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Various volatility-managed portfolios and their performance

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Abstract

I evaluate the performance of volatility-managed portfolios and find that none of the portfolios systematically outperforms the unmanaged portfolio in terms of Sharpe ratio. However, estimating conditional variance with a (T)GARCH model consistently generates better results relative to realized variance estimation. This suggests that more research should be done into alternative conditional variance estimators, with the aim of enhancing the performance of volatility-managed portfolios. Consistent with Cederburg et al. (2019), I combine volatility-managed and unmanaged versions within a portfolio. Using a wider variety of combination and mixture portfolios, I find considerable in-sample gains in terms of Sharpe ratio and CER. I adopt an out-of-sample strategy and show that out-of-sample variants suffer a substantial performance deterioration. Therefore, a better out-of-sample design is needed, to convert in-sample gains into out-of-sample earnings for mean-variance investors.

[1] The views stated in this thesis are those of the author and not necessarily those of Erasmus School of Economics or Erasmus University Rotterdam.

1 Introduction

Moreira and Muir (2017) leave readers with the impression that volatility-managed versions of popular equity strategy portfolios enhance the performance relative to the original unmanaged portfolios. They explain that an increase in volatility is not offset by a proportional increase in expected returns. Therefore, these volatility-managed portfolios should take on large levered positions when volatility was recently low and reduce leverage in times of high volatility. Using nine equity factors, they find statistically significant positive intercepts (alphas) when regressing excess returns of volatility-managed portfolios on excess returns of the corresponding unmanaged portfolios. They translate these positive alphas into higher Sharpe ratios for volatility-managed portfolios, following utility gains for mean-variance investors.

Cederburg et al. (2019) counter this improved performance. Using an expanded sample of 104 equity strategies, they find no statistical evidence that volatility-managed portfolios outperform the unmanaged portfolios in terms of Sharpe ratio and certainty equivalent return (CER). Cederburg et al. (2019) state that the positive alphas, as a result of the spanning regression, are a minimum requirement and do not guarantee an increased performance for the volatility-managed portfolios. On the other hand, they do argue that the positive alphas suggest that the ex post combination of the volatility-managed and unmanaged portfolios outperforms the unmanaged portfolio. However, because the optimal in-sample weights are unknown prior to the end of the sample, these 'combination' portfolios are not feasible in real time. Cederburg et al. (2019) investigate an out-of-sample strategy in which they estimate portfolio weights using an expanding window. Their conclusion is that the in-sample gains in Sharpe ratio and CER, as a result of combining volatility-managed and unmanaged portfolios, can not be converted to an enhanced performance for their out-of-sample strategy. In the volatility-managed portfolio, examined by Moreira and Muir (2017) and Cederburg et al. (2019), the investment position is proportional to the inverse of the conditional variance. As estimator for this conditional variance, they use the realized variance of daily portfolio returns in the preceding month.

Realized variance estimates exhibit a 'ghost feature', they increase sharply whenever a sudden large (daily) excess return occurs, and decrease sharply in the following month. Therefore, I consider alternative methods to estimate the conditional variance of the unmanaged portfolio. As volatility might change only gradually over time, I examine the generalized autoregressive conditional heteroskedasticity (GARCH) model and Threshold GARCH (TGARCH) model to estimate the conditional variance. Moreover, as Hansen and Huang (2016) state that conventional GARCH models are slow at 'catching up' in times of high volatility, the properties of these (T)GARCH estimates seem opposite to the properties of realized variance estimates. Because of this, I explore whether it is advantageous to construct a 'mixture' portfolio that combines these different conditional variance estimators.

Cederburg et al. (2019) argue that the positive alphas in Moreira and Muir's (2017) spanning regression are a minimum requirement and do not guarantee an improved performance for the volatility-managed portfolios. On the other hand, they do claim that these positive alphas imply that the in-sample allocation of the volatility-managed and unmanaged portfolios outperforms the unmanaged portfolio. I provide the link between the positives alphas and these statements. However, as an increase in Sharpe ratio and certainty equivalent return is more directly linked to utility gains for mean-variance investors, I focus on these performance measures.

Following Cederburg et al. (2019), I use three methods to compare (i) Direct comparison between the (individual) volatility-managed portfolios and the unmanaged portfolios (ii) In-sample comparison between the (individual) combination/mixture portfolios and the unmanaged portfolios (iii) Comparison of the out-of-sample performance of the (individual) combination/mixture portfolios relative to their (in-sample) benchmarks and the unmanaged portfolios. To accomplish this, I first replicate the volatility-managed portfolio, in-sample portfolio and out-of-sample portfolio of Cederburg et al. (2019), using the realized variance as conditional variance estimator. For the data, I use seven of the nine equity factors examined by Moreira and Muir (2017).

I contribute to the existing literature in two primary ways. First, using both realized variance and (T)GARCH models to estimate the conditional variance of the unmanaged portfolios, I confirm that none of the volatility-managed portfolios systematically outperforms the unmanaged portfolios. However, I do find that estimating conditional variance with a (T)GARCH model leads to a consistently improved performance relative to realized variance estimation. This shows that the performance of volatility-managed portfolios can be improved by more accurate conditional variance estimation. Second, using a variety of combination and mixture portfolios, I confirm that there are considerable in-sample gains in terms of Sharpe ratio and CER. In particular the impressive performance for the mixture portfolios, that earn substantial higher Sharpe ratios and CERs than the combination portfolios. I adopt an out-of-sample strategy similar to Cederburg et al. (2019), as the in-sample portfolios are not feasible in real time. However, I find that the out-of-sample variants of the combination and mixture portfolios suffer a substantial performance deterioration. To summarize, I identify two opportunities for follow-up research. First, more research should be done into alternative conditional variance estimators, to enhance the performance of the volatility-managed portfolios. Second, it is advantageous to design better out-of-sample strategies, to convert in-sample gains into out-of-sample earnings for mean-variance investors.

The rest of the paper is organized as follows. Section 2 describes the data. My methodology to construct the various combination and mixture portfolios as well as comparing their performance is given in Section 3. Section 4 contains results for this comparison and Section 5 concludes.

2 Data

Because I replicate some of the results of Cederburg et al. (2019), I consider (almost) the same equity factors, they are listed in table 1 [2]. Because data on the profitability (ROE) and investment (IA) factors from the Hou-Xue-Zhang (2015) q-factor model are not available, I remove these datasets. Furthermore, I add a quality-minus-junk (QMJ) factor.

Table 1: List of factors considered

The table lists the various equity factors considered, the origin of the factors as well as their spanned time period.

#	Factor	Origin	Time Period
(1)	Market (MKT)		
(2)	Size (SMB)	Fama and French (1993) three-factor model	08/1926 - 12/2016
(3)	Value (HML)		
(4)	Momentum (MOM)	-	01/1927 - 12/2016
(5)	Profitability (RMW)	Fama and French (2015) five-factor model	08/1963 - 12/2016
(6)	Investment (CMA)		
(7)	Betting-against-beta (BAB)	Frazzini and Pedersen (2014)	08/1946 - 12/2016
(8)	Quality-minus-junk (QMJ)	Asness et al. (2019)	07/1957 - 12/2016

The factors represent returns in excess over the risk-free rate for zero-cost portfolios. Investing in a factor is equal to going long (short) in a certain position and short (long) in another position. For instance, a positive investment in the market factor indicates going long in the market and short in the risk-free rate. By this fact, the net value of the investment is zero, and therefore no initial investment is needed. This results in levered positions when investing in these factors. Although true zero-cost portfolios are not achievable in the real world, for example because of broker costs or short selling regulations, this assumption simplifies the problem. I collect daily and monthly data on excess returns for the time periods listed in table 1. The first month of each sample is only used for realized variance estimation.

3 Methodology

In this section, I first consider the replication of Cederburg et al. (2019). This replication consists of the volatility-managed portfolio, the in-sample 'combination' portfolio and the out-of-sample 'combination' portfolio. Next, I introduce the alternative methods to estimate the conditional variance of the unmanaged portfolio. Thereafter, I explain the 'mixture' portfolio that combines the realized variance volatility-managed portfolio with the (T)GARCH volatility-managed portfolio. Finally, I list my comparison methods and introduce the performance measures.

[2] Data on the MKT, SMB, HML, MOM, RMW and CMA factors are from Kenneth French's website. Data on the BAB factor are from Andrea Frazzini's website. Data on the QMJ factor are from AQR's website.

