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Strategic Open Routing With Different Disutilities Between Waiting Lines

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Abstract

In this thesis we examine the occurrence of "herding" in open network services described in the paper by Arlotto et al. (2019). We analyze a simulation where customers arrive at different times and choose their route before they enter the system. We find that, under certain restrictions, customers learn to choose the same route as all the other customers, instead of preferring a strategy where they avoid the route that most other customers take. We further analyze what impact a change in disutility between the waiting time for the first line and for the second line has on the prevalence of "herding" in the simulation.

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1 Introduction

Strategic routing is a part of time management optimization that is interesting and useful for the advancement of economics and economic theory, and is often a part of everyday life. When choosing which line to wait in first at an amusement park, or deciding which store to go to first on a trip, we often look for a strategy to decide which route to take. The desire is to choose the route that is “the best”, the route with the shortest total waiting time. These two examples of routing networks have in common that there is not necessarily a preference in the order that the two stations are visited, and the customer has the choice of which route to take. In the work of Arlotto et al. (2019), as well as in this thesis, this is referred to as “open routing”.

Arlotto et al. (2019) show that it can be analytically proven that under certain conditions there is a Nash equilibrium where customers show “herding” behavior in systems that have two service stations and customers that need to visit both stations. In this thesis we analyze the simulation presented by Arlotto et al. (2019) that shows herding behavior for a network where the customers all choose their route at the same time and the arrival times of the customers vary. In this thesis, the terms “customer” and “player” are used interchangeably.

We further analyze a network where customers experience a decrease or increase in disutility for the waiting line at the second station. We analyze by simulation if there is still herding behavior where all customers end up choosing to take the same route. We look at whether there is a difference in whether or not herding occurs when the waiting time at the second station weighs more heavily than at the first station (for example, in a fitness center where a player might prefer waiting a longer time for the first machine as they feel that waiting in the second line after working at the first machine is more exhausting). We also look at instances where the waiting time at the first station weighs more heavily than at the second station (for example, in a park with a food truck and an ice cream truck one might expect that the customer will be partly satisfied in the first station and thus will not experience the waiting time for the second line as poorly as with the first line). We introduce a weighing factor w (which is constant for each customer) for being served at the second station. We assume that the waiting time for the second line weighs w times heavier than the waiting time for the first line. In this thesis, weighing factor is sometimes shortened to weight.

The thesis is structured as follows: Section 2 reviews the existing literature on similar open routing networks. Section 3 defines the characteristics of the service network that we analyze, and some of the propositions of the system are defined in Section 4. In Section 5 we explain the simulation model of Arlotto et al. (2019) as well as the extension with different waiting disutilities. The results of the simulation are analyzed in Section 6, and we give our conclusion, as well as possibilities for further research, in Section 7.

2 Literature Review

There are other examples of open routing networks that have been the subject of studies in the past. Baron et al. (2016) showcase an example of a hospital service that consists of tests that patients have to undertake. Theirs is an example of open routing where the company and not the customers get to choose the routes for each customer. This is an open routing network in which the goal of the optimal routing is defined by the company, for example the routing that has the shortest accumulative waiting time.

Arlotto et al. (2019) reference Hassin and Haviv (2003) and Hassin (2016), which respectively give a summary of what had been written on the subject of queuing systems and game theory and a summary of more recently written research on the subject. Their work most closely resembles the work of Parlaktürk and Kumar (2004) which handles a two-station queuing network where customers go through two operations and they choose which operation they begin with. The first operation takes on average a shorter time. There are two stations that can both perform each operation that the customer needs, and both stations have two queues, one for Operation 1 and another for Operation 2. The system overseer decides which queue is helped next, so it draws a distinction between the customers. Unlike the work of Parlaktürk and Kumar (2004), this thesis and the paper of Arlotto et al. (2019) use a network in which the two stations can only perform one operation, and each station has only one queue that works on a First-Come-First-Serve basis (i.e., the first person to join the queue is the first person to be served), so it can make no distinctions between what type of customer is being served.

3 Service Network Definition

We consider a service network with two stations in which customers visit both stations, like in the work of Arlotto et al. (2019). Suppose there are two stations, station A and station B, both of which need to be visited by N customers in no particular order. The stations work on a FCFS basis. We focus on the case in which the stations have a different service rate. We define μ_A and μ_B as the service rates of station A and station B, respectively, with $\mu_A < \mu_B$. Since $\mu_A < \mu_B$ the expected service time at station A is longer than at station B.

A new customer gets introduced into the system by joining the queue at station A or B. Once in the queue, the customer has to wait until everyone ahead of them in the queue is done, at which point they begin being served. After service at the first station is over, the customer joins the queue at the other station and waits for service. The customer leaves the system after they have been served at both stations. For two stations, customers have a choice between two options: they first go to station A and then go to station B (this route will be referred to as route AB), or they first go to station B and then go to station A (this route will be referred to as route BA).

Arlotto et al. (2019) assume that customers prefer to minimize the total time that they spend in the system. However, customers might prefer to wait longer in the line for the second station they visit if that means that they do not have to wait as long in the line for the first station they visit. In our extension, we use a weighing factor w to describe the difference in disutility for the waiting times, where the waiting time for the second line that the customers wait in weigh w times heavier than the waiting time for the first line. We assume that the weighing factor only applies for the time that a customer has to wait to be served, but it does not apply to the time that the customer is being served themselves. Even though we use the word disutility, we still refer to the total system time as the variable that customers want to minimize when we include the weighing factor in the system.

4 Analysis

4.1 Analysis 1: (Arlotto et al. 2019)

Arlotto et al. (2019) prove that for cases where the players all choose their strategy, and then get a randomly assigned place in line before the system starts, certain Nash equilibria can be found. For this, they assume a deterministic service time at station A with service rate μ_A and station B with service rate μ_B. For a network with these properties, one might imagine the optimal strategy to be to avoid the other customers and go to the route that is less traveled. However, Arlotto et al. (2019) find that under certain restrictions customers will “herd” to the same route.

Proposition 1 *(i) If $N \geq 2\mu_A/\mu_B + 1$, then there is a Nash equilibrium where all players take route AB. Also, (ii) if $\mu_B < 2\mu_A$ and $N \geq max\{\mu_B/\mu_A + 1, (2\mu_A + \mu_B)/(2\mu_A - \mu_B)\}$, then there is also a Nash equilibrium where all players take route BA. (Proposition 1 of Arlotto et al. 2019).*

Arlotto et al. (2019) give an intuitive explanation for this proposition. Say that all customers take route AB. On average, a customer will have half of the customers in front and half behind them if they visit station A first. However, if the customer chooses to visit station B first they will also visit station A last, and since the service time at station A is longer, it holds that as long as N is large enough, (since $\mu_A/\mu_B < 1$, this always holds when $N \geq 3$), the customer will be better off also visiting station A first like the others. Similarly, if all the other customers visit station B first, and N is large, it is then optimal for the customer to choose route BA as well. If they choose route AB they will then have to wait for all the customers at station B to be done, which takes longer than waiting for on average half of the customers at station B. A proof of an extended version of this proposition is given in Section 4.2.

