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# US Tail Risk and International Stock Return Predictability

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## **Abstract**

In this paper, we aim to investigate whether lagged U.S. tail risk can predict non-U.S. returns and whether lagged non-U.S. tail risk can predict U.S. returns. We measure the U.S. tail risk by constructing a portfolio that long the CBOE Put Protection Index (PPUT) and short the S&P 500 index. We find that lagged U.S. tail risk displays strong predictive ability for non-U.S. returns by means of predictive regression model, pairwise Granger causality test, adaptive elastic-net estimation, variable importance and out-of-sample forecast gains. Whereas, non-U.S. tail risk exhibits limited ability in predicting U.S. returns.

**Keywords:** tail risk; Granger causality; LASSO; variable importance

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# 1 Introduction

With the development of economic globalization, the interdependence of world economies has increased rapidly. U.S. is the world's largest goods importer and third largest exporter based on World FactBook of CIA (" Central Intelligence Agency ") in 2018. The connection between U.S. economy and the world has become closer and closer. According to the World Bank's global prospect report in January, 2019, during 2018, approximately 2.5 percent of global goods trade has been affected by new U.S. tariffs and trading countries response on global imports. Changing in the U.S. economy influences the world's economy at the mean time. [Wongswan \(2008\)](#) shows that the U.S. monetary policy has an essential impact on more than 10 foreign equity indexes in Europe, Asia and Latin America. Moreover, U.S. offers the largest equity market in the world. As the growing scale of cross-border trade in the equity market, U.S. equity market takes a leading role in pricing international assets. [Rapach et al. \(2013\)](#) show that lagged U.S. returns improves the returns predictability in 10 non-U.S. industrialized countries, whereas lagged non-U.S. returns hardly improve U.S. returns predictability. The strong link in equity markets between U.S. and non-U.S. countries may cause severe problems. The fluctuation in U.S equity market will simultaneously affect worldwide equity markets. [Kelly and Jiang \(2014\)](#) find that U.S. tail risk displays strong ability in predicting its own aggregate market returns. However, questions regarding the role of lagged tail risk in return predictability remain unknown for us.

In this paper, our aim is to investigate the role of tail risk in pricing the returns. More precisely, it is to figure out whether the lagged U.S. tail risk improves returns predictability in non-U.S. industrialized countries and whether lagged non-U.S. tail risks improve U.S. returns predictability.

We measure the U.S. tail risk by constructing a zero-investment portfolio. That is taking a long position in the CBOE Put Protection Index (PPUT) and a short position in the S&P 500 index. The log portfolio return is defined as the tail risk factor. It pays off when the S&P 500 prices decrease. The CBOE PPUT index tracks the performance of a hypothetical strategy that takes a long position to the SP 500 Index and a rolling position in monthly 5% Out-of-the-Money (OTM) SPX Put options. To measure non-U.S. tail risk, we replicate PPUT index for non-U.S. countries by following the methodology of CBOE, and then together with the country index to construct the tail risk portfolio. The advantage of our option-implied tail risk measurement with respect to other approaches is that it is easy to construct using publicly available data, such as option spot prices, strike prices, and the interpretation is intuitive, in particular, compared to [Kelly and Jiang \(2014\)](#), they use extreme value theory to estimate the tail risk. The disadvantage

of this approach is that it requires a large amount of stock returns by pooling different stocks in the same period with assuming that stock returns are i.i.d distributed. However, in reality, it is hard to believe that all stock returns in the cross-section share the same distribution, which contradicts the assumption.

Before studying the role of lagged tail risk, we re-investigate the role of lagged U.S. returns (Rapach et al. (2013)) using the data from July 1986 to November 2015. We conduct pairwise Granger causality test, that is testing whether one country returns can be predicted by the other country returns. Further, we combine it with least absolute shrinkage and selection operation ("LASSO") by Tibshirani and Robert (1996). For each country, we have its own country returns, interest rate, dividend yield and the other country returns in previous month as candidate variables. In an out-of-sample test, we compute the Campbell and Thompson (2008)  $R_{OS}^2$  statistics and Clark and West (2007) adjusted  $MSFE$  by computing the forecasting gains of the model which uses lagged U.S. returns as additional predictor, comparing with historical average forecasts. We find that lagged U.S. returns improve the predictability of non-U.S. returns, while non-U.S. returns cannot improve the predictability of U.S. returns, which agrees with the finding in Rapach et al. (2013).

Based on this finding, we examine the role of lagged U.S. tail risk in return predictability. First, for nine industrialized countries, the benchmark predictive regression model regresses the monthly excess return of the country on the interest rate, U.S. returns and dividend yield in previous month based on the finding in Ang and Bekaert (2007) who argues that both nominal interest rate and dividend yield are economically important predictors in an asset pricing model. Second, to further determine the role of lagged U.S. tail risk, we construct a general model that includes all country returns, U.S. tail risk, country dividend yield and interest rate, then use LASSO to select variables. We find that most of the countries select lagged U.S. tail risk as a significant predictor. Third, we implement random forests to obtain the variable importance, which gives the insight at non-parametric level. We find that for most of the countries, lagged U.S. tail risk belongs to the top 3 important variable. Fourth, we conduct out-of-sample tests. We compute the Campbell and Thompson (2008)  $R_{OS}^2$  statistics and Clark and West (2007) adjusted  $MSFE$  by computing the forecasting gains of the model which uses lagged U.S. tail risk comparing with historical average forecasts. We show that lagged tail risk of the U.S. exhibits great out-of-sample gains, especially during the 2000 dot-com bubble and 2008 Global Financial Crisis.

To investigate the predictive power of non-U.S. tail risk, we utilize the same approach as Rapach et al. (2013) and use the sample from February 2006 to November 2015 due to the

limited access to the non-U.S. tail risk factors. The main difference is that in the benchmark model we include lagged U.S. returns. And to continue exploring the role of lagged international tail risk, we study their pairwise lead-lag relationships using Granger causality tests. We use augmented predictive regressions that uses the lagged tail risk of a country and that of another country as additional predictors. According to the wild bootstrap  $p$ -values, we only find two of five countries outside the U.S. are significantly Granger caused by U.S. tail risk in the past month. In general, our results show that lagged U.S. tail risk plays a lead role on predicting non-U.S. countries returns, whereas lagged tail risk in non-U.S. countries hardly capture returns U.S. returns predictability. The predictive power of U.S. tail risk decreases when we use short sample period. However, the predictive ability is still stronger, compared to lagged U.S. returns.

This paper contributes to the literature in three aspects. First, this paper contributes to the literature on examining the predictive ability of lagged U.S. tail risk for non-U.S. industrialized countries returns. [Kelly and Jiang \(2014\)](#) provide the evidence of strong predictive power of U.S. tail risk on U.S. market returns. The role of U.S. tail risk with respect to non-U.S. returns remain unknown. In this paper, we shows that lagged U.S. tail risk improves non-U.S. returns' predictability, which is supported by predictive regression model, pairwise Granger causality test and machine learning techniques (adaptive elastic-net estimation by [Zou and Zhang \(2009\)](#), variable importance by [Breiman \(2002\)](#)). Second, this paper contributes to the literature on constructing tail risk factor. [Kelly and Jiang \(2014\)](#) construct the conditional tail risk from cross-sectional stocks in spirit of extreme value theory. Our paper differs from this approach. We construct option-implied tail risk factor from a zero-investment portfolio. Third, this paper contributes to the literature on power of lagged U.S. returns on returns' predictability of non-U.S. countries. [Rapach et al. \(2013\)](#) show that lagged U.S. return is a significant predictor for returns in non U.S. industrialized countries. Our paper differs from this paper in two aspects. One is that we use the longer period of the dataset. Second is that we utilize a machine learning approach, namely variable importance, to further investigate the lagged U.S. returns' predictive power.

The rest of the paper proceeds as follows. Section 2 introduces the methodology for investigating the role of lagged U.S. returns in returns predictability. Section 3 describes the methodology for examining the predictive power of lagged tail risk and constructing the tail risk factor. Section 4 describes the data used in the paper. Section 5 discusses the results. Section 6 concludes the paper.

## 2 Investigating Predictive Ability of Lagged U.S. Returns

Rapach et al. (2013) show the strong predict power of lagged U.S. returns on returns in many non-US industrialized countries. In this section, we re-investigate the role of lagged U.S. returns in predicting non-U.S. returns and introduce the approaches in Rapach et al. (2013).

### 2.1 Benchmark Predictive Regression

At first, we build up a benchmark predictive model to determine the predictability of excess returns. Following Ang and Bekaert (2007), we consider a predictive regression model that regresses monthly excess returns on a lagged normal interest rate and log dividend yield:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \epsilon_{i,t+1}, \quad i = 1, \dots, N, \quad (1)$$

where  $r_{i,t+1}$  stands for the country  $i$ 's monthly return in excess of the nominal interest rate at month end  $t + 1$  and are measured in national currency.  $Bill_{i,t}$  represents the country nominal interest rate, which is measured by 3-month Treasury bill rate.  $Yield_{i,t}$  is the natural logarithm of dividend yield.  $\epsilon_{i,t+1}$  represents a disturbance term that follows the standard normal distribution

### 2.2 Predictive Ability of Lagged Returns

In this section, we examine the predictive ability of lagged international returns by means of two aspects, namely pairwise comparison and general specification.

#### 2.2.1 Pairwise Granger Causality Tests

We follow Rapach et al. (2013) and conduct the augmented prediction regression is shown below:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,i}r_{i,t} + \beta_{i,j}r_{j,t} + \epsilon_{i,t+1}, \quad i \neq j. \quad (2)$$

We extend the benchmark model by including  $r_{i,t}$  and  $r_{j,t}$  as regressors. By doing so, we can test the hypothesis that lagged returns in country  $j$  can predict the lagged returns in country  $i$ . If we accept the hypothesis, that means that country  $j$  returns Granger cause country  $i$  returns.

#### 2.2.2 General Model Specification

We consider general model specification approaches instead of pairwise comparison. For each country, we have an augmented VAR(1) model as shown below:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,i}r_{i,t} + \sum_{j \neq i} \beta_{i,j}r_{j,t} + \epsilon_{i,t+1}, \quad i = 1, \dots, N. \quad (3)$$

We include all international monthly return factors. It may reduce the reliability of statistical outcomes, since it may result in multicollinearity by a plethora of regresses. Therefore, we consider two approaches for improving the accuracy of parameter estimates and tests, namely the pooled model specification and machine learning methods. More details are in Section 2.2.2.1 and 2.2.2.2

### 2.2.2.1 Pooled Model Specification

The first approach is on the basis of [Ang and Bekaert \(2007\)](#) and [Hjalmarsson \(2010\)](#). We restrict parameters to incorporate homogeneity by imposing the restrictions:  $\beta_{i,i} = \bar{\beta}_{AR}$ ,  $\beta_{i,j} = \bar{\beta}_j$ ,  $\beta_{i,b} = \bar{\beta}_b$  and  $\beta_{i,d} = \bar{\beta}_d$  for  $i = 1, \dots, N$ . Such restriction scarifies the estimation biasness, but improves the efficiency, since it helps to reduce the mean squared error. The model takes the following form:

$$r_{i,t+1} = \beta_{i,0} + \bar{\beta}_b Bill_{i,t} + \bar{\beta}_d Yield_{i,t} + \bar{\beta}_{AR} r_{i,t} + \sum_{j \neq i} \bar{\beta}_j r_{j,t} + \epsilon_{i,t+1}, \quad i = 1, \dots, N. \quad (4)$$

### 2.2.2.2 LASSO

The second approach is a seminal machine learning method, namely least absolute shrinkage and selection operation ("LASSO") created by [Tibshirani and Robert \(1996\)](#). It absorbs the good feature of subset selection and ridge regression by means of performing parameter shrinkage and variable selection at the same time. Therefore, it provides more interpredictable and stable models. The multiple predictive regression model for country  $i$  can be expressed as

$$r_{i,t+1} = x_t' \beta_i + \epsilon_{i,t+1}, \quad i = 1, \dots, N. \quad (5)$$

where  $x_t$  represents a  $K \times 1$  predictors vector and  $\beta_i = (\beta_{i,1}, \dots, \beta_{i,k})'$  is a  $K \times 1$  parameters vector from equation (3).

The [Tibshirani and Robert \(1996\)](#) LASSO estimator is defined by solving the  $\ell_1$  penalized least squares problem

$$\min_{\beta_i^{LASSO}} \left[ \sum_{t=1}^{T-1} (r_{i,t+1} - x_t' \beta_i)^2 + \lambda_1 \sum_{k=1}^K |\beta_{i,k}| \right], \quad (6)$$

where  $\lambda_1$  is the parameter corresponding to  $\ell_1$  penalty terms that shrinks some parameters to zero. However, the performance of LASSO estimator is not stable and less informative with strong correlated predictors, since it tends to randomly choose one predictor out of a set of highly related predictors. [Zou and Zhang \(2009\)](#) propose an improved version of the LASSO, namely the adaptive elastic-net estimator and define as follows:

$$\min_{\beta_i^{enet}} \left[ \sum_{t=1}^{T-1} (r_{i,t+1} - x_t' \beta_i)^2 + \lambda_1 \sum_{k=1}^K \omega_k |\beta_{i,k}| + \lambda_2 \sum_{k=1}^K \beta_{i,k}^2 \right], \quad (7)$$



where  $\lambda_1$  and  $\lambda_2$  are parameters corresponding to  $\ell_1$  and  $\ell_2$  penalty terms, respectively.  $\omega = (\omega_1, \dots, \omega_k)'$  is a  $K \times 1$  vector is the adaptive data-driven weights of parameters and one can define  $\omega_k = |\hat{\beta}_{i,k}|^{-\gamma}$  in  $\ell_1$  penalty where  $\gamma$  is positive and  $\hat{\beta}_{i,k}$  is the OLS parameter estimate in equation (5).

Following [Rapach et al. \(2013\)](#), we use [Friedman et al. \(2010\)](#) algorithm to solve the minimization problem in equation (7) and use five-fold cross-validation to select  $\lambda_1$ ,  $\lambda_2$  and  $\gamma$ .

### 2.3 Out-Of-Sample Tests

[Goyal and Welch \(2008\)](#) show that economic variables usually fails to beat the native historical average forecast when forecasting excess returns, although these variables have significant in-sample predictability. Therefore, instead of using relatively weak in-sample tests, we determine whether the out-of-sample performance of the models that based on U.S. returns in the past month to forecast country excess returns is better than historical average forecasts.

The historical average baseline forecasts take the form

$$r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1}, \quad (8)$$

which is identical to a baseline model without predictability. The forecast of country  $i$  excess return in month  $t + 1$  equals to the averaging of the excess return in country  $i$  over the sample period (till month  $t$ ). In comparison, the competitive model includes U.S. returns in the past month:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1}, \quad (9)$$

To forecast the country  $i$  excess return in month  $t + 1$ , we first apply OLS using the data in month  $t$ . Afterwards, we plug  $r_{US,t}$  in equation (9), as shown below:

$$\hat{r}_{i,t+1} = \hat{\beta}_{i,0} + \hat{\beta}_{i,US}r_{US,t}, \quad (10)$$

where  $\hat{\beta}_{i,0}$  and  $\hat{\beta}_{i,US}$  are the OLS estimates of  $\beta_{i,0}$  and  $\beta_{i,US}$ .

### 2.4 Out-of-Sample Evaluation

To compare the forecasting performance of baseline model and the competitive model, we measure the proportion of reduction in mean-squared forecast error (*MSFE*) for the competing model relative to the baseline forecasts, which are labeled as out-of-sample  $R^2$  statistics,  $R_{OS}^2$  ([Campbell and Thompson \(2008\)](#)),

$$R_{OS}^2 = 1 - \frac{\sum_{t=1}^T (r_t - \hat{r}_t)^2}{\sum_{t=1}^T (r_t - \bar{r}_t)^2}, \quad (11)$$

where  $\hat{r}_t$  is the prediction in competing model and  $\bar{r}_t$  is the historical average return. Positive  $R_{OS}^2$  means that competing model generates a lower *MSFE*.

