Robustness of distribution-sensitive multidimensional poverty and the Shapley decomposition

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Abstract

Multidimensional poverty measurement is still developing. Recently, Datt proposed an updated version of the widely-used method from Alkire and Foster, which adds a sensitivity to the severity of deprivation. Additionally, Shorrocks proposed a new method for decomposing the multidimensional poverty index, using the Shapley value. In this paper, I apply both Datt's and Shorrocks's method and test for robustness. Datt's method is theoretically better than the Alkire-Foster method, but does not seem to deliver significantly different results. Shorrocks's method has potential, but the results are strongly dependent on the choice for a weighting scheme, on which there is still little reliable data or scientific research available. At this point, decomposition does therefore not offer any substantial benefit.

Table of contents

1. Introduction

In 2015, all UN member states signed the Sustainable Development Goals, in which a set of targets is formulated, with the aim of ending poverty, protecting the planet and ensuring prosperity for all (UN, 2015a). Regarding poverty, the main target is for each nation to reduce poverty by 2030, with at least half the number of poor people in 2015 (according to national poverty definitions). However, an effective way of achieving this is not that clear-cut.

Despite a massive increase in average GDP per capita worldwide and for most individual countries (World Bank, 2017a), poverty is still a problem. Although a lot of high-income countries have been able to eradicate extreme poverty, as measured by the worldwide poverty line of $1.90 per day (World Bank, 2017b), there is no country that has zero poverty when considering national poverty lines based on income (OECD, 2018).[1]

To be able to track the development of poverty, some questions are first to be answered: What is poverty? What defines poverty? And at what point can we recognize an individual as poor?

The past decades, the concept of 'multidimensional poverty' has increased in popularity as an answer to these questions. Multidimensional poverty revolves around the idea that poverty is not solely about income (Alkire & Foster, 2011). Someone can have an average income, but still be poor due to, for example, debt, a large family, disease or insufficient access to proper facilities. Poverty measures should therefore be based on more variables than just income. In multidimensional poverty all these variables, commonly called 'poverty indicators', are considered. All these indicators that can signify poverty can then be combined into a single index, to allow for comparison and policy analysis.

Although the idea behind multidimensional poverty is quite generally accepted in the literature, empirical research on reducing multidimensional poverty is still relatively scarce (Ferrone & Chzhen, 2017). A widely accepted base model for measuring multidimensional poverty has been developed by Alkire and Santos in 2010 (Alkire & Santos, 2010). Some researchers have tried to improve on elements of the model. Datt is one of them: in 2017, he proposed an improved version of the multidimensional poverty index. In this paper, I apply his method to Ireland to see the development of poverty there and I test his method on robustness and usefulness.

Composing a multidimensional poverty index starts with choosing the general dimensions that are considered relevant in experiencing poverty. These dimensions are then filled with specific indicators based on the available data, to provide a way of actually measuring if someone is deprived in this dimension.

As for what dimensions to use that could indicate poverty, I mainly draw on a previous paper from Alkire et al. to set up a basic framework consisting of three dimensions: (1) health, (2) education, and (3) standard of living (Alkire, et al., 2014). However, adjusting to recent developments in scientific literature and European policy, I make two changes. First of all, considering the increased focus of the European Commission on social exclusion and its relation to poverty, I have added a fourth dimension: (4) social exclusion. Secondly, within the standard of living dimension, I have decided to use a 'material deprivation' indicator, which serves as a proxy for the commonly used indicator 'income'.

[1] The poverty line used by the OECD is taken as half the median household income of the total population.

After calculating the multidimensional poverty index, I use a recently developed method by Shorrocks to decompose the index into the contribution of each specific indicator to the poverty index and test for robustness of this method as well (Shorrocks, 2013). This decomposition allows for comparison of the different indicators to determine which indicators have a relatively stronger impact on poverty and whether this changes over time. Shorrocks's decomposition method can deal with some limitations that previous decomposability approaches had and has been applied a few times in multidimensional poverty research, but to the best of my knowledge there is no research yet on robustness of the results.

All in all, the purpose of this paper is to apply the most recent developments in multidimensional poverty measurement to a European, high-income country and to test these new methods on robustness, as there is no research on that yet as far as I can find.

I perform all calculations on data from Irish citizens using the EU Survey on Income and Living Conditions (EU-SILC) between the years 2003 and 2016. I chose a high-income country, because the EU has had a strong focus on reducing poverty the past few years, but research on multidimensional poverty in high-income countries is lacking. I chose Ireland specifically mostly due to data availability. The results of this paper can afterwards be applied to other countries that have participated in the EU-SILC to compare poverty over time and between countries.

The paper starts with a theoretical framework in which the theory behind multidimensional poverty, the model and the required choices are discussed (chapter 2). Then, the data is described (chapter 3), followed by the methodology based on the theoretical framework (chapter 4). Finally, the results are presented and discussed (chapter 5 and 6).

2. Theoretical framework

Poverty is still an issue, not only affecting developing countries, but also high-income countries. Most countries have therefore put policies in place to try to reduce poverty (UN, 2017). In order to do this, measuring poverty is a necessary prerequisite. This theoretical framework will give an overview of the evolution of measuring poverty in the past few decades and will look into the development in monitoring poverty in the European Union.

2.1 Multidimensional poverty

The idea of multidimensional poverty originated around the late 1970s. Sociologist Townsend was one of the first to advocate the concept of multidimensional poverty, by stating that poverty should not just be based on the amount of income or consumption but should instead revolve around a minimum standard of living (Townsend, 1979). Using his definition, a person is considered poor if they do not have access to resources that are "customary, approved or widely encouraged in society". This includes resources that cannot be bought, for instance social relations. Another way of looking at multidimensional poverty is proposed by philosopher Sen. He does not use the theoretical possibility of achieving a certain standard of living, but instead uses the actual satisfaction of these basic needs (Sen, 1985).

Empirical research followed and the ensuing evidence supported the suspicion that multidimensional poverty is more accurate than an income-based approach. For example, Perry found that when comparing an income approach and directly measuring lifestyle deprivation, the overlap in the identified poor is usually between 40 to 50 percent (Perry, 2002). Layte et al. compared income- and lifestyle deprivation in European countries and also found that the overlap was fairly low, from overlaps of 52 percent in Portugal, to overlaps as low as 17 percent in Denmark (Layte, et al., 2001). This means that around half (or even more) of the population that is poor when measuring lifestyle deprivation (the 'actually poor'), are not identified as being poor when using a poverty line based on solely income. (More examples on this are provided in paragraph 2.3 [identification method] below.) The empirical research indicates that income, while important, disregards a number of factors that are believed to have an influence on the total level of disadvantage. Alkire and Foster think this is because many aspects of poverty are ignored by only paying attention to things that can be bought, including health, community participation and a feeling of safety (Alkire & Foster, 2011). Additionally, being theoretically able to afford something does not mean that the person will also be able to actually convert this income into the necessary resources. As an example, a citizen of a rural area might have enough income to theoretically be able to afford proper health care, but in practice this may not be an option if the facilities are far away. Using multidimensional poverty can resolve these measuring problems.

2.2 Multidimensional poverty index

Although multidimensional poverty as a concept is better than a unidimensional income approach, it is a lot more difficult to apply in practice. Composing an index for measuring multidimensional poverty consists of a few steps. All these steps revolve around the question: when should someone be considered poor?

To be able to adequately measure this, the first step is to choose what indicators are relevant for measuring poverty. The goal of this is to decide which aspects of life are important, and would contribute to a sense of poverty if someone is deprived in that aspect. Common indicators are for example someone's education level or health. After deciding on the relevant indicators, the second step is to decide at what point exactly someone is considered deprived in each indicator. Each indicator requires a 'poverty line' that specifies at what level you can speak of deprivation. These two steps together are commonly called the 'identification method' (method of identifying the poor), which is explained further in paragraph 2.3.

The identification method generally leads to a very large dataset, which contains the information on deprivation for all indicators. However, these indicators might not all be equally important. If, for example, the dataset includes an indicator on safety of the neighbourhood, one might say that being 'deprived' in that indicator (living in an neighbourhood that has a certain amount of crime, more than the poverty line) not necessarily leads to that individual being poor, but it would add to the overall level of poverty. To account for this, a third and fourth step are necessary.

The third step is deciding on a potential poverty cut-off. With a poverty cut-off, it is possible to select in how many indicators someone should be deprived before being considered poor. This way, being deprived in only one indicator does not automatically lead to this person being considered poor, but it does add to the overall level of poverty, so it might push someone past the poverty cut-off if that person is deprived in other indicators as well. I explain this step further in paragraph 2.4.

The fourth step is adding weights. Where some indicators will have a significant influence on a person's standard of living, others might only be of little importance. To allow for differentiation between the importance of indicators, weights can be added. I explain this step further in paragraph 2.5.

Once these four steps are finished, all the necessary data is available to start composing the multidimensional poverty index. In paragraph 2.6 to 2.9, I explain the mathematical derivation of the final formula, that combines all the collected data, poverty lines, poverty cut-off and weights into a single index.

2.3 Identification method

For measuring multidimensional poverty, it is necessary to decide on an identification method (Sen, 1976). The identification method describes how the poor individuals are identified. The first step is choosing which dimensions are necessary for measuring poverty and which indicators can be used to measure these dimensions. The second step is determining at what point someone is considered deprived in each indicator.

2.3.1 Three generally accepted dimensions

The usefullness of the dimensions and indicators can differ throughout countries and continents, and should be adjusted as such. For example, Tsui mentions malaria prevention as an important element for developing countries, while this is generally not an issue and thus not a relevant indicator for developed countries (Tsui, 2002). Still, in the literature from the past decades, there are a few basic dimensions that are almost always included, no matter what country. Alkire et al. developed a general mainframe that has been adopted by many other researchers, and by the UN.[2] This framework consists of three main dimensions:

[2] This set of dimensions has evolved over a few papers (Alkire, 2002) (Alkire, et al., 2014).

1. Education;
2. Health;
3. Standard of living.

These three dimensions are then filled with indicators, based on the available data. This general set of dimensions has been used in empirical work for both developing countries and for developed countries (Garcia-Diaz & Prudencio, 2017; Datt, 2017; Weziak-Bialowolska & Dijkstra, 2014). The choice for these three dimensions is generally accepted and there is little explanation necessary for why these dimensions are important. In this paper, I also use these three dimensions. However, adjusting to recent developments in scientific literature and European policy, I will also add a social exclusion dimension.

2.3.2 Social exclusion dimension

A recent development is the focus on social exclusion. The European Commission has lumped poverty and social exclusion together in their goals and is targeting both at the same time, as "millions of Europeans are still on the side-lines, both from the labour market and from social inclusion and integration. Their numbers are increasing, as witnessed by the statistics from 2011" (European Commission, 2015).

Social exclusion is still mostly overlooked in multidimensional poverty measurement, but this is slowly changing. The social exclusion dimension has already been added in a few research papers and with the introduction of EU-SILC, there is more and more data available on social exclusion and its relation to poverty. Neuborg et al. already used the EU-SILC data for instance and added a social component to their set of dimensions for measuring multidimensional child poverty (Neuborg, et al., 2012). They found that for all EU countries on average 11% of all children were deprived in the social exclusion dimension, with some countries even exhibiting deprivation rates of 60% of all children. Overall, countries with high poverty (in the other dimensions) generally also showed high deprivation rates in the social exclusion dimension, which might mean that social exclusion is linked to poverty and contributes significantly to total poverty. Martinez and Perales also added a large social dimension to their variable set and found, after decomposition of their poverty index, that the indicators from the social exclusion dimension had a relatively large contribution to the value of the poverty index, compared to other indicators (Martinez & Perales, 2015). Their social exclusion indicators measured for example if someone had the opportunity and time to practice their hobbies, how much social contact they had with others and if they were able to participate in their neighbourhood.

Based on this information, it seems sensible to add a fourth dimension regarding social exclusion. I will mostly measure this dimension with indicators similar to the ones used by Neuborg et al. and Martinez and Perales, concerning among others the ability to have hobbies and the ability to go on activities with friends – if desired (Neuborg, et al., 2012; Martinez & Perales, 2015).

Another common method for measuring social exclusion in Europe so far is by adding an indicator for joblessness. In research performed in cooperation with the European Union, joblessness is seen as an important indicator for someone being socially excluded from society and it has been included in a few empirical papers (Fusco, et al., 2010; Copeland & Daly, 2012). I therefore also use joblessness as one of the social exclusion indicators.

2.3.3 Income indicator

After deciding on the dimensions, it is often fairly obvious which indicators are most appropriate to fill these dimensions with. The indicators are chosen based on the available data, so there is usually not much room for discussion, as there are simply not that many indicators available. The one indicator that does cause a lot of discussion is income.

It is not unusual for researchers to add an income indicator in the standard of living dimension. Adding income seems like a logical choice, as usually the primary association with poverty is a low income. This is exemplified in the fact that income used to be the base in measuring poverty (and still is for a lot of poverty indices). However, lately it has been argued that it is better to capture not income itself, but the ability to convert this income into resources.

Ringen was one of the first critics of adding income to multidimensional poverty, as he thought that income fails to identify those who cannot participate in society due to a lack of resources (Ringen, 1988). Regarding the EU specifically, Guio and Maquet advocated using an 'absolute' indicator based on material deprivation instead of the 'relative' income indicator (Guio & Maquet, 2006). Absolute material deprivation can be determined by using questions regarding for example the quality of housing or the ability to afford a car or a meal containing meat. Income is relative, since an equal level of income can lead to a highly different level of welfare between countries or even different areas within the same country. Adding material deprivation therefore gives the option to compare European countries that are at different levels of welfare by paying attention to resources, instead of income.

Research on the match between material deprivation and low income is not only theoretical. An example is the paper from Iceland and Bauman, where they examined how material hardship and income poverty are related. Here, material hardship was divided in the categories food insecurity, difficulty meeting basic needs, lack of consumer durables, housing problems, neighborhood problems and fear of crime. They found that a short spell of income poverty often has a long-lasting negative effect on material hardship (Iceland & Bauman, 2007). Using material deprivation would identify poverty therefore better than income, as a household that is no longer 'income poor' can still experience long-lasting poverty due to material deprivation, as a result of the short low-income period. Berthoud and Bryan came to similar conclusions after a longitudinal analysis of income poverty and material deprivation. They saw a strong underlying link: people with long-term low incomes also report long-term material deprivation, but a weak dynamic link: people with an increase in income did not necessarily report a decrease in material deprivation (Berthoud & Bryan, 2011).

Furthermore, Short compared different poverty measures in the US and found, by comparing the composition of the groups based on a number of demographic indicators, that income-based measures identify a different group of people as poor than material hardship-based measures (Short, 2005). Possible reasons for this difference include for example elderly that have low income, but still a high material satisfaction; large families that have a high income but also high expenses; or people that borrowed in the past and now have a high income, but are still materially deprived due to their debt. As another example, Parish et al. found that disabled women in the US showed consistently lower hardship scores than non-disabled women at the same income level, showing again that income alone does not necessarily transfer to material hardship (Parish, et al., 2009).

The papers described above show that those that experience material deprivation are not necessarily 'income poor'. Following the general idea of multidimensional poverty, that poverty is about whether someone has the ability to meet the minimum standard of living in its society, I would say that measuring material deprivation directly offers a much better identification of the materially poor than income. It would therefore make sense to not implement income at all in the poverty measure.

Neuborg et al. followed this reasoning and used the basic dimensions from Alkire et al., but with material deprivation as a proxy for permanent income (Neuborg, et al., 2012). In 2016, Alkire and Apablaza extended their set of dimensions as well by adding more material deprivation indicators (Alkire & Apablaza, 2016).

All in all, even though a lot of researchers do add income as an indicator, it seems that using material deprivation as a proxy is the better choice. I therefore do not include an income indicator in the standard of living dimension, but I include material deprivation directly.

2.3.4 Conclusion

To sum up, the first step: deciding on an identification method, leads to me using the following dimensions for this paper:

1. Education;
2. Health;
3. Standard of living;
4. Social exclusion.

These dimensions then have to be filled with indicators. The following step is deciding on the poverty line for each indicator. Deciding on indicators and their poverty lines is highly dependent on the available data. Therefore, I discuss the indicators and poverty lines further in chapter 3.

2.4 Poverty cut-off

The third step is deciding on a poverty cut-off. The cross-dimensional poverty cut-off k is the value that represents in how many indicators a person has to be deprived to be identified as poor. When using d different poverty indicators, the poverty cut-off value can range from $k = 1$ (commonly called the 'union approach') to $k = d$ (the 'intersection approach').

Defining a specific value for k is difficult. Intuitively, the union approach of $k = 1$ may not seem like the best choice. It might distort the poverty values by identifying too many people as poor. A solution would be to reduce the poverty indicators to merely the most critical factors in determining poverty, or to use a higher poverty cut-off value (Roche, 2013). Researchers therefore often divert to a higher cut-off value, to ensure that only the genuinely poor individuals are identified, while still being able to incorporate all the relevant indicators (Atkinson, 2003). Naturally, the follow-up question is which poverty cut-off value would be appropriate to most accurately identify the poor.

Since the choice for a specific poverty cut-off value is somewhat arbitrary, it is generally considered best to use several cut-off values and examine the different effects (Alkire & Foster, 2011b). A popular method is to compare a couple of consecutive integer values, such as $k = 1, k = 2$ and $k = 3$ [e.g. (Roche, 2013; Garcia-Diaz & Prudencio, 2017)]. Research shows that often the poverty cut-off that produces poverty rates most similar to what you would expect (based on for example the percentage of citizens that consider themselves poor) is a cut-off value roughly equal to 1/3 of the measured poverty indicators. As a result, if it is necessary to pick one cut-off value, "popular understandings of multidimensional poverty" dictate a cut-off value of around 1/3 of all indicators

(Alkire & Apablaza, 2016; Alkire & Santos, 2010). So for instance, when measuring 9 indicators in total, a popular cut-off value would be 3 indicators.

Although using multiple cut-off values would probably be the best way to get a complete view of poverty, it also results in many different indices. For clarity purposes it would therefore be more useful to select one cut-off and in that case, a cut-off value of 1/3 of all indicators appears to be the best choice. A cut-off value of 1/3 of all indicators will therefore also be my main poverty cut-off value k, as examining the result of a change in k is not the main objective of this paper.

2.5 Weights

The fourth step is deciding on a weighting scheme. It can be very useful to assign a relative weight to each indicator. Intuitively, it would not make sense for each indicator to have the exact same effect on someone's level of poverty. Certain things have a bigger impact on one's quality of life than others, even though they can all be considered an indicator of poverty. As such, with weights it is possible to include indicators that are of relatively small importance but can still be valuable to account for in the overall poverty measure. In the literature, it is generally agreed upon that assigning weights provides a significant improvement on the equal-weight measures, as long as the weights are reasonably accurate.

However, finding a weighting scheme that is reasonably accurate proves to be difficult. Multidimensional poverty is supposed to represent the extent to which the population can reach a minimum standard of living, as generally accepted by society. This means that the weights should be representative of society's opinion. The weights should reflect how the typical individual in that society feels about the different dimensions and what they feel has the largest impact on 'being poor'. This weighting purpose is hard to achieve. It has proven to be quite difficult to construct a set of weights that can represent the (average) feeling of the population regarding the importance of different aspects of life.

In the literature, a range of weighting schemes has been laid out, that all have their advantages and disadvantages, and choosing between these is largely a matter of personal preference. Decancq and Lugo discussed all options and identified seven different weighting schemes (Decancq & Lugo, 2012).

Data-driven weights:

1. Frequency-based weights
   In this method, weights are based on how often deprivation occurs in each indicator. Usually, an inverse relation is assumed: the more often a deprivation is found, the less weight it gets (Deutsch & Silber, 2005). This is based on the idea that if a deprivation is rare, the individual will feel worse about being deprived in that area than when a lot of its peers have the same deprivation.
2. Statistical weights
   This method is used to solve 'double counting'. Double counting is what might happen when there are multiple indicators in the same dimension, since there is often a large correlation between being deprived in one of the indicators and being deprived in the others. Statistical weights solve this by giving each dimension an equal weight and then dividing this weight over all indicators in that dimension. This method is often used in recent literature, for example in (Alkire & Santos, 2010; Alkire, et al., 2014; Salazar, et al., 2013).

Normative weights:

3. Equal weights
   This weighting scheme gives an equal weight to every indicator. This weighting scheme is mostly used by people that consider it unjust to choose different weights for indicators, as all indicators are equally important, unless one can scientifically support why they are not. Examples are the papers from Martinez & Perales, Datt and Garcia-Diaz & Prudencio (Martinez & Perales, 2015; Datt, 2017; Garcia-Diaz & Prudencio, 2017).
4. Expert opinion weights
   With this method, weights are decided on by experts, who are supposed to be objective and informed and are thus the best option for ranking importance.
5. Price-based weights
   This method is based on the marginal rates of substitution between the different dimensions. It is not a very popular method, since it is difficult to obtain implied marginal rates of substitution.

Hybrid weights:

6. Stated preference weights
   In this method, the population is asked in a survey to rank the dimensions, to base the weights directly on the opinion of the people. This method is gaining popularity, but so far it has not been used much. The most notable example is a survey by Kruijk and Rutten in 2007 at the Maldives where they asked random citizens to rank dimensions and then used this data to build a weighting scheme. They actually found the results surprising, as the weighting was quite different from the one used before in measuring multidimensional poverty, showing that stated preferences research is very helpful in achieving accurate weighting (Kruijk & Rutten, 2007). Decancq and Lugo consider the stated preferences method to be the best one, as it avoids most problems that the other methods have, where decisions on weights are mostly arbitrary (Decancq & Lugo, 2012). However, collecting the necessary data is quite difficult compared to some of the other methods and the data is not always available.
7. Hedonic weights
   This method uses self-reported life satisfaction to determine the weights. By regressing all dimensions linearly on life satisfaction, a weight can be identified. The disadvantage of this method is that life satisfaction is not always available and that there is a high risk of a strong correlation between the different dimensions. So far, it has been rarely used.

As explained above, the main problem in picking weights is that it is very difficult to find out how much people value each dimension. Considering all these possible weighting schemes, the best one appears to be stated preference weights, as these do in fact give as good a representation of the population's preferences as possible, which is the main goal of the weighting. However, to be able to apply this weighting scheme, the data has to be available, preferably for a large population sample and with years corresponding to the deprivation data timeframe. Since this is often not the case, the second-best option seems to be statistical weights, considering the fact that this is a widely-used weighting method in previous research on multidimensional poverty: it allows for some form of ranking, using generally agreed-upon broad dimensions and is applicable in almost any dataset, without the need for an extra dataset on preferences.

In this paper, I use both stated preference weights and statistical weights, based on the information above, which I will expand upon further in chapter 3.

2.6 Unidimensional poverty index

After deciding on the identification method, the cut-off value and the weighting scheme, an aggregate measure can be constructed to convert this data into a poverty index, so that changes in poverty can be examined (Sen, 1976). In this paragraph, I will explain the derivation of these poverty indices. To do this, I will start with the derivation of a popular unidimensional poverty index, which I will then transform to a multidimensional index, largely following Alkire and Foster (Alkire & Foster, 2011).

Until the 1970s, the only method used to identify the poor was by looking at their income and comparing this to a prespecified poverty line (UN, 2015b). This unidimensional index is called the headcount ratio. Here the number of poor is divided by the total population n. Let $i = 1,2, \dots, n$ represent the individuals and let $y = \{y_1, y_2, \dots, y_n\}$ be the set of incomes. To identify someone as poor, their income has to be below poverty line z: a prespecified value that separates the poor from the nonpoor. For this, I use an 'indicator function' $I_i = I(y_i < z)$, leading to a value of 1 if the individual's income y_i is below z, and a value of 0 otherwise:

$$I_i = I(y_i < z) = \begin{cases} 1 \text{ if } y_i < z \\ 0 \text{ if } y_i \geq z \end{cases}$$

The sum of this indicator $\sum_{i=1}^{n} I_i$ is then the total number of poor within a population.

The headcount ratio is then the number of poor divided by the total population:

$$Headcount\ ratio(z; y) = \frac{1}{n}\sum_{i=1}^{n} I_i \tag{1}$$

The headcount ratio thus represents the percentage of poor individuals in the entire population.

This method is still very popular: many widely-used poverty indices are constructed using this procedure. An example is the World Bank, one of the main sources for information on global poverty. Their statistics are obtained by comparing income to either their international poverty line ($1.90 per day, indicating extreme poverty) or to a national poverty line, which is defined as the line below which a person's minimum nutritional, clothing and shelter needs cannot be met in that specific country (World Bank, 2015).

The advantage of this way of measuring is that it allows for easy calculation and can be obtained for most countries. The problem with this headcount ratio is that it ignores certain aspects of poverty, such as the severity of poverty (Lambert, 2001). Using only the headcount ratio might for instance stimulate poverty policies to focus on improving the situation for the least poor to push them to just above the poverty line, ignoring the poorest share of the population.

Adding the income gap ratio offers a solution to some of the limitations of the headcount ratio, by taking the depth of poverty into account (Foster, 2005). This ratio represents the average shortfall of the poor, by taking the average gap between their income and poverty line z, as a share of the maximum shortfall z. Individual i's income gap is then $(z - y_i)/z$. The income gap ratio is the average income gap of the poor, so the sum of all income gaps divided by the number of poor:

$$Income\ gap\ ratio(z; y) = \frac{\sum_{i=1}^{n}\left(\frac{z - y_i}{z} I_i\right)}{\sum_{i=1}^{n} I_i}$$

The advantage of this measure is that it takes into account the severity of poverty, so there is an incentive to reduce severy poverty. However, since it does not take into account how many people are poor, increasing the number of slightly poor individuals would improve the income gap ratio as the average income gap of the poor would decrease.

A solution is to multiply the headcount ratio with the income gap ratio, to create an index that is sensitive to both the number of poor and the severity of the poverty. It is the result of multiplying the share of the population that is poor with the average income gap of the poor, which means that both a change in the number of poor and a change in the severity of poverty affects the outcome. This is the poverty gap ratio:

$$Poverty\ gap\ ratio(z;y) = headcount\ ratio(z;y) * income\ gap\ ratio(z;y)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(\frac{z-y_i}{z}I_i\right) \tag{2}$$

Foster, Greer and Thorbecke adjusted this index further by squaring the income gap ratio to generate an index that reacts stronger to severely poor people than to slightly poor people (Foster, et al., 2010). This is the FGT measure. They generalized this index to the Foster-Greer-Thorbecke index:

$$FGT(\alpha, z; y) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{z-y_i}{z}\right)^{\alpha} I_i \tag{3}$$

$\alpha = 0$ gives the headcount ratio, $\alpha = 1$ the poverty gap and $\alpha = 2$ the FGT measure.

2.7 Multidimensional poverty index

The unidimensional FGT-index is very popular for unidimensional measuring, but it is not suited for multidimensional poverty data. To allow for multidimensional data, the measure must be adjusted.

2.7.1 Notation

First, some general multidimensional notation will be introduced. Let $j = 1,2, \dots, d$ represent the indicators. Generally, survey answers are used as data for measuring poverty. Information matrix $Y = [y_{ij}]$ is then an n x d matrix of all relevant survey answers of individual i regarding indicator j.

Whether or not an individual i is deprived in an indicator j is determined by constructing a poverty line z_j for that specific indicator. If y_{ij} is lower than poverty line z_j, the individual is considered deprived in indicator j. As such, a new 'indicator function' can be utilized, where again the variable equals 1 if the condition is satisfied and zero otherwise. In this case, the variable should be equal to 1 if individual i is deprived in indicator j. Using poverty line z_j, the 'indicator function' is defined as $I_{ij} = I(y_{ij} < z_j)$, so that if the individual is deprived in that indicator, the value will be 1:

$$I_{ij} = I(y_{ij} < z_j) = \begin{cases} 1 \text{ if } y_{ij} < z_j \\ 0 \text{ if } y_{ij} \geq z_j \end{cases}$$

Applying this 'indicator function' to all values in matrix Y results in the deprivation matrix, thus consisting of only binary variables. This n x d deprivation matrix can then be defined as $G = [g_{ij}]$ where the values for g_{ij} are the result of the 'indicator function' I_{ij} on the values in information matrix Y.

2.7.2 Derivation

Now that the notation for measuring multidimensional poverty is defined, the unidimensional poverty index can be transformed to a multidimensional poverty index, following the same steps as before: set up the headcount ratio, multiply this with the average poverty gap and add the possibility to square the depth of poverty to increase its impact.

The most basic index is again based on the headcount ratio. To set up a headcount ratio for multidimensional poverty, an individual is considered poor if deprived in more indicators than the cut-off value k. c_i is then the number of deprived indicators for individual i, or the sum of row i in G:

$$c_i = \sum_{j=1}^{d} g_{ij}$$

Now suppose I_i^k is an 'indicator function' with $I_i^k = I(c_i \geq k)$, leading to a value of 1 if the number of deprived indicators c_i is larger than or equal to cut-off value k, or a value of 1 if individual i is poor:

$$I_i^k = I(c_i \geq k) = \begin{cases} 1 \text{ if } c_i \geq k \\ 0 \text{ if } c_i < k \end{cases}$$

The number of poor is then the sum of this 'indicator function', leading to the <u>multidimensional headcount ratio</u>:

$$H(k, z; y) = \frac{1}{n} \sum_{i=1}^{n} I_i^k \tag{4}$$

Alkire and Foster adjusted the headcount ratio by adding a measure for the depth of deprivation in terms of indicators. The deprivation share of an individual is c_i/d. The average deprivation share across the poor is then the sum of all deprivations of the poor divided by all possible deprivations for the poor:

$$A(k, z; y) = \frac{\sum_{i=1}^{n} I_i^k c_i}{d \sum_{i=1}^{n} I_i^k}$$

Combining these, the <u>adjusted multidimensional headcount ratio</u> can be defined:

$$M_0(k, z; y) = HA = \frac{1}{nd} \sum_{i=1}^{n} I_i^k c_i \tag{5}$$

This ratio responds to both the frequency and the depth of poverty (in terms of indicators).

