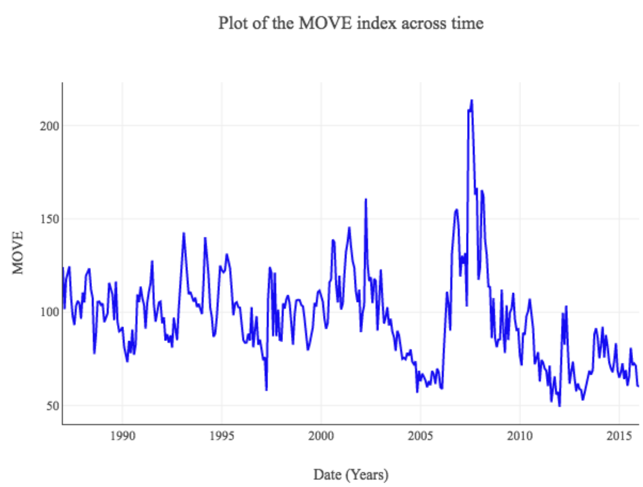


Figure 4: Plot of the Merrill Lynch Option Volatility Estimate across time. Monthly data are used.

5 Results

5.1 Standard Adrian et al. (2013) three-step procedure

We first discuss the standard Adrian et al. (2013) 3-step procedure and subsequently evaluate the results of the two extensions proposed in Section 3, that deal with the possible bias and high persistence in yield time series.

Figure 5 presents the yield fitting and term premium estimates for the 1 and 10-year bonds of the five-factor model, respectively. We document similar findings to Adrian et al. (2013). The 5-factor model close to perfectly fits Nelson-Siegel-Svensson yields. From the figure, one can observe more widely varying short term interest rates when compared to long term rates, this is as expected, as short rates are almost entirely determined by monetary policy, with long rates smoothing across expectations of these short rates and the term premium. The lower zero bound is clearly attained following the Great Recession with rates slowly cropping up after the year 2015.

In terms of the term premium, short term bonds exhibit almost none, with the term premium sometimes falling in the negative range. As the term premium is the reward for holding long term bonds, rather than rolling over short term debt, this is a natural result. A clear downwards trend can be observed from the 10-year bond term premium. The risk associated with holding long-dated paper is thus becoming increasingly poorly rewarded. It seems large scale asset purchasing programmes (quantitative easing) in the United States, Europe, and Japan have put further downward pressure on the term premium. An explanation of this phenomenon is a global shortage of safe assets and an increased demand by part of international bond investors for Treasury bonds, following uncertainty in the growth prospects of other developed economies in the Euro-zone and Japan.

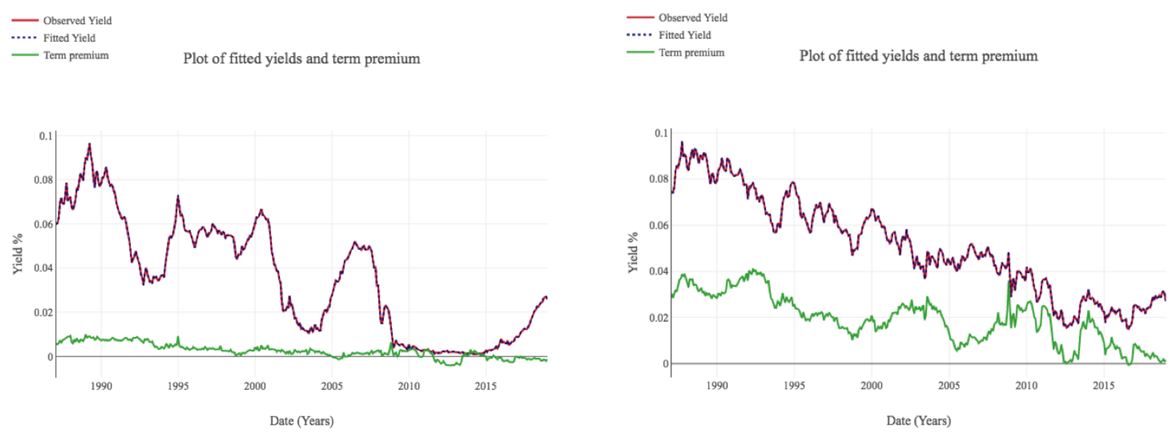


Figure 5: Plots of the Adrian et al. (2013) 3-step procedure yields and term premium

