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Bachelor Thesis for Econometrics and Operations Research

An evaluation of scale-specific predictive systems

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Abstract

Predictions of excess market returns with market proxies seem to have hump-shaped behavior, as found by Bandi, Perron, Tamoni, and Tebaldi (2019). Firstly, Bandi et al. (2019) found that traditional predictive systems are tightly restricted and a novel predictive framework is developed. Time series are decomposed into scale specific components, each allowing for processes operating at specific frequencies. With the components, new predictive regressions are applied to decimated components (filtered observations). Their work is replicated and extended. Bandi et al. (2019) use observations exclusively from the U.S. In this paper, the framework is applied to different markets, namely the Asian-Pacific and European markets. Lastly, the framework is expanded, allowing for more freedom in the explanatory and predicted variables. This is found to have a positive impact on predictive power, up to an R^2 of 99.6%. Although, it is also found that the scale-specific predictive systems are often not able to generate reliable results.

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1 Introduction

Predicting financial events is a matter of great interest, as it is for different types of events. A prevalent relationship to predict is the one between excess market returns and market volatility (see, for instance, French, Schwert, & Stambaugh, 1987). Predictions are stronger over more extended periods, say a few years, as opposed to less extended periods, such as a few weeks (Bandi et al., 2019; see also the introduction of the 2013 Nobel for Economic Sciences[1]). Short-term predictions are difficult because the amount of noise relative to the signal is large. A possible approach, in that case, is to filter the data to make this signal visible. A recent paper in this field of research is by Bandi et al. (2019). It firstly performs two-way aggregated regressions, i.e., regressing backward aggregated (or, past) market uncertainty proxies (market variance, consumption variance, and economic policy uncertainty) on forward aggregated (or, future) excess market returns. The R^2s from these regressions are hump-shaped across the aggregation horizon, confirming that correctly predicting the next few years becomes easier. However, it seems that after 16 years, it becomes too far into the future, and the level of uncertainty increases again. Hence Bandi et al. (2019) introduce a new stylized fact: predictability is hump-shaped across the aggregation horizon.

Bandi et al. (2019) also introduce a novel predictive system, in which the time series is decomposed into transitory and persistent components. It is hypothesized that predictability is scale-specific, substantiated by the results of the two-way aggregated regressions. A scale refers to the length of a particular cycle that is assumed, where a higher scale implies a longer cycle or a lower frequency. The scale regressions with scale-specific components can replicate the hump-shaped behavior found in the data.

One of the purposes of this paper is to replicate the findings of Bandi et al. (2019) and to function as an evaluation of their work. In light of this, the methodology is also applied to different datasets. These contain the excess returns and market volatility proxies of the markets of different regions; namely Hong Kong and South Korea in the Asian-Pacific region, and the U.K. and Spain in Europe. Bandi et al. (2019) put notably much emphasis on the fact that the peak of predictability is at 16 years, which could very well be different for other regions. Indeed, it is found that hump-shaped behavior cannot be assumed to be a universal stylized fact.

Moreover, the second purpose of this paper is to expand the model from univariate scale regressions to multivariate scale regressions. These regressions use three different proxies for market volatility as predictors. Even though the predictors are proxies for the same variable, it is found that the variables tell a different story: economic policy uncertainty is derived from different information in the market than the market variance based on daily returns. Lastly, the restriction that all scales must be the same is removed. Standard theory says that economic phenomena operate across different frequencies. Thus, allowing the scales to differ means that the most informative scale can be used. This results in R^2s up to about 95%. Although, another substantial result of this research

[1] *"There is no way to predict whether the price of stocks and bonds will go up or down over the next few days or weeks. But it is quite possible to foresee the broad course of the prices of these assets over longer time periods, such as the next three to five years ..."*

is that the reliability of the scale-specific predictive systems is questionable, as the regressions use very few observations.

2 Literature

This paper taps into a deeply researched field. Namely the predictability of stock returns. In the literature, one could find predictors such as financial ratios (Campbell & Shiller, 1988; Lewellen, 2004), bond and stock prices (Keim & Stambaugh, 1986), interest rate variables (Lee, 1992), macroeconomic variables, such as consumption (Campbell & Cochrane, 1999; Lettau & Ludvigson, 2001), and business-cycle fluctuations (Rapach & Zhou, 2013). This paper uses market variance (Bandi & Perron, 2008), consumption variance (Tamoni, 2011), and economic policy uncertainty (EPU; Baker, Bloom, & Davis, 2016).

In Bandi et al. (2019) it is proven that traditional predictive systems impose tight restrictions on scale-specific predictability. Nonetheless, scale-specific predictability is included in these systems. The new model they introduce is mainly relevant because it frees up those restrictions and is, therefore, a generalization. Subsequently, they also introduce a proposition, linking "mild risk-return trade-offs at high frequencies" with "strong risk-return trade-offs at low frequencies". With this, they theoretically link the traditional systems to their new model.

The decomposition of a time-series into different stochastic trends and transitory components was brought to widespread recognition by Beveridge and Nelson (1981). It gives important insights as to which different processes operating at different frequencies together establish the economic variable. These different processes, or "layers", of the data are filtered once more into non-overlapping points. The non-overlapping points are used for the predictions. This could also be compared with the low-frequency information extraction of Müller and Watson (2008), who use weighted averages of the raw data to perform this procedure. Moreover, scale-specific regressions have also been proven useful in consumption models to forecast the market risk premium (Bandi, Perron, Tamoni, & Tebaldi, 2016; Ortu, Tamoni, & Tebaldi, 2013; Tamoni, 2011).

In this paper, the same two-way aggregated analysis is done twice, once with yearly observations, which is part of the replication, and once with monthly observations, which is part of the extension. For most results that will follow in this paper, it is difficult to set up hypotheses. However, for this concept, it is possible. It is already known that yearly observations (in the U.S.) exhibit hump-shaped behavior in terms of predictability. When performing these same predictions with monthly observations, one would expect to find the same hump-shaped behavior. Namely, those factors that cause poor predictability for the first few years are trivially also incorporated in the monthly observations. Those factors that cause reliable predictability during the "hump period" are also influencing the monthly observations. However, as opposed to yearly observations, monthly observations can be influenced by seasonality. This provides additional information. Additional information usually leads to better predictions (Heij et al., 2004). Thus, one would expect to find a hump around the same location with monthly observations of at least the size as with yearly observations.

Another implication of standard theory stating that economic phenomena operate according to a certain frequency is that different phenomena do not have to operate with the same frequency. For example, corporate announcements including quarterly earnings and dividend policy (MacKinlay, 1997) and macroeconomic announcements (Andersen, Bollerslev, Diebold, & Vega, 2007; Faust, Rogers, Wang, & Wright, 2007) operate at relatively high frequencies, political cycles (Nordhaus, 1975; Santa-Clara & Valkanov, 2003) operate at medium-high frequencies, and technological innovation (Greenwood & Jovanovic, 1999; Pástor & Veronesi, 2009; Rosenberg & Frischtak, 1984) and demographic changes (Abel, 2003; Favero, Gozluklu, & Tamoni, 2011) operate at low frequencies. That is why, in the last part of the extension, the returns are predicted while varying scales for the components are allowed, i.e., the first predictor does not have to follow the same frequency as the second predictor. As said before, in this research, the predictors are market variance, consumption variance, and economic policy uncertainty. Following the literature, one would expect the frequency of the market variance and consumption variance to be more on the high side, while for the policy uncertainty this is likely lower, as it is linked to political cycles. However, this could vary among the different regions.

3 Data

Data for four variables are needed, namely for the excess market returns, the market variance, the consumption variance, and for the economic policy uncertainty. The replication uses yearly observations from the U.S., and the extension uses monthly observations from the U.S., Hong Kong, South Korea, the U.K., and Spain. An overview with the sources of the observations, the periods, and where they were obtained from is recorded in Appendix A.

I received the dataset from the authors. It was, however, not entirely clear which observations were used for the results in Bandi et al. (2019) as the dataset contained much more than was needed for that paper. When it was possible to find the data independently, those observations were used.

The h-horizon continuously-compounded excess market returns are calculated as

$$r_{t+1,t+h} = r^e_{t+1} + \cdots + r^e_{t+h} \ ,$$

where $r^e_{t+j} = \ln(1 + R_{t+j}) - \ln(1 + R^f_{t+j})$ is the 1-year or 1-month excess logarithmic market return between dates $t + j - 1$ and $t + j$, R_{t+j} is the simple gross yearly or monthly market return, and R^f_{t+j} is the gross yearly or monthly risk-free rate. For the extension, the monthly observations were not directly available, but were computed from the daily returns with

$$R_t + 1 = \prod_{d=t_{first}}^{t_{last}} (1 + R_d) \ ,$$

with $[t_{first}, \ldots, t_{last}]$ the days in month t.

The market's realized variance over a year or month (i.e., between t and $t + 1$) is obtained by computing

$$v_{t,t+1} = \sum_{d=t_{first}}^{t_{last}} r_d^2 \ ,$$

where r_d is the market's logarithmic return on day d and $[t_{first}, \ldots, t_{last}]$ denotes the days in the year or month.

The measure of economic policy uncertainty (EPU) is based on Baker et al. (2016). The authors have a website on which they update the EPU measure for different counties. The variable used in this research is defined as

$$\text{EPU}_t = \left(\text{EPU}_t^*/100\right)^2 \ ,$$

where EPU* is the value as found in the dataset on the Baker et al. (2016) website. Here, the monthly observations are recorded. For the extension, these were used. For the replication, for a particular year, the observation from December in that year was used.

4 Hump-shaped behavior

Firstly, we use OLS regressions, as in Eq. (1), to show that the relation between forward-aggregated excess market returns and backward-aggregated market uncertainty in terms of predictability is hump-shaped when plotted against an aggregation horizon.

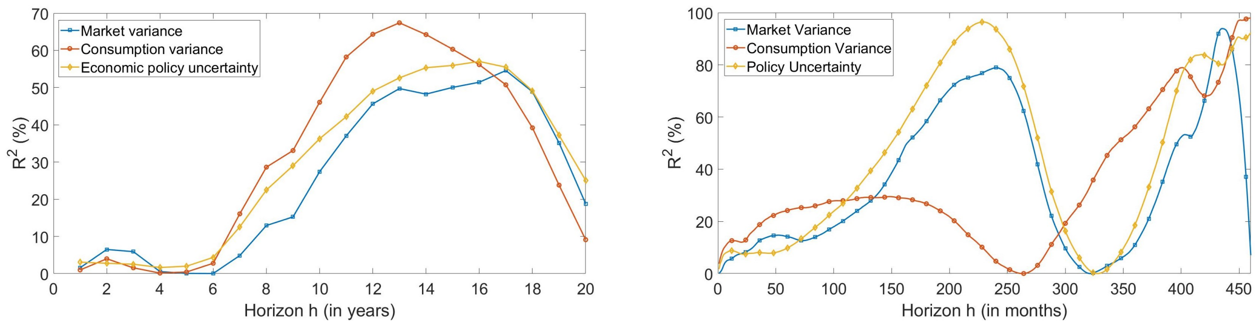
$$r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h} \ , \tag{1}$$

where $r_{t+1,t+h}$ and $v_{t-h+1,t}$ are non-overlapping sums of logarithmic excess market returns (forward aggregation) and market variances (backward aggregation) over an horizon of h years or months.

The regressions are evaluated with Newey-West t-statistics with horizon h lags, which account for consistency regarding heteroskedasticity and autocorrelations. The Valkanov (2003) $t/\sqrt{T}$-test statistics are also applied. Valkanov‘s methods have become standard tools in the predictability literature. For the replication, tables with a complete overview of the outputs are included in Appendix D. Moreover, for all regressions - these and the scale regressions in Section 5 (except for extension 1) - confidence intervals (CIs) of the R^2s are computed. These results can be found in Appendix G. The CIs are computed as $R^2 \pm t(0.95, n-k-1) \cdot SE_{R^2}$, with s

$$SE_{R^2} = \sqrt{\frac{4R^2(1-R^2)(n-k-1)^2}{(n^2-1)(n+3)}} \ .$$

The hump-shaped behavior is confirmed by Figure 1. It displays the R^2s across the aggregation horizon up until 20 years. The peak is at 16 years and reaches about 50-70%. The R^2s increase between 6 and 12 years and decline equally fast after 16 years, making the structure roughly tent-



(a) Replication with United States

(b) Extension 1 with United States

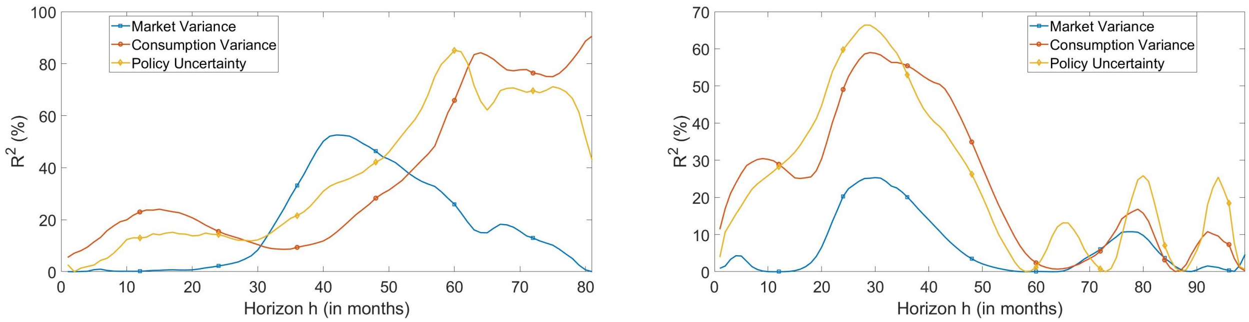
Figure 1: R^2 values obtained by regressing forward-aggregated excess market returns on backward-aggregated market variance (blue line, with squares), consumption variance (red line, with circles), and (squared) economic policy uncertainty (yellow line, with diamonds) for different levels of aggregation (on the horizontal axis). The markers are spaced one year apart. **Left:** Results from the replication with yearly observations from the U.S. **Right:** Results from the first extension with monthly observations from the U.S.

shaped.