Like the unidimensional headcount ratio, the measure can be improved by adding the shortfall in each indicator, instead of only using binary variables. For this, the index M_0 can be multiplied with the average poverty gap for the poor, to create an index that is sensitive to the frequency of poverty, the depth of poverty in terms of the number of deprived indicators and to the depth of poverty within each deprived indicator.

The adjusted multidimensional poverty gap is then:

$$M_1(k, z; y) = \frac{1}{nd} \sum_{i=1}^{n} \sum_{j=1}^{d} \frac{z_j - y_{ij}}{z_j} g_{ij} I_i^k \tag{6}$$

Finally, the measure can be updated to make sure that there is a larger weight on the severely poor (with a relatively large gap), by taking the square of the depth of poverty. This leads to the adjusted multidimensional FGT measure:

$$M_2(k, z; y) = \frac{1}{nd} \sum_{i=1}^{n} \sum_{j=1}^{d} \left(\frac{z_j - y_{ij}}{z_j} \right)^2 g_{ij} I_i^k \tag{7}$$

These measures can altogether be generalized to a class of multidimensional poverty measures:

$$M(\alpha, k, z; y) = \frac{1}{nd} \sum_{i=1}^{n} \sum_{j=1}^{d} \left(\frac{z_j - y_{ij}}{z_j} \right)^\alpha g_{ij} I_i^k \text{ for } \alpha \geq 0 \tag{8}$$

With this formula, $\alpha = 0$ gives the adjusted multidimensional headcount ratio, $\alpha = 1$ gives the adjusted multidimensional poverty gap and $\alpha = 2$ gives the adjusted multidimensional FGT measure.

Finally, weights can be attributed to each indicator by defining a set of weights $w = \{w_1, w_2, \dots, w_d\}$, where $\sum_{j=1}^{d} w_j = d$. The weighted adjusted multidimensional FGT measure then becomes:

$$M(\alpha, k, w, z; y) = \frac{1}{nd} \sum_{i=1}^{n} \sum_{j=1}^{d} w_j \left(\frac{z_j - y_{ij}}{z_j} \right)^\alpha g_{ij} I_i^k \tag{9}$$

This is also the measure that Alkire and Foster use in their poverty research, which is why I will from now on refer to it as the Alkire-Foster measure (Alkire & Foster, 2011).

2.8 Properties

To ensure that the measure responds to changes in poverty the right way, it has to satisfy a number of axioms. These axioms represent the properties the poverty measure would ideally have: if the data is changed in a certain way, how would we want the poverty measure to respond? Should it increase, decrease or remain the same? An axiom defines the wanted change. If the poverty measure satisfies the axiom, it responds the way it should.

These axioms can be categorized in a few groups. Foster has summarized the categories of commonly required axioms for poverty (Foster, 2005). The main categories are: invariance axioms, dominance axioms, and transfer axioms.

The Alkire-Foster measure (equation 9) fits most of them, yet not all, as I will show in the following section.[3]

[3] There are quite a few other popular poverty measures, not used in this paper. Their satisfaction of the axioms and the reason for not using them are discussed in Appendix, part A.

Invariance axioms

The first invariance axiom requires the measure to make sure that a reordering of the deprivation data leaves the poverty value M unchanged. If the data of two individuals is switched, the poverty value should remain the same.

Say that deprivation dataset x is obtained from deprivation dataset y by a permutation of data if $x = Py$, where P is a permutation matrix that switches the data rows. This leads to:

> AXIOM 1 (Symmetry): If x is obtained from y by a permutation of deprivation data, then $M(x;z) = M(y;z)$.

The second invariance axiom requires that poverty is measured on a per capita basis, to ensure comparability across groups with different population sizes. Say that x is obtained from y by a replication of deprivation data if $x = (y, y, \dots, y)$, where x is constructed by cloning the original distribution y a number of times. This leads to:

> AXIOM 2 (Replication invariance): If x is obtained from y by a replication of deprivation data, then $M(x;z) = M(y;z)$.

The third invariance axiom requires that the poverty measure is independent of the deprivation data of the nonpoor. So if the deprivation data of a nonpoor person changes, without that person becoming poor, the poverty value should remain the same. This axiom can be applied multidimensionally by separating deprivation focus and poverty focus.

Deprivation focus requires that any change to the non-deprived that does not cause deprivation should not affect the poverty value. This leads to:

> AXIOM 3A (Deprivation focus): If x is obtained from y by a change in welfare for a nonpoor person in a non-deprived indicator, not causing deprivation in that indicator, then $M(x;z) = M(y;z)$.

Poverty focus requires that any change to the nonpoor that does not cause that person to become poor should not affect the poverty value. This leads to:

> AXIOM 3B (Poverty focus): If x is obtained from y by a change in welfare for a nonpoor person in any indicator, not causing poverty, then $M(x;z) = M(y;z)$.

These two axioms combined also ensure that a decrement that causes a nonpoor person to be deprived in a previously nondeprived indicator, without causing that person to be poor (the number of deprived indicators is still below the cut-off) will not change the poverty value. I discuss changes to a poor person in the dominance axiom section.

Finally, the fourth invariance axiom requires that the poverty value remains the same if both the poverty line and the deprivation data double, so the poverty value is relative. Say that $(x';z')$ is obtained from $(x;z)$ by a proportional change, if $(x';z') = (\alpha x; \alpha z)$ where $\alpha > 0$. This leads to:

> AXIOM 4 (Scale invariance): If $(x';z')$ is obtained from $(x;z)$ by a proportional change, then $M(x';z') = M(x;z)$.

Applying the axioms mentioned above to the Alkire-Foster measure (equation 9), Alkire and Foster show that their poverty measure satisfies all invariance axioms (Alkire & Foster, 2011).

Dominance axioms

The second category concerns the dominance axioms. They were first proposed by Sen and require the poverty value to follow the intuition of what is supposed to happen if something changes for a poor individual (Sen, 1976).

First of all, the function should produce a higher poverty value if a nonpoor person becomes poor and a lower poverty value if a poor person becomes nonpoor. Furthermore, it should respond in the right way if something changes for a poor person. This person can become more deprived or less deprived, both in terms of deprived indicators and in terms of deprivation within an indicator.

If a nonpoor person becomes poor or if a poor person becomes deprived in an additional indicator, the poverty value should increase:

> AXIOM 6A (Monotonicity): If x is obtained from y by a decrement for a poor individual, with the amount of deprived indicators increasing, or by a decrement for a nonpoor individual, with the number of poor increasing, then $M(x;z) > M(y;z)$.

If a poor person is no longer deprived in an indicator (no matter if that person becomes nonpoor or remains poor), the poverty value should decrease:

> AXIOM 6B (Monotonicity): If x is obtained from y by an increment for a poor individual, with the amount of deprived indicators decreasing, then $M(x;z) < M(y;z)$.

Dominance axioms also apply to changes in deprivation within an indicator.

If a poor person becomes more deprived, the poverty value should by all means not decrease. Say x is obtained from y by a decrement among the poor, so one already poor person becomes more deprived in one indicator. This should lead to a higher, or at least equal, poverty value:

> AXIOM 7A (Dimensional monotonicity): If x is obtained from y by a decrement for a poor individual, while the amount of deprived indicators remains the same, then $M(x;z) \geq M(y;z)$.

Ideally, the poverty value would not remain equal if a poor person becomes more deprived, it would increase. This is referred to as the strong version of the dimensional monotonicity axiom:

> AXIOM 8A (Strong dimensional monotonicity): If x is obtained from y by a decrement for a poor individual, while the amount of deprived indicators remains the same, then $M(x;z) > M(y;z)$.

If a poor person becomes less deprived, but not so much that he/she is no longer considered deprived, the poverty value should decrease, or at least remain the same. This is equal to the dimensional monotonicity axiom:

> AXIOM 7B (Monotonicity): If x is obtained from y by an increment for a poor individual, while the amount of deprived indicators remains the same, then $M(x;z) \leq M(y;z)$.

Ideally, the poverty value would decrease. This is equal to the strong monotonicity axiom:

> AXIOM 8B (Strong dimensional monotonicity): If x is obtained from y by an increment for a poor individual, while the amount of deprived indicators remains the same, then $M(x;z) < M(y;z)$.

Applying the axioms mentioned above to the Alkire-Foster measure, Alkire and Foster are able to show that their poverty measure satisfies monotonicity, dimensional monotonicity, and strong dimensional monotonicity if $\alpha > 0$ (Alkire & Foster, 2011). If $\alpha = 0$, a change in deprivation for the already-deprived that does not lead to nondeprivation will not lead to a change in poverty value, as deprivation is only measured in 1 and 0.

Transfer axioms

The third category concerns the transfer axioms. First, consider a progressive transfer among the poor, so a poor person transfers a part of its income to an even poorer person, leading to a leveling of incomes. Say x is obtained from y by a progressive transfer in income among the poor, if for individuals i and j and $y_i < y_j < z$, we have $y_i < x_i \leq x_j < y_j$ while $x_q = y_q$ for all $q \neq i, j$. This should lead to a lower or equal poverty value:

> AXIOM 9 (Transfer): If x is obtained from y by a progressive transfer among the poor, then $M(x; z) \leq M(y; z)$.

The stronger version of the transfer axiom concerns a regressive transfer. This transfer of income should increase poverty, even if one of the individuals turns nonpoor as a result (Hagenaars, 1991). A poor person transfers a part of its income to a less poor person. Say x is obtained from y by a regressive transfer in income among the poor, if for individuals i and j and $y_i \leq y_j < z$, we have $x_i < y_i \leq y_j < x_j$ while $x_q = y_q$ for all $q \neq i, j$. This should increase poverty:

> AXIOM 10 (Strong transfer): If x is obtained from y by a regressive transfer among the poor, then $M(x; z) > M(y; z)$.

Alkire and Foster show that their poverty measure satisfies the weak version of the transfer axiom. The Alkire-Foster measure does not satisfy the strong transfer axiom however (Alkire & Foster, 2011). Datt proves that, except for a poverty cut-off value of $k = 1$ (the 'union approach'), the strong transfer axiom is violated (Datt, 2017). Take for example a situation with 4 indicators and a cut-off value of 2 indicators. The population is 3. In this example, x is obtained from y by a regressive transfer, where poor individual 2 (the second row) transfers part of its income to less poor individual 1, thereby becoming even poorer. Imagine the following situation:

$$G(y) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow G'(x) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So in situation y, both individual 1 and 2 are deprived in at least 2 indicators, leading to them both being identified as poor (row 1 shows 2 deprivations, row 2 shows 3 deprivations). Then, individual 2, who is already more deprived than individual 1, transfers part of its income to individual 1, leading to situation x, where individual 1 is now nondeprived in the third indicator. Since in situation x, individual 1 is only deprived in 1 indicator, it is no longer considered poor.

Using the Alkire-Foster measure (equation 9), the poverty value $M(\alpha, k, z; y)$ can be calculated. The poverty value of situation y is $M(0,2,z; y) = \frac{5}{12} = 0.417$, whereas the poverty value of situation x is $M(0,2,z; x) = \frac{4}{12} = 0.333$. Poverty thus appears to have decreased as a result of this transfer, even though intuitively it should have increased, since an already poor person has transferred part of its welfare to a less poor individual. (In this paper, I assume it is worse for someone to go from 'medium' poor to severely poor than to go from slightly poor to 'medium' poor, which is a fairly common assumption in measuring poverty.) The poverty index thus violates the strong transfer axiom.

Cross-dimensional convexity axiom
Datt then introduces an additional axiom that he believes should be added. This axiom relies on the idea that the more deprived indicators an individual has, the stronger the effect on poverty should be if he gets deprived in an additional indicator or gets more deprived in an already deprived indicator.

> AXIOM 11 (Cross-dimensional convexity): The greater individual i's deprivation in any other indicators $j \neq j'$, the greater the increase in $M(y; z)$ induced from an increment in deprivation j' for individual i.

The Alkire-Foster measure does not satisfy this axiom, even when using the union approach. An example for this is a situation with a cut-off value of 1 indicator, a population of 3 individuals and 4 indicators. Assume in situation y, individual 1 is deprived in 2 indicators and individual 2 is deprived in the 2 other indicators. Then, in situation x, individual 1 becomes nondeprived in the third indicator, while individual 2 becomes deprived in the third indicator:

$$G(y) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow G'(x) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, the poverty value of the Alkire-Foster measure $M(\alpha, k, z; y)$ remains the same: $M(0,1, z; y) = M(0,1, z; x)$. The distribution of deprived indicators does not matter. However, many would argue that getting deprived in an additional indicator affects someone who is already strongly deprived more than someone who is just below the poverty line [e.g. (Stiglitz, et al., 2009)]. Intuitively, the poverty value should therefore increase if the number of poor remains the same and the number of deprived indicators remains the same, but the inequality within the poor increases.

As a solution, Datt proposes an improved version of the Alkire-Foster measure: the 'distribution-sensitive' measures, which will be explained further in the following paragraph.

2.9 Distribution-sensitive measures

The distribution-sensitive measures are an adjusted version of the Alkire-Foster poverty measures. Since the Alkire-Foster measure (equation 9) does not satisfy the cross-dimensional convexity axiom, and only satifies the strong transfer axiom when using the union approach, Datt alters the formula by adding an inequality parameter β to the sum of all indicators and eliminating cut-off value k (Datt, 2017). This alteration leads to a higher weight for the 'more deprived'. This <u>distribution-sensitive multidimensional poverty measure</u> is as follows:

$$M(\alpha, \beta, w, z; y) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{d} \sum_{j=1}^{d} w_j \left(\frac{z_j - y_{ij}}{z_j} \right)^{\alpha} g_{ij} \right)^{\beta} \tag{10}$$

This way, there is no cut-off value, so anyone who is deprived in any way is considered poor. However, by adding a power of β, a higher weight is placed on those who are deprived in more indicators. Applying this measure to the previous example gives the following results:

$$G(y) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow G'(x) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1

β	Poverty value for y	Poverty value for x	Relative change y to x
1	0.333	0.333	+0%
2	0.167	0.208	+25%
3	0.083	0.146	+75%

With this measure, the absolute poverty value decreases as the inequality parameter β increases. This is because the part that is powered to β is generally below 1, or at most equal to 1 if someone is deprived in all indicators. Multiplying this number by itself thus decreases the absolute value. However, the relative change from situation y to x increases as β increases. So with a value of $\beta > 1$, the poverty measure is also sensitive to the distribution of deprived indicators, leading to an increased poverty value if the inequality within the poor increases.

In his paper, Datt proves that this measure satisfies symmetry, replication invariance, deprivation focus, poverty focus and scale invariance, subgroup decomposability, monotonicity, dimensional monotonicity, strong monotonicity for $\alpha > 0$ and $\beta \geq 1$, cross-dimensional convexity for $\beta > 1$, and strong transfer for $\alpha > 1, \beta \geq 1$ or $\alpha \geq 1, \beta > 1$.

Since Datt's method does satisfy all the necessary axioms, it seems that Datt's distribution-sensitive method is an improvement on the Alkire-Foster measure. Datt applies his method to an Indian dataset, but mentions that this is mostly for illustrative purposes. He has not yet examined the robustness of his measure and only compares his measure to the Alkire-Foster measure using the union approach, which is not the most common choice when using the Alkire-Foster measure. Therefore, it would be interesting to test his measure on robustness and to see how it performs compared to the Alkire-Foster measure.

2.10 Shapley Decomposition

2.10.1 Derivation

An attractive feature of the poverty measures above is their decomposability. By decomposing the poverty measure into each indicator's contribution, it will be possible to see how they compare to each other and which indicator proves to have the most important effect on the level of poverty.

Shorrocks has presented a couple of properties a decomposition rule should have (Shorrocks, 2013). The rule should be (1) symmetric, (2) exact and (3) interpretable. These properties have been affirmed by a number of authors (Garcia-Diaz & Prudencio, 2017; Martinez & Perales, 2015; Datt, 2017). As far as I know, the formula presented by Shorrocks is the only one that satisfies all these criteria for now. I will now discuss the three properties and use these to derive Shorrocks's decomposition rule.

First of all, a rule should be symmetric, meaning that the contribution of an indicator should be independent of the way the indicators are listed. For poverty measurement, this requires the contribution of each indicator to be the same, even if the deprivation sets for two individuals are switched (Lustig & Higgins, 2018).

Secondly, the decomposition should be exact and additive, meaning that the sum of all contributions should be equal to the value of the index.

Thirdly, the contributions of each indicator should be interpretable in an intuitive way.

With regard to this last property, the simplest method would be to take the index value and then subtract the value the index would have without that indicator. Say there is a general formula that computes the poverty index, M, that is dependent on a set of indicators P. Then the contribution C of indicator p would be the poverty index including indicator p minus the poverty index excluding indicator p:

$$C_p(P, M) = M(P) - M(P \setminus \{p\}),\ \ p \in P$$

A problem with this method is that is often not exact. For instance, when using a cut-off value of 2 indicators, someone who is deprived in 2 indicators will become nonpoor as soon as one of those indicators is eliminated. Being nonpoor means the other indicator does not count anymore as well, so the effect of eliminating this indicator increases, leading to a relatively large contribution. However, when eliminating the other deprived indicators the same happens. The sum of all contributions then becomes larger than the original index.

The same is true when using the distribution-sensitive measure with $\beta > 1$. Since someone with many deprived indicators has a larger weight than someone with few deprived indicators, this method will not yield an exact decomposition.

A solution is to eliminate each indicator after each other, in a certain sequence. The contribution of each indicator would then be the impact of eliminating that indicator. This method is for example used by Chakravarty and Silber (Chakravarty & Silber, 2008).

To analyze this rule, let there be d indicators and let $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_d)$ be the order in which the indicators are removed.

As an example, let there be 4 indicators: A, B, C and D. If the order of removal is ACBD, then $\sigma_1 =$ A, $\sigma_2 =$ C, $\sigma_3 =$ B and $\sigma_4 =$ D.

Let $F(\sigma_r, \sigma) = \{\sigma_i | i > r\}$ be the set of indicators that are still remaining, where σ_r is the eliminated indicator.

If $r = 1$, then A is eliminated, so F consists of $F = \{\sigma_2, \sigma_3, \sigma_4\} = \{C, B, D\}$.

If $r = 2$, then C is eliminated, so F consists of $F = \{\sigma_3, \sigma_4\} = \{B, D\}$.

The contribution of indicator p, given order σ, is the difference between the poverty index value using set F supplemented with $\{p\}$ and the index value using only set F:

$$C_p^\sigma = M(F(p, \sigma) \cup \{p\}) - M\big(F(p, \sigma)\big),\ \ p \in P$$

Using the example, the contribution of indicator C is:

$$C_C^\sigma = M(F(C, \sigma) \cup \{C\}) - M\big(F(C, \sigma)\big) = M(\{C, B, D\}) - M(\{B, D\})$$

This decomposition is exact, which can be shown using the fact that $F(\sigma_{r-1},\sigma) = [F(\sigma_r,\sigma) \cup \sigma_r]$, for $r = 2,3,\dots,d$:

$$\begin{aligned}
\sum_{r=1}^{d} C_{\sigma_r}^{\sigma} = C_{\sigma_1}^{\sigma} + \sum_{r=2}^{d} C_{\sigma_r}^{\sigma} \\
&= M(F(\sigma_1,\sigma) \cup \{\sigma_1\}) - M\big(F(\sigma_1,\sigma)\big) + \sum_{r=2}^{d}\big[M(F(\sigma_r,\sigma) \cup \{\sigma_r\}) - M\big(F(\sigma_r,\sigma)\big)\big] \\
&= M(F(\sigma_1,\sigma) \cup \{\sigma_1\}) - M\big(F(\sigma_1,\sigma)\big) + \sum_{r=2}^{d}\big[M\big(F(\sigma_{r-1},\sigma)\big) - M\big(F(\sigma_r,\sigma)\big)\big] \\
&= M(F(\sigma_1,\sigma) \cup \{\sigma_1\}) - M\big(F(\sigma_1,\sigma)\big) - \sum_{r=2}^{d}\big[M\big(F(\sigma_r,\sigma)\big) - M\big(F(\sigma_{r-1},\sigma)\big)\big] \\
&= M(F(\sigma_1,\sigma) \cup \{\sigma_1\}) - M\big(F(\sigma_1,\sigma)\big) - \big[M\big(F(\sigma_d,\sigma)\big) - M\big(F(\sigma_1,\sigma)\big)\big] \\
&= M(F(\sigma_1,\sigma) \cup \{\sigma_1\}) - M\big(F(\sigma_d,\sigma)\big) \\
&= M(P) - 0 = M(P)
\end{aligned}$$

So the sum of all contributions is equal to the poverty index using all indicators, as it should be. A problem with this decomposition however is that the contribution is dependent on the order of elimination. This 'path-dependence' problem is now solved by Shorrocks, by considering each possible sequence of elimination (Shorrocks, 2013).

If there are d indicators, there are $d!$ possible elimination sequences. Consider for example the indicators A, B and C, then there are $3! = 6$ possible sequences of elimination: ABC, ACB, BAC, BCA, CAB, CBA.

Let the set E represent all possible elimination sequences σ. The contribution of an indicator p is the expected value of the contribution C_p^{σ}: the contribution for each elimination sequence, taken over all possible elimination sequences:

$$\begin{aligned}
C_p^G(P,M) &= \frac{1}{d!}\sum_{\sigma\in E} C_p^{\sigma} \\
&= \frac{1}{d!}\sum_{\sigma\in E} M(F(p,\sigma) \cup \{p\}) - M(F(p,\sigma))
\end{aligned}$$

Now, consider again the example with indicators A, B and C. When examining the contribution of indicator C, there is no difference between elimination sequence ABC and BAC. For the effect of C, it does not matter in which order the previous indicators have been eliminated – First A, then B leads to the same result as first B, then A.

See the table below for a summary of this process:

Summary: Elimination of indicator C

A	B	C
1	1	1
1	1	0
1	0	1
0	1	1
1	0	0
0	1	0
0	0	1
0	0	0

Above: All possible combinations of indicators A, B and C. 1=present, 0=eliminated.
The yellow rows indicate all combinations where C is already eliminated (C=0).

Below are all combinations where C is eliminated, with all orders of elimination for that combination. For example, in the first row only C is eliminated, while A and B are still present. This leaves two possible orders of elimination: first C, then A, then B, or first C, then B, then A.

Indicators still present	A	B	C	**Possible orders of elimination**
AB	1	1	0	CAB, CBA
A	1	0	0	BCA
B	0	1	0	ACB
-	0	0	0	BAC, ABC

The same is true for all indicators that are eliminated after: the order of elimination of these indicators is irrelevant. The contribution of C does not change if DE is added afterwards, or ED.

For this example with A, B, C, D and E, there are 2! possible ways of adding in A and B before C. There are then $(5-2-1)!$ possible ways of adding in D and E after C (Or: the total number of indicators minus the number of already added minus the examined indicators C). By multiplying the result of one of these sequences with $(5-2-1)!\,2!$, all these sequences can be added at once, instead of calculating each elimination sequence separately, since they all have the same result.

The formula can then be simplified, because it is no longer necessary to calculate each possible elimination sequence. It suffices to calculate every possible moment of elimination (for example, C when nothing is eliminated yet, C when A is already eliminated, C when B is already eliminated, C when A and B are already eliminated). This will allow for faster calculation of larger samples.

The new formula is then:

$$C_p^F(P, M) = \frac{1}{d!} \sum_{\substack{F \subseteq P \setminus \{p\} \\ |F| = f}} (d - 1 - f)!\, f!\, [M(F \cup \{p\}) - M(F)] \tag{11}$$

Here $|F|$ represents the number of elements in the set, which for poverty measures corresponds to the number of indicators that are 'counted' (not yet eliminated).

Equation 11 is equal to calculating the Shapley value, which is why this method is also referred to as the Shapley decomposition.

2.10.2 Empirical findings

Since its publication in 2013, the Shapley decomposition has been applied a few times to multidimensional poverty. Garcia-Diaz and Prudencio applied it to chronic poverty, derived from a household survey from Argentina (using the FGT measure, decomposed by household subgroup, with equal weights and a cross-dimensional poverty cut-off of 4 indicators) and found that the contributions varied greatly across time, not allowing for any clear policy recommendations (Garcia-Diaz & Prudencio, 2017). However, it must be noted that they used a somewhat small sample, due to their wish to examine chronic poverty, which might affect the volatility of the results and therefore make it harder to deduce policy implications from the decomposition.

Martinez and Perales applied the decomposition to Australia (using the SEM poverty index, a particular Australian poverty index, with equal weights) and found that the main indicators affecting poverty were health, personal safety, material resources, and social support (Martinez & Perales, 2015).

Datt applied to decomposition to India, using his distribution-sensitive poverty index and found that the main contributors to poverty were the indicators of nutrition, sanitation, fuel and assets (Datt, 2017).

It is interesting to see the difference here between Australia and India, where the main contributors in India are more 'basic needs', such as nutrition and sanitation, while in Australia elements like safety and social support play a large role in the development of poverty. These results demonstrate the usefulness of measuring multidimensional poverty instead of unidimensional poverty, since poverty in different types of countries covers a different set of problems.

2.10.3 Application to multidimensional poverty

For the application of equation 11 (the Shapley decomposition) to multidimensional poverty, a straightforward way is provided by Datt (Datt, 2017). Assume there are d indicators. He considers every possible variation of $G \subseteq P \setminus \{p\}$, by first creating a 2^d x d matrix that contains all possible combinations of indicators that are either 1 (counted) or 0 (eliminated – everyone is considered nondeprived), and then separates this into 2 matrices: one that contains all the rows that show a value of 1 for indicator p and one that contains all the rows that show a value of 0 for indicator p

As an example, consider the situation with 3 indicators ($d = 3$). The possible combinations of the values 1 and 0 are then:

$$S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This matrix S has $2^3 = 8$ rows and $d = 3$ columns, containing all possible combinations of the values 1 and 0. Now consider the specific indicator $j = 2$ (column 2). In matrix S, there are 4 rows where this column has the value 1 and 4 rows where this column has the value 0.

Dividing the matrix S in two submatrices then gives:

$$S_1^{(2)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } S_0^{(2)} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow S^{(2)} = (S_1^{(2)} | S_0^{(2)}) = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

Where $S_1^{(2)}$ is the submatrix of S consisting of all rows where the value in column 2 is equal to 1, and $S_0^{(2)}$ is the submatrix of S consisting of all rows where the value in column 2 is equal to 0. The submatrices form the full matrix S when combined: $(S_1^{(2)} | S_0^{(2)}) = S^{(2)}$.

In general: for any indicator j, the left half of the matrix S will have a value in column j equal to 1 and the right half will have a value in column j equal to 0. All other values are the same on the left as they are on the right. In the example above, in row 1, indicator 2 is eliminated first, the other indicators are still counted (their value is 1). In row 2, indicator 2 is eliminated second, after indicator 3.

For each combination, the number of indicators to be eliminated is the sum of the row in $S_0^{(j)}$ and the number of indicators already eliminated is the total number of indicators minus the sum of the row in $S_0^{(j)}$ minus 1.

Let $S_1^{(j)}$ be the matrix of all possible combinations t and let $S_0^{(j)}$ be the matrix of all possible combinations t'. Let the sum of row t be $\delta(t)$. Adjusting equation 11 gives:

$$C_j(S, M) = \sum_{t \in S_1^{(j)}} \frac{\big(d - \delta(t)\big)!\,(\delta(t) - 1)!}{d!} M(t) - \sum_{t \in S_0^{(j)}} \frac{\big(d - 1 - \delta(t')\big)!\,\delta(t')!}{d!} M(t') \quad (12)$$

Note that in the first part of the equation, f is substituted by $\delta(t) - 1$, since in f the examined indicator is not counted, while in $\delta(t)$ it is.

The contribution of indicator j to M, denoted as $C_j(M)$, is then the weighted sum of the average poverty value <u>before elimination</u> minus the weighted sum of the poverty value <u>after elimination</u>, for each possible ordering of elimination. The proportional contributions can then be calculated using $C_j(M)/M$.

3. Data

3.1 Dataset

When the concept of multidimensional poverty started to emerge, the interest in household surveys increased. A few large-scale surveys were set up, starting in the late sixties, to get more insight in the standard of living of its citizens. Examples include the Swedish Level of Living Study 1968, the French *Les Indicateurs Sociaux* 1971 and the German *Materialien zum Bericht zur Lage der Nation* 1971 (Alkire, et al., 2014). In 2003, the European Union introduced a new survey on income and living conditions throughout the EU, starting with six member states: Belgium, Denmark, Greece, Ireland, Luxembourg and Austria. In 2004, almost all remaining EU member states joined, providing the EU with a new set of statistics that allows for easy comparison between countries (Eurostat, 2017).

