ERASMUS UNIVERSITY ROTTERDAM

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Effects of Admission Programs of Universities on Admission Rates of Students

Name: R.C. de Winter

Student ID number: 437984

Supervisor: S.H. Bijkerk

Second reader: ……

Date final version: ……

Abstract

Universities in the United States can choose to use SAT scores or motivation letters for their admissions. However, which of the two should be preferred is a highly debated topic and former theoretical literature has not provided a conclusive answer to the question of which admission program should be chosen. Therefore, this theoretical model is set up to research the optimal admission program for universities and the underlying effects of the different admission programs on student admissions. The optimal combination of admission programs differs per student enrolment. University of Chicago decided, presumably for ethical reasons, to no longer require SAT scores from their applicants but only motivation letters, whereas low-quality universities require SAT scores. Rich students mostly benefit from the universities choosing SAT scores for their admissions in tight markets or when one university uses SAT scores and the other university chooses motivation letters. Poor students benefit in loose markets from both universities choosing SAT scores. More motivated students benefit from a university basing their admissions on motivation letters.

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1. Introduction

Universities in the United States can use hard information, like test scores, or soft information, like motivation letters and personality, for the admission of students to universities. However, the choice of which information to use for the admission is highly debated in the United States.[1]

In most situations, SAT scores are used as hard information for the admission. It is known that low-income students in general perform worse on these tests and less often decide to retake these tests (Goodman, Gurantz, & Smith, 2018). Therefore, provided that universities only use hard information for the admission of students, the poor students are being discriminated against because this particular group of students performs worse and does less retakes and, consequently, gathers less information about their ability.

When universities use soft information for the admissions, it is argued that students can be discriminated against as well. For example, Asian-American applicants feel like they are being discriminated against by Harvard University because of the difficulties around objectivity for college admission officers in ranking intangible criteria like personality, courage and likeability.[2]

The aim of this thesis is to investigate what the effects are of different admission programs on admission rates, as former literature did not provide a conclusive answer to this highly debated topic yet. On the one hand, this theoretical model will analyse the effects of the usage of SAT scores on the admission rates of low-income or high-income groups, of high ability or low ability groups and of high motivation or low motivation groups. On the other hand, these effects are assessed when motivation letters are used. For instance, University of Chicago no longer requires SAT scores.[3] The results of this theoretical analysis can be used to understand why University of Chicago decided on this. Therefore, a theoretical model will be designed in order to answer the following research question of this thesis:

What is the effect of admission programs of universities on admission rates of students?

This research question will be answered by investigating a model of a market in which there are two universities, either of a low or a high quality. They have the same number of spots

[1] https://www.forbes.com/sites/christeare/2015/12/26/the-debate-over-the-college-admissions-process/#1426d12c1a7e
[2] https://www.nytimes.com/2018/06/15/us/harvard-universities-personality-criteria-admissions.html
[3] https://www.insidehighered.com/admissions/article/2019/07/15/chicago-sees-success-dropping-testing-requirement-admissions & https://eu.usatoday.com/story/news/2018/06/14/university-chicago-sat-act-optional/701153002/

available, and thus only symmetric markets are analysed. The universities can decide to base their admissions on SAT scores or on motivation letters. It is assumed that they select the optimal combination of admission programs that maximizes the total utility of the universities. The students differ from each other in their ability, motivation and wealth and, consequently, they will have a different optimal decision regarding their applications. Therefore, different combinations of admission programs result in different admission rates of students.

From this research it is obtained that in case there are two universities with the same quality, the universities are often best off by choosing to have one university base their admissions on SAT scores and the other university on motivation letters. However, for some qualities of the universities and for some numbers of spots available at the universities both universities decide to base their admissions on SAT scores or on motivation letters. Rich students and more motivated students will benefit when the universities choose different admission programs. Provided that both universities use SAT scores, rich students will mostly benefit in tight markets and poor students will benefit in loose markets. More motivated students will also benefit in case both universities use motivation letters.

In case the two universities are of a different quality, this thesis only compares both universities using SAT scores and both universities using motivation letters for their admissions. It is often optimal for both universities to base their admissions on SAT scores, but for some combinations of qualities of the universities it is optimal for both universities to base their admissions on motivation letters in some loose markets. Rich students will benefit from both universities using SAT scores and more motivated students benefit in case both universities use motivation letters.

This thesis will be structured as follows. In section 2 the related literature will be discussed. Section 3 will describe the model after which this model will be analysed in different settings in the sections 4, 5 and 6. Finally, section 7 will conclude and discuss this thesis by summarizing the results and explaining limitations and possible extensions for future research.

2. Related literature

A vast amount of literature that relates to the topic of this thesis has already been published. To find out where this model fits into and adds to the existing literature, eight different topics will be elaborated on. First, existing empirical literature will be discussed in order to formulate the results that should be explained in this thesis in a theoretical manner. Second, theoretical literature will be summarized to formulate what this thesis adds to the existing literature and how the model that is used in this thesis differs from former models. [4]

2.1. Empirical literature

Admission processes at universities

Hurwitz, Smith, Niu and Howell (2015) analyse the effects of mandating the SAT for all juniors of public schools on university enrolments by empirically investigating the enrolment-rate as well as the SAT scores. They obtain that the overall enrolment-rate increases as a consequence of the implemented policy and that some of the students who would not have participated in the test without the policy scored quite well. Dennis, Phinney and Chuateco (2005) however, analyse the motivation of students as an important predictor of university outcomes. With empirical analyses they attempt to obtain the predictors of university adjustments and find that personal and career related motivation are of high importance for this. The lack of peer support however has a negative effect and predicts a lower spring GPA. This literature leads to results that should be explained in this thesis. Namely, students' wealth influences the enrolment rate of students and more motivated students do better in university. Therefore, universities would be better off by admitting more motivated students.

Differences between rich and poor students

Hoxby and Avery (2012) empirically investigate the application rates for selective universities of high-achieving, low-income students. Those students who act upon their income, are less likely to be encouraged to apply and go to highly selective universities. Hence, this thesis should provide a theoretical framework that explains the observation that students with a high ability but with a low wealth will possibly not apply to high-quality universities.

Determinants for university choice and university major choice

Zafar (2013) investigates how university majors are selected by students, focussing on the underlying gender gap and obtains that the gender gap is mainly due to gender differences in preferences and tastes. Bobba and Frisancho (2014) attain empirically that the lack of full information about academic potential leads to a misallocation of students and therefore it can be concluded that academic potential is an important determinant of school-track choice. Furthermore, Wiswall and Zafar (2014) empirically investigate determinants of university major choices. They search for an answer to the question whether students have biased beliefs

[4] Appendix 1 chronologically displays the titles, authors, topics and results of the empirical literature elaborated and appendix 2 chronologically presents the titles, authors, topics, results and the structure of the models used in the theoretical literature section.

regarding their expected labour market outcomes and find that indeed the beliefs about future earnings and the perceived ability are important for university major choice. In addition to that, Delavande and Zafar (2014) empirically research the determinants of university choice of students, focussing on future monetary returns and financial constraints of students. They find that future earnings are not very important for the students and that if students were not financially constrained, 37% of the students would choose differently. When there are expanding university opportunities for poor students, like application guidance and no application fees, 48% more students apply to highly selective universities and they are admitted more often (Hoxby & Turner, 2015). This thesis will therefore try to explain that poor students prefer high-quality universities but may not be able to go because of financial constraints. Furthermore, students who do know they have a high ability but are unable to signal this because they have a low wealth will possibly not apply to high-quality universities.

Student distributions at universities

Dynarski, Libassi, Michelmore, and Owen (2018) write about closing the gap between low- and high-income students' enrolments at highly selective universities. They offer free tuition and no fees upon admission to close the gap. Additionally, high-achieving, low-income students do not apply to selective universities and therefore widen the gap (Hoxby & Avery, 2012). Hoxby and Turner (2015) not only assessed the determinants of university choice, but also how this influences the student distributions at universities and obtain that treated students were admitted to highly selective universities more often. This thesis will try to theoretically explain the gap between rich and poor students and why poor students are less often admitted to highly selective universities.

Promotion tournaments and tournaments in firms

DeVaro (2006) investigates promotion tournaments, mostly empirically. He finds evidence that suggests that relative work performance determines promotions. If the same holds for universities, the difference between the rewards for students will motivate them. Therefore, this thesis will theoretically explain that when the quality of the two universities differs more, the students will put in more effort in applying to a university.

Discrimination in firms

Gino and Pierce (2010) empirically investigate a form of discrimination in firms, namely the employee's perception of customer wealth and the likelihood of engaging in illegal behaviour. It is found that there is indeed some discrimination in firms. In this thesis it will be investigated whether wealth-based discrimination is also potentially present in universities and whether this will lead to poor students being more often or less often admitted to a university based on wealth alone.

Persuasion games

Anderson et al. (2010) write about persuasion in experimental ultimatum games. They let the proposer send persuasive messages to the responder and find that those persuasive messages indeed lead to a higher payoff for the proposer as a result of higher acceptance rates of the responder and lower offers by the proposer. This mostly happens in situations in which the subjects were already experienced in the game. This thesis will theoretically explain what the effects are of persuasive messages sent by students on their payoffs.

Signalling games

Banks, Camerer and Porter (1994) write about the refinements of Nash equilibrium in two-person signalling game experiments. They find that the combination of message and action of the sender and the receiver is a Nash equilibrium in 70% of the situations. Some subjects want to choose a more refined equilibrium strategy action, but they mostly appear in case of a mismatch between the sender and the receiver. Lastly, it is predicted that pooling equilibria will arise in which all players choose the same message, but separating equilibria arise because senders of different types choose different messages. This thesis will therefore investigate whether pooling or separating equilibria arise in case students want to signal their type to the university.

2.2. Theoretical literature

There is also a lot of theoretical literature related to the topic of this thesis. However, there has not yet been theoretical research on the different admission programs at universities and on the determinants of college and college major choice. Therefore, this thesis will theoretically investigate the different admission programs in universities. The differences between using hard and soft information for the admissions will be investigated and their effects on the student distributions. Lastly, the determinants of college choice are researched.

Differences between rich and poor students

Differences between rich and poor students are not only researched empirically, but also theoretically. The impact of credit constraints on the accumulation of human capital is an often-investigated topic. Lochner and Monge-Naranjo (2011) obtain a strong positive correlation between schooling and the family income. They do so by designing a model with multiple individuals who simultaneously need to decide on the level of investments in schooling in the first period. Their utility is determined by the consumption in the first period and their discounted consumption in the second period. The model in this thesis differs from the model of Lochner and Monge-Naranjo (2011) in the sense that application and admission to universities is investigated instead of investment levels, hereby taking into account the wealth of the students.

Promotion tournaments and tournaments in firms

Chen (2003) theoretically investigates the nature, determinants and impact of positive and negative effort during promotion tournaments and finds that negative effort is inefficient and leads to the people with the highest ability not having the highest chance of being promoted. He does so by using a model in which n agents simultaneously decide on their levels of positive and negative effort. After this, the principal decides who will be promoted and the agents receive their payoff, which depends on the probability of promotion and the rewards and costs of effort. This is also the case in this thesis, where universities decide who is admitted and the students receive their payoff. Compensation schemes that pay according to relative performance, tournament schemes, are researched by Lazear and Rosen (1981). They find that depending on whether the workers are risk neutral or risk-averse, tournaments lead to efficient allocation of resources. They do so by setting up a model in which two workers compete for the prize of winning the tournament by simultaneously choosing their level of effort. The firm needs to select the compensation scheme that determines the utility for the workers. The utility for the workers depends on the probability of winning, the prizes of winning or not and on the costs of putting in effort. Furthermore, tournaments among inequity averse agents are researched. The agents in this tournament are also motivated by the wellbeing of other agents and those agents therefore exert higher efforts (Grund & Sliwka, 2005). They model this by letting two agents simultaneously choose their output and the winner of the tournament receives the tournament prize. The utility that the agents receive from this tournament depends on the wage that the agents receive, the costs of effort and the inequity costs. The model in this thesis differs from this literature on promotion tournaments and tournament schemes in the exact sense that this model does not use inequity costs and negative effort, but does make use of a tournament scheme to find out whether this also results in an efficient allocation of resources in a model with universities instead of with firms.

Discrimination in firms

Schwab (1986) is an early researcher on statistical discrimination and its (in)efficiency. He finds, by using a theoretical model, that this discrimination is positive for the favoured group and negative for the disfavoured group. He uses a model with employers, and employees with either a high or low ability. The employees must decide to work in a standardized or individual market and the employer decides on the wages in both markets. The employer decides that the wage equals the average ability of the people in the standardized markets. The total payoff for the employers is the sum of production in both markets. In this thesis it is investigated which students are admitted by the university based on their ability, wealth and motivation and whether the favoured group is admitted more often.

Persuasion games

Milgrom and Roberts (1986) write about trying to influence the decision maker in a game where providing verifiable information elicits all relevant information. By using a theoretical model, they attain that competition is useful in some settings but unnecessary in others. This model consists of two players, who, for simplicity, are a seller and a buyer. The seller provides the buyer with product quality information and the buyer decides how much to purchase by taking the information of the seller into account. The utility for the buyer is maximal for a particular quantity, depending on the seller's information. The utility for the seller however is always increasing in the quantity. Seidmann and Winter (1997) generalize results of persuasion games by adding more general conditions on the sender's preferences to the theoretical model and thereby obtain that the ideal action for a sender varies with its type. They use a sender and receiver model in which the receiver chooses an action after he receives a message from the sender. Both the utility for the sender and for the receiver depend on the type of the sender and on the action chosen by the receiver. Additionally, the favourableness of news is introduced to find out that buyers expect that product information that is not shared by a salesman is unfavourable to the product (Milgrom, 1981). Milgrom models this by using a salesman who decides to report or conceal pieces of information to the buyer. The buyer than has to decide on his purchasing strategy. The utility that the buyer receives, positively depends on the quality of the product and negatively depends on the price of the product. The salesman however has a utility that increases in the quantity, as he receives commission per unit sold. Lastly, Nash and perfect Bayesian equilibrium payoffs achievable in persuasion games are found in one-shot and multistage communication games (Forges & Koessler, 2008). To find these equilibria they use a model with an expert and a decision maker. The expert's type is chosen after which there is communication between the expert and the decision maker. The decision maker chooses an action based on this information that results into a utility for the decision maker. His utility depends on the type of the expert and on the action taken by the decision maker. This literature on persuasion games is important for this thesis in the way that it explains persuasion games and their results under different assumptions. These models on persuasion games use only two players whereas the model in this thesis will use different universities and multiple students applying to these universities. Furthermore, the models used in the related literature have in common that an informed player and an uninformed player are present. In this thesis however it is the case that poor students cannot correctly transfer their information, because poor students receive a signal about their ability that is random with probability $\frac{1}{2}$ and that is accurate with probability $\frac{1}{2}$. Lastly, this thesis will use persuasion games in universities instead of in firms or in markets as students try to persuade universities that they have a high ability even though it is unknown to the university what the ability and the wealth of a student are.

Signalling games

Spence (1973) investigates the characteristics of a basic signalling model. He obtains that signalling games result in equilibria that can mainly be used to investigate admission procedures and promotions. To do this he uses a model in which there are two players, an employer and an employee. The employee is meant to send a signal to the employer in order to signal that he should be hired by the employer. The employer sequentially decides on hiring the employee or not, based on the signal of the employee. In case the employee is hired the employer must decide on which wage to offer the employee. This results in a signalling equilibrium where the utility for the employee depends on his wage and his signalling costs, whereas the utility for the employer depends on the marginal product of the employee. Furthermore, Cho & Kreps (1987) investigate the general criteria of equilibria in signalling games and the theory of stability. They attain that different tests can be used to find reasonable equilibria. They use a model with two players where the first player is either a wimp or surly and the first player meets the second player. The first player decides to have beer or quiche for breakfast and the second players decides to duel the first player or not, based on the breakfast of the first player. This results in a sequential equilibrium in which the utility for the players depends on the combination of wimp or surly, beer or quiche and duel or no duel. The model provided in this thesis adds to the existing literature in the sense that a university must decide to admit a student or not in different situations, based on the quality of the university and the admission programs of the university, where signalling games are used when the university bases their admissions on motivation letters. A signalling game is used because students can write a motivation letter to signal to the universities that they are more motivated.

Summarizing, the model of this thesis adds to the existing literature in the way that the empirical results found in former literature will be explained in this thesis by using a theoretical model. Furthermore, the model in this thesis will add to the theoretical literature considering there is no fully conclusive theoretical literature on university admission programs yet. Lastly, the theoretical models already designed will be combined and altered to a model that can be used in universities and to acquire results that extend the already present results. A persuasion game is used in case the universities base their admissions on SAT scores and a signalling game is used when the universities base their admissions on motivation letters.

3. Model

A theoretical model will be developed to answer the research question of this thesis. This model section consists of a description of the players in the game, their payoffs and the assumptions that are made. Furthermore, the utility functions and the strategies of the players will be explained in different possible settings and it will be explained whether these strategies should be chosen simultaneously or sequentially. This section will lastly elaborate which equilibrium strategy is relevant for this model and will therefore be used in the analysis.

3.1. Players

There are two players in this model. There are multiple students, and every student $i \in I = \{1, \dots, n\}$ wants to apply to a university. The students are not symmetric because they differ in their ability which is denoted by $a_i \in [0,1]$. Ability a_i is assumed to be uniformly distributed, hence $a_i \sim u(0,1)$. Furthermore, students differ in their motivation to study, $m_i \in [0,1]$. Motivation m_i is also assumed to be uniformly distributed, $m_i \sim u(0,1)$. This assumes that the ability a_i and the motivation m_i can take any value between 0 and 1 and the expected ability a_i and expected motivation m_i are $E[a_i] = \frac{1}{2}$ and $E[m_i] = \frac{1}{2}$ respectively. It is assumed that students know their ability and motivation, however, both are unknown to the university. Furthermore, it is assumed that the ability a_i is not correlated with the motivation m_i of the student as is stated in assumption 1:

Assumption 1: *The ability a_i and the motivation m_i of the student are uncorrelated.*

There is a third source of asymmetry which is the wealth of the students, $w_i \in \{p, r\}$, where p is used when a student is poor and r is used to denote a rich student. Students know whether they are rich or poor, but this is unobservable for the university. It is assumed that the probability that a student is poor equals the probability that a student is rich, hence $\Pr(w_i = p) = \Pr(w_i = r) = \frac{1}{2}$ and the universities know these probabilities. Therefore, every student i is determined by (a_i, m_i, w_i).

The second player in this model is the university. There are two universities that can differ from each other in their quality. The quality is however common knowledge for both the students and the universities. The quality of university j can either be low or high, hence $q_j = \{L, H\}$. Both universities have x spots available for students and only symmetric markets where $x_1 = x_2$ are analysed. Also, it is assumed that there is a number of students greater than or equal to the number of spots available at universities, hence $n \geq 2x$. Students receive a higher utility from being admitted to a high-quality university than from being admitted to a low-quality university. Markets in which $\frac{2x}{n} < \frac{1}{2}$ are referred to as tight markets in which there is a lot of competition between the students because there are relatively few spots available. Markets in which $\frac{2x}{n} > \frac{1}{2}$ are referred to as loose markets with less competition between the students, as there are relatively more spots available for the students.

3.2. Assumptions

Ability a_i is known to the students, but unobservable for the universities. The utility for the students and the universities is however influenced by the ability of the student. Therefore, students can choose to obtain a signal s_i of their ability through an SAT. This test does not always provide an accurate signal. For the rich students, it does give an accurate signal of their ability, hence $s_i(w_i = r) = a_i$, whereas for poor students the SAT gives a signal of the ability that is accurate with probability $\frac{1}{2}$ and that is random with probability $\frac{1}{2}$ as well, hence $s_i(w_i = p) = \frac{1}{2}a_i + \frac{1}{2} * \frac{1}{2} = \frac{1}{2}a_i + \frac{1}{4}$. This assumes that with a probability of $\frac{1}{2}$ the signal truthfully reveals the ability of student i, whereas with probability $\frac{1}{2}$ the student receives a random signal that is expected to be $\frac{1}{2}$ due to the uniform distribution of ability a_i. This is summarized in assumption 2:

Assumption 2: *An SAT gives a signal s_i that is an accurate signal of the ability for rich students, whereas poor students receive a signal of their ability s_i that is accurate with probability $\frac{1}{2}$ and that is random with probability $\frac{1}{2}$ as well.*

Students know their own ability and their own wealth, and therefore know which signal they can expect from taking an SAT. The students can thus determine in expectation how many students will receive a signal that is higher than their own signal or that is lower than their own signal.

Taking an SAT comes with a cost of $c(t_i) = \frac{1}{2}t_i$, where $t_i \in \{0,1\}$. Whenever a student decides to take an SAT, t_i takes on the value of 1, hence $t_i = 1$ whereas $t_i = 0$ in case a student decides not to take an SAT. Taking an SAT always leads to a verifiable result for the university. Therefore, an SAT score is hard information for the university.

Students can also choose to write a motivation letter, $l_i = \{0,1\}$. Writing a motivation letter is soft information for the university as they can only observe whether a student wrote a motivation letter or not. The universities can however not deduct the exact motivation of the student. Exerting effort to write a motivation letter comes with costs. These costs of writing a motivation letter decrease in the motivation of the student, because the costs are $c(l_i) = (1 - m_i)l_i$. If a student decides to write a motivation letter, then $l_i = 1$ whereas $l_i = 0$ in case a student decides not to write a motivation letter. This is summarized in assumption 3 as follows:

Assumption 3: *The costs of writing a motivation letter are higher for less motivated students and these costs decrease when the motivation of a student increases.*

Whenever the university chooses to base their admissions on SAT scores and a student did not take an SAT, he will not be admitted even if the university still has spots available. The same holds for a university that chooses to base their admissions on motivation letters and a student does not have a motivation letter. A student thus only has a chance to be admitted to a university when he meets the requirements of the university.

3.3. Utility functions

The utility for the student positively depends on the quality of the university. So, when the quality of the university is high, the utility for the student will also be higher and vice versa. The probability of being admitted is k_{ij}. The utility function of the student negatively depends on both the costs for taking an SAT and on the costs of writing a motivation letter. Both costs can be zero in the utility function when the SAT is not taken or whenever the student decides not to write a motivation letter. All students have an outside option with a utility of $U^i = 0$, as not taking an SAT, not writing a motivation letter and not applying to a university leads to a utility of 0. The utility function for student i is denoted as follows:

$$U^i(t_i, l_i) = k_{ij}q_j - c(t_i) - c(l_i)$$

These values of k_{ij}, q_j, $c(t_i)$ and $c(l_i)$ determine whether a student decides to take an SAT or to write a motivation letter or decides to do neither of the two. In case the expected utility of taking an SAT or writing a motivation letter is bigger than or equal to the expected utility of choosing the outside option, the student wants to do so instead of choosing for the outside option and not applying to a university at all. The value of q_j is determined by the quality of the university and is q if $q_j = H$ and is γq in case $q_j = L$, where $0 < \gamma < 1$.

The utility function of the university positively depends on the ability and motivation of the admitted students. Furthermore, it is important for their utility which admission programs the universities choose. The universities announce whether they require SAT scores or motivation letters. The utility function for university j is denoted as follows:

$$U^j(P_j) = \widetilde{a_y} + \widetilde{m_y}$$

In this utility function, $\widetilde{a_y}$ denotes the average ability of the y students who are admitted to the university and $\widetilde{m_y}$ denotes the average motivation of those y students, where $y \leq x$. The P_j stands for the program that the university uses for their admission, being SAT scores or motivation letters. The utility for the university will be larger when the admitted students have a high ability a_i and a high motivation m_i.

3.4. Strategies

The universities have to decide which admission program to use and which students to admit. They announce to the students whether they require SAT scores or motivation letters in their admission program, $P_j = \{(s, 0), (0, l)\}$. The university chooses the admission program that results in the highest average expected utility.

All students simultaneously choose to take an SAT or to write a motivation letter or to do neither of the two and after this they decide to apply to one of the two universities by sending their application. Their application is a message containing their SAT score or their motivation letter. The message is denoted by $r_i \in \{s_i, l_i\}$. Students cannot take an SAT and write a motivation letter as well, but they can use their motivation letter or SAT score twice.

3.5. Timeline of the model

The timeline of this model is as follows:

1. Nature chooses the student's ability a_i, the student's motivation m_i, the student's wealth w_i and the quality of the university q_j.
2. Both universities learn their quality q_j and simultaneously announce their admission program P_j.
3. Students learn the quality of the university q_j, their own ability a_i, their own motivation m_i and their own wealth w_i.
4. Students choose to collect s_i or l_i.

5. Students submit their message, r_i, to the university of their choice.
6. The university decides which students to admit, based on r_i.
7. Both the students and the universities receive their payoff U^i and U^j.

3.6. Equilibrium concept

In games with incomplete information the equilibrium concept used is the Bayesian Nash equilibrium (Tadelis, 2013). The Bayesian Nash equilibrium is defined as an equilibrium in which every player chooses type-contingent strategies that state what a player will do when he learns that he is of a particular type. Perfect Bayesian Nash equilibrium is a refinement of this Bayesian Nash equilibrium in which the players do not want to change their strategy when they learn that they are of a particular type. This thesis will therefore use the Perfect Bayesian Nash equilibrium as equilibrium concept.

The students have an optimal strategy of information collection depending on their types. They decide to gather information or not, in order to maximize their expected utility. Furthermore, they also have an optimal strategy of choosing a university to apply to. Hence, students with different abilities a_i, different motivations m_i and different wealth w_i might have different optimal information collection strategies. In equilibrium, the students choose the combination of information collection and university choice that maximizes their utility.

The universities must choose an admission program that maximizes their expected utility. Hence, in equilibrium there exists an optimal admission program and an optimal strategy of student admission for high-quality universities and for low-quality universities. To end up with the optimal strategies for both the universities and for the students, backward induction will be used in the analysis.

Together, the optimal strategy for information sharing and the action taken by the university leads to a Perfect Bayesian Nash equilibrium that is denoted as follows:

1. For each $i(a_i, w_i)$, where $a_i \sim u(0,1)$ and $w_i \sim u(0,1)$, $\mu(i)$ solves $\underset{r_i}{\operatorname{argmax}}\, U^i\big(q_j, \alpha(r_i), i\big)$;
2. And for each r_i, $\alpha(r_i)$ solves $\underset{P_j}{\operatorname{argmax}} \int_0^1 U^j\big(P_j, i\big)\rho(i, r_i)\,di$;
3. $\rho(i, r_i)$ is derived through Bayes' rule,

where ρ stands for the beliefs about the student's ability or motivation.

