Right Wing Extremism and Political Competition

Capturing the impact of right-wing extremist competition on establishment parties

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Abstract

In this thesis I formulate a coalition formation model in a proportional representation democracy setting. Initially, two flank parties and a centrist party exist, after which I extend to model to allow for the entry of a fourth party that pursues extreme-right beliefs, creating an asymmetric distribution of parties. Afterwards, I extend the model by allowing for party-specific bliss points. The model is inspired by the well known framework developed by Austen-Smith and Banks (1988), but my model generates some interesting additional insights. I find that allowing for a fourth party on the extreme *right* wing of the spectrum has little effect on the policies advocated by the incumbent parties, but yields a more *left-wing* coalition in equilibrium, along with lower volatility surrounding the outcome. Furthermore, by allowing for bliss points, the equilibrium policy becomes considerably more *left-wing*, and incumbent parties tend to polarise, i.e. they move away from the political centre.

1 Introduction

The presence of populism in Europe has become undeniable during the twentieth century. Since the Great Recession hit the continent back in 2008, the disapproval of and the resentment against the incumbent governments have started to take serious forms, resulting not only in the emergence of extremist parties, but also in their staggering growth of popularity in a short amount of time. We have observed the magnitude of the extremist parties in countries such as Italy, the Netherlands, France, the United Kingdom and Germany during their elections at the end of the 2010s. This new, popularised way of performing politics and challenging classic parties seems to have become integrated in the political playing field and should hence also be taken into consideration when analysing politics from a scientific and theoretical perspective. Therefore, it is no longer sufficient to work with models in the field of Political Economy that simply deny the presence of these kind of movements. In this theoretical thesis, I will propose a coalition formation model that takes an extremist party into account. The addition of an extremist party in a three-party framework potentially yields insights that are twofold, since this addition entails an asymmetric distribution of parties on the political framework, as well as a flavour of extremism. The basics of the model are derived from the often-cited and well-known framework as postulated by Austen-Smith and Banks (1988), in the sense that this model includes a legislative stage that is modelled as a bargaining game, resulting in a similar timeline.

I derive the solution of the model I propose in this thesis in a political setting of three parties and in a setting where a fourth, extreme right wing party has entered. Afterwards, I extend the four-party game by adding party bliss points. The derivation of the equilibria in all scenarios yields an opportunity to analyse the addition of a fourth party in my model in isolation, as opposed to comparing this equilibria to existing three-party models. This leaves for a direct comparison, and an assurance that every difference in equilibria is indeed induced by the entry of this fourth party, rather than it being caused by a different set of assumptions or mechanisms.

I find that merely adding a fourth party to the model does not change the ideologies pursued by the incumbent parties in equilibrium, but does change the outcome. In the three party equilibrium, both the left-wing party and the right-wing party have equal chances of making office in a coalition with the centrist party. Analogously, the equilibrium government policy enacted after the election is, with equal probabilities, either a compromise between the left-wing party's preferred policies and centrist policies, or a compromise between the right-wing party's preferred policies and centrist policies. However, in the four-party case, the right-wing party experiences additional competition on its own flank, significantly reducing its bargaining power. Hence, it no longer has a chance of making office, and the equilibrium policy is implemented by the centre-left coalition, yielding a compromise between the left-wing party and the centrist party. If we allow for bliss points, the same coalition forms, but a significantly more left-wing policy package is enacted. This is a result of parties diverging from each other, increasing the distances between parties, and from the centrist party moving leftwards.

The remainder of this thesis is structured as follows: section 2 will elaborate on the motivation of this thesis and the background in which the model takes place, as well as portraying some of the existing literature on this topic. Section 3 will delve deeper into the technicalities of the benchmark model, as well as the proposed extension. Afterwards, in section 4, the equilibrium of the model will be derived and illustrated for all scenarios, succeeded by the interpretation and discussion hereof in section 5. Lastly, a conclusion will be drawn in section 6.

2 Background and Literature

The literature on coalition formation in proportional representation systems is limited, and some of the models that are still widely used have been formulated a long time ago. In this section, I will briefly overview some of the existing literature on coalition formation and on the effects of the penetration of extremist parties into PR democratic systems.

However, before digging into the technical analysis and models, I will briefly motivate the underlying developments that sparked the current phenomena in the political field. The Great Recession was previously mentioned in the introduction as one of the most - if not the most - important driving forces behind the rising popularity of alternative political forces. Such a changing attitude towards politics as a reaction is not uncommon, as a brief look in history reveals many analogous occurrences, such as the rise of national-socialism and communism after the Wall Street Crash of 1929, or the fascist takeover in Italy after the first World War. What is it then, that makes recessions and crises give rise to change in political preferences and the correlated emergence of challenger parties? During these strenuous episodes, governments become financially constrained and governing parties are obliged to implement unpopular (fiscal) measures, which, in the short term, may impair many peoples' fortunes. It is not uncommon for voters to approach general elections as an assessment of the policies implemented by the incumbent administration. Hence, after an unfavourable cycle, many voters may punish the status-quo parties for their perceived defections. One of the fundamental aspects that renders the challenger parties as an attractive alternative, is the fact that these parties can propose policy packages without facing the governmental responsibilities that administrative parties are confronted with during their time in office. Essentially, then, these alternative political forces can propose policy packages which 'speak to the public', or 'advocate the population's *real* needs', even though they might be based on short-sighted desires and can be - especially in the long run - economically harmful (Hobolt & Tilley, 2016). A more specifically influencing matter during the Great Recession for the European countries was the 'burden' of the European Union, as it implied less sovereignty for the national governments. This in turn restricted policy responses to be less curtailed to a distinct country, which was (and still is) received with additional scepticism: during the financially burdensome times, many people felt betrayed by their own government, which caused them to accuse their governments of not prioritising the people they primarily represented. All these considerations taken together, combined with unprecedented communication possibilities, made for easy mobilisation of frustrated voters, giving rise to extreme parties (De Vries & Edwards, 2009).

Knowing then, albeit roughly, how history developed we can continue on reviewing the translation of these mechanisms into formal economic models. However, in line with expectations, encapsulating politics in a scientific framework is extremely ambitious, limiting and challenging. There are numerous factors that influence the processes, of which an important one is the (seemingly) irrational behaviour during general elections, which is complex to capture in a framework such as the ones we are working with. Notwithstanding the difficulties and caveats, many economists and political scientists have imposed this task on themselves, and some of the works which are relevant for my thesis will be outlined hereafter.

The core paper of interest for this thesis is written by Austen-Smith and Banks in 1988 and analyses a three-party proportional representation (PR) democracy, in which two parties will ultimately form a governing coalition. The outcome of the bargaining game, with a government policy package as well as a division of (monetary) benefits at stake for the winners, is that the biggest party will cooperate with the smallest party. The left-wing (L) and right-wing (R) party achieve an equally sizeable vote share in equilibrium as a result of strategic placement and voting, yielding them both a probability of 0.5 of making office The equilibrium ensures a coalition of the winning party with the middle party (M), which loses the election but is sure with a probability of 1 that it will make office. The implemented policy platform is the exact middle point between the two parties forming a coalition (so either the middle point between L and M, or R and M) , and the benefits will be distributed in such a manner that the smallest party (M) will be compensated exactly its loss in utility from having to implement a second-best policy set, whereas the biggest party (R,L) will obtain all possible benefits minus the ones it has to distribute to M[1]. ASB made some interesting advances from most works at the time, which usually only included two parties, resulting in party influences which were monotonic in voter support, as well as placement around the median voter. Another dimension in which ASB differentiate themselves is the modelling of a coalition formation game. In most works in the field, the ultimately implemented policy is a weighted average of the proposed policies, i.e. a parliamentary mean. ASB carefully implement a bargaining phase, to yield a result that is - in most cases - closer to reality than a parliamentary mean concept, especially in cases where there are many (small) parties. (Austen-Smith & Banks, 1988).

One of the most important distinctions between ASB and the classic, fundamental insights proposed by Downs (1957) and Hotelling (1929), lies in the objective of the political parties. In the classic models, parties merely attempt to maximise their vote share, in order to capture the

[1] The specifics regarding the utility payoffs will be discussed later on in the paper.

biggest share of the power at stake. However, in - amongst other works - ASB, parties have a more ideological objective: they want to minimise the political distance between the implemented policy and their own ideal policy platform. Hence, becoming the most influential party is only a tool to achieve the greater goal, as opposed to being the ultimate target. In fact, merely winning the election is not a sufficient condition to maximise a party's utility, as will be discussed later on. Although this is clearly an integral distinction in the theoretical literature, many takeaways from the original works are still widely used and adopted, such as the principle of minimum differentiation - or, less strictly, a principle of converging parties (Hotelling, 1929), (Downs, 1957).

Contrary to many proposed equilibria, one could find that, in a two-party PR framework, parties will not converge, but as a matter of fact face incentives to radicalise. Ortuño-Ortin (1997), like ASB, defines parties to be ideological unities. However, contrarily, he defines the outcome of the election (i.e. the policy set carried out afterwards) as a weighted average of the proposed programmes. This proves to be a vital assumption for the result of polarising parties, as giving up voters by moving toward the radical ends of the spectrum can alter the weighted average in the favour of the party (if the loss in vote share is more than compensated by the gain in ideological alterations of the policy package). The author justifies this approach by claiming that a party winning with a one seat majority would not be able to exert its preferred policy package to the same extent as if it would have won with a majority of e.g. 20 seats. Although this would strike one as rather intuitive, many models do not take this concept into consideration. ASB, for one, do not apply any weight to the winning margin, resulting in a minimised majority outcome at all times. However, to control for the above mentioned factor of dominance Ortuño-Ortin applies weights to the nominated ideals. In his paper, he finds that the adoption of a probabilistic model provokes a diverging reaction from parties, although similar results are also found by other authors by introducing uncertainty regarding the voter distribution (Ortuno-Ortin, 1997).