Turning to observed and model-implied excess returns, these can be observed in Figure 6. The same two maturity bonds are considered. Again, as the case with yields, excess bond returns are close to perfectly captured by the five-factor model. Studying the 1-year maturity bond, wild swings occur in the returns but the more recent data, excluded from the Adrian et al. (2013) paper, exhibit significantly less volatility. For the 10-year maturity bond, this is however not observed, volatility remains quite stable across the full sample. The excess returns exhibit the same patterns as those presented in Adrian et al. (2013), however, in terms of absolute returns, not quite the same results are found. This is true for both fitted and observed returns, which leads us to think there is a degree of mismatch between the yields data used in this study and that of Adrian et al. (2013), in terms of raw data or fitted Nelson-Siegel-Svensson yields. The three-step procedure does produce closely fitted returns which is as expected.

The solid green lines in Figure 6 depict the expected excess holding period returns. These are computed just as fitted returns except setting the innovation term ($\tilde{v}_{t+1}$) from equation (16) to zero. The expected returns capture the risk premium investors demand for holding the 1 and 10-year bonds for one month. The premiums exhibit a large degree of time-variation, but there does not seem to be an identifiable pattern in these premiums. This is also the case in Adrian et al. (2013).

Table 1 presents the pricing errors implied by the five-factor specification. The table reports

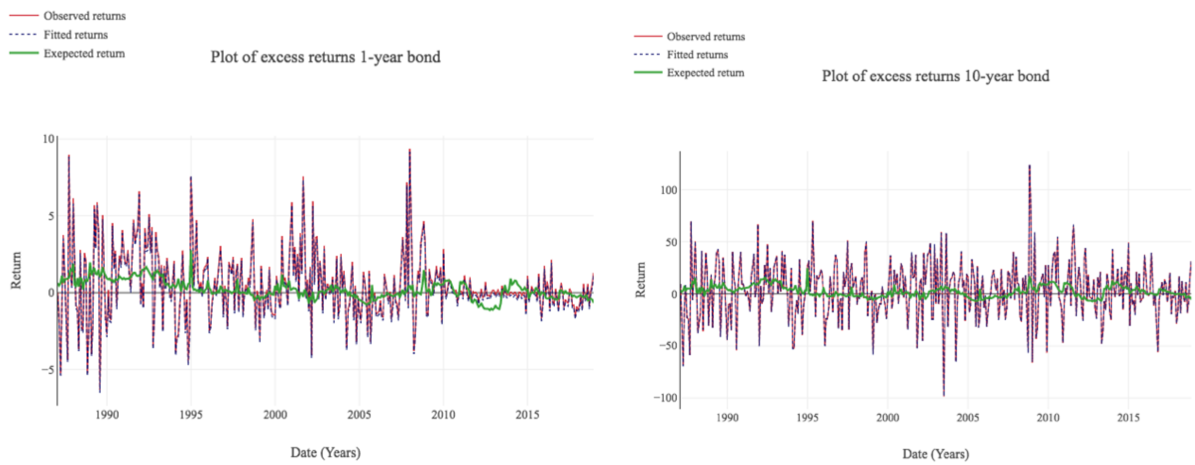


Figure 6: Plots of the Adrian et al. (2013) 3-step procedure excess returns and expected excess return

properties of the yield pricing errors and return pricing errors. Qualitatively, we yield the same results as Adrian et al. (2013). Mean yield pricing errors are very low across the board. We can observe that the highest, in absolute terms, mean yield pricing error is 0.002. Skewness is highly dependant on the maturity considered and the kurtosis of the yield errors is approximately 3 for all maturities. The 12-month bond produces yield errors that are very heavy-tailed, with kurtosis of around 31. In unreported results we find the errors of only the 10-year bond follow the normal distribution, as indicated by the Jarque-Bera test statistic. First-order auto-correlation is however high across all bonds, suggesting the error terms of yield pricing errors are not white noise. The highest attained auto-correlation is that of the yield pricing errors of the 60-month bond, the corresponding value is 0.868. The 12-month bonds' corresponding auto-correlation is only 0.332. This suggests the degree of auto-correlation of the yield pricing errors is maturity dependant. This is different to Adrian et al. (2013), who achieve more similar first-order auto-correlations.

