TWO-STAGE ROBUST OPTIMIZATION PROBLEM FOR OPERATING ROOM PLANNING

Top clinical hospital in the Netherlands

Master Thesis: Operations Research and Quantitative Logistics

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Abstract

The process of operating room planning includes the scheduling of disciplines, patients, surgeons, materials and the availability of the wards. An efficient planning is one of the conditions to keep the patients satisfied and to comply with the principles of the disciplines, patients, surgeons, materials, and wards. The operating room planning is divided into two sub problems. The first problem is the Discipline Assignment (DA) problem in which disciplines are assigned to blocks and the second problem is the Surgery Assignment (SA) problem which assigns surgeries (equivalently patients) to one of the blocks with the corresponding discipline. In this thesis, for each of the sub problems a two-stage robust program is proposed based on the principles of the operating room planning of a top clinical hospital in the Netherlands. The current planning for operating rooms is inefficient and causes problems like overload of wards, overtime of personnel, underutilization of operating rooms while the patient waiting lists are long.

In the SA problem the variability in the number of patients at a ward during a planning cycle is considered in order to decrease the workload at the ward. A column-and-constraint generation approach is used to solve the two-stage robust optimization problems. The proposed DA schedule reduces the impact of the emergency surgeries on the elective surgeries. The number of blocks allocated to the disciplines are divided over the two locations of the hospital Furthermore, the solution method of the SA problem decreases the variability in the ward substantially. Moreover, less patients can be scheduled due to the lack of available beds. This result contributes to a balanced workload. In conclusion, the proposed method improves the efficiency of the operating room planning.

1 Introduction

More hospitals are willing to implement operation research techniques in the planning process of surgeries such as mathematical programming and process analysis. This planning concerns the allocation of a patient to a surgeon, a specific date, and time in a certain operating room.

A top clinical hospital group in the Netherlands experiences long waiting lists and loss of revenue on the operating room planning. Long waiting lists (there is a waiting list for each discipline) result in a decrease of patient satisfaction and cause patients to search for another hospital. Long waiting lists are caused by lack of efficiency in the planning process but also a shortage of staff. Furthermore, the hospital has expressed the desire to automate the planning process. With the opening of a new location with operating rooms the planning of surgeries over these additional operating rooms requires further research in order to decrease the number of unused operating rooms and waiting lists of patients. Therefore, the hospital has asked to evaluate the planning process and give an advice on how to improve the planning process.

It is of interest for the hospital to improve the planning process such that patient satisfaction increases, what can be expressed in a decrease of patients on waiting lists and improved efficiency. To solve these problems a robust optimization model for this planning process is proposed.

Section 2 describes the planning problem and Section 3 presents the literature, summarizing what is already known about this subject and how this differs from the problem proposed in this thesis. Section 4 gives the mathematical formulation of the deterministic and robust optimization model. Section 5 explains the solution method. The results of the numerical experiments are described in Section 6 and a conclusion about the findings, a management advice, and the limitations of the model are given in Section 7. Furthermore, by discussing the limitations an advice is given on how to proceed with this method.

2 Problem description

The hospital group consists of hospitals at different locations in the Netherlands. Operating rooms are available at these different locations. An operating room is used by different disciplines. A discipline is a group of surgeons who perform surgeries of a certain category. An example of a discipline is gynecology. Gynecologists focus on the female reproductive system. This discipline deals with a wide range of surgeries like cesarean sections or removing tumors.

The process of operating room planning can be divided into two phases. These phases are:

- Tactical planning
- Operational planning

How many and which time slots, later referred by blocks, in which operating rooms can be allocated to a discipline is determined in the tactical planning process. A schedule is made for a period of a certain number of weeks. The hospital operates with cyclic schedules, i.e. a schedule repeats itself in every cycle. A few months before the actual period takes place the schedule is determined. To assign a discipline to a time slot and an operating room on a certain day, blocks are constructed by composing a combination of a day, time slot, and an operating room. Thus, the assignment of a discipline to a block is e.g. gynecology on Monday from 8:00 until 12:15 in operating room one. Assigning a discipline to blocks is called block scheduling. After the disciplines are assigned to the blocks, the patients from the different disciplines can be assigned to the blocks of their corresponding discipline. This process is the operational planning. Each day patients arrive at the hospital and receive a date on which the surgery takes place. A standard norm is to schedule the surgery within 6 weeks from arrival of the patient. Therefore, it is of importance to schedule the patients efficiently.

The different phases of planning are shown in Figure 1.

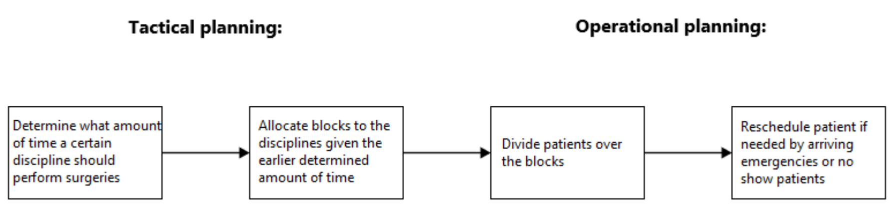


Figure 1: The process of tactical and operational planning in order of time.

The tactical planning, referred as Discipline Assignment problem (DA), determines the capacity given to each discipline. The DA problem results in a basis schedule which is used as foundation to allocate patients to the blocks of their corresponding discipline. First the problem description of the tactical planning is given. After that, the problem of the operational planning, named Surgery Assignment problem (SA), is defined. Both problems are addressed in this thesis.

2.1 Tactical planning: Discipline Assignment

In order to allocate the appropriate amount of time to a discipline, the total time of expected surgeries and the number of patients that can be scheduled in a block must be determined. When this is fixed, a schedule must be made on when and where the discipline performs surgeries.

The problem is rather complicated due to the fact it its hart to give a forecast of the number of arriving patients of each surgery type and the surgery duration. Furthermore, the number of complex surgeries increases causing surgery durations that are difficult to predict. Therefore, the total amount of time required for future surgeries can not be predicted accurately which influences the amount of time assigned to a discipline. If the time assigned to a discipline is too low, less patients are scheduled and thus the waiting lists increase. When the time assigned to the discipline is too broad, the discipline can not cover all time slots which leads to unused operating rooms. Furthermore, the following elements contribute to the level of difficulty of this problem:

- Not all operating rooms are suitable for every surgery type.
- Surgery types that can not occur after each other.
- Surgeries specifically for children until 12 years have to be scheduled in the morning. For example, the discipline ear nose throat inserts tubes in the ears of children. Such a surgery should get a morning time slot.

Each day there is a possibility that emergency patients enter the hospital. For these emergencies a block is reserved in the current schedule. It is difficult to predict the number of emergency surgeries arriving at the hospital each day and what time should be reserved for this. Emergencies have a large impact on the schedule. Therefore, a good estimation is required in order to make the impact on the other surgeries as small as possible.

An example of a DA schedule in which time slots in certain operating rooms are assigned to different disciplines is given in Figure 2.

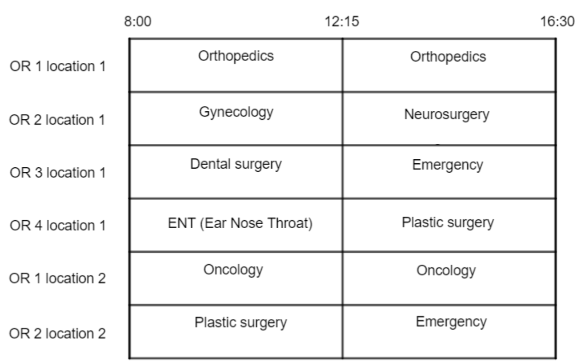


Figure 2: An example of a basis schedule (for disciplines) for a certain day at two locations for in total 6 operating rooms (OR's). In this schedule blocks are reserved for emergencies as done by Wullink et al. (2007).

2.2 Operational planning: Surgery Assignment

There is a broad operational planning process for the entire hospital with different kind of patients. There are two categories of patients, elective and non-elective patients. Elective patients are the patients that do not have the urgency to get a surgery during the arrival date and can wait to be scheduled. In this category of elective patients a distinction can be made between inpatient and outpatient patients. Inpatient means that the patients need admission to the hospital after the surgery (this means a stay in a recovery room for more than one day). Outpatient refers to the patients that have a so-called day-surgery. After the surgery the patient only needs a few hours to recover in a room thus there is no bed needed for an overnight stay at the hospital. The non-elective patients are the emergency patients who enter the hospital and require an immediate surgery. These patients need to be monitored and require at least one night of stay in the hospital. Therefore, the non-elective patients have no distinction between inpatient and outpatient.

The elective patients follow a process from diagnosis to surgery. Figure 3 shows this process.

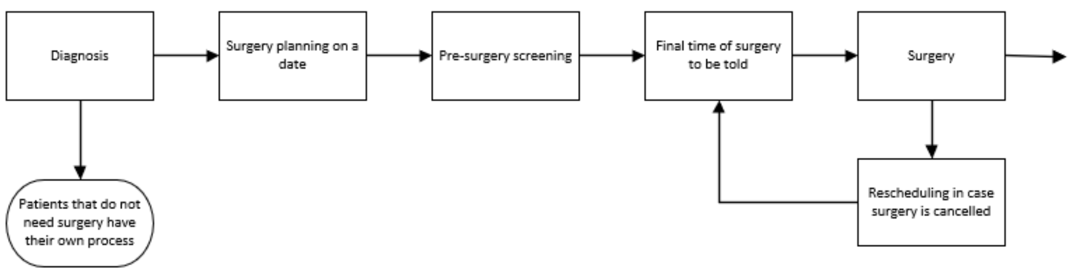


Figure 3: The process flow from diagnosis to surgery for a patient in the hospital.

The doctor or general practitioner (GP) notices a certain disease. After that, the patient goes for a consultation to the doctor to determine the required treatment. During the diagnosis the urgency, surgery type, required material, and duration of the surgery are determined. If no surgery is needed, the patient goes out of this process and follows a different process which is not considered in the operating room planning. When the treatment consists of a surgery the patient needs to be scheduled. The process of scheduling a patient consists of two phases, scheduling the date and scheduling the time. First a date is given to the patient related to the urgency, surgery type, surgeon, and duration of the surgery. Note that it may be possible for a patient to indicate whether a specific surgeon is preferred or not.

Between the moment of receiving the date of the surgery and the moment of receiving the time of the surgery the patient gets a screening. The screening contains the determination of the type of anesthesia, the ASA-class (what health condition the patient has) and other important characteristics of the patient. A few days before the surgery the patient receives the time at which the surgery will take place based on the type of anesthesia, material needed, and type of surgery.

This thesis focuses on the problem where the elective patients are scheduled to the blocks, i.e. a day, time slot, and certain operating rooms. This is also called the Surgery Assignment (SA) problem. Scheduling patients is based on urgency, the surgery duration, the required material for the surgery, and the available space in the recovery rooms. Figure 4 shows an example of a patient schedule for a specific discipline.

The grey areas for the Monday and Tuesday morning indicate the changing periods between surgeries. A changing period is the time between the end of the first surgery and the start of the second. The end of the surgery is the moment that the patient goes out of the operating room and

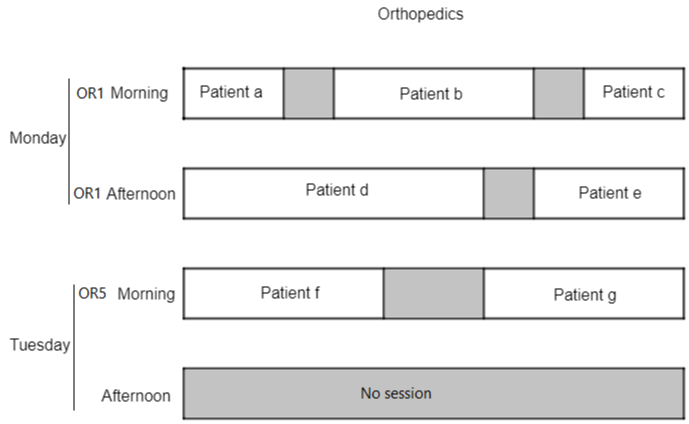


Figure 4: An example of a schedule of patients on Monday and Tuesday of a certain week for Orthopedics. The grey areas are the moments there is no surgery.

the start of the surgery is the moment that the patient arrives in the operating room. This time can vary between surgeries. On Tuesday afternoon there is no session scheduled for Orthopedics, therefore no patients of this discipline are assigned to Tuesday afternoon. Note, in this example the time slots are either morning or afternoon time slots. These two time slots are used by the hospital. Furthermore, it is possible for a discipline to have multiple operating rooms on a day and time slot.

Different surgery types have different surgery durations, Length of Stay (LoS) and required material. Only one or two surgeries can be scheduled on one day caused by long, urgent, and complex surgeries. The operating room may be idle, since no surgery fits in the remaining time of the block. Furthermore, the duration of the short surgeries is often overestimated. Therefore, the operating room sometimes is unused while short surgeries could be executed in this time. Typically, the surgeon finishes his surgeries far before the end of the time slot due to the low accuracy of the prediction of the surgery durations.

When allocating the patients to the blocks the following elements influence the level of difficulty of the problem:

1. The required material of the surgery must be available for the time slot. For example when an intra-thoracic tumour must be removed an x-ray machine is required which is not available in every operating room.

2. Patients need to be scheduled before their 'due'-date (the date before the surgery has to be performed).

3. Day-surgeries, surgeries for which the patient can recover during the day, need to be scheduled in the morning.

4. Children require surgery on specific times. For example children until the age of twelve should be scheduled at the first time slot.

5. Certain surgeries can only be executed a certain number of times per day. For example a navigated biopsy can only be done twice a day.

6. The number of beds available in the ward for the patients that require recovery of more than one day after the surgery.

The recovery room where patients go to after surgery is called a ward. Certain discipline have a special ward. There also is a ward for the day-surgeries where patients of all disciplines go to. In each ward a certain number of beds is available. Furthermore, it is preferred to have a stable patient flow to the ward in order to have a stable working pressure in the ward. This increases the difficulty of the problem in such a way that it is not preferred to have a large group of patients at the ward on Monday and a small group of patients at the ward on Friday. Furthermore, some patients need additional care (or more care than others). The level of required care is determined when the patient arrives at the ward. Therefore, it is hard to take this into account when scheduling of patients.

As time passes the allocation of patients to a block changes dynamically. New patients are assigned to the time slots and some are removed due to different circumstances. Examples of rescheduling patients can be arriving emergencies, a surgeon who is not able to perform surgery or the patient requests to cancel the surgery or asks for another surgery date. This means that the patient can be rescheduled or cancelled depending on the reason of removal of the schedule. Typically, patients are only rescheduled when no other solution for the surgeon or arriving emergency patient can be found. When the surgery date is fixed, the surgery time is determined. One or two days before the surgery day the elective patients get a final confirmation of the surgery time.

During the day emergencies arrive. These emergency patients are scheduled on daily basis and therefore are not considered in this problem. As mentioned, blocks in the DA schedule are reserved for emergency patients.

The main goal of this thesis is to make a model for both the allocating the disciplines and patients to blocks in order to increase the patient satisfaction. To quantify this, objectives like increasing the utilization rate of the operating rooms, decreasing the number of delays and postponements of the surgeries, and decreasing the variability of utilization of the ward are used. The utilization rate of operating rooms and variability in the ward indirectly affects the patient satisfaction. Since a high utilization rate of operating rooms ensures a high throughput and thus a decrease in waiting lists, the patient satisfaction increases. Furthermore, a low variability in the ward ensures enough staff to provide good care for the patients in the ward. Therefore, patients receive the required care faster and this improves the patient satisfaction.

The problem as given above has raised the following research questions:

- How to efficiently schedule the disciplines and patients?
- How to consider the emergencies?
- How to incorporate other uncertainty parameters, such as the surgery duration?
- What is the impact of the DA schedule to the SA schedule?

The first question is addressed by defining a Robust Optimization model and solving this problem to optimality. The deterministic and robust formulation of this problem are proposed in Section 4. The implementation of the emergencies and other uncertainty parameters is explained in Sections 4 and 5. The impact of the uncertainty parameters of both problems are considered in the results in Section 6. The impact of the basis schedule of the DA problem to the SA schedule compared to the basis schedule the hospital is currently using is given in Section 6.

3 Literature review

Many papers tackle the problem of operating room planning. Literature reviews are written by Brecht Cardoen (2009) and Zhu et al. (2019). Brecht Cardoen (2009) evaluates the literature on the fields of decision delineation, patient characteristics and performance measures. Furthermore, the type of analysis, solution method and incorporation of uncertainty are analyzed. Zhu et al. (2019) gives a classification of the examined problems based on the decision level, patient characteristics, problem setting, uncertainty, mathematical models and solution methods. Substantial literature concerning scheduling disciplines and surgeries are reviewed. Furthermore, only a small part integrates the tactical and operational planning of operating rooms. First, a review of all literature of the two phases of planning and the solution method used in this thesis, a robust program, are given. After that, the literature is compared to the problem described in this thesis.

3.1 Combined tactical and operational planning

Guido and Conforti (2017) propose a mixed integer linear program (MILP) with multiple objectives to determine the case mix planning, i.e. the operating room time allocated to each discipline and the set of scheduled surgeries. This model considers elective patients, but can be extended on both elective and non-elective patients. The MILP is solved by a genetic algorithm. Guido and Conforti (2017) made the conclusion that an objective can only be improved by deteriorating another one. The objectives that are used are maximizing the number of operations, the total priority of scheduled surgeries and the total weight of scheduled surgeries, minimizing the overtime and the penalty for overtime. Testi et al. (2007) combine the DA problem with the SA problem in a three-phase approach and only look at the elective patients. This approach is also used by Santibáñez et al. (2007).

3.2 Tactical: Discipline Assignment

Multiple objectives like minimizing the workload, minimizing the number of rooms assigned to each surgical discipline, minimizing the deviation between the shifts assigned to each discipline and minimizing the deviations of the weekly OR time are used by Marques et al. (2019). Marques et al. (2019) propose a mixed integer program (MIP) with the multiple objectives mentioned. Beliën et al. (2008) also introduce a MIP with three objectives. The objectives are optimizing the bed occupancy at the downstream units, minimizing the sharing of operating rooms and minimizing the fluctuations in the DA schedule.

After surgery the patients need to recover. Sometimes patients need intensive treatment. This happens at the Intensive Care Unit (ICU). If there is no intensive treatment needed, the patients go directly to a ward. The flow to the ICU and wards is very important because there must be capacity available at these departments. Furthermore, it is recommended to plan efficiently such that the flow to the ICU and ward does not vary much. Kumar et al. (2018) propose a stochastic program which considers the downstream to the ICU. By sensitivity analysis the conclusion is made that this results in a robust DA schedule. Fügener et al. (2014) consider the downstream to all units but maximizes the revenue as goal.

It is possible to divide the disciplines into different surgical cases. Oostrum et al. (2011) describes a method based on Ward's method to divide the surgical cases. The method was successfully implemented at the Beatrix hospital in the Netherlands.

Non-elective patients are often not considered in the DA schedule. Wullink et al. (2007) introduce a discrete event simulation to model the real life situation with non-elective patients. Two approaches are formulated, one for reserving time for emergencies and one without reserving time for emergencies. The results show that it is more efficient to reserve a time slot for emergencies such that the planning of the elective patients is hardly disturbed.

3.3 Operational: Surgery Assignment

Each day elective and non-elective patients are undergoing surgery. Elective patients need to be scheduled at a specific date and time in the DA schedule. Non-elective patients need to be scheduled during the day. The impact of non-elective patients in the day-schedule is explored. Duma and Aringhieri (2019) provide a hybrid model that uses discrete event simulation to examine the impact of approaches of patient scheduling including the non-elective patients. The analysis shows that a hybrid policy is better while shared (emergencies go between elective patients) and dedicated (emergencies have an own operating room) policies give good results as well. Ravnskjær Kroer et al. (2018) give a daily based two-stage stochastic model to plan elective and non-elective patients. Also Lamiri et al. (2008) propose a stochastic program for planning elective (and non-elective) patients. Both papers use Monte-Carlo simulation but Ravnskjær Kroer et al. (2018) also give two heuristics as solution approach. Abedini et al. (2016) examine the bin packing model for allocating surgeries (patients) to operating rooms (OR) for different rules, the LPT (longest processing time first) and their own multi-step approach, the PTD (priority-type-duration) rule. Their rule outperforms the LPT rule. Summarized, both elective and non-elective patients are included, the uncertainty of surgery duration is taken into account and different methods like the bin packing model with different rules are proposed.

3.4 Robust programming

Neyshabouri and Berg (2016) propose a two-stage robust program to schedule surgeries considering uncertainty in the surgery duration and the downstream to the Surgical Intensive Care Unit (SICU). This robust program is solved by an adapted column-and-constraint generation method. Addis, Carello, and Tanfani (2014) address the problem of the assignment of a subset of patients to the available Operating Room blocks during the considered time horizon. Their goal is to minimize a measure of the waiting time. This is done by introducing a penalty corresponding to tardiness, urgency and waiting time. For this problem a robust optimization model is proposed with uncertain surgery durations. They use a set of randomly generated scenarios, by assuming a lognormal distribution for the surgery durations, in order to asses the performance. Addis, Carello, Grosso, et al. (2015) describe the advantages of a cardinality constrained approach for robust programming. Furthermore, Denton et al. (2010) propose a two-stage stochastic program and its robust counterpart for the assignment of surgeries to ORs. They include the cost of opening an OR. Therefore, the goal is to minimize the cost of opening ORs combined with the variable overtime cost. Their result is that the robust optimization performs as well as an heuristic and even better than the stochastic optimization.