The European Union Statistics on Income and Living Conditions (EU-SILC) is an instrument that is used to collect information on income, labor, education, health, social exclusion and housing conditions. Since it considers all common dimensions of multidimensional poverty, it is a popular dataset for measuring and comparing poverty in the EU and has been the data source for several papers [e.g. (Guio & Maquet, 2006; Maitre, et al., 2013; Alkire, et al., 2014)].

In this paper, the analyzed country is Ireland. Ireland has participated in EU-SILC since 2003, leading to a dataset consisting of the years 2003 to 2016 (Central Statistics Office, 2018). The number of participants ranges from 8,000 to 15,000. Unfortunately, there is no link between consecutive years. Otherwise, the measure could be improved by adding chronic poverty, where a person is only considered poor if it has met the requirements for being poor for a certain amount of subsequent years. However, since the dataset does not link the households over time, this is not possible in this case.[4]

Because roughly half of the respondents only answered half of the questions in the survey, not all observations are used.[5] Respondents that only answered half of the questions were taken out of the sample.

3.2 Subgroups

Some empirical research has been performed on dividing a population in subgroups, to see if there are differences in poverty between these groups. This often turns out to give interesting information. Alkire and Santos analyzed 104 developing countries for example and found that rural areas contain around five times more poor people than urban areas (Alkire & Santos, 2010). They also found that rural areas contain relatively more people that are severely poor than urban areas. Garcia-Diaz and Prudencio found that in Argentina, the composition of a household has a large impact on the probability of them being chronically poor (Garcia-Diaz & Prudencio, 2017). They categorized the households in four groups: households with children and older adults, households with children but without older adults, households with older adults, and households without children and older adults. Households with children but without older adults performed best, whereas households with older adults were more prone to chronic poverty. Also, Alkire et al. divide by subgroup (gender and

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[4] There does exist a panel data set for the SILC, but I do not use this, since it is a lot smaller, containing fewer households and fewer years.

[5] It is not clear from the attached documentation why such a large share of respondents only answered half the survey.

age) when measuring poverty in European countries, where they find women and elderly are significantly more poor than the other groups (Alkire, et al., 2014).

Since the EU-SILC survey offers data on the type of household of each respondent (urban, rural farm and rural non-farm) I will also be able to calculate the multidimensional poverty index for each subgroup separately to see if I also find significant differences.

3.3 Identification method

Based on the theoretical framework, the indicators will be categorized into four broad dimensions: education, health, standard of living and social exclusion. Education, health and standard of living, because they form the basis to most other multidimensional poverty research papers. Social exclusion, since the past few years this dimension has gained a lot of attention, especially in the EU, and is assumed to be a major part of multidimensional poverty.

The indicators that define these dimensions are mostly determined by the availability of data in the EU-SILC. Based on the questions asked in the survey and previous research using the EU-SILC data, I chose a total of twelve indicators (figure 2). The poverty lines are presented in figure 2 as well.

The source type for these indicators is shown in figure 3. For the indicators with a binary source variable, it is obvious when an individual should be considered deprived. I outlined the exact questions asked for the binary source variables in Appendix, part B. Below figure 3, I elaborate on the indicators with an ordinal source variable.

Figure 2

Dimension	Indicator	Poverty line
Education		
	Education level	Has not completed lower secondary education and not currently studying
Standard of living		
	Shelter	Dwelling has leaking roof, damp walls, etc. or rot in the doors, windows frames or floor
	Sanitation	Has no access to at least one of the following: bath or shower, hot water, or running water
	Assets	Cannot afford one of the following: make ends meet, meals with meat/chicken/fish/vegetarian equivalent every second day, keep the house adequately warm or unexpected required expenses AND does not own either one of the following: computer, washing machine, car or phone
	Environment	States that there is noise from neighbors or the street, or that there is pollution, grime or other environmental problems in the area
	Crime	States that there is crime, violence or vandalism in the area
Health		
	General health	Reports health as “Bad” or “Very bad”, or is limited in its activities due to a health problem
	Long-term health	Suffers from a chronic illness or condition
	Nutrition	Had a day in the last fortnight where it did not have a substantial meal due to lack of money
	Unmet medical needs	Needed a medical examination or treatment, but did not receive this due to a lack of money, inability to take off time from work or because of no means of transport
Social exclusion		
	Social life	States to not have hobbies due to lack of money, or to not have a morning, afternoon or evening out in the last fortnight for own entertainment due to lack of money
	Joblessness	Unemployed

Figure 3

Type	Indicator(s)
Binary	Long-term health, nutrition, unmet medical needs, shelter, sanitation, assets, environment, crime, social life, joblessness
Ordinal	Education level, general health, (assets)

Education level

The Irish law states that citizens are required to attend school from the ages of 6 to 16 or until they have completed lower secondary education. A common poverty line is to consider everyone who has not achieved the level of education required by law, deprived. However, other options are possible too.

To decide on the poverty line for education, first consider figure 4, describing the composition of education level in 2009.[6] The first column shows the possible answers, the second column shows the percentage of respondents that gave this answer and the third column shows the cumulative percentage, which is equal to the share of the population that is deprived if the poverty line is equal to that answer.

Figure 4

Highest level of education	Share of population with this answer	Share of population deprived if poverty line equals…
No formal/ primary education	30.67	30.67
Lower secondary	16.62	47.29
Upper secondary	17.49	64.78
Post leaving cert	7.93	72.71
Third level – non degree	6.27	78.99
Third level – degree or above	21.01	100.00

From this, it is clear that adjusting the poverty line one level up or down has a large impact on the level of poverty. Choosing a poverty line of no more than primary education leaves 30.7% deprived and choosing a poverty line of no more than lower secondary leaves 47.3% deprived. This indicator is therefore very sensitive to the poverty line choice.

The education level required by law is a completed lower secondary education. Therefore, it seems sensible to choose a poverty line of a completed level of education of no formal/primary education. This way, every individual that has not gone completed lower secondary education is considered deprived.

[6] For clarity, I only provided the values of one year. I chose 2009 as it is the middle year of the available data.

In the Appendix, part C,Robustness making ends meet (3.3) I have checked for robustness of this poverty line. It appears that the multidimensional poverty index is quite robust to the choice for the poverty line. Changing the poverty line to ‘lower secondary’ does not result in significantly different results. Therefore, I feel safe choosing the poverty line that I feel makes most sense, the one of ‘no formal/primary education’.

As such, to be considered deprived, respondent answers the question “What is your highest level of education attained?” with “No formal/ primary education”.

General health

General health is composed of two ordinal variables. The first is the reported health status, the second being limited in activities due to a health problem. Intuitively, I would set the poverty line at someone reporting a (very) bad health status and/or being (strongly) limited in activities due to a health problem.

Consider the compositions in figure 5, for 2009:

Figure 5

Health status	**Share of population with this answer**	**Share of population deprived if poverty line equals…**
Very bad	0.60	0.60
Bad	3.60	4.20
Fair	19.36	23.56
Good	41.90	65.46
Very good	34.54	100.0

Limited in activities due to a health problem	**Share of population with this answer**	**Share of population deprived if poverty line equals…**
Strongly limited	7.37	7.37
Limited	20.33	27.70
Not limited	72.30	100.0

The share of the population that reports a bad or very bad health status is quite small, with a combined percentage of 4% of the population. Changing the poverty line from bad to very bad will probably only have a small effect. However, around 20% of the population states being limited in activities due to a health problem, compared to around 7% being strongly limited. This is quite a big difference. To see if the results change significantly when changing the poverty line for this aspect, I checked on robustness in Appendix, part C. I found that the multidimensional poverty index is very robust to changes in the poverty line for being limited in activities due to a health problem.

I therefore keep my original choice in poverty line and consider someone deprived if their reported health status is "bad" or "very bad" and/or they are "limited" or "strongly limited" in their activities due to a health problem.

Assets

For assets, all questions are binary, except the one on ability to make ends meet. Consider the composition in figure 6, for 2009:

Figure 6

Ability to make ends meet	**Share of population with this answer**	**Share of population deprived if poverty line equals…**
With great difficulty	9.21	9.21
With difficulty	13.11	22.33
With some difficulty	35.65	57.98
Fairly easily	26.85	84.84
Easily	11.09	95.93
Very easily	4.07	100.00

Since the differences between the different options are quite large, I tested on robustness, which I have shown in Appendix, part C. The poverty index turns out to be very robust to the choice in poverty line for the ability to make ends meet. I therefore use the poverty line which I feel makes most sense intuitively, which means that someone is deprived in the ability to make ends meet if they have 'difficulty' or 'great difficulty'. Having 'some difficulty' would include too many people, whereas only including people with 'great difficulty' would mean people with 'difficulty' making ends meet are not considered deprived, while I feel it would be more reasonable to consider these people deprived.

To sum up, to be considered deprived, respondent answers one of the following yes/no-questions with "No": "Is your household able to keep the house adequately warm?", "Can your household afford to eat meals with meat, fish or a vegetarian equivalent every second day?", "Can your household afford to pay unexpected required expenses?" or it answers the question "Are you able to make ends meet?" with "With great difficulty" or with "With difficulty", and subsequently it also answers one of the following yes/no-questions with "No": "Is there a computer in the household?", "Is there a washing machine in the household?", "Do you have a car?" or "Do you own a mobile phone?".

3.4 Sample weights

The EU-SILC database includes a sample weight variable: 'Euroweight', which indicates how many households a particular observation represents (Central Statistics Office, 2017). The sum of all

Euroweight values is equal to the total population of Ireland. The database should therefore be adjusted by multiplying each observation with:

$$\frac{Euroweight_i}{\sum_{i=1}^{n} Euroweight_i} * n$$

Its Euroweight divided by the sum of all euroweights indicates what share of the population this observation represents. I multiplied it with the total number of observations to generate the relative weight, with an average of 1, to allow for a more intuitive interpretation of the weights. If the sample weight is equal to 1, that observation represents exactly $1/n$ percent of the households.[7] If the sample weight is smaller than one, it represents fewer households than that, whereas if it is larger than 1, it represents more households.

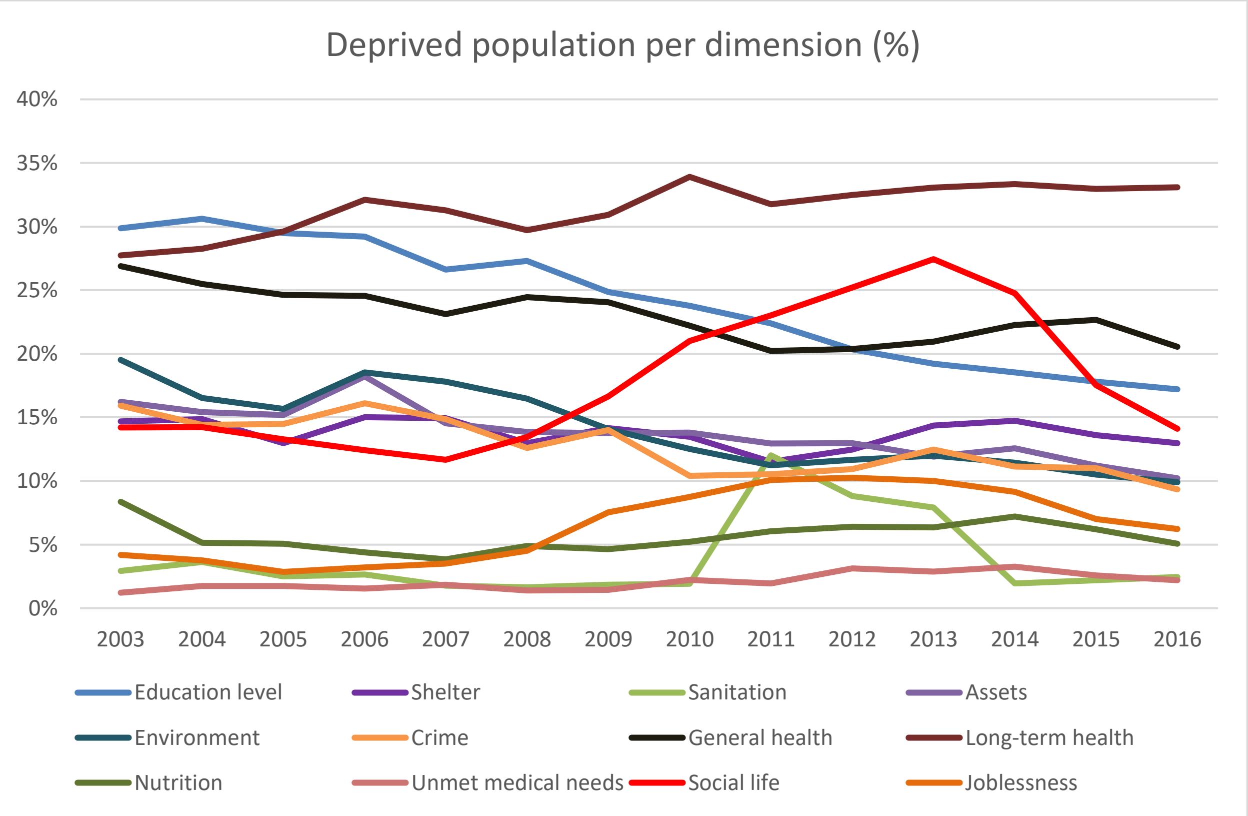
I multiplied each value in the deprivation matrix for individual i with the sample weight of individual i.

3.5 Descriptive statistics

Figure 7 summarizes the average value of all indicators.

Since all indicators consist of values 1 (deprived) and 0 (not deprived), the average value represents the percentage of respondents that are deprived in that indicator. The absolute values and the number of observations (the final column 'n') are presented in Appendix, part D.

Figure 7



[7] As an example, if there are 100 households in the population and 20 observations in the dataset, that observation represents 5% of all households

Some of these indicators have a fairly high level of deprivation. For example, the indicator education level starts with a deprivation level of approximately 30%, but then decreases over time to approximately 17%. This happens rather gradually, and considering the large sample size this points to an overall improvement in education level, probably because the younger are better educated. Two of the health indicators (general health and long-term health) also exhibit a quite high level of deprivation. Both general health and long-term health start at a deprivation level of around 27%, with a growing deprivation in long-term health and a declining deprivation in general health over time. It could be that the poverty line is not strict enough or it could just mean that a large share of the population has health problems, which could very well be true. Health has an important relationship with poverty, so the poverty line should not be adjusted, because then poverty would only be associated with severe diseases, while intuitively poverty is also related to a lower health status in general. The unmet medical needs indicator shows a relatively low level of deprivation, which is not unsurprising for a West-European country, as it has a well-developed health care system that (should) also take care of the poor. Regarding the indicators in the standard of living dimensions, all indicators have a similar level of deprivation. This could indicate that they are all connected: someone who is not able to make ends meet or afford a car might also not be able to maintain a house in a safe neighborhood, or it could be coincidental. The indicator sanitation shows a significantly lower deprivation level than the other standard of living indicators, as expected since most houses in Ireland have access to running water, even the cheaper homes. Limitations in social life due to finances are not uncommon, with approximately 15% of the population experiencing this. This appears to affect social exclusion more than joblessness, which only impacts around 5% of the population. Joblessness does increase after 2008 and starts decreasing again after 2013, presumably caused by the economic crisis in those years.

The two indicators that are not consistent in their development are sanitation and social life. As for sanitation, the years 2011, 2012 and 2013 have clearly higher deprivation levels than all other years. I cannot think of an explanation for this. All other indicators are quite consistent over time and the phrasing of the EU-SILC questions has not changed, so I cannot explain these outliers unfortunately. The social life indicator is also unstable, but it is declining until 2008 and after 2014 and increasing between 2008 and 2014. This inclination is probably due to the economic crisis as well and it started recovering after 2014 (joblessness also decreases for the first time from 2014 to 2015).

Since more indicators are decreasing in deprivation level than increasing, I expect poverty to have decreased overall.

3.6 Weights

As mentioned in the theoretical framework, a weighting scheme based on societal preferences would be the best option. However, these are not always available. For the case of Ireland, there seems to be no proper research yet that investigates societal preferences regarding dimensions of poverty. However, for England there is. Watson et al. used a discrete choice experiment to deduct a weighting scheme for the seven poverty dimensions used by the Oxford Index of Multiple Deprivation (IMD) (Watson, et al., 2008). The original weighting scheme used in the IMD was based on value judgments of the researchers and on theoretical assumptions (and the reliability of the data was accounted for as well). As an alternative, Watson et al. tried an empirical approach and asked people a series of questions in which they had to choose between two deprivation states and select which was worse. From these answers, they derived a set of weights. The most notable differences were that respondents placed a greater weight on housing and health than in the original weighting scheme, and less weight on employment than assumed in the IMD.

The United Kingdom and Ireland, although very similar, are not the same and it cannot automatically be assumed that the societal preferences of the British are the same as those of the Irish. However, I think it is safe to say that they are comparable, considering the resemblances in culture, climate and history. To add to that, a paper by Maitre et al. compared poverty in many European countries and found a large similarity between the English and the Irish results (Maitre, et al., 2013). When comparing three types of poverty defined by the EU (at risk of poverty, severe material deprivation and low work intensity households) they found very similar percentages for each of the categories for Ireland and the UK, while other countries showed clearly different values. Secondly, in the Eurobarometer of 2007 and 2010, citizens of the European Union were asked several questions regarding poverty and exclusion (Eurobarometer, 2007; Eurobarometer, 2010). Irish and British citizens gave very similar answers on what they value most in life, what government should pay most attention to and when they consider someone to be poor, while most other countries give clearly different answers. This similarity in preferences is why I apply the weights of England to the Irish poverty index.

The dimensions used in the IMD are not exactly the same as the ones in this paper. However, they do show a great resemblance and can therefore also be applied to the dimensions from this paper. One dimension from the IMD is not used in this paper (Barriers to housing and service deprivation – which concerns the distance to commonly used facilities and services), so I do not consider this percentage of 9%. The remaining dimensions thus add up to 91%. I adjusted them to sum up to 100%. Furthermore, I combined the IMD dimensions income, living environment and crime to get the weight for my standard of living dimension, since Standard of Living covers all these areas at once. Figure 8 shows the final weighting scheme.

Figure 8

Stated preference weights			
Dimensions	Indicator	Weight (%)	Weight (absolute)
Education			
	Education level	13.0	1.57
Standard of Living			
	Shelter	12.4	1.49
	Sanitation	12.4	1.49
	Assets	12.4	1.49
	Environment	12.4	1.49
	Crime	12.4	1.49
Health			
	General health	5.7	0.68
	Long-term health	5.7	0.68
	Nutrition	5.7	0.68
	Unmet medical needs	5.7	0.68
Social exclusion			
	Social life	1.1	0.13
	Joblessness	1.1	0.13

Of course, this weighting scheme is still far from ideal. First of all, because it is derived from research performed in England, and secondly, because the proposed dimensions in the experiment are not exactly the same as the ones used in this paper. So although societal preferences can be somewhat approximated using this approach, there is still a lot of room for improvement.

Because of this, I will use a second weighting scheme to see how much this affects the results. For this, I use the second-best option, which is statistical weights, where all dimensions are assigned an equal weight and all indicators are assigned an equal part of this weight. This is considered the second-best option, as it is always applicable and appears to offer better results than no weighting. It is therefore a very popular method in research from the past decade. The resulting weights are in figure 9.

Figure 9

	Statistical weights		
Dimensions	Indicator	Weight (%)	Weight (absolute)
Education			
	Education level	25.0	3.00
Standard of Living			
	Shelter	5.0	0.60
	Sanitation	5.0	0.60
	Assets	5.0	0.60
	Environment	5.0	0.60
	Crime	5.0	0.60
Health			
	General health	6.3	0.75
	Long-term health	6.3	0.75
	Nutrition	6.3	0.75
	Unmet medical needs	6.3	0.75
Social exclusion			
	Social life	12.5	1.50
	Joblessness	12.5	1.50

These two weighting schemes are quite different from each other. The first, based on societal preferences, is more evenly spread over the indicators and places a relatively large weight on standard of living. The second one is more evenly spread over the dimensions, but places a relatively large weight on certain indicators (like education level and social life). I will compare the differences and check for robustness between these weighting schemes to see how this affects the results.

4 Methodology

As discussed in the theoretical framework, the most complete multidimensional poverty measures available so far are the Alkire-Foster measure and the distribution-sensitive multidimensional poverty measure. These are therefore the ones I will use (with a preference for the distribution-sensitive measure).

Both measures contain a number of parameters, for which there is not necessarily a single best option. In chapter 3, I have explained my choices for the indicators and their poverty lines. In this chapter, I will describe which values I will use for the remaining parameters.

4.1 Sensitivity to severity of deprivation

Both the Alkire-Foster measure and the distribution-sensitive multidimensional poverty measure require choices on the sensitivity to the severity of deprivation. For the Alkire-Foster measure, this concerns the parameter α, which specifies how sensitive the measure is to the severity of deprivation within an indicator. For the distribution-sensitive multidimensional poverty measure, this concerns both the parameter α and the parameter β, where β specifies how sensitive the measure is to the severity of total deprivation.

Considering that most source variables are binary, the only way to use the SILC-data for measuring multidimensional poverty is by creating binary indicators, with a value of 1 if the individual is deprived. It is therefore only possible to use a value of $\alpha = 0$. Adjusting equation 9 accordingly leads to the following Alkire-Foster measure:

$$M(k, w, z; y) = \frac{1}{nd} \sum_{i=1}^{n} \sum_{j=1}^{d} w_j \, g_{ij} I_i^k \qquad (17)$$

Applying $\alpha = 0$ to the distribution-sensitive multidimensional poverty measure in equation 10 leads to the following:

$$M(\beta, w, z; y) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{d} \sum_{j=1}^{d} w_j g_{ij} \right)^{\beta} \qquad (18)$$

For the inequality parameter β, different values are possible. Following Datt, I will use the values $\beta = 1$, $\beta = 2$, and $\beta = 3$, and compare the results (Datt, 2017).

Note that using a poverty cut-off of $k = 1$ in the Alkire-Foster measure is equal to using an inequality parameter of $\beta = 1$ in the distribution-sensitive multidimensional poverty measure.

From here on, when I mention the multidimensional poverty index (MPI), I refer to these equations.

4.2 Poverty cut-off

I will use two cut-off values. Primarily, the value of $k = 1$, since this does not violate the strong transfer axiom and allows for adding a parameter β to the formula.

As mentioned in the theoretical framework, a common poverty cut-off is a third of all indicators. I will therefore also use a poverty cut-off value of $k = 4$, since I use 12 indicators in total, to compare the results and see what the effect is of choosing this popular value of k.

4.3 Poverty value calculation

After deciding on all parameters, the actual calculation can begin. To start, I create the deprivation matrix, containing all deprivation data. I create this using STATA, where I generate a variable for each indicator, with a value of 1 when the individual falls below the poverty line in that indicator and 0 otherwise.

The proceeding part is performed in Matlab. For the Alkire-Foster measure, I generate the indicator I_i^k by using a function that creates a variable that is equal to 1 if the individual is deprived in k or more than k indicators and 0 otherwise (using the sum of each row in the deprivation matrix). Next, each value in the deprivation matrix is multiplied by the weight given to that indicator in the weighting scheme. I compile the resulting values in a weighted deprivation matrix.

Now that all the elements of the formula are defined, I can calculate the final poverty value in Matlab using equations 9 and 10.

4.4 Statistical inferences

In order to draw conclusions from the found poverty indices, it would be good to be able to apply some statistics to the index. However, each sample only leads to one poverty index, making it impossible to calculate the standard errors. A way of approximating the standard errors is by using the bootstrap method (Alkire & Santos, 2014). Bootstrapping means that from the existing sample, a large number of samples is drawn with replacement ('with replacement' indicates that the same individual can appear in the sample more than once) (Housseini, 2017). From all these samples, I will calculate the poverty index, and all these poverty indices can then be used to infer the confidence interval (Orloff & Bloom, 2014). Alkire and Santos applied this method to their poverty index, using 2,000 samples, and found that the bootstrapped confidence interval is a good approximation of the actual confidence interval by comparing it to the actual standard errors (Alkire & Santos, 2014). They calculated the actual standard errors using an elaborate mathematical procedure, that I do not use as it is beyond the scope of this thesis and the bootstrapped errors provide a sound alternative. I therefore apply the bootstrap method to my poverty indices, to examine if a possible change in poverty is significant or not, using the same number of 2,000 samples. Garcia-Diaz and Prudencio use the same method (Garcia-Diaz & Prudencio, 2017).

The interpretation of this bootstrapped confidence interval is such that an overlap in confidence intervals means it is not possible to say anything about the statistical difference in poverty, but if the confidence intervals do *not* overlap, they are statistically significantly different (Alkire & Santos, 2014).

4.5 Decomposition

I perform the decomposition using the method developed by Shorrocks, as described in paragraph 2.7 (Shorrocks, 2013). I base my methodology largely on the methodology used by Datt, as summarized in equation 12.

First of all, I generate a matrix that contains all possible combinations of 1 and 0. Since I use 12 indicators, this matrix will have 12 columns and $2^{12} = 4{,}096$ rows (each cell has two options: 1 and 0, so there are 2^d combinations possible). Then, for each indicator, I generate the two matrices S_1^j and S_0^j, by splitting up the original matrix into S_1^j that contains all rows where the value in column j is 1, and into S_0^j that contains all rows where the value in column j is 0. These are the 'elimination matrices'. Here, I make sure that for every row t in S_1^j, all values are equal to those in row t' in S_0^j, except for the value in column j.

For every row t and t', I can now calculate the poverty index. Every column that contains a value 0 in the elimination matrix indicates that this indicator is eliminated, meaning that everyone is considered nondeprived in that indicator. As such, I will multiply each value in the weighted deprivation matrix with the value in row t and t' in the corresponding column.

As an example, consider the deprivation matrix with 2 individuals ($n = 2$) and 3 indicators ($d = 3$):

$$G(y) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

If the value in a specific row t of the elimination matrix is $[1 \quad 1 \quad 0]$, only the first two indicators are counted, whereas everyone is considered nondeprived in the third column, leading to the following eliminated deprivation matrix:

$$G'(y) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Everyone is considered nondeprived in indicator $j = 3$, so the column consists of only zeroes. This new matrix is called the 'eliminated deprivation matrix'.

Since there are two elimination matrices (S_1^j and S_0^j), I end up with two eliminated deprivation matrices as well (one including the examined indicator, and one excluding it).

Then, I will calculate the poverty value twice, once with the first eliminated deprivation matrix and once with the second. After calculating the poverty value, I will multiply this value with the weight of respectively $\frac{(d-\delta(t))!(\delta(t)-1)!}{d!}$ and $\frac{(d-1-\delta(t'))!\delta(t')!}{d!}$.

Finally, I repeat this procedure for all rows t and t', so every possible combination is considered, and sum all the results for the set including the examined indicator (the first part of the equation, $\sum_{t \in S_1^j} \frac{(d-\delta(t))!(\delta(t)-1)!}{d!} F(t)$) and I sum all the results for the set excluding the examined indicators (the second part of the equation, $\sum_{t' \in S_0^j} \frac{(d-1-\delta(t'))!\delta(t')!}{d!} F(t')$). The difference is then the contribution of indicator j. Finally, the relative contribution can be calculated using $C_j(M)/M$.

For this whole procedure, I use the computer program Matlab.

4.6 Robustness of parameters

The purpose of such a multidimensional poverty measure is mainly to be able to rank and order different years, countries or other groups in terms of poverty (Alkire & Santos, 2014). For composite indices like this, the main methods for assessing robustness to different parameter values is to calculate the rank correlation coefficient (Foster, et al., 2012). Two common measures for this are

Kendall's tau and Spearman's rho (Housseini, 2017). These two measures are for example used by Alkire and Santos in assessing the robustness of their multidimensional poverty index for subgroups, poverty lines and weighting schemes (Alkire, et al., 2010; Alkire & Santos, 2014). They found that their index is very robust to these changes. In addition to this research, I think it would be interesting to use this same method to assess the robustness of the distribution-sensitive measure, to see how this improved version of the multidimensional poverty index responds to changes in parameters and weights and how the two measures compare.

This same method can be applied to the decomposition results. As far as I could find, the Shapley decomposition has been applied a couple of times to multidimensional poverty indices, but there is nothing known about the robustness of the measure yet. Martinez and Perales actually mention research on robustness of the decomposition as a suggestion for further research (Martinez & Perales, 2015). A robust poverty index does not automatically mean that the decomposition is robust as well. The benefit of decomposition of a poverty index is to identify the contribution of each indicator, so that policy can be adjusted and its efficiency in reducing poverty can be optimized (Martinez & Perales, 2015; Datt, 2017). This means that the ranking of the indicators is of great importance, and the main goal of decomposing. I therefore think that analyzing rank correlation for different decomposition results would be a good method for assessing robustness, and could be a good addition to the current literature on decomposition of poverty indices.

Both Kendall's tau and Spearman's rho are a variation of the 'standard' correlation coefficient (Kruskal, 1958). For rank correlation, consider there to be two ranks: x and y, with r_u and s_u as rank of object u. Kendall's tau only looks at whether two pairs are concordant or not. A pair is concordant if the rank of object v is in both rankings higher than the rank of object u or the other way around. Kendall's tau is then[8]

$$Kendall's\ tau = \frac{2((number\ of\ concordant\ pairs) - (number\ of\ disconcordant\ pairs))}{\sqrt{n(n-1)n(n-1)}}$$

Spearman's rho also takes into account the distance of the two rankings (Kruskal, 1958):

$$Spearman's\ rho = 1 - \frac{6\sum_{u=1}^{w}(r_u - s_u)^2}{w^3 - w}$$

The null hypothesis of both these coefficients is that the two rankings are statistically independent (Croux & Dehon, 2010).