As discussed in the literature review, one would expect to find hump-shaped behavior for the monthly analysis of the U.S. data with the maximum of the hump located around 16 years, which is equal to about 190 months. As in Figure 1, this corresponds well with the behavior of the market variance and policy uncertainty. However, the predictability using the consumption variance does not reach its top until about 400 months ($\pm$ 33 years). Note here that the consumption variance used for the yearly analysis is relatively different from those used for the monthly analysis. For the yearly analyses, the variance is modeled according to a HARCH model, while for the monthly analysis, this is a GARCH(1,1) model. Bandi et al. (2019) say that the results should be similar, but in these results, they are not. As Bandi et al. (2019) do not indicate which specification of the HARCH model was used, this paper used the more conventional GARCH(1,1) model.

Moreover, the R^2s reach about 90%, which is a lot higher than the 55% for yearly observations. This could be an indication that the monthly observations carry more and valuable information. Lastly, the aggregation horizon goes further for the extension than for the replication. As can be seen in Figure 1, the hump is nicely shaped for the first twenty years. After that, there seems to be a second hump, which means that either predictability is not hump-shaped but comes in waves or the R^2s increase due to the decreasing amount of observations. Although, the maximum horizon h is determined such that after aggregation, 51 observations are still available, leaving a predictive regression with 50 observations, in line with Cramer (1987). Thus, the R^2 should be reliable.

The hump-shaped behavior is most clean and clear for the U.S. (Figure 1) compared to the other countries. Hong Kong (Figure 2) is not exhibiting hump-shaped behavior. Though, note here that aggregations of a maximum of only 81 months were possible due to limited observations. Eighty-one months is equal to almost seven years. For the U.S. the hump was not clear until 6 or 7 years. Unexpectedly, the R^2s reach 80% for consumption variance and policy uncertainty. It is about 50% for the market variance and declines after that. If in fact, the hump does not yet occur within this time frame, the graph should only show the behavior leading up to the hump, in which one would



(a) Hong Kong

(b) South Korea

Figure 2: Extension 1. R^2 values obtained by regressing forward-aggregated excess market returns on backward-aggregated market variance (blue line, with squares), consumption variance (red line, with circles), and (squared) economic policy uncertainty (yellow line, with diamonds) for different levels of aggregation (on the horizontal axis). The markers are spaced one year apart. **Left:** Results from Hong Kong. **Right:** Results from South Korea.

not expect the R^2s to become this high. Given that it does, could also mean that the predictability does not exhibit hump-shaped behavior at all.

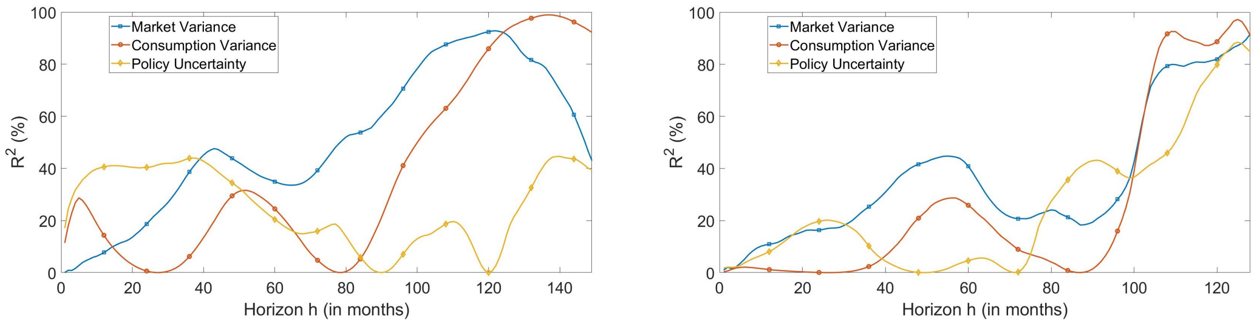
South Korea (Figure 2), on the other hand, does seem to exhibit hump-shaped behavior. It is striking here that the hump appears to be straight away, causing the top of the R^2s to be in the third year (30 months). The hump is not as smooth as the hump observed in the U.S. analyses. The predictability based on the consumption variance and the policy uncertainty perform quite similar. Predictability based on the market variance performs more poorly. After this hump, predictability flattens and oscillates between 0% and 20%.

The United Kingdom (Figure 3) does not clearly show hump-shaped behavior. Notably, in the R^2s from policy uncertainty, it is difficult to extinguish an apparent hump. The predictability of excess market returns using consumption variance and market variance could have hump-shaped behavior, but it is again not a smooth and evident as with the U.S. analyses. The R^2s of the market variance and consumption variance strikingly reach almost 100%.

Lastly, the Spanish market seems to exhibit hump-shaped behavior. The hump is again not as clean and smooth, but certainly visible. The R^2s leading up to the hump are not as flat either, however, after eight years (about 100 months) there is a definite increase.

Ultimately, the U.S. data seems to exhibit the strongest hump-shaped behavior. The South Korean and Spanish markets also seem to have some hump-shaped behavior, but the Hong Kong and U.K. market do not show enough evidence to confirm the presence of hump-shaped behavior. A more encompassing theory would be to say that predictability has a wave-like structure.

Subsequently, the paper continues to show that traditional predictive systems cannot model this hump-shaped behavior. This is explained theoretically and by simulations. Traditional systems are h-step ahead point forecasts, and aggregating forward and backward means that the slope β of the regression gets multiplied by the first lag autocorrelation ρ each time an aggregation takes place. As a result, the slope becomes $\beta\rho^h$, where it holds that $\beta\rho^h \to 0$ as $h \to \infty$ (see also Section 5.1).



(a) United Kingdom

(b) Spain

Figure 3: Extension 1. R^2 values obtained by regressing forward-aggregated excess market returns on backward-aggregated market variance (blue line, with squares), consumption variance (red line, with circles), and (squared) economic policy uncertainty (yellow line, with diamonds) for different levels of aggregation (on the horizontal axis). The markers are spaced one year apart. **Left:** Results from the United Kingdom. **Right:** Results from Spain.

5 New model framework: scale components

Hence, there is a need for a predictive system that can describe the data at different frequencies, thereby enabling enhanced predictability. In short, in this framework, the data is modified, where a variable is split up in a 'transitory' component and a 'persistent' component. The persistent component can be split up again in a transitory and a persistent component. These two components are at a higher scale - and thus a lower frequency - than the first two components. This decomposition can be repeated infinitely until one has a decomposition of an infinite number of transitory components. This way, one could say that the observations are split up into layers (the components), where each layer is the movement across a different frequency.

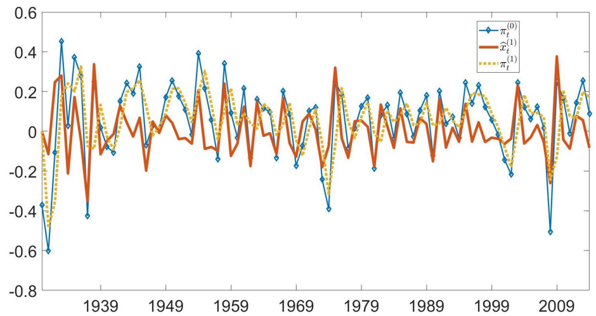
Following is a more detailed description. We start with the time series $\{x_{t-i}\}_{i\in\mathbb{Z}}$ and suppose we want to decompose this series into components with scales $j = 1, \ldots, J$. Let us first set $J = 1$, then we have

$$x_t = \frac{x_t - x_{t-1}}{2} + \frac{x_t + x_{t-1}}{2} ,$$

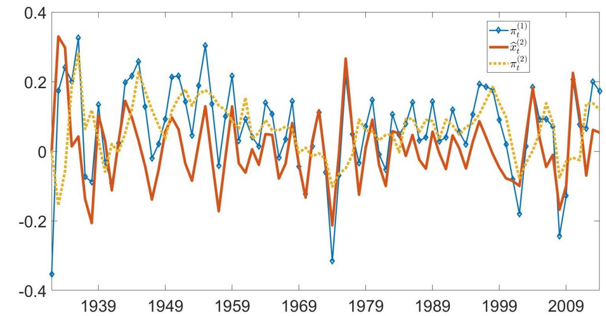
where the first term is the first transitory component $\widehat{x}_t^{(1)}$, and the second term is the first persistent component $\pi_t^{(1)}$. The fist transitory component $\widehat{x}_t^{(1)}$ is the component that operates across scale one. Now, set $J = 2$. That gives us

$$x_t = \frac{x_t - x_{t-1}}{2} + \frac{x_t + x_{t-1} - x_{t-2} - x_{t-3}}{4} + \frac{x_t + x_{t-1} + x_{t-2} + x_{t-3}}{4} ,$$

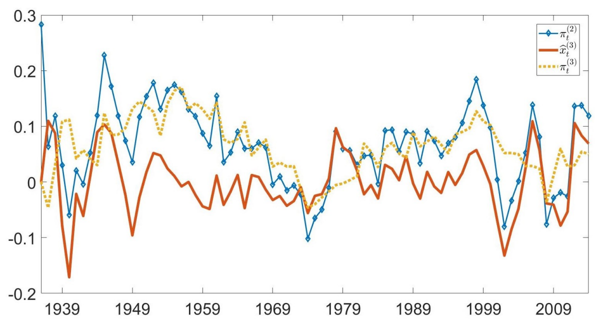
where you can see the persistent component is further split up into the next transitory component $\widehat{x}_t^{(2)}$ and the next persistent component $\pi_t^{(2)}$. The second transitory component is the part of x_t that operates across the second scale, which is a lower frequency than the first scale.



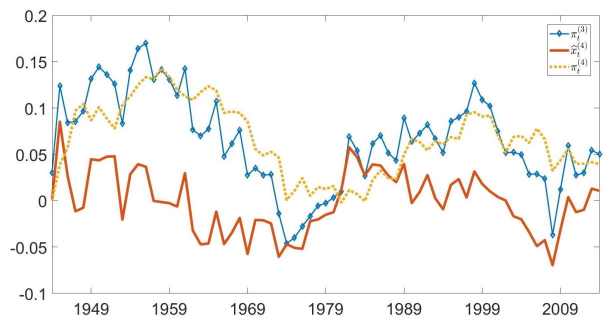
(a) Decomposition to scale $j = 1$



(b) Decomposition to scale $j = 2$



(c) Decomposition to scale $j = 3$



(d) Decomposition to scale $j = 4$

Figure 4: The decomposition of the excess market returns from the yearly observations of the United States. The blue line with diamonds shows the persistent component $\pi_t^{(j-1)}$. This persistent component can be decomposed into a transitory component $\widehat{x}_t^{(j)}$ (solid red line) and a persistent component $\pi_t^{(j)}$ (dashed yellow line) of a higher scale.

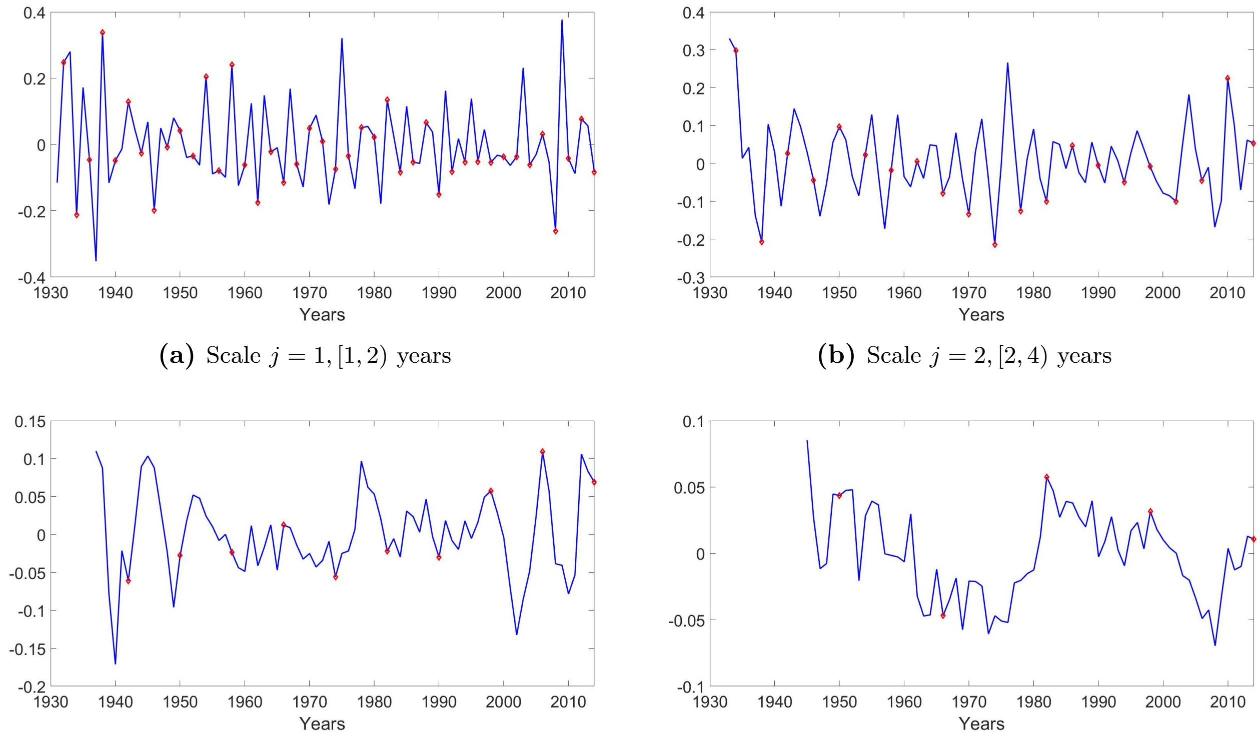
This procedure can be iterated, which gives us the general expression for the component $\widehat{x}_t^{(j)}$.

$$\widehat{x}_t^{(j)} = \frac{\sum_{i=0}^{2^{(j-1)}-1} x_{t-i}}{2^{(j-1)}} - \frac{\sum_{i=0}^{2^j-1} x_{t-i}}{2^j} = \pi_t^{(j-1)} - \pi_t^{(j)} \ , \tag{2}$$

where it then holds that $\pi_t^{(j)}$ is the average of the past 2^j observations $x_{t-2^j+1}, \ldots, x_t$,

$$\pi_t^{(j)} = \frac{1}{2^j} \sum_{i=0}^{2^j-1} x_{t-i} \ .$$

The procedure of decomposition is also presented in Figure 4 for the yearly excess market return series of the United States. The blue line with diamonds represents the persistent component at scale $j-1$. The red line shows the new transitory component and the yellow dashed line represents the new persistent component at scale j. The filtered components can be viewed as changes in information between scale $j-1$ and scale j. The innovations $\widehat{x}_t^{(j)}$ become smoother, and more persistent in calendar time, as j increases (Bandi et al., 2019).



