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An inventory model for slow moving items subject to obsolescence with limited customer patience

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Abstract

Finding a good strategy for dealing with a lower demand at a known future time is critical for the competitiveness of service providers. In this research slow-moving products that are subject to obsolescence are considered. Those products are controlled using a continuous inventory system with fixed lead times and the system is controlled using the one-for-one replenishment policy. Furthermore, limited customer patience is taken into consideration. The impact of the strategy in which service providers start adapting to the new base stock level before the drop in demand occurs on the total cost using natural attrition is analyzed for different situations. Using simulation and a simulation optimization technique for different instances, optimal times to start the removal process of the excess stock are proposed. Eventually this research concludes that the policy strategy results in significant cost reductions for both the cases with limited and unlimited customer patience.

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1 Introduction

In 2017 Scott Kirby, the president of United Airlines announced the retirement of the airplane model Boeing 747: The Queen of the Skies. He states that nowadays there are more fuel-efficient, cost-effective and reliable widebody aircraft, that provide an updated inflight experience for their customers[1]. With this announcement, spare parts service providers for Boeing 747 parts knew that demand would be lower at a known time in the future. Products such as airplane parts are expensive, slow-moving and risk obsolescence. Therefore, controlling inventory and timely adjustment for demand changes is critical and has a big impact on the competitiveness of the service providers. Similar situations arise when contracts are about to expire, or when buyers of products are relocated to other regions.

With more advanced computers and technology, more data is available. This means that companies are better capable of tracking their inventory levels and adjusting for future changes. Adjusting for a foreseen change in demand is critical because the products discussed here result in high cost when this is not done efficiently. Dealing with lower demand is a difficult issue since removing excess stock depends on the demand process. Adjusting too late, the service providers risk having a high number of products in stock and a lower number of customers arriving. Therefore, the service providers risk high inventory holding cost and obsolescence. On the other hand, adjusting too early the service providers risk stock outs. Solving this with backorders is expensive, time-consuming and can be bad for the reputation of the service provider. From this it can be concluded that having a good strategy for dealing with a lower demand in the future is crucial.

In this paper the research is carried out by extending Pinçe and Dekker (2011). In the first part their results are replicated. In this part research is carried out about what impact timely adjustment can have on the total cost. For different parameter combinations the best time to start the stock removal process is determined. These results are compared to the total cost incurred in the situation in which there is no policy change. In this situation the stock removal process starts at the moment the drop in demand occurs.

In the second part the extension is carried out. For this extension it is assumed that there are two types of customers. The first type are the regular customers that sign contracts with the producers. Therefore they will always wait for their product to arrive. However, the second type of customer, the non-regular customers that do not sign a contract with the producers, can get impatient when waiting in the queue for the product to arrive. They will either leave the system with or without product. The impact of limited customer patience on the starting time of the excess removal phase is determined.

The goal of this paper is to determine the effect policy change can have on the total cost under limited and unlimited customer patience. The considered products are slow-moving, subject to obsolescence and are controlled with a continuous review inventory system. Because of the drop in demand, the demand rate is considered to be lower at a known time in the future. For the inventory policy the one-for-one replenishment policy is used. This policy orders a product whenever demand occurs. In order to deal with the lower demand, early adaption to the new base stock level is used. Using this policy the excess stock can be taken away by customers. In other words, natural attrition is used before the drop in demand has occurred.

This paper is organised as follows: In section 2 the literature review is given on earlier done research. Section 3 contains the theoretical background of the problem and the econometric and operational methods that are used. Section 4 contains the data that will be used in this research. In section 5 results will be given followed by a conclusion in section 6.

[1] https://www.luchtvaartnieuws.nl/nieuws/categorie/2/airlines/united-airlines-stelt-boeing-747s-eerder-buiten-dienst

2 Literature review

2.1 Inventory model without limited customer patience

In the papers from Hadley and Whitin (1963), Pierskalla (1969) and Brown et al. (1964) relevant research is done about inventory models that were subject to obsolescence. Hadley and Whitin (1963) discuss different ways to determine optimal operating doctrines. They especially focus on two questions: when to replenish inventory and how much to order. In order to answer these questions they consider different mathematical models. One of these models contains a change in the mean rate of demand over time. They studied this model using dynamic modelling. Furthermore, they discussed that obsolescence is important and should be considered.

Pierskalla (1969) determines an inventory model in which the probability that the product can become obsolete is taken into consideration. However, the model differs from Pinçe and Dekker (2011) because the demand rate over time has not changed and lead times are not considered. The conclusion in this research is, that obsolescence probabilities play a crucial role and should be taken into consideration.

Brown et al. (1964) proposes a model of the (s, S) type that optimizes inventory cost while taking into account stochastic obsolescence. The model discussed in this paper is a Markov model with underlying demand generating states. Using this model, the demand process is generated stochastically. In order to do this, the model incorporates a Bayesian process for the stochastic demands. Furthermore, they propose a model for the case in which at an unknown future moment the demand totally vanishes. Compared to the above mentioned model, Pinçe and Dekker (2011) made the assumption that the obsolescence time is deterministic. Using this assumption the model is able to combine the policy change with the $(S-1, S)$ policy. The $(S-1, S)$ policy means that whenever demand occurs, an order is placed for a new product. With this assumption the model differs from the models described in the above mentioned papers because the model used in this research is more simple and therefore better to use in practice.

Stock disposal models are related to this research. Simpson (1955) did research about the decision to either keep or sell products that are currently in stock. For example when companies have many products in stock, they have to decide to either pay for the holding cost, or for the disposal cost. Eventually, the paper proposes formulas that calculate the maximum time that products should be held in stock. For determining those formulas the assumption was made that demand was known and constant. This assumption does not hold in the research carried out in this paper. Stulman (1989) did similar research, however instead of making the decision to either keep or sell products, the optimal inventory level is determined in this research. These two situations are closely related. In order to determine the optimal inventory level, natural removal of excess stock is taken into account. Also, taken into consideration is whether it is cheaper to dispose the products now and backorder them later instead of holding on to the inventory now. In this paper stochastic demand is taken into consideration. This means that the problem of a sudden drop in the historical demand rate is considered, however obsolescence is not considered here. Rosenfield (1989) did research containing slow-moving objects that are subject to obsolescence and perishability. Eventually, the ultimate goal of this paper is to determine how many products to keep in stock.

In the above mentioned papers, the problem of too many products in stock resulted in the past. Therefore, at time zero the inventory level is too high and in order to lower the inventory level, products are disposed or not re-ordered for certain time units. The problem considered in Pinçe and Dekker (2011) differs from the above described models by starting the excess removal phase earlier. This means that the adaption to different demand rates is done earlier in the process.

2.2 Inventory model with limited customer patience

Hadley and Whitin (1963) mentioned the issue of limited customer patience. In this paper, two possibilities are discussed for the situation in which customers arrive while there are no products in stock. Either the customer leaves the system immediately, or the customer waits until the product arrives. If the customer decides to wait until the product arrives, backorder costs are incurred. According to Hadley and Whitin (1963) these backorder costs should incur customers' goodwill, which is for example telling the customer that he or she has to wait for the product. However, if the customer decide to leave, the company risks losing customers that go to the competition. Furthermore, the service provider risks losing customers spreading complaints.

In later research written by Pinçe et al. (2015) on the same topic as discussed in this research, they discussed the issue of lost sales. They stated that for slow-moving objects it is sufficient to assume that customers will wait for their products, because finding different suppliers for these specific products can be challenging. However, in some cases it might happen that customers find a different supplier and thus move to the competitor. In Pinçe et al. (2015) they did a sensitivity analysis to this assumption, using a fill rate target. However, they concluded that the lost sales case did not have great impact on the cost and the optimal policy.

Compared to Hadley and Whitin (1963) and Pinçe et al. (2015), a different situation is considered in this research. Instead of the case in which customers either leave the system immediately or wait until the product arrives is not considered. Here, it is considered that customers either wait and leave without product, or wait and leave with product. Therefore, there is no fill rate considered.

3 Problem formulation

In this paper, a single product at a single location controlled with a continuous review inventory system is considered. The product is slow-moving and subject to obsolescence. The demand process is considered to be a stochastic non-homogeneous Poisson process. The demand is non-homogeneous because of the drop in demand. Therefore, two rates are considered: λ_0 and λ_1, such that $\lambda_0 > \lambda_1 \geq 0$. Inventory is controlled with the one-for-one replenishment policy. This policy is defined as follows: for every arriving customer, a replenishment order is placed with lead time L.

For the demand process a stochastic non-homogeneous Poisson process is considered. This means that the interarrival times between demands are exponentially distributed with rate λ. The Poisson process has the memoryless property and this property is used here because customers arrive independently of each other.

In order to adapt earlier to the lower demand in the future, the following strategy is used: up to time $T - X$ the base stock level is S_0. During this period, the one-for-one replenishment policy $(S_0 - 1, S_0)$ is applied. After time $T - X$ the new base stock level is S_1. During this period the one-for-one replenishment policy $(S_1 - 1, S_1)$ is applied. After time $T - X$ the inventory level S_0 adapts to the lower inventory level S_1 by not giving replenishment orders for $N = S_0 - S_1$ demand arrivals. Here T denotes the end of the stock removal process and X gives the length of this process. The goal is to find the optimal policy such that the optimal time is found to start the excess removal process.

The timeline is divided into three periods. The first period starts at time $t = 0$. In this period the base stock level is S_1 and the one-for-one replenishment policy $(S_0 - 1, S_0)$ is applied. Here the assumption is made that the first period is long enough for the inventory to reach the steady-state level S_0. This assumption can be made because the objects discussed here have a long life time. The first period ends at time $T - X$, this time is defined as the time at which the excess removal phase starts. An example situation is given in Figure 1.

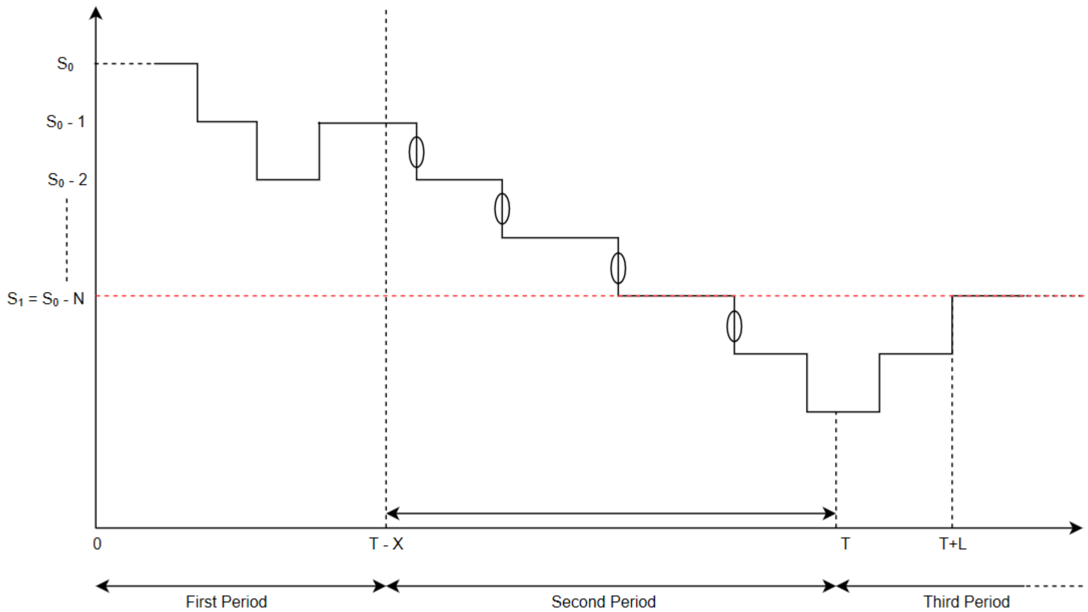


Figure 1: Example situation of the net inventory level

The second period, which is referred to as the transient period, is the period in which no replenishment orders are placed for the first $N = S_1 - S_0$ arriving customers. This is done by not applying the one-for-one replenishment policy for the first N arriving customers. Using this strategy, the excess stocks are removed using natural attrition. Eventually, the inventory level reaches the new base stock level S_1. When this new base stock level is reached before time T, the system continues with the one-for-one replenishment policy when the new base stock level S_1 is reached. In this case, the second period ends at time T. Therefore $T - X$ is the time at which the excess removal phase starts. In the case where there are still customers waiting for their product, the second period ends later. The second period ends when all customers in the queue are served and the inventory level S_1 again. This situation is described in Figure 1.

The third period starts when the transient phase is finished, this happens either at time T or later. This depends on the time at which the transient phase is initiated. The third period only exists when there is still demand, in other words when $\lambda_1 > 0$. In order to keep the inventory level at the new base stock level, the $(S_1 - 1, S_1)$ is applied. During this period the assumption is made that the inventory level reaches S_1 in steady state.

For the case in which $\lambda_0 > 0$, there is no real obsolescence. In this case partial obsolescence is considered. It can happen that the service provider has much more products in stock than the preferred base stock level S_1. Holding on to these products results in costs. These costs are referred to as partial obsolescence costs.

For the case in which $\lambda_1 = 0$, no third period is considered. The products that are left in stock at the end of the second period cannot be sold anymore because no customers are arriving anymore. Therefore after time T real obsolescence costs are taken into account. The products that are left have to be disposed or relocated and this results in costs.

In this research, the goal is to minimize the total cost incurred in the transient phase. In order to do this, the optimal value for X is determined. This means that the optimal time to initiate the excess removal phase is estimated. In order to do so, the costs incurred only in the second period are considered. The reason for doing this is that the holding costs and obsolescence costs can only be decreased during this specific period. Holding cost are denoted by h and are incurred per time unit. Backorder cost per unit time are denoted by π. Obsolescence cost per unit time after time T in the case where $\lambda_1 = 0$ are denoted by c_0.

3.1 Stock removal phase

The most important part of the model is the excess removal phase. In order to calculate the costs incurred in this period, different situations that can occur are defined. In order to look at these different situation, first the assumption is made that the net inventory level at time $T - X$ is equal to S_0. Furthermore, the beginning of the stock removal phase is shifted from time $T - X$ to 0. This shift makes T become X. This is done for computational and comparing use. The different situations that can occur during the stock removal phase depend on the arrival of the N^{th} customer, A_N ($A_N = \sum_{i=1}^{N} \tau_i$, where $\tau_i, i = 1, ..., N$ denotes the interarrival times between demands) and the time X at which the demand rate changes.

3.1.1 Case: $X \leq A_n$

First, the situation in which $\lambda_0 > \lambda_1 \geq 0$ and $A_n \geq X$ is considered. In this situation the arrival of the N^{th} customer happens after time X. The drop in the demand rate therefore happens during the stock removal phase. So, when the N^{th} customer arrives, all outstanding orders are processed and the inventory level has reached the new base stock level S_1. The second period ends at the time of the N^{th} arrival and the third period starts. An example of this situation is given in Figure 2.

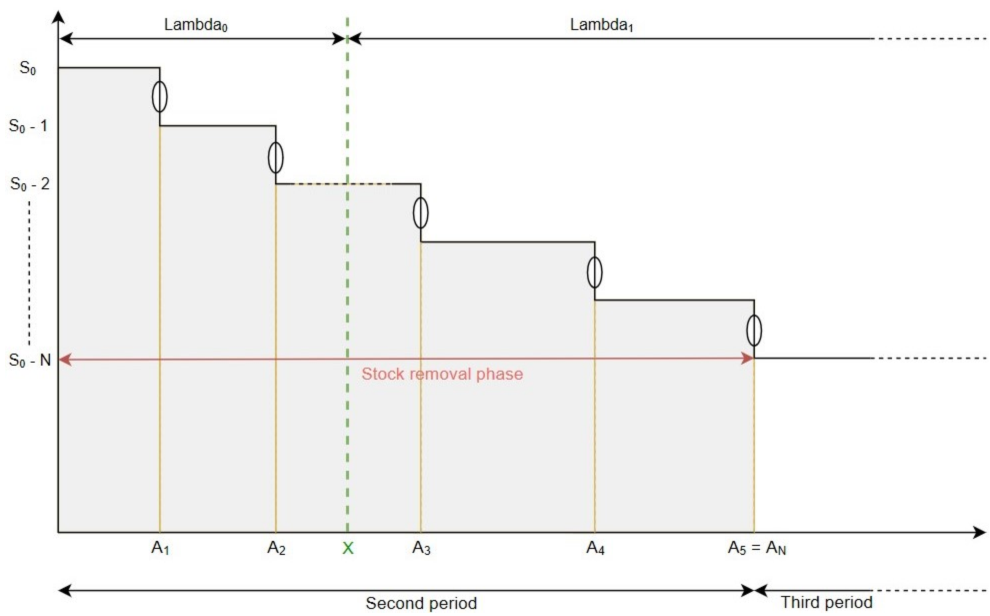


Figure 2: Example situation in which $X \leq A_N$

When $X \leq A_n$, the total cost consists only of holding cost. The inventory level costs is determined by multiplying the positive area under the inventory level with h, the holding cost incurred per unit per time. In figure 2, the grey area should be multiplied with h to determine the total cost incurred in the second period.

3.1.2 case: $X > A_n$

Here, the situation in which $\lambda_0 > \lambda_1 \geq 0$ and $A_n < X$ is considered. Here, the arrival of the N^{th} customer happens after time X. In this situation, the stock removal phase is finished and the inventory system is operating in its regular operation phase again. Therefore, the timeline can be divided into two periods. The first period is the stock removal phase. During this period, the excess stock is removed by not placing replenishment orders (natural attrition). The stock removal phase starts at time 0 and ends at the time of the arrival of the N^{th} customer. Then the regular operation phase starts. During this period, the inventory level is controlled with the $(S_1 - 1, S_1)$ policy. What can happen here is that customers arrive when there is no stock available. Those customers are placed in a first-in-first-out (FIFO) queue. For those customers, backorder costs are considered. The second period ends when all outstanding orders are processed and the inventory level reached the new base stock level S_1 again. An example of this situation is given in Figure 3.