[8] For the derivation of Kendall's tau and Spearman's rho, see Appendix, part E.

5 Results

5.1 Multidimensional Poverty Index (MPI) findings

In this chapter, I will summarize the findings regarding the multidimensional poverty index. Since I use a number of different parameter values, I have separated the findings for each parameter to allow for easy comparison and evaluation of the changes.

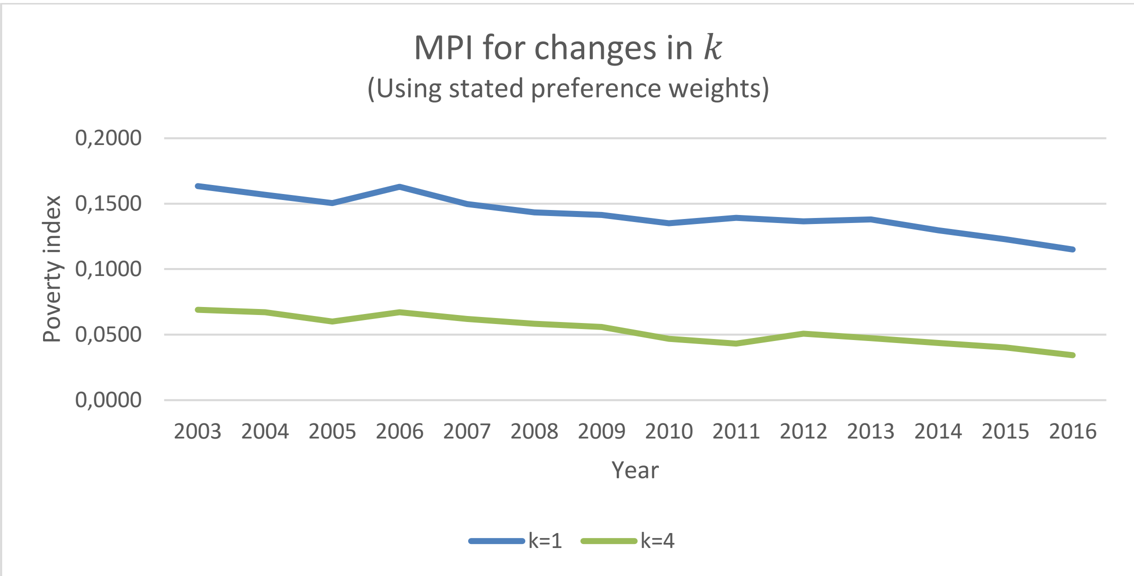
5.1.1 Findings regarding cut-off value k

When using a cut-off value k larger than 1, the value for inequality parameter β can only be equal to 1. I used two different weighting schemes: stated preference and statistical weights.

Stated preference weights

Figure 10 shows the results for the different cut-off values when using stated preference weights. The absolute values are presented in Appendix, part F.

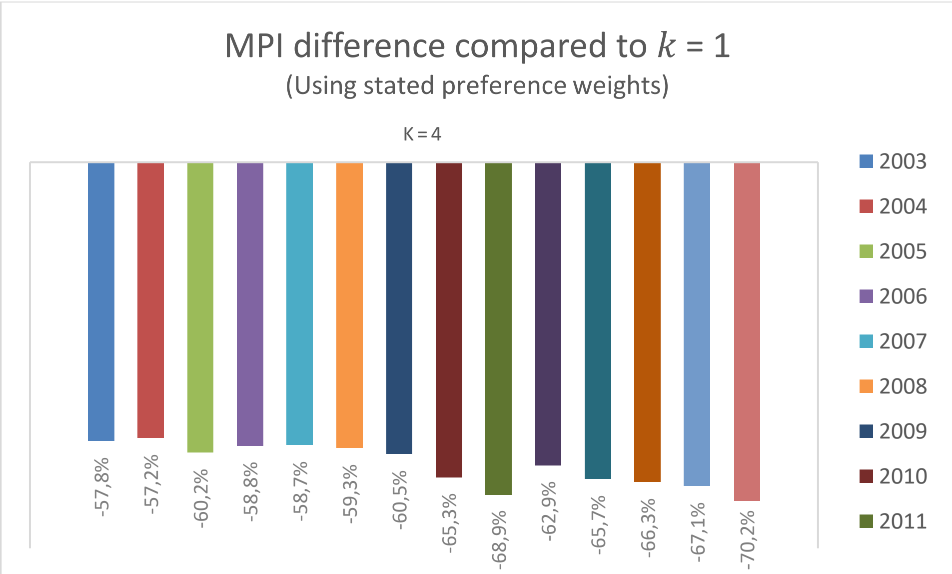
Figure 10



A higher cut-off value k should lead to lower poverty since fewer people are considered poor. The results confirm this: the higher k, the lower the value of the poverty index. For all cut-off values a decrease in poverty is visible over time. This suggests that for everyone, from those deprived in a single indicator to those deprived in ≥ 4 indicators, poverty on average went down.

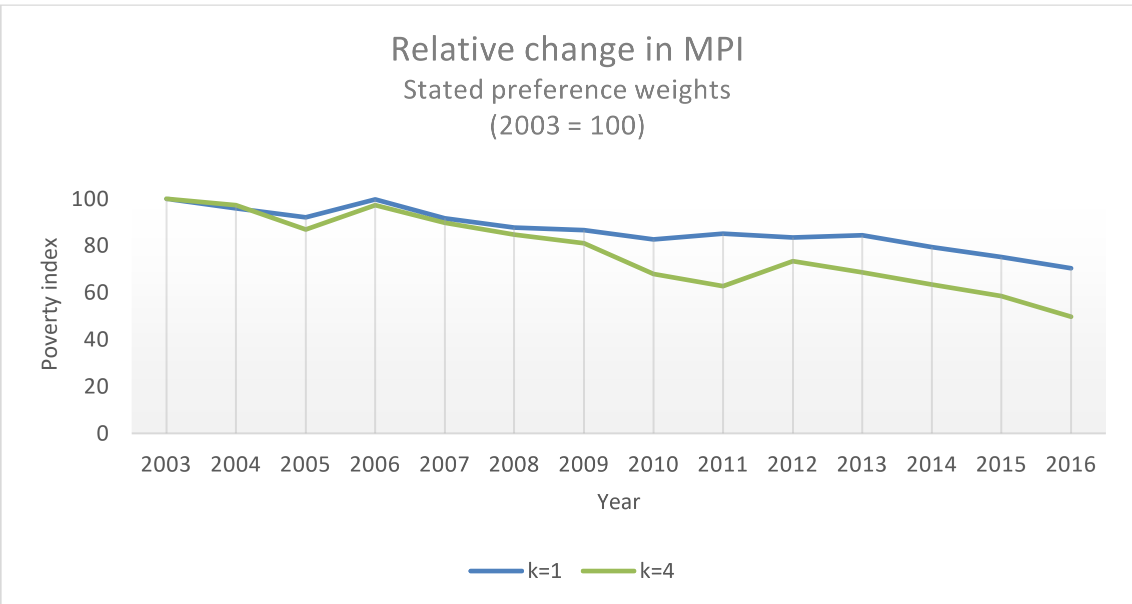
What is interesting is the size of the difference, as shown in figure 11. When using a cut-off value of 4 indicators, the poverty index reduces with more than 50%, even showing a difference of 70% in 2016. This shows that a large proportion of the population that is deprived in at least 1 indicator, is deprived in less than 4 indicators, and this proportion increases over time. A finding that should be kept in mind when deciding on the parameter values: using a cut-off value of one third of all indicators, a common choice, has (at least for Ireland) an enormous impact on the outcome of the poverty index. Excluding this large group could be a conscious choice, depending on the indicators and their poverty line. For example in my sample, a fairly large share of the population is deprived in education and long-term health. Excluding people that are only deprived in one of these indicators could therefore be a necessary step to accurately measure poverty. However, it does significantly impact the poverty outcome and excludes a large group of people, without knowing in what indicators they are deprived. Therefore, these results show the benefit of using a distribution-sensitive poverty measure, which includes everyone that is deprived in any indicator, but weighs them according to their number of deprivations.

Figure 11



To compare the different cut-off values, I additionally created figure 12 that shows the relative change of the poverty index, using 2003 as the base year. In this graph, it is visible that the poverty index using a cut-off value of 4 indicators has decreased relatively more than the one using a cut-off value of 1 indicator. This shows that for the group with 4 or more deprivations, poverty has decreased relatively more than for the group with a deprivation in at least 1 indicator. These results suggest that not only has poverty gone down overall, but more severe poverty has decreased most (in relative terms). This is in line with the findings from figure 11, where the change in poverty from augmenting the cut-off value increases over time.

Figure 12



Statistical weights

Figures 13 to 15 show that using the 'statistical weights' weighting scheme produces similar results. Although the differences between the different cut-off values are less pronounced than when using stated preference weights, the difference is still substantial, with a decrease of around 50% in the poverty index when going from a cut-off of 1 to 4 indicators. I discuss the changes between the weighting schemes further in paragraph 5.1.3.

Figure 13

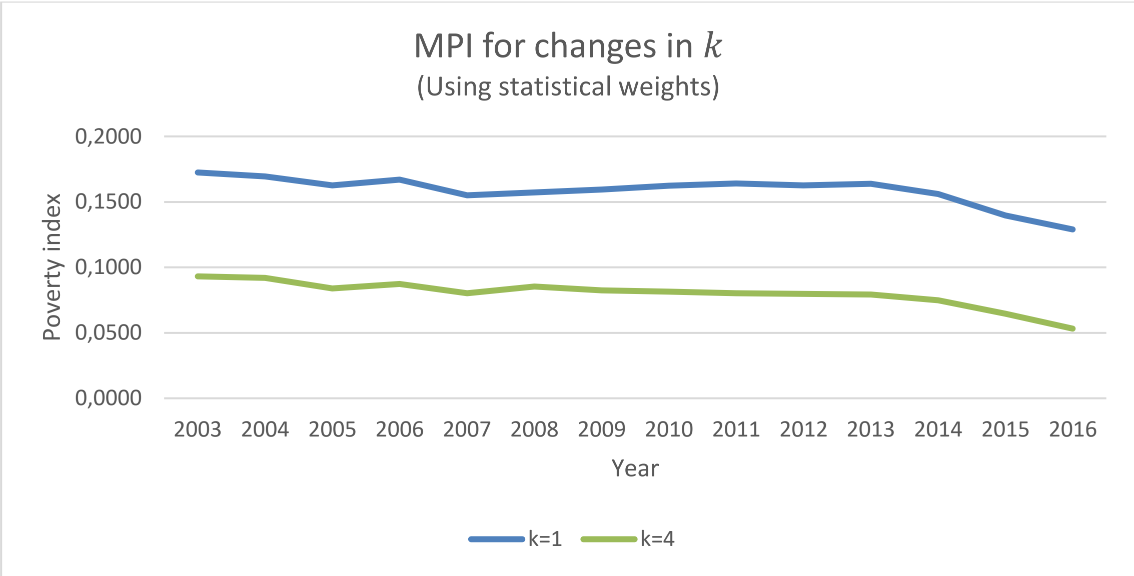


Figure 14

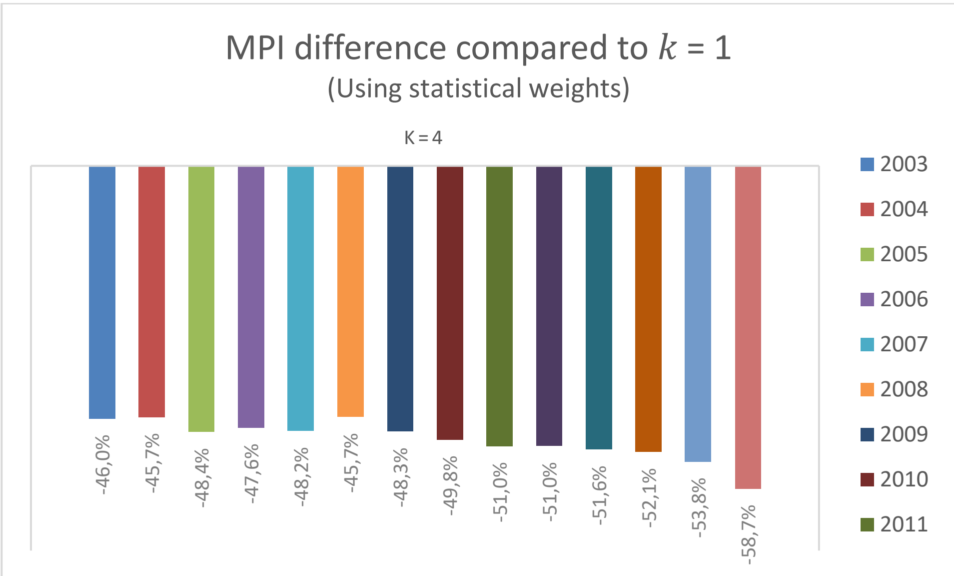
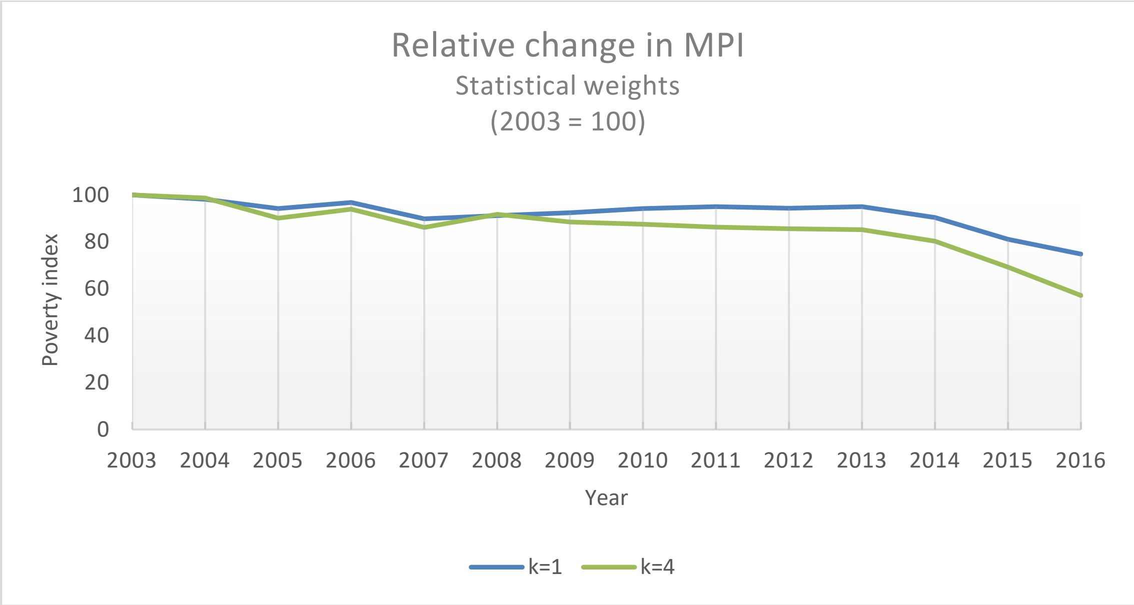


Figure 15



5.1.2 Findings regarding inequality parameter β

As mentioned before, using an inequality parameter of $\beta > 1$ leads to a smaller index value, since the value that is squared by β is always a value in the range from 0 to 1. Comparing the absolute values is therefore not very useful. Figure 16 shows the relative changes when using different values of β. The absolute values are shown in Appendix, part G.

A higher inequality parameter β, so an index that is more sensitive to severe poverty, shows a relatively stronger decrease in poverty. This is positive, since it means that severe poverty has decreased. This result is in line with the findings regarding different cut-off values. Ireland seems to have been able to reduce poverty overall, and severe poverty especially, between the years 2003 and 2016. Using statistical weights leads to the same conclusion (figure 17).

Figure 16

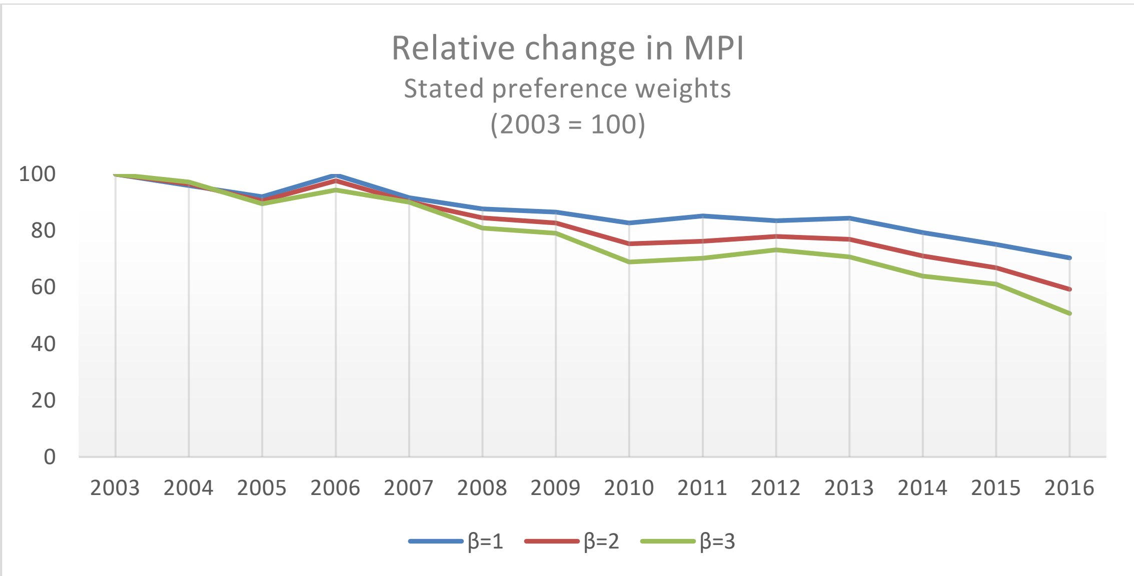
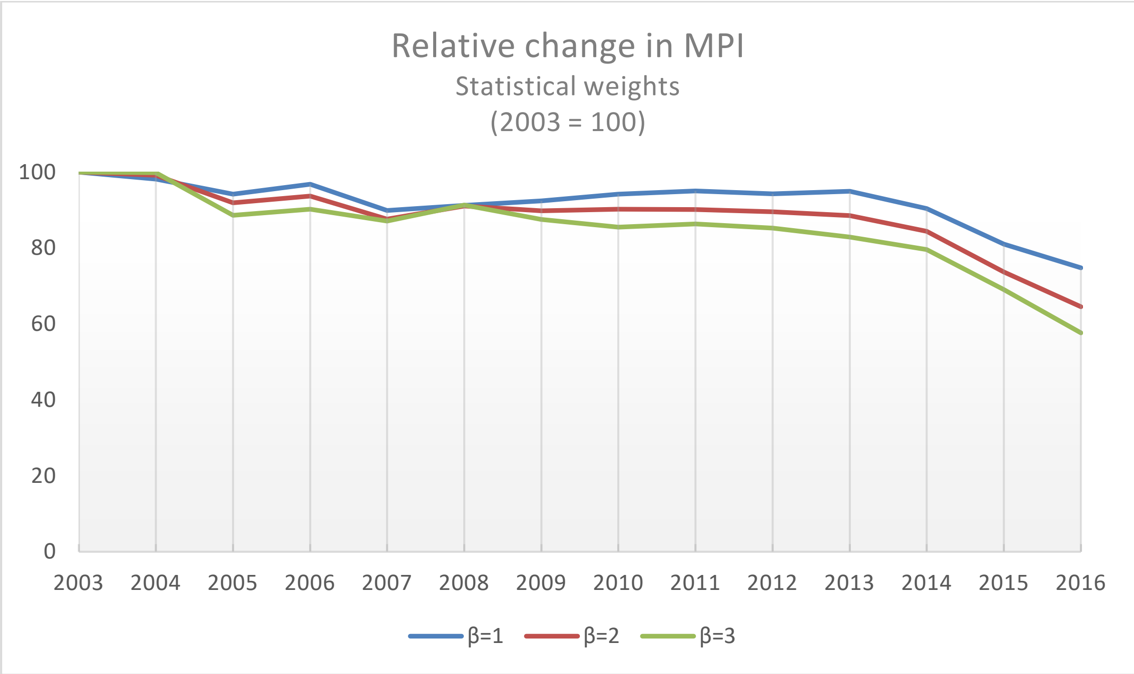


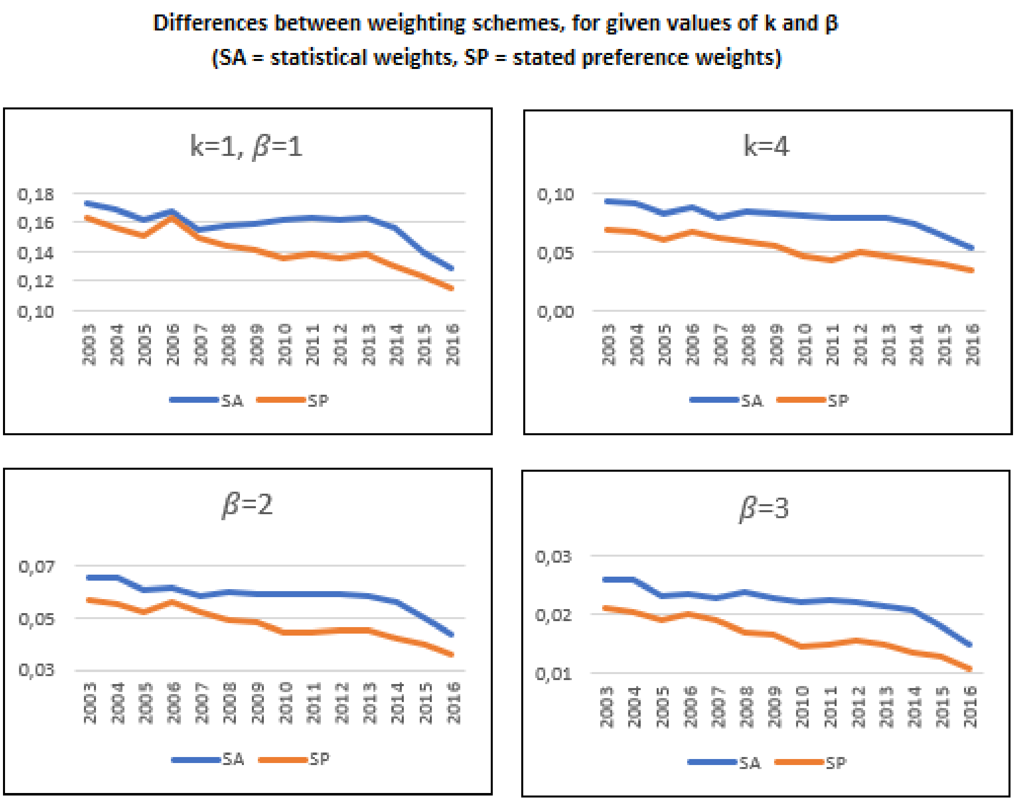
Figure 17



5.1.3 Findings regarding the weighting schemes

To compare the differences between using different weighting schemes, all other parameters should be the same. Figure 18 shows the difference for each value of poverty cut-off k and inequality parameter β. Figure 19 shows the relative differences.

Figure 18



Using stated preference weights always leads to lower poverty. This is not surprising, since stated preference weights assign a higher weight to the standard of living indicators than statistical weights, while these indicators generally show fewer deprivations within the Irish population. Using statistical weights, the other indicators carry more weight than in stated preference weights (education, health and social exclusion), while these indicators tend to show a higher number of deprivations, leading to a higher poverty value.

Using stated preference weights also leads to a higher *relative* decrease (figure 19). Statistical weights place a relatively high weight on education and social exclusion, whereas stated preference weights place a relatively high weight on education and standard of living. Looking at the descriptive statistics, the deprivation share of education has clearly decreased, together with most of the 'standard of living'-indicators, whereas the deprivation shares in the 'social exclusion'-dimension have increased. This explains the difference in the evolution of the poverty values between stated preference weights and statistical weights, where stated preference weights show a relatively steeper decline in poverty.

Figure 19

5.2 Robustness parameters

I have used two tests to assess robustness: Kendall's tau and Spearman's rho. Figure 20 shows the correlation values for each possible pair.

Even when comparing all possible sets of parameters with each other, most correlation values are very high (>0.7) and all are significant, often at a significance level of 1%, indicating that there is a strong correlation between the ranking of years for all different choices I made for the parameters. The lowest correlation values are those of the poverty indices for statistical weights with a cut-off value of 1 and an inequality parameter 1, compared to any of the other parameters. In statistical weights a lot of weight is given to education, which has a relatively high level of deprivation, and changing either the parameters or the weighting schemes reduces the effect of education, which might explain why statistical weights with both parameters set to 1 shows a lower correlation with the other indices. Nevertheless, even those correlation values are still significant at a significance level of 5%.

Since comparing poverty indices over time and between regions or subgroups is the main use of a poverty index, its ranking is of crucial importance. These results seem to suggest that it actually does not matter that much which parameter values are chosen. Whether poverty increased or decreased over time is mostly the same for all parameter values. Although the distribution-sensitive measure (equation 10) seems better theoretically, since it also satisfies the cross-dimensional convexity axiom, in contrast to the Alkire-Foster measure (equation 9), the ranking of years is very similar to those generated by the Alkire-Foster measure.

Figure 20

Kendall's tau (SP = stated preference weights, SA = statistical weights)										
Version	**SP, k = 1**	**SP, k = 4**	**SP, β = 2**	**SP, β = 3**	**SA, k = 1**	**SA, k = 4**	**SA, β = 2**	**SA, β = 3**	**Highest**	**Lowest**
SP, k=1		0.846**	0.934**	0.890**	0.560**	0.780**	0.714**	0.824**	**0.934**	**0.560**
SP, k=4			0.912**	0.956**	0.451*	0.758**	0.692**	0.758**	**0.956**	**0.451**
SP, β=2				0.956**	0.495*	0.758**	0.956**	0.802**	**0.956**	**0.495**
SP, β=3					0.495*	0.758**	0.692**	0.802**	**0.802**	**0.495**
SA, k=1						0.560**	0.670**	0.516**	**0.670**	**0.516**
SA, k=4							0.890**	0.912**	**0.912**	**0.890**
SA, β=2								0.802**	**0.802**	**0.802**
SA, β=3										
* = significant at 5%, ** = significant at 1%										

Spearman's rho (SP = stated preference weights, SA = statistical weights)										
Version	**SP, k = 1**	**SP, k = 4**	**SP, β = 2**	**SP, β = 3**	**SA, k = 1**	**SA, k = 4**	**SA, β = 2**	**SA, β = 3**	**Highest**	**Lowest**
SP, k=1		0.943**	0.982**	0.974**	0.697**	0.903**	0.851**	0.938**	**0.982**	**0.697**
SP, k=4			0.978**	0.987**	0.613*	0.859**	0.789**	0.899**	**0.987**	**0.613**
SP, β=2				0.991**	0.679*	0.886**	0.833**	0.921**	**0.991**	**0.679**
SP, β=3					0.662*	0.873**	0.807**	0.921**	**0.921**	**0.662**
SA, k=1						0.719**	0.815**	0.653*	**0.815**	**0.653**
SA, k=4							0.969**	0.978**	**0.978**	**0.969**
SA, β=2								0.916**	**0.916**	**0.916**
SA, β=3										
* = significant at 5%, ** = significant at 1%										

For completeness, I also calculated the correlation with the produced indices using equal weights. The exact coefficients are in Appendix, part H. Here, the correlation values are much lower and often unsignificant at a significance level of 5%. This shows that the weighting scheme is important. Even though comparing stated preference weights and statistical weights showed a significant correlation of results, this is not necessarily true when comparing with equal weights.

Overall, these results show that the poverty index is very robust to the choice in parameter values and that it happens to be robust to the choice between stated preference weights and statistical, but the index is not that robust to a different choice in weighting scheme in general.

As the results appear to be quite robust for the parameter choices, I will reduce the chosen values to allow for easier comparison and evaluation. Regarding inequality parameter β, I eliminate the value $\beta = 3$, since the added benefits of using a value of a parameter $\beta > 1$ are already reaped when using $\beta = 2$, and using an additional measure with $\beta = 3$ is therefore not necessary.

5.3 Statistical inferences

To get an approximation of the 95%-confidence interval, I used the bootstrap technique, where I drew 2,000 samples with replacement for each year, with sample size n, following Alkire and Santos (Alkire & Santos, 2014). Figures 21 and 22 show the results when using inequality parameter $\beta = 2$ for both stated preference and statistical weights.