(a) Scale $j = 1, [1, 2)$ years

(b) Scale $j = 2, [2, 4)$ years

(c) Scale $j = 3, [4, 8)$ years

(d) Scale $j = 4, [8, 16)$ years

Figure 5: Decomposition into components of excess market returns. The calendar-time observations, made with Eq. (2), are solid blue lines and the scale-time or "decimated" observations are red diamonds.

Ultimately, x_t can be written as

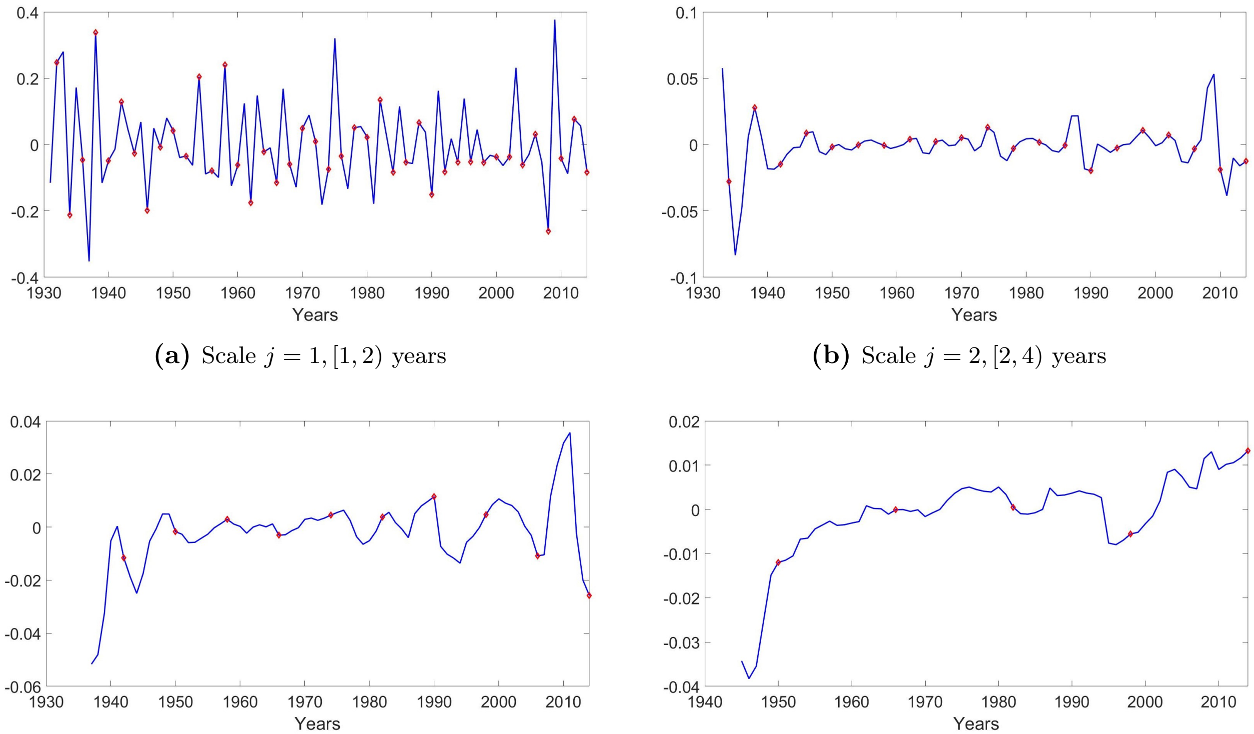
$$x_t = \sum_{j=1}^{J} \widehat{x}_t^{(j)} + \pi_t^{(J)} = \sum_{j=1}^{J} \left\{ \pi_t^{(j-1)} - \pi_t^{(j)} \right\} + \pi_t^{(J)} = \pi_t^{(0)} \ ,$$

in which the filtered components are naturally viewed as changes in information between scale 2^{j-1} and scale 2^j (Bandi et al., 2019).

We can also write a convenient representation of the filter using a suitable projection operator. Here it is given for $J = 2$. In this paper, the replication part (the part for which this is relevant) works with $J = 4$. That case is shown in Appendix B. Thus, for $J = 2$ it is as follows

$$\begin{pmatrix} \pi_t^{(2)} \\ \widehat{x}_t^{(2)} \\ \widehat{x}_t^{(1)} \\ \widehat{x}_{t-2}^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \end{pmatrix} . \tag{3}$$

We denote by $\mathcal{T}^{(2)}$ the (4×4) matrix. Note here that $\mathcal{T}^{(2)}$ is orthogonal, which means that $\left(\mathcal{T}^{(2)}\right)^{-1}$ is well-defined. Thus, by matrix-inversion we can reconstruct the original process, given the filtered



(a) Scale $j = 1, [1, 2)$ years

(b) Scale $j = 2, [2, 4)$ years

(c) Scale $j = 3, [4, 8)$ years

(d) Scale $j = 4, [8, 16)$ years

Figure 6: Decomposition into components of market variance. The calendar-time observations, made with Eq. (2), are solid blue lines and the scale-time or "decimated" observations are red diamonds.

components, as follows

$$\begin{pmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \end{pmatrix} = \left(\mathcal{T}^{(2)}\right)^{-1} \begin{pmatrix} \pi_t^{(2)} \\ \widehat{x}_t^{(2)} \\ \widehat{x}_t^{(1)} \\ \widehat{x}_{t-2}^{(1)} \end{pmatrix} . \tag{4}$$

The last step is decimation. That is the process of selecting *non-overlapping* points. This is quite simple. Take for instance $x_{t-1}^{(1)}$, this component uses observations x_{t-1} and x_{t-2}, but as you can see in Eq. (3), those observations are also used to construct $x_t^{(1)}$ and $x_{t-2}^{(1)}$. Thus, decimation is the process of selecting only $x_t^{(1)}$ and $x_{t-2}^{(1)}$. If we then take the series of decimated observations, the time t will vary over the set $\left\{t = k2^2 \text{ with } k \in \mathbb{Z}\right\}$. Generally, the decimated observations are $\left\{\widehat{x}_t^{(j)}, t = k2^j \text{ with } k \in \mathbb{Z}\right\}$.

The scale components and their decimated counterparts for the yearly observations of excess market returns and market variance of the United States market are shown in Figure 5 and 6 respectively. For completeness, the same graphs for the consumption variance and economic policy uncertainty are included in Appendix C. The pairwise correlations between the *redundant* scale series (the scales series before decimation) are presented in Table 1. This is in line with the correlation

Table 1: Pairwise correlations. Replication of Table 5 in Bandi et al. (2019). Pairwise correlations between the components of excess market returns (Panel A), consumption variance (Panel B), market variance (Panel C), and (squared) EPU (Panel D). The pair-wise correlations are obtained by using *redundant* (i.e. calendar time) observations on the components rather than their decimated counterparts. The correlation between the decimated observations are presented between brackets.

	Panel A:				**Panel B:**			
	Market excess returns				Consumption variance			
Scale $j =$	1	2	3	4	1	2	3	4
1		-0.03 (0.36)	0.01 (-0.15)	0.09 (0.61)		0.21 (0.21)	0.09 (-0.32)	-0.06 (-0.42)
2			-0.18 (-0.52)	0.16 (0.68)			0.28 (-0.71)	-0.08 (-0.39)
3				0.17 (0.02)				0.10 (0.25)
	Panel C:				**Panel D:**			
	Market variance				Economic policy uncertainty			
Scale $j =$	1	2	3	4	1	2	3	4
1		0.17 (-0.68)	-0.12 (-0.22)	-0.04 (-0.43)		0.11 (-0.07)	0.00 (-0.28)	-0.02 (-0.26)
2			-0.07 (0.04)	-0.05 (0.31)			0.21 (-0.24)	0.03 (-0.12)
3				0.19 (0.89)				0.40 (0.07)

table in Bandi et al. (2019). It does, however, make more sense to find the correlations between the decimated observations. The components of different scales should be independent of each other. Hence, the correlation between the decimated observations is presented between brackets. However, these are not independent. The correlations are often constructed with very few observations. For example, when the fourth scale is compared to another, only five observations are available. Moreover, there are sixteen possibilities as to which observations those five are; one can select observations at time 1, 17, . . ., or at time 2, 18, . . ., as the decimated observations. Furthermore, one has to select the five decimated observations from one of the other scale components to compare with the fourth scale. Then, taking the correlations between the redundant observations avoids this variation, as it is like taking the average of all possibilities. Thus, both approaches have their advantages and disadvantages.

5.1 Simulations

Besides explaining theoretically that traditional predictive systems are not able to model hump-shaped behavior, Bandi et al. (2019) show the same with simulations. Here, the details of the simulations will be explained. Full tables are reported in Appendix E. The parameters estimates found in the data are the same as the findings of Bandi et al. (2019), as the aim is to replicate their findings as closely as possible.

Firstly, the data are simulated under the assumption of predictability and an AR(1) process for v_t,

$$r_{t+1} = \beta v_t + u_{t+1} \ , \tag{5}$$

$$v_{t+1} = \rho v_t + e_{t+1} \ , \tag{6}$$

where the parameters are the estimates from the data of excess market returns and consumption variance (yearly observations from the U.S.). These are $\beta = 1.80, \rho = 0.734, \sigma_e = 0.180, \sigma_u = 0.0095$, and $\rho_{u,e} = -0.045$.

With the simulated data (100,000 replications with 85 observations), forward/backward aggregated regressions are run, as in Eq. (1). As noted before, when $|\rho| < 1$, the slopes become $\beta\rho^h \to 0$, hence they should decrease with the horizon h. Simulations indeed confirm this. Instead, the percentages of simulations that find monotonically increasing β estimates between 6 and 12 years, monotonically decreasing estimates between 16 and 20 years, and hump-shaped β estimates (both metrics) are 13.30%, 38.58%, and 5.85% respectively. In the data, the R^2s also exhibit hump-shaped behavior. If we apply the same three metrics to the R^2 estimates, we find percentages equal to 18.04%, 18.76%, and 7.05%, respectively. However, we find hump-shaped behavior of both the slope and R^2 estimates in the data.

Subsequently, we set $\beta = 0$. In this case, the estimates of the parameters from the data are $\rho = 0.734, \sigma_u = 0.195, \sigma_e = 0.0095$, and $\rho_{u,e} = -0.045$. The observations are simulated following the model

$$r_{t+1} = u_{t+1} \ , \tag{7}$$

$$v_{t+1} = \rho v_t + e_{t+1} \ . \tag{8}$$

If the data generating process does not incorporate predictability, i.e., $\beta = 0$, the returns only consist of shocks. When aggregating the returns, one is simply summing up shocks. The influence of these shocks on the following ones does not decrease with time, causing unit-root behavior. One would expect the estimated slopes to be slightly increasing (Bandi et al., 2016), which is inconsistent with the hump-shaped structure found in the data. Indeed, the percentages of simulations that generate hump-shaped slopes and R^2s are 8.05% and 7.64% respectively. When we additionally require the maximum of the R^2s to be higher than 50%, the latter percentage becomes 3.33%. If we were to require the estimates for the slopes and R^2s to be hump-shaped, $\max(R^2) > 50\%$, and the decline of R^2 after the hump to be larger than 30%, which is all consistent with the data, the percentage of simulations showing this behavior becomes 1.22%.

The following simulations are to prove that scale-wise predictive systems can generate hump-shaped behavior. Doing so consists of three parts. The first one is showing that scale-specific predictability implies predictability when aggregating forward and backward. Secondly, it is shown that, if the data is generated under predictability, trying to explain it at the same time, that is, using forward/forward aggregated regressions, will lead to insignificant results. Then thirdly, if predictability of the components is not incorporated, forward/backward aggregation leads to insignificant outcomes as well.

As before, we first simulate observations (128 observations with 100,000 replications) with scale-specific predictability, with scales $j = 1, \ldots, J$. Predictability is only applied on the j^*-th component. In the data, predictability is found strongest at 16 years, which corresponds with the fourth scale.

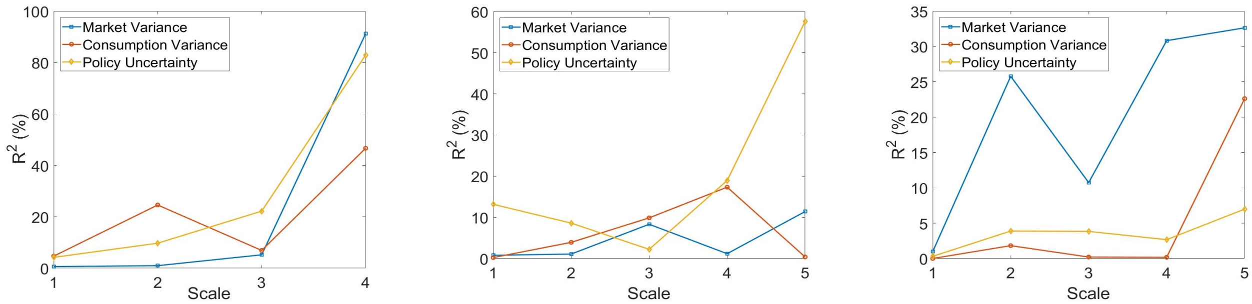
Hence, we set $j^* = 4$. Then the model is as follows:

$$r^{(j^*)}_{k2^{j^*}+2^{j^*}} = \beta_{j^*} v^{(j^*)}_{k2^{j^*}} \qquad\qquad r^{(j)}_{k2^j+2^j} = u^{(j)}_{k2^j+2^j} \ ,$$
$$v^{(j^*)}_{k2^{j^*}+2^{j^*}} = \rho_{j^*} v^{(j^*)}_{k2^{j^*}} + e^{(j^*)}_{k2^{j^*}+2^{j^*}} \qquad\qquad v^{(j)}_{k2^j+2^j} = e^{(j)}_{k2^j+2^j} \ .$$

The shocks $u^{(j)}_t$ and $e^{(j)}_t$ satisfy $\text{corr}\big(u^{(j)}_t, e^{(j)}_t\big) = 0 \ \forall \ t, j$. The parameters are equal to $\beta_{j^*} = 2.8, \rho_{j^*} = 0.20, \sigma^{(1)2}_u = 0.02, \sigma^{(2)2}_u = 0.012, \sigma^{(3)2}_u = 0.005, \sigma^{(1)2}_e = 0.276, \sigma^{(2)2}_e = 0.301, \sigma^{(3)2}_e = 0.218$, and lastly $\sigma^{(4)2}_e = (1-\rho^2_4)0.180$ to represent the unconditional variance of the shocks of the variance series. The variance estimates for the error term e are not the same as the one used by Bandi et al., because these values are unclear. These values are equal to the variance found in the scale components of consumption variance. Also, the simulated data are decimated points. Calendar time observations (demeaned) are obtained with the inverse of the Haar matrix, see Eq. (4) and Appendix B.

