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Master Thesis Quantitative Finance

Empirical Pricing Analysis of Caps and Swaptions using Multi-Factor Models

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Abstract

The aim of this paper is to price caps and swaptions using multi-factor term structure models in a low interest rate environment. We price these products within the Heath-Jarrow-Morton framework, condition on the term structure, and investigate the pricing errors to detect possible modeling biases. The volatility functions are modeled using principal component analysis of up to three factors using interest-rate or option data with either constant or time-varying parameters. We use weekly data of US swaps, caps and swaptions from 2013-2019. The results show that option-based estimation outperforms interest-rate based estimation, and that the multi-factor models generally outperform one-factor models in pricing. We find that pricing errors vary over time, and are related to the term structure shape and level, and moneyness, but not to time-to-maturity, option expiry, and swap tenor. We further find that the low interest rate environment has a negative influence on pricing errors.

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1 Introduction

The market for interest rate derivatives is very large and has rapidly expanded over the last decades. Among these interest rate derivatives are caps and swaptions. According to the International Swaps and Derivatives Association (ISDA), the notional outstanding for swaptions and interest rate options such as caps reached $30 and $12 trillion US Dollar (USD) respectively in 2014.[1] These derivatives can be used for hedging or speculating on interest rates. Swaptions are often used by investors that have a large portion of liabilities, such as insurers and pension funds, because they provide a hedge against rising rates whilst still being able to profit from declining rates. Interest rate caps are, among others, useful for borrowers paying a floating LIBOR that wish to limit their risk exposure to increasing rates. It is important for investors to properly price and hedge these products in order to successfully manage their risk exposure.

In the existing literature, research has been done on the modeling of interest rate derivatives. This followed after the extensive literature on term structure models. A good term structure model should be able to price and hedge interest rate derivatives. The majority of the interest rate derivatives research has been done around the 1990s, this was mainly theoretical due to the difficulty in obtaining data.[2] In the early 2000s more empirical research came to light.[3] The empirical papers distinguish themselves from each other in that they use a different modeling framework, estimation approach, underlying instrument, and/or evaluation criteria.

The direction of this paper is similar to Driessen et al. (2003), who analyze the pricing and hedging performance of caps and swaptions in the Heath et al. (1992) (HJM) framework. They use both an interest-rate-based and option-based estimation method and assume that the interest rates follow either a Gaussian or log-normal distribution. The parameters of the volatility functions are estimated with either Principal Component Analysis (PCA) or Generalized Method of Moments, using a time-series data set from 1995 to 1999. The pricing and hedging performance are both analyzed in an out-of-sample setting. Our approach for pricing purposes is similar, except we only use the models that assume a Gaussian distribution on interest rates, and only use PCA to estimate volatility functions.

In this paper we investigate which combination of volatility function model and estimation method has a good pricing and hedging performance for caps and swaptions within the HJM framework. We investigate up to three factor models for the volatility functions, estimated

[1]Source: https://www.isda.org/a/qJEDE/isda-final-2014.pdf.
[2]See, among others, Amin and Morton (1994), Brace and Musiela (1994a) and Heath et al. (1992).
[3]For example Driessen et al. (2003), Gupta and Subrahmanyam (2005), and Falini (2010).

interest-rate-based or option-based, with constant or time-varying parameters. We further investigate which economic factors (e.g. time-to-maturity, option maturity (swaptions), moneyness (caps), and interest rate level) influence the pricing and hedging errors of models using an error regression.

The main contribution of this paper is the application of an existing pricing framework for caps and swaptions using a recent data set that is characterized by a low interest rate environment. This paper extends on Driessen et al. (2003) in various ways. First, we analyze caps both at-the-money (ATM) and out-of-the-money (OTM), whereas Driessen et al. (2003) only looks at ATM caps. Second, we perform a model forecast comparison test using a novel panel data approach based on Diebold and Mariano (2002) tests using a bootstrapped sampling distribution.[4] Third, we further analyze the pricing errors using an approach similar to Gupta and Subrahmanyam (2005), which has not yet been done on the models used by Driessen et al. (2003). Fourth, we investigate the effect of the low interest rate environment on the pricing errors by linear regressions using OLS, LASSO, and ridge estimation methods. Finally, we perform an in-sample test to investigate the model restrictions.

The main ingredient for pricing derivatives within the HJM framework is the volatility function. The volatility functions in this paper are estimated using PCA (interest-rate-based estimation), with an additional scaling factor for option-based estimation that uses cross-sectional option data. This estimation procedure can use constant parameters, in which we use the first half of the sample as estimation window, or time-varying parameters, in which we use a rolling window of 40 weeks. We subsequently price the caps and swaptions out-of-sample, conditional on the term structure. Caps and swaptions are priced using the derivative pricing formulas of Brace and Musiela (1994a), the implementation of these formulas is not necessarily straightforward. We compare the forecasting performance of the models using a variation on Diebold and Mariano (2002) tests. We perform a Diebold-Mariano test on every product (cap or swaption) for each combination of models and assess the percentage of products in which one model forecast outperforms the other. This allows us to find the model that has the best forecasting performance for the largest amount of products. The Diebold-Mariano statistics are tested against a bootstrapped sampling distribution. The pricing errors in terms of Black implied volatility are regressed on option properties and economic variables in order to find possible sources of modeling bias. We further regress absolute pricing errors on the interest rate level and control variables to assess the effect of the low interest rate environment on pricing

[4] I have tried to find precedents of a similar approach in the literature, but did not find any.

errors; this regression uses OLS, LASSO, and ridge estimation methods. In this research we use a balanced panel data set obtained from Bloomberg that ranges from January 2013 to March 2019 with weekly intervals.

We find that the option-based estimation method outperforms the interest-rate estimation method in most situations. The multi-factor models generally outperform the one-factor models in pricing. The conditional pricing forecasts for caps are quite accurate, but not so much for swaptions. The pricing errors of all products are mainly caused by possible model deficiencies with respect to the term structure shape and level. We find that pricing errors vary over time, however, the time-to-maturity, option expiry, and swap tenor do not influence the pricing errors. The low interest rate environment has a negative influence on pricing errors. The in-sample pricing analysis reveals that a near-perfect pricing prediction is unlikely in this sample using PCA based volatility functions.

This paper is organized as follows. Section 2 contains an overview of existing literature regarding the pricing of interest rate caps and swaptions. In Section 3, the research methodology is discussed. In Section 4, the data used in this research is described. Section 5 contains the results of this research. In Section 6 the conclusions are discussed.

2 Literature

In the current literature a large part of interest-rate-derivative pricing models are modeled with the Heath et al. (1992) framework based on forward rates. This framework has the ability to model the entire yield curve exactly. Amin and Morton (1994) use this framework to price Eurodollar futures and options. They use various term structure models with an option-based estimation method. They find that two-parameter models provide a better fit to prices, both in-sample and out-of-sample, but their estimates are less stable. Their one-parameter models, however, are more robust and profitable in a trading strategy exercise. Flesaker (1993) also uses the HJM framework to price Eurodollar futures and options but with focus on cross-sectional pricing. Bühler et al. (1999) investigate various interest-rate option valuation models with an interest-rate-based estimation method for warrants in the German market within the HJM framework. Their one-factor forward rate model with linear proportional volatility has the most robust pricing performance and outperforms all of their other models. Similar to these papers, we also take the HJM framework as starting point for our pricing exercise.

Option pricing is not limited to the forward rate based HJM framework. Moraleda and

Pelsser (2000) compare the performance of spot and forward rate models in pricing caps and floors. They find that traditional spot rate models, such as Black and Karasinski (1991) and Hull and White (1994), provide the best fit to market prices both in- and out-of-sample.[5] Longstaff et al. (2001) price swaptions and caps using a string market model. Their paper is cross-sectional in essence but still provides useful theoretical results. They estimate their model based on swaption prices which are then, in turn, used to price caps. They use the property that a cap can be seen as a portfolio of options on forward rates, and a swaption can be seen as an option on a portfolio of individual forward rates. Using swaption prices, and the aforementioned property, they estimate an implied covariance matrix that is used to price caps. They find that caps and swaptions are not always priced correctly towards each other which might imply arbitrage opportunities. Collin-Dufresne and Goldstein (2001) argue that these mispricings (but also the mispricings of e.g. Jagannathan et al. (2003) and Driessen et al. (2003)) can be explained due to modeling restrictions regarding the correlation structure. Further, multiple papers use LIBOR market models, among which Brace et al. (1997) and Gupta and Subrahmanyam (2005). The latter, especially, shows that LIBOR market models calibrated on at-the-money options work well in cross-sectionally pricing out-of-the-money options.

This paper takes the same direction as Driessen et al. (2003) and Gupta and Subrahmanyam (2005). Driessen et al. (2003) test various HJM factor models for pricing and hedging ATM caps and swaptions. They use an option-based and interest-rate-based estimation method and find that the option-based prediction results are better overall. A regression-based hedging technique using a nonlinear regression model with time-varying parameter estimates yields the best hedging results. In terms of hedging performance, they find that the amount of available instruments (in their case zero coupon bonds) is more important than the pricing model used, which they test using bucket hedging. They further find that when predicting prices, a multi-factor PCA model with time-varying parameters works best. Gupta and Subrahmanyam (2005) test the pricing and hedging of caps and floors using various spot rate and forward rate models. In terms of out-of-sample pricing accuracy their one-factor log-normal forward rate model performs best. They find that one-factor models are better in pricing due to more parameter stability compared to two-factor models. In terms of hedging, they find that two-factor models work well, they claim this is due to the second factor representing the yield curve dynamics over time, which is important for hedging. They further find that time-varying parameters work

[5] The authors note that their out-of-sample tests are not very formal and unable to capture slow mispricing corrections.

well in pricing, where the time-varying aspect helps fit the volatility term structure, but not in hedging, where stable parameter estimates are more important.

A sizable part of the literature focuses on the amount of factors necessary to properly price and hedge interest-rate derivatives. Fan et al. (2001) find that one and two-factor models are capable of accurately pricing swaptions, whereas multi-factor models are better for hedging. These results are in line with Gupta and Subrahmanyam (2005) and Driessen et al. (2003). Jagannathan et al. (2003) analyze multi-factor Cox et al. (1985) (CIR) models. They find that three-factor models provide a good fit to the term structure. However, in pricing caps and swaptions they find that their three-factor model is misspecified and does not perform well. Heidari and Wu (2001) find that three principal components from both the yield curve and interest-rate-option data (yielding six factors total) are needed to explain movements in the implied volatility surface of swaptions. Subsequently, Heidari and Wu (2002) propose a framework which first models the yield curve dynamics, and in turn prices interest rate derivatives. The yield curve residuals, which are neglectable in yield curve modeling, prove to be important for pricing interest rate caps. De Jong et al. (2004) find that option prices imply a covariance matrix that differs from the covariance matrix in the underlying data. This difference results in the mispricing of caps and swaptions. In this paper we also compare the pricing performances of our single-factor and multi-factor models.

Empirical results show that interest rate derivatives contain a humped shape volatility structure. Moraleda and Vorst (1997) and Ritchken and Chuang (2000) price American interest rate options using a single factor Gaussian model that allows for a humped shape volatility structure. Mercurio and Moraleda (2000) derive an HJM-based interest rate model that also allows for a humped shape volatility. Their model has a good out-of-sample performance when using an option-based estimation method in pricing caps and floors. Falini (2010) also prices caps using multi-factor HJM models containing a humped volatility. He uses the Kalman filter to estimate his models, this has the advantage of using both the cross-sectional correlations between yield curves as the autocorrelation within each yield curve. All of his results are obtained by merely using interest-rate-based estimation. He finds that humped volatility models perform well in pricing caps.

3 Methodology

This section contains the methodology of the research. In Section 3.1, we introduce the HJM framework along with the volatility function models and pricing formulas. In Section 3.2, we explain the empirical implementation of the yield curve and parameter estimation methods. Section 3.3 contains the setup of the pricing exercise and evaluation.

3.1 Heath-Jarrow-Morton Framework

In this section we introduce the HJM framework. Section 3.1.1 contains the theoretical aspects of the HJM framework starting from the bond price function to the forward rate process. In Section 3.1.2 we introduce the volatility function used in the pricing model. And finally, Section 3.1.3 contains pricing formulas for caps and swaptions specific for the HJM framework.

3.1.1 Theoretical Framework

The pricing of caps and swaptions are modeled within the HJM (Heath et al., 1992) framework. The HJM framework has the advantage that it is able to model the entire curve. The essence of this framework will be repeated in this section based on the notation of Brace and Musiela (1994b).

Denote $f(t,T)$ as the forward rate at time t with forward maturity T, and $P(t,T)$ as the price of a zero coupon bond at time t with maturity T, where $t \leq T$. We can write the price of a zero coupon bond in terms of forward rates as

$$P(t,T) = \exp\left\{ - \int_t^T f(t,u)du \right\}, \tag{1}$$

and taking the natural logarithm and differentiating both sides yields the instantaneous forward rate

$$f(t,T) = -\frac{\partial \log P(t,T)}{\partial T}. \tag{2}$$

Under the true probability distribution (physical measure $\mathbb{P}$) it can be shown that the bond price process is as

$$dP(t,T) = P(t,T)\bigg(f(t,t)dt - \bigg(\int_t^T \sigma(t,u)'du \bigg) dW(t) \bigg), \tag{3}$$

and the instantaneous forward rate process is as

$$df(t,T) = \left(\sigma(t,T)'\left(\int_t^T \sigma(t,s)ds\right)\right)dt + \sigma(t,T)'dW(t). \tag{4}$$

Taking the drift separately and rewriting the second term yields

$$df(t,T) = \alpha(t,T)dt + \sum_{i=1}^{K} \sigma_i(t,T)dW_i(t), \tag{5}$$

in which

$$\alpha(t,T) = \sigma(t,T)'\left(\int_t^T \sigma(t,s)ds\right), \tag{6}$$

where $\sigma_i(t,T)$ is the volatility function of factor i, K the amount factors modeled, and $W_i(t)$ a $\mathbb{P}$-Brownian motion. It follows from equation (4) that the forward curve dynamics only depend on the volatility functions. Note that the drift and volatility functions need to satisfy weak regularity conditions that can be found in for example Baxter and Rennie (1996). The volatility function $\sigma(t,T)$ in a K-factor model looks as follows

$$\sigma(t,T) = \begin{pmatrix} \sigma_1(t,T) \\ \vdots \\ \sigma_K(t,T) \end{pmatrix}, \tag{7}$$

and can be modeled as explained in Section 3.1.2.

3.1.2 Volatility Functions

As mentioned earlier, in the HJM-framework we only need to model the volatility function $\sigma_i(t,T)$. For simplicity, let the volatilty functions be deterministic functions that only depend on the time to maturity such that $\sigma_i(t,T) = \sigma_i(T-t)$. This simplification is common in the literature and able to provide reliable prices for at-the-money European options, given that the volatility functions are well-designed (Brace and Musiela, 1994b).

The volatility function used is taken from Driessen et al. (2003). This is a principal component analysis (PCA) based factor model, which is defined as

$$\sigma_i(T-t) = g_i(T-t), \quad i = 1,2,3. \tag{8}$$

This model is selected based on its pricing and hedging performance in Driessen et al. (2003) (for

$i = 2, 3$). The volatility function can be estimated using PCA with either interest-rate-based or option-based estimation, as discussed in Section 3.2.2 and 3.2.3.

3.1.3 Pricing Formulas

Caps and swaptions within the Gaussian HJM-framework can be priced with the formulas as defined in Brace and Musiela (1994a). The notation here is similar to Brace and Musiela (1994b) and reported for convenience.

