# ERASMUS UNIVERSITY ROTTERDAM ERASMUS SCHOOL OF ECONOMICS <br> MSc Economics \& Business <br> Master Specialization Financial Economics 

## Innovation and Transfer of Technology

Decomposing the Benefits of Cost Reducing Innovation

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## ABSTRACT

This paper analyzes interaction of two identical firms in a price reciprocating competition and introduces value of unpredictability in a mixed strategy game. Further, it examines the sensitivity of firm values to different idiosyncratic shocks in preinnovation stage. In the post innovation stage, the study finds that the price and quantity effect of cost reducing innovation are linear in cost reduction and depends only on price elasticities of demand. The study also finds that proprietary benefits of innovation induces switching effect and demand innovation premium. At the end of this paper we suggest a simple license pricing scheme for sharing the benefits of innovation both for complete and partial transfer of technology. In our analysis, shared benefits of nondrastic innovation provide a win-win situation for both firms.

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## 1. INTRODUCTION

There is a consensus between management researchers e.g (Hayes and Garvin 1982) and financial economist e.g (Myers 1984) that traditional valuation techniques are unable to capture the management's ability to adapt to market changes and to cope with technical uncertainty. Managers take strategic or tactical decisions to capitalize on opportunities from ever changing markets and employ necessary safe guards for limiting their losses resulting from adverse market conditions. Valuation techniques based on Real options theory provide a more systematic approach because it takes into account a firm's ability of influencing ex post outcomes.

This study deals with two stage game in which two firms engage in a price reciprocating competition. Smit and Trigeorgis (2004) decomposed the expanded NPV into static NPV, preemption and flexibility value. In absence of dominant strategy, both firms have an incentive to remain unpredictable. Firms use unpredictability as a rational choice to influence the decisions of its rival and therefore, it has an intrinsic value. This study introduces the value of unpredictability to Smit and Trigeorgis (2004) model.

Section 2 is related to literature review which covers selected contributions to the field of real options and provides a theoretical framework for this study. Section 3 of this paper covers the valuation framework employed by Smit and Trigeorgis (2004). However, this paper allows for randomization of moves when none of the firms have a dominant strategy to make or defer investment. This section also provides analytical solutions for calculating equilibrium prices under different market structures.

Section 4 is divided into three different subsections. Section 4.1 introduces base case and analyzes price reciprocating competition between two identical firms. In the sensitivity analysis, one of the firms has been exposed to idiosyncratic shocks in cost, interest rates, uncertainty in market demand and initial demand parameter.

Section 4.2 analyzes the proprietary benefits of innovation. In the proprietary case, one firm makes a successful R\&D investment in cost reduction technology and
gets competitive cost advantage over its rival. Proprietary innovation induces asymmetric changes in prices and sold quantities (or market share) through price and quantity effects of innovation. The changes in market share were then decomposed into switching effect and market demand premium of innovation.

Section 4.3 presents the case of shared benefits of innovation. After charging a fixed license fee, innovator agrees to share innovation benefits through transfer of technology. This section also discusses the role of fixed license fee in complete as well as partial technology transfer. Section 5 concludes the paper.

## 2. LITERATURE REVIEW

Generally all decisions at the firm level involve financial commitments to tap opportunities in uncertain future. Initial research in the area of strategic management documented the role of uncertainty e.g (Cyert and March 1963,Thompson 1967); however, uncertainty was presented as a restrictive factor in limiting firm's potential. Resolution of uncertainty through different options, known as real options, changed the conventional thinking about strategic management. Real option theory dictates that firms can actually benefit from uncertainty just like financial options if the investment is made sequentially e.g (Dixit and Pindyck 1994,Kogut 1991,Kogut and Kulatilaka 2001).

The ground breaking work of Black and Scholes (1973) and Merton(1973) on options pricing paved the way for subsequent research on asset valuation and its applications in other fields including real options. Research in the area of real options started with Myers (1977) seminal paper where he applied option pricing theory and valued investment opportunities as a call option on real assets. Since then, real options theory has been widely used in valuation of strategic investments under uncertainty.

An option represents a choice which can be exercised on the counterparty as a right; however, the exercise of an option is not obligatory. Decision makers prefer a 'certain' outcome over 'uncertain' ones, therefore, firms are willing to buy instruments which give them option to exploit uncertainty in their favor. In general options can be categorized into Put and Call options. Holder of a call option has the right but no obligation to buy and the holder of a put option has the right but no obligation to sell. The 'Right' clause of the option captures all the benefits of upside potential and the 'No obligation' clause makes the losses limited in a downside scenario.

Countless studies have shown that real options provide a systematic framework for valuation of strategic investments in uncertain future. Uncertainty
may provide opportunities as well as threats. Research e.g (Bowman and Hurry 1993,N. Kulatilaka and Kogut 1994,Mcgrath 1997,Smit and Trigeorgis 2004,Trigeorgis 1996) has shown that just like financial options, real options can be used to benefit from upside potential while protecting firms from the hazards of downside risk.

Real options are mainly derived from operational and financial flexibility of a firm (Smit and Trigeorgis 2004). Among the operational flexibility, the most common real options are (i) option to defer, (ii) option to grow, (iii) option to abandon, (iv) option to expand, contract or extend the life of a facility, (v) the option to temporarily shutdown and (vi) the option to switch. In the sphere of financial flexibility, the most common real options are (i) option to default and (ii) option of staged financing. (Baldwin and Trigeorgis 1993,Trigeorgis 1993) provides a list of studies carried out in each category of real options. Reuer and Tong (2007) provides concise discussion on theoretical advances and empirical studies in the area of real options.

Recently, research on real options has witnessed development on two fronts. Firstly, research on valuations techniques now takes into account the competitive environment. (Smit and Ankum 1993) were the first ones to introduce game theory in real options research. They valued option to defer an investment under perfect competition. Smit and Trigeorgis (2004) introduces another dimension by modeling for asymmetric information in a competition. Kulatilaka \& Perotti (1998) and Grenadier (2000) also explored the implications of competition in strategic decisions. The second strand of research focuses on valuation of strategic resources such as R\&D, advertisement campaigns, acquisitions, mergers and diversification e.g. (Bernardo and Chowdhry 2002,Childs and Triantis 1999,Smit and Trigeorgis 2004).

The process of investment in uncertain future carries an inherent risk. The risk is minimized if firms have operational flexibility to resolve these uncertainties by waiting until more information are available. Real option analysis recognizes the
value in waiting. Option to wait derives its value from resolution of market uncertainties (Brennan and Schwartz 1985). Brennan and Swartz used continuous arbitrage and stochastic control theory to value natural resource projects. The model treated stochastic output prices and provided an important insight into embedded value in option to wait. Option to wait provides flexibility to firms to benefit from investments at any stage during the life of the option. Committing an irreversible investment results in loss of flexibility and hence it needs option premium as compensation (Robert Mcdonald and Siegel 1986).

Majd and Pindyck (1987) valued the option of time-to-build in which the life of the option is in essence traded against the preemption value. This study provided a conceptual framework for compound options in which incremental investment creates an option for further investment. The decision to expedite or slowdown the completion of a project depends upon the prevailing market conditions. This type of managerial flexibility in which the production is adjusted to demand were also covered by Brennan and Schwartz (1985) Trigeorgis and Mason(1987).

Mcdonald and Siegel (1985) considers firm as a risk-neutral, price taking value maximizer which is owned by risk-averse investors. This papers models uncertainty in output prices and values the risky investment projects where there is an option to temporarily shutdown the operations. Pindyck (1988) and Majd \& Pindyck (1989) also recognized firms option to alter operating scale in different market conditions.

## 3. VALUATION FRAMEWORK

Valuation of strategic investments in a dynamic environment based on real options and game theoretic framework necessitates explicit recognition of firm's ability to influence ex post outcomes. Smit and Ankum (1993) valued option to defer investment in production facilities similar to a call option on a dividend-paying stock. This was a pioneering study which laid a foundation for use of game theory in quantification of real options (Option to defer/expand etc) under perfect competition. The following sections describe outcomes under various market structures resulting from combination of different competitive strategies of both firms.

### 3.1 Competitive Environment

This paper follows Smit and Trigeorgis (2004) and presents the interaction of two identical firms 'A' and ' B ' under price reciprocating competition. The outcomes of Firm A are influenced not only by its own decisions but also by the strategic responses of its rival (Firm B). Combination of various competitive strategies results in various equilibrium prices under different market structures. If one of the firms, say Firm A, invests in a process e.g. advertisement campaign or R\&D (referred to as strategic investment in this paper), then it may favorably change the market demand ( $\theta$ ) and/or reduce its production cost. This gives a competitive advantage to Firm A over its competitor. Without strategic investment both firms are identical in all respect and this situation is referred to as Base Case. When Firm A makes a strategic investment, then the benefits can either be proprietary or shared. The following paragraphs describe the environment of the price reciprocating competition game;

- Two Firms A and B are competing with each other under price reciprocating competition.
- The market dynamics are represented in a two stage game with initial demand parameter $\theta$. The market demands in stage I and II are uncertain and are
determined by randomized moves of nature with up (u) and down (d) representing good and bad states of the world respectively. The market demand volatility/ uncertainty ( $\sigma$ ) are captured by ' $u$ ' and ' $d$ ' and is given by;

$$
\begin{gathered}
u=e^{E v E} \\
d=\frac{1}{u}=e^{-\sigma v}
\end{gathered}
$$

Where $\sigma$ is the demand uncertainty and $\Delta t$ is the number of jumps in a year.

- In complete markets, the risk-neutral probability $\phi$ can be obtained from the following equation (Smit and Trigeorgis 2004);

$$
\phi=\begin{gathered}
(1+r)-\lambda-d \\
u-d
\end{gathered}
$$

Where $u$ and $d$ represent the moves in the good and bad state of the world, $r$ is the risk-free rate and $\lambda$ the constant asset payout yield and for perpetual project it is equal to $\frac{k}{1+k}$.

- At the beginning, if Firm A makes a strategic investment $\left(\mathrm{K}_{\mathrm{A}}\right)$, the benefits are either proprietary (appropriable to A only) or shared (appropriable to both A and B) through spillover effects. If Firm A does not make strategic investment then both firms are identical.
- In stage I and stage II, both firms A and B can decide on whether to make follow on invest (I) or defer (D) investment in the production phase. If Firm A invests in first stage and Firm B waits until the next period, then Firm A becomes Stackelberg leader. Firm A may even become Monopolist in the second stage if Firm B does not invest in the second stage as well. Figure 1shows the two stage game in extensive form.


## Figure 1: Two Stage Game in Extensive Form ${ }^{1}$



- The decision of one firm in combination with its competitor's response results in the following investment outcomes;

Outcome 1: when both firms invest $(I, I)$, then the market outcomes are known as Nash equilibrium $\left(N_{A}, N_{B}\right)$.

Outcome 2: when Firm A invests and Firm B defers its investment decision until the next stage when the market demand is revealed, then A becomes Stackelberg Leader $\left(\mathrm{L}_{\mathrm{A}}\right)$ and B becomes Stackelberg Follower $\left(F_{B}\right)$.

Outcome 3: when Firm B invests and Firm A defers its investment decision until the next stage until the market demand is revealed, then $B$

[^1]becomes Stackelberg Leader $\left(L_{B}\right)$ and A becomes Stackelberg Follower $\left(F_{A}\right)$.

Outcome 4: when both firms defers (D,D), then the nature moves again and the game is repeated. In duopoly competitions, in order to meet the demands of the market, both firms can not stay out of the market, therefore, the game does not end here and is repeated.

- Value of the firm is determined by using the binomial valuation over risk neutral probabilities and risk free rate. The continuation value of the firm is calculated by using same cost of capital for both firms which is a strong assumption. However, in sensitivity analysis, one firm has been exposed to idiosyncratic shocks in cost of capital.
- The main objective of each firm is to maximize its value. However, the value of one firm depends not only on its own moves but also on the strategic moves of its competitor. To capture the competitive dynamics, game theoretic approach is embedded into the valuation process. In each state of the world, equilibrium can be reached through pure or mixed strategies.