4. Markets with only high-quality universities

In this section the model will be analysed in multiple steps in markets where both universities are of a high quality. As the Perfect Bayesian Nash equilibrium is used as equilibrium concept, backward induction will be used for the analysis. Therefore, first the optimal admission strategy for the universities will be determined. Second, the communication by the students is investigated to answer the questions on how the students will communicate, when the students will communicate and when they will not communicate. Third and last, the optimal admission program for the universities is researched.

4.1. Admission programs consist of an SAT

In this first subsection it is assumed that both universities choose the admission program that is purely based on SAT scores, $p_1 = p_2 = (s, 0)$. Furthermore, there are n students in the market and both universities have x spots available at their university. Therefore, there is a total of $2x$ spots available for the n students. Every possible point in figure 1 below depicts a student with a particular ability a_i and a particular motivation m_i.

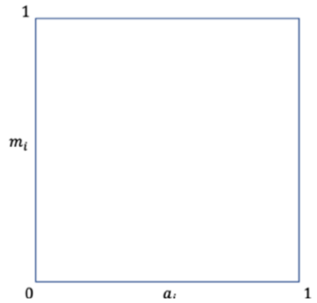

Figure 1 – Students' ability and motivation

The SAT provides a signal about the ability of the student. The signal of a rich student is determined by $s_i = a_i$. Rich students can therefore have a signal $s_i \in [0,1]$ and the expected ability for a rich student who sends signal s_i is $E[a_i|w_i = r] = s_i$. However, poor students receive a signal that is not always correct, $s_i = \frac{1}{2}a_i + \frac{1}{4}$. The expected ability for a poor student who sends signal s_i is therefore $E[a_i|w_i = p] = 2s_i - \frac{1}{2}$. In case poor students have an ability $a_i = 0$, they expect a signal of $s_i = \frac{1}{2} * 0 + \frac{1}{4} = \frac{1}{4}$ and in case $a_i = 1$, they expect a signal of $s_i = \frac{1}{2} * 1 + \frac{1}{4} = \frac{3}{4}$, hence the possible expected signals for a poor student are $s_i \in \left[\frac{1}{4}, \frac{3}{4}\right]$. Figure 2 below depicts for both rich and poor students which signal s_i is expected for each ability a_i, where the expected signal for a rich student is $E[s_i|w_i = r] = a_i$ and the expected signal for a poor student is $E[s_i|w_i = p] = \frac{1}{2}a_i + \frac{1}{4}$. The signal for a poor student is lower than the true ability of the student when $a_i > \frac{1}{2}$ and higher than the true ability of the student when $a_i < \frac{1}{2}$.[5]

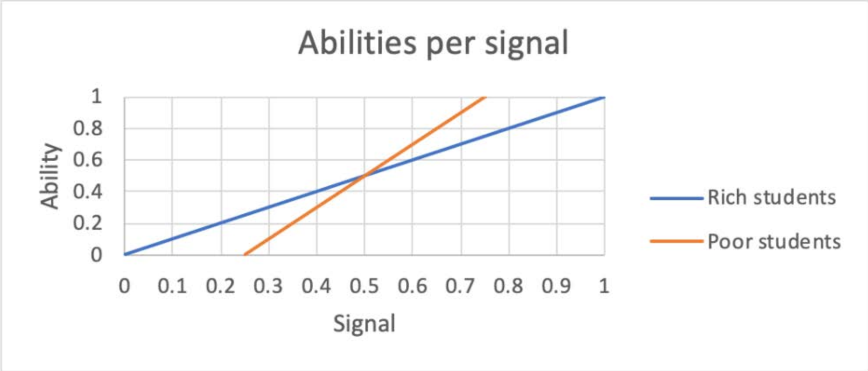

Figure 2 – Abilities per signal

Admission policy of the universities

The universities together have $2x$ spots available. The expected utility for university j, $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$, positively depends on the average ability of the students who are admitted and therefore, to maximize the utility of the universities, they want to admit the $2x$ students with the highest ability. However, the universities only observe the signal s_i of a student and not his ability a_i or his wealth w_i. The signal of a rich student is always increasing in the ability of a student because $\frac{\partial E[a_i|w_i = r]}{\partial s_i} = 1 > 0$, and the signal of a poor student is also always increasing in the ability of a student because $\frac{\partial E[a_i|w_i = p]}{\partial s_i} = 2 > 0$. A higher signal s_i therefore always indicates a higher expected ability to the universities and the universities will thus admit the $2x$ students with the highest signal s_i in order to maximize their expected utility.

Information collection of the student

In case a student is admitted after taking an SAT he receives an expected utility of $E[U^i] = q - \frac{1}{2}$, which is bigger than or equal to the expected utility that he receives from choosing the outside option, $E[U^i] = 0$, when $q \geq \frac{1}{2}$. Would this not be the case, the student would not choose to take an SAT. Therefore, the participation constraint is $q \geq \frac{1}{2}$. However, if the student is not admitted after taking an SAT, he receives an expected utility of $E[U^i] = -\frac{1}{2}$, which is lower than the expected utility that he receives from choosing the outside option. Therefore, students only choose to take an SAT in case they expect that they will be admitted. Rich and poor students know which signal s_i they can expect from taking an SAT, because their ability

[5] All formulas that belong to the figures can also be found in appendix 3.

and their wealth are both known to them. Therefore, students can decide to take an SAT or not. Rich and poor students will take an SAT in case their ability is high enough to expect that they receive a signal s_i that will belong to the $2x$ students with the highest s_i. The ability of a rich student should thus be $a_i \geq \overline{s_i}$, where $\overline{s_i}$ is the signal that is needed to be admitted. However, the signal of a poor student is either accurate or random, both with probability $\frac{1}{2}$. The ability of the poor student should therefore be $a_i \geq 2\overline{s_i} - \frac{1}{2}$.

All n students decide to take an SAT or not, taking into account that the universities admit the $2x$ students with the highest signal s_i. In case $\frac{2x}{n}$ is very small, meaning that there are just a few spots available, all the available spots can be filled with rich students who have a signal $s_i > \frac{3}{4}$, as there are no poor students with a signal $s_i > \frac{3}{4}$. The number of rich students with $s_i > \frac{3}{4}$ is $\frac{1}{2}n * \frac{1}{4} = \frac{1}{8}n$, because half of the students in the market is rich and one out of four of those students has an ability $a_i > \frac{3}{4}$, because of the uniform distribution of a_i. Therefore, when $\frac{2x}{n} = \frac{1}{8}$, only for rich students with an ability $a_i > \frac{3}{4}$ is the expected utility of taking an SAT bigger than from not taking an SAT, as they are the only students who can receive a signal $a_i > \frac{3}{4}$:

$$E\left[U^i(t_i = 1)\middle|a_i > \frac{3}{4}, w_i = r\right] > E\left[U^i(t_i = 0)\middle|a_i > \frac{3}{4}, w_i = r\right]$$

$$q - \frac{1}{2} > 0$$

When $\frac{2x}{n} = \frac{1}{8}$, then for all rich students with $a_i \leq \frac{3}{4}$, the expected utility of taking an SAT is smaller than from not taking an SAT and they will thus never take an SAT:

$$E\left[U^i(t_i = 1)\middle|a_i \leq \frac{3}{4}, w_i = r\right] < E\left[U^i(t_i = 0)\middle|a_i \leq \frac{3}{4}, w_i = r\right]$$

$$-\frac{1}{2} < 0$$

All poor students also do not want to take an SAT when $\frac{2x}{n} = \frac{1}{8}$, as they expect to receive a signal of $s_i = \frac{3}{4}$ at most. Therefore, the expected utility of taking an SAT is always smaller than the expected utility of not taking an SAT:

$$E\left[U^i(t_i = 1)\middle|a_i, w_i = p\right] < E\left[U^i(t_i = 0)\middle|a_i, w_i = p\right]$$

$$-\frac{1}{2} < 0$$

In case $\frac{2x}{n} > \frac{1}{8}$ there are more spots available and therefore there are more students who want to take an SAT and compete for a spot. In this case, poor students will take an SAT as well and the $2x$ students with the highest signal s_i will be admitted. Figure 3 below depicts the minimal signal needed to be admitted, depending on the value of $\frac{2x}{n}$.

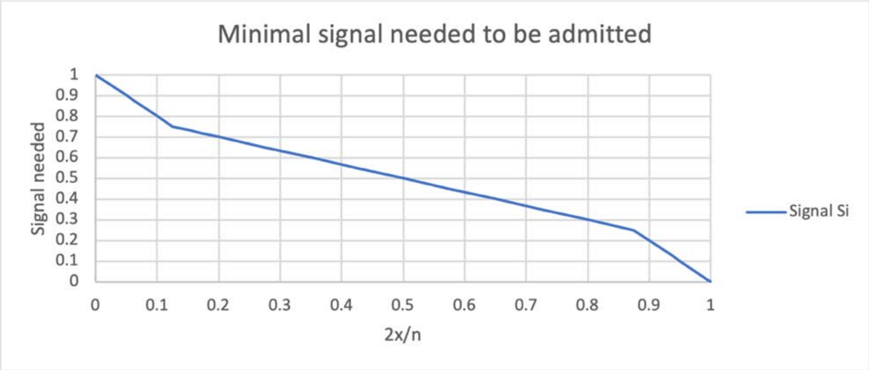

Figure 3 – Minimal signal needed to be admitted

From figure 3 above it can be seen that the minimal signal needed to be admitted if $\frac{2x}{n} < \frac{1}{8}$ is $\overline{s_i} = 1 - 2 * \frac{2x}{n}$, because only rich students will apply. The minimal signal needed to be admitted is $\overline{s_i} = \frac{5}{6} - \frac{2}{3} * \frac{2x}{n}$ if $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$, as rich and poor students will both apply. Lastly, if $\frac{2x}{n} > \frac{7}{8}$, some rich students and all poor students will apply. The minimal signal needed to be admitted is therefore $\overline{s_i} = 2 - 2 * \frac{2x}{n}$. Figure 4 below furthermore depicts the percentage of students taking an SAT.

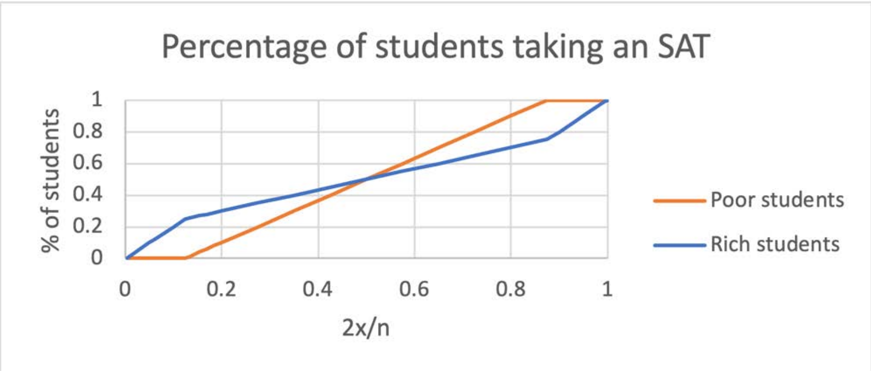

Figure 4 – Percentage of students taking an SAT

Figure 4 above shows which students will be admitted for different values of $\frac{2x}{n}$, taking into account the signal s_i that is needed to be admitted which is depicted in figure 3 above. The percentage of rich students taking an SAT is $1 - \overline{s_i}$. For $\frac{2x}{n} < \frac{1}{8}$ this results in $2 * \frac{2x}{n}$, for $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ this is $\frac{1}{6} + \frac{2}{3} * \frac{2x}{n}$ and for $\frac{2x}{n} > \frac{7}{8}$ this percentage is $2 * \frac{2x}{n} - 1$. The percentage of poor students taking an SAT, is the percentage of students for whom $\frac{1}{2}a_i + \frac{1}{4} > \overline{s_i}$, hence $1 -$

$\left(2\overline{s_i} - \frac{1}{2}\right)$. In case $\frac{2x}{n} < \frac{1}{8}$ there are no poor students taking an SAT, for $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ the percentage is $\frac{4}{3} * \frac{2x}{n} - \frac{1}{6}$ and when $\frac{2x}{n} > \frac{7}{8}$ all poor students take an SAT.

There is a clear cut-off at $\frac{2x}{n} = \frac{1}{2}$. In tight markets, thus $\frac{2x}{n} < \frac{1}{2}$, rich students will take an SAT more often than poor students. However, in loose markets, hence $\frac{2x}{n} > \frac{1}{2}$, rich students will less often take an SAT compared to poor students. For a number of spots $\frac{2x}{n} < \frac{1}{8}$, there are no poor students taking an SAT. However, when $\frac{2x}{n} > \frac{7}{8}$, all poor students take an SAT. In case there are no spots available, hence $\frac{2x}{n} = 0$, no student will take an SAT as no student is admitted. However, when $\frac{2x}{n} = 1$, all students will take an SAT, as all students are admitted for sure. Both in case $\frac{2x}{n} = 0$ and in case $\frac{2x}{n} = 1$, a Perfect Bayesian Nash equilibrium arises in which no student or all students respectively decide to take an SAT.

Expected utility for the universities

The expected utility for the universities equals $E\left[U^j\right] = \widetilde{a_y} + \widetilde{m_y}$. The expected average motivation of the students who are admitted is $E[m_i] = \frac{1}{2}$. However, the expected ability of the students can be computed out of the number of spots available and the number of students who take an SAT. The weighted average of the abilities of the admitted students is needed to compute the expected utility for the universities. For instance, in case $\frac{2x}{n} = \frac{1}{5}$, the minimal signal s_i that is needed for a student to be admitted is $\overline{s_i} = \frac{5}{6} - \frac{2}{3} * \frac{1}{5} = \frac{7}{10}$. All rich students with $a_i \geq \frac{7}{10}$ and all poor students with an ability $a_i \geq \frac{9}{10}$ will take an SAT because they expect to receive a signal $s_i \geq \frac{7}{10}$. Thus, 30% of the rich students and 10% of the poor students will take an SAT because of the uniform distribution of ability a_i, which is 20% of the total number of students. The expected average ability of the admitted students is therefore $\widetilde{a_y} = \frac{3}{4} * \frac{1+\frac{7}{10}}{2} + \frac{1}{4} * \frac{1+\frac{9}{10}}{2} = \frac{7}{8}$ when $\frac{2x}{n} = \frac{1}{5}$. The total expected utility for the university is then $E\left[U^j(s,0)\right] = \widetilde{a_y} + \widetilde{m_y} = \frac{1}{2} + \frac{7}{8} = \frac{11}{8}$. The expected utility for the university is only determined by rich students in case $\frac{2x}{n} < \frac{1}{8}$. In case $\frac{2x}{n} = \frac{1}{8}$, the minimal signal needed to be admitted for a student is $\overline{s_i} = \frac{5}{6} - \frac{2}{3} * \frac{1}{8} = \frac{3}{4}$. The expected average ability of the admitted students is therefore $\widetilde{a_y} = \frac{1+\frac{3}{4}}{2} = \frac{7}{8}$ and the total expected utility is thus $E\left[U^j(s,0)\right] = \widetilde{a_y} + \widetilde{m_y} = \frac{1}{2} + \frac{7}{8} = \frac{11}{8}$, which is the same as when $\frac{2x}{n} =$

$\frac{1}{5}$. In case $\frac{1}{8} < \frac{2x}{n} < \frac{1}{5}$, the expected utility for the university is higher than when $\frac{2x}{n} = \frac{1}{8}$ or when $\frac{2x}{n} = \frac{1}{5}$, as now the poor students with a signal $\frac{7}{10} \leq s_i \leq \frac{3}{4}$ also choose to take an SAT and these students have an ability that is close to 1. This leads to a higher average ability of the admitted students. When $\frac{2x}{n} > \frac{1}{5}$, the expected average ability decreases to $\widetilde{a_y} < \frac{7}{8}$, because there are more students with a lower ability taking an SAT. Lastly, when $\frac{2x}{n} > \frac{7}{8}$, all poor students take an SAT. The average ability of the admitted students is then only influenced by the change of the average ability of the rich students who are admitted and by the distribution of rich and poor students and therefore the average ability decreases slower. Figure 5 below denotes the average ability and the average motivation of the admitted students and the expected utility for the university, depending on the value of $\frac{2x}{n}$.

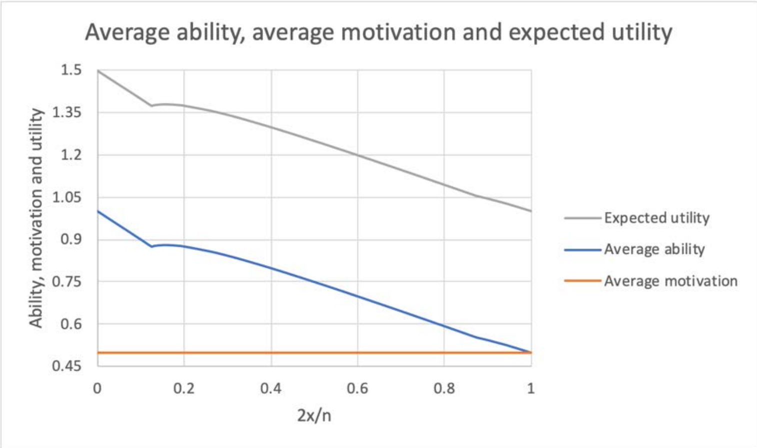

Figure 5 – Average ability, average motivation and expected utility

The expected average motivation is $\widetilde{m_y} = \frac{1}{2}$, the expected average ability is $\widetilde{a_y} = \frac{\frac{1}{2} * \%rich}{\frac{2x}{n}} * \frac{1+(1-\%rich)}{2} + \frac{\frac{1}{2} * \%poor}{\frac{2x}{n}} * \frac{1+(1-\%poor)}{2}$, where % rich and % poor denote the percentages of rich and poor students taking an SAT, as depicted in figure 4 above. For $\frac{2x}{n} < \frac{1}{8}$ this results in $\widetilde{a_y} = 1 - \frac{2x}{n}$, and for $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ this leads to $\widetilde{a_y} = \frac{76 * \frac{2x}{n} - 40 * \left(\frac{2x}{n}\right)^2 - 1}{72 * \frac{2x}{n}}$. Lastly, for $\frac{2x}{n} > \frac{7}{8}$ this results in $\widetilde{a_y} = \frac{4 * \frac{2x}{n} - 2 * \left(\frac{2x}{n}\right)^2 - 1}{2 * \frac{2x}{n}}$.[6]

Hence, in case both universities are of a high quality, $q_1 = q_2 = H$, and both universities choose to use SAT scores for their admission, $p_1 = p_2 = (s, 0)$, rich students more often choose

[6] The formulas for the expected utility are in appendix 3.

to take an SAT than poor students in case of a tight market, $\frac{2x}{n} < \frac{1}{2}$, and poor students more often choose to take an SAT in case of a loose market, $\frac{2x}{n} > \frac{1}{2}$.

Conclusion subsection 4.1

On the one hand, rich students are overrepresented at the universities in tight markets. On the other hand, poor students are overrepresented at universities in loose markets. Furthermore, students only take an SAT in case the quality of the university is high enough, hence only in case the participation constraint, $q \geq \frac{1}{2}$, is satisfied.

4.2. Admission programs consist of a motivation letter

In this second subsection it is assumed that both universities choose the admission program that is based on motivation letters, $p_1 = p_2 = (0, l)$.

Admission policy of the universities

The expected utility for a student writing a motivation letter is $E[U^i | l_i = 1] = kq - (1 - m_i)$, where k denotes the probability of being admitted. This expected utility for a student who writes a motivation letter increases when the motivation of a student increases, $\frac{\partial E[U^i | l_i = 1]}{\partial m_i} = 1 > 0$. Hence, writing a motivation letter more often results in a higher expected utility compared to not writing a motivation letter for students who have a higher motivation, when keeping k constant. Irrespective of being admitted or not, more motivated students will more often decide to write a motivation letter, $l_i = 1$. As the expected utility for the universities is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$ and the ability of the students is unknown, the universities want to admit the students with the highest expected motivation. The universities expect that the motivation of a student writing a motivation letter is higher than the motivation of a student who did not write a motivation letter and they will therefore admit as many students who wrote a motivation letter as possible.

Information collection of the students

The students decide to write a motivation letter or not. Therefore, pooling and separating equilibria might arise. In case of a pooling equilibrium, all students choose the same strategy. However, in case of a separating equilibrium, students with a high motivation choose a different strategy than students with a low motivation. There is one student who is indifferent between writing a motivation letter and not writing a motivation letter as for this student the probability of receiving a positive payoff and the costs of writing a motivation letter are equal. These equilibria exist in case no student wants to deviate.

Pooling equilibrium with $l_i = 1$ for all students

First, it could be the case that a pooling equilibrium arises in which all students decide to write a motivation letter. Consequently, the university does not learn anything about the motivation of the student. The expected utility in this pooling equilibrium is $E\left[U^i \middle| l_i = 1\right] = \frac{2x}{n}q - (1 - m_i)$. Only if this expected utility is bigger than or equal to the expected utility of deviating and choosing not to write a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$, this pooling equilibrium can exist. This should happen even for a student with a motivation of $m_i = 0$, hence for $q \geq \frac{n}{2x}$. In case $q < \frac{n}{2x}$, there is at least one student with a lower motivation who wants to deviate and not write a motivation letter, as the probability of being admitted and receiving q does not outweigh the costs of writing a motivation letter for this student anymore. There is also no pooling equilibrium when $\frac{2x}{n} = 0$, as there are no students admitted and therefore no student wants to write a motivation letter. Therefore, there exists a pooling equilibrium in which all students choose $l_i = 1$ if there are enough spots available and if the quality of the universities is high enough, hence $q \geq \frac{n}{2x}$ and $\frac{2x}{n} > 0$.

Pooling equilibrium with $l_i = 0$ for all students

The other pooling equilibrium that might arise is the pooling equilibrium in which all students decide not to write a motivation letter. In that case, the expected utility of writing a motivation letter should be smaller compared to the expected utility of choosing to write a motivation letter, even for a student with $m_i = 1$. The expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$, whereas the expected utility of deviating and writing a motivation letter is $E\left[U^i \middle| l_i = 1\right] = q - (1 - m_i)$. The expected utility of writing a motivation letter for a student with $m_i = 1$ is $E\left[U^i(l_i = 1) \middle| m_i = 1\right] = q$ and is bigger compared to the expected utility of not writing a motivation letter $E\left[U^i(l_i = 1) \middle| m_i = 1\right] = 0$, when $q > 0$. Thus, there only exists a pooling equilibrium when $q = 0$ and $\frac{2x}{n} \geq 0$, as being admitted does not lead to a higher expected utility than not being admitted, or when $\frac{2x}{n} = 0$ and $q \geq 0$, as no students are admitted and no student therefore wants to write a motivation letter. For all combinations of $\frac{2x}{n} > 0$ and $q > 0$ there is no pooling equilibrium in which no student writes a motivation letter, as there is at least one student with a high motivation who wants to deviate and write a motivation letter.

Expected utility for the universities – pooling equilibria

The expected utility for the universities in a pooling equilibrium with $l_i = 1$ and $q \geq \frac{n}{2x}$ is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. In a pooling equilibrium, the universities learn nothing about the motivation or the ability of the students and therefore $\widetilde{m_y} = E[m_i] = \frac{1}{2}$ and $\widetilde{a_y} = E[a_i] = \frac{1}{2}$. Combined, this leads to an expected utility for the universities of $E[U^j(0, l)] = \frac{1}{2} + \frac{1}{2} = 1$. In the pooling equilibrium with $l_i = 0$ for all students, the expected utility for the universities is $E[U^j(0, l)] = 0$, as no students are admitted.

Separating equilibrium with high m_i choosing $l_i = 1$ and low m_i choosing $l_i = 0$

In this first separating equilibrium, students with a high motivation decide to write a motivation letter whereas students with a lower motivation do not write a motivation letter. They will do so because the expected utility of writing a motivation letter is never smaller than the expected utility of not writing a motivation letter. The student who is indifferent has a motivation $m_i = \widehat{m}$. All students for whom the expected utility of writing a motivation letter is bigger than from not writing a motivation letter decide to write a motivation letter. This number of students is denoted by z and those are all students with a motivation $m_i > \widehat{m}$, hence $z = (1 - \widehat{m})n$.

First, in case $z < 2x$, the expected utility of writing a motivation letter is $E[U^i | l_i = 1] = q - (1 - m_i)$, because there are less students writing a motivation letter than there are spots available and therefore all students will be admitted. In case this turns out to be a separating equilibrium this implies that even though a student with a low motivation will be admitted with certainty, he will not write a motivation letter. The expected utility of not writing a motivation letter is $E[U^i | l_i = 0] = 0$. There is one student who is indifferent, and this is the student for whom the expected utility of writing a motivation letter equals the expected utility of not writing a motivation letter:

$$q - (1 - \widehat{m}) = 0$$

$$\widehat{m} = 1 - q$$

When the quality of the university is higher, $\widehat{m}$ is lower as there are more students who want to write a motivation letter to have a chance to receive the higher payoff q.

The student with a motivation that is somewhat lower than the motivation of the indifferent student, $m_i = \widehat{m} - \varepsilon$, wants to deviate in case the expected utility of deviating, $E[U^i | l_i = 1] = q - (1 - (\widehat{m} - \varepsilon))$, is bigger than the expected utility of not writing a motivation letter, $E[U^i | l_i = 0] = 0$. For the student with $m_i = \widehat{m} - \varepsilon$ this never happens. Therefore, students with $m_i < \widehat{m}$ do not want to deviate, even though writing a motivation letter leads to being admitted with certainty. The student with $m_i = \hat{m} + \varepsilon$ also does not want to deviate because his expected utility of deviating, $E\left[U^i \middle| l_i = 0\right] = 0$, never exceeds his expected utility of writing a motivation letter, $E\left[U^i \middle| l_i = 1\right] = q - \left(1 - (\hat{m} + \varepsilon)\right)$. Therefore, there is a separating equilibrium in which the students with $m_i > \hat{m} = 1 - q$ write a motivation letter and students with a lower motivation do not, when $z < 2x$. The number of students writing a motivation letter is than $z = (1 - \hat{m})n < 2x$, and therefore this separating equilibrium exists when $q < \frac{2x}{n}$ and $\frac{2x}{n} > 0$ and for $q \in [0,1]$ because $\hat{m} \in [0,1]$.