Figure 21

95% confidence interval ($\beta = 2$, stated preference weights)			
Year	**Lower bound**	**MPI**	**Upper bound**
2003	0.0492	0.0519	0.0548
2004	0.0478	0.0502	0.0526
2005	0.0449	0.0470	0.0490
2006	0.0484	0.0507	0.0532
2007	0.0440	0.0469	0.0498
2008	0.0411	0.0439	0.0467
2009	0.0396	0.0429	0.0465
2010	0.0367	0.0392	0.0417
2011	0.0369	0.0396	0.0429
2012	0.0380	0.0405	0.0430
2013	0.0380	0.0399	0.0420
2014	0.0351	0.0369	0.0389
2015	0.0329	0.0347	0.0368
2016	0.0291	0.0308	0.0325
Confidence interval approximated with the bootstrap method, using 2000 samples			

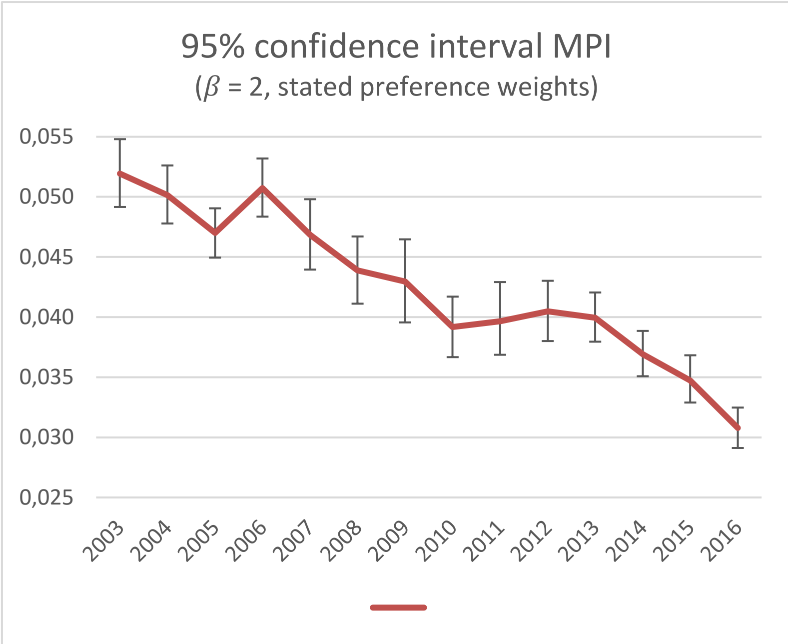
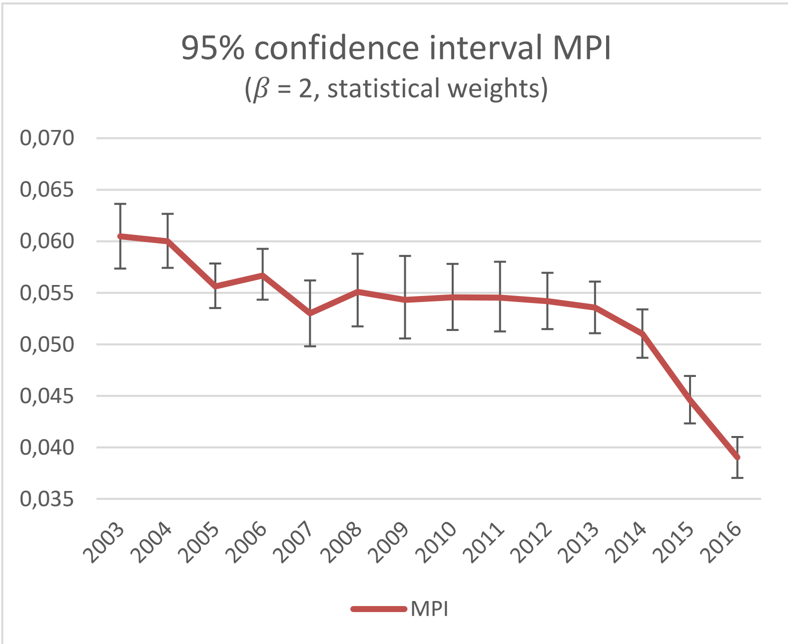


Figure 22

95% confidence interval ($\beta = 2$, statistical weights)			
Year	**Lower bound**	**MPI**	**Upper bound**
2003	0.0573	0.0605	0.0636
2004	0.0574	0.0600	0.0627
2005	0.0535	0.0556	0.0578
2006	0.0543	0.0567	0.0593
2007	0.0498	0.0530	0.0562
2008	0.0517	0.0551	0.0588
2009	0.0506	0.0543	0.0586
2010	0.0514	0.0545	0.0578
2011	0.0513	0.0545	0.0580
2012	0.0515	0.0542	0.0569
2013	0.0511	0.0536	0.0561
2014	0.0487	0.0510	0.0534
2015	0.0423	0.0446	0.0469
2016	0.0370	0.0390	0.0410
Confidence interval approximated with the bootstrap method, using 2000 samples			



The approximated 95%-confidence interval can be interpreted as such that if two confidence intervals do not overlap, their difference is statistically significant (Alkire & Santos, 2014). However, this does not necessarily mean that if they do overlap they are not statistically significantly different.

These results clearly show a significant decrease in poverty over time, for both weighting schemes. The decline when using stated preference weights is more consistent, with for example a significant decrease in poverty from 2003 to 2008, from 2008 to 2014, or from 2014 to 2016. This is consistent with the descriptive statistics, which show a gradual decline in deprivation for education and standard of living indicators, which carry a relatively high weight in stated preference weights.

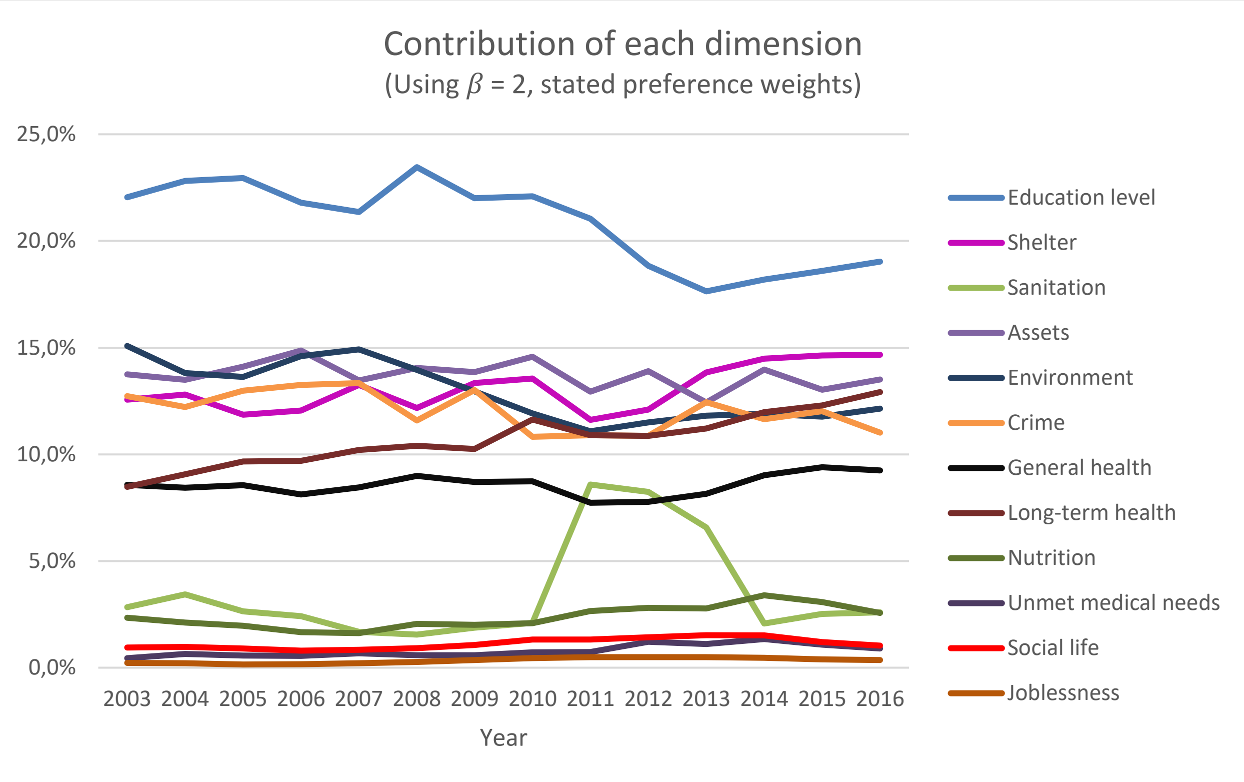
The decline when using statistical weights is more sudden, with a small significant decrease in poverty between the years 2003 and 2014, and a clear significant decrease in poverty from 2014 to 2015 and from 2015 to 2016. Statistical weights assign a relatively high weight to education and social exclusion. Over the years 2003 to 2014, education declined in deprivation levels, but social exclusion deprivation increased. Only after 2014 do both indicators show a decline in deprivation levels.

The other parameter values show a similar pattern, with in all cases a clearly significant decrease in poverty between 2003 and 2016. (Appendix, part I)

5.4 Decomposition

Decomposing leads to a percentual contribution of each indicator, adding up to a total of exactly 100% for each year. Appendix, part J, shows the full results for each parameter value. The percentages are fairly stable over time. An example of this is visible in figure 23, showing the results for an inequality parameter of $\beta = 2$ and stated preference weights. Most contributions remain roughly the same.

Figure 23



Noteworthy is the contribution of education level, which increases first, then decreases. This finding corresponds with the deprivation share of the education level indicator, as this decreases as well. The outlier in the contribution of the sanitation indicator also matches the descriptive statistics, which also show an outlier in those years.

The other parameter values and weighting schemes show a similar level of stability.

Since the contributions are fairly stable over time, for both stated preference and statistical weights, figures 24 and 25 show the average contribution for each indicator over the years 2003 to 2016, to allow for easier comparison.

Contribution compared to weight

A few noteworthy things can be seen in this table. First of all, education level has the highest contribution to poverty in all cases. Although it carries the highest weight for both weighting schemes, its contribution exceeds the given weight, often by large, indicating that it is a very important factor for the poverty index. The same is true for general and long-term health, which both have a contribution in both weighting schemes that clearly exceeds the weighting given to these indicators. From these results, education level, general health and long-term health can therefore be identified as the largest contributors to poverty in Ireland.

The finding that health is one of the most important contributors to poverty in Ireland is in line with the findings from Martinez and Perales in Australia (Martinez & Perales, 2015). However, in Ireland, health appears to be more of an issue than in Australia, whereas material deprivation is not as important as in Australia.

Quite unimportant are sanitation, nutrition and unmet medical needs as these indicators have a lower contribution than their assigned weight for both weighting schemes. Joblessness has a clearly lower contribution than its assigned weight when using statistical weights. With stated preference weights, it has a very low assigned weight, so it is possible that it has a relatively lower contribution there as well, but this cannot be seen, since the numbers are so small.

All other indicators have a contribution that is roughly equal to their given weight.

The results can be matched to the deprivation levels presented in the descriptive statistics, where education level, general health and long-term health show the highest deprivation levels and sanitation, nutrition, unmet medical needs and joblessness the lowest.

Overall, the differences in contribution when shifting parameter values are quite small and the ranking of the indicators seems to mostly remain the same. When changing the weighting scheme however, the contributions shift significantly.

Change in contribution when shifting parameters

Changing the cut-off value k or changing β does not shift the contributions much. The only notable change is in the contribution level of the education level indicator.

For both stated preference and statistical weights, the contribution of education is larger when using $\beta = 2$ than when using $k = 4$. This might indicate that for the more severely deprived, education level is less important compared to the other indicators, than for those with fewer deprivations. When increasing the inequality parameter β, the more severely deprived have a larger influence on the index, but everyone that is deprived in any indicator is counted. A fairly large share of the population is deprived in education level, so chances are high that for those that are only deprived in less than 4 indicators are deprived in education level. This would explain why education level is a more important contributor when also counting those that are only deprived in at least 1, but less than 4 indicators, than when only counting those that are deprived in 4 or more indicators. For the severely poor, being deprived in other indicators might be just as common as being deprived in education level.

Figure 24

Average contribution of each indicator, for differing parameter values, years 2003-2016								
Indiator	**Weight SP**	$\boldsymbol{k = 1, \beta = 1}$	$\boldsymbol{k = 4}$	$\boldsymbol{\beta = 2}$	$\boldsymbol{k = 1, \beta = 1}$	$\boldsymbol{k = 4}$	$\boldsymbol{\beta = 2}$	**Weight SA**
Education level	13%	22%	16%	21%	38%	38%	43%	25%
Shelter	12%	12%	14%	13%	4%	4%	4%	5%
Sanitation	12%	3%	4%	4%	1%	1%	1%	5%
Assets	12%	12%	15%	14%	4%	5%	4%	5%
Environment	12%	12%	13%	13%	4%	3%	3%	5%
Crime	12%	11%	13%	12%	4%	3%	3%	5%
General health	6%	9%	11%	9%	9%	12%	9%	6%
Long-term health	6%	13%	12%	11%	13%	14%	11%	6%
Nutrition	6%	2%	2%	2%	2%	2%	2%	6%
Unmet medical needs	6%	1%	1%	1%	1%	1%	1%	6%
Social life	1%	1%	0%	1%	14%	12%	14%	13%
Joblessness	1%	1%	0%	0%	5%	5%	5%	13%
SP = stated preference weights, SA = statistical weights								

Figure 25

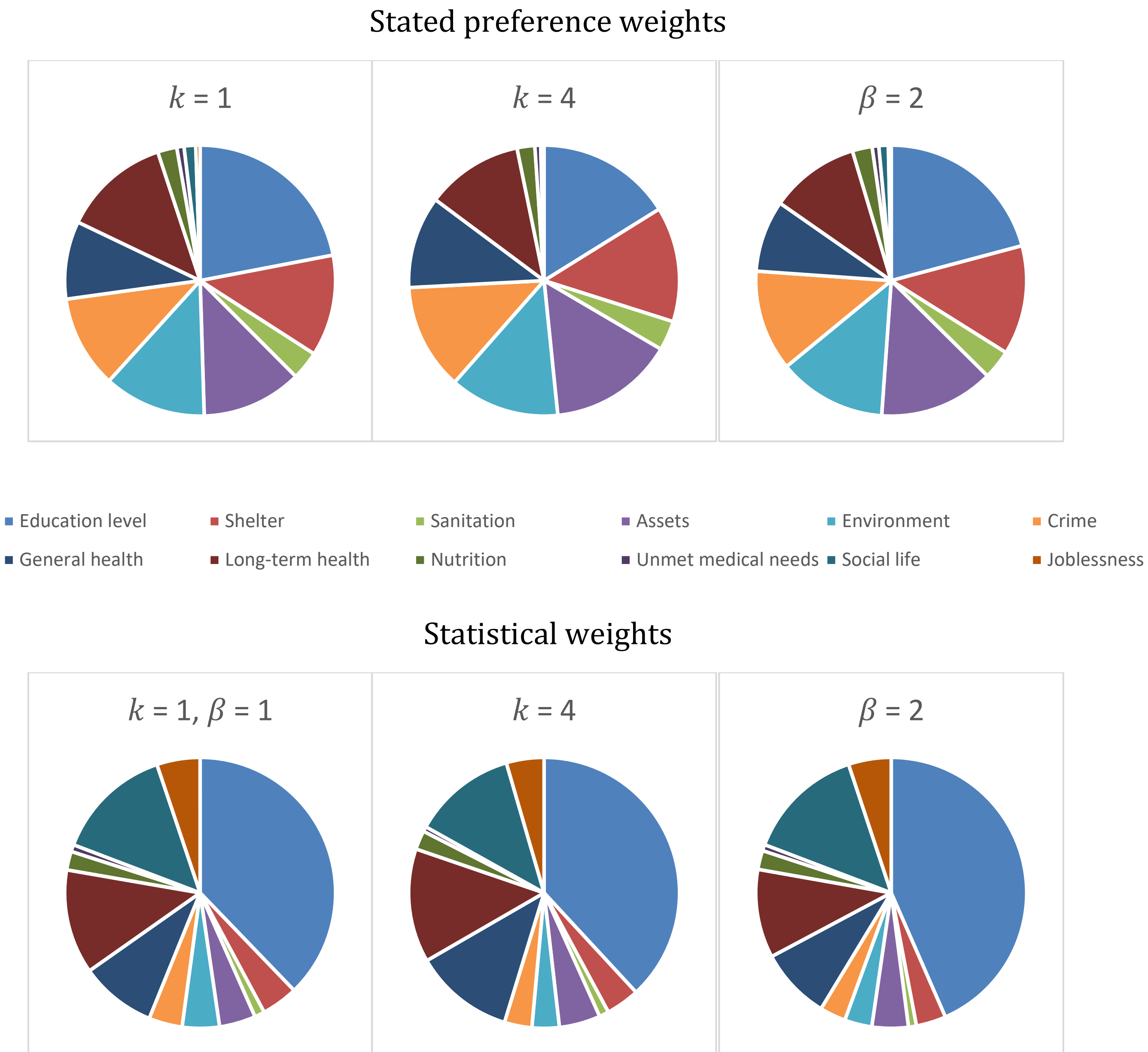

Stated preference weights
k = 1
k = 4
β = 2
Education level
Shelter
Sanitation
Assets
Environment
Crime
General health
Long-term health
Nutrition
Unmet medical needs
Social life
Joblessness
Statistical weights
k = 1, β = 1
k = 4
β = 2

Figure 26

Kendall's tau						
	$k = 1$, SP	$k = 4$, SP	$\beta = 2$, SP	$k = 1$, SA	$k = 4$, SA	$\beta = 2$, SA
$k = 1$, SP		.758**	.788**	0.394	0.333	0.303
$k = 4$, SP			.970**	0.152	0.273	0.242
$\beta = 2$, SP				0.182	0.303	0.273
$k = 1$, SA					.818**	.909**
$k = 4$, SA						.909**
$\beta = 2$, SA						
Same weighting scheme			Different weighting scheme			
*=significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Spearman's rho						
	$k = 1$, SP	$k = 4$, SP	$\beta = 2$, SP	$k = 1$, SA	$k = 4$, SA	$\beta = 2$, SA
$k = 1$, SP		0.895**	0.902**	0.406	0.462	0.378
$k = 4$, SP			0.993**	0.182	0.308	0.210
$\beta = 2$, SP				0.252	0.371	0.280
$k = 1$, SA					0.923**	0.972**
$k = 4$, SA						0.979**
$\beta = 2$, SA						
Same weighting scheme			Different weighting scheme			
*=significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Robustness

To compare the robustness for the different weighting schemes, I decided to use the same method for measuring robustness as I did for the poverty index. The decomposition of the poverty index is mostly aimed at optimizing policy, by finding out which indicators are most important for reducing poverty. Therefore, the ranking of the indicators is a crucial aspect of the decomposition. Using Kendall's tau and Spearman's rho, I compared the rankings for both the stated preference weights and the statistical weights to check for robustness in figure 26.

Both Kendall's tau and Spearman's rho show a significant correlation within the same weighting scheme, with high correlation coefficients, leading to a rejection of the null hypothesis that the rankings are independent. Using a different weighting scheme leads to an insignificant result however, with quite low coefficients. The null hypothesis cannot be rejected, meaning that there is a high chance that the rankings are independent.

This shows that the decomposition values are largely dependent on which weighting scheme is used and are not robust for changes in weighting, contrary to the poverty index itself. It raises the question of how useful decomposition can then be? Conclusions about which indicators present the highest deprivation shares can be drawn already from just the descriptive statistics. Decomposition has the added benefit of ranking the indicators on their contribution, by showing not only the deprivation shares, but also their relative importance in the calculation of the (weighted) poverty index. However, the decomposition values turn out to be largely influenced by the weighting scheme. This means that decomposition is only useful when you are fairly certain that the weights are correct. However, the choice for a weighting scheme is often made without a proper scientific basis and not necessarily an accurate representation of what people value most in life. As explained more thoroughly in the theoretical framework, stated preference weights can be rather different from statistical weights or equal weights, which are the most common choices for a weighting scheme when measuring multidimensional poverty. Furthermore, data on stated preferences is still scarce. So even though the Shapley decomposition solves the main mathematical problems that were existent in previous decomposition methods, the results still seem to not be very useful, as there is little proper data for accurate weights so far, and the decomposition outcomes are largely dependent on this.

6 Subgroup results

As previous empirical work showed significant differences in poverty when dividing the population into subgroups, I applied the same method as before to three different population groups: households that fall in the category 'rural farm', 'rural non-farm' and 'urban'.

6.1 Multidimensional Poverty Index (MPI) findings

To be able to accurately compare the findings for the poverty index, I immediately present the poverty values with their 95%-confidence intervals in figures 26 and 27 and Appendix, part K (calculated using the bootstrap method with 2,000 samples).

All weighting schemes and parameter values exhibit equalization in terms of poverty between the subgroups over time, with a significant decrease in poverty for urban and rural non-farm households. Poverty for rural farm households is quite volatile with a relatively broad confidence interval, due to the low number of rural farm households in the sample. There is no clear trend of the poverty index and, using the confidence interval overlap criterium, no significant increase or decrease is visible.

For urban and rural non-farm households, the results differ per weighting scheme.

Stated preference weights
In 2006, urban households demonstrate the highest poverty index, followed by rural non-farm households, followed by farm households. Urban poverty and rural non-farm poverty both decrease significantly, leaving rural farm and rural non-farm households to be equally poor in 2016, both with a significantly lower level of poverty than urban households. Overall, the difference between the subgroups has decreased, indicating that equality between the subgroups improved compared to 2006.

All parameter values ($k = 1$ with $\beta = 1, k = 4$, and $\beta = 2$) lead to the same conclusions, although rural non-farm poverty only overlaps with urban poverty (until 2010) when using $k = 1$ and $\beta = 1$. A possible explanation for this difference is that there might be relatively fewer strongly deprived rural non-farm households than urban, leading to a relatively lower poverty value for rural non-farm households when using a cut-off value of $k = 4$ or an inequality parameter of $\beta = 2$.

Statistical weights
With statistical weights, the poverty ranking changes. In 2006, rural non-farm households exhibit the highest poverty, followed by urban households, followed by rural farm households. Both rural non-farm and urban poverty decrease significantly, with an especially steep decline for rural non-farm households, finally leading to an overlap in all confidence intervals in 2016. The conclusions are the same for all parameter values.

Figure 27

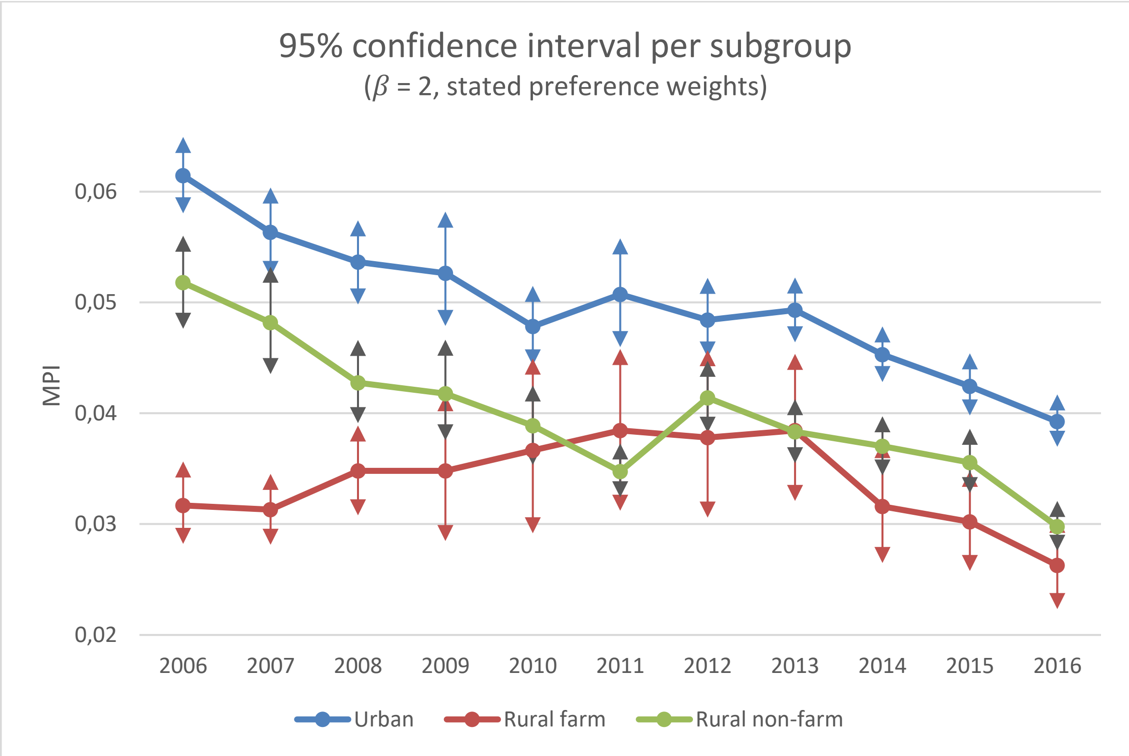
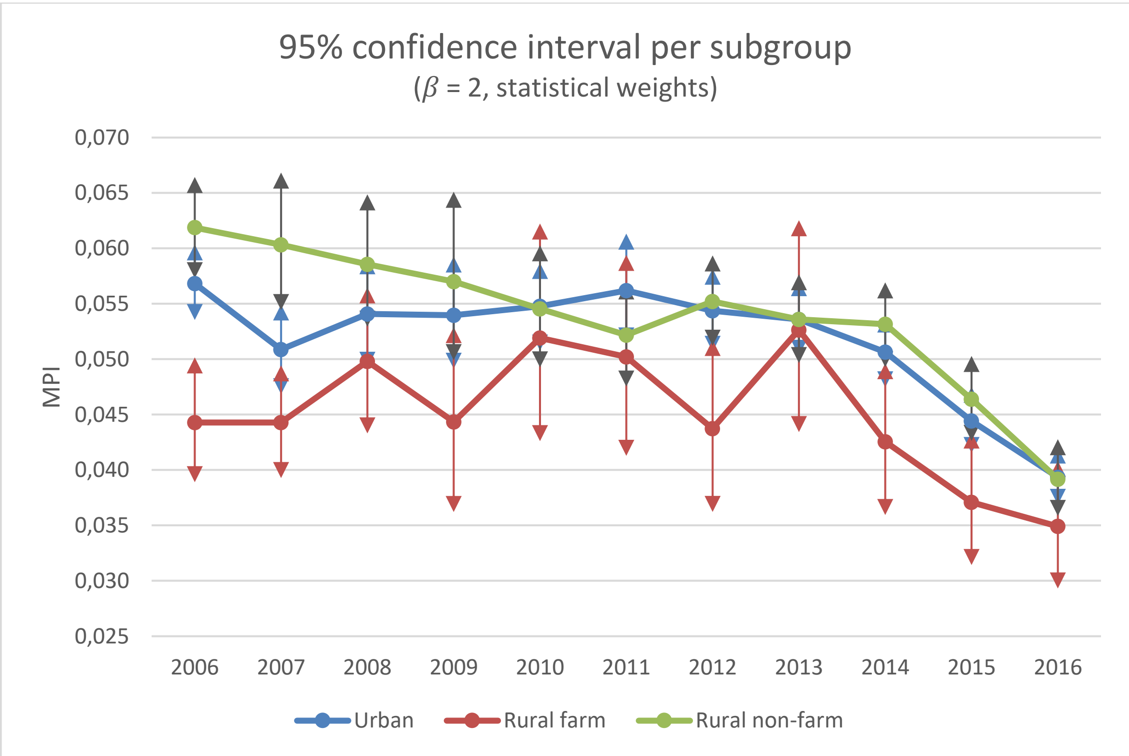


Figure 28



Difference between weighting schemes

Urban poverty is higher than rural non-farm poverty when using stated preference weights, while rural non-farm poverty is higher than urban poverty when using statistical weights. For both weighting schemes, rural non-farm poverty declines especially fast. A possible explanation could be that urban households are more deprived in the standard of living indicators, which carry a higher weight in the stated preference weighting scheme. For example, crime and environmental problems are more likely to be an issue for urban households than for rural households. Another likely explanation could be that rural households are more deprived in education level than urban households, as education level exhibits a clear decline in the descriptive statistics and it is likely that the education level of urban households is higher on average than that of rural households.

Examination of the deprivation levels for each subgroup confirms both these thoughts (Appendix, part L).

6.2 Decomposition

Figures 29 to 31 (and Appendix, part M) show the average decomposition results per subgroup. Similar to the nationwide decomposition results in figures 24 and 25, the indicators with a higher contribution than their weight are education level, general health and long-term health, for all subgroups and weighting schemes. The indicators with a lower contribution than their weight are again sanitation, nutrition and unmet medical needs (and joblessness when using statistical weights, similar to the nationwide decomposition).

Decomposition differences between subgroups

The main differences between the subgroups are that the contributions of assets, environment and crime are lower for rural households than for urban households. Urban households show a relatively lower contribution for education, although still well above the assigned weight. Most of these findings are not unsurprising, it makes sense that urban households experience more often deprivations in environment or crime. A larger contribution for education level for rural households is also not a surprise, as rural households are often less educated on average than urban households.

Since in the stated preferences weighting scheme, the social indicators have a very low weight assigned, it is difficult to see differences in that aspect. However, in the statistical weighting scheme, differences are visible. Joblessness has a very low contribution to poverty of rural farm households, compared to the contribution to poverty in urban and rural non-farm households. This makes sense, as having a farm household generally implies that that person has a job as a farmer. Social life also has a clearly lower contribution to poverty for rural farm households than for urban and rural non-farm households. This is interesting. Perhaps social activities are more expensive in urban areas. However, this does not explain the difference between rural farm and rural non-farm households. It seems that the social lifes or farm and non-farm households are different, which might for example be because of a stronger need to have hobbies as a rural non-farmer or a preference for more expensive hobbies, but other explanations are possible too.