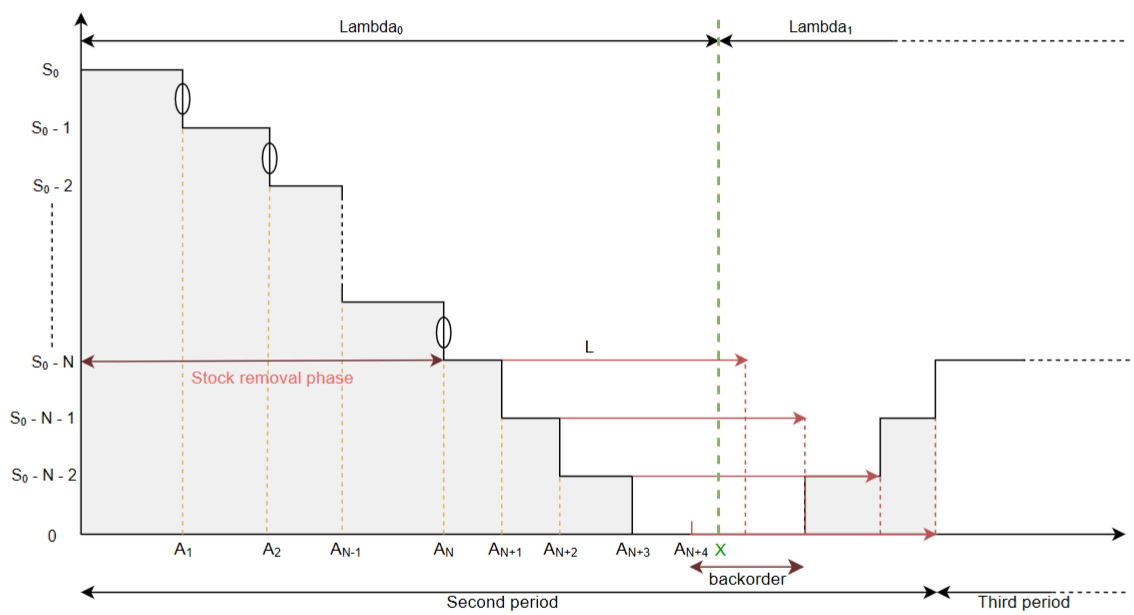


Figure 3: Example situation in which $X > A_N$

In order to determine the total cost incurred in this situation, both backorder and holding cost have to be taken into account. The holding cost is determined in the same way as in the situation in which $X \leq A_n$. But, now holding costs after the arrival of the N^{th} customer have to be accounted for as well. Furthermore the backorder costs when customers are waiting in the queue have to be accounted for. This backorder cost incurred per unit per time is denoted by π.

3.2 Problem formulation with limited customer patience

In this subsection, limited customer patience is introduced. When the inventory level is 0 and customers are waiting in the FIFO queue, they can get impatient and leave without a product. When this happens, the total costs are updated and backorder costs are incurred for the time the customer had spend in the queue.

For the problem formulation, two types of customers are considered. The first type of customers are for example regular or bigger companies that signed a contract with the service provider. This type of customer will always wait for the product to arrive. The second type of customers are for example smaller, non-regular clients that did not sign a contract with the service provider. This type of customer can get impatient and leave the queue without receiving the ordered product. For this customer backorder cost are taken into consideration for the time this client had spend in the queue. For the two types of customers the same waiting cost rate π is charged. The parameter p is introduced here, which gives the percentage of clients that are customer type 1. The other $1-p$ are the percentage of clients that are customer type 2.

The assumption is made here that the waiting times of the customers are distributed according to the exponential distribution with parameter θ. This exponential distribution is used more often to model the waiting time, for example Perry and Posner (1998) considered this waiting time distribution. In this paper, they also adapt the $(S-1, S)$ policy with constant lead times and a Poisson process for the demand process

3.3 Calculation of the base stock level

In order to determine the optimal time to initiate the excess removal phase, first the steady state base stock levels S_0 and S_1 are calculated. From these values, $N = S_0 - S_1$ is calculated. Shenas et al. (2009) determines the optimal solution for the base stock model. They consider a single-item continuous review inventory system, controlled with the $(S-1, S)$ policy. Customers arrive according to the Poisson process asking for one product. Shenas et al. (2009) concludes that the optimal value for S that minimizes the total costs (inventory holding costs and backorder costs) is the minimal value for S that satisfies:

$$P(S, \lambda(L + L_0)) \geq \frac{\beta}{\beta + h}. \tag{1}$$

Here P(μ, λ) is the CDF of Poisson. The inventory position is denoted by S, the demand intensity by λ, the transportation time from the supplier to the retailer by L, the transportation time from the outside source to the supplier by L_0, the shortage cost per unit per time unit by β and the holding cost per unit per time unit by h. In this research, L_0 is considered to be 0 and β in the formula is π.

3.4 Response Surface Method

In order to find the optimal time to start the excess removal phase simulation is used. However, the demand process follows the Poisson process, which results in a noisy objective function. Finding the optimal value for X such that the total cost is minimized has become a challenge. In order to deal with this issue, a simulation optimization technique is used. The technique used here is the Automated Response Surface Methodology (RSM), as described in Myers and Montgomery (1995) and Nicolai and Dekker (2009). According to Nicolai and Dekker (2009), RSM is a collection of mathematical and statistical techniques that is useful for the approximation and optimization of stochastic models. The framework for the RSM method that is used for this paper is given in Figure 4. This figure has changed a bit compared to the RSM method used in Nicolai and Dekker (2009).

3.4.1 Phase 1

The first step of the RSM method is initialization. During this step, an initial starting point is determined (centre point). Furthermore, the range is determined, which is used to determine the region of interest (ROI). The bounds of the ROI are referred to as the lower- and higher point.

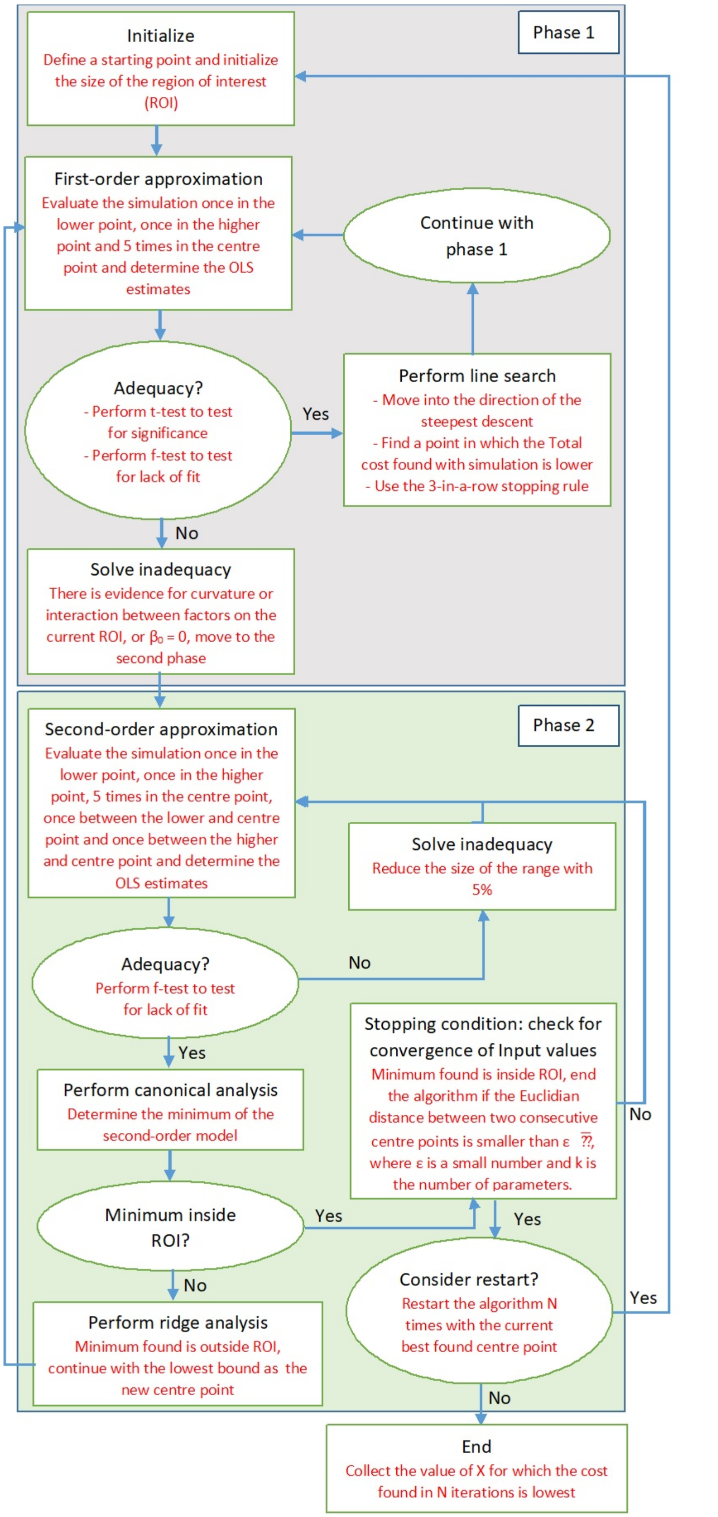


Figure 4: Framework for the Automated Response Surface Methodology

Then the algorithm moves to the second step. In this step, a first-order model is approximated with $y = \beta_0 + \beta_1 x_1 + \epsilon$. the y vector here contains the costs determined with simulation for the real values of X. The objective function here is calculated once in the lower point, once in the higher point and five times in the centre point. This is done in order to prevent curvature from the lower- and higher point. The x vector that is used in OLS contains transformations of the real values. This transformations transforms the lower point into -1, the centre points into 0 and the higher point into 1. The reason for doing this transformation is that it is easier to extrapolate. After both the y and x vectors are determined, OLS is performed and the coefficients of the first-order model are determined.

After the second step, the algorithm tests for model adequacy. Here, both a t-test for significance of regression and a f-test for lack of fit are performed. Both tests, test under a 5% significance level. We consider the model to be adequate if both tests test for model adequacy.

The t-test that is used to test for significance on the individual regression coefficient β_1 is described in section 2.4.2 in Myers and Montgomery (1995). Under the null hypothesis $\beta_1 = 0$. Under the alternative hypothesis β_1 is not equal to 0. If the null hypothesis is rejected, we can conclude that the linear estimated model is adequate. The test statistic used here is: $t(0.025, 5)$.

In order to test for lack of fit, a f-test is performed for which the variance is unknown. The test used here is described in section 5.3 in Weisberg (2005). Under the null hypothesis there is no lack of fit and under the alternative hypothesis there is indeed lack of fit. When the null hypothesis is rejected, we can assume the model is inadequate. The test statistic used here is: $F(0.05, 1, 4) = 7.71$.

Whenever both the tests test for model adequacy, the algorithm performs a line search. If β_1 is negative, the steepest descent is in the direction of the higher point. The algorithm starts in the higher point and moves away from the centre with steps of size one. However, if β_1 is positive, the steepest descent is in the direction of the lower point and the algorithm moves from the lower point with steps of size one into this direction. Because an automated optimization is used here, a consistent n-in-a-row stopping rule is used. The line search ends whenever n consecutive values of the objective function are higher than the preceding value. In this research, the 3-in-a-row stopping rule is used. Whenever the line search is finished, a better area of interest is found and the algorithm moves to step 2 again.

If the first-order model is inadequate Nicolai and Dekker (2009) states that there is some evidence of curvature or interaction between factors on the current ROI, or the regression coefficients are all equal to zero. Whenever this happens, the algorithm moves to the second phase.

3.4.2 Phase 2

During the first step in the second phase, RSM estimates the coefficients of the second-order model with OLS. The estimated second-order model is: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$. The y vector again contains the costs from the simulation for the real values of X. Here, the objective function is calculated once in the lower point, once 1/8 part of the range from the centre point to the lower point (centre point - $0.5 * 0.5 * 0.5*$range), once 1/8 part of the range from the centre point to the higher point (centre point + $0.5 * 0.5 * 0.5*$range), once in the higher point and 5 times in the centre point. Two point are added in order to test the model for adequacy. This is explained in the next paragraph. The x value that is used in OLS contains again the transformations of the real values. After both the y and x vectors are determined, OLS is performed and the coefficients of the second-order model are estimated.

In the second step in the second phase RSM tests the second-order model for adequacy. Here a test for lack of fit is performed in the same way as described in phase 1 and in section 5.3 in Weisberg (2005). Under the null hypothesis, there is no lack of fit and under the alternative hypothesis there is indeed lack of fit. When the null hypothesis is rejected, we can assume the model is inadequate. The f-test statistic is calculated as follows: $F = \frac{SS_{lof}/dflof}{SS_{pe}/dfpe}$. Here, SS_{pe} describes the sum of squared for pure error and SS_{lof} describes the sum of squared for lack of fit. For the formula holds that $df_{lof} = n - p - dfpe$. Here, n gives the number of observations and p the number of variables. In order to get a positive number of degrees of freedom for lof, the two numbers as described in the previous paragraph are added. Eventually $n = 9$, $p = 3$ and $df_{pe} = 4$, thus the test statistic used here is: $F(0.05, 2, 4)$.

Whenever the f-test tests for lack of fit, the region of interest is too large. In order to solve this issue, ROI is decreased. Therefore, the range is decreased with 5% and the algorithm moves again to the estimation of the second-order model.

After an adequate second-order model is found, the canonical analysis is performed. During this step the optimal point according to the second-order estimated model is determined, such that $X_{opt} = -\frac{\beta_1}{2*\beta_2}$.

If X_{opt} is outside the current ROI, ridge analysis is performed. Because it is known that the function of the objective function is unimodal, we continue with either the lower point or the higher point, for which the total cost found with simulation is the new centre point and return to phase 1.

If X_{opt} is inside the current ROI, the stopping condition is checked. This condition checks for convergence of the input values. This means that whenever the euclidean distance between two consecutive optimal points is smaller than $\epsilon * \sqrt{k}$, where ϵ is a small number and k is the number of parameter (in the second-order model $k = 3$), the algorithm is ended. Otherwise, the algorithm moves back to the estimation of the second-order model. Whenever this happens, the new centre point is the current optimal found value and the range is decreased with 5%.

If the distance between the two consecutive optimal points is small, the algorithm is ended. But because the objective function is noisy, we can't be sure that this is a global or local optimum. This is due to the fact that the first centre point is randomly chosen. In order to solve for this issue, RSM is repeated a few times. The new centre point will be the current optimal value found for X and the range is changed to its initial value. In this study the RSM method is repeated $N = 5$ times and eventually the optimal X, which minimizes the objective function, is chosen.

4 Data

4.1 Problem without limited customer patience

In order to find a good approximate for the optimal value of X that minimizes the total costs, the behavior of X in the total cost function is determined. This is done by generating a number of instances with equal intervals on a given range. For example X lies in the range $0 - 1$. Then generating 21 number of instances results in $X \in \{0, 0.05, 0.10, ..., 1\}$. The behavior of X is determined using the data provided in Pinçe and Dekker (2011), which is provided in Table 1.

Table 1: Experiment instances problem without limited customer patience

$h = 1$						
λ_0	λ_1	L	π	c_0	X	Number of inst.
0.5	0	0.75	20	10	0-5	21
1	0	0.75	20	10	0-5	21
5	0	0.15	10	10	0-1	21
5	0	0.50	10	10	0-2	21
10	0	0.15	10	10	0-1	21
10	0	0.50	10	10	0-2	21
0.5	0.2	0.75	20	-	0-5	21
1	0.2	0.75	20	-	0-5	21
5	2	0.15	10	-	0-1	21
5	2	0.50	10	-	0-1	21
10	2	0.15	10	-	0-1	21
10	2	0.50	10	-	0-1	21

Source: Pinçe and Dekker (2011).

For all 252 $(12 * 21)$ experiments, 5000 simulations are performed. From these simulations the average total cost is determined. The average total cost is plotted for the range of X and from these plots the behavior of X is visualized. Each plot contains the belonging 95% confidence interval.

After the behavior of X is determined, the optimal value for X is calculated with RSM as described in the model section. This method is applied for different parameter instances, which are provided in Table 1. However, the situations in which $N = 0$ and $S_1 = 0$ are not considered. This means that eventually 281 number of instances are performed instead.

Table 2: Parameter values used in RSM problem without limited customer patience

h=1					
λ_0	λ_1	c_0	π	L	Number of inst.
0.5, 0.7, 1	0	5, 10	50, 75, 150, 300	0.25, 0.50, 0.75, 1	96
5, 7, 10	0	5, 10	5, 15, 25, 50	0.05, 0.15, 0.25, 0.50	96
0.5, 0.7, 1	0.2	-	50, 75, 150, 300	0.25, 0.50, 0.75, 1	96
5, 7, 10	2	-	5, 15, 25, 50	0.05, 0.15, 0.25, 0.50	96
Total					281

Source: Pinçe and Dekker (2011).

4.2 Problem with limited customer patience

To determine an optimal policy under limited customer patience, again the behavior of X in the total cost function is determined. This is done using the same approach that is used in the case with no limited customer patience. However, here the behavior of X also depends on the variables p and θ. The data used to determine the optimal policy is given in Table 3.

For the parameter p the following values are considered: $p = (0, 0.25, 0.50, 0.75)$. The value $p = 1$ is not taken into account, because this would imply the same situation as when there is no limited customer patience.

For the waiting time distribution it holds that the lower the value of θ, the longer people are willing to wait and then the probability then that a customer leaves with a product is relative higher. For the parameter θ the following values are considered: $\theta = 10, 100, 1000$. The value $\theta = 0$ is not considered, because this would imply again the same situation as when there is no limited customer patience.