## Firm B



Firm A
Defer $\quad \mathrm{F}_{\mathrm{A}}, L_{B} \quad \mathrm{D}_{\mathrm{A}}, D_{A}$

Where N, F, L and D refers to Nash, Leader, Follower and Deferment

- Firm A has dominant strategy to invest if $N_{A}>F_{A}$ and $L_{A}>D_{A}$ and dominant strategy to defer if $N_{A}<E_{A}$ and $L_{A}<D_{D}$.
- Similarly Firm B has dominant strategy to invest if $N_{Q}>F_{B}$ and $L_{B}>D_{B}$ and dominant strategy to defer if $N_{E} \subset F_{S}$ and $E_{E}<D_{E}$.
- In a situation when both firms do not have any dominant strategy or where firms have incentive to deviate for each strategy of its competitor, then the game ends in a mixed equilibrium. In a mixed strategy game, each firm maximizes their payoffs regardless of the moves of its competitor. In this setup, Firm A tries to optimize its payoffs on the probability distribution of moves of Firm B. Suppose that Firm A and B makes investment with probability ' $p$ ' and ' $q$ ' respectively. Equivalently, Firm A and B defers investment with probability ' $1-p$ ' and ' $1-q$ ' respectively.


## Firm B

|  |  |  | Invest | Defer |
| :---: | :---: | :---: | :---: | :---: |
| Firm A | Probability | $q$ | $1-q$ |  |
| Invest | p | $\mathrm{N}_{\mathrm{A}}, N_{B}$ | $\mathrm{~L}_{\mathrm{A}}, F_{B}$ |  |
|  |  |  |  |  |
| Defer | $1-\mathrm{p}$ | $\mathrm{F}_{\mathrm{A}}, L_{B}$ | $\mathrm{D}_{\mathrm{A}}, D_{A}$ |  |

- If Firm A invests, then its payoff is $q N_{A}+(1-q) E_{A}$. If it defers investment, then the payoff is $q F_{A}+(1-q) D_{A}$. Firm A will be indifferent between making investment or deferring its investment if;

$$
q N_{A}+(1-q) L_{A}=q E_{A}+(1-q) D_{A}
$$

Solving for ' $q$ ' results in

$$
\begin{equation*}
q=\frac{D_{A}-L_{A}}{\left(N_{A}-E_{E}\right)-\left(D_{Z}-E_{A}\right)} \tag{3-1}
\end{equation*}
$$

Similarly,

$$
F=\begin{gather*}
D_{E}-L_{E}  \tag{3-2}\\
\left(N_{B}-F_{B}\right)-\left(D_{2}-L_{B}\right)
\end{gather*}
$$

p and q are the probabilities of making investment by Firm A and B. In this paper, p and q will be referred to as Firm A's and B's inclination for investment.

- To standardize the use of notion of unpredictability, following criteria will be used;

| Term | Probability Range for $p$ and $q$ | Firm Preference |
| :---: | :---: | :---: |
| Highly Unpredictable | [0.50, 0.65] | Investment |
|  | [0.35, 0.49] | Deferment |
| Moderately | [0.66, 0.80] | Investment |
| Unpredictable | [0.20, 0.34] | Deferment |
| Less Unpredictable | [0.81, 0.95] | Investment |
|  | [0.05, 0.19] | Deferment |
| (Almost) Predictable | > 0.95 | Investment |
|  | < 0.05 | Deferment |

### 3.2 Outcomes under Different Market Structures

Consider two firms ' A ' and ' B ' which are making homogenous products. The basis of competition is such that firms set their prices and consumer decides about its demand. The demand of a firm's product is influenced not only by it own price setting but also by the price of its competitor. Price has an inverse relationship with demand. If a firm keeps the price high, then its demand goes down and it looses its market share to competitor. If one firm reduces the price then its competitor reciprocates by reducing its price to a level where the price equals to the marginal cost. This kind of price competition hurts both the firms. By increasing the prices, the profit margin increases but the demand goes down which makes this strategy unsustainable in long run. However, lowering prices induces reciprocating reaction from competitors. Therefore, for a sustainable profit streams, management face a daunting challenge of striking a fine balance between the price and its market share.

The detailed analytical proofs of different outcomes in this section are given in Smit and Trigeorgis (2004). Suppose that in addition to producing homogenous products, the demand for goods is linear in prices.

$$
\begin{equation*}
Q_{t}=\theta_{t, t}-b P_{t}+d P_{t} \tag{3-3}
\end{equation*}
$$

Equation (3-3) shows that the demand for Firm A's product decreases with increase in its own price and increases with increase in competitor's price. The subscripts $i$ and $j$ refers to Firm A and B respectively. $e_{i, k}$ is the demand shift parameter for Firm A at time $t$ which is stochastic and captures the market uncertainty. The coefficient ' $b$ ' measures the responsiveness of A's demand to its own price changes. Similarly, ' $d$ ' measures the reaction of A's demand to changes in its competitor prices. Both firms have quadratic cost function which is by;

$$
\begin{equation*}
c\left(Q_{i}\right)=c_{1} Q_{i}+\frac{1}{2} q_{i} Q_{?}^{?} \tag{3-4}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial C\left(Q_{i}\right)}{\partial Q_{i}}=\varepsilon_{i}+q_{i} Q_{i} \tag{3-5}
\end{equation*}
$$

Equation (3-5) shows the marginal cost function. $c_{i}$ and $q_{i}$ represent the fixed and variable components of the total cost that contributes to one additional unit of production. For the given cost structure and price setting, the profit of each firm is given by;

$$
\begin{aligned}
& \pi_{i}=P_{i} Q_{i}-\Omega\left(Q_{i}\right) \\
& \pi_{i}=\left(P_{i}-q_{i}\right) Q_{i}-\frac{1}{2} q_{i} q_{i}
\end{aligned}
$$

For the sake of simplicity, I assume that variable component does not contribute to total cost i.e. $\mathfrak{q}_{4}=0$. The above equation becomes as follows;

$$
\begin{equation*}
\pi_{i}=\left(P_{i}-c_{i}\right)\left(\theta_{i t}-b P_{i}+d P_{i}\right) \tag{3-6}
\end{equation*}
$$

Each firm plays this duopoly game in a way that it will maximize their profits. In a price competition, firms set its own price and the consumers decide about the demand in a competitive market. The profit function is not only influenced by its own prices but also by competitive reaction of its rival and the market uncertainty. Let $P_{i}$ is the price which maximizes the profit of Firm A. The optimal price response $P_{i}$ is obtained by differentiating equation (3-6) w.r.t. to $P_{i}$;

$$
\frac{\partial \pi_{i}}{\partial R_{i}}=\left(R_{i}-s_{i}\right)(-b)+\left(\theta_{i t}-b R_{i}+d R_{i}\right)
$$

$$
\frac{\partial \pi_{i}}{\partial R_{i}}=-b R_{i}+b c_{i}+\theta_{i t}-b R_{i}+d P_{i}
$$

$$
\frac{\partial \pi_{t}}{\partial R_{i}}=-2 b F_{i}+b c_{i}+\theta_{i, k}+d P_{i}
$$

Putting $\frac{\theta_{B_{i}}}{\partial B_{i}}=0$ to arrive at price $P_{i}$ which maximizes $\pi_{i}$;

$$
\begin{align*}
& \frac{\partial \pi_{i}}{\partial R_{i}}=-2 b P_{i}+b c_{i}+\theta_{i, t}+d P_{i}=0 \\
& P_{i}=\frac{\theta_{i, t}+d P_{i}+b c_{i}}{2 b} \\
& E_{i}\left(P_{i}\right)=P_{i}=\frac{\theta_{i k}+d P_{i}+b c_{i}}{2 b} \quad \ldots \ldots \ldots \ldots \tag{3-7}
\end{align*}
$$

Similarly the optimal price reaction of the competitors is;

$$
\begin{equation*}
A_{f}\left(P_{i}\right)=P_{f}=\frac{\theta_{S z}+d P_{i}+b c_{f}}{2 b} \tag{3-8}
\end{equation*}
$$

Where $R_{t}\left(P_{f}\right)$ is the most profitable price reaction of firm (Firm A) when the rival (Firm B) sets its price at $P_{j}$. When both firms invest simultaneously (I,I), the game
ends in Nash equilibrium. In Nash equilibrium, each firm plays its best move and none of the firms benefits from deviation. Substituting the optimal price of the competitor $P_{j}$ from equation (3-8) into equation (3-7), we get the Nash equilibrium price $\boldsymbol{P}_{i}^{P}$,

$$
P_{i}^{N V}=\frac{1}{2 b}\left\{\theta_{i, t}+\frac{d}{2 b}\left(\theta_{i, t}+d P_{i}^{N}+b c_{i}\right)+b c_{i}\right\}
$$

$$
2 b E_{i}^{N}=\theta_{i t}+\frac{d}{2 b}\left(\theta_{i k}+b c_{i}\right)+\frac{d}{2 b} d F_{i}^{N}+b c_{i}
$$

$$
2 b F_{i}^{N}-\frac{\sigma^{2}}{2 b} F_{i}^{N}=\theta_{i t z}+b c_{i}+\frac{\sigma}{2 b}\left(\theta_{i t}+b c_{i}\right)
$$

$$
\left(\frac{4 b^{2}-d^{2}}{2 b}\right) P_{i}^{N}=\left(\theta_{i k}+b c_{i}\right)+\frac{d}{2 b}\left(\theta_{V i}+b c_{j}\right)
$$

$$
\begin{equation*}
F_{i}^{N}=\frac{2 b\left(e_{i, t}+t c_{i}\right)+d\left(e_{i t}+t c_{i}\right)}{4 b^{2}-d^{2}} \tag{3-9}
\end{equation*}
$$

If both firms are identical in terms of cost of production, demand elasticities and market uncertainty then the Nash equilibrium prices reduces to;

Putting $\theta_{i, t}=\theta_{f, t}=\theta_{v}$ and $c_{i}=c_{i}=c$ in equation (7), we get;

$$
F^{N}=\frac{2 b\left(\theta_{i}+b c\right)+a^{I}\left(\theta_{z}+b c\right)}{4 b^{2}-a^{2}}
$$

$$
\begin{align*}
& F^{N}=\frac{\left(2 b+d^{2}\right)\left(\theta_{8}+b c\right)}{\left(2 b+d^{\prime}\right)\left(2 b-d^{d}\right)} \\
& F^{N}=\frac{\left(\theta_{8}+b c\right)}{\left(2 b-d^{d}\right)} \ldots \ldots \ldots \tag{3-10}
\end{align*}
$$

When Firm A invests in the first period and Firm B waits until the next period, then Firm A becomes a Stackelberg leader $\left(S^{L}\right)$ and Firm B becomes Stackelberg follower $S^{F}$. Firm A assumes the leading role and sets a price which maximizes its own profit. However, this choice is influenced by follower's price. Substituting the value of $\Gamma_{j}$ from equation (3-8) into equation (3-6), we get;

$$
\begin{aligned}
& \pi_{i}=\left(P_{i}-c_{i}\right)\left\{e_{i t}+\frac{d}{2 b}\left(\theta_{s i}+d P_{i}+b c_{i}\right)+b c_{i}\right\} \\
& \frac{\partial \pi_{i}}{\partial P_{i}}=\left(P_{t}^{2}-c_{i}\right)\left(-b+\frac{d^{2}}{2 h}\right)+\theta_{i, t}-b P_{i}^{2}+\frac{d}{2 h}\left(\theta_{i t t}+d_{i}^{2} P_{i}^{2}+b c_{j}\right)=0
\end{aligned}
$$