3.1 Replication of Cederburg et al. (2019)

3.1.1 Volatility-managed portfolio

Let T denote the total number of observations, the T-dimensional vector of excess returns of the volatility-managed portfolio in month $t = 1, .., T$ is estimated via

$$r_{v,t} = \frac{s^*}{\hat{\sigma}_{c,t}^2} r_t, \tag{1}$$

in which r_t is the excess return in month t of the unmanaged portfolio, $r_{v,t}$ the excess return of the volatility-managed portfolio and $\hat{\sigma}_{c,t}^2$ is the estimate of the conditional variance of the unmanaged portfolio. Moreira and Muir (2017) argue that an increase in volatility is not offset by a proportional increase in expected returns. A mean-variance investor should, therefore, decrease leverage in the unmanaged portfolio in times of high volatility. This motivates the investment position in the unmanaged portfolio in month t equal to $s^*/\hat{\sigma}_{c,t}^2$, as volatility is persistent and changes only gradually over time. To achieve the same unconditional volatility, the excess returns of the volatility-managed portfolio are multiplied (scaled) by a constant s^*

$$\hat{\sigma}_{uc} = \hat{\sigma} = s^* \hat{\sigma}_v, \tag{2}$$

where $\hat{\sigma}_{uc}$ is the unconditional volatility, $\hat{\sigma}$ is the volatility of the unmanaged portfolio over the full-sample and $\hat{\sigma}_v$ is the volatility of the volatility-managed portfolio over the full-sample. Moreira and Muir (2017) use the realized variance of the preceding month to estimate the conditional variance, as construction is simple and does not rely on parameter estimation

$$\hat{\sigma}_{c,t}^2 = \hat{\sigma}_{RV,t-1}^2 = \frac{22}{TR_{t-1}} \sum_{i=1}^{TR_{t-1}} (r_{t-1}^i)^2. \tag{3}$$

Here, $i = 1, ..., TR_{t-1}$ denotes the trading days in month $t-1$ and r_{t-1}^i denotes the excess return on trading day i of month $t-1$. Moreira and Muir (2017) argue that scale factor s^* does not affect the Sharpe ratio of the volatility-managed portfolio. Therefore, results would be the same if implemented in real time, instead of using the full-sample.

3.1.2 Combination portfolio (in-sample)

Moreira and Muir (2017) use a spanning regression as evidence that volatility-managed portfolios enhance the performance relative to the original unmanaged portfolios. They translate the statistically significant positive alphas in

$$r_{v,t} = \alpha + \beta r_t + \epsilon_t, \tag{4}$$

into higher Sharpe ratios resulting in utility gains for mean-variance investors. However, Cederburg et al. (2019) state that a positive alpha is a minimum requirement for an increased performance. The volatility-managed portfolio gains a higher Sharpe ratio than the unmanaged portfolio if $\bar{r}_v > \bar{r}$, as the portfolios have the same unconditional volatility. A positive intercept in Eq. (4), on the other hand, indicates that $\hat{\alpha} = \bar{r}_v - \hat{\beta}\bar{r} > 0$. Cederburg et al. (2019) prove that $\hat{\beta} = \hat{\rho}$, in which $\hat{\rho}$ is the correlation between the volatility-managed and the unmanaged portfolio. Therefore, a positive alpha implies that $\bar{r}_v > \hat{\rho}\bar{r}$. As $\hat{\rho}$ decreases, it becomes more likely that a positive intercept still yields a lower Sharpe ratio for the volatility-managed portfolios relative to the unmanaged portfolios.

Although a positive alpha does not guarantee an increased performance for the volatility-managed portfolio, Cederburg et al. (2019) argue that it does imply that the performance of the unmanaged portfolio can be improved by the ex post combination of the volatility-managed and unmanaged portfolio. Let $\hat{\mu} = [\bar{r}_v, \bar{r}]^T$ denote the 2-dimensional vector with mean excess returns. Moreover, the 2×2 covariance matrix is given by

$$\hat{\Sigma} = \hat{\sigma}_{uc}^2 \begin{bmatrix} 1 & \hat{\rho} \\ \hat{\rho} & 1 \end{bmatrix}, \tag{5}$$

where $\hat{\rho}$ is the estimate for the correlation between the volatility-managed and unmanaged portfolio. Furthermore, the 2-dimensional vector $x = [x_v^*, x^*]^T$ denotes the optimal in-sample portfolio weights for the volatility-managed and unmanaged portfolio, respectively. Moreira and Muir (2017) translate the positive alphas in Eq. (4) to utility gains for mean-variance investors, and therefore x should maximize the expected utility

$$\max_x U(x) = x^T\hat{\mu} - \frac{\gamma}{2}x^T\hat{\Sigma}x, \tag{6}$$

in which γ denotes the level of risk aversion of the investor. Because the optimization problem is expressed in terms of excess returns, the risk-free rate is inherently accessible. Solving this optimization problem gives the optimal in-sample portfolio weights

$$\begin{bmatrix} x_v^* \\ x^* \end{bmatrix} = \frac{1}{\gamma}\hat{\Sigma}^{-1}\hat{\mu} = \frac{1}{\gamma\hat{\sigma}_{uc}^2(1-\hat{\rho}^2)} \begin{bmatrix} \bar{r}_v - \hat{\rho}\bar{r} \\ \bar{r} - \hat{\rho}\bar{r}_v \end{bmatrix}. \tag{7}$$

The weight for the volatility-managed portfolio in the optimal in-sample portfolio is positive for $\bar{r}_v - \hat{\rho}\bar{r} > 0$. As stated in Cederburg et al. (2019), $\hat{\beta}$ in Eq. (4) is equal to $\hat{\rho}$. This suggests a positive weight for the volatility-managed portfolio, and therefore an increased performance, if $\bar{r}_v - \hat{\beta}\bar{r} = \hat{\alpha} > 0$.

Following Cederburg et al. (2019), the in-sample portfolio weights x_v^* and x^*, that are estimated at the end of the sample, are used to compute the dynamic in-sample portfolio weights d_t^* and returns $r_{d,t}^*$

in month $t = 1, ..., T$ via

$$d_t^* = x_v^* \frac{s^*}{\hat{\sigma}_{c,t}^2} + x^*, \tag{8}$$

$$r_{d,t}^* = d_t^* r_t. \tag{9}$$

The performance of the (in-sample) combination portfolios is compared to the performance of the (in-sample) unmanaged portfolios. An investor that does not have access to the volatility-managed portfolio optimally invests

$$u^* = \frac{1}{\gamma} \frac{\bar{r}}{\hat{\sigma}_{uc}^2}, \tag{10}$$

in the unmanaged portfolio. 'Scaling' factor u^* does not affect the Sharpe ratio.

3.1.3 Combination portfolio (out-of-sample)

Cederburg et al. (2019) describe a problem with the construction of their (in-sample) combination portfolio. The estimates $\hat{\mu}$ and $\hat{\Sigma}$ are unknown until the end of the sample, and therefore the optimal scaling factor s^* and portfolio weights x_v^*, x^* are also unknown prior to the end of the sample. This makes the portfolio construction method not implementable in real time. I follow Cederburg et al. (2019) in their strategy to construct an (out-of-sample) portfolio using an expanding window. Initializing K = 120 months as training sample, I estimate optimal allocation to the volatility-managed and unmanaged portfolios. Let $\hat{\mu}_t = [\bar{r}_{v,t}, \bar{r}_t]^T$ denote the estimated mean excess return using only the training sample up to month t. Furthermore, the sample covariance matrix is

$$\hat{\Sigma}_t = \hat{\sigma}_{uc,t}^2 \begin{bmatrix} 1 & \hat{\rho}_t \\ \hat{\rho}_t & 1 \end{bmatrix}, \tag{11}$$

in which $\hat{\sigma}_{uc,t}^2$ is the unconditional variance over the expanding window, using s_t to scale the volatility of the volatility-managed portfolio. The estimates $\hat{\mu}_t$ and $\hat{\Sigma}_t$ are used to compute the dynamic out-of-sample portfolio weights d_t and returns $r_{d,t}$ in month $t = K + 1, ..., T$ via

$$\begin{bmatrix} x_{v,t} \\ x_t \end{bmatrix} = \frac{1}{\gamma} \hat{\Sigma}_t^{-1} \hat{\mu}_t, \tag{12}$$

$$d_t = x_{v,t} \frac{s_t}{\hat{\sigma}_{c,t}^2} + x_t, \tag{13}$$

$$r_{d,t} = d_t r_t. \tag{14}$$

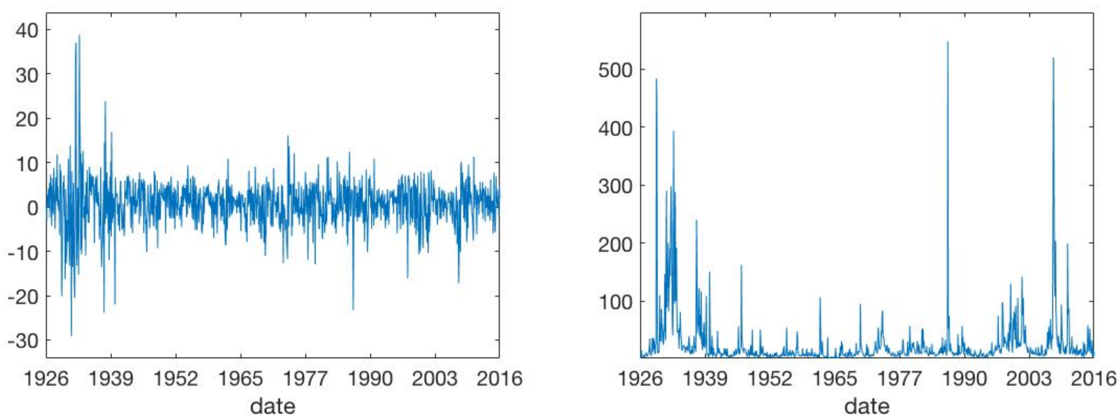
Cederburg et al. (2019) argue that the dynamic out-of-sample portfolio weights d_t are a measure of leverage in the unmanaged portfolio. In line with Cederburg et al. (2019), I impose a leverage constraint of $|d_t| \leq 5$, to avoid extreme weights. As Cederburg et al. (2019) confirm that there findings are robust to different leverage constraints as well as the unconstrained problem, I do not investigate this any further.