Table 3: Experiment instances problem with limited customer patience

$h = 1$, $p = (0, 0.25, 0.50, 0.75)$							
λ_0	λ_1	L	π	c_0	X	θ	Number of inst.
0.5	0	0.75	20	10	0-5	10	21
						100	21
						1000	21
10	0	0.15	10	10	0-1	10	21
						100	21
						1000	21
0.5	0.2	0.75	20	-	0-5	10	21
						100	21
						1000	21
10	2	0.15	10	-	0-1	10	21
						100	21
						1000	21
Total						4*252 = 1260	

The total costs are calculated for the four different values of p, therefore the total number of experiments is $4 * 252 = 1260$. For each of these experiments 5000 simulations are performed and from the simulations the average total cost is calculated. The average total cost is plotted for the range of X along with their 95% confidence intervals. From the plots the behavior of X in the total cost is determined and is discussed in section 5.

After the behavior of X is determined for the problem with limited customer patience, the optimal value for X that minimized the total cost is determined. This is done again using the RSM optimization technique. This method is applied using the parameter combinations given in Table 4. In total 480 instances are performed.

Table 4: Parameter values used in RSM problem with limited customer patience

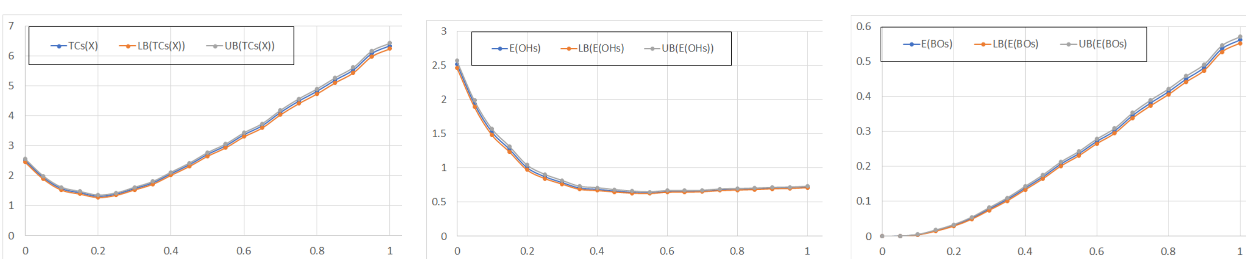
$h = 1$, $p = (0, 0.25, 0.50, 0.75)$						
λ_0	λ_1	c_0	π	L	θ	Number of inst.
0.5	0	5, 10	50, 300	0.25	10, 100, 1000	12
1	0	5, 10	50, 300	0.25	10, 100, 1000	12
5	0	5, 10	5, 50	0.25	10, 100, 1000	12
10	0	5, 10	5, 50	0.25	10, 100, 1000	12
0.5	0.2	-	50, 300	0.50, 0.75, 1	10, 100, 1000	18
1	0.2	-	50, 300	0.50, 0.75, 1	10, 100, 1000	18
5	2	-	50, 300	0.05, 0.15, 0.25	10, 100, 1000	18
10	2	-	50, 300	0.05, 0.15, 0.25	10, 100, 1000	18
Total					120*4 = 480	

5 Numerical studies

5.1 The behavior of the objective function in X

The first step in finding an optimal policy is determining the behavior of X in the expected total cost function. Here the case with unlimited customer patience is considered. In order to do so, the values given in Table 1 are used. In figures 5 and 6 the behavior of X is described for two specific sets of parameter values.

5.1.1 The behavior of the objective function in X with no full obsolescence



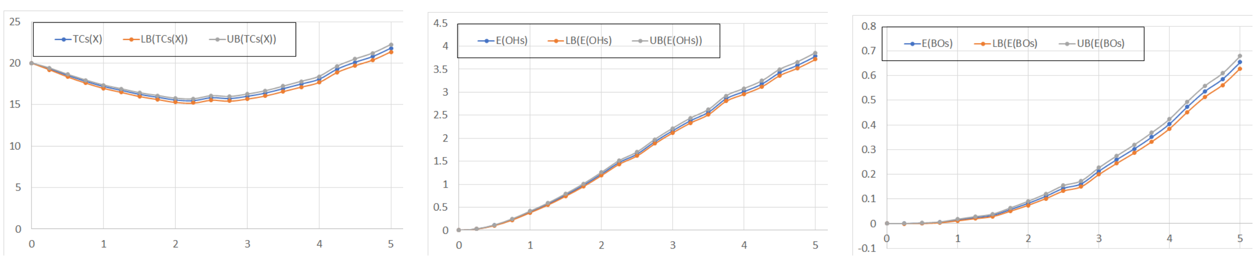
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 5: Behavior of X in total cost functions ($\lambda_0 = 10, \lambda_1 = 2, L = 0.15, \pi = 10, S_0 = 3, N = 2$)

From figure 5a follows that the expected total cost function is unimodal in X. This function is made from combining Figures 5b and 5c. From the figures results a trade-off between the cost related to holding inventory and cost for placing backorders. This trade-off is a result from the following: When the value for X is low, the probability that the event X happens before the arrival of the N^{th} customer is relative high. This means that the event X happens during the excess removal phase, and therefore this phase has not finished yet. With a lower demand rate during this excess removal phase, the company risks having excess stock for a longer period. This situation results in high inventory holding cost. Therefore, it can be concluded that the available time for the demand to take away the excess stock is too short. In the case where full obsolescence occurs ($\lambda_1 = 0$), no customers arrive after time X. With a low value for X, the service provider risks ending up with inventory that has to be either relocated or disposed. This results in high disposal cost and therefore high total cost.

In the case where the value of X is high, the probability that the event X appears after the arrival of the N^{th} customer is relatively high. The excess removal phase is finished and the system is now operating in the regular operation phase again, while using the $(S_1 - 1, S_1)$ policy. Because this policy is not capable of serving demand with rate λ_1, the inventory level can reach 0. When this happens, arriving customers are backordered and therefore the total cost increases due to backorder cost. It can be concluded that the expected backorders is increasing in X and therefore the total cost as well. From this trade-off it can be concluded that it is important to find a good balance between the expected on hand inventory and the expected backorders.

5.1.2 The behavior of the objective function in X with full obsolescence



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 6: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $S_0 = 2$, $N = 2$)

For the full obsolescence case, the behavior in X is different. This difference results from Figure 6b. Compared to Figure 5b the expected on hand inventory here is increasing in X instead of decreasing. This difference is subject to the following: Inventory holding cost are only included during the stock removal phase. If the value of X is low, the excess removal phase ends before the arrival of the N^{th} customer and therefore inventory holding cost are only incurred for a short period. However, in the figures 6a and 6b disposal cost are not taken into account. The expected total cost function here does take into consideration disposal cost (c_0). Therefore in the full obsolescence case the expected total cost function does become unimodal because when X is small the expected total cost increases because of disposing a high number of stock.

In the Appendix section 8.1, the figures for the different instances in Table 1 are given. From these figures, it can be concluded that for all the different parameter combinations the objective function is unimodal in X.

5.2 Simulation results

In this section, the impact of the advanced policy strategy is analyzed. In order to do so, the parameter combinations given in Table 2 are used to determine the optimal values for X (X^*). The values X^* are generated with the RSM method and can be found in Tables 5 and 6.

5.2.1 Simulation results with no full obsolescence

First, X^* found in this research is compared to the optimal X_s^* found by Pinçe and Dekker (2011). In order to compare these values, the difference in percentage $\Delta_X(\%)$ is used. This is calculated as follows: $\frac{X^*-X_s^*}{X_s^*} \times 100$. Then, the costs belonging to X^*, $TC(X^*)$ are compared to the total cost $TC_s(X_s^*)$ found by Pinçe and Dekker (2011) using the difference in percentage $\Delta_{TC}(\%)$. This difference is calculated as follows: $\frac{TC(X^*)-TC_s(X_s^*)}{TC_s(X_s^*)} \times 100$. These values can be found in Tables 5 and 6.

Table 5: Result for X^* for unlimited customer patience and $\lambda_1 > 0$

$h = 1$												
(λ_0, λ_1)	π	L	(S_0, N)	X_s^*	X^*	$\Delta_X(\%)$	$TC_s(X_s^*)$	$TC(X^*)$	$TC(X_s^*)$	$\Delta_{TC}(\%)$	$TC(0)$	$\Delta_0(\%)$
(0.5, 0.2)	50	0.50	(2, 1)	1.80	1.57	14·65	7·66	7·57	7.84 (7.58; 8.01)	−1·19	10.16 (9.88; 10.44)	34·21
		0.75	(2, 1)	1.29	1.25	3·20	8·35	8·23	8.55 (8.26; 8.85)	−1·46	10.02 (9.75;10.30)	21·75
		1.00	(2, 1)	0.96	0.94	2·13	9·06	8·59	8.85 (8.54; 9.16)	−5·47	10.10 (9.82; 10.38)	17·58
	300	0.50	(2, 0)	-	-	-	-	-	-	-	-	-
		0.75	(3, 1)	1.65	1.48	11·49	11·73	11·51	11.99 (11.48; 12.51)	−1·91	15.52 (15.09; 15.96)	34·84
		1.00	(3, 1)	1.24	1.09	13·76	12·52	12·31	12.48 (11.93; 13.03)	−1·71	15.28 (14.87; 15.69)	24·13
(1, 0.2)	50	0.50	(2, 1)	1.02	0.92	10·87	6·88	6·60	6.98 (6.71; 7.26)	−4·24	10.07 (9.79; 10.35)	52·58
		0.75	(3, 2)	1.71	1.56	9·62	14·47	14·34	14.77 (14.31; 15.24)	−0·91	25.29 (24.79; 25.80)	80·89
		1.00	(3, 2)	1.40	1.36	2·94	16·22	15·76	16.10 (15.57; 16.63)	−2·92	25.34 (24.83; 25.85)	60·79
	300	0.50	(3, 1)	1.16	1.08	7·41	9·20	9·08	9.19 (8.73; 9.66)	−1·32	14.81 (14.40; 15.21)	63·11
		0.75	(4, 2)	1.74	1.62	7·41	19·86	19·87	20.30 (19.38; 21.23)	0·05	34.57 (33.89; 35.25)	73·98
		1.00	(5, 3)	2.26	2.10	7·62	32·32	32·38	32.47 (31.09; 33.85)	0·19	59.12 (58.15; 60.09)	82·58
(5, 2)	5	0.05	(1, 1)	0.18	0.17	5·88	0·39	0·39	0.40 (0.39; 0.42)	0·00	0.52 (0.50; 0.53)	33·33
		0.15	(2, 1)	0.24	0.22	9·09	0·70	0·73	0.74 (0.72; 0.76)	4·11	0.98 (0.96; 1.01)	34·25
		0.25	(2, 1)	0.16	0.15	6·67	0·84	0·82	0.83 (0.81; 0.86)	−2·44	1.02 (0.99; 1.04)	24·39
	50	0.05	(2, 1)	0.19	0.19	0·00	0·78	0·77	0.77 (0.75; 0.80)	−1·30	1.00 (0.98; 1.03)	29·87
		0.15	(3, 1)	0.18	0.17	5·88	1·15	1·51	1.18 (1.14; 1.22)	23·84	1.46 (1.43; 1.50)	−3·31
		0.25	(4, 2)	0.29	0.26	11·54	2·51	2·50	2.55 (2.47; 2.62)	−0·40	3.45 (3.39; 3.52)	38·00
(10, 2)	5	0.05	(1, 1)	0.12	0.10	20·00	0·34	0·33	0.35 (0.34; 0.36)	−3·03	0.52 (0.50; 0.53)	57·58
		0.15	(3, 2)	0.29	0.24	20·83	1·02	1·12	1.13 (1.11; 1.16)	8·93	2.46 (2.41; 2.51)	119·64
		0.25	(4, 3)	0.35	0.32	9·37	1·81	2·02	2.05 (2.01; 2.10)	10·40	4.47 (4.40; 4.55)	121·29
	50	0.05	(2, 1)	0.11	0.10	10·00	0·68	0·66	0.69 (0.67; 0.72)	−3·03	1.00 (0.97; 1.03)	51·52
		0.15	(4, 2)	0.21	0.19	10·53	1·88	1·86	1.94 (1.87; 2.01)	−1·08	3.45 (3.38; 3.51)	85·48
		0.25	(6, 4)	0.33	0.30	10·00	4·27	4·53	4.41 (4.28; 4.54))	5·74	8.90 (8.77; 9.03)	96·47

In columns 10 and 12 in the brackets the 95% confidence interval is given.

From Table 5, it follows that the percent error for X differs between 0.00 and 20.83. This means that in all cases $X^* \leq X_s^*$. From this it can be concluded that the simulation and RSM method used in this research underestimates the X value found by Pinçe and Dekker (2011). This difference is due to the stochastic nature of the demand process which results in a noisy objective function.

From Table 5, it follows that the percent error in cost differs between −4.24 and 23.84. What can be seen in the results is that in most cases $\Delta_{TC}(\%) < \Delta_X(\%)$. The expected total cost function thus is less sensitive to changes in X. Above that, in most cases the percent error for the total cost function appear to be small. From the above results it can be concluded that the model used in this paper gives comparable results and the differences that do appear are due to the stochastic nature of the demand process.

The above conclusion is verified by the column $TC(X_s^*)$. In this column the optimal X_s^* found by Pinçe and Dekker (2011) is put in the simulation used in this paper. In the brackets, the 95% confidence intervals are given. Comparing the columns $TC(X^*)$ and $TC(X_s^*)$ one can observe that in most cases $TC(X^*)$ falls inside the 95% confidence intervals of $TC(X_s^*)$. The values that do not fall in the 95% confidence intervals, appear to be close to the confidence intervals.

Now that it is verified that the model used in this research is correct, the effect of changes in the parameters L and π on the variable X are analyzed. Visible from the data is that an increase in the parameters L and π results in a decrease in X given that the values for S_0 and S_1 are constant. These results are explained as follows: when the values for L and π increase, placing backorders becomes relative more expensive. Therefore, holding inventory has become relative cheaper and it is therefore cost efficient to have a lower value for X, because then the probability that backorders are placed is lower.

5.2.2 Simulation results with full obsolescence

Table 6: Result for X^* for unlimited customer patience and $\lambda_1 = 0$

$L = 0.25, h = 1$													
λ_0	π	c_0	S_0	N	X_s^*	X^*	$\Delta_X(\%)$	$TC_s(X_s^*)$	$TC(X^*)$	$TC(X_s)$	$\Delta_{TC}(\%)$	$TC(0)$	$\Delta_0(\%)$
0.5	50	5	1	1	0.44	0.43	−2·33	4·73	4·66	4.68 (4.62; 4.75)	−1·48	5	7·30
		10	1	1	1.00	1.04	3·85	8·22	8·13	8.12 (7.97; 8.27)	−1·09	10	23·00
	300	5	2	2	0.36	0.36	0·00	9·88	9·85	9.86 (9.78; 9.94)	−0·30	10	1·52
		10	2	2	0.93	0.84	−10·71	18·11	18·05	18.18 (17.89; 18.46)	−0·33	20	10·80
1	50	5	2	2	0.86	0.87	1·15	8·26	8·25	8.18 (8.05; 8.31)	−0·12	10	21·21
		10	2	2	1.40	1.29	−8·53	13·31	13·12	13.16 (12.92; 13.40)	−1·43	20	52·44
	300	5	2	2	0.29	0.29	0·00	9·44	9·33	9.42 (9.26; 9.58)	−1·17	10	7·18
		10	2	2	0.50	0.59	15·25	17·40	17·14	17.23 (16.93; 17.53)	−1·49	20	16·69
5	5	5	2	2	0.59	0.58	−1·72	3·37	3·29	3.37 (3.30; 3.45)	−2·37	10	203·95
		10	2	2	0.75	0.77	2·60	4·32	4·24	4.33 (4.22; 4.44)	−1·85	20	371·70
	50	5	4	4	0.52	0.50	−4·00	11·77	11·76	11.88 (11.66; 12.10)	−0·08	20	70·07
		10	4	4	0.67	0.67	0·00	18·33	18·45	18.31 (17.96; 18.67)	0·65	40	116·80
10	5	5	4	4	0.56	0.57	1·75	4·74	4·78	4.82 (4.73; 4.92)	0·84	20	318·41
		10	4	4	0.66	0.64	−3·13	5·88	6·03	5.90 (5.76; 6.04)	2·55	40	563·35
	50	5	6	6	0.43	0.43	0·00	14·74	14·67	14.87 (14.59; 15.15)	−0·47	30	104·50
		10	6	6	0.53	0.53	0·00	22·75	22·26	22.86 (22.41; 23.31)	−2·15	60	169·54

In column 11 in the brackets the 95% confidence interval is given.

From Table 6, it follows that the percent error for X differs between −10.71 and 15.25. Here, it does not always hold that $X^* \leq X_s^*$. Therefore, the conclusion made in the case where there is no full obsolescence does not hold here. Looking at the values for X themselves, the absolute difference is at most 0.11. This difference again follows from the stochastic nature of the demand process.

The percent error in cost differs between −2.37 and 2.55. What can be seen in the results is that in most cases $\Delta_{TC}(\%) < \Delta_X(\%)$. Therefore the same conclusion holds as holds in the case with no full obsolescence.

This conclusion is again verified by the column $TC(X_s^*)$. Comparing the columns $TC(X^*)$ and $TC(X_s^*)$, one can observe that in almost all the cases $TC(X^*)$ falls in the 95% confidence intervals of $TC(X_s^*)$. For the values that do not fall in the 95% confidence intervals, most of the cases are near the bounds.