Furthermore, we can simplify the restriction on N in part (ii) of Proposition 1. Note that

$$N \geq \mu_B/\mu_A + 1 = \frac{(\mu_B/\mu_A + 1)(2\mu_A - \mu_B)}{2\mu_A - \mu_B} = \frac{2\mu_A + \mu_B - \mu_B^2/\mu_A}{2\mu_A - \mu_B} = \frac{2\mu_A + \mu_B}{2\mu_A - \mu_B} - \frac{\mu_B^2/\mu_A}{2\mu_A - \mu_B}.$$

Since we already assume that $\mu_B < 2\mu_A$ and $\mu_B > \mu_A > 0$ we have $\frac{\mu_B^2/\mu_A}{2\mu_A - \mu_B} > 0$. We get $max\{\mu_B/\mu_A + 1, (2\mu_A + \mu_B)/(2\mu_A - \mu_B)\} = (2\mu_A + \mu_B)/(2\mu_A - \mu_B)$. Thus, the restriction of N can be simplified and we get the following: (ii) If $\mu_B < 2\mu_A$ and $N \geq (2\mu_A + \mu_B)/(2\mu_A - \mu_B)$, then there is also a Nash equilibrium where all players take route BA. Arlotto et al. (2019) also prove that, when N is big enough, these "herding" equilibria are the only equilibria in this system in their Proposition 4:

Proposition 2 *If $N > 2\mu_B/\mu_A + 1$, then there is no Nash equilibrium where some players have a mixed strategy and other players have a pure strategy, and any Nash equilibrium where all players use a mixed strategy is unstable, i.e., a small disturbance in the strategy of any of the players will result in other players preferring a pure strategy. (Proposition 4 of Arlotto et al. 2019).*

Arlotto et al. (2019) also find that under specific circumstances—specifically, if N is large enough and if the service at station B is more than twice as fast on average as the service at station A—then AB is a strictly dominant strategy for every player. Together with Proposition 1 and 2 this indicates that "herding" behavior is the preferred strategy in open routing service networks where customers arrive in the system at the same time.

4.2 Analysis 2: Extension

We analyze what happens to Proposition 1 when a weighing factor w gets added to the disutility of the waiting time at the second station. Again, we assume that the weighing factor only influences the time that players have to wait to be served at the second station, and has no influence on the time that the player is served themselves. We first follow the first part of the proof of Proposition 1 by Arlotto et al. (2019). We assume a position where all players take route AB. Arlotto et al. (2019) prove that player i will always find station B empty when they finish at station A. Thus, when player i is j-th in line, the system time of player j is the time that they have to wait to be served at station A, plus the serving time at station B. Let $S^A(j)$ denote the total system time when customer i takes route AB at priority j. We get

$$S^A(j) = \frac{j}{\mu_A} + \frac{1}{\mu_B}, \qquad\qquad j = 1, ..., N.$$

Note that since customer i finds the second station empty, the weighing factor has no influence on the total system time here.

Now, let $T(1, m)$ denote the expected system time that player i experiences if they choose route AB and m other players also choose route AB, and let $T(0, m)$ denote the expected system time that player i experiences if they choose route BA and m other players choose route AB. We get

$$T(1, N-1) = \frac{1}{N}\sum_{j=1}^{N} S^A(j) = \frac{1}{\mu_B} + \frac{N+1}{2\mu_A}.$$

Again, since $S^A(j)$ does not depend on w, $T(1, N-1)$ also does not depend on w.

Now we analyze what happens to $T(0, m)$ when w gets introduced. If player i takes route BA, they will be served immediately at station B, and then they will have to wait until all the other players are finished at station A. Thus, the total system time $T(0, m)$ is given by

$$T(0, N-1) = \frac{1}{\mu_B} + w(\frac{N-1}{\mu_A} - \frac{1}{\mu_B}) + \frac{1}{\mu_A} = (1-w)(\frac{1}{\mu_B} + \frac{1}{\mu_A}) + w\frac{N}{\mu_A}.$$

We note that when $w = 1$, we get the standard case of $T(0, m) = N/\mu_A$ from Arlotto et al. (2019). We note that when $N/\mu_A \geq 1/\mu_B + 1/\mu_A \Rightarrow N \geq \mu_A/\mu_B + 1$, we have a system time $T(0, m)$ that increases when $w > 1$ and decreases when $w < 1$. Since we already assume that $N \geq 2\mu_A/\mu_B + 1$ we do not have to adjust our restrictions for N.

We note that when $w = 1$, since we assume that $N \geq 2\mu_A/\mu_B + 1$, we get

$$\frac{1}{\mu_B} + \frac{1}{2\mu_A} \leq \frac{N}{2\mu_A} \implies \frac{1}{\mu_B} + \frac{1}{2\mu_A} + \frac{N}{2\mu_A} \leq \frac{N}{2\mu_A} + \frac{N}{2\mu_A} \implies \frac{1}{\mu_B} + \frac{N+1}{2\mu_A} \leq \frac{N}{\mu_A}.$$

This implies, for $w = 1$, that the system time for player i will be lower when they also choose route AB first, so we have a Nash equilibrium. As $w > 1$ increases $T(0, N-1)$, this Nash equilibrium will not change when w increases. However, as $w < 1$ decreases $T(0, N-1)$, we get a Nash equilibrium if it holds that

$$\frac{1}{\mu_B} + \frac{N+1}{2\mu_A} \leq \frac{1}{\mu_B} + w(\frac{N-1}{\mu_A} - \frac{1}{\mu_B}) + \frac{1}{\mu_A} \iff \frac{N-1}{2\mu_A} \leq w(\frac{N-1}{\mu_A} - \frac{1}{\mu_B}) \iff$$
$$w \geq \frac{1}{2 - \frac{2\mu_A}{\mu_B(N-1)}}.$$

We can rewrite the assumption $N \geq 2\mu_A/\mu_B + 1$ to get $2\mu_A/(\mu_B(N-1)) \leq 1$. We note that for $2\mu_A/(\mu_B(N-1)) = 1$, the condition of a Nash equilibrium at AB becomes $w \geq 1/(2-1) = 1$. This result suggests that for some values of $2\mu_A/(\mu_B(N-1))$, we can decrease w to a point where there is no Nash equilibrium at station AB anymore.

Now we follow the second part of the proof of Proposition 1. We assume that $\mu_B < 2\mu_A$ and $N \geq (2\mu_A + \mu_B)/(2\mu_A - \mu_B)$, and that all players go for route BA. Because $N \geq \mu_B/\mu_A + 1$, if player i takes route AB, station B will still be serving customers when player i finishes at station A. Now we can write the expected system time when a player takes route AB and the other players take route BA as

$$T(1,0) = \frac{1}{\mu_A} + w(\frac{N-1}{\mu_B} - \frac{1}{\mu_A}) + \frac{1}{\mu_B} = (1-w)(\frac{1}{\mu_A} + \frac{1}{\mu_B}) + w\frac{N}{\mu_B}.$$

Again, we note that $w = 1$ gives the standard case of $T(1,0) = N/\mu_B$ from Arlotto et al. (2019).