3.5 Conclusion

In this thesis the DA problem and SA problem are formulated by introducing a two-stage robust optimization model. These models are solved with a column-and-constraint generation approach which uses the budget of uncertainty. Both problems are solved sequentially which means this approach is different from Guido and Conforti (2017), Testi et al. (2007) and Santibáñez et al. (2007). The problem description of this thesis concerns the following objectives for the DA problem: maximizing the throughput to the operating room and minimizing the penalty for unsatisfied demand for both emergencies and disciplines. The multiple objectives that are chosen in the paper of Marques et al. (2019) are minimizing the workload combined with minimizing the number of rooms assigned to each surgical discipline, the deviation between the shifts assigned to each discipline, and the deviations of the weekly OR time. The objectives from the paper of Beliën et al. (2008) are optimizing the bed occupancy at the downstream units, minimizing the sharing of operating rooms, and the fluctuations in the DA schedule. These objectives differ from the objectives proposed in this thesis. The objectives proposed in this thesis also differ from the objective used by Fügener et al. (2014). The mathematical model in this thesis does not use the optimization of the downstream to the ICU in the DA problem. Therefore, this thesis differs from Kumar et al. (2018). As shown by Wullink et al. (2007), blocks are reserved for emergency in this thesis.

Abedini et al. (2016) assume that patients/surgeries can be scheduled in any operation room which is not the case for this thesis. This thesis combines all results of the papers of Duma and Aringhieri (2019), Lamiri et al. (2008) and Ravnskjær Kroer et al. (2018).

The papers of Neyshabouri and Berg (2016), Addis, Carello, and Tanfani (2014) and Denton et al. (2010) only propose a robust program for the operational planning problem from which non of the problems includes the variability in the ward. Moreover, the uncertain parameter in Denton et al. (2010) and Addis, Carello, and Tanfani (2014) is the surgery duration which means that that the uncertainty in the Length of Stay is not included. Furthermore, Neyshabouri and Berg (2016) considers the uncertainty Length of Stay in the SICU which is only one unit where the patients can recover. This thesis covers all units where patients can recover. The solution method which is described in the paper of Neyshabouri and Berg (2016) is used in this thesis.

Many papers address one of the problems, determining the DA schedule or scheduling the patients but this thesis focuses on both problems and the relation between these models.

In Table 1 the literature is summarized.

Article	Strategical, Tactical or Operational planning	Kind of patients	Objective	Solution method	Mathematical model
Beliën et al., 2008	Tactical planning	Elective	Three objectives (bed occupancy, surgeons concentrated, repetitive schedule)	Decision support system	MIP (multi-objective linear and quadratic OP)
Guido and Conforti, 2016	Strategic, tactical and operational planning	Elective	Multiple objectives (number of operations, overtime)	Genetic Algorithms	MIP
Tanfani, 2007	Tactical and operational planning	Elective	Minimize sum weighted waiting times	Construct heuristic	Binary LP
Santibáñes et al., 2009	Tactical and operational planning	Elective	tradeoffs between OR availability, bed capacity, surgeons' booking privileges, and wait lists	Insight in capacity	MIP
Marques et al., 2018	Tactical planning	Elective	Multiple objectives (workload, number of rooms, deviation shifts and OR time)	Different solutions for different runs	MIP
Kumar et al., 2018	Tactical planning	Elective	Optimize downstream to ICU	Good approach for Robust DA schedule	Stochastic program
Fügener, 2015	Tactical planning	Elective	Maximize revenue	Significant result in rev. by opt. DA schedule	MIP
Oostrum et al., 2008	Tactical planning	Elective and non-elective	Minimizing dummy clusters	Clusters of surgery types	Ward's hierarchical cluster method
Wullink et al., 2017	Tactical planning	Elective and non-elective	Minimize cancelled patients	Approach with block for emergencies	Discrete event simulation

Article	Strategical, Tactical or Operational planning	Kind of patients	Objective	Solution method	Mathematical model
Duma and Aringhieri, 2018	Operational planning	Elective and non-elective	Impact on elective schedule	Discrete event simulation	- (Straight to algorithm)
Kroer et al., 2018	Operational planning	Elective and non-elective	Minimizing overtime	Two solution heuristics	Two-stage stochastic model
Lamiri et al., 2006	Operational planning	Elective (and non-elective)	Minimizing cost for elective surgeries and overtime	Monte Carlo simulation for non-elective cases	Two-stage stochastic model for elective cases
Abedini et al., 2016	Operational planning	Elective	Minimizing the total cost (overtime, idle, regular and set-up cost)	PTD rule performs well	Bin packing model
Denton et al., 2010	Operational planning	Elective	Minimizing overtime and cost of opening OR	Exact solving	Two-stage stochastic program and Robust counterpart
Addis et al., 2014	Operational planning	Elective	Minimizing measures of waiting time	Use randomly generated scenarios	Robust program
Neyshabouri and Berg, 2016	Operational planning	Elective	Minimizing cost of patient admission, overtime and cost of unsatisfying capacity	Column-and-constrained generation method	Robust program

Table 1: Description of characteristics used in literature.

4 Mathematical program

In this section, the mathematical formulation of the deterministic and two-stage robust program of the Discipline Assignment (DA) problem (Section 4.1) and the Surgery Assignment (SA) problem (Section 4.2) are described.

4.1 Discipline Assignment problem

First, Section 4.1.1 gives the notation which is used in both the deterministic and the two-stage robust program. After that, Section 4.1.2 explains the deterministic formulation, in which the uncertain parameter is fixed. In this formulation the objective and all constraints are described. The objective and constraints are used in the two-stage robust optimization problem. Moreover, Section 4.1.3 gives the formulation of the two-stage optimization problem which considers the uncertainty by proposing a set of possible values the uncertain parameter can take.

4.1.1 Notation

The following notation is used in the formulation of the MIP for the DA problem.

Table 2: Notation of the DA problem.

	Sets with indices
$b_1 \in B_1$	the set of days in a planning cycle
$b_2 \in B_2$	the set of possible time slots; 1 = morning, 2 = afternoon
$b_3 \in B_3$	the set of ORs
$\mathbf{b} = (b_1, b_2, b_3) \in B$	the set of blocks, i.e. day b_1, time slot b_2, and operating room b_3
$c \in C$	the set of disciplines except the emergency discipline
	Deterministic parameters
$q_{\mathbf{b},c}$	compatibility of a block $\mathbf{b}$ with discipline c
$h_{b_2,c}$	the number of times discipline c has to be assigned to time slot b_2
c_c^{T}	the cost of allocating a block to discipline c
$c_{b_1}^{\mathrm{E}}$	the cost of not satisfying the number of required blocks for emergencies on day b_1
c_c^{P}	the cost of not being able to schedule all patients during a planning cycle per discipline c
	Stochastic parameters
em_{b_1}	the number of patients that received surgery in day b_1
pt_c	the required number of blocks for the patients that require surgery in a planning cycle per discipline c
	Decision variables
$x_{\mathbf{b},c}$	$= \begin{cases} 1 & \text{if discipline } c \text{ is allocated to block } \mathbf{b} \\ 0 & \text{otherwise} \end{cases}$
α_c	the number of blocks the schedule lacks for discipline c
β_{b_1}	the number of blocks the schedule lacks for emergencies on day b_1

4.1.2 Deterministic formulation

In the following part, the description of the (stochastic) parameters and decision variables is given. The parameters and decision variables are divided into subgroups: disciplines and emergencies.

4.1.2.1 Disciplines

Different disciplines with certain specialities are available in the hospital with each elective and non-elective patients. The elective patients are scheduled by their discipline and the non-elective patients are scheduled by the emergency discipline which is indicated by e. The set of disciplines C consists of all disciplines except the emergency discipline. In the following description of the parameters, variables and constraints a distinction is made between including and excluding the emergency discipline.

One should decide on which block is assigned to which discipline. Therefore, the decision variable is:

$$x_{\mathbf{b},c} = \begin{cases} 1 & \text{if discipline } c \in C \cup \{e\} \text{ is allocated to block } \mathbf{b} \in B \\ 0 & \text{otherwise} \end{cases}$$

Certain disciplines require specific instrumentation or a special construction of equipment in an operating room. Therefore, some operating rooms are specially designed for these disciplines. Furthermore, not every discipline performs surgery on every location. Hence, the following parameter is introduced:

$$q_{\mathbf{b},c} = \begin{cases} 1 & \text{if discipline } c \in C \cup \{e\} \text{ is qualified or compatible for block } \mathbf{b} \in B \\ 0 & \text{otherwise} \end{cases}$$

To ensure every discipline performs surgery in an operating room which is suitable, the following constraint is introduced:

$$x_{\mathbf{b},c} \leq q_{\mathbf{b},c} \; \forall \; \mathbf{b} \in B, c \in C \cup \{e\} \tag{1}$$

Furthermore, some disciplines perform surgeries especially for children. For example, the discipline Ear Nose Throat places tubes in the ears of children. Recall that children need to receive surgery in the morning. Therefore, the discipline Ear Nose Throat requires a block in the morning. In addition, it has advantages to schedule patients, that will recover during the day, in the morning. Since every discipline has patients that can recover during the day, every discipline needs a block in the morning. This results in the following parameter:

$$h_{b_2,c} = \text{ the number of times discipline } c \in C \text{ has to be assigned to time slot } b_2 \in B_2$$

In order to satisfy the required number of blocks for each time slot, this constraint is proposed:

$$\sum_{b_1 \in B_1, b_3 \in B_3} x_{\mathbf{b},c} \geq h_{b_2,c} \; \forall \; c \in C, b_2 \in B_2 \tag{2}$$

Note that the condition that a discipline should be assigned to at least one block is covered by the fact that $h_{1,c}$ is greater or equal to one since each discipline has patients that recover during the day. Every block should be assigned to one discipline. Therefore, the following constraint is introduced:

$$\sum_{c \in C \cup \{e\}} x_{\mathbf{b},c} \leq 1 \; \forall \; \mathbf{b} \in B \tag{3}$$

The required number of blocks for a discipline depends on the number of patients who require surgery in a planning cycle. The number of patients that require surgery is uncertain. Therefore, the required number of blocks for each discipline is a stochastic parameter. We denote this stochastic parameter, in this case deterministic, by:

$$pt_c = \text{ the required number of blocks for the patients that require surgery in a planning cycle per discipline } c \in C$$

Moreover, the total number of patients that arrive in the planning cycle might be too high. Hence, the patients cannot fit into the available blocks in the planning cycle. To avoid infeasibility, the auxiliary parameter α_c, which is a decision variable, is introduced in order to penalize the number of patients that cannot be scheduled. To take the required number of blocks per discipline into account and give a value to the insufficient number of blocks per discipline, the following constraint is introduced:

$$\sum_{\mathbf{b}\in B} x_{\mathbf{b},c} + \alpha_c \geq pt_c \ \forall \ c \in C \tag{4}$$

4.1.2.2 Emergencies

The arriving emergencies require an operating room. Therefore, blocks are reserved for emergencies each day. The number of arriving emergency patients is uncertain. For the deterministic model, the number of blocks reserved for emergencies per day is fixed. The following parameter is introduced:

$$em_{b_1} = \text{ the number of blocks required for emergencies on day } b_1$$

An insufficient number of blocks reserved for emergencies on day b_1 must be penalized. The decision variable corresponding to the number of insufficient emergency blocks per day is defined as β_{b_1}. The shortage of blocks for emergencies is given by the constraint:

$$\sum_{b_2\in B_2, b_3\in B_3} x_{\mathbf{b},e} + \beta_{b_1} \geq em_{b_1} \ \forall \ b_1 \in B_1 \tag{5}$$

4.1.2.3 Objective

The first part of the objective is maximizing the throughput. Maximizing the throughput, i.e. maximizing the number of patients that can receive surgery, can be achieved by maximizing a reward for each discipline. Since the other two parts of the objective are minimization goals, the maximization of the reward for each discipline is described as a minimization as well. This results in minimizing a cost penalty based on a priority of the allocation of the disciplines. The priority depends on the number of patients that can be scheduled during a block and the urgency of surgeries of that discipline. The penalty is denoted by c_c^{T}. Note that this penalty can take negative values since this is actually a reward per discipline. The resulting first part of the objective is:

$$Z_1 = \sum_{\mathbf{b}\in B, c\in C\cup\{e\}} c_c^{\mathrm{T}} x_{\mathbf{b},c}$$

The second part of the objective is a minimization of not fulfilling the blocks that are necessary for each discipline. The penalty costs for not satisfying the number of blocks required per discipline in a planning cycle is denoted by c_c^{P}. The second part of the objective is formulated as:

$$Z_2 = \sum_{c\in C} c_c^{\mathrm{P}} \cdot \alpha_c$$

At last, an insufficient number of blocks reserved for emergencies on day b_1 must be penalized. When the blocks assigned to emergencies do not satisfy the required number of blocks for emergencies on day b_1, the penalty costs $c_{b_1}^{\mathrm{E}}$ are incurred. The last part of the objective is:

$$Z_3 = \sum_{b_1\in B_1} c_{b_1}^{\mathrm{E}} \cdot \beta_{b_1}$$

The complete objective is formulated as:

$$\min \sum_{i=1}^{3} Z_i = \min \sum_{\mathbf{b} \in B, c \in C \cup \{e\}} c_c^{\mathrm{T}} x_{\mathbf{b},c} + \sum_{c \in C} c_c^{\mathrm{P}} \cdot \alpha_c + \sum_{b_1 \in B_1} c_{b_1}^{\mathrm{E}} \cdot \beta_{b_1}$$

Each objective is analyzed and an order between the objectives is determined, i.e. the importance of the objective is evaluated. The hospital determined that an order could be given to the objectives by giving values to the penalties that reflect this order. The hospital determined the following order of the objectives:

1. Minimizing the lack of blocks per discipline.
2. Minimizing the lack of blocks for emergencies.
3. Minimizing the the cost of allocating a block to a discipline.

Recall that proposing a negative cost penalty for minimizing the costs of allocating a block to a discipline for the third objective implies maximizing the reward per discipline. Therefore, the penalties take values by satisfying $|c_c^{\mathrm{T}}| \leq c_{b_1}^{\mathrm{E}} \leq c_c^{\mathrm{P}}$.

4.1.2.4 Constraints

Based on the situation of the hospital, the problem has the constraints (1)-(5). Furthermore, the following binary and integer constraints of the decision variables are introduced:

$$x_{\mathbf{b},c} \in \{0,1\} \ \forall \ \mathbf{b} \in B, c \in C \cup \{e\} \tag{6}$$

$$\alpha_c \in \mathbb{N}_0 \ \forall \ c \in C \tag{7}$$

$$\beta_{b_1} \in \mathbb{N}_0 \ \forall \ b_1 \in B_1 \tag{8}$$

with $\mathbb{N}_0$ the set of positive integers where zero is included.

4.1.3 Two-stage robust optimization problem formulation

In the deterministic formulation all uncertain parameters take a fixed value. However, in practice the value of the uncertain parameter can vary. One should make a decision before the value of the uncertain parameters is known. This uncertain parameter can take different values in a set. A robust optimization model ensures that a decision is made such that it is feasible for all values of the uncertain parameter and optimal for the worst-case objective. The robust model that is proposed, is a two-stage robust optimization model. In the first stage, one should make a decision based on the parameters that are known in the current state, i.e. the parameters that are not uncertain. The first stage decision corresponds to the main result that the problem must give. This decision variable is called a *here and now variable.* In the second stage the uncertain parameters become known. In this stage, the value of the decision variables corresponding to the uncertain parameters can be established based on the first stage decision variables and the value of the uncertain parameter. These second stage decision variables are called *wait and see variables.* For the DA problem this means that in the first stage the decision about the assignment of the disciplines to the blocks has to be made. In the second stage the two uncertain parameters for the emergencies (em_{b_1}) and arriving patients (pt_c) become known, i.e. the uncertain parameters take values in an uncertainty set. The goal is to minimize the worst-case scenario for the unsatisfied demand of emergencies and elective patients. In the next part, the *budget of uncertainty* and the uncertainty sets are introduced.

4.1.3.1 Uncertainty sets

The uncertainty set is the set with possible values for the corresponding uncertain parameter. For the DA problem, the deterministic formulations of the uncertain parameters are em_{b_1} for the emergency blocks and pt_c for the blocks for elective patients. The stochastic parameters that take values in the uncertainty set of the uncertain parameters for the emergency block and block for elective patients are defined as $\tilde{em}_{b_1}$ and $\tilde{pt}_c$ respectively. The number of emergency blocks that are required to be reserved per day can deviate at most $\hat{em}_{b_1}$ from its minimum value $\bar{em}_{b_1}$. This results in the following set of possible values that the uncertain parameter $\tilde{em}_{b_1}$ can take: $\{\bar{em}_{b_1}, ..., \bar{em}_{b_1} + \hat{em}_{b_1}\}$, i.e. $\tilde{em}_{b_1}$ has minimum value $\bar{em}_{b_1}$ and maximum value $\bar{em}_{b_1} + \hat{em}_{b_1}$. The number of blocks reserved for emergencies is integer valued. The number of blocks required for arriving patients can deviate at most $\hat{pt}_c$ from its minimum value $\bar{pt}_c$. Therefore, the uncertain parameter $\tilde{pt}_c$ takes values in the set $\{\bar{pt}_c, ..., \bar{pt}_c + \hat{pt}_c\}$. Note that $\tilde{pt}_c$ only takes integer values as well. The maximum value of the uncertain parameter gives the worst solution. Since this maximum value does not occur often, one should prefer that a certain number of parameters takes the maximum value. In order to define which value the uncertain parameter takes, the budget of uncertainty is introduced and the uncertainty set is adjusted.

4.1.3.2 Budget of uncertainty

The budget of uncertainty is defined as the number of parameters that can deviate from the expected value of the uncertain parameter, i.e. the budget of uncertainty represents the total maximum deviation compared to the minimum value of the uncertain parameter. The value of the budget of uncertainty has to be proposed by the decision maker. Each uncertain parameter is assumed to deviate with a finite number of the chosen value. The budget of uncertainty of the emergencies and arriving patients per discipline are Γ_{em} and Γ_{pt} respectively. To ensure that the total maximum deviation compared to the expected value is at most the budget of uncertainty, the following constraints are introduced:

$$\sum_{b_1 \in B_1} \frac{\tilde{em}_{b_1} - \bar{em}_{b_1}}{\hat{em}_{b_1}} \leq \Gamma_{em}$$

$$\sum_{c \in C} \frac{\tilde{pt}_c - \bar{pt}_c}{\hat{pt}_c} \leq \Gamma_{pt}$$

These constraints corresponding to the budget of uncertainty are used to define the uncertainty sets of $\tilde{em}_{b_1}$ and $\tilde{pt}_c$ respectively.

The budget of uncertainty determines the size of the uncertainty set. When the budget of uncertainty is high, the uncertainty set is large since the uncertain parameter can take many different values. We assume that the uncertainty in the number of emergency blocks and the number of arriving patients are independent. Hence, the following uncertainty sets for the uncertain parameters using the budget of uncertainty are defined:

$$U_{em} = \left\{ \tilde{em} : \tilde{em}_{b_1} \in \{\bar{em}_{b_1}, ..., \bar{em}_{b_1} + \hat{em}_{b_1}\right\} \forall b_1, \sum_{b_1 \in B_1} \frac{\tilde{em}_{b_1} - \bar{em}_{b_1}}{\hat{em}_{b_1}} \leq \Gamma_{em}\}$$

$$U_{pt} = \left\{ \tilde{pt} : \tilde{pt}_c \in \{\bar{pt}_c, ..., \bar{pt}_c + \hat{pt}_c\} \;\; \forall c, \; \sum_{c \in C} \frac{\tilde{pt}_c - \bar{pt}_c}{\hat{pt}_c} \leq \Gamma_{pt} \right\}$$

The uncertain sets are polyhedral by definition. Besides defining the uncertainty sets, the first and second stage are formulated by the method explained in 4.1.3. The first stage of the two-stage robust optimization problem of the DA problem is:

$$
\begin{aligned}
\min \quad & \sum_{\mathbf{b} \in B, c \in C \cup \{e\}} c_c^{\mathrm{T}} x_{\mathbf{b},c} + OPT[P(x, \Gamma_{pt}, \Gamma_{em})] & \\
s.t.\ & x_{\mathbf{b},c} \leq q_{\mathbf{b},c} & \forall\ \mathbf{b} \in B, c \in C \cup \{e\} \\
& \sum_{b_1 \in B_1, b_3 \in B_3} x_{\mathbf{b},c} \geq h_{b_2,c} & \forall\ c \in C, b_2 \in B_2 \\
& \sum_{c \in C \cup \{e\}} x_{\mathbf{b},c} \leq 1 & \forall\ \mathbf{b} \in B \\
& x_{\mathbf{b},c} \in \{0,1\} & \forall\ \mathbf{b} \in B, c \in C \cup \{e\}
\end{aligned}
$$

where $OPT[P(x, \Gamma_{pt}, \Gamma_{em})]$ is the optimal solution to the second stage problem. The first stage of the two-stage robust optimization problem consists of the part of the objective and constraints which do not contain or are dependent of the uncertain parameters. Despite of the dependence of the first stage variable $x_{\mathbf{b},c}$ on the uncertain parameters, the variable can get a value by the first stage constraints. The second stage contains the part of the objective and constraints that are left over. In this stage, the first stage decision variable $x_{\mathbf{b},c}$ is a fixed value, i.e. the outcome of the first stage. The second stage is formulated as:

$$
\begin{aligned}
\max_{\tilde{em} \in U_{em}, \tilde{pt} \in U_{pt}} \quad \min & \left(\sum_{c \in C} c_c^{\mathrm{P}} \cdot \alpha_c + \sum_{b_1 \in B_1} c_{b_1}^{\mathrm{E}} \cdot \beta_{b_1} \right) & \\
s.t. & \sum_{b_2 \in B_2, b_3 \in B_3} x_{\mathbf{b},e} + \beta_{b_1} \geq \tilde{em}_{b_1} & \forall\ b_1 \in B_1 \\
& \sum_{\mathbf{b} \in B} x_{\mathbf{b},c} + \alpha_c \geq \tilde{pt}_c & \forall\ c \in C \\
& \beta_{b_1} \in \mathbb{N}_0 & \forall\ b_1 \in B_1 \\
& \alpha_c \in \mathbb{N}_0 & \forall\ c \in C
\end{aligned}
$$

4.2 Surgery Assignment problem

The result of the first problem is the input of the problem where the surgeries are assigned to blocks, i.e. the decision variable $x_{\mathbf{b},c}$ is now used as input parameter for the SA problem. The SA problem is solved for a smaller planning cycle than the DA problem. If the planning cycle of the DA and SA problem are the same, patients with an urgency of one week that arrive after determining the schedule of the upcoming month, do not receive surgery on time. Given that they cannot be scheduled in that month, since the schedule is already determined, these patients have to wait another month. Therefore, a new planning cycle $\hat{B_1}$ is introduced. We define the input parameter as $x^{\mathrm{I}}_{\hat{\mathbf{b}}}$ with $\hat{\mathbf{b}} = (\hat{b_1}, b_2, b_3, c)$. Furthermore, the Length of Stay of patients can be longer than the planning cycle. In order to schedule the patients on time, a second planning cycle, B_1', is introduced in order to ensure that the Length of Stay can be larger than the current planning cycle. Since only the elective patients are being scheduled in advance, the emergency discipline is not taken into account in this problem. This means that $c \in C$ for all parameters, variables, and constraints in the SA problem. Furthermore, the planning cycle of the SA problem differs from the planning cycle of the DA problem.