Overall, rural non-farm and rural farm households have quite similar decomposition results, indicating that they are generally deprived in the same indicators. Urban households show a clearly different distribution.

Figure 29

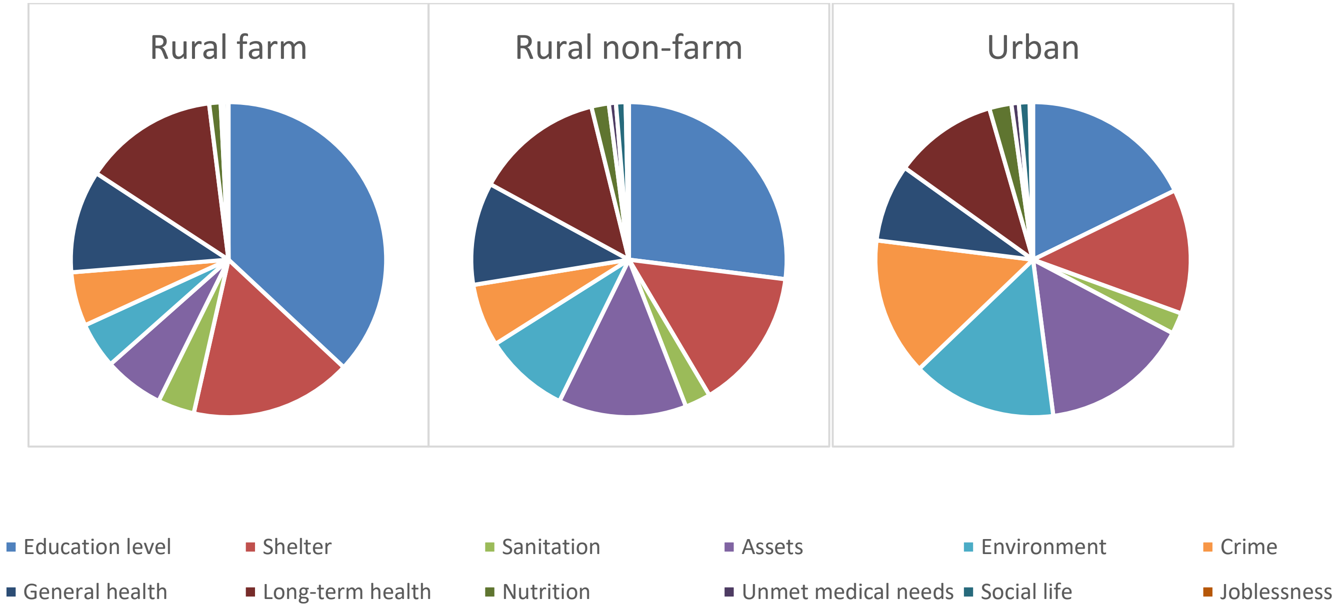
Average contribution of each indicator, per subgroup, for 2003-2016				
Indicator	**Weight**	**Rural farm**	**Rural non-farm**	**Urban**
Education level	13%	37%	27%	18%
Shelter	12%	17%	15%	13%
Sanitation	12%	4%	3%	2%
Assets	12%	6%	13%	15%
Environment	12%	5%	9%	15%
Crime	12%	6%	6%	14%
General health	6%	11%	11%	8%
Long-term health	6%	14%	13%	11%
Nutrition	6%	1%	2%	2%
Unmet medical	6%	0%	1%	1%
Social life	1%	0%	1%	1%
Joblessness	1%	0%	0%	0%
Parameters: $\beta = 2$, stated preference weights				

Figure 30

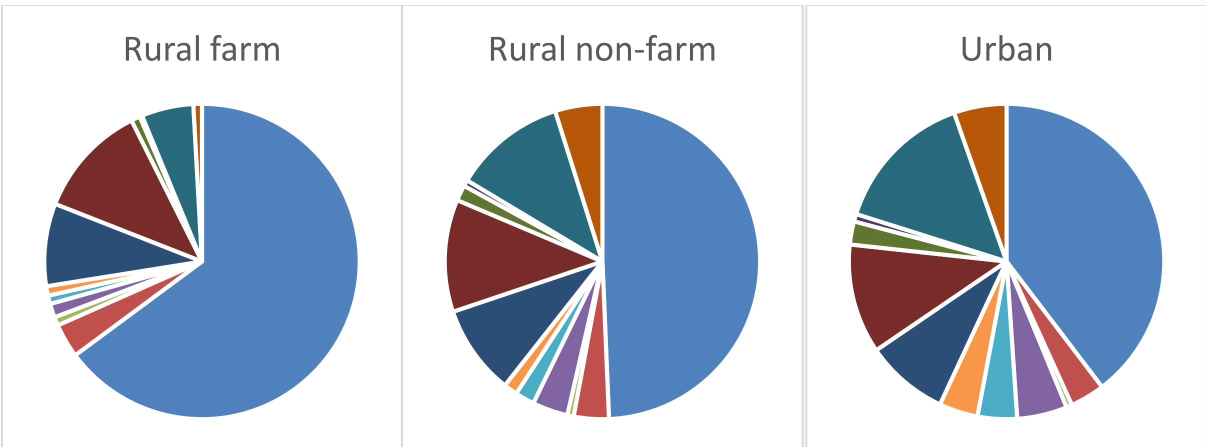
Average contribution of each indicator, per subgroup, for 2003-2016				
Indicator	**Weight**	**Rural farm**	**Rural non-farm**	**Urban**
Education level	25%	65%	49%	40%
Shelter	5%	4%	4%	4%
Sanitation	5%	1%	1%	1%
Assets	5%	1%	4%	5%
Environment	5%	1%	2%	4%
Crime	5%	1%	1%	4%
General health	6%	8%	9%	9%
Long-term health	6%	12%	12%	11%
Nutrition	6%	1%	2%	2%
Unmet medical	6%	0%	1%	1%
Social life	13%	5%	12%	15%
Joblessness	13%	1%	5%	5%
Parameters: $\beta = 2$, statistical weights				

Figure 31

Stated preference weights
$(\beta = 2)$



Statistical weights
$(\beta = 2)$



6.3 Robustness

Again, I calculated the rank correlation coefficients to compare the effect of a different weighting scheme and correlation between subgroups. The results are presented in figure 32 and Appendix, part N. Especially between rural non-farm and farm households the correlation is very high and significant. Between urban and rural households, the correlation is also significant and fairly high (often >0.7) when using stated preference weights, indicating that the decomposition ranking is quite robust to changes in subgroup. For statistical weights however, the rankings of urban and rural households are not significantly correlated. Whether there is a significant difference between these types of households, and if policy should be different per subgroup, is therefore not yet clear. More accurate information on the weighting schemes is necessary.

Changes in weighting schemes lower the ranking coefficients and lead in some cases to no significant correlation. It seems that for rural households, the decomposition ranking is somewhat robust to a change in weighting scheme (although the correlation coefficients are not very high) whereas for urban households the rankings are usually not statistically significantly correlated. Perhaps this is because for rural households, the deprivations are more concentrated in a few indicators, leading to a more similar decomposition for different weighting schemes, whereas for urban households the deprivations are more evenly distributed across indicators.

Overall this confirms the finding from before, that a decomposition ranking has only limited value, as it is not robust to weighting choices in many cases.

Figure 32

Kendall's tau ($\beta = 2$)						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		.667**	.939**	.606*		
Urban - SP			.727**		0.273	
Rural non-farm - SP						.364
Rural farm - SA					.576**	.636**
Urban - SA						.879**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Spearman's rho ($\beta = 2$)						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		.797**	.986**	.748**		
Urban - SP			.839**		0.273	
Rural non-farm - SP						0.420
Rural farm - SA					0.727**	0.797**
Urban - SA						0.958**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

7 Conclusion

The central purpose of this paper has been to apply the most recent developments in multidimensional poverty measurement to Ireland, a developed country, using the EU-SILC data, and to test for the robustness of these new methods. The general dimensions used are education, standard of living, health and social exclusion (based on the framework from Alkire et al. and the poverty reduction goals of the EU).

In 2017, Datt proposed an improved version of the widely-used multidimensional poverty index from Alkire et al., by adding an inequality parameter β, that has the added quality of being sensitive to the distribution of deprivations. The underlying idea is that if two poor people trade part of their income, making one of them less poor and one of them more poor (but both still poor), the value of the poverty index should increase. I applied this poverty measure to Ireland and compared the results with those of the 'traditional' poverty measure from Alkire et al., that uses a dimensional cut-off. Although Datt's distribution-sensitive measure is theoretically better, the index did not produce significantly different results from the other poverty measures. The measure appears to be very robust to changes in the dimensional cut-off or in the inequality parameter β, with very high rank correlation coefficients between all measures. The measure also appeared to be quite robust to a change in weighting scheme, although not as strongly as to changes in parameter values.

Regarding the actual results, Ireland is on the right track in reducing poverty. All results pointed towards a significant decrease in poverty over the years 2003 to 2016 for Ireland. Additionally, severe poverty also decreased significantly, even more than total poverty (in relative terms).

I also applied a fairly new decomposition method to the poverty indices, as proposed by Shorrocks in 2013. This decomposition method has already been applied to a few developing countries since, as it overcomes a couple of technical problems previous decomposition methods had. With this method, the contributions of the different indicators can be identified. For Ireland, the main contributors to poverty are education level, general health and long-term health. However, applying the same robustness tests to the decomposition results, I found that the results are very sensitive to the choice in weighting scheme. Where the poverty index was fairly robust to a switch between stated preference and statistical weights, the decomposition is not, as the rank correlation coefficients were insignificant for a change in weights. This means that changing the weighting schemes usually leads to a quite different order of indicators when sorted by contribution. Since the choice for a weighting scheme is one of the more difficult choices in multidimensional poverty and, at least for now, hard to support scientifically, the decomposition results are not that useful in my opinion. To find out which indicators show the highest deprivation rates, decomposition is not necessary as simply calculating what share of the population is deprived in each indicator will provide the same information. Decomposition is useful to find a ranking of indicators, but since this is very dependent on the weighting scheme, it does not have much use if the weighting scheme is not accurate. All in all, I think the method from Shorrocks has great potential, but as long as the research on weighting schemes is as scarce as it is today, it does not yet have much use.

As previous research found some significant differences when separating subgroups, I also applied the same methodology to urban, rural farm and rural non-farm households. I found that in 2006, there were significant differences in poverty between these types of households, but equalized over time, due to a decrease in poverty for the poorer households. Applying decomposition to the subgroups showed that for rural farm and rural non-farm households, the decomposition was very similar, and quite robust for changes in weighting scheme. I suspect this is because the rural households demonstrated a few clear deprivations in some indicators, whereas this was more evenly distributed over all indicators for urban households (where a change in weighting scheme led to a significant change in indicator ranking). Robustness checks showed a significantly dependent ranking for urban and rural households when using stated preference weights, but an insignificant result for statistical weights. Separating by subgroup (and adjusting policy accordingly) can thus be useful in certain cases, depending on country and weights.

This analysis shows where I believe the main opportunities for improvement are for multidimensional poverty measurement. More data and research on (stated preference) weighting is necessary for developing multidimensional poverty measurement further. Getting more insight in what people actually think is important and what constitutes poverty is a critical element of being able to accurately measure poverty. Furthermore, data overall could be improved. The EU-SILC is a very good start, as it gives the opportunity to compare different EU countries and contains some fairly good data for poverty measuring, but there is still improvement possible, both in the type of questions asked and the scale of the survey. For example, more demographic questions could be implemented, so that a more specific poverty-reducing policy can be designed for each subgroup. Another possibility is to extend the survey to non-EU countries. For now it seems that the methodology is there, it is the data availability that is the weak point and the main roadblock in measuring multidimensional poverty accurately.

8 Appendix

A. Other indices (2.8)

Other well-known poverty measuring methods are the measures by Tsui, Bourguignon & Chakravarty ('BC') and the Human Development Index ('HDI'). They all have its pros and cons. Duclos and Tiberti have examined all axioms and all measures mentioned above and concluded that for multidimensional measures specifically, the most important axioms are the monotonicity and the dimensional monotonicity axiom (Duclos & Tiberti, 2016). Of the three measures discussed here the HDI does not satisfy dimensional monotonicity and is therefore not the best option. Tsui and BC do satisfy dimensional monotonicity and are considered good options by Duclos and Tiberti (Duclos & Tiberti, 2016). However, they both do not satisfy cross-dimensional convexity. This axiom was added by Datt and is a reasonable axiom to add to the requirements for a poverty measure (Datt, 2017). Since both Tsui's and BC's measures do not satisfy this axiom, Datt's distribution-sensitive measure appears to be the best option.

B. Data (3.3)

Shelter
To be considered deprived, respondent answers the yes/no-question "Does your dwelling have a leaking roof, damp walls etc. or rot in the doors, window frames or floor?" with "Yes".

Sanitation
To be considered deprived, respondent answers one the following yes/no-questions with "No": "Does your housing have a bath or shower?", "Does your housing have hot water?" or "Does your housing have running water?".

Environment
To be considered deprived, respondent answers at least one of the following yes/no-questions with "Yes": "Is there pollution or grime or are there other environmental problems in your area?" and "Is there noise from neighbors on the street?"

Crime
To be considered deprived, respondent answers the yes/no-question "Is there crime, violence or vandalism in your area?" with "Yes".

Long-term health
To be considered deprived, respondent answers the yes/no-question "Do you suffer from a chronic illness or condition?" with "Yes".

Nutrition
To be considered deprived, respondent answers the yes/no-question “Did you have a day in last fortnight where you did not have a substantial meal due to a lack of money?” with “Yes”.

Unmet medical needs
To be considered deprived, respondent says to have needed a medical examination or treatment in the last twelve months, but did not receive it. Only those respondents are considered deprived that give a reason that might be related to poverty. Those reasons are: Could not afford it, could not take time off work or having no means of transport. Reasons that are not considered are: Waiting list, fear, or wanting to wait it out to see if the problems fixed itself, as I do not consider these to be related to poverty.

Social life
To be considered deprived, respondent says to not have hobbies or leisure activities because it cannot afford these or respondent says to not have had a morning, afternoon or evening out in the last fortnight for own entertainment because it cannot afford these. I specifically chose only the answers where the respondent mentioned not being able to afford these things, to filter out those that did not do any social activities due to their own preferences.

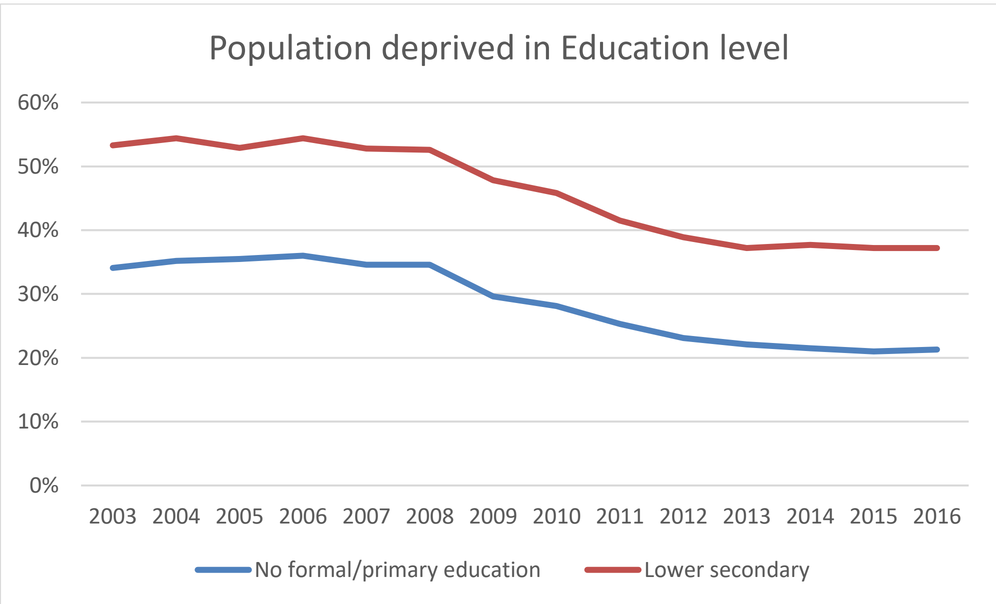
Joblessness
To be considered deprived, a respondent considers its principal economic status “Unemployed”. Ideally, also the respondents with a job with a very low number of hours (<20% of total work capacity) are considered, but this question is only added in the later years of the data, so I do not use this. Other possible principal economic statuses, which do not lead to deprivation, are “At work”, “Student”, “Home duties”, “Retired”, “Ill/disabled” or “Other inactive person”.

C. Robustness making ends meet (3.3)

Education level

The possible answers in the survey for education level are: no formal/primary education, lower secondary, upper secondary, post leaving cert, third level non degree and third level degree or above. Since it would not make sense to say that people that have finished upper secondary are deprived in their education level, there are two possible poverty lines: a highest education level of 'no formal/primary education' or of 'lower secondary'. In this section, I check for robustness of these poverty lines. The averages are as follows:

Percentage of the population deprived in Education level for each poverty line		
	No formal/primary	Lower secondary
2003	34%	53%
2004	35%	54%
2005	36%	53%
2006	36%	54%
2007	35%	53%
2008	35%	53%
2009	30%	48%
2010	28%	46%
2011	25%	42%
2012	23%	39%
2013	22%	37%
2014	22%	38%
2015	21%	37%
2016	21%	37%



The difference between the lines is fairly large, but it does follow the same trend. However, based on this information, it is not possible to say if the measure is robust for a choice between these poverty lines. Therefore, I check the rank correlation coefficient of the computed MPI's, just like in the rest of this paper. The figures below show the rank correlation coefficients of the different parameter combinations:

Rank correlation coefficient of MPI using different poverty lines		
	Stated preference weights	
	Kendall's tau	**Spearman's rho**
$k = 1, \beta = 1$	0.93**	0.98**
$k = 4, \beta = 1$	0.98**	1.00**
$k = 1, \beta = 2$	0.98**	1.00**
$k = 1, \beta = 3$	0.96**	0.99**
Comparing the poverty lines of 'no formal/primary education' and 'lower secondary education' **significant at a 1% level		

Rank correlation coefficient of MPI using different poverty lines		
	Stated preference weights	
	Kendall's tau	**Spearman's rho**
$k = 1, \beta = 1$	0.65**	0.77**
$k = 4, \beta = 1$	0.76**	0.91**
$k = 1, \beta = 2$	0.87**	0.94**
$k = 1, \beta = 3$	0.65**	0.77**
Comparing the poverty lines of 'no formal/primary education' and 'lower secondary education' **significant at a 1% level		

All coefficients are significant at a 1% significance level and have fairly high absolute values. Therefore, I conclude that the measure is robust to the choice in education level. As a result, I choose the poverty line that according to me makes most sense: if someone has not received education for the amount of time required by law, they are deprived in the 'education level'-indicator. This corresponds to a poverty line of 'no formal/primary education'.

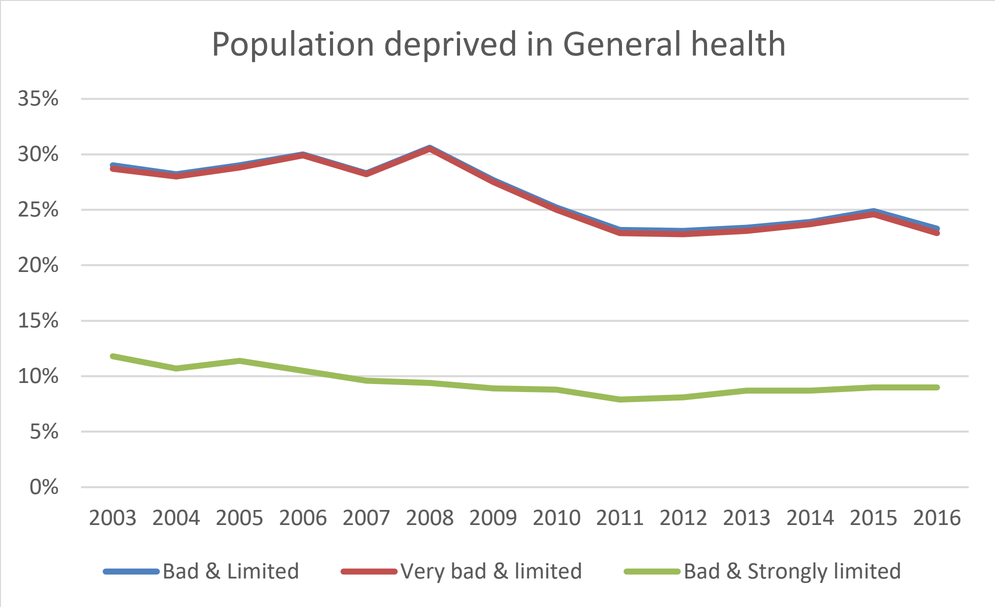
General health

For general health, I calculated the same statistics as for education level. There are five possible answers regarding health status: very bad, bad, fair, good, and very good. The most logical choice in my opinion would be to set the poverty line at a reported health status of ‘bad’ or ‘very bad’. However, perhaps only choosing ‘very bad’ would be better.

The ‘General health’-indicator is also determined by a possible limitation in activities due to a health problem. The possible choices are: strongly limited, limited, and not limited. In my opinion, the logical choice for the poverty line would be ‘limited’ and ‘strongly limited’, but again, perhaps only ‘strongly limited’ would be better.

My base choice is a poverty line of a ‘bad’ health status and/or ‘limited’ in activities due to a health problem. In this section, I check for robustness on changing the health status poverty line to ‘very bad’ and on changing the activities poverty line to ‘strongly limited’. This leads to the following results:

Percentage of the population deprived in General health for each poverty line			
	Very bad & limited	Bad & limited	Bad & strongly limited
2003	29%	29%	12%
2004	28%	28%	11%
2005	29%	29%	11%
2006	30%	30%	11%
2007	28%	28%	10%
2008	31%	31%	9%
2009	28%	28%	9%
2010	25%	25%	9%
2011	23%	23%	8%
2012	23%	23%	8%
2013	23%	23%	9%
2014	24%	24%	9%
2015	25%	25%	9%
2016	23%	23%	9%



Apparently, changing the health status poverty line barely changes the number of deprived. It seems that a large share of the deprived in ‘General health’ are deprived due to being limited in their activities due to a health problem. The rank correlation coefficient of comparing ‘very bad’ and ‘bad’ is therefore not that interesting. The change in poverty line for the activity limitation is, however. The results are shown in the figures below:

Rank correlation coefficient of MPI using different poverty lines						
	Stated preference weights					
	Kendall’s tau			**Spearman’s rho**		
	VB & B	B & SL	VB & SL	VB & B	B & SL	VB & SL
$k = 1, \beta = 1$	1.00**	0.89**	1.00**	1.00**	1.00**	1.00**
$k = 4, \beta = 1$	0.98**	0.85**	0.96**	1.00**	1.00**	0.99**
$k = 1, \beta = 2$	1.00**	0.93**	1.00**	1.00**	1.00**	1.00**
$k = 1, \beta = 3$	1.00**	0.96**	0.98**	1.00**	1.00**	1.00**

VB = poverty line of ‘Very bad & limited’; B = poverty line of ‘Bad & Limited’; SL = poverty line of ‘Bad & Strongly limited’

**significant at a 1% level

Rank correlation coefficient of MPI using different poverty lines						
	Stated preference weights					
	Kendall’s tau			**Spearman’s rho**		
	VB & B	B & SL	VB & SL	VB & B	B & SL	VB & SL
$k = 1, \beta = 1$	1.00**	0.93**	0.93**	1.00**	0.99**	0.99**
$k = 4, \beta = 1$	0.98**	0.80**	0.78**	1.00**	0.90**	0.90**
$k = 1, \beta = 2$	0.98**	0.82**	0.85**	1.00**	0.92**	0.93**
$k = 1, \beta = 3$	1.00**	0.91**	0.91**	1.00**	0.98**	0.98**

VB = poverty line of ‘Very bad & limited’; B = poverty line of ‘Bad & Limited’; SL = poverty line of ‘Bad & Strongly limited’

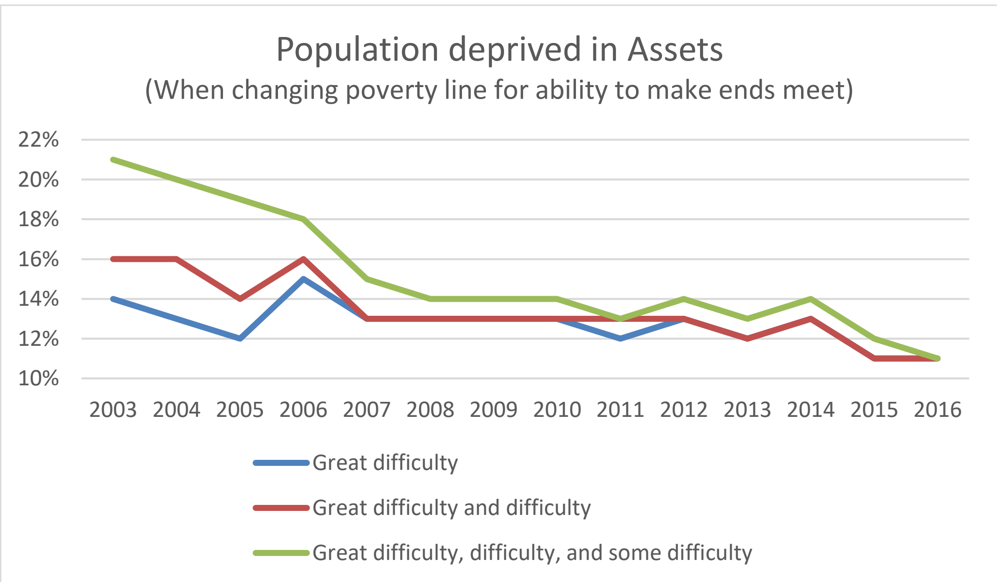
**significant at a 1% level

All coefficients are significant at a 1% significance level and have very high absolute values. Therefore, I conclude that the measure is robust to the choice in both health status and limitation in activities. As a result, I choose the poverty line that according to me makes most sense: someone is deprived if it reports a health status of ‘bad’ or ‘very bad’ and/or is ‘limited’ or ‘strongly limited’ in activities due to a health problem.

Making ends meet

The possible answers in the survey for ability in making ends meet are: with great difficulty, with difficulty, with some difficulty, fairly easily, easily and very easily. This means there are three possible poverty lines for the assets sub indicator ‘making ends meet’: only those with ‘great difficulty’ are deprived, or the deprived include those that experience ‘difficulty’, or also those with ‘some difficulty’. I check for robustness in this section. The averages are as follows:

Percentage of the population deprived in Assets for each poverty line			
	Great difficulty	Difficulty	Some difficulty
2003	14%	16%	21%
2004	13%	16%	20%
2005	12%	14%	19%
2006	15%	16%	18%
2007	13%	13%	15%
2008	13%	13%	14%
2009	13%	13%	14%
2010	13%	13%	14%
2011	12%	13%	13%
2012	13%	13%	14%
2013	12%	12%	13%
2014	13%	13%	14%
2015	11%	11%	12%
2016	11%	11%	11%



The differences between the deprivation levels are somewhat large in the first few years, but converge over time, until they are almost equal after 2011.

As for robustness of the poverty measure, I used the same method as in the rest of paper, namely Kendall's tau and Spearman's rho. The following graphs show the rank correlation coefficient for each parameter combination.

Rank correlation coefficient of MPI using different poverty lines						
	Stated preference weights					
	Kendall's tau			**Spearman's rho**		
	GD & D	D & SD	GD & SD	GD & D	D & SD	GD & SD
$k=1, \beta=1$	0.96**	0.98**	0.93**	0.99**	1.00**	0.98**
$k=4, \beta=1$	0.93**	0.96**	0.93**	0.98**	0.99**	0.98**
$k=1, \beta=2$	0.98**	0.98**	0.96**	1.00**	1.00**	0.99**
$k=1, \beta=3$	1.00**	0.96**	0.96**	1.00**	0.99**	0.99**
GD = poverty line of 'Great difficulty'; D = poverty line of 'Difficulty'; SD = poverty line of 'Some difficulty' **significant at a 1% level						

Rank correlation coefficient of MPI using different poverty lines						
	Statistical weights					
	Kendall's tau			**Spearman's rho**		
	GD & D	D & SD	GD & SD	GD & D	D & SD	GD & SD
$k=1, \beta=1$	0.98**	0.91**	0.89**	1.00**	0.97**	0.95**
$k=4, \beta=1$	1.00**	0.98**	0.98**	1.00**	1.00**	1.00**
$k=1, \beta=2$	1.00**	1.00**	1.00**	1.00**	1.00**	1.00**
$k=1, \beta=3$	0.98**	1.00**	0.98**	1.00**	1.00**	1.00**
GD = poverty line of 'Great difficulty'; D = poverty line of 'Difficulty'; SD = poverty line of 'Some difficulty' **significant at a 1% level						

All values are significant at a 1% significance level. As these values are all very high (>0.93), the index is very robust to the choice in poverty line for the ability to make ends meet. I therefore choose to use the poverty line that seems best to me, which is that someone is deprived if he/she has 'great difficulty' or 'difficulty' making ends meet.