Now looking at the return pricing errors in the lower part of Table 1, again, the results follow the patterns observed in Adrian et al. (2013). Mean return errors and standard deviations are small. The highest, in absolute terms, mean return error is 0.04. Skewness, as in the case of yield errors, vary depending on the maturity considered, ranging from 0.065 to 0.025. Kurtoses of the return errors are approximately three for the lower maturity bonds and increase starting at the 60-month bond. We also document low first-order auto-correlations at all maturities. Given that serially correlated return pricing errors generate excess return predictability not captured by the pricing factors, our obtained low first-order auto-correlations can be deemed desirable. An implicit assumption in the three-step procedure is that the auto-correlation of return pricing errors is zero, these results then satisfy this assumption.

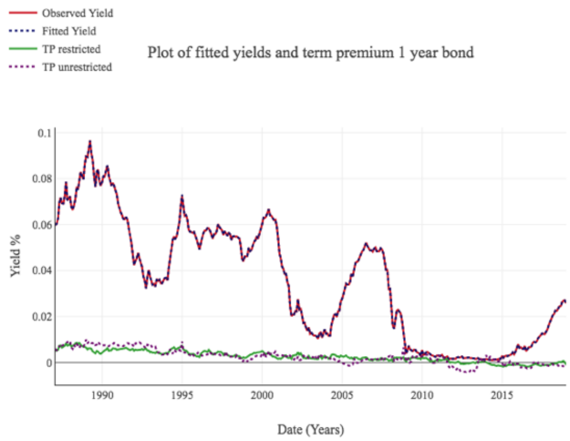
Table 1: Summary of the time series properties of the pricing errors implied by the five-factor specification. AR(1) refers to first-order auto-correlation of the errors. "n" refers to the n month maturity bond that is considered

Summary Statistics	n = 12	n = 24	n = 36	n = 60	n = 96	n = 120
Panel A : Yield Pricing errors						
Mean	0.000	0.002	0.000	-0.002	0.000	-0.002
Standard deviation	0.005	0.005	0.004	0.003	0.003	0.008
Skewness	-3.339	0.286	0.282	0.030	0.701	-0.075
Kurtosis	31.45	3.500	7.186	3.993	3.872	3.252
AR(1)	0.332	0.786	0.625	0.868	0.826	0.785
Panel B: Return Pricing errors						
Mean	-0.004	0.004	-0.004	-0.003	-0.002	-0.004
Standard deviation	0.025	0.026	0.030	0.030	0.025	0.065
Skewness	-0.104	0.064	0.152	0.049	0.346	-0.020
Kurtosis	3.947	3.264	3.781	5.013	5.015	10.469
AR(1)	0.177	0.103	0.040	0.056	-0.110	-0.125

5.2 Extending the Adrian et al. (2013) methodology

5.2.1 Imposing a random walk on the first principal component

We now turn to the results obtained when the first principal component of yields is modelled as a random walk, to account for the high persistence is yields. Figure 7 shows the yield fitting and term premium estimates for the 1 and 10-year bonds, for comparison purposes the term premium estimates of the unrestricted model are also plotted. We can see the yields are close to perfectly fit by the restricted model, as the case with the unrestricted model. The term premium estimates are quite different though. Modelling the levels of yields as a random walk produces more stable term premium estimates, this is an expected result because the unrestricted system is generally estimated to be less persistent than the data-generating process, we model the first principal component as an I(1) process so persistence increases. The higher persistence causes expected future short rates to be less stable and as such the term premia are less volatile. In the long term bond, we can also see that the levels of term premiums vary substantially between the restricted and unrestricted model. The term premium is generally lower at the start of the sample period in the restricted model, while this is reversed after the 2008 crisis.