Now that it is verified that the model used in this research is correct, the effect of changes in parameters L and π on the variable X are analyzed for the case with full obsolescence. This effect is the same as concluded in the case with no full obsolescence. However, now the parameter c_0 also has an effect on X. This effect is opposite from the L and π. An increase in c_0 results therefore in an increase in the variable X. This results from the following: The higher the value of c_0, the more expensive disposal cost have become. A higher value for X results in a higher chance that the excess stock removal phase has finished and the number of products in stock is lower. This results in a lower number of products that have to be disposed and therefore the cost are relative cheaper.

5.2.3 Effect of control policy change

Finally, the effect of the proposed control policy change is analyzed. With this policy change, the excess removal process starts before the drop in demand has occurred ($X > 0$), instead that is starts at the time the drop in demand occurs ($X = 0$). In order to analyze this policy change, the optimal X^* found in this research is compared with the simulated value for $X = 0$. This is done using the difference in percentage in the expected total cost $\Delta_0(\%) = \frac{TC(0)-TC(X^*)}{TC(X^*)} * 100$. Here, $TC_s(0)$ denotes the cost obtained with no advance policy change using simulation.

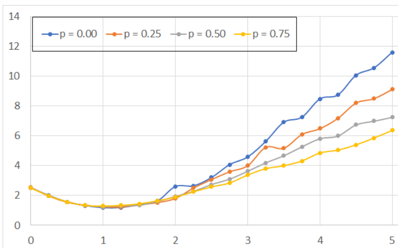
For the case where there is no full obsolescence, $\Delta_0(\%)$ differs between 17.58 and 121.29 percent. From this it can be concluded that the effect of a policy change is significant. However, there is one case in which this difference is -3.31. This is due to the stochastic nature of the demand process. The conclusion that the effect of a policy change is significant follows from the fact that the inventory holding cost have decreased significantly. When the excess removal process starts at the time the drop in demand occurs, it takes much more time to remove the excess stock because there are fewer customers arriving. Therefore, products are longer in stock, which results in high inventory holding cost. When the excess removal process can start before the drop in demand occurs, there are less products in stock when the drop actually occurs and therefore it takes less time to adapt to the new inventory level. It can thus be concluded that the policy change is effective. Also interesting is that the higher the value for both λ_0 and λ_1, the more cost efficient this policy change is.

For the case where there is full obsolescence, $\Delta_0(\%)$ differs between 1.52 and 563.35 percent. The same conclusion holds as for the case where there is no full obsolescence. This is explained as follows: the higher the value for X, the lower the number of products in stock and therefore the lower the products that have to be disposed. For the column where $X = 0$, the products in stock are disposed for the price c_0. Therefore, $TC(0)$ reaches the steady state case in which $TC(0) = S_0 * c_0$. This steady state level is reached because Pinçe and Dekker (2011) made the assumption that at time 0 the inventory level is equal to S_0, however in Pinçe and Dekker (2011) there is some unexplained deviation. Also interesting is that for the parameter π it follows that whenever π is higher, the difference in percentage is smaller. This is explained as follows: the higher the value of π, the more expensive backordering becomes. Therefore disposing products has become relative cheaper. This results in a lower value for X and therefore the difference in percentage is lower.

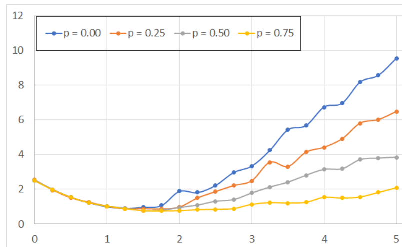
5.3 The behavior of the objective function in X with limited customer patience

In this section, the behavior of X in the expected total cost function is again determined, however now the case with limited customer patience is considered. In order to do so, the values given in Table 3 are used. In figures 7,8 and 9 the behavior of X is described for two specific sets of parameter values.

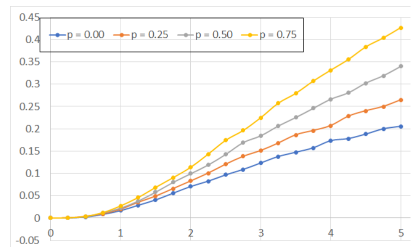
5.3.1 The behavior of the objective function in X with no full obsolescence and limited customer patience



(a) Expected total cost function

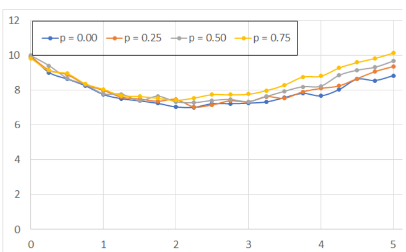


(b) Expected on hand inventory

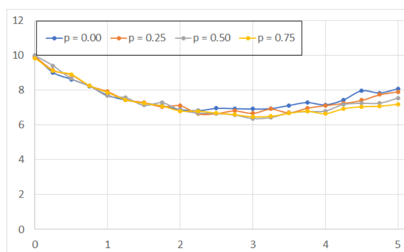


(c) Expected backorders

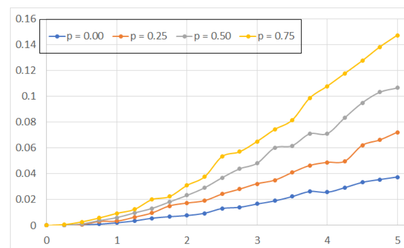
Figure 7: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 2$, $L = 0.15$, $\pi = 10$, $\theta = 10$, $S_0 = 3$, $N = 2$)



(a) Expected total cost function



(b) Expected on hand inventory



(c) Expected backorders

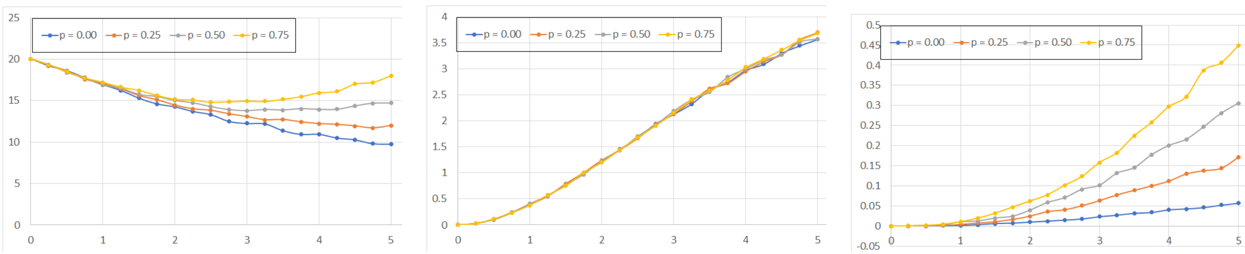
Figure 8: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $L = 0.75$, $\pi = 20$, $\theta = 10$, $S_0 = 2$, $N = 1$)

From both the Figures 7a and 8a, it can be concluded that the objective function is unimodal in X for the four different values of the parameter p. Figure 7a is made from combining figures 7b and 7c. Figure 8a is made from combining figures 8b and 8c.

Following from Figure 7a, it can be concluded that the lower the percentage of clients with contracts (p), the higher the expected total cost function becomes when the value of X increases after the minimum. This behavior can be explained from looking at the expected on hand inventory. However, from Figure 7a it follows that the lower the percentage of clients with contracts, the less fast the expected total cost function increases after the minimum. This difference is subject to the difference in λ_0 and λ_1. This pattern is also visible from the figures in the Appendix section 8.2.2. However, this pattern is not visible in the area around the minimum value of X and therefore this will not play an important role in determining the optimal value of X.

The trade-off discussed in the case for unlimited customer patience is also present here. This trade-off is however less visible for higher values of λ. This is mainly due to the lower expected on hand inventory for lower values of X.

5.3.2 The behavior of the objective function in X with full obsolescence and limited customer patience



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 9: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $\theta = 10$, $S_0 = 2$, $N = 2$)

Same as in the case for the unlimited patience rate, the behavior in X is different for the full obsolescence case. This difference again results from the expected on hand inventory. The explanation for this is the same as in the unlimited customer patience case. However, for the full obsolescence case Figure 9 and the figures in the appendix section 8.2.1 seem not to be unimodal. In fact it is observed that the figures are unimodal with a higher range, however this is not visible from the figures because the range used for X is too small here. This makes it seem like the total cost function is not unimodal. A higher range is not considered for comparison use with the unlimited customer patience case.

5.4 Simulation results with limited customer patience

In this section, the impact of the advanced policy strategy is analyzed. In order to do so, the parameter combinations given in Table 4 are used to determine the optimal values for X (X_{ext}) in situations with limited customer patience. The values X_{ext} are generated again with the RSM method and can be found in Tables 7 and 8.

5.4.1 Simulation results with no full obsolescence and limited customer patience

Table 7: Result for X^* for unlimited customer patience and $\lambda_1 > 0$

$h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\pi = 50$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.5	2	1	0	10	2.28	1.57	45·35	7.15	7.57	−5·61
				100	2.69	1.57	71·31	6.85	7.57	−9·48
				1000	3.46	1.57	120·57	6.85	7.57	−9·54
			0.25	10	2.10	1.57	33·74	7.28	7.57	−3·80
				100	2.60	1.57	65·53	7.00	7.57	−7·56
				1000	2.23	1.57	42·35	6.90	7.57	−8·84
			0.5	10	1.88	1.57	19·49	7.36	7.57	−2·78
				100	1.87	1.57	19·18	7.20	7.57	−4·83
				1000	1.95	1.57	24·14	7.37	7.57	−2·64
			0.75	10	1.58	1.57	0·34	7.35	7.57	−2·86
				100	2.20	1.57	40·34	7.57	7.57	0·00
				1000	2.12	1.57	34·83	7.39	7.57	−2·32

Firstly, the effect of changes in p on the variable X_{ext} and on the expected total cost are analyzed for the case where there is no full obsolescence. From Table 7, it follows that in most cases when the percentage of customers that sign contracts increases, the value for X_{ext} decreases. This is visible from the decrease in $\Delta_X(\%)$, which is calculated as follows: $\frac{X_{ext}-X^*}{X^*} \times 100$. This result is explained as follows: when the percentage of customers that sign contracts increases, the more customers will wait for their product to arrive and therefore spend more time in the queue. This results in higher backorder costs and therefore it has become relative cheaper to hold products in inventory. This results in a lower value for X_{ext}. The final conclusion holds that when the percentage of clients that sign contracts increase, the later the excess removal process should start and the shorter this period becomes.

From Table 7, it follows that when the percentage of customers that sign contracts increases, the higher the incurred expected total cost are. This is visible from the lower percentage difference in $\Delta_{TC}(\%)$, that is calculated as follows: $\frac{TC(X_{ext})-TC(X^*)}{TC(X^*)} \times 100$. This result is explained as follows: When the percentage of customers that sign contracts increases, the more customers will wait for their product to arrive and therefore spend more time in the queue. This results in higher backorder cost and therefore the expected total cost increases. Looking at Table 7 the cost decrease is relative small and in some cases limited customer patience does not have a significant effect on the total cost. The final conclusion here is that when the percentage of clients that sign contracts increase, the higher the incurred expected total cost are.

Secondly, the effect of changes in θ on the variable X_{ext} and the expected total cost are analyzed for the case where there is no full obsolescence. From Table 7 it follows that in most cases when customers become more impatient, the value for X increases. This is visible from the increase in the values for $\Delta_X(\%)$. This result is explained as follows: when customers become more impatient (θ increases), those customers will spend less time waiting in the queue for the product to arrive. This results in a lower backorder cost and therefore backordering has become relative cheaper and this results in a higher value for X_{ext}. The final conclusion holds that when customers become more impatient, the earlier the excess removal process should start and the longer this period becomes.

From Table 7, it follows that in most cases when customers become more impatient, the lower the incurred expected total cost are. This is visible from the higher percentage difference in $\Delta_{TC}(\%)$. This result is explained as follows: When customers become more impatient, fewer customers will wait for their product to arrive and therefore spend less time in the queue. This results in lower backorder cost and therefore the expected total cost decreases. Looking at Table 7, the cost decrease is relative small and in most cases limited customer patience does not have a significant effect on the total cost. The final conclusion here is that when customers become more impatient, the lower the incurred expected total cost are.

From the above results, can be concluded that in general the value of X increases when limited customer patience is considered and that the expected total cost decreases when limited customer patience is considered. However, this pattern is not always visible looking at the values in the appendix section 8.3. In some cases for a higher percentage of clients that sign contract a higher value of X_{ext} is visible and sometimes there is a decrease in expected total cost. In some cases, when customers become more impatient, the value for X increases and sometimes there is an increase in the expected total cost when customers become more impatient. For example, the patterns does not always return for the cases in which $L = 0.05$ and $(\lambda_0,\ \lambda_1) = (5, 2)$ and in which $L = 0.05$ and $(\lambda_0,\ \lambda_1) = (10, 2)$. In this case $\Delta_X(\%)$ is negative in most cases and in some cases $\Delta_{TC}(\%)$ is positive. In most cases the absolute difference between X_{ext} and X^* lies between 0.01 and 0.04. This absolute difference is at most 0.08 and the absolute difference between $TC(X_{ext})$ and $TC(X^*)$ lies between 0.00 and 0.02. The small differences might be explained due to the following: When impatience rate is 100 and 1000, the mean waiting time is 0.01 and 0.001, which means on average backordered customer will leave without product. This leads to an actually lower total demand rate for both before and after the demand drop. Furthermore, these small differences arise also from the stochastic nature of the demand process and therefore are negligible. The final conclusions made therefore still hold.

5.4.2 Simulation results with full obsolescence and limited customer patience

Table 8: Result for X^* for limited customer patience and $\lambda_1 = 0$

$h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 1$, $N = 1$, $\pi = 50$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	1·21	0.43	180·79	4.22	4.66	−9·45
		100	10·69	0.43	2385·71	2.78	4.66	−40·25
		1000	13·01	0.43	2926·52	1.66	4.66	−64·36
	0.25	10	0·93	0.43	116·70	4.42	4.66	−5·05
		100	1·40	0.43	225·40	4.14	4.66	−11·13
		1000	1·43	0.43	231·42	4.02	4.66	−13·74
	0.5	10	0·61	0.43	42·62	4.52	4.66	−2·93
		100	0·75	0.43	73·59	4.43	4.66	−4·91
		1000	0·77	0.43	80·13	4.40	4.66	−5·50
	0.75	10	0·46	0.43	6·93	4.59	4.66	−1·40
		100	0·43	0.43	0·50	4.57	4.66	−1·96
		1000	0·61	0.43	42·83	4.55	4.66	−2·45

In Table 8, the results for X_{ext} are presented for the full obsolescence case where there is limited customer patience. The same pattern for changes in p and θ hold as for the no full obsolescence case. This pattern is also clearly visible in the results given in the appendix.

For the full obsolescence case, $\Delta_X(\%)$ is also much bigger compared to the case with no full obsolescence. It can be concluded that limited customer patience has a relative bigger effect on X in the full obsolescence case compared to the case with no full obsolescence. In some cases this difference reaches almost 3000%, which is a lot. For the full obsolescence case $\Delta TC(\%)$ is also much bigger compared to the case with no full obsolescence. From this can be concluded that the policy change here is even more effective and leads to higher cost decreases. This change is clearly visible from the figures in the appendix section 8.2. From these figures follows that for the full obsolescence case, the minimum of the expected total cost function lies much more to the right than compared with the no full obsolescence case. Also in some cases the figures seem to converge. This is however not true, the expected total cost function increases after the minimum, however this increase is much lower than compared to the case where there is no full obsolescence.

6 Conclusion

In this paper, research is performed to determine the effect policy change can have on the total cost. With this policy change the excess removal process starts before the drop in demand has occurred instead of starting at the time the drop in demand occurs. The considered products are slow-moving, subject to obsolescence and are controlled with a continuous review inventory system. For the inventory policy, the one-for-one replenishment policy is used. In this paper the research is carried out by extending Pinçe and Dekker (2011) by taking into account the effect limited customer patience has on the policy change.

The model used in this research predicts almost the same results as found in Pinçe and Dekker (2011). Small changes are due to the stochastic nature of the demand process. Therefore, it can be concluded that the model used in this research explains correct.

From the results follows that when the excess removal phase starts before the drop in demand instead of starting at the time the drop in demand occurs can lead to significant cost savings. Therefore, the control policy change is effective. This is proven under different circumstances and for both no full and full obsolescence case. Service providers should therefore use this control policy in order to have significant cost reductions and therefore outperform competitors.

Comparing the situation described in Pinçe and Dekker (2011) and the situation with limited customer patience, it follows that limited customer patience results in a longer stock removal process and lower total cost. Furthermore, it can be concluded that when the percentage of clients that sign contracts increases, the later the excess removal process should start and the higher the incurred expected total cost are. It is also concluded that the more impatient customers become, the earlier the excess removal process should start and the lower the incurred total cost are. Eventually, it can be concluded that the policy change here leads to even more significant cost savings. Companies with customers that have limited patience should therefore even adapt earlier to the new base stock level. Because customers spend less time in the queue, backorders have become relative cheaper than holding inventory. Therefore, the stock removal process can start earlier and become longer.

Service providers of slow-moving products that are subject to obsolescence and for which the demand in the future will be lower can use the results in this paper. Timely adjustment has significant effect on the total cost and therefore adjusting at a good time is critical for the competitiveness of the service provider. The model used in this research can be used to forecast the optimal moment for the service provider, with clients that sign contracts and clients that don't, for when to adjust for a lower demand in the future.

7 Discussion and future research

During this research several assumptions have been made to simplify the model and those assumptions limit the generality of the model used in this research. However, the model still provides insight into the impact the policy change can have on the total cost and therefore can be used for further research in which some of the assumptions made can be changed.

One of the assumptions made is that the waiting time is distributed according to the Exponential distribution. For some specific products, this might not be the case and therefore other distributions should be considered. Different distributions can have different effects on the length of the excess removal process and on the incurred total cost.

In this research, the two types of customers are charged the same waiting cost rate. However, it could be interesting to research the policy in which customers that sign contracts with the service provider get charged a lower waiting cost rate. The reason for doing this, is that then more customer might be willing to sign contracts and then the service provider has more security in the products that are going to be sold and less customers might go to competitors.

