Currency Hedging for Long Term Investors with Liabilities

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Abstract

This thesis considers a long term investor with liabilities that invests internationally and therefore runs currency risk and has to decide on how much of this risk it wants to hedge. The purpose of this thesis is to optimize this currency hedging decision. To achieve this, a mean-variance framework is derived that includes exchange rates and liabilities. It will be found that the investment horizon of the investor will play an important role in the optimal currency decision and that this decision also depends on the characteristics of the investor. Furthermore, it will be found that the total exposure to foreign currency in the portfolio is not important in determining the optimal currency decision but that this decision is mainly determined by the correlations of the exchange rates with the assets and the liabilities.

1 Introduction

Long term investors like pension funds usually hold internationally diversified portfolios. Among others, Solnik (1974) shows that investors can reduce total portfolio risk by including foreign securities, rather than diversifying their portfolios domestically.

By holding foreign securities, an investor faces currency risk. The traditional view is that investors should fully hedge their currency exposure, because from the perspective of long-run policy, currency exposure should be seen as having zero expected return. Hedging therefore does not lower total expected portfolio return but it does reduce the risk and can therefore be seen as a "free lunch" as it is called by Perold and Schulman (1988). This point is also made by Jorion (1989), Kaplanis and Schaefer (1991) and Glen and Jorion (1992).

Froot (1993) however argues that this is a short term argument and that it generally applies only if real exchange rates follow a random walk. He argues that when real exchange rates and asset prices display mean reversion, the optimal hedging decision of an investor will generally depend on the investment horizon. He shows that for longer horizons unhedged assets are less volatile then hedged assets. Because a pension fund is an investor with a relatively long investment horizon, it may therefore prefer to hedge less than fully. A drawback is that Froot focuses on how hedging effects the variance of an individual asset, where hedge ratios should be determined according to the effects on the entire portfolio.

Campbell et al. (2003) give another argument for holding foreign currency. They argue that short term debt, which is usually seen as a riskless asset in a mean-variance framework isn't riskless in real terms in the long term[1]. In the short term, the only risk arises from shocks to the price level, which are modest over short periods. In the long term it is no longer riskless, because the real interest rate varies over time and a long-term investor must roll over short term debt at uncertain future real interest rates. This risk can be hedged by holding foreign currency if the domestic currency tends to depreciate when the domestic real interest rate falls.

Campbell et al. (2007) show that currency positions can be effective in managing the risk of

[1] In fact, it is also not riskless in nominal terms.

the total portfolio due to correlations between currency returns and equity returns. They consider seven countries and find that for some countries the currency exposure should be (partly) hedged or even over hedged and for other countries that the currency exposure should even be increased.

The literature mentioned above considers the currency hedging policy for asset-only investors. Pension funds have for a long time been considered as asset only investors, because regulatory frameworks and accounting standards did not require fair valuation of pension liabilities. Recently, there has been a shift to fair valuation of liabilities and these should therefore also be considered in determining the optimal portfolio choices, since liabilities are also subject to risk like inflation risk and interest rate risk. Considering assets and liabilities together in a total balance sheet approach is called asset and liability management (ALM).

Leibowitz (1987) and Sharpe and Tint (1990) determine the pension asset allocation through surplus management, thereby considering the pension liabilities. Sundaresen and Zapatero (1997) provide a framework in which they link the valuation and asset allocation policies of a pension plan with the lifetime marginal productivity schedule of the workers in the firm. Van Binsbergen en Brandt (2006) study the impact of regulations on the investment decisions of a pension plan by explicitly modeling the tradeoff between the long-term objectives and the short-term constraints. Hoevenaars et al. (2007) study the added value of alternative asset classes in an ALM-framework by maximizing the funding ratio return, a concept introduced by Leibowitz et al. (1994).

The contribution of this thesis lies in combining the fields of currency hedging and asset and liability management. Current literature on currency hedging has not yet considered investors with liabilities and current literature on asset and liability management has not yet looked at exchange rate risk. The main purpose of this thesis will be to determine the optimal currency policy for an investor with liabilities for different horizons, given the characteristics of the investor and the asset allocation of the investor. Furthermore this thesis aims to give insight in how this optimal currency policy changes when the characteristics of the investor change or when the asset allocation of the investor changes. The insights obtained from this thesis will therefore be relevant for pension funds and other investors with liabilities like insurance companies, because it will be helpful in reducing the total balance sheet risk.

The perspective of this thesis will be that of a Dutch and a Swiss pension fund. The pension systems in those countries have a lot of similarities. Moreover, the Swiss perspective is interesting because of the development of the Swiss Franc, which has not moved according to theoretical models. Furthermore, the US market and the British market will be used as foreign investment markets.

A useful tool in studying the variances and covariances of different time series is the term structure of the risk-return tradeoff, introduced by Campbell and Viceira (2005). They estimate the covariance-matrix of a VAR (1) model for different horizons to see how the variances and covariances change with investment horizon. This method will be used in this thesis to model the variances and covariances for different horizons and these will be used in an optimization framework to determine the optimal currency policy.

The set up of this thesis will be as follows. In section 2, an optimization framework will be set up. First the portfolio return will be derived for a hedged and unhedged portfolio. Next, the portfolio will be expanded with liabilities and at the end of that section an analytical expression will be derived for the optimal currency policy.

Section 3 first discusses the VAR(1) model. Next a description of the data and of the applied data transformations will be presented as well as summary statistics of the data. The last subsection presents the parameter estimates for the VAR(1) model.

Section 4 studies the term structure of the risk-return tradeoff. This will be done by looking at the correlation between the exchange rate returns and the different asset returns, liability returns and price inflation for different investment horizons.

Section 5 presents the empirical results. First the optimal currency policy for a specified base scenario will be determined. It will be found that contrary to the findings of Froot (1993) for asset-only investors, investors with liabilities will hedge their currency positions and will even over hedge their positions for all investment horizons. Next a number of sensitivity analyses will be performed on the characteristics of the investor and on its asset allocation. The primary finding will be that the exposure in the portfolio to foreign currency does have little influence on the currency positions but that these are primarily determined by the correlations with the asset and liability returns. Section 6 concludes.

2 Mean-Variance analysis

This thesis considers the problem of a pension fund that invests in domestic and foreign bonds and stocks and must decide how much of the currency exposure it wants to hedge. This problem best reflects the situation of a pension fund that in general first determines the strategic asset allocation and then decides how much currency exposure it wants to run. The exposure to foreign currencies can be adjusted by entering into forward exchange rate contracts. First the portfolio return will be defined where the return definition of Campbell et al. (2007) will be followed. Because a pension fund also faces liabilities, these need to be taken into account. Therefore the framework will be expanded with liabilities following Hoevenaars et al. (2008). The framework will also be expanded to a multi-period framework. Finally the mean-variance optimization will be defined.

2.1 Portfolio returns with currencies

In this section the portfolio return will be defined along the lines of Campbell et al. (2007). Let $R_{i,c,t+1}$ denote the gross return on asset i in currency c from holding asset i for one period from time t to time $t+1$, where asset i can be domestic or foreign bonds or stocks. Let $S_{c,t}$ denote the spot exchange rate in domestic currency per unit of foreign currency c at time t. The domestic country is indexed by $c = 1$ and of course, the domestic exchange rate is constant over time and equal to 1 so $S_{1,t} = 1$ for all t.

An investor exchanges at time t one unit of domestic currency for $1/S_{c,t}$ units of foreign currency c in the spot market and invests the proceeds in the bond or stock market of country c to earn a return of $R_{i,c,t+1}$. These returns can be exchanged at a rate of $S_{c,t+1}$ so the unhedged returns can be written as $R_{i,c,t+1}S_{c,t+1}/S_{c,t}$. When the returns are stacked in the vector $\boldsymbol{R}_{t+1}$which denotes the gross returns in local currency, the unhedged portfolio return can be written as:

$$R_{p,t+1}^{uh} = \boldsymbol{R}_{t+1}'\boldsymbol{\omega}_t\boldsymbol{D}(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t), \tag{2.1}$$

where $\boldsymbol{\omega}_t$ is a $(n \times n)$ diagonal matrix with the weights of the n assets in the portfolio $(\omega_{1,t}, \omega_{2,t}, \dots, \omega_{n,t})$ on the diagonal. These portfolio weights always add up to one. The portfolio is build such that the first two elements correspond to domestic bonds and stocks.

The following two elements correspond to bonds and stocks in the first foreign currency and so on. The number of currencies under consideration m, including domestic currency, is therefore half the number of assets in the portfolio: $m = \frac{n}{2}$. The portfolio weights always add up to 1. Pension funds usually only have long positions and therefore it is assumed that the portfolio weights are positive. $\boldsymbol{R}_{t+1}$ is the $(n \times 1)$ vector of gross returns in local currency. $\boldsymbol{D}$ is a distribution matrix which has in each row at position c a 1 when the asset is denoted in currency c and 0 otherwise. In the case of three currencies under consideration and given the structure of the portfolio as described above, this distribution matrix would be given by:

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The $(m \times 1)$ vector $\boldsymbol{S}_t$ has as its elements the spot exchange rates and ÷ denotes the element-by-element ratio operator so that when the i-th element of the vector $(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t)$ corresponds to currency c, then it is given by $S_{c,t+1}/S_{c,t}$.

To hedge part of the currency exposure, the pension fund can enter into forward exchange rate contracts. Let $F_{c,t}$ denote the one-period forward exchange rate in domestic currency per unit of foreign currency c and let $\theta_{c,t}$ denote the domestic currency value of the amount of forward exchange rate contracts for the investments denoted in currency c the pension fund enters into at time t per unit of domestic currency invested in the portfolio. Because $\theta_{c,t}$ is the value per unit invested in the portfolio, $\theta_{c,t}$ will be referred to as the hedge position. At time $t+1$ the pension fund can exchange $\theta_{c,t}/S_{c,t}$ of the returns denoted in currency c $R_{i,c,t+1}\omega_{i,t}/S_{c,t}$ at an exchange rate of $F_{c,t}$. The remaining part, $R_{i,c,t+1}\omega_{i,t}/S_{c,t} - \theta_{c,t}/S_{c,t}$ will be exchanged at the spot exchange rate $S_{c,t+1}$. The vector with hedge positions will have a length that is half that of the number of assets in the portfolio.

Putting everything together the partly hedged portfolio return can be written as:

$$R_{p,t+1}^{h} = \boldsymbol{R}_{t+1}'\boldsymbol{\omega}_t\boldsymbol{D}(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) - \boldsymbol{\theta}_t'(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) + \boldsymbol{\theta}_t'(\boldsymbol{F}_t \div \boldsymbol{S}_t), \quad (2.2)$$

where $\boldsymbol{F}_t$ is the $(m \times 1)$ vector of forward exchange rates, and $\boldsymbol{\theta}_t = (\theta_{1,t}, \theta_{2,t}, \ldots, \theta_{m,t})'$. Because $F_{1,t} = S_{1,t} = 1$ for all t, the value of the hedge position of the domestic assets is arbitrary. Therefore it is set such that the sum of all hedge positions is 1. Therefore it holds that:

$$\theta_{1,t} = 1 - \sum_{c=2}^{m} \theta_{c,t} \tag{2.3}$$

When covered interest parity holds, the forward contract for currency c trades at $F_{c,t} = S_{c,t}(1 + I_{1,t})/(1 + I_{c,t})$, where $I_{1,t}$ denotes the domestic short-term interest rate and $I_{c,t}$ is the short-term interest rate of country c. When this expression for the forward contract is substituted in equation (2.2), the hedged portfolio return can be written as:

$$R_{p,t+1}^{h} = \boldsymbol{R}_{t+1}'\boldsymbol{\omega}_t\boldsymbol{D}(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) - \boldsymbol{\theta}_t'(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) + \boldsymbol{\theta}_t'\left[\left(\boldsymbol{1} + \boldsymbol{I}_t^d\right) \div (\boldsymbol{1} + \boldsymbol{I}_t)\right], \tag{2.4}$$

where $\boldsymbol{1}$ is the $(m \times 1)$ vector of ones, $\boldsymbol{I}_t$ is the $(m \times 1)$ vector with foreign short-term interest rates and $\boldsymbol{I}_t^d$ is the $(m \times 1)$ vector with as each element the domestic short-term interest rate. From equation (2.4) it can be seen that selling currency forward is analogous to a strategy of going short in foreign cash and invest the proceeds in domestic cash or selling foreign currency and lending domestically.

The pension fund is said to have fully hedged its currency exposure when it sets the hedge position $\theta_{c,t}$ equal to the weights of the investments denoted in currency c. This is given by $\omega_{2c-1,t} + \omega_{2c,t}$. It under-hedges its currency exposure when it sets $\theta_{c,t} < \omega_{2c-1,t} + \omega_{2c,t}$ and it over-hedges its currency exposure when it sets $\theta_{i,c,t} > \omega_{2c-1,t} + \omega_{2c,t}$. When the pension fund chooses to fully hedge, it must be noted that the position is not exactly hedged, because the position fluctuates with realized return.

By using forward contracts, a pension fund can choose how much currency exposure it wants to run. Therefore, the portfolio return can also be written in terms of exposure instead of hedge positions. To this end, a new variable is introduced: $\psi_{c,t} \equiv \omega_{2c-1,t} + \omega_{2c,t} - \theta_{i,c,t}$, so when the pension fund does not want to have any exposure at all, it sets $\psi_{c,t} = 0$. A positive value of $\psi_{c,t}$ means that the pension fund is not fully hedging its position and that it wants to have a currency exposure or equivalently, it has a demand for currency. Now equation (2.4) can be written in terms of currency demands:

$$R^h_{p,t+1} = \boldsymbol{R}'_{t+1}\boldsymbol{\omega}_t\boldsymbol{D}(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) - \boldsymbol{1}'\boldsymbol{\omega}^*_t\left[(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) - \left(\boldsymbol{1} + \boldsymbol{I}^d_t\right) \div (\boldsymbol{1} + \boldsymbol{I}_t)\right] + \boldsymbol{\Psi}'_t\left[(\boldsymbol{S}_{t+1} \div \boldsymbol{S}_t) - \left(\boldsymbol{1} + \boldsymbol{I}^d_t\right) \div (\boldsymbol{1} + \boldsymbol{I}_t)\right], \tag{2.5}$$

where $\boldsymbol{\omega}^*_t = diag\left(\omega_{1,t} + \omega_{2,t}, \omega_{3,t} + \omega_{4,t}, \dots, \omega_{2m-1,t} + \omega_{2m,t}\right)$ and $\boldsymbol{\Psi}_t = \left(\psi_{1,t}, \psi_{2,t}, \dots, \psi_{m,t}\right)$. It follows that $\boldsymbol{\Psi}_t = \boldsymbol{\omega}^*_t\boldsymbol{1} - \boldsymbol{\theta}_t$. Because the portfolio weights add up to one, equation (2.3) implies:

$$\psi_{1,t} = -\sum_{c=2}^{m} \psi_{c,t}, \tag{2.6}$$

or $\boldsymbol{\Psi}'_t\boldsymbol{1} = \boldsymbol{0}$, so that $\psi_{1,t}$ represents domestic currency exposure. It is also easily seen that the currency portfolio is a zero investment portfolio. Since the portfolio weights add up to one, the portfolio is fully invested in the i assets. Therefore, the only way to achieve an exposure to a currency is to go short in another currency and the result is a zero investment portfolio.