D. Descriptive statistics (3.4)

Year	Education level	Shelter	Sanitation	Assets	Environment	Crime	General health
2003	30%	15%	3%	16%	20%	16%	27%
2004	31%	15%	4%	15%	17%	14%	25%
2005	29%	13%	3%	15%	16%	14%	25%
2006	29%	15%	3%	18%	19%	16%	25%
2007	27%	15%	2%	15%	18%	15%	23%
2008	27%	13%	2%	14%	16%	13%	24%
2009	25%	14%	2%	14%	14%	14%	24%
2010	24%	13%	2%	14%	13%	10%	22%
2011	22%	12%	12%	13%	11%	11%	20%
2012	20%	12%	9%	13%	12%	11%	20%
2013	19%	14%	8%	12%	12%	12%	21%
2014	19%	15%	2%	13%	11%	11%	22%
2015	18%	14%	2%	11%	10%	11%	23%
2016	17%	13%	2%	10%	10%	9%	21%

Year	Long-term health	Nutrition	Unmet medical needs	Social life	Joblessness	n
2003	28%	8%	1%	14%	4%	3,090
2004	28%	5%	2%	14%	4%	5,477
2005	30%	5%	2%	13%	3%	6,085
2006	32%	4%	2%	12%	3%	5,836
2007	31%	4%	2%	12%	4%	5,607
2008	30%	5%	1%	13%	5%	5,246
2009	31%	5%	1%	17%	8%	5,183
2010	34%	5%	2%	21%	9%	4,642
2011	32%	6%	2%	23%	10%	4,333
2012	32%	6%	3%	25%	10%	4,562
2013	33%	6%	3%	27%	10%	4,922
2014	33%	7%	3%	25%	9%	5,484
2015	33%	6%	3%	18%	7%	5,443
2016	33%	5%	2%	14%	6%	5,218

E. Derivation Kendall's tau and Spearman's rho (4.6)

Assume there are two sets with w objects: x and y, that are to be compared.

For the general correlation coefficient, say a_{uv} is the x-score assigned to a pair of objects u and v and b_{uv} is the y-score assigned to a pair of objects u and v. The general correlation coefficient is then (Croux & Dehon, 2010)

$$\rho = \frac{\sum_{u=1}^{w}\sum_{v=1}^{w} a_{uv}b_{uv}}{\sqrt{\sum_{u=1}^{w}\sum_{v=1}^{w} a_{uv}^2 \sum_{u=1}^{w}\sum_{v=1}^{w} b_{uv}^2}}$$

These two scores, a_{uv} and b_{uv}, are then defined to come to a specific correlation coefficient. Now assume these two sets, x and y, are two rankings, with r_u and s_u as rank of object u (r_u and s_u are thus integer values).

Kendall's tau

Kendall's tau only considers the sign of the difference between the two objects within a rank.

Define: $a_{uv} = sgn(r_v - r_u)$ and $b_{uv} = sgn(s_v - s_u)$, where $sgn(x) = \begin{cases} 1\ if\ x > 0 \\ 0\ if\ x = 0 \\ -1\ if\ x < 0 \end{cases}$

Then $a_{uv}b_{uv} = 1$ if the u is in both cases ranked higher than v or in both cases ranked lower, $a_{uv}b_{uv} = -1$ if u is ranked higher (lower) in one ranking and lower (higher) in the other, and $a_{uv}b_{uv} = 0$ if the rank is the same. This leads to Kendall's tau:

$$Kendall's\ tau = \frac{2((number\ of\ concordant\ pairs) - (number\ of\ disconcordant\ pairs))}{\sqrt{n(n-1)n(n-1)}}$$

Spearman's rho

Spearman uses a similar approach, but also incorporates the distance between the ranks: $a_{uv} = (r_v - r_u)$ and $b_{uv} = (s_v - s_u)$. With regard to the denominator, the sums of a_{uv} and b_{uv} are equal, leading to the correlation coefficient:

$$Spearman's\ rho = \frac{\sum_{u=1}^{w}\sum_{v=1}^{w}(r_v - r_u)(s_v - s_u)}{\sum_{u=1}^{w}\sum_{v=1}^{w}(r_v - r_u)^2}$$

For the numerator: $\sum_{u=1}^{w}\sum_{v=1}^{w}(r_v - r_u)(s_v - s_u)$

$$= \sum_{u=1}^{w}\sum_{v=1}^{w}[r_v s_v - r_v s_u - r_u s_v + r_u s_u]$$
$$= \sum_{u=1}^{w}\sum_{v=1}^{w}[r_v s_v + r_u s_u] + \sum_{u=1}^{w}\sum_{v=1}^{w}[-r_v s_u - r_u s_v]$$
$$= 2w\sum_{v=1}^{w} r_v s_v - 2\sum_{u=1}^{w}\sum_{v=1}^{w}[r_v s_u]$$
$$= 2w\sum_{v=1}^{w} r_v s_v - 2\left(\frac{1}{2}w(w+1)\right)^2$$
$$= 2w\sum_{v=1}^{w} r_v s_v - \frac{1}{2}w^2(w+1)^2$$

For the denominator:

$$\sum_{u=1}^{w}\sum_{v=1}^{w}(r_v - r_u)^2$$

$$= \sum_{u=1}^{w}\sum_{v=1}^{w} r_v^2 - 2r_v r_u + r_u^2$$

$$= 2w\sum_{v=1}^{w} r_v^2 - 2\sum_{u=1}^{w}\sum_{v=1}^{w} r_v r_u$$

$$= 2w\sum_{v=1}^{w} r_v^2 - 2\left(\sum_{v=1}^{w} r_v\right)^2$$

$$= 2w\left(\frac{w(w+1)(2w+1)}{6}\right) - 2\left(\frac{1}{2}w(w+1)\right)^2$$

$$= \frac{1}{3}w^2(w+1)(2w+1) - \frac{1}{2}w^2(w+1)^2$$

$$= \frac{1}{6}w^2(w+1)[2(2w+1) - 3(w+1)]$$

$$= \frac{1}{6}w^2(w^2 - 1)$$

Leading to Spearman's rho:

$$Spearman's\ rho = 1 - \frac{6\sum_{v=1}^{w}(r_v - s_v)}{w^3 - w}$$

F. Findings regarding cut-off value k (5.1.1)

MPI for different values of k (Using stated preference weights)														
k	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
k = 1	0.1634	0.1568	0.1505	0.1629	0.1498	0.1434	0.1415	0.1351	0.1392	0.1364	0.1379	0.1297	0.1228	0.1150
k = 4	0.0690	0.0671	0.0599	0.0671	0.0619	0.0584	0.0559	0.0469	0.0433	0.0507	0.0474	0.0437	0.0403	0.0343

MPI for different values of k (Using statistical weights)														
k	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
k = 1	0.1724	0.1693	0.1625	0.1669	0.1550	0.1573	0.1594	0.1624	0.1639	0.1626	0.1637	0.1559	0.1397	0.1290
k = 4	0.0931	0.0919	0.0838	0.0874	0.0803	0.0855	0.0824	0.0815	0.0803	0.0797	0.0793	0.0748	0.0646	0.0532

G. Findings regarding β (5.1.2)

MPI for different values of β (Using stated preference weights)														
β	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
β **= 1**	0.1634	0.1568	0.1505	0.1629	0.1498	0.1434	0.1415	0.1351	0.1392	0.1364	0.1379	0.1297	0.1228	0.1150
β **= 2**	0.0519	0.0502	0.0470	0.0507	0.0469	0.0439	0.0429	0.0392	0.0396	0.0405	0.0399	0.0369	0.0347	0.0308
β **= 3**	0.0210	0.0204	0.0188	0.0198	0.0189	0.0170	0.0166	0.0145	0.0148	0.0154	0.0149	0.0134	0.0128	0.0107

MPI for different values of β (Using statistical weights)														
β	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
β **= 1**	0.1724	0.1693	0.1625	0.1669	0.1550	0.1573	0.1594	0.1624	0.1639	0.1626	0.1637	0.1559	0.1397	0.1290
β **= 2**	0.0605	0.0600	0.0556	0.0567	0.0530	0.0551	0.0543	0.0545	0.0545	0.0542	0.0536	0.0510	0.0446	0.0390
β **= 3**	0.0260	0.0259	0.0230	0.0234	0.0226	0.0237	0.0227	0.0222	0.0224	0.0221	0.0215	0.0207	0.0179	0.0150

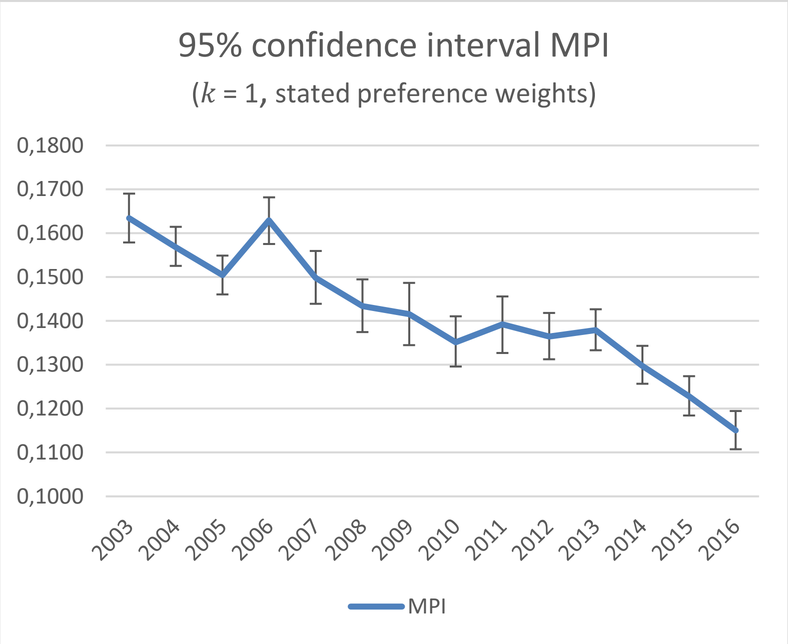
H. Robustness weights (5.2)

Kendall's tau (SP = stated preference weights, SA = statistical weights, WE = equal weights)											
Version	SP, k = 1	SP, k = 4	SP, β = 2	SA, k = 1	SA, k = 4	SA, β = 2	WE, k = 1	WE, k = 4	WE, β = 2	Highest	Lowest
SP, k = 1		0.846**	0.934**	0.560**	0.780**	0.714**	0.363	0.385	0.538**	0.934	0.363
SP, k = 4			0.912**	0.451*	0.758**	0.692**	0.297	0.538**	0.560**	0.912	0.297
SP, β = 2				0.495*	0.758**	0.692**	0.385	0.451*	0.560**	0.758	0.385
SA, k = 1					0.560**	0.670**	0.714**	0.560**	0.670**	0.714	0.560
SA, k = 4						0.890**	0.275	0.429*	0.363	0.890	0.275
SA, β = 2							0.385	0.451*	0.385	0.451	0.385
WE, k = 1								0.626**	0.736**	0.736	0.626
WE, k = 4									0.714**	0.714	0.714
WE, β = 2											
Same weighting scheme						Different weighting scheme					

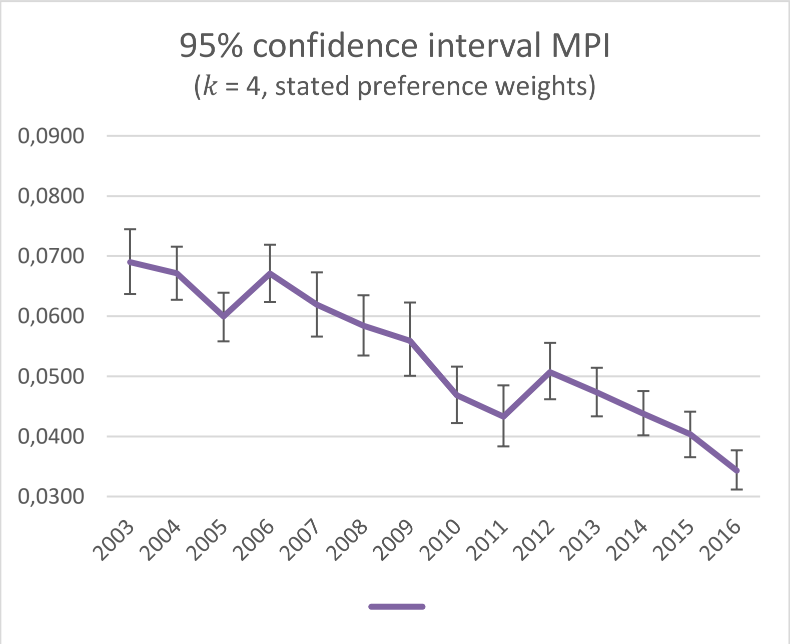
Spearman's rho (SP = stated preference weights, SA = statistical weights, WE = equal weights)											
Version	SP, k = 1	SP, k = 4	SP, β = 2	SA, k = 1	SA, k = 4	SA, β = 2	WE, k = 1	WE, k = 4	WE, β = 2	Highest	Lowest
SP, k = 1		0.943**	0.982**	0.697**	0.903**	0.851**	0.473	0.525	0.640*	0.982	0.473
SP, k = 4			0.978**	0.613*	0.859**	0.789**	0.464	0.670*	0.701**	0.978	0.464
SP, β = 2				0.679*	0.886**	0.833**	0.490	0.604*	0.684**	0.886	0.490
SA, k = 1					0.719**	0.815**	0.877**	0.714**	0.789**	0.877	0.714
SA, k = 4						0.969**	0.433	0.547*	0.508	0.969	0.433
SA, β = 2							0.530	0.582*	0.552*	0.582	0.530
WE, k = 1								0.824**	0.877**	0.877	0.824
WE, k = 4									0.864**	0.864	0.864
WE, β = 2											
Same weighting scheme						Different weighting scheme					

I. Confidence intervals (5.3)

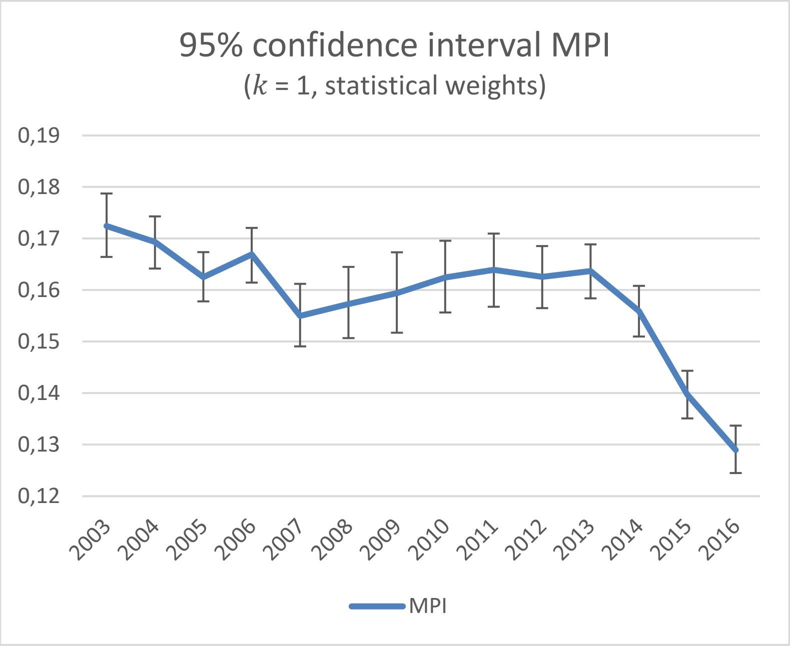
95% confidence interval ($k = 1, \beta = 1$, stated preference weights)			
Year	**Lower bound**	**MPI**	**Upper bound**
2003	0.1579	0.1634	0.1690
2004	0.1525	0.1568	0.1615
2005	0.1460	0.1505	0.1549
2006	0.1575	0.1629	0.1682
2007	0.1439	0.1498	0.1559
2008	0.1375	0.1434	0.1495
2009	0.1345	0.1415	0.1487
2010	0.1296	0.1351	0.1411
2011	0.1327	0.1392	0.1456
2012	0.1313	0.1364	0.1418
2013	0.1333	0.1379	0.1427
2014	0.1257	0.1297	0.1343
2015	0.1184	0.1228	0.1274
2016	0.1107	0.1150	0.1195
Confidence interval approximated with the bootstrap method, using 2,000 samples			



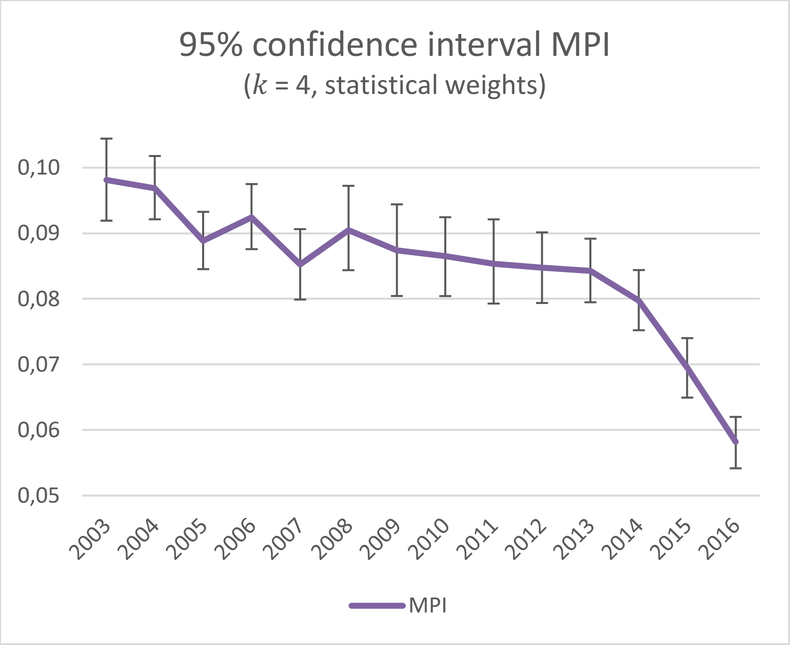
95% confidence interval (k = 4, stated preference weights)			
Year	**Lower bound**	**MPI**	**Upper bound**
2003	0.0637	0.0690	0.0745
2004	0.0627	0.0671	0.0716
2005	0.0558	0.0599	0.0639
2006	0.0624	0.0671	0.0719
2007	0.0566	0.0619	0.0673
2008	0.0535	0.0584	0.0635
2009	0.0501	0.0559	0.0623
2010	0.0422	0.0469	0.0516
2011	0.0383	0.0433	0.0485
2012	0.0462	0.0507	0.0556
2013	0.0433	0.0474	0.0514
2014	0.0402	0.0437	0.0475
2015	0.0365	0.0403	0.0441
2016	0.0312	0.0343	0.0377
Confidence interval approximated with the bootstrap method, using 2,000 samples			



95% confidence interval (k = 1, β = 1, statistical weights)			
Year	**Lower bound**	**MPI**	**Upper bound**
2003	0.1664	0.1724	0.1787
2004	0.1642	0.1693	0.1743
2005	0.1578	0.1625	0.1674
2006	0.1615	0.1669	0.1721
2007	0.1491	0.1550	0.1612
2008	0.1507	0.1573	0.1645
2009	0.1517	0.1594	0.1673
2010	0.1557	0.1624	0.1696
2011	0.1568	0.1639	0.1710
2012	0.1565	0.1626	0.1685
2013	0.1584	0.1637	0.1689
2014	0.1510	0.1559	0.1608
2015	0.1351	0.1397	0.1443
2016	0.1245	0.1290	0.1337
Confidence interval approximated with the bootstrap method, using 2000 samples			



95% confidence interval (k = 4, statistical weights)			
Year	**Lower bound**	**MPI**	**Upper bound**
2003	0.0869	0.0931	0.0994
2004	0.0871	0.0919	0.0968
2005	0.0795	0.0838	0.0883
2006	0.0826	0.0874	0.0925
2007	0.0749	0.0803	0.0856
2008	0.0794	0.0855	0.0922
2009	0.0754	0.0824	0.0894
2010	0.0754	0.0815	0.0874
2011	0.0743	0.0803	0.0871
2012	0.0744	0.0797	0.0851
2013	0.0745	0.0793	0.0842
2014	0.0702	0.0748	0.0794
2015	0.0599	0.0645	0.0690
2016	0.0492	0.0532	0.0570
Confidence interval approximated with the bootstrap method, using 2000 samples			



J. Decomposition (5.4)

Contribution of each indicator (%) ($\boldsymbol{k = 1}, \boldsymbol{\beta = 1}$, stated preference weights)												
Year	Education level	Shelter	Sanitation	Assets	Environment	Crime	General health	Long-term health	Nutrition	Unmet medical needs	Social life	Joblessness
2003	24	11	2	12	15	12	9	10	3	0	1	0
2004	25	12	3	12	13	11	9	10	2	1	1	0
2005	26	11	2	13	13	12	9	11	2	1	1	0
2006	23	11	2	14	14	12	9	11	2	1	1	0
2007	23	12	1	12	15	12	9	12	1	1	1	0
2008	25	11	1	12	14	11	10	12	2	1	1	0
2009	23	12	2	12	12	12	10	12	2	1	1	1
2010	23	12	2	13	11	10	9	14	2	1	2	1
2011	21	10	11	12	10	9	8	13	2	1	2	1
2012	19	11	8	12	11	10	9	14	3	1	2	1
2013	18	13	7	11	11	11	9	14	3	1	2	1
2014	19	14	2	12	11	11	10	15	3	1	2	1
2015	19	14	2	11	11	11	11	15	3	1	2	1
2016	20	14	3	11	11	10	10	16	3	1	1	1

Contribution of each indicator (%) ($\boldsymbol{k = 4}$, stated preference weights)												
Year	Education level	Shelter	Sanitation	Assets	Environment	Crime	General health	Long-term health	Nutrition	Unmet medical needs	Social life	Joblessness
2003	17	14	3	15	15	13	10	10	2	0	0	0
2004	17	14	4	14	14	13	11	11	2	1	0	0
2005	17	13	3	15	14	13	11	11	2	0	0	0
2006	17	12	3	15	15	14	11	11	1	0	0	0
2007	16	14	2	14	15	14	12	12	1	0	0	0
2008	19	13	2	16	13	12	12	12	2	1	0	0
2009	18	15	2	15	14	13	10	11	2	0	0	0
2010	16	15	2	17	11	12	12	12	2	1	0	0
2011	17	13	6	13	12	12	11	12	2	1	0	0
2012	15	12	8	16	12	12	10	11	2	1	0	0
2013	14	14	5	13	13	14	11	12	3	1	0	0
2014	14	14	2	15	12	12	12	13	3	1	0	0
2015	14	15	3	14	12	12	12	13	3	1	0	0
2016	14	15	3	16	14	12	11	12	2	0	0	0

Contribution of each indicator (%) ($\boldsymbol{\beta = 2}$, stated preference weights)												
Year	Education level	Shelter	Sanitation	Assets	Environment	Crime	General health	Long-term health	Nutrition	Unmet medical needs	Social life	Joblessness
2003	22	13	3	14	15	13	9	8	2	0	1	0
2004	23	13	3	13	14	12	8	9	2	1	1	0
2005	23	12	3	14	14	13	9	10	2	1	1	0
2006	22	12	2	15	15	13	8	10	2	1	1	0
2007	21	13	2	13	15	13	8	10	2	1	1	0
2008	23	12	2	14	14	12	9	10	2	1	1	0
2009	22	13	2	14	13	13	9	10	2	1	1	0
2010	22	14	2	15	12	11	9	12	2	1	1	0
2011	21	12	9	13	11	11	8	11	3	1	1	0
2012	19	12	8	14	11	11	8	11	3	1	1	0
2013	18	14	7	12	12	12	8	11	3	1	2	0
2014	18	14	2	14	12	12	9	12	3	1	2	0
2015	19	15	3	13	12	12	9	12	3	1	1	0
2016	19	15	3	14	12	11	9	13	3	1	1	0

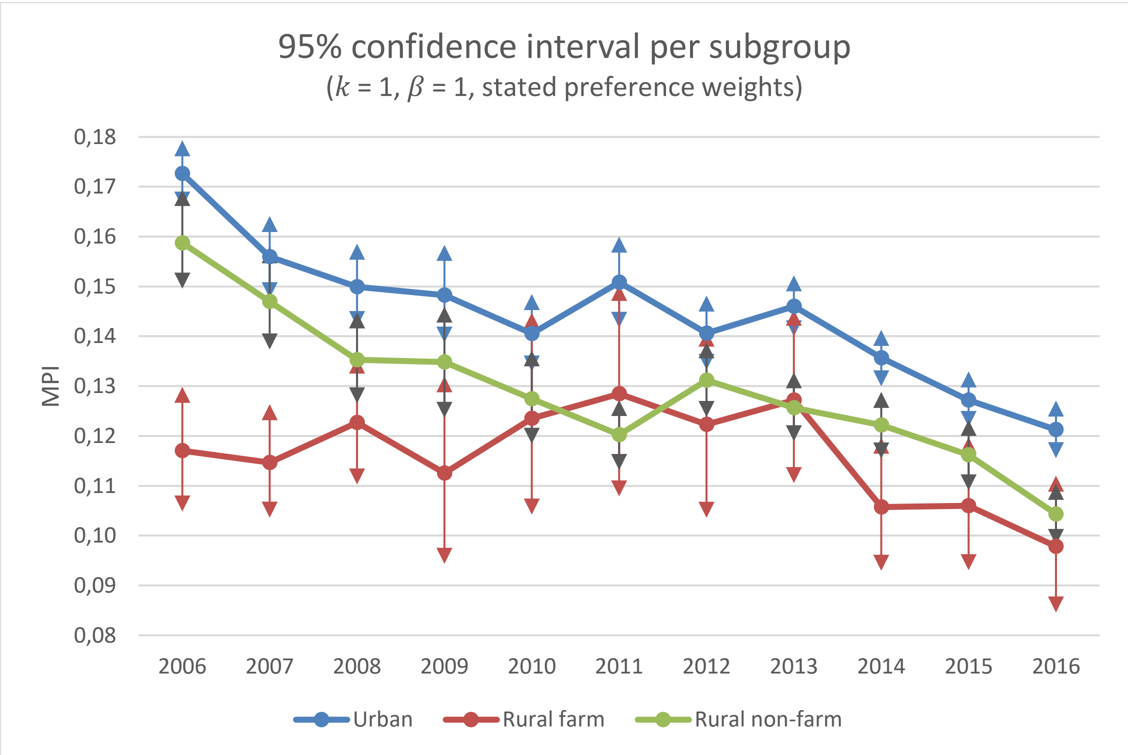
Contribution of each indicator (%) ($\boldsymbol{k = 1}, \boldsymbol{\beta = 1}$, statistical weights)												
Year	Education level	Shelter	Sanitation	Assets	Environment	Crime	General health	Long-term health	Nutrition	Unmet medical	Social life	Joblessness
2003	43	4	1	5	6	5	10	10	3	0	10	3
2004	45	4	1	5	5	4	9	10	2	1	11	3
2005	45	4	1	5	5	4	9	11	2	1	10	2
2006	44	4	1	5	6	5	9	12	2	1	9	2
2007	43	5	1	5	6	5	9	13	2	1	9	3
2008	43	4	1	4	5	4	10	12	2	1	11	4
2009	39	4	1	4	4	4	9	12	2	1	13	6
2010	37	4	1	4	4	3	9	13	2	1	16	7
2011	34	4	4	4	3	3	8	12	2	1	18	8
2012	31	4	3	4	4	3	8	12	2	1	19	8
2013	29	4	2	4	4	4	8	13	2	1	21	8
2014	30	5	1	4	4	4	9	13	3	1	20	7
2015	32	5	1	4	4	4	10	15	3	1	16	6
2016	33	5	1	4	4	4	10	16	2	1	14	6