Another theoretical framework, developed by Casamatta and Den Donder (2005) shows that the reaction of incumbent parties might depend on the type of electoral system in place. They compare the benchmark case with the existence of two parties to a richer game in which two additional parties enter to yield a four-party political framework. Alternative to my addition in this thesis, both of these entrants pursue extremist beliefs, leaving the symmetry of the political spectrum intact. In their paper, they analyse what happens to the incentives of the incumbent parties and what the equilibrium result and party placements look like under plurality rule and under proportional representation. In their proportional representation game, they make use of the parliamentary mean as discussed earlier and adopted by other authors in their works. They find that, under plurality rule, the incumbent centrist parties adopt either the same position as before or more extreme positions, i.e. moving away from the centre to the flanks. However, in the PR framework, they respond differently. Here, in equilibrium, the centrist parties become more centrist (Casamatta & De Donder, 2005).

In a somewhat similar fashion, Merrill and Grofman experimented with the influence of extreme parties in their paper published in 2019. They also considered a PR setting, but used a parliamentary mean instead of a coalition formation mechanism to decide on the equilibrium policy platform implemented by the government. The equilibrium following the specification of their model implied incumbent parties moving away from the extreme entrants, and thus the equilibrium policy actually moving in the opposite direction to the political competition from the new entrant. They also consider a scenario in which the distribution of voters is bi-modal, which they use to simulate a more polarised society. In this alternative specification, they find that the central parties move away from the centre, i.e. the centrist parties polarise as well, following the distributional shift of the voters' preferences (Merrill & Grofman, 2019).

Although the approach is different; an empirical one instead of constructing a theoretical model, Abou-Chadi and Krause also investigate the effect of a new entrant on the behaviour of the incumbent parties, especially if this new contestant advocates an extreme message. The authors of this paper emphasise the difficulty of differentiating between the impact of the gradual altering general public opinion versus the impact of extremist parties. Hence, they apply a regression discontinuity design, using the threshold of representation, attempting to find the causal effect of the extreme parties on the behaviour of the incumbent establishment. Using a sample of Western-European countries between 1980 and 2014, they find significant behavioural changes: both left-wing as well as right-wing incumbent parties develop more anti-immigrant positions, as well as becoming more vocal regarding cultural protectionism. Although the public opinion of course has an influence, their adoption of the RD-design excludes this influence in their research (Abou-Chadi & Krause, 2018). What this means for an analysis such as the one pursued in this thesis, is that incumbent parties, whether they locate at the left- or the right side of the spectrum, move more towards the right as a general reaction to the introduction of right-wing extremist parties. This conclusion is also underwritten by several other sources in the literature of empirical work (Bale, Green-Pedersen, Krouwel, Luther, & Sitter, 2010), (Wagner & Meyer, 2017).

3 Model

The model I formulate is greatly inspired by the model as proposed by Austen-Smith and Banks (1988). Hence, I will not delve too deep into the technicalities of the mechanisms that are adopted from their model, as these can easily be found by consulting the original paper. I will, however, go through the most important aspects and in the second subsection I will thoroughly explain the alterations as proposed in this thesis.

I will start by visualising the setting of the sequential model, which aims to reflect a democratic system built on the principles of proportional representation (PR), in which multiple parties aspire not necessarily to maximise votes, but to obtain the amount of seats in parliament that will secure them participation in the coalition. The timing of the game is such that first the parties announce their chosen policy platforms, after which voters will cast their ballots based on these published policy platforms. Sincere, expressive voting is assumed, which means that voters will support the party which is positioned closest to their preferences on the ideological spectrum. In the last stage, the parties will attempt to form a coalition. This formation takes the form of a sequential non-cooperative bargaining process, in which the election-winning party will propose the first coalition, followed by the runner-up if the proposal is rejected, until ultimately the party with the least votes gets to formulate a proposal. If no coalition is formed after the last stage, some status quo outcome will be implemented. Not only do the parties bargain about the set of policies to be introduced, they also divide a portfolio. This portfolio can be interpreted as benefits of making office for the party. Because these advantages only apply to the party itself, and not the voters, this portfolio is only relevant for the legislative stage, i.e. the voters do not take these into consideration.

All parties can move around freely on the spectrum, i.e. they are free in adapting their preferred policy package before announcing it (with some minor restrictions which will become clear in the next subsection). However, I restrict the extremist party to be statically located at the right-extreme point of the spectrum. I argue that the extremist party that emerges is much more rigid in its placement than the incumbent parties, for multiple reasons. Firstly, most extremist parties are formed on the basis of anti-establishment feelings, or protests against the incumbent parties. This would restrict them to stay far away (on the ideological spectrum) from the parties they are rebelling against. Secondly, extremist parties tend to oppose forming compromises more so than traditional parties, as they are less pragmatic than centrist parties and attach more value to staying true to their ideology. Taking this into account, it could be a possibility to constrain ER to its position, as has been done in previous works (Merrill & Grofman, 2019). Moreover, one can question whether the objective of a challenger party is identical to the incumbent parties. It is not unheard of that these new entrants merely join the political playing field to win over votes from the establishment and cause havoc, rather than actually competing for a governing position (Downs, 1957). Analogously, studies have shown that many votes for these challenger parties arise out of resentment or protest, meaning that many votes for extreme or unorthodox parties are, essentially, anti-establishment and protest votes. This was showed in works for e.g. Eastern-Europe (Pop-Eleches, 2010) and the Netherlands (Schumacher & Rooduijn, 2013). Taking all these arguments in consideration, I am confident that this additional restriction is well-justified.

3.1 Benchmark Model

There exist four parties, L, M, R, ER, representing the Left, Middle, Right and Extreme Right. These parties compete on a finite policy spectrum $P \in [-1, 1]$ to persuade citizens, from a finite and sufficiently large set of N individuals, to vote for them. In the first stage, $t = -1$, the parties disclose their preferred policy platform on P, where $p_k \in P$, k $\in\Omega = (L, M, R, ER)$ denotes each respective preference. Let $S \in \Omega$ denote the possible subsets of Ω. I assume $p_L < p_M < p_R < p_{ER}$ for two reasons; firstly as it eliminates possibilities of parties announcing the exact same policy set and secondly as this reflects reality. At $t = 0$, every citizen casts a single vote, i.e. it cannot vote for multiple parties simultaneously. The results can be summarised by voting shares, denoted as $w_k \in [0, 1]$. A coalition C needs to obtain majority to form a government, that is $w_C > 1/2$. The following periods ($t = 1 \dots t = 4$) consist of the bargaining game as described above, where the policy y as well as the benefits for each party, $g_k \geq 0$ (part of the total G) are distributed. I define the distribution of this portfolio as $\Delta(G) = \{g_k \geq 0, \forall k \in \Omega \text{ and } \sum g_k = G\}$. G is sufficiently large to ensure that a coalition will form at all times. At $t = 1$, the winner of the elections makes a first proposal, containing of a policy platform y_1 and the distribution of the portfolio g_1, where $g_1 = (g_{L1}, g_{M1}, g_{R1}, g_{ER1})$. The intended coalition partner parties can accept the offer, after which a government is formed if $w_C > 1/2$, or reject the offer, after which the game moves to $t = 2$ and the runner-up of the election will propose in an analogous manner. The game, upon rejection, continues until, ultimately, at $t = 4$ the last offer is proposed. If rejected, some status quo is implemented, yielding all parties a utility of 0[2] and the game ends. A party's strategy, then, is thus defined as the combination of three elements, being an electoral position ($p_k \in P$), a proposal $\Gamma_k \in D(w) \times P \times \Delta(G)$, where $D(w)$ essentially captures a winning coalition, i.e. $D(w) = \{C \in S(\Omega) : w_c > 1/2\}$, and a response strategy $r_k : D_k(w) \times P \times \Delta(G)$. Relevant information regarding the population is that they are uniformly distributed, as are their preferences along the political spectrum P. Parties obtain (dis)utility only from the portfolio share they obtain (g_k) and the distance between the preferred and communicated policy platform $p_k \in P$ and the policy ultimately implemented by the government y^*. This form can be defended on the basis of voters punishing parties for deviating from their promises during the elections. Hence, parties will experience disutility from implementing a policy platform that is not exactly their preferred one[3]. The utility function for party k then takes the following form:

$$U_k = g_k - (y - p_k)^2$$

As mentioned before, the population votes in an expressive manner, i.e. they vote for the party that most closely resembles their own preferences. Hence, their objective is to minimise the distance between the party they vote for and their personal political preferences. This can be summarised by a utility function in the following form:

$$u_i = -(p_k - x_i)^2 + \gamma\zeta$$

[2]Due to G being sufficiently big, a coalition will always form as one of the parties will always have a positive utility payoff

[3]See the original ASB paper for a more carefully explained justification (p.409)

Here, x_i is the voter's ideal point on the policy spectrum P. The dummy γ takes the value of 1 if the voter casts a legitimate vote, and 0 otherwise. The scalar ζ is sufficiently large to ensure that everyone in the population casts a vote.