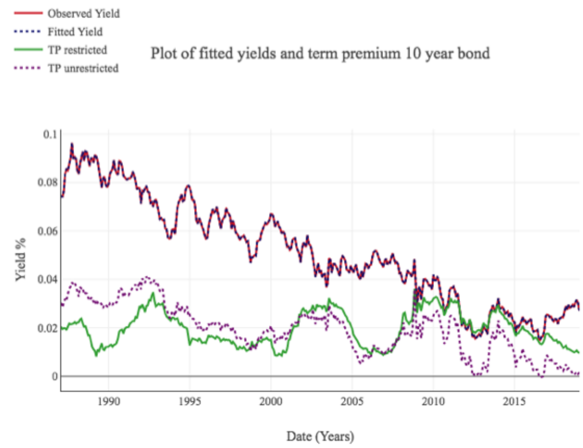


Figure 7: Plots of the term structure. Restricted VAR process with a random walk imposed on the first principal component is modelled in the Adrian et al. (2013) 3-step procedure

Table 2: Summary of the time series properties of the pricing errors implied by the restricted five-factor specification. AR(1) refers to first-order auto-correlation of the errors. "n" refers to the n month maturity bond that is considered

Summary Statistics	n = 12	n = 24	n = 36	n = 60	n = 96	n = 120
Panel A : Yield Pricing errors						
Mean	-0.001	0.001	0.000	-0.002	0.000	-0.002
Standard deviation	0.005	0.005	0.004	0.003	0.003	0.008
Skewness	-4.041	0.311	0.422	-0.036	0.812	-0.084
Kurtosis	40.170	3.214	8.526	4.070	5.544	3.277
AR(1)	0.352	0.773	0.591	0.865	0.823	0.784
Panel B: Return Pricing errors						
Mean	-0.004	-0.003	-0.004	-0.003	-0.001	-0.003
Standard deviation	0.026	0.026	0.032	0.030	0.025	0.064
Skewness	-0.094	0.062	0.155	0.052	0.357	-0.010
Kurtosis	3.885	3.251	4.087	5.041	4.925	10.427
AR(1)	0.055	0.102	0.039	0.172	-0.103	-0.122

Table 2 depicts the time series properties of the pricing errors implied by the restricted model. The model that imposes a random walk on the first principal component performs similarly to the unrestricted model in Adrian et al. (2013). Mean yield and return pricing errors are very low across the board, as expected given the fit observed in Figure 7. The maximum mean yield error is, in absolute terms, 0.002. For returns, this is 0.004. These are thus the same maximum mean errors as in the unrestricted model. The standard deviations and skewness are also similar to those of the unrestricted model. The corresponding values are close to zero, except for the skewness of the 12-month yield pricing error which is approximately -4. Again the 12-month bond yield pricing errors' kurtosis is very large, at 40.17. The first-order auto-correlation is also low for the return errors, they are in fact generally closer to zero than in the unrestricted model, as an assumption of the model is that the correlations are zero, this is a desirable result favouring the restricted model. Overall no significant changes with respect to the previous findings occur. It seems the greatest difference is in the term premiums the two different models obtain for the 10-year bond.

5.2.2 Bootstrapping to alleviate small sample bias

Finally, we present the yield fitting and term premium estimates for the 1 and 10-year bonds of the five-factor model with bias correction in the first step of the three-step Adrian et al. (2013) procedure. The small sample bias that occurs is removed from the VAR process.

Figure 8 shows the dynamics of the term structure. From the figure it can be observed that yields are again fit extremely well by this bias-corrected model; visually, there is no difference between the fit presented and the actual Gurkaynak et al. (2007) yield data. This is true for both the 1 and 10-year Treasury bonds.

Table 3 illustrates the percentage changes of the bias-corrected Phi estimates with respect to the original, Adrian et al. (2013) models' Phi estimates. We can observe that some individual parameters change quite drastically due to the bias-correction. The effect of the lag of the second principal component on the fourth pricing factor changes the most out of all the parameters. The estimate decreases by 92.5 per cent. The, generally, smallest change is in the effects on the level factor, where the largest change is almost 20%, that of the curvature pricing factor. Overall, we can say the bootstrap bias-correction does change the parameter estimates quite substantially. This is an interesting finding, as the eventual fit of both models seems to, as mentioned previously, remain practically constant.

In view of the term premium estimates, it is clear the bootstrap model produces more volatile term premium estimates than the original 3-step procedure. Unlike the restricted model, the bias-corrected term premium estimates do follow the same pattern as the original unconstrained model premium. The added volatility is not as expected, Bauer et al. (2012) highlights that the bias in the estimates of the time series dynamics of the pricing factors produce term premia that are too volatile. In this case, bias-correction produces term premia estimates that are more volatile than the unadjusted premia. The bias correction has thus made the component of long-term yields reflecting expected future short rates more stable than without bias correction. This would indicate market expectations remain quite smooth, however, expectations are known to react quickly to new information. We encourage further research to determine whether the expectations component found in this bootstrap method are plausible, as the expectations data needed to verify this fact is not available to us.