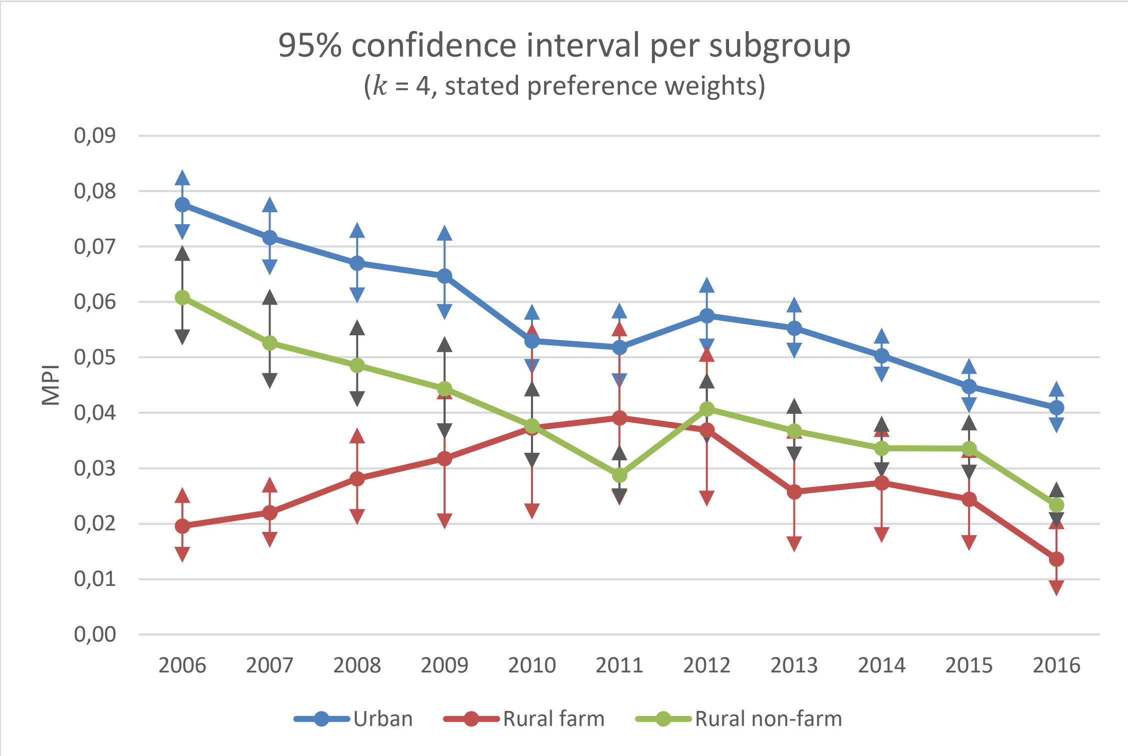
Contribution of each indicator (%) ($\boldsymbol{k} = \boldsymbol{4}$, statistical weights)												
Year	Education level	Shelter	Sanitation	Assets	Environment	Crime	General health	Long-term health	Nutrition	Unmet medical needs	Social life	Joblessness
2003	41	4	1	5	4	3	12	12	2	1	12	3
2004	40	4	2	5	4	3	13	13	2	1	11	3
2005	40	4	1	5	4	4	13	14	2	0	11	2
2006	40	4	1	5	4	4	13	15	2	0	9	2
2007	39	4	1	5	4	4	13	15	2	0	10	3
2008	41	4	0	5	3	3	12	14	2	0	10	4
2009	40	4	1	5	3	3	12	13	2	0	11	5
2010	38	4	1	5	3	3	12	14	2	0	14	5
2011	39	3	2	4	2	3	10	13	2	1	14	7
2012	37	3	3	5	3	3	10	13	2	1	14	6
2013	34	4	2	4	3	4	10	13	3	1	16	6
2014	34	4	1	5	3	3	11	13	3	1	16	6
2015	35	4	1	5	3	3	12	15	3	1	13	5
2016	35	4	1	5	3	3	13	15	3	1	12	6

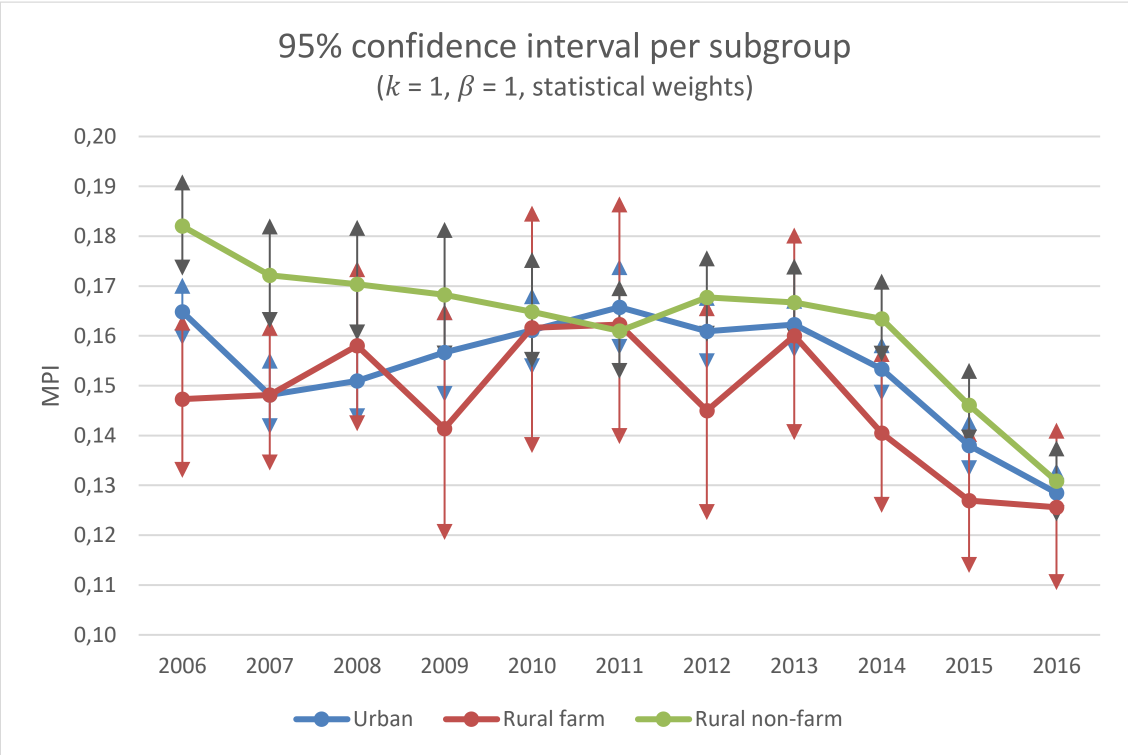
Contribution of each indicator (%) ($\boldsymbol{\beta} = \boldsymbol{2}$, statistical weights)												
Year	**Education level**	**Shelter**	**Sanitation**	**Assets**	**Environment**	**Crime**	**General health**	**Long-term health**	**Nutrition**	**Unmet medical needs**	**Social life**	**Joblessness**
2003	48%	4%	1%	5%	4%	3%	9%	9%	2%	0%	12%	3%
2004	49%	4%	1%	4%	4%	3%	9%	9%	2%	1%	12%	3%
2005	50%	3%	1%	5%	4%	3%	9%	10%	2%	1%	11%	2%
2006	49%	3%	1%	5%	4%	4%	9%	10%	2%	1%	10%	3%
2007	48%	4%	1%	4%	4%	4%	9%	11%	2%	1%	10%	3%
2008	48%	3%	0%	5%	3%	3%	9%	10%	2%	1%	11%	4%
2009	45%	4%	0%	4%	3%	3%	9%	10%	2%	1%	13%	5%
2010	42%	4%	1%	4%	3%	3%	8%	11%	2%	1%	16%	6%
2011	41%	3%	2%	4%	3%	3%	7%	10%	3%	1%	17%	7%
2012	38%	3%	2%	4%	3%	3%	8%	10%	3%	1%	18%	8%
2013	36%	4%	2%	4%	3%	3%	8%	11%	3%	1%	19%	8%
2014	36%	4%	0%	4%	3%	3%	8%	11%	3%	1%	19%	7%
2015	39%	4%	1%	4%	3%	3%	9%	12%	3%	1%	16%	6%
2016	41%	4%	1%	4%	3%	3%	9%	13%	2%	1%	14%	6%

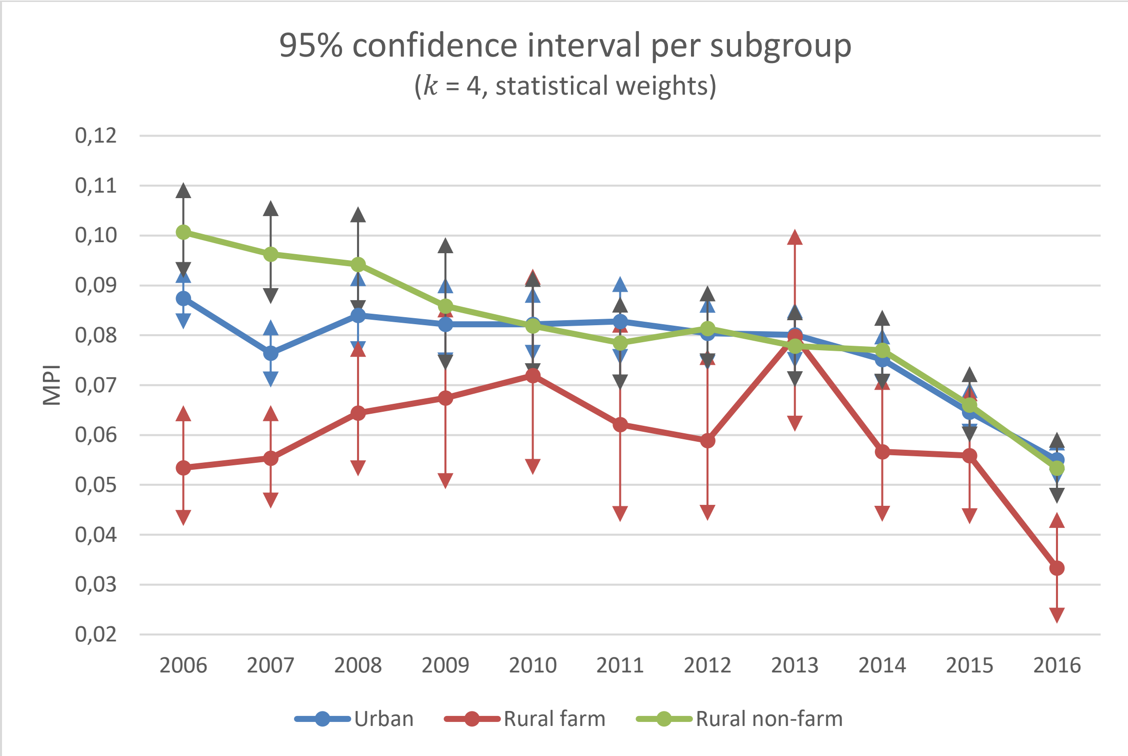
K. Subgroup confidence intervals (6.1)

Year	Number of households		
	Rural farm	*Rural non-farm*	*Urban*
2006	462	1750	3578
2007	454	1747	3406
2008	428	1671	3147
2009	317	1699	3165
2010	252	1647	2743
2011	223	1549	2560
2012	215	1692	2655
2013	240	1772	2910
2014	312	1904	3268
2015	355	1760	3328
2016	366	1650	3202




95% confidence interval per subgroup
(k = 4, stated preference weights)
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0,00
MPI
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
Urban
Rural farm
Rural non-farm


95% confidence interval per subgroup
(k = 1, β = 1, statistical weights)
0,20
0,19
0,18
0,17
0,16
0,15
0,14
0,13
0,12
0,11
0,10
MPI
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
Urban
Rural farm
Rural non-farm


95% confidence interval per subgroup
(k = 4, statistical weights)
MPI
0,12
0,11
0,10
0,09
0,08
0,07
0,06
0,05
0,04
0,03
0,02
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
Urban
Rural farm
Rural non-farm

L. Subgroup descriptive statistics (6.1)

Urban households **Share of households deprived in each indicator**												
Year	**Education level**	**Shelter**	**Sanitation**	**Assets**	**Environment**	**Crime**	**General health**	**Long-term health**	**Nutrition**	**Unmet medical needs**	**Social life**	**Joblessness**
2006	30%	14%	2%	18%	22%	23%	29%	37%	5%	2%	12%	3%
2007	27%	15%	1%	15%	21%	20%	27%	37%	4%	1%	12%	3%
2008	28%	13%	1%	15%	20%	18%	29%	36%	5%	1%	13%	4%
2009	25%	13%	2%	15%	17%	17%	27%	35%	5%	1%	17%	7%
2010	24%	13%	2%	15%	15%	13%	25%	39%	6%	2%	22%	9%
2011	21%	12%	12%	14%	13%	13%	22%	36%	7%	2%	23%	9%
2012	22%	13%	7%	13%	13%	13%	23%	36%	8%	3%	26%	9%
2013	18%	15%	9%	14%	15%	15%	23%	36%	7%	3%	27%	10%
2014	20%	15%	2%	13%	12%	12%	24%	36%	7%	4%	25%	9%
2015	20%	14%	2%	11%	11%	12%	24%	35%	6%	2%	17%	7%
2016	18%	13%	3%	13%	13%	11%	23%	35%	5%	2%	15%	6%

Rural non-farm households **Share of households deprived in each indicator**												
Year	Education level	**Shelter**	**Sanitation**	**Assets**	Environment	**Crime**	General health	Long-term health	**Nutrition**	Unmet medical needs	Social life	**Joblessness**
2006	46%	15%	5%	14%	11%	7%	35%	42%	3%	1%	10%	3%
2007	45%	16%	3%	11%	12%	6%	32%	40%	3%	1%	9%	3%
2008	44%	14%	2%	12%	9%	5%	34%	39%	3%	1%	10%	3%
2009	36%	14%	3%	10%	8%	8%	28%	35%	4%	1%	12%	6%
2010	32%	14%	2%	11%	9%	5%	25%	37%	5%	2%	19%	8%
2011	31%	11%	10%	11%	7%	5%	25%	36%	4%	2%	19%	10%
2012	25%	11%	11%	13%	7%	7%	23%	34%	6%	3%	23%	10%
2013	25%	12%	8%	11%	8%	8%	25%	36%	6%	2%	26%	10%
2014	23%	15%	1%	13%	9%	10%	24%	35%	7%	2%	23%	8%
2015	22%	13%	1%	12%	8%	9%	26%	37%	5%	2%	18%	7%
2016	25%	12%	2%	7%	5%	6%	23%	35%	4%	1%	12%	5%

M. Subgroup decomposition results

Average contribution of each indicator, per subgroup, for 2003-2016				
Indicator	**Weight**	**Rural farm**	**Rural non-farm**	**Urban**
Education level	13%	40%	28%	18%
Shelter	12%	15%	13%	12%
Sanitation	12%	2%	2%	2%
Assets	12%	4%	11%	14%
Environment	12%	5%	8%	14%
Crime	12%	5%	6%	13%
General health	6%	11%	11%	9%
Long-term health	6%	16%	15%	13%
Nutrition	6%	1%	2%	2%
Unmet medical	6%	0%	1%	1%
Social life	1%	1%	1%	1%
Joblessness	1%	0%	1%	1%
(Parameters: cut-off value $k = 1$, $\beta = 1$, stated preference weights)				

Average contribution of each indicator, per subgroup, for 2003-2016				
Indicator	**Weight**	**Rural farm**	**Rural non-farm**	**Urban**
Education level	25%	60%	44%	33%
Shelter	5%	5%	4%	5%
Sanitation	5%	1%	1%	1%
Assets	5%	1%	4%	5%
Environment	5%	1%	3%	6%
Crime	5%	1%	2%	5%
General health	6%	9%	10%	9%
Long-term health	6%	14%	14%	14%
Nutrition	6%	1%	2%	2%
Unmet medical	6%	0%	1%	1%
Social life	13%	5%	12%	14%
Joblessness	13%	1%	5%	5%
(Parameters: cut-off value $k = 1$, $\beta = 1$, statistical weights)				

Average contribution of each indicator, per subgroup, for 2003-2016				
Indicator	**Weight**	**Rural farm**	**Rural non-farm**	**Urban**
Education level	13%	20%	19%	14%
Shelter	12%	16%	15%	14%
Sanitation	12%	7%	3%	2%
Assets	12%	10%	15%	17%
Environment	12%	5%	8%	15%
Crime	12%	7%	6%	15%
General health	6%	16%	15%	10%
Long-term health	6%	16%	15%	10%
Nutrition	6%	2%	2%	2%
Unmet medical	6%	1%	0%	1%
Social life	1%	0%	0%	0%
Joblessness	1%	0%	0%	0%
(Parameters: cut-off value $k = 4$, stated preference weights)				

Average contribution of each indicator, per subgroup, for 2003-2016				
Indicator	**Weight**	**Rural farm**	**Rural non-farm**	**Urban**
Education level	25%	40%	39%	37%
Shelter	5%	4%	5%	4%
Sanitation	5%	2%	1%	1%
Assets	5%	2%	5%	6%
Environment	5%	1%	2%	4%
Crime	5%	2%	2%	4%
General health	6%	19%	15%	11%
Long-term health	6%	20%	16%	13%
Nutrition	6%	1%	2%	2%
Unmet medical	6%	1%	0%	1%
Social life	13%	7%	10%	13%
Joblessness	13%	2%	5%	5%
(Parameters: cut-off value $k = 4$, statistical weights)				

N. Subgroup rank correlation (6.3)

Kendall's tau $(k = 1, \beta = 1)$						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		.667**	0.879**	0.667**	0.333	0.424
Urban - SP			.727**	0.394	0.364	0.273
Rural non-farm - SP				0.545*	0.333	0.485*
Rural farm - SA					0.667**	0.758**
Urban - SA						0.788**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Spearman's rho $(k = 1, \beta = 1)$						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		0.804**	0.951**	0.776**	0.427	0.517
Urban - SP			0.860**	0.559	0.420	0.343
Rural non-farm - SP				0.713*	0.420	0.524
Rural farm - SA					0.853**	0.895**
Urban - SA						0.916**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Kendall's tau ($k = 4$)						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		.576**	0.909**	0.606**	0.394	0.394
Urban - SP			0.606**	0.182	0.212	0.091
Rural non-farm - SP				0.515*	0.364	0.424
Rural farm - SA					0.667**	0.667**
Urban - SA						0.818**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Spearman's rho ($k = 4$)						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		0.664**	0.972**	0.706*	0.448	0.469
Urban - SP			0.734**	0.189	0.196	0.056
Rural non-farm - SP				0.622*	0.455	0.483
Rural farm - SA					0.811**	0.804**
Urban - SA						0.937**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Kendall’s tau ($\beta = 2$)						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		.667**	.939**	.606*		
Urban - SP			.727**		0.273	
Rural non-farm - SP						.364
Rural farm - SA					.576**	.636**
Urban - SA						.879**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

Spearman’s rho ($\beta = 2$)						
Subgroup & weights	Rural farm - SP	Urban - SP	Rural non-farm - SP	Rural farm - SA	Urban - SA	Rural non-farm - SA
Rural farm - SP		.797**	.986**	.748**		
Urban - SP			.839**		0.273	
Rural non-farm - SP						0.420
Rural farm - SA					0.727**	0.797**
Urban - SA						0.958**
Rural non-farm - SA						
Same weighting scheme			Different weighting scheme			
* = significant at 5%, **=significant at 1% SP = stated preference weights, SA = statistical weights						

9 References

Alkire, S., 2002. Dimensions of Human Development. *World Development,* pp. 181-205.

Alkire, S. & Apablaza, M., 2016. Multidimensional Poverty in Europe 2006-2012: Illustrating a Methodology. *Oxford Poverty and Human Development Initiative (OPHI).*

Alkire, S., Apablaza, M. & Jung, E., 2014. Multidimensional Poverty Measurement for EU-SILC Countries. *Oxford Poverty and Health Development Initiative (OPHI).*

Alkire, S. & Foster, J., 2011b. Understandings and Misunderstandings of Multidimensional Poverty Measurement. *Journal of Economic Inequality,* pp. 289-314.

Alkire, S. & Foster, J., 2011. Counting and Multidimensional Poverty Measurement. *Journal of Public Economics,* pp. 476-487.

Alkire, S. et al., 2015. Multidimensional Poverty Measurement and Analysis: Chapter 5 - The Alkire-Foster Counting Methodology. *Oxford Poverty and Health Development Initiative (OPHI).*

Alkire, S. & Santos, M. E., 2010. Acute Multidimensional Poverty: A New Index for Developing Countries. *Oxford Poverty and Human Development Initiative (OPHI).*

Alkire, S. & Santos, M. E., 2014. Measuring Acute Poverty in the Developing World: Robustness and Scope of the Multidimensional Poverty Index. *World Development,* pp. 251-274.

Alkire, S., Santos, M. E., Seth, S. & Yalonetzky, G., 2010. Is the Multidimensional Poverty Index Robust to Different Weights?. *Oxford Poverty and Human Development Index.*

Atkinson, A., 2003. Multidimensional Deprivation: Contrasting Social Welfare and Counting Approaches. *Journal of Economic Inequality,* pp. 51-65.

Berthoud, R. & Bryan, M., 2011. Income, Deprivation and Poverty: A Longitudinal Analysis. *Journal of Social Policy,* pp. 135-156.

Central Statistics Office, 2017. *Standard Report on Methods and Quality for the 2016 Survey on Income and Living conditions (EU-SILC),* Cork: Central Statistics Office.

Central Statistics Office, 2018. *European Union Survey of Income and Living Conditions (EU-SILC).* [Online]
Available at: http://www.ucd.ie/issda/eu-silc

Chakravarty, S. & Silber, J., 2008. Measuring Multidimensional Poverty: The Axiomatic Approach. In: *Quantitative Approaches to Multidimensional Poverty Measurement.* Hampshire: Palgrave MacMillan, pp. 192-209.

Copeland, P. & Daly, M., 2012. Varieties of poverty reduction: Inserting the poverty and social exclusion target into Europe 2020. *Journal of European Social Policy,* pp. 273-287.

Croux, C. & Dehon, C., 2010. Influence Functions of the Spearman and Kendall Correlation Measures. *Statistical Methods and Applications,* pp. 497-515.

Datt, G., 2017. Distribution-Sensitive Multidimensional Poverty Measures with an Application to India. *Monash Business School Working Papers.*

Decancq, K. & Lugo, M. A., 2012. Weights in Multidimensional Indices of Wellbeing: An Overview. *Econometric Reviews,* pp. 7-34.

Deutsch, J. & Silber, J., 2005. Measuring multidimensional poverty: An empirical comparison of various approaches. *Review of Income and Wealth,* pp. 145-174.

Duclos, J.-Y. & Tiberti, L., 2016. Multidimensional Poverty Indices: A Critical Assessment. *CIRPÉE Working Papers.*

Eurobarometer, 2007. *Poverty and Exclusion,* s.l.: European Commission.

Eurobarometer, 2010. *Poverty and Exclusion,* s.l.: European Commission.

European Commission, 2015. *Poverty and Social Exclusion.* [Online]
Available at: http://ec.europa.eu/social/main.jsp?langId=en&catId=751

Eurostat, 2017. *European Union Statistics on Income and Living Conditions.* [Online]
Available at: http://ec.europa.eu/eurostat/web/microdata/european-union-statistics-on-income-and-living-conditions

Ferrone, L. & Chzhen, Y., 2017. How to Reach the Sustainable Development Goal 1.2? Simulating Different Strategies to Reduce Multidimensional Child Poverty in Two Middle-Income Countries. *Child Indicators Research,* pp. 1-18.

Foster, J., 2005. Poverty Indices. In: *Poverty, Inequality and Development - Essays in Honor of Erik Thorbecke.* New York: Springer, pp. 41-65.

Foster, J. E., McGillivray, M. & Seth, S., 2012. Rank Robustness of Composite Indices: Dominance and Ambiguity. *Oxford Poverty and Human Development Initiative Working Paper.*

Foster, J., Greer, J. & Thorbecke, E., 2010. The Foster-Greer-Thorbecke (FGT) Poverty Measures: 25 Years Later. *The Journal of Economic Inequality,* pp. 491-524.

Fusco, A., Guio, A. & Marlier, E., 2010. Characterising the Income Poor and the Materially Deprived in European Countries. In: *Income and Living Conditions in Europe.* Luxembourg: Publications Office of the European Union.

Garcia-Diaz, R. & Prudencio, D., 2017. A Shapley Decomposition of Multidimensional Chronic Poverty in Argentina. *Bulletin of Economic Research,* pp. 69: 23-41.

Guio, A.-C. & Maquet, I., 2006. *Material Deprivation and Poor Housing.* Helsinki, s.n., pp. 1-28.

Hagenaars, A. J. M., 1991. The Definition and Measurement of Poverty. In: *Economic Inequality and Poverty.* New York: M.E. Sharpe, pp. 134-156.

Housseini, B., 2017. *Robustness Analysis and Statistical Inference.* [Online]
Available at: https://www.ophi.org.uk/wp-content/uploads/SS17-D8S1-RobustSE.pdf

Iceland, J. & Bauman, K. J., 2007. Income Poverty and Material Hardship: How Strong is the Association?. *The Journal of Socio-Economics,* pp. 376-396.

Kruijk, H. d. & Rutten, M., 2007. Weighting dimensions of poverty based on people's priorities: Constructing a composite poverty index for the Maldives. *Q-Squared Working Paper.*

Kruskal, W., 1958. Ordinal Measures of Association. *Journal of the American Statistical Association,* pp. 814-861.

Lambert, P. J., 2001. *The Distribution and Redistribution of Income.* Manchester: Manchester University Press.

Layte, R., Maître, B., Nolan, B. & Whelan, C., 2001. Persistent and consistent poverty in the 1994 and 1995 waves of the European Community Household Panel Study. *Review of Income and Wealth,* pp. 427-449.

Lustig, N. & Higgins, S., 2018. Can a Poverty-Reducing and Progressive Tax and Transfer System Hurt the Poor?. In: *Commitment to Equity Handbook: Estimating the Impact of Fiscal Policy on Inequality and Poverty.* Washington: The Brookings Institution, pp. 175-206.

Luzzi, G. F., Flückiger, Y. & Weber, S., 2008. A Cluster Analysis of Multidimensional Poverty in Switzerland. In: *Quantitative Approaches to Multidimensional Poverty Measurement.* Hampshire: Palgrave MacMillan, pp. 63-79.

Maitre, B., Nolan, B. & Whelan, C., 2013. A critical evaluation of the EU 2020 poverty and social exclusion target: an analysis of EU-SILC 2009. *UCD Geary Institute Discussion Papers.*

Martinez, A. & Perales, F., 2015. The Dynamics of Multidimensional Poverty in Contemporary Australia. *Social Indicators Research,* pp. 479-496.

Neuborg, C. d. et al., 2012. Child Deprivation, Multidimensional Poverty and Monetary Poverty in Europe. *Unicef Innocenti Working Paper.*

OECD, 2018. *Poverty Rate (indicator).* [Online]
Available at: https://data.oecd.org/inequality/poverty-rate.htm

Orloff, J. & Bloom, J., 2014. *Bootstrap Confidence Intervals.* [Online]
Available at: https://ocw.mit.edu/courses/mathematics/18-05-introduction-to-probability-and-statistics-spring-2014/readings/MIT18_05S14_Reading24.pdf

Parish, S., Rose, R. & Andrews, M., 2009. Income Poverty and Material Hardship among U.S. Women with Disabilities. *Social Service Review,* pp. 33-52.

Perry, B., 2002. The Mismatch between Income Measures and Direct Outcome Measures of Poverty. *Social Policy Journal of New Zealand,* pp. 101-127.

Ringen, S., 1988. Direct and Indirect Measures of Poverty. *Journal of Social Policy,* pp. 351-366.

Roche, J. M., 2013. Monitoring Progress in Child Poverty Reduction: Methodological Insights and Illustration to the Case Study of Bangladesh. *Social Indicators Research,* pp. 363-390.

Salazar, R. C. A., Diaz, B. & Pinzon, R. P., 2013. A Counting Multidimensional Poverty Index in Public Policy Context: the Case of Colombia. *Oxford Poverty and Human Development Initiative (OPHI).*

Sen, A., 1976. Poverty: An Ordinal Approach to Measurement. *Econometrica,* pp. 219-231.

Sen, A., 1976. Poverty: An Ordinal Approach to Measurement. *Econometrica,* pp. 219-231.

Sen, A., 1985. *Commodities and Capabilities.* Oxford: Oxford University Press.

Shorrocks, A. F., 2013. Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value. *The Journal of Economic Inequality,* pp. 99-126.

Short, K., 2005. Material and Financial Hardship and Income-Based Poverty Measures in the USA. *Journal of Social Policy,* pp. 21-38.

Stiglitz, J., Sen, A. & Fitoussi, J., 2009. *Report by the Commission on the Measurement of Economic Performance and Social Progress,* s.l.: s.n.

Townsend, P., 1979. *Poverty in the United Kingdom: A Survey of Household Resources and Standards of Living.* Harmondsworth: Penguin Publishing.

Tsui, K.-Y., 2002. Multidimensional Poverty Indices. *Social Choice and Welfare,* pp. 19:69-93.

UN, 2015a. *Sustainable Development Goals.* [Online]
Available at: https://www.un.org/sustainabledevelopment/sustainable-development-goals/

UN, 2015b. Multidimensional Poverty. *Development Issues*, 21 October.

UN, 2017. *States Members of the United Nations and States Members of Specialized Agencies.* [Online]
Available at: https://sustainabledevelopment.un.org/memberstates.html

Wagle, U., 2008. *Multidimensional Poverty Measurement. Concepts and Applications..* New York: Springer.

Watson, V., Sutton, M., Dibben, C. & Ryan, M., 2008. Deriving weights for the Index of Multiple Deprivation based on societal preferences: The application of a discrete choice experiment. *Oxford Poverty and Human Development Initiative (OPHI).*

Weziak-Bialowolska, D. & Dijkstra, L., 2014. Monitoring Multidimensional Poverty in the Regions of the European Union. *JRC Science and Policy Reports,* pp. 1-102.

Whelan, C., 2007. Understanding the Implications of Choice and Deprivation Index for Measuring Consistent Poverty in Ireland. *The Economic and Social Review,* pp. 38:211-234.

Whelan, C., Nolan, B. & Maitre, B., 2008. Measuring Material Deprivation in the Enlarged EU. *Economic and Social Research Institute (ESRI) Working Paper,* p. 249.

World Bank, 2015. *FAQs: Global Poverty Line Update.* [Online]
Available at: http://www.worldbank.org/en/topic/poverty/brief/global-poverty-line-faq

World Bank, 2017a. *GDP per capita (Current US$).* [Online]
Available at: https://data.worldbank.org/indicator/NY.GDP.PCAP.CD?year_low_desc=true

World Bank, 2017b. *Poverty Gap at $1.90 a day (2011 PPP) (%).* [Online]
Available at: https://data.worldbank.org/indicator/SI.POV.GAPS?year_high_desc=false

Yalonetzky, G., 2014. Conditions for the most robust multidimensional poverty comparisons using counting measures and ordinal variables. *Social Choice and Welfare,* pp. 773-807.