In addition, we also employ the [Clark and West \(2007\)](#) adjusted MSFE, which tests the null hypothesis of equal *MSFE* ( $R_{OS}^2 = 0$ ) against the alternative hypothesis that the *MSFE* of the predictive regression model is lower than the baseline model ( $R_{OS}^2 > 0$ ).

## 2.5 Wild Bootstrap Procedure

The evaluation of the model is based on computing the wild bootstrap *p*-value and the confidence interval for test statistics, which follows [Rapach et al. \(2013\)](#). Each bootstrap procedure generates the pseudo-sample of country returns and preserves the pattern of conditional heteroskedasticity in error terms. We generate predictive regression and historical average forecasts, respectively. For each pseudo-sample, we obtain the slope estimates and corresponding *t*-statistics as well as  $\chi^2$ -statistics for  $R^2$  estimates. Repeating the procedure 2000 times, we obtain the empirical distribution for *t*-statistics and  $\chi^2$ -statistics, respectively. For example, in benchmark predictive model (equation (1)), the empirical *p*-value of  $\hat{\beta}_{i,b}$  is computed as the proportion of bootstrapped *t*-statistics smaller than the *t*-statistic for the original sample under the null hypothesis of zero  $\hat{\beta}_{i,b}$  against the alternative hypothesis of negative  $\hat{\beta}_{i,b}$ . For out-of-sample tests, in each bootstrap process, we store the largest *MSFE* – *adjusted* statistics for each country. Therefore, we have an empirical distribution of maximum *MSFE* – *adjusted* statistics with 1% bootstrapped critical value of 2.49. For pooled general model, we compute fixed design wild bootstrap [Gouçaves and Lutz \(2004\)](#) and [Clark and McCracken \(2012\)](#). See [Rapach et al. \(2013\)](#) for more details.

## 3 Investigating Predictive Ability of Lagged U.S. Tail Risk

In this section, we investigate the role of lagged tail risk in predicting international stock returns. We incorporate the information in lagged U.S. returns in benchmark model based on finding in [Rapach et al. \(2013\)](#).

### 3.1 Benchmark Predictive Regression

We add lagged U.S. returns in the benchmark predictive model (equation (1)). The benchmark model becomes

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1}, \quad i \neq US, \quad (12)$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \epsilon_{i,t+1}, \quad i = US, \quad (13)$$

where for non-U.S. countries we include lags of its own returns  $r_{i,t}$  and U.S. returns  $r_{US,t}$ , while for the U.S., only its own lagged return  $r_{i,t}$  is included.

### 3.2 Predictive Ability of Lagged Tail Risk

This section introduces approaches used in investigating the role of the lagged tail risk. We apply pairwise Granger causality tests introduced in Section 3.2.1 when non-U.S. tail risk factors are available. In case that only the U.S. tail risk is available, we implement regression in Section 3.2.2. For general model specification (Section 3.2.3), we also distinguish two situations.

#### 3.2.1 Pairwise Granger Causality Tests

We conduct the augmented prediction regressions for  $i \neq j$  and  $i \neq US$  are shown below:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \alpha_{i,i}Tail_{i,t} + \alpha_{i,j}Tail_{j,t} + \epsilon_{i,t+1}, \quad (14)$$

for  $i = US$  and  $j \neq US$ ,

$$r_{US,t+1} = \beta_{US,0} + \beta_{US,b}Bill_{US,t} + \beta_{US,d}Yield_{US,t} + \beta_{US,r}r_{US,t} + \alpha_{US,US}Tail_{US,t} + \alpha_{US,j}Tail_{j,t} + \epsilon_{US,t+1}, \quad (15)$$

where we extend the benchmark model by including  $Tail_{i,t}$  and  $Tail_{j,t}$  as regressors. By doing so, we can analyze the predictability of lagged tail risk of country  $j$  relating to the lagged tail risk of country  $i$ . That is examining whether the country  $j$  tail risk Granger causes country  $i$  returns.

#### 3.2.2 Predictive Regression Model

To examine the predictive ability of lagged U.S. tail risk, we construct the predictive regression model by taking U.S. tail risk factor into account:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1}, \quad i \neq US, \quad (16)$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1}, \quad i = US. \quad (17)$$

#### 3.2.3 General Model Specification

When the international tail risk factors are available, we define the general model by extending the equation (12) in order to examine the predict ability of tail risk lags. The model is defined

as

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \alpha_{i,i}Tail_{i,t} + \sum_{j \neq i} \alpha_{i,j}Tail_{j,t} + \epsilon_{i,t+1}, \quad i = 1, \dots, N-1 \quad \text{and} \quad i \neq US, \quad (18)$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \alpha_{i,i}Tail_{i,t} + \sum_{j \neq i} \alpha_{i,j}Tail_{j,t} + \epsilon_{i,t+1}, \quad i = US. \quad (19)$$

Equation (18) and (19) are the augmented VAR(1) models. We include all international tail risk factors.

When only the U.S. tail risk factor is available, the general model is defined as: for  $i = 1, \dots, N$ ,

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,i}r_{i,t} + \sum_{j \neq i} \beta_{i,j}r_{j,t} + \alpha_{i,US}Tail_{US,t} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \epsilon_{i,t+1}, \quad (20)$$

where we include all international lagged returns and U.S. tail risk. It may reduce the reliability of statistical outcomes, since it may results in multicollinearity by a plethora of regresses. To improve the accuracy of parameter estimates and tests, we extend the pooled model specification and machine learning methods.

### 3.2.3.1 Pooled Model Specification

We restrict parameters in equation (18) and (19) by following [Ang and Bekaert \(2007\)](#) and [Hjalmarsson \(2010\)](#). It incorporates homogeneity by imposing the restrictions:  $\alpha_{i,i} = \bar{\alpha}_{AR}$ ,  $\alpha_{i,j} = \bar{\alpha}_j$ ,  $\beta_{i,b} = \bar{\beta}_b$ ,  $\beta_{i,d} = \bar{\beta}_d$  and  $\beta_{i,r} = \bar{\beta}_r$  and  $\beta_{i,US} = \bar{\beta}_{US}$ . Such restriction scarifies the estimation biasness, but improves the efficiency, since it helps to reduce the mean squared error. Equations become:

$$r_{i,t+1} = \beta_{i,0} + \bar{\beta}_b Bill_{i,t} + \bar{\beta}_d Yield_{i,t} + \bar{\beta}_r r_{i,t} + \bar{\beta}_{US} r_{US,t} + \bar{\alpha}_{AR} Tail_{i,t} + \sum_{j \neq i} \bar{\alpha}_j Tail_{j,t} + \epsilon_{i,t+1}, \quad i = 1, \dots, N-1 \quad \text{and} \quad i \neq US, \quad (21)$$

$$r_{i,t+1} = \beta_{i,0} + \bar{\beta}_b Bill_{i,t} + \bar{\beta}_d Yield_{i,t} + \bar{\beta}_r r_{i,t} + \bar{\alpha}_{AR} Tail_{i,t} + \sum_{j \neq i} \bar{\alpha}_j Tail_{j,t} + \epsilon_{i,t+1}, \quad i = US, \quad (22)$$

Similarly, by posing the restrictions on parameters, equation (20) becomes:

$$r_{i,t+1} = \beta_{i,0} + \bar{\beta}_{AR} r_{i,t} + \sum_{j \neq i} \bar{\beta}_j r_{j,t} + \bar{\alpha}_{US} Tail_{US,t} + \bar{\beta}_b Bill_{i,t} + \bar{\beta}_d Yield_{i,t} + \epsilon_{i,t+1}, \quad (23)$$

### 3.2.3.2 LASSO

The second approach is an application of least absolute shrinkage and selection operation ("LASSO") created by [Tibshirani and Robert \(1996\)](#). More details about estimation procedure can be found in Section 2.2.2.2.

### 3.2.3.3 Variable Importance - Random Forests

So far, methods are on the basis of parametric model. In this section, we implement a non-parametric model to give more insight into the predictive ability of tail risk factor. Breiman (2001) defines the random forests, which consist of a large amount of trees. Groemping (2009) summarizes two key features of random forests: (i) a random subset of the observations is used for constructing an individual tree. (ii) a random subset of variables is used in creating the split within individual tree. Such randomness makes sure the individual trees' diversity. The prediction of the forest equals to the average over all individual trees' predictions. Note that on average 36.8% of observations are not used for any construction of trees, call "out of the bag" (OOB) for the tree.

Breiman (2002) introduces a well-know variable importance metric in random forests, called permutation-based "MSE reduction", which has been widely used in various researches (Groemping (2009); Ishwaran (2007); Strobl et al. (2008)). It is defined as the difference between the baseline model and competing model. In our case, the baseline model is designed as follows: the OOB mean squared error for individual tree  $k$  is equal to the mean of the squared differences between OOB values and the corresponding predictions.:

$$OOB_{MSE_k} = \frac{1}{n_{OOB,k}} \sum_{i=1:i \in OOB_k}^n (r_i - \hat{r}_{i,k})^2, \quad (24)$$

where  $n_{OOB,k}$  is the number of OOB observations in tree  $k$ ,  $r_i$  is the actual value of return  $i$  and  $\hat{r}_{i,k}$  is the corresponding prediction in tree  $k$ .

The competing model is computing the OOB-MSE after permuting the column values of one predictor  $X_j$ . If the predictor  $X_j$  has little predictive power for  $r_i$ , permuting the column values of  $X_j$  in OOB samples makes rarely difference in OOB mean squared error. The competing model is shown as follows:

$$OOB_{MSE_k(X_{j,permutated})} = \frac{1}{n_{OOB,k}} \sum_{i=1:i \in OOB_k}^n (r_i - \hat{r}_{i,k}(X_{j,permutated}))^2, \quad (25)$$

where  $X_j$  could be lagged returns, interest rate and dividend yield of country  $i$  as well as lagged returns in other countries. The difference for predictor  $X_j$  in tree  $k$  is calculated as

$$difference_{j,k} = OOB_{MSE_k(X_{j,permutated})} - OOB_{MSE_k}, \quad (26)$$

the difference is equal to 0 if  $X_j$  does not join any split of tree  $k$ . The average of these difference over all trees is defined as the permutation-based "MSE reduction" for regressor  $X_j$  for the forests.

### 3.3 Out-Of-Sample Tests

The out-of-sample tests are very similar to tests in Section 2.3. The baseline model remains the same. The only difference is that We use lagged U.S. tail risk instead of lagged U.S. returns in the competing model.

The baseline forecasts can be expressed as

$$r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1}, \quad (27)$$

which is identical to a baseline model without predictability. As comparison, we include the tail risk lags of U.S. in the model:

$$r_{i,t+1} = \beta_{i,0} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1}, \quad (28)$$

To forecast the country  $i$  excess return in month  $t + 1$ , we first apply OLS using the data over month  $t$ , then fill  $Tail_{US,t}$  in equation (29), as shown below:

$$\hat{r}_{i,t+1} = \hat{\beta}_{i,0} + \hat{\alpha}_{i,US}Tail_{US,t}, \quad (29)$$

where  $\hat{\beta}_{i,0}$  and  $\hat{\alpha}_{i,US}$  are the OLS estimates of  $\beta_{i,0}$  and  $\alpha_{i,US}$ .

### 3.4 Constructing Tail Risk Factor

To construct tail risk factor, we require historical prices of country index and the Put Protection Index (PPUT) from Chicago Board Options Exchange (CBOE) for the U.S.. PPUT is a benchmark index, which tracks the performance of hypothetical risk management strategy that takes a long position to the S&P 500 Index and buys a monthly 5% Out-of-the-Money (OTM) SPX Put option. Following the methodology of CBOE, we construct PPUT index for the rest of countries.

The index is designed as purchasing a unit of the SPX Index and a unit of a 5% OTM monthly SPX Put option simultaneously. CBOE selects the first available strike below 95% of the last disseminated value of the SPX Index before 11:00 am ET to be the strike of the SPX Put option. The SPX Put option is purchased at a volume weighted average trade price between 11:30 am and 12:00 pm ET (VWAP). If there is no trade in the SPX Put option during the period, instead CBOE uses the last ask quote of the SPX Put option before 12:00 pm ET. Typically, on the third Friday (Roll Day) of every month since the initial roll date, the old SPX Put option settles at 9:30 am ET against the Special Opening Quotation of the S&P 500 Index. A new 5% OTM monthly SPX Put option will be subsequently purchased.

Following the CBOE methodology, the daily return of the index on each trading day (excluding roll dates) is calculated as:

$$R_t = \frac{SPX_t + DIV_t + Put\_5\%_t}{SPX_{t-1} + Put\_5\%_{t-1}}, \quad (30)$$

where  $SPX_t$  is the SPX Index close price on day  $t$ ,  $DIV_t$  is the SPX dividend,  $Put\_5\%_t$  is the average of the last bid-ask quote of the 5% OTM Put option before 4:00 pm ET. The terms with subscript  $t - 1$  refers to indicate the values on the previous day.

In our case, due to the unavailability of the data for the SPX dividend, we ignore the  $DIV_t$  in numerator of equation (30) (also the following equations). To obtain the 5% OTM Put option, we first rank all put options on day  $t$  by moneyness, then select the put option with the nearest value of moneyness to 95% from below. If the volume of selected Put option is zero,  $Put\_5\%_t$  is approximated by the ask quote on the previous day. To make it comparable to historical PPUT index returns, we calculate the historical PPUT index daily return as  $\frac{Price_t}{Price_{t-1}}$ , where  $Price_t$  stands for the historical daily price of PPUT index on day  $t$ .

On Roll Days, the returns are calculated in three steps. First, we calculate the return from the previous day market close to morning settlement of the expiring option (9:30 am ET):

$$R_1 = \frac{SOQ_t + DIV_t + Put\_5\%\_old_{settle}}{SPX_{t-1} + Put\_5\%\_old_{t-1}}, \quad (31)$$

where  $SOQ_t$  is the Special Opening Quotation of the SPX Index on the Roll Day.  $DIV_t$  is the SPX dividend,  $Put\_5\%\_old_{settle} = Max(0, K_{old} - SOQ_t)$  is the settlement value of the old SPX Put option, and  $Put\_5\%\_old_{t-1}$  is the average of the last bid-ask quote of the old SPX Put option before 4:00 pm ET on the previous day.

In our case,  $K_{old}$  in  $Put\_5\%\_old_{settle} = Max(0, K_{old} - SOQ_t)$  is the strike price of the Put option selected on the previous day.

Second, we calculate the return from morning settlement (9:30 am ET) to the moment the new SPX option position is deemed purchased:

$$R_2 = \frac{SOQ_t}{SPX_{vwap}}, \quad (32)$$

where  $SOQ_t$  is the Special Opening Quotation of the SPX Index on the Roll Day, and  $SPX_{vwap}$  is the volume weighted average price of the SPX Index.

Lastly, we calculate the return from the time the new SPX Put option position is deemed purchased to the market close:

$$R_3 = \frac{SPX_t + DIV_t + Put\_5\%\_new_t}{SPX_{vwap} + Put\_5\%\_new_{vwap}}, \quad (33)$$

where  $SPX_t$  is the last disseminated value of the SPX Index on the Roll Day, and  $Put\_5\%\_new_t$  is the average of the last bid-ask quote of the new SPX Put option before 4:pm ET on the Roll Day.  $SPX_{vwap}$  and  $Put\_5\%\_new_{vwap}$  are after volume weighted.