0.37	-10.1	19.98	-1.08	-2.57
33.30	0.75	-3.91	0.04	-1.01
-50.7	-25.1	1.17	3.10	-3.04
-50.2	-22.7	12.38	1.70	-0.30
-26.6	-92.5	3.19	2.17	2.71

Table 3: Table of the percentage changes in Phi estimates after bootstrap bias correction. The corresponding elements in this 5x5 table represent the individual element changes in the Phi matrix (5x5). The formula used is: (Bias-corrected Phi - Original Phi)/ Original Phi.

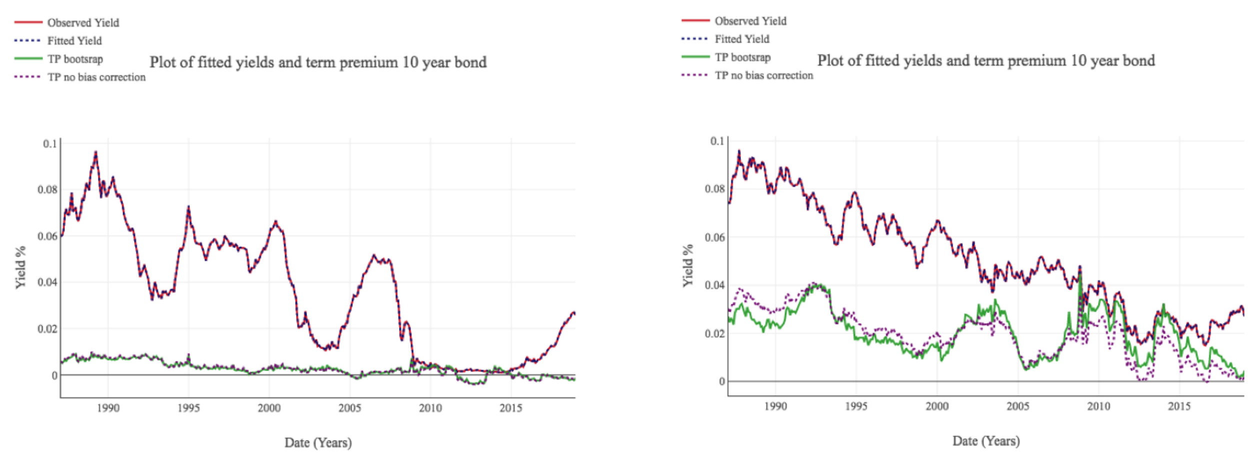


Figure 8: Plots of the term structure. A model that eliminates the small sample bias in the first step VAR process of the Adrian et al. (2013) 3-step procedure

Regarding yield and return pricing errors, Table 4 shows the fit is, on average, as accurate as in the previous two models; across all maturities errors are low and similar to the previous findings. We can for example see that the maximum mean yield error, in absolute terms, is 0.003. For returns this number is 0.004. The remaining central moments are also similar to those of the original and restricted model. Standard deviations and skewness are close to zero. Yet again, the skewness of the 12-month bonds' yield pricing error is an outlier, obtaining a value of -4.04. We again observe low first-order autocorrelations, which is a desirable result as it indicates a high consistency with the decomposition of yield pricing errors of Section 3.

5.2.3 Evaluating the forecasting accuracy

We now consider the forecasting accuracy of the three considered specifications. We estimate the 12-month ahead fitted yields using data up to an including the year 2017. Forecasts are for the year 2018. Figure 9 presents the forecasts obtained for the 1-year and 8-year Treasury bonds. We can see neither of the models very accurately predict the red, dashed, Observed Yield line. Forecasts are too low across the board, this may be due to the extremely low and bounded below by the zero lower bound, interest-rate environment, present following the Great Recession. As such, the

Table 4: Summary of the time series properties of the pricing errors implied by bootstrap, bias-corrected five-factor specification. AR(1) refers to first-order auto-correlation of the errors. "n" refers to the n month maturity bond that is considered