The performance of the (out-of-sample) combination portfolio is compared to the performance of the (out-of-sample) unmanaged portfolio, where the optimal investment in the unmanaged portfolio in month t is given by

$$u_t = \frac{1}{\gamma} \frac{\bar{r}_t}{\hat{\sigma}^2_{uc,t}}. \tag{15}$$

3.2 Alternative conditional variance estimation

Moreira and Muir (2017) and Cederburg et al. (2019) estimate the conditional variance using the realized variance of the preceding month, as it does not rely on parameter estimation and is easy to construct. However, as figure 1 suggests, realized variance estimation might have some drawbacks. For instance, November 1987, when the realized variance estimate sharply increases to approximately 550. The plotted monthly returns indicate that this sudden increase in volatility is not likely to be accurate. Investigating the data learns that the daily excess return of a particular day in this month is -17.44%. Squaring this value results in an excessive realized variance estimate.



(a) Monthly excess returns **(b)** Realized variance estimates

Figure 1: Data on the MKT factor
The figure presents data on the MKT factor. Panel A shows the monthly excess returns over the risk-free rate in percents. Furthermore, panel B displays realized variance estimates of the conditional variance using squared (daily) excess returns of the preceding month. The data span from 1926 to 2016.

This suggests that realized variance estimates exhibit a 'ghost feature'. They increase sharply whenever a sudden large (daily) excess return occurs, and decrease sharply in the following month. Figure 1 indicates this as well, as the realized variance estimate in December 1987 drops to a value below 60. Because of this, I investigate alternative methods to estimate the conditional variance in Eq. (1). I consider the generalized autoregressive conditional heteroskedasticity (GARCH) model and Threshold GARCH (TGARCH) model.

3.2.1 GARCH model

The intuitive idea behind the GARCH model, introduced by Bollerslev (1986), is that volatility changes only gradually over time, and therefore σ_t^2 will be closely related tot σ_{t-1}^2. The GARCH (1,1) model is given by

$$r_t = \mu_t + \epsilon_t, \tag{16}$$

$$\hat{\sigma}_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\hat{\sigma}_{t-1}^2. \tag{17}$$

Parameters μ, ω, α and β can be estimated with maximum likelihood, assuming a normal distribution for $z_t = \epsilon_t/\sigma_t$. Using daily excess returns, I construct volatility-managed portfolios. To do this, I obtain monthly conditional variance estimates as follows

$$\hat{\sigma}_{c,t}^2 = 22 * (\hat{\sigma}_{GARCH,t}^{(1)})^2, \tag{18}$$

in which $(\hat{\sigma}_{GARCH,t}^{(1)})^2$ is the daily conditional variance of the first day of month t, estimated using the GARCH model. I explore multiple out-of-sample strategies to estimate the required parameters. Based on Cederburg et al. (2019), I obtain parameter estimates using an expanding window with an initial training sample of $K = 120$ months. I also consider a rolling window of length $R = 60$ months.

Moreover, I construct in-sample and out-of-sample combination portfolios using monthly returns. The in-sample portfolios estimate the parameters using the full-sample, and therefore these portfolios are not implementable in real time. Out-of-sample portfolios are constructed using an expanding window with an initial training sample of $K = 120$ months.

3.2.2 Threshold GARCH model

In the traditional GARCH model the effect of positive and negative values of ϵ_{t-1}^2 on the variance estimate $\hat{\sigma}_t^2$ is the same. However, this is not backed by empirical evidence, as periods of high volatility often start with large negative returns. The Threshold GARCH (TGARCH) model, proposed by Zakoian (1994), can be an effective way of estimating the variance, because it allows for a different effect of large positive/negative returns

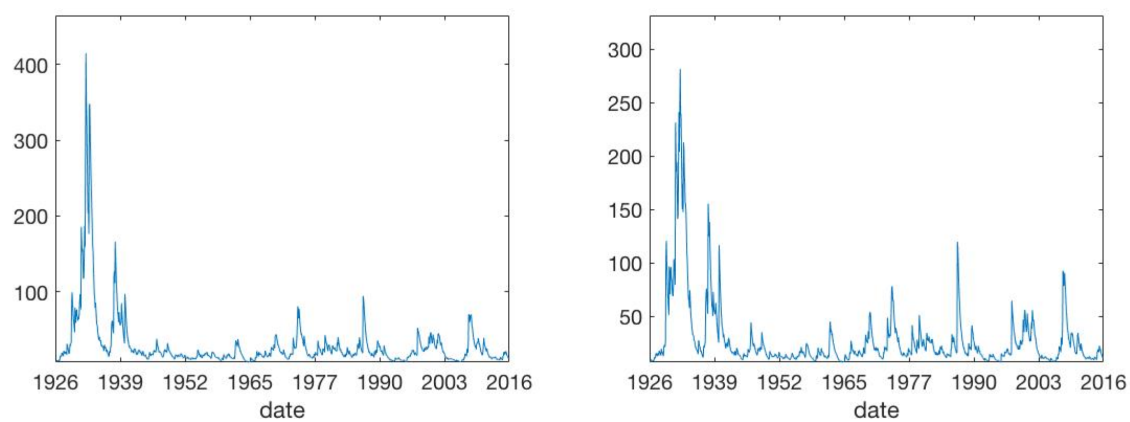
$$r_t = \mu_t + \epsilon_t, \tag{19}$$

$$\hat{\sigma}^2_{c,t} = \hat{\sigma}^2_{TGARCH,t} = \omega + \alpha\epsilon^2_{t-1} + \gamma\epsilon^2_{t-1}I[\epsilon_{t-1} < 0] + \beta\hat{\sigma}^2_{t-1}, \tag{20}$$

where $I[E] = 1$ if E occurs, and 0 otherwise. Parameters $\mu, \omega, \alpha, \beta$ and γ can be estimated with maximum likelihood, following an approach similar to the GARCH model.

3.3 Mixture portfolio

Figure 2 presents conditional variance estimates using a (T)GARCH model. Comparing these estimates with the realized variance estimates of conditional variance in figure 1, indicates a difference. This difference in conditional variance estimation seems primarily substantial during the time period 1938 - 2016. Realized variance estimates, on the one hand, exhibit a 'ghost feature', and therefore might react to excessively to a large daily excess return. On the other hand, Hansen and Huang (2016) state that conventional GARCH models are slow at 'catching up' when volatility is high. Consequently, (T)GARCH models possibly respond too little to large daily excess returns. Therefore, it might be beneficial to combine these different volatility-managed portfolios.



(a) GARCH estimates **(b)** TGARCH estimates

Figure 2: Conditional variance estimates for the MKT factor
The figure presents conditional variance estimates for the MKT factor. Panel A displays conditional variance estimates using a GARCH model. Conditional variance estimated with a TGARCH model is shown in panel B. The data span from 1926 to 2016.

I empirically investigate a portfolio construction method. This 'mixture' portfolio is a combination of the realized variance (RV) volatility-managed portfolio, the (T)GARCH (GA) volatility-managed portfolio and the unmanaged portfolio. Let the 3-dimensional vector $\hat{\mu} = [\bar{r}_{RV}, \bar{r}_{GA}, \bar{r}]^T$ denote the mean excess returns of these portfolios, respectively. Furthermore, let $\hat{\Sigma}$ denote the corresponding covariance matrix. The optimal in-sample portfolio weights are then given by

$$\begin{bmatrix} x^*_{RV} \\ x^*_{GA} \\ x^* \end{bmatrix} = \frac{1}{\gamma}\hat{\Sigma}^{-1}\hat{\mu}. \tag{21}$$

Using these weights, the dynamic in-sample portfolio weights m^*_t and returns $r^*_{m,t}$ for the mixture portfolio in month $t = 1, ..., T$ are computed via

$$m^*_t = x^*_{RV}\frac{s^*_{RV}}{\hat{\sigma}^2_{RV,t-1}} + x^*_{GA}\frac{s^*_{GA}}{\hat{\sigma}^2_{GA,t}} + x^*, \tag{22}$$

$$r^*_{m,t} = m^*_t r_t, \tag{23}$$

in which $\hat{\sigma}^2_{RV,t-1}$ is the realized variance of the preceding month and $\hat{\sigma}^2_{GA,t}$ is the conditional variance estimate using the (T)GARCH model. Furthermore, s^*_{RV} and s^*_{GA} are the constants to scale the unconditional volatility of the volatility-managed portfolios, respectively. Following the same approach, the dynamic out-of-sample portfolio weights m_t and returns $r_{m,t}$ for the mixture portfolio in month $t = K + 1, ..., T$ are given by

$$m_t = x_{RV,t}\frac{s_{RV,t}}{\hat{\sigma}^2_{RV,t-1}} + x_{GA,t}\frac{s_{GA,t}}{\hat{\sigma}^2_{GA,t}} + x_t, \tag{24}$$

$$r_{m,t} = m_t r_t. \tag{25}$$

4 Performance Measures

Consistent with Cederburg et al. (2019), I use three methods to compare (i) Direct comparison between the (individual) volatility-managed portfolios and the unmanaged portfolios (ii) In-sample comparison between the (individual) combination/mixture portfolios and the unmanaged portfolios (iii) Comparison of the out-of-sample performance of the (individual) combination/mixture portfolios relative to their (in-sample) benchmarks and the unmanaged portfolios. Because the utility optimization problem is expressed in terms of excess returns, the risk-free rate is inherently available within each portfolio. To compare, I use the Sharpe ratio and certainty equivalent return as performance measures.