The total return of the Rolling Day:

$$R_t = R1 * R2 * R3. \quad (34)$$

In our case, we do not have intraday data. Therefore, it is impossible to compute the volume weighted average return. We approximate  $R_t = R_1$

### 3.5 Black-Scholes Model

When the option prices are unknown, we consider the Black-Scholes model to price European put option, provided by [Black and Scholes \(1973\)](#). The Black-Scholes formulas are shown below:

$$P(S_t, t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S_t, \quad (35)$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}} \left[ \ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t) \right], \quad (36)$$

$$d_2 = d_1 - \sigma\sqrt{T-t}, \quad (37)$$

where  $S_t$  is the spot price of the underlying option,  $K$  is the strike price,  $r$  is the risk-free rate,  $\sigma$  is the volatility of the option returns and  $T - t$  is the time to maturity in years.  $\sigma$  can be approximated by the implied volatility. Hence, given the volatility, we can back out the spot price of the option.

## 4 Data

This section describes the dataset used to replicate the [Rapach et al. \(2013\)](#) and analyze the role of lagged tail risk.

### 4.1 Predictive Ability of Lagged U.S. Returns

We use the same dataset as [Rapach et al. \(2013\)](#), which is derived from Global Financial Data (GFD) including 9 industrialized countries: United States, France, Germany, Japan, Switzerland, United Kingdom, Australia, Italy and Netherlands . We update the sample to the most recent available period, ranging from July 1986 to November 2015. The dataset contains monthly indices returns, dividend yield and 3-month Treasury bill rates at the country level, where the Treasury bill rates are served to compute the excess returns.



Table 1 shows the descriptive statistic for countries excess returns in monthly frequency. On average the monthly excess returns range from 0.14% (Italy) to 0.61 % (United States). All countries have high volatility of excess returns, which is also in line with the large magnitude of maximum/minimum values. Especially, Italy displays the largest volatility among all countries. Three out of nine countries obtain monthly Sharpe ratios larger or equal to 0.10, among which the United States has the largest Sharpe ratio (0.14). five out of nine countries display fairly substantial positive autocorrelations in their returns, among which Switzerland displays the largest autocorrealtions (0.16). The rest of countries exhibit fairly small autocorrelations, ranging from 0.01 to 0.05.

Table 1: Descriptive Statistics: 1986:07-2015:11

*Table 1 shows the descriptive statistics for country excess returns in monthly frequency for 9 industrialized countries, ranging from July 1986 to November 2015. The excess return is the country index minus the 3-month Treasury bill rate. The autocorrelation is the first-order autocorrelation. Sharpe ratio is the fraction of the average excess return over its volatility.*

Country	Mean (%)	Volatility (%)	Min (%)	Max (%)	Autocorrelation	Sharpe ratio
United States	0.61	4.41	-22.09	12.96	0.04	0.14
France	0.46	5.48	-22.49	21.58	0.11	0.08
Germany	0.45	5.86	-24.09	19.84	0.10	0.08
Japan	0.19	5.59	-21.68	17.51	0.14	0.03
United Kingdom	0.37	4.48	-27.33	12.90	0.05	0.08
Switzerland	0.54	4.65	-24.88	12.22	0.16	0.12
Australia	0.34	4.65	-43.06	14.27	0.01	0.07
Italy	0.14	6.26	-16.16	22.64	0.02	0.02
Netherlands	0.54	5.22	-23.69	13.32	0.12	0.10

## 4.2 Predictive Ability of Lagged U.S. Tail Risk

This section introduces the dataset used for constructing non-U.S. tail risk factor and investigating the predictive power of lagged tail risk.

### 4.2.1 Constructing Tail Risk Factor

To construct the tail risk factor, we require daily option prices listed on the country's Exchange, 1-month Treasury bill rates and implied volatility. Based on the availability of option price, there are 6 industrialized countries left: United States, France, Germany, Japan, United Kingdom and Switzerland, ranging from February 2006 to December 2015. Table 2 introduces the country option indices used to construct PPUT index.

Table 2: Information on Indices used for Tail Risk Factor Construction

Table 2 gives the information on indices used to construct Put Protection Index.

Country	Index Name	Issuer	Ticker
United States	S&P 500	NEW S & P 500 INDEX	SPX
France	CAC 40® index	CAC 40	CAC
Germany	DAX®, the blue chip index of Deutsche Börse AG	DAX IND	DAX
Japan	Nikkei Stock Average (Nikkei 225)	NIKKEI 225	NKY
United Kingdom	FTSE 100 index	FTSE 100	UKX
Switzerland	SMI®, the blue chip index of SIX Swiss Exchange	SMI	SMI

Before constructing the non-U.S. tail risk factor, we replicate the U.S. PPUT index from 2006 to 2015, where the daily returns on Roll Days are approximated by  $R_t = R_1$ . The correlation between constructed PPUT index returns and historical PPUT index returns is 0.68. There are 120 Roll Days, which have limited effect over the whole sample period. If we subtract all Roll Days, the correlation increases to 0.89. Figure 1 are the plots of obtained returns. We see that most of volatile returns are on Roll Days. Therefore, we decide to construct the PPUT index for the rest of the countries without Roll Days.

To measure the tail risk, we construct a zero-investment strategy that long the PPUT index and short a chosen index. The strategy is to hedge the decrease in S & P 500 index. The log return of this strategy refers to the tail risk. For instance, the U.S. tail risk can be interpreted as long the PPUT index and short S&P 500 index and can be calculated as:

$$Tail_{us,t} = \log\left(\frac{PPUT_t}{PPUT_{t-1}}\right) - \log\left(\frac{SPX_t}{SPX_{t-1}}\right), \quad (38)$$

where  $PPUT_t$  and  $SPX_t$  are the prices of PPUT index and S&P 500 index at time  $t$ . In this paper, we use month-end observations of PPUT index and chosen indices.

Figure 2 shows the time series plot of S & P 500 index returns and PPUT index returns from February 2006 to November 2015. We see several spikes in the tail returns, that means PPUT index pays off when there is large negative jump in the S & P 500 returns

Figure 1: PPUT Index Construction: Figure 1 are the plots for PPUT index returns and constructed PPUT index. The daily return is computed as shown in equation (30). In the upper panel,  $PPUT_{rollday}$  is the constructed PPUT index including the Roll days, where daily returns on the Roll Days are approximated by  $R_t = R_1$ . In the lower panel,  $PPUT_{exrollday}$  is the constructed PPUT index excluding the Roll days

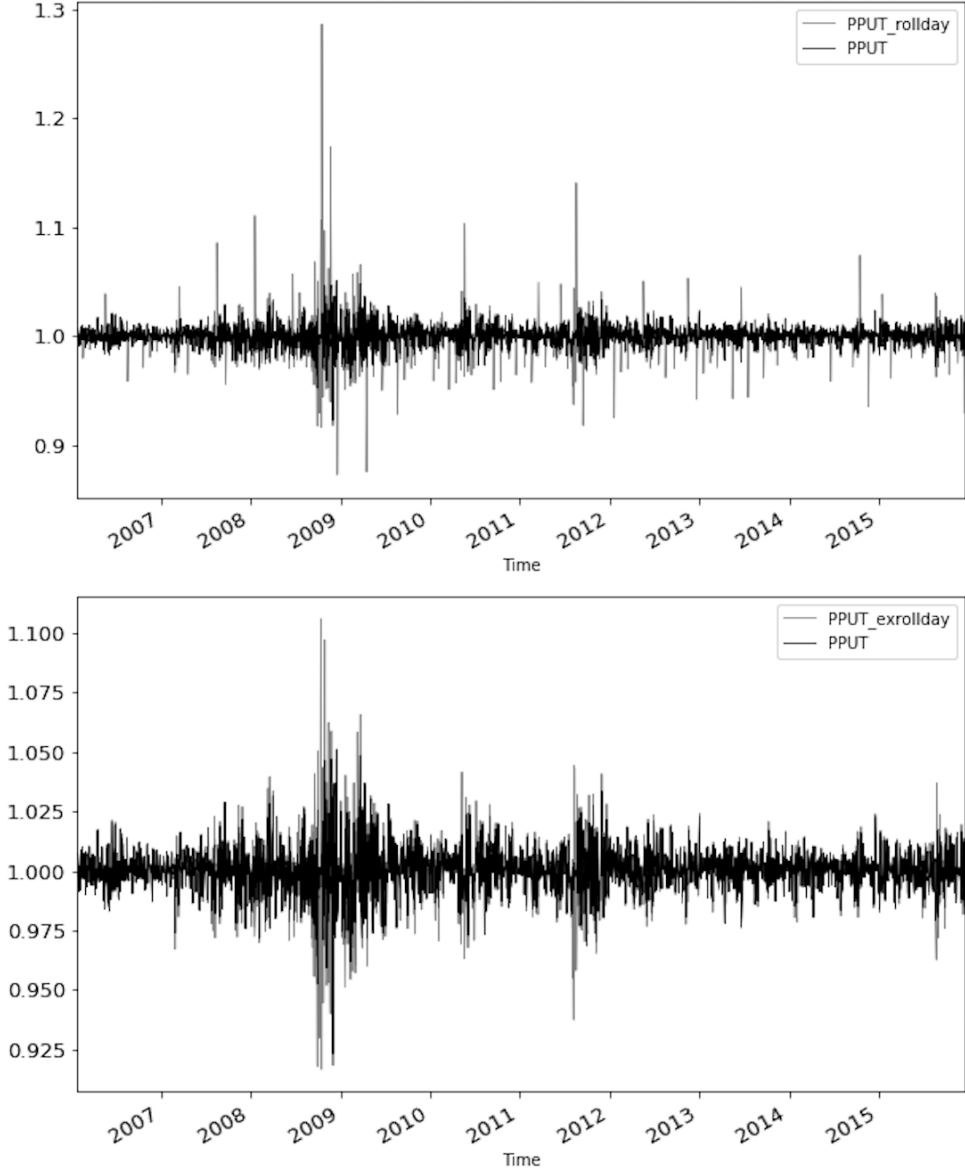


Figure 2: Time Series of SP 500 Return and Tail Return: *Figure 2 shows the time series of S & P 500 returns and constructed tail risk factor from February 2006 to November 2015.*

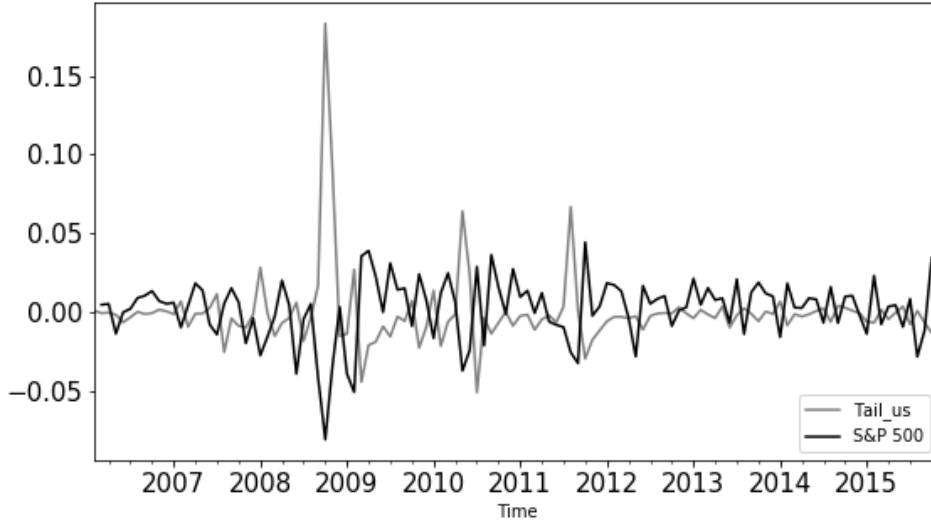


Table 3 shows the descriptive statistics of monthly country tail returns. All of the countries have negative average tail returns, among which the United State has the lowest average tail returns (-0.026%). The kurtosis of the United Kingdom, Switzerland and the United States are above 20%, among which the United Kingdom has the highest kurtosis (45.063%). The distribution of the United States and Japan are skewed towards the right, while the rest is skewed towards the left.

Table 3: Descriptive Statistics - Monthly Country Tail Risk Returns

*Table 3 shows the descriptive statistics for monthly tail risk returns of 6 industrialized countries, ranging from February 2006 to November 2015. The autocorrelation is the first-order autocorrelation.*

Country	Mean (%)	Volatility (%)	Kurtosis(%)	Skewness(%)	Autocorrelation
United States	-0.026	2.349	33.736	4.826	0.307
France	-0.009	0.085	2.894	-0.997	0.093
Germany	-0.009	0.077	3.458	-0.685	0.095
Japan	-0.005	0.127	2.848	0.234	0.019
United Kingdom	-0.007	0.140	45.063	-4.538	-0.051
Switzerland	-0.013	0.082	19.060	-3.561	0.384

Table 4 reports the correlation for countries tail risk returns. U.S. tail risk is negatively correlated with EU countries, among which Switzerland has the largest correlation with the value of -0.556. In contrast, U.S. tail risk is positively correlated with Japan. In addition, we observe tail risk returns of France, Germany and Switzerland are highly correlated with each other, since they develop closely in numerous areas as the members of European Union.

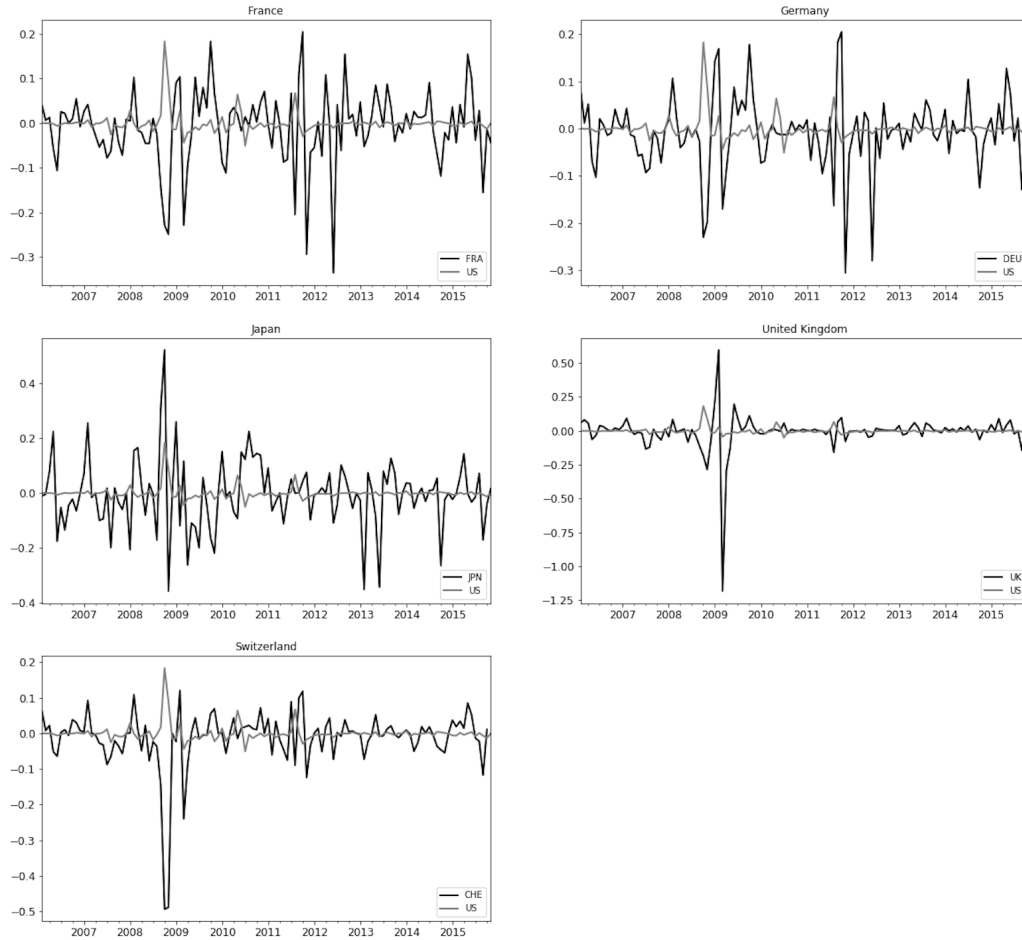
Table 4: Correlation Matrix - Monthly Country Tail Risk Returns

*Table 4 reports the correlations for monthly national tail risk returns for 6 industrialized countries, ranging from February 2006 to November 2015.*

Country	United States	France	Germany	Japan	United Kingdom	Switzerland
United States	1.000	-0.251	-0.238	0.189	-0.003	-0.556
France		1.000	0.932	0.017	0.563	0.743
Germany			1.000	0.049	0.560	0.722
Japan				1.000	-0.027	-0.012
United Kingdom					1.000	0.583
Switzerland						1.000

Table 3 plots the constructed non-U.S. tail risk returns together with the U.S. tail risk (in gray line). Tail risk returns in all countries fluctuate over the sample period, among which France and Germany display similar patterns. We notice that at the end of 2008 Financial Crisis, most of countries generate negative tail risk returns, except the United States and Japan. Because they have world's first and second largest developed economy, respectively. They are more likely to overcome the crisis. United Kingdom and Switzerland display fairly stable patterns over the whole sample period, albeit experience large peaks in 2009. In general, we see that the patter of the U.S. tail risk is more stable than the patterns of non U.S. tail risk returns and they display opposite patterns in returns at the end of the crisis.