Now that we have established the most important parts of the benchmark model, we will move on to the extensions, before we will establish the various equilibria of the game.

3.2 Bliss Points

There are some questionable features about the benchmark model. An example would be the limitless ability of parties to move along the policy spectrum P , without being punished. One would assume that if a party identifies itself as e.g. a left-wing party, that it should stick to this side of the political spectrum. If it becomes too centrist, or too extremist, the party should be punished by its supporters or, alternatively, the members of the party itself should experience some disutility for advocating against their own ideology. The idea postulated above stems from long-lasting critiques on the consensus of coalition-formation games, or political party behaviour models in general: parties are being modelled as (relatively) unconstrained. It is untrue that leading up to elections, party leaders can change the direction of the party drastically or that a party would undergo significant alterations from its ideology. Parties are complex organisations and it is therefore undesirable to model them freely, without serious constraints (Strøm, 1990).

Taking this critique into account, the extension to my model will be an exogenous bliss-point, assigned to each party, based on the assumed preferences of the respective supporters on the spectrum $P \in (-1, 1)$. Bliss points in models in this field are not unorthodox and widely applied, and thus a suitable manner of tackling this caveat. Thus, let $B^L = -0.5, B^M = 0, B^R = 0.5, B^{ER} = 1$, where B^k denotes the bliss-points of each party. These values are the preferred policy platforms of the parties, and if they move away from these points - which they could do for strategic reasons, i.e. to secure more votes - they will experience disutility. This addition means that the new party utility function will take the following form:

$$U_k = g_k - (y - p_k)^2 - \alpha(B^k - p_k)^2$$

Thus, the distance of the communicated policy platform to the bliss point of the party enters negatively and quadratically, and is weighted by the parameter α. Later on, in the next chapter, it will become obvious what this addition means for the behaviour of the parties.

4 Equilibrium

To solve the model, we use backward induction: we start with solving the last period, step by step moving to $t = -1$ to solve the whole model.

4.1 Legislative Stage

4.1.1 Three Party Case

The three party case of the legislative stage is completely identical to the case in ASB. However, for completeness, it will be cited here:

Lemma 1. Legislative Equilibrium (ASB)
let party d offer the proposal at $t = 1$, party f at $t = 2$ and party g at $t = 3$. (1) If d has a majority in the legislature, then $y^* = p_d, g^*_d = G$; (2) If d does not have a majority, then $C^* = [d, g], y^*$ lies between p_d and p_{dg} and $max\ g^*_g = (p_d - p_{dg})^2$ if $y^* = p_{dg}$ and 0 else, $g^*_d = G - g^*_g, g^*_f = 0$
(Austen-Smith & Banks, 1988)

4.1.2 Four Party Case

The last stage of the game is the legislative stage, or the bargaining stage. This stage consists of four stages. In each stage, a party proposes to their preferred partner(s). These will then either accept or reject the proposal after comparing it to their outside option, denoted by u^t_k. This stage remains unaltered, regardless of the usage of the benchmark model or the extended version, as the extension only affects the parties' placements on P. The reason is that this stage only relies on the order of magnitude of the parties, decided by vote shares. Hence, if we solve for all possible outcomes in this stage, the newly generated movements will already be covered and we can simply look up the new equilibrium outcome.

Define $\mathbf{p} = (p_L, p_M, p_R, p_{ER})$ to be the vector of preferred policies, and $\mathbf{w} = (w_L, w_M, w_R, w_{ER})$ as the vector of parties' vote shares. If in some scenario any two parties happen to achieve the exact same vote shares, a coin will be tossed to decide which party ends up proposing first out of the two, i.e. both parties will be allowed a to propose first with a probability of 0.5. G, the portfolio, is assumed to be sufficiently large to secure a coalition at all times. In the case of return to status quo all parties will receive a utility of 0.

Let us move on to the discussion of the outcomes. If a party j manages to achieve an absolute majority, it is fairly straightforward that $y^* = p_j$ and $g^*_j = G$ will be implemented, as party j does not have to compensate any other party or formulate a compromise, and can thus implement its first-best proposal. In all other scenarios, multiple parties will have to collaborate and form a coalition. Let us assume that party j won the election and wants to convince party z to form a coalition. As mentioned before, party z will compare the utility it obtains from this collaboration to its outside option. This means that the objective for party j will be to offer party z just as much as its outside option, so that z is indifferent. This way, j maximises its own utility, constrained to z receiving just enough utility to collaborate with j.

In the last period, $t = 4$, it is obvious that $u^4_k = 0$, i.e. all parties have the same outside option:

the status quo outcome, which yields them all 0 utility. At $t = 3$, however, each party's outside option would be the accepted proposal in $t = 4$: (y_4, g_4). This does not mean that every party has the same opportunity cost, as entering the values of this proposal in each party's utility function does not yield identical results, due to the fact that distances to y_4 as well as the distribution of g_4 differs for each party. Thus, $u_k^3 = U_k(y_4, g_4; p)$, and analogously for every single period. Generally and formally, we can define $\delta(C, y, g, t)$ to be 1 if the proposal is accepted by all parties involved, and 0 otherwise. This brings us to the following definition:

Definition 4.1. Considering the proposals and responses, Γ and r respectively, we can define the outside options the following way:

$u_k^4(\Gamma, r) = 0$
$u_k^3(\Gamma, r) = \delta(C_4, y_4, g_4, 4) \cdot U_k(y_4, g_4; p)$
$u_k^2(\Gamma, r) = \delta(C_3, y_3, g_3, 3) \cdot U_k(y_3, g_3; p) + (1 - \delta(C_3, y_3, g_3, 3)) \cdot u_k^3$
$u_k^1(\Gamma, r) = \delta(C_2, y_2, g_2, 2) \cdot U_k(y_2, g_2; p) + (1 - \delta(C_2, y_2, g_2, 2)) \cdot u_k^2$

Having properly defined the outside options, we can move on to the definition of an equilibrium in this stage of the game. Intuitively, it must be true for an equilibrium to exist that all parties involved in the coalition must be obtaining at least their outside options and hence have no incentive to deviate.

Definition 4.2. An equilibrium consists of proposals Γ_k^* and response strategies $r_k^*(\cdot)$ such that $r_k^*(C, y, g, t) = 1$ if $U_k(y, g; p) \geq u_k^t(\Gamma, r^*)$ and 0 otherwise

Because this game has a finite and well-defined end point, it is possible to solve for the last stage ($t = 4$) and work our way to the first period, since solving for the last stage yields us all outside options for the preceding stage. Applying this method to all periods solves the game. Furthermore, the model takes place in a setting of perfect information, forcing equilibrium proposals to be accepted. The equilibrium proposals are Pareto-optimal for the coalition parties, as y_t will be somewhere between the cooperating parties' ideal points, and the full portfolio will always be distributed. Thus, none of the partners can be made better off without its partner being made worse off.

Now, let us have a look at an equilibrium path of the game at hand. Consider a setting in which $w_L > w_M > w_R > w_{ER}$. We will solve ER's proposal first, as this party secured the least amount of votes. There is a vital difference in this stage between the ASB solution and mine[4]. ASB create a three-party environment with the implicit assumption that all parties can form a two-party coalition at all times to obtain a sufficient amount of votes to form a government. However, when

[4] It is relevant to compare the two outcomes, because the mechanism behind the legislative stage is identical. Hence, the comparison will yield the direct impact of the addition of a fourth party: ER.

dropping the assumption of an electoral threshold, and adding a fourth party, this is no longer the case. In this example solution here, I will assume that ER can only form a two-party coalition with L, but not with all other parties. In other words, ER is relatively small, and L is relatively large. ER then has two options at this stage; either to form a two-party coalition with L, or form a thee-party coalition with R and M. The two options can be weighed off by ER and will prove to be a fairly straight-forward comparison, but before doing so we will need to solve ER's maximisation problem to formulate its proposal. It is known that every single party has an outside option $u_k^4 = 0$, thus, ER cannot implement its first best $y = p_{ER}$ and $g_R = G$. Knowing this, ER will solve, for the three-party solution:

$$\max_{y,g} \quad g_{ER} - (y - p_{ER})^2 + \lambda[G - g_{ER} - (y - p_R)^2 - (y - p_M)^2]$$

The solutions to this optimisation problem are

$$y_4^* = \frac{p_{ER} + p_R + p_M}{3} \equiv p_{ERRM}$$

$$g_{4ER}^* = G - (p_R - p_{ERRM})^2 - (p_M - p_{ERRM})^2$$

$$g_{4R}^* = (p_R - p_{ERRM})^2$$

$$g_{4M}^* = (p_M - p_{ERRM})^2$$

Analogously, the two-party solution for a collaboration with the left-wing party would be

$$y_4^* = \frac{p_{ER} + p_L}{2} \equiv p_{ERL}$$

$$g_{4ER}^* = G - (p_L - p_{ERL})^2$$

$$g_{4L}^* = (p_L - p_{ERL})^2$$

Thus, in these situations, the equilibrium policy is the middle point between the parties involved and the portfolio will be distributed in such a way that the partaking parties will be exactly compensated for their utility loss induced by y, whereas ER will keep the rest, yielding the party a positive utility payoff. A simple comparison between the two options tells us that only if the right-wing party is located sufficiently far away from ER to drag y considerably to the left-wing, ER would prefer the two-party solution[5]. However, in most cases, ER would form the coalition {ER,R,M}, which we will elaborate in this example.