It is easier to work with log returns. Therefore, a log version of equation (2.5) is needed. Campbell et al. (2007) derive this log version and show that it is approximately equal to:

$$r^h_{p,t+1} = \boldsymbol{1}'\boldsymbol{\omega}_t\left(\boldsymbol{r}_{t+1} + \boldsymbol{i}^d_t - \boldsymbol{i}_t\right) + \boldsymbol{\Psi}'_t\left(\Delta\boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}^d_t\right) + \frac{1}{2}\Sigma^h_t, \tag{2.7}$$

where the third term is a Jensen's variance correction term and is equal to

$$\Sigma^h_t = \boldsymbol{1}'\boldsymbol{\omega}_t diag\left(Var_t(\boldsymbol{r}_{t+1} + \Delta\boldsymbol{s}_{t+1})\right) - (-\boldsymbol{\Psi}_t + \boldsymbol{\omega}^*_t\boldsymbol{1})'\, diag\left(Var_t(\Delta\boldsymbol{s}_{t+1})\right) - Var_t\left(\boldsymbol{1}'\boldsymbol{\omega}_t\left(\boldsymbol{r}_{t+1} + \boldsymbol{i}^d_t - \boldsymbol{i}_t\right) + \boldsymbol{\Psi}'_t\left(\Delta\boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}^d_t\right)\right), \tag{2.8}$$

where the operator $diag$ stacks the diagonal elements of a matrix into a vector. For ease of notation the following vectors will be defined:

$$\boldsymbol{r}^h_{t+1} \equiv \boldsymbol{r}_{t+1} + \boldsymbol{i}^d_t - \boldsymbol{i}_t$$

$$\Delta\boldsymbol{s}^u_{t+1} \equiv \Delta\boldsymbol{s}_{t+1} + \boldsymbol{i}_t - \boldsymbol{i}^d_t$$

The intuition behind these variables is as follows. $\boldsymbol{r}^h_{t+1}$ is simply the vector of hedged returns of the different asset classes. Exchange rate returns can be divided in an expected return which is explained by the interest rate difference and an unexpected return. $\Delta\boldsymbol{s}^u_{t+1}$ is the vector with as elements the unexpected part of the exchange rate returns. Substituting these variables in equation (2.7) and (2.8) yields:

$$r_{p,t+1}^{h} = \mathbf{1}'\boldsymbol{\omega}_t \boldsymbol{r}_{t+1}^{h} + \boldsymbol{\Psi}_t' \Delta \boldsymbol{s}_{t+1}^{u} + \frac{1}{2}\Sigma_t^h \tag{2.9}$$

$$\begin{aligned}\Sigma_t^h = \; & \mathbf{1}'\boldsymbol{\omega}_t diag\big(Var_t(\boldsymbol{r}_{t+1} + \Delta \boldsymbol{s}_{t+1})\big) - (-\boldsymbol{\Psi}_t + \boldsymbol{\omega}_t^* \mathbf{1})' \, diag\big(Var_t(\Delta \boldsymbol{s}_{t+1})\big) \\ & - Var_t\big(\mathbf{1}'\boldsymbol{\omega}_t \boldsymbol{r}_{t+1}^{h} + \boldsymbol{\Psi}_t' \Delta \boldsymbol{s}_{t+1}^{u}\big)\end{aligned} \tag{2.10}$$

2.2 Funding ratio returns with currencies

Because a pension fund also faces liabilities, these need to be taken into account. Therefore the framework will be expanded with liabilities following Hoevenaars et al. (2007). They approach asset-liability management from a funding ratio perspective, a concept first introduced by Leibowitz et al. (1994). The advantage of working with the funding ratio return is that it is independent of the initial funding ratio. The funding ratio (F) is defined as the assets (A) of a pension fund divided by its liabilities (L). The funding ratio log-return is then defined as the log return of the assets minus the log return of the liabilities:

$$r_{F,t+1} = r_{A,t+1} - r_{L,t+1} \tag{2.11}$$

This expression can also be interpreted as the relative change in the funding ratio. The log return on the assets is defined in equation (2.9). Substituting in equation (2.11) yields:

$$r_{F,t+1} = \mathbf{1}'\boldsymbol{\omega}_t \boldsymbol{r}_{t+1}^{h} + \boldsymbol{\Psi}_t' \Delta \boldsymbol{s}_{t+1}^{u} + \frac{1}{2}\Sigma_t^h - r_{L,t+1} \tag{2.12}$$

When the portfolio weights are kept constant over different periods, the multi-period funding ratio return can be obtained by simply adding the single-period funding ratio returns. This is what is also seen in practice for pension funds. They usually perform an ALM study to determine the strategic asset allocation for several years and once this has been determined, the portfolio will be rebalanced to the strategic weights on a regular base. Therefore the multi-period funding ratio return is given by:

$$r_{F,t+\tau}^{(\tau)} = \sum_{j=1}^{\tau} r_{F,t+j} = \mathbf{1}'\boldsymbol{\omega}_t \boldsymbol{r}_{t+\tau}^{h,(\tau)} + \boldsymbol{\Psi}_t^{(\tau)\prime} \Delta \boldsymbol{s}_{t+\tau}^{u,(\tau)} + \frac{\tau}{2}\Sigma_t^h - r_{L,t+\tau}^{(\tau)} \tag{2.13}$$

The subscript $t+\tau$ denotes the cumulative return τ periods from period t to $t+\tau$. The vector $\boldsymbol{\Psi}_t^{(\tau)}$ contains the horizon dependent currency exposures. To come to the mean and variance of the multi-period funding ratio returns, first the annualized expected returns and the annualized covariance matrix will be defined. Therefore the returns are stacked in the vector:

$$x_t = \begin{pmatrix} \boldsymbol{r}_t^h \\ \Delta \boldsymbol{s}_t^u \\ r_{L,t} \end{pmatrix} \tag{2.14}$$

The annualized expected returns and the annualized covariance matrix are now given by:

$$\mu_t = \frac{1}{\tau}\mathrm{E}_t\left[x_{t+\tau}^{(\tau)}\right] = \begin{pmatrix} \mu_{\mathbf{r}_t^h}^{(\tau)} \\ \mu_{\Delta s_t^u}^{(\tau)} \\ \mu_{L,t}^{(\tau)} \end{pmatrix} \tag{2.15}$$

$$\Sigma^{(\tau)} = \frac{1}{\tau}\mathrm{Var}_t\left[x_{t+\tau}^{(\tau)}\right] = \begin{pmatrix} \Sigma_{\mathbf{r}^h\mathbf{r}^h}^{(\tau)} & \Sigma_{\mathbf{r}^h\Delta s^u}^{(\tau)} & \sigma_{\mathbf{r}^h L}^{(\tau)} \\ \Sigma_{\Delta s^u\mathbf{r}^h}^{(\tau)} & \Sigma_{\Delta s^u\Delta s^u}^{(\tau)} & \sigma_{\Delta s^u L}^{(\tau)} \\ \sigma_{L\mathbf{r}^h}^{(\tau)} & \sigma_{L\Delta s^u}^{(\tau)} & \sigma_L^{(\tau)2} \end{pmatrix} \tag{2.16}$$

The notation $x_{t+\tau}^{(\tau)}$ denotes the cumulative excess return in the period from t to $t+\tau$. The covariances change as the investment horizon τ changes. This relation between the investment horizon τ and the annualized covariance matrix $\Sigma^{(\tau)}$ is the term structure of the risk-return tradeoff which is introduced by Campbell and Viceira (2005).

The final step is evaluating the mean and variance of the multi-period funding ratio return:

$$\mathrm{E}_t\left(r_{F,t+\tau}^{(\tau)}\right) = \tau\left(\mathbf{1}'\boldsymbol{\omega}_t\mu_{\mathbf{r}_t^h}^{(\tau)} + \boldsymbol{\Psi}_t^{(\tau)\prime}\mu_{\Delta s_t^u}^{(\tau)} + \frac{1}{2}\Sigma_t^h - \mu_{L,t}^{(\tau)}\right) \tag{2.17}$$

$$\mathrm{Var}_t\left(r_{F,t+\tau}^{(\tau)}\right) = \tau\Big(\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^h\mathbf{r}^h}^{(\tau)}\boldsymbol{\omega}_t\mathbf{1} + \boldsymbol{\Psi}_t^{(\tau)\prime}\Sigma_{\Delta s^u\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} + \sigma_L^{(\tau)2} + 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^h\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)}$$
$$-2\cdot\mathbf{1}'\boldsymbol{\omega}_t\sigma_{\mathbf{r}^h L}^{(\tau)} - 2\cdot\boldsymbol{\Psi}_t^{(\tau)\prime}\sigma_{\Delta s^u L}^{(\tau)}\Big) \tag{2.18}$$

2.3 Mean-variance Optimization

The purpose is to determine the optimal currency policy for a long term investor with liabilities given the portfolio and the liabilities. Given the definitions before, this comes down to determining the optimal currency exposure. Therefore it is assumed that the portfolio weights $\boldsymbol{\omega}_t$ are given and that the choice variable is $\boldsymbol{\Psi}_t^{(\tau)}$, the vector with as its elements the horizon dependent currency demands. Note that $\psi_{1,t}^{(\tau)}$ need not to be determined, because it corresponds to the investments in the domestic currency in the portfolio. As can be seen from equation (6), its weight is given once the other currency demands are determined. Therefore this weight is excluded from the choice variable and the adjusted vector is now given by:

$$\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}}^{(\tau)} = \left(\psi_{2,t}^{(\tau)}, \dots, \psi_{m,t}^{(\tau)}\right)'$$

Following Van Binsbergen and Brandt (2006) it is assumed that a pension fund has constant relative risk aversion (CRRA) preferences on the funding ratio at some future date $T = t + \tau$:

$$V_t^{(\tau)} = \max_{\{\omega_t,\dots,\omega_{T-1}\}} E_t\left[\frac{F_T^{1-\lambda}}{1-\lambda}\right], \text{ where } \lambda \geq 0. \tag{2.19}$$

This is a standard power utility function, where the parameter λ can range from zero to infinity. This parameter λ is typically interpreted as a measure of the risk tolerance of an investor. Pension funds will in general be risk averse, because they are bound by certain risk constraints by regulatory authorities. Therefore in this thesis it is assumed that $\lambda > 1$.

The difference with Van Binsbergen and Brandt is that here it is assumed that the portfolio weights are fixed over the investment horizon. Therefore the problem to be solved is similar to that of Hoevenaars et al. (2008). They state that when normality of the excess returns is assumed, the optimization problem of equation (2.19) reduces to:

$$\max \frac{1}{2}(1-\lambda)\mathrm{Var}_t\left(r_{F,t+\tau}^{(\tau)}\right) + E_t\left(r_{F,t+\tau}^{(\tau)}\right) \tag{2.20}$$

After some algebraic manipulation shown in the appendix, this problem leads to the following vector of optimal mean-variance currency demands:

$$\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}}^{(\tau)*}(\lambda) = \frac{1}{\lambda}\left[\left(1-\frac{1}{\lambda}\right)\widetilde{\Sigma_{\Delta s^u \Delta s^u}^{(\tau)}} + \frac{1}{\lambda}\widetilde{\Sigma_{\Delta s^u \Delta s^u}^{(1)}}\right]^{-1}\left[\widetilde{\mu_{\Delta s_t^u}^{(\tau)}} + \frac{1}{2}diag\left(Var_t\left(\widetilde{\Delta \boldsymbol{s}_{t+1}}\right)\right) - \widetilde{\mathbf{1}'\boldsymbol{\omega}_t \Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(1)}} + (1-\lambda)\left(\widetilde{\mathbf{1}'\boldsymbol{\omega}_t \Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(\tau)}} - \widetilde{\sigma_{\Delta s^u L}^{(\tau)}}\right)\right] \quad (2.21)$$

When looking at the term in the second squared brackets, it can be seen that it can be split in a part that is multiplied by $(1-\lambda)$ and a part that is not. The part that is multiplied by $(1-\lambda)$ depends entirely on the horizon dependent covariances between the unexpected exchange rate returns and the asset and liability returns. This part can therefore be seen as the hedge demand and will by itself result in the currency positions that, given the portfolio weights, will minimize the variance of the funding ratio return. The other part also concerns returns and variances and can be seen as a speculative portfolio. This speculative portfolio is therefore given by:

$$\widetilde{\boldsymbol{\Psi}}_{\mathrm{S,t}}^{(\tau)*}(\lambda) = \frac{1}{\lambda}\left[\left(1-\frac{1}{\lambda}\right)\widetilde{\Sigma_{\Delta s^u \Delta s^u}^{(\tau)}} + \frac{1}{\lambda}\widetilde{\Sigma_{\Delta s^u \Delta s^u}^{(1)}}\right]^{-1}\left[\widetilde{\mu_{\Delta s_t^u}^{(\tau)}} + \frac{1}{2}diag\left(Var_t\left(\widetilde{\Delta \boldsymbol{s}_{t+1}}\right)\right) - \widetilde{\mathbf{1}'\boldsymbol{\omega}_t \Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(1)}}\right] \quad (2.22)$$

The hedge demand is given by:

$$\widetilde{\boldsymbol{\Psi}}_{\mathrm{H}}^{(\tau)*}(\lambda) = \left[\left(1-\frac{1}{\lambda}\right)\widetilde{\Sigma_{\Delta s^u \Delta s^u}^{(\tau)}} + \frac{1}{\lambda}\widetilde{\Sigma_{\Delta s^u \Delta s^u}^{(1)}}\right]^{-1}\left(\frac{1}{\lambda}-1\right)\left(\widetilde{\mathbf{1}'\boldsymbol{\omega}_t \Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(\tau)}} - \widetilde{\sigma_{\Delta s^u L}^{(\tau)}}\right) \quad (2.23)$$

When a pension fund would be extremely risk averse, it will let λ go to infinity and it is easily seen that in that case the speculative portfolio will be zero and the only part left is the hedging demand. When looking at the expression in equation (2.23) it can be seen that when the covariance between the unexpected exchange rate returns and the liability return increases, that also the demand for currency increases. The term in squared brackets will not change when this covariance changes. Because it is assumed that $\lambda > 1$ the term $\left(\frac{1}{\lambda}-1\right)$ will have a negative sign and that causes the covariance matrix between the unexpected exchange rate returns and the liability return to have a positive sign. In this case that means that when the liabilities increase, the unexpected exchange rate return tends in the same direction and in that way they have an offsetting effect on the funding ratio. When this tendency becomes stronger, the demand for currency will increase. Along these lines it can be said that when the covariance between the unexpected exchange rate returns and the liability returns has a positive sign, it will contribute to a positive demand for currency and when it has a negative sign, it will contribute to a negative demand or stated otherwise, an over hedged currency position.

By the same reasoning does the horizon dependent covariance matrix between the unexpected exchange rate returns and the hedged asset returns have a negative sign. When the covariance between the unexpected exchange rate returns and the bond and stock returns decreases, the demand for currency will increase. When the covariances decrease, there will be better diversification possibilities, which increases the demand for currency. Also when the sign of the covariance is positive, it will contribute to a negative demand for currency or an over hedged currency position and when the sign of the covariance is negative, it will contribute to a positive demand for currency.

Equation (2.22) also contains the covariance matrix between the unexpected exchange rate returns and the hedged asset returns but this time it is the one year horizon covariance matrix. When the pension fund will not be that risk averse equation (2.22) will also partly attribute to the currency demand and therefore the effect described above will become stronger. From this equation it can also be seen that when the unexpected exchange rate return increases, the demand for currency will increase and that when the variance of the exchange rate returns decreases, the demand for currency will increase.

3 Data description and model specification

This section describes the model that will be used for the modeling of the return dynamics and describes the data that will be used. The model that will be used is a vector-autoregressive (VAR) model and will be described in section 3.1. The data to be used as well as summary statistics of the data will be discussed in section 3.2. This section ends with the estimation results in section 3.3.

3.1 Model

From section 2.2 it became clear that the interest lies in the covariances between the returns on assets and liabilities and the unexpected exchange rate returns for different investment horizons. This covariance structure can be derived by constructing a VAR model of order one. A VAR model is a relatively simple model in which current values of economic variables and asset- and liability returns are linearly related to past values of the same set of variables. In mathematical notation the VAR model of order one is given by:

$$x_{t+1} - \mu = A(x_t - \mu) + \varepsilon_{t+1}, \;\; \varepsilon_{t+1} \sim N(0, \Sigma) \tag{3.1}$$

where x_t is the vector with returns as specified in equation (2.14) and ε_{t+1} is a vector of "residuals" or "one-step forecast errors" which are assumed to be multivariate normally distributed with a zero mean vector and covariance matrix Σ. This covariance matrix is the matrix as specified in equation (2.16). The data that are used are year data, so the covariance matrix Σ corresponds with an investment horizon of one year. It can be computed for different investment horizons using the estimated VAR coefficient matrix A. This coefficient matrix A describes the linear relation between current and past values of the returns.

Although the VAR-model has a relatively simple structure, it is very well able to describe the most important dynamic characteristics of the annually observed returns. These characteristics do not only include the averages and standard deviations of the returns, but also the correlations and auto and cross correlations.

3.2 Data and summary statistics

The perspective of this thesis is that of a Dutch and a Swiss pension fund, investing domestically and also in the United Kingdom and in the United States and are therefore facing currency risk with respect to the British Pound and the US Dollar. For the Dutch case, investing domestically means investing in fixed income in the Netherlands and investing in equity in a European index. Therefore the analysis will be based on the short- and long term interest rates and equity returns in these countries, the exchange rates between these countries and the price inflation in the Netherlands and Switzerland. The price inflation is needed since pension liabilities are (conditionally) indexed with price inflation, depending on the financial situation of a pension fund. This conditionality is expressed as a function of the funding ratio of a pension fund. In practice, some pension funds index their active participants with wage inflation but here it is assumed that both active and inactive participants are indexed with price inflation. An overview of the data that will be used, as well as their source can be found in table 1. All data are end of year data from 1970 to 2007 unless stated otherwise.

First logs of the series are taken. The equity series are already return series so these series need no further transformation. They are turned into hedged series by adding the logarithm of the domestic short term interest rate and subtracting the logarithm of the foreign short term interest rate. The log bond returns are calculated from the long nominal interest rate series by using the approximation of Campbell et al. (1997) which is given by:

$$r_{n,t+1} = y_{n-1,t+1} - D(y_{n-1,t+1} - y_{n,t}) \qquad (3.2)$$

where n is the maturity of the bond and $y_{n,t}$ is the log long nominal interest rate: $\ln(1 + Y_{n,t})$ at time t. $y_{n-1,t+1}$will be approximated by $y_{n,t+1}$. These series are also turned into hedged series in the same way as described before. D is the duration of the bond. In this thesis, the duration will be an input parameter and will therefore not depend on time and on maturity. The 10 year interest rate will be used for the long interest rate.

Panel A: Price Inflation

Region	Source
Netherlands	Bloomberg: OENLC005
Switzerland	Bloomberg: OECHC006

Panel B: Short nominal interest rate

Region	Source
Netherlands	1970-1994: van de Poll (1996)[2]
	1995-2007: Bloomberg: NEC0YL03
Switzerland	Bloomberg: OECHR007
United Kingdom	1970-1991: Bank of England, 1 year nominal rate
	1992-2007: Bloomberg: GUKTB3MO
United States	Bloomberg: USGG3M

Panel C: Long nominal interest rate

Region	Source
Netherlands	Bloomberg: OENLR006
Switzerland	Bloomberg: OECHR006
United Kingdom	Bloomberg: OEGBR006
United States	Bloomberg: USGG10YR

Panel D: Equity returns

Region	Source
Netherlands	1970-2001: MSCI[3] Europe Gross Index – local
	2002-2007: MSCI Europe Gross Index – Euro
Switzerland	MSCI Switzerland Gross Index – local
United Kingdom	MSCI United Kingdom Gross Index – local
United States	MSCI North America Gross Index - USD

Panel E: Exchange rates (in Euro)

Region	Source
Swiss Franc[4] (CHF)	Bloomberg: OECHK003
British Pound (GBP)	Bloomberg: OEGBK004
US Dollar (USD)	Bloomberg: OENLK002 (inverse)

Table 1: Used data series, end of year data from 1970-2007 unless stated otherwise and their source.