Figure 3: Time Series of Country Tail Risk Returns: *Figure 3 shows the time series of U.S. tail risk returns and constructed non-U.S. tail risk returns from February 2006 to November 2015.*



#### 4.2.2 Analyzing the Predictive Ability of Lagged Tail Risk

The dataset consists of 9 industrialized countries: United States, France, Germany, Japan, United Kingdom, Switzerland, Australia, Italy and Netherlands,. The dataset contains indices returns, dividend yield, 3-month Treasury bill rates and tail risk factors in monthly frequency, where the Treasury bill rates are served to compute the excess returns. Note that CBOE PPUT index is available from July 1986, we have U.S. tail risk factor from July 1986 to November 2016. Therefore, we use the sample from July 1986 to November 2015 when investigating the predictive ability of American tail risk. We use the smaller sample from February 2006 to November 2015 (including France, Germany, Switzerland, Japan, United Kingdom and United States) when investigating the predictive ability of non-U.S. tail risk, because the constructed non-U.S. tail risk is available from February 2006.

Table 5 reports the descriptive statistic for excess returns of 6 countries in monthly fre-

quency. The average monthly excess returns range from 0.21% (Japan) to 0.65 % (Germany). All countries have high volatility of excess returns, which is also consistent with the large magnitude of maximum/minimum values. Especially, Germany displays the largest volatility among all countries. 3 out of 6 countries have monthly Sharpe ratios larger or equal to 0.10, among which Sharpe ratio of Germany is the largest(0.12). Most of countries returns display fairly large positive autocorrelations above 0.15.

Table 5: Descriptive Statistics: 2006:02-2015:11

*Table 5 reports the descriptive statistics for monthly country excess returns in national currency for 6 industrialized countries, ranging from February 2006 to November 2015. The excess return is the country index minus the 3-month Treasury bill rate. The autocorrelation is the first-order autocorrelation. Sharpe ratio is the fraction of the mean excess return over its volatility.*

Country	Mean (%)	Volatility (%)	Min (%)	Max (%)	Autocorrelation	Sharpe ratio
United States	0.60	4.39	-16.87	12.96	0.17	0.14
France	0.39	4.98	-14.62	21.58	0.16	0.08
Germany	0.65	5.52	-18.04	19.84	0.18	0.12
Japan	0.21	5.22	-19.97	17.51	0.24	0.04
United Kingdom	0.38	4.14	-13.65	12.90	0.03	0.09
Switzerland	0.40	3.90	-10.28	12.22	0.20	0.10

### 4.3 Data Adjustment

In reality, different national equity markets have different closing times. We need to account for it when investigating the relationships of international returns. Because in the same month up-to-date information is not supposed to be incorporated into all equity market on the final trading day of the month. It means, if the market in country A is closed while at the same time, the information has been spread in country B on the final trading day of the month  $t$ . The stock prices in country A can not incorporate the new information till the first trading day of the month  $t + 1$ . It will result in spurious evidence of lagged relations among monthly country returns. Following Rapach et.al (2013), this problem can be solved by adjusting  $r_{j,t}$  in equation (14) in a way that can exactly reflect the differences in closing times between country  $i$  and country  $j$ . More precisely, if the equity market in country  $j$  close after the market in country  $i$ , we eliminate the final trading day of month  $t$  from  $r_{j,t}$ . The same adjustment applies when investigating lead-lag relationships across countries in returns. Table 6 reports the data adjustment for the selected indices according to their closing time.

Table 6: Lagged Country Excess Stock Returns Adjustment

Table 6 reports the adjustment on  $r_{j,t}$  in the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,i}r_{i,t} + \beta_{i,j}r_{j,t} + \epsilon_{i,t+1}, i \neq j,$$

"All" indicates that  $r_{j,t}$  is calculated as the price of total return index at the end of the last trading day of month  $t$  divided by its price at the end of the last trading day of month  $t - 1$ . "Exclude" indicates that  $r_{j,t}$  is calculated as the price of total return index at the end of the penultimate trading day of month  $t$  divided by its price at the end of the penultimate trading day of month  $t - 1$ .

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
i	$\underset{j}{AUS}$	FRA	DEU	ITA	JPN	NLD	CHE	GBR	USA
AUS		Exclude	Exclude	Exclude	ALL	Exclude	Exclude	Exclude	Exclude
FRA	ALL		Exclude	ALL	ALL	ALL	ALL	ALL	Exclude
DEU	ALL	Exclude		ALL	ALL	ALL	ALL	ALL	Exclude
ITA	ALL	ALL	Exclude		ALL	ALL	ALL	ALL	Exclude
JPN	ALL	Exclude	Exclude	Exclude		Exclude	Exclude	Exclude	Exclude
NLD	ALL	ALL	Exclude	ALL	ALL		ALL	ALL	Exclude
CHE	ALL	ALL	Exclude	ALL	ALL	ALL		ALL	Exclude
GBR	ALL	ALL	Exclude	ALL	ALL	ALL	ALL		Exclude
USA	ALL	ALL	ALL	ALL	ALL	ALL	ALL	ALL	

## 5 Results

In this section, first, we report the results for replication of [Rapach et al. \(2013\)](#). We investigate the role of lagged U.S. returns in predicting non-U.S. returns. Second, we report the results for examining the predictive ability of lagged tail risk. The results are based on two sample periods from July 1986 to November 2015 and February 2006 to November 2015.

### 5.1 Predictive Ability of Lagged U.S. Returns

This section presents the results for the role of lagged U.S. returns. Table 7 reports results of benchmark predictive regression. The signs of  $\hat{\beta}_{i,b}$  and  $\hat{\beta}_{i,d}$  are in accordance with expectations (negative and positive), except Italy and Switzerland (negative  $\beta_{i,d}$  estimates). We observe that the magnitude of  $\hat{\beta}_{i,d}$  for United Kingdom is relatively larger than others. It is in line with the finding from [Kellard et al. \(2010\)](#) that the United Kingdom has greater predictability of dividend yield than the United States.

We noticed that the  $R^2$  statistics are fairly small because to some extent the stock returns are unpredictable. However, [Kandel et al. \(1996\)](#) argue that  $R^2$  statistics close to 0.5% can be



accepted as an economically significant signal for return predictability. The United States, Germany, United Kingdom, the Netherlands and Italy have  $R^2$  statistics larger than 1%. We reject the null hypothesis of zero beta estimates for Germany, United Kingdom and the Netherlands based on the wild bootstrapped  $p$ -values. Besides, we estimate the equation (1) under the pooled restrictions:  $\beta_{i,b} = \bar{\beta}_b$  and  $\beta_d = \bar{\beta}_d$ . We see that  $\bar{\beta}_b$  is significant at 10% level.

Table 7: Estimation of Benchmark Predictive Regression Model - 1986:07 to 2015:11

Table 7 reports the results of the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \epsilon_{i,t+1},$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate and the log country dividend yield, respectively. In column (2), (3), (6) and (7), heteroskedasticity-robust  $t$ -statistics in brackets test for the null hypothesis that  $\beta_{i,b} = 0$  ( $\beta_{i,d} = 0$ ) against alternative hypothesis that  $\beta_{i,b} < 0$  ( $\beta_{i,d} > 0$ ). In column (4) and (8), heteroskedasticity-robust  $\chi^2$  statistics in brackets test for the null hypothesis that  $\beta_{i,b} = \beta_{i,d} = 0$ . The pooled results are estimated under the restrictions that  $\beta_{i,b} = \bar{\beta}_b$  and  $\beta_{i,d} = \bar{\beta}_d$ . Following [Rapach et al. \(2013\)](#), the  $p$ -values are computed by means of wild bootstrap procedures, which accounts for the contemporaneous correlation in the data. \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$i$	$\hat{\beta}_{i,b}$	$\hat{\beta}_{i,d}$	$R^2$	$i$	$\hat{\beta}_{i,b}$	$\hat{\beta}_{i,d}$	$R^2$
United States	-0.070 (-0.659)	1.485 (1.712)	1.049% (2.932)	France	-0.140 (-1.657)	0.922 (0.668)	0.947% (3.555)
Germany	-0.293* (-2.352)	0.938 (0.644)	1.383%* (5.704)	Japan	-0.172 (-0.608)	0.211 (0.224)	0.685% (1.691)
United Kingdom	(-0.128) -1.659	3.613* (2.785)	2.622%* (7.984)	Switzerland	-0.159 (-1.606)	-0.155 (-0.183)	0.763% (2.578)
Australia	-0.072 (-0.630)	1.809 (0.588)	0.488% (0.414)	Italy	-0.144* (-1.703)	-1.126 (-0.981)	1.056% (2.935)
Netherlands	-0.231* (-2.221)	0.647 (0.530)	1.093%* (6.132)	Pooled	-0.110* (-1.701)	0.461 (0.763)	0.643% (3.369)

Table 8 reports the pairwise Granger causality test results. There are 64 out of 72 positive  $\hat{\beta}_{i,j}$  estimates, among which 27 are significant at 10% level. We observe the strongest predictive ability for lags of U.S. returns. Five out of eight  $\hat{\beta}_{i,US}$  are significant, among which 5 are larger than 0.15 when  $i = DEU, UK, AUS, ITA, NLD$ . The pooled  $\hat{\beta}_{i,US}$  estimate is still significant, and the magnitude of average of the  $\hat{\beta}_{i,US}$  (0.138) is larger than 0.10. In contrast, we observe no insignificant  $\hat{\beta}_{US,j}$  for  $j = \text{non-U.S. countries}$ . This reflects the lags of returns in countries outside the United States hardly predict returns of the United States. Overall, large  $\beta_{i,US}$  estimates and small  $\beta_{US,j}$  estimates show that the lagged U.S. returns play an important role in predicting international returns.

In addition, France and Switzerland show strong predictive power for other countries. Returns in five out of eight countries can be significantly predicted by lagged French returns and  $\beta_{i,FRA}$  estimates are larger than 0.15 for  $i = DEU, JPN, AUS, ITA, NLD$ . Lagged Swiss returns can

significantly predict 5 out of 8 countries returns. The  $\beta_{i,CHE}$  estimates are sizable for European countries ( $i = DEU, ITA, NLD$ ). In the last row, pooled beta estimates for France and Switzerland are also relatively sizable at 10 % significant level (0.153 and 0.141, respectively).

Table 8: Results of Pairwise Granger Causality Test - 1986:07 to 2015:11

Table 8 reports the results of the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,i}r_{i,t} + \beta_{i,j}r_{j,t} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \epsilon_{i,t+1}, \quad i \neq j,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate and the log country dividend yield, respectively. Heteroskedasticity-robust  $t$ -statistics in brackets test for the null hypothesis of zero  $\beta_{i,j}$  against the alternative hypothesis of positive  $\beta_{i,j}$ . "Average" is the mean of  $\hat{\beta}_{i,j}$  estimates in column. The pooled results are estimated under the restrictions that  $\beta_{i,j} = \bar{\beta}_j$  for all  $i \neq j$ . The wild bootstrapped  $p$ -value is constructed by following [Rapach et al. \(2013\)](#). \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$i$	$\hat{\beta}_{i,US}$	$\hat{\beta}_{i,FRA}$	$\hat{\beta}_{i,DEU}$	$\hat{\beta}_{i,JPN}$	$\hat{\beta}_{i,UK}$	$\hat{\beta}_{i,CHE}$	$\hat{\beta}_{i,AUS}$	$\hat{\beta}_{i,ITA}$	$\hat{\beta}_{i,NLD}$
United States (US)		0.078 (1.430)	0.034 (0.593)	0.051 (1.194)	0.064 (0.559)	0.048 (0.548)	0.081 (1.098)	0.051 (1.124)	0.037 (0.416)
France (FRA)	0.094 (0.855)		-0.007 (-0.072)	0.022 (0.317)	-0.015 (-0.107)	0.123 (1.046)	0.050 (0.614)	-0.036 (-0.521)	-0.003 (-0.017)
Germany (DEU)	0.192* (1.788)	0.210* (1.773)		0.089* (1.450)	0.080 (0.708)	0.233* (1.967)	0.079 (0.848)	0.046 (0.680)	0.064 (0.458)
Japan (JPN)	0.065 (0.813)	0.157* (2.787)	0.054 (1.013)		0.115* (1.575)	0.107* (1.536)	0.044 (0.640)	0.030 (0.583)	0.039* (2.016)
United Kingdom (UK)	0.156* (1.454)	0.122 (1.420)	0.017 (0.251)	0.081* (1.710)		0.075 (0.960)	0.076 (0.988)	-0.009 (-0.186)	-0.051 (-0.499)
Switzerland (CHE)	0.078 (0.818)	0.064 (0.702)	-0.006 (-0.082)	0.045 (0.980)	0.052 (0.642)		0.066 (0.944)	-0.016 (-0.310)	0.014 (0.145)
Australia (AUS)	0.155* (1.676)	0.181* (2.535)	0.115* (1.867)	0.114* (2.332)	0.086 (1.014)	0.103* (1.554)		0.075* (1.882)	0.008 (0.328)
Italy (ITA)	0.182* (1.911)	0.275* (3.037)	0.182* (2.240)	0.062 (0.913)	0.165* (1.568)	0.286* (2.719)	0.081 (1.049)		0.150 (1.392)
Netherlands (NLD)	0.248* (2.732)	0.219* (2.402)	0.132* (1.385)	0.114* (2.237)	0.148 (1.171)	0.266* (2.154)	0.133* (1.625)	0.041 (0.740)	
Average	0.130	0.145	0.058	0.064	0.077	0.138	0.068	0.020	0.029
Pooled	0.138* (2.125)	0.153* (3.332)	0.064* (1.497)	0.073* (1.842)	0.089 (1.372)	0.141* (2.394)	0.071 (1.228)	0.025 (0.732)	0.028 (1.211)

The main reason why lagged U.S. returns play a leading role in predicting international returns could be that the United States has the world's largest GDP and world's largest equity market in terms of market capitalization. As for France and Switzerland, they have relatively smaller equity markets, comparing to the United States. Market concentration is an essential issue. The top ten firms in terms of capitalization in France and Switzerland comprise over half of total country capitalization according to the World Federation of Exchanges. By the concept of information frictions, when shocks occur, a small amount of large companies are more likely to incorporate information in stock prices.