To find Stackelberg leader price $P^{2}$, we put $\frac{\partial \overrightarrow{i n t}}{\theta D_{t}}=0$;

$$
\left(F_{t}^{k}-c_{t}\right)\left(-b+\frac{d^{2}}{2 b}\right)+a_{t, z}-b F_{t}^{k}+\frac{d}{2 b}\left(a_{k, z}+d F_{t}^{k}+b c_{j}\right)=0
$$

$$
\begin{gather*}
\left(P_{i}^{2}-c_{i}\right)\left(\frac{d^{2}-2 b^{2}}{2 b}\right)+\theta_{i, t}-b P_{i}^{2}+\frac{d}{2 b}\left(\theta_{i, t}+d P_{i}^{2}+b c_{i}\right)=0 \\
E_{i}^{2}\left(\frac{d^{2}-2 b^{2}-2 b^{2}+d^{2}}{2 b}\right)-\left\{\frac{c_{i}\left(d^{2}-2 b^{2}\right)-2 b \theta_{i, t}-d\left(\theta_{i t}+b c_{i}\right)}{2 b}\right\}=0 \\
E_{i}^{2}\left(2 d^{2}-4 b^{2}\right)-c_{i}\left(d^{2}-2 b^{2}\right)-2 b \theta_{i t}-d^{I}\left(\theta_{h, t}+b c_{i}\right)=0 \\
P_{i}^{2}=\frac{c_{i} d^{2}-2 b^{2} c_{i}-2 b \theta_{i, t}-d\left(\theta_{j, t}+b c_{j}\right)}{2 d^{2}-4 b^{2}} \\
E_{t}^{2}=\frac{2 b\left(\theta_{i, t}+b c_{i}\right)-d^{I}\left(\theta_{i t}+b c_{j}-d c_{i}\right)}{4 b^{2}-2 d^{2}} \ldots \ldots \ldots \ldots . . \tag{3-12}
\end{gather*}
$$

Stackelberg leader may fetch a higher price than Nash equilibrium price (Smit and Trigeorgis 2004). The profit function of the leader in the first stage not only takes into account the $\boldsymbol{P}_{\boldsymbol{t}}^{\boldsymbol{2}}$ but also captures the first stage monopoly profits. If the competing firm invests in the second stage, then Firm A stays as a leader but if Firm B does not invest even in the second stage, then Firm A becomes monopolist. A monopolist firm will maximize its profits over price $P_{m}$ using $Q_{m}=Q_{A}+Q_{\mathrm{E}}$. In monopolist setting $\theta_{t, t}=\theta_{i, t}=\theta_{i v}, c_{t}=c_{f}=c$ and $p_{i}=p_{i}=p_{m}$. Equation (3-4) becomes;

$$
Q_{m}=2\left\{\theta_{r}-P_{m n}(b-d!\}\right.
$$

Profit function in equation (3-6) implies;

$$
\pi_{i}=2\left(P_{m}-c\right)\left\{\theta_{t}-P_{m}(b-d)\right\}
$$

To get the price at which the monopolist firm maximizes the profits, put $\frac{\partial \sqrt{x}}{\partial P_{92}}=0$

$$
\begin{aligned}
& \frac{\partial \pi_{i}}{\partial P_{m}}=-\left(F_{m}-c\right)(b-d)+\theta_{2}-E_{m}(b-d)=0 \\
& \Rightarrow\left(F_{m}-c\right)(b-d)=\theta_{t}-P_{m}(b-d) \\
& \Rightarrow 2 P_{m}(b-d)=\theta_{t}+c(b-d) \\
& \Rightarrow E_{m}=\frac{\theta_{v}+c(b-d)}{2(b-d)}
\end{aligned}
$$

## 4. RESULTS

### 4.1 Base Case with no Innovation

In base case, we assume that both firms are identical in all respects. In the base case $\theta_{i, t}=\theta_{\gamma, t}=10, u=1.25, b=2, d=1, c_{i}=c_{y}=1$, Rtsk free rate $=10 \%$, cost of capital of each firm $k_{a}=k_{z}=13 \%$ and the follow up investment is I=100. Price reaction of the firm is given by equation (3-7);

$$
\begin{gathered}
E_{i}=\frac{\theta_{i}+d P_{i}+b g_{i}}{2 b} \\
\frac{\partial E_{i}}{\partial R_{i}}=\frac{\partial E_{i}}{\partial F_{i}}=\frac{d}{2 b}=\theta
\end{gathered}
$$

The above result is quite significant in interpretation of results. It shows that the price of a firm reacts positively to any changes in competitor's price. Firm increases (decrease) its price in reaction to increase (decrease) in its rival price. The reciprocating reaction depends on sensitivities of demand to its own price $b$ and to its competitor's price $d$.

$$
\begin{align*}
& Q_{i}=\theta_{i}-b P_{i}+d P_{i} \\
& \frac{\partial Q_{i}}{\partial \varepsilon_{i}}=-\frac{\partial P_{i}}{\partial \varepsilon_{i}}+d \frac{\partial P_{i}}{\partial s_{i}} \tag{4-1}
\end{align*}
$$

and

$$
\begin{equation*}
\frac{\partial Q_{i}}{\partial c_{i}}=-b \frac{\partial R_{i}}{\partial c_{i}}+d \frac{\partial R_{i}}{\partial c_{i}} \tag{4-2}
\end{equation*}
$$

From Firm B's reaction function we know that;

$$
\begin{gathered}
E_{z}=\frac{\theta_{i t}+d F_{i}+b c_{j}}{2 t} \\
\frac{\partial E_{i}}{\partial g_{i}}=\frac{d}{2 b}\left(\frac{\theta P_{i}}{\partial g_{i}}\right)
\end{gathered}
$$

Similarly from equation (3-8), we have;

$$
\frac{\partial P_{i}}{\partial c_{i}}=\frac{d}{2 \phi}\left(\frac{\partial P_{i}}{\partial c_{i}}\right)+\frac{1}{2}
$$

$\Rightarrow \frac{\partial E_{i}}{\partial s_{i}}=\frac{d^{2}}{4 b^{2}}\left(\frac{\partial E_{i}}{\partial s_{t}}\right)+\frac{1}{2}$

$$
\begin{aligned}
& \Rightarrow \frac{\partial P_{t}}{\partial s_{t}}\left(\frac{4 b^{2}-d^{2}}{4 b^{2}}\right)=\frac{1}{2} \\
& \Rightarrow \frac{\partial P_{i}}{\partial c_{t}}=\frac{2 b^{2}}{4 b^{2}-d^{2}}=0 \\
& \text { and } \\
& \frac{\partial P_{i}}{\partial c_{t}}=\frac{b d}{4 b^{2}-d^{2}}=0
\end{aligned}
$$

Substituting values of $\frac{\partial \theta_{4}}{\partial_{4}}$ and $\frac{\partial B_{i}}{\partial_{\mathrm{q}}}$ into equation (4-1) and (4-2), we get;

$$
\frac{\partial Q_{i}}{\partial g_{t}}=\frac{b\left(d^{2}-2 b^{2}\right)}{4 b^{2}-d^{2}} \quad \text { Q for }\left(b_{i} d^{2}\right)=(2,1)
$$

$\frac{\partial q_{i}}{\partial q_{t}}=\frac{b^{2} d}{4 b^{2}-d^{2}}: \theta$ for $\left(b, d_{1}\right)=(2 ; 1)$

The marginal effects of proprietary cost reduction on price and demand depends on competitors response coefficient $b$ and $d$. In this game we assume $b=2$ and $d=1$. The price and demand sensitivities to production cost are as follows;

$$
\begin{aligned}
& \frac{\partial p_{i}}{\partial q_{i}}=\frac{3}{15}=53.33 \% \\
& \frac{\partial Q_{i}}{\partial v_{i}}=-\frac{14}{15}=-93.33 \% \\
& \frac{\partial p_{i}}{\partial s_{t}}=\frac{2}{15}=13.33 \%
\end{aligned}
$$

$$
\frac{\partial Q_{i}}{\partial \varphi_{i}}=\frac{4}{15}=26.67 \%
$$

Due to technological innovation, if the cost of production of Firm A is decreased by $20 \%$, then Firm A will reduce its price by $10.7 \%$ ( $53.33 \% \times-20 \%$ ), and its demand will increase by $18.7 \%(-93.33 \% \times-20 \%)$. However, since the analysis assumes no spill over effects i.e. the innovation is kept proprietary, in response to decrease in price by Firm A, Firm B will decrease its price by $3.7 \%$ ( $13.33 \% \times 20 \%$ ). After this innovation takes place, Firm A will start selling its products at much lower prices than Firm B. As a result, Firm B demand will go down by $5.3 \%$ ( $26.33 \% \times-20 \%$ ) despite of reduction in price.

In the base case scenario when demand is favorable in the first stage, none of the firms has dominant strategy and the game results in a mixed equilibrium. End of the first period payoffs of each firm are as follows;

## Firm B

|  |  | Invest | Defer |
| :---: | :---: | :---: | :---: |
| Firm A | Invest | $(126,126)$ | $(170,127)$ |
|  | Defer | $(127,170)$ | $(120,120)$ |

Figure 2 shows that in base case firms prefer to randomize their moves because none of the firms has a dominant strategy. The moves of firms are (almost) predictable and favor investment i.e. $(p, q)=(0,97,0.97)$. $p$ and $q$ are the probabilities of investment of Firm A and B. If A invests in the fist stage, then its payoff will be $126 \mathrm{q} / 170(1) q)$ and in case $A$ defers investment then its payoff will be $127 q+120(1-q)$. A will be indifferent in making or deferring investment if $126 q+170(1-q)=127 q+120(1-q) \Rightarrow q=0.97$, similarly, Firm $B$ will be indifferent if $\quad \mathrm{p}=0.97$.

Figure 2: Base Case Competitive Strategies under Different Market Structures


In the good state, mixed equilibrium price is $(127,127)^{2}$.

Similarly, none of the firms has a dominant strategy when demand develops unfavorably and hence the game is played with mixed strategy. In the bad state, both firms are highly unpredictable but somehow showing an inclination for deferring investment. The mixed equilibrium prices are $(15,15)$ as result of $(p, q)=(0.37,0,37)$. Firm value of $(56,56)^{3}$ is obtained by discounting at risk-free rate the first period values in good and bad state over risk-neutral probabilities $\phi$ and $\mathbb{I}-\phi$. Since the model in this paper allows for mixed strategy, the expanded net present value is decomposed of (i) Static NPV, (ii) Strategic Preemption, (iii) Flexibility and (iv) Unpredictability value from investment/waiting. If in a mixed equilibrium, $p<0.50$, then Firm $A$ is unpredictable but favors waiting and hence the unpredictability value comes from waiting.

Table 1 shows base case outcomes for different initial demand parameters. In the base case we assume that both firms are identical in all respect. $A_{i}$ refers to initial demand parameter and $\theta_{M}$ and $\theta_{i}$ show good and bad realization of market demand. $P_{i}$; $Q_{i}, \pi_{i}$ and NPV show price, quantity sold, profit and net present value of Firm A respectively. The firm value (NPV) when it is a Stackelberg leader is higher than the Nash value i.e. $39.08>38.46$. First period monopoly profits also contribute partly to the value of the leading firm. The firm gets maximum benefit when its rival never enters the market. The firm value in this case is much higher than what can be achieved under other market structures.

The embedded options in the investment decision process mainly depend on all those parameters which are relevant in pricing of financial options using Black-Scholes option pricing model. Table 2 shows components of firm value under different market structures. In the base case firm does not have preemption value and prefers to stay

[^2]${ }^{3} 56-\frac{0.45: 5+(6-0.4152}{14}$, where 0.41 and 0.59 are the probabilities in good and bad state of nature
flexible and unpredictable. However, in the post innovation stage, if benefits are kept proprietary, firm preempts competitors but stays unpredictable. When the benefits are shared, then it compromises a major portion in preemption.