Summary Statistics restricted model	n = 12	n = 24	n = 36	n = 60	n = 96	n = 120
Panel A : Yield Pricing errors						
Mean	-0.001	0.001	0.000	-0.003	0.000	-0.002
Standard deviation	0.005	0.005	0.004	0.003	0.003	0.008
Skewness	-4.041	0.311	0.422	-0.037	0.812	-0.084
Kurtosis	40.17	3.213	8.526	4.070	5.544	3.277
AR(1)	0.352	0.773	0.591	0.865	0.823	0.785
Panel B: Return Pricing errors						
Mean	-0.004	-0.004	-0.004	-0.003	-0.002	-0.003
Standard deviation	0.026	0.027	0.032	0.030	0.025	0.065
Skewness	-0.094	0.067	0.155	0.052	0.357	-0.009
Kurtosis	3.885	3.251	4.087	5.041	4.925	10.428
AR(1)	0.172	0.102	0.039	0.055	-0.104	-0.123

VAR model for the principal components of yields may have been over-fitted resulting in yields that grow only slightly, as in the case of the 1-year bond. In the 1-month (left) bond yield predictions, the restricted model is clearly outperformed by both the original and bias-corrected model. The 12-month (right) graph shows that the original Adrian et al. (2013) 3-step procedure without bias correction or restrictions performs best, it yields increase to a larger degree than the other two, which mirrors the increase in actual yields to a higher degree.

These results are also confirmed by Table 5, which depicts the Root Mean Square Prediction Errors of the three considered models. We clearly see that at all maturities the original model performs, by far, the best. The coefficient restriction model overall produces the largest RMSPE's. Overall, it seems that alleviating the problems that yield time series dynamics give rise to, reduces the forecasting capabilities of the model. We attribute this to a more sample dependent model. In case of the bias-corrected model, the bias correction is based on the full sample, which may not be representative of the current dynamics in yields, especially given the unprecedented Central Banks' intervention since 2008. The Coefficient restriction model remains too stable due to the level component's forecast of the level component being equal to the previous month level and as such produces yield forecasts that remain too low.

The analysis is, of course, limited to the 2018 yield forecasts, the sample selected for both the training period and the actual forecast sample largely affect what the results from this section show. We encourage further research to determine if these findings are translated to other training and forecasting periods.

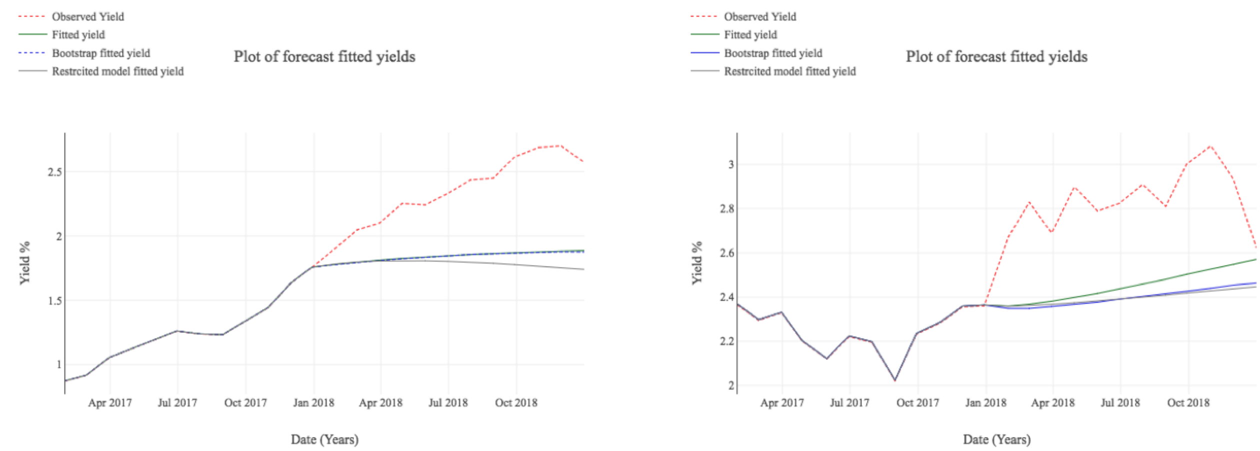


Figure 9: Plot of the different models 12-month ahead, iterative, forecasts for yields in the year 2018 for the 12-month (left) and 96-month (right) Treasury bonds. Also plotted is the actual fitted yields from the Adrian et al. (2013) 3-step procedure