Moving on to $t = 3$, R can now propose. R knows the outside options: $u_{ER}^3 > 0, u_M^3 = 0, u_L^3 = -(p_L - p_{ERRM})^2$. Intuitively, one could assume that R would propose to its nearest competitor.

[5] Which of the two outcomes prevails here is of no relevance for the further path of the game. If {ER,L} would form, then in the second period, {M,L} would still form with the same outcomes.

However, this need not be the case. R's nearest, feasible[6] competitor is M, but L has a considerably worse outside option. Hence, R will propose the coalition {R,L}. The proposition R makes depends on its own location. If $p_R \leq p_{ERRM}$, then R can implement its first-best and this proposition will yield L at least its outside option in utility: $y_3 = p_R, g_{3R} = G, g_{3L} = 0$. However, if $p_R > p_{ERRM}$, R will have to solve a new maximisation problem and will propose $y_3 = p_{ERRM}, g_L = 0, g_R = G$, i.e. it will propose the same policy as the last period, and capture the whole portfolio itself. Which of the two scenarios happens is irrelevant for the path of the game.

Moving on to $t = 2$, it is M's turn to propose. Again, the outside options are known because of complete information: $u^2_{ER} = -(p_{ER} - p_{ERRM})^2, u^2_R > 0, u^2_L = -(p_L - p_{ERRM})^2$. Looking at these opportunity costs, it should immediately become clear what M's preferred option is. M knows that it can offer its first-best to L, not having to compensate its partner at all. Why is this the case? By assumption, we know that $p_M < p_R$, meaning that the distance between M's and L's ideal policies must at all times be sufficiently small to grant L at least the utility it would receive from its current outside option. Thus, $y_2 = p_M, g_{2M} = G, g_{2L} = 0$.

Lastly, we will solve the first period of the legislative stage, $t = 1$. L can now propose, knowing the following outside options: $u^2_{ER} = -(p_{ER} - p_M)^2, u^2_R = -(p_R - p_M)^2, u^2_M > 0$. L also knows that it can form a majority coalition with every single party. What matters now for L's proposal is the relative distance to M. If $d_L \equiv (p_L - p_M) \geq d_R \equiv (p_R - p_M)$, then L will offer $y_1 = p_{LR}, g_L = G - (p_{LR} - p_M)^2, g_R = (p_{LR} - p_M)^2$. If, however, $d_L < d_R$, then, in a similar fashion to the second option in the third period, L can offer the same policy package to either R or ER as the previous period, and capture all of the transferable benefits, i.e. $y = p_M, g_L = G, g_k = 0, k = (R, ER)$. This result is driven by the fact that, in this occasion, the indifference condition for the other party is binding and L does not have to go as far as offering the middle point between the two parties. As one probably noticed from the notation, L is now indifferent between collaborating with R or ER. This is due to the fact that both parties are situated further away from M than L is, and whether L proposes to ER or to R does not change its proposal and hence its utility outcome.

To illustrate an alternative path, let us consider the situation in which $w_M > w_L > w_R > w_{ER}$, and where w_M is sufficiently big to form a coalition with any party. Now, in stage $t = 4$, the coalition {ER,M} will form, and $y_4 = p_{ERM}, g_{4ER} = G - (p_M - p_{ERM})^2, g_{4M} = -(p_M - p_{ERM})^2$. In the previous period, the coalition {R,L} would form, and $y_3 = p_{ERM}, g_{3R} = G, g_{3L} = 0$. In $t = 2$, {L,M} would form, with $y_2 = p_{LM}, g_{2L} = G, g_{2M} = 0$, and, finally, in $t = 1$, we will observe the coalition {M,k}, $k = (R, ER)$, with $y^*_1 = p_M, g^*_{1M} = G, g^*_{1k} = 0$

The rest of the equilibria - there are 24 of them - can be found in the appendix and will not be solved here. As we can, however, already infer from the two paths sketched here, there seem to be a few patterns occurring. Firstly, it seems that the second-biggest party will never make it to the

[6] Since ER has a positive outside option, it is not possible for R to formulate a proposition that makes R better off than any other proposition, while at the same time guaranteeing ER at least its outside option

equilibrium stage 1 coalition. This is the case, because it will always have a positive outside option in the most important stage and will hence not be considered by the proposing party in $t = 1$. Furthermore, it seems that if party L and ER win the elections, they try to collaborate with the party closest to their ideology, conditional on this party not being runner-up. The middle party would prefer to form a coalition with the left-wing party if possible, whereas the right-wing party would like to form a coalition with the extreme right party. This difference in preferences between L and ER on the one hand and M and R on the other hand can be explained by their position. Because M and R have parties on both sides of them, they have a stronger bargaining position. This is because they could always threaten to collaborate with a party that has contrasting beliefs to the desired partner. In the spirit of this model, it can be explained as follows: the middle party could make an offer to the left-wing party. However, because it also has the option available to work with the right-wing party, it has a strong bargaining position and can make an offer to L that is much more beneficial to M. If the roles were reversed, L would not have as strong of a bargaining position. Why? If L threatens to collaborate with the right-wing party, it will hurt its own utility payoff as well, because it has to sacrifice more policies in the proposed platform. Because we operate in a perfect-information world, the other parties know that, by threatening to collaborate with an alternative party, L would hurt its own payoff. Therefore, this threat is not credible and hence will not improve the bargaining position of party L (and, analogously, for party ER). This is not so much the case for the centre-party, as the 'ideological distance' would not increase as much if it would switch between the two possible coalitions. This discrepancy between bargaining strengths is the reason why the 'central' parties (i.e. M and R) would, generally, prefer to make a proposal to the 'outside' parties, (i.e. L and ER respectively) as opposed to making a proposal to each other. After having chosen their partners, the proposals will be formulated in such a manner, that the collaborating party will be made off exactly equal to its outside option. In this manner, the proposal will be accepted and the offering party will maximise its own utility.

Lemma 2. Legislative Equilibrium
Define parties d, f, g, h to make proposals at $t = 1, t = 2, t = 3, t = 4$ respectively. If party d has an absolute majority, i.e. $w_d > 0.5$, then $y^* = p_d, g_d^* = G, C^* = \{d\}$. If not the case, and $w_{d+k} > 0.5$, then y^* will lie somewhere between p_d and p_{dk}, $C^* = (d, k)$ where $k \in (g, h)$. If $y^* = p_{dk}$, then $g_k^* = (p_k - p_{dk})^2$ if $u_k^1 = 0$. If $u_k^1 < 0$, then $g_k^* = (p_k - p_{dk})^2 + u_k^1$, with a limit of $g_k^* = 0$. If $u_k^1 > -(p_k - p_{dk})^2$, then y^* is closer to p_d than to p_k, and $g_k^* = 0$. In all cases, $g_d^* = G - g_k^*,\ g_{-k}^*,\ g_f^* = 0$. If $w_{d+g} < 0.5$, then $C^* = d, g, h$, and $y^* = p_{dgh},\ g_d^* = G - g_g^* - g_h^*,\ g_g^* = (p_g - p_{dgh})^2 + u_g^1,\ g_h^* = (p_h - p_{dgh})^2 + u_h^1,\ g_f^* = 0$ If $u_k^1 > -(p_k - p_{dgh})^2$, then y^* is closer to p_d than to p_k, and $g_k^* = 0$.

4.2 Voting Stage

Since voters are restricted to behave in a sincere way, i.e. there is no room for strategic voting, all voters will behave similarly: they will vote for the party which closest resembles their ideological stances. If the voter is located at precisely the middle-point between two parties on the political spectrum, it will flip a fair coin to decide for which party it votes, yielding both parties a probability of 0.5 of achieving its vote. This means, essentially, that voters care initially and foremost about the actual vote they cast, and not about the ultimate policy enacted[7].

Definition 4.3. Let $\sigma_i^k(p)\{0, 0.5, 1\}$ be the probability that i will vote for party k. Then, a voting equilibrium consists of a vector σ^*, such that $\forall p, \forall i, \forall \sigma_i(p)$: $u_i(p_k) > u_i(p_{k'})$

Essentially then, the vote distribution can be directly derived from the party positions on the political spectrum. Voters will cast a vote based on the party closest to them on the spectrum, meaning that the tipping point between voting for party j or party z is situated at the middle point between their locations on P. The voting equilibria can then be summarised as follows:

4.2.1 Three Party Case

Lemma 3. Three Party Voting Equilibrium
Let $p_L < p_M < p_R$ on a policy spectrum $P \in (-1, 1)$. In this case, the vote shares are the following:
$w_L = 0.5(1 + p_L + \frac{p_M - p_L}{2})$
$w_M = 0.5(\frac{p_R - p_M}{2} + \frac{p_M - p_L}{2})$
$w_R = 0.5(1 - p_R + \frac{p_R - p_M}{2})$

4.2.2 Four Party Case

Lemma 4. Voting Equilibrium
Let $p_L < p_M < p_R < p_{ER}$ on a policy spectrum $P \in (-1, 1)$. In this case, the vote shares are the following:
$w_L = 0.5(1 + p_L + \frac{p_M - p_L}{2})$
$w_M = 0.5(\frac{p_R - p_M}{2} + \frac{p_M - p_L}{2})$
$w_R = 0.5(\frac{p_{ER} - p_R}{2} + \frac{p_R - p_M}{2})$
$w_{ER} = 0.5(\frac{p_{ER} - p_R}{2})$

[7] The justification for this decision will be discussed at length in the discussion part at the end of the thesis.