Now, say that all customers take route BA and customer i has a $1/N$ chance of getting priority j, where $j = 1, ..., N$. When the system begins, station A has no players, and will only start when the first player has been served at station B, which is $1/\mu_B$. After that, player i will be served at station A after the $j-1$ customers have finished before them at station A. We get

$$S^B(j) = \frac{j}{\mu_B} + w(\frac{j-1}{\mu_A} - \frac{j-1}{\mu_B}) + \frac{1}{\mu_A}.$$

Again, we note that with $w = 1$ we get the standard case of $S^B(j) = j/\mu_A + 1/\mu_B$ from Arlotto et al. (2019). Now we get an expected total system time $T(0,0)$ of

$$\begin{aligned} T(0,0) = \frac{1}{N}\sum_{j=1}^{N} S^B(j) = \frac{1}{N}\sum_{j=1}^{N}(\frac{j}{\mu_B} + w(\frac{j-1}{\mu_A} - \frac{j-1}{\mu_B}) + \frac{1}{\mu_A}) \\ = \frac{N+1}{2\mu_B} + w(\frac{N-1}{2\mu_A} - \frac{N-1}{2\mu_B}) + \frac{1}{\mu_A}. \end{aligned}$$

Again, we note that $w = 1$ gives the standard case of $T(0,0) = 1/\mu_B + (N+1)/2\mu_A$ from Arlotto et al. (2019).

When $w = 1$ we can use the assumptions that $\mu_B < 2\mu_A$ and $N \geq (2\mu_A + \mu_B)/(2\mu_A - \mu_B)$ to get

$$2\mu_A + \mu_B \leq (2\mu_A - \mu_B)N \implies \frac{1}{\mu_B} + \frac{N+1}{2\mu_A} \leq \frac{N}{\mu_B}.$$

Thus, for $w = 1$, we have a Nash equilibrium where player i will be better off going to route BA with all the other players than to switch. In general, this equilibrium holds when

$$\begin{aligned} \frac{N+1}{2\mu_B} + w(\frac{N-1}{2\mu_A} - \frac{N-1}{2\mu_B}) + \frac{1}{\mu_A} \leq \frac{1}{\mu_A} + w(\frac{N-1}{\mu_B} - \frac{1}{\mu_A}) + \frac{1}{\mu_B} \iff \\ \frac{N-1}{2\mu_B} \leq w(\frac{N-1}{\mu_B} - \frac{1}{\mu_A}) \iff w \geq \frac{1}{2 - \frac{2\mu_B}{\mu_A(N-1)}}. \end{aligned}$$

Given these results, we can now extend Proposition 1 to

Proposition 3 *(i) In a system with weighing factor* $w \geq 1/(2 - 2\mu_A/\mu_B(N-1))$ *and* $N \geq 2\mu_A/\mu_B + 1$*, there is a Nash equilibrium where all players take route AB. Also, (ii) in a system with weighing factor* $w \geq 1/(2 - 2\mu_B/\mu_A(N-1))$, $\mu_B < 2\mu_A$ *and* $N \geq (2\mu_A + \mu_B)/(2\mu_A - \mu_B)$*, there is also a Nash equilibrium where all players take route BA.*

5 Simulation

We are now interested in service networks where the players have different arrival times. As Arlotto et al. (2019) point out, it is hard to define a Nash equilibrium in a service network with arrival times on varying intervals. Arlotto et al. (2019) use a simulation where all the customers arrive at a different time and all customers choose a route before the system starts to show that, subject to certain restrictions, "herding" behavior takes place. In the simulation, the customers go through the system a certain number of times and learn from the process, and update their strategy according to their previous results. They do this by first picking a route. After all the customers pick a route, they then go through the system, and they observe the time they spent in the system. They also observe the time they would have spent in the system had they chosen the other route and all the other customers kept taking the same route. In the first round the moves are randomly generated, and the next round, every customer chooses the route that would have given them a shorter expected running time. The expected running time for the route is based on the average running time of that route in all the previous rounds for that customer.

5.1 Simulation 1: (Arlotto et al. 2019)

Arlotto et al. (2019) use two parameters—γ and ϕ, respectively—to denote the mean of the times between customer arrivals and the variance of the arrival time of the players, respectively. For every player i the arrival time of the player will follow the uniform distribution over the region $[i\gamma-\phi, i\gamma+\phi]$. Arlotto et al. (2019) use exponential service times with a fixed service rate of 1 for station B, and simulate all possible combinations for $\gamma \in \{0.001, 0.1, 0.25, 0.5, 0.75, 1\}$, $\phi \in \{0, 0.25, 0.5, 0.75, 1\}$ and $\mu_A \in \{0.1, 0.25, 0.5, 0.75\}$, a total of 120 experiments. For each set of parameters Arlotto et al. (2019) run 100 independent trials, with 250 learning rounds per trial, and a total number of $N = 50$ customers. This is the base case of the study, in our extension this case would have weight $w = 1$.

5.2 Simulation 2: Extension

For the extension we look at a network where the waiting time for the first station that the player visits has a different weight than the second station. We simulate all combinations for the weighing factor $w \in \{0.5, 0.75, 0.95, 1.05, 1.25, 1.5\}$, and the other parameters $\gamma \in \{0.001, 0.1, 0.250.5, 0.75, 1\}$, $\phi \in \{0, 0.25, 0.5, 0.75, 1\}$ and $\mu_A \in \{0.5, 0.75\}$, and we check if there are any notable differences in the prevalence of herding compared to the standard case in Simulation 5.1 where $w = 1$.

Intuitively, the hypothesis is that the simulation will show that a weighing factor larger than 1 will increase the likelihood that a customer chooses route AB while a weighing factor smaller than 1 will decrease the likelihood. This is because customers have a longer waiting line at station A than station B for the same number of customers in front of them. In other words, the expected waiting time at station A is longer than station B. When $w > 1$ the utility at route AB will increase based on the waiting time at station B while the utility at route BA will increase based on the waiting time at station A. As a result, while w is a constant, the impact will be greater for route BA than for route AB, thus making route AB more attractive. Similarly, when $w < 1$ the utility at route AB will decrease based on the waiting time at station B while the utility at route BA will decrease based on the waiting time at station A, thus making route BA more attractive. Another reason why $w > 1$ will encourage herding is that if $N - 1$ customers choose route AB and a new customer starts at station B, they will join the back of the line at station A. When $w > 1$, the punishment for joining the back of the line at the second station is greater, thus it might be more important to limit the amount of customers in front of you at the busier station, and thus all customers might be more tempted to herd.