Then for the first part, we expect to find hump-shaped behavior, when running the regression as in Eq. (1) on the simulated data. Since the observations are demeaned, the intercept is not included. The hump-shaped behavior found by Bandi et al. also appears in the simulations here, but they differ in magnitude. Then, very importantly, the hump-shaped behavior found in the R^2s does not appear in the same way. One could call it hump-shaped behavior, but there are two humps. The medians of the adjusted R^2s for horizon 8 to 11 are aberrant. Then, again in agreement with behavior found in the data, the R^2 reaches its maximum at horizon 16. Due to this decrease between 8 and 10 years, the percentage of simulations that have a monotonically increasing R^2 between 6 and 12 years is only 0.33%. This results in only 0.29% of simulations being able to generate hump-shaped behavior. This would mean that the core of the evidence in support of scale-specific predictability being better able than traditional systems to generate the hump-shaped behavior found in the data, is not valid. As there has previously also been some ambiguity surrounding consumption variance (at least for the U.S.), these simulations were also run with the parameter estimates from economic policy uncertainty. These results can also be found in Appendix E. The parameter estimates for that simulation are from the replication in this paper, to see if the scale-specific data generating process and scale regressions would generate the desired effects. These are hump-shaped R^2 and slope estimates, with for the R^2s specifically, monotonically increasing estimates between 6 and 12 years, monotonically decreasing estimates between 16 and 20 years, a R^2 maximum of at least 50%, and a decline in estimates of at least 30% between 16 and 20 years. Again, there is a small hump at seven years, with a local minimum at ten years, before increasing to the big hump at 16 years. Hence, the strange results with consumption variance are not caused by the ambiguity surrounding consumption variance, but rather because the model seems to generate different results than expected. It is still hump-shaped, but not monotonically increasing between 6 and 12 years; it seems more often to be monotonically increasing between 12 and 16 years.

However, the results from the second part with forward/forward regressions and the third part with $\beta_{j^*} = 0$, i.e., no predictability is assumed, show nothing out of the ordinary. The medians



(a) Replication: U.S. **(b)** Extension 2: Hong Kong **(c)** Extension 2: South Korea

Figure 7: R^2 values obtained by scale specific regressions (scales along x-axis) of excess market returns on market variance (blue line with squares), on consumption variance (red line with circles), and on (squared) economic policy uncertainty (yellow line with diamonds). **Left:** Results from the replication with yearly observations from the U.S. **Middle:** Results from the second extension with monthly observations from Hong Kong **Right:** Results from the second extension with monthly observations from South Korea.

of the slope estimates are decreasing for the second part and remain zero for the third part. The medians of the R^2s show slightly hump-shaped behavior, although much smaller in magnitude for the second part and they are slightly increasing for the third.

5.2 Scale Regressions

The scale regressions are given by

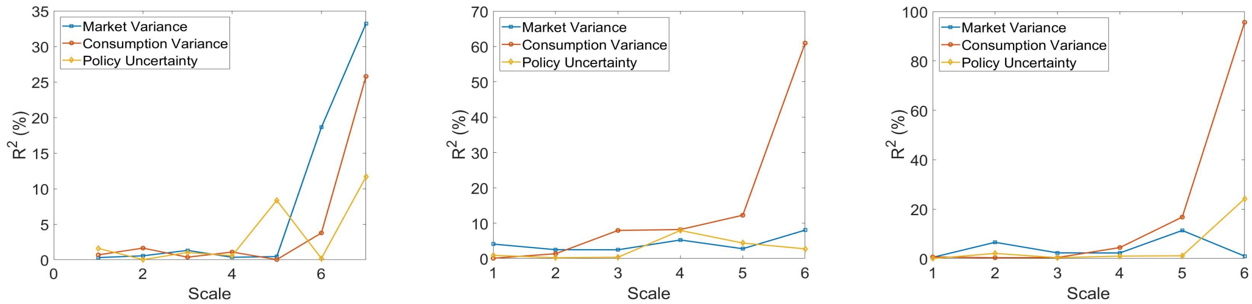
$$r^{(j)}_{k2^j+2^j} = \beta_j v^{(j)}_{k2^j} + u^{(j)}_{k2^j+2^j} \, , \tag{9}$$

where $r^{(j)}_{k2^j}$ are the decimated observations of the scale components of the logarithmic excess market returns and $v^{(j)}_{k2^j}$ are the decimated observations of the scale components of the market variance (proxy).

The results of the replication are shown in Figure 7, in the left graph. As expected, the R^2s for the fourth scale are highest and much higher than for the other scales. The R^2s even reach higher values than the ones found in Bandi et al. (2019), but more notably, they differ from the findings from the two-way aggregation regressions. In those regressions, the predictability of consumption variance is highest, while it is lowest in the scale regressions. This again confirms the ambiguity surrounding the consumption variance for the yearly observations of the U.S. Moreover, for the consumption variance, the R^2 of the fourth scale is bounded by $[5.13\%, 88.19\%]$ with 90% certainty. Hence, the value of 46.66% is not reliable.

This is linked to a significant disadvantage of low-frequency analyses. The predictive regressions in the higher scales use very few observations. Bandi et al. (2019) state that they do not believe that this difficulty implies the inference of low frequency is any less important, but it is required that such inference is conducted carefully.

A general note on the confidence intervals of the R^2s: if the R^2 becomes close to zero or one, the CI becomes tight. This is caused by the SE of the R^2 because the term $R^2(1 - R^2) \to 0$ when



(a) United States **(b)** United Kingdom **(c)** Spain

Figure 8: Extension 2. R^2 values obtained by scale specific regressions (scales along x-axis) of excess market returns on market variance (blue line with squares), consumption variance (red line with circles), and on (squared) economic policy uncertainty (yellow line with diamonds) for monthly data. **Left:** the United States. **Middle:** the United Kingdom. **Right:** Spain.

$R^2 \to 0$ or $R^2 \to 1$. Intuitively, it can be explained as follows; when the R^2 is low, there is almost no correlation between the variables. One can be relatively certain of that. Namely, if there were just many outliers, but also a lot of correlated observations, the R^2 would have been slightly higher. Hence, a tight CI makes sense. Same goes if the R^2 is high: the variables are highly correlated. If there are many outliers, the R^2 would not have been this high. Hence, again, one can be relatively certain that the R^2 is, in fact, this high. Moreover, the CI is symmetric. One might not expect this, because the R^2 is bounded by $[0, 1]$. Thus, there is also reason to doubt this method. A different approach could be bootstrapping. Although, that is just for the two-way aggregated regressions. The scale regressions use too little observations for bootstrapping to be possible. This also relates to the disadvantage of the scale regressions model. One can never be certain about the reliability of it. Even though the regressions use highly informed observations, meaning many factors caused the observations to be exactly how they are, there is no statistically substantiated way to confirm the correctness of the model.

For the monthly observations of the five countries, scale regressions were also run (extension 2), see Figure 7 (middle and right graph) and Figure 8. To assess the performance of the scale-specific predictions, the results of the scale regressions are compared with the ones of the two-way aggregated regressions. For Hong Kong, five scales could be included, which means the predictive power of at most 32 months can be assessed. From Figure 2, one would expect the find rather small R^2s. It is then surprising that the R^2 of the fifth scale for policy uncertainty is almost 60%. The other market within the Asian-Pacific market is South-Korea, where one would, in contrast to Hong Kong, expect to find higher R^2s, especially in the fifth scale. This is not confirmed by the scale regressions, as the R^2s reach values of only 35% at most, while these are around 60% in the aggregated regressions.

In the European market, the R^2s for consumption variance are unexpectedly high, certainly compared to the two other market proxies. Again, this is not entirely supported by the results from the scale regressions. For both Spain and the U.K., it was possible to include a sixth scale, which accounts for processes operating at frequency lengths between 32 and 64 years. During this period, the predictability of excess market returns using consumption variance is somewhat high but not as high as indicated by the scale regressions. Also, it is especially not as high as the predictability during that period using the market variance as a proxy. This is, however, not found in the data.

Lastly, the R^2s for the U.S. do not grow as high, up till 35% at the seventh scale (64 to 128 years). Although, an increase is visible and presumably when taking Figure 1 into account, the R^2s in the eighth scale would likely be much higher. Unfortunately, this was not possible due to a lack of monthly data spanning such an extended period.

5.3 Extended Framework

In the extension, there are four models. The first two models are the two different predictor models from Bandi et al. (2019) explained above. The third model is the model in Eq. (10) with $j = l = m = n$. The fourth model is the same as model three, but without the restriction on the scales to be the same.

$$y^{(j)}_{k2^j+2^j} = \beta_{1,l} x^{(l)}_{1,k2^l} + \beta_{2,m} x^{(m)}_{2,k2^m} + \beta_{3,n} x^{(n)}_{3,k2^n} + u^{(j)}_{k2^j+2^j} \ , \tag{10}$$

The fourth model is quite tricky because one needs to make sure the times of the explanatory variables are never further than the time of the explained variable; otherwise, it would not be predicting. The constructing of the variables is done according to the following procedure.

Assume that we have monthly observations, all lined up such observation t for each variable corresponds to the same time. For example, we have data from 1990 to 2010, which means we have $20 \times 12 = 240$ observations for variable p_t and variable q_t. Then observations p_1 and q_1 correspond to January 1990, p_2 and q_2 correspond to February 1990, and so forth. When constructing the scale components, we already know the scales belonging to each variable, i.e., j, l, m, n are known. Then we go along the y observations and create the components such that they are independent, that is, the observations used to create the component do not overlap. For the first component, we use the first 2^j observations; for the second component, we use the second 2^j observations. In this way, the components are already decimated. Then we construct the variables x_1, x_2, and x_3. The explanation is with x_1; the other two variables are constructed in the same manner. Say we are at time $t = k2^j$, then $t + 1 = k2^j + 1$, and observation y_{t+1} is the first observation used to construct the next component, which we call c for now. Then the observations from time $t - 2^l + 1$ to t are used to construct the component of variable x_1. Then in the regression, this component is used to predict the component c. As a result, the consecutive components from the x_1 variable are possibly constructed with overlapping observations. Essentially, the y components are decimated and the x_1, x_2, and x_3 components are not decimated. The logic behind this is that ultimately, we want to predict y. It does not make sense to predict y twice, which is why y is decimated. It is possible however that a certain observation of x_t influences two different decimated y observations, which is why it is not a problem that the x_1, x_2, and x_3 observations are not decimated.

The univariate scale regressions go to the sixth scale for the U.K. and Spain, which is different in the multivariate scale regressions, where the highest scale for these countries is the fifth. The sixth

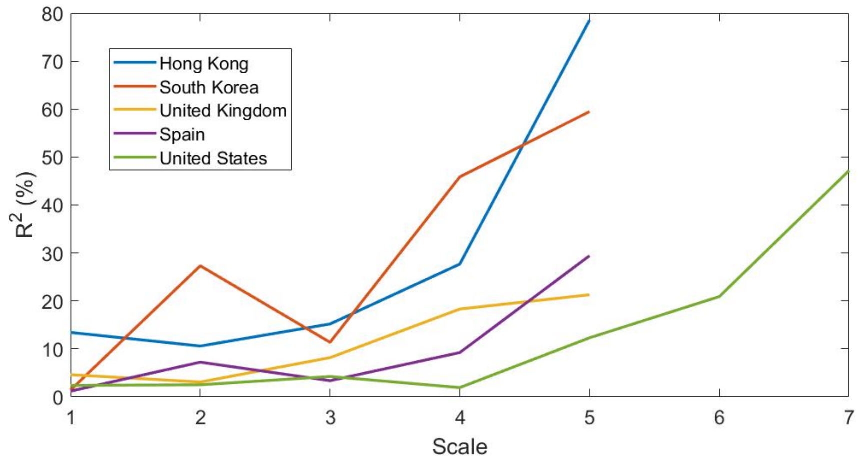


Figure 9: Extension 3. R^2 values obtained by scale specific regressions (scales along x-axis) of excess market returns on market variance, consumption variance, and on (squared) economic policy uncertainty for monthly data from Hong Kong (blue line), South Korea (red line), the United Kingdom (yellow line), Spain (purple line), and the United States (green line).

scale cannot be included in the multivariate regressions, because there are four observations for the U.K. and three observations for Spain. In the univariate regressions, this leads to an X matrix of 4×2 and 3×2, while in the multivariate regressions, one gets an X matrix of 4×4 and 3×4. Hence, for the U.K., there is an exact solution to the equation $y = \beta X$, and there are no error terms needed. Hence the R^2 is equal to 1, and there is no confidence interval. For Spain, the X matrix is rank deficient, and there are infinitely many solutions and also no error terms.

As for the results, the scale regressions using multiple explanatory variables (extension 3; Figure 9) performs much better than the regressions from extension 2. It shows that the R^2 are nicely increasing across the scales, although not always monotonically. Again, for the U.S., predictability is most likely much better for an eighth scale, if that were possible. An eight scale would display frequency lengths between 128 and 256 months, which is between about 10 and 21 years. This is consistent with the findings of the yearly observations. Moreover, the regressions for the Asian-Pacific markets, i.e., Hong Kong and South Korea, show quite good results. The R^2 at the highest scale, which is the fifth scale, reaches 59.5% and 78.5% in resp. South Korea and Hong Kong.