The structure of describing the DA problem is used to explain the SA problem. The notation is explained in Section 4.2.1. Section 4.2.2 describes the deterministic formulation of the SA problem and Section 4.2.3 gives the two-stage robust optimization problem formulation.

4.2.1 Notation

To following notation is used for the sets, (stochastic) parameters and decision variables.

Table 3: Notation of the Surgery Assignment problem

	Sets
$a \in A$	the set of materials
$\hat{b_1} \in \hat{B_1}$	the set of days in a planning cycle
$s \in S$	the set of surgeons
$w \in W$	the set of wards (recovery rooms)
$W^{\mathrm{day}} \subset W$	the set of day recovery wards
$p \in P$	the set of patients that need to be scheduled
$\hat{\mathbf{b}} = (\hat{b_1}, b_2, b_3, c) \in \hat{B} \cup \{\mathbf{b}'\}$	the set of blocks, i.e. day $\hat{b_1}$, time slot b_2, operating room b_3, and discipline c ($\mathbf{b}'$ is a dummy block for postponed patients)
	Deterministic parameters
ψ_a	the number of items of material a
tb_w	the total number of beds in ward w
$ob_{b_1',w}$	the number of occupied beds at day b_1' in ward w
γ	duration of changing period
τ	the duration of a block
$mt_{a,p}$	$= \begin{cases} 1 & \text{if patient } p \text{ needs material } a \\ 0 & \text{otherwise} \end{cases}$
$cp_{c,s}$	$= \begin{cases} 1 & \text{if surgeon } s \text{ is from discipline } c \\ 0 & \text{otherwise} \end{cases}$
k_p	$= \begin{cases} 1 & \text{if patient } p \text{ is a child (zero till twelve years)} \\ 0 & \text{otherwise} \end{cases}$
$sr_{s,p}$	$= \begin{cases} 1 & \text{if surgeon } s \text{ can perform the surgery of patient } p \\ 0 & \text{otherwise} \end{cases}$
$wr_{p,w}$	$= \begin{cases} 1 & \text{if patient } p \text{ should go to ward } w \\ 0 & \text{otherwise} \end{cases}$

	Deterministic parameters	
ld_p	latest day patient p can be scheduled	
$x^{\mathrm{I}}_{\hat{\mathbf{b}}}$	$=\begin{cases}1 & \text{if discipline } c \text{ is allocated to day } \hat{b_1}\text{, time slot } b_2\text{, and operating room } b_3\\ 0 & \text{otherwise}\end{cases}$	
$c^{\mathrm{L}}_{\hat{\mathbf{b}},p}$	the penalty if patient p is scheduled in block $\hat{\mathbf{b}}$	
$c^{\mathrm{O}}_{\hat{\mathbf{b}}}$	cost for the minutes overtime of block $\hat{\mathbf{b}}$	
$c^{\mathrm{W}}_{b'_1,w}$	cost for the difference in occupied beds per day at day b'_1 in ward w	
	Stochastic parameters	
d_p	the duration of the surgery of patient p in minutes	
l_p	the number of days patient p should stay in the hospital	
	Decision variables	
$y_{\hat{\mathbf{b}},p}$	$=\begin{cases}1 & \text{if patient } p \text{ is allocated to block } \hat{\mathbf{b}}\\ 0 & \text{otherwise}\end{cases}$	
$i_{\hat{\mathbf{b}},s}$	$=\begin{cases}1 & \text{if surgeon } s \text{ performs surgeries in block } \hat{\mathbf{b}}\\ 0 & \text{otherwise}\end{cases}$	
$v_{b'_1,p,w}$	$=\begin{cases}1 & \text{if patient } p \text{ requires to be at ward } w \text{ on day } b'_1\\ 0 & \text{otherwise}\end{cases}$	
$o_{\hat{\mathbf{b}}}$	= the duration that exceeds the total duration of the block $\hat{\mathbf{b}}$	
$r_{b'_1,w}$	= the absolute difference of the number of patients at ward w between day b'_1 and b'_1+1	

4.2.2 Deterministic formulation

In order to define all constraints, an explanation about the parameters and decision variables is given. The explanation is divided in five classes, patients, materials, surgeons, wards, and blocks.

4.2.2.1 Patient information

Each patient has the following information:

$$k_p = \begin{cases}1 & \text{if patient } p \text{ is a child (zero till twelve years)}\\ 0 & \text{otherwise}\end{cases}$$
$$ld_p = \text{ latest day patient } p \text{ can be scheduled}$$
$$d_p = \text{ the duration of the surgery of patient } p \text{ in minutes}$$
$$l_p = \text{ the number of days patient } p \text{ should stay in the hospital}$$

To ensure that children receive surgery in the morning, the parameter k_p is given. The date ld_p indicates the latest date before which the patient needs to receive surgery. In this formulation the duration of a surgery is deterministic but in practice the surgery duration is uncertain and depends on a lot of factors. The same holds for the Length Of Stay which is uncertain and depends on how the surgery went, i.e. did complications occur, how does the patient reacts on the surgery, and how quickly the patient recovers.

The decision for the patient is the block to which the patient is assigned, i.e. a surgery of a patient is assigned to a block. This decision is denoted by:

$$y_{\hat{\mathbf{b}},p} = \begin{cases}1 & \text{if patient } p \in P \text{ is allocated to block } \hat{\mathbf{b}} \in \hat{B} \cup \{\mathbf{b}'\}\\ 0 & \text{otherwise}\end{cases}$$

A dummy block $\mathbf{b}'$ is introduced for all patients that cannot be scheduled in the current planning cycle. These patients are postponed to the consecutive planning cycle. Each patient has to be assigned to a block or the dummy block, which means the patient will be scheduled in the current planning cycle or in one of the consecutive planning cycles. Furthermore, children should receive surgery in the morning. The following constraints take this into account.

$$\sum_{\hat{\mathbf{b}} \in \hat{B} \cup \{\mathbf{b}'\}} y_{\hat{\mathbf{b}},p} = 1 \qquad \forall\ p \in P \tag{9}$$

$$\sum_{\hat{b_1} \in \hat{B}_1, b_3 \in B_3, c \in C} y_{\hat{\mathbf{b}},p} \geq k_p \qquad \forall\ p \in P, b_2 = 1 \tag{10}$$

4.2.2.2 Materials

For each surgery a basic set of material is used, but for certain surgeries some special equipment is needed, e.g. an x-ray. Certain equipment is limited. The available number of material a is defined as ψ_a. Besides, each patient or surgery requires different materials. The parameter corresponding to the required material for the patient is:

$$mt_{a,p} = \begin{cases} 1 & \text{if patient } p \in P \text{ needs material } a \in A \\ 0 & \text{otherwise} \end{cases}$$

To ensure that the materials are available for the patients that require the material and that the total amount of materials used at a certain time does not exceed the limit of materials, the following constraints are proposed.

$$\sum_{b_3 \in B_3} mt_{a,p} y_{\hat{\mathbf{b}},p} \leq \psi_a \forall\ a \in A, \hat{b_1} \in \hat{B}_1, b_2 \in B_2, c \in C \tag{11}$$

4.2.2.3 Surgeons

The surgeons belong to a certain discipline. This is given with the parameter:

$$cp_{c,s} = \begin{cases} 1 & \text{if surgeon } s \in S \text{ is from discipline } c \in C \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, it is not necessary for the surgeon that makes the diagnosis to perform the surgery on the patient. It often occurs that patients appear on a waiting list and receive surgery from the first surgeon that is available to perform the surgery. But it is possible that a patient wants a specific surgeon. Therefore, $sr_{s,p}$ is introduced. This parameter describes whether surgeon s may or may not perform the surgery of patient p.

$$sr_{s,p} = \begin{cases} 1 & \text{if surgeon } s \in S \text{ can perform the surgery of patient } p \in P \\ 0 & \text{otherwise} \end{cases}$$

In the DA problem the disciplines are assigned to a block. In the SA problem a surgeon is assigned to a block of its corresponding discipline. The assignment of surgeons to a block is defined by the decision variable:

$$i_{\hat{\mathbf{b}},s} = \begin{cases} 1 & \text{if surgeon } s \in S \text{ performs surgeries in block } \hat{\mathbf{b}} \in \hat{B} \\ 0 & \text{otherwise} \end{cases}$$

Each surgeon can only be assigned to a block of its own discipline. Furthermore, a surgeon can not be assigned to multiple blocks at the same time. Each patient should be assigned to a block in which the surgeon is available. Therefore the following constraints are introduced:

$$i_{\hat{\mathbf{b}},s} \leq cp_{c,s} \qquad \forall\ \hat{\mathbf{b}} \in \hat{B}, s \in S \tag{12}$$

$$\sum_{b_3 \in B_3} i_{\hat{\mathbf{b}},s} \leq 1 \qquad \forall\ \hat{b_1} \in \hat{B}_1, b_2 \in B_2, c \in C, s \in S \tag{13}$$

$$sr_{s,p} y_{\hat{\mathbf{b}},p} \leq i_{\hat{\mathbf{b}},s} \qquad \forall\ \hat{\mathbf{b}} \in \hat{B}, s \in S, p \in P \tag{14}$$

4.2.2.4 Wards

The number of beds per ward is limited. Therefore, the parameter tb_w is introduced for each ward to indicate their capacity in the ward. Some patients recover in a ward where they can stay over night and others recover in a ward where they should leave at the end of the day or earlier. To ensure a patient goes to a ward for which the patient is intended, the following parameter is introduced:

$$wr_{p,w} = \begin{cases} 1 & \text{if patient } p \in P \text{ should go to ward } w \in W \\ 0 & \text{otherwise} \end{cases}$$

As there is a certain number of beds per day in a ward, tb_w, it is necessary to limit the surgeries for each type of ward. By determining which day(s) a patient stays at a ward, the number of beds that are being used per ward can be established. Note that it is possible for a patient to stay at the ward at a day after the planning cycle. Therefore, $ob_{b'_1,w}$ is determined in each planning cycle in order to keep track of the number of patients in the ward w at day b'_1. The decision variables are constructed in order to determine the number of beds that remain available on day $\hat{b_1}$ at ward w and the difference between the number of patients at a ward between two consecutive days. Since the number of patients in a ward at a certain day is not fully known for days after the end of the planning cycle, these days are not taken into account in the objective.

$$v_{b'_1,p,w} = \begin{cases} 1 & \text{if patient } p \in P \text{ needs to be at ward } w \in W \text{ on day } b'_1 \in \hat{B}_1 \\ 0 & \text{otherwise} \end{cases}$$

$$r_{b'_1,w} = \text{ the absolute difference of the number of patients at ward } w \in W \text{ between day } b'_1 \text{ and } b'_1 + 1$$

Each patient is assigned to a ward to which it is intended to recover at. Moreover, a patient is assigned to a ward from the day the surgery of the patient has been executed until the Length of Stay of the patient. Besides, the number of patients at a ward cannot exceed the total number of available beds. At last, the absolute difference of the number of patients at a ward between two consecutive days should be taken into account with two constraints.

$$\sum_{\hat{b_1} \in \hat{B}_1, b_3 \in B_3, c \in C} y_{\hat{\mathbf{b}},p} \geq wr_{p,w} \qquad \forall\ p \in P, b_2 = 1, w \in W_{day} \tag{15}$$

$$v_{b'_1,p,w} \geq y_{\hat{\mathbf{b}},p} \qquad \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \tag{16}$$

$$b'_1 = \hat{b_1}, ..., \hat{b_1} + l_p - 1$$

$$\sum_{p \in P} wr_{p,w} v_{b'_1,p,w} \leq tb_w - ob_{b'_1,w} \qquad \forall\ w \in W, b'_1 \in B'_1 \tag{17}$$

$$r_{b'_1,w} \geq (ob_{b'_1,w} + \sum_{p\in P} v_{b'_1,p,w}) - (ob_{b'_1-1,w} + \sum_{p\in P} v_{b'_1-1,p,w}) \qquad \forall\ b'_1 \in \hat{B}_1\backslash\{0\}, w \in W \tag{18}$$

$$r_{b'_1,w} \geq (ob_{b'_1-1,w} + \sum_{p\in P} v_{b'_1-1,p,w}) - (ob_{b'_1,w} + \sum_{p\in P} v_{b'_1,p,w}) \qquad \forall\ b'_1 \in \hat{B}_1\backslash\{0\}, w \in W \tag{19}$$

4.2.2.5 Block characteristics

The total surgery duration of patients that are scheduled in a block should not exceed the duration of a block. For each block in this problem the duration of the block is the same. Therefore, the notation τ is used for the duration of a block. Between surgeries, the operating room is cleaned. The duration of this changing period is defined as γ. Furthermore, the amount of time of surgery duration that exceeds the total duration of a block is determined and proposed by decision variable:

$$o_{\hat{\mathbf{b}}} = \text{ the duration that exceeds the total duration of the block } \hat{\mathbf{b}} \in \hat{B}$$

The total surgery duration in a block cannot exceed the length of a block. If this occurs, overtime is collected. This is described by the following constraint:

$$\sum_{p\in P}(d_p + \gamma) y_{\hat{\mathbf{b}},p} \leq \tau + o_{\hat{\mathbf{b}}} \qquad \forall\ \hat{\mathbf{b}} \in \hat{B} \tag{20}$$

4.2.2.6 Objective

The objective consists of three minimization problems. One of the goals is to maximize the throughput. In order to maximize the throughput the total number of patients scheduled in the blocks during a cycle is maximized. The other two objectives are minimization problems. Hence, the maximization of the throughput is written into a minimization problem. The objective becomes a minimization of the number of patients that have to be scheduled in the dummy block, since this maximizes the patients that are being scheduled. Furthermore, it often happens that a delay of a surgery occurs. A patient should not be scheduled after the latest day the patient could be scheduled. So, each day that a patient is too late is penalized. In order to schedule patients on time, the objective value is minimized. The penalties listed above can be combined into one objective. The cost parameter is denoted by $c^{\mathrm{L}}_{\hat{\mathbf{b}},p}$ and equals the cost of postponement to the next planning cycle if $\hat{\mathbf{b}} = \mathbf{b}'$ and otherwise it is a linearly increasing penalty over the days from the latest day a patient should be scheduled. Hence, the objective is denoted by:

$$Z_1 = \sum_{\hat{\mathbf{b}}\in\hat{B}\cup\{\mathbf{b}'\},p\in P} c^{\mathrm{L}}_{\hat{\mathbf{b}},p} y_{\hat{\mathbf{b}},p}$$

Minimizing the variability to the ward can be written as an absolute value function:

$$\min \sum_{w\in W} \sum_{\hat{b_1}\in\hat{B}_1\backslash\{0\}} |(ob_{b'_1,w} + \sum_{p\in P} v_{b'_1,p,w}) - (ob_{b'_1-1,w} \sum_{p\in P} v_{b'_1-1,p,w})|$$

The absolute value function is not linear which results in a non-linear objective function and thus in a non-linear problem. To avoid this a new auxiliary variable is introduced:

$$r_{b'_1,w} = |(ob_{b'_1,w} + \sum_{p\in P} v_{b'_1,p,w}) - (ob_{b'_1-1,w} \sum_{p\in P} v_{b'_1-1,p,w})|$$

This auxiliary variable is used in the objective and the two constraints (18) and (19) are proposed to give $r_{b'_1,w}$ the absolute value of the difference between the patients at a ward between day b'_1 and b'_1+1. A penalty cost is given for each ward for which the number of patients differs from the consecutive day which is denoted by $c^{\mathrm{W}}_{b'_1,w}$. These penalty cost can differ per day since the difference between Friday and Saturday is less import than the difference between weekdays. Moreover, the ward where patients arrive and leave within one day is closed during the weekends, thus a difference is needed between wards. The resulting objective is:

$$Z_2 = \sum_{w \in W} \sum_{b'_1 \in \hat{B}_1 \setminus \{0\}} c^{\mathrm{W}}_{b'_1,w} r_{b'_1,w}$$

The overtime of a block $o_{\hat{\mathbf{b}}}$ should be minimized. If a sequence of surgeries takes longer than originally planned the staff members need to work longer or a consecutive surgery (patient) can be cancelled. If this happens during a block in the morning, the start of the consecutive block is delayed. Thus, there is a higher probability that this block exceeds the scheduled time as well. Therefore, a penalty is given for the extra time per block that is needed to satisfy all surgeries. The penalty costs are $c^{\mathrm{O}}_{\hat{\mathbf{b}}}$. The objective for minimizing the penalty of overtime is:

$$Z_3 = \sum_{\hat{\mathbf{b}} \in \hat{B}} c^{\mathrm{O}}_{\hat{\mathbf{b}}} o_{\hat{\mathbf{b}}}$$

The total objective equals the minimization over the summation of all three objectives that are introduced:

$$\min \sum_{i=1}^{3} Z_i = \sum_{\hat{\mathbf{b}} \in \hat{B} \cup \{\mathbf{b}'\}, p \in P} c^{\mathrm{L}}_{\hat{\mathbf{b}},p} y_{\hat{\mathbf{b}},p} + \sum_{w \in W} \sum_{b'_1 \in \hat{B}_1 \setminus \{0\}} c^{\mathrm{W}}_{b'_1,w} r_{\hat{b_1},w} + \sum_{\hat{\mathbf{b}} \in \hat{B}} c^{\mathrm{O}}_{\hat{\mathbf{b}}} o_{\hat{\mathbf{b}}}$$

Each objective is analyzed and an order between the objectives is determined. The order is used to give a value to the penalty of the corresponding objective. The hospital determined that the order of the objectives is as follows:

1. Minimizing the postponed patients and minimizing the number of days that a patients is scheduled after the latest day the patient should receive surgery.
2. Minimizing the difference of the number of patients in the ward between consecutive days.
3. Minimizing the overtime of each block.

Therefore, the penalties take values by satisfying the following condition:

$$c^{\mathrm{O}}_{\hat{\mathbf{b}}} \leq c^{\mathrm{W}}_{b'_1,w} \leq c^{\mathrm{L}}_{\hat{\mathbf{b}},p}$$

4.2.2.7 Constraints

The summarize, the constraints corresponding with the patient problem are constraints (9)-(20). Furthermore, constraint (21), (22), and (23) are the binary constraints for the decision variables. Constraint (24) is the continuous constraint of the decision variable $o_{\hat{\mathbf{b}}}$. At last, constraint

(25) ensures that the decision variable $r_{b'_1,w}$ is a positive integer.

$$y_{\hat{\mathbf{b}},p} \in \{0,1\} \qquad \forall\ \hat{\mathbf{b}} \in \hat{B}, p \in P \quad (21)$$

$$i_{\hat{\mathbf{b}},s} \in \{0,1\} \qquad \forall\ \hat{\mathbf{b}} \in \hat{B}, s \in S \quad (22)$$

$$v_{b'_1,p,w} \in \{0,1\} \qquad \forall\ b'_1 \in B'_1, p \in P, w \in W \quad (23)$$

$$o_{\hat{\mathbf{b}}} \geq 0 \qquad \forall \hat{\mathbf{b}} \in \hat{B} \quad (24)$$

$$r_{b'_1,w} \in \mathbb{N}_0 \qquad \forall b'_1 \in \hat{B}_1 \backslash \{0\}, w \in W \quad (25)$$

4.2.3 Two-stage robust optimization problem formulation

In the deterministic formulation of the Surgery Assignment problem all parameters take a fixed value. In practice, the surgery duration and Length of Stay are uncertain. Like the robust model proposed for the DA problem, this problem can also be modeled in two stages in which the second stage corresponds to the stochastic, uncertain, parameters. In the first stage, the decision has to be made to which block the patient is assigned. This decision depends on the decision of which surgeon performs the surgery and what kind of materials are available to a block. Two decisions are determined in the first stage, the allocation of the patient and the surgeon. In the second stage, the uncertain parameters, the surgery duration (d_p) and the Length of Stay (l_p) of a patient, take a certain value of a set. The decisions corresponding to these parameters have to be made. These decisions are the duration of the overtime of a block, the assignment of a patient to a ward at a certain day and the absolute difference of patients in a ward between two consecutive days. The goal of the second stage is to minimize the worst-case scenario for the overtime per block and the variation in the ward. In order to formulate the first and second stage, the uncertain sets and budget of uncertainty need to be defined.