Another interesting policy could be placing a penalty on customers for leaving the system without a product. Products are backordered for those customers which results in high cost. When impatient customers leave the system without a product they ordered, they should therefore be punished with a penalty. Doing this might lead to less customers leaving without a product.

References

Brown, G. W., Lu, J. Y., and Wolfson, R. J. (1964). Dynamic modeling of inventories subject to obsolescence. *Management Science*, 11(1):51–63.

Hadley, G. and Whitin, T. (1963). Analysis of inventory systems. *Prentice Hall, Inc., Englewood Cliffs, Nj.*

Myers, R. and Montgomery, D. (1995). The analysis of response surfaces. *Response Surface Methodology. Wiley Series In Probability And Mathematical Statistics. Wiley, New York.*

Nicolai, R. and Dekker, R. (2009). Automated response surface methodology for simulation optimization models with unknown variance. *Quality Technology & Quantitative Management*, 6(3):325–352.

Perry, D. and Posner, M. J. (1998). An (s- 1, s) inventory system with fixed shelf life and constant lead times. *Operations Research*, 46(3-supplement-3):S65–S71.

Pierskalla, W. P. (1969). An inventory problem with obsolescence. *Naval Research Logistics Quarterly*, 16(2):217–228.

Pinçe, Ç. and Dekker, R. (2011). An inventory model for slow moving items subject to obsolescence. *European Journal of Operational Research*, 213(1):83–95.

Pinçe, Ç., Frenk, J., and Dekker, R. (2015). The role of contract expirations in service parts management. *Production and Operations Management*, 24(10):1580–1597.

Rosenfield, D. B. (1989). Disposal of excess inventory. *Operations Research*, 37(3):404–409.

Shenas, N. Y., Jahromi, A. E., and Modarres, M. (2009). A new approach to find the optimal solution for base stock policies. *Journal of Applied Sciences*, 9(4):789–793.

Simpson, J. (1955). A formula for decisions on retention or disposal of excess stock. *Naval Research Logistics Quarterly*, 2(3):145–155.

Stulman, A. (1989). Excess inventory with stochastic demand: Continuous reporting model. *Journal of the Operational Research Society*, 40(11):1041–1047.

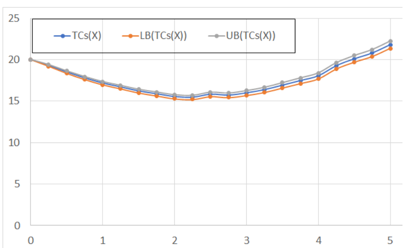
Weisberg, S. (2005). *Applied linear regression*, volume 528. John Wiley & Sons.

8 Appendix

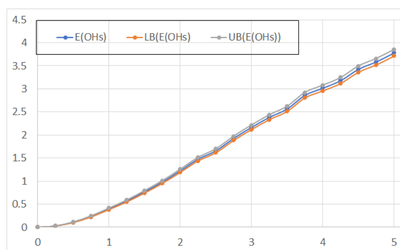
8.1 The behavior of the objective function and operating characteristics for the values given in Table 1

In this section the behavior of the objective function in X with and without full obsolescence is plotted. For each different parameter combinations, the expected total cost function, the expected on hand inventory and the expected backorders are plotted. The blue line describes the TC, the orange line the lower bound and the green line the upper bound.

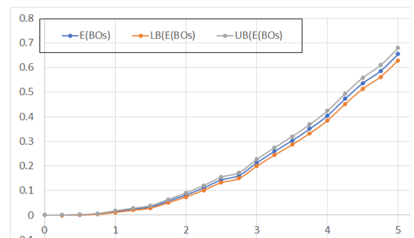
8.1.1 Full obsolescence case



(a) Expected total cost function

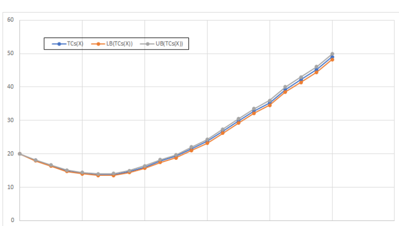


(b) Expected on hand inventory

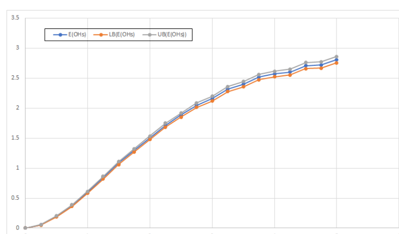


(c) Expected backorders

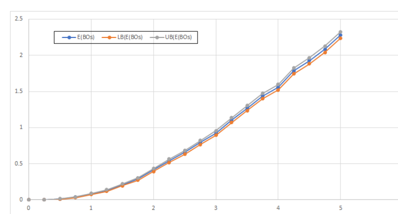
Figure 10: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $S_0 = 2$, $N = 2$)



(a) Expected total cost function

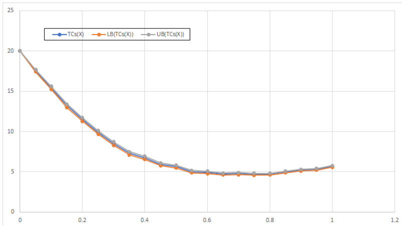


(b) Expected on hand inventory

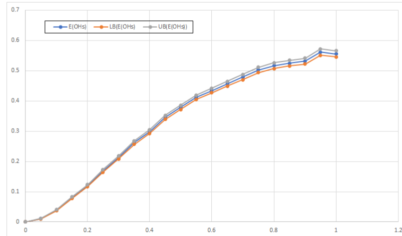


(c) Expected backorders

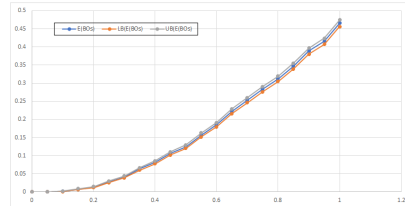
Figure 11: Behavior of X in the cost functions ($\lambda_0 = 1$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $S_0 = 2$, $N = 2$)



(a) Expected total cost function

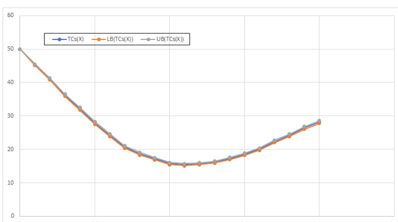


(b) Expected on hand inventory

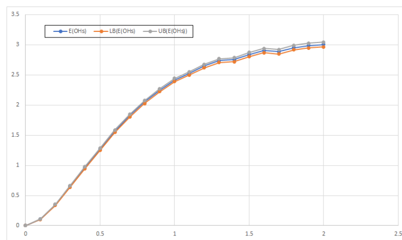


(c) Expected backorders

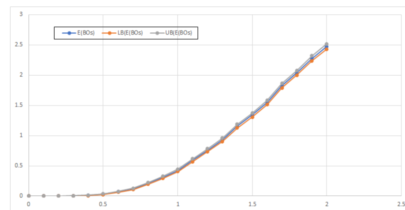
Figure 12: Behavior of X in the cost functions ($\lambda_0 = 5$, $\lambda_1 = 0$, $L = 0.15$, $\pi = 10$, $c_0 = 10$, $S_0 = 2$, $N = 2$)



(a) Expected total cost function

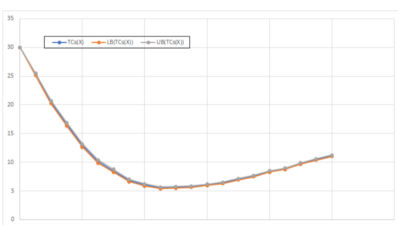


(b) Expected on hand inventory

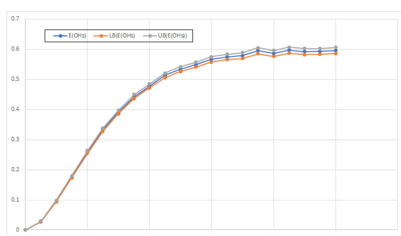


(c) Expected backorders

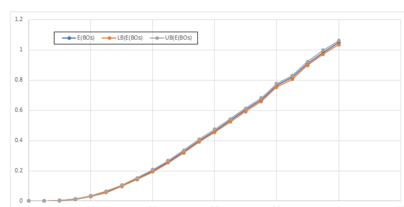
Figure 13: Behavior of X in the cost functions ($\lambda_0 = 5$, $\lambda_1 = 0$, $L = 0.50$, $\pi = 10$, $c_0 = 10$, $S_0 = 5$, $N = 5$)



(a) Expected total cost function

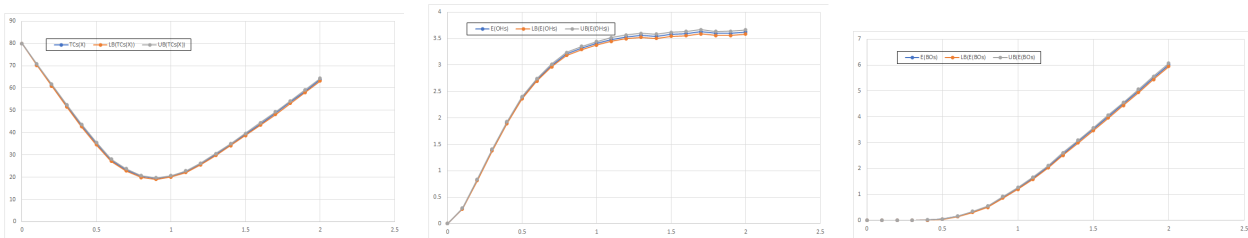


(b) Expected on hand inventory



(c) Expected backorders

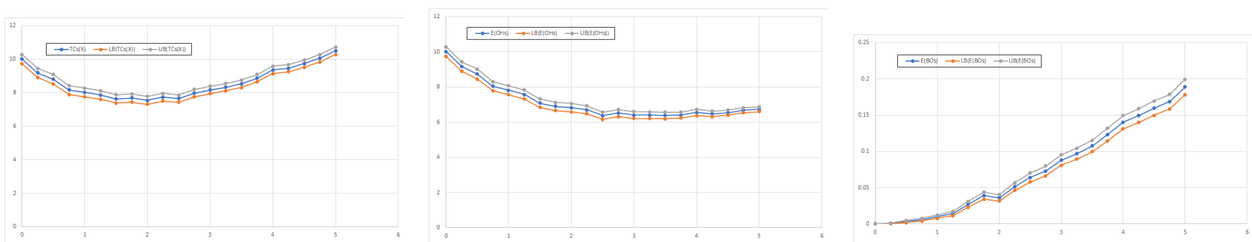
Figure 14: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.15$, $\pi = 10$, $c_0 = 10$, $S_0 = 3$, $N = 3$)



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

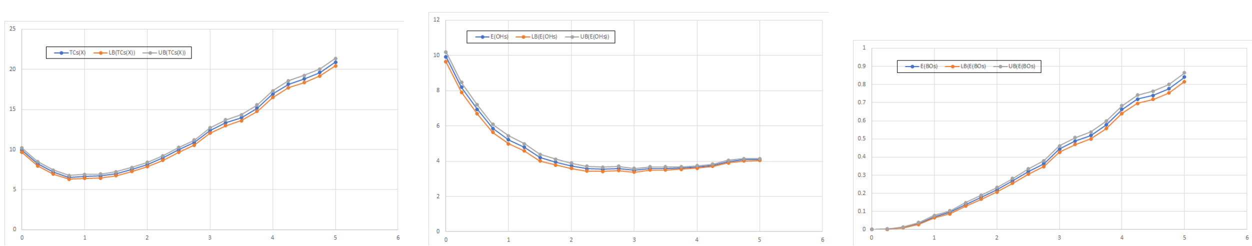
Figure 15: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.50$, $\pi = 10$, $c_0 = 10$, $S_0 = 8$, $N = 8$)

8.1.2 No full obsolescence case



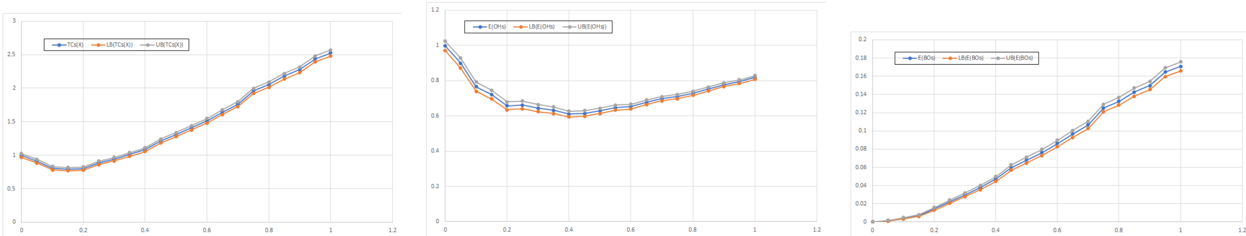
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 16: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $L = 0.75$, $\pi = 20$, $S_0 = 2$, $N = 1$)



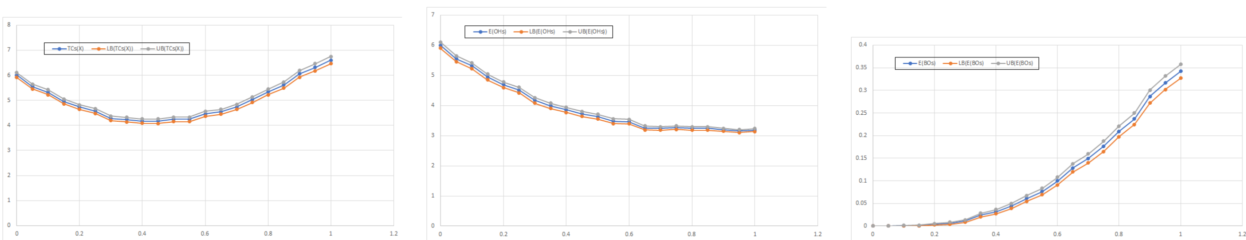
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 17: Behavior of X in the cost functions ($\lambda_0 = 1$, $\lambda_1 = 0.2$, $L = 0.75$, $\pi = 20$, $S_0 = 2$, $N = 1$)



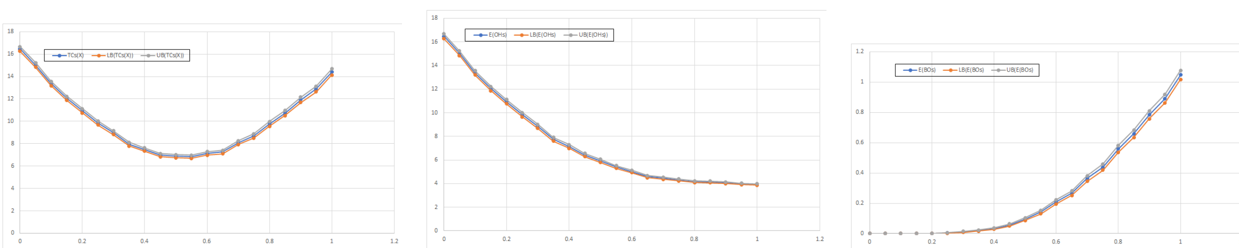
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 18: Behavior of X in the cost functions ($\lambda_0 = 5$, $\lambda_1 = 2$, $L = 0.15$, $\pi = 10$, $S_0 = 2$, $N = 1$)



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 19: Behavior of X in the cost functions ($\lambda_0 = 5$, $\lambda_1 = 2$, $L = 0.50$, $\pi = 10$, $S_0 = 5$, $N = 3$)



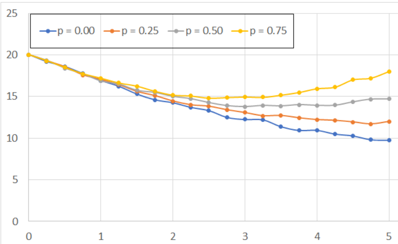
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 20: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 2$, $L = 0.50$, $\pi = 10$, $S_0 = 8$, $N = 6$)

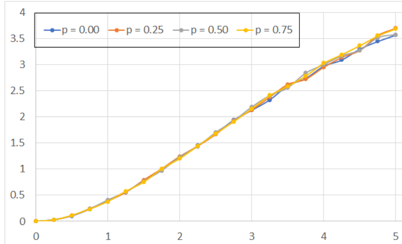
8.2 The behavior of the objective function and operating characteristics for the values given in Table 3

In this section the behavior of the objective function in X with limited customer patience for the cases with and without full obsolescence is plotted. For each different parameter combinations, the expected total cost function, the expected on hand inventory and the expected backorders are plotted. The blue line describes the TC, the orange line the lower bound and the green line the upper bound. In this section the blue line belongs to $p = 0\%$, the orange line belongs to $p = 25\%$, the grey line belongyus to $p = 75\%$ and the yellow line belongs to $p = 75\%$.