To see if the data shows hump-shaped behavior when multiple predictors are used, auxiliary two-way aggregated regressions have been run. The results are presented in Appendix F. The R^2s do not exhibit hump-shaped behavior. Instead, predictability just seems to increase with the aggregation horizon, or, i.e., with a declining number of observations.

The last part of the extension (extension 4) is the framework in its most expanded form, where both multiple explanatory variables and multiple scales are allowed. For every country, the number of regressions run is equal to the number of scales possible, say w, to the power three and multiplied by $(w-1)$, such that every possible combination of scales is run, except for those where all scales of the explanatory variables are equal to the scale of the predicted variable, since this is extension 3. For the U.S., this means that $6 \times 7^3 = 2058$ regressions were run. As this is too much to present here,

Table 2: Extension 4. The R^2s for the fourth model in the extension for the U.S., Hong Kong, South Korea, the U.K., and Spain. The R^2s are presented in decreasing order from left to right. The 15 highest values are reported with appurtenant scales, where the scales refer to the scale that a specific variable from the regression has.

Panel A: The United States															
scale y	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
scale x_1	3	3	3	2	3	3	3	3	2	3	2	3	7	7	3
scale x_2	2	1	6	6	5	3	5	7	3	7	7	3	3	3	5
scale x_3	7	7	7	4	1	6	4	7	4	4	4	7	6	3	7
R^2 (%)	94.7	94.7	94.4	94.2	93.9	93.7	93.7	92.3	91.7	91.1	91.0	91.0	90.7	90.4	89.3
Panel B: Hong Kong															
scale y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
scale x_1	5	1	3	2	4	5	1	3	2	5	4	5	5	5	5
scale x_2	4	4	4	4	4	3	3	3	3	1	3	4	2	3	4
scale x_3	3	3	3	3	3	3	3	3	3	2	3	2	2	2	4
R^2 (%)	99.6	99.6	99.6	99.5	99.3	98.5	98.4	98.3	98.2	97.6	97.5	97.2	96.2	93.3	89.7
Panel C: South Korea															
scale y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
scale x_1	2	3	2	2	3	3	3	1	1	2	5	5	4	4	4
scale x_2	3	4	2	1	1	2	3	1	2	5	1	2	1	2	4
scale x_3	2	2	2	2	2	2	2	5	5	2	5	5	5	5	2
R^2 (%)	97.6	93.9	93.0	93.0	89.2	89.2	89.1	88.7	88.7	88.6	88.5	88.5	88.4	88.4	87.2
Panel D: The United Kingdom															
scale y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
scale x_1	3	3	5	1	4	2	3	4	5	2	3	5	1	5	2
scale x_2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4
scale x_3	4	1	1	1	1	1	5	5	5	5	5	1	5	4	4
R^2 (%)	95.0	94.7	90.5	90.0	89.0	88.9	88.0	78.6	78.1	76.1	75.9	75.4	75.2	74.2	72.4
Panel E: Spain															
scale y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
scale x_1	5	5	2	2	5	4	4	4	4	1	3	4	3	3	2
scale x_2	2	1	1	2	3	2	1	1	2	5	4	3	1	2	3
scale x_3	1	1	1	1	1	4	4	1	1	2	1	1	1	1	1
R^2 (%)	74.2	74.2	71.1	71.1	68.5	67.9	67.9	67.7	67.7	67.0	66.6	66.0	65.8	65.8	65.7

the 15 regressions with the highest R^2s are reported in Table 2. The results of the other regressions are available upon request.

The first thing to notice is that for all countries, every regression in the top 15 is for the highest possible scale of the predicted variable. Secondly, the R^2s can become extraordinarily high, especially in the Asian markets with 99.6% in Hong Kong and 97.6% in South Korea. Even though it is sometimes apparent that a variable gives the best predictions for one particular scale, this is not always the case. Thus for actual predicting, it is difficult to determine which scales will be most informative. Though, most notably, the scales which seem to be most informative, are often not equal to the scale of the predicted variable.

Moreover, in only 3% of the five top-15 regressions are the scales of the explanatory variables equal to each other. In 51% of the regressions, there are no similar scales, in 47% two scales the same. Some interesting findings are that market variance has the highest predictive power in scale 3 for the U.S., while this seems to be arbitrary for the other countries. The consumption variance is most informative in the third and fourth scale for Hong Kong and the U.K., while this is again arbitrary for the other countries. Policy uncertainty has a very country-specific predictive power in terms of scales. The fourth and seventh scale is most informative for the U.S., the second and third scale have the highest predictive power in the Asian countries, and lastly, in the European markets, the first scale seems to be quite valuable. Still, there do not seem to be universal rules for the frequencies of different economic phenomena. Still, for a specific country, it is possible to construct a set of rules, although not for every variable.

The results in Table 2 are also presented in Appendix G, but with CIs for the R^2s. They are tightly bounded by their CIs, as they are usually above 80%. To assess reliability in a second way, the best version of extension four is run with auxiliary two-way aggregated regressions. These give R^2s between 25% and 50%. This is not bad, but the auxiliary two-way aggregated regressions for extension three can perform better. As the best performing predictions are always the ones with the highest scale possible for the predicted variable, it seems that, again, the low number of observations seems to cause excessively high R^2s.

6 Conclusion

In terms of predictability, hump-shaped behavior is found in yearly data from the United States. Early on, there is too much uncertainty, compared to the signal we are interested in, to make useful predictions. The predictive power of the last 16 years of market volatility (market variance, consumption variance, and economic policy uncertainty) to predict the next 16 years of excess market returns, is strongest. After that, it is too far into the future, and the uncertainty compared to the actual signal increases again, i.e., the R^2 decreases.

As traditional h-step ahead predictive systems are strictly parameterized upon aggregation, Bandi et al. introduces a new methodological framework in which scale components of the time series of interest are used to predict one-another. Scale components are parts or layers of the data that operate with certain frequencies, depending on the specific scale. With simulations, it was shown that the behavior found in the data could be modeled reasonably well with the new methodological framework and much better than with traditional systems. The simulations in this paper differ from those in Bandi et al. (2019). Here it is shown that the hump-shaped behavior is modeled mostly as expected. However, there is a first, much smaller hump before the hump at 16 years. The results from Bandi et al. (2019) seem to be valid and confirm the hump-shaped behavior in the yearly U.S. data with two-way aggregated and scale regressions. Though, it remains unclear how the consumption variance was modeled, and whether it is informative since it is not for the monthly data where it was modeled differently.

The hump-shaped behavior is treated as a new stylized fact, implying that it is universal. This paper examines whether that is true by applying the new framework to different markets in the Asian-Pacific region and Europe. With forward/backward aggregated regressions, it is found that the hump-shaped behavior cannot be treated as a universal stylized fact with certainty. For every country, predictability seems instead to behave in a wave-like structure, than that there is one clean, smooth hump. The univariate scale regressions confirm this. Only in the yearly analysis of the U.S. is there a scale (the fourth) for which the R^2s are substantially high for all three explanatory variables.

For all monthly analyses, there is a tendency for the R^2s to increase, but hump-shaped behavior cannot be confirmed. Instead, it is probable that with monthly analyses, the calendar steps grow too fast with the scales. Take, for example, the fifth scale. It accounts for frequencies in the data between 16 and 32 months. Predictability could be both low and high within this time frame. The scale regressions will assign only one R^2 to this period, which means one could miss important information. Moreover, the hump found at 16 years in the U.S. is difficult to compare with the other countries, since these countries do not have 32+ years of data accessible. Thus, it is not possible to prove whether other countries have a hump at 16 years. Besides, the practical use of analyses should be considered as well, which is somewhat weak here.

Adding all explanatory variables together in one scale regression improves predictability, although there is room for even more improvement. Allowing for different scales can improve predictive power drastically. Predictability is most reliable at the highest scale for the excess market returns. Predictability benefits from the absence of the restriction that scales should be the same for all variables. Namely, in the absence of this restriction, it is found that in 3% of the cases, the scales of all three explanatory variables are the same. Furthermore, the scales that are most informative for each variable across the top-15 show some constancies, but no universal laws are governing the frequencies of market variance, consumption variance, and economic policy uncertainty. That is disappointing for actual predictions because it is difficult to determine beforehand which combination of scales will be most informative. However, if one were to focus on a particular country or market, there are tendencies of the variables that can guide us in the right direction.

Lastly, throughout this paper, it is confirmed multiple times that the scale regressions yield unreliable results. The confidence intervals for the R^2s are often too wide, which makes it impossible to assess the performance of the new model correctly. It firstly needs to be improved. One suggestion would be to incorporate more information in the regressions. For instance, one could include multiple scales of the same variable.

For future research, it would be exciting to look at the implications of seasonality. In monthly observations, it is possible that data exhibits seasonal trends. Although typical seasonal trends are not identified, there may be structures operating at seasonal levels. Next to that, the scale-specific predictability framework could be useful, but one would need to develop a stronger methodological foundation for the expanded model, as the work that is done in this paper is mostly empirical.

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A Data Overview

For the replication, yearly observations from 1930 to 2014 were used. For the extension, the monthly observations from Hong Kong are from April 1998 to December 2015; from South Korea, they are from May 1995 to December 2015; from the U.K. they are from January 1987 to December 2015; and finally, for Spain, they are from June 1990 to December 2015 as well. To be able to compare results from between analyses with yearly and monthly observations, the extension was also performed for monthly observations from the U.S., which are from January 1934 to October 2014.

The excess market returns need either the yearly or monthly returns from a particular stock in the country‘s market and a risk-free rate. For the replication, the returns are the yearly observations of the NYSE/Amex value-weighted return indices with dividends, which are obtained from the Chicago Center for Research in Security Prices through the Wharton Research Data Services online database. The risk-free rate is the yield of the 3-Month Treasury Bill: Secondary Market Rate from FRED, Federal Reserve Bank of St. Louis. It records the monthly observations. For each year, the observation from December was used. The observations from 1930 to 1933 are missing in the FRED database, but for this research, they were retrieved from the dataset received from the authors. Furthermore, for the extension, the returns and risk-free rate were obtained through Datastream. For Hong Kong, South-Korea, the U.K., and Spain the following stock indices were used respectively: the Hang Seng index, the KOSPI index, the FTSE 100 index, and the IBEX 35 index. The indices were retrieved as price indices. The gross returns are calculated as $R_t = (P_t - P_{t-1})/P_{t-1}$. The risk-free rate is, for Hong Kong, the U.K., and Spain, the yield of the 3-Month Treasury Bill from the particular country. For South Korea, it is the yield of the 3-Year Treasury Bond.

For the replication, the annual consumption is from the Bureau of Economic Analysis, series 7.1, and is defined as consumer expenditures on non-durables and services. The measure of consumption variance is based on modeling consumption growth as an AR(1) with an error variance evolving as a heterogeneous ARCH model. The HARCH dynamics accommodate heterogeneous information arrival. The exact HARCH data was used from the dataset I received from the authors. For the extension, the consumption data is obtained from the World Bank. The World Bank records yearly observations on Households and NPISHs (non-profit institutions serving households) Final consumption expenditure per capita. The variance is modeled as the variance of the error terms of an AR(1) model of consumption growth, thus following an AR(1)-GARCH(1,1) model. The variance is then turned into monthly observations using linear interpolation.

The measure of economic policy uncertainty (EPU) is based on Baker et al. (2016). The authors have a website on which they update the EPU measure for different counties. The variable used in this research is defined as $\mathrm{EPU}_t = \left(\mathrm{EPU}_t^*/100\right)^2$ where EPU^* is the value as found in the dataset on the Baker et al. (2016) website. These observations are identical to the ones used by Bandi et al. (2019).

B Haar Transform

$$
\begin{pmatrix}
\pi_t^{(4)} \\
\widehat{x}_t^{(4)} \\
\widehat{x}_t^{(3)} \\
\widehat{x}_t^{(2)} \\
\widehat{x}_t^{(1)} \\
\widehat{x}_{t-8}^{(3)} \\
\widehat{x}_{t-12}^{(2)} \\
\widehat{x}_{t-8}^{(2)} \\
\widehat{x}_{t-4}^{(2)} \\
\widehat{x}_{t-14}^{(1)} \\
\widehat{x}_{t-12}^{(1)} \\
\widehat{x}_{t-10}^{(1)} \\
\widehat{x}_{t-8}^{(1)} \\
\widehat{x}_{t-6}^{(1)} \\
\widehat{x}_{t-4}^{(1)} \\
\widehat{x}_{t-2}^{(1)}
\end{pmatrix}
= \mathcal{T}^{(4)}
\begin{pmatrix}
x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ x_{t-4} \\ x_{t-5} \\ x_{t-6} \\ x_{t-7} \\ x_{t-8} \\ x_{t-9} \\ x_{t-10} \\ x_{t-11} \\ x_{t-12} \\ x_{t-13} \\ x_{t-14} \\ x_{t-15}
\end{pmatrix}
$$

Table 3: The Haar matrix $\mathcal{T}^{(4)}$

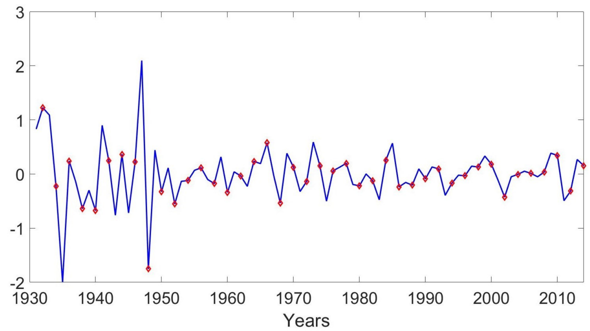
$$
\begin{bmatrix}
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} & -\frac{1}{16} \\
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} & -\frac{1}{8} \\
0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\
0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}
\end{bmatrix}
$$

$$
\begin{pmatrix}
x_t - \pi_t^{(4)} \\
x_{t-1} - \pi_t^{(4)} \\
x_{t-2} - \pi_t^{(4)} \\
x_{t-3} - \pi_t^{(4)} \\
x_{t-4} - \pi_t^{(4)} \\
x_{t-5} - \pi_t^{(4)} \\
x_{t-6} - \pi_t^{(4)} \\
x_{t-7} - \pi_t^{(4)} \\
x_{t-8} - \pi_t^{(4)} \\
x_{t-9} - \pi_t^{(4)} \\
x_{t-10} - \pi_t^{(4)} \\
x_{t-11} - \pi_t^{(4)} \\
x_{t-12} - \pi_t^{(4)} \\
x_{t-13} - \pi_t^{(4)} \\
x_{t-14} - \pi_t^{(4)} \\
x_{t-15} - \pi_t^{(4)}
\end{pmatrix}
= \left(\mathcal{T}^{*(4)}\right)^{-1}
\begin{pmatrix}
\widehat{x}_t^{(4)} \\
\widehat{x}_t^{(3)} \\
\widehat{x}_t^{(2)} \\
\widehat{x}_t^{(1)} \\
\widehat{x}_{t-8}^{(3)} \\
\widehat{x}_{t-12}^{(2)} \\
\widehat{x}_{t-8}^{(2)} \\
\widehat{x}_{t-4}^{(2)} \\
\widehat{x}_{t-14}^{(1)} \\
\widehat{x}_{t-12}^{(1)} \\
\widehat{x}_{t-10}^{(1)} \\
\widehat{x}_{t-8}^{(1)} \\
\widehat{x}_{t-6}^{(1)} \\
\widehat{x}_{t-4}^{(1)} \\
\widehat{x}_{t-2}^{(1)}
\end{pmatrix}
$$