For a long time, liabilities have been discounted with a fixed interest rate and would not fluctuate with changes in the interest rate and were therefore not subject to interest rate risk. Recently, there has been a shift to fair valuation of the liabilities and these will therefore change when a change in the discount rate occurs. The duration of liabilities is in general much larger than the duration of the bond portfolio of a pension fund. The pension fund therefore has a duration mismatch. A lot of pension funds apply the principle of duration

[2] Van de Poll (1996), "Bronbeschrijving gegevens voor onderzoek risicopremie Nederlandse aandelen", Onderzoeksrapport WO&E nr 465/9615, DNB

[3] http://www.mscibarra.com

[4] The Bloomberg series for CHF and GBP are expressed in USD and have been converted to Euro values using the Euro/USD series

matching where the duration of the bond portfolio is increased by adding long maturity bonds to the portfolio or by adding interest rate swaps. By bringing the duration of the bond portfolio more in line with the duration of the liabilities, the pension fund will be less sensitive to changes in the interest rate. An interesting question therefore will be if a pension fund who has matched the duration of the bond portfolio with the duration of the liabilities will have another demand for currency.

The exchange rate series are calculated using the following steps. First the exchange rate returns are calculated using $\Delta s_{c,t+1} = \ln S_{c,t+1} - \ln S_{c,t}$. Then the unexpected part of the exchange rate return is calculated as:

$$\Delta s_{c,t+1}^{u} = \Delta s_{c,t+1} + \ln(1 + I_{c,t}) - \ln(1 + I_{1,t}) \tag{3.3}$$

The liability returns are also constructed by the approximation of Campbell et al. (1997) but also an inflation term is added to account for the (conditional) indexation of the liabilities. Here it is assumed that indexation is granted conditionally and therefore the liabilities have to be discounted by the nominal interest rate. The liability returns are therefore constructed in the following way:

$$r_{L,t+1} = y_{n-1,t+1} - D_L\left(y_{n-1,t+1} - y_{n,t}\right) + \varphi\pi_{t+1} \tag{3.4}$$

where π_t is the log price inflation and φ is the indexation ambition of the pension fund. Since indexation is granted conditionally, indexation will in general not be granted fully at all times and therefore an ambition is specified of the percentage of indexation a pension fund want to reach over a certain period. Most pension funds set their ambition equal to around 80%.

The assumption of conditional indexation best reflects a Dutch pension fund, because the majority of Dutch pension funds are granting indexation conditionally. Swiss pension funds however grant indexation unconditional. For sake of comparability it is assumed that a Swiss pension fund will also grant indexation conditionally. In one of the analysis, there will also be looked at a situation where unconditional indexation is the case.

There are some conditions to assume that the liabilities of a pension fund can be described as a constant maturity (indexed-linked) bond. Hoevenaars et al. (2008) state that a sufficient condition for this to be true is that the distribution of the age cohorts and the accrued pension rights are constant through time and that the inflow from contributions are equal to the net present value of the new liabilities.

For the base scenario it is assumed that the pension fund has no active duration matching policy and has a duration of its bond portfolio of 5. The duration of the liabilities is set equal to 20. The indexation ambition is set equal to 80%. There will also be looked at the currency demand for pension funds that have matched the duration of the assets and the liabilities and for pension funds that have a higher or lower duration of their liabilities.

The series for the hedged bond and stock returns, the unexpected exchange rate returns and the liability returns as described in equations (3.2) – (3.4) are constructed using the input parameters of the base scenario. Summary statistics of these series containing data from 1970–2007 can be found in table 2. Also presented are the summary statistics for the two sub periods 1970-1982 and 1983-2007. The first period is known for its high inflation and interest rate levels. Also at the beginning of the 1980's the central banks changed their monetary policy to using the interest rates as policy instruments for controlling inflation. Also shown are liability returns in case that pensions would not be indexed.

The average return on bonds in the Netherlands and on UK and US bonds, hedged with respect to the Euro is around 7%. Swiss bonds and UK and US bonds, hedged with respect to the Swiss Franc have an average return of around 5%-5,5%. The standard deviation of the returns on bonds in the Netherlands and in Switzerland is 4,43% and 3,84% which is lower than the standard deviation of bond returns in the UK and US, irrespectively to which currency they have been hedged and range from 6%-8%. When looking at the different sub periods it can be seen that except for US bonds, the returns are higher in the first period. Also the standard deviation for all bonds is higher in the first sub period.

The average stock returns are also higher for the Netherlands and the UK and US hedged with respect to the Euro than for Switzerland and the UK and US hedged with respect to the Swiss Franc. The first range from 9,6% to 11,1% and the second range from 7,9% to 9,2%.

Period	1970-2007		1970-1982		1983-2007	
	Mean	Stdev	Mean	Stdev	Mean	Stdev
Hedged bond Returns						
Netherlands	7,09%	4,43%	8,25%	4,73%	6,53%	4,26%
Switzerland	4,47%	3,84%	5,54%	4,42%	3,96%	3,51%
United Kingdom (EUR)	6,89%	7,90%	8,34%	12,36%	6,19%	4,70%
United States (EUR)	7,22%	6,01%	6,74%	7,25%	7,45%	5,47%
United Kingdom (CHF)	5,21%	7,54%	6,27%	11,69%	4,70%	4,66%
United States (CHF)	5,54%	6,44%	4,67%	7,86%	5,96%	5,77%
Hedged Stock Returns						
Netherlands	11,14%	18,82%	8,69%	19,35%	12,31%	18,85%
Switzerland	9,19%	21,31%	3,54%	19,35%	11,90%	22,04%
United Kingdom (EUR)	9,56%	16,92%	9,20%	22,95%	9,74%	13,71%
United States (EUR)	10,46%	15,90%	7,58%	19,04%	11,85%	14,38%
United Kingdom (CHF)	7,89%	16,21%	7,13%	21,43%	8,25%	13,54%
United States (CHF)	8,78%	15,56%	5,51%	18,04%	10,36%	14,35%
Unexp Exchange Rate Returns						
GBP-EUR	-0,37%	10,29%	-3,22%	11,91%	1,01%	9,37%
USD-EUR	-2,26%	12,71%	-2,52%	10,87%	-2,14%	13,72%
GBP-CHF	0,03%	11,78%	-4,96%	14,07%	2,43%	9,94%
USD-CHF	-1,86%	13,51%	-4,26%	13,23%	-0,71%	13,75%
Liability Returns						
Netherlands	11,25%	17,76%	12,98%	19,72%	10,42%	17,11%
Netherlands (not indexed)	8,40%	17,82%	7,52%	20,03%	8,82%	17,08%
Switzerland	7,76%	15,45%	11,76%	17,47%	5,84%	14,36%
Switzerland (not indexed)	5,48%	15,84%	7,82%	18,82%	4,36%	14,50%

Table 2: Summary statistics for the different series containing yearly data from 1970-2007 and for the sub periods 1970-1982 and 1983-2007, constructed under the assumptions of the base scenario. For the hedged series is stated in parentheses with respect to which currency the series is hedged.

The standard deviation of the stock returns, contrary to bond returns, is higher in the Netherlands and in Switzerland than in the UK and US, irrespectively to which currency they have been hedged. For the Netherlands and Switzerland it is equal to 18,82% and 21,31% and for the UK and US it ranges from 15,5% to 17%. What is remarkable is that for the second sub period, compared to the first sub period, all the average hedged stock returns are higher and that except for Switzerland, all the standard deviation are lower. For US and UK equity, irrespectively to which currency they have been hedged it is even lower by 4 to 8 percentage point.

The average unexpected exchange rate returns are negative for the GBP-EUR, USD-EUR and USD-CHF series and is approximately zero for the GBP-CHF series. In these currency pairs, the former is the foreign currency and the latter is the domestic currency. The USD series have an average unexpected exchange rate return close to -2% and the average return for the GBP-EUR series is equal to -0,37%. The standard deviation of these series ranges from 10,3%

to 13,5%. Therefore none of the unexpected exchange rate series has a mean significantly different from zero. When looking at the sub periods it can be seen that only the USD-EUR series has approximately the same mean for the two sub periods. The mean for the British Pound series even changes sign and has a difference of 4% with respect to the Euro and even 7% with respect to the Swiss Franc.

The average liability return in the Netherlands is equal to 11,25% and has a standard deviation of 17,76%. The average liability return in Switzerland equals 7,76% and has a standard deviation of 15,45%. This return is higher for the Netherlands because the long nominal interest rate has been higher in the Netherlands over the sample period and also the price inflation has been higher. These liability returns show that a pension must hold a portfolio of bonds and stocks to accomplish a higher average expected return on the assets than the return on the liabilities. It can also be seen by looking at the average returns for the full period that for a Dutch pension fund an indexation ambition of 80% and at the same time keeping the funding ratio stable cannot be realized unless extra contributions are made because the indexation cannot be financed by the excess return. None of the asset classes has an average return that is at least equal to the average return on the liabilities.

When looking at the sub periods it can be seen that an ambition of 80% can be reached in the second period due to lower price inflation and in the Swiss case also due to lower interest rates. The standard deviation is 2.5 to 4 percentage point lower in the second sub period compared to the first sub period.

3.3 Estimation results

This section reports the parameter estimates of the VAR model as described in equation (3.1) as well as the correlation matrix of the residuals. Section 3.3.1 reports the estimates for the Dutch series and section 3.3.2 reports the estimates for the Swiss series.

3.3.1 Dutch series

The parameter estimates of the coefficient matrix A of the VAR model as described in equation (3.1) for the Dutch series, as well as the correlation matrix of the residuals can be found in table 3. Also stated are the t-statistics of the parameter estimates.

Panel A: Parameter estimates of the coefficient matrix A of the VAR model

	$r_{B,NL,t-1}$	$r_{S,NL,t-1}$	$r_{B,UK,t-1}$	$r_{S,UK,t-1}$	$r_{B,US,t-1}$	$r_{S,US,t-1}$	$\Delta s^u_{UK,t-1}$	$\Delta s^u_{US,t-1}$	$r_{L,t-1}$	R^2
$r_{B,NL,t}$	1,92	-0,07	0,07	0,01	-0,04	0,00	-0,13	0,10	-0,51	0.42
	(2,92)	(-0,71)	(0,39)	(0,12)	(-0,20)	(0,02)	(-1,38)	(1,49)	(-3,49)	
$r_{S,NL,t}$	5,04	-0,58	-1,04	0,53	-0,60	0,24	0,75	-0,13	-0,35	0.32
	(1,67)	(-1,33)	(-1,30)	(1,00)	(-0,63)	(0,67)	(1,80)	(-0,42)	(-0,52)	
$r_{B,UK,t}$	2,94	-0,30	-0,35	0,10	0,15	0,13	-0,10	0,22	-0,63	0.40
	(2,52)	(-1,77)	(-1,13)	(0,50)	(0,42)	(0,97)	(-0,61)	(1,82)	(-2,46)	
$r_{S,UK,t}$	4,68	-0,48	-0,51	0,27	0,08	0,30	0,43	-0,02	-0,73	0.22
	(1,65)	(-1,16)	(-0,68)	(0,54)	(0,09)	(0,92)	(1,08)	(-0,08)	(-1,18)	
$r_{B,US,t}$	1,67	-0,02	-0,08	0,06	0,26	-0,10	-0,09	0,18	-0,53	0.35
	(1,77)	(-0,12)	(-0,31)	(0,37)	(0,87)	(-0,95)	(-0,66)	(1,81)	(-2,54)	
$r_{S,US,t}$	3,15	-0,29	-0,39	0,33	0,03	0,05	0,50	-0,32	-0,40	0.17
	(1,12)	(-0,71)	(-0,53)	(0,66)	(0,03)	(0,14)	(1,27)	(-1,07)	(-0,64)	
$\Delta s^u_{UK,t}$	0,30	-0,59	-0,88	0,42	-0,20	0,42	0,16	0,04	0,23	0.33
	(0,18)	(-2,45)	(-2,02)	(1,45)	(-0,39)	(2,18)	(0,71)	(0,21)	(0,63)	
$\Delta s^u_{US,t}$	2,69	-0,31	0,37	-0,30	-1,03	0,62	0,29	0,24	-0,36	0.47
	(1,51)	(-1,19)	(0,79)	(-0,96)	(-1,84)	(2,98)	(1,18)	(1,29)	(-0,91)	
$r_{L,t}$	3,97	-0,32	0,30	0,06	-0,17	0,02	-0,55	0,47	-1,12	0.24
	(1,31)	(-0,73)	(0,38)	(0,12)	(-0,18)	(0,06)	(-1,31)	(1,46)	(-1,69)	

Panel B: Correlation matrix of the residuals

	$r_{B,NL}$	$r_{S,NL}$	$r_{B,UK}$	$r_{S,UK}$	$r_{B,US}$	$r_{S,US}$	Δs^u_{UK}	Δs^u_{US}	r_L
$r_{B,NL}$	1								
$r_{S,NL}$	0,365	1							
$r_{B,UK}$	0,581	0,451	1						
$r_{S,UK}$	0,396	0,896	0,675	1					
$r_{B,US}$	0,730	0,400	0,460	0,332	1				
$r_{S,US}$	0,273	0,807	0,413	0,753	0,376	1			
Δs^u_{UK}	-0,386	-0,018	-0,010	0,029	-0,519	0,034	1		
Δs^u_{US}	-0,002	0,283	0,116	0,253	-0,240	0,058	0,453	1	
r_L	0,998	0,360	0,566	0,383	0,727	0,269	-0,355	0,020	1

Table 3: Panel A contains the parameter estimates of the coefficient matrix A of the VAR model for the Dutch series. In parenthesis are the t-statistics of these estimates. The last column contains the regression R^2. Panel B contains the correlation matrix of the residuals.

The first thing that becomes clear from examining these statistics is that the majority of the parameter estimates are not significant and that the hedged stock return series and the liability return series have no significant parameter estimates at all. Still these estimates are useful for constructing the "Term structures of risk" which will be discussed in section 4.

Also remarkable is the almost perfect correlation between the domestic bond returns and the liability returns. This can be explained by the fact that both series are constructed using the same underlying series, the long term interest rate. They only differ in duration and in the inflation part.

What is also of interest is the conditional correlation of the unexpected exchange rate returns with the other returns. Desirable is a negative and preferably low correlation with the bond and stock return series because this will give rise to diversification possibilities and a positive and preferably high correlation with the liability return, because than the unexpected exchange rate returns serve as a hedge. These will be examined more closely in section 4 when also is looked at these correlations at different investment horizons.

3.3.2 Swiss series

The parameter estimates of the VAR model as described in equation (3.1) for the Swiss series, as well as the correlation matrix of the residuals can be found in table 4. Also stated are the t-statistics of the parameter estimates.

There are a lot of similarities when comparing these results to the results for the Dutch series as discussed in the previous section. Here also the majority of the parameter estimates is not significant and the hedged stock return series have no significant parameter estimates at all. The liability return series, contrary to the Dutch case, does have one significant parameter estimate. This is the domestic bond return series. Also in the Swiss case there is an almost perfect correlation between the domestic bond return and the liability return.

Panel A: Parameter estimates of the coefficient matrix A of the VAR model

	$r_{B,CH,t-1}$	$r_{S,CH,t-1}$	$r_{B,UK,t-1}$	$r_{S,UK,t-1}$	$r_{B,US,t-1}$	$r_{S,US,t-1}$	$\Delta s^u_{UK,t-1}$	$\Delta s^u_{US,t-1}$	$r_{L,t-1}$	R^2
$r_{B,CH,t}$	3,92	0,01	0,10	-0,02	-0,12	-0,03	-0,07	0,02	-0,97	0.56
	(4,09)	(0,12)	(0,69)	(-0,29)	(-1,02)	(-0,50)	(-1,06)	(0,49)	(-4,51)	
$r_{S,CH,t}$	10,50	-0,50	-1,30	0,62	0,57	-0,16	0,89	0,27	-1,76	0.22
	(1,46)	(-1,40)	(-1,14)	(1,07)	(0,64)	(-0,35)	(1,73)	(0,76)	(-1,10)	
$r_{B,UK,t}$	6,09	-0,14	-0,46	0,07	0,30	-0,01	-0,05	0,22	-1,42	0.45
	(2,88)	(-1,38)	(-1,36)	(0,42)	(1,15)	(-0,05)	(-0,32)	(2,10)	(-3,02)	
$r_{S,UK,t}$	7,31	-0,24	-0,77	0,29	0,51	-0,03	0,55	0,06	-1,38	0.13
	(1,30)	(-0,85)	(-0,86)	(0,64)	(0,72)	(-0,08)	(1,36)	(0,24)	(-1,10)	
$r_{B,US,t}$	5,29	0,01	-0,10	0,08	0,21	-0,16	-0,03	0,15	-1,45	0.53
	(3,12)	(0,14)	(-0,37)	(0,62)	(1,00)	(-1,53)	(-0,26)	(1,81)	(-3,83)	
$r_{S,US,t}$	6,40	-0,09	-0,42	0,32	0,47	-0,18	0,56	-0,26	-1,35	0.13
	(1,15)	(-0,33)	(-0,47)	(0,72)	(0,68)	(-0,51)	(1,40)	(-0,97)	(-1,08)	
$\Delta s^u_{UK,t}$	-7,06	-0,44	-0,82	0,30	0,33	0,37	0,05	0,08	1,98	0.36
	(-1,97)	(-2,46)	(-1,43)	(1,03)	(0,74)	(1,63)	(0,19)	(0,46)	(2,47)	
$\Delta s^u_{US,t}$	-5,37	-0,32	0,82	-0,32	-0,67	0,62	-0,05	0,31	1,35	0.38
	(-1,33)	(-1,61)	(1,27)	(-0,97)	(-1,32)	(2,44)	(-0,16)	(1,56)	(1,50)	
$r_{L,t}$	13,57	-0,01	0,35	-0,04	-0,67	-0,12	-0,32	0,10	-3,29	0.45
	(3,14)	(-0,03)	(0,51)	(-0,10)	(-1,23)	(-0,44)	(-1,05)	(0,48)	(-3,40)	

Panel B: Correlation matrix of the residuals

	$r_{B,CH}$	$r_{S,CH}$	$r_{B,UK}$	$r_{S,UK}$	$r_{B,US}$	$r_{S,US}$	Δs^u_{UK}	Δs^u_{US}	r_L
$r_{B,CH}$	1								
$r_{S,CH}$	0,352	1							
$r_{B,UK}$	0,638	0,383	1						
$r_{S,UK}$	0,293	0,759	0,697	1					
$r_{B,US}$	0,539	0,434	0,282	0,206	1				
$r_{S,US}$	0,169	0,811	0,401	0,760	0,255	1			
Δs^u_{UK}	-0,097	0,243	0,218	0,276	-0,215	0,446	1		
Δs^u_{US}	0,030	0,457	0,379	0,500	0,065	0,440	0,535	1	
r_L	0,994	0,353	0,622	0,293	0,544	0,182	-0,087	0,019	1

Table 4: Panel A contains the parameter estimates of the coefficient matrix A of the VAR model for the Swiss series. In parenthesis are the t-statistics of these estimates. The last column contains the regression R^2. Panel B contains the correlation matrix of the residuals.