Table 9 reports the pooled OLS results in equation (4), which measures the relations on average. The lagged returns of the United States and France still significantly improve the

return predictability of other countries. We see that the U.S. returns in the past month continue to play a leading role in forecasting international returns. The  $\beta_{US}$  estimate (0.124) is very close to the pooled estimate in Table 8. The main difference is that Swiss returns do not display significant predictive ability for European countries any more. Therefore, its predictive power is not robust.

Table 9: Pooled General Model Specification Estimation Results - 1986:07 to 2015:11

Table 9 reports the estimates of  $\bar{\beta}_{i,j}$  (denoted by  $\hat{\beta}_j$ ) in the model:

$$r_{i,t+1} = \beta_{i,0} + \bar{\beta}_b Bill_{i,t} + \bar{\beta}_d Yield_{i,t} + \bar{\beta}_{AR} r_{i,t} + \sum_{j \neq i} \bar{\beta}_j r_{j,t} + \epsilon_{i,t+1}, \quad i = 1, \dots, N,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate and dividend yield, respectively. Following [Rapach et al. \(2013\)](#), we compute the bias-corrected wild bootstrapped 90 % confidence intervals. \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\hat{\beta}_{US}$	$\hat{\beta}_{FRA}$	$\hat{\beta}_{DEU}$	$\hat{\beta}_{JPN}$	$\hat{\beta}_{UK}$	$\hat{\beta}_{CHE}$	$\hat{\beta}_{AUS}$	$\hat{\beta}_{ITA}$	$\hat{\beta}_{NLD}$
0.124*	0.121*	-0.064	0.012	-0.012	0.073	0.008	-0.015	-0.021
[0.009,0.241]	[0.020,0.218]	[-0.150,0.022]	[-0.053,0.073]	[-0.149,0.123]	[-0.052,0.199]	[-0.028,0.043]	[-0.078,0.048]	[-0.068,0.027]

Table 10 reports the results of adaptive elastic-net estimation. To some extent, we see some similarities in results with Table 8 and 9. Five of eight countries select returns lags of U.S. as predictors for their returns by LASSO, among which the Netherlands and the United Kingdom can be significantly predicted by returns lags of U.S.. In contrast, none of returns lags could significantly predict returns of the United States. This agrees with the conclusion in [Rapach et al. \(2013\)](#). In addition, lagged French returns display strongest predictive ability. However, according to the finding in [Rapach et al. \(2013\)](#), when the data ranges from February 1980 to December 2010, France displays limited predictive ability for non-FRA countries. Therefore, the robustness of the results for France needs to be further examined.

Table 10: Adaptive Elastic-Net Estimation Results - 1986:07 to 2015:11

Table 10 shows the estimates of  $\beta_{i,j}$  (denoted by  $\hat{\beta}_{i,j}^*$ ) from the adaptive elastic-net model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,t}r_{i,t} + \sum_{j \neq i} \beta_{i,j}r_{j,t} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \epsilon_{i,t+1}$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate and the log country dividend yield, respectively. "Average" is the average  $\hat{\beta}_{i,j}^*$  estimates in the column. Following [Rapach et al. \(2013\)](#), we compute the bias-corrected wild bootstrapped 90 % confidence intervals in brackets. \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$i$	$\hat{\beta}_{i,US}^*$	$\hat{\beta}_{i,FRA}^*$	$\hat{\beta}_{i,DEU}^*$	$\hat{\beta}_{i,JPN}^*$	$\hat{\beta}_{i,UK}^*$	$\hat{\beta}_{i,CHE}^*$	$\hat{\beta}_{i,AUS}^*$	$\hat{\beta}_{i,ITA}^*$	$\hat{\beta}_{i,NLD}^*$
United States (US)		0.033 [-0.017,0.107]	0	0	0	0	0.010 [-0.043,0.067]	0	0
France (FRA)	0.038 [-0.030,0.124]		0	0		0.074* [0.010,0.191]	0	0	0
Germany (DEU)	0.036 [-0.033,0.122]	0.030 [-0.057,0.110]		0	0	0.093* [0.008,0.228]	0	0	0
Japan (JPN)	0	0.117* [0.058,0.218]	0		0	0	0	0	0
United Kingdom (UK)	0.081* [0.023,0.209]	0.138* [0.077,0.300]	0	0.035* [0.0003,0.097]		0	0	-0.013 [-0.062,0.016]	-0.150 [-0.335,-0.107]
Switzerland (CHE)	0.011 [-0.011,0.040]	0.053 [-0.005,0.127]	0	0.004 [-0.009,0.018]	0	0	0.015 [-0.019,0.057]	0	0
Australia (AUS)	0	0.125* [0.057,0.243]	0	0.038* [0.009,0.104]	0	0		0	0
Italy (ITA)	0	0.205* [0.086,0.445]	0	0	0	0.157 [-0.007,0.392]	-0.003 [-0.114,0.096]		-0.072 [-0.294,0.058]
Netherlands (NLD)	0.121* [0.001,0.280]	0.133 [-0.007,0.311]	0	0.064 [-0.006,0.149]	0	0.147 [-0.032,0.354]	0.006 [-0.088,0.098]	-0.015 [-0.090,0.041]	
Average	0.037	0.108	0	0.017	0	0.059	0.005	-0.0001	-0.034

Table 11 reports the out-of sample results. The in-sample period ranges from July 1986 to December 1989. By doing so, we balance the number of observations for estimation and evaluation. There are five of eight non-U.S. countries having positive  $R_{OS}^2$  in column (2) and (5). This means that the competitive model that incorporates information in lagged returns of the U.S. reduces the  $MSFE$  comparing to the historical average return model. Among these positive  $R_{OS}^2$  statistics, France, Switzerland, Germany and the Netherlands has economically sizable  $R_{OS}^2$  statistics (above 0.5%). The 1% bootstrapped critical value under the null hypothesis that country returns cannot be predicted is equal to 2.49. The Netherlands has the largest critical value of 2.014 in column (2) and (5). Column (3) and (6) reports the pooled  $R_{OS}^2$  statistics under the restriction:  $\beta_{i,US} = \bar{\beta}_{US}$  for non-U.S. countries. It improves the efficiency, albeit with the sacrifice in biasness. In truth, we see the pooled  $R_{OS}^2$  statistics in column (3) and (6) larger than  $R_{OS}^2$  statistics in column (2) and (5). There are six of eight positive  $R_{OS}^2$  statistics, among which five are significant at the conventional level. It provides further evidence of the better out-of-sample performance of the competing model relative to the historical average model.

Table 11: Out-of-Sample Tests for Lagged U.S. Return - 1990:01 to 2015:11

Table 11 reports out-of-sample  $R^2$  and  $R_{OS}^2$  statistics (Campbell and Thompson (2008)) by measuring the difference in mean-squared forecast error between constant expected excess return model and a competing model that includes lagged U.S. returns. Baseline model and competing model are shown as follows:

$$r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1},$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1},$$

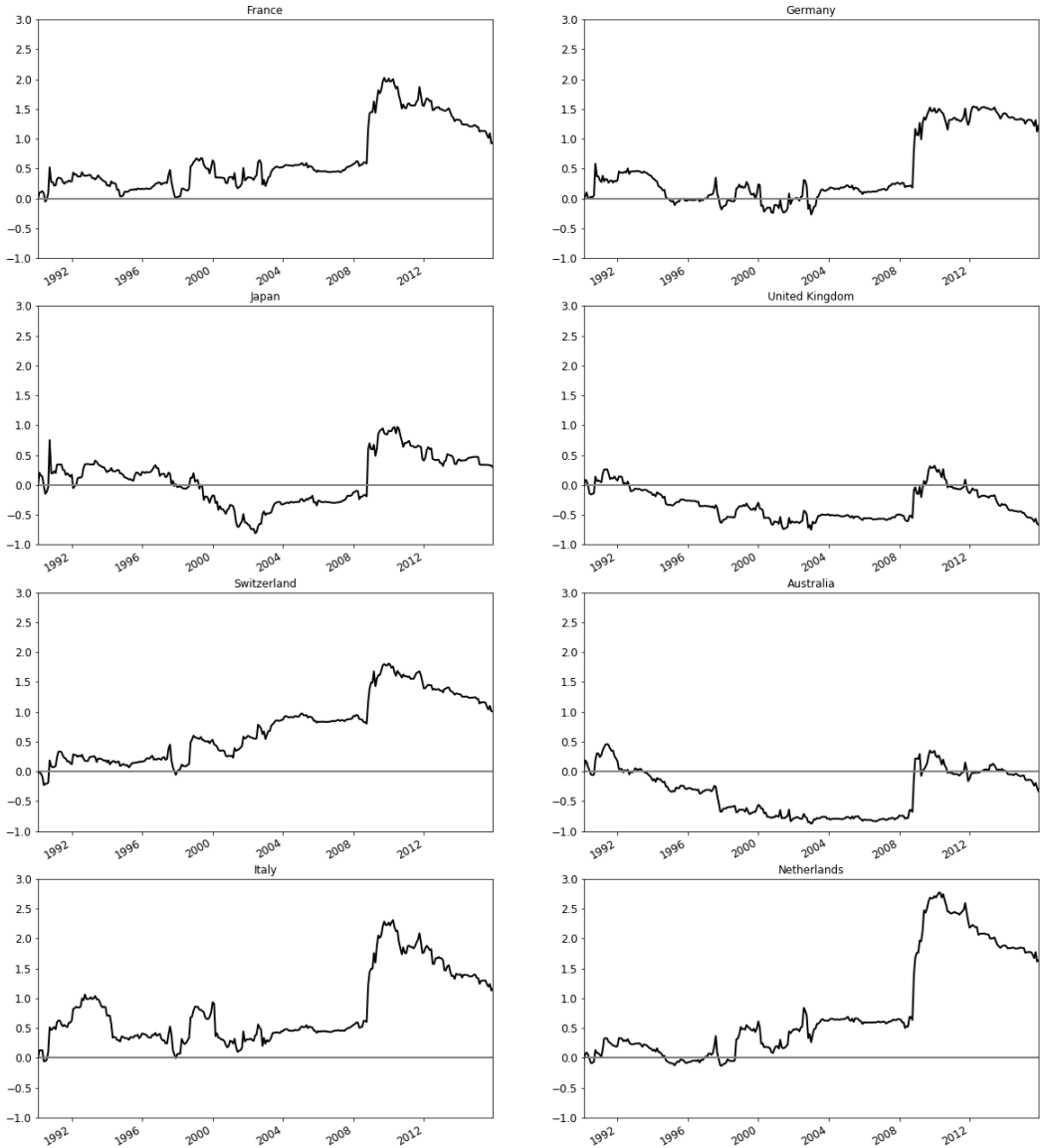
where  $r_{i,t+1}$  is the country monthly excess returns at month  $t$ . The results in column (3) and (6) are obtained by imposing the restriction on the competing model:  $\beta_{i,US} = \bar{\beta}_{US}$  for non-U.S. countries.  $MSFE$  – adjusted statistics (Clark and West (2007)) in brackets test for the null hypothesis of zero  $R_{OS}^2$  against positive  $R_{OS}^2$ . \* presents the 10% significance of the estimate. "Average" is the average of  $R_{OS}^2$  across 8 countries. The in-sample period is from July 1986 to December 1989.

(1)	(2)	(3)	(4)	(5)	(6)
$i$	$R_{OS}^2$	$R_{OS,pooled}^2$	$i$	$R_{OS}^2$	$R_{OS,pooled}^2$
France	0.808%* (1.395)	1.100%* (1.586)	Germany	0.961%* (1.589)	1.182%* (1.666)
Japan	-0.145% (0.619)	0.292% (1.016)	United Kingdom	-0.820% (0.625)	-1.248% (0.683)
Switzerland	1.659%* (2.224)	1.622%* (2.112)	Australia	-0.425% (0.655)	-0.703% (0.925)
Italy	0.233% (0.930)	0.945%* (1.571)	Netherlands	1.836%* (2.014)	1.964%* (2.004)
Average	0.513%	0.644%			

Figure 4 depicts the cumulative differences in  $MSFE$  for the baseline model relative to the competing forecasts. This graphical approach is provided by Goyal and Welch (2008) to

determine the out-of-sample forecasting performance. The height of the curve at the end of the period higher than the height at the beginning of the period reveals  $MSFE$  of the competing model is smaller than that of the baseline model. For most of the countries, the competing model consistently beats the baseline forecasts. In particular, there are sizable forecast gains in the competing model during the 2008 global financial crisis. Given the importance of U.S. economy in the world, shocks in the United States simultaneously affects the market outside the United States. Together with the information friction, the predictive ability of lagged U.S. returns becomes stronger during American recessions.

Figure 4: Out-Sample forecasting results - 1990:01-2015:11 *Figure 4 illustrates cumulative squared differences of forecasts errors for monthly excess return between historical average baseline model ( $r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1}$ ) and the competitive model that uses lagged U.S. return ( $r_{i,t+1} = \beta_{i,0} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1}$ ).*



## 5.2 Predictive Ability of Lagged Tail Risk

This section presents the results based on two different sample periods. For the long sample period (July 1986 to November 2015), we report the results for investigating the role of lagged U.S. tail risk in returns predictability of the non-U.S. countries. For the short sample period (February 2006 to November 2015), we check the consistency of predictive power of U.S. tail risk. In addition, we report the results for investigating the role of the lagged non-U.S. tail risk in returns predictability of the U.S..

### 5.2.1 Predictive Ability of Lagged U.S. Tail risk: 1986:07-2015:11

In this section, we report the results using sample from July 1986 to November 2015 for testing the predictive power of lagged U.S. tail risk in pricing non-U.S. industrialized countries. In section 5.1, we conclude that returns predictability is significantly improved by lagged U.S. returns, which agrees with [Rapach et al. \(2013\)](#). Therefore, we extend the benchmark predictive model by adding lagged U.S. returns.

Table 12 reports the OLS estimates in the predictive regression model that additionally includes lagged returns and lagged U.S. returns. Results in the fifth column are consistent with the pairwise Granger causality test results in the second column of Table 8. Thus, based on the replication results in the previous section, it is reasonable to include the lagged U.S. returns in the benchmark model. Indeed, lagged returns of the U.S. display sizable predictive ability for international returns, and the positive sign of  $\hat{\beta}_{i,US}$  agrees with our expectation. The reason why we expect positive sign is due to the positive first-order autocorrelation of U.S. returns (shown in Table 1). That means large returns today display a positive signal for tomorrow returns.

Table 13 reports the OLS estimates of U.S. tail risk lags in the predictive regression model. In the column (6), seven of eight non-U.S. countries have significant  $\alpha_{i,US}$  estimates at 10% level with (absolute) value above 0.25. That means lagged U.S. tail risk improve the returns predictability. In addition, we notice that lagged U.S. tail risk can significantly predict the U.S. returns while its lagged returns cannot. Besides, we find that the negative sign of  $\hat{\alpha}_{i,US}$  agrees with our expectation. That means that if a stock suffers from shocks today, its returns will decrease tomorrow. Indeed, it happens in reality, especially during the financial crisis.

In the last column of Table 13, the number of significant  $R^2$  statistics increases from 3 to 7, comparing to the last column of Table 12, in which we do not incorporate the information in U.S. tail risk. These  $R^2$  statistics can be seen as economically significant signal for return predictability, since they are much larger than 1.5%. In particular, United Kingdom, Switzerland and the Netherlands have  $R^2$  statistics above 5%. In the last row of Table 13, the pooled results

are obtained under the restriction:  $\beta_{i,b} = \bar{\beta}_b$ ,  $\beta_{i,d} = \bar{\beta}_d$ ,  $\beta_{i,r} = \bar{\beta}_r$ ,  $\beta_{i,US} = \bar{\beta}_{US}$  and  $\alpha_{i,US} = \bar{\alpha}_{US}$  for all  $i$ . By doing so, we scarify the biasness but gain the efficiency. The magnitude of pooled  $\hat{\alpha}_{i,US}$  is more than three times as large as other significant pooled predictors. Overall, lagged U.S. tail risk shows strongest predictive power on returns in non-U.S. areas.