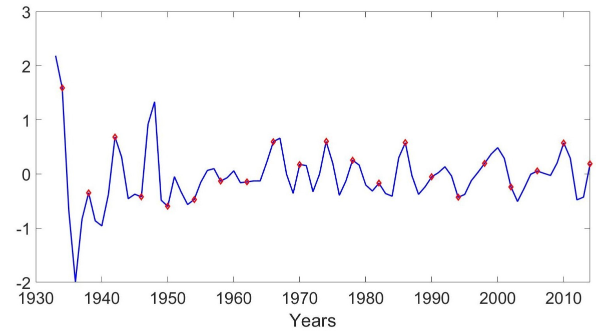
Table 4: The inverse of the Haar matrix $\left(\mathcal{T}^{*(4)}\right)^{-1}$ (without first column)

1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0
1	1	-1	0	0	0	0	0	1	0	0	0	0	0	0
1	1	-1	0	0	0	0	0	-1	0	0	0	0	0	0
1	-1	0	0	0	1	0	0	0	1	0	0	0	0	0
1	-1	0	0	0	1	0	0	0	-1	0	0	0	0	0
1	-1	0	0	0	-1	0	0	0	0	1	0	0	0	0
1	-1	0	0	0	-1	0	0	0	0	-1	0	0	0	0
-1	0	0	0	1	0	1	0	0	0	0	1	0	0	0
-1	0	0	0	1	0	1	0	0	0	0	-1	0	0	0
-1	0	0	0	1	0	-1	0	0	0	0	0	1	0	0
-1	0	0	0	1	0	-1	0	0	0	0	0	-1	0	0
-1	0	0	0	-1	0	0	1	0	0	0	0	0	1	0
-1	0	0	0	-1	0	0	1	0	0	0	0	0	-1	0
-1	0	0	0	-1	0	0	-1	0	0	0	0	0	0	1
-1	0	0	0	-1	0	0	-1	0	0	0	0	0	0	-1

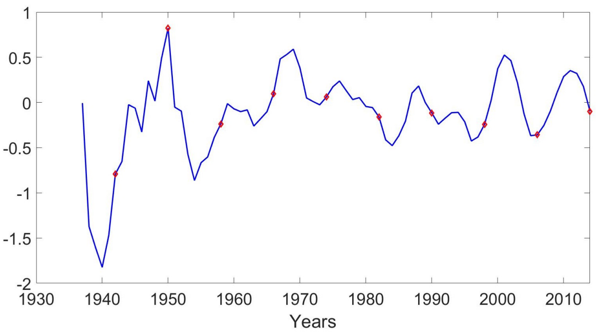
C Replication Decimation Graphs



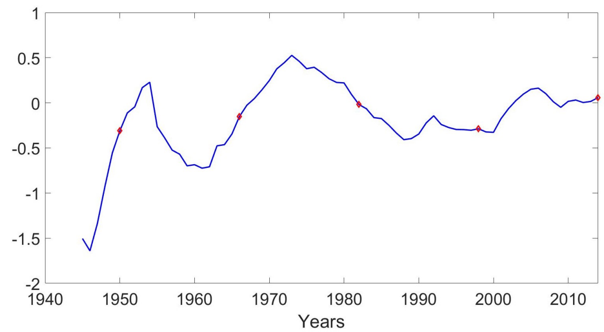
(a) Scale $j = 1, [1, 2)$ years



(b) Scale $j = 2, [2, 4)$ years

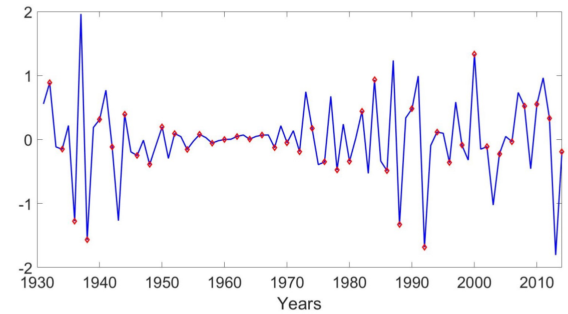


(c) Scale $j = 3, [4, 8)$ years

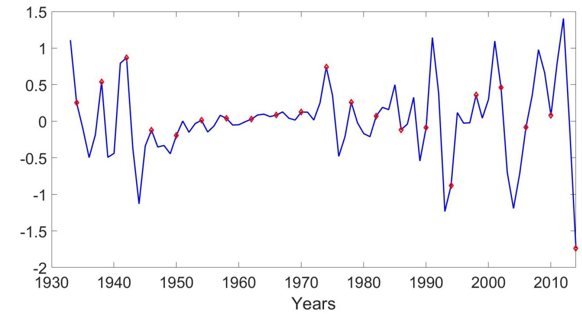


(d) Scale $j = 4, [8, 16)$ years

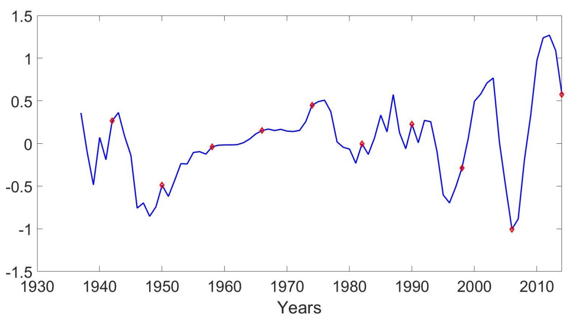
Figure 10: Decomposition into components of consumption variance. The calendar-time observations, made with Eq. (2), are solid blue lines and the scale-time or "decimated" observations are red diamonds.



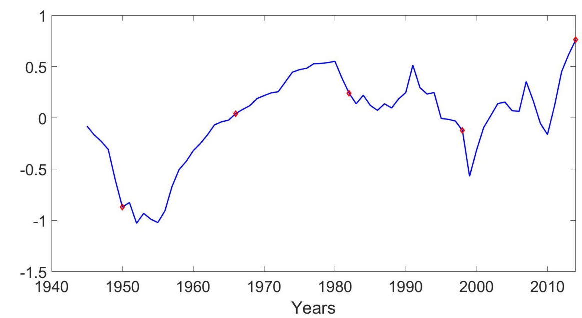
(a) Scale $j = 1, [1, 2)$ years



(b) Scale $j = 2, [2, 4)$ years



(c) Scale $j = 3, [4, 8)$ years



(d) Scale $j = 4, [8, 16)$ years

Figure 11: Decomposition into components of economic policy uncertainty. The calendar-time observations, made with Eq. (2), are solid blue lines and the scale-time or "decimated" observations are red diamonds.

D Replication Results Tables

Table 5: Market variance. **Panel A:** Replication of Table 1 in Bandi et al. (2019). Results from linear regressions of h-period continuously-compounded market returns on the CRSP value-weighted returns in excess of a 3-month Treasury bill rate on h-perdiod past market variance. For each regression, the table report OLS estimates, Newey-West t-statistics with h lags (one row beneath slope estimates)), the $t/\sqrt{T}$ tests suggested by Valkanov (2003) (two rows beneath slope estimates) and R^2s. Ninety-five percent confidence intervals for the R^2s are also reported (symmetrically computed with $\pm t(0.95, n-k-1) \cdot SE$). Significance at the 5%, 2.5%, and 1% level of the $t/\sqrt{T}$ test using Valkanov (2003) critical values is denoted by *, **, and ***, respectively. **Panel B:** Component-wise predictive regressions of the components of excess stock market returns on the components of market variance. For each regression, the table reports OLS estimates, Newey-West t-statistics with 2^j lags and R^2 values. The sample is annual and spans the period 1930-2014.

Panel A1: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$										
	horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
$v_{t-h+1,t}$	0.62	0.85	0.66	0.19	0.07	0.05	0.59	0.98	1.08	1.54
	1.43	2.89	2.08	0.60	0.15	0.10	1.31	2.24	2.07	2.91
	0.13	0.26	0.25	0.07	0.02	0.02	0.22	0.38	0.42*	0.60**
R^2 (%)	1.59	6.49	5.91	0.55	0.06	0.03	4.86	12.93	15.24	27.39
5^{th}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.06	2.52	12.58
95^{th}	5.92	14.89	14.06	3.21	0.96	0.71	12.71	24.81	27.96	42.20
Panel A2: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$										
	horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
$v_{t-h+1,t}$	1.87	2.14	2.24	2.13	2.13	2.06	2.03	1.84	1.50	0.99
	3.83	4.54	4.89	4.43	4.60	4.58	5.19	5.41	5.10	3.37
	0.75***	0.90***	0.98***	0.95***	0.98***	1.01***	1.07***	0.96***	0.72***	0.47**
R^2 (%)	37.02	45.64	49.74	48.25	50.06	51.45	54.59	48.88	35.14	18.79
5^{th}	21.87	30.91	35.31	33.39	35.21	36.57	40.01	33.07	17.81	0.00
95^{th}	52.17	60.37	64.18	63.12	64.91	66.33	69.17	64.69	52.46	34.96

Panel B: $r^{(j)}_{k2^j+2^j} = \beta^{(j)}_j v^{(j)}_{k2^j} + u^{(j)}_{k2^j+2^j}$				
	Scale j			
	1	2	3	4
$v^{(j)}_t$	-0.47	0.81	1.68	7.29
	-0.81	0.35	1.43	23.28
R^2 (%)	0.62	0.98	5.20	91.21
$[5^{th}, 95^{th}]$	[0.00, 4.30]	[0.00, 7.38]	[0.00, 23.71]	[81.64, 100.00]

Table 6: Consumption variance. **Panel A:** Replication of Table 1 in Bandi et al. (2019). Results from linear regressions of h-period continuously-compounded market returns on the CRSP value-weighted returns in excess of a 3-month Treasury bill rate on h-perdiod past consumption variance. For each regression, the table report OLS estimates, Newey-West t-statistics with h lags (one row beneath slope estimates)), the $t/\sqrt{T}$ tests suggested by Valkanov (2003) (two rows beneath slope estimates) and R^2s. Ninety-five percent confidence intervals for the R^2s are also reported (symmetrically computed with $\pm t(0.95, n-k-1) \cdot SE$). Significance at the 5%, 2.5%, and 1% level of the $t/\sqrt{T}$ test using Valkanov (2003) critical values is denoted by *, **, and ***, respectively. **Panel B:** Component-wise predictive regressions of the components of excess stock market returns on the components of consumption variance. For each regression, the table reports OLS estimates, Newey-West t-statistics with 2^j lags and R^2 values. The sample is annual and spans the period 1930-2014.

Panel A1: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$										
	horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
$v_{t-h+1,t}$	0.01	0.02	0.01	0.00	0.00	0.01	0.02	0.03	0.03	0.04
	1.01	1.67	0.80	0.24	0.35	0.79	1.96	2.78	2.86	4.06
	0.10	0.20	0.12	0.04	0.07	0.17	0.43**	0.62***	0.69***	0.91***
R^2 (%)	0.99	3.99	1.56	0.14	0.44	2.78	16.07	28.62	33.09	46.07
5^{th}	0.00	0.00	0.00	0.00	0.00	0.00	3.48	14.14	18.30	31.80
95^{th}	4.41	10.74	5.94	1.50	2.86	8.77	28.66	43.10	47.88	60.33
Panel A2: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$										
	horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
$v_{t-h+1,t}$	0.04	0.04	0.05	0.04	0.04	0.04	0.03	0.03	0.02	0.01
	5.24	6.08	6.72	6.99	7.64	7.82	8.63	7.78	5.20	2.51
	1.16***	1.32***	1.41***	1.32***	1.21***	1.11***	0.99***	0.79***	0.55**	0.31
R^2 (%)	58.22	64.33	67.42	64.25	60.32	56.17	50.75	39.19	23.78	9.11
5^{th}	45.62	52.86	56.52	52.40	47.36	42.14	35.50	22.35	7.03	0.00
95^{th}	70.82	75.81	78.31	76.10	73.27	70.21	66.00	56.03	40.32	21.72

Panel B: $r^{(j)}_{k2^j+2^j} = \beta^{(j)}_j v^{(j)}_{k2^j} + u^{(j)}_{k2^j+2^j}$				
	Scale j			
	1	2	3	4
$v^{(j)}_t$	-0.06	-0.10	-0.03	0.22
	-2.00	-5.01	-1.21	5.18
R^2 (%)	4.69	24.59	6.91	46.66
$[5^{th}, 95^{th}]$	[0.00, 14.66]	[0.22, 48.96]	[0.00, 27.85]	[5.13, 88.19]

Table 7: Economic Policy Uncertainty. **Panel A:** Replication of Table 1 in Bandi et al. (2019). Results from linear regressions of h-period continuously-compounded market returns on the CRSP value-weighted returns in excess of a 3-month Treasury bill rate on h-perdiod past consumption variance. For each regression, the table report OLS estimates, Newey-West t-statistics with h lags (one row beneath slope estimates)), the $t/\sqrt{T}$ tests suggested by Valkanov (2003) (two rows beneath slope estimates) and R^2s. Ninety-five percent confidence intervals for the R^2s are also reported (symmetrically computed with $\pm t(0.95, n-k-1) \cdot SE$). Significance at the 5%, 2.5%, and 1% level of the $t/\sqrt{T}$ test using Valkanov (2003) critical values is denoted by *, **, and ***, respectively. **Panel B:** Component-wise predictive regressions of the components of excess stock market returns on the components of consumption variance. For each regression, the table reports OLS estimates, Newey-West t-statistics with 2^j lags and R^2 values. The sample is annual and spans the period 1930-2014.