The pricing formula for a cap at time t paying n caplets at time T_j for $j = 0, ..., n-1$ with a notional equal to 1 is

$$\text{Cap}(t) = \sum_{j=0}^{n-1} \Big(P(t, T_j)\Phi(-h_j(t)) - (1 + k\delta)P(t, T_{j+1})\Phi(-h_j(t) - \zeta_j(t)) \Big), \tag{9}$$

which is essentially the sum of discounted caplets written as put options on zero coupon bonds. Here $\delta = T_{j+1} - T_j$ denotes the period between payments, k denotes the strike price, $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution, and payments are settled in arrears. Further,

$$h_j(t) = \left(\log \frac{(1 + k\delta)P(t, T_{j+1})}{P(t, T_j)} - \frac{1}{2}\zeta_j(t)^2 \right) / \zeta_j(t), \tag{10}$$

and

$$\zeta_j(t)^2 = \text{Var}(\log P(T_j, T_{j+1}) | \mathcal{F}_t) = \int_0^{T_j - t} \left[\sum_{i=1}^{K} \left(\int_s^{s+\delta} \sigma_i(u) du \right)^2 \right] ds, \tag{11}$$

where $\mathcal{F}_t$ denotes the filtration at time t. The inner integral can be approximated with a Riemann sum as the volatility functions are linearly interpolated between quarterly estimations which is explained later in Section 3.2.2. The outer integral can be approximated using numerical integration.

The price of a payer swaption, again derived from Brace and Musiela (1994b), at time t is

$$\text{Swaption}_t = \int_{R^K} \max\Big\{ 0, P(t, T)\phi_K(x) - \sum_{i=1}^{n} C_i P(t, T_i)\phi_K(x + \gamma_i) \Big\} dx, \tag{12}$$

where

$$C_i = k\delta \text{ for } i = 1, ..., n-1, \quad C_n = 1 + k\delta, \tag{13}$$

and

$$\gamma_i'\gamma_j = \text{cov}(\log P(T,T_i), \log P(T,T_j)|\mathcal{F}_t) \\ = \int_0^{T-t}\left[\sum_{k=1}^{K}\left(\int_s^{T_i-T+s}\sigma_k(u)du\right)\left(\int_s^{T_j-T+s}\sigma_k(u)du\right)\right]ds, \tag{14}$$

with $\phi_K(x)$ the K-dimensional standard normal probability density function. Further, γ_i can be estimated by performing an eigendecomposition on the covariance matrix of log bondprices of varying maturities.[6] Equation (14) can be approximated similarly to equation (11). Equation (12) is approximated using numerical integration over the area between -5 and 5 for each dimension, which is sufficient for standard normal probability density functions.

3.2 Empirical Implementation

In this section we explain the empirical implementation of the model introduced in the previous section. In Section 3.2.1 we introduce the smoothed yield curve used for discounting cash flows. Section 3.2.2 and Section 3.2.3 contain the interest-rate-based and option-based estimation techniques of the volatility functions respectively.

3.2.1 Yield Curve Smoothing

In order to construct a forward interest rate curve we start by constructing a yield curve. The yield curve is smoothed based on Bliss (1997) with parameters similar to Driessen et al. (2003) which yields

$$P(t,T) = \exp\left\{\beta_1(T-t)+\beta_2(T-t)^2+\beta_3(T-t)^3+\beta_4\max(0,T-t-2)^3+\beta_5\max(0,T-t-4)^3\right\}. \tag{15}$$

This function provides a good fit to the actual yield curve, especially on a medium- and long-horizon. The inclusion of the last two maximization terms within the exponent allow the model to provide a smooth fit with a low error for these horizons. The beta values are estimated by performing a linear regression on the logarithm, where the dependent variable is based on observed yields. This regression is performed for each time t and is therefore cross-sectional in nature. This yields a bond price function that is defined for maturities for which there is no observed data or financial product. The instantaneous forward rate function can be constructed

[6]The eigenvalues resulting from the eigendecomposition are sometimes negative in practice, which is in contradiction with the positive semi-definiteness of a covariance matrix. These eigenvalues are multiplied by negative one to avoid imaginary numbers and only has a small effect on the swaption pricing performance.

by substituting equation (15) in equation (2), which yields

$$f(t,T) = -\Big(\beta_1 + 2\beta_2(T-t) + 3\beta_3(T-t)^2 + \mathbb{1}_{\{T-t>2\}}3\beta_4(T-t-2)^2 + \mathbb{1}_{\{T-t>4\}}3\beta_5(T-t-4)^2\Big). \tag{16}$$

3.2.2 Interest-Rate-Based Parameter Estimation of Volatility Functions

The interest-rate-based parameter estimation method is similar to that of Driessen et al. (2003). We approximate the covariance matrix of instantaneous forward rate changes by starting with equation (5) and setting the drift $\alpha(t,T) = 0$, this yields

$$\operatorname{cov}\big[df(t,T_i), df(t,T_j)\big] \approx \sum_{k=1}^{K} \sigma_k(T_i - t)\sigma_k(T_j - t)dt = \sum_{k=1}^{K} g_k(T_i - t)g_k(T_j - t)dt. \tag{17}$$

This approximation is justified as for weekly forward rate changes the drift is relatively small (Driessen et al., 2003). The first equality follows from the simplification introduced in Section 3.1.2, and assuming independent Brownian motions; the dt term arises due to the quadratic variation property of Brownian motions. The second equality follows from equation (8).

In order to estimate the $g_i(T-t)$ functions we start by taking weekly changes of instantaneous forward rates

$$\widehat{df}(t,T) = f(t,T) - f(t-1,T), \tag{18}$$

where the instantaneous forward rates are calculated as in equation (16). Next, we perform an eigendecomposition on the sample covariance matrix of weekly changes of instantaneous forward rates, $\hat{\Sigma}$, where the columns contain forward rate maturities, varying between 3 months and 15 years with quarterly intervals, and the rows contain observations. We sort the eigenvectors and eigenvalues in descending order, and select K factors, similar to the use of PCA. This yields

$$\hat{\Sigma} = \sum_{i=1}^{n} \delta_i v_i v_i' \approx \sum_{i=1}^{K} \delta_i v_i v_i', \quad K = 1, 2, 3, \tag{19}$$

where v_i is the eigenvector corresponding to eigenvalue δ_i, such that $\delta_1 > \delta_2 > ... > \delta_n$. This covariance matrix has element ij equal to

$$\operatorname{cov}\big[\widehat{df}(t,T_i), \widehat{df}(t,T_j)\big] = \hat{\Sigma}_{ij} \approx \sum_{k=1}^{K} \delta_k v_{k,i} v_{k,j}, \tag{20}$$

such that

$$\widehat{\sigma_k}(T-t) = \widehat{g_k}(T-t) = \sqrt{\delta_k} v_{k,T-t} \sqrt{252}, \tag{21}$$

where the last term comes from the Euler discretization of weekly data using daily observed implied volatilities. This function is made continuous by linearly interpolating between time to maturity values $T-t$.

Yield curves, but also forward curves, are often characterized by high correlations between yields. This property makes these curves suitable for data reduction techniques such as PCA. PCA has mainly been used on yield curves, e.g. Bühler et al. (1999), but in some cases also on (changes in) forward curves, e.g. Driessen et al. (2003). Litterman and Scheinkman (1991), who apply PCA on yield curves, interpret the first three factors as level, steepness, and curvature. This interpretation is not necessarily one-to-one with the factors of the forward curve, which is discussed in Section 5.1.

3.2.3 Option-Based Parameter Estimation of Volatility Functions

The volatility models can also be estimated using an option-based estimation, this method is similar to that of Driessen et al. (2003). The goal of the option-based estimation is to use observed option prices to improve volatility functions. This method is implemented due to volatility functions often having the right shape but not the right magnitude, a scale parameter can help improve the price predictions. We define the volatility function, as introduced in equation (8), for option-based estimation as

$$g_i(T-t) = \alpha_i \hat{g}_i(T-t), \quad i = 1, 2, 3, \tag{22}$$

where $\hat{g}_i(T-t)$ is estimated using interest-rate-based estimation as defined in equation (21). We estimate α_i, a scale parameter, such that the sum of squared residuals between observed option prices and model implied option prices is minimized. An α_i value equal to 1 for all i indicates that the interest-rate-based estimation is equal to the option-based estimation and therefore optimal in-sample.

The constant parameter models estimate the α_i's using panel data of the constant estimation period. This yields α_i values that provide a good on-average fit. We define the loss function as

$$\min_{\alpha} \sum_{t \in W} \sum_{j \in C} \left(P_{t,j}^{mkt} - P_{t,j}^{model}(\alpha) \right)^2, \tag{23}$$

which is minimized with respect to vector α which contains up to three elements. Here set W consists of the constant estimation window times, set C of ATM caps, $P_{t,j}^{mkt}$ is the observed market price in basis points, and $P_{t,j}^{model}(\alpha)$ the modeled option price in basis points as a function of α. The time-varying parameter models estimate the α_i's by calibrating the model to the observed option prices for each week and we therefore have an $\alpha^{(t)}$ vector for each week t. This is similar to regular calibration and has a loss function defined as

$$\min_{\alpha^{(t)}} \sum_{j \in C} \left(P_{t,j}^{mkt} - P_{t,j}^{model}(\alpha^{(t)}) \right)^2, \quad \forall t \in C, \tag{24}$$

which is minimized with respect to $\alpha^{(t)}$ and contains up to three elements for each time t.

All α_i's are optimized to ATM caps as we conjecture these to be the most stable. Using OTM caps could lead to biased scale parameter estimates as cap volatilities as a function of their strike price are usually not constant. Further, using caps and swaptions combined to optimize α_i's provided optimization problems that did not necessarily converge when minimized.

3.3 Pricing and Evaluation

In this section we discuss the pricing of caps and swaptions and its further evaluation. Section 3.3.1 contains the setup of the conditional pricing out-of-sample. In Section 3.3.2 we discuss the Diebold-Mariano based model forecast comparison test to compare the forecast accuracy of pricing models. Section 3.3.3 contains the methodology regarding the evaluation of pricing errors, and in Section 3.3.4 we introduce an in-sample pricing exercise to explore the limitations of the HJM framework regarding cap pricing.

3.3.1 Conditional Pricing Setup

In this section the setup of the out-of-sample h-period ahead conditionally pricing exercise is described. The setup is similar to that of Driessen et al. (2003). For all pricing predictions we take $h = 2$ weeks which is a commonly used horizon in bank risk management (Driessen et al., 2003).

The first step is to estimate the parameters of the volatility functions, as defined earlier, using yield data and/or option data up to time t, this yields volatility functions. A rolling window of 40 weeks of yield and option data is used to estimate the time-varying volatility function parameters, whereas a fixed estimation period is used to estimate the constant volatility function parameters. The fixed estimation period is equal to the first half of the sample (January

2013 up to January 2016), and the out-of-sample period spans the second half of the sample (January 2016 up to and including March 2019). The second step is to estimate the parameters of the bond pricing function using yield data at time $t+h$, as explained in Section 3.2.1. The volatility function and bond pricing function are plugged into the pricing formulas which results in a cap or swaption price estimate at time $t+h$. To summarize, given the new yield curve, what is our prediction for the prices of caps and swaptions.

The estimated prices are compared to the observed prices. A relative measure, the mean absolute percentage error, is used for ATM caps and swaptions. An absolute measure, the mean absolute error, is used for OTM caps due to observed OTM cap prices often being relatively close to zero.

3.3.2 Model Forecast Comparison Test

In order to test which model has the best out-of-sample pricing performance, we use Diebold and Mariano (2002) tests. This test is useful to compare predictive accuracy of model forecasts using time series data of forecast errors. The loss differential of this test is defined as follows

$$d_t = e_{1t}^2 - e_{2t}^2, \tag{25}$$

where e_{it} is the forecast error of model i at time t. The null hypothesis H_0 is the two forecasts have the same accuracy, and the alternative hypothesis H_1 is method 2 (model that provides e_{2t}) provides more accurate forecasts than method 1. As this is a one-sided test, the results will not be symmetric, however, this allows us to assess whether one model has a higher forecasting accuracy than another. It is important to note that the Diebold-Mariano test compares forecasts, and not models (Diebold, 2015). It is possible that the model that has more accurate forecasts, is not the model that is more likely to have generated the data.

As we use a panel data set, we cannot apply this test on our data directly, hence we make a small alteration. We perform a test on each individual product for each combination of models (constant or time-varying parameters, 1-3 factors, interest-rate-based or option-based), this yields 132 ($= 12 \cdot 11$) tests per product. A product is defined as a cap or swaption with certain properties. When comparing model forecasts we evaluate how often H_0 is rejected relative to the total amount of products. This allows us to assess the percentage of products in which one model outperforms the other.

The Diebold-Mariano test requires that the loss differential, d_t, is covariance stationary, this is also known as “Assumption DM” (Diebold, 2015). If this assumption holds then the test statistic is standard normal distributed. As the sampling distribution of the test statistic is unknown in practice, we use a bootstrapping procedure to estimate this distribution. This procedure is as follows: first we de-mean the loss differential data for the selected models and product, this ensures that the mean is zero which is the case given H_0 is true. Second, we apply the bootstrapping procedure by random sampling with replacement from the loss differential data and subsequently calculate the Diebold-Mariano test statistic, this provides us with an estimated sampling distribution of the test statistic. Lastly, we calculate the p-value of the Diebold-Mariano test statistic calculated using the full sample against the estimated sampling distribution. The advantage of bootstrapping is that it is asymptotically more accurate compared to assuming, in this case, the standard normal distribution (DiCiccio and Efron, 1996).

The Diebold-Mariano test further requires models to be non-nested as nested models can exhibit unwanted correlations between the model errors that can cause the test statistic to explode. This is only problematic in case of perfect correlation, nevertheless, the test remains asymptotically valid.

3.3.3 Pricing Prediction Error Evaluation

The pricing errors are further investigated with an approach similar to Amin and Morton (1994) and Gupta and Subrahmanyam (2005). This step is not performed by Driessen et al. (2003) and might therefore provide new findings. We linearly regress the pricing errors on option properties to examine the (possible) source of error. The regression for caps is almost equivalent to the one Gupta and Subrahmanyam (2005) use on pricing errors of caps and floors and is as follows

$$(IV_{mkt} - IV_{model})_t = \beta_0 + \beta_1 \text{LMR}_t + \beta_2 \text{MAT}_t + \beta_3 \text{ATMVol}_t + \beta_4 \text{r}_t + \beta_5 \text{Slope}_t + \epsilon_t, \tag{26}$$

for swaptions a slight alteration is made and looks as follows

$$(IV_{mkt} - IV_{model})_t = \beta_0 + \beta_1 \text{OPT}_t + \beta_2 \text{TEN}_t + \beta_3 \text{r}_t + \beta_4 \text{Slope}_t + \epsilon_t, \tag{27}$$

where IV_{mkt} is the Black implied volatility according to the market and IV_{model} the Black implied volatility according to our model. The first regressor, LMR, is the logarithm of moneyness ratio (par rate divided by strike rate) and measures the effect of being more in- or out-of-the-money. The first regressor for swaptions, OPT, measures the effect of the option maturity. The second regressor, MAT, measures the cap maturity effect, for swaptions this is replaced with, TEN, which measures the effect of the swap tenor. The third regressor, ATMVol, contains the implied volatility of an option with similar characteristics but ATM moneyness.[7] The fourth regressor, r_t, contains the observed 3-Month LIBOR rate percentage at time t. The fifth regressor, Slope, is defined as the difference in zero coupon bond price between a 5-year and 3-month maturity. We estimate both regressions using OLS and use White standard errors to draw inference on the significance of the coefficient estimates.