Second, in case $z = 2x$, the expected utility of writing a motivation letter is $E\left[U^i \middle| l_i = 1\right] = q - (1 - m_i)$ as all students with a motivation letter will be admitted and the expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$. The student who is indifferent between writing a motivation letter and not writing a motivation letter is the student with the following motivation:

$$q - (1 - \hat{m}) = 0$$
$$\hat{m} = 1 - q$$

Again, when the quality of the university is higher, there are more students willing to write a motivation letter because being admitted leads to a higher payoff.

The student with $m_i = \hat{m} - \varepsilon$ does want to deviate in case deviating and writing a motivation letter, $E\left[U^i \middle| l_i = 1\right] = \frac{2x}{2x+1} q - \left(1 - (\hat{m} - \varepsilon)\right)$, leads to a higher expected utility compared to not writing a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$. This student however does not want to deviate because the probability of being admitted is even smaller than in the equilibrium case and therefore this expected utility is never bigger than 0. Students with $m_i < \hat{m}$ therefore do not want to deviate. This separating equilibrium exists when the student with $m_i = \hat{m} + \varepsilon$ does not want to deviate as well. Deviating by not writing a motivation letter results in an expected utility of $E\left[U^i \middle| l_i = 0\right] = 0$, whereas the expected utility of not deviating is $E\left[U^i \middle| l_i = 1\right] = q - \left(1 - (\hat{m} + \varepsilon)\right)$. This is never smaller than 0 for this student and students with $m_i > \hat{m}$ thus do not want to deviate. Therefore, there is a separating equilibrium in which the number of students writing a motivation letter is $z = (1 - \hat{m})n = 2x$. The conditions for this separating equilibrium are that $q = \frac{2x}{n}$ and $\frac{2x}{n} > 0$, and $q \in [0,1]$ because $\hat{m} \in [0,1]$.

Third and last, it can be the case that there exists a separating equilibrium in which $z > 2x$, the students with $m_i > \hat{m}$ choose $l_i = 1$ and the students with $m_i < \hat{m}$ choose $l_i = 0$. The expected utility for a student with a higher motivation is than $E\left[U^i \middle| l_i = 1\right] = \frac{2x}{z} q - (1 - m_i)$, because the $2x$ spots will be randomly divided over the z students with a motivation letter. The expected utility for a student with a lower motivation is $E\left[U^i \middle| l_i = 0\right] = 0$. The student who is indifferent between writing a motivation letter and not writing a motivation letter has a motivation of:

$$\frac{2x}{(1-\widehat{m})n}q - (1-\widehat{m}) = 0$$

$$\widehat{m} = 1 - \sqrt{\frac{2x}{n}q}$$

Whenever $\frac{2x}{n}$ or q increases, $\widehat{m}$ decreases ceteris paribus. This happens because the probability of being admitted, k, is higher and the payoff from being admitted is higher.

The student with $m_i = \widehat{m} - \varepsilon$ wants to deviate in case deviating leads to an expected utility that is higher than the expected utility of not writing a motivation letter. The expected utility of deviating is $E\left[U^i \middle| l_i = 1\right] = \frac{2x}{\sqrt{\frac{2x}{n}qn+1}}q - \left(1 - (\widehat{m} - \varepsilon)\right)$ and the expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$. The expected utility of deviating is never bigger than from not deviating because the probability of being admitted is lower than $\frac{2x}{z}$. Therefore, students with $m_i < \widehat{m}$ do not want to deviate.

In case the student with $m_i = \widehat{m} + \varepsilon$ does not want to deviate as well, a separating equilibrium arises. Deviating and not writing a motivation letter leads to an expected utility of $E\left[U^i \middle| l_i = 0\right] = 0$, whereas deciding to write a motivation letter and not deviating leads to an expected utility of $E\left[U^i \middle| l_i = 1\right] = \frac{2x}{z}q - \left(1 - (\widehat{m} + \varepsilon)\right)$. This expected utility of writing a motivation letter is always bigger than the expected utility of deviating for students with $m_i > \widehat{m}$ and these students therefore do not want to deviate.

A separating equilibrium arises in which students with $m_i > \widehat{m} = 1 - \sqrt{\frac{2x}{n}q}$ write a motivation letter whereas students with $m_i < \widehat{m}$ do not write a motivation letter. The student with $m_i = \widehat{m} = 1 - \sqrt{\frac{2x}{n}q}$ is indifferent between writing a motivation letter and not writing a motivation letter. The number of students with a motivation letter is thus $(1-\widehat{m})n = \sqrt{\frac{2x}{n}q}n$ and this number of students is bigger than the number of spots available at the universities. Therefore, the quality of the university should be $q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$ to have a separating equilibrium in which students with a high motivation separate from students with a lower motivation, as highly motivated students choose $l_i = 1$ and students with a lower motivation choose $l_i = 0$. Furthermore, the quality of the university is $q \in [0,1]$ because $\hat{m} \in [0,1]$.

Separating equilibrium with $m_i < \hat{m}$ choosing $l_i = 1$ and $m_i > \hat{m}$ choosing $l_i = 0$

It could be the case that highly motivated students decide not to write a motivation letter and students with a lower motivation will decide to write a motivation letter. The expected utility of writing a motivation letter for students with a low motivation should then be bigger than from not writing a motivation letter. It is assumed that the indifferent student has a motivation of $\hat{m}$. In case a student with $m_i < \hat{m}$ chooses $l_i = 1$, then his expected utility of writing a motivation letter should be bigger than from not writing a motivation letter, $E\left[U^i(l_i = 1)\middle|m_i < \hat{m}\right] > \left[U^i(l_i = 0)\middle|m_i < \hat{m}\right]$. A student with $m_i = 1$ always wants to deviate from not writing a motivation letter to writing a motivation letter, as his motivation is bigger than from a student with $m_i < \hat{m}$ and therefore his expected utility of writing a motivation letter is also bigger than from not writing a motivation letter, $E\left[U^i(l_i = 1)\middle|m_i = 1\right] > \left[U^i(l_i = 0)\middle|m_i = 1\right]$. Therefore, there is never a separating equilibrium in which only students with a motivation $m_i < \hat{m}$ write a motivation letter, $l_i = 1$, and students with a motivation $m_i > \hat{m}$ do not write a motivation letter, $l_i = 0$.

Pooling and separating equilibria exist in different situations. A pooling equilibrium in which all students choose $l_i = 1$, exists for $q \geq \frac{n}{2x}$ and $\frac{2x}{n} > 0$. A pooling equilibrium in which all students choose $l_i = 0$ exists when the quality of the university is $q = 0$ and $\frac{2x}{n} \geq 0$ or when there are no spots available at the university, $\frac{2x}{n} = 0$ and $q \geq 0$. A separating equilibrium in which all students with $m_i > \hat{m} = 1 - q$ choose $l_i = 1$ and all other students choose $l_i = 0$ exists for $q \leq \frac{2x}{n}$ and $\frac{2x}{n} > 0$, hence for $q \in [0,1]$ as $\hat{m} \in [0,1]$. Lastly, a separating equilibrium in which all students with $m_i > \hat{m} = 1 - \sqrt{\frac{2x}{n}q}$ choose $l_i = 1$ and all other students choose $l_i = 0$ exists for $q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$, thus for $q \in [0,1]$ as $\hat{m} \in [0,1]$.

The percentage of students writing a motivation letter is 100% in case there is a pooling equilibrium in which all students write a motivation letter. In case of a separating equilibrium this percentage of students is $(1 - \hat{m}) = \sqrt{\frac{2x}{n}q}$ for qualities of the university $q > \frac{2x}{n}$ and $(1 - \hat{m}) = q$ for $q \leq \frac{2x}{n}$. This percentage of students writing a motivation letter in case of a separating equilibrium is depicted in figure 6 below for different values of $\frac{2x}{n}$ and q and these percentages are needed to derive the expected utility for the universities.

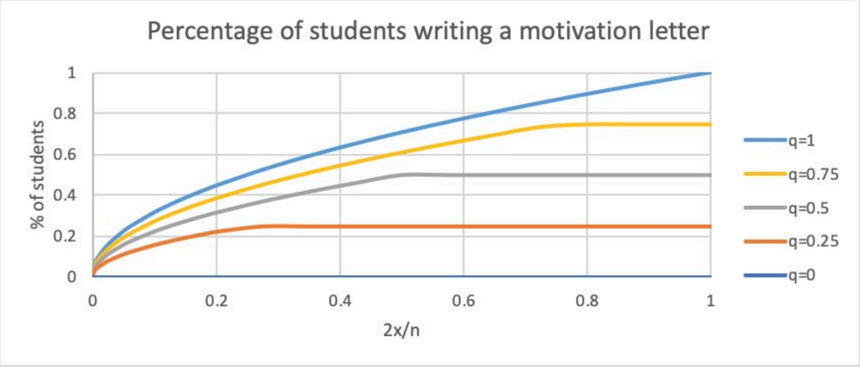

Figure 6 – Percentage of students writing a motivation letter

<u>*Expected utility for the universities – separating equilibrium*</u>

The expected utility for the universities is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. All students who have a motivation $m_i > \hat{m}$, will hand in a motivation letter and will possibly be admitted. Therefore, the average motivation of the admitted students is $\widetilde{m_y} = \frac{1+\hat{m}}{2}$. The average ability of these students is unknown to the universities, because the motivation letters written by the students do not say anything about their ability. Therefore, the expected average ability of the admitted students is $\widetilde{a_y} = E[a_i] = \frac{1}{2}$. Combined, this leads to an expected utility for the universities of $E[U^j(0,l)] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{2x}{n}q}$ if $q > \frac{2x}{n}$ and $E[U^j(0,l)] = \frac{3}{2} - \frac{1}{2}q$ if $q \leq \frac{2x}{n}$. This average motivation and ability of the admitted students and the expected utility for the universities are depicted in figure 7 below for different values of q and $\frac{2x}{n}$.

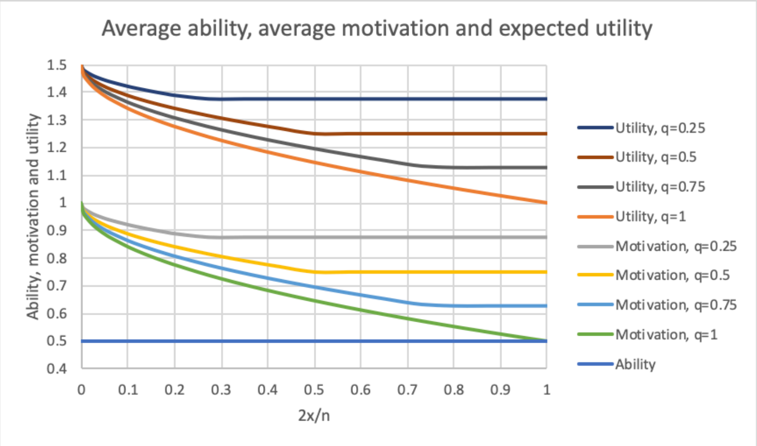

Figure 7 – Average ability, average motivation and expected utility

<u>*Conclusion subsection 4.2*</u>

In case both universities are of a high quality, $q_1 = q_2 = H$, and both universities choose to use motivation letters for their admissions, $p_1 = p_2 = (0, l)$, a pooling equilibrium arises *where all students choose to write a motivation letter if* $q \geq \frac{n}{2x}$ *and* $\frac{2x}{n} > 0$. *A pooling equilibrium arises where all students choose not to write a motivation letter in case* $q = 0$ *and* $\frac{2x}{n} \geq 0$ *or in case* $\frac{2x}{n} = 0$ *and* $q \geq 0$. *Furthermore, a separating equilibrium arises in which all students with* $m_i > \widehat{m}$ *decide to write a motivation letter and all students with* $m_i < \widehat{m}$ *do not write a motivation letter, for* $q \in [0,1]$, *with* $\widehat{m} = 1 - \sqrt{\frac{2x}{n}q}$ *when* $q > \frac{2x}{n}$ *and* $\frac{2x}{n} > 0$ *and* $\widehat{m} = 1 - q$ *when* $q \leq \frac{2x}{n}$ *and* $\frac{2x}{n} > 0$. *More motivated students will thus always be overrepresented.*

4.3. University 1 demands an SAT score whereas university 2 demands a motivation letter

In this third subsection it is assumed that university 1 chooses the admission program that is based on SAT scores, $p_1 = (s, 0)$, and university 2 chooses the admission program that is based on motivation letters, $p_2 = (0, l)$. The results from this section are also accurate in situations in which university 1 uses motivation letters for their admissions and university 2 chooses SAT scores for their admissions.

Admission policy of the universities

University 1 has x spots available for students who take an SAT and receive signal s_i and therefore university 1 wants to admit the x students who apply to university 1 with the highest signal s_i. University 2 also has x spots available but only admits students with a motivation letter.

Information collection of the students

University 1 and 2 together have $2x$ spots available with $\frac{2x}{n} \leq 1$. Only symmetric markets in which $x_1 = x_2$ are investigated and therefore $\frac{x}{n} \leq \frac{1}{2}$. For all students with a particular ability a_i, it is expected that their motivation is on average $m_i = \frac{1}{2}$. When the participation constraint is satisfied, students take an SAT and being admitted to university 1 after taking an SAT results in an expected utility of $E[U^i] = q - \frac{1}{2}$. Being admitted to university 2 after writing a motivation letter leads however to an expected utility of $E[U^i] = kq - (1 - m_i)$. The expected utility of being admitted to university 1 after taking an SAT is always bigger than or equal to the expected utility of being admitted to university 2 after writing a motivation letter, $E[U^i | l_i = 1] = kq - \frac{1}{2}$. Therefore, all students who expect to be admitted to university 1 will take an SAT and apply to university 1.

All students who expect not to be admitted to university 1 decide to write a motivation letter or not. Pooling and separating equilibria arise, as in subsection 4.2. A pooling equilibrium in which all students decide not to write a motivation letter exists when $q = 0$ and $\frac{x}{n} \geq 0$ or when $\frac{x}{n} = 0$ and $q \geq 0$. Furthermore, a pooling equilibrium arises in which all students write a motivation letter when $q \geq \frac{n}{x}$. Lastly, a separating equilibrium arises in which all students with a motivation $m_i > \widehat{m} = 1 - \sqrt{\frac{x}{n}q}$ write a motivation letter and all students with a lower motivation do not write a motivation letter, where $q \in [0,1]$ as $\widehat{m} \in [0,1]$. In this separating equilibrium the threshold is $\widehat{m} = 1 - \sqrt{\frac{x}{n}q}$ and not $\widehat{m} = 1 - q$ because $q > \frac{x}{n}$ for all values of $\frac{x}{n}$, except for $\frac{x}{n} = \frac{1}{2}$. For $\frac{x}{n} = \frac{1}{2}$ a pooling equilibrium arises in which all students write a motivation letter, as all students who are not yet admitted to university 1 will be admitted to university 2. Combined with the condition $q \in [0,1]$, the quality of the universities must be $q \in \left[\frac{1}{2}, 1\right]$. Figure 8 below depicts the percentage of rich or poor students taking an SAT, denoted by % rich and % poor, and the percentage of students writing a motivation letter, denoted by % motivation, for $q = 1$.

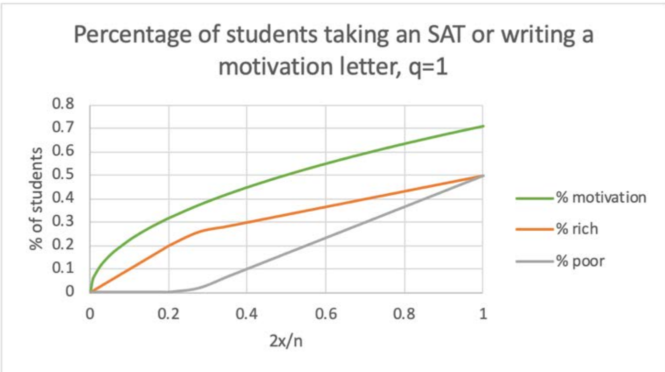

Figure 8 – Percentage of students taking an SAT or writing a motivation letter, q=1

The percentage of rich students taking an SAT is $1 - \overline{s_i}$, whereas the percentage of poor students taking an SAT is $1 - \left(2\overline{s_i} - \frac{1}{2}\right)$, hence this percentage is $2 * \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\frac{1}{6} + \frac{2}{3} * \frac{x}{n}$ for $\frac{x}{n} \geq \frac{1}{8}$. Poor students only take an SAT for $\frac{x}{n} \geq \frac{1}{8}$ and this percentage is $\frac{4}{3} * \frac{x}{n} - \frac{1}{6}$. The percentage of students writing a motivation letter is $\sqrt{\frac{x}{n}q}$. These percentages are needed to derive the expected utility for the universities and there are always more rich students than poor students taking an SAT.

Expected utilities for the universities

The expected utility for the universities is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. The expected average ability is the weighted average of the abilities of the admitted students, hence $\widetilde{a_y} = \frac{\frac{1}{2}*\%rich}{\frac{2x}{n}} * \frac{1+(1-\%rich)}{2} + \frac{\frac{1}{2}*\%poor}{\frac{2x}{n}} * \frac{1+(1-\%poor)}{2}$, where the percentages are as in figure 8, hence for university 1, $\widetilde{a_y} = 1 - \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ for $\frac{x}{n} \geq \frac{1}{8}$. The expected average motivation is $\widetilde{m_y} = E[m_i] = \frac{1}{2}$ and therefore is the expected utility for university 1 $E[U^j] = \frac{3}{2} - \frac{x}{n}$ when $\frac{x}{n} < \frac{1}{8}$ and $E[U^j] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ when $\frac{x}{n} \geq \frac{1}{8}$.

There is a selection effect, as the rich students take an SAT more often than poor students and rich students will thus be admitted more often. The average motivation and average ability of the admitted students and the expected utility for the universities are depicted in figure 9 below.

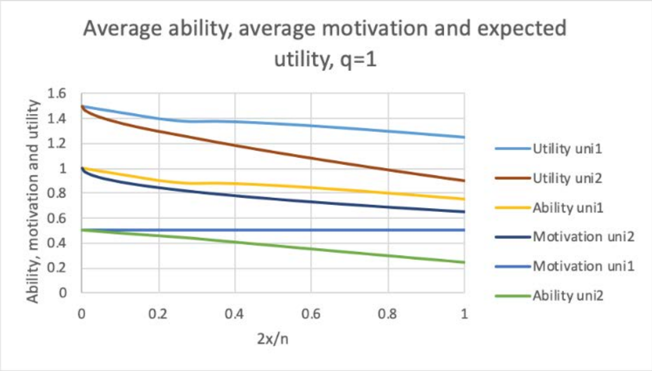

Figure 9 – Average ability, average motivation and expected utility, q=1

The expected average ability for university 2 is the weighted average of the abilities of the students who will not be admitted to university 1 and is $\widetilde{a_y} = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}}$ for $\frac{x}{n} < \frac{1}{8}$ and $\widetilde{a_y} = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}}$ for $\frac{x}{n} \geq \frac{1}{8}$. Furthermore, the expected average motivation of those students can be computed by taking into account the indifferent motivation $\widehat{m} = 1 - \sqrt{\frac{x}{n}}q$. The expected average motivation is therefore $\widetilde{m_y} = \frac{1+\widehat{m}}{2} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}}q$.[7]

[7] The formulas of the expected utilities can be found in appendix 3.

<u>*Conclusion subsection 4.3*</u>

In both tight and loose markets, all students who expect to be admitted to university 1 take an SAT and apply to university 1 when $q \geq \frac{1}{2}$. All other students decide to write a motivation letter or not. A pooling equilibrium arises in which all students write a motivation letter when $q \geq \frac{n}{x}$, and a pooling equilibrium arises in which all students decide not to write a motivation letter when $q = 0$ and $\frac{x}{n} \geq 0$ or when $\frac{x}{n} = 0$ and $q \geq 0$. A separating equilibrium arises in which all students with a motivation $m_i > \widehat{m} = 1 - \sqrt{\frac{x}{n}q}$ write a motivation letter when $q > \frac{x}{n}$ and $\frac{2x}{n} > 0$. Rich students will be overrepresented at university 1 in case of a tight market and poor students will be overrepresented at university 1 in case of a loose market. More motivated students will be overrepresented at university 2.

As it is assumed that the universities choose the combination of admission programs that leads to the highest average expected utility for the universities, subsections 4.1, 4.2 and 4.3 can be compared. The results of these subsections are summarized in proposition 1 below:

Proposition 1: *When both universities are of a high quality, $q_1 = q_2 = H$, then for $q = \frac{1}{2}$, there arise Perfect Bayesian Nash equilibria in which the universities choose different combinations of admission programs for different values of $\frac{2x}{n}$. For $\frac{2x}{n} < 0.03966$ and $0.18954 < \frac{2x}{n} < 0.3277$, the universities decide to base their admissions on SAT scores. However, when $0.03966 < \frac{2x}{n} < 0.18954$ or $0.3277 < \frac{2x}{n} < 0.60338$, one university bases their admissions on SAT scores whereas the other university chooses motivation letters. Lastly, for $\frac{2x}{n} > 0.60338$, both universities base their admissions on motivation letters. Figure 10 below depicts the optimal combination of admission programs, where the average expected utility for the two universities is depicted.*[8]

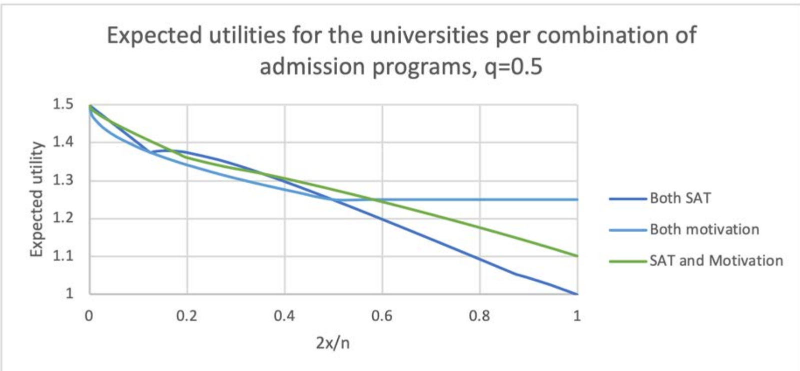

Figure 10 – Expected utilities for the universities per combination of admission programs, q=0.5

[8] *The formulas for the average expected utilities are in appendix 3.*

For $q = \frac{3}{4}$*, a Perfect Bayesian Nash equilibrium arises in which one university chooses to base their admissions on SAT scores, whereas the other university chooses motivation letters for* $0.05926 < \frac{2x}{n} < 0.1612$ *and for* $0.42926 < \frac{2x}{n} < 0.91436$*. However, for* $\frac{2x}{n} > 0.91436$*, both universities choose to base their admissions on motivation letters and for* $\frac{2x}{n} < 0.05926$ *and* $0.1612 < \frac{2x}{n} < 0.42926$ *both universities base their admissions on SAT scores. These different optimal combinations of admission programs are depicted in figure 11 below, where the average expected utility for the two universities is depicted.*

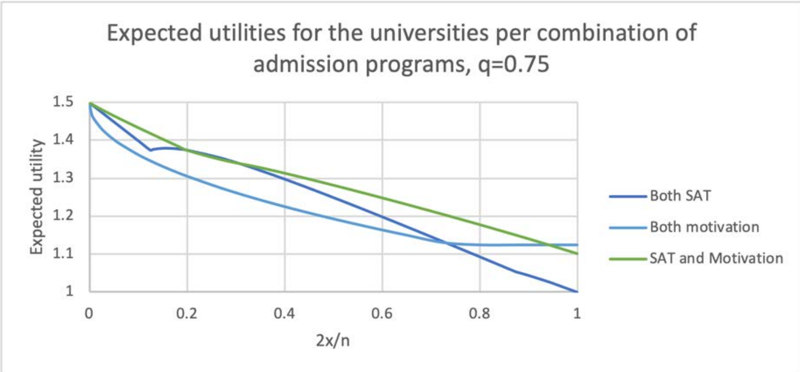

Figure 11 – Expected utilities for the universities per combination of admission programs, q=0.75

If $q = 1$*, for* $\frac{2x}{n} < 0.0787$ *and for* $0.14574 < \frac{2x}{n} < 0.52878$*, a Perfect Bayesian Nash equilibrium arises in which the universities use SAT scores, whereas for* $0.0787 < \frac{2x}{n} < 0.14574$ *and for* $\frac{2x}{n} > 0.52878$*, one university uses SAT scores and the other university chooses motivation letters for their admissions. The optimal combinations of admission programs are depicted in figure 12 below, which depicts the average expected utility for the two universities.*

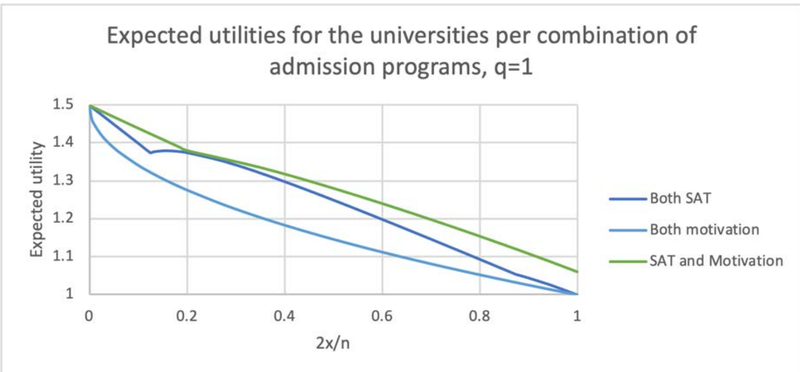

Figure 12 – Expected utilities for the universities per combination of admission programs, q=1

Rich students and more motivated students benefit from one university choosing to base their admissions on SAT scores and the other university basing their admissions on motivation letters. Poor students however will always receive a signal that is lower than their true ability, *because $\frac{x}{n} \leq \frac{1}{2}$. In case both universities choose to base their admissions on SAT scores, rich students mostly benefit in tight markets, and poor students benefit in loose markets. Lastly, in case both universities base their admissions on motivation letters, the motivated students, both rich and poor, benefit from this. The expected utilities for the universities and the different conditions for equilibria to exist are summarized in table 1 below for different markets.*

Market	Expected utility	Conditions
$\boldsymbol{p_1 = (s, 0)}$ **and** $\boldsymbol{p_2 = (s, 0)}$	$E[U^j] = \frac{3}{2} - \frac{2x}{n}$	$\frac{2x}{n} < \frac{1}{8}$ and $q \geq \frac{1}{2}$
	$E[U^j] = \frac{112*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$	$\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ and $q \geq \frac{1}{2}$
	$E[U^j] = \frac{5*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$	$\frac{2x}{n} > \frac{7}{8}$ and $q \geq \frac{1}{2}$
$\boldsymbol{p_1 = (0, l)}$ **and** $\boldsymbol{p_2 = (0, l)}$	Pooling with $l_i = 1$: $E[U^j] = 1$	$q \geq \frac{n}{2x}$ and $\frac{2x}{n} > 0$
	Pooling with $l_i = 0$: $E[U^j] = 0$	$q = 0$ and $\frac{2x}{n} \geq 0$ or $\frac{2x}{n} = 0$ and $q \geq 0$
	Separating with $\widehat{m} = 1 - q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}q$	$q \leq \frac{2x}{n}, \frac{2x}{n} > 0$ and $q \in [0,1]$
	Separating with $\widehat{m} = 1 - \sqrt{\frac{2x}{n}q}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{2x}{n}q}$	$q > \frac{2x}{n}, \frac{2x}{n} > 0$ and $q \in [0,1]$
$\boldsymbol{p_1 = (s, 0)}$ **and** $\boldsymbol{p_2 = (0, l)}$	• University 1: $E[U^1] = \frac{3}{2} - \frac{x}{n}$ • University 2: $E[U^2] = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}$	$\frac{x}{n} < \frac{1}{8}$ and $q \in \left[\frac{1}{2}, 1\right]$
	• University 1: $E[U^1] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ • University 2: $E[U^2] = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}$	$\frac{x}{n} \geq \frac{1}{8}$ and $q \in \left[\frac{1}{2}, 1\right]$

Table 1 – Expected utilities and conditions in different markets

5. Markets with only low-quality universities

In this section the model will be analysed in multiple steps in markets where both universities have a low quality. This section is very similar to section 4, but the only difference is that both universities are of a low quality instead of a high quality.