Table 1: Base case outcomes under different market structures and initial demand parameter for reciprocating price competition

| $\theta$ |  | Nash |  |  |  | Stackelberg Leader |  |  |  | Monopolist |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $F_{i}$ | $Q:$ | $\pi_{1}$ | NPV | $P_{i}$ | $Q$ | $\pi_{i}$ | NPV | $P_{i}$ | $Q$ | $\pi_{i}$ | NFV |
| 8 | $\theta_{u}$ | 4.00 | 6.00 | 18.00 | 38.46 | 4.21 | 5.63 | 18.08 | 39.08 | 5.50 | 9.00 | 40.50 | 211.54 |
|  | $e_{6}$ | 2.80 | 3.60 | 6.48 | -50.15 | 2.93 | 3.38 | 6.51 | -49.93 | 3.70 | 5.40 | 14.58 | 12.15 |
| 9 | $\theta_{4}$ | 4.42 | 6.83 | 23.35 | 79.59 | 4.66 | 6.41 | 23.45 | 80.40 | 6.13 | 10.25 | 52.53 | 304.09 |
|  | $\hat{\theta}_{a}$ | 3.07 | 4.13 | 8.54 | -34.29 | 3.21 | 3.88 | 8.58 | -34.00 | 4.10 | 6.20 | 19.22 | 47.85 |
| 10 | $\theta_{u}$ | 4.83 | 7.67 | 29.39 | 126.07 | 5.11 | 7.19 | 29.52 | 127.08 | 6.75 | 11.50 | 66.13 | 408.65 |
|  | $\theta_{a}$ | 3.33 | 4.67 | 10.89 | -16.24 | 3.50 | 4.38 | 10.94 | -15.87 | 4.50 | 7.00 | 24.50 | 88.46 |
| 11 | $\theta_{u}$ | 5.25 | 8.50 | 36.13 | 177.88 | 5.55 | 7.97 | 36.29 | 179.13 | 7.38 | 12.75 | 81.28 | 525.24 |
|  | $\theta$ | 3.60 | 5.20 | 13.52 | 4.00 | 3.79 | 4.88 | 13.58 | 4.46 | 4.90 | 7.80 | 30.42 | 134.00 |
| 12 | $\theta_{4}$ | 5.67 | 9.33 | 43.56 | 235.04 | 6.00 | 8.75 | 43.75 | 236.54 | 8.00 | 14.00 | 98.00 | 653.85 |
|  | $\theta_{d}$ | 3.87 | 5.73 | 16.44 | 26.43 | 4.07 | 5.38 | 16.51 | 26.99 | 5.30 | 8.60 | 36.98 | 184.46 |

Table 2: Value Components under Different Market Structures

| Value Component | Base Case | Proprietary Case | Shared Case |
| :---: | :---: | :---: | :---: |
| Static NPV | 38 | 91 | 109 |
| Strategic Preemption | 0 | 28 | 6 |
| Flexibility | 11 | 0 | 0 |
| Unpredictability | 7 | 15 | 1 |
| Total NPV | 56 | 134 | 116 |
| In each case following parameters were used |  |  |  |
| $\theta_{i}$ | 10 | 12 | 12 |
| $\theta_{i}$ | 10 | 9 | 12 |


| $\bar{\epsilon}_{i}$ | 0 | 0.2 | 0.2 |
| :--- | :--- | :---: | :--- |
| $\bar{\varepsilon}_{j}$ | 0 | 0 | 0.2 |

The most important variables to which the option value is very sensitive are interest rates and market uncertainty. Besides these variables, the firm value is also very vulnerable to asymmetric cost structures and idiosyncratic shocks in demand. The following paragraphs explain sensitivity of firm's value to changes in different input parameters of the model. Figure 3 shows the sensitivity of firms' value to incremental changes in cost of production of one firm under price reciprocating competition. In this sensitivity analysis, the cost of Firm B has been fixed at $c_{j}=1$ and cost of Firm A was exposed to idiosyncratic shocks. The expanded NPV (represented by NPV_A and NPV_B) captures values in both stages as well as the continuation value of the firm. During continuation period, it is assumed that the firm will follow a static path i.e. nothing will change after the second period. Firm A has a competitive cost advantage if $c_{k}<c_{f}$ and competitive cost disadvantage if $c_{i}>c_{j}$.

Figure 3: Firm's Sensitivity to Idiosyncratic Shocks to Cost of Production


One interesting observation is that the firm with a competitive cost advantage gains an immediate benefit of 19.98 when the cost is reduced from 0.8 to 0.7 . On the other hand, Firm B gets a major benefit of 14.57 if its rival Firm A increase its cost from 1.5 to 1.6 . The logical conclusion is firm's own efficiency pays more than inefficiency of its rival. Therefore, the benefits arising from cost cuttings through innovations are larger than the benefits which are derived from rival's fat costs/inefficiencies.

Table 3 shows tradeoff between strategic preemption and flexibility value of a firm. The preemption value of a firm jumps from 0 to 23 when the cost of production is reduced from 0.80 to 0.70 .

Table 3: Effect of decrease in Cost of Production on Value Components of Firm

| Value Component | $\varepsilon=0$ | - $=0.1$ | $\mathbf{c}^{=}=0,2$ | $c=0,3$ | ビ=0,4 | $\mathrm{t}^{=}=0,3$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Static NPV | 38 | 42 | 46 | 50 | 56 | 58 |
| Strategic | 0 | 0 | 0 | 23 | 23 | 23 |
| Preemption |  |  |  |  |  |  |
| Flexibility | 11 | 9 | 9 | 0 | 0 | 0 |
| Unpredictability | 7 | 7 | 6 | 8 | 7 | 6 |
| Total NPV | 56 | 58 | 61 | 81 | 85 | 88 |

Figure 4 shows that in base case if demand is favorable, rival Firm B exits the market in the first stage for $c_{t} \leq \llbracket .70$. If demand remains favorable in the second stage, then Firm B enters again (Annexure 1). In case of unfavorable demand realization, both firms do not have a dominant strategy and they prefer to randomize their moves. In bad state, both firms are highly unpredictable i.e. $(p, q) \in[0.35,0.65]$ for $c_{t} \leq 1$. If Firm A has a competitive cost disadvantage, then it slowly and steadily loses market to its rival. For $c_{i} \geq 1.60$ i.e. producing at $60 \%$ higher cost, it is no more feasible for Firm A to stay in the market. If the market turn out favorable in the second stage, then Firm A enters again, but if the market remains sluggish, then Firm A stays out.

Figure 4: Probability of Investment of Each Firm in Good State under Base Case


Cost of capital serves as a consistent discount rate which accounts for the risks arising from the capital structure of the firm. It combines the risk of all investors in one rate which can be used as a discount factor in firm valuation. In the base case we use one cost of capital in our analysis which assumes that the cash flows of both firms bear same risk. In reality, even firms active in the same industry have different risk profile. Figure 5 shows the competitive response of both firms when their relative risk profile is changing. The cost of capital of Firm B is kept constant i.e. $k_{2}=13 \%$ whereas $k_{a}$ is given an idiosyncratic shock. By keeping $k_{h}$ constant, the difference between firm values due to changes in $k_{a}$ captures only the idiosyncratic risk. The figure shows that Firm A's value is relatively more sensitive to changes in $k_{a}$. The rival Firm B does not react strongly to changes in $k_{a}$ because the shock is firm specific.

Figure 5: Sensitivity of firms' value to idiosyncratic changes in interest rates (Firm B interest rate fixed at 13\% and Firm A experience idiosyncratic changes in interest rates)


Figure 6 shows the sensitivities of both firms to changes in the initial demand parameter $\theta_{i}$ of Firm A. In the whole analysis the demand of Firm B is fixed at $\theta_{S}=10$. For initial demand of $\theta_{t}=8.5$ if the market develops unfavorably then Firm A exits the market in the first stage. The rival Firm B benefits from this development and gains a major benefit. This shows that it is hard for smaller firms (starting with a comparatively $20 \%$ lower demand than its competitor) to survive if the market enters into a bad state i.e. $\ominus$ - duwn. The probabilities of investment are given in Annexure 2. On the other hand, if the initial demand for Firm A is $\theta_{d}-11$ as compared to Firm B demand $\theta_{f}=10$ i.e. $10 \%$ higher, then if the market develops favorably, the bigger Firm A preempts Firm B and influence it to quit the market.

Figure 6: Effect of Varying Initial Demand Parameter (theta) on Firm Value


Figure 7 shows the effect of idiosyncratic volatility (volatility of Firm B kept constant at $\mathrm{u}=1.25$, and volatility of Firm A is allowed to change) on value of firms. Options become more valuable with increasing volatility, therefore, value of Firm A experience a direct increase with increase in its volatility and the rival Firm B (whose volatility is fixed) follows the trend in reciprocation.

Figure 7: Effect of Idiosyncratic Demand Volatility on Firms Value


Figure 8 shows the effect of idiosyncratic volatility on prices and compares it with systemic shocks to volatility under Nash equilibrium. Panel A and Panel B illustrate this phenomenon when market moves up and down respectively. When market moves up, firms A's prices monotonically increase with increase in volatility (both idiosyncratic and systemic). On the other hand, when market turns out to be unfavorable, Firm A reduces its prices with increase in volatility (both idiosyncratic and systemic). If Firm A is less volatile than its competitor i.e. $\Phi_{i} x_{j} \sigma_{j}$, then in the good state, Firm A charges a higher price in case of idiosyncratic volatility than the price if the shock would have been systemic. If Firm A is comparatively more volatile i.e. $\sigma_{i} \geqslant \sigma_{f}$, then it charges a lower price in case of idiosyncratic shock as compared to systemic shock.

Panel B shows that the whole trend reverses if the market moves down. Regardless of the nature of shock, Firm A decreases its prices with increase in volatility. However, if $\sigma_{f} x_{f} \sigma_{f}$, i.e. Firm A is less volatile then it charges a comparatively lower price than its competitor and for $\sigma_{i}>\sigma_{i}$, it charges a higher price. Similar contrasting patterns can be witnessed in quantities as well (Annexure 3).

Figure 8: Effect of Idiosyncratic Volatility on Prices and Quantities Sold of Firm A when Market Moves Up and Down under Nash Outcomes


Panel B: When Market Moves Down


### 4.2 Proprietary Benefits of Innovation

In proprietary case, we consider a situation in which Firm A makes a strategic Research and Development (R\&D) investment in a cost reduction technology. Figure 9 shows outcomes of proprietary innovation under different market structures. If the investment is successful, then it can result in non-drastic as well as drastic innovations. The innovation is termed as non-drastic if the rival firm continues to compete even after the innovation benefits are kept proprietary. In case of drastic innovations, the rival firm can no more profitably compete with the innovator. The rival firm either quits the market or pays a license fee to stay profitable in the market and shares the benefits of innovation. In proprietary case, the study takes following input parameters ${ }^{4}$;

```
0}=1
0}=
nast recluntime= }=0=0.
c
c}=
k}=\mp@subsup{k}{C}{}=13
risk free rate =r'= 10%
u=1.25
```

In the base case, we considered that both firms are identical and assumed that both firms have symmetric cost $\varepsilon_{i}=q_{j}$ where $c_{i}$ and $c_{j}$ are the per unit cost of production. For simplicity, we assume that the variable component of cost does not contribute to overall cost of production. Let us assume that R\&D investment results in a success innovation which reduces the cost of production of Firm $\mathrm{A} \in$ such that

[^3]Figure 9: Proprietary Case Competitive Strategies under Different Market Structures

$0<\epsilon<c_{i}$. If the benefits of innovation are kept proprietary, the cost of production of Firm A reduces from $c_{i}$ to $c_{i}-\epsilon$ and the cost of production of Firm B remains at $c_{j}$. It gives a competitive advantage to innovator on its rival. The innovations turns out to be drastic if $\varepsilon$ is large enough to force its competitor out of the market. In this paper, we consider only non-drastic innovations.