6 Results

6.1 Arlotto et al. (2019): $w = 1$

The results of the simulation described by Arlotto et al. (2019) with $\gamma = 0.001$ for several levels of μ_A and ϕ are shown in Figure 1 for the standard case where $w = 1$. The left panel in Figure 1 shows the case where the arrival times of the customers are nearly equal, and consistent (e.g. customer 1 will always arrive first). We notice that there is a preference for the AB route for all three levels of μ_A, with the total number of AB customers being more consistently close to 50 when the difference between μ_A and μ_B increases. In other words, a higher expected service time at station A compared to station B means that the route where station A gets chosen first gets a higher average preference. In the right graph of Figure 1, we examine the case where the arrival times have a high variation, and the order of arrivals are not very dependent on the index of the customer (e.g. customer 1 can be in any position in every round). We note that for a service rate at station A that is 25 or 50 percent of the service rate at station B, the customers all herd to the AB route. For a service rate of 75 percent we see that herding takes place at route AB in 98% of all cases, whereas the other 2% of the trials see the customers herding at route BA. This aligns with Proposition 1 and 2 which states the customers herd to one of the two stations when N is sufficiently large, and that there is only one dominant strategy (i.e. all customers herd to station AB) when the service rate of station B is bigger than the service rate of station A by a factor of at least 2.

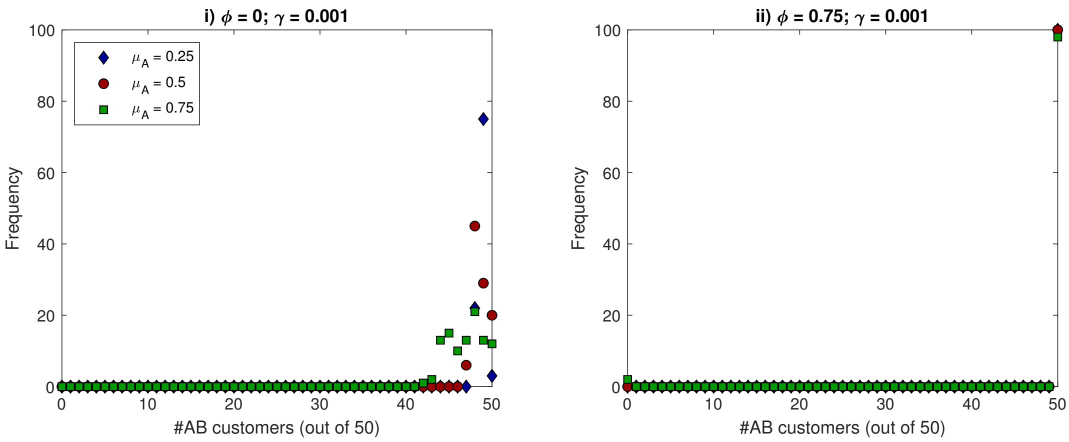

Figure 1: Number of customers who chose route AB in the 250-th round after 100 trials, with $w = 1$.

Table 1 and 2 show the average and first quartile of AB customers in the simulation for $\mu_A = 0.75$ and $\mu_A = 0.5$ respectively. We use a Student's t-test to compare the new results to the results of Arlotto et al. (2019). We assume the same variance in both distributions. Since we do not know the total number of AB customers per trial of Arlotto et al. (2019), we calculate the unbiased estimators of the variances of the new sample and we assume that this estimator is the same for the sample by Arlotto et al. (2019). Since $N = 50$, we have a total of $2 \cdot 50 - 2 = 98$ degrees of freedom. For a significance level of 0.05 and a two-sided test we get a critical value of 1.984. Table 3 shows the t-statistics for comparing these new results with the results of Arlotto et al. (2019). Since all values in the table are smaller than the critical value, we do not find a significant difference between the new values and the values by Arlotto et al. (2019). Note that some values have no t-statistic because in these cases the customers completely herd to AB (i.e. after 250 rounds all customers herd to AB in all 100 trials), thus the variation is 0.

Like in the results by Arlotto et al. (2019), we find that complete herding to AB, only occurs when γ is small, and does not occur when $\gamma > 0.1$. However, we see for all three choices of μ_A that most of the customers end up choosing the route AB, (since $\mu_A = 0.25$ sees customers herding to AB very consistently, we abandon this value in our further analysis). We also notice that, when $w = 1$, complete herding at BA never occurs when $\gamma > 0.1$. In both tables we observe a decrease in the number of AB customers as γ increases, and a bigger ϕ does not seem to have a great impact on the prevalence of herding. When we look at the difference between the results of Table 1 and Table 2 we see that for $\mu_A = 0.5$ the values are all greater than the values for the larger $\mu_A = 0.75$. This indicates that the "herding" behavior is driven greatly by how much longer extra they have to wait for being behind one more player.

Table 1: Summary of results with $\mu_A = 0.75$ and $w = 1$ from simulation for number of AB customers after all 100 trials

Sample mean						Sample first quartile					
$w = 1$			ϕ			$w = 1$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.1	46.55	46.6	45.55	47.73	49.26	0.1	45	44.5	44	47	49
0.25	45.89	45.72	46.29	45.9	46.26	0.25	44.5	44	44	44	44
0.5	44.88	45.17	44.9	44.81	44.49	0.5	43	43	42	42	42
0.75	41.77	42.42	42.08	41.62	41.98	0.75	39.5	40	39.5	39.5	40
1	35.97	36.89	37.48	37.33	37.36	1	32.5	35	35	34.5	35

Table 2: Summary of results with $\mu_A = 0.5$ and $w = 1$ from simulation for number of AB customers after all 100 trials

Sample mean						Sample first quartile					
$w = 1$			ϕ			$w = 1$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.1	48.67	48.5	50	50	50	0.1	48	48	50	50	50
0.25	48.21	48.36	48.01	48.75	49.23	0.25	47	48	47	48.5	49
0.5	48.02	48.23	48.16	48.26	47.72	0.5	47	47.5	47	47	47
0.75	47.8	47.62	47.67	47.68	47.81	0.75	47	47	47	47	47
1	47.39	47.5	47.27	47.41	47.49	1	47	47	46	47	47

Table 3: Summary of t-statistics for the distributions of the new results and the results from Arlotto et al. (2019)

$\mu_A = 0.75$						$\mu_A = 0.5$					
$w = 1$			ϕ			$w = 1$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.1	0.8436	0.0227	0.4221	1.8027	0.4891	0.1	1.0849	0.2681	-	-	-
0.25	1.5021	0.2292	0.1382	0.3325	1.1484	0.25	0.6448	0.1618	1.2537	1.6085	0.5603
0.5	0.4773	0.1420	0.1510	0.2646	0.2033	0.5	0.1398	0.7531	0.6219	0.7616	0.5557
0.75	0.0795	0.3032	0.4940	0.1004	0.4445	0.75	0.3207	1.2431	0.6202	0.5124	0.2316
1	0.0733	0.5558	1.1840	1.3811	0.9652	1	0.3542	0.3504	0.8243	0.4407	0.7009

6.2 Extension: $w \neq 1$

Figure 2 shows the number of AB customers after 100 trials for different weights. On the top left panel we see the results with a small arrival interval, $\gamma = 0.001$, no variation in the arrival times, $\phi = 0$ and a weight of 0.95 (i.e. the second line weighs 0.95 times heavier than the first line). We see that for all cases of μ_A there is a significant decrease in the number of times the total number of AB customers nears 50.