5.1. Admission programs consist of an SAT

In this first subsection it is assumed that both universities choose to use the admission program that is based on SAT scores only, thus $p_1 = p_2 = (s, 0)$. Again, the $2x$ students with the highest signal s_i will be admitted.

Information collection of the student

In case a student is admitted, he receives an expected utility that is different from the expected utility in subsection 4.1. When a student is admitted after taking an SAT, he will receive an expected utility of $E[U^i] = \gamma q - \frac{1}{2}$ and the expected utility of choosing the outside option is $E[U^i] = 0$. The new participation constraint is therefore $\gamma q \geq \frac{1}{2}$.

The students need to decide on taking an SAT or not, taking into account that the universities admit the $2x$ students with the highest signal s_i. The minimal signal needed to be admitted, depending on the value of $\frac{2x}{n}$, does not change compared to subsection 4.1 and the same holds for the percentage of students taking an SAT. The percentage of rich students taking an SAT is therefore $1 - \overline{s_i}$ and the percentage of poor students is $\frac{3}{2} - 2\overline{s_i}$. Figure 13 below depicts these percentages for different values of $\frac{2x}{n}$, assuming that the participation constraint is satisfied, and these percentages are needed to derive the expected utility for the universities. For $\frac{2x}{n} < \frac{1}{2}$ there are always more rich students taking an SAT than poor students and vice versa for $\frac{2x}{n} > \frac{1}{2}$.

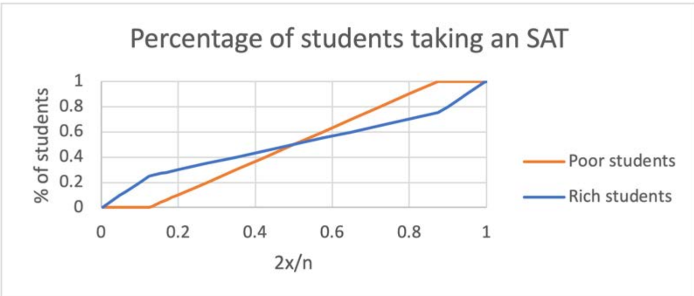

Figure 13 – Percentage of students taking an SAT

Again, there is a clear cut-off at $\frac{2x}{n} = \frac{1}{2}$. In tight markets, rich students will more often take an SAT than poor students and in loose markets poor students will more often take an SAT than rich students. No poor student takes an SAT for $\frac{2x}{n} < \frac{1}{8}$ and all poor students take an SAT for $\frac{2x}{n} > \frac{7}{8}$. A Perfect Bayesian Nash equilibrium arises for $\frac{2x}{n} = 0$ in which no student takes an SAT because no student will be admitted and a Perfect Bayesian Nash equilibrium arises for $\frac{2x}{n} = 1$ in which all students take an SAT and will be admitted.

Expected utility for the universities

The expected utility for the universities does not change compared to subsection 4.1, as this is the expected average ability added to the expected average motivation of the admitted students. The expected average motivation is $E[m_i] = \frac{1}{2}$ and the expected average ability is $\widetilde{a_y} = 1 - \frac{2x}{n}$

for $\frac{2x}{n} < \frac{1}{8}$, $\widetilde{a_y} = \frac{76*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$ for $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ and $\widetilde{a_y} = \frac{4*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$ for $\frac{2x}{n} > \frac{7}{8}$, depicted in figure 14 below.

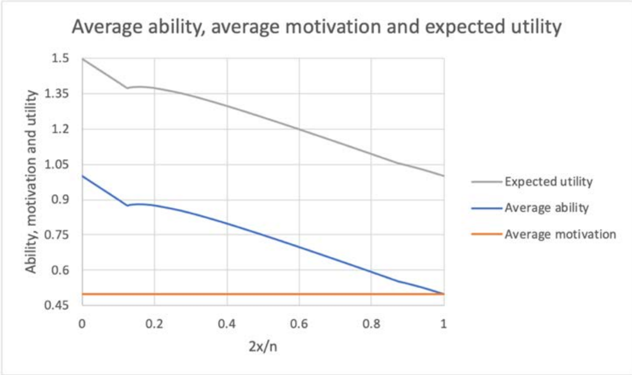

Figure 14 – Average ability, average motivation and expected utility

Conclusion subsection 5.1

Rich students will be overrepresented in case of a tight market, $\frac{2x}{n} < \frac{1}{2}$, because they take an SAT more often than poor students. Poor students however will be overrepresented in a loose market, $\frac{2x}{n} > \frac{1}{2}$, because their signal will be higher than their true ability. Furthermore, students will only choose to take an SAT when the participation constraint is satisfied.

5.2. Admission programs consist of a motivation letter

In this second subsection it is assumed that both universities base their admissions purely on motivation letters, $p_1 = p_2 = (0, l)$. This subsection is closely related to subsection 4.2, however the quality of the universities is different. Nevertheless, both universities will again admit as many students with a motivation letter as possible.

Information collection of the students

Pooling equilibria and separating equilibria might arise, as the students decide to write a motivation letter or not. These equilibria only arise in case no student wants to deviate from the equilibrium strategy.

Pooling equilibrium with $l_i = 1$ for all students

A pooling equilibrium might arise in which all students decide to write a motivation letter. As in subsection 4.2, this pooling equilibrium arises in case there are enough spots available compared to the quality of the university, hence when $\gamma q \geq \frac{n}{2x}$ and $\frac{2x}{n} > 0$.

Pooling equilibrium with $l_i = 0$ for all students

The other pooling equilibrium that might arise is the one in which all students choose not to write a motivation letter. As in subsection 4.2, there is only a pooling equilibrium in which all students decide not to write a motivation letter when $\gamma q = 0$ and $\frac{2x}{n} \geq 0$ or when $\frac{2x}{n} = 0$ and $\gamma q \geq 0$.

Expected utility for the universities – pooling equilibria

In the first pooling equilibrium all students hand in a motivation letter and therefore the universities do not learn anything about the motivation or the ability of the students. The expected utility for the university is therefore $E\left[U^j(l,0)\right] = E[a_i] + E[m_i] = \frac{1}{2} + \frac{1}{2} = 1$. In the pooling equilibrium where all students choose $l_i = 0$, the expected utility for the universities is $E\left[U^j(0,l)\right] = 0$, as no students are admitted.

Separating equilibrium with $m_i > \hat{m}$ choosing $l_i = 1$ and $m_i < \hat{m}$ choosing $l_i = 0$

In this first separating equilibrium, students with a higher motivation write a motivation letter and students with a lower motivation do not write a motivation letter. As in subsection 4.2, 2 separating equilibria might arise.

First, in case $z \leq 2x$, the student who is indifferent has a motivation of $\hat{m} = 1 - \gamma q$. When γq increases, the indifferent student has a lower motivation because the payoff from being admitted increases. As in subsection 4.2, no student wants to deviate and therefore, there is a separating equilibrium when $z \leq 2x$. The number of students writing a motivation letter is $z = (1 - \hat{m})n \leq 2x$, and therefore this separating equilibrium exists when $\gamma q \leq \frac{2x}{n}$ and $\frac{2x}{n} > 0$, hence for $\gamma q \in [0,1]$ as $\hat{m} \in [0,1]$.

Second, there can exist a separating equilibrium in which $z > 2x$. For the indifferent student the expected utilities from writing a motivation letter or not writing a motivation letter are equal and therefore this student has a motivation of $\hat{m} = 1 - \sqrt{\frac{2x}{n}\gamma q}$. In case $\frac{2x}{n}$ or γq increases, the indifferent student has a lower motivation. The probability of being admitted increases due to the increase in the number of spots available and the higher payoff from being admitted leads to more students wanting to write a motivation letter. As in subsection 4.2, there are no students who want to deviate and therefore there is a separating equilibrium in which students with $m_i > \hat{m} = 1 - \sqrt{\frac{2x}{n}\gamma q}$ want to write a motivation letter and all other students do not write a motivation letter. The number of students writing a motivation letter is $z = \sqrt{\frac{2x}{n}\gamma q}n$ and this is bigger than the number of spots available at the universities. The quality of the university should be $\gamma q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$ to have a separating equilibrium in which students with $m_i > \hat{m}$ choose $l_i = 1$ and students with $m_i < \hat{m}$ choose $l_i = 0$, and $\gamma q \in [0,1]$ as $\hat{m} \in [0,1]$.

Separating equilibrium with $m_i < \hat{m}$ choosing $l_i = 1$ and $m_i > \hat{m}$ choosing $l_i = 0$

It could be the case that students with a high motivation choose not to write a motivation letter when students with a low motivation choose to write a motivation letter. The student with $m_i = 1$ however always wants to deviate and write a motivation letter as his motivation is bigger than for a student with $m_i < \hat{m}$ and therefore his expected utility of writing a motivation letter is also bigger than from not writing a motivation letter. Hence, this separating equilibrium will never exist.

A pooling equilibrium in which all students choose $l_i = 1$ only exists when $\gamma q \geq \frac{n}{2x}$. However, all students decide to pool together and choose $l_i = 0$ when the quality of the university is $\gamma q = 0$ and $\frac{2x}{n} \geq 0$ or when there are no spots available at the university, $\frac{2x}{n} = 0$ and $\gamma q \geq 0$. A separating equilibrium in which all students with $m_i > \hat{m} = 1 - \gamma q$ hand in a motivation letter and all less motivated students choose not to write a motivation letter exists when $\gamma q \leq \frac{2x}{n}$ and $\frac{2x}{n} > 0$. Lastly, a separating equilibrium in which all students with $m_i > \hat{m} = 1 - \sqrt{\frac{2x}{n}\gamma q}$ write a motivation letter and all other students choose not to, exists when $\gamma q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$.

When a separating equilibrium arises, the percentage of students writing a motivation letter is $(1 - \hat{m}) = \sqrt{\frac{2x}{n}\gamma q}$ when the quality of the universities is $\gamma q > \frac{2x}{n}$ and $(1 - \hat{m}) = \gamma q$ when the quality of the universities is $\gamma q \leq \frac{2x}{n}$. The percentage of students writing a motivation letter is depicted in figure 15 and is needed to derive the expected utility for the universities.

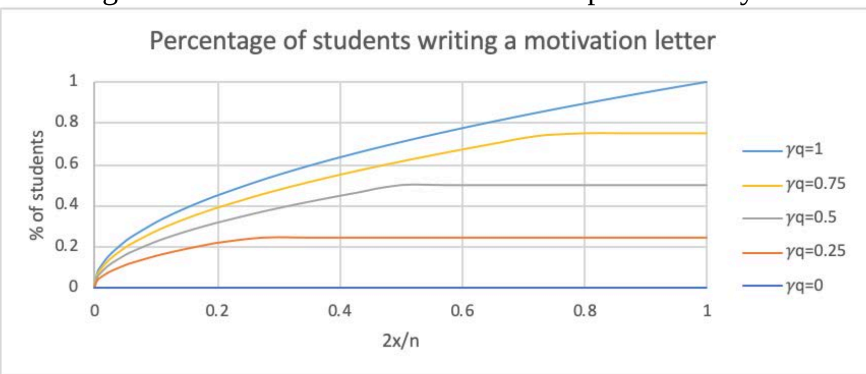

Figure 15 – Percentage of students writing a motivation letter

Expected utility for the universities – separating equilibrium

All students who have a motivation $m_i > \hat{m}$, will hand in a motivation letter and will possibly be admitted. Therefore, the average motivation of all admitted students is $\widetilde{m_y} = \frac{1+\hat{m}}{2}$, which is $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{2x}{n}\gamma q}$ for $\frac{2x}{n} < \gamma q$ and $\widetilde{m_y} = 1 - \frac{1}{2}\gamma q$ for $\frac{2x}{n} \geq \gamma q$. The average ability of the students who are admitted is however unknown to the universities and is therefore $\widetilde{a_y} = E[a_i] = \frac{1}{2}$. Combined, this leads to an expected utility for the universities of $E\left[U^j(0,l)\right] = 1 + \frac{1}{2}\hat{m}$. The expected average ability and motivation of the admitted students and the expected utility for the universities is depicted in figure 16 below.

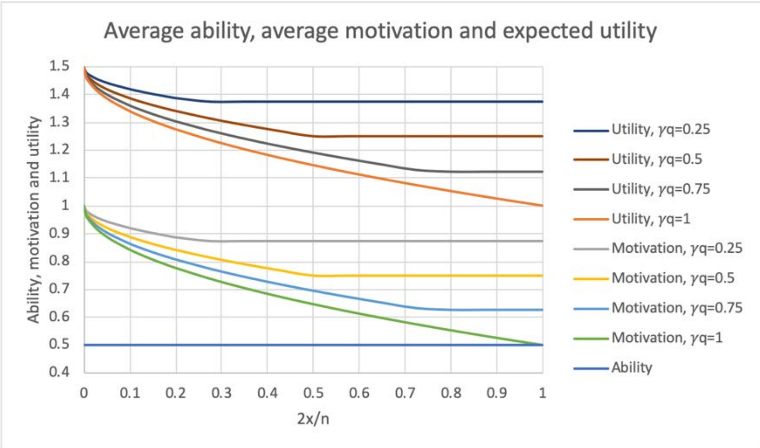

Figure 16 – Average ability, average motivation and expected utility

Conclusion subsection 5.2

In case both universities are of a low quality, $q_1 = q_2 = L$, and both universities choose to use motivation letters for their admissions, $p_1 = p_2 = (0, l)$, a pooling equilibrium arises in which all students choose to write a motivation letter when $\gamma q \geq \frac{n}{2x}$. The other pooling equilibrium arises in which no student writes a motivation letter in case $\gamma q = 0$ and $\frac{2x}{n} \geq 0$ or when $\frac{2x}{n} = 0$ and $\gamma q \geq 0$. Furthermore, there may arise a separating equilibrium in which all students with a motivation $m_i > \hat{m}$ decide to write a motivation letter and all students with $m_i < \hat{m}$ do not write a motivation letter, with $\hat{m} = 1 - \sqrt{\frac{2x}{n}\gamma q}$ for $\gamma q > \frac{2x}{n}$ and $\hat{m} = 1 - \gamma q$ for $\gamma q \leq \frac{2x}{n}$. More motivated students will therefore always be overrepresented at the universities.

5.3. University 1 demands an SAT score whereas university 2 demands a motivation letter

This third subsection is closely related to subsection 4.3 in the way that both universities have the same quality and the universities choose to use a different admission program. University 1 chooses the admission program that is based on SAT scores, $p_1 = (s, 0)$, and university 2 chooses the admission program that is based on motivation letters, $p_2 = (0, l)$. The results from this subsection are also applicable to the situation in which university 1 uses motivation letters and university 2 chooses to use SAT scores for their admissions.

Admission policies of the universities

University 1 has x spots available for students with an SAT score and therefore admits the x students with the highest signal s_i. University 2 however has x spots available for students with a motivation letter and admits as many students who wrote a motivation letter as possible.

Information collection of the students

All students with an ability a_i are expected to have an average motivation of $E[m_i] = \frac{1}{2}$. When the student is admitted to university 1 after taking an SAT, he receives an expected utility of $E\left[U^i | t_i = 1\right] = \gamma q - \frac{1}{2}$. However, when the student is admitted to university 2, he receives an expected utility of $E\left[U^i | l_i = 1\right] = k\gamma q - \frac{1}{2}$. This expected utility is always smaller than or equal to the expected utility of being admitted to university 1 and therefore all students who expect to be admitted to university 1 will take an SAT and apply to university 1.

All other students who do not apply to university 1 must decide on writing a motivation letter or not. As can be seen in subsection 5.2, a pooling equilibrium arises in which all students decide to write a motivation letter in case $\gamma q \geq \frac{n}{x}$ and a pooling equilibrium in which all students decide not to write a motivation letter in case $\gamma q = 0$ and $\frac{x}{n} \geq 0$ or when $\frac{x}{n} = 0$ and $\gamma q \geq 0$. A separating equilibrium arises in which all students with $m_i > \widehat{m} = 1 - \sqrt{\frac{x}{n}\gamma q}$ write a motivation letter and all other students decide not to write a motivation letter, when $\gamma q > \frac{x}{n}$. Figure 17 below depicts the percentages of students taking an SAT and the percentage of students writing a motivation letter. This percentage of rich students taking an SAT is $2 * \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\frac{1}{6} + \frac{2}{3} * \frac{x}{n}$ for $\frac{x}{n} \geq \frac{1}{8}$. For poor students this percentage is 0 for $\frac{x}{n} < \frac{1}{8}$ and $\frac{2}{3} * \frac{x}{n} - \frac{1}{6}$ for $\frac{x}{n} \geq \frac{1}{8}$. The percentage of students writing a motivation letter is $\sqrt{\frac{x}{n}\gamma q}$. These percentages are needed to derive the expected utilities for the universities and there are always more rich students taking an SAT than poor students.

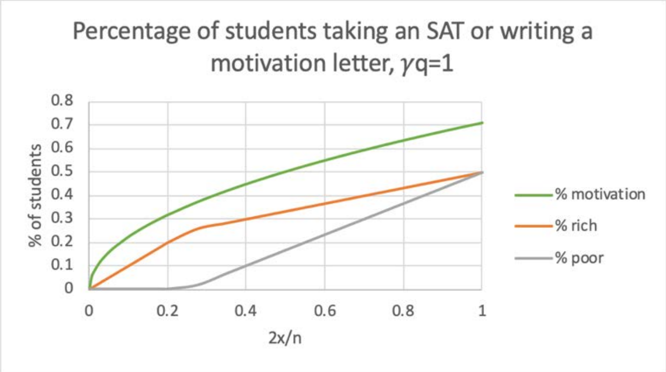

Figure 17 – Percentage of students taking an SAT or writing a motivation letter, γq=1

<u>*Expected utilities for the universities*</u>

There is a selection effect because the rich students will take an SAT more often than poor students. This results in $\widetilde{a_y} = 1 - \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\widetilde{a_y} = \frac{76*\frac{x}{n}-40*\left(\frac{x}{n}\right)^2-1}{72*\frac{x}{n}}$ for $\frac{x}{n} \geq \frac{1}{8}$. However, the average motivation of the admitted students is unknown and is therefore $E[m_i] = \frac{1}{2}$. For university 2 the average ability is the weighted average of the ability of the students who are not admitted to university 1. This average ability is lower, as all students with a high enough signal apply to university 1. This results in $\widetilde{a_y} = \frac{2*\left(\frac{x}{n}\right)^2-2*\frac{x}{n}+1}{2-2*\frac{x}{n}}$ for $\frac{x}{n} < \frac{1}{8}$ and $\widetilde{a_y} = \frac{40*\left(\frac{x}{n}\right)^2-76*\frac{x}{n}+37}{72-72*\frac{x}{n}}$ for $\frac{x}{n} \geq \frac{1}{8}$. The expected average motivation of the admitted students is $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}}\gamma q$. This average motivation and average ability of the admitted students and the expected utilities for the universities are depicted in figure 18.

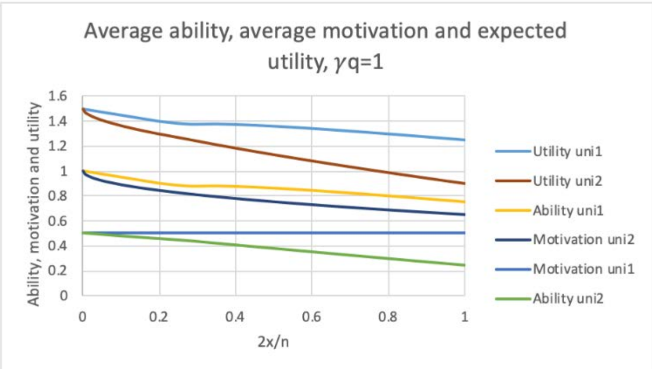

Figure 18 – Average ability, average motivation and expected utility, γq=1

<u>*Conclusion subsection 5.3*</u>

In both tight and loose markets, students will take an SAT when they expect that they will be admitted and apply to university 1 when $\gamma q \geq \frac{1}{2}$*. All other students who will not apply to university 1 must decide on writing a motivation letter or not. In case* $\gamma q \geq \frac{n}{x}$ *and* $\frac{x}{n} > 0$*, a pooling equilibrium arises with* $l_i = 1$*, whereas a pooling equilibrium with* $l_i = 0$ *arises when* $\gamma q = 0$ *and* $\frac{x}{n} \geq 0$ *or when* $\frac{x}{n} = 0$ *and* $\gamma q \geq 0$*. A separating equilibrium will arise in which all students with* $m_i > \widehat{m} = 1 - \sqrt{\frac{x}{n}\gamma q}$ *write a motivation letter and all other students do not when* $\gamma q > \frac{x}{n}$ *and* $\frac{x}{n} > 0$*. Rich students will thus be overrepresented at university 1 in case of a tight market and poor students will be overrepresented at university 1 in case of a loose market. More motivated students will always be overrepresented at university 2.*

The universities choose a combination of admission programs that results in the highest average expected utility for the universities. Subsections 5.1, 5.2 and 5.3 can therefore be compared. The results of these subsections are summarized in proposition 2 below:

Proposition 2: *When both universities are of a low quality,* $q_1 = q_2 = L$*, then for* $\gamma q = \frac{1}{2}$*, a Perfect Bayesian Nash equilibrium arises in which one university chooses to base their admissions on motivation letters, whereas the other university uses SAT scores for* $0.03966 < \frac{2x}{n} < 0.18954$ *and for* $0.3277 < \frac{2x}{n} < 0.60338$*. However, both universities choose to base their admissions on SAT scores when* $\frac{2x}{n} < 0.03966$ *or when* $0.18954 < \frac{2x}{n} < 0.3277$*. Lastly, both universities use motivation letters for their admission programs when* $\frac{2x}{n} > 0.60338$*. Figure 19 below depicts the optimal combination of admission programs, where the average expected utility for the two universities is shown.*[9]

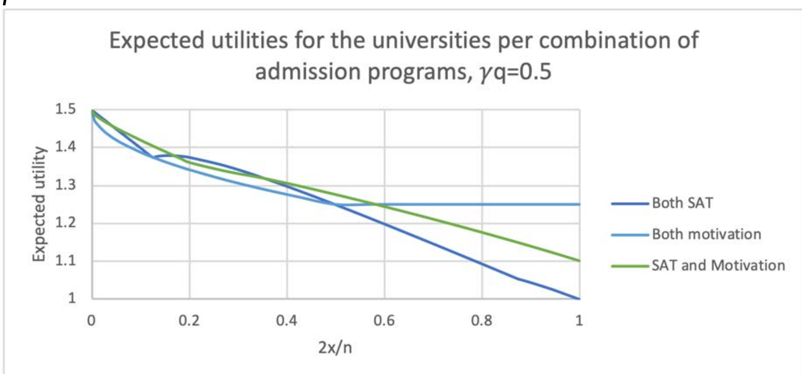

Figure 19 – Expected utilities for the universities per combination of admission programs, γq=0.5

[9] *The formulas for the average expected utility are in appendix 3.*

However, when $\gamma q = \frac{3}{4}$*, there is a Perfect Bayesian Nash equilibrium in which one university bases their admissions on SAT scores and the other uses motivation letters, when* $0.05926 < \frac{2x}{n} < 0.1612$ *and when* $0.42926 < \frac{2x}{n} < 0.91436$*. However, both universities base their admissions on SAT scores when* $\frac{2x}{n} < 0.05926$ *and when* $0.1612 < \frac{2x}{n} < 0.42926$*. Lastly, both universities use motivation letters for their admissions when* $\frac{2x}{n} > 0.91436$*. Figure 20 below depicts the optimal combination of admission programs, where the lines depict the average expected utility for the two universities.*

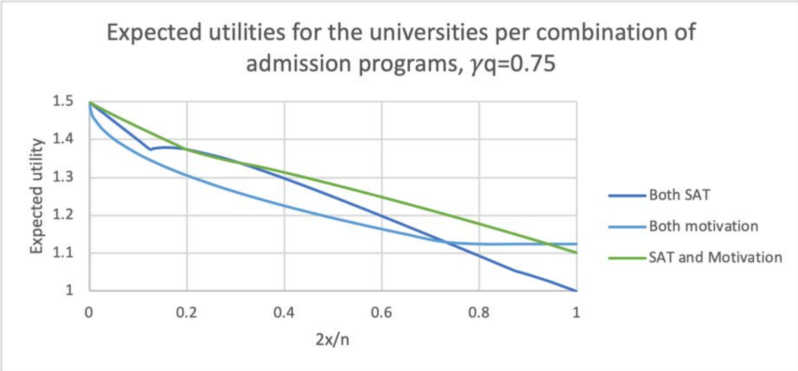

Figure 20 – Expected utilities for the universities per combination of admission programs, γq=0.75