4.2.3.1 Budget of uncertainty and uncertainty sets

For the SA problem, the deterministic formulation of the uncertain parameters are d_p for the surgery duration and l_p Length of Stay in the ward. The stochastic formulation of the uncertain parameters are defined as $\tilde{d_p}$ and $\tilde{l_p}$ respectively. Each uncertain parameter is assumed to deviate with a finite number from its expected value. The surgery duration can deviate at most $\hat{d_p}$ from its minimum value $\bar{d_p}$. This results in the following interval $\tilde{d_p} \in [\bar{d_p}, \bar{d_p} + \hat{d_p}]$. In contrary to the uncertain parameters of the DA problem, the surgery duration does not only take integer values but is continuous. In the worst-case the surgery duration takes longer than expected. Therefore, the surgery duration can take any value in the interval where $\bar{d_p}$ is the minimum and the maximum is $\bar{d_p} + \hat{d_p}$. The Length of Stay is the number of days a patient requires to stay in the ward, i.e. an integer. Hence, this parameter cannot take all values in a continuous interval. The value of this uncertain parameter lies in the set $\tilde{l_p} \in \{\bar{l_p}, ..., \bar{l_p} + \hat{l_p}\}$ where $\bar{l_p}$ is the minimum number of days a patient stays in a ward. The budget of uncertainty for the surgery duration and Length of Stay are Γ_d and Γ_l respectively.

In the case of the surgery duration the normalized deviation of the expected value equals $\bar{z_p} = \frac{\tilde{d_p} - \bar{d_p}}{\hat{d_p}}$ with $0 \leq \bar{z_p} \leq 1$. With this information the following uncertain sets for the uncertain parameters are defined:

$$U_d = \{\tilde{d} : \tilde{d_p} = \bar{d_p} + \bar{z_p}\hat{d_p} \ \forall p,\ 0 \leq \bar{z_p} \leq 1, \sum_{p \in P} \bar{z_p} \leq \Gamma_d^c\}$$

$$U_l = \{\tilde{l} : \tilde{l_p} \in \{\bar{l_p}, ..., \bar{l_p} + \hat{l_p}\} \ \forall p, \sum_{p \in P} \frac{\tilde{l_p} - \bar{l_p}}{\hat{l_p}} \leq \Gamma_l^c\}$$

The uncertainty sets are polyhedral by definition. Besides defining the uncertainty sets, the first and second stage are formulated by the method explained in 4.2.3. The first and second stage of the two-stage robust optimization problem are similarly constructed as in Section 4.1.3 for the DA problem. In Appendix A.1 the first and second stage of the two-stage robust optimization problem are given.

5 Solution method

Both the DA and SA problem are solved by a column-and-constraint generation approach. Section 5.1 gives a general description of the steps in the column-and-constraint generation approach based on the paper of Neyshabouri and Berg (2016). In the column-and-constraint generation approach the first and second stage of the robust optimization problem are solved iteratively. The second stage problems have to be adjusted in order to be able to solve the problems. Hence, Section 5.2 gives an elaborate description of the revision of the second stage problems of each problem.

5.1 Column-and-constraint generation approach

Column generation is a solution method based on the idea that not all variables need to be taken into account. The algorithm is based on the simplex method where most of the variables are non-basis variables and take value zero. Therefore, only valuable variables are considered in column generation. Since including constraints for all possible values of the uncertain parameters results in a large problem, each iteration only constraints for valuable values of the uncertain parameters are considered in the problem. The global steps of the column-and-constraint generation approach proposed by Neyshabouri and Berg (2016) are:

1. Initialize the lower- and upper bound.
2. Solve the Master problem.
3. Set the value of the lower bound to the objective value of the optimal solution of the Master problem.
4. Solve the sub problems.
5. Set the value of the upper bound to the minimum of the upper bound and the objective value including the objective value of the sub problems.
6. If the gap between the lower- and upper bound is less than a fixed threshold; STOP. Else, continue with the next step.
7. Add the constraints corresponding to the optimal solution value of the sub problems.
8. Continue to the next iteration $\rightarrow$ Start again with Step 2.

In the first iteration the Master problem equals the first stage problem in which no uncertain parameters are included. The second stage problem is divided into two sub problems, each corresponding to one uncertain parameter. The solution of the two sub problems of the second stage generates the scenarios which are added to the Master problem. The DA problem is solvable by this method. The description of the algorithm of the DA problem is given in Algorithm C.1 in the Appendices. Each iteration of the DA problem $|C| + |B_1| + 1$ constraints are added to the problem. When the size of the number of disciplines and the length of the planning cycle is high, the Master problem is large. Since the length of the planning cycle is chosen, one could influence the size of the Master problem.

The uncertainty in the Length of Stay (LoS) makes the Surgery Assignment problem more complex. The uncertainty in the LoS is defined as an index parameter in the second stage problem. The uncertainty is passed to a new decision variable that is defined as one for the days since the patient is released from the ward given their admission time. Section 5.2.2.2 introduces this decision variable. The new variable is used in the sub problem in order to make the decision on what the value of the uncertain parameter should be in the constraint that is appended to the problem. When the sub problem is solved, the Length of Stay is calculated using this variable. The Length of Stay is then used in the scenarios (constraints) which are added to the Master problem. The solution method is adjusted for the LoS and is presented in Algorithm C.2. When the number of patients or the level of uncertainty is high, the size of the Master problem is large. Since the number of patients is high when the two locations are combined, the patients are divided over the two locations and each location is solved separately. The small location with the least number of patients can be solved by this method. The large location with a high number of patients requires a different approach. Section 5.3 describes the approach for the large location.

Since there are two uncertain parameters in each problem with a finite uncertainty set, the solution is found in a finite number of iterations.

5.2 Reformulation sub problems

The second stage of the robust optimization problem is a max-min problem. The maximization is over two uncertainty sets, which is complex to solve. Therefore, the second stage problem is divided into two sub problems, one for each uncertain parameter. In order to solve the second stage problem, the sub problems are rewritten into a single maximization LP problem.

The following steps turn the second stage problem in a single maximization LP problem.

1. First the second-stage problem is divided into two sub problems, each corresponding to a decision variable that obtains its value from an uncertain parameter.

2. The sub problem is written to its dual in order to get a max-max problem.

3. The constraints of the uncertainty set are inserted into the sub problem in order to get a single maximization problem. The solution of the single maximization problem provides an upper bound which is used in the column-generation approach.

4. In this sub problem, a bilinear term appears by multiplying the dual variable to the variable of the uncertain parameter. The bilinear term is replaced by a single decision value which gets its value from McCormick constraints that are added to the problem.

These steps are performed on both the DA and SA problem.

5.2.1 Discipline Assignment problem

The second stage problem can be divided into two sub problems, the Emergency Capacity problem (EC) which is defined as $P_1(x, \Gamma_{em})$ and the Demand Satisfaction problem (DS) which is defined as $P_2(x, \Gamma_{pt})$. The first sub problem, $P_1(x, \Gamma_{em})$ is formulated as follows:

$$
\begin{aligned}
\max_{\tilde{em} \in U_{em}} \min & \left(\sum_{b_1 \in B_1} c^{\mathrm{E}}_{b_1} \cdot \beta_{b_1} \right) & \\
s.t. \quad & \sum_{b_2 \in B_2, b_3 \in B_3} x_{\mathbf{b},e} + \beta_{b_1} \geq \tilde{em}_{b_1} & \forall\, b_1 \in B_1 \\
& \beta_{b_1} \in \mathbb{N}_0 & \forall\, b_1 \in B_1
\end{aligned}
$$

The second sub problem, $P_2(x, \Gamma_{pt})$, is:

$$\begin{aligned}
\max_{\tilde{pt} \in U_{pt}} \min & \left(\sum_{c \in C} c_c^{\mathrm{P}} \cdot \alpha_c \right) \\
s.t. \ & \sum_{\mathbf{b} \in B} x_{\mathbf{b},e} + \alpha_c \geq \tilde{pt}_c && \forall\, c \in C \\
& \alpha_c \in \mathbb{N}_0 && \forall\, c \in C
\end{aligned}$$

This is possible because the variables β_{b_1} and α_c are independent. The first sub problem is rewritten according to the steps in Section 5.2.

5.2.1.1 Emergency Capacity problem

The inner optimization of the Emergency Capacity problem is a linear program. The minimization problem can be substituted by its dual based on strong duality to transform the sub problem in a single maximization problem. Once the dual of the minimization problem is constructed, the problem results in a double maximization; one over the uncertainty set and one for the objective. The maximization problem over the set U_{em} corresponds to optimizing em_{b_1} within the uncertainty set where $e\bar{m}_{b_1}$ and $e\hat{m}_{b_1}$ are fixed. The resulting single maximization problem is:

$$\begin{aligned}
\max & \sum_{b_1 \in B_1} \left(e\tilde{m}_{b_1} - \sum_{b_2 \in B_2, b_3 \in B_3} x_{\mathbf{b},e} \right) u_{b_1} \\
s.t. & \sum_{b_1 \in B_1} \frac{e\tilde{m}_{b_1} - e\bar{m}_{b_1}}{e\hat{m}_{b_1}} \leq \Gamma_{em} \\
& u_{b_1} \leq c_{b_1}^{\mathrm{E}} && \forall\, b_1 \in B_1 \\
& u_{b_1} \geq 0 && \forall\, b_1 \in B_1 \\
& e\tilde{m}_{b_1} \geq 0 && \forall\, b_1 \in B_1
\end{aligned}$$

where u_{b_1} is the dual variable. Note that for any $b_1 \in B_1$ the optimal solution $u_{b_1}^*$ lies in the interval $\{0, c_{b_1}^{\mathrm{E}}\}$. If $e\tilde{m}_{b_1} - \sum_{b_2 \in B_2, b_3 \in B_3} x_{\mathbf{b},e} \geq 0$ then the dual variable u_{b_1} equals $c_{b_1}^{\mathrm{E}}$ since it is a maximization problem.
If $e\tilde{m}_{b_1} - \sum_{b_2 \in B_2, b_3 \in B_3} x_{\mathbf{b},e} < 0$ then the dual variable u_{b_1} equals zero.

Since $e\tilde{m}_{b_1} u_{b_1}$ is a bilinear term, McCormick constraints are added in order to make the problem linear. The variable $\tilde{u_{b_1}} = e\tilde{m}_{b_1} u_{b_1}$ is introduced by the following constraints:

$$\begin{aligned}
& \tilde{u_{b_1}} \leq e\bar{m}_{b_1} u_{b_1} && \forall\, b_1 \in B_1 \\
& \tilde{u_{b_1}} \leq c_{b_1}^{\mathrm{E}} e\tilde{m}_{b_1} + (e\bar{m}_{b_1} - e\hat{m}_{b_1}) u_{b_1} && \forall\, b_1 \in B_1 \\
& \tilde{u_{b_1}} \geq (e\bar{m}_{b_1} - e\hat{m}_{b_1}) u_{b_1} && \forall\, b_1 \in B_1 \\
& \tilde{u_{b_1}} \geq c_{b_1}^{\mathrm{E}} e\tilde{m}_{b_1} + e\bar{m}_{b_1} u_{b_1} && \forall\, b_1 \in B_1
\end{aligned}$$

The complete LP problem is given in Appendix B.1.

5.2.1.2 Demand Satisfaction problem

Since $\tilde{pt}_c$ is integer, the (DS) problem is formulated in the same way as the (EC) problem. The resulting sub problem in which the inner maximization is substituted by the dual and the McCormick constraints are added, is given in Appendix B.2

5.2.2 Surgery Assignment problem

The second stage problem of the SA problem can be divided into two sub problems as well. The variables $o_{\hat{\mathbf{b}}}$ and $r_{b'_1,w}$ are independent. Therefore, the sub problems are $R_1(y, \Gamma_d)$ and $R_2(y, \Gamma_l)$. The first sub problem, the Block Capacity problem (BC), $R_1(y, \Gamma_d)$ is formulated as:

$$
\begin{aligned}
\max_{d \in U_d} \min \; & \sum_{\hat{\mathbf{b}} \in \hat{B}} c^{\mathrm{O}}_{\hat{\mathbf{b}}} o_{\hat{\mathbf{b}}} \\
s.t. \; & \sum_{p \in P} (d_p + \gamma) y_{\hat{\mathbf{b}},p} \leq \tau + o_{\hat{\mathbf{b}}} && \forall \; \hat{\mathbf{b}} \in \hat{B} \\
& o_{\hat{\mathbf{b}}} \geq 0 && \forall \; \hat{\mathbf{b}} \in \hat{B}
\end{aligned}
$$

The second sub problem, the Downstream Capacity problem (DC), becomes:

$$
\begin{aligned}
\max_{l \in U_l} \min \; & \sum_{w \in W} \sum_{b'_1 \in \hat{B}_1 \backslash \{0\}} c^{\mathrm{W}}_{b'_1,w} r_{b'_1,w} \\
s.t. \; & v_{b'_1,p,w} \geq y_{\hat{\mathbf{b}},p} && \forall \; p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \\
& && b'_1 = \hat{b_1}, ..., \hat{b_1} + l_p - 1 \\
& \sum_{p \in P} w r_{p,w} v_{b'_1,p,w} \leq tb_w - ob_{b'_1,w} && \forall \; w \in W, b'_1 \in B'_1 \\
& r_{b'_1,w} \geq (ob_{b'_1,w} + \sum_{p \in P} v_{b'_1,p,w}) - (ob_{b'_1-1,w} + \sum_{p \in P} v_{b'_1-1,p,w}) && \forall \; b'_1 \in \hat{B}_1 \backslash \{0\}, w \in W \\
& r_{b'_1,w} \geq (ob_{b'_1-1,w} + \sum_{p \in P} v_{b'_1-1,p,w}) - (ob_{b'_1,w} + \sum_{p \in P} v_{b'_1,p,w}) && \forall \; b'_1 \in \hat{B}_1 \backslash \{0\}, w \in W \\
& \sum_{p \in P} (d_p + \gamma) y_{\hat{\mathbf{b}},p} \leq \tau + o_{\hat{\mathbf{b}}} && \forall \; \hat{\mathbf{b}} \in \hat{B} \\
& v_{b'_1,p,w} \in \{0, 1\} && \forall \; b'_1 \in B'_1, p \in P, w \in W \\
& r_{b'_1,w} \in \mathbb{N}_0 && \forall \; b'_1 \in \hat{B}_1 \backslash \{0\}, w \in W
\end{aligned}
$$

5.2.2.1 Block Capacity problem

First, the dual of the minimization problem is determined to transform the sub problem in a single maximization problem. Once the dual of the minimization problem is constructed, the problem results in a double maximization; one over the uncertainty set and one for the objective. The maximization problem over the set u_d corresponds to optimizing $\bar{z}$ for a fixed $\hat{d_p}$ and $y_{\mathbf{b},p}$. The constraints corresponding to the uncertainty are inserted in the dual problem which results in the single maximization problem:

$$
\begin{aligned}
\max \; & \sum_{\hat{\mathbf{b}} \in \hat{B}} \sum_{p \in P} (\bar{d}_p y_{\hat{\mathbf{b}},p} + \hat{d}_p \bar{z}_p y_{\hat{\mathbf{b}},p} + \gamma y_{\hat{\mathbf{b}},p} - \tau) \nu_{\hat{\mathbf{b}}} \\
s.t. \; & \sum_{p \in P} \bar{z}_p \leq \Gamma_d \\
& \bar{z}_p \leq 1 && \forall \; p \in P \\
& \nu_{\hat{\mathbf{b}}} \leq c^{\mathrm{O}}_{\hat{\mathbf{b}}} && \forall \; \hat{\mathbf{b}} \in \hat{B} \\
& \bar{z}_p, \nu_{\hat{\mathbf{b}}} \geq 0 && \forall \; \hat{\mathbf{b}} \in \hat{B}
\end{aligned}
$$

where $\nu_{\hat{\mathbf{b}}}$ denotes the dual. The bilinear term $\bar{z}_p\nu_{\hat{\mathbf{b}}}$ is substituted by a single variable $\tilde{\nu}_{\hat{\mathbf{b}},p}$. Since $\bar{z}_p$ is binary, the following constraints are added for $\tilde{\nu}_{\hat{\mathbf{b}},p}$.

$$
\begin{aligned}
\tilde{\nu}_{\hat{\mathbf{b}},p} &\leq c^{\mathrm{O}}_{\hat{\mathbf{b}}}\bar{z}_p \ \forall\ \hat{\mathbf{b}} \in \hat{B}, p \in P \\
\tilde{\nu}_{\hat{\mathbf{b}},p} &\leq \nu_{\hat{\mathbf{b}}} \ \forall\ \hat{\mathbf{b}} \in \hat{B}, p \in P
\end{aligned}
$$

The complete LP problem is given in Appendix B.3.

5.2.2.2 Downstream Capacity problem

The Downstream Capacity problem is more complex due to the fact that the parameter Length of Stay is present in the set of indexes and not in the inequality. Therefore, the problem is rewritten in order to formulate the LoS in the inequality. First, two new variables are introduced:

$$
\theta_{b'_1,p,w} = \begin{cases} 1 & \text{if patient } p \text{ enters ward } w \text{ by day } b'_1 \\ 0 & \text{otherwise} \end{cases}
$$

$$
\chi_{b'_1,p,w} = \begin{cases} 1 & \text{if patient } p \text{ leaves ward } w \text{ by day } b'_1 \\ 0 & \text{otherwise} \end{cases}
$$

Note that 'by day b'_1' is present in the definition of the variables. This means that variable $\theta_{b'_1,p,w}$ is 1 for the day that the patient arrives at the ward and the days thereafter. Furthermore, $\chi_{b'_1,p,w}$ is 1 for the day that the patient leaves the ward and the days thereafter. By definition $\tilde{l}_p = \sum_{b'_1=1}^{T_2}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w})$. Figure 5 shows the construction of the parameters.

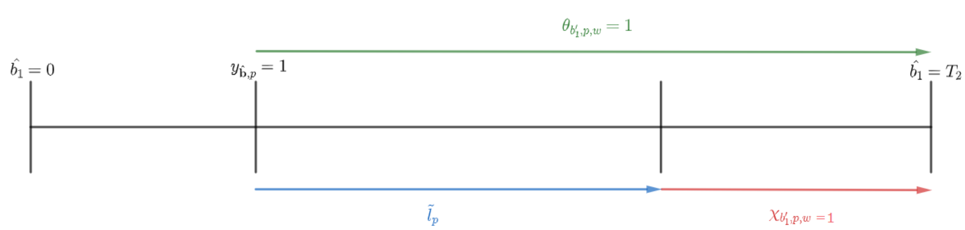


Figure 5: The construction of the parameter $\theta_{b'_1,p,w}$ and decision variable $\chi_{b'_1,p,w}$.

Note that $\theta_{\hat{b_1},p,w}$ can be determined after the first stage since $y_{\hat{\mathbf{b}},p}$ is known. The following conditions are used to give $\theta_{\hat{b_1},p,w}$ its value:

$$
\begin{aligned}
\theta_{b'_1,p,w} &\geq wr_{p,w}y_{\hat{\mathbf{b}},p} && b'_1 = \hat{b_1},\ \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \\
\theta_{b'_1,p,w} &\leq wr_{p,w}(1 - y_{\hat{\mathbf{b}},p}) && b'_1 = 1, ..., \hat{b_1} - 1,\ \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \\
\theta_{b'_1,p,w} &\leq \theta_{b'_1+1,p,w} && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
\sum_{w \in W} \theta_{b'_1,p,w} &= 1 && \forall\ b'_1 \in B'_1, p \in P
\end{aligned}
$$

The first constraint ensures that the patient goes to the ward at the day of the surgery. Every day before the day of surgery is set to zero by the second constraint. Furthermore, the third constraint makes sure that every parameter is bigger or equal than the day before in order to set the parameter every day after the day of surgery to one. At last, the final constraint ensures that each patient is allocated to at most at one ward at a certain day.