The goal of the regressions explained above is to assess whether certain models contain a modeling bias. We perform a second regression to assess the influence of the interest rate level on absolute modeling errors. In this situation we take the observed 5-year swap rate as proxy for interest rate level. The regression for caps looks as follows

$$|IV_{mkt} - IV_{model}|_t = \beta_0 + \beta_1 \mathrm{LMR}_t + \beta_2 \mathrm{MAT}_t + \beta_3 \mathrm{ATMVol}_t + \beta_4 \mathrm{IRLvl}_t + \epsilon_t, \tag{28}$$

and for swaptions we have

$$|IV_{mkt} - IV_{model}|_t = \beta_0 + \beta_1 \mathrm{OPT}_t + \beta_2 \mathrm{TEN}_t + \beta_3 \mathrm{IRLvl}_t + \epsilon_t, \tag{29}$$

where IRLvl_t is the 5-year observed swap rate at time t and $|\cdot|$ is the absolute value function. We estimate this regression using OLS with White standard errors, and other linear estimations with LASSO (least absolute shrinkage and selection operator) and ridge penalties. The LASSO penalty is defined as $||\beta||_1$ and the ridge penalty as $\frac{1}{2}||\beta||_2^2$, where $||\cdot||_1$ denotes the Manhattan norm and $||\cdot||_2$ denotes the Euclidian norm. These penalized estimation techniques reduce the variance of the estimators by introducing a bias. The advantage of these penalties is that the estimators often have a higher predictive accuracy. The LASSO penalty contains a regressor selection that makes the model more interpretable. The ridge penalty contains a form of shrinkage which reduces overfitting. In our case the penalized estimates can be compared to the OLS estimates, which gives us an indication of the predictive power of the estimates, combined with a significance test from OLS. The LASSO and ridge estimators are not tested for significance as they are biased by nature.

In a low interest rate environment rising rates are more dangerous to asset valuations than in a high rate environment. This is due to the often observed concave shape of the yield curve. A 1% increase in interest rates if the current rate is 1% decreases future cash flows (due to "heavier" discounting) more than an identical increase if the current rate is 5%. We conjecture

[7] Swaptions are only considered ATM, hence the moneyness and ATM volatilty regressors are omitted.

that this results in a higher demand for interest rate derivatives in a low rate environment, and therefore affect prices and thus mispricing. Further, a low interest rate level is more likely to reach negative values than a high interest rate level. A negative interest rate level, and especially negative forward rates, are more consequential as most of the pricing models used in practice are not suitable for this.[8] We conjecture that the usage of different models, due to rates being close to zero, influences the mispricing of caps and swaptions.

3.3.4 In-Sample Pricing

The usage of pricing formulas for caps within the HJM framework will not necessarily provide accurate prices. The full pricing potential of this framework can be shown with an in-sample pricing exercise. The goal of this exercise is to identify sources of mispricing from within the model. The in-sample pricing exercise aims to minimize the squared differences of observed prices and modeled prices (in basis points) by changing the volatility functions. Specifically, the optimizer can change the percentage volatility for each quarterly forward maturity between 0 and 10 years. The forward maturities in between these quarterly points are linearly interpolated. This set-up most closely resembles the optimal potential of the PCA models. The difference with the earlier mentioned option-based estimation is that in this setup the pricing is done in-sample, and the volatility function can take any shape and not just the shape provided by the interest-rate-based PCA models. This exercise is merely done for ATM caps as convergence is not guaranteed when increasing the amount of products and forward maturities, which is necessary for OTM caps and swaptions.

4 Data

The data of this research is split up into two parts: yield curve data and (interest rate) derivatives data. The yield curve data set consists of LIBOR rates with maturities varying between 1 and 6 months, and US swap rates with maturities varying between 1 and 15 years. The derivative data set consists of US caps and swaptions. The caps have a maturity between 1 and 10 years. The swaptions have an option duration between 1 month and 5 years, and a swap duration between 1 and 10 years. The combined data set contains daily observations ranging from January 2013 to March 2019, and is obtained from Bloomberg (Bloomberg Tickers in Appendix A). The daily data is converted to weekly data by taking the values on Friday of each

[8]For example the Black model or any other model using a log-normal distributed forward rate or natural logarithm of the forward rate.

Table 1: Yield Curve Fit

Rates	Avg. Error	Avg. Abs. Error	Avg. of Weekly Maximum Error
Money Market	-10.1 bps	14.6 bps	7.4 bps
Swap	-0.1 bps	0.7 bps	1.8 bps

Note: Yield curve fit using weekly data ranging from January 2013 to March 2019 (326 observations). An error is defined as the difference between the observed yield and the modeled yield. The money market rates consists of LIBOR-based products with maturities of 1, 2, 3, and 6 months. The swap rates consist of US Swap rates with maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 15 years.

week. The data is not based on actual market trades as caps and swaptions are over-the-counter (OTC) derivatives, instead the rates are generated by the Bloomberg Generic Composite pricing algorithm.[9] This algorithm produces an indication of a quote based on actual OTC deals.[10]

The buyer of a cap receives a payment if the interest rate exceeds the agreed upon strike price at the end of a pre-specified quarterly period. These individual (potential) cashflows are called caplets. The caps considered have the 3-month LIBOR rate as underlying index, which is plotted in Appendix B Figure 9. The caps used are both at-the-money and out-of-the-money. The OTM caps contain strike premia between 1 and 5 percent. The strike price of a cap equals the swap rate of corresponding maturity. The ATM caps are quoted in Black (1976) implied volatility, and the OTM caps are quoted in a basis point (bps) premium. The ATM caps are converted to basis point premium by using the Black (1976) formula for caps as defined in Appendix C.

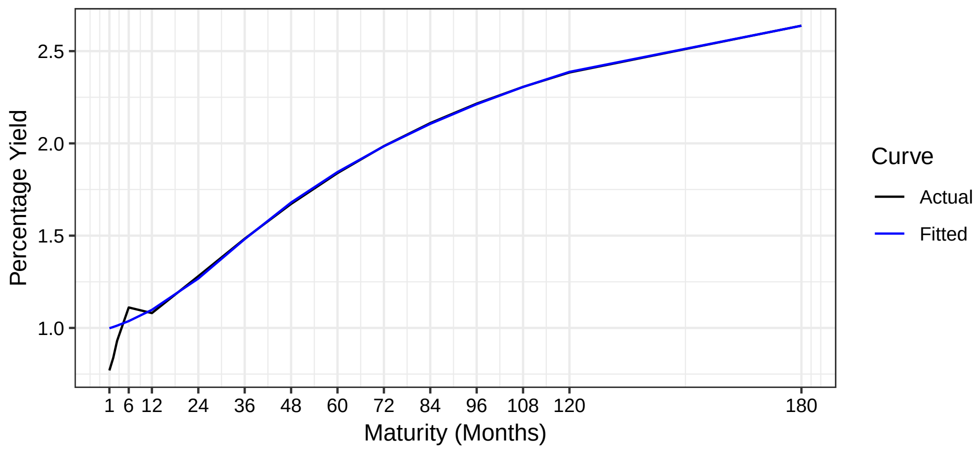
The buyer of a payer (receiver) swaption has the option to enter an payer (receiver) interest rate swap with a certain strike rate at a pre-specified date. Swaptions have varying option durations (expiry) and swap durations (tenor). The swaptions considered have an option expiration date that equals the start of the underlying swap. The strike rate is equal to the swap rate that corresponds with the maturity of the tenor. In our data set all floating legs are based on the 3-month LIBOR rate. The ATM swaptions are quoted in Black implied volatility based on a straddle (long position in both a payer and receiver swaption with identical properties), and converted to payer swaptions in basis points for pricing purposes.

Table 1 shows the fit of the yield curve smoothing from equation (15). The money market rates (maturities less than 1 year) have an average absolute error of 14.6 bps, which is relatively high but not problematic as the average of weekly maximum errors is 7.4 bps. The swap rates (maturities between 1 and 15 years) have a good fit with an average absolute error of 0.7 bps.

[9] Source: https://www.bloomberg.com/notices/financial-data.

[10] The dataset contains four observations that are considered as measurement errors where daily prices change with more than 10,000% only to revert the day after. These observations have been removed and replaced by a linear interpolation of the next and previous daily observation.

Figure 1: Yield Curve Comparison



Note: Average actual yield curve and average fitted yield curve using weekly data ranging from January 2013 to March 2019 (326 observations). The fitted yield curve is constructed using equation (15). Both curves are linearly interpolated between observed maturities.

Figure 1 shows the average percentage yield of the observed/actual yield curve and the fitted curve. The relatively poor fit of money market rates is again visible, however, the error is on average both positive and negative with reasonable magnitude.

Table 2 shows the descriptive statistics of interest rate caps where Panel A shows the average prices and Panel B the standard deviation. Note that the ATM options are quoted in implied volatility (IV), whereas the OTM options are quoted in basis points, this is similar to the quotations on Bloomberg. The ATM caps indicate a hump shaped volatility structure which has also been found by, among others, Amin and Morton (1994), Driessen et al. (2003), and Falini (2010). Note that the OTM cap prices increase for longer maturities, due to a larger amount of caplets and hence more potential cash flows, and decrease for higher strike prices, due to a lower probability of the underlying rate to exceed the strike price.

Table 3 shows the descriptive statistics of swaptions where Panel A shows the average prices and Panel B the standard deviation. Note that, in general, both the average and standard deviation of implied volatility decrease when either the option duration or swap tenor increase. There is minor evidence of a hump shaped volatility structure. The standard deviations of both caps and swaptions are approximately 5 times higher than standard deviations found in research dealing with option data of the late 1990s.[11]

[11]See for example Longstaff et al. (2001) and Driessen et al. (2003).

Table 2: Interest Rate Cap Descriptive Statistics

Panel A: Averages of Cap Prices

		OTM Premium (bps)				
Maturity	ATM (IV)	1%	2%	3%	4%	5%
1	44.2	-	9.5	0.6	0.1	0.1
2	50.2	100.2	31.9	5.7	1.1	-
3	50.9	204.6	80.1	24.7	8.8	-
4	48.6	340.8	155.5	62.7	27.6	14.7
5	46.2	491.9	256.8	119.1	58.6	32.6
6	43.1	667.8	369.8	188.9	99.6	56.9
7	42.1	853.9	501.6	267.8	150.6	87.8
8	39.6	1046.6	629.0	352.7	204.1	125.9
9	38.2	1242.8	766.2	442.1	262.1	166.1
10	37.6	1438.7	904.7	534.1	321.6	207.3

Panel B: Standard Deviations of Cap Prices

		OTM Premium (bps)				
Maturity	ATM (IV)	1%	2%	3%	4%	5%
1	22.5	-	19.5	1.3	0.3	0.2
2	23.4	107.5	49.5	8.9	0.9	-
3	20.8	148.6	70.7	19.0	5.5	-
4	17.1	172.8	85.0	32.8	16.3	10.8
5	14.5	187.8	99.9	53.3	33.3	24.0
6	12.7	205.5	120.6	80.3	55.6	39.7
7	11.3	227.3	149.1	110.9	80.5	57.5
8	10.6	253.5	178.6	143.1	106.0	78.1
9	10.0	282.0	211.6	175.3	132.1	96.5
10	9.4	312.1	245.1	207.6	158.1	115.7

Note: Average weekly cap prices with varying maturities and strike prices over the period January 2013 to March 2019 (326 observations). ATM caps are given in Black Implied Volatility (IV), OTM caps are given in basis point (bps) price. The three omitted products were not available for this period. The strike price of a cap is equal to the swap rate of the same maturity plus a premium for OTM caps.

Table 3: Swaption Descriptive Statistics

Panel A: Averages of Swaption Black Implied Volatilities (%)

Option	Swap Tenor									
Expiry	1	2	3	4	5	6	7	8	9	10
1 MO	40.8	44.5	43.4	41.3	39.6	36.8	35.4	33.8	32.2	30.3
3 MO	43.8	45.3	43.9	41.7	39.5	36.9	35.0	34.0	32.7	31.2
6 MO	45.9	45.3	43.4	41.2	39.1	37.3	35.2	35.1	33.3	31.5
1 YR	47.1	44.9	42.5	40.0	37.9	36.4	34.5	33.8	32.9	31.5
2 YR	45.2	41.9	39.2	37.0	35.4	34.2	32.9	32.5	31.5	30.6
3 YR	41.5	38.6	36.4	34.7	33.4	32.7	31.5	31.5	30.6	29.6
4 YR	38.0	35.7	34.3	32.9	31.9	31.3	30.3	30.3	29.8	28.7
5 YR	35.1	33.5	32.4	31.5	30.6	30.2	29.3	29.4	29.0	28.0

Panel B: Standard Deviations of Swaption Black Implied Volatilities (%)

Option	Swap Tenor									
Expiry	1	2	3	4	5	6	7	8	9	10
1 MO	22.7	21.0	18.0	15.7	14.4	12.3	11.2	10.4	9.7	9.5
3 MO	22.9	19.2	16.4	14.6	13.3	11.5	10.9	10.0	9.5	9.1
6 MO	22.3	18.3	15.5	13.3	12.3	10.9	10.1	10.7	9.2	8.7
1 YR	20.4	16.1	13.6	11.8	10.8	9.7	9.2	9.2	9.8	8.0
2 YR	15.7	12.2	10.4	9.5	9.0	8.1	8.0	9.3	7.7	7.2
3 YR	11.6	9.8	8.9	8.3	7.9	7.5	7.3	7.9	7.1	6.8
4 YR	9.6	8.6	7.9	7.6	7.3	7.1	6.8	7.2	7.2	6.4
5 YR	8.3	7.8	7.4	7.2	7.0	6.8	6.6	6.8	6.6	6.2

Note: Average weekly US swaption prices with varying swap tenors and (European) option expirations over the period January 2013 to March 2019 (326 observations). All prices are in Black Implied Volatility. The strike price of a swaption is equal to the swap rate of the same maturity.

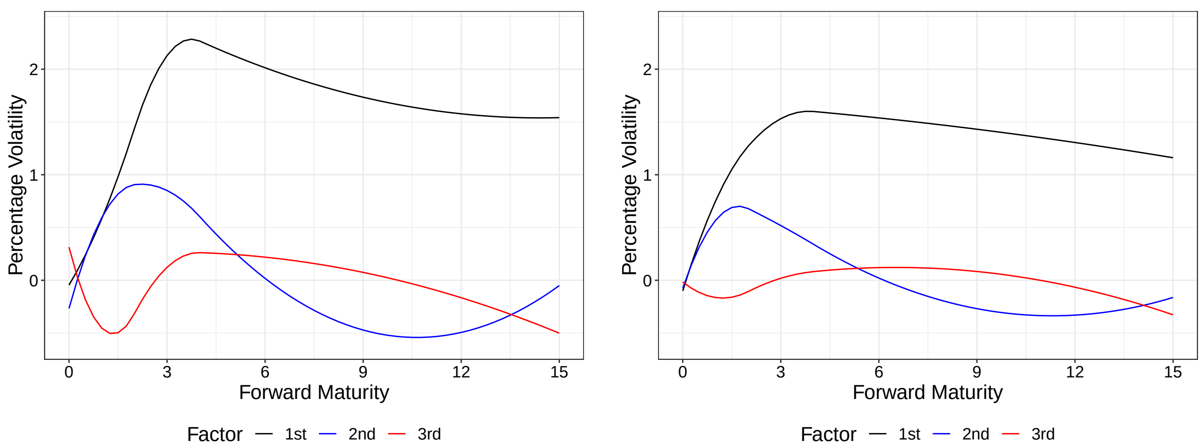
5 Results

This section contains the results of this paper. In Section 5.1, we analyze the results of the parameter estimation techniques. Section 5.2 contains the pricing prediction results of caps and swaptions. In Section 5.4, we further analyze the pricing errors with an error regression. Section 5.5 contains the results of the in-sample pricing exercise.