4 Term structure of assets and liabilities

From section 2 and 3 it became clear that of particular interest are the correlations of the unexpected exchange rate returns with the asset returns and the liability returns and therefore also with the price inflation since that is a part of the liability return. Even more of interest is how these correlations change with the investment horizon. A useful tool to study how the correlations change over time is the term structure of the risk return tradeoff which is introduced by Campbell and Viceira (2005). They estimate a VAR(1) model and construct the variance covariance matrix for different investment horizons as described in appendix A.2. This term structure gives the correlation between cumulative returns over different horizons. A drawback of this method is that short term data (year data) are used to estimate the long-term characteristics. However, since a dataset containing 200 year of data is not available, it is an acceptable method to work with. Section 4.1 studies the term structure of the unexpected exchange rate returns with the hedged asset returns. Section 4.2 studies the term structure of the unexpected exchange rate returns with the liability returns and price inflation.

4.1 Correlation with asset returns

This section studies the term structure of the unexpected exchange rate returns with the hedged asset returns. This will be done for the base-scenario as defined in section 3.2. Also attention will be paid to the correlation of the unexpected exchange rate returns with long duration bonds. The duration will be set such that it equals the duration of the liabilities and is therefore equal to 20. Section 4.1.1 will study the term structure for the Dutch case and section 4.1.2 will study the term structure for the Swiss case. The long duration bonds will be studied in section 4.1.3.

4.1.1 Dutch case

Figure 1 shows the term structures of the unexpected exchange rate returns with the hedged asset returns. The solid line represents the British Pound and the dashed line the US Dollar.

When looking at the Dutch bonds it can be seen that the correlation with the British Pound decreases at first and after six years starts increasing again and stabilizes at its short term level again after 20 years. Diversification possibilities and therefore demand for currency will be strongest for an investment horizon of six years. The correlation with the US Dollar is slightly decreasing at first, but starts increasing from an investment horizon of six years from -0.05 to 0.4 and therefore diversification possibilities are decreasing after five years with the investment horizon.

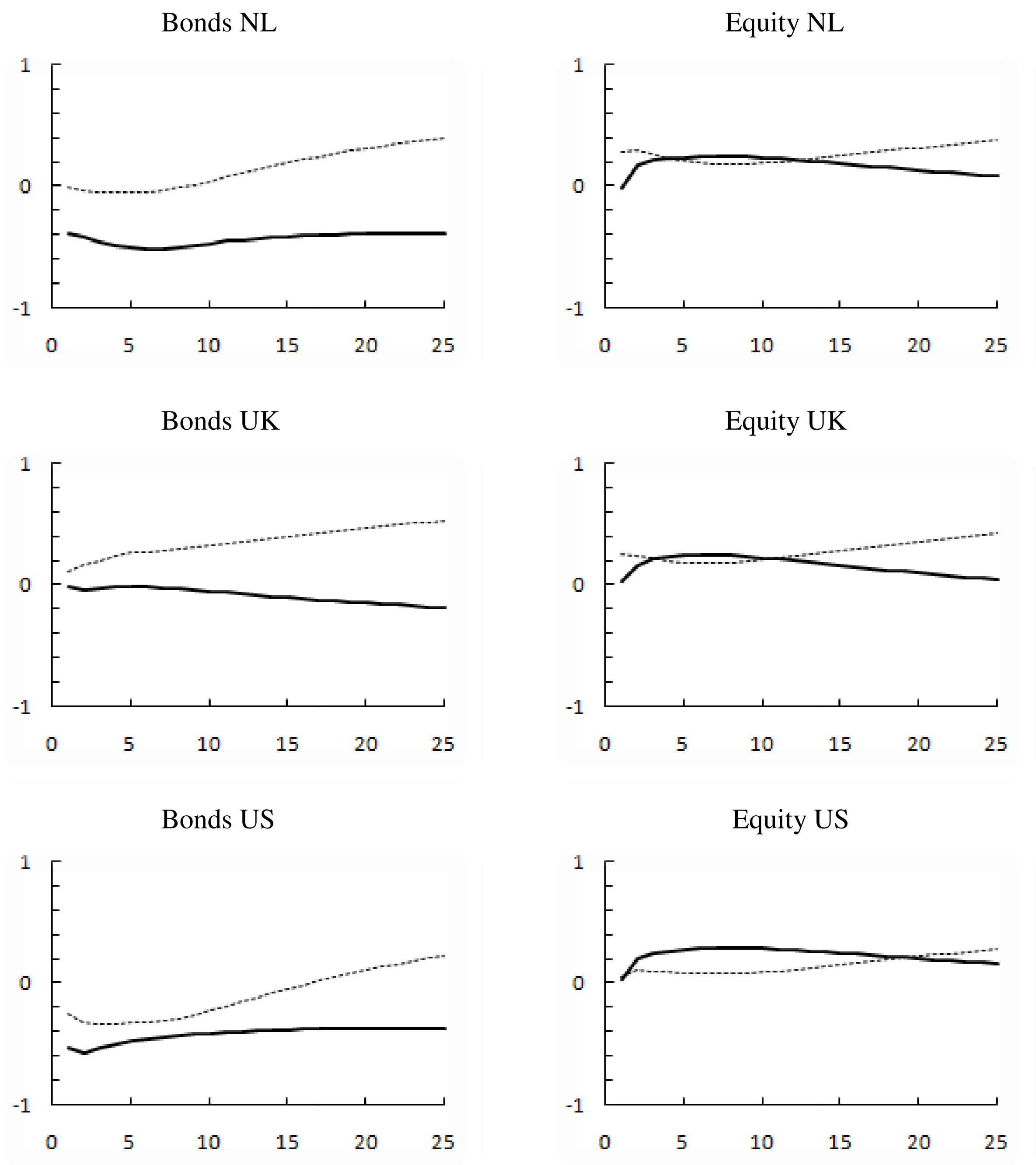

Figure 1: Correlations between the returns of the different hedged asset categories and the unexpected exchange rate returns across different investment horizons in years. The solid line represents the British Pound and the dashed line represents the US Dollar. The asset classes are hedged with respect to the Euro.

The correlations of the British Pound and US Dollar with British bonds show contrary patterns. They both start around 0, but the correlation with the British Pound decreases with the investment horizon and therefore diversification possibilities are increasing and the correlation with the US Dollar increases with the investment horizon.

The term structure of the US Dollar and US Bonds has similar characteristics as that of the US Dollar with Dutch bonds. It only starts at a lower value. The term structure of the British Pound with US Bonds dips at an investment horizon of two years, but is furthermore increasing with investment horizon and is relatively flat after an investment horizon of 15 years but still at reasonably low values and it therefore gives rise to diversification possibilities.

The correlations of the unexpected exchange rate returns with the hedged equity returns show very similar term structures for the different countries. The correlation with the British pound is approximately zero for an investment horizon of 1 year. It increases with the investment horizon to an horizon of six to seven years and then starts decreasing again. The correlation with the US Dollar shows the opposite pattern. It first decreases up to an investment horizon of six to seven years and then starts increasing.

4.1.2 Swiss case

Figure 2 shows the term structures of the unexpected exchange rate returns with the hedged asset returns. The solid line represents the British Pound and the dashed line the US Dollar.

When looking at the Swiss bonds it can be seen that both the correlation with the British Pound and the US Dollar have a value of near zero for an investment horizon of 1 year, are decreasing up to an investment horizon of 5 years and are onwards relatively stable where the correlation with the US Dollar is slightly increasing and with the British Pound is slightly decreasing some more. Remarkable however is the sharp decrease in the term structure of the British Pound with Swiss bonds and that it remains at these low values for longer investment horizons. It has approximately the same value as Dutch bonds for an horizon of 4 years, but the correlation with Dutch Bonds start increasing again where in this case it is decreasing even further.

The term structure of British bonds with the UK pound shows the same pattern as that of Swiss bonds, but it is shifted upward, so it start at a higher value. The term structure of British bonds and US Dollar is more or less flat, except for a small hump for short investment horizons of two to six years.

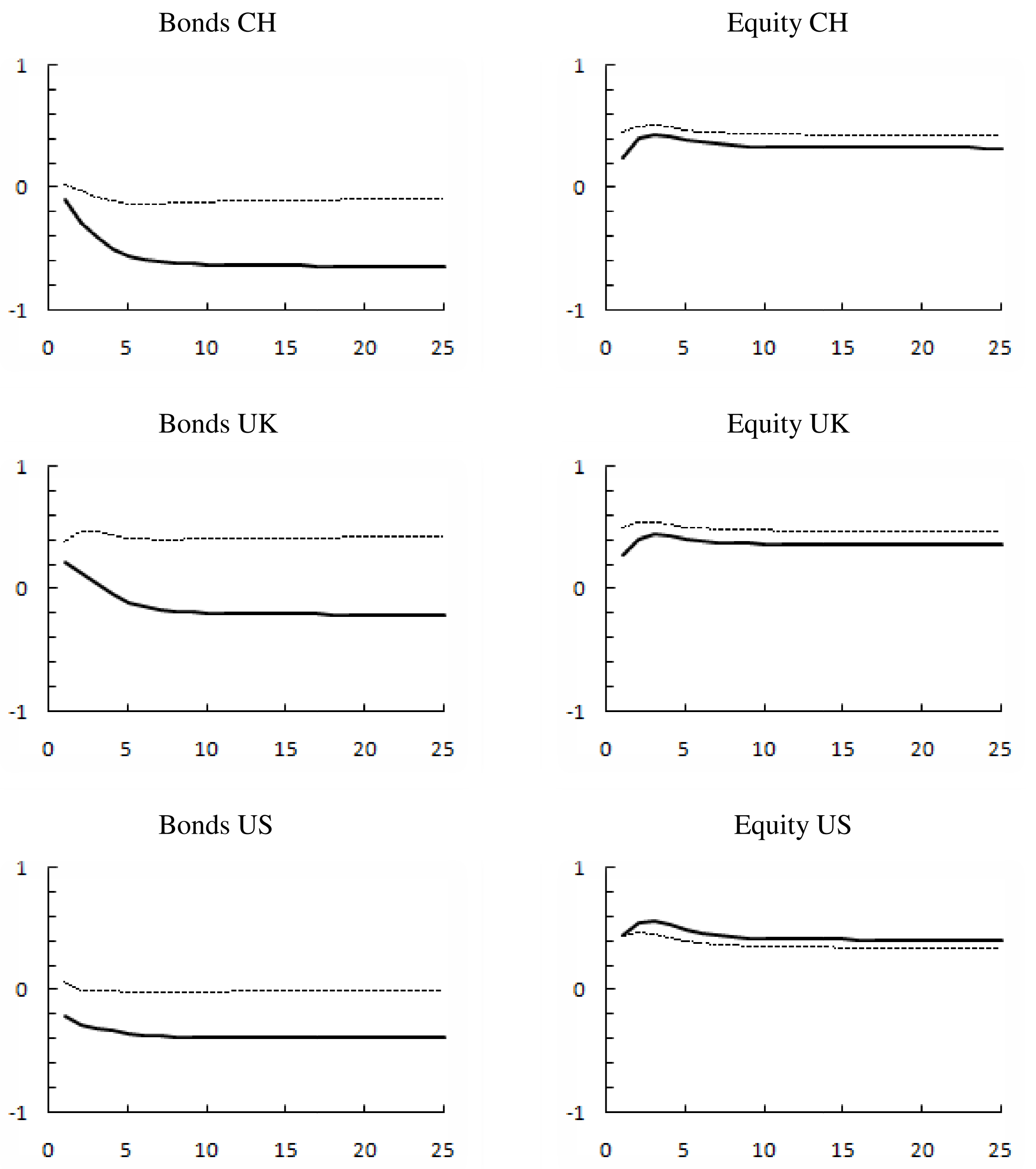

Figure 2: Correlations between the returns of the different hedged asset categories and the unexpected exchange rate returns across different investment horizons in years. The solid line represents the British Pound and the dashed line represents the US Dollar. The asset classes are hedged with respect to the Swiss Franc.

The picture for US Bonds is very similar to that of Swiss bonds, but both term structures and especially that of the British Pound are far less declining then in the Swiss bond case. The term structure of the US Dollar is almost entirely flat at a level of zero. That of the British Pound is entirely flat after an investment horizon of seven years.

As in the Dutch case the correlations of the unexpected exchange rate returns with the hedged equity returns show very similar term structures for the different countries. The only difference in the three countries under consideration is the level of the term structure and that for US equity the term structure of the British Pound lies under that of the US Dollar where for Swiss equity and British equity it lies above. All term structures have a small hump for short investment horizons and are from a horizon of about five years slightly declining but more or less flat. The correlations are at a relatively high level. They are all positive whereas the term structures for the bond classes show a lot of negative values. They are also large when compared to the term structures in the Dutch case.

4.1.3 Long duration bonds

This section studies the term structure of the unexpected exchange rate returns with long duration bonds. The duration of the bonds equals that of the liabilities and is equal to 20. These term structures are shown in figure 3. The solid line represents the British Pound and the dashed line the US Dollar.

When looking at the Dutch long duration bonds it can be seen that both term structures are increasing with the investment horizon. The correlation with the British Pound increases from -0.4 for an investment horizon of one year to 0.2 for an investment horizon of 25 years and the correlation with the US Dollar increases from 0.05 to 0.35. When the term structures are compared with those for regular Dutch bonds it can be seen that the term structures for the US Dollar have more or less the same starting point and end point, but the term structure for the British Pound remains flat for regular bonds and is increasing for long duration bonds and the diversification possibilities will therefore be smaller for the latter case.

The term structure of Swiss long duration bonds and US Dollar is as in the Dutch case very similar to that of regular bonds. It has more or less the same correlation across different investment horizons. The picture for the British Pound however is very different for regular and long duration bonds. Where it was sharply decreasing for regular bonds and remaining at low levels for longer investment horizons, it is slightly increasing up to an investment horizon of ten years and relatively flat for the longer investment horizons.

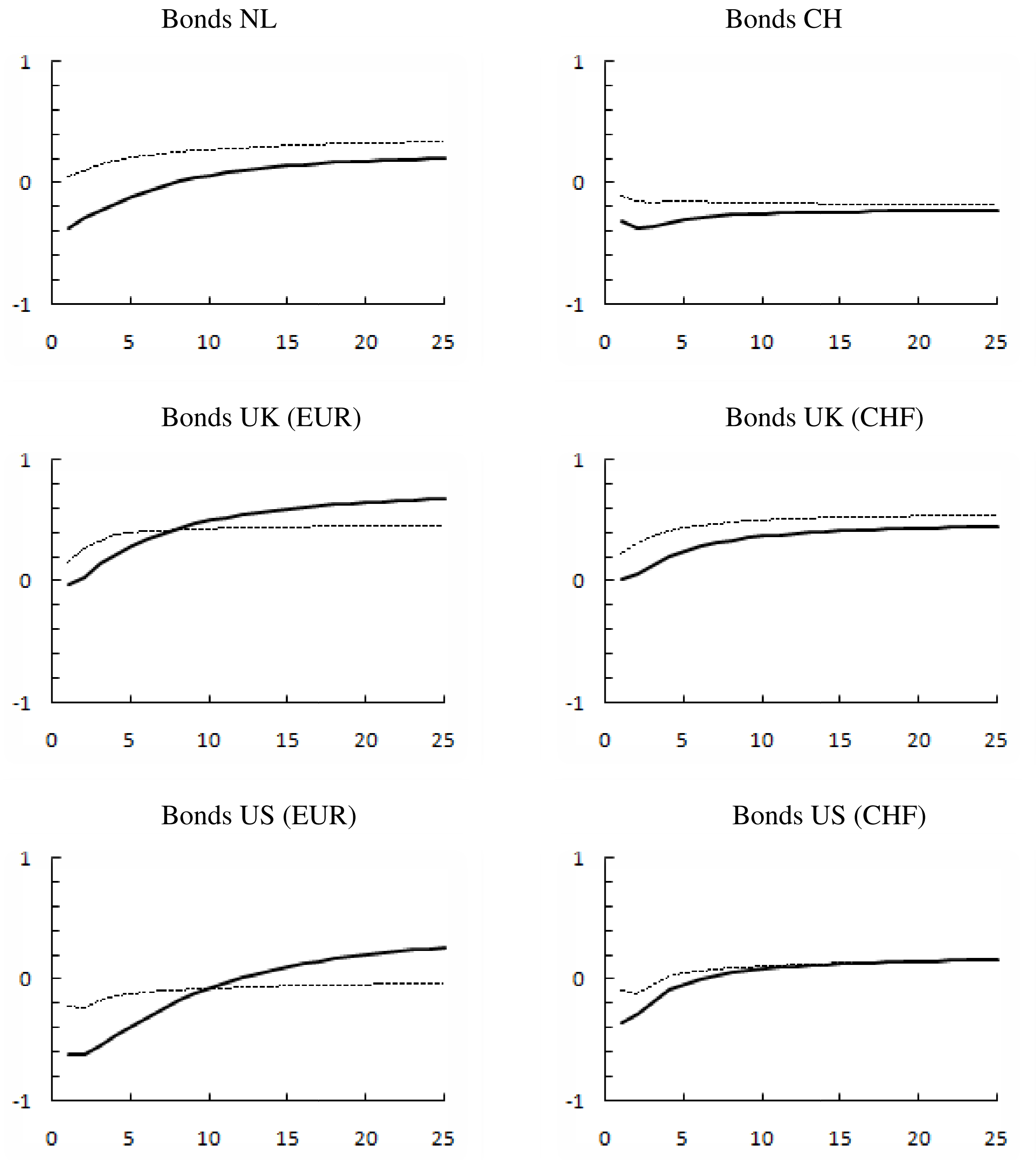

Figure 3: Correlations between the returns of hedged long duration bonds and the unexpected exchange rate returns across different investment horizons in years. The solid line represents the British Pound and the dashed line represents the US Dollar. In parenthesis is stated with respect to which currency the returns have been hedged.