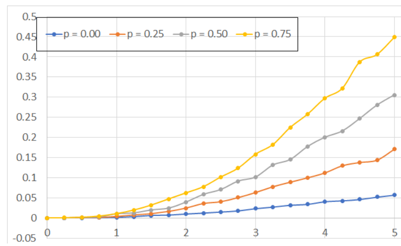
8.2.1 Full obsolescence case



(a) Expected total cost function

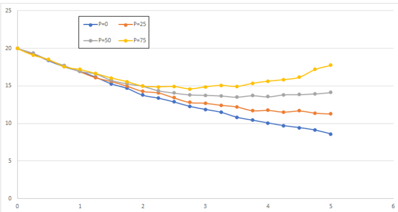


(b) Expected on hand inventory

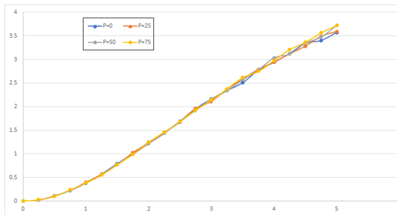


(c) Expected backorders

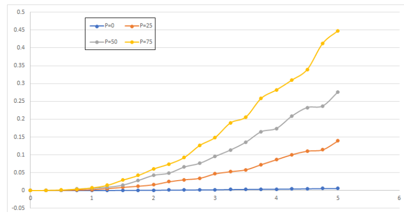
Figure 21: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $\theta = 10$, $S_0 = 2$, $N = 2$)



(a) Expected total cost function

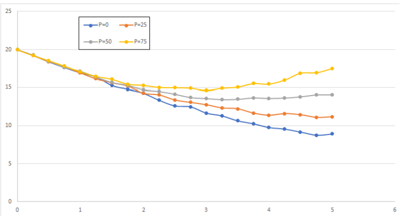


(b) Expected on hand inventory

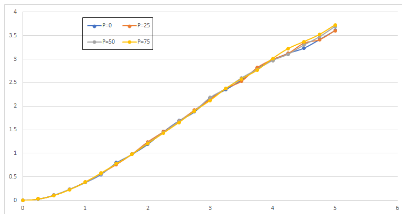


(c) Expected backorders

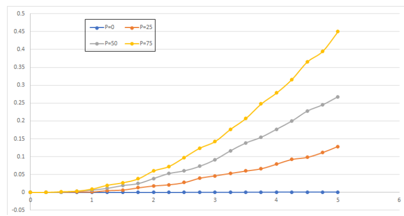
Figure 22: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $\theta = 100$, $S_0 = 2$, $N = 2$)



(a) Expected total cost function

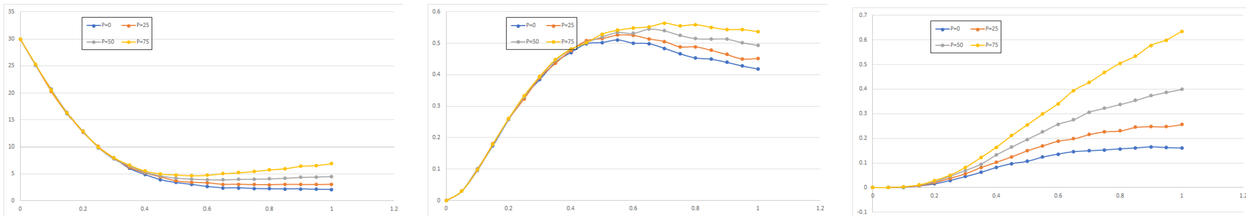


(b) Expected on hand inventory



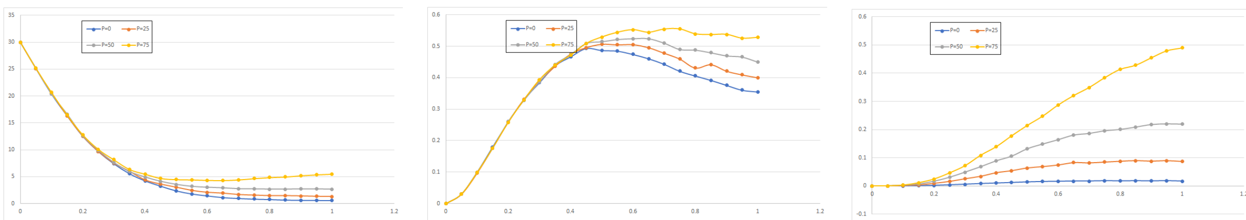
(c) Expected backorders

Figure 23: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.75$, $\pi = 20$, $c_0 = 10$, $\theta = 1000$, $S_0 = 2$, $N = 2$)



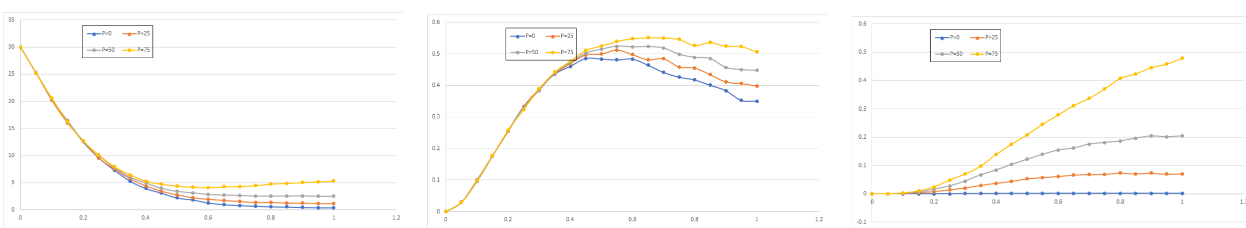
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 24: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.15$, $\pi = 10$, $c_0 = 10$, $\theta = 10$, $S_0 = 3$, $N = 3$)



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

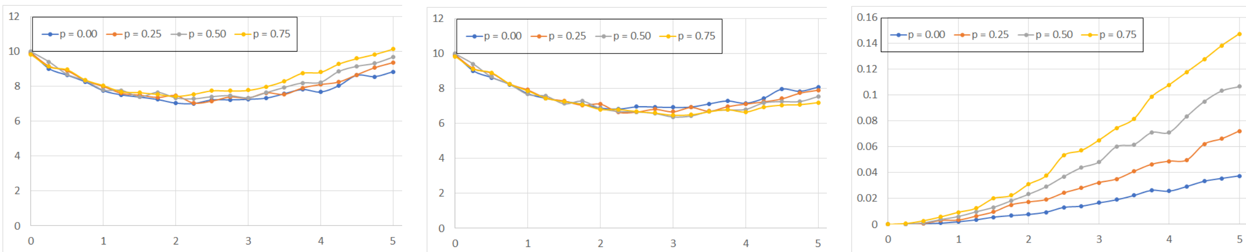
Figure 25: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.15$, $\pi = 10$, $c_0 = 10$, $\theta = 100$, $S_0 = 3$, $N = 3$)



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

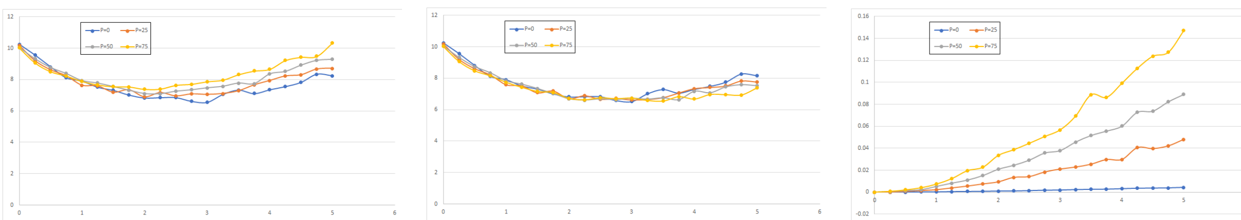
Figure 26: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.15$, $\pi = 10$, $c_0 = 10$, $\theta = 1000$, $S_0 = 3$, $N = 3$)

8.2.2 No full obsolescence case



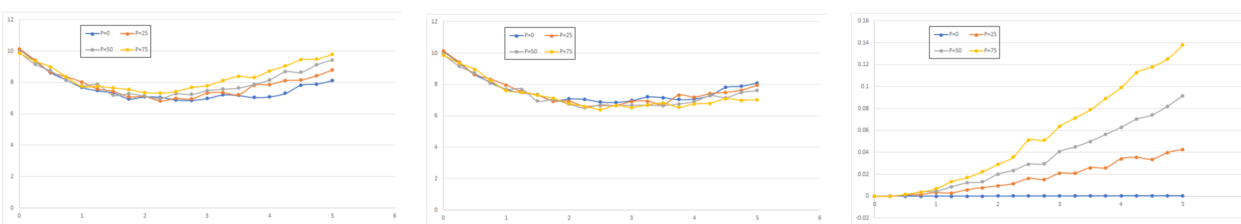
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 27: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $L = 0.75$, $\pi = 20$, $\theta = 10$, $S_0 = 2$, $N = 1$)



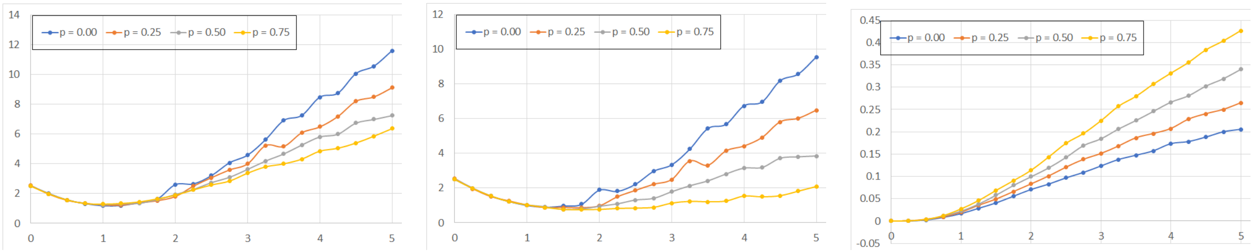
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 28: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $L = 0.75$, $\pi = 20$, $\theta = 100$, $S_0 = 2$, $N = 1$)



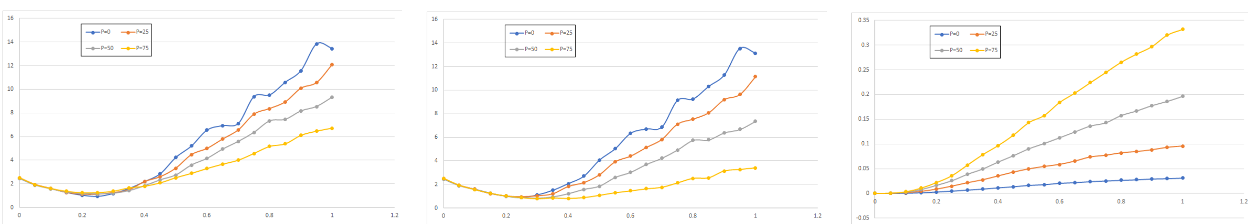
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 29: Behavior of X in the cost functions ($\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $L = 0.75$, $\pi = 20$, $\theta = 1000$, $S_0 = 2$, $N = 1$)



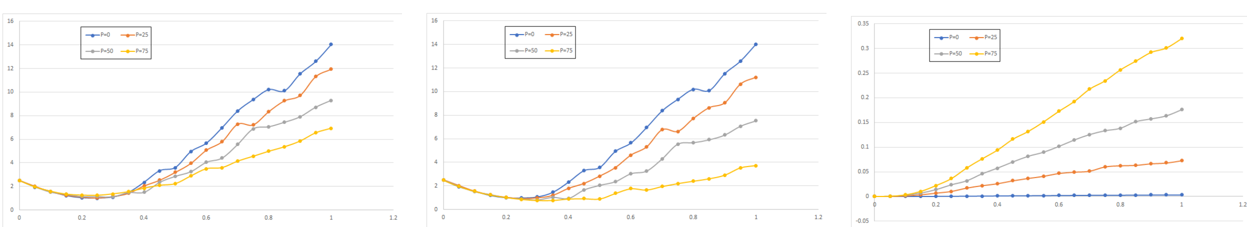
(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 30: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 2$, $L = 0.15$, $\pi = 10$, $\theta = 10$, $S_0 = 3$, $N = 2$)



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 31: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 2$, $L = 0.15$, $\pi = 10$, $\theta = 100$, $S_0 = 3$, $N = 2$)



(a) Expected total cost function (b) Expected on hand inventory (c) Expected backorders

Figure 32: Behavior of X in the cost functions ($\lambda_0 = 10$, $\lambda_1 = 2$, $L = 0.15$, $\pi = 10$, $\theta = 1000$, $S_0 = 3$, $N = 2$)

8.3 Performance of X_{ext} and comparison with X^* for $\lambda_1 > 0$

In this section the impact of the advanced policy strategy with limited customer patience for different parameter combinations is given. The results are obtained with RSM and simulation.

Table 9: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\pi = 50$

$h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\pi = 50$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.5	2	1	0	10	2.28	1.57	45·35	7.15	7.57	−5·61
				100	2.69	1.57	71·31	6.85	7.57	−9·48
				1000	3.46	1.57	120·57	6.85	7.57	−9·54
			0.25	10	2.10	1.57	33·74	7.28	7.57	−3·80
				100	2.60	1.57	65·53	7.00	7.57	−7·56
				1000	2.23	1.57	42·35	6.90	7.57	−8·84
			0.5	10	1.88	1.57	19·49	7.36	7.57	−2·78
				100	1.87	1.57	19·18	7.20	7.57	−4·83
				1000	1.95	1.57	24·14	7.37	7.57	−2·64
			0.75	10	1.58	1.57	0·34	7.35	7.57	−2·86
				100	2.20	1.57	40·34	7.57	7.57	0·00
				1000	2.12	1.57	34·83	7.39	7.57	−2·32
0.75	2	1	0	10	2.21	1.25	77·20	7.25	8.23	−11·86
				100	2.28	1.25	82·20	6.86	8.23	−16·59
				1000	2.44	1.25	95·21	6.69	8.23	−18·75
			0.25	10	1.80	1.25	44·02	7.56	8.23	−8·11
				100	1.98	1.25	58·42	7.30	8.23	−11·35
				1000	1.83	1.25	46·16	7.16	8.23	−12·99
			0.5	10	1.56	1.25	24·62	7.82	8.23	−4·98
				100	1.71	1.25	37·12	7.66	8.23	−6·87
				1000	1.93	1.25	54·53	7.58	8.23	−7·87
			0.75	10	1.23	1.25	−1·78	8.10	8.23	−1·52
				100	1.61	1.25	28·97	7.87	8.23	−4·43
				1000	1.32	1.25	5·82	8.21	8.23	−0·22
1	2	1	0	10	1.96	0.94	108·00	7.28	8.59	−15·22
				100	2.55	0.94	171·63	6.58	8.59	−23·37
				1000	2.81	0.94	198·42	6.54	8.59	−23·86
			0.25	10	1.49	0.94	58·18	7.78	8.59	−9·47
				100	2.10	0.94	123·49	7.52	8.59	−12·50
				1000	2.17	0.94	130·76	7.41	8.59	−13·78
			0.5	10	1.42	0.94	51·52	8.11	8.59	−5·62
				100	1.16	0.94	22·93	7.98	8.59	−7·14
				1000	1.04	0.94	10·34	8.04	8.59	−6·35
			0.75	10	1.01	0.94	7·88	8.33	8.59	−3·08
				100	1.11	0.94	17·67	8.41	8.59	−2·10
				1000	1.06	0.94	12·78	8.37	8.59	−2·61

Table 10: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\pi = 300$

$h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0.2$, $\pi = 300$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.5	2	0	-	-		-	-	-	-	-
0.75	3	1	0	10	2.27	1.48	53.21	10.55	11.51	-8.38
				100	2.53	1.48	71.02	10.26	11.51	-10.86
				1000	2.49	1.48	68.06	10.07	11.51	-12.55
			0.25	10	2.36	1.48	59.58	10.95	11.51	-4.89
				100	2.79	1.48	88.23	10.73	11.51	-6.75
				1000	2.33	1.48	57.41	10.71	11.51	-6.92
			0.5	10	1.66	1.48	12.23	11.09	11.51	-3.68
				100	2.32	1.48	56.95	11.47	11.51	-0.39
				1000	2.46	1.48	66.31	11.11	11.51	-3.44
			0.75	10	1.97	1.48	33.33	11.26	11.51	-2.18
				100	1.52	1.48	2.49	11.40	11.51	-0.92
				1000	2.48	1.48	67.42	11.47	11.51	-0.35
1	3	1	0	10	1.51	1.09	38.86	10.94	12.31	-11.12
				100	3.05	1.09	180.26	10.19	12.31	-17.22
				1000	1.95	1.09	79.29	10.25	12.31	-16.71
			0.25	10	1.93	1.09	77.50	11.43	12.31	-7.17
				100	2.51	1.09	130.09	10.88	12.31	-11.61
				1000	2.35	1.09	115.17	10.60	12.31	-13.89
			0.5	10	1.80	1.09	65.58	11.49	12.31	-6.69
				100	1.67	1.09	53.26	11.14	12.31	-9.48
				1000	1.44	1.09	31.68	11.36	12.31	-7.72
			0.75	10	1.54	1.09	41.52	11.66	12.31	-5.25
				100	1.16	1.09	6.25	11.89	12.31	-3.43
				1000	1.34	1.09	23.38	11.94	12.31	-2.98

Table 11: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 1$, $\lambda_1 = 0.2$, $\pi = 50$

$h = 1$, $\lambda_0 = 1$, $\lambda_1 = 0.2$, $\pi = 50$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.5	2	1	0	10	1.34	0.92	46·00	5·37	6·6	−18·66
				100	1.56	0.92	69·47	4·56	6·6	−30·97
				1000	1.57	0.92	70·71	4·67	6·6	−29·20
			0.25	10	1.31	0.92	42·07	5·75	6·6	−12·87
				100	1.43	0.92	55·60	5·35	6·6	−18·99
				1000	1.40	0.92	51·99	5·23	6·6	−20·69
			0.5	10	1.10	0.92	19·64	6·11	6·6	−7·38
				100	1.32	0.92	43·91	5·87	6·6	−11·04
				1000	1.25	0.92	35·37	5·95	6·6	−9·87
			0.75	10	0.87	0.92	−5·66	6·47	6·6	−1·98
				100	0.96	0.92	4·53	6·24	6·6	−5·52
				1000	1.12	0.92	22·17	6·34	6·6	−4·01
0.75	3	2	0	10	2.54	1.56	62·87	10·68	14·34	−25·53
				100	3.09	1.56	98·21	9·03	14·34	−37·05
				1000	2.99	1.56	91·85	8·84	14·34	−38·38
			0.25	10	2.08	1.56	33·17	12·04	14·34	−16·05
				100	2.72	1.56	74·06	10·99	14·34	−23·34
				1000	2.63	1.56	68·72	10·82	14·34	−24·57
			0.5	10	1.94	1.56	24·24	13·00	14·34	−9·37
				100	1.96	1.56	25·33	12·62	14·34	−11·99
				1000	2.12	1.56	36·12	12·29	14·34	−14·29
			0.75	10	1.70	1.56	8·92	13·70	14·34	−4·44
				100	1.73	1.56	10·92	13·65	14·34	−4·81
				1000	1.91	1.56	22·56	13·80	14·34	−3·79
1	3	2	0	10	2.47	1.36	81·54	10·57	15·76	−32·95
				100	3.10	1.36	128·11	8·88	15·76	−43·68
				1000	3.04	1.36	123·62	8·48	15·76	−46·18
			0.25	10	2.08	1.36	52·69	12·73	15·76	−19·20
				100	2.23	1.36	63·81	11·68	15·76	−25·86
				1000	2.16	1.36	58·60	11·80	15·76	−25·13
			0.5	10	1.79	1.36	31·44	14·15	15·76	−10·20
				100	1.82	1.36	33·77	13·83	15·76	−12·27
				1000	1.64	1.36	20·24	13·59	15·76	−13·76
			0.75	10	1.55	1.36	13·97	15·26	15·76	−3·19
				100	1.62	1.36	18·86	15·15	15·76	−3·89
				1000	1.66	1.36	22·19	15·04	15·76	−4·57