5.1 Parameter Estimates

Figure 2 shows the volatility functions used in the constant parameter models (left) and the average of the volatility functions used in the time-varying parameter models (right) as explained in Section 3.2.2. There is an overlap of at most 40 weeks between the two samples, the volatility functions are of similar shape and magnitude. In the constant parameter case the first factor explains 89.5%, the second factor 7.6%, and the third factor 1.9% of the variation. In the time-varying parameter case the first factor explains on average 91.3%, the second 6.9%, and the third 1.1% of the variation. The factors in the time-varying case have an average standard deviation of 0.2%, 0.1%, and 0.1% respectively, and a maximum standard deviation of 0.3%, 0.2%, and 0.3% respectively. Figure 10 in Appendix D shows the interest-rate-based volatility function of the 3-factor model over time for forward maturities of 1 month, and 1 and 5 years. This figure indicates that the volatility functions vary over time and this variation differs per forward maturity.

Figure 2: Interest Rate Based Volatility Functions: constant (left) and time-varying (right)



Note: Percentage volatility of the volatility functions per maturity estimated using PCA on the weekly changes in instantaneous forward rates using equation (21). The left figure contains constant parameter estimates based on the estimation period from January 2013 to January 2016 (161 observations). The right figure contains average time-varying parameter estimates based on a rolling window of 40 weeks on the out-of-sample period from January 2016 to March 2019 (163 windows). The forward maturities are expressed in years.

Table 4: Option-Based Alpha Estimates Optimized with at-the-money Caps

Panel A: Constant Parameters

Constant	α_1	α_2	α_3
1-factor	0.8	-	-
2-factor	1.0	0.8	-
3-factor	1.5	2.4	1.4

Panel B: Time-Varying Average Parameters

Time-Varying	α_1	α_2	α_3
1-factor	5.1 (5.7)	-	-
2-factor	1.6 (2.0)	3.2 (3.7)	-
3-factor	2.1 (1.9)	3.7 (4.2)	3.5 (7.6)

Note: Alpha estimates used for option-based estimation of volatility functions optimized with ATM Caps as explained in Section 3.2.3. Panel A contains estimates based on the estimation period from January 2013 to January 2016 (161 observations). Panel B contains average estimates and standard deviations between parenthesis based on a rolling window of 40 weeks on the out-of-sample period from January 2016 to March 2019 (163 windows).

The volatility function shapes in Figure 2 are not uncommon and relatively similar shapes are found in e.g. Driessen et al. (2003). The shape of the factors are, however, different than those of yield curve factors in e.g. Litterman and Scheinkman (1991). The first factor is only a true 'level' factor for forward maturities larger than approximately 2.5 years, this is possibly caused due to the usage of changes in forward rates instead of forward rates. The second factor corresponds with the shape of a 'curvature' factor, it increases the volatility changes for maturities smaller than 6 years, and decreases for maturities larger than 6 years; this is the case for both constant and time-varying estimates. The third factor resembles the 'steepness' or 'slope' factor for forward maturities larger than 3 years. As opposed to yield curve factors, the curvature factor explains more variation than the slope factor. Using a three factor model implies using the sum of the first three factors, this results in a shape that increases up to approximately 3 years forward maturity and decreases afterwards (for reference see black and blue line in Figure 8).

Table 4 contains estimates of α values used for option-based estimation optimized using ATM caps as explained in Section 3.2.3. The 1-factor constant α_1 value of 0.8 indicates that when using this model, the volatility function values are multiplied by 0.8 when used for pricing. The α estimates in the constant parameter models are relatively small and close to 1, this means that we can expect the interest-rate-based and option-based constant parameter models to be relatively close to each other in the pricing prediction exercise. On the contrary, the average α estimates of the time-varying model are larger with relatively high standard deviations. This would imply that interest-rate-based and option-based time-varying parameter models can have very different results when used in a pricing prediction exercise. The high standard deviations have multiple causes. First, the optimization problem is quite large and it is therefore difficult to guarantee a global minimum, this can cause parameter estimates to differ substantially. Second, multi-factor models have multiple α values to estimate, and as each routine starts from scratch this causes an almost label-switching-like behavior, which increases standard deviations.

5.2 Pricing Prediction

Table 5 contains relative pricing prediction errors of ATM caps. The standard deviation of all models is on average 25.6% (unreported) and similar between models. The constant parameter models outperform the time-varying parameter models for every combination of factors and estimation technique, this is not in line with the findings of e.g. Driessen et al. (2003). These results indicate that a longer, more stable estimation window is more favorable for caps pricing in this sample. Option-based estimated models generally outperform the interest-rate-based estimated models, except in the 1-factor constant parameter case. As expected, the improvement of option-based estimated models compared to interest-rate-based estimated models is larger for time-varying parameters than constant parameters. The 3-factor models generally outperform the 1- and 2-factor models, which is in line with e.g. Driessen et al. (2003).

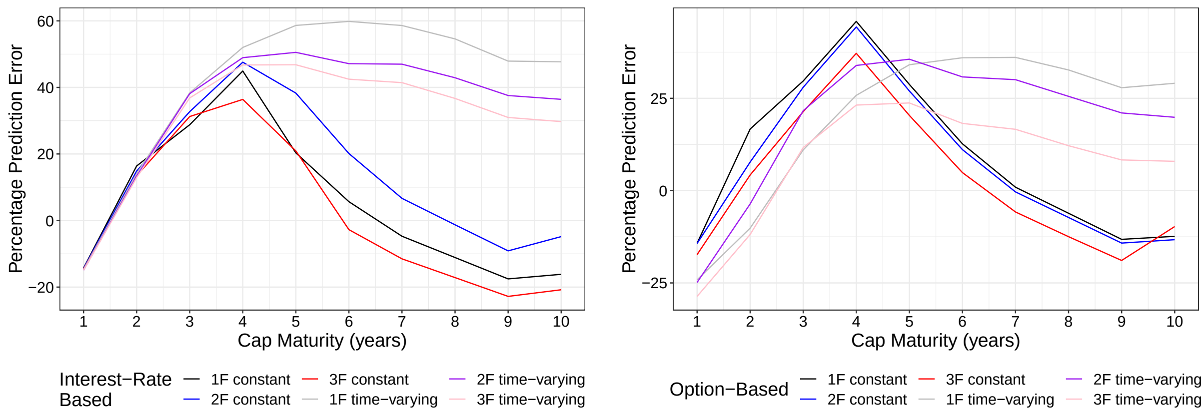
Figure 3 shows the relative prediction errors of ATM caps for interest-rate-based models (left) and option-based models (right) against cap maturity. The figures indicate that there seems to be a relation between the cap maturity and the percentage prediction error. The prediction error increases up to a cap maturity of 4 years after which the constant parameter model rapidly decreases and the time-varying model stays at roughly that level with a slight decrease. In Section 5.4 we see that this possible maturity effect disappears when performing an error regression on multiple regressors.

Table 5: Relative Prediction Errors at-the-money Caps

Parameters:	Constant		Time-Varying	
Estimation/ Model	Interest-Rate-Based	Option-Based	Interest-Rate-Based	Option-Based
1-Factor	19.1%	19.3%	45.3%	35.6%
2-Factor	20.4%	18.6%	38.3%	29.1%
3-Factor	20.3%	16.7%	34.9%	20.9%

Note: Relative 2-week ahead mean absolute prediction errors for all ATM caps in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions). A relative prediction error is defined as the observed price minus the modeled price divided by the observed price.

Figure 3: Relative Prediction Errors at-the-money Caps by Maturity



Note: Relative 2-week ahead mean prediction errors for all ATM caps in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions). The left (right) figure plots the interest-rate-based (option-based) models. A relative prediction error is defined as the observed price minus the modeled price divided by the observed price.

Table 6 shows the absolute pricing prediction errors of both ATM and OTM caps in terms of basis points. The standard deviation of all models is on average 222 basis points (unreported) and similar between models. The option-based estimation is done using the parameters estimated using ATM caps. The time-varying models generally perform better than the constant parameter models and the option-based estimated models perform better than the interest-rate-based estimated models. The amount of factors necessary for pricing OTM caps is not clear, this is possibly due to prediction errors arising from high strike prices and not from erroneous volatility functions.

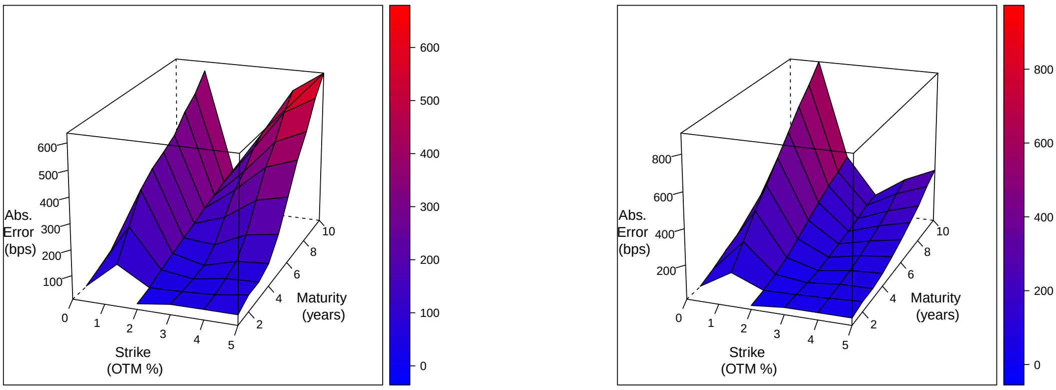
Figure 4 shows plots of absolute pricing prediction errors of ATM and OTM caps against strike price and cap maturity. Both plots are based on the 3-factor interest-rate-based model, with constant parameters in the left panel and time-varying parameters in the right panel. The plots indicate that the absolute pricing error increases as the cap maturity increases. Further,

Table 6: Absolute Prediction Errors ATM and OTM Caps

Parameters:	Constant		Time-Varying	
Estimation/ Model	Interest-Rate-Based	Option-Based	Interest-Rate-Based	Option-Based
1-Factor	200.4 bps	195.5 bps	198.9 bps	193.5 bps
2-Factor	190.8 bps	196.6 bps	191.3 bps	189.3 bps
3-Factor	207.0 bps	199.3 bps	189.3 bps	190.2 bps

Note: Mean absolute 2-week ahead prediction errors for all caps (both ATM and OTM) in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions) in basis points. An absolute prediction error is defined as the absolute value of the observed price minus the modeled price. Option-based estimation parameters are optimized with ATM caps.

Figure 4: Absolute Prediction Errors ATM and OTM Caps

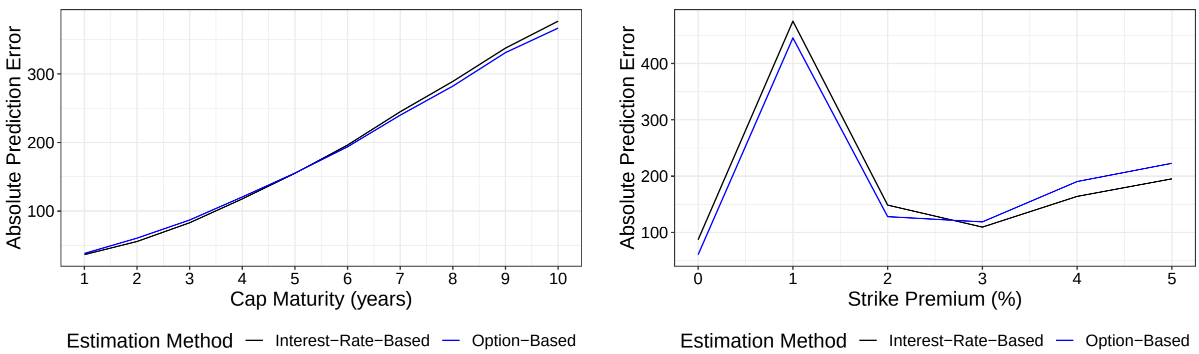


Note: Mean absolute 2-week ahead prediction errors for all caps (both ATM and OTM) in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions) in basis points. Both figures are based on the 3-factor interest-rate-based model with constant parameters (left), and time-varying parameters (right). An absolute prediction error is defined as the absolute value of the observed price minus the modeled price. Option-based estimation parameters are optimized with ATM caps.

the 1% OTM strike proves troublesome for all models, excluding this result the absolute pricing error increases as the strike price increases. The time-varying model's pricing prediction errors are smaller as the strike price increases, which is especially visible for the 5% OTM cap. Appendix E Figure 11 and Figure 12 contain the plots of the other models, however, their shapes are similar.

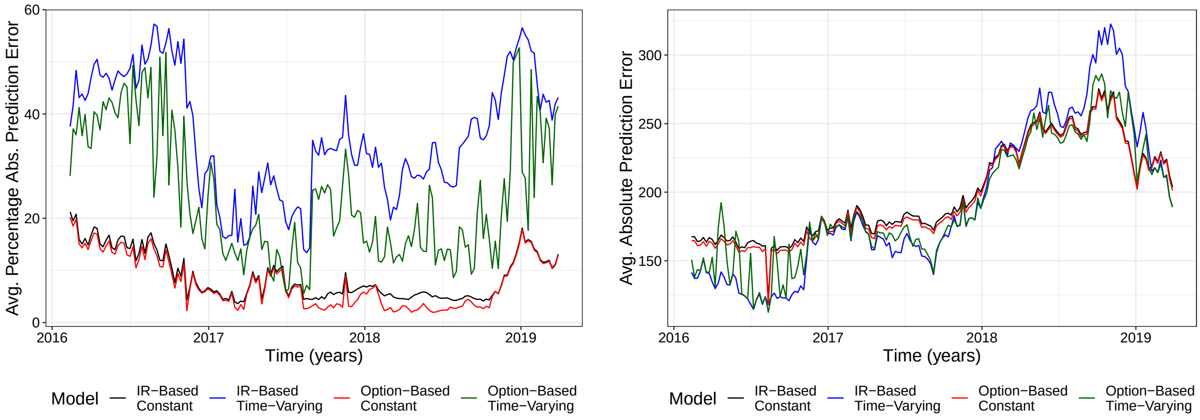
Figure 5 shows the average absolute prediction errors averaged over all factor models using both constant and time-varying parameters plotted against cap maturity (left) and strike premium (right). The averaging over models and parameters has little impact on the shape of the curves as they are all similar. The purpose of these figures is to amplify the results found in Figure 4. Figure 5 indicates that the absolute prediction error increases as the cap maturity

Figure 5: Average Absolute Prediction Error ATM and OTM Caps by Maturity and Strike



Note: Mean absolute 2-week ahead prediction errors in bps over all six pricing models (time-varying, constant, 1-3 factors) for all ATM and OTM caps in the out-of-sample period from January 2016 to March 2019. An absolute prediction error is defined as the absolute value of the observed price minus the modeled price.

Figure 6: Average Prediction Error ATM (left) and OTM Caps (right) over time



Note: Mean 2-week ahead prediction errors over all factor models (1-3) for ATM caps in percentage absolute error (left), and for ATM and OTM caps in absolute basis point error (right). Calculated in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions).

increases, however, the prices of caps also increase with maturity, hence the relative effect is dampened. The relatively high prediction error of the 1% strike premium (compared to other OTM caps) can be traced back to a steeper slope for low strike prices when regressing cap prices against cap maturity. These pricing error effects are further examined in Section 5.4.