The correlations of the British Pound and US Dollar with British long duration bonds show an increasing pattern with investment horizon, irrespectively to which currency they have been hedged. The correlation with the US Dollar increases for both base currencies with approximately the same size over the investment horizon whereas the correlation with the British Pound shows a stronger increase over the investment horizon when it is hedged with respect to the Euro. When looking at the hedge with respect to the Euro again it can be seen that when the term structures are compared to that of regular UK bonds, the term structure of the US Dollar shows more or less the same pattern but the term structure of the British Pound is decreasing for regular bonds and increasing for long duration bonds. The same can be said for the term structure for the British Pound when it is hedged with respect to the Swiss Franc. The term structure for the US Dollar is flat for regular bonds and the correlation is approximately equal for the longer investment horizons.

The correlation of US long duration bonds hedged with respect to the Euro with the British Pound has a large negative correlation for the short investment horizons but increases with investment horizon and turns positive for investment horizons longer than 12 years. The correlation for regular bonds is also large negative for short investment horizons but remains negative for longer investment horizons. The correlation with the US Dollar is less negative for short investment horizons but remains negative over the term structure. This in contrary to regular bonds where the correlation becomes positive for investment horizons larger than 17 years. The term structures of US long duration bonds hedged with respect to the Swiss Franc are increasing with the investment horizon and coincide for longer horizons. The long duration bonds have a positive correlation with the currencies for investment horizons over five years whereas this correlation is negative for regular bonds except for the US Dollar with an investment horizon of one year.

4.2 Correlation with price inflation and liability returns

This section studies the term structure of the unexpected exchange rate returns with price inflation and liability returns. There will be looked at the liability returns as defined in section 3.2 but also at liability return series that only depend on the interest rate and not on the inflation term or stated otherwise with an indexation ambition of zero. There will also be looked at the term structure of only price inflation with the unexpected exchange rate returns. These term structures are presented in figure 4.

In the previous section it was the case that when the correlation was lower, the diversification possibilities where higher and therefore the demand for currency would increase. In this section a higher correlation means a better hedge and therefore an increase in the demand for currency.

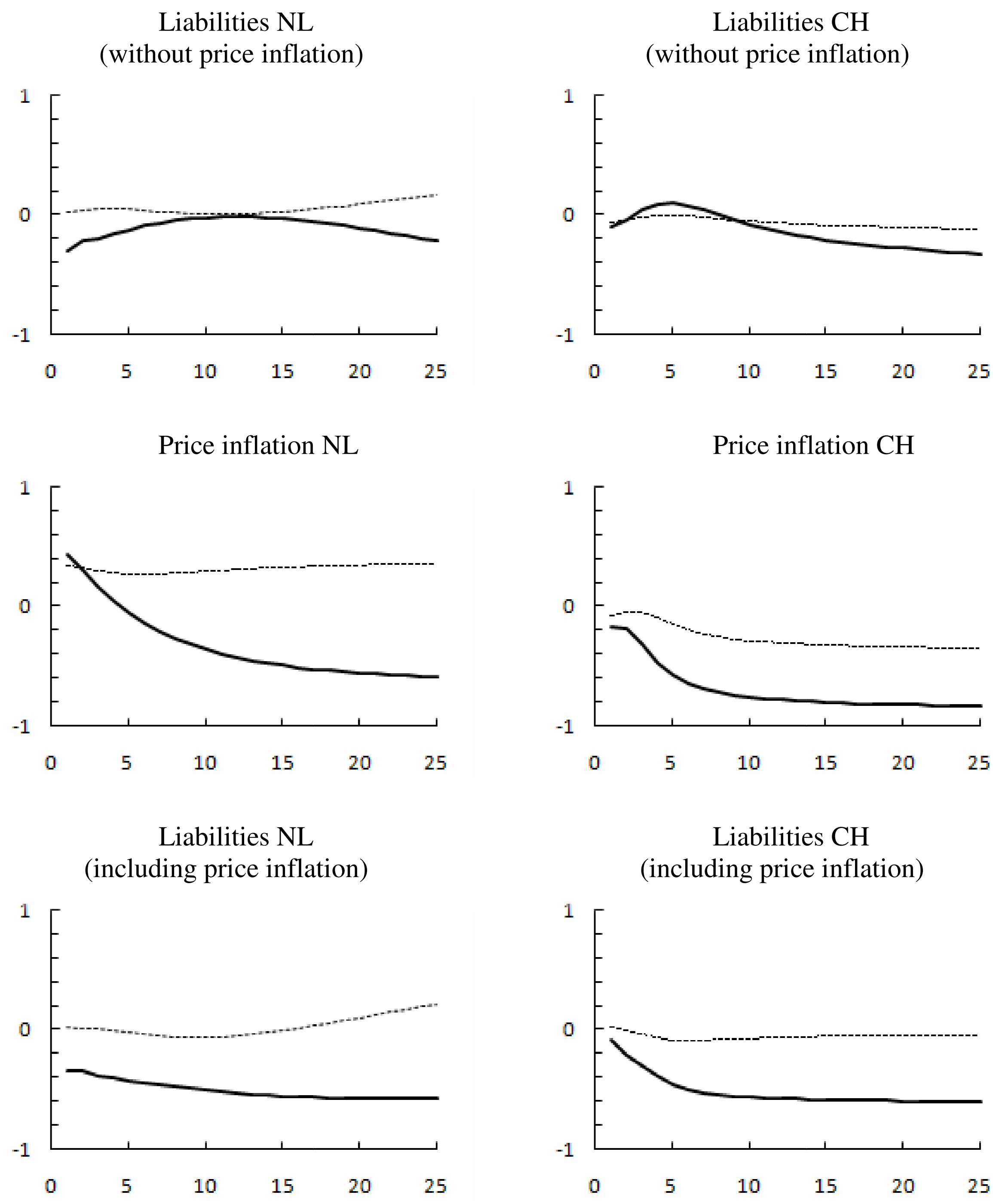

Figure 4: Correlations between the liability returns (without and including a price inflation term) and price inflation and the unexpected exchange rate returns across different investment horizons in years. The solid line represents the British Pound and the dashed line represents the US Dollar.

When looking at the term structures of the liability return without price inflation component in the Netherlands with the unexpected exchange rate returns it can be seen that the term structure for the British Pound is negative for all investment horizons and the term structure for the US Dollar is positive for all investment horizons. They are both close to zero for investment horizons from 10 to 15 years. The correlation with the British Pound starts decreasing again for longer investment horizons and the correlation with the US Dollar starts increasing again. The correlation of the liability return without price inflation in Switzerland has, contrary to the Dutch case, a negative correlation with the US Dollar for all investment horizons. It is almost zero for the short investment horizons and is slightly decreasing with investment horizon. The correlation with the British Pound is negative for almost all investment horizons except for horizons from three to seven years.

It can also be noted that the series for the liabilities without price inflation are constructed in the same way as the long duration bonds. The term structures in this section however are different from the term structures presented in section 4.1.3. This is caused by the fact that although the series are the same, they are part of a different dataset. These different datasets will therefore result in different VAR models. In Appendix A.2 it can be seen that the parameters of the VAR model are used to determine the term structures. Because the VAR parameters are different, the term structures are also different.

The term structure of Dutch price inflation and the US Dollar is positive for all investment horizons and is relatively stable. The term structure for the British Pound however shows a very different pattern. It has a correlation of 0.44 for an investment horizon of one year but starts decreasing very rapidly when the investment horizon increases and has correlations for investment horizons of over 20 years of about -0.55. This pattern is similar for the term structure of Swiss price inflation and the British Pound, but in this case the correlation is already negative for short investment horizons and reaches even lower correlations for investment horizons of over 15 years of about -0.8. The correlation with the US Dollar is also negative for all investment horizons but is less negative and has a correlation of about -0.35 for the longer investment horizons.

The term structure of the Dutch liability returns including a price inflation term and the US Dollar looks very similar to that where no inflation term is included. It has the same value for an investment horizon of one year and has approximately the same values for the longer investment horizons. The correlation of the liability returns including price inflation with the British Pound is also approximately equal for an investment horizon of one year to that of the liability returns without a price inflation term. However, in this case the term structure is decreasing with the investment horizon and has a correlation of about -0.55 for the longer investment horizons whereas for the liability returns without inflation term, the correlation first increases with investment horizon and starts decreasing again for the longer investment horizons to a correlation of about -0.2.

The correlation between the Swiss liability returns including a price inflation term and the US Dollar is relatively stable for different investment horizons. It is positive for an investment horizon of one year and negative for the other investment horizons but close to zero. The correlation with the British Pound is slightly negative for an investment horizon of one year but starts decreasing rapidly with investment horizon, similar to the term structure of Swiss price inflation and the British Pound. It is relatively stable for investment horizons longer than ten years and approximately equal to -0.6.

From the two pictures containing the term structures for the liability returns including a price inflation component it can be concluded that the US Dollar has for both countries and all investment horizons a higher correlation with the liability series than the British Pound and will therefore have better hedge qualities. Based on these term structures, the demand for US Dollar will be stronger than for British Pound.

5 Optimal currency demands for investors with liabilities

This section presents the optimal currency demands for an investor with liabilities. The currency demands will be determined for different investment horizons and different values of the risk aversion parameter λ. Section 5.1 discusses the outcomes for the base scenario as presented in section 3.2. The other sections will be sensitivity analyses on the different input parameters. Section 5.2 will discuss the effect of a different duration of the liabilities on the currency demands and section 5.3 will discuss the effect of a duration matching strategy. In section 5.4 an analysis will be done on the impact of a different asset allocation. There will be looked at a different allocation between bonds and equity and portfolios with a different currency exposure. Section 5.5 will study the impact of the indexation ambition on the currency demands and section 5.6 discusses the case when there will be looked at the sub period 1983-2007. Finally in section 5.7 some attention will be paid to parameter uncertainty.

5.1 Base scenario

This section presents the currency demands under the assumptions of the base scenario. It is assumed that the duration of the bond portfolio equals 5, that the duration of the liabilities equals 20 and that the indexation ambition of the pension fund equals 80%. Furthermore for the base scenario it is assumed that the pension fund holds a portfolio of 60% bonds and 40% equity. The bond portfolio is equally weighted over the home country, the UK and the US. The equity portfolio has 20% in the home country and 10% in British Equity and 10% in US equity.

Base Scenario									
		The Netherlands				**Switzerland**			
λ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05

Table 5: Currency demands under the assumptions of the base scenario. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

The results will be presented for The Netherlands and for Switzerland for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20. The results for the base scenario can be found in table 5.

The results for the Netherlands show that the demand for Euro is in a range of 0.50 to 0.60 for all investment horizons and all values of the risk aversion parameter. This demand is simply minus the sum of the demand for British Pound and US Dollar since the currency portfolio is a zero investment portfolio. In almost all cases the demand for British Pound and US Dollar is negative, which means that the investor will over hedge its exposures. The demand for British Pound and US Dollar is different however for different investment horizons and levels of risk aversion. It can be seen that for all levels of risk aversion, the demand for British Pound decreases at first from an investment horizon of one year to five years and start increasing again for horizons of 10 and 25 years or stated otherwise, the short position increases first and the British Pound is shorted less for longer investment horizons. The opposite pattern is visible for the demand for US Dollar. The demand first increases and then starts decreasing again. This pattern can be explained by looking at the term structures as presented in figure 1 and figure 4 in section 4. It can be seen that for all asset classes, except for UK bonds, the correlation of the asset with the US Dollar unexpected exchange rate return is decreasing from an investment horizon of one year to five years and therefore giving rise to better diversification possibilities and so the demand for US Dollar will increase. The correlation with the liabilities is also decreasing as can be seen in figure 4, causing the demand for US Dollar also to decrease, but the decrease is small and not enough to offset the increase in the currency demand caused by the declining correlation with the assets. The patterns are reversed for investment horizons longer than five years. From this horizon the correlation with all six asset classes is increasing with investment horizon which lowers the diversification possibilities and therefore lowers the demand for US Dollar. Again also the correlation with the liabilities is increasing for horizons over five years, but again not enough to offset the decrease in currency demand caused by the increasing correlation with the asset classes.

The same can be said about the pattern in the demand for British Pound for different investment horizons. In this case for most asset classes the correlation with the British Pound unexpected exchange rate return is increasing first from a horizon of one year to five years and is then decreasing causing the demand for British Pound first to decrease and then to increase again. The correlation with the liabilities is also decreasing for horizons of one to five years making the decrease in demand for British Pound even stronger. It remains decreasing for horizons over five years but the decrease is much smaller and not enough to offset the effect of the decreasing correlations with the asset classes.

It can also be seen that for all investment horizons, the demand for British Pound is decreasing when the value of the risk aversion parameter is increasing and the demand for US Dollar is increasing. This effect can be explained by looking at equation (2.21). It was shown that the currency demand can be split in a speculative part (equation (2.22)) and a hedge demand (equation 2.23)). When looking at table 2 in section 3, it can be seen that the mean of the British Pound unexpected exchange rate returns is higher than that of the US Dollar. Although not shown here, it is in such a way that the speculative part will cause the demand for British Pound to be positive and the demand for US Dollar to be negative. When the value of the risk parameter increases, the speculative part becomes smaller which causes the demand for the British Pound to decrease and the demand for US Dollar to increase or to become less negative.

The results for Switzerland show that the demand for Swiss Franc and US Dollar is increasing with investment horizon and the demand for British Pound is decreasing with investment horizon for all levels of risk aversion. The demand for Swiss Franc is always positive, the demand for US Dollar is always negative and the demand for British Pound is almost always negative meaning that also a Swiss investor will mainly over hedge its currency exposures.

Again the patterns can be explained by looking at the term structures in section 4 in figure 2 and 4. The most important cause for the decrease in demand for British Pound is the term structure with the liabilities. This term structure shows a sharp decrease from a horizon of one year to a horizon of ten years. The correlation with the bonds is also somewhat decreasing but not enough to offset the effect on the currency demand caused by the decreasing term structure with the liabilities. The correlation of the US Dollar with the different asset classes is somewhat decreasing for the short investment horizons and is furthermore relatively stable with investment horizon, causing the demand for US Dollar to increase when the investment horizon increases from one to five year and the demand is still somewhat increasing for the longer investment horizons but only small.

When looking at different values of the risk aversion parameter the same can be concluded for Switzerland as for the Netherlands. The demand for British Pound is decreasing when the level of risk aversion increases and the demand for US Dollar is increasing. The same holds as for the Netherlands that the speculative part is positive for the British Pound and therefore becoming smaller when the risk aversion parameter increases and the speculative part is negative for the US Dollar and becomes less negative when the risk aversion parameter increases.

5.2 Duration of the liabilities

This section studies the effect of the duration of the liabilities on the currency demands. The duration of the liabilities depends among others on the average age of the participants in the pension fund. When the participants are younger, the payments have to be made at a later time in the future and therefore the duration will be higher. In the base scenario it was assumed that the duration of the liabilities is 20. In this analysis there will be looked at a duration of 15 of the liabilities and a duration of 25. The results are presented in table 6.

The patterns of the base scenario for both countries are still present when the liabilities have a different duration. The currency demand for British Pound is decreasing with an increasing value for the risk aversion parameter and the demand for US Dollar is increasing. Also the pattern for different investment horizons is the same for both countries.

The levels at which these patterns are visible however are different. When looking at the Dutch situation it can be seen that when the liabilities have a duration of 15 the currency demand for British Pound is somewhat less negative for the short investment horizons and more negative for the larger investment horizons. This is the opposite for the demand for US Dollar which is smaller for the short horizons and larger for the longer horizons.

Looking at the demands when the duration of the liabilities equals 25 the patterns described above are the opposite. The demand for British Pound is in this case more negative for the short horizons and less negative for the large horizons and the demand for US Dollar is now larger for short horizons and smaller for longer horizons.