Table 12: Benchmark Predictive Regression Model Estimation - 1986:07: to 2015:11

Table 12 reports the results of the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1}, \quad i \neq US,$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \epsilon_{i,t+1}, \quad i = US,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate and country dividend yield, respectively. In column (2), (3), (4) and (5), heteroskedasticity-robust  $t$ -statistics in brackets test for  $H_0: \beta_{i,b} = 0$  against  $H_A: \beta_{i,b} < 0$ ;  $H_0: \beta_{i,d} = 0$  against  $H_A: \beta_{i,d} > 0$ ;  $H_0: \beta_{i,r} = 0$  against  $H_A: \beta_{i,r} > 0$ ;  $H_0: \beta_{i,US} = 0$  against  $H_A: \beta_{i,US} > 0$ . In column (6), heteroskedasticity-robust  $\chi^2$  statistics in brackets test for  $H_0: \beta_{i,b} = \beta_{i,d} = \beta_{i,r} (= \beta_{i,US}) = 0$ . The pooled results are estimated under the restrictions that  $\beta_{i,b} = \bar{\beta}_b$ ,  $\beta_{i,d} = \bar{\beta}_d$ ,  $\beta_{i,r} = \bar{\beta}_r$  and  $\beta_{i,US} = \bar{\beta}_{US}$  for all  $i$ . Based on wild bootstrapped  $p$ -value, constructing by following [Rapach et al. \(2013\)](#), \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)
$i$	$\hat{\beta}_{i,b}$	$\hat{\beta}_{i,d}$	$\hat{\beta}_{i,r}$	$\hat{\beta}_{i,US}$	$R^2$
United States	-0.071 (-0.673)	1.522 (1.772)	0.048 (0.688)	-	1.277% (4.003)
France	-0.124 (-1.486)	1.308 (0.973)	0.056 (0.569)	0.094 (0.855)	2.404% (7.853)
Germany	-0.276* (-2.244)	1.225 (0.866)	-0.003 (-0.043)	0.192* (1.788)	3.353%* (10.100)
Japan	-0.111 (-0.391)	0.401 (0.436)	0.107* (1.455)	0.065 (0.813)	2.582% (5.693)
United Kingdom	-0.127 (-1.654)	3.620* (2.854)	-0.072 (-0.655)	0.156* (1.454)	3.734%* (10.974)
Switzerland	-0.133 (-1.366)	0.100 (0.117)	0.103 (1.009)	0.078 (0.818)	3.432% (8.876)
Australia	-0.068 (-0.605)	1.590 (0.532)	-0.077 (-1.043)	0.155* (1.676)	1.755% (2.996)
Italy	-0.151* (-1.826)	-1.134 (-1.016)	-0.067 (-1.033)	0.182* (1.911)	2.189% (7.527)
Netherlands	-0.210* (-2.057)	0.783 (0.683)	-0.041 (-0.483)	0.248* (2.732)	4.244%* (14.608)
Pooled	-0.107* (-1.240)	0.452 (0.244)	0.010 (0.205)	0.138* (2.125)	2.108%* (8.974)

Table 14 reports the results of the adaptive elastic-net estimation. For each country, we build up a predictive regress model that contains a number of predictors: own nominal interest rate, own dividend yield, lagged returns in all countries and U.S. lagged tail risk. Six of eight countries select lagged U.S. tail risk as predictors for their return (except Australia), among which France, Germany, United Kingdom, Switzerland, Italy and the Netherlands can be significantly predicted by lagged U.S. tail risk. In addition, we observe that predictive ability of lagged French returns



remain strong, while that of returns lags of the United States declines.

Compared to Table 10, the number of countries that select lagged U.S. returns as predictor decreases from five to three. And lagged U.S. returns are not as a significant predictor anymore for the United Kingdom and the Netherlands. We see that in table 14 most of countries tend to select lagged U.S. tail risk rather than U.S. returns. That means the U.S. tail risk is more important in return predictability than U.S. returns.

Table 13: Predictive Regression Model Estimation - 1986:07: to 2015:11

Table 13 reports the results of the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1}, \quad i \neq US,$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1}, \quad i = US,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$ ,  $Yield_{i,t}$  and  $Tail_{US,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate, the log country dividend yield and the U.S. tail risk, respectively. In column (2), (3), (4), (5) and (6), heteroskedasticity-robust  $t$ -statistics in brackets test for  $H_0: \beta_{i,b} = 0$  against  $H_A: \beta_{i,b} < 0$ ,  $H_0: \beta_{i,d} = 0$  against  $H_A: \beta_{i,d} > 0$ ,  $H_0: \beta_{i,r} = 0$  against  $H_A: \beta_{i,r} > 0$ ,  $H_0: \beta_{i,US} = 0$  against  $H_A: \beta_{i,US} > 0$  and  $H_0: \alpha_{i,US} = 0$  against  $H_A: \alpha_{i,US} < 0$ , respectively. In column (7), heteroskedasticity-robust  $\chi^2$  statistics in brackets test for  $H_0: \beta_{i,b} = \beta_{i,d} = \beta_{i,r} = \alpha_{i,US} (= \beta_{i,US}) = 0$ . The pooled results are estimated under the restrictions that  $\beta_{i,b} = \bar{\beta}_b$ ,  $\beta_{i,d} = \bar{\beta}_{i,d}$ ,  $\beta_{i,r} = \bar{\beta}_r$ ,  $\beta_{i,US} = \bar{\beta}_{US}$  and  $\alpha_{i,US} = \bar{\alpha}_{US}$  for all  $i$ . Based on wild bootstrapped  $p$ -value, computing by following [Rapach et al. \(2013\)](#), \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
$i$	$\hat{\beta}_{i,b}$	$\hat{\beta}_{i,d}$	$\hat{\beta}_{i,r}$	$\hat{\beta}_{i,US}$	$\hat{\alpha}_{i,US}$	$R^2$
United States	-0.088 (-0.836)	1.709 (1.967)	-0.071 (-0.880)	-	-0.443* (-2.563)	3.598% (9.024)
France	-0.127 (-1.549)	1.461 (1.099)	0.042 (0.549)	0.127* (1.982)	-0.392* (-1.918)	4.773%* (16.379)
Germany	-0.298* (-2.424)	1.577 (1.114)	0.028 (0.414)	0.070 (1.038)	-0.431* (-2.165)	4.191%* (12.760)
Japan	-0.099 (-0.358)	0.468 (0.508)	0.107* (1.492)	0.124* (1.922)	-0.250* (-1.695)	4.124%* (13.721)
United Kingdom	-0.142 (-1.849)	3.929* (3.024)	-0.037 (-0.513)	0.121* (2.550)	-0.335* (-2.216)	5.840%* (18.526)
Switzerland	-0.138 (-1.449)	0.214 (0.261)	0.064 (0.929)	0.053 (1.128)	-0.474* (-3.016)	6.610%* (21.398)
Australia	-0.075 (-0.652)	1.939 (0.633)	-0.024 (-0.419)	0.075* (1.429)	-0.187 (-1.178)	1.511% (3.857)
Italy	-0.146* (-1.753)	-1.060 (-0.953)	-0.040 (-0.700)	0.099 (1.191)	-0.392* (-2.403)	2.928%* (12.751)
Netherlands	-0.232* (-2.289)	1.082 (0.934)	0.020 (0.250)	0.056 (0.975)	-0.501* (-3.045)	5.241%* (19.676)
Pooled	-0.110* (-1.303)	0.552 (0.330)	0.025 (0.516)	0.088* (1.789)	-0.357* (-2.695)	3.359%* (16.030)

Table 14: Adaptive Elastic-Net Estimation Results - 1986:07 to 2015:11

Table 14 reports the estimates of  $\beta_{i,j}$  and  $\alpha_{i,US}$  (denoted by  $\hat{\beta}_{i,j}^*$  and  $\hat{\alpha}_{i,US}^*$ ) in adaptive elastic-net model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,i}r_{i,t} + \sum_{j \neq i} \beta_{i,j}r_{j,t} + \alpha_{i,US}Tail_{US,t} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \epsilon_{i,t+1}, \quad \text{for } i = 1, \dots, N,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$ ,  $Yield_{i,t}$  and  $Tail_{US,t}$  represents the monthly country excess returns, the 3-month Treasury bill rate, the log country dividend yield and the U.S. tail risk, respectively. "Average" is the mean of  $\hat{\beta}_{i,j}^*$  estimates in the column. Following Rapach et al. (2013), we compute the bias-corrected wild bootstrapped 90% confidence intervals and report in the brackets. \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$i$	$\hat{\beta}_{i,US}^*$	$\hat{\beta}_{i,FRA}^*$	$\hat{\beta}_{i,DEU}^*$	$\hat{\beta}_{i,JPN}^*$	$\hat{\beta}_{i,UK}^*$	$\hat{\beta}_{i,CHE}^*$	$\hat{\beta}_{i,AUS}^*$	$\hat{\beta}_{i,ITA}^*$	$\hat{\beta}_{i,NLD}^*$	$\hat{\alpha}_{i,US}^*$
United States (US)		0	0	0	0	0	0	0	0	-0.269 [-0.538,-0.117]
France (FRA)	0		0	0	-0.016 [-0.142,0.058]	0.070 [-0.015,0.198]	0	0	0	-0.344* [-0.694,-0.128]
Germany (DEU)	0.059 [-0.099,0.236]	0.119 [-0.048,0.340]		0.033 [-0.046,0.119]	-0.072 [-0.028,0.088]	0.185* [0.069,0.410]	0	0	-0.133 [-0.396,0.082]	-0.368* [-0.738,-0.059]
Japan (JPN)	0	0.130* [0.048,0.260]	-0.005 [-0.090,0.053]		0	0	0	-0.006 [-0.070,0.035]	0	-0.083 [-0.277,0.056]
United Kingdom (UK)	0.039 [-0.023,0.128]	0.133* [0.081,0.292]	0	0.043* [0.001,0.111]		0	0	-0.018 [-0.070,0.013]	-0.164* [-0.357,-0.120]	-0.247* [-0.520,-0.104]
Switzerland (CHE)	0	0.011 [-0.074,0.085]		0	0	0	0	0	0	-0.464* [-0.756,-0.254]
Australia (AUS)	0	0.132* [0.055,0.254]	0	0.038* [0.008,0.105]		0	0	0	0	0
Italy (ITA)	0	0.221* [0.092,0.446]	0	0	0	0.176* [0.021,0.414]	-0.032 [-0.160,0.064]		-0.129 [-0.392,0.022]	-0.229* [-0.538,-0.018]
Netherlands (NLD)	0.016 [-0.062,0.099]	0.089* [0.002,0.237]	0	0.058* [0.008,0.140]		0.127* [0.012,0.317]	0	0		-0.372* [-0.684,-0.185]
Average	0.014	0.104	-0.001	0.022	-0.011	0.070	-0.004	-0.003	-0.053	-0.297

Table 15 reports the results out-of-sample tests. In general, results agree with out-of-sample results in Table 11. Half of countries have positively sizable  $R^2$  statistics in the column (2) and (5) (France, Switzerland, Italy and the Netherlands). This means that competing model that uses lagged tail risk of the U.S. has smaller  $MSFE$ , comparing to the baseline model. The 1% bootstrapped critical value under the null hypothesis that country returns can not be predicted is equal to 2.49. The Netherlands still has the largest critical value of 2.394 in column (2) and (5). Column (3) and (6) reports the pooled  $R^2$  statistics under the restriction:  $\alpha_{i,US} = \bar{\alpha}_{i,US}$ . Again it improves the efficiency, albeit with the sacrifice in biasness. In truth, we see the pooled statistics in column (3) and (6) are larger than those in column (2) and (5). Six of eight  $R^2$  statistics are positively significant at conventional level. It provides further evidence of the better out-of-sample performance of the competing model.

Compared to Table 11, the sign of out-of-sample  $R_{OS}^2$  and  $R_{OS,pooled}^2$  for all countries remain the same. However, the number of significantly positive  $R_{OS}^2$  and  $R_{OS,pooled}^2$  increases from nine to ten. The U.S. tail risk has better out-of-sample performance than U.S. returns.

Table 15: Out-of-Sample Tests for Lagged U.S. Tail Risk - 1990:01 to 2015:11

Table 15 shows out-of-sample  $R^2$  and  $R_{OS}^2$  statistics (Campbell and Thompson (2008)) by measuring the difference in mean-squared forecast error between constant expected excess return model and a competing model that includes lagged U.S. returns. Baseline model and competing model are shown as follows:

$$r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1},$$

$$r_{i,t+1} = \beta_{i,0} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1},$$

where  $r_{i,t+1}$  represents the monthly excess returns at month  $t$ . Results in column (3) and (6) are obtained by imposing the restriction on the competing model:  $\beta_{i,US} = \hat{\beta}_{US}$  for non-U.S. countries.  $MSFE$  - adjusted statistics (Clark and West (2007)) in brackets test for the null hypothesis of zero  $R_{OS}^2$  against positive  $R_{OS}^2$ . \* presents the 10% significance of the estimate. "Average" is the average of  $R_{OS}^2$  across 8 countries. The in-sample period is from July 1986 to December 1989.

(1)	(2)	(3)	(4)	(5)	(6)
$i$	$R_{OS}^2$	$R_{OS,pooled}^2$	$i$	$R_{OS}^2$	$R_{OS,pooled}^2$
France	0.614%* (1.651)	1.493%* (1.743)	Germany	-0.409%* (1.635)	0.317%* (1.538)
Japan	-0.238%* (1.845)	0.075%* (1.872)	United Kingdom	-0.331%* (1.650)	-1.679%* (1.681)
Switzerland	2.553%* (2.748)	5.131%* (2.740)	Australia	-0.432% (-0.489)	-3.663% (0.841)
Italy	0.555%* (1.631)	0.788%* (1.810)	Netherlands	1.644%* (2.394)	2.093%* (2.326)
Average	0.495%	0.569%			

Figure 5 depicts the cumulative differences in  $MSFE$  for the baseline forecasts relative to the competing forecasts. The curve above 0 represents a smaller  $MSFE$  for the competing model. we see that for most of the countries, the competing model outperforms the baseline

model. In particular, there are sizable forecast gains during the 2000 dot-com bubble. Given the importance of the U.S. economy in the world together with the information friction, the predictive ability of lagged tail risk of the United States becomes stronger during the recessions.