Lastly, when $\gamma q = 1$*, for* $\frac{2x}{n} < 0.0787$ *and for* $0.14574 < \frac{2x}{n} < 0.52878$*, a Perfect Bayesian Nash equilibrium arises in which both universities choose to base their admissions on SAT scores. However, one university chooses to base their admissions on SAT scores whereas the other university chooses to base their admissions on motivation letters when* $0.0787 < \frac{2x}{n} < 0.14574$ *and when* $\frac{2x}{n} > 0.52878$*. The optimal combination of admissions programs is depicted in figure 21 below, which depicts the average expected utility for the two universities.*

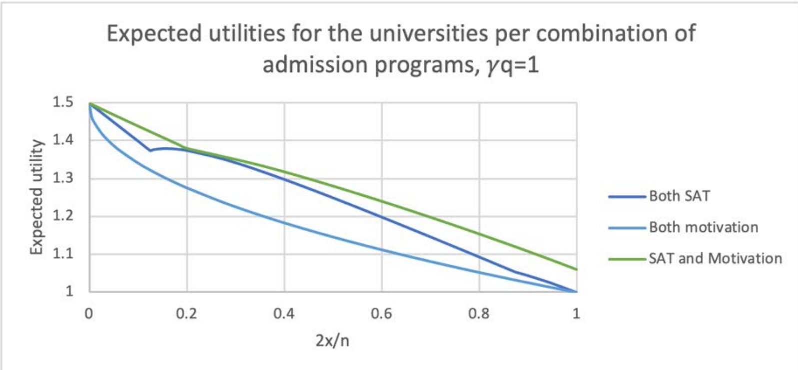

Figure 21 – Expected utilities for the universities per combination of admission programs, γq=1

When one university chooses to base their admissions on SAT scores and the other university chooses motivation letters, rich students and more motivated students benefit. Poor students however will always receive a signal that is lower than their true ability for $\frac{x}{n} \leq \frac{1}{2}$*. Rich students* *mostly benefit in tight markets and poor students benefit in loose markets when the universities base their admissions on SAT scores. Lastly, more motivated students benefit in case both universities base their admissions on motivation letters. The expected utilities for the universities and the different conditions for equilibria to exist are summarized in table 2 below for different markets.*

Market	Expected utility	Conditions
$\boldsymbol{p_1 = (s,0)}$ **and** $\boldsymbol{p_2 = (s,0)}$	$E[U^j] = 1 - \frac{2x}{n} + \frac{1}{2}$	$\frac{2x}{n} < \frac{1}{8}$ and $\gamma q \geq \frac{1}{2}$
	$E[U^j] = \frac{112*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$	$\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ and $\gamma q \geq \frac{1}{2}$
	$E[U^j] = \frac{5*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$	$\frac{2x}{n} > \frac{7}{8}$ and $\gamma q \geq \frac{1}{2}$
$\boldsymbol{p_1 = (0,l)}$ **and** $\boldsymbol{p_2 = (0,l)}$	Pooling with $l_i = 1$: $E[U^j] = 1$	$\gamma q \geq \frac{n}{2x}$ and $\frac{2x}{n} > 0$
	Pooling with $l_i = 0$: $E[U^j] = 0$	$\gamma q = 0$ and $\frac{2x}{n} \geq 0$ or $\frac{2x}{n} = 0$ and $\gamma q \geq 0$
	Separating with $\widehat{m} = 1 - \gamma q$: $E[U^j] = 1 + \frac{1}{2}\widehat{m}$	$\gamma q \leq \frac{2x}{n}, \frac{2x}{n} > 0$ and $\gamma q \in [0,1]$
	Separating with $\widehat{m} = 1 - \sqrt{\frac{2x}{n}\gamma q}$: $E[U^j] = 1 + \frac{1}{2}\widehat{m}$	$\gamma q > \frac{2x}{n}, \frac{2x}{n} > 0$ and $\gamma q \in [0,1]$
$\boldsymbol{p_1 = (s,0)}$ **and** $\boldsymbol{p_2 = (0,l)}$	• University 1: $E[U^1] = \frac{3}{2} - \frac{x}{n}$ • University 2: $E[U^2] = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}$	$\frac{x}{n} < \frac{1}{8}$ and $\gamma q \in \left[\frac{1}{2}, 1\right]$
	• University 1: $E[U^1] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ • University 2: $E[U^2] = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}$	$\frac{x}{n} \geq \frac{1}{8}$ and $\gamma q \in \left[\frac{1}{2}, 1\right]$

Table 2 – Expected utilities and conditions in different markets

6. Diverse markets with high-quality and low-quality universities

In this section the model will be analysed in multiple steps in markets where both universities are of a different quality. For simplicity the universities are referred to as the low-quality university and the high-quality university.

6.1. Admission programs consist of an SAT score

In this first subsection it is assumed that both universities use the admission program that is purely based on SAT scores, $p = (s, 0)$. Both universities again admit the x students with the highest signal s_i.

Information collection of the students

Being admitted to the low-quality university after taking an SAT leads to an expected utility for the student of $E[U^i] = \gamma q - \frac{1}{2}$, whereas being admitted to the high-quality university leads to an expected utility of $E[U^i] = q - \frac{1}{2}$. Both expected utilities are bigger than or equal to the expected utility of choosing the outside option, $E[U^i] = 0$, in case $\gamma q \geq \frac{1}{2}$. The participation constraint is therefore $\gamma q \geq \frac{1}{2}$. Not being admitted after taking an SAT leads however to an expected utility of $E[U^i] = -\frac{1}{2}$. Students will thus only take an SAT in case they expect that they will be admitted and when the quality of the university is high enough.

The signal that is needed to be admitted is depicted in figure 22 below and is needed to derive the percentage of students applying to the low-quality and the high-quality university. The minimal signal needed to be admitted to the low-quality university is the same as in subsections 4.1 and 5.1, hence $\overline{s_{i_1}} = 1 - 2 * \frac{2x}{n}$ for $\frac{2x}{n} < \frac{1}{8}$, $\overline{s_{i_1}} = \frac{5}{6} - \frac{2}{3} * \frac{2x}{n}$ for $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$ and $\overline{s_{i_1}} = 2 - 2 * \frac{2x}{n}$ for $\frac{2x}{n} > \frac{7}{8}$. However, for the high-quality university, the minimal signal needed to be admitted is $\overline{s_{i_2}} = 1 - 2 * \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\overline{s_{i_2}} = \frac{5}{6} - \frac{2}{3} * \frac{x}{n}$ for $\frac{x}{n} \geq \frac{1}{8}$. The x students, rich and poor, with the abilities that lead to the highest signals will thus apply to the high-quality university and the x students with the subsequent highest signals will apply to the low-quality university when the participation constraint is satisfied.

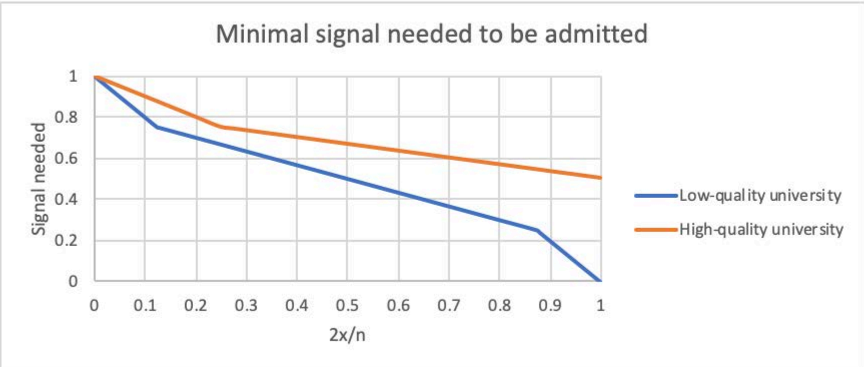

Figure 22 – Minimal signal needed to be admitted

Students know their own ability and their own wealth and therefore know which signal they can expect from taking an SAT. The percentage of students taking an SAT is depicted in figure 23 below. This total percentage of students is the same as in subsection 4.1 and 5.1, hence the percentage of rich students is $1 - \overline{s_i}$ and the percentage of poor students is $\frac{3}{2} - 2\overline{s_i}$.

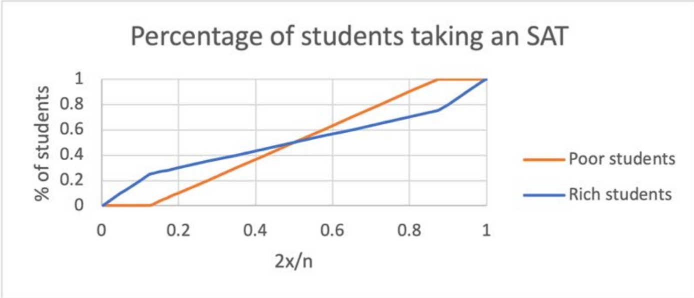

Figure 23 – Percentage of students taking an SAT

Furthermore, the percentage of students applying to the universities is depicted in figure 24 below. The percentage of rich students applying to the low-quality university is $\overline{s_{i_2}} - \overline{s_{i_1}}$, so $2 * \frac{x}{n}$ for $\frac{2x}{n} < \frac{1}{8}$, $\frac{2}{3} * \frac{x}{n}$ for $\frac{1}{8} \le \frac{2x}{n} \le \frac{7}{8}$ and $\frac{10}{3} * \frac{x}{n} - \frac{7}{6}$ for $\frac{2x}{n} > \frac{7}{8}$. The percentage of poor students applying to the low-quality university is 0 for $\frac{2x}{n} < \frac{1}{8}$, $\frac{8}{3} * \frac{x}{n} - \frac{5}{3}$ for $\frac{1}{8} \le \frac{2x}{n} \le \frac{1}{4}$, $\frac{4}{3} * \frac{x}{n}$ for $\frac{1}{4} < \frac{2x}{n} \le \frac{7}{8}$ and lastly $\frac{7}{6} - \frac{4}{3} * \frac{x}{n}$ for $\frac{2x}{n} > \frac{7}{8}$. However, for the high-quality university the percentage of rich students applying is $2 * \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\frac{1}{6} + \frac{2}{3} * \frac{x}{n}$ for $\frac{x}{n} \ge \frac{1}{8}$ and the percentage of poor students is 0 for $\frac{2x}{n} \le \frac{1}{4}$ and is $\frac{4}{3} * \frac{x}{n} - \frac{1}{6}$ for $\frac{2x}{n} > \frac{1}{4}$. Figure 24 below takes into account the minimal signal needed to be admitted for different values of $\frac{2x}{n}$ as depicted in figure 22 above. Rich students thus more often take an SAT in case of a tight market, $\frac{2x}{n} < \frac{1}{2}$, whereas in case of a loose market poor students more often take an SAT. Rich students more often apply to the low-quality university in case the poor students did not reach the point of wanting to take an SAT and applying to the low-quality university yet, hence when $\frac{2x}{n} < \frac{1}{5}$. Furthermore, there are always more rich students applying to the high-quality university than there are poor students applying to this university. There is thus a selection effect as the rich students are always overrepresented at the high-quality university. These percentages of students applying to the universities are needed to derive the expected utilities for the universities.

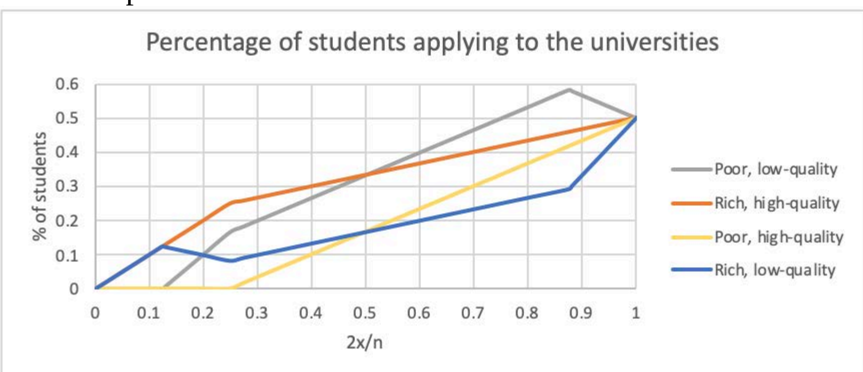

Figure 24 – Percentage of students applying to the universities

Expected utilities for the universities

The expected utility for the universities is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. The signal s_i does not say anything about the average motivation of a student and therefore is the expected average motivation of the admitted students $E[m_i] = \frac{1}{2}$. The expected average ability for the low-quality university is $\widetilde{a_y} = 1 - \frac{3}{2} * \frac{2x}{n}$ for $\frac{2x}{n} < \frac{1}{8}$, $\widetilde{a_y} = \frac{80*\frac{x}{n} - 88*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ for $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{1}{4}$, $\widetilde{a_y} = \frac{19}{18} - \frac{5}{3} * \frac{x}{n}$ for $\frac{1}{4} < \frac{2x}{n} \leq \frac{7}{8}$ and lastly $\widetilde{a_y} = \frac{212*\frac{x}{n} - 248*\left(\frac{x}{n}\right)^2 - 35}{72*\frac{x}{n}}$ for $\frac{2x}{n} > \frac{7}{8}$. The expected average ability for the high-quality university however is $\widetilde{a_y} = 1 - \frac{x}{n}$ for $\frac{2x}{n} < \frac{1}{4}$ and $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ for $\frac{2x}{n} \geq \frac{1}{4}$. These expected average abilities and motivations and the expected utilities are depicted in figure 25 below.

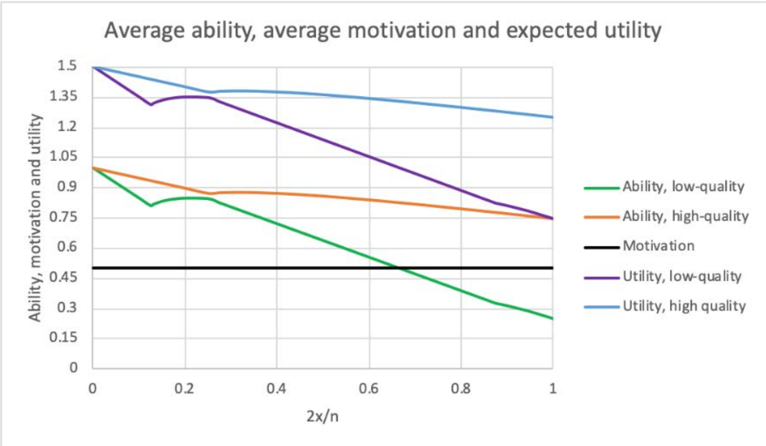

Figure 25 – Average ability, average motivation and expected utility

Conclusion subsection 6.1

Rich students are overrepresented at the low-quality university and at the high-quality university in case $\frac{2x}{n} < \frac{1}{5}$. For all other values of $\frac{2x}{n}$, the poor students are overrepresented at the low-quality university. Rich students are always overrepresented at the high-quality university, because the minimal signal needed to be admitted to the high-quality university is always bigger than $\frac{1}{2}$. Rich students more often obtain this signal than poor students with the same ability.

6.2. Admission programs consist of a motivation letter

In this second subsection it is assumed that both universities choose the admission program that is purely based on motivation letters, $p = (0, l)$. This subsection is closely related to subsections 4.2 and 5.2. The universities will admit all the students who wrote a motivation letter in case there are enough spots available for them.

Information collection of the students

All students decide to write a motivation letter or not. In case the student decides to write a motivation letter, he will first apply to the high-quality university as being admitted to this university leads to a higher payoff than being admitted to the low-quality university. In case the student is admitted, he will not apply to the low-quality university. In case the student is not admitted to the high-quality university he will also apply to the low-quality university with the same motivation letter, as there are no additional costs. Again, pooling and separating equilibria may arise. In previous sections students did not want to apply to two universities, because when the qualities of the universities are equal, students do not want to apply to both universities as being admitted to a university then always results in the same utility. Also, when both universities base their admissions on SAT scores, students do not want to apply to both universities because they expect that they will be admitted after taking an SAT and applying to one university.

Pooling equilibrium with $l_i = 1$ for all students

In case all students write a motivation letter, the probability of being admitted to the high-quality university is $\frac{x}{n}$. Furthermore, when a student is not admitted to the high-quality university, he will apply to the low-quality university and will be admitted with probability $\left(1-\frac{x}{n}\right) * \frac{x}{n-x} = \frac{x}{n}$. Therefore, the expected utility for the students is $E\left[U^i \middle| l_i = 1\right] = \frac{x}{n}q + \frac{x}{n}\gamma q - (1-m_i)$. The students do not want to deviate and decide not to write a motivation letter in case this expected utility is bigger than the expected utility of deviating, even for the student with $m_i = 0$. The expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$. All students therefore decide to write a motivation letter in case $q + \gamma q \geq \frac{n}{x}$ and when $\frac{x}{n} > 0$. Thus, there is a pooling equilibrium in which all students choose $l_i = 1$ when there are enough spots available at the universities and when the qualities of the universities are high enough.

Pooling equilibrium with $l_i = 0$ for all students

This pooling equilibrium in which all students decide not to write a motivation letter exists in case the expected utility of writing a motivation letter is equal to or smaller than the expected utility of not writing a motivation letter, even for a student with $m_i = 1$. The expected utility of not writing a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$, is bigger than or equal to the expected utility of deviating and writing a motivation letter in case $q = 0$ or when $\frac{2x}{n} = 0$. Therefore, a pooling equilibrium arises with $l_i = 0$ for all students when $q = 0$ and $\frac{2x}{n} \geq 0$ or when $\frac{2x}{n} = 0$ and $q \geq 0$.

Expected utilities for the universities – pooling equilibrium

The expected utility for the universities is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. In the pooling equilibrium in which all students write a motivation letter, the expected utility for the universities is $E[U^j(0,l)] = E[a_i] + E[m_i] = \frac{1}{2} + \frac{1}{2} = 1$. However, in the pooling equilibrium in which all students decide not to write a motivation letter, the expected utility for the universities is $E[U^j(0,l)] = 0$, as no student is admitted.

Separating equilibrium with $m_i > \hat{m}$ choosing $l_i = 1$ and $m_i < \hat{m}$ choosing $l_i = 0$

A separating equilibrium may arise in which students with a higher motivation decide to write a motivation letter, whereas students with a lower motivation do not write a motivation letter.

First, the number of students writing a motivation letter can be smaller than x. All students with a motivation letter will then be admitted to the high-quality university. The expected utility of writing a motivation letter is $E[U^i|l_i = 1] = q - (1 - m_i)$ and the expected utility of not writing a motivation letter is $E[U^i|l_i = 0] = 0$. The student who is indifferent between writing a motivation letter and not writing a motivation letter has a motivation of:

$$q - (1 - \hat{m}) = 0$$

$$\hat{m} = 1 - q$$

When the quality of the university is higher, the indifferent student will thus have a lower motivation.

The student with a motivation $m_i = \hat{m} - \varepsilon$ does not want to deviate, as deviating leads to an expected utility of $E[U^i|l_i = 1] = q - (1 - (\hat{m} - \varepsilon))$ and this expected utility is never bigger than the expected utility of not writing a motivation letter, $E[U^i|l_i = 0] = 0$. The student with a higher motivation, $m_i = \hat{m} + \varepsilon$ also does not want to deviate as the expected utility of writing a motivation letter, $E[U^i|l_i = 1] = q - (1 - (\hat{m} + \varepsilon))$ is bigger than the expected utility of deviating and not writing a motivation letter, $E[U^i|l_i = 0] = 0$. Therefore, a separating equilibrium arises in which students with $m_i > \hat{m} = 1 - q$ write a motivation letter and all other students choose not to. This number of students, $z = (1 - \hat{m})n$ is smaller than x and therefore this separating equilibrium exists when $q < \frac{x}{n}$ and $\frac{x}{n} > 0$, and $q \in [0,1]$ as $\hat{m} \in [0,1]$.

Second, the number of students writing a motivation letter can be exactly equal to the number of spots available at the high-quality university. In that case, all students writing a motivation letter will be admitted to the high-quality university. The expected utility of writing a motivation letter is therefore $E\left[U^i \middle| l_i = 1\right] = q - (1 - m_i)$ and the expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$. The indifferent student has a motivation of:

$$q - (1 - \widehat{m}) = 0$$
$$\widehat{m} = 1 - q$$

Again, when the quality of the university increases, there are more students who want to write a motivation letter and the indifferent motivation decreases.

The student with a motivation $m_i = \widehat{m} - \varepsilon$ does not want to deviate as their expected utility of deviating, $E\left[U^i \middle| l_i = 1\right] = \frac{x}{x+1} q + \frac{1}{x+1} \gamma q - \left(1 - (\widehat{m} - \varepsilon)\right)$, is never bigger than the expected utility of not writing a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$. The student with $m_i = \widehat{m} + \varepsilon$ also does not want to deviate because the expected utility of deviating, $E\left[U^i \middle| l_i = 0\right] = 0$ is never bigger than the expected utility of writing a motivation letter, $E\left[U^i \middle| l_i = 1\right] = q - \left(1 - (\widehat{m} + \varepsilon)\right)$. Therefore, there is also a separating equilibrium when $z = x$. The number of students writing a motivation letter is exactly equal to x and therefore this separating equilibrium exists when $q = \frac{x}{n}$, $\frac{x}{n} > 0$ and $q \in [0,1]$ as $\widehat{m} \in [0,1]$.

Third, the number of students writing a motivation letter can be bigger than the number of spots available at one university, but smaller than the number of spots available at both universities. All students with a motivation letter first apply to the high-quality university and in case they are not admitted they will be admitted to the low-quality university for sure. The expected utility of writing a motivation letter is therefore $E\left[U^i \middle| l_i = 1\right] = \frac{x}{z} q + \frac{z-x}{z} \gamma q - (1 - m_i)$ and the expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$. There is one indifferent student with a motivation of:

$$\frac{x}{(1-\widehat{m})n} q + \frac{(1-\widehat{m})n - x}{(1-\widehat{m})n} \gamma q - (1 - \widehat{m}) = 0$$
$$\widehat{m} = 1 - \frac{1}{2}\gamma q - \frac{1}{2}\sqrt{(\gamma q)^2 + \frac{4x}{n} q - \frac{4x}{n}\gamma q}$$

The indifferent student will have a lower motivation when the quality of both universities or when the number of spots available increases, as more students want or can be admitted.

The student with $m_i = \widehat{m} - \varepsilon$ does not want to deviate as the expected utility of deviating, $E\left[U^i \middle| l_i = 1\right] = \frac{x}{z+1} q + \frac{z+1-x}{z+1} \gamma q - \left(1 - (\widehat{m} - \varepsilon)\right)$, is never bigger than the expected utility of not writing a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$. The student with a motivation $m_i = \widehat{m} + \varepsilon$ also does not want to deviate as the expected utility of deviating, $E\left[U^i \middle| l_i = 0\right] = 0$, is never bigger than the expected utility of writing a motivation letter, $E\left[U^i \middle| l_i = 1\right] = \frac{x}{z}q + \frac{z-x}{z}\gamma q - \left(1 - (\widehat{m} + \varepsilon)\right)$. Therefore, there is a separating equilibrium in which $x < z < 2x$ when $q > \frac{x}{n}$, $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$, $\frac{2x}{n} > 0$ and $\frac{1}{2}q + \frac{1}{2}\gamma q \in [0,1]$. The indifferent student is the student with a motivation of $\widehat{m} = 1 - \frac{1}{2}\gamma q - \frac{1}{2}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$.

Fourth, the number of students writing a motivation letter can be exactly equal to the total number of spots available at the universities, $z = 2x$. The expected utility of writing a motivation letter is $E\left[U^i \middle| l_i = 1\right] = \frac{1}{2}q + \frac{1}{2}\gamma q - (1 - m_i)$, whereas the expected utility of not writing a motivation letter is $E\left[U^i \middle| l_i = 0\right] = 0$. The student who is indifferent between writing a motivation letter and not writing a motivation letter is the student with:

$$\frac{1}{2}q + \frac{1}{2}\gamma q - (1 - \widehat{m}) = 0$$

$$\widehat{m} = 1 - \frac{1}{2}q - \frac{1}{2}\gamma q$$

When the quality of the universities increases, the indifferent motivation decreases.

The student with a motivation $m_i = \widehat{m} - \varepsilon$ does not want to deviate as the expected utility of deviating, $E\left[U^i \middle| l_i = 1\right] = \frac{x}{2x+1}q + \frac{x}{2x+1}\gamma q - \left(1 - (\widehat{m} - \varepsilon)\right)$, is smaller than the expected utility of not writing a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$. Furthermore, the expected utility of deviating for the student with $m_i = \widehat{m} + \varepsilon$ is $E\left[U^i \middle| l_i = 0\right] = 0$, which is never bigger than the utility from writing a motivation letter for this student, $E\left[U^i \middle| l_i = 1\right] = \frac{1}{2}q + \frac{1}{2}\gamma q - \left(1 - (\widehat{m} + \varepsilon)\right)$. Therefore, there is a separating equilibrium when $z = 2x$, for $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$, $\frac{2x}{n} > 0$ and $\frac{1}{2}q + \frac{1}{2}\gamma q \in [0,1]$.

Fifth and last, in case the number of students writing a motivation letter is bigger than the total number of spots available at the universities, not all students with a motivation letter can be admitted to one of the two universities. The expected utility of writing a motivation letter is therefore $E\left[U^i \middle| l_i = 1\right] = \frac{x}{z}q + \frac{x}{z}\gamma q - (1 - m_i)$. All students who do not write a motivation letter will not be admitted to one of the two universities and the expected utility of not writing a motivation letter is therefore $E\left[U^i \middle| l_i = 0\right] = 0$. The indifferent student has a motivation of:

$$\frac{x}{(1 - \widehat{m})n}q + \frac{x}{(1 - \widehat{m})n}\gamma q - (1 - \widehat{m}) = 0$$

$$\hat{m} = 1 - \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$$

Again, when $\frac{x}{n}$ or q increases, the indifferent motivation decreases and there will be more students writing a motivation letter.