Base Scenario									
		The Netherlands				Switzerland			
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05
Duration of the liabilities of 15									
5	Euro/Swiss Franc	0.52	0.48	0.54	0.60	0.42	0.57	0.60	0.63
	British Pound	-0.08	-0.34	-0.37	-0.34	0.21	-0.19	-0.26	-0.31
	US Dollar	-0.44	-0.14	-0.17	-0.26	-0.63	-0.38	-0.34	-0.32
10	Euro/Swiss Franc	0.45	0.49	0.54	0.63	0.42	0.60	0.64	0.66
	British Pound	-0.28	-0.55	-0.57	-0.52	0.01	-0.38	-0.45	-0.49
	US Dollar	-0.17	0.06	0.03	-0.11	-0.43	-0.22	-0.19	-0.17
20	Euro/Swiss Franc	0.41	0.50	0.55	0.65	0.42	0.61	0.65	0.67
	British Pound	-0.37	-0.66	-0.67	-0.61	-0.09	-0.46	-0.53	-0.57
	US Dollar	-0.04	0.16	0.12	-0.04	-0.33	-0.15	-0.12	-0.10
Duration of the liabilities of 25									
5	Euro/Swiss Franc	0.67	0.55	0.55	0.56	0.38	0.51	0.57	0.59
	British Pound	-0.36	-0.43	-0.29	-0.29	0.25	-0.15	-0.29	-0.34
	US Dollar	-0.31	-0.12	-0.26	-0.27	-0.63	-0.36	-0.28	-0.25
10	Euro/Swiss Franc	0.62	0.58	0.57	0.57	0.39	0.54	0.60	0.61
	British Pound	-0.60	-0.70	-0.52	-0.42	0.05	-0.33	-0.46	-0.51
	US Dollar	-0.02	0.12	-0.05	-0.15	-0.44	-0.21	-0.14	-0.10
20	Euro/Swiss Franc	0.60	0.60	0.58	0.57	0.41	0.56	0.61	0.62
	British Pound	-0.72	-0.83	-0.63	-0.48	-0.06	-0.42	-0.54	-0.58
	US Dollar	0.12	0.23	0.05	-0.09	-0.35	-0.14	-0.07	-0.04

Table 6: Currency demands for different durations of the liabilities. The top panel shows the currency demands under the assumptions of the base scenario. The second and third panel show the currency demands when the duration of the liabilities is 15 respectively 25. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

Looking at the Swiss case there is not such a clear pattern visible that given an investment horizon and a risk aversion that the demand tends in one direction when the duration is smaller and tends in the other direction when the duration is larger. Also it looks like that when a direction is visible, it is in the opposite direction as in the Dutch case. For example the demand for Euro is decreasing for almost all investment horizons when the duration of the liabilities decreases whereas the demand for Swiss Franc increases for almost all investment horizons when the duration of the liabilities decreases. Furthermore, the absolute change in demand is much smaller for a Swiss pension fund then for a Dutch pension fund.

It is therefore difficult to say something in general about the effect of a change in the duration of the liabilities because the effect is different for different countries and different investment horizons and also the size of the change is different. It can be concluded that the impact of a different duration of the liabilities is minor on the currency decision.

5.3 Duration matching

Pension funds have an interest rate risk on both sides of their balance sheet. Both the bond returns and the liability returns are sensitive to changes in the interest rate. A decrease in the interest rate causes the value of the bond portfolio and of the liabilities to increase and an increase in the interest rate causes the value of the bond portfolio and the liabilities to decrease. In general the duration of the liabilities will be greater than the duration of the bond portfolio and the effect of a change in the interest rate on the liabilities will therefore also be greater than the effect on the bond portfolio. Therefore pension funds bring the duration of their bond portfolio in line with the duration of the liabilities by investing in interest rate swaps or in long duration bonds to increase the duration of the bond portfolio. This is called duration matching.

Some pension funds fully match the duration of the bond portfolio with the duration of the liabilities. The disadvantage of fully matching is that the pension fund will no longer profit from an increase in the interest rate. Therefore some pension funds partly match the duration. In this analysis there will be looked at a pension fund that has a duration of its bond portfolio equal to 10 and a duration equal to 20. The latter is the same duration as the liabilities. It must be noted however that the pension fund is in this case still not fully matched, because the pension fund is not fully invested in bonds but only for 60%. The results of this analysis are presented in table 7.

For a Dutch pension fund, when looking at the effect of a higher level of risk aversion it can be seen that this is still the same for the longer duration bonds compared to the base scenario. The demand for the British Pound decreases and the demand for US Dollar increases when the values of risk aversion parameter increases. The pattern across different investment horizons is different from the base scenario. The demand for British Pound is less negative for an investment horizon of one year but furthermore it is decreasing with investment horizon and it is decreasing more rapidly for the case where the duration of the bond portfolio equals

20 than when it equals 10. This sharp decrease can be explained by looking at the term structures of the unexpected exchange rate returns with the long duration bonds as presented in figure 3 in section 3. Here it can be seen that the correlation with the different long duration bonds is rapidly increasing with the investment horizon whereas the term structures for bonds with a low duration are or only slowly increasing or even decreasing. The sharp increase in the term structures lowers the diversification possibilities and therefore the demand for currency.

Base Scenario										
		The Netherlands				**Switzerland**			
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05
Duration bond portfolio 10									
5	Euro/Swiss Franc	0.50	0.47	0.61	0.81	0.45	0.65	0.71	0.74
	British Pound	-0.16	-0.43	-0.58	-0.77	0.22	-0.23	-0.31	-0.35
	US Dollar	-0.34	-0.04	-0.03	-0.04	-0.67	-0.42	-0.40	-0.39
10	Euro/Swiss Franc	0.46	0.52	0.67	0.87	0.48	0.72	0.76	0.79
	British Pound	-0.37	-0.67	-0.83	-1.02	-0.03	-0.46	-0.52	-0.56
	US Dollar	-0.09	0.15	0.16	0.15	-0.45	-0.26	-0.24	-0.23
20	Euro/Swiss Franc	0.43	0.55	0.69	0.90	0.49	0.75	0.79	0.81
	British Pound	-0.47	-0.79	-0.95	-1.14	-0.15	-0.57	-0.63	-0.66
	US Dollar	0.04	0.24	0.26	0.24	-0.34	-0.18	-0.16	-0.15
Duration bond portfolio 20									
5	Euro/Swiss Franc	0.24	0.22	0.52	0.83	0.32	0.61	0.72	0.78
	British Pound	0.28	0.02	-0.45	-0.88	0.52	-0.02	-0.15	-0.22
	US Dollar	-0.52	-0.24	-0.07	0.05	-0.84	-0.59	-0.57	-0.56
10	Euro/Swiss Franc	0.21	0.34	0.64	0.90	0.38	0.69	0.78	0.83
	British Pound	0.02	-0.31	-0.76	-1.11	0.23	-0.28	-0.38	-0.43
	US Dollar	-0.23	-0.03	0.12	0.21	-0.61	-0.41	-0.40	-0.40
20	Euro/Swiss Franc	0.20	0.41	0.70	0.93	0.41	0.73	0.81	0.85
	British Pound	-0.11	-0.49	-0.92	-1.22	0.09	-0.40	-0.49	-0.52
	US Dollar	-0.09	0.08	0.22	0.29	-0.50	-0.33	-0.32	-0.33

Table 7: Currency demands for different durations of the bond portfolio. The top panel shows the currency demands under the assumptions of the base scenario. The second and third panel show the currency demands when the duration of the bond portfolio is 10 respectively 20. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

Also the demand for US Dollar for a Dutch pension fund shows a different patterns compared to the base scenario. For the base scenario it was first increasing with investment horizon and then decreasing. In the case where the duration of the bond portfolio is 10, the demand is increasing from a horizon of one to five years and it is relatively stable for longer investment horizons. In the case where the duration of the bond portfolio is 20, the demand remains increasing with investment horizon. Whereas the term structures of the long duration bonds with the British Pound are increasing with the investment horizon are the term structures with the US Dollar more or less flat for investment horizons longer than five years. The increase in the correlation with the liabilities causes the currency demand to increase.

When looking at a Swiss pension fund holding long duration bonds, the same thing about the demand for British Pound can be said although the decrease in demand with investment horizon is less sharp. This is because the term structures for the bonds of a Swiss investor are not increasing as sharp as is the case for a Dutch investor. They are stable for investment horizons longer than ten years. The demand for US Dollar shows the same pattern as for the case for a Dutch investor with the duration of the bonds equal to 10. It is first increasing and more or less stable for horizons longer than five years. This same pattern can be seen for bonds with a duration equal to 20 whereas for a Dutch investor the demand for US Dollar kept increasing with investment horizon.

For both countries it holds for most situations that at very short investment horizons the demand for the home currency is lower and for long horizons it is higher than in the base scenario. The horizon at which the demand for home currency turns from less lower to higher than in the base scenario is different for both countries. For the Netherlands it is at a horizon of about ten years and for Switzerland it is at a horizon of about five years.

5.4 Asset allocation

This section will study the effect of different asset allocations on the demand for currency. In section 5.4.1 there will be looked at asset allocations where the percentage of bonds and equity in the portfolio differs but where the exposure to foreign currency is kept constant and in section 5.4.2 there will be looked at portfolios where the percentage of bonds and equity are kept constant, but where the exposure to foreign currency differs.

5.4.1 Different percentage of bonds and equity

The base scenario considers a pension fund that invests 60% in bonds and 40% in equity. In this section there will be looked at an allocation of 90% in bonds and 10% in equity and an allocation of 30% in bonds and 70% in equity. The results are presented in table 8.

When looking at the results it can be seen that for both countries it holds for almost all investment horizons and all levels of risk aversion that when the percentage of bonds in the portfolio increases, the demands for foreign currency become greater, or less negative and

Base Scenario										
		The Netherlands				**Switzerland**				
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05
Portfolio consisting of 90% bonds									
5	Euro/Swiss Franc	0.49	0.39	0.40	0.43	0.17	0.31	0.37	0.37
	British Pound	-0.21	-0.25	-0.18	-0.24	0.29	-0.04	-0.15	-0.18
	US Dollar	-0.28	-0.14	-0.22	-0.19	-0.46	-0.27	-0.22	-0.19
10	Euro/Swiss Franc	0.43	0.40	0.42	0.44	0.18	0.35	0.39	0.40
	British Pound	-0.43	-0.47	-0.39	-0.38	0.09	-0.22	-0.31	-0.35
	US Dollar	0.00	0.07	-0.03	-0.06	-0.27	-0.13	-0.08	-0.05
20	Euro/Swiss Franc	0.41	0.41	0.42	0.44	0.19	0.36	0.40	0.40
	British Pound	-0.54	-0.59	-0.49	-0.44	-0.01	-0.30	-0.39	-0.42
	US Dollar	0.13	0.18	0.07	0.00	-0.18	-0.06	-0.01	0.02
Portfolio consisting of 30% bonds									
5	Euro/Swiss Franc	0.69	0.66	0.70	0.72	0.61	0.80	0.84	0.85
	British Pound	-0.21	-0.52	-0.48	-0.38	0.16	-0.37	-0.46	-0.50
	US Dollar	-0.48	-0.14	-0.22	-0.34	-0.77	-0.43	-0.38	-0.35
10	Euro/Swiss Franc	0.63	0.70	0.72	0.74	0.62	0.84	0.86	0.88
	British Pound	-0.43	-0.79	-0.72	-0.54	-0.04	-0.56	-0.64	-0.68
	US Dollar	-0.20	0.09	0.00	-0.20	-0.58	-0.28	-0.22	-0.20
20	Euro/Swiss Franc	0.61	0.70	0.73	0.75	0.63	0.85	0.87	0.88
	British Pound	-0.54	-0.91	-0.84	-0.61	-0.14	-0.65	-0.72	-0.76
	US Dollar	-0.07	0.21	0.11	-0.14	-0.49	-0.20	-0.15	-0.12

Table 8: Currency demands for different allocations between bonds and equity. The top panel shows the currency demands under the assumptions of the base scenario. The second panel shows the currency demands when there is 90% allocated to bonds and 10% to equity and the third panel shows the currency demands when there is 30% allocated to bonds and 70% allocated to equity. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

when the percentage of equity increases, the demands for foreign currency become smaller or more negative. It can be seen from figure 1 and figure 2 in section 3 that the correlation of the unexpected exchange rate returns with the bond returns are especially for the short term somewhat lower than the correlations with equity returns. A portfolio that is invested more in bonds offers therefore better diversification possibilities which increases the demand for foreign currency. This effect is stronger for the British Pound that has lower correlations with bonds.

Base Scenario										
		The Netherlands				**Switzerland**				
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05
60% allocated in home currency and 40% in foreign currency									
5	Euro/Swiss Franc	0.60	0.52	0.54	0.57	0.38	0.55	0.59	0.60
	British Pound	-0.20	-0.37	-0.32	-0.31	0.23	-0.21	-0.31	-0.35
	US Dollar	-0.40	-0.15	-0.22	-0.26	-0.61	-0.34	-0.28	-0.25
10	Euro/Swiss Franc	0.54	0.54	0.56	0.58	0.39	0.59	0.61	0.63
	British Pound	-0.42	-0.62	-0.55	-0.46	0.03	-0.40	-0.48	-0.52
	US Dollar	-0.12	0.08	-0.01	-0.12	-0.42	-0.19	-0.13	-0.11
20	Euro/Swiss Franc	0.53	0.55	0.57	0.59	0.39	0.60	0.63	0.64
	British Pound	-0.53	-0.74	-0.66	-0.53	-0.07	-0.49	-0.57	-0.60
	US Dollar	0.02	0.19	0.09	-0.06	-0.32	-0.11	-0.06	-0.04
20% allocated in home currency and 80% in foreign currency									
5	Euro/Swiss Franc	0.58	0.52	0.56	0.58	0.39	0.55	0.59	0.60
	British Pound	-0.24	-0.41	-0.35	-0.32	0.20	-0.21	-0.31	-0.35
	US Dollar	-0.34	-0.12	-0.21	-0.28	-0.59	-0.34	-0.28	-0.25
10	Euro/Swiss Franc	0.52	0.54	0.57	0.60	0.40	0.58	0.62	0.63
	British Pound	-0.46	-0.65	-0.57	-0.47	0.00	-0.39	-0.48	-0.52
	US Dollar	-0.06	0.11	0.00	-0.13	-0.40	-0.19	-0.14	-0.11
20	Euro/Swiss Franc	0.49	0.55	0.58	0.60	0.41	0.60	0.64	0.64
	British Pound	-0.57	-0.77	-0.68	-0.54	-0.10	-0.48	-0.57	-0.60
	US Dollar	0.08	0.22	0.10	-0.06	-0.31	-0.12	-0.07	-0.04

Table 9: Currency demands for different currency exposures. The top panel shows the currency demands under the assumptions of the base scenario. The second and third panel show the currency demands when 40% respectively 80% is allocated to assets denoted in foreign currency. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

5.4.2 Different exposures to foreign currency

The base scenario considers a pension fund that has 40% of its investments denoted in the home currency and 60% in the foreign currencies. In this section there will be looked at a pension fund that has an exposure of 40% to foreign currency and to a pension fund that has an exposure of 80% to foreign currency. The percentage of bonds and equity in the portfolio remains constant in this analysis. The results are presented in table 9.

The results show that when the exposure to foreign currency increases or decreases, the demand for currency is more or less the same. For almost all investment horizons and levels of risk aversion the difference in demand for currency is not more than 0.02. It means however that for example the exposure to British Pound is 30% and the demand equals -0.20 that the position will be over hedged by 66.7% and that when the exposure is 20% that the position will be over hedged by 100%. Apparently, the exposure to foreign currency does not have much influence on the demand for currency.

5.5 Indexation ambition

The indexation ambition is very different among pension funds. Some pension funds strive to always give full indexation to their participants and others have no indexation ambition at all and will see each year how much is available for indexation. This section will study the effect of the indexation ambition on the currency demands. In the base scenario it was assumed that the pension fund has an ambition of 80%. There will also be looked at a pension fund that has no indexation ambition and a pension fund that has the ambition to always give full indexation. The results are presented in table 10. The results for a Dutch pension fund show that when a pension fund has no indexation ambition that for all investment horizons and all levels of risk aversion the demand for British Pound increases and the demand for US Dollar decreases. Except for an investment horizon of 1 year and values of the risk aversion parameter of 10 and 20, this is also the case for Switzerland.

Base Scenario									
		The Netherlands				**Switzerland**			
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05
No indexation ambition									
5	Euro/Swiss Franc	0.55	0.32	0.24	0.24	0.35	0.22	0.28	0.31
	British Pound	-0.15	-0.04	0.17	0.15	0.34	0.32	0.21	0.15
	US Dollar	-0.40	-0.28	-0.41	-0.39	-0.69	-0.54	-0.49	-0.46
10	Euro/Swiss Franc	0.50	0.35	0.25	0.21	0.41	0.26	0.29	0.31
	British Pound	-0.38	-0.30	-0.03	0.06	0.09	0.14	0.07	0.03
	US Dollar	-0.12	-0.05	-0.22	-0.27	-0.50	-0.40	-0.36	-0.34
20	Euro/Swiss Franc	0.47	0.37	0.26	0.19	0.43	0.27	0.29	0.31
	British Pound	-0.49	-0.43	-0.13	0.03	-0.03	0.06	0.01	-0.03
	US Dollar	0.02	0.06	-0.13	-0.22	-0.40	-0.33	-0.30	-0.28
Full indexation ambition									
5	Euro/Swiss Franc	0.61	0.57	0.62	0.67	0.42	0.62	0.66	0.68
	British Pound	-0.24	-0.48	-0.47	-0.44	0.18	-0.29	-0.38	-0.43
	US Dollar	-0.37	-0.09	-0.15	-0.23	-0.60	-0.33	-0.28	-0.25
10	Euro/Swiss Franc	0.55	0.59	0.63	0.69	0.43	0.27	0.68	0.71
	British Pound	-0.46	-0.72	-0.69	-0.61	-0.03	-0.49	-0.53	-0.61
	US Dollar	-0.09	0.13	0.06	-0.08	-0.40	-0.18	-0.13	-0.10
20	Euro/Swiss Franc	0.52	0.60	0.65	0.71	0.43	0.68	0.72	0.73
	British Pound	-0.57	-0.84	-0.81	-0.70	-0.13	-0.58	-0.66	-0.70
	US Dollar	0.05	0.24	0.16	-0.01	-0.30	-0.10	-0.06	-0.03

Table 10: Currency demands for different indexation ambitions. The top panel shows the currency demands under the assumptions of the base scenario. The second panel shows the currency demands when the pension fund has no indexation ambition and the third panel shows the currency demands when the pension fund wants to index the pension rights fully. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

When looking at the Dutch case, the demand for British Pound is increasing with the investment horizon whereas it was first decreasing for short investment horizons in the base scenario. This can be explained by looking at the term structures in figure 4 in section 4. Here it can be seen that the term structure between the liabilities without price inflation and the British Pound unexpected exchange rate return lies entirely above the term structure of the liability return in the base scenario and is therefore providing a better hedge which causes the demand for currency to increase. Furthermore the term structure is increasing with investment horizon up to an horizon of about 12 years, causing the demand for British Pound to increase with investment horizon.