Figure 5: Results of Out-of-Sample Forecasting - 1990:01 - 2015:11 *Figure 5 presents cumulative squared differences of forecasts errors for monthly excess return between the historical average baseline model ( $r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1}$ ) and the competitive model that uses lagged U.S. tail risk ( $r_{i,t+1} = \beta_{i,0} + \alpha_{i,US}Tail_{US} + \epsilon_{i,t+1}$ ).*

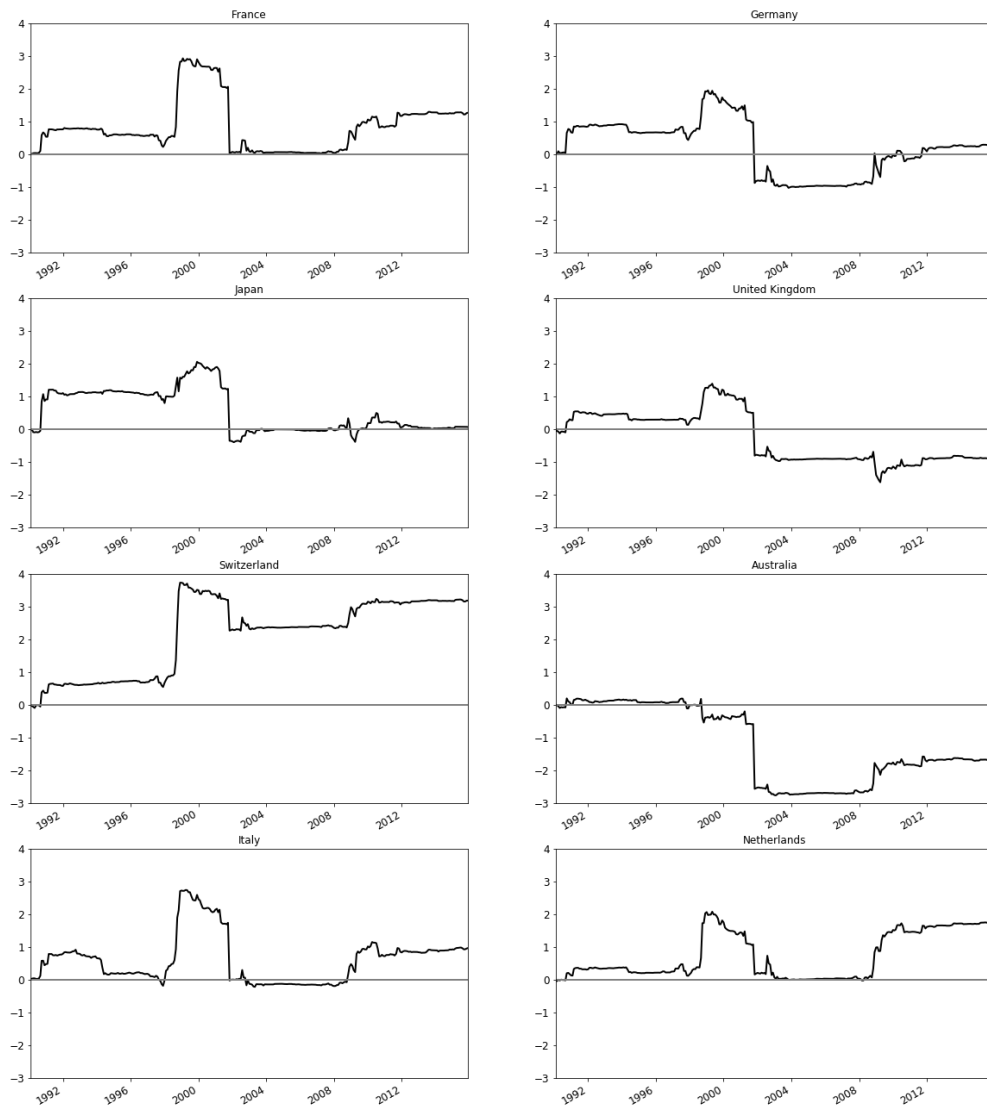
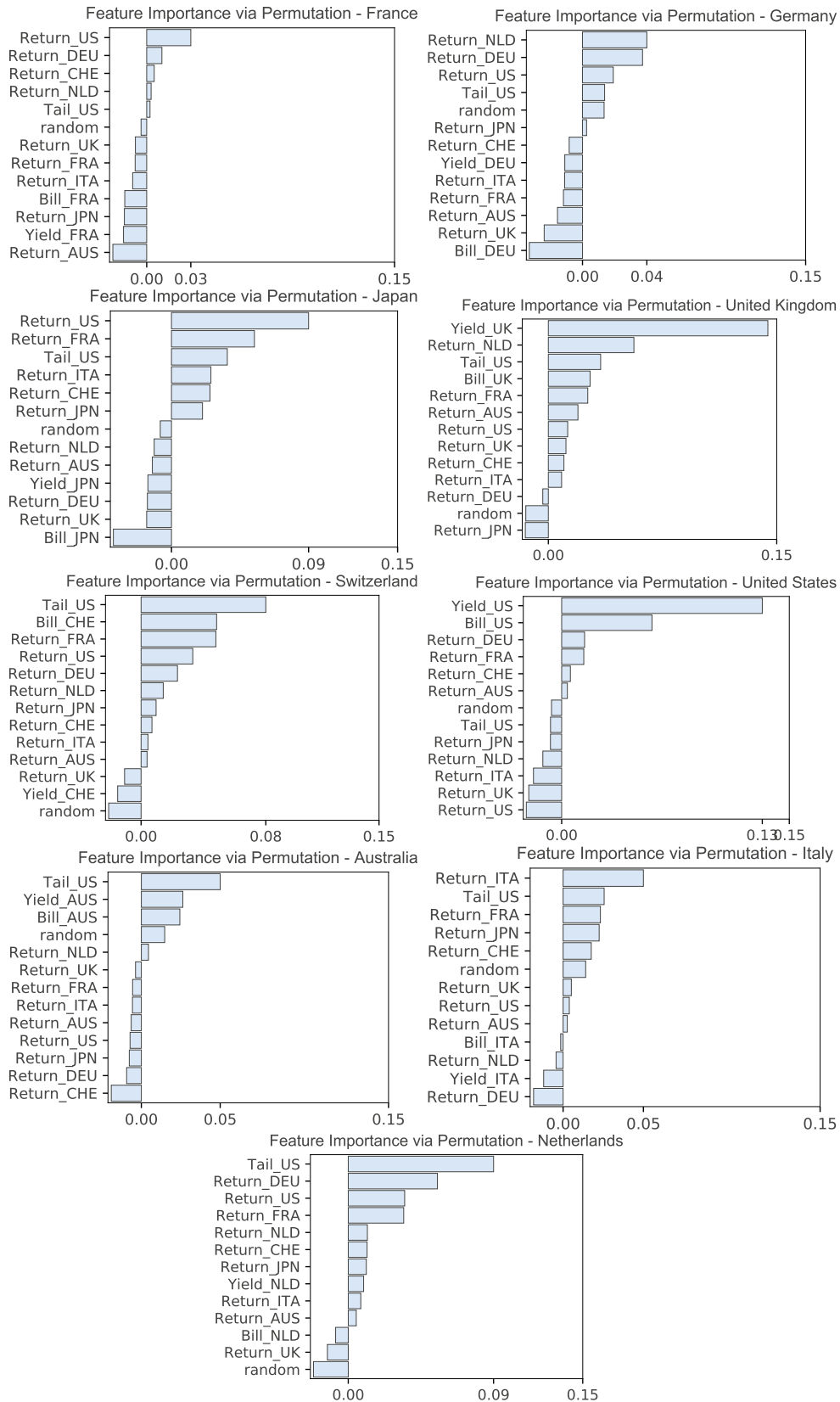


Figure 6 plots the permutation-based variable importance. We add a random column, which is generated from random numbers. Any features with negative importance or less important than the random column should be tossed out. All non-U.S. countries select U.S. tail risk as regressors, since it is more important than the random column. In most of the countries, U.S. tail risk lies in the top three important variables. In particular, for Switzerland, Australia and the Netherlands, it is the most important variable. In addition, six of nine countries select U.S.

return as regressors, which agrees with Rapach et al. (2013).

Figure 6: Variable Importance Figure 6 plots the permutation-based variable importance (Breiman (2002)). "Random" is a series of random number. Any features less important than the random column could be tossed out.



There are two main findings compared to U.S. returns. First, U.S. tail risk is more important than U.S. returns in the United Kingdom, Switzerland, the United States, Australia and Italy. In another word, U.S. returns are more important than U.S. tail risk in only three countries (France, Germany, Japan). To some extent, we can say that the predictive power of U.S. tail risk is stronger than that of U.S. returns. Second, We find that lagged U.S. tail risk is the most important variable for Australian returns. However, we do not see any significant predictive power of lagged U.S. tail risk in predictive regression model and adaptive elastic net estimation. The possible explanation could refer to the intuition behind the approaches. The predictive regression and adaptive elastic-net estimation are parametric models. However, the permutation-based variable importance is constructed at non-parametric level. Only a random subset of variables and observations is used to construct each individual tree. In total, we have many "trees". Therefore, the importance is expected to be rather stable..

## 5.2.2 Predictive Ability of Lagged International Tail Risk: 2006:02 - 2015:11

This section reports the results for testing the lagged international tail risk using data from February 2006 to November 2015 based on the availability of international tail risk. In addition, by using different sample periods we can test the robustness of predictive power for lagged tail risk of the U.S..

Table 16 reports the OLS estimates in the benchmark predict regression model using a small dataset from February 2006 to November 2015. Because we use fewer observations, results might be not very robust. We see that the  $R^2$  statistics become larger than before and the sign of  $\hat{\beta}_{i,d}$  does not consistent any more.

Table 17 reports the pairwise Granger causality test results. There are 22 out of 30 negative  $\alpha_{i,j}$  estimates, Among which six are significant at 10% level. In particular, returns in France and Switzerland can be significantly predicted by lagged U.S. tail risk. However, we observe stronger predictive power for lagged Japanese tail risk in terms of the magnitude of estimates. Two out of five  $\hat{\alpha}_{i,JPN}$  are significant with absolute value above 5.5 at conventional level  $i = \text{FRA}, \text{DEU}$ . The pooled  $\alpha_{i,US}$  estimate is still significant, and the magnitude of average of the  $\hat{\alpha}_{i,US}$  (-0.275) is larger than 0.25. In contrast, we observe no insignificant  $\hat{\alpha}_{US,j}$  for  $j = \text{countries other than the United States}$ . This reflects the lagged tail risk of countries other than the United States hardly predict its returns. Overall, the lagged U.S. tail risk consistently dominates the predictability of international returns in small dataset, while U.S. returns can not be predicted by lagged tail risk of non-U.S. countries.

Table 16: Benchmark Predictive Regression Model Estimation - 2006:02: to 2015:11

Table 16 reports the results of the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1}, \quad i \neq US,$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \epsilon_{i,t+1}, \quad i = US,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, 3-month Treasury bill rate and log country dividend yield, respectively. In column (2), (3), (4) and (5), heteroskedasticity-robust  $t$ -statistics in brackets test for  $H_0: \beta_{i,b} = 0$  against  $H_A: \beta_{i,b} < 0$ ;  $H_0: \beta_{i,d} = 0$  against  $H_A: \beta_{i,d} > 0$ ;  $H_0: \beta_{i,r} = 0$  against  $H_A: \beta_{i,r} > 0$ ;  $H_0: \beta_{i,US} = 0$  against  $H_A: \beta_{i,US} > 0$ . In column (6), heteroskedasticity-robust  $\chi^2$  statistics in brackets test for  $H_0: \beta_{i,b} = \beta_{i,d} = \beta_{i,r} = \beta_{i,US} = 0$ . The pooled results are estimated under the restrictions that  $\beta_{i,b} = \bar{\beta}_b$ ,  $\beta_{i,d} = \bar{\beta}_d$ ,  $\beta_{i,r} = \bar{\beta}_r$  and  $\beta_{i,US} = \bar{\beta}_{US}$  for all  $i$ . Based on wild bootstrapped  $p$ -value, constructing by following [Rapach et al. \(2013\)](#), \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)
$i$	$\hat{\beta}_{i,b}$	$\hat{\beta}_{i,d}$	$\hat{\beta}_{i,r}$	$\hat{\beta}_{i,US}$	$R^2$
United States	-0.136 (-0.675)	2.194 (0.472)	0.180* (1.492)	-	3.912% (6.257)
France	-0.709* (-2.113)	0.141 (0.046)	0.114 (0.650)	0.854 (0.041)	7.409% (9.805)
Germany	-0.682 (-1.775)	-0.209 (-0.057)	0.064 (0.408)	12.426 (0.592)	7.434% (9.378)
Japan	-5.018 (-1.882)	0.924 (0.458)	0.188* (1.712)	-0.522 (-0.036)	9.884%* (10.626)
United Kingdom	-0.376 (-1.667)	0.745 (0.187)	-0.189 (-0.958)	21.107 (1.253)	5.475% (6.550)
Switzerland	-1.202* (-2.211)	-2.756 (-1.264)	0.208 (1.152)	-10.356 (-0.736)	10.661% (9.137)
Pooled	-0.561* (-1.852)	0.647 (0.116)	0.136* (0.766)	1.374 (0.067)	6.155%* (10.501)

Table 18 reports the pooled OLS results by means of measuring the relations on average. However, we see no significant beta estimates. Note that the pooled estimation uses observations across six countries, which reduces the impact of "small  $T$ " problem and tests for all countries at the same time. In addition, we are able to incorporate the information in high variability of data, which is often ignored or even not exist in individual time-series ([Hicks and Janoski \(1994\)](#)).

Table 19 reports the adaptive elastic-net estimation results. Four of five countries select lagged U.S. tail risk as predictors for their return (except Japan), among which France, Germany and Switzerland can be significantly predicted by lagged U.S. tail risk. In addition, we observe that lagged Japanese and Swiss tail risk are statistically significant predictors for returns of the United States. The adaptive elastic net estimation results are different from pooled estimation results. The possible explanation could be in two ways. The first one is that we use individual time series when applying LASSO for each countries. However, we incorporate the homogeneity, which is often ignored or even not exist in individual time-series when pooling the data. The second one is that two approaches impose different constrains. LASSO shrinks some parameters

to zero while pooled estimation restrict the parameters of different countries to be the same.

Table 17: Results of Pairwise Granger Causality Test - 2006:02 to 2015:11

Table 17 shows the results for the model:

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \alpha_{i,i}Tail_{i,t} + \alpha_{i,j}Tail_{j,t} + \epsilon_{i,t+1}, \quad i \neq US,$$

$$r_{i,t+1} = \beta_{i,0} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \alpha_{i,i}Tail_{i,t} + \alpha_{i,j}Tail_{j,t} + \epsilon_{i,t+1}, \quad i = US,$$

where  $r_{i,t+1}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  represents the monthly country excess returns, 3-month Treasury bill rate and log country dividend yield, respectively. Heteroskedasticity-robust  $t$ -statistics in brackets test for  $H_0: \alpha_{i,j} = 0$  against  $H_A: \alpha_{i,j} < 0$ . "Average" is the mean of  $\alpha_{i,j}$  estimates in the column. For all  $i \neq j$ , the pooled results are estimated under the restrictions that  $\alpha_{i,j} = \bar{\alpha}_j$ . We construct the wild bootstrapped  $p$ -value by following Rapach et al. (2013), \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
$i$	$\hat{\alpha}_{i,US}$	$\hat{\alpha}_{i,FRA}$	$\hat{\alpha}_{i,DEU}$	$\hat{\alpha}_{i,JPN}$	$\hat{\alpha}_{i,UK}$	$\hat{\alpha}_{i,CHE}$
United States (US)		1.923 (0.364)	2.284 (0.420)	-5.145 (-1.356)	-0.390 (-0.089)	5.010 (0.638)
France (FRA)	-0.404* (-1.788)		4.184 (0.227)	-5.582* (-1.588)	-7.990 (-1.553)	5.367 (0.796)
Germany (DEU)	-0.342 (-1.335)	-28.688 (-1.376)		-7.671* (-2.100)	-8.213 (-1.277)	-0.064 (-0.006)
Japan (JPN)	-0.022 (-0.095)	-5.548 (-0.976)	-4.578 (-0.862)		-4.731* (-1.983)	-3.251 (-0.633)
United Kingdom (UK)	-0.249 (-1.314)	0.936 (0.177)	2.538 (0.455)	-3.360 (-1.150)		4.157 (0.662)
Switzerland (CHE)	-0.350* (-1.637)	-9.976* (-1.951)	-8.590 (-1.424)	-3.330 (-1.330)	-5.947 (-1.598)	
Average	-0.228	-9.739	-1.055	-5.018	-6.401	2.244
Pooled	-0.275* (-1.589)	-1.956 (-0.140)	0.398 (0.022)	-4.892* (-0.463)	-3.067 (-0.303)	1.678 (0.154)