In the bottom left panel we see the results with $\gamma = 0.001$ and $\phi = 0$ and $w = 1.05$. Here we see that the customers all herd at AB for $\mu_A = 0.25$ and $\mu_A = 0.5$, while the number of customers who choose AB greatly increases compared to no weighing factor for $\mu_A = 0.75$.

The top right and bottom right panels denote the observations of AB customers with a close arrival rate but with a big variation, $\phi = 0.75$, with a weight of 0.95 and 1.05, respectively. In both cases we have once again that all customers herd to AB for $\mu_A = 0.25$ and $\mu_A = 0.5$, while $\mu_A = 0.75$ sees customers herding to BA in 5% of all trials for a weight of 0.95, and in 2% of all trials for a weight of 1.05.

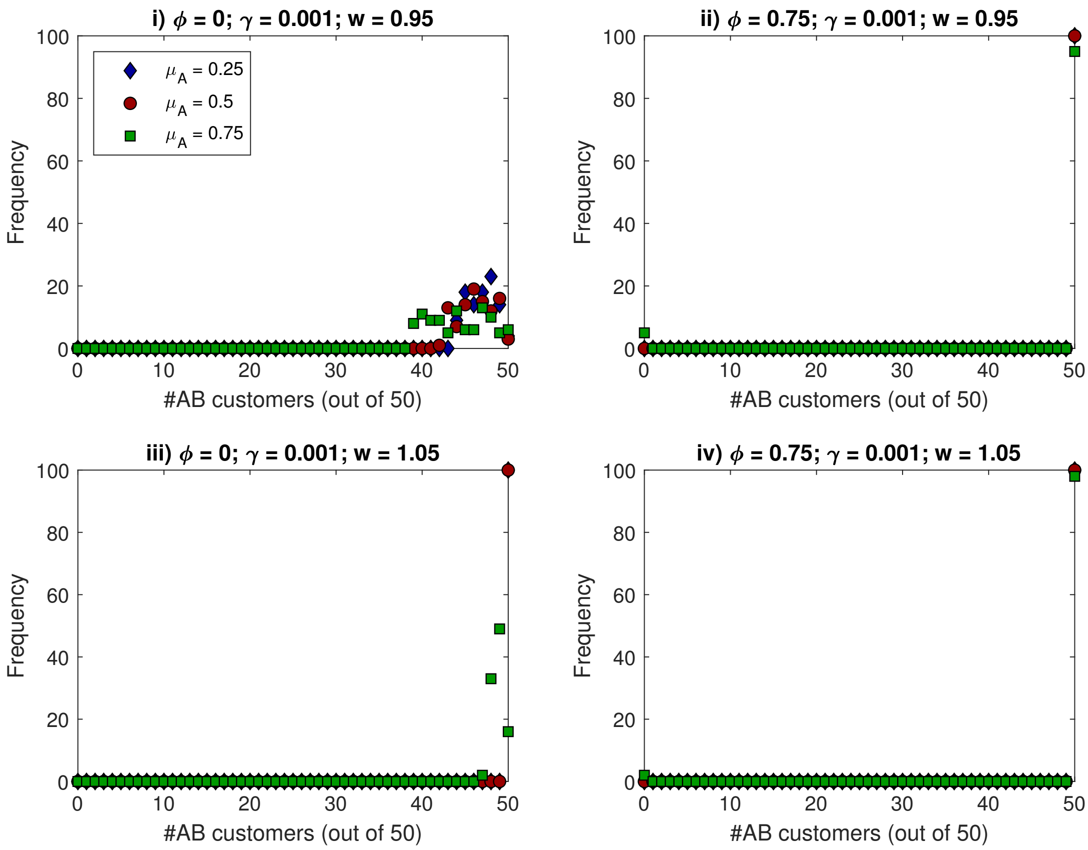

Figure 2: Number of customers who chose route AB in the 250-th round after 100 trials with $w = 0.95$ (top) and $w = 1.05$ (bottom).

The results of the extension for $\mu_A = 0.75$ with $\gamma \geq 0.001$ are shown in Table 4. The six panels at the top represent the results for the sample mean and sample first quartile for $w = 0.5, w = 0.75$ and $w = 0.95$, respectively. Compared to the results in Table 1, we can see that lowering the weight for visiting the second station greatly decreases the number of AB customers. For $w = 0.5$ the difference is far bigger than the difference for $w = 0.75$ and $w = 0.95$, which indicates that increasing the disutility difference has an increasingly powerful effect on whether or not herding will occur. As in the standard weighing factor $w = 1$, we see that a smaller γ leads to more customers choosing route AB, and we notice that the instance where the arrival times happen almost simultaneously (i.e. $\gamma = 0.001$ and $\phi = 0$) there is a great decrease in the number of AB customers compared to cases with bigger variances in arrival times. When $w = 0.5$ we see that when $\gamma = 1$, almost all customers herd to BA. We also observe that when γ is small, customers will still herd to AB more than BA, which indicates that the waiting disutility only has a big influence when the weighing factor gets further away from 1.

The six panels at the bottom represent the results for the sample mean and sample first quartile for $w = 1.05, w = 1.25$ and $w = 1.5$, respectively. They all show a noticeable increase in the number of AB customers compared to the results in Table 1. Similarly to the lower weights, the further the weighing factor gets away from $w = 1$, the larger the difference is between the average number of AB customers. At $w = 1.05$ we find that when $\gamma = 0.1$ and $\phi > 0.5$ all customers will herd to the AB route. We note that in every instance except for $\gamma = 0.1$ and $\phi = 0.5$, setting $w = 1.05$ causes around a 50% decrease in the number of customers who do not choose the AB route (i.e. customers who choose the BA route). As w increases further, more customers herd to AB. When $w = 1.25$, we notice that all customers herd to AB for every instance of γ except for $\gamma = 0.001$ and $\gamma = 1$. However, in the case of $\gamma = 0.1$ we notice some instances where all the customers herd to BA, whereas in the case of $\gamma = 1$ we always see all or almost all of the customers herding at AB. As w further increases to 1.5, we notice that customers now also herd at AB when $\gamma = 1$, and we notice an increase in the number of times customers herd at BA when $\gamma = 0.001$.

In Table 5 the results are shown when $\mu_A = 0.5$. The weights again show similar increases and decreases compared to the results in Table 2. Every observation where $w < 1$ shows a decrease in the average and first quartile of every observation (except for some cases where the customers continue to all herd at AB) compared to when $w = 1$. However, unlike the standard case of Arlotto et al. (2019), when $w < 1$ we observe some instances where the number of AB customers is higher when $\mu_A = 0.75$ compared to when $\mu_A = 0.5$. For every observation where $w > 1$ we see no more customers herding at BA, and almost every observation has all customers herding to AB (except for one instance where it is very close to all of them).