Table 12: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 1$, $\lambda_1 = 0.2$, $\pi = 300$

$h = 1$, $\lambda_0 = 1$, $\lambda_1 = 0.2$, $\pi = 300$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.5	3	1	0	10	1.49	1.08	37·86	7·69	9·08	−15·32
				100	2.12	1.08	96·50	6·56	9·08	−27·79
				1000	1.92	1.08	77·71	6·27	9·08	−30·94
			0.25	10	1.64	1.08	51·49	8·12	9·08	−10·59
				100	1.56	1.08	44·30	7·35	9·08	−19·04
				1000	1.95	1.08	80·20	6·92	9·08	−23·77
			0.5	10	1.38	1.08	27·61	8·38	9·08	−7·75
				100	1.31	1.08	20·93	7·84	9·08	−13·71
				1000	1.63	1.08	51·35	8·14	9·08	−10·40
			0.75	10	1.12	1.08	4·02	8·92	9·08	−1·77
				100	1.21	1.08	12·48	8·42	9·08	−7·31
				1000	1.35	1.08	25·11	8·74	9·08	−3·70
0.75	4	2	0	10	2.85	1.62	76·00	15·36	19·87	−22·69
				100	3.35	1.62	106·62	12·23	19·87	−38·45
				1000	3.44	1.62	112·56	11·67	19·87	−41·26
			0.25	10	2.22	1.62	36·88	16·36	19·87	−17·66
				100	2.49	1.62	53·54	14·88	19·87	−25·11
				1000	2.80	1.62	73·00	14·81	19·87	−25·49
			0.5	10	2.06	1.62	27·28	17·41	19·87	−12·39
				100	2.57	1.62	58·88	17·61	19·87	−11·38
				1000	2.05	1.62	26·32	16·62	19·87	−16·35
			0.75	10	2.16	1.62	33·19	18·60	19·87	−6·37
				100	2.21	1.62	36·70	18·59	19·87	−6·43
				1000	1.70	1.62	4·81	18·81	19·87	−5·31
1	5	3	0	10	3.37	2.1	60·45	23·90	32·38	−26·20
				100	4.67	2.1	122·21	18·69	32·38	−42·29
				1000	5.35	2.1	154·63	17·51	32·38	−45·91
			0.25	10	2.78	2.1	32·50	27·17	32·38	−16·09
				100	3.26	2.1	55·09	24·60	32·38	−24·04
				1000	3.24	2.1	54·09	24·42	32·38	−24·59
			0.5	10	2.73	2.1	30·03	28·73	32·38	−11·27
				100	2.90	2.1	37·98	28·30	32·38	−12·59
				1000	2.82	2.1	34·21	27·73	32·38	−14·35
			0.75	10	2.58	2.1	22·95	30·04	32·38	−7·22
				100	2.36	2.1	12·19	30·84	32·38	−4·75
				1000	2.51	2.1	19·57	30·57	32·38	−5·59

Table 13: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 5$, $\lambda_1 = 2$, $\pi = 5$

$h = 1$, $\lambda_0 = 5$, $\lambda_1 = 2$, $\pi = 5$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.05	1	1	0	10	0.13	0.17	−21·20	0·38	0·39	−2·58
				100	0.09	0.17	−49·75	0·39	0·39	−0·06
				1000	0.10	0.17	−39·16	0·41	0·39	5·56
			0.25	10	0.12	0.17	−27·19	0·40	0·39	2·17
				100	0.09	0.17	−45·30	0·39	0·39	0·30
				1000	0.10	0.17	−41·15	0·39	0·39	0·25
			0.5	10	0.12	0.17	−26·99	0·41	0·39	4·54
				100	0.12	0.17	−29·19	0·40	0·39	1·36
				1000	0.10	0.17	−38·72	0·40	0·39	1·49
			0.75	10	0.11	0.17	−32·48	0·39	0·39	−0·06
				100	0.12	0.17	−29·67	0·39	0·39	−0·78
				1000	0.14	0.17	−17·43	0·39	0·39	0·78
0.15	2	1	0	10	0.23	0.22	3·04	0·71	0·73	−2·07
				100	0.25	0.22	13·05	0·67	0·73	−8·52
				1000	0.23	0.22	3·21	0·64	0·73	−12·41
			0.25	10	0.21	0.22	−4·42	0·71	0·73	−3·28
				100	0.24	0.22	8·87	0·67	0·73	−8·61
				1000	0.26	0.22	18·19	0·68	0·73	−7·11
			0.5	10	0.22	0.22	−1·89	0·71	0·73	−3·16
				100	0.24	0.22	9·14	0·69	0·73	−5·67
				1000	0.24	0.22	8·24	0·69	0·73	−5·58
			0.75	10	0.23	0.22	4·92	0·73	0·73	−0·01
				100	0.23	0.22	5·70	0·71	0·73	−2·37
				1000	0.23	0.22	3·74	0·72	0·73	−1·49
0.25	2	1	0	10	0.21	0.15	39·01	0·75	0·82	−8·36
				100	0.31	0.15	108·15	0·65	0·82	−20·26
				1000	0.31	0.15	104·01	0·61	0·82	−25·28
			0.25	10	0.17	0.15	15·36	0·76	0·82	−6·85
				100	0.27	0.15	79·06	0·70	0·82	−14·47
				1000	0.30	0.15	97·59	0·70	0·82	−14·22
			0.5	10	0.17	0.15	11·32	0·79	0·82	−3·62
				100	0.19	0.15	25·82	0·76	0·82	−6·91
				1000	0.21	0.15	40·76	0·75	0·82	−8·40
			0.75	10	0.17	0.15	13·93	0·81	0·82	−1·25
				100	0.15	0.15	2·94	0·78	0·82	−5·40
				1000	0.16	0.15	8·19	0·79	0·82	−3·46

Table 14: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 5$, $\lambda_1 = 2$, $\pi = 50$

$h = 1$, $\lambda_0 = 5$, $\lambda_1 = 2$, $\pi = 50$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.05	1	1	0	10	0.15	0.19	−18·76	0·77	0·77	−0·57
				100	0.19	0.19	−0·59	0·73	0·77	−5·22
				1000	0.21	0.19	10·47	0·69	0·77	−9·96
			0.25	10	0.17	0.19	−10·76	0·74	0·77	−4·40
				100	0.22	0.19	14·51	0·75	0·77	−2·63
				1000	0.22	0.19	14·63	0·73	0·77	−5·47
			0.5	10	0.18	0.19	−5·62	0·75	0·77	−2·33
				100	0.16	0.19	−16·55	0·77	0·77	−0·11
				1000	0.24	0.19	25·71	0·73	0·77	−4·83
			0.75	10	0.17	0.19	−8·31	0·77	0·77	−0·50
				100	0.18	0.19	−6·40	0·74	0·77	−4·07
				1000	0.20	0.19	4·13	0·76	0·77	−1·17
0.15	3	1	0	10	0.17	0.17	0·60	1·09	1·51	−27·88
				100	0.25	0.17	46·99	1·02	1·51	−32·72
				1000	0.27	0.17	56·38	0·98	1·51	−35·27
			0.25	10	0.19	0.17	12·39	1·11	1·51	−26·45
				100	0.25	0.17	45·27	1·05	1·51	−30·35
				1000	0.24	0.17	40·71	1·06	1·51	−29·56
			0.5	10	0.20	0.17	19·39	1·11	1·51	−26·24
				100	0.18	0.17	4·43	1·10	1·51	−27·04
				1000	0.26	0.17	53·61	1·09	1·51	−27·71
			0.75	10	0.19	0.17	9·41	1·13	1·51	−25·09
				100	0.18	0.17	7·63	1·13	1·51	−25·48
				1000	0.18	0.17	4·30	1·10	1·51	−26·84
0.25	4	2	0	10	0.36	0.26	37·90	2·33	2·50	−6·73
				100	0.49	0.26	89·51	2·08	2·50	−16·72
				1000	0.54	0.26	106·81	1·96	2·50	−21·56
			0.25	10	0.34	0.26	30·02	2·42	2·50	−3·07
				100	0.39	0.26	49·78	2·26	2·50	−9·76
				1000	0.44	0.26	69·89	2·18	2·50	−12·81
			0.5	10	0.30	0.26	14·56	2·48	2·50	−0·63
				100	0.39	0.26	48·94	2·38	2·50	−4·69
				1000	0.35	0.26	34·27	2·37	2·50	−5·08
			0.75	10	0.28	0.26	7·79	2·51	2·50	0·33
				100	0.26	0.26	1·81	2·45	2·50	−2·01
				1000	0.33	0.26	27·77	2·46	2·50	−1·64

Table 15: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 10$, $\lambda_1 = 2$, $\pi = 5$

$h = 1$, $\lambda_0 = 10$, $\lambda_1 = 2$, $\pi = 5$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.05	1	1	0	10	0.07	0.10	−26·79	0·33	0·33	0·30
				100	0.07	0.10	−28·46	0·30	0·33	−9·91
				1000	0.07	0.10	−31·29	0·31	0·33	−6·41
			0.25	10	0.09	0.10	−10·49	0·33	0·33	−1·38
				100	0.07	0.10	−26·99	0·31	0·33	−5·41
				1000	0.07	0.10	−32·64	0·31	0·33	−6·42
			0.50	10	0.08	0.10	−24·14	0·33	0·33	−0·95
				100	0.08	0.10	−21·29	0·32	0·33	−3·03
				1000	0.07	0.10	−27·04	0·30	0·33	−8·07
			0.75	10	0.10	0.10	−0·74	0·33	0·33	1·09
				100	0.10	0.10	−3·72	0·33	0·33	0·71
				1000	0.09	0.10	−13·92	0·32	0·33	−2·64
0.15	3	2	0	10	0.26	0.24	9·92	1·05	1·12	−6·62
				100	0.24	0.24	−1·36	0·93	1·12	−17·10
				1000	0.25	0.24	2·17	0·88	1·12	−21·24
			0.25	10	0.25	0.24	3·96	1·04	1·12	−7·41
				100	0.25	0.24	3·62	0·95	1·12	−15·61
				1000	0.30	0.24	24·69	0·94	1·12	−15·79
			0.50	10	0.28	0.24	15·59	1·07	1·12	−4·62
				100	0.25	0.24	5·88	1·00	1·12	−10·49
				1000	0.28	0.24	15·16	0·97	1·12	−13·07
			0.75	10	0.25	0.24	4·00	1·07	1·12	−4·62
				100	0.26	0.24	9·04	1·00	1·12	−10·49
				1000	0.25	0.24	5·80	0·97	1·12	−13·07
0.25	4	3	0	10	0.38	0.32	19·34	1·64	2·02	−18·89
				100	0.41	0.32	29·44	1·34	2·02	−33·73
				1000	0.38	0.32	19·75	1·34	2·02	−33·53
			0.25	10	0.36	0.32	13·12	1·73	2·02	−14·22
				100	0.42	0.32	31·87	1·57	2·02	−22·41
				1000	0.41	0.32	28·86	1·50	2·02	−25·75
			0.50	10	0.35	0.32	10·23	1·84	2·02	−8·71
				100	0.37	0.32	14·49	1·70	2·02	−16·04
				1000	0.36	0.32	13·71	1·70	2·02	−15·97
			0.75	10	0.32	0.32	0·55	1·92	2·02	−4·97
				100	0.33	0.32	3·13	1·87	2·02	−7·30
				1000	0.34	0.32	5·18	1·85	2·02	−8·42

Table 16: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 10$, $\lambda_1 = 2$, $\pi = 50$

$h = 1$, $\lambda_0 = 10$, $\lambda_1 = 2$, $\pi = 50$										
L	S_0	N	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
0.05	2	1	0	10	0.09	0.10	−8·20	0·63	0·66	−4·47
				100	0.09	0.10	−6·06	0·57	0·66	−14·28
				1000	0.10	0.10	−0·27	0·52	0·66	−21·65
			0.25	10	0.10	0.10	−4·49	0·64	0·66	−3·32
				100	0.09	0.10	−11·00	0·60	0·66	−9·03
				1000	0.09	0.10	−6·45	0·56	0·66	−14·74
			0.50	10	0.08	0.10	−18·78	0·65	0·66	−1·91
				100	0.10	0.10	1·68	0·61	0·66	−7·16
				1000	0.10	0.10	3·35	0·60	0·66	−8·35
			0.75	10	0.09	0.10	−7·05	0·67	0·66	1·03
				100	0.11	0.10	6·04	0·63	0·66	−4·42
				1000	0.09	0.10	−5·83	0·63	0·66	−3·97
0.15	4	2	0	10	0.21	0.19	12·08	1·69	1·86	−9·16
				100	0.29	0.19	50·97	1·35	1·86	−27·22
				1000	0.30	0.19	57·30	1·24	1·86	−33·40
			0.25	10	0.19	0.19	0·07	1·77	1·86	−5·06
				100	0.26	0.19	34·50	1·51	1·86	−18·73
				1000	0.26	0.19	36·09	1·45	1·86	−21·86
			0.50	10	0.20	0.19	4·97	1·78	1·86	−4·18
				100	0.22	0.19	17·59	1·65	1·86	−11·03
				1000	0.22	0.19	15·13	1·62	1·86	−13·00
			0.75	10	0.21	0.19	10·35	1·83	1·86	−1·45
				100	0.20	0.19	6·97	1·70	1·86	−8·44
				1000	0.20	0.19	5·73	1·77	1·86	−4·69
0.25	6	4	0	10	0.37	0.30	22·99	3·85	4·53	−15·11
				100	0.53	0.30	75·63	2·91	4·53	−35·86
				1000	0.55	0.30	83·04	2·59	4·53	−42·92
			0.25	10	0.32	0.30	8·31	4·02	4·53	−11·24
				100	0.42	0.30	38·47	3·49	4·53	−22·92
				1000	0.44	0.30	45·34	3·36	4·53	−25·86
			0.50	10	0.35	0.30	16·08	4·22	4·53	−6·91
				100	0.36	0.30	21·08	3·91	4·53	−13·77
				1000	0.38	0.30	25·62	3·89	4·53	−14·02
			0.75	10	0.34	0.30	11·69	4·32	4·53	−4·67
				100	0.33	0.30	9·32	4·23	4·53	−6·58
				1000	0.34	0.30	13·03	4·16	4·53	−8·12

8.4 Performance of X_{ext} and comparison with X^* for $\lambda_1 = 0$

Table 17: Performance of X_{ext} and comparison with X^* for $h = 1,\ \lambda_0 = 0.5,\ \lambda_1 = 0,\ L = 0.25,\ S_0 = 1,\ N = 1,\ \pi = 50$

$h = 1,\ \lambda_0 = 0.5,\ \lambda_1 = 0,\ L = 0.25,\ S_0 = 1,\ N = 1,\ \pi = 50$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	1·21	0.43	180·79	4.22	4.66	−9·45
		100	10·69	0.43	2385·71	2.78	4.66	−40·25
		1000	13·01	0.43	2926·52	1.66	4.66	−64·36
	0.25	10	0·93	0.43	116·70	4.42	4.66	−5·05
		100	1·40	0.43	225·40	4.14	4.66	−11·13
		1000	1·43	0.43	231·42	4.02	4.66	−13·74
	0.5	10	0·61	0.43	42·62	4.52	4.66	−2·93
		100	0·75	0.43	73·59	4.43	4.66	−4·91
		1000	0·77	0.43	80·13	4.40	4.66	−5·50
	0.75	10	0·46	0.43	6·93	4.59	4.66	−1·40
		100	0·43	0.43	0·50	4.57	4.66	−1·96
		1000	0·61	0.43	42·83	4.55	4.66	−2·45
10	0	10	2·52	1.04	142·73	6.18	8.13	−24·02
		100	8·19	1.04	687·44	2.81	8.13	−65·44
		1000	21·12	1.04	1930·77	1.68	8.13	−79·36
	0.25	10	1·59	1.04	53·25	6.95	8.13	−14·45
		100	3·00	1.04	188·10	5.84	8.13	−28·14
		1000	2·74	1.04	163·58	5.54	8.13	−31·85
	0.5	10	1·46	1.04	40·41	7.50	8.13	−7·76
		100	1·70	1.04	63·49	6.91	8.13	−15·04
		1000	1·76	1.04	69·41	6.89	8.13	−15·20
	0.75	10	1·11	1.04	6·71	7.90	8.13	−2·84
		100	1·30	1.04	24·95	7.73	8.13	−4·86
		1000	1·40	1.04	34·77	7.68	8.13	−5·57

Table 18: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 1$, $N = 1$, $\pi = 300$