## Nash Equilibrium Outcomes

Suppose that after the non-drastic innovation both firms invest in the first stage (I,I). Firm A enters the first stage with a production cost of $c_{i}-\varepsilon$ as compared to $c_{j}$ for Firm B. If both firms have symmetric cost in the pre-innovation stage; then Firm A has a competitive cost advantage of $\varepsilon$ over Firm B in the post innovation stage in case benefits are kept proprietary. The cost reduction innovation affects prices as well as the market share of both firms and hence induces price as well quantity effects. The Nash equilibrium price $P_{i}^{V^{8}}$ is given by;

$$
\begin{align*}
& P_{i}^{H^{k}}=\frac{2 b\left[\theta_{i s i}+b\left(\theta_{i}-a\right)\right]+d^{2}\left(\theta_{s i}+b c_{i}\right)}{4 b^{2}-d^{2}} \ldots \ldots \ldots . .  \tag{4-3}\\
& \quad \Rightarrow E_{i}^{N^{2}}=\frac{2 b\left(\theta_{i s}+b c_{i}\right)+d^{2}\left(\theta_{i s}+b c_{i}\right)}{4 b^{2}-d^{2}}-\frac{2 b^{2} a}{4 b^{2}-d^{2}} \\
& \Rightarrow E_{i}^{N^{2}}=E_{i}^{N}-\frac{2 b^{2} a}{4 b^{2}-d^{2}}
\end{align*}
$$

$E_{i}^{N^{8}}$ is the Nash equilibrium price in post innovation stage in which the benefits are proprietary and $P_{i}^{H}$ is the pre-innovation Nash equilibrium price.

$$
\begin{equation*}
\Delta F_{i}^{N}=F_{i}^{V^{t}}-F_{i}^{N}=-\frac{2 b^{2} \varepsilon}{4 b^{2}-d^{2}}<0 \tag{4-4}
\end{equation*}
$$

Similarly,

$$
E j^{N^{t}}=\frac{2 b\left(\theta_{i s}+b c_{i}\right)+d\left[\theta_{i \theta}+b\left(c_{i}-a\right)\right]}{4 b^{2}-d^{2}}
$$

$$
\Rightarrow F_{i}^{j^{i}}=\frac{2 b\left(\theta_{\pi s}+b G_{f}\right)+d\left(\theta_{i t}+b C_{i}\right)}{4 b^{2}-d^{2}}-\frac{b d^{2}}{4 b^{2}-d^{2}}
$$

$\Rightarrow F_{i}^{V^{i}}=F_{i}^{V}-\frac{b d a}{4 b^{2}-d^{[2}}$
$\Leftrightarrow \Delta F_{i}^{W}=F_{i}^{H^{i}}-F_{i}^{N}=-\frac{b d^{H}}{4 b^{2}-d^{2}} \pi_{n} \theta$
$\Delta F_{i}^{N}$ and $\Delta P_{f^{\prime}}^{\prime}$ in equation (4-4) and equation (4-5) captures the price effect of innovation. The negative sign of $\Delta F_{i}^{N H}$ shows that successful cost reduction innovation allows Firm A to reduce its price by $\frac{2 b^{2}}{4 \varepsilon^{2}-a^{2}}$. Similarly, the sign of $\Delta F_{i}^{N}$ is negative which shows that Firm B (rival) will reciprocate by reducing its price by $\frac{b d i}{4 h^{2}-d^{2}} \cdot \Delta E_{i}^{W}$ and $\Delta P_{i}^{N}$ depend on level of cost reduction $\epsilon$ and price sensitivity coefficients ' $b$ ' and ' $d$ ' where in our case we assume that $b>d>0$. Both equation (4-4) and equation (4-5) show that price reduction does not depend on the initial demand parameter and this decision is taken independent of the market uncertainty.

The quantity effect of innovation can be decomposed into two parts. The first part provides a competitive gain in demand through reduction in Firm's own price. Customers favorably respond to it and the quantity sold is increased. The second part of quantity effect results from partial loss in demand due to rival's price reduction. The rival Firm B reduces its price by $\Delta F_{j}^{N F}$ when Firm A reduces its price by $\Delta F_{i}^{V F}$ which
negatively affects the demand for Firm A. $\frac{\partial Q_{i}}{\partial A_{i}}=-i$ and $\frac{\partial Q_{i}}{\partial \theta_{i}}=d$ shows the price elasticities of Firm A's demand to changes in its own and rival's price respectively. The combined effect of innovation on the quantity is as follows;

$$
\begin{gather*}
\Delta Q_{t}^{N}=-b\left(\frac{-2 b^{2} \varepsilon}{4 b^{2} a^{2}}\right)+d^{I}\left(\frac{-b d^{2} \epsilon}{4 b^{2} a^{2}}\right) \\
\Rightarrow \Delta Q_{t}^{H}=\frac{2 b^{2} a}{4 b^{2}-d^{2}}-\frac{b a^{2} a}{4 b^{2}-d^{2}}=\frac{b\left(2 b^{2}-d^{2}\right) c}{4 b^{2}-d^{2}} \cdot Q \tag{4-6}
\end{gather*}
$$

The first part of equation (4-6) $\frac{2^{3} \varepsilon}{4 b^{5}-a^{5}}=0$ shows the direct gain (in market share) from price reduction facilitated by innovation and the second part $-\frac{b d^{2} i}{4 b^{2}-a^{2}}{ }^{4} 0$ shows loss (in market share) from rival's price reciprocating strategy. The overall quantity effect is positive i.e. $\frac{b\left(2 b^{2}-d^{2} y^{2}\right.}{4 b^{2}-d^{2}} Q$ for $b>d>a$.

If both firm invests, then the change in profit value $\pi_{i}$ of Firm $A$ in the first stage is given by;

$$
\pi_{t}^{t}=\left(P_{t}^{t}-c_{t}^{t}\right) Q_{t}^{t}
$$

$\Rightarrow \pi_{i}^{\prime}=\left[\left(F_{i}^{N}+\Delta F_{i}^{N}\right)-\left(\varsigma_{t}-\varepsilon\right)\right]\left(Q_{i}^{N}+\Delta Q_{i}^{H}\right)$

$$
\Rightarrow \pi_{i}^{i}=\left(P_{i}^{N}-\frac{2 b^{2}}{4 b^{2}-d^{2}} \varepsilon-c_{i}+\varepsilon\right)\left(Q_{i}^{N}+\frac{b\left(2 b^{2}-d^{2}\right)}{4 b^{2}-d^{2}} \varepsilon\right)
$$

$$
\Rightarrow \pi_{i}^{t}=\left(\left(F_{i}^{N}-c_{i}\right)+\frac{2 b^{2}-d^{2}}{4 b^{2}-d^{2}} \epsilon\right)\left(Q_{t}^{N}+\frac{b\left(2 b^{2}-d^{2}\right)}{4 b^{2}-d^{2}} \epsilon\right)
$$

$$
\pi_{i}^{i}=\left(P_{i}^{N}-c_{i}\right) Q_{i}^{N}+\left(\frac{2 b^{2}-d^{2}}{4 b^{2}-d^{2}}\right) Q_{i}^{N} f+\left(\frac{2 b^{2}-d^{2}}{4 b^{2}-d^{2}}\right)\left(P_{i}^{N}-c_{i}\right) k \sigma+\left(\frac{2 b^{2}-d^{d^{2}}}{4 b^{2}-d^{2}}\right)^{2} 2 f^{2}
$$

$\Rightarrow \pi_{i}^{t}=\pi_{i}+\left(\frac{2 b^{2}-\alpha^{2}}{4 b^{2}-\alpha^{2}}\right)\left\lfloor Q_{i}^{N}+\left(P_{i}-c_{i}\right) \hat{b}\right] \epsilon+\left(\frac{2 b^{2}-d^{2}}{4 b^{2}-d^{2}}\right)^{2} b \epsilon^{2}$

$$
\Rightarrow \Delta \pi_{i}=G\left[Q_{i}^{N}+\left(E_{i}^{N}-\varsigma_{i}\right) b\right] a+G^{2} b q
$$

Where $G=\frac{2 b^{2}-d^{2}}{4 b^{2}-d^{2}} \geq 0$ and $\Delta \pi_{i}$ shows the change in profit due to change in price resulting from post innovation cost reduction by $\epsilon$. Equation (4-5) and (4-6) indicate that price and quantity reactions are linear in cost reduction. Equation (4-7) shows that $\Delta \pi_{i}$ is a quadratic and convex function because $\frac{\partial \Delta \pi_{i}^{2}}{\partial e^{2}}=2 G^{2} \delta>0$.

Since Firm B is at competitive disadvantage, it reacts differently to any proprietary cost cuts by Firm A. $\frac{\partial Q_{i}}{\partial B_{i}}=d$ and $\frac{\partial Q_{A}}{\partial Q_{f}}--\frac{t}{}$ represent the price elasticities of Firm B demand (quantity sold). The total change in Firm B quantity sold in reaction to price cuts by Firm A can be obtained from equation (4-4) and (4-5).

$$
\Delta Q_{V}^{V}=-\frac{2 b^{2} d^{d}}{4 b^{2}-d^{2}} q+\frac{b^{2} d}{4 b^{2}-d^{2}} q=-\frac{b^{2} d^{2}}{4 b^{2}-d^{2}}, \ldots \ldots \ldots \ldots
$$

The first part of equation (4-8) i.e. $-\frac{2 b^{7} \pi}{4 b^{2}-a^{2}}$ indicates loss due to its rival price reduction and the second part i.e. $\frac{b^{2}}{4 b^{2}-a^{2}}$ enows direct gain in quantity (market share) when Firm B makes a forced price reduction in response to Firm A post innovation pricing strategy. The overall quantity reaction of Firm B is negative i.e. $-\frac{b_{0}^{2}}{4 b^{2}-d^{2}}{ }^{2}$. 0 . Equation (4-6) and (4-8) show that the firm with cost disadvantage (Firm B) looses its market share to a firm with cost advantage (Firm A). However, in the post innovation stage, the innovator gains more than what its rival looses in the market i.e $\left|\frac{b\left(2 b^{2}-d^{2}\right)}{4 b^{2}-d^{2}} e\right| \geqslant\left|-\frac{b^{2} a}{4 b^{2}-d^{2}} e\right|$. Therefore, in addition to punishing cost inefficient firm, the market responds favorably to innovations which bring about cost efficiency to firms.

$$
\begin{equation*}
\left|\Delta Q_{i}^{W}\right|=\left|\Delta Q_{i}^{W}\right|+\delta_{i}^{F} \tag{4-9}
\end{equation*}
$$

$$
\begin{gathered}
\Rightarrow \delta_{i}^{N F}=\left|\Delta Q_{i}^{A F}\right|-\left|\Delta Q_{F}^{N F}\right| \\
\Rightarrow \delta_{i}^{N}=\left|\frac{b\left(2 b^{2}-d^{2}\right)}{4 b^{2}-d^{2}} \epsilon\right|-\left|-\frac{b^{2} d}{4 b^{2}-d^{2}} \varepsilon\right|
\end{gathered}
$$

$$
\begin{aligned}
& \Rightarrow \delta_{t}^{N}=\left|\frac{b\left(2 b^{2}-d^{2}\right)}{4 b^{2}-d^{2}} \epsilon\right|-\left|-\frac{b^{2} d^{2}}{4 b^{2}-d^{2}} \epsilon\right| \\
& \Rightarrow \delta_{i}^{N}=\frac{\delta\left[2 b^{2}-d^{2}\left(b+d^{2}\right)\right]}{4 b^{2}-d^{2}} q
\end{aligned}
$$

Equation (4-9) provides an alternative decomposition of quantity effect of innovation. Table 4 indicates that the total gain in innovator's (Firm A) quantity is made of two components. First component captures the switching effect $\left(Y_{i}^{N}\right)$. For innovator, this shows the gain in market share of $\left|\Delta Q_{j}^{N}\right|$ when customers switch from cost inefficient Firm B to cost efficient Firm A. The remaining part $\mathcal{E}_{i}^{8}$ contributes to innovation premium and comes from boost in product demand which mainly stems from positive externalities of innovation. In this paper we call $\mathscr{C}_{i}^{N}$ as innovation demand premium under Nash equilibrium.