4.1 Sharpe ratio

The (annualized) Sharpe ratio is used in direct comparison, in-sample comparison as well as out-of-sample comparison and is given by

$$SR = \frac{\hat{\mu}_{PT}}{\hat{\sigma}_{PT}}, \tag{26}$$

in which $\hat{\mu}_{PT}$ and $\hat{\sigma}_{PT}$ are the (annualized) mean and standard deviation of the excess returns of a certain portfolio. To asses whether the difference between the Sharpe ratio of two portfolios is statistically significant, I use the test proposed by Jobson and Korkie (1981). A description of the test is given in Appendix A.

4.2 Certainty equivalent return

Following Cederburg et al. (2019), the (annualized) certainty equivalent return for the in-sample (CER_{in}) and out-of-sample (CER_{out}) portfolio are computed as follows

$$CER_{in} = \frac{SR^2}{2\gamma}, \tag{27}$$

$$CER_{out} = \hat{\mu}_{PT} - \frac{\gamma}{2} * \hat{\sigma}^2_{PT}, \tag{28}$$

in which $\hat{\mu}_{PT}$ and $\hat{\sigma}^2_{PT}$ are the (annualized) mean and variance of the excess returns of a certain portfolio, SR is the Sharpe ratio of the portfolio and γ is the level of risk aversion. Consistent with Cederburg et al. (2019), I examine the statistical significance of the CER_{out} difference between two portfolios using a test defined in DeMiquel et al. (2009). The test is described in Appendix A.

5 Results

5.1 Direct comparison

The (T)GARCH-managed portfolios, require an initial training sample to estimate parameters. Therefore, the results for the direct comparison in table 2 are computed over the test sample of $T - K$ observations, initializing an expanding window of $K = 120$ months. The table also lists results for the (T)GARCH model using a rolling window of length $R = 60$ months as well as results for the RV-managed portfolios over the same test sample. These results for the RV-managed portfolios are comparable to the results over the full-sample, which are reported in table 6 in Appendix C. My findings over the full-sample are similar to Cederburg et al. (2019), there are no substantial differences for the seven equity factors that they also consider. However, because I shorten the time period for the BAB factor, these results do differ somewhat.

I compare the performance of the RV-managed and (T)GARCH-managed portfolios relative to the unmanaged portfolios and find that none of the volatility-managed portfolios systematically outperforms the unmanaged portfolios. The Sharpe ratio of the RV-managed portfolio is higher than the Sharpe ratio of the unmanaged portfolio for five of the eight factors, with two of these Sharpe ratio differences statistically significant positive. Performance enhancement by realized variance estimation seems centered around the MOM factor (Sharpe ratio [average excess return] gain of 0.45 [6.24%]) and the BAB factor (Sharpe ratio [average excess return] gain of 0.33 [3.66%]), with improvements relative to the unmanaged portfolios that are significant at the 1% level. For the other factors, the performance enhancement ranges from 0.00 to 0.18, whereas the performance deterioration ranges from -0.06 to -0.14.

Estimating conditional variance with a (T)GARCH model does not give contrasting results, as it neither leads to a consistently improved performance. Results for the (T)GARCH model are quite similar for both the expanding and moving window. The (T)GARCH-managed portfolio earns a higher Sharpe ratio than the unmanaged portfolio for the MKT, MOM, RMW, BAB and QMJ factor. The improved performance is statistically significant at the 1% level for the MOM factor (Sharpe ratio [average excess return] gains range from 0.49 to 0.51 [6.83% to 7.10%]) and the BAB factor (Sharpe ratio [average excess return] gains range from 0.33 to 0.37 [3.64% to 4.12%]). On the other hand, the unmanaged portfolio outperforms the (T)GARCH-managed portfolio for the SMB, HML and CMA factor. However, these differences in Sharpe ratio seem less substantial, as they (mostly) range between -0.02 and -0.09. A notable exception is the performance deterioration of the GARCH-managed portfolio (using an expanding window) for the SMB factor (Sharpe ratio [average excess return] loss of -0.18 [-1.82%]), this deterioration is statistically significant at the 5% level.

Although both the RV-managed and (T)GARCH-managed portfolios do not succeed in systematically outperforming the unmanaged portfolios, performance enhancement (deterioration) seems more (less) substantial for the (T)GARCH-managed portfolios. I, therefore, compare the performance of the (T)GARCH-managed portfolios relative to the RV-managed portfolios. The results of this comparison are in table 2. I find that the (T)GARCH-managed portfolios earn higher Sharpe ratios for all the equity factors, with the exception of the RMW factor. The Sharpe ratio gains range from 0.00 to 0.09, although generally not statistically significant. However, because the enhanced performance is consistent, it does seem that modelling the conditional variance of the unmanaged portfolios with a (T)GARCH model results in more precise estimates, and therefore in volatility-managed portfolios that exhibit on average higher Sharpe ratios. This shows that the performance of volatility-managed portfolios can be improved by more accurate conditional variance estimation. Because of this, it could be advantageous to investigate alternative estimators, with the aim of outperforming the unmanaged portfolios.

5.2 In-sample comparison

Table 3 reports the results for the in-sample comparison. I compare the performance of the different (in-sample) combination and mixture portfolios relative to the unmanaged portfolios. My findings of the RV-combination portfolios are consistent with Cederburg et al. (2019), as there are no notable differences with the exception of the BAB factor. These differences in the BAB factor are due to differences in spanned time period. Both the results for the Sharpe ratios and the certainty equivalent returns confirm that all the combination and mixture portfolios outperform the unmanaged portfolios, although some improvements are more impressive than others.

The most substantial gains in Sharpe ratio are achieved for the MOM factor (ranges from 0.41 to 0.57), the BAB factor (ranges from 0.16 to 0.32), the RMW factor (ranges from 0.07 to 0.15) and the MKT factor (ranges from 0.10 to 0.13). On the other hand, the performance of some equity factors can not be enhanced by incorporating volatility modelling into portfolio allocation. For instance, the RMW factor, with Sharpe ratio gains that range from 0.01 to 0.02. However, consistent with Cederburg et al. (2019), I generally find substantial in-sample gains, that is, when there is no estimation error. This motivates the need for research in exploring good out-of-sample strategies, to convert these gains to portfolios that are feasible in real time. The improved performance is also confirmed by notable gains in CER. The most impressive improvements, using a risk aversion level $\gamma = 5$, are for the MOM factor (ranges from 5.74% to 8.76%) and the BAB factor (ranges from 3.04% to 7.01%).

Comparing the results for the Sharpe ratios and CERs of the combination and mixture portfolios, indicates a strong performance for the mixture portfolios relative to combination portfolios. Designing an out-of-sample strategy, that converts the strong in-sample performance of the mixture portfolios to out-of-sample gains, could therefore greatly benefit mean-variance investors.

5.3 Out-of-sample comparison

Table 4 reports results for the out-of-sample comparison. Using an expanding window of initial length $K = 120$ months, I compare the performance of the different (out-of-sample) combination and mixture portfolios relative to the (out-of-sample) unmanaged portfolios. My findings of the RV-combination portfolios are similar to Cederburg et al. (2019). Differences in the BAB factor are caused by differences in spanned time period. I find that neither the combination portfolios nor the mixture portfolios systematically outperform the unmanaged portfolios in terms of Sharpe ratio and certainty equivalent return. The out-of-sample strategy only seems to enhance the performance of the unmanaged portfolios for the MOM and BAB factors, as these are the only factors with statistically significant positive Sharpe ratio differences. Furthermore, performance for the SMB and CMA factors is consistently worsened.