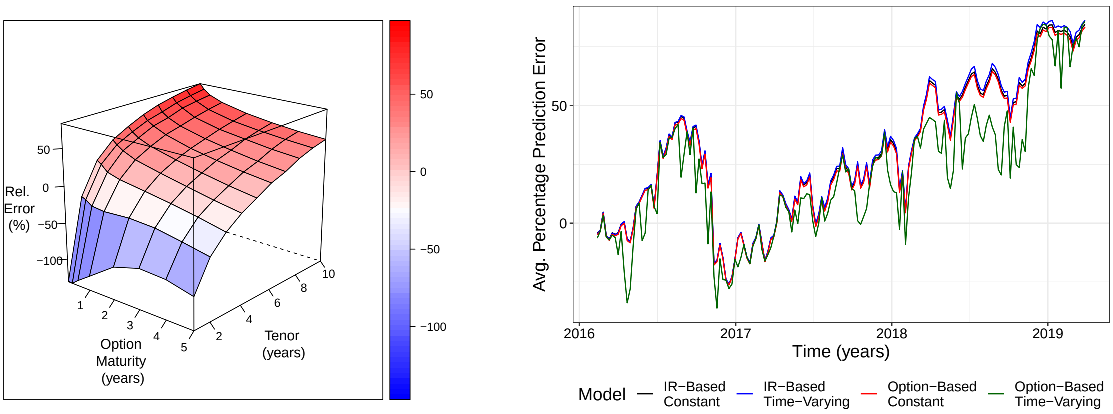
Figure 6 plots the average percentage absolute prediction error of ATM caps (left) and the average absolute prediction error of ATM and OTM caps (right) against time. The shapes of the factor models are all quite similar, hence the average of all factor models (1-3 factors) is plotted. The left panel shows that the prediction errors vary over time, further, the time-varying models show much more volatile errors which possibly points to an estimation window that is too narrow. The right panel shows that when adding OTM caps to the mix, the prediction errors also vary over time. This is, however, only a conjecture as the right panel uses absolute prediction errors. These findings indicate that time-homogeneous volatility functions might not be able to fully capture the dynamics of the forward rate volatility.

Table 7: Relative Prediction Error Swaptions

Parameters:	Constant		Time-Varying	
Estimation/ Model	Interest-Rate-Based	Option-Based	Interest-Rate-Based	Option-Based
1-Factor	60.8%	61.1%	61.2%	59.2%
2-Factor	60.5%	60.7%	61.0%	60.9%
3-Factor	60.6%	59.6%	61.0%	60.1%

Note: Relative 2-week ahead mean absolute prediction errors for all swaptions in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions). A relative prediction error is defined as the observed price minus the modeled price divided by the observed price. Option-based estimation parameters are optimized with ATM caps.

Figure 7: Relative (left) and Average over time (right) Swaption Prediction Errors



Left: Relative 2-week ahead mean absolute prediction errors for swaptions using the 1-factor option-based model with time-varying parameters. The option-based estimation parameters are optimized with ATM caps. Right: The average 2-week ahead prediction error for swaptions over time. The plotted lines are averaged over the factor models (1-3). Both: Calculated in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions).

Table 7 shows the relative pricing prediction errors of swaptions. The standard deviation of all models is on average 66.6% (unreported) and similar between models. The option-based parameters are estimated using ATM caps. The time-varying option-based 1-factor model performs best with a pricing error of 59.2%. The prediction errors are all close to 61% which is somewhat alarming and indicative of more fundamental pricing problems.

Figure 7 (left) shows a plot of relative pricing prediction errors for swaptions using the 1-factor option-based model with time-varying parameters. The option maturity has an ambiguous effect on the pricing prediction errors. The prediction errors increase as the tenor increases. The plots of other models, which are quite similar, can be found in Appendix F on Figure 13 and Figure 14.

Figure 7 (right) shows the average pricing prediction errors of swaptions over time. The shapes of the factor models are all quite similar, hence the average of all factor models (1-3 factors) is plotted. The figure indicates that swaption errors vary over time, in fact, the swaptions are priced relatively well in some periods. The average pricing prediction errors seem to be increasing in a linear fashion from 2017 until the end of the sample. The caps and swaption pricing errors are further analyzed in the next section.

5.3 Model Forecast Comparison Test

This section contains the results of the model comparison tests as explained in Section 3.3.2. Table 8, 9, and 10 contain the results of ATM caps, OTM caps, and swaptions respectively.

Table 8: Model Forecast Comparison Test ATM Caps

Estimation			Interest-Rate-Based						Option-Based						
Parameters			Constant			Time-Varying			Constant			Time-Varying			
Factors			1	2	3	1	2	3	1	2	3	1	2	3	Avg.
Interest-Rate-Based	Constant	1	-	50%	60%	80%	80%	80%	60%	20%	50%	90%	90%	80%	67%
		2	50%	-	40%	80%	80%	70%	50%	40%	40%	90%	90%	70%	64%
		3	40%	60%	-	90%	80%	80%	50%	40%	30%	100%	100%	80%	68%
	Time-Var.	1	20%	20%	10%	-	0%	0%	20%	10%	10%	30%	20%	20%	15%
		2	20%	20%	20%	100%	-	20%	20%	10%	10%	30%	20%	20%	26%
		3	20%	30%	20%	100%	80%	-	20%	10%	10%	60%	20%	20%	35%
Option-Based	Constant	1	40%	50%	50%	80%	80%	80%	-	30%	40%	90%	90%	80%	65%
		2	80%	60%	60%	90%	90%	90%	70%	-	40%	90%	90%	80%	76%
		3	50%	60%	70%	90%	90%	90%	60%	60%	-	100%	100%	90%	78%
	Time-Var.	1	10%	10%	0%	70%	70%	40%	10%	10%	0%	-	0%	0%	20%
		2	10%	10%	0%	80%	80%	80%	10%	10%	0%	100%	-	10%	35%
		3	20%	30%	20%	80%	80%	80%	20%	20%	10%	100%	90%	-	50%
Average			33%	36%	32%	85%	74%	65%	35%	24%	22%	80%	65%	50%	-

Note: Percentage of rejected H_0, using a 5% significance level, for ATM Caps of Diebold-Mariano tests where H_1 states that the model in the row has greater accuracy than the model in the column. Sampling distribution of test statistic estimated using bootstrapping. Out-of-sample period ranges from January 2016 to March 2019 (163 weekly predictions).

The tables contain the percentage of rejected H_0 of equal forecasting accuracy between models of Diebold-Mariano tests, using a bootstrapped sampling distribution, where H_1 states that the model in the row has greater accuracy than the model in the column. This means that a high percentage indicates that the model in the row has a greater statistical accuracy for that percentage of products than the model in the column. For example, in Table 10 we see that the H_0 of equal predictive accuracy between the option-based constant parameters 3-factor (column) and option-based time-varying 1-factor (row) model is rejected 75% of the time against the H_1 that option-based time-varying 1-factor model has a greater predictive accuracy than the option-based constant parameter 3-factor model. This implies that, on average, the option-based time-varying 1-factor model is more accurate for 75% of the swaptions. A model with relatively good forecasts is characterized by a high percentage in the average column, and a low percentage in the average row. Appendix G contains the same test without bootstrapping and thus assuming a standard normal test distribution.

Table 8 contains the results of the model comparison test for ATM caps. The option-based

Table 9: Model Forecast Comparison Test ATM and OTM Caps

Estimation			Interest-Rate-Based						Option-Based						
Parameters			Constant			Time-Varying			Constant			Time-Varying			
Factors			1	2	3	1	2	3	1	2	3	1	2	3	Avg.
Interest-Rate-Based	Constant	1	-	49%	61%	54%	54%	54%	40%	51%	56%	58%	60%	56%	54%
		2	51%	-	60%	56%	58%	54%	51%	53%	60%	60%	61%	58%	56%
		3	39%	40%	-	53%	53%	53%	39%	39%	46%	60%	63%	56%	49%
	Time-Var.	1	46%	44%	47%	-	44%	44%	46%	46%	54%	53%	51%	56%	48%
		2	46%	42%	47%	56%	-	54%	46%	46%	54%	58%	56%	56%	51%
		3	46%	46%	47%	56%	46%	-	44%	46%	54%	67%	56%	56%	51%
Option-Based	Constant	1	60%	49%	61%	54%	54%	56%	-	60%	58%	63%	60%	58%	58%
		2	49%	47%	61%	54%	54%	54%	40%	-	60%	61%	60%	58%	55%
		3	44%	40%	54%	46%	46%	46%	42%	40%	-	60%	65%	60%	49%
	Time-Var.	1	42%	40%	40%	47%	42%	33%	37%	39%	40%	-	37%	44%	40%
		2	40%	39%	37%	49%	44%	44%	40%	40%	35%	63%	-	54%	44%
		3	44%	42%	44%	44%	44%	44%	42%	42%	40%	56%	46%	-	44%
Average			46%	44%	51%	52%	49%	49%	42%	45%	51%	60%	56%	56%	-

Note: Percentage of rejected H_0, using a 5% significance level, for ATM and OTM Caps of Diebold-Mariano tests where H_1 states that the model in the row has greater accuracy than the model in the column. Sampling distribution of test statistic estimated using bootstrapping. Out-of-sample period ranges from January 2016 to March 2019 (163 weekly predictions).

constant parameter 2- and 3-factor models have a statistically significant outperformance for on average 76% and 78% of the products, and they only get outperformed 24% and 22% of the time respectively. This result is in line with the low prediction errors found in Table 5. The results in this table further confirm the outperformance of constant parameter models over time-varying parameter models, and the outperformance of 3-factor models over smaller factor models.

Table 9 contains the results of the model comparison test for ATM and OTM caps. The ATM and OTM caps performances are much closer together than for the ATM caps only. We find that the interest-rate-based constant parameter 2-factor and option-based constant parameter 1-factor models marginally outperform the rest as they both significantly outperform the other models 56% and 58% of the time, and are outperformed 44% and 42% of the time respectively. Further, the constant parameter models perform better than the time-varying parameter models. These findings are not perfectly in line with Table 6, however, the results are relatively close together in both tables and a clear winner does not seem present.

Table 10: Model Forecast Comparison Test Swaptions

Estimation			Interest-Rate-Based						Option-Based						
Parameters			Constant			Time-Varying			Constant			Time-Varying			
Factors			1	2	3	1	2	3	1	2	3	1	2	3	Avg.
Interest-Rate-Based	Constant	1	-	6%	6%	93%	74%	71%	99%	28%	8%	21%	18%	10%	39%
		2	94%	-	15%	96%	94%	94%	96%	94%	10%	21%	21%	13%	59%
		3	94%	85%	-	96%	93%	93%	96%	94%	8%	23%	20%	14%	65%
	Time-Var.	1	8%	4%	4%	-	1%	3%	21%	4%	6%	19%	11%	6%	8%
		2	26%	6%	8%	99%	-	9%	50%	10%	9%	19%	15%	10%	24%
		3	29%	6%	8%	98%	91%	-	53%	10%	8%	19%	15%	10%	31%
Option-Based	Constant	1	1%	4%	4%	79%	50%	48%	-	5%	8%	19%	14%	10%	22%
		2	73%	6%	6%	96%	90%	90%	95%	-	9%	21%	19%	11%	47%
		3	93%	90%	93%	94%	91%	93%	93%	91%	-	25%	59%	29%	77%
	Time-Var.	1	79%	79%	78%	81%	81%	81%	81%	79%	75%	-	79%	76%	79%
		2	83%	79%	80%	89%	85%	85%	86%	81%	41%	21%	-	9%	67%
		3	90%	88%	86%	94%	90%	90%	90%	89%	71%	24%	91%	-	82%
Average			61%	41%	35%	92%	76%	69%	78%	53%	23%	21%	33%	18%	-

Note: Percentage of rejected H_0, using a 5% significance level, for swaptions of Diebold-Mariano tests where H_1 states that the model in the row has greater accuracy than the model in the column. Sampling distribution of test statistic estimated using bootstrapping. Out-of-sample period ranges from January 2016 to March 2019 (163 weekly predictions).

Table 10 contains the results of the model comparison test for swaptions. Whereas the results in Table 7 were inconclusive, this table is more pronounced. The option-based 3-factor constant, 1-factor time-varying, and 3-factor time-varying model outperform the rest of the models on average. Especially the option-based time-varying 1-factor model has promising results as it outperforms other models 79% of the time, and gets outperformed 21% of the time on average. The option-based time-varying 3-factor model has slightly better average statistics, however, when they are tested against each other, the 1-factor variant comes out on top. We further find that multi-factor models generally outperform single-factor models.

5.4 Error Analysis

This section contains the results of the error regressions as explained in Section 3.3.3. Table 11 contains the results of the error regression on ATM and OTM caps pricing errors. The beta estimates for each model are all statistically significant at a 1% level except $\hat{\beta}_2$ (Maturity). An F-test for each model (unreported) indicates that the null hypothesis that all coefficients jointly

Table 11: Error Regressions ATM and OTM Caps

Model	Const.	LMR ($x10^2$)	MAT ($x10^4$)	ATMVol	r	Slope	Adj. R^2
Interest-rate-based – constant parameters							
1-factor	4.0*	6.1*	-1.2	0.5*	-4.5*	-4.5*	21.1%
2-factor	4.4*	6.1*	-1.3	0.7*	-4.9*	-5.0*	20.6%
3-factor	4.1*	6.1*	-0.9	0.5*	-4.6*	-4.6*	22.2%
Interest-rate-based – time-varying parameters							
1-factor	5.6*	6.2*	-0.7	1.3*	-6.4*	-6.5*	21.2%
2-factor	3.8*	6.2*	-0.8	1.2*	-4.5*	-4.6*	20.6%
3-factor	3.1*	6.2*	-0.9	1.2*	-3.7*	-3.9*	20.6%
Option-based – constant parameters							
1-factor	4.1*	6.1*	-1.3	0.6*	-4.6*	-4.7*	20.7%
2-factor	4.4*	6.1*	-1.3	0.6*	-4.9*	-5.0*	21.6%
3-factor	5.1*	6.1*	-0.3	0.6*	-5.8*	-5.8*	24.0%
Option-based – time-varying parameters							
1-factor	4.3*	6.2*	-0.4	1.3*	-5.1*	-5.2*	18.4%
2-factor	4.5*	6.2*	-0.5	1.0*	-5.2*	-5.3*	20.0%
3-factor	5.5*	6.2*	-0.8	1.1*	-6.4*	-6.5*	22.2%

Note: Estimates of error regressions of ATM and OTM caps as defined in Section 3.3.3. The observed implied volatility on the market minus modeled is regressed on a constant (Const.), logarithm of the moneyness ratio (LMR), maturity (MAT), volatility of a similar ATM product (ATMVol), 3-month observed LIBOR rate (r), and term structure slope (Slope). White standard errors are used, * indicates 1% significance level. Estimated in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions).

equal zero is rejected. The constant estimate, $\hat{\beta}_0$, indicates that, on average, the modeled implied volatility is too low. The significant estimate for the logarithm of the moneyness ratio, $\hat{\beta}_1$, is in line with the findings of Gupta and Subrahmanyam (2005), and indicates that as the strike price increases, the difference in Black implied volatility decreases. The significant positive estimate for ATM Volatility, $\hat{\beta}_3$, confirms the volatility skew (situation in which the implied volatility goes down when the option gets more out-of-the-money) found in the data (unreported). The significant $\hat{\beta}_4$ (r) and $\hat{\beta}_5$ (Slope) values indicate that the prediction errors partly come form the models inability to model the level of the underlying 3-month LIBOR and the slope of the term structure, which can explain the variability in errors over time. The option-based 1-factor model with time-varying parameters has the lowest adjusted R-squared of 18.4%, which implies this model is the least biased. Interestingly, the coefficient estimates between models are roughly the same, possibly indicating that a better pricing prediction requires a different method to estimate the volatility functions.