Panel A1: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$

	horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
$v_{t-h+1,t}$	0.02	0.02	0.01	0.01	0.01	0.01	0.02	0.03	0.03	0.03
	1.82	1.50	1.08	0.73	0.72	0.97	1.58	2.17	2.57	3.03
	0.18	0.17	0.16	0.13	0.14	0.21	0.37	0.53**	0.63***	0.74***
R^2 (%)	3.08	2.78	2.53	1.65	1.99	4.36	12.56	22.48	29.01	36.26
5^{th}	0.00	0.00	0.00	0.00	0.00	0.00	0.96	8.54	14.32	21.30
95^{th}	9.01	8.49	8.05	6.20	7.04	11.74	24.16	36.41	43.71	51.22

Panel A2: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$

	horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
$v_{t-h+1,t}$	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
	3.46	3.95	4.06	4.00	3.85	3.69	3.55	3.46	3.41	3.48
	0.84***	0.96***	1.04***	1.09***	1.11***	1.13***	1.09***	0.96***	0.75***	0.57**
R^2 (%)	42.17	49.05	52.62	55.35	55.97	57.03	55.49	49.12	37.19	25.07
5^{th}	27.32	34.74	38.63	41.61	42.12	43.17	41.08	33.35	19.93	0.00
95^{th}	57.01	63.37	66.62	69.09	69.81	70.90	69.90	64.90	54.45	42.31

Panel B: $r^{(j)}_{k2^j+2^j} = \beta^{(j)}_j v^{(j)}_{k2^j} + u^{(j)}_{k2^j+2^j}$

	Scale j			
	1	2	3	4
$v^{(j)}_t$	-0.04	-0.09	-0.06	0.08
	-1.15	-2.89	-5.73	22.81
R^2 (%)	4.21	9.73	22.20	82.94
$[5^{th}, 95^{th}]$	[0.00, 13.71]	[8.62, 28.07]	[0.00, 53.57]	[65.23, 100.00]

E Simulations

Table 8: Replication of Table A.1 in Online Supplement

Panel A: distribution of slope estimates										
	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Mean	1.84	1.56	1.28	1.03	0.83	0.65	0.51	0.38	0.27	0.17
Median	1.83	1.56	1.29	1.05	0.85	0.67	0.53	0.40	0.30	0.20
SD	1.54	1.62	1.72	1.82	1.93	2.04	2.15	2.27	2.39	2.53
[5th]	-0.68	-1.11	-1.55	-1.97	-2.36	-2.72	-3.03	-3.35	-3.67	-3.96
[95th]	4.36	4.21	4.07	3.98	3.94	3.93	3.97	4.02	4.12	4.21
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Mean	0.07	-0.01	-0.08	-0.15	-0.22	-0.29	-0.35	-0.40	-0.46	-0.51
Median	0.11	0.03	-0.05	-0.12	-0.19	-0.26	-0.32	-0.37	-0.43	-0.48
SD	2.67	2.81	2.97	3.13	3.30	3.47	3.65	3.83	4.01	4.19
[5^{th}]	-4.30	-4.62	-4.92	-5.26	-5.59	-5.92	-6.25	-6.56	-6.92	-7.23
[95^{th}]	4.34	4.50	4.67	4.83	4.99	5.24	5.46	5.70	5.95	6.17
Panel B: Distribution of R^2 estimates										
	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Mean	2.93	4.35	5.17	5.83	6.51	7.28	8.13	9.05	10.07	11.17
Median	1.86	2.55	2.80	3.05	3.37	3.75	4.25	4.83	5.46	6.14
SD	3.21	5.02	6.25	7.21	8.07	8.98	9.92	10.90	11.93	13.01
[5^{th}]	0.02	0.02	0.03	0.03	0.03	0.03	0.04	0.04	0.05	0.06
[95^{th}]	9.48	14.82	18.31	21.10	23.70	26.30	29.18	32.38	35.68	39.13
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Mean	12.36	13.61	14.94	16.32	17.73	19.17	20.60	22.02	23.42	24.78
Median	6.93	7.83	8.80	9.83	10.99	12.31	13.68	15.05	16.42	17.83
SD	14.14	15.28	16.41	17.53	18.59	19.60	20.55	21.45	22.31	23.15
[5^{th}]	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.14	0.16	0.17
[95^{th}]	42.80	46.26	50.05	53.70	56.98	60.10	63.01	65.73	68.38	70.73

Table 9: Additional metrics for simulation A.1

Additional metrics	
β increasing 6-12 (%)	13.3
β decreasing 16-20 (%)	38.58
β hump-shaped (%)	5.85
R^2 increasing 6-12 (%)	18.04
R^2 decreasing 16-20 (%)	18.76
R^2 hump-shaped (%)	7.05
R^2 hump-shaped & $R^2 > 50\%$ (%)	3.05
R^2 hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	2.78
R^2 and β hump-shaped (%)	2.30
R^2 and β hump-shaped & $R^2 > 50\%$ (%)	1.02
R^2 and β hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	0.93

Table 10: Replication of Table A.2 in Online Supplement

Panel A: distribution of slope estimates										
	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Mean	0.03	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.06	0.07
Median	0.03	0.03	0.03	0.04	0.04	0.04	0.05	0.06	0.07	0.08
SD	1.66	1.76	1.87	1.97	2.09	2.20	2.33	2.45	2.59	2.73
[5^{th}]	-2.68	-2.84	-3.02	-3.19	-3.35	-3.55	-3.75	-3.95	-4.15	-4.39
[95^{th}]	2.77	2.94	3.12	3.28	3.45	3.66	3.84	4.06	4.25	4.47
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Mean	0.07	0.08	0.09	0.09	0.10	0.11	0.11	0.12	0.13	0.14
Median	0.09	0.09	0.10	0.11	0.11	0.12	0.12	0.13	0.14	0.15
SD	2.88	3.04	3.20	3.37	3.54	3.72	3.90	4.09	4.28	4.47
[5^{th}]	-4.61	-4.85	-5.09	-5.32	-5.59	-5.87	-6.15	-6.44	-6.74	-7.00
[95^{th}]	4.73	4.97	5.22	5.49	5.80	6.09	6.38	6.67	6.99	7.26
Panel B: Distribution of R^2 estimates										
	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Mean	1.20	2.26	3.34	4.42	5.53	6.66	7.81	8.98	10.19	11.44
Median	0.55	1.07	1.60	2.17	2.74	3.38	4.03	4.77	5.52	6.34
SD	1.67	3.06	4.44	5.76	7.06	8.37	9.61	10.83	12.05	13.25
[5^{th}]	0.00	0.01	0.01	0.02	0.03	0.03	0.04	0.04	0.05	0.06
[95^{th}]	4.58	8.54	12.63	16.57	20.51	24.36	28.34	32.17	36.08	39.91
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Mean	12.72	14.03	15.33	16.64	17.96	19.31	20.66	22.00	23.31	24.57
Median	7.24	8.22	9.22	10.29	11.36	12.50	13.75	14.93	16.32	17.62
SD	14.40	15.51	16.58	17.63	18.64	19.64	20.59	21.48	22.31	23.05
[5^{th}]	0.06	0.07	0.09	0.10	0.10	0.12	0.14	0.15	0.16	0.18
[95^{th}]	43.60	47.17	50.58	53.91	57.15	60.16	63.14	65.98	68.45	70.53

Table 11: Additional metrics for simulation A.2

Additional metrics	
β increasing 6-12 (%)	21.41
β decreasing 16-20 (%)	32.64
β hump-shaped (%)	8.05
R^2 increasing 6-12 (%)	19.51
R^2 decreasing 16-20 (%)	19.21
R^2 hump-shaped (%)	7.64
R^2 hump-shaped & $R^2 > 50\%$ (%)	3.33
R^2 hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	3.11
R^2 and β hump-shaped (%)	2.75
R^2 and β hump-shaped & $R^2 > 50\%$ (%)	1.29
R^2 and β hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	1.22

Table 12: Replication of Table B.1 in Online Supplement

Panel A: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$

	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Median of β_h	0.04	0.00	-0.10	-0.26	-0.50	-0.68	-0.76	-0.67	-0.37	0.03
SD of β_h	0.15	0.21	0.26	0.30	0.32	0.34	0.36	0.39	0.41	0.42
Median of Adj. R^2	-0.07	0.23	0.70	2.15	6.64	11.65	13.24	9.57	2.71	0.92
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Median of β_h	0.47	0.89	1.21	1.44	1.58	1.63	1.59	1.50	1.37	1.17
SD of β_h	0.42	0.43	0.45	0.48	0.51	0.52	0.51	0.49	0.47	0.47
Median of Adj. R^2	4.54	16.86	32.01	45.96	55.67	59.09	56.16	49.37	39.80	28.80

Panel B: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t+1,t+h} + \varepsilon_{t+1,t+h}$

	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Median of β_h	0.08	0.10	0.12	0.14	0.14	0.13	0.11	0.07	0.00	-0.09
SD of β_h	0.18	0.26	0.32	0.38	0.44	0.49	0.53	0.56	0.58	0.59
Median of Adj. R^2	0.29	0.78	1.21	1.59	1.94	2.21	2.41	2.54	2.68	2.88
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Median of β_h	-0.19	-0.29	-0.40	-0.50	-0.57	-0.60	-0.58	-0.51	-0.41	-0.31
SD of β_h	0.60	0.60	0.59	0.59	0.58	0.58	0.58	0.59	0.60	0.62
Median of Adj. R^2	3.23	3.83	4.77	6.07	7.46	8.20	7.49	6.13	4.79	3.84

Panel C: Distribution of slope and R^2 estimates

β increasing 6-12 (%)	30.92
β decreasing 16-20 (%)	59.98
β hump-shaped (%)	23.61
R^2 increasing 6-12 (%)	0.33
R^2 decreasing 16-20 (%)	64.82
R^2 hump-shaped (%)	0.29
R^2 hump-shaped & $R^2 > 50\%$ (%)	0.17
R^2 hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	0.17
R^2 and β hump-shaped (%)	0.26
R^2 and β hump-shaped & $R^2 > 50\%$ (%)	0.16
R^2 and β hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	0.16

Table 13: Replication of Table B.2 in Online Supplement

	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Median of β_h	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD of β_h	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Median of Adj. R^2	-0.42	-0.15	0.17	0.50	0.71	0.83	0.87	0.92	1.00	1.00
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Median of β_h	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD of β_h	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04
Median of Adj. R^2	0.87	0.68	0.63	0.88	1.39	1.86	2.08	2.17	2.22	2.24

Table 14: Replication of Table B.1 in Online Supplement - Economic Policy Uncertainty

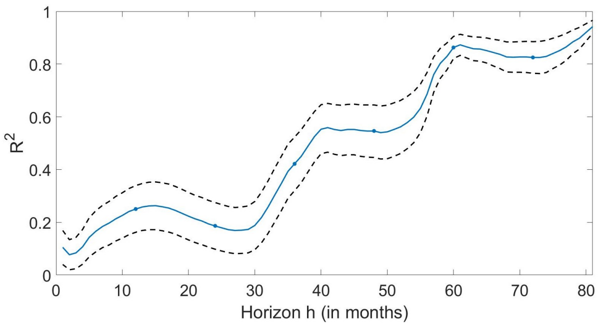
Panel A: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t-h+1,t} + \varepsilon_{t+1,t+h}$										
	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Median of β_h	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02	-0.01	0.00	0.01
SD of β_h	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Median of Adj. R^2	-0.37	0.05	0.54	1.35	2.58	3.46	3.13	1.51	0.49	1.27
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Median of β_h	0.03	0.04	0.05	0.06	0.06	0.06	0.06	0.06	0.05	0.05
SD of β_h	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Median of Adj. R^2	5.54	13.94	23.47	32.39	39.12	41.96	39.78	35.13	28.89	21.98
Panel B: $r_{t+1,t+h} = \alpha_h + \beta_h v_{t+1,t+h} + \varepsilon_{t+1,t+h}$										
	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Median of β_h	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02	-0.03
SD of β_h	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03
Median of Adj. R^2	-0.31	0.05	0.50	1.03	1.62	2.20	2.96	3.88	5.07	6.26
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Median of β_h	-0.03	-0.03	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04	-0.03	-0.03
SD of β_h	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Median of Adj. R^2	7.59	9.36	11.18	12.61	14.32	14.98	14.25	12.82	11.45	10.17

Panel C: Distribution of slope and R^2 estimates	
β increasing 6-12 (%)	53.65
β decreasing 16-20 (%)	50.56
β hump-shaped (%)	32.97
R^2 increasing 6-12 (%)	2.91
R^2 decreasing 16-20 (%)	49.77
R^2 hump-shaped (%)	2.06
R^2 hump-shaped & $R^2 > 50\%$ (%)	0.00
R^2 hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	0.00
R^2 and β hump-shaped (%)	1.74
R^2 and β hump-shaped & $R^2 > 50\%$ (%)	0.00
R^2 and β hump-shaped & $R^2 > 50\%$ (%) & $R^2_{16} - R^2_{20} > 30\%$ (%)	0.00

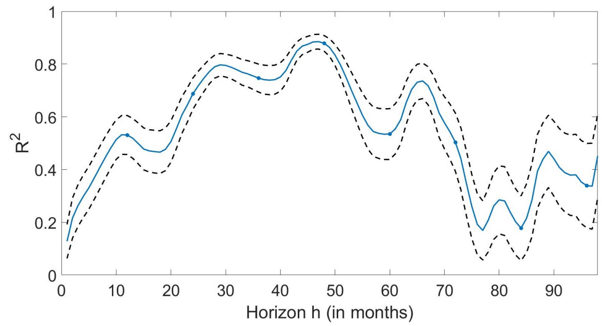
Table 15: Replication of Table B.2 in Online Supplement - Economic Policy Uncertainty

	Horizon h (in years)									
	1	2	3	4	5	6	7	8	9	10
Median of β_h	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD of β_h	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Median of Adj. R^2	-0.51	-0.32	-0.14	0.17	0.48	0.49	0.54	0.52	0.76	1.05
	Horizon h (in years)									
	11	12	13	14	15	16	17	18	19	20
Median of β_h	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SD of β_h	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Median of Adj. R^2	1.16	1.17	1.11	1.06	1.08	1.11	1.19	1.28	1.48	1.45

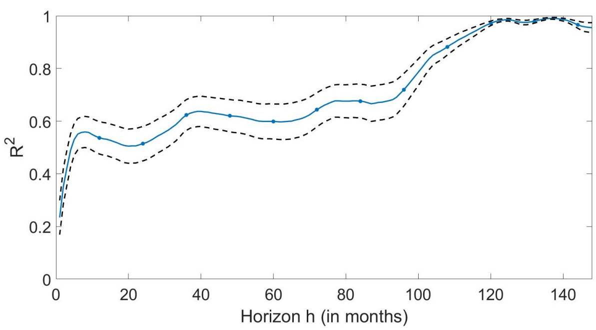
F Two-way aggregated regressions in support of the extensions



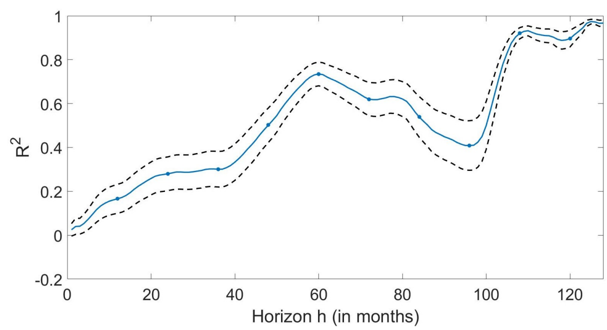
(a) Hong Kong



(b) South Korea



(c) United Kingdom



(d) Spain

Figure 12: Forward/backward aggregated regressions with multiple predictors. They are auxiliary regressions to extension three for Hong Kong, South Korea, the U.K., and Spain.