Although $\theta_{b'_1,p,w}$ can directly be determined after the first stage, $\chi_{b'_1,p,w}$ is dependent on the Length of Stay, $\tilde{l}_p$, and gets its value in the second stage problem. Since $\tilde{l}_p$ can be determined by θ and χ, a new uncertainty set can be constructed in order to leave $\tilde{l}_p$ out of the indices. The new uncertainty set to the corresponding uncertain variable $\chi_{b'_1,p,w}$ consists of a few conditions. The first condition makes sure that each patient goes to one ward each day:

$$\sum_{w\in W} \chi_{b'_1,p,w} \leq 1 \qquad \forall\ b'_1 \in B'_1, p \in P \tag{26}$$

Constraint 27 ensures that every patient leaves the ward after $\bar{l_p} + \hat{l_p}$ days. Furthermore, constraint 28 makes sure that until the Length of Stay $\bar{l_p}$ is passed the patient does not leave the ward.

$$\chi_{b'_1,p,w} \geq wr_{p,w} y_{\hat{\mathbf{b}},p} \qquad b'_1 = \hat{b_1} + \bar{l_p} + \hat{l_p} - 1\ \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \tag{27}$$

$$\chi_{b'_1,p,w} \leq wr_{p,w}(1 - y_{\hat{\mathbf{b}},p}) \qquad b'_1 = 1, ..., \hat{b_1} + \bar{l_p} - 1\ \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \tag{28}$$

Combined with the constraints above, the following constraint sets the parameter every day after the patient leaves the ward to one.

$$\chi_{b'_1,p,w} \leq \chi_{b'_1+1,p,w} \qquad \forall\ b'_1 \in B'_1, p \in P, w \in W \tag{29}$$

The constraint of the budget of uncertainty is rewritten to constraint (30). Furthermore, the uncertainty set can now be written as follows:

$$\sum_{p\in P, \hat{l_p}\geq 0, y_{b',p}=0} \left(\frac{\sum_{b'_1=1}^{T_2} \sum_{w\in W}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w}) - \bar{l_p}}{\hat{l_p}} \right) \leq \Gamma_l \tag{30}$$

$$U_\chi(y,\theta) = \{\chi : (26) - (30)\}$$

The transformed problem is:

$$\begin{aligned}
\max_{\chi\in U_\chi(y,\theta)} \min \sum_{w\in W} \sum_{b'_1\in \hat{B}_1\backslash\{0\}} & c^{\mathrm{W}}_{b'_1,w} r_{b'_1,w} \\
s.t. \sum_{p\in P} & wr_{p,w}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w}) \leq tb_w - ob_{b'_1,w} && \forall\ w \in W, b'_1 \in B'_1 \\
& r_{b'_1,w} \geq (ob_{b'_1,w} + \sum_{p\in P}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w})) \\
& - (ob_{b'_1-1,w} \sum_{p\in P}(\theta_{b'_1-1,p,w} - \chi_{b'_1-1,p,w})) && \forall\ b'_1 \in \hat{B}_1\backslash\{0\}, w \in W \\
& r_{b'_1,w} \geq (ob_{b'_1-1,w} + \sum_{p\in P}(\theta_{b'_1-1,p,w} - \chi_{b'_1-1,p,w})) \\
& - (ob_{b'_1,w} + \sum_{p\in P}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w})) && \forall\ b'_1 \in \hat{B}_1\backslash\{0\}, w \in W \\
& \chi_{b'_1,p,w} \in \{0,1\} && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
& r_{b'_1,w} \in \mathbb{N}_0 && \forall b'_1 \in \hat{B}_1\backslash\{0\}, w \in W
\end{aligned}$$

Like the all sub problems, with strong duality the inner minimization is rewritten to a maximization problem. After that, the max-max problem is transformed into a single minimization problem. The resulting objective can be written as follows:

$$\begin{aligned}\max \sum_{b'_1 \in B_1} \sum_{w \in W} &((ob_{b'_1,w} + \sum_{p \in P}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w})) - (ob_{b'_1-1,w} + \sum_{p \in P}(\theta_{b'_1-1,p,w} - \chi_{b'_1-1,p,w})))\phi^1_{b'_1,w} \\ &- ((ob_{b'_1-1,w} + \sum_{p \in P}(\theta_{b'_1-1,p,w} - \chi_{b'_1-1,p,w})) - (ob_{b'_1,w} + \sum_{p \in P}(\theta_{b'_1,p,w} - \chi_{b'_1,p,w})))\phi^2_{b'_1,w}\end{aligned}$$

The objective consists of a multiplication of two variables. Since $\chi_{b'_1,p,w}\phi^1_{b'_1,w}$, $\chi_{b'_1-1,p,w}\phi^1_{b'_1,w}$, $\chi_{b'_1,p,w}\phi^2_{b'_1,w}$, and $\chi_{b'_1-1,p,w}\phi^2_{b'_1,w}$ are bilinear terms, these terms are each substituted by a single variable, $\tilde{\phi}^1_{b'_1,p,w}$, $\grave{\boldsymbol{\phi}}^1_{b'_1-1,p,w}$, $\tilde{\phi}^2_{b'_1,p,w}$ and $\grave{\boldsymbol{\phi}}^2_{b'_1-1,p,w}$ respectively, in order to transform this problem to a linear problem. Furthermore, the constraints to rewrite the bilinear term are incorporated. The complete formulation is given in Appendix B.4.

5.3 Approximation large location

Since the number of patients for the first and largest location is high, the problem size is big. The large problem size results in a high computation time. Since the hospital cannot wait too long for a schedule, the problem size has to be decreased in order to solve the problem for the largest location. Each patient is from a certain discipline and disciplines are assigned to blocks in the DA problem. Thus, it is possible to schedule patients per discipline. Moreover, the postponed patients and the overtime of a block can be calculated per discipline. Furthermore, the difference of patients in the ward per day can be determined per discipline, but the wards are not divided over the discipline, i.e. patients of different disciplines can go to the same ward. However, when the difference of the patients in the ward is minimized for each discipline, the variability in the ward is minimized over all. By scheduling the patients per discipline separately the problem size decreases since each sub problem only contains the patients of one or a small group of disciplines. First, the group of patients is divided in groups of disciplines. Each group of patients of a certain discipline c (or set of n disciplines $C_n = \{c_1, c_2, ..., c_n\}$) is defined as P_c (or P_{C_n}). The large disciplines are solved separately and the smaller disciplines are joined to solve together. The first discipline that is solved, can be solved by the method explained in 5.1 where now $P = P_c$ ($P = P_{C_n}$). The set C^k is the set with all disciplines that are solved till iteration k where in each iteration k one discipline or a set of disciplines is solved. Since the parameter $ob_{b'_1,w}$ already indicates the number of occupied beds on a day b'_1 in a certain ward w, this parameter is used in order to ensure that the number of patients in the ward does not exceed the capacity. Furthermore, this parameter is taken into account in the objective value. In iteration k the parameter $ob_{b'_1,w}$ receives its value as follows:

$$ob^k_{b'_1 w} = ob^{k-1}_{b'_1 w} + \sum_{p \in P_c} v_{b'_1,p,w} \ \forall \ b'_1 \in B'_1, c \in C, w \in W$$

In the last iteration the patients of all disciplines are taken into account in the ward and therefore $r_{b'_1,w}$ gives the result of the ward for the entire problem. In order to obtain the total outcome of the overtime and postponed patients, these indicators should be summed for all disciplines. Since the first discipline that is scheduled is the least affected by the constraint of the ward capacity, the patients of the most important discipline should be scheduled first.

6 Numerical experiments

To model the DA and SA problem, data from the hospital is used. Section 6.1 describes the used data and Section 6.2 explains the way of cleaning the data. Furthermore, Section 6.3 evaluates the performance of the DA problem and tests are performed by changing penalties, the order of importance of disciplines and the compatibility of disciplines to the operating rooms. The performance of the SA problem is evaluated in Section 6.4 and the result for the schedule proposed by the DA problem is compared to the schedule that is currently used.

6.1 Data description

Each day two options for time slots are given by the hospital: the morning 8:00 am until 12:15 pm and the afternoon 12:16 pm until 16:30 pm. The hospital consists of 10 disciplines: a group of surgeries covering oncology and cardiology (CHI), ear nose throat surgery (ENT), gynecology (GYN), mouth, jaw and face surgery (MJF), neurosurgery (NEU), ophthalmology (OPT), orthopedics (ORT), plastic surgery (PLAS), special dentistry (SDEN), and urology (URO). The hospital currently consists of three locations of which one recently joined the group of cooperating hospitals. Therefore, this location is not included in the numerical experiments. There are fifteen operating rooms at the first location (of which only 13 are being used due to lack of staff) and five at the second location. Not every discipline performs surgery at every location. For example, plastic surgery and special dentistry only perform surgeries at the first location. Furthermore, the emergency surgeries are performed in clinical operating rooms, which are operating rooms that are compatible with those for the complex surgeries. These operating rooms are only located at the first location. The discipline special dentistry and ear nose throat surgery have surgeries especially for children.

There are in total fifteen different wards to recover for multiple days and six wards where patients can recover during the day.

In July 2018, the hospital has switched to another program to register the patient's information. The recorded information differs from the previous program. Therefore, only the data from July 2018 until July 2019 is used. This program is quite new and people can make mistakes when recording the patient's information. Unfortunately, these mistakes are hard to find and often not corrected in the data. Furthermore, doctors, planners and other staff members that need to register patient information do not enter all information that is required. When one of the staff members is required to receive this information, a call is made to the specific department where this information is available. However, this information is frequently used and not recorded.

6.2 Data cleaning

The required information is sometimes missing or recorded incorrectly. The data points of which information is absent are removed. The effect of removing data points is that certain surgeries are not included. As a consequence, in reality the number of patients is higher, i.e. the solution of the problem is underestimated. Furthermore, it may occur that a data point deviates a lot from the other data points caused by incorrect entering of information or complications for this data point. The patients for which the information is missing or incorrect are removed from the data. Note that the missing or incorrect data occurs for the following parameters:

- Latest day the patient can be scheduled (urgency)
- Date of admission at recovery

- Date of discharge
- Surgery duration (from arriving at OR to leaving the OR)

6.3 Performance analysis: DA problem

To evaluate the proposed solution method, multiple tests are generated for the hospitals interest. These tests include changing the input value of certain parameters in the model. The values of penalties according the order of importance of the objectives, the penalties according to the order of importance of the disciplines, and the compatibility of disciplines to operating rooms, are changed. The numerical experiments of the DA and SA problem are implemented on a Windows machine with an Intel Core i7, 2.4 GHz CPU, and 8 GB of RAM.

For each of the input parameters of which the value is changed, Section 6.3.2 gives the results of multiple tests with different values for the input parameters according to the information given by the hospital are performed. Section 6.3.4 shows the value of the second stage variables, objective value, number of iterations and running time are given for a given combination of the budget of uncertainty Γ_{em} or Γ_{pt}.

Furthermore, a schedule is made for a planning cycle of 20 days, which equals four weeks of five weekdays and covers approximately a month. The budget of uncertainty is defined as the number of parameters that can differ from their minimum value. Therefore, Γ_{em} can take all integer values between and equal to zero and twenty. In the hospital, ten disciplines are available, thus Γ_{pt} can take all integer values between and equal to zero and ten.

In order to evaluate the impact of the change of a parameter, one of the budget of uncertainty remains fixed value while the other budget of uncertainty varies. The difference between the impact on the emergencies and the elective patients is determined by varying the corresponding budget of uncertainty.

The objective value, number of iterations, and running time are determined to establish a combination of budget of uncertainty of which the hospital can run the program. A fast and robust solution is required. The value of the second stage decision variables gives insight in the impact of certain parameters on the solution. Furthermore, Section 6.3.5 describes the difference between the deterministic DA problem, the robust DA problem, and the current situation.

6.3.1 Statistics uncertain parameters

Certain parameters do not change in the computational experiments in order to solve the two-stage stochastic problem. First, the values of $\bar{pt}_c$ and $\hat{pt}_c$ are given, which stand for the minimum blocks a discipline requires in a planning cycle and respectively the additional blocks a discipline may need in a planning cycle. These values of the number of required blocks are constructed by summing the duration of surgeries of patients arriving in a month and dividing by the duration of a block. Furthermore, for each patient a changing period of ten minutes is added to the total surgery time per month. Since each discipline performs different surgeries with each a different duration, the percentage of the total surgery duration each discipline is responsible for, is given. Furthermore, the relative importance is to indicate what value should be given to the penalty of the corresponding uncertain parameter. A high importance indicates a high penalty and vice versa. This penalty is based on the priority of the surgeries performed by a discipline, i.e. how critical it is for the patient to receive the surgery. Table 4 shows the values of the parameters explained above.

Moreover, the values of $\bar{em}_{b_1}$ and $\hat{em}_{b_1}$ are determined, which stands for the minimum number of blocks required for emergency surgeries and the additional blocks the hospital needs in order to be able to perform all surgeries in a planning cycle. The values of the minimum required and the

Discipline c	$\bar{pt}_c$	$\hat{pt}_c$	Percentage of surgeries (%)	Relative importance
CHI	97	56	34.1	5
ENT	26	21	6.4	2
GYN	38	26	9.2	4
MJF	7	13	2.5	1
NEU	24	20	0.1	3
OPT	28	19	6.7	2
ORT	101	25	25.8	3
PLAS	22	16	6.5	2
SDEN	6	14	2.0	1
URO	30	15	6.7	2

Table 4: Statistics of parameters $\bar{pt}_c, \hat{pt}_c$, percentage of surgeries and relative in importance per discipline c.

deviation of the minimum number of blocks for emergencies are constructed by rounding up the summation of the duration of the emergency surgeries each day divided by the total duration of a block. In this case no distinction is made between the different weekdays. The minimum number of blocks reserved for emergencies per day, $\bar{em}_{b_1}$, equals one. Furthermore, the extra blocks the hospital may require, $\hat{em}_{b_1}$, equals 12.

In total for 20 days, 2 time slots per day and 18 operating rooms, the number of available blocks equals 720. The maximum number of blocks for emergencies is 260 and for elective surgeries for all disciplines is 604. Hence, at most 144 blocks cannot be fulfilled. This occurs when Γ_{em} and Γ_{pt} equal their maximum or for other high vales of the budget of uncertainty.

Different numbers of blocks are required for emergencies per day. The occurrence of at most two required emergency blocks per day covers almost 75% of the cases. Figure 6 shows to cover 95% of occurrences of required emergency blocks per day five blocks per day must be satisfied per day. To have five blocks per day assigned to emergencies Γ_{em} should at least be equal to nine.

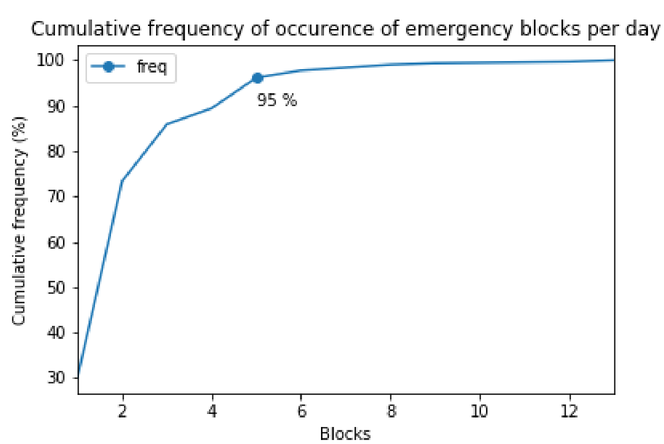


Figure 6: The cumulative frequency of the required emergency blocks per day.

Furthermore, the number of blocks that are required per discipline differs per month. Table 5 shows the maximum and the 95 percentile of the number of required blocks per discipline. Since the total number of additional required blocks equals 225. The average number of extra blocks assigned equals 23. Therefore, to cover 95 percent of the cases for all disciplines the budget of uncertainty Γ_{pt} should equal eight. Hence, in the computational experiments $\Gamma_{em} = 9$ and $\Gamma_{pt} = 8$.

Discipline	CHI	ENT	GYN	MJF	NEU	OPT	ORT	PLAS	SDEN	URO	Total
Maximum	153	47	64	20	44	47	126	38	20	45	604
95%	151	46	59	19	42	46	125	37	18	43	586

Table 5: The maximum and 95% of the required number of blocks per discipline.

6.3.2 Sensitivity analysis penalties

The penalties given to each objective are of high importance to the solution of the problem. The hospital provided the following order of the penalties of the objectives:

$$|c_c^{\mathrm{T}}| \leq c_{b_1}^{\mathrm{E}} \leq c_c^{\mathrm{P}} \tag{31}$$

Each penalty takes a certain value, but it is hard to predict this value. Two tests are performed where the order of inequality (31) is considered for $c_{b_1}^{\mathrm{E}}$ and c_c^{P}. The penalty c_c^{T} is the reward per allocated block per discipline and c_c^{P} the penalty per lack of block per discipline. Assuming that allocating a block to a certain discipline costs the same as having a block too short, the reward per block per discipline is the negative value of the penalty of not satisfying a block per discipline. Thus, $|c_c^{\mathrm{T}}|$ equals c_c^{P}. Given that $\hat{em}_{b_1}$ is the same for each day $b_1 \in B_1$, the maximum objective that can be given for emergencies is $c_{b_1}^{\mathrm{E}} \cdot 12 \cdot 20 = 240 \cdot c_{b_1}^{\mathrm{E}}$. The maximum penalty that can be given for elective surgeries for the disciplines equals $c_c^{\mathrm{P}} \cdot \sum_{c \in C} \hat{pt}_c = c_c^{\mathrm{P}} \cdot 225$. Hence, the blocks for elective surgeries per discipline are not entirely satisfied when $c_{b_1}^{\mathrm{E}} \geq 0.94 \cdot c_c^{\mathrm{P}}$. In order to get results, where not only the required emergency blocks are not satisfied but also the elective surgeries per discipline, a third test is done where the order $c_{b_1}^{\mathrm{E}} \geq c_c^{\mathrm{P}}$ of the penalties of the objectives is considered. Table 6 shows the input values of the three tests. The blocks are allocated by assuming that there is no difference between the disciplines and between emergencies per day,.

Penalty	$c_{b_1}^{\mathrm{E}}$	c_c^{T}	c_c^{P}
Test 1	20	-10	10
Test 2	10	-10	10
Test 3	10	-20	20

Table 6: Input values of the three cost penalties where each index has the same penalty.

Solutions can differ in the number of scheduled emergency blocks and the number of blocks scheduled per discipline. The number of scheduled blocks for emergencies and disciplines are shown for the value of the budget of uncertainty $\Gamma_{eme} = 9$ and $\Gamma_{pt} = 8$ since with these values 95% of the cases in the hospital are covered.

Figure 7 shows for test 1, when $c_c^{\mathrm{P}} \leq c_{b_1}^{\mathrm{E}}$, that the required number of blocks for elective surgeries per discipline are low for certain discipline. Urology (URO) receives only a few blocks as well as neurosurgery (NEU) and orthopedics (ORT). In test 3, the number of blocks for the discipline special dentistry (SDEN) is high, while this is not required (see Table 5). Test 2 in Figure 7 shows that the required number of blocks per discipline is met (see Tables 8 and 9).

Furthermore, Figure 7 shows that the number of blocks reserved for emergencies per day is high for test 1 and low for test 3. In test 3, when $c_c^{\mathrm{P}} \geq c_{b_1}^{\mathrm{E}}$, the number of blocks required for

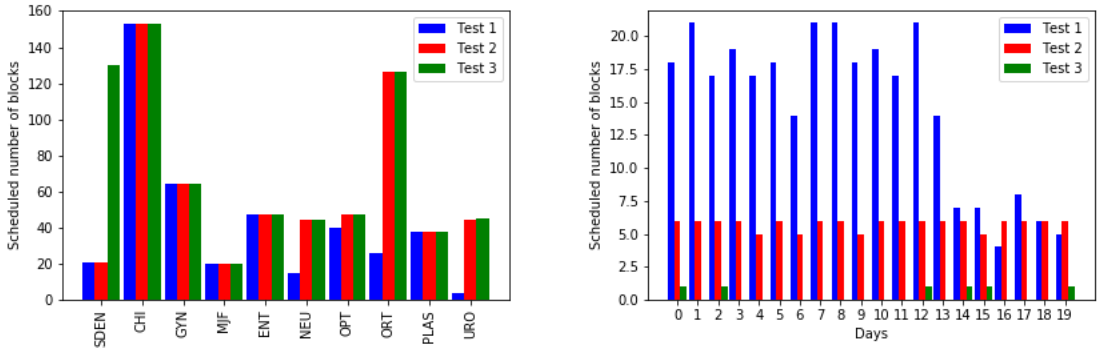


Figure 7: The number of scheduled blocks per discipline and the number of scheduled blocks for emergencies per day for the input values of test 1, 2, and 3 in Table 6 and $\Gamma_{em} = 9$ and $\Gamma_{pt} = 8$.

emergencies given by $\tilde{em}_{b_1}$ is not met for every day. Given that the penalty of not satisfying the emergency blocks is the lowest, it is evident that the number of blocks reserved for emergency surgeries is lower. However, the number of blocks reserved for emergencies is less than currently scheduled. In test 1, a high number of blocks is allocated to emergencies, even higher than the maximum required number of blocks for emergencies per day which equals 13. This is unnecessary since there are not that many emergency patients per day. Each day at least five blocks are given to emergencies in test 2. One can conclude that the order of the penalties is of high impact on the solution. Furthermore, when c_c^{P} equals $c_{b_1}^{\mathrm{E}}$ the solution is better since the number of blocks allocated to disciplines and emergencies is balanced. Moreover, the number of emergency blocks that is required following the 95%, namely five per day, is satisfied.