Panel A shows results for the Sharpe ratios. Both the RV-combination portfolios and the unmanaged portfolios earn a higher Sharpe ratio for four of the eight factors. In line with the direct comparison, the increased performance by modelling volatility is centered around the MOM and BAB factor, both statistically significant at the 1% level. The GARCH-combination portfolio outperforms the unmanaged portfolio for five of the eight factors, whereas the TGARCH-combination portfolio outperforms the unmanaged portfolio for six of the eight factors. Statistically significant positive Sharpe ratio differences are obtained for the MOM factor (at the 1% level) and the BAB factor (at the 10% level). On the other hand, the differences in Sharpe ratio for the MKT, HML, RMW and QMJ factors are close to zero. The findings for the CERs in panel B are similar. The RV-combination portfolios as well as the (T)GARCH-combination portfolios outperform the unmanaged portfolios for four of the eight factors. Statistically significant positive differences in CER belong to the MOM factor (at the 1% level) and the BAB factor (at the 5% level).

There is a strong in-sample performance for the mixture portfolios relative to the combination portfolios. However, this enhanced performance is not converted by the out-of-sample strategy. The (RV, (T)GARCH)-mixture portfolio outperforms the unmanaged portfolio in terms of Sharpe ratio and CER for just three of the eight factors. Again, the MOM and BAB factor exhibit the largest gains, both gains statistically significant at the 1% level. Performance of the (RV, (T)GARCH)-mixture portfolios is the worst for the SMB factor. The (RV, TGARCH)-mixture portfolio earns a lower CER than the unmanaged portfolio that is statistically significant at the 5% level.

Table 4 also compares the performance of the (out-of-sample) combination and mixture portfolios relative to their (in-sample) benchmarks. These results provide better insight into the quality of the out-of-sample strategy, as they measure the performance deterioration relative to the in-sample design. All combination and mixture portfolios exhibit a negative difference in Sharpe ratio and CER. In terms of Sharpe ratio, the deterioration in performance is the most substantial for the (RV, GARCH)-mixture portfolios (ranges from -0.09 to -0.20) and the (RV, TGARCH)-mixture portfolios (ranges from -0.07 to -0.21). The differences in Sharpe ratio are statistically significant negative at the 10% level for all factors, except the RMW and BAB factor. Results for CER are similar, performance deterioration in terms of CER for the mixture portfolios is statistically significant at the 10% level for all factors, except the RMW factor. I also find a consistent loss for the combination portfolios, although not all statistically significant. These findings show that the current out-of-sample strategy, that uses an expanding window to estimate the moments of asset returns, does not efficiently convert the considerable in-sample gains into out-of-sample earnings for mean-variance investors. Therefore, it is advantageous to design better out-of-sample strategies.

Table 2: Results for direct comparison over the test sample

The table compares the performance of the volatility-managed portfolios relative to the unmanaged portfolios over the test sample of $T - K$ observations, initializing $K = 120$ months. Moreover, the rolling window has length $R = 60$ months. In the table, [m]-managed portfolio is the abbreviation for the volatility-managed portfolio that uses method m = realized variance (RV), GARCH, TGARCH to estimate the conditional variance of the unmanaged portfolio. I present the mean and standard deviation of the excess returns in percent per year. Furthermore, I report (annualized) Sharpe ratios as well as the difference between the Sharpe ratio of the (individual) volatility-managed portfolios and the unmanaged portfolios. The table also shows Sharpe ratio differences between the RV-managed and (T)GARCH-managed portfolios. The numbers in brackets are p-values corresponding to the null hypothesis of equal Sharpe ratios, using the test of Jobson and Korkie (1981).

	Factor							
	MKT (1)	SMB (2)	HML (3)	MOM (4)	RMW (5)	CMA (6)	BAB (7)	QMJ (8)
	Panel A: Performance measures for unmanaged portfolios							
Mean	7.66	2.39	4.66	7.67	3.45	4.24	10.27	5.08
Standard deviation	15.71	10.09	9.78	13.98	8.05	6.85	11.07	8.06
[S1] Sharpe ratio	0.49	0.24	0.48	0.55	0.43	0.62	0.93	0.63
	Panel B.1: Performance measures for RV-managed portfolios							
Mean	7.68	0.94	4.06	13.91	4.91	3.48	13.93	5.40
Standard deviation	15.71	10.09	9.78	13.98	8.05	6.85	11.07	8.06
[S2] Sharpe ratio	0.49	0.09	0.42	1.00	0.61	0.51	1.26	0.67
	Panel B.2: Performance comparison							
Correlation unmanaged portfolio	0.71	0.69	0.69	0.54	0.56	0.68	0.61	0.62
Sharpe ratio difference, [S2] - [S1]	0.00	-0.14	-0.06	0.45	0.18	-0.11	0.33	0.04
	[0.99]	[0.10]	[0.49]	[0.00]	[0.21]	[0.36]	[0.01]	[0.75]
	Panel C.1: Performance measures for GARCH-managed portfolios over the expanding window							
Mean	7.72	0.57	4.46	14.71	4.59	4.03	14.40	5.84
Standard deviation	15.71	10.09	9.78	13.98	8.05	6.85	11.07	8.06
[S3] Sharpe ratio	0.49	0.06	0.46	1.05	0.57	0.59	1.30	0.72
	Panel C.2: Performance comparison							
Correlation unmanaged portfolio	0.80	0.74	0.73	0.64	0.65	0.76	0.68	0.72
Sharpe ratio difference, [S3] - [S1]	0.00	-0.18	-0.02	0.50	0.14	-0.03	0.37	0.09
	[0.96]	[0.02]	[0.80]	[0.00]	[0.27]	[0.77]	[0.00]	[0.38]
Correlation RV-managed portfolio	0.95	0.90	0.96	0.92	0.96	0.96	0.97	0.96
Sharpe ratio difference, [S3] - [S2]	0.00	-0.04	0.04	0.06	-0.04	0.08	0.04	0.05
	[0.95]	[0.48]	[0.22]	[0.23]	[0.38]	[0.05]	[0.17]	[0.19]
	Panel D.1: Performance measures for GARCH-managed portfolios over the rolling window							
Mean	8.11	1.22	4.43	14.50	4.66	3.94	14.10	6.03
Standard deviation	15.71	10.09	9.78	13.98	8.05	6.85	11.07	8.06
[S4] Sharpe ratio	0.52	0.12	0.45	1.04	0.58	0.58	1.27	0.75
	Panel D.2: Performance comparison							
Correlation unmanaged portfolio	0.78	0.74	0.75	0.62	0.63	0.75	0.66	0.70
Sharpe ratio difference, [S4] - [S1]	0.03	-0.12	-0.02	0.49	0.15	-0.04	0.35	0.12
	[0.70]	[0.15]	[0.76]	[0.00]	[0.25]	[0.68]	[0.00]	[0.29]
Correlation RV-managed portfolio	0.93	0.92	0.95	0.92	0.90	0.96	0.97	0.95
Sharpe ratio difference, [S4] - [S2]	0.03	0.03	0.04	0.04	-0.03	0.07	0.02	0.08
	[0.53]	[0.54]	[0.30]	[0.38]	[0.66]	[0.14]	[0.65]	[0.11]

Table 2 (continued)

	Factor							
	MKT	SMB	HML	MOM	RMW	CMA	BAB	QMJ
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel E.1: Performance measures for TGARCH-managed portfolios over the expanding window							
Mean	8.15	1.16	4.34	14.77	4.67	3.88	13.91	5.66
Standard deviation	15.71	10.09	9.78	13.98	8.05	6.85	11.07	8.06
[S5] Sharpe ratio	0.52	0.11	0.44	1.06	0.58	0.57	1.26	0.70
	Panel E.2: Performance comparison							
Correlation unmanaged portfolio	0.79	0.74	0.74	0.64	0.66	0.76	0.68	0.72
Sharpe ratio difference, [S5] - [S1]	0.03	-0.12	-0.03	0.51	0.15	-0.05	0.33	0.07
	[0.66]	[0.13]	[0.69]	[0.00]	[0.23]	[0.61]	[0.00]	[0.50]
Correlation RV-managed portfolio	0.94	0.93	0.95	0.92	0.95	0.97	0.97	0.96
Sharpe ratio difference, [S5] - [S2]	0.03	0.02	0.03	0.06	-0.03	0.06	0.00	0.03
	[0.45]	[0.61]	[0.41]	[0.19]	[0.54]	[0.14]	[0.97]	[0.43]
	Panel F.1: Performance measures for TGARCH-managed portfolios over the moving window							
Mean	8.51	1.85	4.05	14.75	4.89	3.63	14.03	5.89
Standard deviation	15.71	10.09	9.78	13.98	8.05	6.85	11.07	8.06
[S6] Sharpe ratio	0.54	0.18	0.41	1.05	0.61	0.53	1.27	0.73
	Panel F.2: Performance comparison							
Correlation unmanaged portfolio	0.77	0.74	0.74	0.63	0.63	0.75	0.67	0.70
Sharpe ratio difference, [S6] - [S1]	0.05	-0.05	-0.06	0.51	0.18	-0.09	0.34	0.10
	[0.47]	[0.51]	[0.44]	[0.00]	[0.18]	[0.41]	[0.00]	[0.37]
Correlation RV-managed portfolio	0.90	0.92	0.95	0.92	0.94	0.95	0.96	0.94
Sharpe ratio difference, [S6] - [S2]	0.05	0.09	0.00	0.06	0.00	0.02	0.01	0.06
	[0.29]	[0.05]	[0.98]	[0.21]	[0.96]	[0.64]	[0.81]	[0.22]