Table 18: Pooled General Model Specification Estimation Results - 2006:02 to 2015:11

Table 18 reports the estimates of  $\bar{\alpha}_{i,j}$  (denoted by  $\hat{\alpha}_j$ ) in the predictive regression mode for  $i = 1, \dots, N - 1$  and  $i \neq US$ :

$$r_{i,t+1} = \beta_{i,0} + \bar{\beta}_b Bill_{i,t} + \bar{\beta}_d Yield_{i,t} + \bar{\beta}_r r_{i,t} + \bar{\beta}_{US} r_{US,t} + \bar{\alpha}_{AR} Tail_{i,t} + \sum_{j \neq i} \bar{\alpha}_j Tail_{j,t} + \epsilon_{i,t+1},$$

where  $r_{i,t+1}$ ,  $\beta_{i,i}Tail_{i,t}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  is the monthly excess return, the country tail return, the 3-month Treasury bill rate and the log dividend yield, respectively. Following Rapach et al. (2013), we compute the bias-corrected wild bootstrapped 90 % confidence intervals in brackets. \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)
$\hat{\alpha}_{US}$	$\hat{\alpha}_{FRA}$	$\hat{\alpha}_{DEU}$	$\hat{\alpha}_{JPN}$	$\hat{\alpha}_{UK}$	$\hat{\alpha}_{CHE}$
-0.187 [[-0.557,0.181]	-3.227 [-9.441,3.059]	4.170 [-4.352,12.713]	-4.822 [-9.864,0.152]	-4.253 [-10.467,2.104]	3.183 [-8.283,14.879]



Table 19: Adaptive Elastic-Net Estimation Results - 2006:02 to 2015:11

Table 19 reports the estimates of  $\alpha_{i,j}$  (denoted by  $\hat{\alpha}_{i,j}^*$ ) in the adaptive elastic-net model:

$$r_{i,t+1} = \beta_{i,0} + \alpha_{i,t}Tail_{i,t} + \sum_{j \neq i} \alpha_{i,j}Tail_{j,t} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \beta_{i,US}r_{US,t} + \epsilon_{i,t+1}, \quad i \neq US,$$

$$r_{i,t+1} = \beta_{i,0} + \alpha_{i,i}Tail_{i,t} + \sum_{j \neq i} \alpha_{i,j}Tail_{j,t} + \beta_{i,b}Bill_{i,t} + \beta_{i,d}Yield_{i,t} + \beta_{i,r}r_{i,t} + \epsilon_{i,t+1}, \quad i = US,$$

where  $r_{i,t+1}$ ,  $\beta_{i,i}Tail_{i,t}$ ,  $Bill_{i,t}$  and  $Yield_{i,t}$  is the monthly excess return, the country tail return, the 3-month Treasury bill rate and the log dividend yield, respectively. "Average" is the mean of  $\hat{\alpha}_{i,j}^*$  estimates in the column. Following Rapach et al. (2013), we compute the bias-corrected wild bootstrapped 90 % confidence intervals and report in brackets. \* presents the 10% significance of the estimate.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
$i$	$\hat{\alpha}_{i,US}^*$	$\hat{\alpha}_{i,FRA}^*$	$\hat{\alpha}_{i,DEU}^*$	$\hat{\alpha}_{i,JPN}^*$	$\hat{\alpha}_{i,UK}^*$	$\hat{\alpha}_{i,CHE}^*$
United States (US)		0	0	-4.614* [-11.254, -0.841]	-2.090 [-10.119, 3.212]	8.233* [4.886, 18.400]
France (FRA)	-0.208* [-0.474, -0.088]		3.296 [-0.545, 10.079]	-4.015 [-9.933, 0.036]	-4.762 [-12.649, 0.973]	0
Germany (DEU)	-0.188* [-0.454, -0.085]	0		-6.660* [-13.522, -2.206]	-5.709 [-15.914, 1.788]	0
Japan (JPN)	0	-1.221 [-8.817, 6.866]	0		-3.215 [-8.138, 0.106]	0
United Kingdom (UK)	-0.011 [-0.074, 0.033]	0	0	0		0
Switzerland (CHE)	-0.082* [-0.198, -0.019]	-3.002* [-9.444, -0.929]	0	-1.767 [-4.924, 0.348]	-1.794 [-7.324, 1.931]	
Average	-0.098	-0.845	0.653	-3.411	-3.914	1.647

Table 20 reports the results of out-of-sample test. The in-sample period ranges from February 2006 to December 2010. In column (2) and (5), all of non-U.S. countries having positively sizable  $R^2$  statistics. It means that competing model generates the smaller  $MSFE$ , comparing to the historical average model. The 1% bootstrapped critical value under the null hypothesis that country returns can not be predicted is equal to 2.49. Germany has the largest critical value of 1.619 across countries. Column (3) and (6) report the pooled  $R^2$  statistics under the restriction:  $\alpha_{i,US} = \bar{\alpha}_{US}$  for non-U.S. countries. Three of five  $R^2$  statistics are positively significant at the conventional level. It provides evidence of the better out-of-sample performance of the competing model than baseline forecasts.

Table 20: Out-of-Sample Performance of Lagged U.S. Tail Risk - 2011:01 to 2015:11

Table 20 shows out-of-sample  $R^2$  and  $R_{OS}^2$  statistics (Campbell and Thompson (2008)) by measuring the difference in mean-squared forecast error between historical average model and the competitive model which includes U.S. returns in the past month. Baseline model and competing model are shown as follows:

$$r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1},$$

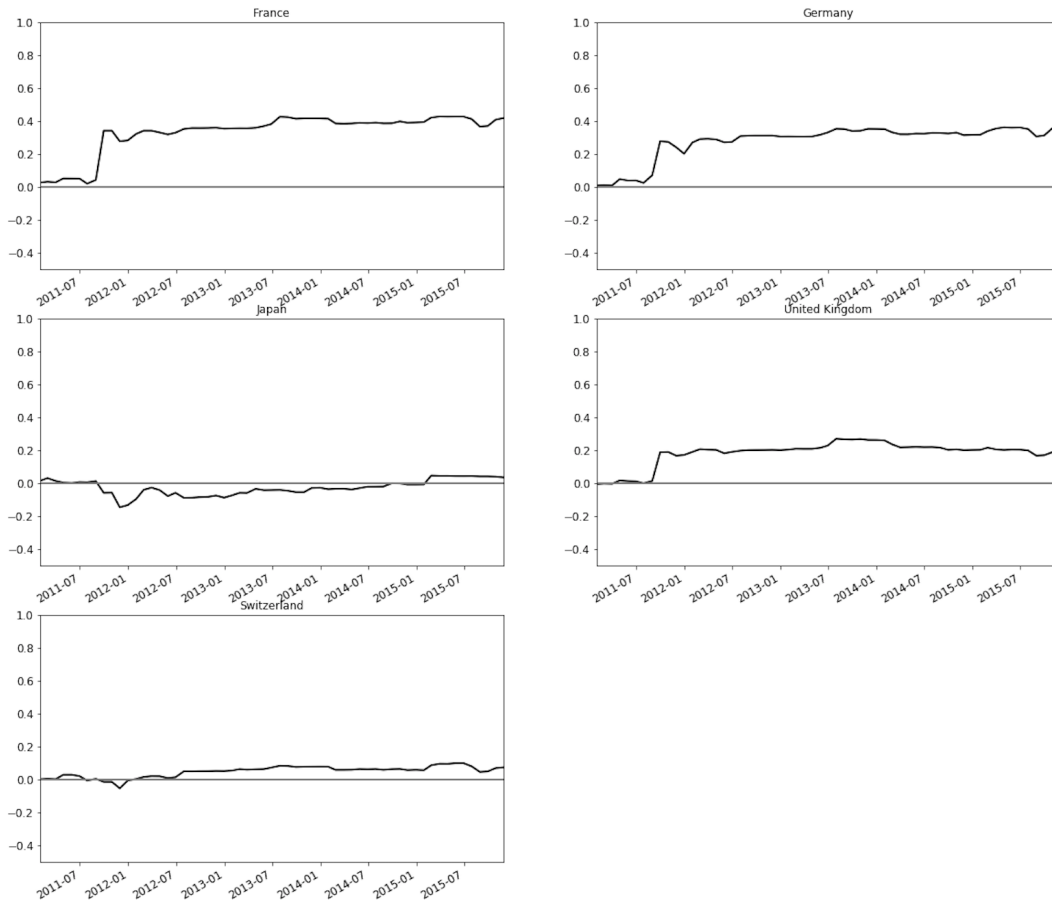
$$r_{i,t+1} = \beta_{i,0} + \alpha_{i,US}Tail_{US,t} + \epsilon_{i,t+1},$$

where  $r_{i,t+1}$  is the country monthly excess returns at month  $t$ . The results in column (3) and (6) are obtained by imposing the restriction on the competing model:  $\alpha_{i,US} = \hat{\alpha}_{US}$  for non-U.S. countries.  $MSFE$  – adjusted statistics in brackets from Clark and West (2007) test for the null hypothesis of zero  $R_{OS}^2$  against positive  $R_{OS}^2$ . \* presents the 10% significance of the estimate. "Average" is the mean of  $R_{OS}^2$  across 5 countries. The in-sample period is from February 2006 to December 2010.

(1)	(2)	(3)	(4)	(5)	(6)
$i$	$R_{OS}^2$	$R_{OS,pooled}^2$	$i$	$R_{OS}^2$	$R_{OS,pooled}^2$
France	4.288%* (1.371)	3.683%* (1.351)	Germany	3.145%* (1.619)	2.608%* (1.605)
Japan	0.329% (0.877)	0.255% (0.791)	United Kingdom	2.015% (1.112)	2.992% (1.125)
Switzerland	0.975%* (1.408)	1.069%* (1.407)	Average	2.151%	2.121%

Figure 7 depicts the cumulative differences in  $MSFE$  for the baseline model relative to the competing forecasts. For all of the countries, the competing model still generates a smaller  $MSFE$  relative to the baseline model. In particular, there are sizable forecast gains at the beginning of 2012.

Figure 7: Out-of-Sample Forecasting Results - 2010:01 - 2015:11 *Figure 7 presents cumulative squared differences of forecasts for monthly excess return between the historical average baseline model ( $r_{i,t+1} = \beta_{i,0} + \epsilon_{i,t+1}$ ) and the competitive model that uses lagged U.S. tail risk ( $r_{i,t+1} = \beta_{i,0} + \beta_{i,US}Tail_{US} + \epsilon_{i,t+1}$ ).*



## 6 Conclusion

This paper re-investigates the finding in [Rapach et al. \(2013\)](#) utilizing the most recent dataset and studies the role of lagged U.S. tail risk in international returns predictability. Following the methods in [Rapach et al. \(2013\)](#), our results show that lagged returns in the United States have robust and consistent predictive ability for non-U.S. returns, whereas lagged returns in non-U.S. countries still exhibit little predictive ability for U.S. returns. Regarding the tail risk, we conclude that U.S. tail risk in the past month displays economically sizable predictive ability for international returns. In the model specification, adaptive elastic net estimation results show that seven of eight non-U.S. countries select lagged tail risk of the United State as predictor, among which six countries returns can be economically predicted by lagged U.S. tail risk. Furthermore, results of variable importance show that for most of the countries, lagged U.S. tail risk belongs to the top three important regressors. In addition, lagged U.S. tail risk exhibits great out-of-

sample forecast gains in  $MSFE$  relative to the historical average forecasts, especially during Global Financial Crisis in 2008. It can be explained by the leading position of the United States' economy and information friction.

We also discuss the predictive power of non-U.S. tail risk. By replicating PPUT index, we construct non-U.S. tail risk from February 2006 to November 2015. The adaptive elastic net estimation results show that four of five countries select lagged U.S. tail risk as predictor, among which returns in France, Germany and the Netherlands can be statistically predicted by lagged U.S. tail risk. In contrast, non-U.S. tail risk does not display significant predictive ability for U.S. returns.

There are several limitations in this paper. First, we exclude the returns on Roll Days when replicating the PPUT index for other countries. Thus, those tail risk factors may be not very accurate to some extent. Second, we use a short sample from February 2006 to November 2015 when investigating the role of non-U.S. tail risk. The results may be plausible because only a small number of observations are used. Third, the predictive ability of tail risk in U.S. is strong using our dataset. However, the robustness remains unknown. The direction of further research could be examining the robustness of the predictive ability of U.S. tail risk factor by means of using different datasets. In addition, we could investigate whether the predictive power differs if we use non-option-implied tail risk.

## References

- Ang, A. and Bekaert, G. (2007). Return predictability: Is it there? *Reviews of Financial Studies*, 20:651–707.
- Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81:637–654.
- Breiman, L. (2001). Random forests. *Machine Learning*, 45:5–32.
- Breiman, L. (2002). Manual on setting up, using, and understanding random forests v3.1. [https://www.stat.berkeley.edu/~breiman/Using\\_random\\_forests\\_v3.1.pdf](https://www.stat.berkeley.edu/~breiman/Using_random_forests_v3.1.pdf).
- Campbell, J. Y. and Thompson, S. B. (2008). Predicting the equity premium out of sample: Can anything beat the historical average? *Review of Financial Studies*, 21:1509–1531.
- Clark, T. E. and McCracken, M. W. (2012). Reality checks and nested forecast model comparisons. *Journal of Business and Economic Statistics*, 30:53–66.

- Clark, T. E. and West, K. D. (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics*, 138:291–311.
- Friedman, Jerome, Hastie, T., and Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, 33:1–22.
- Gouçalves, S. and Lutz, K. (2004). Bootstrapping autoregressions with conditional heteroskedasticity of unknown form. *Journal of Econometrics*, 123:89–120.
- Goyal, A. and Welch, I. (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies*, 21:1455–1508.
- Groemping, U. (2009). Variable importance assessment in regression: Linear regression versus random forest. *The American Statistician*, 63(4):308–319.
- Hicks, A. and Janoski, T. (1994). Pooled time-series and cross-sectional analysis. *The Comparative Political Economy of the Welfare State*, Cambridge: Cambridge University Press:167–168.
- Hjalmarsson, E. (2010). Predicting global stock returns. *Journal of Financial and Quantitative Analysis*, 45:49–80.
- Ishwaran, H. (2007). Variable importance in binary regression trees and forests. *Electronic Journal of Statistics*, 1:519–537.
- Kandel, Shmuel, and F. Stambaugh, R. (1996). On the predictability of stock returns: An asset allocation perspective. *Journal of Finance*, 51:385–424.
- Kellard, Neil, M., Nankervis, J. C., and Papadimitriou, F. I. (2010). Predicting the equity premium with dividend ratios: Reconciling the evidence. *Journal of Empirical Finance*, 17:539–551.
- Kelly, B. and Jiang, H. (2014). Tail risk and asset prices. *Review of Financial Studies*, 27(10):2841–2871.
- Rapach, D. E., Strauss, J. K., and Zhou, G. (2013). International stock return predictability: what is the role of the united states? *Journal of Finance*, 68(4):1633–1662.
- Strobl, C., Boulesteix, A., Kneib, T., Augustin, T., and Zeileis, A. (2008). Conditional variable importance for random forests. *BMC Bioinformatics*, 9:307.
- Tibshirani and Robert (1996). Regression shrinkage and selection via the lasso,. *Journal of the Royal Statistical Society. Series B (Methodological)*, 58(1):267–288.

Wongswan, J. (2008). The response of global equity indexes to us monetary policy announcements. *Journal of International Money and Finance*, 28(2):344–365.

Zou, H. and Zhang, H. (2009). On the adaptive elastic-net with a diverging number of parameters. *Annals of Statistics*, 37:1733–1751.