Given that the priority of the penalties are of effect on the solution, multiple tests are executed in order to evaluate the impact of the order of importance of disciplines. The relative importance is given by the hospital and shown in Table 4. Different priorities can be given by for example the percentage of surgeries the discipline is responsible for. The order of importance given by the percentage of surgeries the discipline is accountable for is:

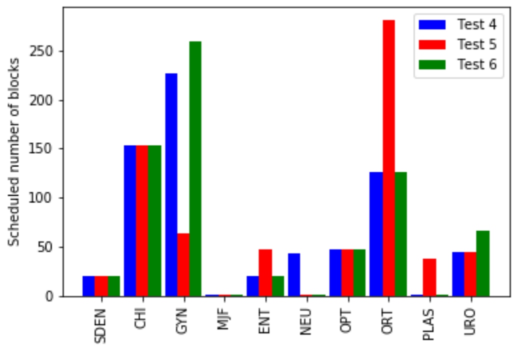
$$c_{CHI}^{\mathrm{P}} \geq c_{ORT}^{\mathrm{P}} \geq c_{GYN}^{\mathrm{P}} \geq c_{URO}^{\mathrm{P}} = c_{OPT}^{\mathrm{P}} \geq c_{PLAS}^{\mathrm{P}} \geq c_{ENT}^{\mathrm{P}} \geq c_{MJF}^{\mathrm{P}} \geq c_{SDEN}^{\mathrm{P}} \geq c_{NEU}^{\mathrm{P}} \tag{32}$$

Three tests are done with different orders of importance for disciplines. The average penalty for the disciplines is the same for each test and equals the penalty of the emergencies, namely 20. Table 7 shows the penalties for the different tests for the disciplines.

Penalty					c_c^{P}					
	CHI	ENT	GYN	MJF	NEU	OPT	ORT	PLAS	SDEN	URO
Test 4	30	15	30	10	25	20	25	15	10	20
Test 5	30	15	25	10	10	25	30	20	10	25
Test 6	30	10	30	10	10	30	30	10	10	30

Table 7: Input values of penalties for the lack of blocks per discipline with different penalties. In test 4 for each discipline a penalty is given according the order of importance in Table 4, in test 5 the order of inequality (32) is considered and in test 6 the disciplines are divided into two groups where each discipline in a group has the same penalty.

First, the importance given by the hospital is considered (see Table 4). Second, the penalty for the disciplines is given based on the order given by the percentage of surgeries performed. In the last test, the disciplines are divided into two groups, one group with high importance and one group with low importance. Since some disciplines can have the same importance, the impact of giving the disciplines in a group the same penalty is evaluated. Recall that the reward per discipline is the negative value of the penalty per discipline.



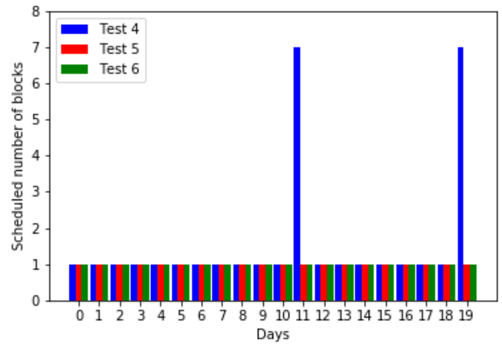


Figure 8: The number of scheduled blocks for disciplines and emergencies for the input values of test 4, 5, and 6 in Table 6 and $\Gamma_{em} = 9$ and $\Gamma_{pt} = 8$.

Figure 8 shows that at least one block is allocated to emergencies each day for all tests. Only in test 4, seven blocks are allocated to emergencies on two of the twenty days. Furthermore, the disciplines MJF does not receive the required blocks per day for all tests. Hence, the penalties used in the tests have a large impact on the discipline MJF which is one of the disciplines with the lowest penalty. Furthermore, the penalties of test 4 and 6 have a large negative impact on the scheduled blocks of the disciplines ENT, ORT, and PLAS. PLAS can only be performed at the first location. Given that the penalty of PLAS is lower in test 4 and 6, it is more complex and less preferred to schedule the blocks for PLAS in these tests. Moreover, the penalties of test 4 and 6 have a large positive impact on the scheduled blocks of GYN. In test 5, the number of scheduled blocks for the disciplines NEU and MJF is low, while the number of blocks allocated to the disciplines ENT, ORT, and PLAS is higher than in test 5. In test 4, the number of block scheduled for neurosurgery is higher since the penalty for neurosurgery is higher. While NEU has the lowest amount of surgeries and therefore the lowest penalty in test 5 and 6, it should not create a situation in which no blocks are assigned to this discipline. Summarized, the impact of the order of discipline is large. One should be careful with proposing an order to the penalties.

6.3.3 Sensitivity analysis flexibility

The hospital is trying to change the profile of each location, i.e. determining which discipline performs surgeries at which location. Since few surgeries of some disciplines are performed at one of the locations, the decision can be made to not let that discipline perform surgeries on that location. Since the discipline neurosurgery performs few surgeries at the second location, a decision can be made to let these disciplines only perform surgeries at the first location. Furthermore, in order to perform surgeries efficiently it is possible to make certain operating rooms predetermined for specific disciplines. Therefore, the following situations are evaluated:

- Neurosurgery is only performed on the first location
- The biggest four operating rooms at the first location are only compatible for the surgeries with the most complex surgeries (CHI and ORT).

These situations are solved by using the penalties from test 2 from Table 6, since these penalties give the best balance between the emergencies and disciplines.

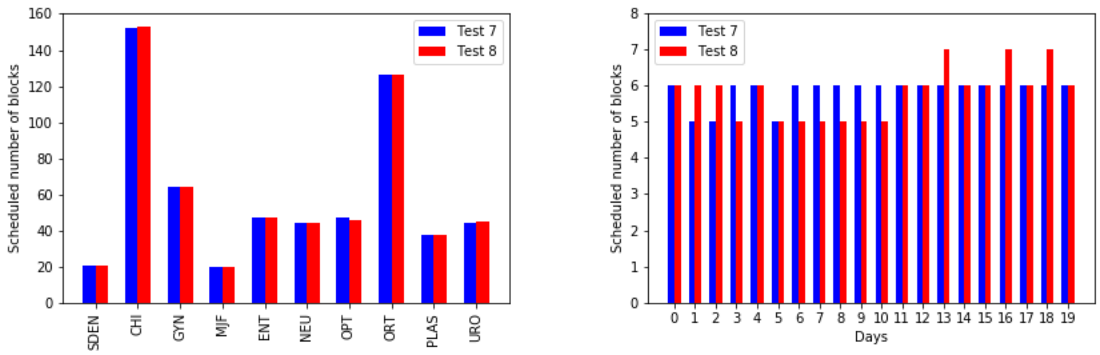


Figure 9: The scheduled blocks for disciplines and emergencies when neurosurgery is only performed at the first location (test 7) and when the compatibility for the disciplines CHI and ORT is changed (test 8) for the penalty values from test 2 in Table 6, $\Gamma_{em} = 9$, and $\Gamma_{pt} = 8$.

Compared to test 2 in Figure 7 where the penalties are equal to the ones used by changing the compatibility, the objective value and the scheduled blocks are the same when neurosurgery is only performed at the first location. Furthermore, emergencies are only performed at the first location. Since it is not possible to schedule neurosurgery at the second location, and the number of required emergency blocks remains the same, the days on which the emergency blocks are not satisfied differ from the result in test 2. This is dependent on the initial solution and the days on which neurosurgery is assigned to the blocks. Furthermore, Figure 9 shows for changing the compatibility of CHI and ORT to the biggest operating rooms that the number of scheduled blocks for the disciplines CHI, OPT, and URO differ a little from the blocks scheduled in test 2. Besides, the number of blocks for emergencies is higher for some days. Summarized, changing the compatibility of neurosurgery to the operating rooms on the second location has little impact on the schedule. It is only favorable to add the flexibility constraint for CHI and ORT if the small effect on the disciplines and emergencies is negligible.

6.3.4 Computational performance

Since the hospital cannot wait too long to make their schedule, it is important that the program runs within an acceptable period of time. For $\Gamma_{em} = 9$ the objective value, number of iterations and running time are determined while Γ_{pt} increases. A similar experiment is performed for $\Gamma_{pt} = 8$, while the budget of uncertainty Γ_{em} is increased. Figure 10 shows the effect of changing one budget of uncertainty to the penalty value of test 4 in Table 7, since in this test the hospitals preferences are considered. One could see by changing Γ_{em} the objective value linearly increases. When Γ_{pt} increases, the objective value increases until $\Gamma_{pt} = 5$ and remains the same when Γ_{pt} increases further. The number of iterations it takes to solve the problem increases when the budget of uncertainty increases. However, the number of iterations decreases for certain values of the budget of uncertainty. When the number of iterations is combined with the running time, one could see that the increase (or decrease) in the number of iterations does not suggest that the running time increases (decreases) with the same ratio. Typically, it takes more iterations to solve the problem when Γ_{pt} changes than solving the problem when Γ_{em} changes. While Γ_{em} increases, the running time shows a peak at $\Gamma_{em} = 19$. Therefore, it is more efficient to solve the problem for $\Gamma_{em} = 20$ or lower values of Γ_{em}. While Γ_{pt} increases, the running time shows a peak at $\Gamma_{pt} = 4$ but the number

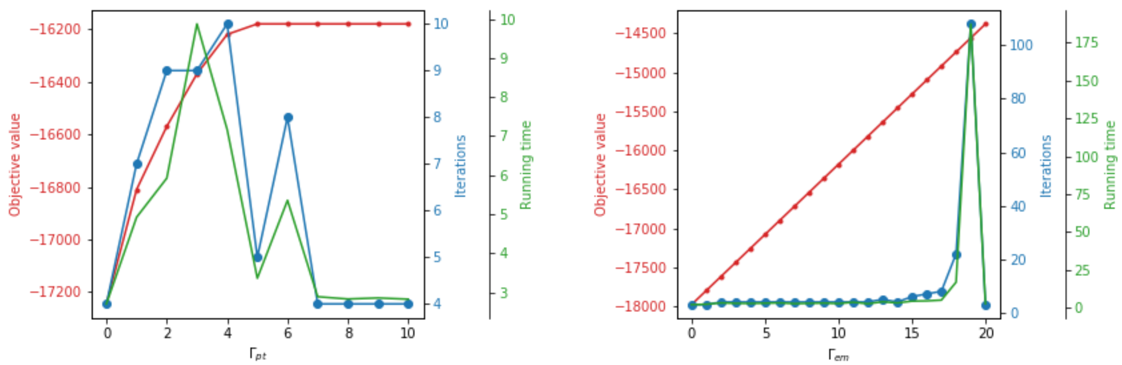


Figure 10: Change of the objective value and the number of iterations it takes to solve the problem by increasing the value of the budget of uncertainty according to the input values of the penalties given in test 4 in Table 7.

of iterations and running time decrease for $\Gamma_{pt} = 5$. Since the five smallest disciplines can increase the required number of blocks to their maximum without having a lack of blocks for a discipline, it is easier to solve the problem for $\Gamma_{pt} = 5$. Given that the penalties differ for the disciplines, it is better to set the uncertain parameter of the discipline with the lowest deviation and penalty to its maximum. Hence, first the uncertain parameter of mouth, jaw and face surgery is set to the maximum number of blocks, given that this discipline has one of the lowest penalties and the lowest deviation in number of blocks. Since the penalty of special dentistry is one of the lowest and the deviation of its minimum is the second lowest (see Table 4), it is optimal to set the number of blocks required for special dentistry to its maximum. It is easier to schedule the blocks of special dentistry than of other disciplines that require a larger number of blocks. Furthermore, the blocks that are given to the discipline with the lowest cost penalty decrease and therefore the objective increases.

In order to evaluate the over-all performance, the objective value, number of iterations, and running time are determined for each combination of the budget of uncertainty, i.e. all combinations of Γ_{em} and Γ_{pt}. The objective value and number of iterations that it takes to solve the problem are determined in order to evaluate which solution is the best for the hospital when the penalty values of test 4 in Table 7 are used. Since the objective is a minimization of cost penalties, the lower the objective value is, the better the solution. Furthermore, the lower the number of iterations and computation time, the easier it is for the hospital to solve the problem. Figure 11 shows the running time, objective value, and number of iterations.

The value of $\tilde{em}_{b_1}$ and $\tilde{pt}_c$ are known for the combination $\Gamma_{em} = 0$ and $\Gamma_{pt} = 0$. The values are $\tilde{em}_{b_1} = \bar{em}_{b_1}$ and $\tilde{pt}_c = \bar{pt}_c$. Therefore, it takes one iteration to solve this problems. The number of iterations depends on all possibilities that can be made of the uncertainty sets. Besides, the running time does not only depend on the number of iterations but at the running time of the master problem of the column-and-constraint generation method as well. If a problem is solved in a few iterations but the master problem takes a long time to be solved, the running time is longer for this problem than for a problem that is solved in a high number of iterations with a low computation time for the master problem. Figure 11 shows that the number of iterations can be higher while the running time does not increase substantially and vice versa. Furthermore, Figure 11 shows that the objective value increases when the budget of uncertainty increases. One can see that the objective value remains the same for $\Gamma_{pt} > 4$, i.e. the objective value goes to its limit at $\Gamma_{pt} = 4$, while Γ_{em} is fixed. While Γ_{em} increases, the objective value increases as well. The number of iterations it takes to solve the problem and the running time are higher for $\Gamma_{em} > 19$. This can

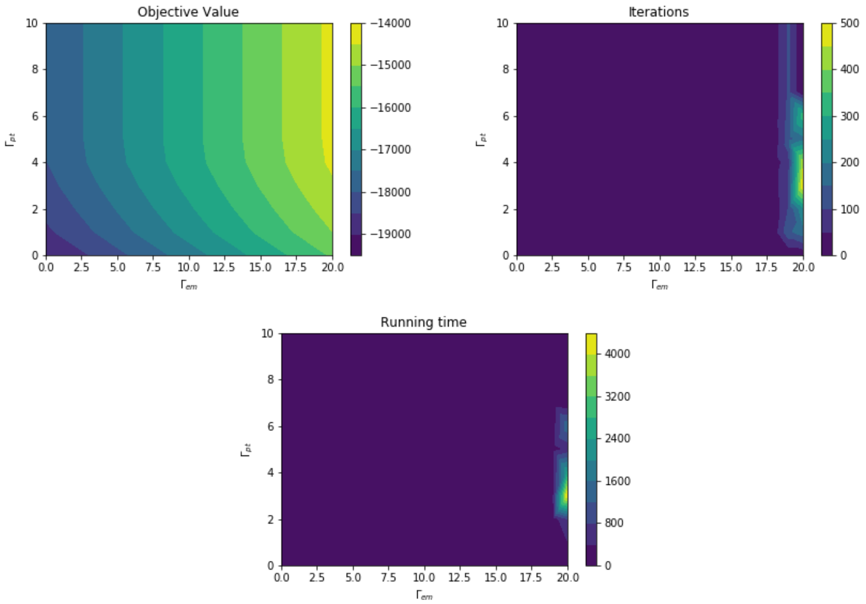


Figure 11: The objective value, number of iterations and running time for solving the problem with penalty values according to test 4 in Table 7 for all combinations of the budget uncertainty.

be caused by the possibilities of the values the uncertain parameter can take. Summarized, the computation time is reasonable and the objective value increases intuitively.

6.3.5 Comparison deterministic solution

The 95% percentile values (see Figure 6 and Table 5) for the number of blocks per discipline pt_c and for emergencies per day em_{b_1} are used as input values for the deterministic problem. In order to evaluate the solution of the stochastic problem and the schedule that is currently used, the schedules of these problems are compared to the outcome of the deterministic problem. The penalties used to solve the deterministic and stochastic problem are those from test 2 in Table 6, since the number of blocks are well divided and balanced like the current schedule.

Discipline	CHI	ENT	GYN	MJF	NEU	OPT	ORT	PLAS	SDEN	URO
Deterministic	99	30	43	13	26	4	87	37	20	35
Stochastic	116	34	44	12	28	4	87	38	21	20
Current schedule	132	43	34	16	40	41	119	41	20	46

Table 8: The number of blocks allocated to the disciplines at the large location for the deterministic problem, the stochastic problem, and the current schedule.

Discipline	CHI	ENT	GYN	MJF	NEU	OPT	ORT	URO
Deterministic	53	17	16	7	16	43	39	9
Stochastic	37	13	20	8	16	43	39	24
Current schedule	57	10	18	1	0	6	55	9

Table 9: The number of blocks allocated to the disciplines at the small location for the deterministic problem, the stochastic problem, and the current schedule.

In general, the blocks allocated to the disciplines at the large location are less than currently scheduled. However, in the stochastic and deterministic DA schedule more blocks are assigned to the disciplines GYN. In the current DA schedule, less blocks are scheduled since the arriving C-sections of semi-elective patients (patients that can be scheduled between 24 hours till three days) are not considered. These patients receive surgery by interrupting or cancelling a scheduled surgery. This implies that the stochastic DA schedule reduces the impact of these patients on the elective schedule and therefore improves patient satisfaction. Since the discipline OPT has a special operating room at the small location, most of the blocks for this discipline are scheduled at this location. However, in practice this location can be used for non-surgical treatments. A restriction can be set to the compatibility of this operating room in order to reserve blocks for these non-surgical treatments. Furthermore, the discipline NEU is not allocated to the operating rooms at the small location in the currently used schedule. This is caused by the fact that it was yet decided to schedule patients of this discipline to the small location. The proposed DA schedule could serve as a good starting point.

Table 12 in Appendix D shows that the total costs of the current DA schedule are higher than costs of the deterministic and stochastic problem. Therefore, the proposed model improves the DA schedule. Furthermore, the costs of the stochastic problem are higher than the costs of deterministic problem. Given the objective value of the stochastic problem is optimal for the worst-case scenario, it is evident that this objective value is higher. Since the number of scheduled blocks for disciplines do not differ a lot between the deterministic and the stochastic problem, the benefit of the stochastic problem is the ability for the user of the program to change the budget of uncertainty. At the moment, during the summer a special schedule is proposed by the hospital in which the capacity is reduced in order to enable the staff to schedule their holidays. By changing the compatibility constraints and changing the budget of uncertainty a better schedule can be proposed.

Compared to the deterministic schedule, more blocks are scheduled for emergencies in the stochastic problem, namely four days of five blocks and sixteen days of six blocks per day. For the deterministic schedule the emergency blocks are five on each day.

6.4 Performance analysis: SA problem

When patients are allocated to blocks, the total surgery duration of patients assigned to a block can exceed the duration of a block, which is represented as overtime. Moreover, the surgery of patients can be postponed and the wards can be full or empty according to which block the patient is assigned. These three indicators measure the performance of the method. The DA schedule where disciplines are assigned to blocks is the input of the SA problem. Therefore, the solution of the SA problem depends on the input solution. Hence, the solution given by the method for the DA problem is compared to the current DA schedule the hospital is using for both the small and the large location.

6.4.1 Statistics parameters

Since the value of the cost penalties give intuitive results, a fixed value is used for the cost parameters. The ratio between the three cost penalties is based on intuitive relations. Two hours of overtime is assumed to be equal to postponing one patient. Since overtime is given in minutes, the values of the cost parameters are $c^O_{\mathbf{\hat{b}}} = 1$ and $c^L_{\mathbf{\hat{b}},p} = 120$. Furthermore, postponing one patient is supposed to be equal to a difference of 12 patients in the ward. Thus, the values of the last cost parameter is $c^W_{b'_1,w} = 12$.

The minimum duration of a surgery and the Length of Stay of a patient and the maximum deviation of the minimum of both uncertain parameters are determined. The minimum surgery duration $\bar{d}_p$ of a patient is established by searching for the minimum duration of the surgery the patient requires that is performed by the surgeons that can perform the surgery of the patient, e.g. patient A requires surgery type I from either surgeon 1 or 2, than $\bar{d}_A$ is the minimum surgery duration of all surgeries from type I performed by surgeon 1 or 2. The maximum deviation $\hat{d}_p$ is established similarly by extracting the minimum surgery duration from the maximum surgery duration. Note that the size of the problem increases when the deviation from the minimum surgery duration is high. Therefore, the deviation from the minimum surgery duration is bounded by 60 minutes, since assumed is that each surgery type deviates at most 60 minutes from the minimum surgery duration.

The Length of Stay of a patient not only depends on the type of surgery but also on the ward the patient is going to. The minimum Length of Stay $\bar{l}_p$ and the maximum deviation $\hat{l}_p$ are determined in the same way as the minimum surgery duration and its deviation.

In certain cases the surgery duration and the Length of Stay do not deviate much from its minimum. In 59.8% of the cases the surgery duration is less than the mean. Thus, 40.2% of the surgeries take longer than the mean surgery duration. This value is used for the budget of uncertainty.

In order to give a value to the budget of uncertainty of the Length of Stay, the number of cases in which the surgery duration deviates less than 5% from its minimum is counted. The Length of Stay deviates less than 5% from the minimum number of days a patient requires to stay in 27.4% of the cases. However, in 65.7% of the cases the patient only requires one day extra than the minimum number of days. Given that the stay of a patient of one day can be expanded with one extra day is an increase of 100% but has less impact than an extension of a Length of Stay with multiple days, the budget of uncertainty Γ_l is set to 34.3% of the number of patients that require to be scheduled.

6.4.2 Impact of DA schedule small location

Since the DA schedule is cyclic, i.e. the schedule repeats itself every month, the schedule is evaluated for the incoming patients one month, supposed that the case-mix, the number of patients per discipline and surgery type, of each month is the same. The data consists of all patients that are scheduled. Assuming that the patients that arrive in a certain month are scheduled within a year, all patients that arrived in the first month of the data set are scheduled. Therefore, the first month of the data set is evaluated. The number of postponed patients, overtime in minutes, and difference in number of patients per day per ward are given per month.