Table 4: Impact of Proprietary Innovation on increase(+)/decrease(-) in Price, Quantity, Switching Effect and Demand Innovation Premium for Nash Outcomes

|  |  | Change in sold quantity of $A$ |  |  |  |  | $\Delta v^{\eta}$ | Change in sold quantity of B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | $\Delta^{\prime \prime}$ | Price reduction effect | Competition effect | Total $2 Q^{7}$ | $\gamma^{-r}$ | $\omega^{\circ}$ |  | Price reduction effect | Competition effect | Total $2 Q_{0}^{7}$ |
| 0.1 | -0.05 | 0.11 | -0.01 | 0.10 | 0.02 | 0.08 | -0.01 | 0.03 | -0.05 | -0.02 |
| 0.2 | -0.11 | 0.21 | -0.03 | 0.18 | 0.06 | 0.12 | -0.03 | 0.05 | -0.11 | -0.06 |
| 0.3 | -0.16 | 0.32 | -0.04 | 0.28 | 0.08 | 0.2 | -0.04 | 0.08 | -0.16 | -0.08 |
| 0.4 | -0.21 | 0.43 | -0.05 | 0.38 | 0.1 | 0.28 | -0.05 | 0.11 | -0.21 | -0.10 |
| 0.5 | -0.27 | 0.53 | -0.07 | 0.47 | 0.14 | 0.33 | -0.07 | 0.13 | -0.27 | -0.14 |
| 0.6 | -0.32 | 0.64 | -0.08 | 0.56 | 0.16 | 0.4 | -0.08 | 0.16 | -0.32 | -0.16 |
| 0.7 | -0.37 | 0.75 | -0.09 | 0.66 | 0.18 | 0.48 | -0.09 | 0.19 | -0.37 | -0.18 |
| 0.8 | -0.43 | 0.85 | -0.11 | 0.74 | 0.22 | 0.52 | -0.11 | 0.21 | -0.43 | -0.22 |
| 0.9 | -0.48 | 0.96 | -0.12 | 0.84 | 0.24 | 0.6 | -0.12 | 0.24 | -0.48 | -0.24 |

Figure 10: Impact of Cost Reduction on Components of Changes in Market Share


For numerical example, this paper uses Smit and Trigeorgis (2004) and introduces the effect of cost reduction innovation into the model. Smit and Trigeorgis considers the effect of successful advertising campaign on demand parameters. It assumes that in the proprietary case, the initial demand increases from 10 to 12 whereas the demand of the rival firm goes down from 10 to 9 . We assume that after successful innovation by Firm A, the demand of Firm increases from 10 to 12 and its cost also reduces by $20 \%$ i.e. $\epsilon=0.2$. The demand for Firm B goes down from 10 to 9 . In a competitive environment, Firm B takes into account both the changes in demand parameters and cost advantage of its rival in its reaction function.

We assume that risk profile of both the firms does not change with this innovation. In the first stage, if the market is favorable, then Firm A invests and the Firm B optimally responds by not investing in the first stage and it waits until the uncertainty is resolved in the next period. If the market demand turns out unfavorable, then none of the firms have a dominant strategy. Both firms prefer to be unpredictable and the game ends in mixed equilibrium. The total value of Firm A is 134.3 decomposed into static NPV of 90.6, strategic preemption value of 28.3 and
unpredictability value from waiting of 15.4 . This premium provides an incentive to the firms to remain unpredictable.

Figure 11 shows the price and quantity effect of proprietary innovation on final outcomes when both firms invest. Both the price and quantity effect are linear in cost reduction $\epsilon$, which depends on the success of innovation. If the innovation is very successful, then it will give more flexibility to the innovator to reduce its prices. The figure shows that price $\bar{P}_{i}$ of innovator firm decreases more rapidly than its rival firm. However, in response to price reduction, the gain in quantity for the innovator $\Delta Q_{i}$ is disproportionately more. The innovator Firm A gains in quantity and it rival Firm B quantity $\Delta Q_{f}$ decreases despite of decrease in price due to reciprocating competition. The overall effect of the proprietary innovation is that the value of innovator firm $\left(N P V_{A}\right)$ increases and the value of its rival $\left(N P V_{B}\right)$ decreases.

Figure 11: Price and Quantity Effect of Proprietary Innovation on Firm Value under Nash Equilibrium


The gain/loss in market share can be decomposed into price $(+)$ and competition effects (-). Figure 12 illustrates that price effect positively and competition effect negatively contributes to gain/loss in market share. For innovator Firm A, gain due to
price effect overcompensates for any loss due to competition. On the other hand the rival firm losses more due to competition than gain due to price effect. As a result if both firms invest in the first stage, Firm B with a competitive cost disadvantage losses its market share to Firm A.

Figure 12: Price and Competition Effect of Proprietary Innovation on gain/loss in Market Share under Nash Equilibrium


However, Table 5 suggests that Firm A gains more than what its rival losses i.e. $0.15>1-0.05 \mid$. Innovator's gain can be decomposed into switching effect of 0.05 and innovation demand premium of $8_{i}^{N}=0,13$.

Table 5: Effect of Proprietary Innovation on increase(+)/decrease(-) in Price and Quantity


## Stackelberg (Leader/Follower) Equilibrium Outcomes

When innovator (Firm A) invests in the first stage and the rival Firm B waits until the second stage, then Firm A becomes the stackelberg leader and Firm B becomes follower. Without innovation, the stackelberg price is given in equation (3-12). In the post-innovation stage, the stackelberg leader price $P^{V^{\dagger}}$ is given by;

$$
F_{i}^{i^{t}}=\frac{2 b\left(\theta_{i s}+b c_{i}-b a\right)-d\left(\theta_{i s}+b c_{i}-d c_{i}+d a\right)}{4 b^{2}-2 d^{2}}
$$

$p_{i}^{k^{i}}=\frac{2 h\left(\theta_{i d}+b c_{i}\right)-d\left(\theta_{j i}+h c_{j}-d c_{i}\right)}{4 b^{2}-2 d^{2}}-\frac{2 h^{2}-d^{2}}{2\left(2 b^{2}-d^{2}\right)} e$
$\Delta F_{t}^{2}=-\frac{1}{2} q$

Firm B as a follower will react to Firm A's price $P^{l^{\prime}}$. From the reaction function we have;

$$
\begin{aligned}
& \partial P_{i}^{F}=\frac{d \partial P_{t}}{\partial c_{t}}=2 b \partial c_{t}
\end{aligned}
$$

The change in followers price $\Delta F^{F}$ in reaction to stackelberg leader's price reduction by $\varepsilon$ is as follows;

$$
\Rightarrow \Delta E_{F}^{F}=\frac{\alpha}{2 b}\left(-\frac{1}{2} a\right)
$$

$$
\begin{equation*}
\Delta F_{F}^{F}=-\frac{d}{4 b} a \tag{4-11}
\end{equation*}
$$

Substituting the price changes from equation (4-10) and equation (4-11) into equation (4-1) gives;

$$
\begin{gather*}
\Delta Q_{t}^{L}=-b\left(-\frac{1}{2}\right) q+d\left(-\frac{d}{4 b}\right) c \\
\Delta Q_{t}^{L}=\frac{b}{2} \epsilon-\left(\frac{d^{2}}{4 b}\right) \epsilon \\
\Delta Q_{t}^{2}=\frac{b}{2} \epsilon-\left(\frac{d^{2}}{4 b}\right) \epsilon=\frac{2 b^{2}-d^{2}}{4 b} \epsilon>0 \tag{4-12}
\end{gather*}
$$

The first part of equation (4-12) shows increase in demand of Firm A by $\frac{v}{2} \in$ due to Firm A's reduction in price by $\frac{1}{2} \epsilon$. This increase results from direct gain from innovation. The second part of the equation shows a loss from competitor's price reciprocation. In response to innovator's price reduction, the rival firm also reduces its price by $\frac{a}{4 \delta} \approx$ which results in innovator's demand decrease of $\frac{d^{a}}{4 b} a$. The combined effect, however, is a net increase of $\frac{2 b^{2}-d^{2}}{4}$. Similarly, equation (4-2) gives;

$$
\begin{equation*}
\Delta Q_{E}^{E}=-\frac{d}{4} x x_{n} 0 \tag{4-13}
\end{equation*}
$$

The price and quantity effects are linear in cost reduction $\epsilon$. The innovation demand premium in this case is;

$$
\begin{align*}
& \sigma_{t}^{2}=\left|\frac{2 b^{2}-d^{2}}{4 b} \epsilon\right|-\left|-\frac{d}{4} \epsilon\right| \\
& -2 t=\frac{2 b^{2}-\alpha^{2}\left(b+a^{2}\right)}{4 b} a \tag{4-14}
\end{align*}
$$

Firm A becomes a Stackelberg Leader with competitive cost advantage if its rival defers investment in the first stage and waits until the next period. Figure 13 shows that decrease in cost allows Firm A to lower its prices and capture the market. On the other hand, rival Firm B lowers its price in reciprocation but despite of price reduction, it losses market share (Annexure 4).

Figure 13: Price and Quantity Effect of Proprietary Innovation on Firm Value under Stackelberg (L/F) Equilibrium


Figure 14 shows the price and competition effect of cost reduction technology on gain/loss in market share. For Firm A, price effect contributes more to gain in the
market share than does the competition effect. However, the rival Firm B with cost disadvantage, losses more to competition than it gains from lowering its prices.

Figure 14: Price and Competition Effect of Proprietary Innovation on gain/loss in Market Share under Stackelberg (L/F) Equilibrium


### 4.3 Shared Benefits of Innovation and License Pricing

Suppose that Firm A makes a strategic investment in a cost reduction technology. The invest results in a successful innovation which reduces the cost of production of Firm A by $\epsilon$ where $0<\varepsilon<\varepsilon_{i}$. When the benefits are shared, it also reduces cost of production of its rival Firm B by $\epsilon$. Figure 15 shows outcomes of shared innovation under different market structures. In shared case, the study uses the following parameters ${ }^{5}$;

```
0
0}=1
cost recoluctime= =f=0.2
```