Table 3: Results for the in-sample comparison

The table reports results for the comparison of the (in-sample) combination/mixture portfolios relative to the (in-sample) unmanaged portfolios over the full-sample of T observations. Let $[m]$-combination portfolio denote the (in-sample) combination portfolio consisting of the $[m]$-managed portfolio, the unmanaged portfolio and the risk-free rate. Here, $[m]$-managed portfolio is the abbreviation for the volatility-managed portfolio that uses method m = realized variance (RV), GARCH, TGARCH to estimate the conditional variance of the unmanaged portfolio. Moreover, (RV, (T)GARCH)-mix portfolio denotes the (in-sample) mixture portfolio that consists of the RV-managed portfolio, the (T)GARCH-managed portfolio, the unmanaged portfolio and the risk-free rate. I report (annualized) Sharpe ratios and certainty equivalent returns (CERs-in), using $\gamma = 5$ as level of risk aversion. The CER-in is computed via Eq. (27). The table also presents differences between the Sharpe ratio and CER-in of the combination/mixture portfolios and the unmanaged portfolios. Appendix B reports the ex post optimization parameters for the combination and mixture portfolios.

	Factor							
	MKT (1)	SMB (2)	HML (3)	MOM (4)	RMW (5)	CMA (6)	BAB (7)	QMJ (8)
	Panel A: Sharpe ratios							
[S1] Unmanaged portfolio	0.42	0.24	0.39	0.49	0.41	0.54	0.89	0.58
[S2] RV-combination portfolio	0.53	0.25	0.43	0.99	0.55	0.54	1.21	0.64
Difference, [S2] - [S1]	0.11	0.01	0.04	0.50	0.14	0.00	0.32	0.06
[S3] GARCH-combination portfolio	0.52	0.29	0.42	0.90	0.46	0.54	1.05	0.66
Difference, [S3] - [S1]	0.10	0.06	0.03	0.41	0.05	0.01	0.16	0.08
[S4] TGARCH-combination portfolio	0.54	0.27	0.45	0.94	0.48	0.54	1.06	0.63
Difference, [S4] - [S1]	0.12	0.04	0.06	0.46	0.07	0.01	0.17	0.05
[S5] (RV, GARCH)-mix portfolio	0.54	0.29	0.43	1.03	0.56	0.55	1.21	0.67
Difference, [S5] - [S1]	0.12	0.06	0.04	0.55	0.15	0.02	0.32	0.09
[S6] (RV, TGARCH)-mix portfolio	0.55	0.27	0.45	1.05	0.55	0.55	1.21	0.65
Difference, [S6] - [S1]	0.13	0.04	0.06	0.57	0.14	0.02	0.32	0.07
	Panel B: Certainty equivalent returns (%) (CERS-in)							
[C1] Unmanaged portfolio	1.75	0.56	1.53	2.36	1.68	2.87	7.90	3.36
[C2] RV-combination portfolio	2.76	0.62	1.88	9.73	3.06	2.89	14.60	4.10
Difference, [C2] - [C1]	1.01	0.07	0.35	7.37	1.38	0.02	6.70	0.74
[C3] GARCH-combination portfolio	2.69	0.86	1.77	8.10	2.13	2.93	10.94	4.41
Difference, [C3] - [C1]	0.94	0.30	0.24	5.74	0.45	0.06	3.04	1.04
[C4] TGARCH-combination portfolio	2.90	0.75	1.99	8.91	2.30	2.93	11.16	4.01
Difference, [C4] - [C1]	1.15	0.19	0.46	6.55	0.62	0.06	3.26	0.65
[C5] (RV, GARCH)-mix portfolio	2.90	0.87	1.89	10.62	3.13	3.07	14.62	4.45
Difference, [C5] - [C1]	1.15	0.31	0.36	8.27	1.45	0.19	6.71	1.09
[C6] (RV, TGARCH)-mix portfolio	3.03	0.75	2.02	11.12	3.07	3.07	14.60	4.19
Difference, [C6] - [C1]	1.28	0.19	0.49	8.76	1.39	0.20	6.70	0.82

Table 4: Results for the out-of-sample comparison

The table summarizes the results for the out-of-sample comparison over the test sample of $T - K$ observations, initializing $K = 120$ months. Let $[m]$-combination portfolio denote the (out-of-sample) combination portfolio consisting of the $[m]$-managed portfolio, the unmanaged portfolio and the risk-free rate. Here, $[m]$-managed portfolio is the abbreviation for the volatility-managed portfolio that uses method m = realized variance (RV), GARCH, TGARCH to estimate the conditional variance of the unmanaged portfolio. Furthermore, (RV, (T)GARCH)-mix portfolio denotes the (out-of-sample) mixture portfolio that consists of the RV-managed portfolio, the (T)GARCH-managed portfolio, the unmanaged portfolio and the risk-free rate. For every portfolio, I report the $[m]$-benchmark portfolio as the corresponding (in-sample) portfolio over the test sample. panel A shows (annualized) Sharpe ratios as well as the difference in Sharpe ratio of the combination/mixture portfolios relative to the unmanaged portfolios and their corresponding benchmarks. Panel B reports the (annualized) certainty equivalent returns (CERs-out) computed via Eq. (28). Furthermore, I present the corresponding differences in CER-out. The numbers in brackets are p-values for the null hypotheses of equal Sharpe ratios and CERs-out, respectively. To compute p-values, I use the tests proposed by Jobson and Korkie (1981) and DeMiquel et al. (2009).

	Factor							
	MKT	SMB	HML	MOM	RMW	CMA	BAB	QMJ
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Panel A.1: Sharpe ratios for combination portfolios							
[S1] Unmanaged portfolio	0.46	0.20	0.43	0.49	0.34	0.56	0.97	0.59
[S2] RV-combination portfolio	0.42	0.14	0.36	0.93	0.49	0.52	1.20	0.59
Difference, [S2] - [S1]	-0.04	-0.05	-0.06	0.45	0.15	-0.04	0.23	0.00
	[0.65]	[0.38]	[0.37]	[0.00]	[0.46]	[0.15]	[0.00]	[0.99]
[S2*] RV-benchmark portfolio	0.52	0.26	0.48	1.00	0.62	0.63	1.27	0.72
Difference, [S2] - [S2*]	-0.10	-0.11	-0.12	-0.06	-0.13	-0.11	-0.08	-0.13
	[0.00]	[0.16]	[0.07]	[0.10]	[0.38]	[0.00]	[0.11]	[0.20]
[S3] GARCH-combination portfolio	0.47	0.18	0.41	0.84	0.41	0.51	1.04	0.60
Difference, [S3] - [S1]	0.01	-0.02	-0.02	0.35	0.06	-0.04	0.07	0.01
	[0.90]	[0.82]	[0.76]	[0.00]	[0.70]	[0.21]	[0.09]	[0.87]
[S3*] GARCH-benchmark portfolio	0.50	0.28	0.46	0.89	0.52	0.62	1.09	0.78
Difference, [S3] - [S3*]	-0.04	-0.11	-0.05	-0.05	-0.11	-0.10	-0.05	-0.18
	[0.28]	[0.07]	[0.16]	[0.23]	[0.43]	[0.00]	[0.30]	[0.03]
[S4] TGARCH-combination portfolio	0.47	0.09	0.45	0.86	0.40	0.53	1.04	0.59
Difference, [S4] - [S1]	0.01	-0.11	0.02	0.38	0.06	-0.03	0.07	0.00
	[0.89]	[0.23]	[0.77]	[0.00]	[0.77]	[0.40]	[0.07]	[1.00]
[S4*] TGARCH-benchmark portfolio	0.52	0.26	0.48	0.93	0.54	0.62	1.10	0.74
Difference, [S4] - [S4*]	-0.05	-0.17	-0.04	-0.07	-0.14	-0.09	-0.06	-0.14
	[0.08]	[0.03]	[0.21]	[0.02]	[0.40]	[0.00]	[0.26]	[0.06]
	Panel A.2: Sharpe ratios for mixture portfolios							
[S5] (RV, GARCH)-mix portfolio	0.42	0.14	0.33	0.92	0.47	0.51	1.19	0.58
Difference, [S5] - [S1]	-0.04	-0.06	-0.10	0.43	0.13	-0.05	0.21	-0.01
	[0.65]	[0.57]	[0.22]	[0.00]	[0.53]	[0.72]	[0.00]	[0.86]
[S5*] (RV, GARCH)-benchmark portfolio	0.52	0.28	0.48	1.03	0.62	0.65	1.28	0.78
Difference, [S5] - [S5*]	-0.10	-0.14	-0.15	-0.11	-0.15	-0.14	-0.09	-0.20
	[0.04]	[0.09]	[0.07]	[0.01]	[0.32]	[0.00]	[0.11]	[0.04]
[S6] (RV, TGARCH)-mix portfolio	0.42	0.05	0.36	0.96	0.42	0.52	1.21	0.57
Difference, [S6] - [S1]	-0.04	-0.15	-0.07	0.47	0.08	-0.04	0.24	-0.02
	[0.64]	[0.10]	[0.39]	[0.00]	[0.72]	[0.44]	[0.00]	[0.76]
[S6*] (RV, TGARCH)-benchmark portfolio	0.53	0.26	0.49	1.05	0.62	0.65	1.27	0.74
Difference, [S6] - [S6*]	-0.11	-0.21	-0.13	-0.10	-0.19	-0.13	-0.07	-0.18
	[0.01]	[0.02]	[0.08]	[0.01]	[0.26]	[0.00]	[0.25]	[0.06]