Table 10 shows the results for the small location for the budget of uncertainty given in Section 6.4.1, namely 40.2% of the patients for the surgery duration and 34.3% of the patients for the Length of Stay.

	Overtime (min)	Ward penalty	Scheduled patients
Stochastic DA, stochastic SA	2577	3144	352
Current DA, stochastic SA	2677	3852	360
Current DA, current SA	2897	3876	388

Table 10: The number of postponed patients, overtime, penalty of the ward and the scheduled patients for the new schedule and the currently used schedule.

The number of scheduled patients is less when the stochastic SA problem is implemented. However, the variability in the ward is considered and therefore certain patients cannot be scheduled since the capacity of the ward is reached. The penalty of the ward is decreased by 18.9% by using the stochastic SA and the stochastic DA schedule. Furthermore, using the current DA and the stochastic DA, the ward penalty decreases as well. Furthermore, the number of scheduled patients is higher for the current DA schedule. Furthermore, the overtime is lower for both DA schedules and the stochastic SA compared to the current DA and current SA. However, in the SA problem the overtime of the morning blocks is considered. Since the small location worked with a break from 12.15 till 12.45 overtime is calculated for patients that arrive before 12.15 and leave the OR before 14.00. This time interval is considered because otherwise overtime is considered for surgeries that take a whole day. On the other hand, in the stochastic DA overtime is counted for surgeries that are scheduled in the morning block and take a whole day. Since the number of postponed patients for the current DA and SA are not known, only the total objective of the sub problems is compared. Table 13 in Appendix D shows the total objective value of the overtime and ward penalty for the three experiments in Table 10. The stochastic SA problem creates a more efficient schedule based on the overtime and ward penalty.

In order to evaluate the SA problem for different values of the budget of uncertainty, one of the budget of uncertainty is changed while the other remains zero. Figure 12 shows the result of the different combinations of the budget of uncertainty.

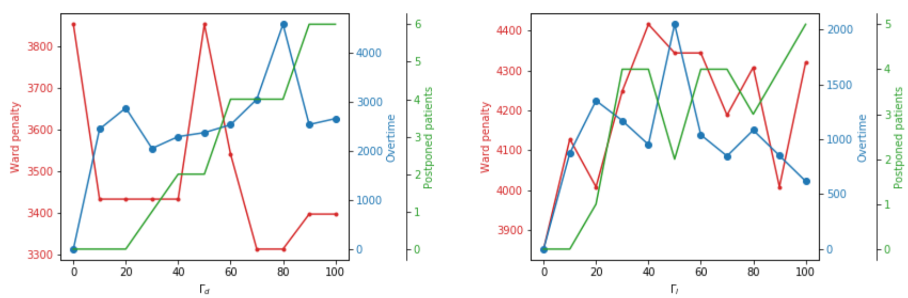


Figure 12: The number of postponed patients, overtime and ward penalty for increasing budget of uncertainty Γ_d and Γ_l.

Figure 12 shows that the overtime increases when the budget of uncertainty of the surgery duration, Γ_d, changes. Since for an increase of Γ_d the surgery duration for certain patients increases, patients are postponed or an increase in overtime occurs. This causes the number of postponed patients to increase and the overtime to fluctuate. Furthermore, the ward penalty varies when the budget of uncertainty of the surgery duration increases. An increase in surgery duration results in more postponed patients which can increase the ward penalty.

Figure 12 shows that the ward penalty increases when the budget of uncertainty of the Length of Stay increases but fluctuates for higher values of the budget of uncertainty. Given that the Length of Stay increases for multiple patients, some patients cannot be scheduled on certain day (on which they were scheduled when Γ_l was lower) since the number of beds are insufficient for the number of patients. The way the patients are allocated to different blocks, causes the difference in the ward penalty. The effect of the overtime and postponed patients cancels each other out. The running time increases when the budget of uncertainty increases. Given that an increase in budget of uncertainty, the possibilities for the surgery duration of each patient increases, it takes longer to solve the problem. Generally, the objective value increases when the uncertainty increases.

6.4.3 Impact of DA schedule large location

The current DA schedule of the large location is compared to the schedule determined by test 2 in Table 6. Such as the small location, the schedule of one month is evaluated. First, the number of patients that are scheduled in the corresponding month and the overtime in that month are established. After that, the overtime and scheduled patients are determined for the current DA schedule by using the current schedule as input for the SA problem. At last, the proposed schedule of the stochastic DA problem is used to schedule the patients. Since the large location is scheduled discipline by discipline, the overtime, and scheduled patients are given per discipline. Table 11 shows the outcome of the experiments.

	Scheduled patients			Overtime (min)		
Discipline	Current DA current SA	Current DA stoch. SA	Stoch. DA stoch. SA	Current DA current SA	Current DA stoch. SA	Stoch. DA stoch. SA
CHI	322	301	297	3592	3930	3296
ENT	151	78	82	356	83	1746
GYN	89	76	78	338	356	529
MJF	24	18	21	202	70	285
NEU	48	40	43	1304	745	456
OPT	139	137	3	164	1550	16
ORT	233	203	173	1981	426	1273
PLAS	70	41	49	712	0	245
SDEN	17	10	18	294	62	104
URO	100	67	65	435	1431	1681

Table 11: Scheduled patients and overtime based on the data, the outcome of the SA problem for the current schedule, and the result of the SA problem for the new schedule, which is determined by the DA problem.

Table 11 shows that the number of patients scheduled by the solution method proposed in Section 5.3 is less than the patients that are actually scheduled. Since patients are currently allocated to blocks based on surgery duration and the variability in the ward is not considered, patients are postponed to the next planning cycle in the stochastic SA problem caused by insufficient number of available bed in the ward. Given that in the new schedule OPT has most blocks at the small location, it is evident that less patients are scheduled and the overtime is less as well. In the case of ENT, PLAS, and URO the number of scheduled patients is significantly lower. The disciplines are scheduled in the order of importance given in Table 4. Given that the disciplines ENT, PLAS and URO have a low relative importance, these disciplines are scheduled at the end. Thus, it is possible that the number of available beds in the ward has reached its maximum and therefore multiple patients cannot be scheduled. It is evident that less patients are scheduled for disciplines where the surgery duration of a certain surgery type deviates a lot. Furthermore, the overtime for the current schedule is determined similarly as for the small location. However, for the large location there is no break for the staff. Therefore, the overtime could be overestimated. Table 11 shows that the total overtime of the current DA and SA is higher than the total overtime of the current DA and stochastic SA. However, the total overtime of the stochastic SA and stochastic DA is higher. The disciplines for which more patients are scheduled contribute to a higher overtime.

Table 14 in Appendix D shows the objective value corresponding to the penalty of the ward. The objective value for the ward is decreased by 60.6% by using the solution method for the stochastic SA problem for the current DA schedule. The ward penalty is lower for the stochastic DA and stochastic SA as well.

Summarized, the objective value of the ward decreases significantly by using the solution method for the stochastic SA problem. The overtime is lower for the current schedule and stochastic SA problem but lower for the stochastic DA schedule and the stochastic SA problem.

7 Conclusion

First, Section 7.1 and 7.2 give a conclusion about the performance of the models. After that, Section 7.3 proposes a recommendation to the hospital in in order to improve the efficiency of their operating room scheduling. At last, Section 7.4 describes the limitations of the model and suggestions on how to proceed with this method are given.

7.1 Discipline Assignment problem

The order of the cost penalties influences the lack of blocks for emergencies and disciplines. Therefore, it is important for the hospital to determine what is preferred. Since the hospital now persists on having one emergency block per day, every additional block for emergencies results in less impact of emergencies on the elective schedule. Furthermore, changing the order of the disciplines ensures that the disciplines with the lowest penalty do not receive the required blocks. When the penalty of the lack of blocks per discipline is greater or equal to the penalty for the lack of emergency blocks, the disciplines get at least their minimum required blocks in the planning cycle. Therefore, it is not sufficient to have the penalty for the disciplines lower than the penalty for the emergencies. Hence, $c_c^{\mathrm{P}} \geq c_{b_1}^{\mathrm{E}}$ fits the best with the vision of the hospital.

The objective value increases when the budget of uncertainty increases since the number of required blocks increases. The objective value reaches a limit when the budget of uncertainty Γ_{pt} is large enough. While the hospital considers uncertainty, it is more efficient to solve the problem for the maximum the budget of uncertainty since the number of iterations it takes to solve the problem is lower for the maximum budget of uncertainty than for a value in between zero and the maximum. The objective value is the same for the maximum as well as lower budget of uncertainty.

Changing the compatibility for operating rooms does not effect the DA schedule if neurosurgery is only assigned to one location. However, changing the compatibility of the four biggest operating rooms is at little expense of another discipline. Therefore, if the hospital prefers this compatibility constraints the choice can be made if the impact on the other discipline is acceptable. In conclusion, the stochastic DA schedule ensures that the impact on elective surgeries is less and is optimal in the worst-case scenario.

7.2 Surgery Assignment problem

The solution method for the SA problem results in less overtime and variability in the ward but the result shows that less patients are scheduled. Since the variability in the ward is included, it sometimes is better to postpone a patient to another planning cycle than scheduling the patient and reserve a place in the ward. Furthermore, it is possible that no beds are available for certain patients and therefore these patients are postponed. For the small location, the result shows a decrease of 18.9% of the objective value of the variability in the ward. Furthermore, for the large location the variability in the ward decreases substantial, namely 60.6%. Moreover, the total objective value of overtime and variability in the ward shows that for the small and large location the surgeries are scheduled more efficiently by using the proposed solution method. At last, changing the budget of uncertainty has intuitive effect on the number of postponed patients, overtime and variability in the ward.

7.3 Recommendations hospital

The results show that the DA schedule that is currently used, consists of enough blocks to schedule the emergency and elective patients. However, the blocks are divided differently over the disciplines and emergencies. First, more blocks are reserved for emergencies to ensure that patients that require a surgery within three days, can be scheduled on time. The disadvantage of these emergency blocks is that staff is scheduled when no patients could arrive. Since patients that require immediate surgery arrive in the morning, the morning blocks could be idle. Therefore, the morning blocks can be assigned to emergencies of a certain discipline in order to make sure a patient can be scheduled in the morning block. The stochastic DA schedule balances the allocated blocks over the disciplines. Momentarily, the small location is often used for the disciplines CHI and ORT. Besides, the number of blocks allocated to the discipline GYN is higher than the currently allocated number of blocks. Since the discipline GYN performs multiple C-sections that are not planned and there are two emergency blocks per day, the impact of this type of patients on the schedule can also be decreased by allocating increasing the number of blocks for emergencies.

Since the blocks per discipline should be sufficient, the patients are not efficiently planned. The results show that there is a decrease in overtime and variability in the ward between the currently used schedule and the schedule proposed by the DA problem. However, the number of patients scheduled in a planning cycle decreases. This is caused by the implementation of the ward variability. The ward has a large impact on the patient schedule. Therefore, in order to divide workload in the ward, the solution method proposed in this thesis gives a proper solution. Given that the large location is solved per discipline of groups of disciplines, the possibility of defining wards per (group of) discipline(s) should be considered. Since it is possible to specify a ward for a discipline if there are enough patients to schedule in this ward, the large disciplines could get their own ward, while smaller disciplines are scheduled on another ward.

The running time for the SA problem is long. Since the number of possible values in the uncertainty set influence the running time, this number should be kept small. In reality, the groups of surgery types for which the minimum surgery duration and deviation of the minimum surgery duration is considered, are large. This causes that the deviation of the minimum surgery duration is high. When the deviation of the minimum surgery duration, $\hat{d}_p$, is high, the cardinality of the uncertainty set increases and thus the running time. In order to decrease $\hat{d}_p$, the groups of surgery types should be chosen such that the deviation of the minimum surgery duration is low, i.e. the surgery durations of a surgery type should be around the same value.

7.4 Limitations

Since the number of patients in each planning cycle is high, the solution method of SA problem is divided up into one method for the small location and one for the large location. A disadvantage of the solution method is the higher the number of patients, the slower the program. The problem of the larger location is solved for each discipline separately. Since each day patients arrive that require a different ward it is possible e.g. for discipline CHI to have five patients at the day ward at day one and zero patients at the day ward at day two but the discipline ORT to have zero patients at the day ward at day one and five patients at the day ward at day two. Hence, it is possible that the optimal solution cannot be found in the method proposed in Section 5.3. In order to decrease the problem size, the instances for which the solution is not possible because of certain restrictions can be removed. However, since the complexity of the problem is dependent on the number of patients, it is more efficient to decrease the number of patients in a planning cycle. Furthermore, the disciplines are solved separately. Therefore, it is possible that none of the patients can be scheduled after the first (or first few) discipline(s) since the capacity of the ward is already met by the first discipline. Hence, it would be more fair to set a capacity on the number of beds in each ward for each discipline. The problem is now solved for one patient group at the time and patients can be rescheduled as a result of arriving patients. However, in practice, patients need to receive a surgery date at a certain amount of time before the date itself in order to be prepared. Therefore, the surgery date needs to be fixed at a certain time. This can be incorporated by adding constraints for the patients for which the surgery date is fixed. Furthermore, not all requirements are added to the model in order to keep the problem size small. In Appendix E, additional constraints are described to add to the model. However, given that the problem size is already large, an heuristic could constructed in order to take those constraints into account.

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Appendix A Two-stage robust optimization problem

A.1 Two-stages of Surgery Assignment problem

The first stage of the two-stage robust optimization problem of the SA problem is:

$$
\begin{aligned}
\min \quad & \sum_{\hat{\mathbf{b}} \in \hat{B} \cup \{\mathbf{b'}\}, p \in P} c^{\mathrm{L}}_{\hat{\mathbf{b}},p} y_{\hat{\mathbf{b}},p} + OPT[R(y, \Gamma_d, \Gamma_l)] & \\
s.t. \quad & \sum_{b_3 \in B_3} mt_{a,p} y_{\hat{\mathbf{b}},p} \leq \psi_a & \forall\, a \in A, \hat{b_1} \in \hat{B}_1, b_2 \in B_2, c \in C \\
& i_{\hat{\mathbf{b}},s} \leq cp_{c,s} & \forall\, \hat{\mathbf{b}} \in \hat{B}, s \in S \\
& \sum_{b_3 \in B_3} i_{\hat{\mathbf{b}},s} \leq 1 & \forall\, \hat{b_1} \in \hat{B}_1, b_2 \in B_2, c \in C, s \in S \\
& \sum_{\hat{\mathbf{b}} \in \hat{B} \cup \{\mathbf{b'}\}} y_{\hat{\mathbf{b}},p} = 1 & \forall\, p \in P \\
& \sum_{\hat{b_1} \in \hat{B}_1, b_3 \in B_3, c \in C} y_{\hat{\mathbf{b}},p} \geq k_p & \forall\, p \in P, b_2 = 1 \\
& sr_{s,p} y_{\hat{\mathbf{b}},p} \leq i_{\hat{\mathbf{b}},s} & \forall\, \hat{\mathbf{b}} \in \hat{B}, s \in S, p \in P \\
& \sum_{\hat{b_1} \in \hat{B}_1, b_3 \in B_3, c \in C} y_{\hat{\mathbf{b}},p} \geq wr_{p,w} & \forall\, p \in P, b_2 = 1, w \in W_{day} \\
& y_{\hat{\mathbf{b}},p} \in \{0, 1\} & \forall\, \hat{\mathbf{b}} \in \hat{B}, p \in P \\
& i_{\hat{\mathbf{b}},s} \in \{0, 1\} & \forall\, \hat{\mathbf{b}} \in \hat{B}, s \in S
\end{aligned}
$$

where $OPT[R(y, \Gamma^c_d, \Gamma^c_l)]$ is the optimal solution to the second stage problem. As specified for the DA problem, the first stage contains the part of the objective and constraints that do not contain or are independent of the uncertain parameters. The first stage variable $y_{\hat{\mathbf{b}},p}$ is fixed in the second stage problem. The objective and constraints that are left over, models the second stage problem.

The second stage of the Robust Surgery Assignment Problem becomes:

$$
\begin{aligned}
\max_{\tilde{d}\in U_d,\tilde{l}\in U_l} \min & \sum_{w\in W} \sum_{\hat{b_1}\in \hat{B}_1\setminus\{0\}} c^{\mathrm{W}}_{b'_1,w} r_{b'_1,w} + \sum_{\hat{\mathbf{b}}\in\hat{B}} c^{\mathrm{O}}_{\hat{\mathbf{b}}} o_{\hat{\mathbf{b}}} && \\
s.t.\ & v_{b'_1,p,w} \geq y_{\hat{\mathbf{b}},p} && \forall\ p\in P, w\in W, \hat{\mathbf{b}}\in\hat{B} \\
& && b'_1 = \hat{b_1},...,\hat{b_1}+l_p-1 \\
& \sum_{p\in P} wr_{p,w} v_{b'_1,p,w} \leq tb_w - ob_{b'_1,w} && \forall\ w\in W, b'_1\in B'_1 \\
& r_{b'_1,w} \geq (ob_{b'_1,w} + \sum_{p\in P} v_{b'_1,p,w}) - (ob_{b'_1-1,w} + \sum_{p\in P} v_{b'_1-1,p,w}) && \forall\ b'_1\in\hat{B}_1\setminus\{0\}, w\in W \\
& r_{b'_1,w} \geq (ob_{b'_1-1,w} \sum_{p\in P} + v_{b'_1-1,p,w}) - (ob_{b'_1,w} + \sum_{p\in P} v_{b'_1,p,w}) && \forall\ b'_1\in\hat{B}_1\setminus\{0\}, w\in W \\
& \sum_{p\in P} (d_p+\gamma) y_{\hat{\mathbf{b}},p} \leq \tau + o_{\hat{\mathbf{b}}} && \forall\ \hat{\mathbf{b}}\in\hat{B} \\
& o_{\hat{\mathbf{b}}} \geq 0 && \forall\ \hat{\mathbf{b}}\in\hat{B} \\
& v_{b'_1,p,w} \in \{0,1\} && \forall\ b'_1\in B'_1, p\in P, w\in W \\
& r_{b'_1,w} \in \mathbb{N}_0 && \forall b'_1\in\hat{B}_1\setminus\{0\}, w\in W
\end{aligned}
$$

Appendix B Dual problems

B.1 Emergency Capacity problem

$$
\begin{aligned}
\max \sum_{b_1 \in B_1} & \left(\tilde{em}_{b_1} - \sum_{b_2 \in B_2, b_3 \in B_3} x_{\mathbf{b},e} \right) u_{b_1} & \\
s.t. \sum_{b_1 \in B_1} & \frac{\tilde{em}_{b_1} - \bar{em}_{b_1}}{\hat{em}_{b_1}} \leq \Gamma_{em} & \\
& \tilde{u_{b_1}} \leq \bar{em}_{b_1} u_{b_1} & \forall\ b_1 \in B_1 \\
& \tilde{u_{b_1}} \leq c^{\mathrm{E}}_{b_1} \tilde{em}_{b_1} + (\bar{em}_{b_1} - \hat{em}_{b_1}) u_{b_1} & \forall\ b_1 \in B_1 \\
& \tilde{u_{b_1}} \geq (\bar{em}_{b_1} - \hat{em}_{b_1}) u_{b_1} & \forall\ b_1 \in B_1 \\
& \tilde{u_{b_1}} \geq c^{\mathrm{E}}_{b_1} \tilde{em}_{b_1} + \bar{em}_{b_1} u_{b_1} & \forall\ b_1 \in B_1 \\
& u_{b_1} \leq c^{\mathrm{E}}_{b_1} & \forall\ b_1 \in B_1 \\
& u_{b_1} \geq 0 & \forall\ b_1 \in B_1 \\
& \tilde{em}_{b_1} \geq 0 & \forall\ b_1 \in B_1
\end{aligned}
$$

where $\tilde{u_{b_1}} = \tilde{em}_{b_1} u_{b_1}$.

B.2 Demand Satisfaction problem

$$
\begin{aligned}
\max \sum_{c \in C} & \tilde{\pi}_c - \sum_{\mathbf{b} \in B} x_{\mathbf{b},c} \pi_c & \\
s.t. \sum_{c \in C} & \frac{\bar{pt}_c - \tilde{pt}_c}{\hat{pt}_c} \leq \Gamma_{pt} & \\
& \tilde{\pi}_c \leq \bar{pt}_c \pi_c & \forall\ c \in C \\
& \tilde{\pi}_c \leq c^{\mathrm{P}}_c \tilde{pt}_c + (\bar{pt}_c - \hat{pt}_c) \pi_c & \forall\ c \in C \\
& \tilde{\pi}_c \geq (\bar{pt}_c - \hat{pt}_c) \pi_c & \forall\ c \in C \\
& \tilde{\pi}_c \geq c^{\mathrm{P}}_c \tilde{pt}_c + \bar{pt}_c \pi_c & \forall\ c \in C \\
& u_c \leq c^{\mathrm{P}}_c & \forall\ c \in C \\
& u_c \geq 0 & \forall\ c \in C \\
& \tilde{pt}_c \geq 0 & \forall\ c \in C
\end{aligned}
$$

where π denotes the dual for the corresponding constraint and $\tilde{\pi}_c = \tilde{pt}_c \pi_c$.