$h = 1$, $\lambda_0 = 0.5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 1$, $N = 1$, $\pi = 300$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	0·85	0.36	136·03	9·67	9·85	−1·79
		100	3·64	0.36	910·69	8·44	9·85	−14·34
		1000	28·72	0.36	7877·48	4·16	9·85	−57·76
	0.25	10	0·70	0.36	93·85	9·75	9·85	−1·04
		100	0·94	0.36	162·34	9·57	9·85	−2·86
		1000	0·88	0.36	143·40	9·52	9·85	−3·39
	0.5	10	0·45	0.36	23·74	9·81	9·85	−0·41
		100	0·71	0.36	96·00	9·67	9·85	−1·81
		1000	0·67	0.36	87·36	9·76	9·85	−0·94
	0.75	10	0·51	0.36	40·85	9·79	9·85	−0·65
		100	0·43	0.36	18·90	9·78	9·85	−0·73
		1000	0·41	0.36	12·56	9·78	9·85	−0·68
10	0	10	1·61	0.84	91·86	16·52	18·05	−8·47
		100	6·48	0.84	671·44	10·06	18·05	−44·24
		1000	29·09	0.84	3363·33	4·21	18·05	−76·66
	0.25	10	1·27	0.84	51·52	17·04	18·05	−5·58
		100	2·26	0.84	169·28	15·84	18·05	−12·27
		1000	2·31	0.84	175·55	15·93	18·05	−11·75
	0.5	10	1·28	0.84	52·44	17·57	18·05	−2·67
		100	1·32	0.84	57·13	17·10	18·05	−5·26
		1000	1·24	0.84	48·13	17·19	18·05	−4·76
	0.75	10	1·07	0.84	27·37	17·87	18·05	−0·99
		100	1·03	0.84	23·20	17·56	18·05	−2·72
		1000	1·10	0.84	30·36	17·69	18·05	−1·97

Table 19: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 1$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 2$, $N = 2$, $\pi = 50$

$h = 1,\ \lambda_0 = 1,\ \lambda_1 = 0,\ L = 0.25,\ S_0 = 2,\ N = 2,\ \pi = 50$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	1.96	0.87	125.39	6.82	8.25	−17.31
		100	10.19	0.87	1071.05	2.74	8.25	−66.75
		1000	19.26	0.87	2114.01	2.01	8.25	−75.61
	0.25	10	1.50	0.87	72.04	7.37	8.25	−10.68
		100	2.02	0.87	132.08	6.40	8.25	−22.39
		1000	2.00	0.87	129.33	6.36	8.25	−22.89
	0.5	10	1.14	0.87	31.48	7.77	8.25	−5.79
		100	1.24	0.87	42.32	7.47	8.25	−9.43
		1000	1.31	0.87	50.45	7.42	8.25	−10.02
	0.75	10	0.93	0.87	6.89	8.07	8.25	−2.18
		100	1.00	0.87	14.53	7.91	8.25	−4.14
		1000	1.09	0.87	25.12	7.96	8.25	−3.55
10	0	10	2.92	1.29	126.43	8.69	13.12	−33.80
		100	10.48	1.29	712.53	2.76	13.12	−78.98
		1000	15.09	1.29	1069.94	1.87	13.12	−85.71
	0.25	10	2.07	1.29	60.37	10.52	13.12	−19.80
		100	3.19	1.29	147.45	8.14	13.12	−37.92
		1000	3.03	1.29	134.97	7.84	13.12	−40.22
	0.5	10	1.81	1.29	40.65	11.50	13.12	−12.32
		100	2.19	1.29	69.40	10.60	13.12	−19.22
		1000	2.19	1.29	69.46	10.50	13.12	−19.99
	0.75	10	1.63	1.29	26.68	12.26	13.12	−6.58
		100	1.77	1.29	37.25	12.10	13.12	−7.76
		1000	1.72	1.29	33.07	11.98	13.12	−8.67

Table 20: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 1$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 2$, $N = 2$, $\pi = 300$

$h = 1,\ \lambda_0 = 1,\ \lambda_1 = 0,\ L = 0.25,\ S_0 = 2,\ N = 2,\ \pi = 300$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	0·61	0.29	111·40	8·79	9·33	−5·79
		100	2·53	0.29	771·36	6·02	9·33	−35·44
		1000	16·03	0.29	5428·47	2·43	9·33	−73·91
	0.25	10	0·50	0.29	73·34	8·96	9·33	−4·00
		100	0·69	0.29	136·39	8·63	9·33	−7·51
		1000	0·71	0.29	146·10	8·52	9·33	−8·65
	0.5	10	0·42	0.29	44·03	9·09	9·33	−2·58
		100	0·46	0.29	57·44	9·06	9·33	−2·86
		1000	0·42	0.29	43·12	9·01	9·33	−3·46
	0.75	10	0·33	0.29	14·21	9·18	9·33	−1·61
		100	0·35	0.29	20·81	9·22	9·33	−1·15
		1000	0·45	0.29	54·71	9·12	9·33	−2·27
10	0	10	1·07	0.59	80·81	14·87	17·14	−13·23
		100	4·02	0.59	581·76	6·97	17·14	−59·33
		1000	13·70	0.59	2222·16	2·24	17·14	−86·91
	0.25	10	0·73	0.59	23·38	15·63	17·14	−8·81
		100	1·11	0.59	88·51	14·24	17·14	−16·91
		1000	1·09	0.59	85·10	13·79	17·14	−19·57
	0.5	10	0·77	0.59	30·00	16·24	17·14	−5·24
		100	0·72	0.59	22·49	15·83	17·14	−7·63
		1000	0·82	0.59	39·15	15·64	17·14	−8·74
	0.75	10	0·57	0.59	−3·93	16·83	17·14	−1·83
		100	0·62	0.59	4·61	16·76	17·14	−2·23
		1000	0·61	0.59	3·44	16·80	17·14	−1·98

Table 21: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 2$, $N = 2$, $\pi = 5$

$h = 1$, $\lambda_0 = 5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 2$, $N = 2$, $\pi = 5$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	8·75	0.58	1407·97	1·12	3·29	−65·82
		100	2·57	0.58	342·31	0·46	3·29	−86·14
		1000	2·25	0.58	288·19	0·40	3·29	−87·93
	0.25	10	1·12	0.58	92·68	1·84	3·29	−43·92
		100	3·25	0.58	460·93	0·92	3·29	−71·91
		1000	4·07	0.58	601·32	0·86	3·29	−73·78
	0.5	10	0·88	0.58	51·24	2·46	3·29	−25·19
		100	1·04	0.58	78·91	2·06	3·29	−37·32
		1000	9·86	0.58	1599·98	1·71	3·29	−48·03
	0.75	10	0·68	0.58	17·91	2·94	3·29	−10·54
		100	0·74	0.58	26·93	2·78	3·29	−15·35
		1000	0·72	0.58	24·76	2·74	3·29	−16·72
10	0	10	4·56	0.77	492·56	1·10	4·24	−73·94
		100	2·13	0.77	177·00	0·45	4·24	−89·31
		1000	2·44	0.77	217·29	0·38	4·24	−90·97
	0.25	10	3·04	0.77	294·31	1·78	4·24	−57·94
		100	7·63	0.77	890·42	0·95	4·24	−77·62
		1000	4·74	0.77	515·63	0·85	4·24	−79·90
	0.5	10	1·13	0.77	46·68	2·80	4·24	−34·01
		100	1·20	0.77	55·58	2·14	4·24	−49·41
		1000	1·77	0.77	129·83	2·13	4·24	−49·72
	0.75	10	0·90	0.77	16·24	3·52	4·24	−17·04
		100	0·87	0.77	13·40	3·27	4·24	−22·91
		1000	0·93	0.77	21·27	3·25	4·24	−23·45

Table 22: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 4$, $N = 4$, $\pi = 50$

$h = 1$, $\lambda_0 = 5$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 4$, $N = 4$, $\pi = 50$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	1·00	0.5	99·98	8·23	11·76	−30·03
		100	5·63	0.5	1025·89	1·23	11·76	−89·54
		1000	4·95	0.5	889·37	0·74	11·76	−93·72
	0.25	10	0·77	0.5	54·47	9·63	11·76	−18·12
		100	1·02	0.5	104·95	7·31	11·76	−37·84
		1000	1·16	0.5	132·19	6·81	11·76	−42·06
	0.5	10	0·68	0.5	36·19	10·46	11·76	−11·02
		100	0·71	0.5	41·67	9·79	11·76	−16·76
		1000	0·76	0.5	52·97	9·46	11·76	−19·52
	0.75	10	0·58	0.5	16·28	11·21	11·76	−4·67
		100	0·62	0.5	24·59	10·78	11·76	−8·33
		1000	0·60	0.5	20·90	10·74	11·76	−8·71
10	0	10	14·97	0.67	2134·06	3·61	18·45	−80·43
		100	25·11	0.67	3648·27	1·11	18·45	−94·00
		1000	14·86	0.67	2118·64	0·82	18·45	−95·55
	0.25	10	1·06	0.67	57·53	12·78	18·45	−30·72
		100	26·86	0.67	3908·87	2·09	18·45	−88·65
		1000	9·27	0.67	1284·21	2·82	18·45	−84·74
	0.5	10	0·86	0.67	27·86	15·15	18·45	−17·91
		100	0·98	0.67	46·39	13·06	18·45	−29·23
		1000	0·97	0.67	45·44	12·81	18·45	−30·56
	0.75	10	0·74	0.67	11·19	16·72	18·45	−9·38
		100	0·74	0.67	10·24	16·23	18·45	−12·04
		1000	0·76	0.67	13·79	15·90	18·45	−13·80

Table 23: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 4$, $N = 4$, $\pi = 5$

$h = 1$, $\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 4$, $N = 4$, $\pi = 5$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	5·30	0.57	830·59	1·00	4·78	−79·09
		100	3·81	0.57	568·66	0·45	4·78	−90·63
		1000	2·45	0.57	329·85	0·41	4·78	−91·47
	0.25	10	6·25	0.57	996·41	1·33	4·78	−72·23
		100	5·18	0.57	809·62	0·72	4·78	−85·03
		1000	7·71	0.57	1252·27	0·64	4·78	−86·65
	0.5	10	0·75	0.57	30·91	3·62	4·78	−24·17
		100	14·33	0.57	2413·65	1·17	4·78	−75·46
		1000	5·94	0.57	941·61	1·24	4·78	−74·14
	0.75	10	0·57	0.57	0·64	4·18	4·78	−12·47
		100	0·64	0.57	11·64	4·02	4·78	−15·91
		1000	0·67	0.57	17·49	4·03	4·78	−15·66
10	0	10	23·78	0.64	3615·17	1·18	6·03	−80·37
		100	16·60	0.64	2494·13	0·46	6·03	−92·41
		1000	2·41	0.64	276·27	0·39	6·03	−93·59
	0.25	10	10·25	0.64	1501·29	1·43	6·03	−76·35
		100	8·22	0.64	1184·13	0·70	6·03	−88·33
		1000	3·63	0.64	466·95	0·70	6·03	−88·35
	0.5	10	3·00	0.64	367·98	3·52	6·03	−41·64
		100	7·02	0.64	996·91	1·24	6·03	−79·41
		1000	15·15	0.64	2267·77	1·24	6·03	−79·40
	0.75	10	0·68	0.64	6·13	4·98	6·03	−17·38
		100	0·80	0.64	24·34	4·74	6·03	−21·37
		1000	0·75	0.64	17·49	4·60	6·03	−23·67

Table 24: Performance of X_{ext} and comparison with X^* for $h = 1$, $\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 4$, $N = 4$, $\pi = 50$

$h = 1$, $\lambda_0 = 10$, $\lambda_1 = 0$, $L = 0.25$, $S_0 = 4$, $N = 4$, $\pi = 50$								
c_0	p	θ	X_{ext}	X^*	$\Delta_X(\%)$	$TC(X_{ext})$	$TC(X^*)$	$\Delta_{TC}(\%)$
5	0	10	0·68	0.43	57·42	10·39	14·67	−29·15
		100	9·57	0.43	2126·43	0·76	14·67	−94·84
		1000	5·87	0.43	1265·50	0·54	14·67	−96·32
	0.25	10	0·56	0.43	30·33	11·97	14·67	−18·42
		100	8·02	0.43	1764·84	2·31	14·67	−84·23
		1000	8·27	0.43	1824·24	1·62	14·67	−88·94
	0.5	10	0·51	0.43	17·96	13·20	14·67	−10·04
		100	0·56	0.43	30·52	11·88	14·67	−19·05
		1000	0·57	0.43	31·68	11·79	14·67	−19·63
	0.75	10	0·46	0.43	7·62	13·96	14·67	−4·85
		100	0·48	0.43	10·68	13·45	14·67	−8·32
		1000	0·48	0.43	11·77	13·41	14·67	−8·57
10	0	10	12·49	0.53	2257·03	3·17	22·26	−85·75
		100	11·97	0.53	2158·38	0·75	22·26	−96·65
		1000	6·48	0.53	1122·65	0·54	22·26	−97·59
	0.25	10	0·70	0.53	32·76	15·86	22·26	−28·73
		100	10·96	0.53	1967·12	1·66	22·26	−92·55
		1000	13·49	0.53	2444·70	1·26	22·26	−94·33
	0.5	10	0·64	0.53	20·17	18·74	22·26	−15·79
		100	0·75	0.53	40·83	16·35	22·26	−26·53
		1000	0·70	0.53	32·20	15·85	22·26	−28·81
	0.75	10	0·57	0.53	8·46	20·73	22·26	−6·86
		100	0·59	0.53	11·83	19·93	22·26	−10·45
		1000	0·58	0.53	9·65	19·90	22·26	−10·59

8.5 Code

8.5.1 Code for Simulation and RSM

The code used for this research is rather extensive/long, therefore the code is provided separately in a Zip file. In this section a listing of all programs/files included in the zip archive is given with a one-sentence explanation of what the program does. The software used in this research is Eclipse. Furthermore Maven Apache is used to perform OLS in the RSM method.

- Class: AbstractMultipleLinearRegression;
  This class is Licensed to the Apache Software Foundation[2] and gives an abstract base for implementations of MultipleLinearRegression.

- Class: Arrival;
  This class describes the event of a customer arriving at the service provider.

- Class: Customers;
  This class constructs a customer and decides whether the customer has limited or no limited customer patience.

- Class: Departure;
  This class describes the event of a customer departing from the system.

- Class: Event;
  This class defines what an event is.

- Class: EventX;
  This class defines the event X.

- Class: InventorySimulation;
  This class defines the simulation of customer in the inventory process and defines what happens during each event.

- Class: InventorySimulationRunner;
  This class performs RSM with simulation and returns the best X that minimizes the objective cost function.

- Class: InventoryUpdate;
  This class describes the event for when the inventory is updates after lead time L.

- Class: LostSales;
  This class describes what happens in the situation with full obsolescence ($\lambda_1 = 0$).

- Class: MultipleLinearRegression;
  This class is Licensed to the Apache Software Foundation[3] and performs multiple linear regression.

[2]https://commons.apache.org/proper/commons-math/javadocs/api-3.6/src-html/org/apache/commons /math3/stat/regression/MultipleLinearRegression.htmlline.32.
[3]https://commons.apache.org/proper/commons-math/javadocs/api-3.6/src-html/org/apache/commons /math3/stat/regression/AbstractMultipleLinearRegression.htmlline.37

- Class: OLSMultipleLinearRegression;
  This class is licensed to the Apache Software Foundation [4] and this class implements ordinary least squares (OLS) to estimate the parameters of a multiple linear regression model.

- Class: Simulation;
  This class constructs a discrete event simulation.

8.5.2 Code for base stock levels

The following code is used to determine the base stock levels. This is done using the Software Matlab.

```
% lambda01 = [0.5 0 0.7 0 1 0];
lambda02 = [5 0 7 0 10 0];
lambda03 = [0.5 0.2 0.7 0.2 1 0.2];
lambda04 = [5 2 7 2 10 2];
pi_1 = [50 75 150 300];
pi_2 = [5 15 25 50];
L_1 = [0.25 0.5 0.75 1];
L_2 = [0.05 0.15 0.25 0.50];
x = 0;
result1 = [];
result2 = [];
result3 = [];
result4 = [];

% line 1
for i = 1:6 %%lambda
    for j = 1:4 %% pi
        for k = 1:4 %% L
            while poisscdf(x,lambda01(i)*L_1(k)) < pi_1(j)/(pi_1(j)+1)
                x = x+1;
            end
            result1 = [result1; lambda01(i) pi_1(j) L_1(k)  x];
            x = 0;
        end
    end
end

% line 2
for i = 1:6 %%lambda
    for j = 1:4 %% pi
        for k = 1:4 %% L
            while poisscdf(x,lambda02(i)*L_2(k)) < pi_2(j)/(pi_2(j)+1)
                x = x+1;
            end
            result2 = [result2; lambda02(i) pi_2(j) L_2(k) x];
```

[4] https://commons.apache.org/proper/commons-math/javadocs/api-3.6/src-html/org/apache/commons /math3/stat/regression/OLSMultipleLinearRegression.htmlline.54

```
            x = 0;
        end
    end
end

% line 3
for i = 1:6 %%lambda
    for j = 1:4 %% pi
        for k = 1:4 %% L
            while poisscdf(x,lambda03(i)*L_1(k)) < pi_1(j)/(pi_1(j)+1)
                x = x+1;
            end
            result3 = [result3; lambda03(i) pi_1(j) L_1(k) x];
            x = 0;
        end
    end
end

% line 4
for i = 1:6 %%lambda
    for j = 1:4 %% pi
        for k = 1:4 %% L
            while poisscdf(x,lambda04(i)*L_2(k)) < pi_2(j)/(pi_2(j)+1)
                x = x+1;
            end
            result4 = [result4; lambda04(i) pi_2(j) L_2(k) x];
            x = 0;
        end
    end
end
```