Table 4 (continued)

	Factor							
	MKT (1)	SMB (2)	HML (3)	MOM (4)	RMW (5)	CMA (6)	BAB (7)	QMJ (8)
	Panel B.1: CERs-out (%) for combination portfolios							
[C1] Unmanaged portfolio	1.75	0.39	1.55	2.31	1.12	3.10	7.54	3.31
[C2] RV-combination portfolio	1.60	0.03	1.30	8.62	2.41	2.72	14.33	3.40
Difference, [C2] - [C1]	-0.14	-0.36	-0.25	6.31	1.29	-0.38	6.80	0.09
	[0.87]	[0.28]	[0.67]	[0.00]	[0.47]	[0.17]	[0.00]	[0.93]
[C2*] RV-benchmark portfolio	2.67	0.64	2.28	9.89	3.75	3.84	16.22	5.19
Difference, [C2] - [C2*]	-1.07	-0.61	-0.98	-1.27	-1.33	-1.12	-1.89	-1.79
	[0.00]	[0.14]	[0.07]	[0.13]	[0.35]	[0.00]	[0.11]	[0.20]
[C3] GARCH-combination portfolio	1.65	0.22	1.67	6.86	1.63	2.64	9.99	3.29
Difference, [C3] - [C1]	-0.09	-0.17	0.12	4.55	0.51	-0.47	2.45	-0.02
	[0.93]	[0.73]	[0.84]	[0.01]	[0.72]	[0.21]	[0.04]	[0.99]
[C3*] GARCH-benchmark portfolio	2.51	0.79	2.09	7.87	2.63	3.75	11.89	5.95
Difference, [C3] - [C3*]	-0.86	-0.57	-0.42	-1.01	-1.00	-1.12	-1.90	-2.65
	[0.13]	[0.07]	[0.20]	[0.18]	[0.43]	[0.00]	[0.17]	[0.03]
[C4] TGARCH-combination portfolio	1.73	-0.23	1.98	7.36	1.22	2.80	9.85	3.07
Difference, [C4] - [C1]	-0.01	-0.62	0.43	5.05	0.10	-0.30	2.31	-0.24
	[0.99]	[0.16]	[0.52]	[0.01]	[0.96]	[0.40]	[0.04]	[0.81]
[C4*] TGARCH-benchmark portfolio	2.66	0.69	2.30	8.65	2.91	3.76	12.14	5.32
Difference, [C4] - [C4*]	-0.93	-0.92	-0.32	-1.29	-1.69	-0.95	-2.29	-2.26
	[0.05]	[0.03]	[0.26]	[0.02]	[0.36]	[0.00]	[0.13]	[0.06]
	Panel B.2: CERs-out (%) for mixture portfolios							
[C5] (RV, GARCH)-mix portfolio	0.93	-0.35	0.87	8.34	2.22	2.55	13.87	2.91
Difference, [C5] - [C1]	-0.81	-0.74	-0.68	6.03	1.10	-0.55	6.33	-0.39
	[0.50]	[0.28]	[0.36]	[0.00]	[0.53]	[0.27]	[0.00]	[0.76]
[C5*] (RV, GARCH)-benchmark portfolio	2.71	0.78	2.27	10.62	3.75	4.05	16.23	5.97
Difference, [C5] - [C5*]	-1.87	-1.13	-1.40	-2.28	-1.53	-1.50	-2.36	-3.06
	[0.02]	[0.05]	[0.07]	[0.02]	[0.30]	[0.00]	[0.11]	[0.04]
[C6] (RV, TGARCH)-mix portfolio	1.10	-0.78	0.98	9.16	1.52	2.71	14.41	2.72
Difference, [C6] - [C1]	-0.65	-1.17	-0.57	6.85	0.40	-0.40	6.88	-0.58
	[0.57]	[0.04]	[0.51]	[0.00]	[0.86]	[0.44]	[0.00]	[0.64]
[C6*] (RV, TGARCH)-benchmark portfolio	2.79	0.68	2.35	11.11	3.72	4.06	16.22	5.47
Difference, [C6] - [C6*]	-1.69	-1.46	-1.37	-1.95	-2.20	-1.35	-1.81	-2.75
	[0.01]	[0.01]	[0.07]	[0.02]	[0.25]	[0.00]	[0.24]	[0.07]

6 Conclusion

Moreira and Muir (2017) claim that the positive alphas in their spanning regressions imply that volatility-managed versions of popular equity factors enhance their performance relative to the unmanaged portfolios. This statement is somewhat nullified by Cederburg et al. (2019), as they argue that the positive alphas are a minimum requirement for an improved performance. Using both realized variance and a (T)GARCH model to estimate conditional variance of the unmanaged portfolios, I confirm that none of the volatility-managed portfolios systematically outperform the unmanaged portfolios. The enhanced performance of the volatility-managed portfolios seems centered around the BAB and MOM factor. However, I do find that estimating conditional variance with a (T)GARCH model leads to a consistently improved performance relative to realized variance estimation. This shows that the performance of volatility-managed portfolios can be improved by more accurate conditional variance estimation. Because of this, it could be advantageous to investigate alternative estimators, with the aim of outperforming the unmanaged portfolios.

Positives alphas may not guarantee an improved performance for the volatility-managed portfolios, however, they do imply that the in-sample allocation between the volatility-managed and unmanaged portfolios outperforms the unmanaged portfolios. Using a variety of combination and mixture portfolios, I confirm that there are considerable in-sample gains in terms of Sharpe ratio and CER. In particular, the impressive performance for the mixture portfolios, that earn substantial higher Sharpe ratios and CERs than the combination portfolios.

The problem with these in-sample portfolios is that estimated moments of asset returns are unknown prior to the end of the sample. This makes the portfolio construction method not implementable in real time. Therefore, I adopt an out-of-sample strategy similar to Cederburg et al. (2019). Using an expanding window to estimate the required parameters, I find no statistical evidence that the out-of-sample variants of the combination and mixture portfolios systematically outperform the unmanaged portfolio. Comparing the results with the in-sample benchmarks provides insight in the quality of the out-of-sample strategy. I find statistically significant performance deterioration, especially for the mixture portfolios. These findings show that the current out-of-sample strategy does not efficiently convert the considerable in-sample gains into out-of-sample earnings for mean-variance investors. Therefore, it is important to explore better out-of-sample strategies.

In conclusion, I identify two opportunities for follow-up research. First, more research should be done into alternative conditional variance estimators, to enhance the performance of the volatility-managed portfolios. Second, it is advantageous to design better out-of-sample strategies, to convert in-sample gains into out-of-sample earnings for mean-variance investors.

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Appendix A: Statistical tests

Jobson and Korkie (1981) test

To test the null hypothesis of equal Sharpe ratios for portfolios i and j, I use the following test. Let $\hat{\mu}_i$, $\hat{\sigma}_i$ ($\hat{\mu}_j$, $\hat{\sigma}_j$) denote the mean and standard deviation of excess returns of portfolio i (portfolio j). Furthermore, let $\hat{\sigma}_{i,j}$ be the covariance between excess returns of the two portfolios and let T denote the number of observations. The Jobson and Korkie (1981) test statistic, which is asymptotically distributed as a standard normal, is then computed via

$$\hat{z}_{JK} = \frac{\hat{\sigma}_j \hat{\mu}_i - \hat{\sigma}_i \hat{\mu}_j}{\sqrt{\hat{\theta}}}, \tag{29}$$

in which

$$\hat{\theta} = \frac{1}{T}(2\hat{\sigma}_i^2\hat{\sigma}_j^2 - 2\hat{\sigma}_i\hat{\sigma}_j\hat{\sigma}_{i,j} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_j^2 + \frac{1}{2}\hat{\mu}_j^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_j}{\hat{\sigma}_i\hat{\sigma}_j}\hat{\sigma}_{i,j}^2). \tag{30}$$

DeMiquel et al. (2009) test

To test the null hypothesis of equal certainty equivalent returns for portfolios i and j, I use the following test. Let $\hat{\sigma}_i$ ($\hat{\sigma}_j$) be the standard deviation of excess returns of portfolio i (portfolio j) and let $\hat{\sigma}_{i,j}$ be the covariance between excess returns of the two portfolios. Moreover, let T denote the total number of observations and γ the level of risk aversion. The DeMiquel et al. (2009) test statistic, which is asymptotically distributed as a standard normal, is then computed via

$$\hat{z}_{DM} = \frac{(\hat{\mu}_i - \frac{\gamma}{2}\hat{\sigma}_i^2) - (\hat{\mu}_j - \frac{\gamma}{2}\hat{\sigma}_j^2)}{\sqrt{\hat{\theta}}}, \tag{31}$$

in which

$$\hat{\theta} = \frac{1}{T}((\hat{\sigma}_i^2 - \hat{\sigma}_{i,j}) + (\hat{\sigma}_j^2 - \hat{\sigma}_{i,j}) + \frac{\gamma^2}{2}((\hat{\sigma}_i^4 - \hat{\sigma}_{i,j}^2) + (\hat{\sigma}_j^4 - \hat{\sigma}_{i,j}^2))). \tag{32}$$

Appendix B: Ex post optimization parameters

Table 5: Results for the ex post optimization parameters

The table presents the ex post optimization parameters for the in-sample combination and mixture portfolios. I report the scaling factor s^* that scales the volatility and the optimal in-sample portfolio weights for the volatility-managed portfolios x_v^* and the unmanaged portfolios x^*. I also list the correlations ρ between volatility-managed and unmanaged portfolios within the combination portfolios.