4.3 Electoral Stage

At last, we arrive at the first stage of the four-party game, in which the parties announce their preferred policy platforms. Essentially, this stage defines the outcome of the game, as the policy placement ensures the vote weights and thus the outcome of the election, due to the way voters behave. Parties know the outcomes from any possible placement due to the full information about all equilibria in the legislative stage. Hence, we can pin down the equilibrium here. Define $\psi_k(p) = E_\pi[U_k(y^*(w(p)), p), g^*(w(p); p)]$ to be the indirect utility gained from party k, when situating at $p \in P$, given the equilibrium in the legislative stage. Now, we can define an equilibrium in the following manner:

Definition 4.4. An electoral equilibrium consists of a vector $p^*{=}(p^*_L, p^*_M, p^*_R, p^*_{ER})$, such that $\forall$ k$\in \Omega$, $\forall$ p$\in P$: $\psi_k(p^*) \geq \psi_k(p_k, p^*_{-k})$.

Parties will position themselves (i.e. announce a certain policy package on the policy spectrum $P \in (-1, 1)$) which maximises their own expected utility. They can do this, because the perfect-information setting allows them to perfectly predict the ensuing pattern, conditional on the placement they choose. In other words, they know exactly which vote shares and equilibrium legislative outcome follow from a set of party positions.

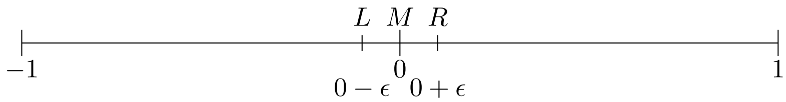
Before continuing with the four party equilibrium, it would be wise to have a look at the equilibrium in my proposed model in absence of the fourth party. All in all, the contribution of my thesis is the addition of an extremist party, which introduces an asymmetry in the distribution of parties in the electoral platform. Hence, we need to know what would happen in the original setting, in order to compare the four-party case. Although the mechanics of this model are inspired by the ASB case, it is adapted in a non-negligible manner, meaning that simply comparing it to their solution would not suffice, and a newly derived solution to the three-party equilibrium is necessary.

4.3.1 Three Party Case

Taking into consideration the strategies of all the parties involved, the following equilibrium will unfold:

Lemma 5. Three Party Electoral Equilibrium
Considering the equilibrium vote shares and legislative outcomes, the parties will position themselves such that:
$p_L = 0 - \epsilon$
$p_M = 0$
$p_R = 0 + \epsilon$
Where ϵ is a very small but non-negative margin.

Figure 1: **Three Party Electoral Equilibrium**



In this equilibrium, the election outcome will be $w_L = w_R > w_M$. Thus, the left-wing and the right-wing party capture an equal amount of votes and will make office at probability 0.5 each. They will form a coalition with M, yielding them a minimum majority. The legislative outcome is $C^* = (k, M),\ y^* = p_{kM},\ g_k^* = G - g_M^*,\ g_M^* = (p_M - p_{kM})^2,\ k \in (L, R)$. This will yield the following utility outcomes to the parties:

$$U_M = 0$$
$$U_L = \begin{cases} 0.5 \cdot \left[G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2\right] \\ 0.5 \cdot \left[-(p_L - p_{RM})^2\right] \end{cases}$$
$$U_R = \begin{cases} 0.5 \cdot \left[G - (p_M - p_{RM})^2 - (p_R - p_{RM})^2\right] \\ 0.5 \cdot \left[-(p_R - p_{LM})^2\right] \end{cases}$$

Clearly, no party can be made better off by deviating from their current position. M would only be able to improve its payoff by winning the election, which it cannot do within the limitations of the game. Any deviation would lead to an outright majority of a flank party, which yields M a negative utility and is hence a worse option. It is important to notice that, even if ϵ is big enough to yield moving room, M would not want to deviate. By doing so, it would ensure that $d_L \neq d_R$, yielding M a negative payoff.

L and R will receive the worst of the two possible payoffs ($U_K = -(p_k - p_{k'M}), k \in (L, R)$) with certainty by deviating on the spectrum $\langle 0 + \epsilon; 1 \rangle$ for R or $\langle -1; 0 - \epsilon \rangle$ for L. If they were to deviate even more, i.e. to move to $p_R = 1$ or $p_L = -1$ for R and L respectively, it would obtain a payoff ($U_k = -(p_k - p_M)^2$). Thus, all parties would make themselves worse off by deviating, which proves this is the equilibrium of the game. The proof of uniqueness for this equilibrium can be found in the appendix.

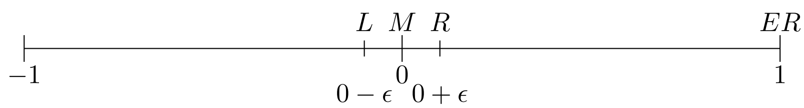
4.3.2 Four Party Case

Considering the response strategies of all parties involved, the following equilibrium will unfold:

Lemma 6. Electoral Equilibrium
Considering the equilibrium vote shares and legislative outcomes, the parties will position themselves such that:
$p_L = 0 - \epsilon$

$p_M = 0$
$p_R = 0 + \epsilon$
$p_{ER} = 1$

Figure 2: **Four Party Electoral Equilibrium**



The full outcome of the election under the assumptions of the four-party model is that we will observe the party placement (or the political stances) as illustrated above, which results in the following relative vote weights: $w_L > w_R > w_{ER} > w_M$, leading to a legislative outcome that can be described as $C^* = (L, M),\ y^* = p_{LM}(= -0.5\epsilon),\ g^*_L = G - (p_M - p_{LM})^2,\ g^*_M = (p_M - p_{LM})^2$. The coalition of L and M has a minimum majority and the implemented policy package will leave the parties with the following utility payoffs:

$U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2 = G - 0.5\epsilon^2$
$U_M = 0$
$U_R = -(p_R - p_{LM})^2 = -0.5625\epsilon^2$
$U_{ER} = -(p_{ER} - p_{LM})^2 = (1 + 0.5\epsilon)^2$

We can check whether this is an equilibrium by verifying whether any party has an incentive to deviate after observing the other parties' positions. Let us start with party ER. ER obviously has no incentive to deviate, as it is assumed to be fixed to its position. Party R can only move to the right, and by doing so it would only make itself worse off. R has currently optimised its utility function and can hence only become better off in an alternative order of party magnitudes. Conditional on the current placement set, it cannot enforce an outcome where R achieves fewer votes than M with certainty. The best thing it can do, is ensure that $w_M = w_R$, if $p_R = p_{ER} - \epsilon$. If R positions itself in this manner, it will receive a more negative payoff than it currently does. R, then, cannot feasibly induce any different policy outcome. Furthermore, even if it would be able to do so, it would surely make itself worse off as the current disutility R experiences is very limited ($1.5\epsilon^2$) and moving further away from the middle will only increase this disutility figure.

Moving over to party M, it finds itself in the position where $d_L = d_R$. In this case, its best response is to situate in the middle, which is currently the case. If M would move, it would upset the equality of distances and obtain a worse utility outcome. Moving to the right would give L the opportunity to obtain an outright majority outcome, which yields M a negative utility outcome. If

M were to move to the left, R could ensure that $w_{L+M} < 0.5$, which would change the coalition and yield M a negative payoff as well.

Lastly, party L does not experience any incentive to deviate, as it has currently maximised its utility. Moving to the left until $p_L = -1 + 2\epsilon$ would induce the outcome $w_L > w_R > w_M > w_{ER}$ ceteris paribus, which would yield L a payoff of $U_L = G - (p_M - p_{LM}) + (p_M - p_{LR})^2 - (p_L - p_{LM})^2$. However, taking into account the distance that L needs to cover for this to occur, it would not be an improvement[8]. If L would move to the left to an extent that $p_L \in \langle -1; -1 + 2\epsilon; \rangle$, the outcome $w_L > w_M > w_R > w_{ER},\ d_L > d_R$ would occur, and would grant L fewer utils. If L were to move in between its current position and the above mentioned positions, the outcome would remain unchanged, but L would experience additional disutility from increasing the distance between p_L and y^*. Thus, we can conclude that the positions as described in lemma 6 do indeed form an equilibrium, as no party has an incentive to deviate. It is a Pareto-optimal equilibrium, as no party can make itself better off without making another party worse off. Now that the equilibria of both games are known, the impact of the additional fourth party can be analysed. The interpretation of the equilibria can be found in the next section, and the proof of uniqueness can be found in the appendix.

Although there are some observable differences between the equilibria, which will be expanded upon later on, there appear to be some striking, or perhaps undesirable outcomes. Firstly, and most obviously, there would be the peculiar distribution of communicated party programmes. In the equilibrium described above, some striking results emerge. The incumbent parties all position themselves very close to the middle-point of the spectrum, with the obvious outcast being ER (although this is driven by assumptions). This leaves a lot of room on the political spectrum uncovered and would surely invite new parties to emerge. For example, the whole left flank is practically abandoned, as the left-wing party represents very centrist beliefs in this equilibrium.

An additional problem, which could be considered the cause of the situation, is that parties are unrestricted in their positioning choice, i.e. without being punished for any movement away from the ideals which their name would suggest they represent. This assumption is somewhat unrealistic and a core driver of the equilibrium outcome. It is common that parties have continuity in the policy package they propose, an observation that could be modelled by e.g. using bliss-points. Moving away from these party bliss points will cause the parties to experience disutility (e.g. because voters lose faith in them or because the party in itself would no longer be comfortable with its own proposals). An obvious example from the emerged equilibrium is the left-wing party, as they essentially become a centrist party. The left-wing voters, however, would surely be discouraged by this movement, and it would not be unrealistic that they would either abstain from voting (Adams & Merrill III, 2003), (Thurner & Eymann, 2000) or form a separate party, representing their beliefs

[8] The additional positive utility part would always be smaller than the additional loss in the two negative utility segments.