[^4]```
c
c}=0.
ka}-\mp@subsup{k}{c}{}-13
rtsk free rate =r=10%
u=1.25
```


## Nash Equilibrium Outcomes

When both firms invest in the first stage, then the game ends in a Nash equilibrium. In the post-innovation stage, when the benefits are shared, Firm A Nash equilibrium price $E_{i}^{M^{f}}$ is given by;

$$
E_{i}^{t^{s}}=\frac{2 b\left[\theta_{i s}+b\left(\varepsilon_{i}-a\right)\right]+d\left[\theta_{g k}+b\left(\varepsilon_{i}-a\right)\right]}{4 b^{2}-d^{2}}
$$

$P_{i}^{N^{t}}=P_{i}^{N}-\frac{\left(2 b+d^{2}\right) b \varepsilon}{4 b^{2}-d^{2}}$

Figure 15: Shared Case Competitive Strategies under Different Market Structures


$$
\begin{equation*}
\Delta F_{i}^{W}=-\frac{b q}{\left(2 b-d^{d}\right)} \sigma_{n} 0 \tag{4-15}
\end{equation*}
$$

Similarly, Firm B Nash equilibrium price $\boldsymbol{P}_{j}^{\beta^{\prime}}$ is given by;

$$
\begin{align*}
& E_{i}^{N^{i}}=\frac{2 b\left[\theta_{i s}+b\left(c_{i}-q\right)\right]+d^{d}\left[\theta_{i s}+b\left(c_{i}-q\right)\right]}{4 b^{2}-d^{2}} \\
& E_{i}^{N^{i}}-P_{i}^{N}-\frac{(2 b+d) b \varepsilon}{4 b^{2}-d^{2}} \\
& \quad \Delta E_{i}^{N}=-\frac{b a}{\left(2 b-d^{2}\right)} \epsilon_{i} \theta \ldots \ldots \ldots \ldots \ldots . . \tag{4-16}
\end{align*}
$$

Equation (4-15) and (4-16) show that if the innovation benefits are completely shared then the price of both firms reacts symmetrically to cost cuttings. Similarly, the quantities sold of both firms react the same way if benefits are completely shared.

We know that

$$
\begin{align*}
& \Delta Q_{t}=-b\left(-\frac{v \varepsilon}{2 b-\frac{b}{d}}\right)+d\left(-\frac{b \varepsilon}{2 b-b^{t}}\right) \\
& \Delta \psi_{t}=\frac{b(b-d) \varepsilon}{2 b-d}>0 \quad \ldots \ldots \ldots \ldots \ldots . \tag{4-17}
\end{align*}
$$

Similarly,

$$
\begin{equation*}
\Delta Q_{i}=\frac{b(b-d) \varepsilon}{2 b-\alpha}>0 \tag{4-18}
\end{equation*}
$$

## Stackelberg Outcomes

Firm A becomes Stackelberg leader when it invests in the first stage and its rival Firm B waits until the next stage. In a practical situation, it may happen when the markets have entry barriers. In the post-innovation stage, the Stackelberg leader price of Firm A is given by $\boldsymbol{E P}^{\boldsymbol{l}^{i}}$;

$$
\begin{align*}
& P \psi^{f}=\frac{2 b\left(\theta_{i, k}+b c_{i}-b a\right)-d\left(\theta_{i, i}+b c_{f}-b a-d c_{i}+d q\right)}{4 b^{2}-2 d^{2}} \\
& E_{i} b^{\prime}=\frac{2 b\left(\theta_{i s}+b c_{i}\right)-d\left(\theta_{i s}+b c_{g}-d c_{i}\right)}{4 b^{2}-2 d^{2}}-\frac{\left(2 b^{2}+b d^{2}-d^{2}\right) d}{4 b^{2}-2 d^{2}} \\
& E f^{f}=P^{2}-\frac{\left(2 b^{2}+b d-d^{2}\right) d}{4 b^{2}-2 d^{2}} \\
& \Delta F_{i}^{k}=-\frac{\left(2 b^{2}+5 d^{2}-d^{2}\right) d}{4 b^{2}-2 d^{2}} \tag{4-19}
\end{align*}
$$

Firm B will react to Firm A's price $P_{l}^{l^{\prime}}$ which maximizes payoff in case it stays out of the market in the first stage. From the reaction function of Firm B, it is evident that $P_{i}^{F}=f\left(P_{i}^{L}, c_{i}\right)$.

$$
\begin{equation*}
\frac{\partial P_{F}}{d^{L} c_{t}}=\frac{\partial P_{i}}{\partial P_{i}^{L}} \frac{\partial \Gamma_{t}^{L}}{\partial c_{t}}+\frac{\partial P_{F}}{\partial c_{F}} \frac{\partial c_{i}}{\partial c_{t}} \tag{4-20}
\end{equation*}
$$

Technology is fully transferred if $\frac{\theta_{0}}{\theta_{9}}=1$. However, it is also possible to transfer partial technology if the innovation can be divided into transferable components and hence $00 \frac{\partial \sigma_{i}}{\partial \sigma_{i}} \frac{\varepsilon_{n}}{} 1$. In this case, we assume complete transfer of technology. From the reaction function, we have $\frac{\partial B_{i}^{K}}{\partial E_{i}}=\frac{d}{\partial v}$ and $\frac{\partial D_{i}^{E}}{A n_{j}}=\frac{1}{2}$. From equation (4-19) and (4-20), we get;

$$
\frac{d P_{F}^{F}}{d c_{t}}=\frac{d}{2 b}\left(\frac{2 b^{2}+b d-d^{2}}{4 b^{2}-2 d^{2}}\right)+\frac{1}{2}
$$

For a cost reduction of $\epsilon$, the change in Firm B (follower) price is given by;

$$
\begin{equation*}
\Delta F_{j}^{F}=-\left[\frac{d}{2 b}\left(\frac{2 b^{2}+b d-d^{2}}{4 b^{2}-2 d^{2}}\right)+\frac{1}{2}\right] \epsilon \quad \quad \ldots \ldots \tag{4-21}
\end{equation*}
$$

The change in quantity of Firm A and Firm B is given by;

$$
\Delta Q_{t}^{2}=b\left(\frac{2 b^{2}+b d^{d}-d^{2}}{4 b^{2}-2 d^{2}}\right) \varepsilon-\frac{d^{2}}{2 b}\left(\frac{2 b^{2}+b d-d^{2}}{4 b^{2}-2 d^{2}}\right) \varepsilon-\frac{d}{2} \varepsilon
$$

$$
\begin{gather*}
\Delta Q_{i}^{L}=\left(\frac{\left(2 b^{2}+b d-d^{2}\right)}{2\left(2 b^{2}-d^{2}\right)} \frac{\left(2 b^{2}-d^{2}\right)}{2 h}-\frac{d}{2}\right) \in \\
\Delta Q_{i}^{L}=\left(\frac{2 b^{2}+b d-d^{2}}{4 b}-\frac{d}{2}\right) \in \\
\Delta Q_{i}^{L}=\left(\frac{2 b^{2}-b d^{2}-d^{2}}{4 b}\right) \in
\end{gather*}
$$

Similarly, for Firm B;

$$
\begin{align*}
& \Delta Q_{i}^{E}=b\left[\frac{d}{2 b}\left(\frac{2 b^{2}+b d^{2}-d^{2}}{4 b^{2}-2 d^{2}}\right)+\frac{1}{2}\right] \varepsilon-d^{d}\left(\frac{2 b^{2}+b d-d^{2}}{4 b^{2}-2 d^{2}}\right) \varepsilon \\
& \Delta Q_{f}=\left[\left(\frac{d}{2}-d^{d}\right)\left(\frac{2 b^{2}+b d-d^{2}}{4 b^{2}-2 d^{2}}\right)+\frac{b}{2}\right] \varepsilon \\
& \Delta Q_{F}^{F}=\left[\frac{b}{2}-\frac{d}{2}\left(\frac{2 b^{2}+b d-d^{2}}{4 b^{2}-2 d^{2}}\right)\right] \varepsilon{ }^{2} \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots
\end{align*}
$$

Table 6 summarizes the effect of proprietary and shared innovation on changes in prices and market share under different market structures.

Table 6: Effect of Proprietary and Shared Innovation on changes in Prices and Market Shares under different Market Structures

| Market Structure | Change due to Innovation | Proprietary Innovation | Shared Innovation |
| :---: | :---: | :---: | :---: |
| Nash | $\Delta R^{Y \prime}$ | $=\frac{2 a^{2} a}{4 b^{2}-a^{2}} \operatorname{cit}$ | $-\frac{b}{\left(2 h-a^{2}\right)} \geq<0$ |
|  | $\Delta R^{Y}$ | $=\frac{b d a}{4 b^{\prime}=d^{\prime}}=0$ | $-\frac{h}{\left(2 b-d^{2}\right)}:<0$ |
|  | $2 Q^{W}$ | $\frac{\Delta\left(2 b^{2}-u^{2}\right)}{4 b^{2}-d^{2}} \geqslant 0$ | $\frac{b(b-a)}{2 b-a}+0$ |
|  | $2 Q\}^{3}$ | $-\frac{b a}{4 b^{2}-a^{2}} e<0$ | $\frac{b, b-a)}{2 b-a}=0$ |
| Stackelberg <br> (Leader/Follower) | $\Delta R^{*}$ | $-\frac{1}{2} 1 \times 0$ | $-\frac{\left(2 a^{2}+a d^{2}-a^{2}+\right.}{4 b^{2}-2 a^{2}} \approx<0$ |
|  | $\Delta P F$ | $-\frac{d}{4 b}+8$ | $-\left[\frac{a}{2 b}\left(\frac{2 b^{2}+b d^{2}-a^{2}}{4 b^{2}-2 d^{2}}\right)+\frac{1}{2}\right]+a 0$ |
|  | $2 Q ?$ | $\frac{2 b^{2}-a^{2}}{4 b}=0$ | $\frac{2 b^{*}-b a^{2}-a^{*}}{4 b}=0$ |
|  | $\Delta Q \%$ | $-\frac{d}{4}$ ¢ 0 | $\left[\frac{a}{2}-\frac{a}{2}\left(\frac{2 b^{2}+a d-a^{2}}{4 b^{2}-2 a^{2}}\right)\right]=0$ |

## License Pricing

In a duopoly setting, Firm B can benefit from this innovation by paying a fixed license fee ' $F$ ' to the innovator. The license fee should be large enough to adequately compensate Firm A which will forgo its competitive advantage in case it transfers this technology to its rival. However, the licensee (Firm B) will not be willing to pay more than the benefit that it derives from the license. When the technology is transferred, it will reduce the cost of production of Firm B by $\epsilon$ as well.

Suppose, in proprietary setting the value of innovators Firm A is $N P V_{A}^{P}$ and Firm B is $N P V_{B}^{B}$. When the technology is shared, the values of Firm A and B are $N P V_{A}^{S}$ and $N P V_{A}^{\S}$ respectively. The upper and lower bounds of the license fee F is given by;

$$
\left(N P V_{A}^{P}-N P V_{A}^{S}\right) \leq F \leq\left(N P V_{E}^{S}-N P V_{E}^{B}\right)
$$

Investment in R\&D can result in innovation which can give proprietary or shared benefits. In case of shared benefits, both the innovator and the technology recipient get certain benefits. In this paper the overall benefits to the market will be termed as Universal Benefits of Innovation (UBI). Innovation is termed as universally successful if;