The student with a motivation of $m_i = \hat{m} - \varepsilon$ does not want to deviate. His expected utility of deviating, $E\left[U^i \middle| l_i = 1\right] = \frac{x}{z+1}q + \frac{x}{z+1}\gamma q - \left(1 - (\hat{m} - \varepsilon)\right)$, is always smaller than the expected utility of not writing a motivation letter, $E\left[U^i \middle| l_i = 0\right] = 0$. The student with a motivation $m_i = \hat{m} + \varepsilon$ also does not want to deviate, as his expected utility of deviating, $E\left[U^i \middle| l_i = 0\right] = 0$ is also never bigger than the expected utility of writing a motivation letter, $E\left[U^i \middle| l_i = 1\right] = \frac{x}{z+1}q + \frac{x}{z+1}\gamma q - \left(1 - (\hat{m} + \varepsilon)\right)$.

As there are no students who want to deviate, a separating equilibrium arises. In this separating equilibrium the student with a motivation of $\hat{m} = 1 - \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$ is indifferent between writing a motivation letter and not writing a motivation letter. All students with a higher motivation write a motivation letter and all students with a lower motivation do not write a motivation letter. The number of students writing a motivation letter is $z = (1 - \hat{m})n = \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q n}$ and this number of students is bigger than the total number of spots available at the universities. Therefore, the quality of the universities should be $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$, hence $\frac{1}{2}q + \frac{1}{2}\gamma q \in [0,1]$ as $\hat{m} \in [0,1]$.

<u>*Separating equilibrium with $m_i < \hat{m}$ choosing $l_i = 1$ and $m_i > \hat{m}$ choosing $l_i = 0$*</u>

A separating equilibrium in which all students with a lower motivation choose $l_i = 1$ and all other students choose $l_i = 0$ can exist. The expected utility of writing a motivation letter should then be bigger than the expected utility of not writing a motivation letter for students with $m_i < \hat{m}$. If this is the case, the expected utility of $l_i = 1$ is also bigger than the expected utility of $l_i = 0$ for more motivated students. Therefore, there is no separating equilibrium in which students with $m_i < \hat{m}$ choose $l_i = 1$ and students with $m_i > \hat{m}$ choose $l_i = 0$.

Pooling equilibria and separating equilibria arise in different situations. A pooling equilibrium with $l_i = 1$ arises when $q + \gamma q \geq \frac{n}{x}$ and $\frac{2x}{n} > 0$. Furthermore, a pooling equilibrium with $l_i = 0$ exists when $q = 0$ and $\frac{2x}{n} \geq 0$ or when $\frac{2x}{n} = 0$ and $q \geq 0$. Separating equilibria arise in which all students with $m_i > \hat{m}$ write a motivation letter. Hence, $\hat{m} = 1 - q$

when $q \leq \frac{x}{n}$ and $\frac{x}{n} > 0$ and $\widehat{m} = 1 - \frac{1}{2}\gamma q - \frac{1}{2}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$ for $q > \frac{x}{n}$, $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$ and $\frac{2x}{n} > 0$. Furthermore, $\widehat{m} = 1 - \frac{1}{2}q - \frac{1}{2}\gamma q$ when $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$ and $\frac{2x}{n} > 0$ and lastly, $\widehat{m} = 1 - \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$ for $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$.

In case the separating equilibrium arises, not all students decide to write a motivation letter. The percentage of students writing a motivation letter is $(1 - \widehat{m})$ and is depicted in figure 26 below for different combinations of q and γq. These percentages will be used to derive the expected utilities for the universities.

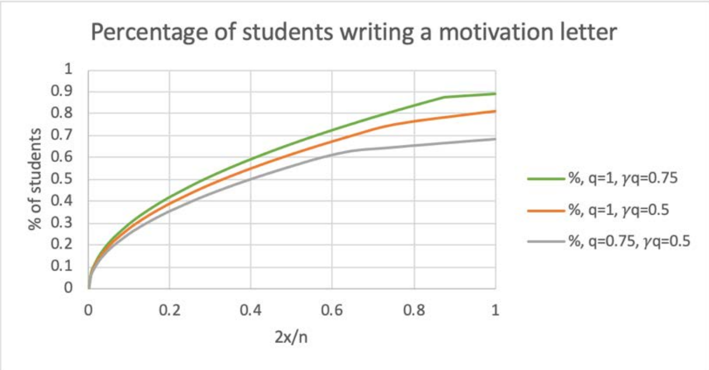

Figure 26 – Percentage of students writing a motivation letter

Expected utilities for the universities – separating equilibrium

The expected utilities for the universities are $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. In this separating equilibrium only students with a motivation $m_i > \widehat{m}$ will write a motivation letter and will be admitted to the universities, and therefore the expected average motivation of the admitted students is $\widetilde{m_y} = \frac{1+\widehat{m}}{2}$. However, the average ability of the admitted students is unknown and is therefore $\widetilde{a_y} = E[a_i] = \frac{1}{2}$. The expected utility for the universities is thus $E[U^j(0, l)] = 1 + \frac{1}{2}\widehat{m}$, where $\widehat{m}$ differs in the different separating equilibria. For $q \leq \frac{x}{n}$ the average motivation is $\widetilde{m_y} = 1 - \frac{1}{2}q$, whereas $\widetilde{m_y} = 1 - \frac{1}{4}\gamma q - \frac{1}{4}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$ for $q > \frac{x}{n}$ and $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$. The average motivation is $\widetilde{m_y} = 1 - \frac{1}{4}q - \frac{1}{4}\gamma q$ for $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$ and lastly $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$ for $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$. The low-quality university has an expected utility of $E[U^j(0, l)] = 0$ in case $(1 - \widehat{m}) \leq \frac{x}{n}$, as all students will apply to the high-quality university and no student is admitted to the low-quality university. The expected average ability and motivation of the students and the expected utilities for the universities are depicted in figure 27 below.

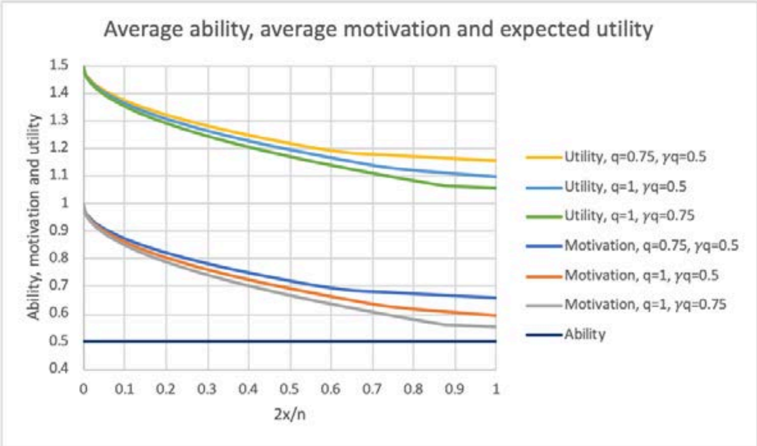

Figure 27 – Average ability, average motivation and expected utility

Conclusion subsection 6.2

In tight markets, a pooling equilibrium in which all students choose $l_i = 1$ only exists for $q + \gamma q \geq \frac{n}{x}$. A pooling equilibrium with $l_i = 0$ arises when $q = 0$ or when $\frac{2x}{n} = 0$. A separating equilibrium arises in which all students with $m_i > \hat{m}$ write a motivation letter and all students with $m_i < \hat{m}$ do not write a motivation letter, with $\hat{m} = 1 - q$ when $q \leq \frac{x}{n}$ and $\frac{x}{n} > 0$ and $\hat{m} = 1 - \frac{1}{2}\gamma q - \frac{1}{2}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$ for $q > \frac{x}{n}, \frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$ and $\frac{2x}{n} > 0$. Furthermore, $\hat{m} = 1 - \frac{1}{2}q - \frac{1}{2}\gamma q$ when $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$ and $\frac{2x}{n} > 0$ and lastly, $\hat{m} = 1 - \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$ for $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$ and $\frac{2x}{n} > 0$. Therefore, more motivated students are always overrepresented at both universities. Furthermore, when the number of students writing a motivation letter is smaller than or equal to the number of spots available at one university, all students writing a motivation letter will apply and will be admitted to the high-quality university and no students will apply and will be admitted to the low-quality university.

The universities choose the combination of admission programs that results in the highest average expected utility for the universities. Subsections 6.1 and 6.2 can therefore be compared. The results of these subsections are summarized in proposition 3 below:

Proposition 3: *For $q = \frac{3}{4}$ and $\gamma q = \frac{1}{2}$, a Perfect Bayesian Nash equilibrium arises in which both universities should always base their admissions on SAT scores, irrespective of the value of $\frac{2x}{n}$. Figure 28 below depicts the optimal combination of admission programs, where the average expected utility for the two universities is depicted.*

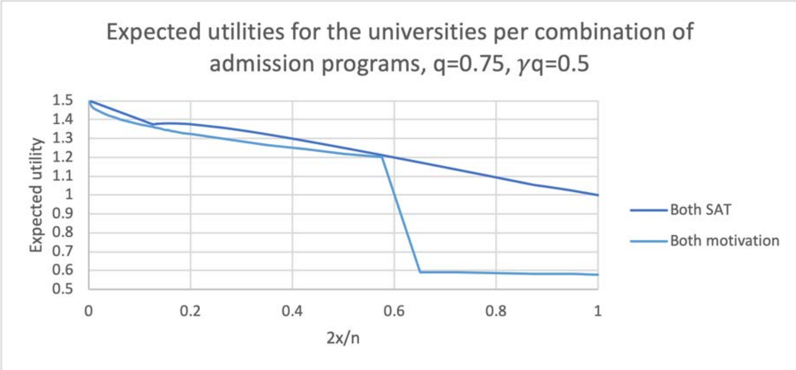

Figure 28 – Expected utilities for the universities per combination of admission programs, q=0.75, γq=0.5

For $q = 1$ *and* $\gamma q = \frac{1}{2}$*, a Perfect Bayesian Nash equilibrium arises in which again both universities should base their admissions on SAT scores, irrespective of the value of* $\frac{x}{n}$*. Figure 29 below depicts the optimal combination of admission programs, where the average expected utility for the universities is depicted.*

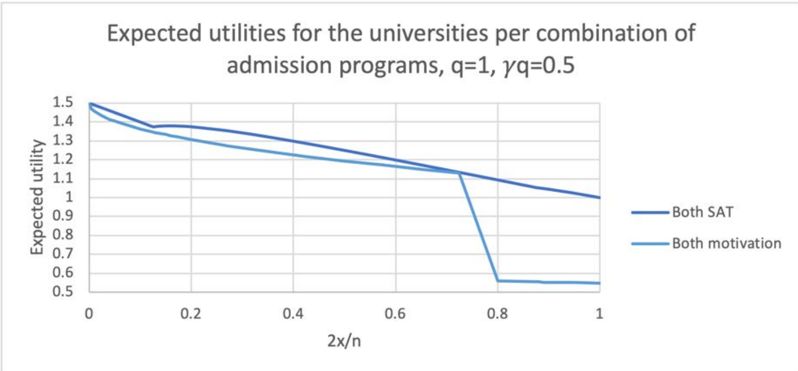

Figure 29 – Expected utilities for the universities per combination of admission programs, q=1, γq=0.5

For $q = 1$ *and* $\gamma q = \frac{3}{4}$*, a Perfect Bayesian Nash equilibrium arises in which both universities base their admissions on SAT scores for* $\frac{2x}{n} < 0.85616$ *and both universities base their admissions on motivation letters for* $0.1612 < \frac{2x}{n} < 0.85616$*. Figure 30 below depicts the optimal combination of admission programs, where the average expected utilities of the universities are depicted.*

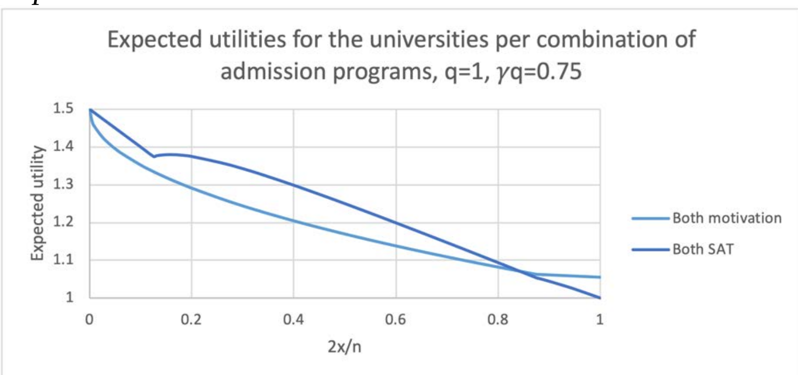

Figure 30 – Expected utilities for the universities per combination of admission programs, q=1, γq=0.75

6.3. The low-quality university demands a motivation letter whereas the high-quality university demands an SAT score

In this third subsection it is assumed that the low-quality university chooses to base their admissions on motivation letters, $p = (0, l)$ whereas the high-quality university bases their admissions on SAT scores, $p = (s, 0)$. This subsection is closely related to subsections 4.3 and 5.3. However, in this subsection the qualities of the universities differ from each other. Subsection 6.3 is to illustrate the scope of the situation in which there are two universities with different qualities: the high-quality university uses SAT scores for their admissions and the low-quality university uses motivation letters for their admissions. This subsection is however not taken into account in proposition 3.

Admission policies of the universities

The low-quality university bases their admissions on motivation letters, whereas the high-quality university bases their admissions on SAT scores. The low-quality university therefore wants to admit as many students with a motivation letter as possible, whereas the high-quality university wants to admit the x students with the highest signal s_i.

Information collection of the students

Students prefer being admitted to the high-quality university with certainty over being admitted to the low-quality university with a particular probability. Therefore, all students who will be admitted to the high-quality university, thus all students with $s_i > \overline{s_i}$, apply to the high-quality university after taking an SAT, with $\overline{s_i} = 1 - 2 * \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\overline{s_i} = \frac{5}{6} - \frac{2}{3} * \frac{x}{n}$ for all other values of $\frac{x}{n}$. All students who do not apply to the high-quality university decide to write a motivation letter or not. Again, pooling equilibria and separating equilibria might arise.

Pooling equilibrium with $l_i = 1$ for all students

A pooling equilibrium might arise in which all students who are not yet admitted to the high-quality university write a motivation letter. For this pooling equilibrium to exist it must be that $\frac{x}{n-x}\gamma q - (1 - m_i) \geq 0$, even for a student with $m_i = 0$, hence $\frac{x}{n-x}\gamma q - 1 \geq 0$. This happens for $\gamma q \geq \frac{n-x}{x}$ and $\frac{x}{n-x} > 0$. The probability of being admitted is $\frac{x}{n-x}$ because there are x spots available at the university and those spots will be divided over the $n - x$ students who are not yet admitted to the high-quality university. Therefore, there is only a pooling equilibrium with $l_i = 1$ for $\gamma q \geq \frac{n-x}{x}$ and $\frac{x}{n-x} > 0$.

Pooling equilibrium with $l_i = 0$ for all students

A pooling equilibrium in which no student writes a motivation letter might arise. This happens if $\frac{x}{z}\gamma q - (1 - m_i) \leq 0$, even for a student with $m_i = 1$, hence $\frac{x}{z}\gamma q \leq 0$. Just as in subsection 5.2 this pooling equilibrium with $l_i = 0$ only exists when $\gamma q = 0$ and $\frac{x}{n} \geq 0$ or when $\frac{x}{n} = 0$ and $\gamma q \geq 0$.

Expected utility for the universities – pooling equilibria

In the pooling equilibrium in which all students who are not yet admitted to the high-quality university hand in a motivation letter and apply to the low-quality university, the low-quality university does not learn anything about the motivation of the students. The expected average ability however can be computed by taking the weighted average of the abilities of the students who are not yet admitted to the high-quality university. Therefore, the expected utility for the low-quality university is $E\left[U^j(l, 0)\right] = \widetilde{a_y} + E[m_i] = \widetilde{a_y} + \frac{1}{2}$. In the second pooling equilibrium no student applies to the low-quality university and therefore no student is admitted. Consequently, the expected utility for the low-quality university is $E\left[U^j(0, l)\right] = 0$. For the high-quality university the expected average ability is the weighted average of the abilities of the admitted students, hence $\widetilde{a_y} = 1 - \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ for $\frac{x}{n} \geq \frac{1}{8}$. The expected average motivation is however unknown and is therefore $E[m_i] = \frac{1}{2}$.

Separating equilibrium with $m_i > \hat{m}$ choosing $l_i = 0$ and $m_i < \hat{m}$ choosing $l_i = 1$

A separating equilibrium might arise in which the students with a low motivation apply to the low-quality university after writing a motivation letter and the students with a higher motivation do not write a motivation letter. It must be that there is an indifferent student with a motivation of $\underline{m_i}$. The expected utility of writing a motivation letter is bigger than the expected utility of not applying to a university at all for the students with the lower motivation, hence $\frac{x}{z}\gamma q - (1 - m_i) > 0$. This inequality results in a threshold of $\underline{m_i}$. For a student with $m_i = \underline{m_i} + \varepsilon$ the expected utility of writing a motivation letter should be smaller than the expected utility of not applying to a university at all. However, when this expected utility of writing a motivation letter is bigger than the expected utility of not applying to a university at all for a student with $m_i < \underline{m_i}$, than this is also true for a student with $m_i = \underline{m_i} + \varepsilon$. Therefore, the student with $m_i = \underline{m_i} + \varepsilon$ wants to deviate to writing a motivation letter and consequently this separating equilibrium in which a student with a low motivation writes a motivation letter and a student with a higher motivation does not write a motivation letter does not exist, as in subsections 4.2, 5.2 and 6.2.

Separating equilibrium with $m_i > \widehat{m}$ choosing $l_i = 1$ and $m_i < \widehat{m}$ choosing $l_i = 0$

A separating equilibrium might arise in which the students with a higher motivation apply to the low-quality university after writing a motivation letter and the students with a lower motivation do not write a motivation letter and do not apply to the low-quality university. This happens when $\frac{x}{z}\gamma q - (1 - m_i) > 0$ for all students with a higher motivation. There is a student who is indifferent between writing a motivation letter and not applying to a university at all. This student has the following motivation for $\gamma q > \frac{x}{n}$ and $\frac{x}{n} > 0$:

$$\frac{x}{(1-\widehat{m})n}\gamma q - (1 - \widehat{m}) = 0$$

$$\widehat{m} = 1 - \sqrt{\frac{x}{n}\gamma q}$$

In a market with $\gamma q \leq \frac{x}{n}$ and $\frac{x}{n} > 0$ the indifferent student has a different motivation:

$$\gamma q - (1 - \widehat{m}) = 0$$

$$\widehat{m} = 1 - \gamma q$$

Therefore, depending on the values of γq and $\frac{x}{n}$ a separating equilibrium arises in which all students with a motivation $m_i > \overline{m}$ write a motivation letter and apply to the low-quality university and all other students do not write a motivation letter.

Expected utilities for the universities

The expected utility for the universities is $E[U^j] = \widetilde{a_y} + \widetilde{m_y}$. There is a selection effect, as the rich students take an SAT more often than poor students. The expected average ability for the high-quality university is $\widetilde{a_y} = 1 - \frac{x}{n}$ for $\frac{x}{n} < \frac{1}{8}$ and $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ for $\frac{x}{n} \geq \frac{1}{8}$. The expected average motivation is however unknown and is therefore $E[m_i] = \frac{1}{2}$.

Furthermore, the expected average ability for the low-quality university is the weighted average of the rich and the poor students who are not admitted to the high-quality university. The expected average motivation of the admitted students is computed by taking the indifferent motivation $\widehat{m}$ into account, hence $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}$ for $\gamma q > \frac{x}{n}$ and $\frac{x}{n} > 0$ or $\widetilde{m_y} = 1 - \frac{1}{2}\gamma q$ for $\gamma q \leq \frac{x}{n}$ and $\frac{x}{n} > 0$.

<u>Conclusion subsection 6.3</u>

All students with a high enough signal to be admitted to the high-quality university will take an SAT and apply to the high-quality university. All other students decide to write a motivation letter or not. A pooling equilibrium arises in which all students write a motivation letter when $\gamma q \geq \frac{n-x}{x}$ *and* $\frac{x}{n-x} > 0$*, and a pooling equilibrium arises in which all students do not write a motivation letter when* $\gamma q = 0$ *and* $\frac{x}{n} \geq 0$ *or when* $\frac{x}{n} = 0$ *and* $\gamma q \geq 0$*. A separating equilibrium in which all students with* $m_i > \hat{m}$ *write a motivation letter exists with* $\hat{m} = 1 - \sqrt{\frac{x}{n}\gamma q}$ *when* $\gamma q > \frac{x}{n}$ *and* $\frac{x}{n} > 0$ *and with* $\hat{m} = 1 - \gamma q$ *when* $\gamma q \leq \frac{x}{n}$ *and* $\frac{x}{n} > 0$*. As a consequence, more motivated students will always be overrepresented at the low-quality university and rich students will always be overrepresented at the high-quality university.*

6.4. The low-quality university demands an SAT score whereas the high-quality university demands a motivation letter

This fourth and last subsection is closely related to subsections 4.3, 5.3 and 6.3. However, in this subsection it is assumed that the low-quality university bases their admissions on SAT scores, $p = (s, 0)$ and the high-quality university bases their admissions on motivation letters, $p = (0, l)$. However, subsection 6.4 is to illustrate the scope of the situation in which there are two universities with a different quality. The low-quality university uses SAT scores for their admissions whereas the high-quality university uses motivation letters. Again, this subsection is not taken into account in proposition 3.

<u>Admission policies of the universities</u>

The low-quality university bases their admissions on SAT scores whereas the high-quality university bases their admissions on motivation letters. The low-quality university therefore wants to admit the x students with the highest signal s_i whereas the high-quality university wants to admit as many students with a motivation letter as possible.

<u>Information collection of the students</u>

Students can apply to the low-quality university after taking an SAT and they can apply to the high-quality university after writing a motivation letter. Students with a high enough ability to be admitted to the low-quality university can prefer being admitted with certainty over being admitted to the high-quality university with a particular probability. At the same time, students with a very high motivation can prefer applying to the high-quality university and not being admitted with certainty over applying to the low-quality university and being admitted with certainty. Again, pooling equilibria and separating equilibria might arise.

Pooling equilibrium with $l_i = 1$ for all students

A pooling equilibrium may arise in which all students decide to write a motivation letter. In that case students should prefer writing a motivation letter and being admitted with a particular probability over being admitted to the low-quality university or not applying to a university at all. When the expected utility of applying to the high-quality university, $E[U^i] = \frac{x}{z}q - (1 - m_i)$, is bigger than or equal to the expected utility of being admitted to the low-quality university, $E[U^i] = \gamma q - \frac{1}{2}$, than it must also be the case that the expected utility of applying to the high-quality university is bigger than or equal to the expected utility of not applying to a university at all, $E[U^i] = 0$. Therefore, in case $\frac{x}{z}q - (1 - m_i) \geq \gamma q - \frac{1}{2}$, even for a student with $m_i = 0$, a pooling equilibrium arises in which all students write a motivation letter.

When there is a pooling equilibrium in which all students write a motivation letter, then $z = n$. Therefore, the expected utility of writing a motivation letter for a student with $m_i = 0$ is $E[U^i] = \frac{x}{n}q - 1$. This expected utility must be bigger than or equal to the expected utility of being admitted to the low-quality university:

$$\frac{x}{n}q - 1 \geq \gamma q - \frac{1}{2}$$

$$\frac{x}{n}q - \gamma q \geq \frac{1}{2}$$

This only happens for $\frac{1}{2} + \left(\gamma - \frac{x}{n}\right) q \leq 0$ and $\frac{x}{n} > 0$. Therefore, a pooling equilibrium arises in which all students write a motivation letter when $\frac{1}{2} + \left(\gamma - \frac{x}{n}\right) q \leq 0$ and $\frac{x}{n} > 0$.

Pooling equilibrium with $l_i = 0$ for all students

A pooling equilibrium in which no student writes a motivation letter may arise as well. This pooling equilibrium only arises when the expected utility of writing a motivation letter, $E[U^i] = \frac{x}{z}q - (1 - m_i)$, is smaller than the expected utility of being admitted to the low-quality university, $E[U^i] = \gamma q - \frac{1}{2}$, and smaller than the expected utility of not applying to a university at all, $E[U^i] = 0$. When the expected utility of writing a motivation letter is smaller than the expected utility of not applying to a university at all than this utility is also smaller than the expected utility of being admitted to the low-quality university. Hence, this pooling equilibrium arises when $\frac{x}{z}q - (1 - m_i) \leq 0$. Therefore, a pooling equilibrium in which no student writes a motivation letter arises for $q = 0$ and $\frac{x}{n} \geq 0$ or $\frac{x}{n} = 0$ and $q \geq 0$.

Expected utility for the universities – pooling equilibria

When all students write a motivation letter, the high-quality university does not learn anything about the motivation of the students. Therefore, the expected average motivation of the students is $\widetilde{m_y} = E[m_i] = \frac{1}{2}$. The expected average ability of the students is also unknown. The expected average ability is therefore $\widetilde{a_y} = E[a_i] = \frac{1}{2}$ and the expected utility for the high-quality university is $E[U^j] = \frac{1}{2} + \frac{1}{2} = 1$. Furthermore, when no student writes a motivation letter, there are no students admitted to the high-quality university. The expected utility for the high-quality university is therefore $E[U^j] = 0$.

Separating equilibrium with $m_i > \hat{m}$ choosing $l_i = 0$ and $m_i < \hat{m}$ choosing $l_i = 1$

A separating equilibrium might arise in which the students with a low motivation apply to the high-quality university after writing a motivation letter and the students with a higher motivation do not write a motivation letter. Therefore, there must be an indifferent student. For a student with a low motivation, writing a motivation letter should lead to a higher expected utility than not applying to a university at all and also to a higher expected utility than applying to the low-quality university, hence $\frac{x}{z}q - (1 - m_i) > 0$ and $\frac{x}{z}q - (1 - m_i) > \gamma q - \frac{1}{2}$. The second inequality is more stringent, and this inequality results in a threshold of $\underline{m_i}$. A student with $m_i = \underline{m_i} + \varepsilon$ wants to deviate to writing a motivation letter as his expected utility of writing a motivation letter is also bigger than the expected utility of being admitted to the low-quality university, because this is also the case for students with a motivation $m_i < \underline{m_i}$. Consequently, this separating equilibrium in which a student with a low motivation writes a motivation letter and a student with a higher motivation does not write a motivation letter does not exist, as in subsections 4.2, 5.2, 6.2 and 6.3.