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RMSPE:	n=12	n=24	n=36	n=60	n=96	n=120
Original model	0.5612	0.6557	0.6521	0.5555	0.4055	0.3370
Bias-corrected	0.5649	0.6716	0.6760	0.5921	0.4571	0.3917
Coefficient restriction	0.6340	0.7423	0.7389	0.6291	0.4586	0.3803

Table 5: Table of Root Mean Squared Prediction Errors (RMSPE) of the 12-month ahead, iterative, expanding window forecasts produced by the three considered models. “n” refers to the n month maturity bond that is considered

5.2.4 Economic significance of the term premia obtained

The linear regression results of the term premia on economic variables are presented in Table 6. All regressions have all their variables significant at least the 5% significance level. The F-statistic is versus only the constant model, indicating the independent variables fit the data better than an intercept-only model. The table shows all term premia we obtain have a negative level component. The results show unemployment has a positive effect on the term premium, suggesting that indeed the term premium is counter-cyclical. Inflation volatility has a negative impact on the term premium suggesting the demand for long term bond increases with volatile inflation. Investors thus park their assets in safe havens when inflation uncertainty rises. The MOVE index is shown to have a positive effect on term premia, this means that an increase in the current level of fear in the bond market increases the term premia and thus the yields of the 10-year bonds. This is then due to the demand for these long-term bonds dropping.

The bias-corrected model obtains a level component more than twice as large as the original model. The restricted model produces a term premium that is more stable, which shows in the coefficients being more modest (absolute terms) as compared to the other two. It is interesting to see that the coefficients vary widely between the three considered models. In theory, there is one "true" term premium, but the results presented in Table 6 show macro-economic variables have a different effect on the produced premia. The premia produced cannot at the same time represent the actual underlying premium and be affected to different degrees by certain variables. This suggests one of the premia produced must most closely resemble the "true" term premium.

Turning to the ratio of explained variance to the sample variance of the dependent variable, we can see the most of the variance is explained by the restricted model, followed by the bias-corrected model and subsequently the original model of Adrian et al. (2013). These results show that the term premium obtained, when the first principle component of yields is restricted to follow a random walk, is explained almost twice as well by the considered macro-variables than the original models' term premium.

The regressions are certain to encounter econometric inaccuracies, however, we simply compare term premia applying the same methods across the board and compare the predictive power of these variables. The R^2 is an indication of how economically sensical the obtained premia are because the premia form from certain macro-economic conditions which are represented by the variables considered. Omitted variable bias is of course expected to affect the coefficients and $R^{2\prime}s$ in the regressions, however, we deem the premium obtained in the restricted model more economically plausible due to the almost duplication of R^2.

Considering the underlying term premia refer to theoretically the same phenomenon - the only difference is how the premia are constructed, we find the restricted models' term premium more economically viable. This is true because the term premium is inherently a macro-economic phenomenon and should thus be explained by the variables we consider, to the same degree. A difference in the means of constructing the premia (the three considered models) should not affect to what degree the macro-economic variables explain the premia. The bias-corrected model also produces a more economically explainable term premium than the original model, however, the gain in explanatory power is not as significant, at around 12%. These results are based on the fact that another variable, not included in the regressions, does not distort the signal given by the R^2. It is possible that the inclusion of an omitted variable raises the R^2 of either of the two less explained term premia, rendering the results shown here obsolete. We encourage further research to determine if the inclusion of other variables can alter the interpretation we find in this study.

If the obtained term premium in the restricted model is more economically valid than the premium of Adrian et al. (2013), we cast doubt on not considering the first principle component a random walk. This is true especially considering the fit of both models is nearly identical and almost perfectly match the true yields.

In summary, in determining the term premium it may be beneficial, in terms of representing the "true" premium, to model the first principal component a random walk. Evidently, the forecasting performance of the original model is deemed better and for this application, no modifications are suggested.