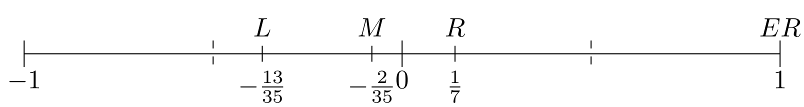
(Lago & Martínez, 2011). Furthermore, in equilibrium, all parties behave opportunistically as opposed to ideologically. Although opportunistic moves by parties are definitely not unprecedented, they are clearly over represented in this model.

4.3.3 Four Party Case with Bliss Points

A solution to tackle the problem postulated above would be to add bliss points. The logic behind it, as well as the modelling of the bliss points have been clarified in the previous section. In this section the impact of the addition of bliss points will be analysed. In the previously shown adjusted utility function, the bliss points were scaled by a parameter α. In this section, it is assumed that this weight is equal to one. Further down in the discussion I will discuss the impact of alterations to α. Before we delve deeper into the incentives each party faces, it should be clear that adding bliss points has no impact on the legislative stage or on the voting stage. Voters will still behave in the exact same manner, i.e. they will vote for the party reflecting their political stances the closest. If the inclusion of bliss points alters the political policies as proposed by parties, then the voting weights will adjust accordingly. Because of complete information, parties know this and take this into consideration when they announce their policy position. Hence, we do not have to revisit the voting stage. Similarly, the electoral stage is unaffected. We know that the bargaining power (i.e. the sequence of the bargaining game) is affected by the relative voting weights as well. These weights are, as explained above, derived from the electoral positions. Thus, the behaviour of voting as well as the outcome of the bargaining stage is already well-known and taken into consideration in the electoral stage. Hence, it will suffice to only derive the electoral stage.

Lemma 7. Electoral Equilibrium in the presence of bliss points
$p_L = -\frac{13}{35}$
$p_M = -\frac{2}{35}$
$p_R = \frac{1}{7}$
$p_{ER} = 1$

Figure 3: **Four Party Electoral Equilibrium with Bliss Points**



The equilibrium as described in lemma 7 would yield the following voting stage outcome:
$w_L = \frac{55}{140} > w_R = \frac{37}{140} > w_{ER} = \frac{30}{140} > w_M = \frac{18}{140}$
The order of magnitude is identical to the four party case, yielding roughly the same equilbrium

proposal. However, contrary to the previous equilibrium, it no longer holds that $d_L = d_R$, as R is now positioned closer to M. Therefore, L can get away with transferring fewer benefits to M in equilibrium: $C^* = (L, M),\ y^* = p_{LM},\ g_L^* = G - g_M^*,\ g_M^* = (p_M - p_{LM})^2 - (p_M - p_{LR})^2$.
This will ensure the following payoffs for each party:
$U_L = G - (p_M - p_{LM})^2 + (p_M - p_{LR})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2 = G - 0.036$
$U_M = -(p_M - p_{LR})^2 - (p_M - 0)^2 = -0.085$
$U_R = -(p_R - p_{LM})^2 - (p_R - 0.5)^2 = -0.255$
$U_{ER} = -(p_{ER} - p_{LM})^2 - (p_{ER} - 1)^2 = -1.474$

Now let us verify whether this is an equilibrium by checking if any party has an incentive to deviate. As always, ER cannot deviate, because this is assumed to be the case. Party R now faces a significantly worse utility payoff than in the benchmark model, but still does not have any power to improve it. Since R has optimised its position, it follows that moving to the left or to the right without altering the outcome of the election will only yield R a worse payoff. Furthermore, R is not in a position to feasibly alter the outcome: it cannot improve its own vote share and can only have an impact on M's vote share in the way described before: increasing M's vote share such that $w_M > w_R$ would not be a viable strategy, in the sense that the utility payoff would be less.

Party M has optimised its placement as well and finds itself in the same boat as R. M would only be able to improve its payoff if $d_L = d_R$, but moving to the left in order to induce this would yield M a lower utility payoff than it currently has (due to the bliss point). If M would move to the right to get closer to its bliss point, it would lose utility by moving away from the equilibrium policy and by being transferred fewer benefits. M currently receives the fewest votes of all parties, but cannot alter this outcome. Hence, it is impossible for M to induce a different coalition outcome by deviating from its current position. Still, one might wonder why M now gets a negative payoff as opposed to a zero payoff in equilibrium. This is due to the fact that $d_L \neq d_R$, which means that L can now get away with transferring fewer benefits to M in equilibrium. M will accept this, because its outside option $u_M^1 = -(p_M - p_{LR})^2$. This means that its outside option is 0 if $d_L = d_R$, but it is negative otherwise (as is the case here).

Moving on to L, this party has maximised its payoff and can also not improve currently. It is already facing its preferred outcome, a coalition with M (the only better option would, obviously, be absolute majority) and has optimised the bliss-point versus policy-gap trade-off. Hence, any movement will surely make L worse off. Once more, the proof for uniqueness can be found in the appendix, and the interpretation of this equilibrium and its implications will be discussed in the next section.

5 Discussion

5.1 Interpretation

First and foremost we should have a careful look at how the two baseline equilibria (three and four parties). We can treat the three-party case as the original benchmark, whereas the asymmetric four-party equilibrium is the new addition. This comparison can prove to be very useful, as the model assumptions are identical and any alteration in the outcome will thus be induced purely by the addition of a fourth party. It seems that, at the first sight, not a lot has changed: the incumbent parties L,M,R adopt the same policy placement as they did in the benchmark model. However, this does not mean that the outcome is identical, as the entry of ER adds a new dimension of political competition on the right wing of the spectrum. This new competition means that the incumbent right-wing party, R, loses voters in the four-party case and finds itself in a less powerful position than before. This results in a difference in outcomes: L winning the election and forming a coalition with M for sure, as opposed to L and R having an equal probability of forming a coalition with M. This also means that the expected utility in equilibrium has decreased for R, whereas it has increased for L (and remains unaltered for M, as it is indifferent between governing with L or R in the case where they are both equally distant from M). Furthermore, the imposed policy package is now for certain a compromise between the ideologies of L and M, whereas before it would, analogously to the vote weights, be a compromise between L and M or R and M both at equal probabilities. Thus, also the expectation and realisation of the enacted equilibrium policy package is more left-wing than in the three party case (albeit slightly). To conclude, the addition of a fourth party, situated on the extreme end of the right flank, produces a result in which the left-wing party becomes favoured at the expense of the right-wing party. The reason is a polarised right-wing population, which weakens the position of the separate parties. This, in turn, leads to a greater bargaining position for the left-wing party, which can now conveniently form a coalition with the centrist party. The outcome is a more left-wing government and policy package, and a worse outcome for the right-wing party.

The addition of bliss points significantly altered the outcome of the game. Not only did all parties (except for ER) deviate in a sizeable manner, the equilibrium policy package was also greatly adjusted. We can conclude that all parties moved away from the centre, L and M both towards the left flank and R towards the right flank. The majority margin of the coalition in terms of vote shares has increased marginally ($w_{LM} = 0.52$ vs $w'_{LM} = 0.5 + 0.5\epsilon$) and the equilibrium policy platform has become much more left-wing oriented ($y^* = -\frac{3}{14}$ vs $y^{*\prime} = -\frac{\epsilon}{2}$). These deviations leave both parties in the coalition now worse off in terms of payoff than in the four-party benchmark game, although its relative payoff compared to the opposition has increased drastically. This is not in the least part due to the fact that parties are much more spread out now, as the limitless alterations of communicated policy platforms have been restricted. The equilibrium outcome now reflects reality much more closely, which is a step in the right direction of this model. It seems that the addition of bliss points was a sensible one and has improved the game in a non-negligible manner. Although the centrist party is now no longer located in the absolute middle point of the political spectrum, it is still very close and the current equilibrium does not facilitate other parties to pursue centrist beliefs. It also seems that the addition of bliss points has increased the welfare of voters. Remember that voters' utility is a function of the distance between its own preferred policy and the communicated policy of the party it voted for. Because the voters are uniformly distributed, and the parties are now more widely spread, many voters will have seen their utility increased, whereas only a few experienced a utility decrease. Hence, the addition of bliss points is welfare increasing for the voters in this particular model.

The value of the scalar α in the model defines the weight of the bliss point in the trade off between policy gap and bliss point gap when parties position themselves. Hence, increasing the value of α will make the parties want to position themselves closer to the bliss point, whereas decreasing the value of α will ensure that parties position closer to the equilibrium outcome. We know that the order of magnitude outcome, and hence the coalition outcome, is the same if $\alpha = 0$ and $\alpha = 1$. This will partly remain to be the case if we increase α. The reason is that the bliss point adds disutility, which makes parties want to move less towards the equilibrium policy than they would in absence of bliss points. However, at some point, this will mean that $w_M > w_{ER}$, which will still yield the same outcome, but a different order of magnitude. Thus, the bliss points are important for the policy-gap versus bliss point gap trade-off, rendering it important for placement. However, it is not important enough to alter the equilibrium coalition parties.