Table 12 contains the results of the error regression on swaption pricing errors. An F-test

Table 12: Error Regressions Swaptions

Model	Const.	OPT ($\times 10^4$)	TEN ($\times 10^4$)	r	Slope	Adj. R^2
Interest-rate-based – constant parameters						
1-factor	-4.2*	-12.3	-4.0	4.7*	4.4*	9.5%
2-factor	-4.2*	-11.9	-3.4	4.6*	4.3*	9.3%
3-factor	-4.1*	-11.5	-2.5	4.5*	4.3*	8.9%
Interest-rate-based – time-varying parameters						
1-factor	-4.3*	-12.1	-3.4	4.7*	4.5*	9.9%
2-factor	-4.2*	-11.9	-2.1	4.6*	4.4*	9.6%
3-factor	-4.2*	-11.9	-2.0	4.6*	4.4*	9.5%
Option-based – constant parameters						
1-factor	-4.3*	-12.5	-3.9	4.7*	4.5*	9.6%
2-factor	-4.2*	-11.9	-3.6	4.6*	4.4*	9.3%
3-factor	-3.8*	-11.0	-0.7	4.2*	4.0*	8.6%
Option-based – time-varying parameters						
1-factor	-5.2*	-13.2	-4.0	5.6*	5.4*	9.6%
2-factor	-3.9*	-6.9	-3.5	4.3*	4.1*	9.2%
3-factor	-3.7*	-11.0	-2.9	4.0*	3.8*	8.4%

Note: Estimates of error regressions of swaptions as defined in Section 3.3.3. The observed implied volatility on the market minus modeled is regressed on a constant (Const.), option expiry (OPT), option tenor (TEN), 3-month observed LIBOR rate (r), and term structure slope (Slope). White standard errors are used, * indicates 1% significance level. Estimated in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions).

for each model (unreported) indicates that the null hypothesis that all coefficients jointly equal zero is rejected. The beta estimates of the constant, $\hat{\beta}_0$, LIBOR rate, $\hat{\beta}_3$, and term structure slope, $\hat{\beta}_4$, are statistically significant at a 1% level for all models; the latter two coefficients can help explain the variability of prediciton errors over time found earlier. The betas estimates of the option expiry, $\hat{\beta}_1$, and swap tenor, $\hat{\beta}_2$, are not significantly different from zero, indicating the earlier conjectured tenor relation in e.g. Figure 7 (left panel) disappears when converting to Black implied volatility. In contrast with caps, the intercepts for swaption error regressions are all negative, indicating that, on average, the modeled implied volatility is too high. The option-based 3-factor model with time-varying parameters has the lowest adjusted R-squared of 8.4%, which implies that this model is the least biased. The coefficient estimates are all quite similar between models and estimation techniques.

Table 13 contains estimates of the interest rate level coefficient, β_4, of all our models on various linear regressions for caps. The OLS estimates are (almost) all negative and mostly statistically significant (at the 1% level) except for the interest-rate-based time-varying models

Table 13: Error Regressions Interest Rate Level ATM and OTM Caps

	β_4 - Interest Rate Level - coefficient (x10^2)		
Model	LASSO	Ridge	OLS White
Interest-rate-based – constant parameters			
1-factor	-2.0	-1.9	-2.2*
2-factor	-1.9	-1.7	-2.0*
3-factor	-2.4	-2.3	-2.6*
Interest-rate-based – time-varying parameters			
1-factor	0.9	0.9	0.9
2-factor	-0.7	-0.7	-0.7
3-factor	-1.4	-1.3	-1.4
Option-based – constant parameters			
1-factor	-2.0	-1.8	-2.1*
2-factor	-2.2	-2.0	-2.4*
3-factor	-4.3	-3.6	-4.4*
Option-based – time-varying parameters			
1-factor	-6.0	-5.8	-6.1*
2-factor	-6.3	-5.8	-6.4*
3-factor	-9.6	-8.2	-9.8*

Note: β_4 estimates of error regressions for interest rate level of ATM and OTM caps as defined in Section 3.3.3 and as follows $|IV_{mkt} - IV_{model}|_t = \beta_0 + \beta_1 \text{LMR}_t + \beta_2 \text{MAT}_t + \beta_3 \text{ATMVol}_t + \beta_4 \text{IRLvl}_t + \epsilon_t$. White standard errors are used for OLS, * indicates 1% significance level (OLS Only). Estimated in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions).

(which all have p-values that exceed 10%). The negative sign indicates that as the interest rates increase, the absolute modeling error decreases. The option-based time-varying models have the highest sensitivity to interest rate changes as their values are largest in magnitude. The LASSO model estimates are all close to the OLS estimates, this further emphasizes the predictability of the OLS results. The ridge estimates all slightly shrink towards zero. We conclude based on this table that a lower interest rate environment leads to on average larger modeling errors for caps.

Table 14 contains estimates of the interest rate level coefficient, β_3, of all our models on various linear regressions for swaptions. The OLS estimates are all negative and statistically significant at the 1% level. The LASSO model estimates are again close to the OLS estimates, and the ridge estimates show very little shrinkage. These results emphasize that the unbiased OLS estimates also have a strong predictability. The results of these models again indicate that as the interest rate level increases, the absolute errors decrease. The magnitude of the estimators is, on average, slightly larger for swaptions than caps. We conclude that also for

Table 14: Error Regressions Interest Rate Level Swaptions

	β_3 - Interest Rate Level - coefficient (x10^2)		
Model	LASSO	Ridge	OLS White
Interest-rate-based – constant parameters			
1-factor	-6.2	-6.0	-6.2*
2-factor	-6.2	-6.0	-6.2*
3-factor	-6.3	-6.1	-6.3*
Interest-rate-based – time-varying parameters			
1-factor	-6.0	-5.8	-6.0*
2-factor	-6.0	-5.9	-6.1*
3-factor	-6.1	-5.9	-6.1*
Option-based – constant parameters			
1-factor	-6.2	-6.0	-6.2*
2-factor	-6.2	-6.1	-6.3*
3-factor	-6.2	-6.0	-6.2*
Option-based – time-varying parameters			
1-factor	-7.3	-7.1	-7.4*
2-factor	-6.3	-6.1	-6.3*
3-factor	-6.9	-6.7	-6.9*

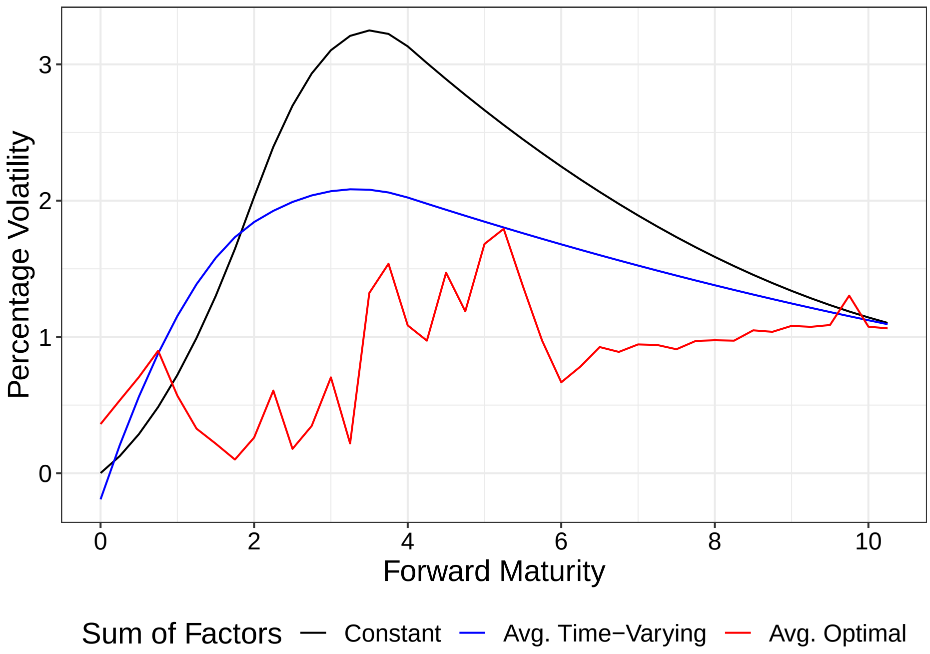
Note: β_3 estimates of error regressions for interest rate level of swaptions as defined in Section 3.3.3 and as follows $|IV_{mkt} - IV_{model}|_t = \beta_0 + \beta_1 \text{OPT}_t + \beta_2 \text{TEN}_t + \beta_3 \text{IRLvl}_t + \epsilon_t$. White standard errors are used for OLS, * indicates 1% significance level (OLS Only). Estimated in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions).

swaptions, a lower interest rate environment leads to on average larger modeling errors.

5.5 In-Sample Pricing

This section contains the results of the in-sample pricing exercise as explained in Section 3.3.4. Using the time-varying optimized volatility function for ATM caps yields a absolute pricing prediction error of 7.6%. The out-of-sample pricing prediction model with the best performance is the constant parameter option-based 3-factor model with an error of 16.7% (Table 5). Figure 8 shows the sum of the first three volatility functions for the interest-rate-based constant (black) and time-varying model (blue), and the average volatility function that minimizes the in-sample fit of ATM cap prices (red). On average, the interest-rate-based out of sample volatility functions are higher than the optimal volatility function. These results indicate that there is room for improvement in the estimation of volatility functions, however, there will always remain some mispricing. These findings are in line with e.g. Driessen et al. (2003) and other pricing papers. Collin-Dufresne and Goldstein (2001) state that these mispricings are due to restrictions on

Figure 8: Volatility Function Comparison



Note: Percentage volatility of various PCA-based volatility functions. The black line is the sum of the first three volatility functions of the constant interest-rate-based model. The blue line is the average sum of the first three volatility functions of the time-varying interest-rate-based model. The red line is the average time-varying volatility function that minimizes the in-sample fit of ATM cap prices. Based on weekly data ranging from January 2013 to March 2019 (326 observations).

the term structure, volatility structure and correlation structure, and that in order to better price these products, models that account for interest rate jumps and stochastic volatility are required.

6 Conclusion

The aim of this paper is to conditionally price caps and swaptions in an empirical low-interest-rate environment using multi-factor term structure models based on PCA with constant or time-varying parameters and an interest-rate-based or option-based estimation method. The pricing errors are further investigated to find possible sources of bias. Additionally, we investigate which model has the best forecasting accuracy and what the effect of a low interest rate environment is on pricing errors.

For the pricing of ATM caps, the constant parameter models outperform the time-varying parameter models, and the option-based estimation method outperforms the interest-rate estimation method. The best result is obtained using the 3-factor constant parameter option-based estimation method, which is confirmed in the model forecast comparison test. The pricing errors show a maturity effect, this however disappears when performing an error regression (combined with OTM caps) using black implied volatility. The pricing errors further vary over time which indicates the need for time-dependent volatility models. As a robustness test, an in-sample pricing exercise is performed for ATM caps. This test showed that, on average, the shape of the volatility functions is in the right direction, however, the model can never exceed a pricing error of 7.6%.

In terms of prediction errors, the OTM caps can be priced relatively well using time-varying parameters and an option-based estimation method (calibrated to ATM caps). Multi-factor models generally have a smaller pricing error than single-factor models. The model forecast comparison test, however, shows that the constant parameter models have a stronger on average forecasting accuracy compared to the time-varying parameter models. We conclude that for OTM caps the pricing results are not very pronounced and more research might be required. The pricing errors show a visible maturity effect that, again, disappears when performing an error regression. The error regression further shows a significant relation between the pricing errors and the moneyness, volatility skew, underlying rate, and yield curve shape. Also for OTM caps, the pricing errors vary over time.

The swaption pricing predictions in terms of prediction errors are less pronounced than for caps. The swaption pricing errors are all relatively high. The best results are obtained when using option-based estimation. The model forecast comparison test reveals a relatively accurate on average forecasting accuracy for option-based time-varying parameter models and we further find that multi-factor models, on average, outperform single-factor models. For swaptions, we also find that the errors vary over time. The pricing error regression shows that the errors are mainly caused by time-dependent variables such as the LIBOR rate and term structure slope.

The error regressions on interest rate level reveal that as interest rates decrease, absolute modeling errors increase. We conclude that a part of our pricing errors is possibly caused by the low interest-rate level. This is partially emphasized by smaller pricing errors found in earlier research in higher interest rate level environments.

The results of this paper come with its limitations. These last few paragraphs contain said limitations and ideas for future research. In Section 3.2.2 the drift of the instantaneous forward rate process is set to zero. The justification of this approximation is based on an argument given by Driessen et al. (2003), they state that this approximation is justified as the drift is small relative to the volatility of forward rate changes. We did, however, not check whether this was also the case in our sample. Further, a model that is also capable of modeling this drift can possibly achieve better pricing results.

The ATM caps are given in Black implied volatility and converted to basis points, however, this requires several input parameters such as a discount curve, a certain forward rate, and a strike price. The discount curve we choose is based on swaps, which are very liquid products, whereas Bloomberg uses an overnight-indexed-swap curve. We take the other parameters as close as possible to the Bloomberg parameters, however, the resulting basis point prices imply arbitrage opportunities between ATM and 1% OTM caps. Further, as the caps and swaption quotes are based on the mentioned Bloomberg Generic Composite pricing algorithm, no information about trading volume and liquidity is available, which could influence the pricing predictions.

The conditional pricing predictions in this paper are all based on a random walk forecast. The 2-week ahead forecasts could also be predicted using a more sophisticated method that benefits from the time series data of volatility function parameters such as an autoregressive model. Further, the volatility functions in this paper are all time-homogeneous deterministic functions, better pricing results can possibly be obtained by using time-dependent or stochastic volatility functions. This has already been done, by e.g. Falini (2010), for caps, but not yet for swaptions.

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A Bloomberg Tickers

Table 15: Bloomberg Tickers: Yield Curve

Bloomberg Ticker	Type	Maturity
US0001M Index	LIBOR Rate	1 month
US0002M Index	LIBOR Rate	2 months
US0003M Index	LIBOR Rate	3 months
US0006M Index	LIBOR Rate	6 months
USSW1 Curncy	Interest Rate Swap	1 year
USSW2 Curncy	Interest Rate Swap	2 years
USSW3 Curncy	Interest Rate Swap	3 years
USSW4 Curncy	Interest Rate Swap	4 years
USSW5 Curncy	Interest Rate Swap	5 years
USSW6 Curncy	Interest Rate Swap	6 years
USSW7 Curncy	Interest Rate Swap	7 years
USSW8 Curncy	Interest Rate Swap	8 years
USSW9 Curncy	Interest Rate Swap	9 years
USSW10 Curncy	Interest Rate Swap	10 years
USSW15 Curncy	Interest Rate Swap	15 years

Note: Tickers as per April 2019.