Table 6: Regression results for the term premia obtained on macro factors. "O" refers to the original model in Adrian et al. (2013), "R" refers to the restricted model, "BC" refers to the bias-corrected model. Standard errors in parenthesis. Regressions are estimated using data from 04-1988 to 04-2017. * $p<0.05$, **$p<0.01$

	Term Premium - O	Term Premium - R	Term Premium - BC
Constant	-0.5195* (0.2301)	-0.7702** (0.125)	-1.1562** (0.1898)
Unemployment	0.4423** (0.0541)	0.4137**(0.0294)	0.4878**(0.0446)
S.D. of inflation	-0.8933** (0.1211)	-0.2168** (0.0657)	-0.5445** (0.0999)
MOVE index	0.0115 **(0.0019)	0.0062**(0.001)	0.0110** (0.0015)
R^2	0.351	0.627	0.474
Adjusted R^2	0.345	0.624	0.470
N^o Obs	349	349	349
F-statistic	62.2	194	104

6 Conclusion

This paper replicates and extends the Adrian et al. (2013) three-step procedure that estimates the term structure of US Treasuries. Using a data-set that covers unprecedented monetary policy and the period of economic recovery that followed, we study the yield fit and term premia obtained by the original authors in this more up-to-date setting. Our methodology firstly replicates that of the original authors, after which we model the yield time series dynamics differently. Specifically, we model the VAR process in the first step of the procedure as consisting partly of a component that follows a random walk; the level component. We also eliminate the possible bias that is caused in the first step due to the high persistence of yields and the small sample size considered. The obtained term structures are evaluated in terms of how well they fit the observed yield data and subsequently how well the different models can predict one-month ahead yields. In addition, the resulting differently constructed term premia are explained by macro-economic variables that, in theory, should explain the same amount of the total variation in the term premium. This is done to proxy for the economic plausibility of the obtained term premia.

The evidence we present suggests the original Adrian et al. (2013) model best forecasts future yields. That is, the yield persistence dynamics and bias corrections proposed, decrease the forecasting accuracy of the model. The Gurkaynak et al. (2007) yields are found to be fit equally well by all the three considered models. We also produce term premia, from the two extensions to the original work, that are more economically justified, given that they are explained to a larger degree by some of the macro-factors causing them. The term premium that is best explained, in terms of R^2, is obtained by the model that imposes a random walk on the first pricing factor, followed by that of the bias-corrected model and finally the term premium of the original Adrian et al. (2013) model. We encourage further research to determine if omitted variables are causing the higher R^2 or whether the term premium obtained by the restricted model is indeed more economically justified.

Notwithstanding, this paper is prone to some limitations. The used data-set spans decades and is characterised by the Great Recession and other significant economic downturns and expansions. Forecasting results incorporate the extreme observations in these periods that may have biased parameter estimates, all the considered data may then not be appropriate in the current setting of a more stable economic environment. Further research is encouraged to study the structural breaks that may be present. The bootstrap bias correction also encounters non-stationarity at some points and thus only incorporates a part of the bias-correction, such that the process remains stationary. The Adrian et al. (2013) procedure is also not intended for forecasting purposes, more typical yield forecasting models do not solely rely on linear regressions that are used in this study. Such models include state-space models that lie outside of the scope of this paper. Of course, the entire Adrian et al. (2013) three-step estimation approach is prone to the same limitations as the original paper. For example, the models are all over-parameterised and internally inconsistent. Finally, the economic significance conclusions of the obtained term premia, rely on omitted variables not altering the explanatory power of the considered variables, therefore it should be studied further whether these conclusions persist when more variables are incorporated.

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7 Appendix

Explanation of the code in Zip file:

The following program finds the standard Adrian et al. (2013) term structure and term premium, as well as the bond excess returns: Thesis_Code_28_05_noextensions

The following program finds bootstrap, bias-corrected term structure and term premium, as well as bond the excess returns: Thesis_Code_28_05_Bootstrap_Corrected

The following program finds the coefficient restricted term structure and term premium, as well as the bond excess returns: Thesis_Code_28_05_Coeff_Restriction

The following code performs the 12 step ahead forecasts of the standard Adrian et al. (2013) model: Thesis_Code_28_05_noextensions-forecasts

The following code performs the 12 step ahead forecasts of the bias-corrected model: Thesis_Code_28_05-Bootstrap-forecast-corrected-Copy1

The following code performs the 12 step ahead forecasts of the coefficient restricted model: Thesis_Code_28_05-Coeff_restriction-foreacsts