The term structure for Swiss liabilities without price inflation and the British Pound unexpected exchange rate return is also entirely above the term structure of the liability return of the base scenario. This term structure is first increasing with investment horizon and after five years decreasing and the same pattern can be seen when looking at the demand for British Pound for different investment horizons.

The results for a pension fund that has a full indexation ambition are comparable to the base scenario. This is mainly because an ambition of 80% is already close to full indexation. For both countries it holds that the demand for British Pound is slightly decreasing and the demand for US Dollar and the home currency is slightly increasing.

5.6 Sub period 1983-2007

The period from 1970-1982 is known for high inflation and high interest rates. This section will look at the demand for currency when this period is not taken into account and when the data used will only cover the period 1983-2007. The results of this analysis are presented in table 11.

Only taking into account the sub period 1983-2007 changes the results drastically. There are two things that play an important role here. The first one is that the mean of the unexpected exchange rate returns have changed. The second one is that the term structures of the unexpected exchange rate returns and the asset and liability returns have changed. These changed term structures can be found in Appendix A.3.

In table 11 it can be seen that when the value of the risk parameter becomes larger, the currency demands in absolute value become smaller very rapidly. From this it can be concluded that when the pension funds are not that risk averse, high speculative positions will be taken. From table 2 in section 3 it becomes clear that the mean of the unexpected exchange rate returns are greater for the sub period and especially for the British Pound. The demand for British Pound is therefore large and positive. When the value of the risk aversion parameter increases, the speculative demand becomes smaller and therefore the currency demands in absolute value become smaller.

Base Scenario									
		The Netherlands				**Switzerland**			
γ	Horizon (years)	1	5	10	25	1	5	10	25
5	Euro/Swiss Franc	0.59	0.53	0.55	0.57	0.39	0.55	0.60	0.61
	British Pound	-0.21	-0.39	-0.33	-0.31	0.23	-0.20	-0.30	-0.34
	US Dollar	-0.38	-0.14	-0.22	-0.26	-0.62	-0.35	-0.30	-0.27
10	Euro/Swiss Franc	0.53	0.54	0.57	0.59	0.40	0.59	0.63	0.63
	British Pound	-0.43	-0.63	-0.56	-0.46	0.03	-0.39	-0.48	-0.51
	US Dollar	-0.10	0.09	-0.01	-0.13	-0.43	-0.20	-0.15	-0.12
20	Euro/Swiss Franc	0.51	0.55	0.58	0.60	0.40	0.61	0.64	0.64
	British Pound	-0.54	-0.75	-0.67	-0.53	-0.07	-0.48	-0.56	-0.59
	US Dollar	0.03	0.20	0.09	-0.07	-0.33	-0.13	-0.08	-0.05
Sub Period 1983-2007									
5	Euro/Swiss Franc	0.18	0.61	-0.85	-1.04	-0.73	-1.02	-1.07	-1.01
	British Pound	0.81	1.71	2.14	2.51	1.65	2.01	1.84	1.53
	US Dollar	-0.99	-1.10	-1.29	-1.47	-0.92	-0.99	-0.77	-0.52
10	Euro/Swiss Franc	0.39	0.15	-0.36	0.58	0.07	-0.24	-0.35	-0.35
	British Pound	-0.07	0.56	0.93	1.34	0.63	0.79	0.78	0.64
	US Dollar	-0.32	-0.41	-0.57	-0.76	-0.56	-0.53	-0.43	-0.29
20	Euro/Swiss Franc	0.53	0.11	0.52	0.31	0.25	0.13	0.02	0.03
	British Pound	-0.51	-0.07	0.24	0.67	0.12	0.16	0.24	0.21
	US Dollar	-0.02	-0.04	-0.18	-0.36	-0.37	-0.29	-0.26	-0.18

Table 11: Currency demands for the sub period 1983-2007. The top panel shows the currency demands under the assumptions of the base scenario. The second panel shows the currency demands when the analysis will be done on the sub period 1983-2007. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20.

When looking at the term structures in Appendix A.3 it becomes clear that for the Dutch case the term structure of the liabilities with the unexpected exchange rate returns shows the most clear difference with the term structure in section 4. The term structure of the British Pound in the sub period lies almost entirely above the same term structure for the full period. Higher correlations offer better hedging possibilities and will therefore increase the demand for currency. The term structure of the US Dollar in the sub period on the other hand lies almost entirely below the same term structure for the full period.

When looking at the Swiss case, it can be seen that the term structures of the British Pound with the different asset classes in the sub period lie in general below the same term structures in the full period resulting in better diversification possibilities and therefore an increase in the demand for currency. The term structure of the British Pound with the liabilities in the sub period lies almost entirely above the term structure of the full period. Both effects contribute to a higher demand for British Pound. The term structures of the US Dollar with the different asset categories have not changed much. Also the term structure of the US Dollar with the liability returns has not changed much. The decrease in demand for US Dollar can be explained by the financing of the increased demand for British Pound since the currency demands must add up to zero.

It can be concluded that it is important for pension funds to consider which period they think is representative for the future to determine the correlations, because it has a major influence on the demand for currency.

5.7 Parameter uncertainty

In this section attention will be paid to parameter uncertainty. From section 5.6 it became already clear that the results can change drastically when the sample period is changed. To give some insight, the distribution of the currency demands in the base scenario has been simulated by bootstrap. This has been done by using the residuals of the VAR model in equation (3.1) to construct new datasets. The VAR model is estimated on these new sets again and the estimated parameters are used as input in equation (2.21) to determine the currency demands.

In table 12 the currency demands of the base scenario are presented as well as their 5% percentile and their 95% percentile. These are stated in parenthesis. The result is a 90% confidence interval. The intervals have an average length of about 0.26. When compared to the change in currency demand when only the sub period 1983 – 2007 is taken into account, these intervals are not that large.

The average interval length of the currency demands for the Dutch case is somewhat greater than that for the Swiss case. The results for Switzerland can said to be more stable. Another thing that can be seen is that the average interval length increases with the investment horizon. This makes sense. A longer investment horizon creates more uncertainty. The level of the risk parameter has no effect on the average interval length.

Base Scenario					
		The Netherlands			
γ	Horizon (years)	1	5	10	25
5	Euro	0.59	0.53	0.55	0.57
		(0.47 0.72)	(0.40 0.66)	(0.40 0.69)	(0.43 0.73)
	British Pound	-0.21	-0.39	-0.33	-0.31
		(-0.31 -0.09)	(-0.52 -0.20)	(-0.50 -0.14)	(-0.48 -0.14)
	US Dollar	-0.38	-0.14	-0.22	-0.26
		(-0.54 -0.29)	(-0.31 -0.01)	(-0.39 -0.08)	(-0.41 -0.13)
10	Euro	0.53	0.54	0.57	0.59
		(0.39 0.67)	(0.40 0.67)	(0.41 0.72)	(0.42 0.74)
	British Pound	-0.43	-0.63	-0.56	-0.46
		(-0.54 -0.30)	(-0.77 -0.45)	(-0.72 -0.37)	(-0.65 -0.28)
	US Dollar	-0.10	0.09	-0.01	-0.13
		(-0.22 0.00)	(-0.07 0.20)	(-0.16 0.13)	(-0.26 0.03)
20	Euro	0.51	0.55	0.58	0.60
		(0.36 0.65)	(0.42 0.68)	(0.43 0.73)	(0.43 0.75)
	British Pound	-0.54	-0.75	-0.67	-0.53
		(-0.68 -0.40)	(-0.89 -0.59)	(-0.82 -0.50)	(-0.73 -0.35)
	US Dollar	0.03	0.20	0.09	-0.07
		(-0.07 0.13)	(0.05 0.30)	(-0.05 0.22)	(-0.19 0.09)
		Switzerland			
5	Swiss Franc	0.39	0.55	0.60	0.61
		(0.32 0.47)	(0.46 0.66)	(0.48 0.70)	(0.49 0.72)
	British Pound	0.23	-0.20	-0.30	-0.34
		(0.16 0.32)	(-0.32 -0.06)	(-0.43 -0.15)	(-0.48 -0.17)
	US Dollar	-0.62	-0.35	-0.30	-0.27
		(-0.73 -0.53)	(-0.49 -0.25)	(-0.43 -0.18)	(-0.42 -0.14)
10	Swiss Franc	0.40	0.59	0.63	0.63
		(0.33 0.48)	(0.49 0.69)	(0.51 0.73)	(0.51 0.75)
	British Pound	0.03	-0.39	-0.48	-0.51
		(-0.01 0.12)	(-0.50 -0.25)	(-0.60 -0.34)	(-0.64 -0.36)
	US Dollar	-0.43	-0.20	-0.15	-0.12
		(-0.54 -0.34)	(-0.33 -0.10)	(-0.27 -0.04)	(-0.25 -0.01)
20	Swiss Franc	0.40	0.61	0.64	0.64
		(0.33 0.48)	(0.51 0.69)	(0.53 0.75)	(0.52 0.76)
	British Pound	-0.07	-0.48	-0.56	-0.59
		(-0.16 0.01)	(-0.59 -0.34)	(-0.67 -0.42)	(-0.73 -0.45)
	US Dollar	-0.33	-0.13	-0.08	-0.05
		(-0.45 -0.23)	(-0.25 -0.03)	(-0.20 0.02)	(-0.17 0.06)

Table 5: Currency demands under the assumptions of the base scenario. Results are presented for investment horizons of 1, 5, 10 and 25 years and with values for the risk aversion parameter of 5, 10 and 20. In parenthesis are stated the 5% percentile and 95% percentile of the distribution of the currency demands.

6 Conclusion

This thesis has considered the problem of determining the optimal currency position of a long term investor with liabilities. Due to current Dutch regulation, liabilities need to be valued at market value. Decisions concerning the assets of investors with liabilities can no longer be determined in an asset only context but the liabilities also need to be taken into account. This also holds for the currency policy. In this thesis a mean variance framework has been set up to determine the optimal currency decision. First a base scenario has been defined which considered a pension fund that has characteristics that are common under pension funds in the Netherlands. Contrary to what Froot (1993) has found for asset only investors, investors with liabilities under the assumptions of the base scenario will for almost all investment horizons and levels of risk aversion hedge their currency exposures or even over hedge these exposures. Still investors with liabilities need to be aware of the length of their investment horizon, because the positions taken in currency do change with investment horizon. This is mainly caused by the fact that the correlations of the unexpected exchange rate returns and the asset and liability returns change with the investment horizon.

Also some sensitivity analyses have been performed to see how the currency decision changes when the characteristics of the pension fund changes or when the asset allocation of the pension fund changes. The effect of a change in the duration of the liabilities is minor. The demand for currency for pension funds that apply duration matching and therefore have a higher duration of their bond portfolio is decreasing in absolute value for short investment horizons and increasing in absolute value for longer investment horizons. The effect of a different asset allocation on the currency demands was most clear. A higher percentage allocated to equity causes the demands for currency in absolute value to increase. A different exposure to foreign currency however did hardly change the demand for currency. The demand for currency is mainly determined by the correlation with the asset and liability returns and the mean of the unexpected exchange rate returns and hardly by the exposure in the portfolio. Pension funds that have no indexation ambition have a higher demand for British Pound than pension funds that strive to give a certain percentage of indexation to their participants. This is primarily caused by the large negative correlation between the British Pound and price inflation. Also the period of the historical data that are used to estimate the correlations are of importance. It became clear that the currency demands can change drastically when another historical period is being used for the estimation of the model.

In this thesis a VAR model is estimated on year data and the estimates of this model are used to construct long term characteristics. Preferably, these long term characteristics would have been estimated on long term data but unfortunately these were not available.

It was also assumed that the liabilities only had interest and inflation risk and that the age cohorts and the accrued pension rights are constant through time and that the inflow from contributions is equal to the net present value of the new liabilities. If this would not be the case, a pension fund would also have actuarial risk on its liabilities.

This thesis considered a pension fund that had already determined its asset allocation and then determines the optimal currency positions. A logical next step for further research would be to optimize the weights of the assets in the portfolio and the currency demands simultaneously.

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Appendices

A.1 Derivation of the optimal mean-variance currency demands

$E_t\left(r_{F,t+\tau}^{(\tau)}\right)$ and $\mathrm{Var}_t\left(r_{F,t+\tau}^{(\tau)}\right)$ are as defined in equation (2.17) and (2.18). Substituting these into the Lagrangian as defined in equation (19) yields:

$$\mathcal{L}\left(\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}}^{(\tau)}\right) = \frac{1}{2}(1-\lambda)\tau\left(\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}}^{(\tau)}\boldsymbol{\omega}_t\mathbf{1} + \boldsymbol{\Psi}_t^{(\tau)'}\Sigma_{\Delta s^u\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} + \sigma_L^{(\tau)2} + 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} - 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\sigma_{\mathbf{r}^{\mathrm{h}}L}^{(\tau)} - 2\cdot\boldsymbol{\Psi}_t^{(\tau)'}\sigma_{\Delta s^u L}^{(\tau)}\right) + \mathbf{1}'\boldsymbol{\omega}_t\mu_{r_t^h}^{(\tau)} + \boldsymbol{\Psi}_t^{(\tau)'}\mu_{\Delta s_t^u}^{(\tau)} + \frac{1}{2}\Sigma_t^h - \mu_{L,t}^{(\tau)} \tag{A1.1}$$

Substituting for Σ_t^h as defined in equation (10), this expression is equivalent to:

$$\mathcal{L}\left(\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}}^{(\tau)}\right) = \frac{1}{2}(1-\lambda)\tau\left(\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}}^{(\tau)}\boldsymbol{\omega}_t\mathbf{1} + \boldsymbol{\Psi}_t^{(\tau)'}\Sigma_{\Delta s^u\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} + \sigma_L^{(\tau)2} + 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} - 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\sigma_{\mathbf{r}^{\mathrm{h}}L}^{(\tau)} - 2\cdot\boldsymbol{\Psi}_t^{(\tau)'}\sigma_{\Delta s^u L}^{(\tau)}\right) + \tau\left(\mathbf{1}'\boldsymbol{\omega}_t\mu_{r_t^h}^{(\tau)} + \boldsymbol{\Psi}_t^{(\tau)'}\mu_{\Delta s_t^u}^{(\tau)} - \mu_{L,t}^{(\tau)}\right) + \frac{\tau}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(Var_t(\boldsymbol{r}_{t+1} + \Delta\boldsymbol{s}_{t+1})\right) - \left(-\boldsymbol{\Psi}_t^{(\tau)} + \boldsymbol{\omega}_t^*\mathbf{1}\right)' diag\left(Var_t(\Delta\boldsymbol{s}_{t+1})\right) - Var_t\left(\mathbf{1}'\boldsymbol{\omega}_t\boldsymbol{r}_{t+1}^h + \boldsymbol{\Psi}_t^{(\tau)'}\Delta\boldsymbol{s}_{t+1}^u\right)\right] \tag{A1.2}$$

$$\mathcal{L}\left(\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}}^{(\tau)}\right) = \frac{1}{2}(1-\lambda)\tau\left(\boldsymbol{\Psi}_t^{(\tau)'}\Sigma_{\Delta s^u\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} + 2\cdot\left(\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} - \boldsymbol{\Psi}_t^{(\tau)'}\sigma_{\Delta s^u L}^{(\tau)}\right)\right) - \tau\boldsymbol{\Psi}_t^{(\tau)'}\mu_{\Delta s_t^u}^{(\tau)} + \frac{\tau}{2}\left[\boldsymbol{\Psi}_t^{(\tau)'}\, diag\left(Var_t(\Delta\boldsymbol{s}_{t+1})\right) - \boldsymbol{\Psi}_t^{(\tau)'}\Sigma_{\Delta s^u\Delta s^u}^{(1)}\boldsymbol{\Psi}_t^{(\tau)} - 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(1)}\boldsymbol{\Psi}_t^{(\tau)}\right] + \frac{1}{2}(1-\lambda)\tau\left(\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}}^{(\tau)}\boldsymbol{\omega}_t\mathbf{1} + \sigma_L^{(\tau)2} - 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\sigma_{\mathbf{r}^{\mathrm{h}}L}^{(\tau)}\right) + \tau\left(\mathbf{1}'\boldsymbol{\omega}_t\mu_{r_t^h}^{(\tau)} - \mu_{L,t}^{(\tau)}\right)\frac{\tau}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(Var_t(\boldsymbol{r}_{t+1} + \Delta\boldsymbol{s}_{t+1})\right) - \mathbf{1}'\boldsymbol{\omega}_t^* diag\left(Var_t(\Delta\boldsymbol{s}_{t+1})\right) - \mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}}^{(1)}\boldsymbol{\omega}_t\mathbf{1}\right] \tag{A1.3}$$

$$\mathcal{L}\left(\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}}^{(\tau)}\right) = \frac{1}{2}(1-\lambda)\tau\left(\boldsymbol{\Psi}_t^{(\tau)'}\Sigma_{\Delta s^u\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} + 2\cdot\left(\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(\tau)}\boldsymbol{\Psi}_t^{(\tau)} - \boldsymbol{\Psi}_t^{(\tau)'}\sigma_{\Delta s^u L}^{(\tau)}\right)\right) - \tau\boldsymbol{\Psi}_t^{(\tau)'}\mu_{\Delta s_t^u}^{(\tau)} - \frac{\tau}{2}\left[\boldsymbol{\Psi}_t^{(\tau)'}\, diag\left(Var_t(\Delta\boldsymbol{s}_{t+1})\right) - \boldsymbol{\Psi}_t^{(\tau)'}\Sigma_{\Delta s^u\Delta s^u}^{(1)}\boldsymbol{\Psi}_t^{(\tau)} - 2\cdot\mathbf{1}'\boldsymbol{\omega}_t\Sigma_{\mathbf{r}^{\mathrm{h}}\Delta s^u}^{(1)}\boldsymbol{\Psi}_t^{(\tau)}\right] + K(\lambda) \tag{A1.4}$$

where

$$K(\lambda) = \frac{1}{2}(1-\lambda)\tau\left(\mathbf{1}'\boldsymbol{\omega}_t\Sigma^{(\tau)}_{\mathbf{r}^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}}\boldsymbol{\omega}_t\mathbf{1} + \sigma^{(\tau)2}_L - 2\cdot\ \mathbf{1}'\boldsymbol{\omega}_t\sigma^{(\tau)}_{\mathbf{r}^{\mathrm{h}}L}\right) + \tau\left(\mathbf{1}'\boldsymbol{\omega}_t\mu^{(\tau)}_{\mathbf{r}^h_t} - \mu^{(\tau)}_{L,t}\right) + \frac{\tau}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(Var_t(\boldsymbol{r}_{t+1} + \Delta\boldsymbol{s}_{t+1})\right) - \mathbf{1}'\boldsymbol{\omega}^*_t diag\left(Var_t(\Delta\boldsymbol{s}_{t+1})\right) - \mathbf{1}'\boldsymbol{\omega}_t\Sigma^{(1)}_{\mathbf{r}^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}}\boldsymbol{\omega}_t\mathbf{1}\right] \quad \text{(A1.5)}$$