Table 16: Bloomberg Tickers: Interest Rate Caps

		OTM Premium				
Maturity	ATM	1%	2%	3%	4%	5%
1	USCV1	-	USCV201	USCV301	USCV401	USCV501
2	USCV2	USCV102	USCV202	USCV302	USCV402	-
3	USCV3	USCV103	USCV203	USCV303	USCV403	-
4	USCV4	USCV104	USCV204	USCV304	USCV404	USCV504
5	USCV5	USCV105	USCV205	USCV305	USCV405	USCV505
6	USCV6	USCV106	USCV206	USCV306	USCV406	USCV506
7	USCV7	USCV107	USCV207	USCV307	USCV407	USCV507
8	USCV8	USCV108	USCV208	USCV308	USCV408	USCV508
9	USCV9	USCV109	USCV209	USCV309	USCV409	USCV509
10	USCV10	USCV1010	USCV2010	USCV3010	USCV4010	USCV5010

Note: Tickers as per April 2019. All tickers are followed by a space and “Curncy”.

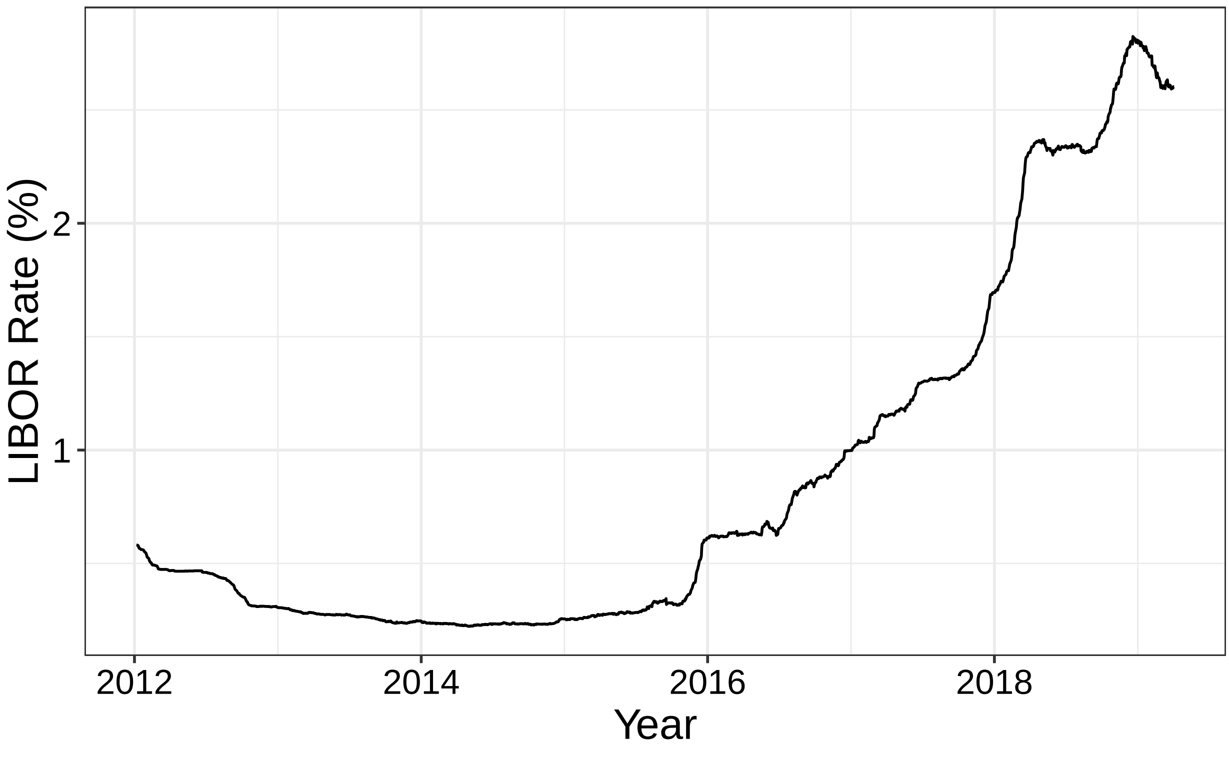
Table 17: Bloomberg Tickers: Swaptions

Option	Swap Tenor								
Expiry	1	2	3	4	5	6	7	8	9	10
1 MO	0A1	0A2	0A3	0A4	0A5	0A6	0A7	0A8	0A9	0A10
3 MO	0C1	0C2	0C3	0C4	0C5	0C6	0C7	0C8	0C9	0C10
6 MO	0F1	0F2	0F3	0F4	0F5	0F6	0F7	0F8	0F9	0F10
1 YR	011	012	013	014	015	016	017	018	019	0110
2 YR	021	022	023	024	025	026	027	028	029	0210
3 YR	031	032	033	034	035	036	037	038	039	0310
4 YR	041	042	043	044	045	046	047	048	049	0410
5 YR	051	052	053	054	055	056	057	058	059	0510

Note: Tickers as per April 2019. All tickers are preceded with "USSV", followed by the code from the table, and ending with "Curncy"; e.g. "USSV0F5 Curncy"

B Libor Rate

Figure 9: LIBOR Rate



Note: 3-month LIBOR Rates in %.

Figure 9 shows the movement of the LIBOR rate over time. The LIBOR rate is important since it is the underlying floating rate for all of the caps and swaptions in our sample.

C Black's Formula Implementation

This section provides the pricing formulas of Black (1976) used to convert Black implied volatility prices into actual prices and our implementation of it. All the formulas are retrieved from Filipovic (2009). For caplet i with a notional of 1, reset date T_{i-1} and payment date T_i the time t value is

$$\text{Caplet}(t; T_{i-1}, T_i) = \delta P(t, T_i)\Big(F(t; T_{i-1}, T_i)\Phi(d_1(i;t)) - k\Phi(d_2(i;t))\Big), \tag{30}$$

where

$$d_1(i;t) = \frac{\log\big(\frac{F(t;T_{i-1},T_i)}{k}\big) + \frac{1}{2}\sigma_{iv}(t)^2(T_{i-1} - t)}{\sigma_{iv}(t)\sqrt{T_{i-1} - t}}, \tag{31}$$

and

$$d_2(i;t) = d_1(i;t) - \sigma_{iv}(t)\sqrt{T_{i-1} - t}, \tag{32}$$

where $\delta = T_i - T_{i-1}$, $F(t; T, T+\delta)$ the δ-period forward rate at time t, $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, k the strike price, and σ_{iv} the Black option implied volatility. In our implementation we take the strike price equal to the swap rate with corresponding maturity defined in Filipovic (2009) as

$$k = \frac{P(0, T_0) - P(0, T_n)}{\delta \sum_{i=1}^{n} P(0, T_i)}, \tag{33}$$

and the δ-period forward rate as

$$F(t; T, T+\delta) = \frac{1}{\delta}\Big(\frac{P(t,T)}{P(t, T+\delta)} - 1\Big). \tag{34}$$

The caplet price becomes *undefined* in case the forward rate is negative, in this situation we set the respective caplet price to zero.

For a payer swaption with swap cashflows at T_i for $i = 1, ..., n$, and option maturity $T_0 - t$, the time t value is

$$\text{Swaption}_t = \delta\Big(R_{swap}(t)\Phi(d_1(t)) - K\Phi(d_2(t))\Big) \sum_{i=1}^{n} P(t, T_i), \tag{35}$$

where

$$d_1(t) = \frac{\log\big(\frac{R_{swap}(t)}{K}\big) + \frac{1}{2}\sigma_{iv}(t)^2(T_0 - t)}{\sigma_{iv}(t)\sqrt{T_0 - t}}, \tag{36}$$

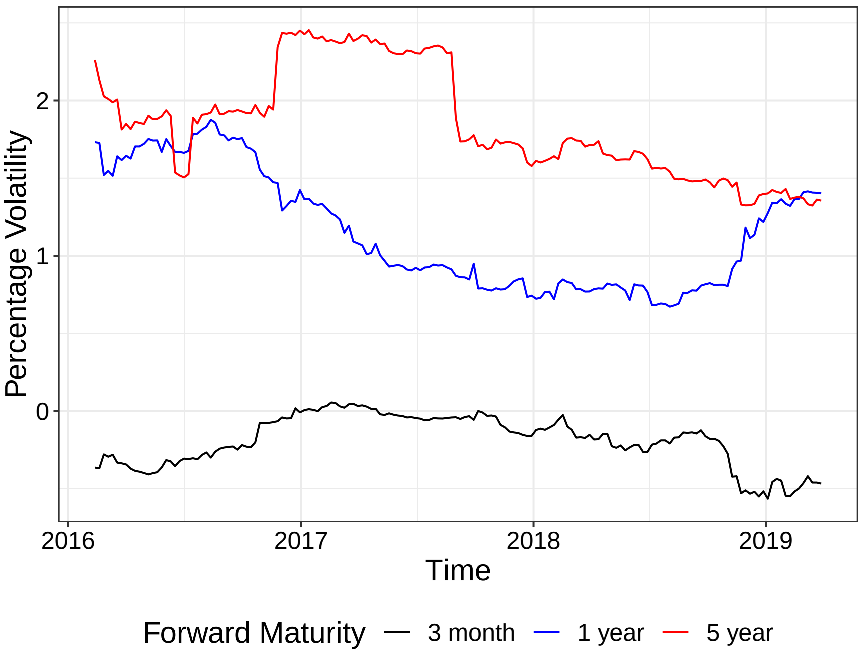
and

$$d_2(t) = d_1(t) - \sigma_{iv}(t)\sqrt{T_0 - t}, \tag{37}$$

which is again taken from Filipovic (2009). Here $\delta = T_i - T_{i-1}$, $\Phi(\cdot)$ again denotes the standard normal cumulative distribution function, K the strike price, and σ_{iv} the Black option implied volatility.

D Supplementary Results

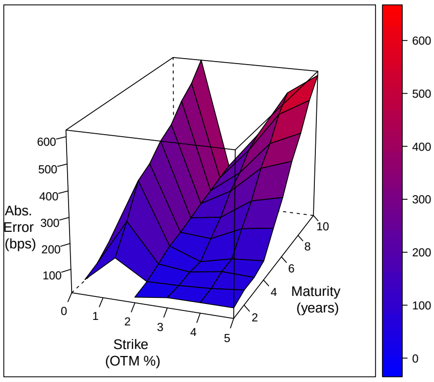
Figure 10: Volatility Functions Over Time



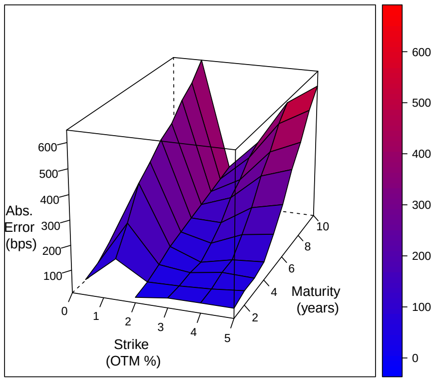
Note: Volatility functions (sum of factors) of the 3-factor interest-rate-based model for various forward maturies over time in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions)

E Cap Prediction Errors

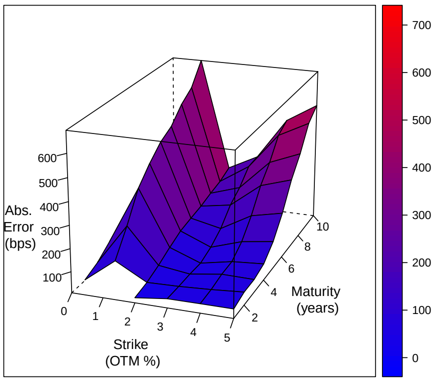
Figure 11: Absolute Prediction Errors ATM and OTM Caps (Constant Parameters)



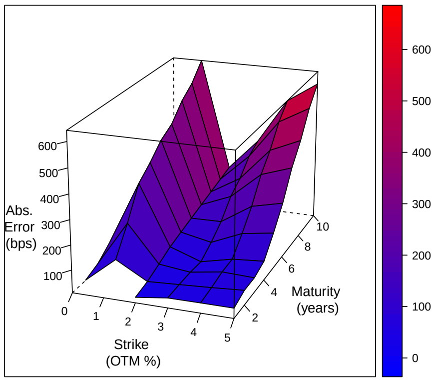
(a) 1-factor Interest-Rate-Based



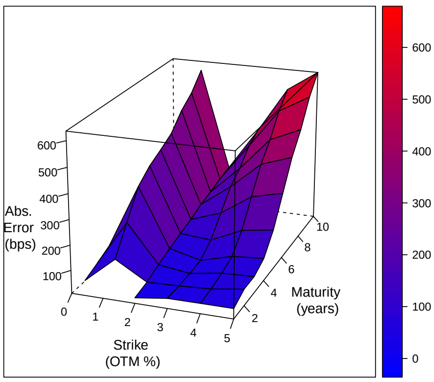
(b) 1-factor Option-Based



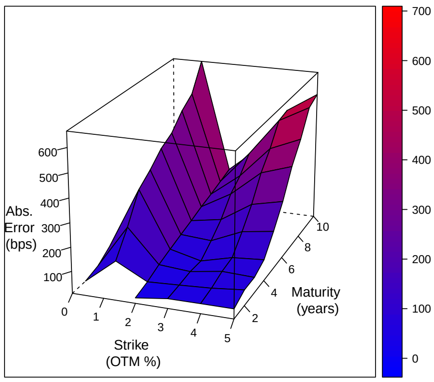
(c) 2-factor Interest-Rate-Based



(d) 2-factor Option-Based



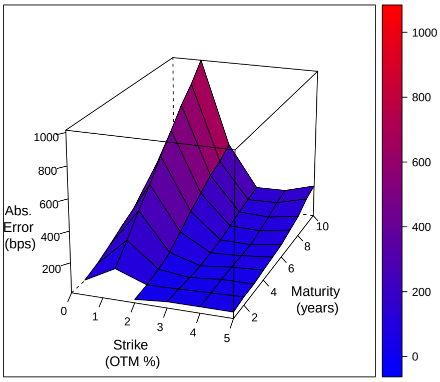
(e) 3-factor Interest-Rate-Based



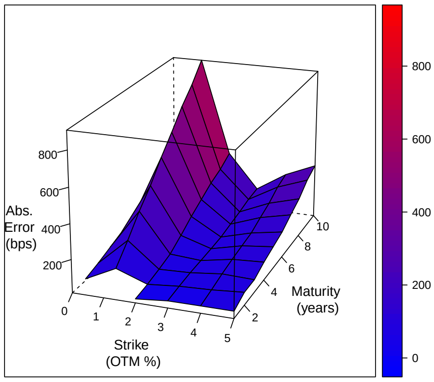
(f) 3-factor Option-Based

Note: Mean absolute 2-week ahead prediction errors for all caps (both ATM and OTM) in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions) in basis points for constant parameter models. An absolute prediction error is defined as the absolute value of the observed price minus the modeled price. Option-based estimation parameters are optimized with ATM caps.