```
\(U D I=\left(N P V_{A}^{S}+N P V_{E}^{S}\right)\) or \(\left(N P V_{A}^{F}+N P V_{E}^{F}\right)-\left(N P V_{A}^{l}+N P V_{B}^{l}\right) \geq I\)
```

Where $I$ is the R\&D investment, $N P V_{A}^{t}$ and $N P V_{E}^{t}$ are values of Firm A and B in the pre-innovation stage. In our case the R\&D investment is not universally successful if it is kept proprietary i.e. Total value of both firms in Proprietary case-Total value of both firms in base case $<$ R\&D Investment.

$$
(134+45)-(56+56)=67<85
$$

However, if the innovation is shared then innovation becomes universally successful.

$$
(116+116)-(56+56)=120>8.5
$$

The license fee depends on the bargaining power of both firms. If Firm B is likely to get a higher benefit from this deal, then it is likely that Firm B will be willing to pay a lucrative fee to the innovator to induce it to strike a licensing deal. Similarly, if sharing this technology results in higher loss of competitive advantage, then the innovator will require a fee which at least makes good for such a loss.

In duopoly bargaining problem, proprietary innovations in our analysis result in Pareto-inefficient outcomes. Both firms engage in negotiation to mutually agree on licensing fee. Bargaining solution is a typical case of equilibrium selection among feasible set of agreement payoffs. Each point in this set gives a better outcome compared to disagreement payoffs. In case of disagreement, the bargaining ends without reaching any conclusion and status quo is maintained which results in proprietary outcomes. Nash provided an axiomatic solution to the bargaining game. Kalai and Smorodinsky (1975) provided an analytical solution to the bargaining game and replaced one of the Nash axioms i.e. Independence of Irrelevant Alternatives with monotonicity condition. According to this solution, the agreement point is one which maintains the ratio of maximum gains to both players in a duopoly setting. After the license fee F has been paid, gains of both players are as follows;

Gatre of player 1 (with help of player2 $)=g_{1}=F-\left(N P V_{A}^{F}-N P V_{A}^{\delta}\right)$

Similarly,

Gatns of player $2($ with help of player 1$)=g_{2}=\left(N P V_{E}^{S}-N P V_{B}^{B}\right)-F$

### 4.3.1 Complete Transfer of Technology

Both firms were identical in the base case. In the post-innovation stage, both firms remain identical if technology is completely transferred. Both firms will help each other in a way (innovator in terms of sharing the technology and the technology recipient in terms of license fee payment) such that $\frac{a_{2}}{a_{2}}=\mathbb{1}$, which means that both will derive the
same level of benefit from each other. License Fee can be obtained from the following expression;

$$
\begin{align*}
& \frac{g_{1}}{g_{2}}=\frac{F-\left(N P V_{A}^{B}-N P V_{A}^{S}\right)}{\left(N P V_{B}^{S}-N P V_{B}^{B}\right)-F}=1 \\
& \quad \Rightarrow F=\frac{\left(N P V_{A}^{B}-N P V_{A}^{S}\right)+\left(N P V_{B}^{S}-N P V_{B}^{B}\right)}{2} \tag{4-24}
\end{align*}
$$

Equation (4-24) shows that the License Fee is sensitive to gains/losses of both firms. In example $1 \& 2$, the universal gain to both firms resulting from innovation is 50 in each case. However, in example 1, Firm A losses 20 as compared to a loss of 30 in example 2. Similarly, Firm B gains 70 in the example 1 and gains 80 in example 2. Therefore, logically Firm A would require a higher compensation for its loss in example 2 than in example 1. Similarly, due to increase in gains, Firm B will be willing to pay more for the license. Both arguments logically imply that $F_{2}$ should be greater than $F_{1}$. It is also pertinent to mention here that both $F_{1}$ and $F_{2}$ fall within the lower and upper bounds i.e. $20 \leq E_{1} \leq 70$ and $30 \leq F_{2} \leq 50$. The range of both bounds (50 in this case) is the universal gain of innovation which in essence defines the payoff space of feasible bargaining points. Both examples are summarized in the following tables;

Example 1


Example 2

| Firm A <br> Firm B | Proprietary Case | Shared Case | Gain |
| :---: | :---: | :---: | :---: |
|  | 130 | 100 | (30) |
|  | 40 | 120 | 80 |
|  | Universal Gain/Loss of Innovation |  | 50 |
| Lets Licensing Fee $=F_{2}$ <br> Gain to Ftrm $A=g_{A}=F_{2}-30$ <br> and |  |  |  |

### 4.3.2 Partial Transfer of Technology

It is also possible to partially transfer the technology if innovation is divisible into transferable components. Innovators can decide on which components can be shared and which components to be kept proprietary for strategic uses. Similarly, the potential buyers of technology can shop around for a suitable component of technology to get best value for their money. The firm value of innovator firm A declines with increase in degree of technology transfer and on the other hand the technology recipient Firm B benefits from it. Figure 16 shows the effect of degree of partial technology transfer on the value of competing firms. The figure shows that gain in value of Firm B is more than the loss in value of Firm A. Hence, sharing of technology is not a zero sum game and it results in overall market premium.

Figure 16: The Effect of Partial Technology Transfer on Firm Value


Table 7 shows the effect of partial technology transfer on firm values and license fees. Increase in degree of technology transfer reduces value of firm A which is compensated by increase in license fees.

Table 7: Effect of Partial Technology Transfer on Firm Values and License Fees

| $\begin{aligned} & e=0.2 \\ & g_{l}=0.8 \end{aligned}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Tech. Transfer | $c_{j}$ | $\mathrm{NFP}_{4}$ | $\mathrm{NFP}_{2}$ | Focs $(\bar{\prime})$ |
| 10\% | 0.98 | 117.48 | 109.10 | 36.10 |
| 20\% | 0.96 | 117.27 | 109.82 | 36.56 |
| 30\% | 0.94 | 117.06 | 110.54 | 37.02 |
| 40\% | 0.92 | 116.86 | 111.26 | 37.49 |
| 50\% | 0.90 | 116.65 | 111.98 | 37.95 |
| 60\% | 0.88 | 116.44 | 112.70 | 38.42 |
| 70\% | 0.86 | 116.24 | 113.43 | 38.88 |
| 80\% | 0.84 | 116.03 | 114.16 | 39.35 |
| 90\% | 0.82 | 115.82 | 114.89 | 39.82 |
| 100\% | 0.80 | 115.62 | 115.62 | 40.29 |

Table 8 shows the effect of partial technology transfer on firm values adjusted for license. It also shows that shared benefits in each case provide a better outcome than
proprietary case i.e. $(134,45)$. Technology transfer results in loss of competitive advantage to innovator Firm A, and therefore, decreases its firm value. Rival Firm B pays an appropriate fee to compensate the innovator for any losses resulting from licensing deal. Figure 17 shows that license fee under Kalai-Smorodinsky bargaining solution smoothes out firm values and provides a win-win situation both for innovator and technology recipient. Although negligible, but the data suggests that values of both firms monotonically increase with the degree of technology transfer which represents market premium resulting from technology sharing.

Table 8: Effect of Partial Technology Transfer on Firm Values Adjusted for License Fees

| $\begin{aligned} & a=0.2 \\ & 8=0.8 \end{aligned}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Tech. Transfer | $c_{i}$ | ${ }_{M P} V_{\text {d }}$ | $\triangle P V_{B}$ |
| 10\% | 0.98 | 153.58 | 73.00 |
| 20\% | 0.96 | 153.83 | 73.26 |
| 30\% | 0.94 | 154.09 | 73.51 |
| 40\% | 0.92 | 154.34 | 73.77 |
| 50\% | 0.90 | 154.60 | 74.03 |
| 60\% | 0.88 | 154.86 | 74.29 |
| 70\% | 0.86 | 155.12 | 74.54 |
| 80\% | 0.84 | 155.38 | 74.81 |
| 90\% | 0.82 | 155.64 | 75.07 |
| 100\% | 0.80 | 155.90 | 75.33 |

Figure 17: Firm Value Adjusted for Fixed License Fee under Different Degree of Technology Transfer


## 5. CONCLUSION

This study analyzed three different cases in price reciprocating competition.

First, the base case considered competitive interaction of two identical firms. In the absence of pure Nash equilibrium, firms choose to randomize their moves and derive certain value from unpredictability. The preemption value of firm increases with increase in competitive cost advantage. Sensitivity analysis of production cost suggests that benefits due to cost reducing innovations are larger than the benefits which are derived from cost inefficiencies of competitor. Idiosyncratic shocks to risk adjusted discount rate show that value of the firm decreases sharply with increase in discount rate. On the other hand, idiosyncratic demand uncertainty has a positive impact on firm value. In favorable market conditions, firms set higher prices with incremental increase in both idiosyncratic and systemic demand uncertainty. In the down state of the market, firms set lower prices with increase in both idiosyncratic and systemic demand uncertainty. Firms with higher initial demand have higher tendency to preempt its rival.

Second, in proprietary case, one firm makes a strategic R\&D investment in a cost reducing technology and keeps its benefits proprietary. However, benefits of proprietary innovation did not justify R\&D cost. Changes in prices and quantities resulting from proprietary innovation are linear in cost reduction and depend on price sensitivities of demand. For innovator, the quantity effect (or change in market share) of innovation mainly comes from switching and demand innovation premium. The cost inefficient firm losses market share despite reducing prices in reciprocation.

Third, in shared case, the innovator shares the benefits of innovation with its rival through transfer of technology for a fixed license fee. As a result, innovator losses competitive advantage but in return gets compensation commensurate with its bargaining power. After complete transfer of technology, both firms become identical again in the post innovation stage. Firms' values increase with increase in degree of
technology transfer. Sharing the benefits of non-drastic innovation is mutually beneficial for innovator and technology recipient in this setting.

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## ANNEXURES

Annexure 1: Probability Distribution of making an Investment when one of the firms experience an idiosyncratic shock to its cost of production

| Cost of Firm A | $\mathbf{0 . 3 0}$ | $\mathbf{0 . 4 0}$ | $\mathbf{0 . 5 0}$ | $\mathbf{0 . 6 0}$ | $\mathbf{0 . 7 0}$ | $\mathbf{0 . 8 0}$ | $\mathbf{0 . 9 0}$ | $\mathbf{1 . 0 0}$ | $\mathbf{1 . 1 0}$ | $\mathbf{1 . 2 0}$ | $\mathbf{1 . 3 0}$ | $\mathbf{1 . 4 0}$ | $\mathbf{1 . 5 0}$ | $\mathbf{1 . 6 0}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability <br> of invest | Up | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.97 | 0.97 | 0.97 | 0.98 | 0.98 | 0.99 | 0.99 | 1.00 | 0.00 |
| of Firm A | Down | 0.52 | 0.51 | 0.49 | 0.47 | 0.45 | 0.43 | 0.40 | 0.37 | 0.34 | 0.30 | 0.26 | 0.21 | 0.15 | 0.07 |
| Probability <br> of invest <br> of Firm B | Up | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.99 | 0.98 | 0.97 | 0.96 | 0.95 | 0.94 | 0.93 | 0.92 | 1.00 |

Annexure 2: Effect of Idiosyncratic Demand Shock on Firm's Probability of Investment in Good and Bad State

| Theta_of Firm A | 7.5 | 8.0 | 8.5 | 9.0 | 9.5 | 10.0 | 10.5 | 11.0 | 11.5 | 12.0 | 12.5 | 13.0 | 13.5 |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Probability <br> of invest <br> of Firm A | Down | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Probability <br> of invest <br> of Firm B | Up | 0.53 | 0.00 | 0.00 | 0.27 | 0.28 | 0.37 | 0.38 | 0.38 | 0.39 | 0.39 | 0.40 | 0.41 | 0.41 |

## Annexure 3: Effect of Idiosyncratic Volatility on Prices and Quantities Sold of Firm A when Market Moves Up and Down

| Volatility of Firm A | 1.16 |  | 1.19 |  | 1.22 |  | 1.25 |  | 1.28 |  | 1.31 |  | 1.34 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Market Movement | U | D | U | D | U | D | U | D | U | D | U | D | U | D |
| Price due to | 4.53 | 3.54 | 4.63 | 3.47 | 4.73 | 3.40 | 4.83 | 3.33 | 4.93 | 3.27 | 5.03 | 3.21 | 5.13 | 3.15 |
| Systematic Shock |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Price due to | 4.59 | 3.50 | 4.67 | 3.44 | 4.75 | 3.39 | 4.83 | 3.33 | 4.91 | 3.28 | 4.99 | 3.24 | 5.07 | 3.19 |
| Idiosyncratic Shock |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Quantity due to | 7.07 | 5.08 | 7.27 | 4.94 | 7.47 | 4.80 | 7.67 | 4.67 | 7.87 | 4.54 | 8.07 | 4.42 | 8.27 | 4.31 |
| Systematic Shock |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Quantity due to | 7.19 | 5.00 | 7.35 | 4.88 | 7.51 | 4.77 | 7.67 | 4.67 | 7.83 | 4.57 | 7.99 | 4.47 | 8.15 | 4.38 |
| Idiosyncratic Shock |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Annexure 4: Price and Competition Effect of Proprietary Innovation on Changes in Market Share under Stackelberg Outcomes

| * | $\Delta{ }^{2}$ | Change in sold quantity of $A$ |  |  | $\Delta P_{F}$ | Change in sold quantity of B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Price reduction effect | Competition effect | Total -Q |  | Price reduction effect | Competition effect | Total -4. |
| 0.1 | -0.05 | 0.10 | -0.01 | 0.09 | -0.01 | 0.03 | -0.05 | -0.02 |
| 0.2 | -0.10 | 0.20 | -0.03 | 0.17 | -0.03 | 0.05 | -0.10 | -0.05 |
| 0.3 | -0.15 | 0.30 | -0.04 | 0.26 | -0.04 | 0.07 | -0.15 | -0.08 |
| 0.4 | -0.20 | 0.40 | -0.05 | 0.35 | -0.05 | 0.10 | -0.20 | -0.10 |
| 0.5 | -0.25 | 0.50 | -0.06 | 0.44 | -0.06 | 0.12 | -0.25 | -0.13 |
| 0.6 | -0.30 | 0.60 | -0.08 | 0.52 | -0.08 | 0.15 | -0.30 | -0.15 |
| 0.7 | -0.35 | 0.70 | -0.09 | 0.61 | -0.09 | 0.17 | -0.35 | -0.18 |
| 0.8 | -0.40 | 0.80 | -0.10 | 0.70 | -0.10 | 0.20 | -0.40 | -0.20 |
| 0.9 | -0.45 | 0.90 | -0.11 | 0.79 | -0.11 | 0.22 | -0.45 | -0.23 |


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[^1]:    ${ }^{1}$ Adapted from Smit and Trigeorgis (2004)

[^2]:    ${ }^{2} 126 \times 0.97+170 \times 0.97=127$

[^3]:    ${ }^{4}$ Same parameters were also used by SMIT, but it did not consider the effect of cost reduction technology

[^4]:    ${ }^{5}$ Same parameters were also used by SMIT, but it did not consider the shared effect of cost reduction technology