Although I will not derive and proof the three-party bliss point case, it is sufficient to exercise a thought experiment to notice that the four-party bliss point equilibrium is unique, and adding bliss points in the three party case would not yield the same outcome. In the three party case, two parties (L and R) have a realistic chance of winning the election and capturing most of the benefits G. Because the amount of benefits is undefined, but sufficiently large, it will always be able to outweigh the additional costs that moving away from bliss points imposes on moving. Hence, it is always utility enhancing to attempt to win the election. Because parties L and R face symmetric incentives, they will attempt to undercut each other until they no longer can, which occurs at the positions as described in lemma 5. Hence, the same outcome will occur in the three-party model, even in the presence of bliss points. The critical difference between this outcome and the four-party model is that party R does not have a realistic opportunity of winning the election. Only L does now, which means it does not have to undercut R at all cost to capture the winning benefits. Hence, it can balance the utility gained from winning the election with the disutility experienced from moving away from its bliss point.

5.2 Limitations and Future Research

The most obvious caveat of this model lies in the important assumption of expressive voting, and such a simplification requires a justification. Although there is no proper consensus on the magnitude of impact of strategic voting in the existing literature; the estimates for e.g. Britain and Canada lie between 3% and 17% of voters voting strategically (Blais, Nadeau, Gidengil, & Nevitte, 2001), (Bowler & Lanoue, 1992), (Niemi, Written, & Franklin, 1992), many multi-candidate models attempt to include the possibility of strategic voting - just like ASB. Although the central reason for assuming explicit voting in this thesis lies in the need for simplification - the three-party equivalent as established by ASB is already sizeable and extensive, and the four-party extension would simply be beyond the scope of this thesis - this argument alone is of course not sufficient. The main reasoning behind the inclusion of strategic voting in models is the assumption that voters primarily care about the coalition formed in the end, instead of the party they vote for. Not only is this assumption quite strong in the sense that it relies on fully informed voters, it is also debatable to assume that all voters would reason in this manner. The justification of applying expressive voting lies in the view that, although there surely exists a non-negligible amount of people that vote on the basis of ante-election expectations of outcomes, this amount is rather small compared to the mass that supports a party or politician because they feel represented and backed by them or their ideas, or because they sympathise with an underlying ideology. Furthermore, it is rather difficult to tell before the elections what the outcome will be and even if one would be able to predict the outcome of the election perfectly, this leaves no guarantee for which coalitions will be formed. E.g. in the model postulated in this thesis, any theoretical coalition can be a legislative outcome, which might not necessarily resemble the mass' expectations of coalition formation. It is rather difficult to credibly suggest that voters will be as informed and knowledgeable about the elections and its outcomes as the partaking agents (Ortuno-Ortin, 1997). This is especially the case in models in which an extensive legislative stage is modelled, such as in this thesis. Alternatively, in models where e.g. a parliamentary mean is applied instead of a bargaining game, strategic voting already becomes much more defensible. Furthermore, many more factors enter the decision to cooperate with another party which are not necessarily captured by formal models, such as previous experiences, personal relationships and ideological differences. Thus, the core rationale relies on the hypothesis that not only most voters are not sophisticated or invested in politics enough to look much further than which party represents their needs most, and that it remains extremely difficult to tell the outcome of the election on forehand. Another aspect to take into consideration is the background of the elections. Not all elections are the same and the electoral system, as well as the amount of parties have an influence on the salience of strategic voting. As an example, if the election consists of multiple stages (two-round system), there is a much higher certainty regarding the outcome of the election than with the use of a single voting round. In the latter system peoples' expectations commonly are based on polls, which can regularly turn out to be rather inaccurate or biased (Martin, Traugott, & Kennedy, 2005), (Campbell, 2008), leaving the voters not only with incomplete, but also incorrect information to base their predictions on (a phenomenon which is also often not accounted for in the strategic voting models). Although I am confident that the decision to opt for an adaptation of strict expressive voting is well-justified, I would still advice further experimentation being conducted with the inclusion of strategic voting, so one can compare the outcomes and analyse whether differences arise, and how much of an impact strategic voting would have in this specific four-case setting.

Although the justification of the assumption on the extreme-right party has already been discussed extensively, we are now in a position to have a look at what-could-have-been situations. If ER would not have been restricted to its current position, it would have been possible for the party to adopt less-extremist stances in order to increase its vote share. However, just as is the case with the centrist and the centre-right party, it would never have been able to win the election. This means that ER would only have been able to support a status-quo based party in its coalition, in the hope to make the equilibrium policy platform more right-wing. Thus, ER would have been able to extend the current addressed possibilities to the ones in which ER becomes the runner-up. Quickly consulting the appendix teaches us that alternative options are $y^* = p_{LM}, p_L, p_{LR}$. The latter might sound interesting for ER, but requires that $w_L + w_M < 0.5$, which will only happen if $p_M < 0$ and p_R=0[9]. In this case, R practically becomes a new centrist party (what M used to be) and the policy outcome is by definition not more right-wing than the previous p_{LM} was. Thus, although ER could increase its vote share, it cannot obtain a position in which it could influence the outcome to be more right-wing. The only option, then, that ER has left to increase its utility payoff is by closing the gap between the expected outcome and its own policy stances. However, as argued before, it would be against the nature of an anti-establishment, extremist party to essentially mirror status-quo positions. Moreover, as mentioned before, it is ex-ante known that, because of the political competition, ER cannot win the election but could merely play a support role. It would be highly unlikely that an anti-establishment party would moderate its extremist stances, just to support an incumbent party in a coalition. Taking this into consideration, we can conclude that the assumption is relatively harmless.

This thesis was specifically intended to explore the implications of adding a fourth party to an existing model. The number of parties, then, in this framework was chosen exogenously. However, it could be an interesting exercise to implement an endogenous mechanism of entry and exit of multiple parties. This way, one no longer has to assume an exogenous number of parties and there is more at stake than merely the government policy and the portfolio for the participants. Parties have to take into consideration that they can be punished to the extent of being forced out of the parliament if they do not behave sufficiently strategic, which includes some additional incentives

[9] If R would not position itself in 0, M could always go back to force $w_L + w_M > 0.5$, which it would because this yields M a higher expected utility outcome.

for parties to perhaps modify their behaviour. Especially considering the equilibria outcomes I find in my three party and four party settings, it would not be far-fetched to expect additional parties to attempt to enter the political field, as a great share of the population would probably feel underrepresented.

To build upon the point raised above regarding the equilibria party positions being very similar, there might be room for improvement or for experimenting with the party utility functions. In the current model, parties care about the outcome of the bargaining phase (y^*, g^*), and the distance between their own announced ideology and these outcomes. The fact that this is the only manner in which the party position enters the utility function, yields incentives to minimise this difference. Primarily, this means attempting to have enough bargaining power to influence the policy outcome sufficiently to be beneficial to the concerning party. However, if this influence proves to be too little or none at all, parties face the incentive to adjust their policy stances to minimise the difference between the equilibrium outcome and their own advocated stances. Essentially, the secondary incentive as described here is pure opportunism and seems to have a more dominant role in this game than what would be observed in common PR democracies. Furthermore, the utility function only takes a party's size into consideration in the form of what a party's relative size directly means for its outcome in the bargaining stage. In other words, whether a party wins the election by a one vote margin or by, say, a 200,000 votes margin makes no difference in this model. Of course, this result was a conscious choice of adopting a bargaining stage into the model, as opposed to e.g. a parliamentary mean. However, one could potentially experiment with the idea of the portfolio benefits g_k being dependent on absolute vote shares. In this model, the winning party divides the portfolio benefits, whereas it would reflect reality more if this power was more evenly split if the parties approach similar vote shares. In other words, it is not very realistic to assess complete power over the distribution of the benefits to the winning party, if its majority over the cooperating party is small or negligible. The last point I will make regarding the utility function and the derived objection function for parties touches upon this point as well. In the current form, as has been shown for the centrist party, it can be that a compromise outcome between two competitor parties yields a higher payoff than partaking in a coalition. Of course, it could be that the compromise between the left-wing and the right-wing party leads to centrist policies. That being said, it is more likely that left-wing and right-wing policies will be implemented on various topics, leading to a more centrist package on average rather than centrist policies in all areas. However, even if the policy enacted is rather centrist, in reality the centrist party would surely prefer to partake in this government anyway. This means that there are improvements to be made regarding the form and characteristics of the portfolio benefits that are used in this game. The model would reflect reality more closely if being part of the governing coalition yields more utility than not taking part in the government, regardless of the policy implemented. Perhaps that the previously mentioned experiment, in which portfolio benefits are (partly) being directly influenced by vote weights, could already improve the model regarding this caveat.

6 Conclusion

In this thesis I formulated a model of coalition formation which attempts to capture the mechanisms behind, ultimately, the decision making progress on the nature of the policy programmes political parties advocate. I did this by establishing a model which consists of an electoral stage, expressive voting and an extensive non-cooperative bargaining game which shapes the legislative stage. I solved the equilibria for a three-party model, a four-party model and a four-party model with party bliss points. The aim of this thesis was to analyse the impact of the addition of a fourth party that pursues extremist beliefs in my three-party model. This new political competition on the right flank of the spectrum did not induce the establishment parties to deviate from the policies they also advocated in the three-party model. Nonetheless, the outcome of the election changed. The coalition in this new model is now with certainty one consisting of the left and centrist party, whereas before it could be either a coalition of the centrist party with the left or the right party. This change in coalition outcome also means that the implemented policy package is more left-wing than it used to be in the three-party model, and it is also much less volatile. Afterwards, I extended the model to include party bliss points, which changed the outcome considerably. All establishment parties now advocate different political stances than they used to in the other two equilibria. The common trend is that they all move away from the centre: the right-wing party moves to the right, whereas the left-wing and the centrist party move to the left. The outcome is a centre-left coalition with certainty and an equilibrium policy package that is significantly more left-wing than before. Although there is some room for improvement of the model, the interesting conclusion arises that allowing for a fourth party on the right wing yields a more left-wing coalition and policy package in equilibrium. If we allow for bliss points, this conclusion is amplified as a result of polarising parties. This particular conclusion is, although possibly counter-intuitive, not unheard of. The particular conclusion of the equilibrium policy package moving away from extremist entrants is e.g. shared by (Merrill & Grofman, 2019). All in all then, this thesis proposes an interesting addition to the literature of political competition featuring right-wing extremism. It seems, however, that the entry of the extremist party has achieved the exact opposite of what it intended, as the government becomes more left-wing oriented in equilibrium.