Separating equilibrium with $m_i > \hat{m}$ choosing $l_i = 1$ and $m_i < \hat{m}$ choosing $l_i = 0$

A separating equilibrium might arise in which only students with a high motivation write a motivation letter. Students know their own ability and their own motivation and there are some students with a high motivation and a high ability. These students can choose to write a motivation letter, apply to the high-quality university and being admitted with a particular probability over taking an SAT and being admitted to the low-quality university for sure when $\frac{x}{z}q - (1 - m_i) > \gamma q - \frac{1}{2}$. This results in a threshold $\overline{m}$ for which high ability students will apply to the high-quality university after writing a motivation letter instead of applying to the low-quality university after taking an SAT:

$$\frac{x}{z}q - (1 - m_i) > \gamma q - \frac{1}{2}$$

$$\overline{m} = \frac{1}{2} + \gamma q - \frac{x}{z}q$$

$$\overline{m} = \frac{1}{2} + \gamma q - \frac{x}{(1-\overline{m})n}q$$

$$\overline{m} = \frac{3}{4} + \frac{1}{2}\gamma q - \frac{\sqrt{n^2 - 4\gamma q n^2 + 4\gamma^2 q^2 n^2 + 16qxn}}{4n}$$

This threshold depends on the values of γ and q. By taking the derivative $\frac{\partial \overline{m}}{\partial q}$, it can be stated whether there is a positive or a negative relationship between $\overline{m}$ and q. This derivative results in $\frac{\partial \overline{m}}{\partial q} = \frac{1}{2}\gamma - \frac{2n\gamma^2 q - n\gamma + 4x}{2\sqrt{4n^2\gamma^2 q^2 + 16nxq - 4n^2\gamma q + n^2}}$, which is always bigger than 0 as $\gamma q \geq \frac{1}{2}$. Therefore, there is a positive relationship between $\overline{m}$ and q. Hence, if q and thus γq increase, $\overline{m}$ will increase as well and therefore there are less students who decide to write a motivation letter and apply to the high-quality university. More students prefer being admitted to the low-quality university with certainty after taking an SAT over writing a motivation letter and not being admitted to the high-quality university for sure, because there is more at stake with the higher q and γq. Only students with a motivation $m_i > \overline{m}$ decide to write a motivation letter and not to be admitted with certainty, as their costs for writing a motivation letter are lower.

Furthermore, by taking the derivative $\frac{\partial \overline{m}}{\partial \gamma}$, it can be stated whether the relationship between $\overline{m}$ and γ is positive or negative. This derivative $\frac{\partial \overline{m}}{\partial \gamma} = \frac{1}{2}q - \frac{2n\gamma q^2 - nq}{2\sqrt{4n^2\gamma^2 q^2 + 16nxq - 4n^2\gamma q + n^2}}$ is again always bigger than 0. Therefore, there is a positive relationship between $\overline{m}$ and γ. This means that when γ increases $\overline{m}$ will increase as well. This is the case because an increase of γ leads to a more equal γq and q. Consequently, it is less attractive for students to take the risk of not being admitted to the high-quality university after writing a motivation letter. Students will therefore less often write a motivation letter and will less often apply to the high-quality university after an increase of γ.

All students with a motivation $m_i > \overline{m}$, irrespective of their ability, will apply to the high quality-university after writing a motivation letter. This is depicted in area 1 and 2 in figure 31 below. All students with a motivation $m_i < \overline{m}$ will not apply to the high-quality university but decide whether to apply to the low-quality university after taking an SAT or not to apply to a university at all. Only the students who are sure that they will receive a signal s_i that is high enough to be admitted will take an SAT. These students are depicted in area 3 in figure 31 below. Lastly, all students with $m_i < \overline{m}$ who expect to receive a signal $s_i < \overline{s_i}$ will not apply to a university at all, as depicted in area 4 in figure 31 below.

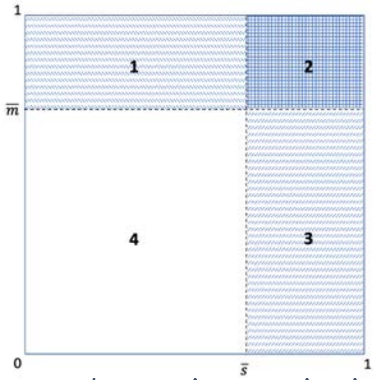

Figure 31 – Areas where students write a motivation letter or take an SAT

However, because there are some students with $s_i > \overline{s}$ who rather apply to the high-quality university after writing a motivation letter than apply to the low-quality university after taking an SAT, the threshold $\overline{s}$ will move down. This leaves more room for students with a lower ability to apply to the low-quality university.

The $\overline{s}$ in figure 31 is such that exactly all students in area 2 and 3 can be admitted to the low-quality university. However, the students in area 2 do not want to apply to the low-quality university. The $\overline{s}$ will move to the left and therefore also students with a lower signal can be admitted. The threshold $\overline{s}$ will move to the left such that there are exactly as many students with a signal between $\hat{s}$ and $\overline{s}$ and a motivation $m_i < \overline{m}$ as there are students with a signal $s_i > \overline{s}$ who have already applied to the high-quality university, hence area 5 in figure 32 below must be equal to area 2. Furthermore, exactly $\frac{x}{n}$ percent of the students should want to apply to the low-quality university:

$$(1 - \hat{s})\overline{m} = \frac{x}{n}$$

$$\hat{s} = 1 - \frac{x}{n} * \frac{1}{\overline{m}}$$

In such a way, all students with a motivation $m_i > \overline{m}$ write a motivation letter and apply to the high-quality university, as in area 1 and 2 in figure 32 below. All other students who have a high enough ability to receive a signal $s_i > \overline{s_i}$ to be admitted to the low-quality university will take an SAT and will apply to the low-quality university, as in area 3. All students in area 5 take an SAT and apply to the low-quality university as well and all students in area 4 do not apply to one of the two universities. This results in substitution for the low-quality university between students with a high ability and a high motivation (area 2) and students with a lower ability and a lower motivation (area 5). Consequently, the low-quality university loses students with a high ability and a high motivation to the high-quality university. Rich students will be overrepresented at the low-quality university for most values of $\frac{x}{n}$, as these students receive higher signals than poor students in tight markets.

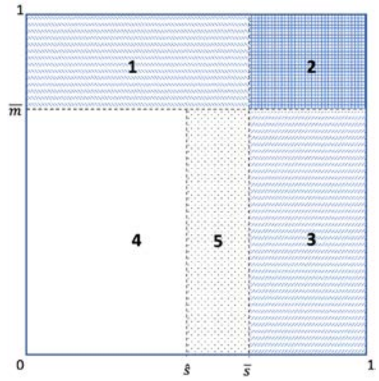

Figure 32 – New areas where students write a motivation letter or take an SAT

The high-quality university can decide to base their admissions on motivation letters when the low-quality university still bases their admissions on SAT scores. In case both the low-quality university and the high-quality university decide to use SAT scores, the high-quality university always attracts more rich students than poor students. Therefore, the poor students with a high ability will less often be admitted to the high-quality university, because they receive a signal s_i that is always smaller than their ability a_i for $a_i > \frac{1}{2}$. High-quality universities can choose to change their admission program from SAT scores to motivation letters for mostly ethical reasons, because students with a lower signal will now be admitted as well and those are mostly poor students. Consequently, low-income students with a high motivation who would not be admitted to the high-quality university in the situation in which there are two universities who both use SAT scores for their admissions, students in area 2 on the left side of figure 33, will now have a chance to be admitted to the high-quality university after writing a motivation letter, area 6 on the right side of figure 33 below. Figure 33 below compares the results of subsection 6.1 in which both universities use SAT scores for their admissions and subsection 6.4 in which the high-quality university switches to using motivation letters for their admissions.

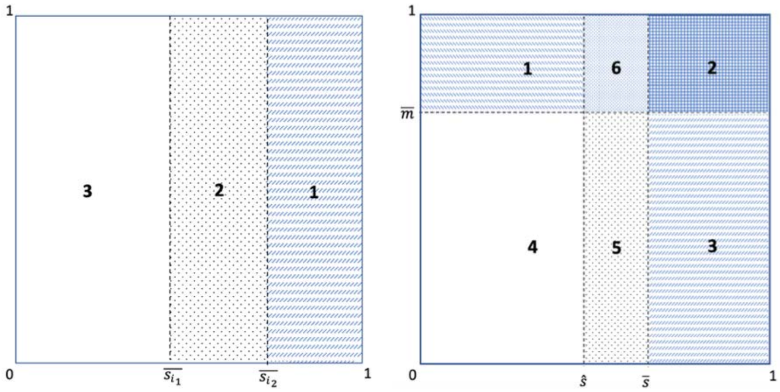

Figure 33 – Results from subsection 6.1 and 6.4 compared

<u>*Expected utilities for the universities*</u>

The expected utility for the universities is $E\left[U^j\right] = \widetilde{a_y} + \widetilde{m_y}$. For the high-quality university the expected average motivation can be computed out of the threshold $\overline{m}$. The expected average motivation of the admitted students is $\widetilde{m_y} = \frac{1+\frac{3}{4}+\frac{1}{2}\gamma q-\frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{4n}}{2} = \frac{7}{8} + \frac{1}{4}\gamma q - \frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{8n}$. The average ability of the admitted students is however unknown. Therefore, the expected utility for the high-quality university is $E\left[U^j\right] = \frac{1}{2} + \frac{7}{8} + \frac{1}{4}\gamma q - \frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{8n}$. Furthermore, the expected average motivation of the students who are admitted to the low-quality university is the average motivation of the students who are not admitted to the high-quality university, $\widetilde{m_y} = \frac{\frac{3}{4}+\frac{1}{2}\gamma q-\frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{4n}}{2} = \frac{3}{8} + \frac{1}{4}\gamma q - \frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{8n}$. The average ability of those students is the weighted average of the abilities of the admitted students. The expected utility for the low-quality university is therefore $E\left[U^j\right] = \widetilde{a_y} + \frac{3}{8} + \frac{1}{4}\gamma q - \frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{4n}$.

<u>*Conclusion subsection 6.4*</u>

There only arises a pooling equilibrium in which all students write a motivation letter when $\frac{1}{2} + \left(\gamma - \frac{x}{n}\right) q \leq 0$ *and* $\frac{x}{n} > 0$ *and a pooling equilibrium in which no student writes a motivation letter arises when* $q = 0$ *and* $\frac{x}{n} \geq 0$ *or* $\frac{x}{n} = 0$ *and* $q \geq 0$. *Furthermore, a separating equilibrium arises in which all students with* $m_i > \widehat{m}$ *write a motivation letter for* $\gamma q \in \left[0, \frac{1}{2}\right]$, $n > 0$, $x > 0$, *with* $\overline{m} = \frac{3}{4} + \frac{1}{2}\gamma q - \frac{\sqrt{n^2-4\gamma q n^2+4\gamma^2 q^2 n^2+16qxn}}{4n}$. *This* $\overline{m}$ *increases when* γ *or* q *increases. For a higher* γ *there is a smaller difference between the two universities and therefore less students want to take the risk of not being admitted with certainty and for a higher q there is more at stake, so students again prefer being admitted with certainty. Rich students will be overrepresented at the low-quality university and more motivated students are overrepresented at the high-quality university.*

University of Chicago, a high-quality university, decided to base their admissions on motivation letters while the low-quality universities still base their admissions on SAT scores. Subsection 6.4 illustrates that this results in a substitution of students with a high ability and a high motivation with students with a lower ability and an average motivation who apply to the low-quality university. Furthermore, all students with a high motivation will apply to the high-

quality university. University of Chicago might have decided to base their admissions on motivation letters for ethical reasons because due to this change in admission programs more low-income students can apply to the high-quality university compared to when both universities use SAT scores for their motivations. The expected utilities and the conditions for equilibria to exist in different markets are summarized in table 3 below.

Market	**Expected utility**	**Conditions**
$\boldsymbol{p_l = (s, 0)}$ **and** $\boldsymbol{p_h = (s, 0)}$	• Low quality: $E\left[U^j\right] = \frac{3}{2} - \frac{3}{2} * \frac{2x}{n}$ • High quality: $E\left[U^j\right] = \frac{3}{2} - \frac{x}{n}$	$\frac{2x}{n} < \frac{1}{8}$ and $\gamma q \geq \frac{1}{2}$
	• Low quality: $E\left[U^j\right] = \frac{116*\frac{x}{n} - 88*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$ • High quality: $E\left[U^j\right] = \frac{3}{2} - \frac{x}{n}$	$\frac{1}{8} \leq \frac{2x}{n} \leq \frac{1}{4}$ and $\gamma q \geq \frac{1}{2}$
	• Low quality: $E\left[U^j\right] = \frac{14}{9} - \frac{5}{3} * \frac{x}{n}$ • High quality: $E\left[U^j\right] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$	$\frac{1}{4} < \frac{2x}{n} \leq \frac{7}{8}$ and $\gamma q \geq \frac{1}{2}$
	• Low quality: $E\left[U^j\right] = \frac{248*\frac{x}{n} - 248*\left(\frac{x}{n}\right)^2 - 35}{72*\frac{x}{n}}$ • High quality: $E\left[U^j\right] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$	$\frac{2x}{n} > \frac{7}{8}$ and $\gamma q \geq \frac{1}{2}$
$\boldsymbol{p_l = (0, l)}$ **and** $\boldsymbol{p_h = (0, l)}$	Pooling with $l_i = 1$: $E\left[U^j\right] = 1$	$q + \gamma q \geq \frac{n}{x}$ and $\frac{2x}{n} > 0$
	Pooling with $l_i = 0$: $E\left[U^j\right] = 0$	$q = 0$ and $\frac{2x}{n} \geq 0$ or $\frac{2x}{n} = 0$ and $q \geq 0$
	Separating with $\widehat{m} = 1 - q$: • High quality: $E\left[U^j\right] = 1 + \frac{1}{2}\widehat{m}$ • Low quality: $E\left[U^j\right] = 0$	$q \leq \frac{x}{n}$, $\frac{x}{n} > 0$ and $q \in [0,1]$
	Separating with $\widehat{m} = 1 - \frac{1}{2}\gamma q - \frac{1}{2}\sqrt{(\gamma q)^2 + 4 * \frac{x}{n} q - 4 * \frac{x}{n}\gamma q}$: $E\left[U^j\right] = 1 + \frac{1}{2}\widehat{m}$	$q > \frac{x}{n}$, $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$, $\frac{2x}{n} > 0$ and $\frac{1}{2}q + \frac{1}{2}\gamma q \in [0,1]$
	Separating with $\widehat{m} = 1 - \frac{1}{2}q - \frac{1}{2}\gamma q$: $E\left[U^j\right] = 1 + \frac{1}{2}\widehat{m}$	$\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$, $\frac{2x}{n} > 0$ and $\frac{1}{2}q + \frac{1}{2}\gamma q \in [0,1]$
	Separating with $\widehat{m} = 1 - \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$: $E\left[U^j\right] = 1 + \frac{1}{2}\widehat{m}$	$\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$, $\frac{2x}{n} > 0$ and $\frac{1}{2}q + \frac{1}{2}\gamma q \in [0,1]$
$\boldsymbol{p_l = (0, l)}$ **and** $\boldsymbol{p_h = (s, 0)}$	Pooling with $l_i = 1$	$\gamma q \geq \frac{n-x}{x}$ and $\frac{x}{n-x} > 0$
	Pooling with $l_i = 0$	$\gamma q = 0$ and $\frac{x}{n} \geq 0$ or $\frac{x}{n} = 0$ and $\gamma q \geq 0$
	Separating with $\widehat{m} = 1 - \gamma q$	$\gamma q \in \left[0, \frac{1}{2}\right]$, $\gamma q \leq \frac{x}{n}$ and $\frac{x}{n} > 0$
	Separating with $\widehat{m} = 1 - \sqrt{\frac{x}{n}\gamma q}$	$\gamma q \in \left[0, \frac{1}{2}\right]$, $\gamma q > \frac{x}{n}$ and $\frac{x}{n} > 0$
$\boldsymbol{p_l = (0, l)}$ **and** $\boldsymbol{p_h = (s, 0)}$	Pooling with $l_i = 1$	$\frac{1}{2} + \left(\gamma - \frac{x}{n}\right) q \leq 0$ and $\frac{x}{n} > 0$
	Pooling with $l_i = 0$	$q = 0$ and $\frac{x}{n} \geq 0$ or $\frac{x}{n} = 0$ and $q \geq 0$
	Separating with $\overline{m} = \frac{3}{4} + \frac{1}{2}\gamma q - \frac{\sqrt{n^2 - 4\gamma q n^2 + 4\gamma^2 q^2 n^2 + 16 q x n}}{4n}$	$\gamma q \in \left[0, \frac{1}{2}\right]$, $n > 0$, $x > 0$

Table 3 – Expected utilities and conditions in different markets

7. Conclusion and discussion

In this section this thesis will be concluded by summarizing the results and thereby answering the research question. Furthermore, it will be elaborated whether the results of the former empirical literature are theoretically explained in this thesis and for the former theoretical literature it will be elaborated what this thesis adds to that literature. Lastly, the limitations of this thesis and the recommendations for future research will be stated.

7.1. Summary and main results

The goal of this thesis was to provide an answer to the question "*What is the effect of admission programs of universities on admission rates of students?*". It is found that in case both universities have the same quality, one university should often base their admissions on SAT scores and the other university should base their admissions on motivation letters to maximize the average expected utility for the universities. In that case, rich students will always be overrepresented in the university that bases their admissions on SAT scores and more motivated students will be overrepresented in the university that bases their admissions on motivation letters. However, for some qualities of the universities and for some number of spots available at the universities, the universities both choose to base their admissions on SAT scores or on motivation letters. Rich students and more motivated students respectively benefit from this. Poor students will however benefit in loose markets where both universities use SAT scores for their admissions, because the SAT scores that poor students receive in those markets will be higher than their true ability.

In case the universities differ in quality, the optimal combination of admission programs differs depending on the exact qualities of the universities. It is often optimal for both universities to base their admissions on SAT scores, but for some combinations of qualities of the universities and in some loose markets it is optimal for both universities to base their admissions on motivation letters. Rich students will benefit from both universities using SAT scores and more motivated students benefit in case both universities use motivation letters.

The results from former empirical literature on admission processes stated that SAT scores were influenced by the wealth of a student (Hurwitz, Smith, Niu, & Howell, 2015) and that universities should admit more motivated students (Dennis, Phinney, & Chuateco, 2005). The signal obtained by a student after taking an SAT in this model was therefore accurate for a rich student and accurate or random, both with probability $\frac{1}{2}$, for a poor student. Furthermore, universities admit those students who write a motivation letter. Empirical literature on the differences between rich and poor students stated that students with a high ability but with a low wealth would less often apply to a university (Hoxby, & Avery, 2012; Dynarski, Libassi, Michelmore, & Owen, 2018). This result is theoretically explained in this thesis, because poor students with a high ability are less often admitted to a university compared to rich students with the same ability, because their signal is lower than their true ability in tight markets. Poor students thus lack the money to have the same opportunities as rich students for some values of x and n and therefore there is a gap between rich and poor students. Furthermore, in case the difference in payoffs between two options is bigger, people put in more effort (DeVaro, 2006). In this thesis the threshold of writing a motivation letter depends on the quality of the universities and when those qualities differ more students write a motivation letter more often. This theoretical model also shows that persuasive messages lead to higher payoffs for the senders of these messages and separating equilibria arise in signalling games. Furthermore, this thesis adds to the former theoretical literature in the way that the problem of poor students lacking money is added to the literature on the difference between rich and poor students. In this thesis there is a difference between students based on wealth, motivation and ability. These differences have not yet been investigated before.

University of Chicago, a high-quality university in this thesis, recently decided to no longer require the SAT and ACT scores for their admissions.[10] However, a motivation letter is still required. University of Chicago decided on this, as it is stated that low income students and minorities are admitted less often. By not requiring the SAT scores for admissions, it is possible to admit outstanding students from all backgrounds. In the model of this thesis it is the situation as in subsection 6.4. The low-quality university demands the SAT scores, and the high-quality university demands motivation letters. Changing to motivation letters is presumably not beneficial in maximizing the expected utility for University of Chicago, but it is an ethical matter to only base admissions on motivation letters, because low income students get the chance to be admitted more often.

In the model of this thesis the signal from taking an SAT is accurate for rich students and accurate or random for poor students, whereas the motivation letter is not different for rich or poor students. This assumption creates inequity between rich and poor students who take the SAT, whereas there is no separation between rich and poor students who write a motivation letter.

[10] https://www.insidehighered.com/admissions/article/2019/07/15/chicago-sees-success-dropping-testing-requirement-admissions & https://eu.usatoday.com/story/news/2018/06/14/university-chicago-sat-act-optional/701153002/

7.2. Limitations and future research

The limitations of this thesis are used as recommendations for future research. The first limitation is that the universities can only choose to base their admissions on SAT scores or motivation letters. It could be investigated how the results change when universities can use different forms of hard and soft information for the admission programs. Furthermore, universities should be able to base their admissions not only on hard or soft information, but also to use neither of the two or both for their admissions. This is also the University of Chicago case, in which the high-quality university decided to change from both hard and soft information to only soft information for their admissions. Furthermore, this leads to the possibility of researching whether students from minorities will be admitted to universities more often when the university changes from both using SAT scores and motivation letters to only using SAT scores for their admissions.

Second, in this thesis it is researched what universities can do best when there are two universities in the market. There are however many markets with more than two universities. Therefore, future research can investigate what the best combination of admission programs is and what the effects are on admission rates, but in markets with more than two universities.

Third, it is assumed that choosing the outside option leads to an expected utility of $E[U^i] = 0$. However, it can be argued that choosing not to apply to a university still leads to a positive utility because students can still choose to go to a community college and be admitted for sure. This utility will be smaller than from going to a university but might be big enough to change the actions of some students, because students dislike applying to a university and not being admitted. Future research could therefore take this into account.

Fourth, the utility function of the universities is assumed to positively depend on the expected average ability and expected average motivation of the admitted students. However, it can be assumed that the utility for the universities and the students also depends on the number of students who are admitted to the university. In case there are too many students admitted to the university, there can be dissatisfaction among the students as the quality of the education decreases, which influences the utility for both the university and the student. Furthermore, professors at universities might prefer teaching to smaller groups than to bigger groups, which results in more dissatisfaction at the university and a lower utility for the university in case there are a lot of students admitted to the university. A different utility function for the universities and for the students can therefore be used in future research.