B.3 Block Capacity problem

$$
\begin{aligned}
\max \sum_{\hat{\mathbf{b}} \in \hat{B}} \sum_{p \in P} & (\bar{d}_p y_{\hat{\mathbf{b}},p} + \hat{d}_p \bar{z}_p y_{\hat{\mathbf{b}},p} + \gamma y_{\hat{\mathbf{b}},p} - \tau) \nu_{\hat{\mathbf{b}}} && \\
s.t. \ \sum_{p \in P} & \bar{z}_p \leq \Gamma_d && \\
& \bar{z}_p \leq 1 && \forall\ p \in P \\
& \nu_{\hat{\mathbf{b}}} \leq c^{\mathrm{O}}_{\hat{\mathbf{b}}} && \forall\ \hat{\mathbf{b}} \in \hat{B} \\
& \tilde{\nu}_{\hat{\mathbf{b}},p} \leq c^{\mathrm{O}}_{\hat{\mathbf{b}}} \bar{z}_p && \forall\ \hat{\mathbf{b}} \in \hat{B}, p \in P \\
& \tilde{\nu}_{\hat{\mathbf{b}},p} \leq \nu_{\hat{\mathbf{b}}} && \forall\ \hat{\mathbf{b}} \in \hat{B}, p \in P \\
& \bar{z}_p, \nu_{\hat{\mathbf{b}}} \geq 0 && \forall\ \hat{\mathbf{b}} \in \hat{B}
\end{aligned}
$$

B.4 Downstream Capacity problem

$$
\begin{aligned}
\max \sum_{b'_1 \in \hat{B}_1, w \in W} & \left((ob_{b'_1,w} + \sum_{p \in P} \theta_{b'_1,p,w}) \phi^1_{b'_1,w} - \sum_{p \in P} \tilde{\phi}^1_{b'_1,p,w} \right) - \left((ob_{b'_1-,w} + \sum_{p \in P} \theta_{b'_1-1,p,w}) \phi^1_{b'_1,w} - \sum_{p \in P} \grave{\boldsymbol{\phi}}^1_{b'_1-1,p,w} \right) \\
& + \left((ob_{b'_1-1,w} + \sum_{p \in P} \theta_{b'_1-1,p,w}) \phi^2_{b'_1,w} - \sum_{p \in P} \grave{\boldsymbol{\phi}}^2_{b'_1,p,w} \right) - \left((ob_{b'_1,w} + \sum_{p \in P} \theta_{b'_1,p,w}) \phi^2_{b'_1,w} - \sum_{p \in P} \tilde{\phi}^2_{b'_1,p,w} \right)
\end{aligned}
$$

$$
\begin{aligned}
s.t. \quad & \sum_{p \in P, \hat{l_p} \geq 0, y_{b',p}=0} \left(\frac{\sum_{b'_1 \in B'_1} \sum_{w \in W} (\theta_{b'_1,p,w} - \chi_{b'_1,p,w}) - \bar{l_p}}{\hat{l_p}} \right) \leq \Gamma_l && \\
& \sum_{p \in P} wr_{p,w} (\theta_{b'_1,p,w} - \chi_{b'_1,p,w}) \leq tb_w - ob_{b'_1,w} && \forall\ w \in W, b'_1 \in B'_1 \\
& \chi_{b'_1,p,w} \geq y_{\hat{\mathbf{b}},p} && b'_1 = \hat{b_1} + \bar{l_p} + \hat{l_p} - 1 \\
& && \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \\
& \chi_{b'_1,p,w} \leq 1 - y_{\hat{\mathbf{b}},p} && b'_1 = 1, ..., \hat{b_1} + \bar{l_p} - \hat{l_p} - 1 \\
& && \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \\
& \chi_{b'_1,p,w} \leq \chi_{b'_1+1,p,w} && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
& \sum_{w \in W} \chi_{b'_1,p,w} \leq 1 && \forall\ b'_1 \in B'_1, p \in P \\
& \tilde{\phi}^1_{b'_1,p,w} \leq c^W_{b'_1,w} \chi_{b'_1,p,w} && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
& \tilde{\phi}^2_{b'_1,p,w} \leq c^W_{b'_1,w} \chi_{b'_1,p,w} && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
& \grave{\boldsymbol{\phi}}^1_{b'_1-1,p,w} \leq c^W_{b'_1,w} \chi_{b'_1-1,p,w} && \forall\ b'_1 \in B'_1 \backslash \{0\}, p \in P, w \in W \\
& \grave{\boldsymbol{\phi}}^2_{b'_1-1,p,w} \leq c^W_{b'_1,w} \chi_{b'_1-1,p,w} && \forall\ b'_1 \in B'_1 \backslash \{0\}, p \in P, w \in W
\end{aligned}
$$

$$
\begin{aligned}
&\tilde{\phi}^1_{b'_1,p,w} \leq \phi^1_{b'_1,w} - c^W_{b'_1,w}(1 - \chi_{b'_1,p,w}) && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
&\tilde{\phi}^2_{b'_1,p,w} \leq \phi^2_{b'_1,w} - c^W_{b'_1,w}(1 - \chi_{b'_1,p,w}) && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
&\grave{\boldsymbol{\phi}}^1_{b'_1-1,p,w} \leq \phi^1_{b'_1,w} - c^W_{b'_1,w}(1 - \chi_{b'_1-1,p,w}) && \forall\ b'_1 \in B'_1 \backslash \{0\}, p \in P, w \in W \\
&\grave{\boldsymbol{\phi}}^2_{b'_1-1,p,w} \leq \phi^2_{b'_1,w} - c^W_{b'_1,w}(1 - \chi_{b'_1-1,p,w}) && \forall\ b'_1 \in B'_1 \backslash \{0\}, p \in P, w \in W \\
&\phi^1_{b'_1,w} + \phi^2_{b'_1,w} \leq c^{\mathrm{W}}_{b'_1,w} && \forall\ b'_1 \in B'_1, w \in W \\
&\chi_{b'_1,p,w} \in \{0,1\} && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
&\phi^1_{b'_1,w}, \phi^2_{b'_1,w}, \geq 0 && \forall\ b'_1 \in B'_1, w \in W \\
&\tilde{\phi}^1_{b'_1,p,w}, \tilde{\phi}^2_{b'_1,p,w} \geq 0 && \forall\ b'_1 \in B'_1, p \in P, w \in W \\
&\grave{\boldsymbol{\phi}}^1_{b'_1,p,w}, \grave{\boldsymbol{\phi}}^2_{b'_1,p,w} \geq 0 && \forall\ b'_1 \in B'_1, p \in P, w \in W
\end{aligned}
$$

Appendix C Algorithms

C.1 Master Surgery Schedule problem

Algorithm 1 Column-and-constraint generation algorithm MSS problem

LB $= -\infty$, UB $= \infty$, $I = \emptyset$, $K = 0$

1: Solve Master Problem.

$$
\begin{aligned}
\min \quad & \sum_{\mathbf{b}\in B, c\in C} c_c^T x_{\mathbf{b},c} - \Omega \\
s.t.\ \Omega \geq & \left(\sum_{c\in C} c_c^{\mathrm{P}} \cdot \alpha_c^{\iota} + \sum_{b_1\in B_1} c_{b_1}^{\mathrm{E}} \cdot \beta_{b_1}^{\iota} \right) && \iota \in I \\
& x_{\mathbf{b},c} \leq q_{\mathbf{b},c} && \forall\ \mathbf{b} \in B, c \in C \\
& \sum_{b_1\in B_1, b_3\in B_3} x_{\mathbf{b},c} \geq h_{b_2,c} && \forall\ c \in C, b_2 \in B_2 \\
& \sum_{c\in C} x_{\mathbf{b},c} \leq 1 && \forall\ \mathbf{b} \in B \\
& \sum_{b_2\in B_2, b_3\in B_3} x_{\mathbf{b},e} + \beta_{b_1}^{\iota} - \tilde{em}_{b_1}^{\iota} \geq 0 && \forall\ b_1 \in B_1, \iota \leq K \\
& \sum_{\mathbf{b}\in B} x_{\mathbf{b},c} + \alpha_c^{\iota} - \tilde{pt}_c^{\iota} \geq 0 && \forall\ c \in C, \iota \leq K \\
& \beta_{b_1}^{\iota} \in \mathbb{N}_0 && \forall\ b_1 \in B_1, \iota \leq K \\
& \alpha_c^{\iota} \in \mathbb{N}_0 && \forall\ c \in C, \iota \leq K \\
& x_{\mathbf{b},c} \in \{0, 1\} && \forall\ \mathbf{b} \in B, c \in C
\end{aligned}
$$

Solution value is $(x^*_{K+1}, \Omega^*_{K+1}, \alpha^{1*}, ..., \alpha^{K*}, \beta^{1*}, ..., \beta^{K*})$

2: LB is the optimal objective value of the Master problem.

3: Solve both sub problems, the EC and DS problem, with objective values E^*_{K+1} and A^*_{K+1} respectively.

4: UB $= \min\{UB, \sum_{\mathbf{b}\in B, c\in C} re_c x^*_{K+1} + E^*_{K+1} + A^*_{K+1}\}$

5: **if** UB $-$ LB $\leq \epsilon$ **then**

STOP

6: **else**

Add a cut following Step 8

7: **end if**

8: Add second stage variables with the corresponding constraints of the optimal value of the sub problem.

$$
\begin{aligned}
& \Omega \geq \left(\sum_{c\in C} c_c^{\mathrm{P}} \cdot \alpha_c^{I+1} + \sum_{b_1\in B_1} c_{b_1}^{\mathrm{E}} \cdot \beta_{b_1}^{I+1} \right) \\
& \sum_{b_2\in B_2, b_3\in B_3} x_{\mathbf{b},e} + \beta_{b_1}^{I+1} - \tilde{em}_{b_1}^{I+1} \geq 0 && \forall\ b_1 \in B_1 \\
& \sum_{\mathbf{b}\in B} x_{\mathbf{b},c} + \alpha_c^{I+1} - \tilde{pt}_c^{I+1} \geq 0 && \forall\ c \in C
\end{aligned}
$$

where $\tilde{em}^{I+1}$ is the optimal solution (worst-cast scenario for surgery duration) obtained by solving the EC problem and $\tilde{pt}^{I+1}$ is the optimal solution solving the DS problem.

9: Set $I = I + 1$ and $K = K \cup \{I + 1\}$ and continue to the next iteration with step 1.

C.2 Surgery Assignment problem

Algorithm 2 Column-and-constraint generation algorithm Surgery Assignment problem

LB $= -\infty$, UB $= \infty$, $I = \emptyset$, $K = 0$
Solve Master Problem

$$
\begin{aligned}
&\min \sum_{p \in P} c^{\mathrm{l}}_{\hat{\mathbf{b}},p} y_{\hat{\mathbf{b}},p} + \Delta && \\
&s.t.\ \Delta \geq \sum_{w \in W} \sum_{b'_1 \in \hat{B_1} \setminus \{0\}} c^{\mathrm{W}}_{b'_1,w} r^{\iota}_{b'_1,w} + \sum_{\hat{\mathbf{b}} \in \hat{B}} c^{\mathrm{O}}_{\hat{\mathbf{b}}} o^{\iota}_{\hat{\mathbf{b}}} && \iota \in I \\
&\sum_{p \in P} (d^{\iota}_p + \gamma) y_{\hat{\mathbf{b}},p} \leq \tau + o^{\iota}_{\hat{\mathbf{b}}} && \forall\ \hat{\mathbf{b}} \in \hat{B}, \iota \leq K \\
&v^{\iota}_{b'_1,p,w} \geq y_{\hat{\mathbf{b}},p} && \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B} \\
& && b'_1 = \hat{b_1}, \ldots, \hat{b_1} + l^{\iota}_p - 1, \iota \leq K \\
&\sum_{p \in P} wr_{p,w} v^{\iota}_{b'_1,p,w} \leq tb_w - ob_{b'_1,w} && \forall\ w \in W, b'_1 \in B'_1, \iota \leq K \\
&r^{\iota}_{b'_1,w} \geq (ob_{b'_1,w} + \sum_{p \in P} v^{\iota}_{b'_1,p,w}) - (ob_{b'_1-1,w} + \sum_{p \in P} v^{\iota}_{b'_1-1,p,w}) && \forall\ b'_1 \in \hat{B_1} \setminus \{0\}, w \in W, \iota \leq K \\
&r^{\iota}_{b'_1,w} \geq (ob_{b'_1-1,w} + \sum_{p \in P} v^{\iota}_{b'_1-1,p,w}) - (ob_{b'_1,w} + \sum_{p \in P} v^{\iota}_{b'_1,p,w}) && \forall\ b'_1 \in \hat{B_1} \setminus \{0\}, w \in W, \iota \leq K \\
&\sum_{b_3 \in B_3} mt_{a,p} y_{\hat{\mathbf{b}},p} \leq \psi_a && \forall\ a \in A, \hat{b_1} \in \hat{B_1}, b_2 \in B_2, c \in C \\
&i_{\hat{\mathbf{b}},s} \leq cp_{c,s} x^{\mathrm{I}}_{\hat{\mathbf{b}}} && \forall\ \hat{\mathbf{b}} \in \hat{B}, s \in S \\
&\sum_{b_3 \in B_3} i_{\hat{\mathbf{b}},s} \leq 1 && \forall\ \hat{b_1} \in \hat{B_1}, b_2 \in B_2, c \in C, s \in S \\
&\sum_{\hat{\mathbf{b}} \in \hat{B} \cup \{b'\}} y_{\hat{\mathbf{b}},p} = 1 && \forall\ p \in P \\
&\sum_{\hat{b_1} \in \hat{B_1}, b_3 \in B_3, c \in C} y_{\hat{\mathbf{b}},p} \geq k_p && \forall\ p \in P, b_2 = 1 \\
&sr_{s,p} y_{\hat{\mathbf{b}},p} \leq i_{\hat{\mathbf{b}},s} && \forall\ \hat{\mathbf{b}} \in \hat{B}, s \in S, p \in P \\
&\sum_{\hat{b_1} \in \hat{B_1}, b_3 \in B_3, c \in C} y_{\hat{\mathbf{b}},p} \geq wr_{p,w} && \forall\ p \in P, b_2 = 1, w \in W_{day} \\
&v^{\iota}_{b'_1,p,w} \in \{0,1\} && \forall\ b'_1 \in B'_1, p \in P, w \in W, \iota \leq K \\
&r^{\iota}_{b'_1,w} \in \mathbb{N}_0 && \forall b'_1 \in \hat{B_1} \setminus \{0\}, w \in W, \iota \leq K \\
&y_{\hat{\mathbf{b}},p} \in \{0,1\} && \forall\ \hat{\mathbf{b}} \in \hat{B}, p \in P \\
&i_{\hat{\mathbf{b}},s} \in \{0,1\} && \forall\ \hat{\mathbf{b}} \in \hat{B}, s \in S \\
&o^{\iota}_{\hat{\mathbf{b}}} \geq 0 && \forall\ \hat{\mathbf{b}} \in \hat{B}
\end{aligned}
$$

Solution value is $(y^*_{K+1}, z^*_{K+1}, i^*_{K+1}, \Delta^*_{K+1}, o^{1*}, \ldots, o^{K*}, r^{1*}, \ldots, r^{K*})$
LB is the optimal objective value of the Master problem.
Create an arrival parameter $\theta_{b'_1,p,w}$ such that $\theta_{b'_1,p,w} = 1$ if $y_{\hat{\mathbf{b}},p} \cdot wr_{p,w} = 1$ for all $b'_1 \geq \hat{b_1}$ and zero otherwise. This parameter is used for the DC sub problem.

4: Solve both sub problems, the BC and DC problem, with objective values S^*_{K+1} and D^*_{K+1} respectively.
5: UB $= \min\{UB, \sum_{p \in P} c^{1}_{\hat{\mathbf{b}},p} y^*_{K+1} + S^*_{K+1} + D^*_{K+1}\}$
6: **if** UB $-$ LB $\leq \epsilon$ **then**
STOP
7: **else**
Add a cut following Step 10
8: **end if**
9: Calculate the LoS for each patient at iteration $K+1$ by $\tilde{l}_p = \sum_{w \in W, b'_1 \in V'_1} (\theta_{b'_1,p,w} - \chi_{b'_1,p,w})$.
10: Add second stage variables with the corresponding constraints of the optimal value of the sub problem.

$$\Delta \geq \sum_{w \in W} \sum_{b'_1 \in \hat{B_1} \setminus \{0\}} c^{\mathrm{W}}_w r^{K+1}_{b'_1,w} + \sum_{\hat{\mathbf{b}} \in \hat{B}} c^{\mathrm{O}}_{\hat{\mathbf{b}}} o^{K+1}_{\hat{\mathbf{b}}}$$

$$\sum_{p \in P} (d^{K+1}_p + \gamma) y_{\hat{\mathbf{b}},p} \leq \tau + o^{K+1}_{\hat{\mathbf{b}}} \qquad \forall\ \hat{\mathbf{b}} \in \hat{B}$$

$$v^{K+1}_{b'_1,p,w} \geq y_{\hat{\mathbf{b}},p} \qquad \forall\ p \in P, w \in W, \hat{\mathbf{b}} \in \hat{B}$$

$$b'_1 = \hat{b_1}, \ldots, \hat{b_1} + l^{K+1}_p - 1$$

$$\sum_{p \in P} wr_{p,w} v^{K+1}_{b'_1,p,w} \leq tb_w - ob_{b'_1,w} \qquad \forall\ w \in W, b'_1 \in B'_1$$

$$r^{K+1}_{b'_1,w} \geq (ob_{b'_1,w} + \sum_{p \in P} v^{K+1}_{b'_1,p,w}) - (ob_{b'_1-1,w} + \sum_{p \in P} v^{K+1}_{b'_1-1,p,w}) \qquad \forall\ b'_1 \in \hat{B_1} \setminus \{0\}, w \in W$$

$$r^{K+1}_{b'_1,w} \geq (ob_{b'_1-1,w} + \sum_{p \in P} v^{K+1}_{b'_1-1,p,w}) - (ob_{b'_1,w} + \sum_{p \in P} v^{K+1}_{b'_1,p,w}) \qquad \forall\ b'_1 \in \hat{B_1} \setminus \{0\}, w \in W$$

where d^{I+1} is the optimal solution (worst-cast scenario for surgery duration) obtained by solving the BC sub problem and l^{K+1} the optimal solution obtained by the DC sub problem.
11: Set $I = I + 1$ and $K = K \cup \{I+1\}$ and continue to the next iteration with step 1

Appendix D Results

	Total objective value
Deterministic DA	-7200
Stochastic DA	-6540
Current DA	-6280

Table 12: The total objective value of the deterministic DA problem, the stochastic DA problem, and the current schedule according to the experiments in Section 6.3.5.

	Objective value overtime and ward
Stochastic DA, stochastic SA	5721
Current DA, stochastic SA	6673
Current DA, current SA	6773

Table 13: The total objective value of overtime and ward penalty of the stochastic DA and stochastic SA problem, the current DA and stochastic SA problem, and the current DA and current SA for the small location.

	Objective value ward penalty
Stochastic DA, stochastic SA	4188
Current DA, stochastic SA	3744
Current DA, current SA	9492

Table 14: The objective value corresponding to the ward penalty of the stochastic DA and stochastic SA problem, the current DA and stochastic SA problem, and the current DA and current SA for the large location.

Appendix E Scope: additional constraints

Due to the size of Surgery Assignment problem and the lack of data, some conditions are not taken into account. For further research the constraints corresponding to the conditions are formulated in this section.

The first condition does not allow for surgeons to be scheduled in a block in which they are not available. The following parameter describes the availability of the surgeons:

$$\rho_{\hat{\mathbf{b}},\mathbf{s}} = \begin{cases} 1 & \text{if surgeon } s \text{ is available at block } \hat{\mathbf{b}} \\ 0 & \text{otherwise} \end{cases}$$

The corresponding constraint equals:

$$i_{\hat{\mathbf{b}},s} \leq \rho_{\hat{\mathbf{b}},s} \ \forall \ \hat{\mathbf{b}} \in \hat{B}, s \in S$$

The second condition takes the patients that want to be scheduled after the current planning cycle into account. First, the penalty corresponding to the latest date the patient should be scheduled should not be given. Second, the patients need to be scheduled in the dummy block which means the patient goes to the next planning cycle. The parameter that specifies the desire of the patient to be postponed to the next planning cycle equals:

$$\eta_p = \begin{cases} 1 & \text{if patient } p \text{ wants to postpone the surgery to the consecutive planning cycle} \\ 0 & \text{otherwise} \end{cases}$$

The objective changes to:

$$\min \sum_{\hat{\mathbf{b}} \in \hat{B}, p \in P} \sigma C^L_{\hat{\mathbf{b}},p} (1 - \eta_p) y_{\hat{\mathbf{b}},p}$$

The constraint corresponding to the postponement is the following:

$$y_{b',p} \geq 1 - \eta_p \ \forall \ p \in P$$

Furthermore, the changing period differs between types of surgeries and the effective workflow and the availability of staff. Therefore, the changing period is uncertain and can be changed to the uncertain parameter:

$$\tilde{\gamma_p} = \text{ the amount of time the staff needs to clean the OR after patient } p$$

At last, the deviation of level of care of a patient in a ward is high. One day 10 patients are present in the ward which need a high level of care and the other day 10 patients find themselves in the ward which need a low level of care. One wishes to set a fixed total level a care which the ward can manage and make sure the level of care does not exceed this level. In order to formulate the constraint, the uncertain parameter $\tilde{LC_p}$ is defined:

$$\tilde{LC_p} = \text{ the level of care of patient } p$$

In order to define the constraint, the norm of level of care of ward is determined and formulated as TC_w. The corresponding constraint equals:

$$\sum_{p \in P} \tilde{LC_p} v_{b'_1,p,w} \leq TC_w \ \forall \ b'_1 \in B'_1, w \in W$$