Table 6 gives a summary of the sample means for the first and last 10 customers when the customers all arrive with no variation in the arrivals (i.e. $\phi = 0$). We see that the first 10 customers tend to show more herding behavior at both stations than the last 10 customers. We see that even with a weighing factor smaller than 1, the first 10 customers often either all herd at AB or BA. Even at the standard case when $w = 1$ we see that the last 10 customers have a sample mean that is closer to the middle, signifying there are more customers in this group that choose not to herd to AB. We notice that for both groups of customers, raising the weighing factor above 1 increases the number of AB customers and lowering the weighing factor below 1 decreases the number of AB customers in almost every case. It seems that the addition of a weighing factor has a similar impact on the first group of players in the system as on the last group of players who enter the system.

Table 4: Summary of results with $\mu_A = 0.75$ and varying weights from simulation for number of AB customers after all 100 trials

Sample mean						Sample first quartile					
$w = 0.5$			ϕ			$w = 0.5$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	27.73	40.13	41.13	40.53	39.69	0.001	18	39	40	39	39
0.1	26.82	23.83	26.9	26.6	25.89	0.1	16	15	18.5	19.5	21
0.25	23.68	24.35	23.82	22.13	25.62	0.25	16.5	17	17	16	20
0.5	18.15	18.6	18.85	18.1	18.41	0.5	13.5	14	15	15	15
0.75	5.81	5.71	6.03	6.85	6.2	0.75	3	3	3	4	3
1	0	0.01	0.05	0.04	0.06	1	0	0	0	0	0

Sample mean						Sample first quartile					
$w = 0.75$			ϕ			$w = 0.75$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	34.2	50	50	50	50	0.001	26	50	50	50	50
0.1	32.87	32.76	36.73	32.91	32.8	0.1	22	24	27	23	23
0.25	31.55	33.84	31.24	33.93	32.78	0.25	20.5	25.5	23	25	23
0.5	28.55	29.47	29.53	30.66	27.92	0.5	21.5	26	23	25	22
0.75	19.22	18.73	19.39	20.05	19.72	0.75	16	15	15	16	16
1	1.39	1.91	2.05	2.3	2.58	1	0	0	0	1	1

Sample mean						Sample first quartile					
$w = 0.95$			ϕ			$w = 0.95$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	44.76	49	48.5	48.5	50	0.001	42	50	50	50	50
0.1	43.74	44.24	44.41	43.17	42.82	0.1	41	41	42	41	42
0.25	43.82	43.32	43.06	43.9	42.99	0.25	41	40	40.5	41	40
0.5	41.2	41.45	41.14	41.08	41.44	0.5	38	38	38	38	39
0.75	37.51	38.45	38.65	37.99	37.28	0.75	34	35	36	35	33.5
1	27.07	27.85	28.55	28.38	28.44	1	24	25	25	25	25

Sample mean						Sample first quartile					
$w = 1.05$			ϕ			$w = 1.05$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	48.78	47.5	47.5	46.5	47	0.001	48	50	50	50	50
0.1	48.51	49.27	49.99	50	50	0.1	48	49	50	50	50
0.25	48.25	48.33	48.06	48.6	49.39	0.25	47	47	47	48	49
0.5	47.36	47.51	47.35	47.74	47.4	0.5	46	46	46	46.5	46
0.75	45.32	45.43	45.68	44.75	45.39	0.75	43	44	43.5	43	43.5
1	42.52	42.03	42.1	42.02	41.48	1	41	39	40	39.5	38

Sample mean						Sample first quartile					
$w = 1.25$			ϕ			$w = 1.25$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	50	49	48.5	48	49	0.001	50	50	50	50	50
0.1	50	50	50	50	50	0.1	50	50	50	50	50
0.25	50	50	50	50	50	0.25	50	50	50	50	50
0.5	50	50	50	50	50	0.5	50	50	50	50	50
0.75	50	50	50	50	50	0.75	50	50	50	50	50
1	49.17	49.31	49.21	49.49	49.41	1	49	49	49	49	49

Sample mean						Sample first quartile					
$w = 1.5$			ϕ			$w = 1.5$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	50	45.5	45	44.5	46.5	0.001	50	50	50	50	50
0.1	50	50	50	50	50	0.1	50	50	50	50	50
0.25	50	50	50	50	50	0.25	50	50	50	50	50
0.5	50	50	50	50	50	0.5	50	50	50	50	50
0.75	50	50	50	50	50	0.75	50	50	50	50	50
1	50	50	50	50	50	1	50	50	50	50	50

Table 5: Summary of results with $\mu_A = 0.5$ and varying weights from simulation for number of AB customers after all 100 trials

Sample mean						Sample first quartile					
$w = 0.5$			ϕ			$w = 0.5$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	27.69	42.22	42.68	41.79	40.89	0.001	20.5	41	42	41	40
0.1	27.46	27.17	27.37	26.24	26.01	0.1	23	23	25.5	24	24
0.25	23.04	22.21	22.2	22.13	21.07	0.25	18	17	18	18	18
0.5	11.2	11.6	12.43	12.4	11.46	0.5	7	8	7	10	8
0.75	0.67	0.45	0.49	0.76	0.54	0.75	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0	0
Sample mean						Sample first quartile					
$w = 0.75$			ϕ			$w = 0.75$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	37.25	50	50	50	50	0.001	32.5	50	50	50	50
0.1	36.03	36.8	36.83	36.26	36.68	0.1	34	34.5	34	35	35
0.25	33.21	33.01	33.67	33.26	33.89	0.25	31	30	31.5	31	31.5
0.5	25.57	26.21	26.78	26.68	27.28	0.5	22	23	23	24	24
0.75	12.59	12.91	14.14	14.33	14.22	0.75	9	9	11	11	11
1	0	0	0	0.01	0.01	1	0	0	0	0	0
Sample mean						Sample first quartile					
$w = 0.95$			ϕ			$w = 0.95$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	46.03	50	50	50	50	0.001	44	50	50	50	50
0.1	45.95	46.36	45.49	46.29	47.76	0.1	44	45	45	46	47
0.25	45.63	45.69	45.61	45.88	45.38	0.25	44	44	45	45	44
0.5	44.83	44.79	45.03	45.05	44.62	0.5	43.5	44	44	44	43
0.75	42.17	42.44	42.6	42.74	42.74	0.75	41	41	41	41	41
1	34.74	35.04	35.88	36.26	36.78	1	33	33	34	34	35
Sample mean						Sample first quartile					
$w = 1.05$			ϕ			$w = 1.05$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	50	50	50	50	50	0.001	50	50	50	50	50
0.1	50	50	50	50	50	0.1	50	50	50	50	50
0.25	50	50	50	50	50	0.25	50	50	50	50	50
0.5	50	50	50	50	50	0.5	50	50	50	50	50
0.75	50	50	50	50	50	0.75	50	50	50	50	50
1	50	49.99	50	50	50	1	50	50	50	50	50
Sample mean						Sample first quartile					
$w = 1.25$			ϕ			$w = 1.25$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	50	50	50	50	50	0.001	50	50	50	50	50
0.1	50	50	50	50	50	0.1	50	50	50	50	50
0.25	50	50	50	50	50	0.25	50	50	50	50	50
0.5	50	50	50	50	50	0.5	50	50	50	50	50
0.75	50	50	50	50	50	0.75	50	50	50	50	50
1	50	50	50	50	50	1	50	50	50	50	50
Sample mean						Sample first quartile					
$w = 1.5$			ϕ			$w = 1.5$			ϕ		
γ	0	0.25	0.5	0.75	1	γ	0	0.25	0.5	0.75	1
0.001	50	50	50	50	50	0.001	50	50	50	50	50
0.1	50	50	50	50	50	0.1	50	50	50	50	50
0.25	50	50	50	50	50	0.25	50	50	50	50	50
0.5	50	50	50	50	50	0.5	50	50	50	50	50
0.75	50	50	50	50	50	0.75	50	50	50	50	50
1	50	50	50	50	50	1	50	50	50	50	50