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Appendix 1 – Empirical Literature Overview

Title	Authors	Topic/Results	
An Experimental Analysis of Nash Refinements in Signaling Games	Banks & Camerer & Porter (1994)	-Refinements of Nash equilibrium in two-person signalling games. -The combination of message and action was in 70% of the situations Nash equilibria. Some subjects wanted to choose the more refined equilibrium action and all games predict pooling equilibria. However, senders of different types often chose different messages, hence a separating equilibrium.	8
The Role of Motivation, Parental Support, and Peer Support in the Academic Success of Ethnic Minority First-Generation College Students	Dennis & Phinney & Chuateco (2005)	-Motivation is an important predictor of college outcomes. -Personal and career related motivation was a positive predictor of college adjustment in the following spring, whereas the lack of peer support was a negative predictor and predicted a lower spring GPA.	1
Internal promotion competitions in firms	DeVaro (2006)	-Presenting evidence suggesting that relative work performance determines promotions and estimating a model of promotion tournaments. -Employers indeed set wage spreads to make sure the optimal performance levels are reached, and workers will be more motivated by larger spreads.	5
Persuasion in experimental ultimatum games	Andersson & Galizzi & Hoppe & Kranz & van der Wiel & Wengström (2010)	-Persuasion effects in experimental ultimatum games where Proposers could send a message to the responder before he made a decision or after he made a decision. -Persuasive messages indeed lead to a higher payoff for the proposer and mostly happened in situations where the subjects were experienced in the game. This was the results of higher acceptance rates and lower offers.	7
Robin Hood Under the Hood: Wealth-Based Discrimination in Illicit Customer Help	Gino & Pierce (2010)	-Does an employee's perception of customer wealth affect his likelihood of engaging in illegal behaviour? -Wealth-based discrimination is present in employee-customer relationships and envy towards wealthy customers and empathy towards the customers with the same economic status drive much of the illegal behaviour.	6
The missing "one-offs": The hidden supply of high-achieving, low income students	Hoxby & Avery (2012)	-High-achieving, low-income students do not apply to any selective college or university. -High-achieving, low-income students who act upon their income do not come from families or neighbourhoods that are more disadvantaged than those students who act upon their ability. The students who act upon their income, however, are unlikely to be encouraged to go to highly selective colleges.	2/4
College Major Choice and the Gender Gap	Zafar (2013)	-How college majors are chosen, focussing on the underlying gender gap. -Enjoying coursework and gaining parents' approval are the most important determinants. Males and females differ, because males care more about the monetary outcomes in the workplace. The gender gap is mainly due to gender differences in preferences and tastes, and not because females are underconfident about their ability.	3
Learning About Oneself: The Effects of Signaling Ability on School Choice.	Bobba & Frisancho (2014)	-A lack of full information about academic potential leads to misallocation of students. -Expected and realized performance becomes more equal. More academic-oriented schools are chosen more often by higher ability students. Students choosing a school-track do not take into account the negative feedback they might have received.	3

University Choice: The Role of Expected Earnings, Non-pecuniary Outcomes and Financial Constraints.	Delavande & Zafar (2014)	-Determinants of the university choice of students with a focus on beliefs about the future monetary returns and on financial constraints of students. -Expected future earnings are not very important, unlike non-pecuniary factors. If students were not financially constrained, 37% of students would choose differently. Expected utility for students would increase by 21% when the financial constraints could be relaxed by policies.	3
Determinants of College Major Choice: Identification using an Information Experiment.	Wiswall & Zafar (2014)	-Determinants of college major choices. Whether students have biased beliefs regarding their expected labour market outcomes. -There is a positive correlation between unobserved tastes and expected earnings. Important determinants for college major choice are students' beliefs about future earnings and perceived ability. Younger students are more likely to switch majors, because of lower switching costs.	3
What High-Achieving Low-Income Students Know About College	Hoxby & Turner (2015)	-The effects of Expanding College Opportunities on students' knowledge and decision-making regarding colleges. -48% percent more students applied for highly selective colleges. Treated students were admitted more often and enrolled in more highly selective colleges.	3/4
The Maine Question: How is 4-Year College Enrollment Affected by Mandatory College Entrance Exams?	Hurwitz & Smith & Niu & Howell (2015)	-Estimating the consequences on college enrolments by mandating the SAT for all juniors of public schools. -Overall, the policy increased the enrolment-rate and some students who normally would not do take the SAT scored quite well.	1
Closing the Gap: The Effect of a Targeted, Tuition-Free Promise on College Choices of High-Achieving, Low-Income Students.	Dynarski, Libassi, Michelmore, & Owen (2018)	-Closing the gap between low-income and high-income students' enrolments at highly selective colleges. -Smaller application and enrolment gaps at colleges can be achieved by offering free tuition and fees upon admission to high-achieving, low-income students.	4

1. Admission processes at universities (hard vs soft information)
2. Differences between rich and poor students
3. Determinants of college (major) choice
4. Student distributions at colleges
5. Promotion tournaments
    a. Tournaments
6. Discrimination
7. Persuasion games
8. Signalling games

Appendix 2 – Theoretical Literature Overview

Title	**Authors**	**Topic/Results**	**Players**	**Actions**	**Sequential / Simultaneous**	**Equilibrium concept**	**Communication game**	**Utility functions**	**Information collection?**	
Job Market Signaling	Spence (1973)	-The characteristics of a basic signalling model. -An equilibrium that can be used to investigate admission procedures and promotions.	2 players, an employer and an employee.	The employee must decide on the signal that he sends to the employer, and the employer must decide on hiring the employee or not and on which wage to offer the employee.	Sequential	Signalling equilibrium, Perfect Bayesian equilibrium	Yes	The employee receives a wage that depends on signals and indices minus the signalling costs for the employee. The employer receives a marginal product that depends on the attributes of employees.	The employee provides information to the employer.	8
Rank-Order Tournaments as Optimum Labor Contracts	Lazear & Rosen (1981)	-Compensation schemes that pay according to relative performance. -Risk neutral agents lead to efficient allocation of resources. Risk-averse workers sometimes prefer a tournament.	2 workers that compete for the prize of winning the tournament.	The workers need to choose their level of effort. The firm needs to choose the competitive prize structure.	Simultaneous	Nash equilibrium	No	Worker: $PW_1 + (1 - P)W_2 - C(\mu)$	No	5a
Relying on the information of interested parties	Milgrom & Roberts (1986)	-Trying to influence a decision maker by providing verifiable information elicits all relevant information. -Competition may be unnecessary when the decision maker is sophisticated and well informed, and competition is mostly insufficient when the decision maker is	2 players, a seller and a buyer.	The seller provides the buyer with product quality information. Based on this information the buyer decides how much to purchase.	Sequential	Sequential equilibrium	Yes	Buyer: $u(x, q)$ where x is the seller's information and q is quantity. For each x there is a unique $q^*(x)$ that maximizes u. Seller: $v(w, q)$ which is increasing in q.	The seller provides verifiable information about product quality to the buyer.	7

		unsophisticated or not well informed. Competition can be helpful in other situations.								
Is Statistical Discrimination Efficient?	Schwab (1986)	-The efficiency effects of statistical discrimination are examined. -Statistical discrimination increases efficiency of labor supply for the favored group but decreases efficiency for the disfavored group.	Employers, and employees with either a high or low ability.	Individuals choose in which market to work: standardized or individual markets. Employers choose the wage.	Sequential	Nash equilibrium	No	Wage equals the average ability of the people in the standardized markets. The overall social product is the sum of production in both markets.	No	6
Signaling Games and Stable Equilibria	Cho & Kreps (1987)	-The general criteria of equilibria in signalling games and the theory of stability. -Different tests can be used to find reasonable equilibrium outcomes and stability cannot be guaranteed in case it is unintuitive.	2 players. Player 1 is either a wimp or surly and player 1 meets player 2.	Player 1 chooses to have beer or quiche for breakfast and player 2 decides to duel or not to duel player 1.	Sequential	Sequential Equilibrium	Yes	The utility for player 1 and player 2 depends on the combination of surly or wimp, beer or quiche and duel or no duel.	Player 2 wants to see whether player 1 had beer or quiche for breakfast.	8
Good news and bad news: representation theorems and applications	Milgrom (1991)	-Modelling methods in information economics where the “favorableness” of news is introduced and applied in four models. -Four equilibria of these models are found, where the most relevant one is that buyers expect that product information that is not shared by a	2 players, a salesman and a buyer.	The salesman decides to report or conceal the pieces of information he has. The buyer decides on his purchasing strategy.	Sequential	Sequential equilibrium	Yes	Buyer: $\tilde{\theta}F(q) - pq$, where θ is the quality of the product, q are the units and p is the price. The salesman receives commission which is an increasing function of q.	The salesman reports or conceals information for the buyer.	7

		salesman is unfavourable to the product.								
Strategic information transmission with verifiable messages	Seidmann & Winter (1997)	-Generalizing results by assuming more general conditions on the sender's preferences. -The ideal action for a sender varies with its type.	2 players, a sender (S) and a receiver (R).	S chooses a message and R chooses an action after observing the message by S.	Sequential	Sequential equilibrium	Yes	S: $u(a;t)$ R: $v(a;t)$ Where t is S's type and a is the action of R.	S provides R with information.	7
Sabotage in Promotion Tournaments	Chen (2003)	-Investigating the nature, determinants, and impact of positive and negative effort in organizations. -More able people more often get attacked by negative effort of their co-workers. Therefore, the negative effort is inefficient and leads to the most able people not having the highest chance of being promoted.	A principal and n agents.	The agents need to decide on their level of positive and negative effort. The principal needs to decide which agent will be promoted.	Simultaneous	Nash equilibrium	No	$p_i(W_1, \dots, W_n, Z)u - v\left(e_i + \sum_{j \neq i} a_{ij}\right) + q(W_1, \dots, W_n, Z)$ where $p_i(\cdot)$ is the probability of promotion, Z is a vector of other variables that influence pay and probability of promotion, $q(\cdot)$ is the reward for effort and $v(\cdot)$ are the costs of effort.	No	5
Envy and Compassion in Tournaments	Grund & Sliwka (2005)	-Tournaments among inequity averse agents, because most individuals are not purely motivated by material self-interest but also by the wellbeing of others. -Those agents exert higher efforts and inequity costs have to be traded off against	2 agents	Agents decide on their individual output and the winner of the tournament receives the prize.	Simultaneous	Nash equilibrium	No	$u_i = w_i - \alpha \max\{w_j - w_i; 0\} - \beta \max\{w_i - w_j; 0\} - C(e_i)$ where the 2^{nd} and 3^{rd} term are inequity costs and the $C(e_i)$ denotes the costs of effort.	No	5a

		losses in human capital.								
Long persuasion games	Forges & Koessler (2008)	-Nash and perfect Bayesian equilibrium payoffs that are achievable in persuasion games with unmediated communication. -The results state all Nash equilibrium and perfect Bayesian equilibrium payoffs of one-shot and multistage communication games.	2 players, an informed (expert) and an uninformed player (decision maker)	The expert's type is chosen, there is communication between the players and then the decision maker chooses an action.	Sequential	Nash equilibrium	Yes	$u(k, j)$ where k denotes the type of the expert and j denotes the decision by the decision maker.	The informed player shares his information with the uninformed player.	7
Credit Constraints in Education	Lochner & Monge-Naranjo (2011)	-The impact of credit constraints on the accumulation of human capital. -There is a strong positive correlation between schooling and the family income.	Multiple individuals	The individuals decide on the levels of investment in the first period.	Simultaneous	Nash equilibrium	No	$U = u(c_0) + \beta u(c_1)$ where c_0 is the consumption in the first period and c_1 is the discounted consumption in the second period.	No	2/4

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Appendix 3 – Formulas Per Figure

Figure 2: Abilities per signal

- Rich students: $s_i = a_i$
- Poor students: $s_i = \frac{1}{2}a_i + \frac{1}{4}$

Figure 3: Minimal signal needed to be admitted

- $\frac{2x}{n} < \frac{1}{8}$: $\overline{s_i} = 1 - 2 * \frac{2x}{n}$
- $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\overline{s_i} = \frac{5}{6} - \frac{2}{3} * \frac{2x}{n}$
- $\frac{2x}{n} > \frac{7}{8}$: $\overline{s_i} = 2 - 2 * \frac{2x}{n}$

Figure 4: Percentage of students taking an SAT

- Rich students, $a_i \geq \overline{s_i}$
  - $\frac{2x}{n} < \frac{1}{8}$: $\% = 2 * \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{1}{6} + \frac{2}{3} * \frac{2x}{n}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\% = 2 * \frac{2x}{n} - 1$
- Poor students, $\frac{1}{2}a_i + \frac{1}{4} \geq \overline{s_i}$
  - $\frac{2x}{n} < \frac{1}{8}$: $\% = 0$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{4}{3} * \frac{2x}{n} - \frac{1}{6}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\% = 1$

Figure 5: Average ability, average motivation and expected utility

- Average motivation: $\widetilde{m_y} = \frac{1}{2}$
- Average ability:
  - $\frac{2x}{n} < \frac{1}{8}$: $\widetilde{a_y} = 1 - \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\widetilde{a_y} = \frac{76*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\widetilde{a_y} = \frac{4*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$
- Expected utility:
  - $\frac{2x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $E[U^j] = \frac{112*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$
  - $\frac{2x}{n} > \frac{7}{8}$: $E[U^j] = \frac{5*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$

Figure 6: Percentage of students writing a motivation letter

- $\frac{2x}{n} < q$: $\% = \sqrt{\frac{2x}{n}q}$
- $\frac{2x}{n} \geq q$: $\% = q$

Figure 7: Average ability, average motivation and expected utility

- Average ability: $\widetilde{a_y} = \frac{1}{2}$
- Average motivation:
  - $\frac{2x}{n} < q$: $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{2x}{n}q}$
  - $\frac{2x}{n} \geq q$: $\widetilde{m_y} = 1 - \frac{1}{2}q$
- Expected utility:

- $\frac{2x}{n} < q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{2x}{n}q}$
- $\frac{2x}{n} \geq q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}q$

Figure 8: Percentage of students taking an SAT or writing a motivation letter

- Rich students, $1 - \overline{s_i}$:
  - $\frac{x}{n} < \frac{1}{8}$: $\% = 2 * \frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\% = \frac{1}{6} + \frac{2}{3} * \frac{x}{n}$
- Poor students, $\frac{3}{2} - 2\overline{s_i}$:
  - $\frac{x}{n} < \frac{1}{8}$: $\% = 0$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\% = \frac{4}{3} * \frac{x}{n} - \frac{1}{6}$
- Motivation letters: $\% = \sqrt{\frac{x}{n}q}$

Figure 9: Average ability, average motivation and expected utility

- Average ability university 1:
  - $\frac{x}{n} < \frac{1}{8}$: $\widetilde{a_y} = 1 - \frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
- Average motivation university 1: $\widetilde{m_y} = \frac{1}{2}$
- Expected utility university 1:
  - $\frac{x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - \frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $E[U^j] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
- Average ability university 2:
  - $\frac{x}{n} < \frac{1}{8}$: $\widetilde{a_y} = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\widetilde{a_y} = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}}$
- Average motivation university 2: $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}$
- Expected utility university 2:
  - $\frac{x}{n} < \frac{1}{8}$: $E[U^j] = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $E[U^j] = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}$

Figure 10, 11, 12: Expected utilities for the universities per combination of admission programs

- Both SAT:
  - $\frac{2x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $E[U^j] = \frac{112*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$
  - $\frac{2x}{n} > \frac{7}{8}$: $E[U^j] = \frac{5*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$
- Both motivation letters:
  - $\frac{2x}{n} < q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{2x}{n}q}$
  - $\frac{2x}{n} \geq q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}q$

- SAT and Motivation letters:
  - $\frac{x}{n} < \frac{1}{8}$: $E[U^j] = \left(\frac{3}{2} - \frac{x}{n}\right) * \frac{1}{2} + \left(\frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}\right) * \frac{1}{2}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $E[U^j] = \left(\frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}\right) * \frac{1}{2} + \left(\frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}q}\right) * \frac{1}{2}$

Figure 13: Percentage of students taking an SAT

- Rich students, $a_i \geq \overline{s_i}$
  - $\frac{2x}{n} < \frac{1}{8}$: $\% = 2 * \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{1}{6} + \frac{2}{3} * \frac{2x}{n}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\% = 2 * \frac{2x}{n} - 1$
- Poor students, $\frac{1}{2}a_i + \frac{1}{4} \geq \overline{s_i}$
  - $\frac{2x}{n} < \frac{1}{8}$: $\% = 0$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{4}{3} * \frac{2x}{n} - \frac{1}{6}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\% = 1$

Figure 14: Average ability, average motivation and expected utility

- Average motivation: $\widetilde{m_y} = \frac{1}{2}$
- Average ability:
  - $\frac{2x}{n} < \frac{1}{8}$: $\widetilde{a_y} = 1 - \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\widetilde{a_y} = \frac{76*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\widetilde{a_y} = \frac{4*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$
- Expected utility:
  - $\frac{2x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $E[U^j] = \frac{112*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$
  - $\frac{2x}{n} > \frac{7}{8}$: $E[U^j] = \frac{5*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$

Figure 15: Percentage of students writing a motivation letter

- $\frac{2x}{n}\gamma q$: $\% = \sqrt{\frac{2x}{n}\gamma q}$
- $\frac{2x}{n} \geq \gamma q$: $\% = \gamma q$

Figure 16: Average ability, average motivation and expected utility

- Average ability: $\widetilde{a_y} = \frac{1}{2}$
- Average motivation:
  - $\frac{2x}{n} < \gamma q$: $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{2x}{n}\gamma q}$
  - $\frac{2x}{n} \geq \gamma q$: $\widetilde{m_y} = 1 - \frac{1}{2}\gamma q$
- Expected utility:
  - $\frac{2x}{n} < \gamma q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{2x}{n}\gamma q}$
  - $\frac{2x}{n} \geq \gamma q$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\gamma q$

Figure 17: Percentage of students taking an SAT or writing a motivation letter

- Rich students, $1 - \overline{s_i}$:

  - $\frac{x}{n} < \frac{1}{8}$: $\% = 2 * \frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\% = \frac{1}{6} + \frac{2}{3} * \frac{x}{n}$
- Poor students, $\frac{3}{2} - 2\overline{s_i}$:
  - $\frac{x}{n} < \frac{1}{8}$: $\% = 0$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\% = \frac{2}{3} * \frac{x}{n} - \frac{1}{6}$
- Motivation letters: $\% = \sqrt{\frac{x}{n}\gamma q}$

Figure 18: Average ability, average motivation and expected utility

- Average ability university 1:
  - $\frac{x}{n} < \frac{1}{8}$: $\widetilde{a_y} = 1 - \frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
- Average motivation university 1: $\widetilde{m_y} = \frac{1}{2}$
- Expected utility university 1:
  - $\frac{x}{n} < \frac{1}{8}$: $E\left[U^j\right] = \frac{3}{2} - \frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $E\left[U^j\right] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
- Average ability university 2:
  - $\frac{x}{n} < \frac{1}{8}$: $\widetilde{a_y} = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\widetilde{a_y} = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}}$
- Average motivation university 2: $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}$
- Expected utility university 2:
  - $\frac{x}{n} < \frac{1}{8}$: $E\left[U^j\right] = \frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $E\left[U^j\right] = \frac{40*\left(\frac{x}{n}\right)^2 - 76*\frac{x}{n} + 37}{72 - 72*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}$

Figure 19, 20, 21: Expected utilities for the universities per combination of admission programs

- Both SAT:
  - $\frac{2x}{n} < \frac{1}{8}$: $E\left[U^j\right] = \frac{3}{2} - \frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $E\left[U^j\right] = \frac{112*\frac{2x}{n} - 40*\left(\frac{2x}{n}\right)^2 - 1}{72*\frac{2x}{n}}$
  - $\frac{2x}{n} > \frac{7}{8}$: $E\left[U^j\right] = \frac{5*\frac{2x}{n} - 2*\left(\frac{2x}{n}\right)^2 - 1}{2*\frac{2x}{n}}$
- Both motivation letters:
  - $\frac{2x}{n} < q$: $E\left[U^j\right] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{2x}{n}\gamma q}$
  - $\frac{2x}{n} \geq q$: $E\left[U^j\right] = \frac{3}{2} - \frac{1}{2}\gamma q$
- SAT and Motivation letters:
  - $\frac{x}{n} < \frac{1}{8}$: $E\left[U^j\right] = \left(1 - \frac{x}{n} + \frac{1}{2}\right) * \frac{1}{2} + \left(\frac{2*\left(\frac{x}{n}\right)^2 - 2*\frac{x}{n} + 1}{2 - 2*\frac{x}{n}} + 1 - \frac{1}{2}\sqrt{\frac{x}{n}\gamma q}\right) * \frac{1}{2}$

- $\frac{x}{n} \geq \frac{1}{8}$: $E[U^j] = \left(\frac{112*\frac{x}{n}-40*\left(\frac{x}{n}\right)^2-1}{72*\frac{x}{n}}\right)*\frac{1}{2}+\left(\frac{40*\left(\frac{x}{n}\right)^2-76*\frac{x}{n}+37}{72-72*\frac{x}{n}}+1-\frac{1}{2}\sqrt{\frac{x}{n}\gamma q}\right)*\frac{1}{2}$

Figure 22: Minimal signal needed to be admitted

- Low-quality university:
  - $\frac{2x}{n} < \frac{1}{8}$: $\overline{s_i} = 1 - 2*\frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\overline{s_i} = \frac{5}{6} - \frac{2}{3}*\frac{2x}{n}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\overline{s_i} = 2 - 2*\frac{2x}{n}$
- High-quality university:
  - $\frac{x}{n} < \frac{1}{8}$: $\overline{s_i} = 1 - 2*\frac{x}{n}$
  - $\frac{x}{n} \geq \frac{1}{8}$: $\overline{s_i} = \frac{5}{6} - \frac{2}{3}*\frac{x}{n}$

Figure 23: Percentage of students taking an SAT

- Rich students, $a_i \geq \overline{s_i}$
  - $\frac{2x}{n} < \frac{1}{8}$: $\% = 2*\frac{2x}{n}$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{1}{6} + \frac{2}{3}*\frac{2x}{n}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\% = 2*\frac{2x}{n} - 1$
- Poor students, $\frac{1}{2}a_i + \frac{1}{4} \geq \overline{s_i}$
  - $\frac{2x}{n} < \frac{1}{8}$: $\% = 0$
  - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{4}{3}*\frac{2x}{n} - \frac{1}{6}$
  - $\frac{2x}{n} > \frac{7}{8}$: $\% = 1$

Figure 24: Percentage of students applying to the universities

- Low-quality university:
  - Rich students: $\% = \overline{s_{i_2}} - \overline{s_{i_1}}$
    - $\frac{2x}{n} < \frac{1}{8}$: $\% = 2*\frac{x}{n}$
    - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{2}{3}*\frac{x}{n}$
    - $\frac{2x}{n} > \frac{7}{8}$: $\% = \frac{10}{3}*\frac{x}{n} - \frac{7}{6}$
  - Poor students:
    - $\frac{2x}{n} < \frac{1}{8}$: $\% = 0$
    - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{1}{4}$: $\% = \frac{8}{3}*\frac{x}{n} - \frac{5}{3}$
    - $\frac{1}{4} < \frac{2x}{n} \leq \frac{7}{8}$: $\% = \frac{4}{3}*\frac{x}{n}$
    - $\frac{2x}{n} > \frac{7}{8}$: $\% = \frac{7}{6} - \frac{4}{3}*\frac{x}{n}$
- High-quality university:
  - Rich students: $\% = 1 - \overline{s_{i_2}}$
    - $\frac{x}{n} < \frac{1}{8}$: $\% = 2*\frac{x}{n}$
    - $\frac{x}{n} \geq \frac{1}{8}$: $\% = \frac{1}{6} + \frac{2}{3}*\frac{x}{n}$
  - Poor students:
    - $\frac{2x}{n} \leq \frac{1}{4}$: $\% = 0$
    - $\frac{2x}{n} > \frac{1}{4}$: $\% = \frac{4}{3}*\frac{x}{n} - \frac{1}{6}$

Figure 25: Average ability, average motivation and expected utility

- Low-quality university:
  - Average motivation: $\widetilde{m_y} = \frac{1}{2}$
  - Average ability:

        - $\frac{2x}{n} < \frac{1}{8}$: $\widetilde{a_y} = 1 - \frac{3}{2} * \frac{2x}{n}$
        - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{1}{4}$: $\widetilde{a_y} = \frac{80*\frac{x}{n} - 88*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
        - $\frac{1}{4} < \frac{2x}{n} \leq \frac{7}{8}$: $\widetilde{a_y} = \frac{19}{18} - \frac{5}{3} * \frac{x}{n}$
        - $\frac{2x}{n} > \frac{7}{8}$: $\widetilde{a_y} = \frac{212*\frac{x}{n} - 248*\left(\frac{x}{n}\right)^2 - 35}{72*\frac{x}{n}}$
    - Expected utility:
        - $\frac{2x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - \frac{3}{2} * \frac{2x}{n}$
        - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{1}{4}$: $E[U^j] = \frac{116*\frac{x}{n} - 88*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
        - $\frac{1}{4} < \frac{2x}{n} \leq \frac{7}{8}$: $E[U^j] = \frac{14}{9} - \frac{5}{3} * \frac{x}{n}$
        - $\frac{2x}{n} > \frac{7}{8}$: $E[U^j] = \frac{248*\frac{x}{n} - 248*\left(\frac{x}{n}\right)^2 - 35}{72*\frac{x}{n}}$
- High-quality university:
    - Average motivation: $\widetilde{m_y} = \frac{1}{2}$
    - Average ability:
        - $\frac{2x}{n} < \frac{1}{4}$: $\widetilde{a_y} = 1 - \frac{x}{n}$
        - $\frac{2x}{n} \geq \frac{1}{4}$: $\widetilde{a_y} = \frac{76*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
    - Expected utility:
        - $\frac{2x}{n} < \frac{1}{4}$: $E[U^j] = \frac{3}{2} - \frac{x}{n}$
        - $\frac{2x}{n} \geq \frac{1}{4}$: $E[U^j] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$

Figure 26: Percentage of students writing a motivation letter

- $q \leq \frac{x}{n}$: $\% = q$
- $q > \frac{x}{n}$ and $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$: $\% = \frac{1}{2}\gamma q + \frac{1}{2}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$
- $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$: $\% = \frac{1}{2}q + \frac{1}{2}\gamma q$
- $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$: $\% = \sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$

Figure 27: Average ability, average motivation and expected utility

- Average ability: $\widetilde{a_y} = \frac{1}{2}$
- Average motivation:
    - $q \leq \frac{x}{n}$: $\widetilde{m_y} = \frac{1+1-q}{2}$
    - $q > \frac{x}{n}$ and $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$: $\widetilde{m_y} = 1 - \frac{1}{4}\gamma q - \frac{1}{4}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$: $\widetilde{m_y} = 1 - \frac{1}{4}q - \frac{1}{4}\gamma q$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$: $\widetilde{m_y} = 1 - \frac{1}{2}\sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$
- Expected utility:
    - $q \leq \frac{x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}q$
    - $q > \frac{x}{n}$ and $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{4}\gamma q - \frac{1}{4}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{4}q - \frac{1}{4}\gamma q$

- $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$

Figure 28, 29, 30: Expected utilities for the universities per combination of admission programs

- Both SAT:
  - Low-quality university:
    - $\frac{2x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - \frac{3}{2} * \frac{2x}{n}$
    - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{1}{4}$: $E[U^j] = \frac{116*\frac{x}{n} - 88*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
    - $\frac{1}{4} < \frac{2x}{n} \leq \frac{7}{8}$: $E[U^j] = \frac{14}{9} - \frac{5}{3} * \frac{x}{n}$
    - $\frac{2x}{n} > \frac{7}{8}$: $E[U^j] = \frac{248*\frac{x}{n} - 248*\left(\frac{x}{n}\right)^2 - 35}{72*\frac{x}{n}}$
  - High-quality university:
    - $\frac{2x}{n} < \frac{1}{4}$: $E[U^j] = \frac{3}{2} - \frac{x}{n}$
    - $\frac{2x}{n} \geq \frac{1}{4}$: $E[U^j] = \frac{112*\frac{x}{n} - 40*\left(\frac{x}{n}\right)^2 - 1}{72*\frac{x}{n}}$
  - Average expected utility:
    - $\frac{2x}{n} < \frac{1}{8}$: $E[U^j] = \frac{3}{2} - 2 * \frac{x}{n}$
    - $\frac{1}{8} \leq \frac{2x}{n} \leq \frac{7}{8}$: $E[U^j] = \frac{224*\frac{x}{n} - 160*\left(\frac{x}{n}\right)^2 - 1}{144*\frac{x}{n}}$
    - $\frac{2x}{n} > \frac{7}{8}$: $E[U^j] = \frac{10*\frac{x}{n} - 8*\left(\frac{x}{n}\right)^2 - 1}{4*\frac{x}{n}}$
- Both motivation letters:
  - Low-quality university:
    - $(1 - \hat{m}) \leq \frac{x}{n}$: $E[U^j] = 0$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{4}q - \frac{1}{4}\gamma q$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$
  - High-quality university:
    - $q \leq \frac{x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}q$
    - $q > \frac{x}{n}$ and $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{4}\gamma q - \frac{1}{4}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{4}q - \frac{1}{4}\gamma q$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$
  - Average expected utility:
    - $q \leq \frac{x}{n}$: $E[U^j] = \frac{3}{4} - \frac{1}{4}q$
    - $q > \frac{x}{n}$ and $\frac{1}{2}q + \frac{1}{2}\gamma q < \frac{2x}{n}$: $E[U^j] = \frac{3}{4} - \frac{1}{8}\gamma q - \frac{1}{8}\sqrt{(\gamma q)^2 + \frac{4x}{n}q - \frac{4x}{n}\gamma q}$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q = \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{4}q - \frac{1}{4}\gamma q$
    - $\frac{1}{2}q + \frac{1}{2}\gamma q > \frac{2x}{n}$: $E[U^j] = \frac{3}{2} - \frac{1}{2}\sqrt{\frac{x}{n}q + \frac{x}{n}\gamma q}$