	Factor							
	MKT (1)	SMB (2)	HML (3)	MOM (4)	RMW (5)	CMA (6)	BAB (7)	QML (8)
	Panel A: Parameters for RV-combination portfolio							
Scaling parameter, s^*	10.33	2.63	2.95	4.60	1.44	1.54	2.95	1.33
RV-managed weight, x_v^*	0.44	-0.19	0.38	1.19	1.21	0.17	1.98	0.93
Unmanaged weight, x^*	0.17	0.54	0.43	0.03	0.36	1.42	0.49	0.94
Risky allocation, $x_v^* + x^*$	0.61	0.36	0.81	1.22	1.57	1.59	2.47	1.87
Correlation between RV-managed and unmanaged portfolio	0.63	0.63	0.57	0.48	0.59	0.68	0.61	0.64
	Panel B: Parameters for GARCH-combination portfolio							
Scaling parameter, s^*	22.44	8.04	8.89	14.51	3.43	3.32	6.66	3.92
GARCH-managed weight, x_v^*	0.52	-0.50	0.37	1.23	0.79	-0.39	1.52	1.48
Unmanaged weight, x^*	0.04	0.82	0.38	-0.23	0.51	1.86	0.61	0.32
Risky allocation, $x_v^* + x^*$	0.57	0.32	0.75	1.01	1.30	1.47	2.13	1.80
Correlation between GARCH-managed and unmanaged portfolio	0.78	0.78	0.72	0.66	0.71	0.83	0.72	0.82
GARCH parameter μ	0.79	0.09	0.39	0.55	0.23	0.21	0.66	0.27
GARCH parameter ω	0.70	0.15	0.37	1.43	0.21	0.17	0.44	0.18
GARCH parameter α	0.13	0.13	0.17	0.42	0.16	0.16	0.21	0.13
GARCH parameter β	0.85	0.87	0.81	0.58	0.78	0.80	0.75	0.84
	Panel C: Parameters for TGARCH-combination portfolio							
Scaling parameter, s^*	22.29	8.04	8.70	13.99	3.43	3.32	6.66	3.88
TGARCH-managed weight, x_v^*	0.58	-0.41	0.50	1.31	0.93	-0.40	1.57	1.15
Unmanaged weight, x^*	0.00	0.75	0.29	-0.27	0.41	1.87	0.57	0.60
Risky allocation, $x_v^* + x^*$	0.58	0.34	0.79	1.04	1.34	1.47	2.14	1.75
Correlation between TGARCH-managed and unmanaged portfolio	0.78	0.79	0.71	0.66	0.72	0.83	0.72	0.81
TGARCH parameter μ	0.70	0.05	0.31	0.79	0.21	0.21	0.66	0.31
TGARCH parameter ω	0.90	0.15	0.23	0.82	0.20	0.17	0.44	0.15
TGARCH parameter α	0.06	0.10	0.07	0.48	0.11	0.16	0.19	0.15
TGARCH parameter γ	0.12	0.07	0.15	-0.41	0.08	-0.01	0.02	-0.10
TGARCH parameter β	0.84	0.87	0.85	0.73	0.79	0.80	0.75	0.86
	Panel D: Parameters for (RV, GARCH)-mixture portfolio							
RV-managed weight, x_{RV}^*	0.28	0.08	0.31	0.88	1.50	0.58	2.03	0.32
GARCH-managed weight, x_{GA}^*	0.28	-0.57	0.11	0.61	-0.46	-0.86	-0.08	1.19
Unmanaged weight, x^*	0.05	0.81	0.39	-0.23	0.51	1.85	0.51	0.35
Risky allocation, $x_{RV}^* + x_{GA}^* + x^*$	0.62	0.33	0.81	1.26	1.55	1.57	2.46	1.86
Correlation between RV-managed and GARCH-managed portfolio	0.83	0.78	0.81	0.71	0.83	0.82	0.82	0.83
	Pandel E: Parameters for (RV, TGARCH)-mixture portfolio							
RV-managed weight, x_{RV}^*	0.22	0.01	0.15	0.81	1.30	0.59	1.98	0.62
TGARCH-managed weight, x_{GA}^*	0.39	-0.42	0.39	0.75	-0.15	-0.87	0.01	0.59
Unmanaged weight, x^*	0.01	0.75	0.28	-0.29	0.41	1.86	0.48	0.66
Risky allocation, $x_{RV}^* + x_{GA}^* + x^*$	0.62	0.34	0.82	1.27	1.56	1.57	2.47	1.87
Correlation between RV-managed and TGARCH-managed portfolio	0.83	0.79	0.80	0.71	0.83	0.82	0.82	0.83

Appendix C: Direct comparison over the full-sample

Table 6: Results for direct comparison over the full-sample

The table compares the performance of the realized variance (RV)-managed portfolios relative to the unmanaged portfolios over the full-sample of T observations. I report the mean and standard deviation of the excess returns in percent per year. Moreover, I present (annualized) Sharpe ratios alongside differences between the Sharpe ratio of the RV-managed portfolios and the unmanaged portfolios. To test the null hypothesis of equal Sharpe ratios, I use the test proposed by Jobson and Korkie (1981), the corresponding p-values are reported in brackets.

	Factor							
	MKT (1)	SMB (2)	HML (3)	MOM (4)	RMW (5)	CMA (6)	BAB (7)	QMJ (8)
	Panel A: Performance measures for unmanaged portfolios							
Mean	7.78	2.62	4.74	7.96	3.13	3.73	9.28	4.39
Standard deviation	18.61	11.10	12.13	16.39	7.62	6.96	10.45	7.57
[S1] Sharpe ratio	0.42	0.24	0.39	0.49	0.41	0.54	0.89	0.58
	Panel B.1: Performance measures for RV-managed portfolios							
Mean	9.49	0.95	4.57	16.17	4.13	2.77	12.45	4.37
Standard deviation	18.61	11.10	12.13	16.39	7.62	6.96	10.45	7.57
[S2] Sharpe ratio	0.51	0.09	0.38	0.99	0.54	0.40	1.19	0.58
	Panel B.2: Performance comparison							
Correlation unmanaged portfolio	0.63	0.63	0.57	0.48	0.59	0.68	0.61	0.64
Sharpe ratio difference, [S2] - [S1]	0.09	-0.15	-0.01	0.50	0.13	-0.14	0.30	0.00
	[0.31]	[0.10]	[0.88]	[0.00]	[0.29]	[0.21]	[0.01]	[0.99]

Appendix D: Programmed Code

The following programs are included in the Zip file 'Codes.zip'

1. RealizedVar.py - Computes the RV-managed portfolios as well as the in-sample and out-of-sample RV-combination portfolios.
2. GARCH.py - Computes the in-sample and out-of-sample GARCH-combination portfolios.
3. TGARCH.py - Computes the in-sample and out-of-sample TGARCH-combination portfolios.
4. GARCH_Expanding.py - Computes the GARCH-managed portfolios using an expanding window to estimate the GARCH parameters.
5. GARCH_Moving.py - Computes the GARCH-managed portfolios using an moving window to estimate the GARCH parameters.
6. TGARCH_Expanding.py - Computes the TGARCH-managed portfolios using an expanding window to estimate the TGARCH parameters.
7. TGARCH_Moving.py - Computes the TGARCH-managed portfolios using an moving window to estimate the TGARCH parameters.
8. RV_GARCH.py - Computes the in-sample and out-of-sample (RV, GARCH)-mixture portfolios.
9. RV_TGARCH.py - Computes the in-sample and out-of-sample (RV, TGARCH)-mixture portfolios.
10. DataThesis.xlsx - Computes Sharpe ratios/CERs and their corresponding p-values for every portfolio, using the collected mean, standard deviation and correlation from Python.