Table 6: Summary of the results with $\phi = 0$ and other varying parameters for number of AB customers in the first and last group of 10 customers after all 100 trials

	Sample mean first 10 customers						Sample mean last 10 customers						
$\mu_A = 0.75$	γ						$\mu_A = 0.75$	γ					
w	0.001	0.1	0.25	0.5	0.75	1	w	0.001	0.1	0.25	0.5	0.75	1
0.5	7.74	5.96	5.06	2.8	0.06	0	0.5	4.65	5.4	3.64	4.14	2.05	0
0.75	10	8.63	7.48	6.1	2.5	0.02	0.75	5.32	5.04	5.04	4.19	3.72	1.56
0.95	10	10	10	9.9	9.05	5.19	0.95	4.93	5.45	5.36	5.18	5.42	5.02
1	10	10	10	10	9.8	8.29	1	6.84	6.05	5.89	6.06	6.7	5.47
1.05	10	10	10	10	10	9.73	1.05	8.7	8.69	8.4	7.32	6.76	6.54
1.25	10	10	10	10	10	10	1.25	10	10	10	10	10	9.17
1.5	10	10	10	10	10	10	1.5	10	10	10	10	10	10
$\mu_A = 0.5$	γ						$\mu_A = 0.5$	γ					
w	0.001	0.1	0.25	0.5	0.75	1	w	0.001	0.1	0.25	0.5	0.75	1
0.5	10	7.56	5.26	1.59	0	0	0.5	4.66	4.59	2.76	2.1	0.4	0
0.75	10	10	9.04	6.55	1.73	0	0.75	4.97	3.83	3.88	3.54	1.91	0
0.95	10	10	10	10	10	8.74	0.95	6.17	6.04	5.9	5.9	5.13	4.38
1	10	10	10	10	10	10	1	8.65	8.76	8.44	7.93	7.78	7.5
1.05	10	10	10	10	10	10	1.05	10	10	10	10	10	9.98
1.25	10	10	10	10	10	10	1.25	10	10	10	10	10	10
1.5	10	10	10	10	10	10	1.5	10	10	10	10	10	10

7 Conclusion

We simulate an open routing service network similar to Arlotto et al. (2019) where all customers choose their own route and adapt their strategy depending on which route they expect to be faster. We extend on this network with a weighing factor for the waiting disutility in the second line, i.e., the second waiting line has a different weight than the first waiting line. We note the analysis by Arlotto et al. (2019) that states there is a dominant strategy in a two-station service network where all customers are in the system as it starts, and try to see via simulation whether this remains the (only) dominant strategy in situations where the system has customers arriving at different times. We observe that customers prefer herding at this strategy under these circumstances, when the network holds to certain restrictions, but as the weighing factor for the second line decreases, the number of times the dominant strategy gets chosen decreases, and when the weighing factor decreases far enough, the other strategy gets preferred and the customers will herd to that strategy. We observe that increasing the weighing factor for the second line increases the number of times the customers choose the dominant strategy. Furthermore, we observe that the influence of the weighing factor is prevalent for customers who arrive early and customers who arrive later in the system.

For this research, we added a weighing factor that is constant for all customers. Further research could look at weighing factors that vary between customers (one customer might prefer waiting in the first line more than the other customer). Analyzing other optimization goals, like minimizing the maximum waiting time in one line could also be a way to expand on this thesis. Finally, expanding this theory of weighing factors to other service networks, like networks with more than two servers, could be of interest in future work.