$K(\lambda)$ is independent of $\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}}$. Now this problem only needs to be solved for $\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}}$, since $\psi^{(\tau)}_{1,c,t}$ is known once the other currency demands are determined. Therefore the Lagrangian is rewritten in terms of $\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}}$:

$$\boldsymbol{\mathcal{L}}\left(\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}}\right) = \frac{1}{2}(1-\lambda)\tau\left(\widetilde{\boldsymbol{\Psi}}^{(\tau)\prime}_{\mathrm{t}}\widetilde{\Sigma^{(\tau)}_{\Delta s^u\Delta s^u}}\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}} + 2\cdot\left(\widetilde{\mathbf{1}'\boldsymbol{\omega}_t}\widetilde{\Sigma^{(\tau)}_{\mathbf{r}^{\mathrm{h}}\Delta s^u}}\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}} - \widetilde{\boldsymbol{\Psi}}^{(\tau)\prime}_{\mathrm{t}}\widetilde{\sigma^{(\tau)}_{\Delta s^u L}}\right)\right) + \frac{\tau}{2}\left[2\widetilde{\boldsymbol{\Psi}}^{(\tau)\prime}_{\mathrm{t}}\widetilde{\mu^{(\tau)}_{\Delta s^u_t}} + \widetilde{\boldsymbol{\Psi}}^{(\tau)\prime}_{\mathrm{t}} diag\left(Var_t\left(\widetilde{\Delta\boldsymbol{s}_{t+1}}\right)\right) - \widetilde{\boldsymbol{\Psi}}^{(\tau)\prime}_{\mathrm{t}}\widetilde{\Sigma^{(1)}_{\Delta s^u\Delta s^u}}\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}} - 2\cdot\widetilde{\mathbf{1}'\boldsymbol{\omega}_t}\widetilde{\Sigma^{(1)}_{\mathbf{r}^{\mathrm{h}}\Delta s^u}}\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}}\right] + K(\lambda) \quad \text{(A1.6)}$$

The First Order Conditions for the optimal $\widetilde{\boldsymbol{\Psi}}^{(\tau)}_{\mathrm{t}}$ gives the following expression:

$$(1-\lambda)\tau\left(\widetilde{\Sigma^{(\tau)}_{\Delta s^u\Delta s^u}}\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}} + \widetilde{\mathbf{1}'\boldsymbol{\omega}_t}\widetilde{\Sigma^{(\tau)}_{\mathbf{r}^{\mathrm{h}}\Delta s^u}} - \widetilde{\sigma^{(\tau)}_{\Delta s^u L}}\right) + \tau\left(\widetilde{\mu^{(\tau)}_{\Delta s^u_t}} + \frac{1}{2}diag\left(Var_t\left(\widetilde{\Delta\boldsymbol{s}_{t+1}}\right)\right) - \widetilde{\Sigma^{(1)}_{\Delta s^u\Delta s^u}}\widetilde{\boldsymbol{\Psi}}_{\mathrm{t}} - \widetilde{\mathbf{1}'\boldsymbol{\omega}_t}\widetilde{\Sigma^{(1)}_{\mathbf{r}^{\mathrm{h}}\Delta s^u}}\right) = 0 \quad \text{(A1.7)}$$

After some rearranging, the optimal vector of currency demand is:

$$\widetilde{\boldsymbol{\Psi}}^{(\tau)*}_{\mathrm{t}}(\lambda) = \frac{1}{\lambda}\left[\left(1-\frac{1}{\lambda}\right)\widetilde{\Sigma^{(\tau)}_{\Delta s^u\Delta s^u}} + \frac{1}{\lambda}\widetilde{\Sigma^{(1)}_{\Delta s^u\Delta s^u}}\right]^{-1}\left[\widetilde{\mu^{(\tau)}_{\Delta s^u_t}} + \frac{1}{2}diag\left(Var_t\left(\widetilde{\Delta\boldsymbol{s}_{t+1}}\right)\right) - \widetilde{\mathbf{1}'\boldsymbol{\omega}_t}\widetilde{\Sigma^{(1)}_{\mathbf{r}^{\mathrm{h}}\Delta s^u}} + (1-\lambda)\left(\widetilde{\mathbf{1}'\boldsymbol{\omega}_t}\widetilde{\Sigma^{(\tau)}_{\mathbf{r}^{\mathrm{h}}\Delta s^u}} - \widetilde{\sigma^{(\tau)}_{\Delta s^u L}}\right)\right] \quad \text{(A1.8)}$$

A.2 Covariance matrix for different investment horizons

Consider equation (3.1). To obtain an expression for $x_{t+2} - \mu$, this equation can be forward substituted to obtain:

$$x_{t+2} - \mu = A(x_{t+1} - \mu) + \varepsilon_{t+2} = A(A(x_t - \mu) + \varepsilon_{t+1}) + \varepsilon_{t+2} = A^2(x_t - \mu) + A\varepsilon_{t+1} + \varepsilon_{t+2} \tag{A2.1}$$

In this way it is also possible to obtain an expression for $x_{t+\tau} - \mu$:

$$x_{t+\tau} - \mu = A(x_{t+\tau-1} - \mu) + \varepsilon_{t+\tau} = A^\tau(x_t - \mu) + \sum_{i=1}^{\tau} A^{\tau-i}\varepsilon_{t+i} \tag{A2.2}$$

The expectation of equation (A.2.2) is given by:

$$\mathrm{E}(x_{t+\tau} - \mu) = A^\tau(x_t - \mu) \tag{A2.3}$$

The advantage of log returns is that they are additive. The expectation of the cumulative return is therefore given by given by:

$$\mathrm{E}\left(x_{t+\tau}^{(\tau)} - \mu\right) = \sum_{i=1}^{\tau} A^i(x_t - \mu) \tag{A2.4}$$

and the forecast error is given by:

$$\sum_{j=1}^{\tau}\sum_{i=0}^{j-1} A^i\varepsilon_{t+j-i} \tag{A2.5}$$

From this expression the τ-period covariance matrix follows as:

$$\Sigma^{(\tau)} = \sum_{j=1}^{\tau}\left(\left(\sum_{i=0}^{j-1} A^i\right)\Sigma\left(\sum_{i=0}^{j-1} A^i\right)'\right) \tag{A2.6}$$

Where Σ is as defined in equation (3.1).

A.3 Term structures for the sub period 1983 - 2007

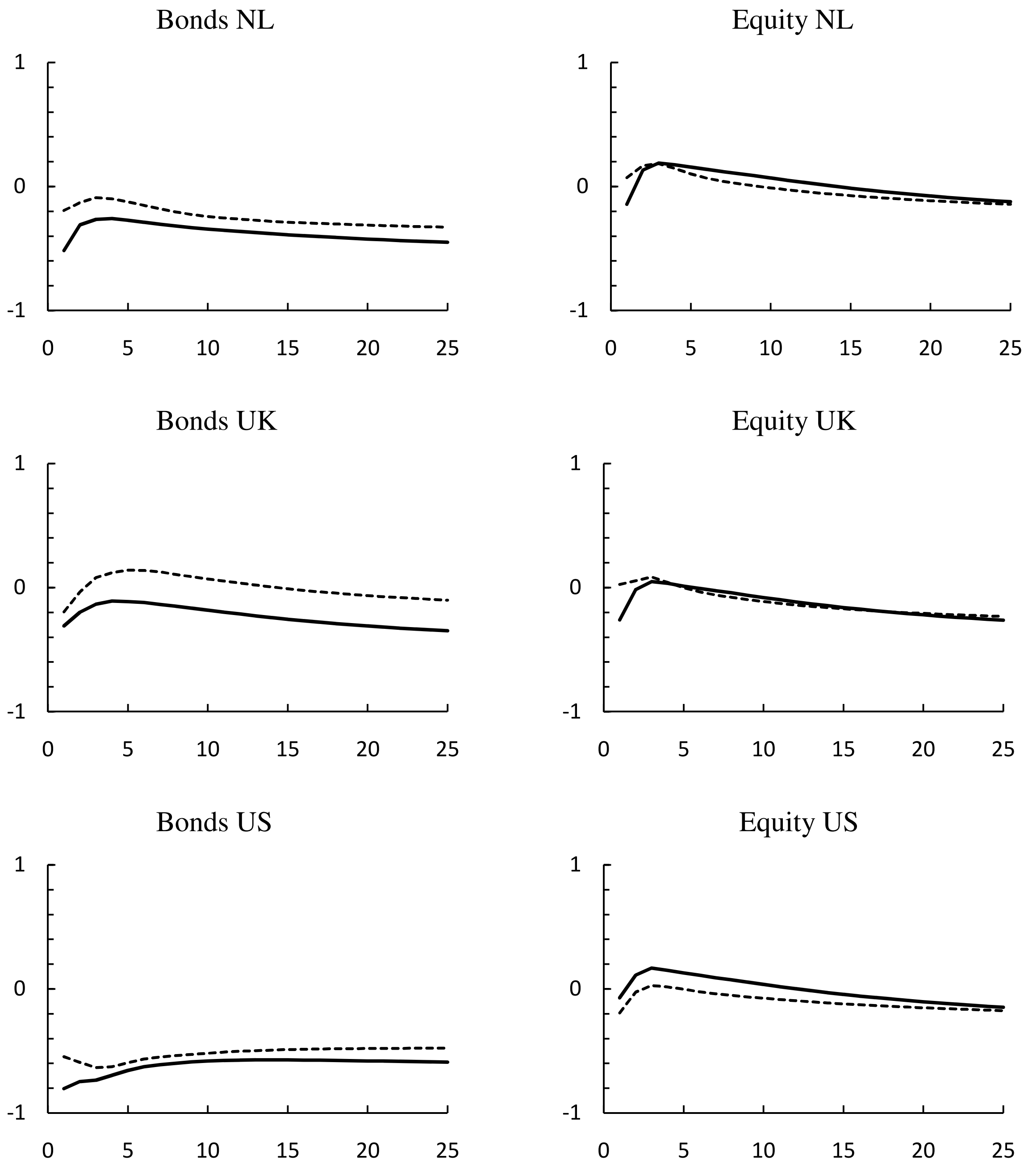

Figure A.1: Correlations between the returns of the different hedged asset categories and the unexpected exchange rate returns across different investment horizons in years for the sub period 1983 - 2007. The solid line represents the British Pound and the dashed line represents the US Dollar. The asset classes are hedged with respect to the Euro.

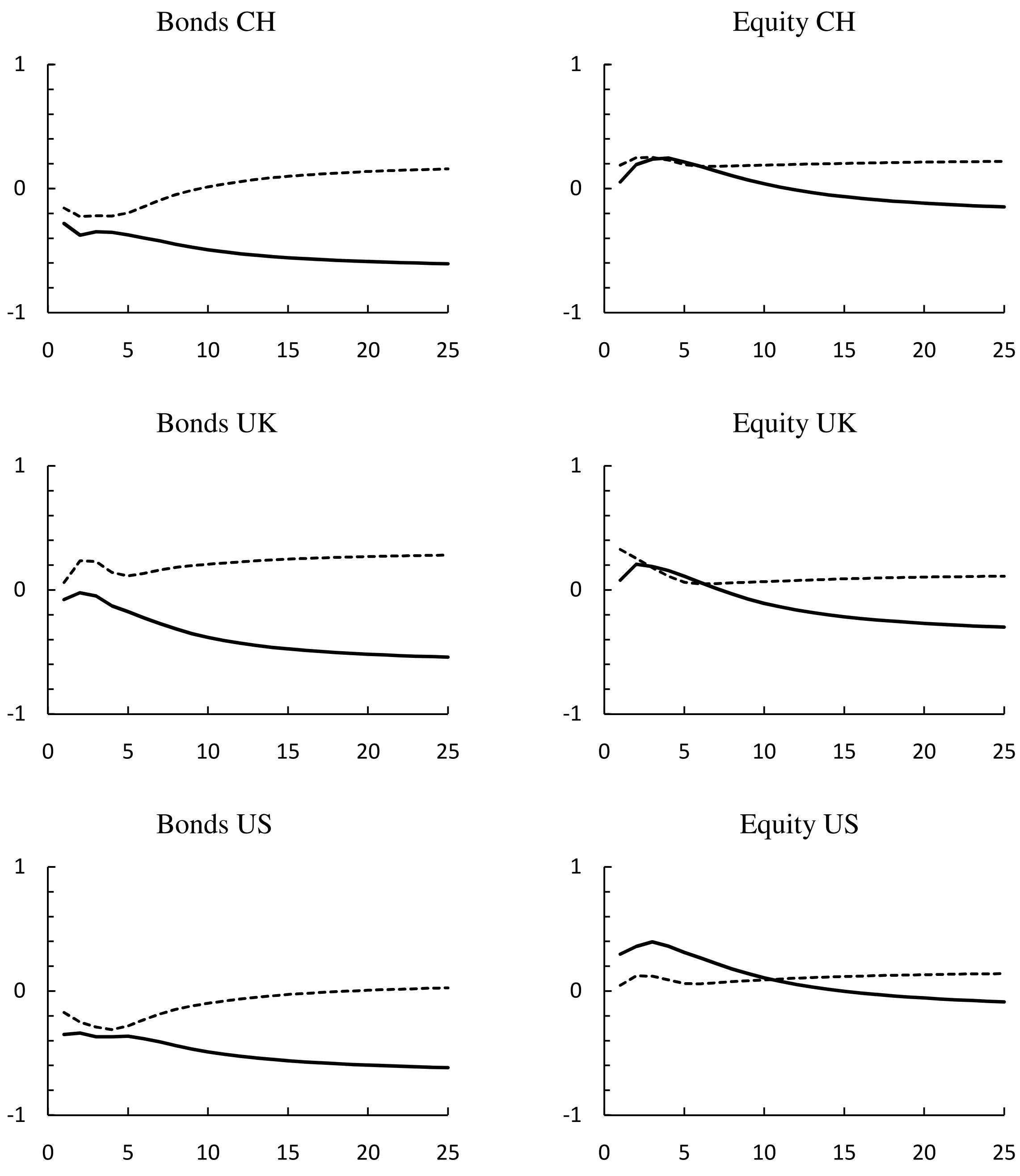

Figure A.2: Correlations between the returns of the different hedged asset categories and the unexpected exchange rate returns across different investment horizons in years for the sub period 1983 - 2007. The solid line represents the British Pound and the dashed line represents the US Dollar. The asset classes are hedged with respect to the Swiss Franc.

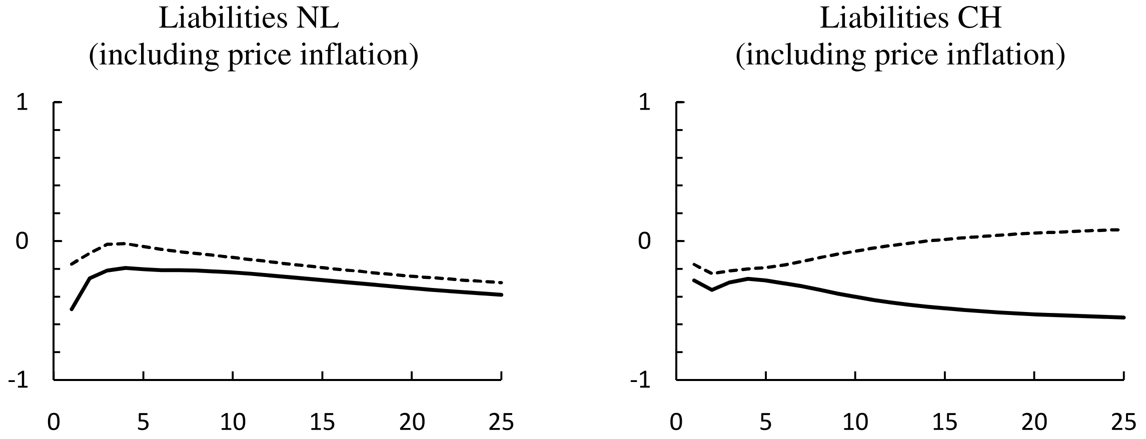

Figure A.3: Correlations between the liability returns including a price inflation term and the unexpected exchange rate returns across different investment horizons in years for the sub period 1983 - 2007. The solid line represents the British Pound and the dashed line represents the US Dollar.