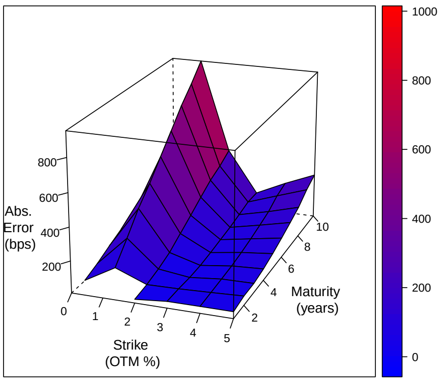
Figure 12: Absolute Prediction Errors ATM and OTM Caps (Time-Varying Parameters)



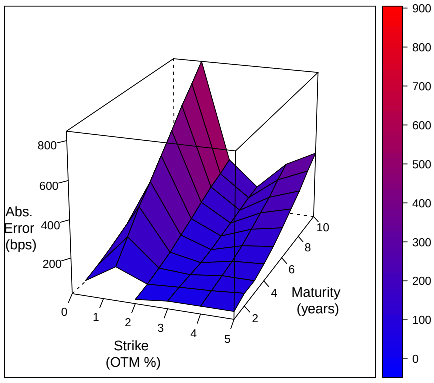
(a) 1-factor Interest-Rate-Based



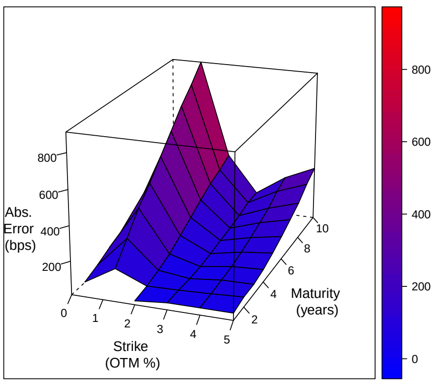
(b) 1-factor Option-Based



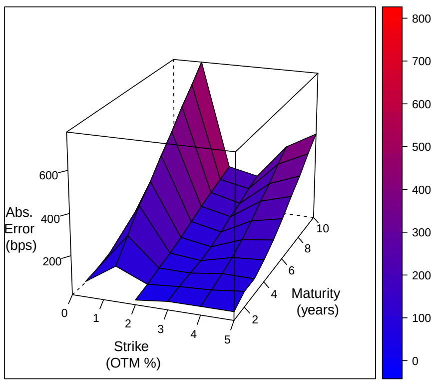
(c) 2-factor Interest-Rate-Based



(d) 2-factor Option-Based



(e) 3-factor Interest-Rate-Based

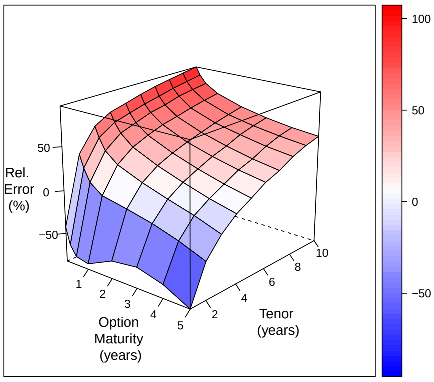


(f) 3-factor Option-Based

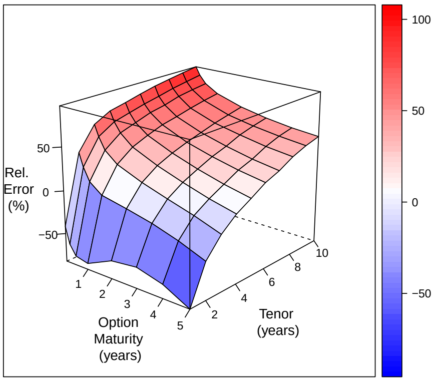
Note: Absolute 2-week ahead prediction errors for all caps (both ATM and OTM) in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions) in basis points for time-varying parameter models. An absolute prediction error is defined as the absolute value of the observed price minus the modeled price. Option-based estimation parameters are optimized with ATM caps.

F Swaption Prediction Errors

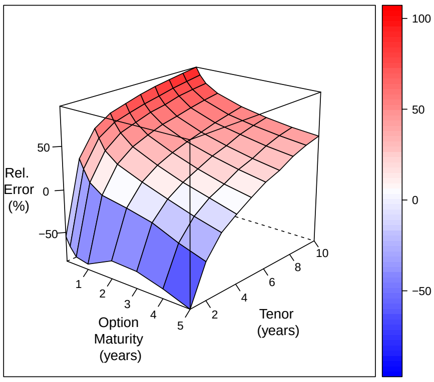
Figure 13: Relative Prediction Errors Swaptions (Constant Parameters)



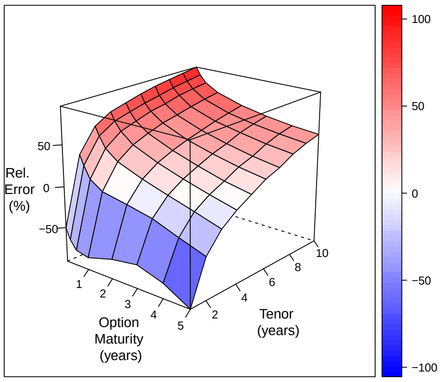
(a) 1-factor Interest-Rate-Based



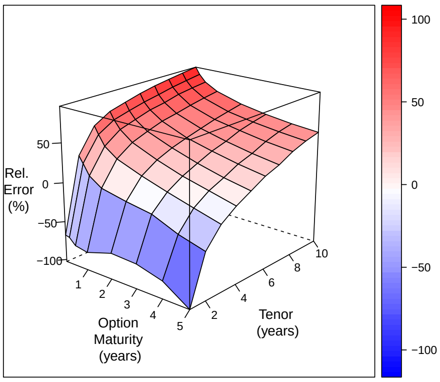
(b) 1-factor Option-Based



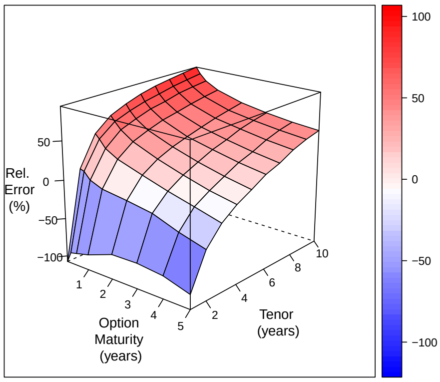
(c) 2-factor Interest-Rate-Based



(d) 2-factor Option-Based



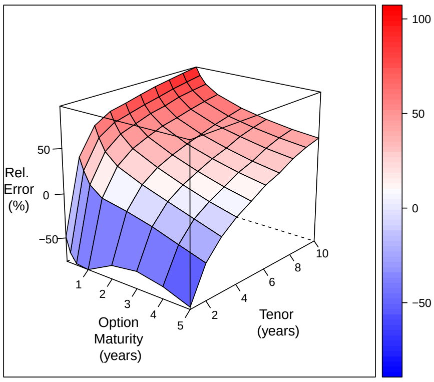
(e) 3-factor Interest-Rate-Based



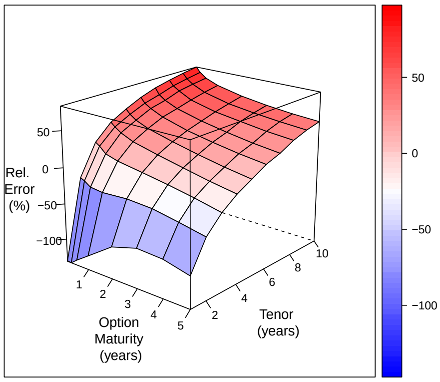
(f) 3-factor Option-Based

Note: Relative 2-week ahead mean absolute prediction errors for swaptions using constant parameter models in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions). A relative prediction error is defined as the observed price minus the modeled price divided by the observed price. The option-based estimation parameters are optimized with ATM caps.

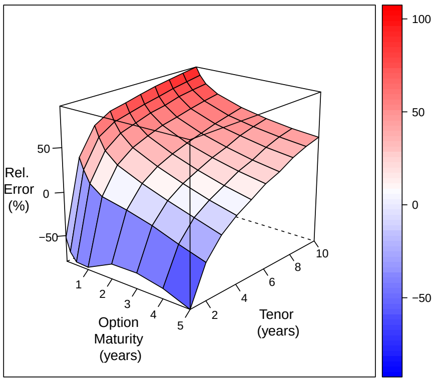
Figure 14: Relative Prediction Errors Swaptions (Time-Varying Parameters)



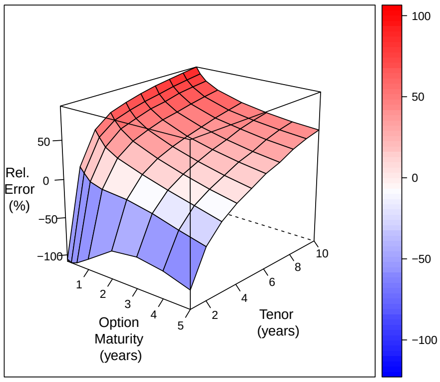
(a) 1-factor Interest-Rate-Based



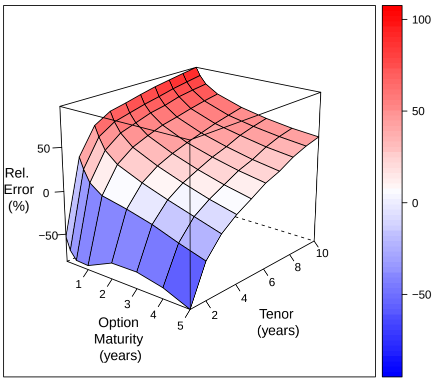
(b) 1-factor Option-Based



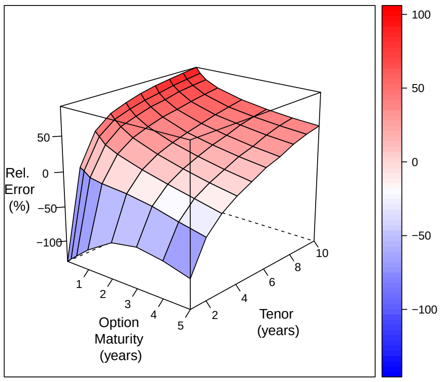
(c) 2-factor Interest-Rate-Based



(d) 2-factor Option-Based



(e) 3-factor Interest-Rate-Based



(f) 3-factor Option-Based

Note: Relative 2-week ahead mean absolute prediction errors for swaptions using time-varying parameter models in the out-of-sample period from January 2016 to March 2019 (163 weekly predictions). A relative prediction error is defined as the observed price minus the modeled price divided by the observed price. The option-based estimation parameters are optimized with ATM caps.

G Model Forecast Comparison Test (Standard Normal)

Table 18: Model Forecast Comparison Test ATM Caps (Standard Normal)

Estimation			Interest-Rate-Based						Option-Based						
Parameters			Constant			Time-Varying			Constant			Time-Varying			
Factors			1	2	3	1	2	3	1	2	3	1	2	3	Avg.
Interest-Rate-Based	Constant	1	-	50%	60%	80%	80%	80%	60%	20%	50%	90%	90%	80%	67%
		2	50%	-	40%	80%	80%	70%	50%	40%	40%	90%	80%	70%	63%
		3	40%	60%	-	80%	80%	80%	50%	40%	30%	100%	100%	60%	65%
	Time-Var.	1	20%	20%	10%	-	0%	0%	20%	10%	10%	30%	20%	20%	15%
		2	20%	20%	20%	90%	-	0%	20%	10%	10%	30%	20%	20%	24%
		3	20%	20%	20%	90%	80%	-	20%	10%	10%	30%	20%	20%	31%
Option-Based	Constant	1	40%	50%	50%	80%	80%	80%	-	30%	40%	90%	90%	70%	64%
		2	80%	60%	60%	90%	90%	90%	60%	-	40%	90%	90%	80%	75%
		3	40%	50%	70%	90%	90%	90%	60%	60%	-	100%	100%	90%	76%
	Time-Var.	1	0%	10%	0%	70%	60%	10%	10%	0%	0%	-	0%	0%	15%
		2	10%	10%	0%	80%	80%	80%	10%	10%	0%	90%	-	0%	34%
		3	20%	30%	20%	80%	80%	80%	20%	10%	10%	90%	80%	-	47%
Average			31%	35%	32%	83%	73%	60%	35%	22%	22%	75%	63%	46%	-

Note: Percentage of rejected H_0, using a 5% significance level, for ATM Caps of Diebold-Mariano tests where H_1 states that the model in the row has greater accuracy than the model in the column. Sampling distribution of test statistic assumed standard normal. Out-of-sample period ranges from January 2016 to March 2019 (163 weekly predictions).

Table 19: Model Forecast Comparison Test ATM and OTM Caps (Standard Normal)

Estimation			Interest-Rate-Based						Option-Based						
Parameters			Constant			Time-Varying			Constant			Time-Varying			
Factors			1	2	3	1	2	3	1	2	3	1	2	3	Avg.
Interest-Rate-Based	Constant	1	-	49%	61%	54%	54%	54%	40%	47%	54%	56%	58%	54%	53%
		2	51%	-	60%	56%	54%	54%	51%	51%	60%	58%	58%	58%	56%
		3	39%	40%	-	51%	53%	53%	39%	39%	44%	58%	60%	49%	48%
	Time-Var.	1	46%	44%	46%	-	39%	40%	46%	46%	54%	49%	49%	56%	47%
		2	46%	42%	47%	53%	-	44%	46%	46%	54%	56%	56%	56%	50%
		3	44%	42%	44%	53%	46%	-	40%	46%	54%	51%	56%	56%	48%
Option-Based	Constant	1	60%	49%	61%	54%	54%	51%	-	58%	56%	53%	58%	56%	56%
		2	49%	47%	60%	53%	53%	53%	39%	-	60%	53%	58%	58%	53%
		3	37%	39%	53%	46%	46%	46%	40%	40%	-	58%	58%	56%	47%
	Time-Var.	1	39%	37%	39%	44%	40%	19%	35%	33%	37%	-	21%	37%	35%
		2	39%	35%	35%	42%	42%	42%	33%	35%	32%	42%	-	44%	38%
		3	37%	37%	37%	44%	44%	42%	39%	37%	35%	37%	42%	-	39%
Average			44%	42%	49%	50%	48%	45%	41%	43%	49%	52%	52%	53%	-

Note: Percentage of rejected H_0, using a 5% significance level, for ATM and OTM Caps of Diebold-Mariano tests where H_1 states that the model in the row has greater accuracy than the model in the column. Sampling distribution of test statistic assumed standard normal. Out-of-sample period ranges from January 2016 to March 2019 (163 weekly predictions).

Table 20: Model Forecast Comparison Test Swaptions (Standard Normal)

Estimation			Interest-Rate-Based						Option-Based						
Parameters			Constant			Time-Varying			Constant			Time-Varying			
Factors			1	2	3	1	2	3	1	2	3	1	2	3	Avg.
Interest-Rate-Based	Constant	1	-	5%	5%	93%	71%	69%	98%	26%	5%	19%	8%	9%	37%
		2	93%	-	11%	96%	93%	94%	95%	91%	8%	20%	13%	10%	57%
		3	91%	84%	-	95%	91%	91%	94%	93%	8%	19%	13%	10%	63%
	Time-Var.	1	6%	4%	3%	-	1%	1%	19%	4%	4%	15%	6%	6%	6%
		2	20%	4%	8%	98%	-	6%	44%	8%	5%	16%	9%	10%	21%
		3	23%	5%	6%	98%	86%	-	48%	9%	5%	16%	8%	9%	28%
Option-Based	Constant	1	1%	3%	3%	73%	48%	43%	-	4%	5%	16%	8%	9%	19%
		2	70%	5%	5%	95%	88%	88%	94%	-	8%	19%	9%	10%	44%
		3	90%	90%	93%	93%	90%	91%	90%	91%	-	21%	41%	19%	74%
	Time-Var.	1	78%	76%	76%	79%	79%	79%	78%	76%	74%	-	76%	70%	76%
		2	78%	71%	63%	83%	80%	80%	81%	76%	21%	19%	-	0%	59%
		3	86%	84%	84%	88%	88%	88%	86%	86%	58%	20%	88%	-	78%
Average			58%	39%	32%	90%	74%	66%	75%	51%	18%	18%	25%	15%	-

Note: Percentage of rejected H_0, using a 5% significance level, for swaptions of Diebold-Mariano tests where H_1 states that the model in the row has greater accuracy than the model in the column. Sampling distribution of test statistic assumed standard normal. Out-of-sample period ranges from January 2016 to March 2019 (163 weekly predictions).