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A Code for Service Network Simulation

```
function [av_AB_customers, fp_AB_customers] = Openroutingextension
tic;
gammas(1) = 0.001;
gammas(2) = 0.1;
gammas(3) = 0.25;
gammas(4) = 0.5;
gammas(5) = 0.75;
gammas(6) = 1;
phis(1) = 0;
phis(2) = 0.25;
phis(3) = 0.5;
phis(4) = 0.75;
phis(5) = 1;
mus(1) = 0.25;
mus(2) = 0.5;
mus(3) = 0.75;
weights(1) = 0.5;
weights(2) = 0.75;
weights(3) = 0.95;
weights(4) = 1;
weights(5) = 1.05;
weights(6) = 1.25;
weights(7) = 1.5;
trials = 100;
mu_B = 1;
N = 50;
rounds = 250;
for c_gam = 1:6
    gam = gammas(c_gam);
    for c_phi = 1:5
        phi = phis(c_phi);
        for c_mu = 1:3
            mu_A = mus(c_mu);
            for c_w = 1:7
                w = weights(c_w);
                for trial = 1:trials
                    route_cust = randi(2,[1,N]); %Route 1 = AB; Route 2 = BA
                    total_time_AB = zeros(1,50);
                    total_time_BA = zeros(1,50);
                    for ronde = 1:rounds
                        rando = rand([1,N]);
                        for i=1:N
                            arr(i) = i*gam + phi*(2*rando(i) - 1); %Arrival time
                                customer i
                        end
                        serve_time_A = exprnd(1/mu_A,[1,N]);
                        serve_time_B = exprnd(1/mu_B,[1,N]);
                        for z=1:N+1
```

```
beginA = zeros(1,N);
beginB = zeros(1,N);
for i=1:N
    eindeA(i) = -100000000; %not yet served at A
    eindeB(i) = -100000000; %not yet served at B
    if route_cust(i) == 1
        arrA(i) = arr(i);
        arrB(i) = 100000000; %arrival time of customer i at B still unknown
    else
        arrA(i) = 100000000; %arrival time customer i at A still unknown
        arrB(i) = arr(i);
    end
end
finished = 0; %number of customers that finished at both stations
while finished < N %while there are still customers in the system
    next_event = arrA(1);
    cust = 1;
    A = 1;
    for i=1:N
        for j=1:2
            if (j == 1)
                if (arrA(i) < next_event)
                    next_event = arrA(i);
                    cust = i; %next event is by customer i
                    A = 1; %next event is at station A
                end
            else
                if (arrB(i) < next_event)
                    next_event = arrB(i);
                    cust = i; %next event is by customer i
                    A = 2; %%next event is at station B
                end
            end
        end
    end

    if (A == 1) %customer arrives at station A
        last_service_A = eindeA(1); %initial number
        for i = 2:N
            if (eindeA(i) > last_service_A)
                last_service_A = eindeA(i); %last helped customer at A after new arrival
            end
```

```
                end
                if (last_service_A > next_event) %if last customer still being served with new arrival
                    beginA(cust) = last_service_A; %new customer waits until line is empty
                else
                    beginA(cust) = next_event; %row is already empty
                end

                eindeA(cust) = beginA(cust) + serve_time_A(cust);
                if (arrB(cust) > 99999999) %not already served at B
                    arrB(cust) = eindeA(cust);
                elseif (z==1) %already served at B
                    total_time(cust) = eindeA(cust) - beginA(cust) + w*(beginA(cust) - eindeB(cust)) + eindeB(cust) - arr(cust);
                    total_time_BA(cust) = total_time_BA(cust) + total_time(cust); %accumulative waiting time for this route after all rounds so far
                    finished = finished + 1;
                else %z>=2
                    if cust == z-1 %the one taking the alternative route
                        alt_total_time(cust) = eindeA(cust) - beginA(cust) + w*(beginA(cust) - eindeB(cust)) + eindeB(cust) - arr(cust);
                        total_time_BA(cust) = total_time_BA(cust) + alt_total_time(cust); %accumulative waiting time for this route after all rounds so far
                    end
                    finished = finished + 1;
                end
                arrA(cust) = 99999998; %no service at station A anymore
            else %customer arrives at station B
                last_service_B = eindeB(1);
                for i = 2:N
                    if (eindeB(i) > last_service_B)
                        last_service_B = eindeB(i); %last helped customer at A after new arrival
                    end
                end
```

```
                if (last_service_B > next_event)
                    beginB(cust) = last_service_B; %new customer waits until line is empty
                    beginB(cust) = next_event; %row is already empty
                end

                eindeB(cust) = beginB(cust) + serve_time_B(cust);
                if (arrA(cust) > 99999999) %not already served at A
                    arrA(cust) = eindeB(cust);
                elseif (z==1) %already served at A
                    total_time(cust) = eindeB(cust) -beginB(cust) + w*(beginB(cust) - eindeA(cust)) + eindeA(cust) - arr(cust);
                    total_time_AB(cust) = total_time_AB(cust) + total_time(cust);
                    finished = finished + 1;
                else %z>=2
                    if cust == z-1
                        alt_total_time(cust) = eindeB(cust) - beginB(cust) + w*(beginB(cust) - eindeA(cust)) + eindeA(cust) - arr(cust);
                        total_time_AB(cust) = total_time_AB(cust) + alt_total_time(cust);
                    end
                    finished = finished + 1;
                end
                arrB(cust) = 99999998; %no service at station B anymore
            end
            if (finished == N)
                if (z<N+1) %alt time
                    route_cust(z) = 3 - route_cust(z);
                    if (z > 1)
                        route_cust(z-1) = 3 - route_cust(z-1); %reset previous route
                    end
                else %z=N+1
                    route_cust(z-1) = 3 - route_cust(z-1); %reset final route
                end
            end
        end
    end


    %now checking if alternate route was better
```

```
                        for x = 1:N
                            if ((total_time_AB(x) < total_time_BA(x) &&
                               route_cust(x) == 2) || (total_time_BA(x) <
                               total_time_AB(x) && route_cust(x) == 1))
                                route_cust(x) = 3 - route_cust(x);
                            end
                        end

                    end
                    AB_customers = 2 - route_cust; %%1 if AB, 0 if BA
                    routes(c_gam,c_phi,c_mu,c_w,trial,:) = AB_customers;
                    H(trial) = sum(AB_customers,'all');
                end
                Histogrs(c_gam,c_phi,c_mu,c_w,:) = H;
                av_AB_customers(c_gam,c_phi,c_mu,c_w) = sum(H,'all')/trials;
                fp_AB_customers(c_gam,c_phi,c_mu,c_w) = prctile(H,25);
            end
        end
    end
end
time = toc;
```

B Code for Figure 1

```
function hist = histog(Histogrs)
%%Draws the histogram of a given set of values. The value Histogrs is taken
   from the results of Appendix A.
deze = zeros(51,7);
for frst = 1:51
    deze(frst, 1) = frst - 1;
    for trial = 1:100
        if Histogrs(1,1,1,4,trial) == frst - 1
            deze(frst, 2) = deze(frst, 2) + 1;
        end
        if Histogrs(1,1,2,4,trial) == frst - 1
            deze(frst, 3) = deze(frst, 3) + 1;
        end
         if Histogrs(1,1,3,4,trial) == frst - 1
             deze(frst, 4) = deze(frst, 4) + 1;
         end
        if Histogrs(1,4,1,4,trial) == frst - 1
            deze(frst, 5) = deze(frst, 5) + 1;
        end
        if Histogrs(1,4,2,4,trial) == frst - 1
            deze(frst, 6) = deze(frst, 6) + 1;
        end
         if Histogrs(1,4,3,4,trial) == frst - 1
             deze(frst, 7) = deze(frst, 7) + 1;
         end
```

```
    end
end

Y1 = deze(:,2:4);
Y2 = deze(:,5:7);
X = deze(:,1);
ax1 = subplot(1,2,1);
plot(X,Y1(:,1),'d', 'MarkerFaceColor', [0,0,0.6], 'Color', 'k')
hold on
plot(X,Y1(:,2),'o', 'MarkerFaceColor', [0.6,0,0], 'Color', 'k')
hold on
plot(X,Y1(:,3),'s', 'MarkerFaceColor', [0,0.6,0], 'Color', 'k')
title(ax1,'i) \phi = 0; \gamma = 0.001')
ylabel(ax1,'Frequency')
xlabel(ax1, '#AB customers (out of 50)')
ylim([0 100])
legend(ax1, '\mu_A = 0.25', '\mu_A = 0.5', '\mu_A = 0.75', 'Location','
    northwest');
ax2 = subplot(1,2,2);
plot(X,Y2(:,1),'d', 'MarkerFaceColor', [0,0,0.6], 'Color', 'k')
hold on
plot(X,Y2(:,2),'o', 'MarkerFaceColor', [0.6,0,0], 'Color', 'k')
hold on
plot(X,Y2(:,3),'s', 'MarkerFaceColor', [0,0.6,0], 'Color', 'k')
title(ax2,'ii) \phi = 0.75; \gamma = 0.001')
ylabel(ax2,'Frequency')
xlabel(ax2, '#AB customers (out of 50)')
end
```

C Code for Calculating t-statistics

```
function tvals = ttests(Histogrs,av_AB_customers,arlottos)
%%Calculates the t-statistics shown in Table 3. The parameter arlottos
    represent the values in Table 2 and 3 of Arlotto et al. (2019).
for q = 1:5
    for r = 1:5
        for s = 1:2
            upp(q,r,s) = (av_AB_customers(q+1,r,s+1,4) - arlottos(q,r,s));
            devs(q,r,s) = std(Histogrs(q+1,r,s+1,4,:));
            tvals(q,r,s) = abs(upp(q,r,s)/(devs(q,r,s)*sqrt(2/50)));
        end
    end
end
devs
end
```