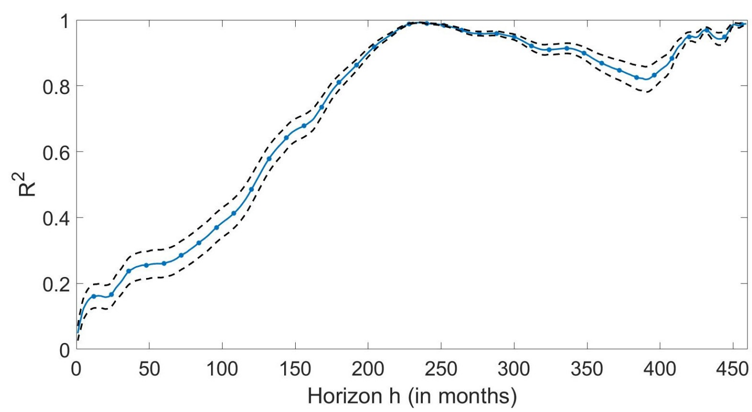


Figure 13: Forward/backward aggregated regressions with multiple predictors. They are auxiliary regressions to extension three for the U.S.

Table 16: Auxiliary two-way aggregated regressions; they use the best performing combination of scales from extension four

	US	HK	SK	UK	SP
R^2	38.9	45.1	49.9	47.1	25.1
$[5^{th}, 95^{th}]$	[34.2, 43.5]	[35.5, 54.8]	[42.0, 57.8]	[40.3, 53.9]	[17.3, 32.8]

G The R^2s with confidence intervals for the extensions

Table 17: R^2s with confidence interval for extension two, for the U.S., Hong Kong, and South Korea.

Panel A: The United States

	Scale j						
	1	2	3	4	5	6	7
Mkt. Var	1.6	0.0	1.0	0.6	8.4	0.2	11.7
	[0.0, 3.4]	[0.0, 0.3]	[0.0, 3.9]	[0.0, 3.6]	[0.0, 22.2]	[0.0, 2.9]	[0.0, 31.5]
Cons. Var	0.7	1.6	0.4	1.1	0.0	3.8	25.8
	[0.0, 1.9]	[0.0, 4.3]	[0.0, 2.1]	[0.0, 5.2]	[0.0, 0.9]	[0.0, 15.5]	[1.0, 50.6]
EPU	0.3	0.6	1.3	0.4	0.5	18.6	33.2
	[0.0, 1.1]	[0.0, 2.1]	[0.0, 4.6]	[0.0, 2.7]	[0.0, 4.0]	[0.0, 40.7]	[7.9, 58.6]

Panel B: Hong Kong

	Scale j				
	1	2	3	4	5
Mkt. Var	0.8	1.1	8.3	1.2	11.4
	[0.0, 3.5]	[0.0, 5.4]	[0.0, 22.8]	[0.0, 8.1]	[0.0, 38.7]
Cons. Var	0.2	3.9	9.9	17.3	0.4
	[0.0, 1.7]	[0.0, 11.9]	[0.0, 25.4]	[0.0, 39.5]	[0.0, 6.0]
EPU	13.2	8.6	2.2	19.0	57.6
	[3.5, 22.9]	[0.0, 19.8]	[0.0, 10.2]	[0.0, 41.6]	[28.3, 86.9]

Panel C: South Korea

	Scale j				
	1	2	3	4	5
Mkt. Var	1.0	25.8	10.7	30.8	32.6
	[0.0, 3.8]	[10.6, 41.0]	[0.0, 26.9]	[2.3, 59.4]	[0.0, 69.6]
Cons. Var	0.0	1.8	0.2	0.2	22.6
	[0.0, 0.1]	[0.0, 7.2]	[0.0, 2.7]	[0.0, 3.3]	[0.0, 58.0]
EPU	0.3	3.9	3.8	2.7	7.0
	[0.0, 1.9]	[0.0, 11.5]	[0.0, 14.3]	[0.0, 14.5]	[0.0, 30.6]

Table 18: R^2s with confidence interval for extension two for the U.K. and Spain.

Panel A: The United Kingdom

	Scale j					
	1	2	3	4	5	6
Mkt. Var	4.1	2.5	2.5	5.2	2.8	8.0
	[0.0, 8.9]	[0.0, 7.8]	[0.0, 9.8]	[0.0, 19.4]	[0.0, 16.6]	[0.0, 37.7]
Cons Var	0.1	1.4	7.9	8.2	12.2	61.0
	[0.0, 0.7]	[0.0, 5.3]	[0.0, 20.4]	[0.0, 25.3]	[0.0, 38.5]	[26.2, 95.7]
EPU	0.9	0.2	0.4	7.9	4.4	2.7
	[0.0, 3.2]	[0.0, 1.8]	[0.0, 3.2]	[0.0, 24.9]	[0.0, 21.5]	[0.0, 21.1]

Panel B: Spain

	Scale j					
	1	2	3	4	5	6
Mkt. Var	0.5	6.6	2.3	2.3	11.4	1.0
	[0.0, 2.3]	[0.0, 15.4]	[0.0, 9.8]	[0.0, 12.3]	[0.0, 37.9]	[0.0, 18.9]
Cons Var	0.7	0.3	0.4	4.5	16.8	95.7
	[0.0, 2.8]	[0.0, 2.4]	[0.0, 3.6]	[0.0, 18.1]	[0.0, 47.0]	[87.9, 100.0]
EPU	0.0	2.1	0.4	1.0	1.1	24.2
	[0.0, 0.2]	[0.0, 7.3]	[0.0, 3.4]	[0.0, 7.6]	[0.0, 10.4]	[0.0, 92.2]

Table 19: R^2s with confidence interval for extension three

	Scale j						
	1	2	3	4	5	6	7
The U.S.	2.4	2.5	4.3	1.9	12.3	20.9	47.2
	[0.2, 4.6]	[0.0, 5.7]	[0.0, 10.0]	[0.0, 7.3]	[0.0, 28.4]	[0.0, 43.7]	[23.3, 71.06]
Hong Kong			13.4	10.6	15.2	27.7	78.6
			[3.7, 23.2]	[0.0, 22.8]	[0.0, 33.3]	[3.2, 52.1]	[61.2, 95.9]
South Korea			1.4	27.3	11.4	45.9	59.5
			[0.0, 4.8]	[12.5, 42.2]	[0.0, 26.8]	[22.8, 69.0]	[38.9, 80.0]
The U.K.			4.6	3.1	8.2	18.3	21.3
			[0.0, 9.6]	[0.0, 8.8]	[0.0, 20.1]	[0.0, 38.7]	[0.0, 44.9]
Spain			1.2	7.2	3.4	9.2	29.4
			[0.0, 3.4]	[0.0, 16.2]	[0.0, 11.9]	[0.0, 25.8]	[4.6, 54.2]

Table 20: R^2s with confidence interval for extension four

Panel A: The United States															
y	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
x_1	3	3	3	2	3	3	3	3	2	3	2	3	7	7	3
x_2	2	1	6	6	5	3	5	7	3	7	7	3	3	3	5
x_1	7	7	7	4	1	6	4	7	4	4	4	7	6	3	7
R^2	94.7	94.7	94.4	94.2	93.9	93.7	93.7	92.3	91.7	91.1	91.0	91.0	90.7	90.4	89.3
5^{th}	91.4	91.4	90.9	90.4	90.1	89.7	89.6	87.4	86.5	85.5	85.4	85.3	84.9	84.3	82.6
95^{th}	98.1	98.1	98.0	97.9	97.8	97.7	97.7	97.2	96.9	96.7	96.7	96.6	96.5	96.4	95.9
Panel B: Hong Kong															
y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
x_1	5	1	3	2	4	5	1	3	2	5	4	5	5	5	5
x_2	4	4	4	4	4	3	3	3	3	1	3	4	2	3	4
x_3	3	3	3	3	3	3	3	3	3	2	3	2	2	2	4
R^2	99.6	99.6	99.6	99.5	99.3	98.5	98.4	98.3	98.2	97.6	97.5	97.2	96.2	93.3	89.7
5^{th}	99.3	99.2	99.2	99.1	98.7	97.1	96.9	96.8	96.5	95.4	95.2	94.7	92.8	87.3	80.8
95^{th}	100.0	100.0	100.0	100.0	99.9	99.9	99.8	99.8	99.8	99.8	99.7	99.7	99.6	99.2	98.6
Panel C: South Korea															
y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
x_1	2	3	2	2	3	3	3	1	1	2	5	5	4	4	4
x_2	3	4	2	1	1	2	3	1	2	5	1	2	1	2	4
x_3	2	2	2	2	2	2	2	5	5	2	5	5	5	5	2
R^2	97.6	93.9	93.0	93.0	89.2	89.2	89.1	88.7	88.7	88.6	88.5	88.5	88.4	88.4	87.2
5^{th}	96.0	90.0	88.5	88.5	82.6	82.6	82.4	81.7	81.7	81.6	81.4	81.4	81.2	81.2	79.4
95^{th}	99.1	97.8	97.4	97.4	95.9	95.9	95.9	95.7	95.7	95.7	95.6	95.6	95.6	95.6	95.1
Panel D: The United Kingdom															
y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
x_1	3	3	5	1	4	2	3	4	5	2	3	5	1	5	2
x_2	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4
x_3	4	1	1	1	1	1	5	5	5	5	5	1	5	4	4
R^2	95.0	94.7	90.5	90.0	89.0	88.9	88.0	78.6	78.1	76.1	75.9	75.4	75.2	74.2	72.4
5^{th}	91.8	91.4	84.5	83.9	82.3	82.2	80.7	66.2	65.5	62.6	62.3	61.5	61.2	59.8	57.2
95^{th}	98.2	98.1	96.4	96.2	95.8	95.7	95.3	90.9	90.7	89.7	89.6	89.3	89.2	88.7	87.7
Panel E: Spain															
y	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
x_1	5	5	2	2	5	4	4	4	4	1	3	4	3	3	2
x_2	2	1	1	2	3	2	1	1	2	5	4	3	1	2	3
x_3	1	1	1	1	1	4	4	1	1	2	1	1	1	1	1
R^2	74.2	74.2	71.1	71.1	68.5	67.9	67.9	67.7	67.7	67.0	66.6	66.0	65.8	65.8	65.7
5^{th}	59.8	59.8	55.4	55.4	51.7	50.8	50.8	50.5	50.5	49.6	48.9	48.2	47.9	47.9	47.7
95^{th}	88.6	88.6	86.9	86.9	85.4	85.0	85.0	84.9	84.9	84.5	84.2	83.9	83.8	83.8	83.7

H Code

The code is in the same zip file Code_TANS. There are two parts: the replication and the extension. For the replication, one needs the files:

- load data: loads the data that is needed for the replication from the excel file 'REPLICATION DATA';
- thesis replication: contains all the code to run the replication results, i.e. two-way aggregated and scale regressions;
- thesis sim A: generates the results for the replication of simulation A.1 and A.2;
- thesis sim B: generates the results for the replication of simulation B.1 and B.2;
- nwest: performs a regression and generates Newey-West t-statistics;
- decimation: decimates calendar-time scale components to get scale-time scale components;
- REPLICATION DATA: the excel file that contains the data needed for the replication and the results of the replication.

For the extension, one needs the files:

- thesis extension: one can select a country and the script contains the code to run all the regressions in the extension;
- scale component: takes a raw time series and generates scale-time scale components;
- excess returns maker: takes the simple gross returns and the risk-free rate to construct the excess market returns;
- market variance maker: takes the daily simple gross returns and constructs the market variance;
- consumption variance maker: takes the consumption levels and models the variance of the consumption growth;
- epu maker: slightly modifies the EPU data as needed;
- dataprep: generates the four needed variables, all correctly aligned to dates, and uses the four variable 'makers';
- nwest: same as before, runs a regressions and generates Newey-West t-statistics;
- EXTENSION DATA: the excel file that contains the data needed for the extension.

Please make sure the excel files are saved in the same location as the code. To replicate the replication results, follow the READ ME sheet in the excel file 'REPLICATION DATA'. Same goes for the extension, in the excel file 'EXTENSION DATA'. The code is also commented. For most runs/parts of the replication, some lines need to be turned on or turned off with '%'. Which lines need be turned on or off is also indicated in the comments, but a full walk through is included in the excel files.

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