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A Appendix

Lemma 2. These are all possible legislative outcomes (proof at the bottom)

1. If $w_L > w_M > w_R > w_{ER}$, then
   a. $y^* = p_{LR}, g_L^* = G - (p_{LR} - p_M)^2, g_R^* = -(p_{LR} - p_M)^2$ if $d_L \geq d_R$, and $C^* = \{L, R\}$
   b. $y^* = p_M, g_L^* = G$ if $d_L < d_R$, and $C^* = \{L, R\}$ if ER is not sufficiently big to form a coalition with L, or $C^* = \{L, k\}, k = (R, ER)$ otherwise.
2. If $w_L > w_M > w_{ER} > w_R$, then
   a. $y^* = p_{LR}, g_L^* = G - g_R^*, g_R^* = (p_M - p_{LR})^2$ if $d_L \geq d_R$, and $C^* = \{L, R\}$
   b. $y^* = p_M, g_L^* = G, g_R^* = 0$ if $d_L < d_R$, and $C^* = \{L, R\}$.
3. If $w_L > w_R > w_M > w_{ER}$, then
   a. $y^* = p_{LM}, g_L^* = G - g_M^*, g_M^* = (p_M - p_{LM})^2 - (p_M - p_{RL})^2$ if $d_L \geq d_R$, and $C^* = \{L, M\}$
   b. $y^* = p_{LM}, g_L^* = G - g_M^*, g_M^* = (p_M - p_{LM})^2$ if $d_L < d_R$, and $C^* = \{L, M\}$.
4. If $w_L > w_R > w_{ER} > w_M$, then
   a. $y^* = p_{LM}, g_L^* = G - g_M^*, g_M^* = (p_M - p_{LM})^2 - (p_M - p_{RL})^2$ if $w_{L+M} > 0.5$, and $C^* = \{L, M\}$
   b. $y^* = p_{LER}, g_L^* = G - g_{ER}^*, g_{ER}^* = (p_{ER} - p_{LER})^-(p_{ER} - p_R)^2$ if otherwise, and $C^* = \{L, ER\}$.
5. If $w_L > w_{ER} > w_R > w_M$, then
   a_1. $y^* = p_{LM}, g_L^* = G - g_M^*, g_M^* = (p_M - p_{LM})^2 - (p_M - p_{RL})^2$ if $w_{L+M} > 0.5$ and $2d_L > d_{ER}$, and $C^* = \{L, M\}$
   a_2. $y^* = p_L, g_L^* = G, g_M^* = 0$ if $w_{L+M} > 0.5$ and $2d_L \leq d_{ER}$, and $C^* = \{L, M\}$
   b. $y^* = p_{LR}, g_L^* = G - g_R^*, g_R^* = (p_R - p_{LR})^-(p_R - p_{ERM})^2$ if otherwise, and $C^* = \{L, R\}$.
6. If $w_L > w_{ER} > w_M > w_R$, then
   $y^* = p_{LM}, g_L^* = G - g_M^*, g_M^* = (p_M - p_{LM})^2 - (p_M - p_{ERL})^2$, and $C^* = \{L, M\}$
7. If $w_R > w_L > w_M > w_{ER}$, then
   a. $y^* = p_R, g_R^* = G, g_{ER}^* = 0$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$
   b_1. $y^* = p_{RM}, g_R^* = G - g_M^*, g_M^* = (p_M - p_{RM})^2$ if $w_{R+ER} \leq 0.5$ and $d_R \geq d_L$, and $C^* = \{R, M\}$.
   b_2. $y^* = p_{RM}, g_R^* = G - g_M^*, g_M^* = (p_M - p_{RM})^2 - (p_M - p_{LR})^2$ if $w_{R+ER} < 0.5$ and $d_R > d_L$, and $C^* = \{R, M\}$.
8. If $w_R > w_L > w_{ER} > w_M$, then
   a. $y^* = p_R, g_R^* = G, g_{ER}^* = 0$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$
   b. $y^* = p_{RM}, g_R^* = G - g_M^*, g_M^* = (p_M - p_{RM})^2 - (p_M - p_{LRER})^2$ if $w_{R+ER} < 0.5$, and $C^* = \{R, M\}$.
9. If $w_R > w_M > w_L > w_{ER}$, then
   a. $y^* = p_R, g_R^* = G, g_{ER}^* = 0$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$
   b_1. $y^* = p_{RL}, g_R^* = G - g_L^*, g_L^* = (p_M - pRL)^2$ if $w_{R+ER} < 0.5$ and $d_R \geq d_L$, and $C^* = \{R, L\}$.
   b_2. $y^* = p_M, g_R^* = G - g_M^*, g_L^* = 0$ if $w_{R+ER} < 0.5$ and $d_R < d_L$, and $C^* = \{R, L\}$.
10. If $w_R > w_M > w_{ER} > w_L$, then

a. $y^* = p_R, g^*_R = G, g^*_{ER} = 0$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$

b. $y^* = p_{RLER}, g^*_R = G - g^*_L - g^*_{ER}, g^*_L = (p_L - p_{RLER})^2 - (p_L - p_M)^2$,
$g^*_{ER} = (p_{ER} - p_{RLER})^2 - (p_{ER} - p_M)^2$ if $w_{R+ER} < 0.5$, and $C^* = \{R, L, ER\}$.

11. If $w_R > w_{ER} > w_M > w_L$, then
$y^* = p_R, g^*_R = G, g^*_M = 0$, and $C^* = \{R, M\}$

12. If $w_R > w_{ER} > w_L > w_M$, then
$y^* = p_R, g^*_R = G, g^*_k = 0$, and $C^* = \{R, k\}, k \in (M, L)$ conditional on $w_{R+k} > 0.5$

13. If $w_M > w_L > w_R > w_{ER}$, then
$y^* = p_M, g^*_M = G, g^*_k = 0$, and $C^* = \{M, k\}, k \in (R, ER)$ conditional on $w_{M+k} > 0.5$

14. If $w_M > w_L > w_{ER} > w_R$, then
$y^* = p_M, g^*_M = G, g^*_k = 0$, and $C^* = \{M, k\}, k \in (R, ER)$ conditional on $w_{M+k} > 0.5$

15. If $w_M > w_R > w_L > w_{ER}$, then
$y^* = p_M, g^*_M = G, g^*_L = 0$, and $C^* = \{M, L\}$

16. If $w_M > w_R > w_{ER} > w_L$, then

a. $y^* = p_M, g^*_M = G, g^*_L = 0$ if $w_{M+L} > 0.5$, and $C^* = \{M, L\}$

b. $y^* = p_{MER}, g^*_M = G - g^*_{ER}, g^*_{ER} = (p_{ER} - p_{MER})^2 - (p_{ER} - p_R)^2$ if $w_{M+L} < 0.5$, and $C^* = \{M, ER\}$.

17. If $w_M > w_{ER} > w_L > w_R$, then
$y^* = p_M, g^*_M = G, g^*_L = 0$, and $C^* = \{M, L\}$

18. If $w_M > w_{ER} > w_R > w_L$, then

a. $y^* = p_M, g^*_M = G, g^*_L = 0$ if $w_{M+L} > 0.5$, and $C^* = \{M, L\}$

b. $y^* = p_{MR}, g^*_M = G - g^*_R, g^*_R = (p_R - p_{MR})^2 - (p_R - p_{ERRM})^2$ if $w_{M+L} < 0.5$, and $C^* = \{M, R\}$

19. If $w_{ER} > w_L > w_M > w_R$, then

a. $y^* = p_{ERR}, g^*_{ER} = G - g^*_R, g^*_R = (p_R - p_{ERR})^2 - (p_R - p_{LER})^2$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$

b. $y^* = p_{ERM}, g^*_{ER} = G - g^*_M, g^*_M = (p_M - p_{ERM})^2$ if $w_{R+ER} < 0.5$, and $C^* = \{ER, M\}$.

20. If $w_{ER} > w_L > w_R > w_M$, then

a_1. $y^* = p_{ER}, g^*_{ER} = G, g^*_R = 0$, if $w_{R+ER} > 0.5$, and $2\Delta(ER - R) \leq \Delta(ER - L)$ and $C^* = \{ER, R\}$

a_2. $y^* = p_{ERR}, g^*_{ER} = G, g^*_R = 0$, if $w_{R+ER} > 0.5$, and $2\Delta(ER - R) > \Delta(ER - L)$ and $C^* = \{ER, R\}$

b. $y^* = p_{ERMR}, g^*_{ER} = G - g^*_M - g^*_R, g^*_M = (p_M - p_{ERMR})^2 - (p_M - p_{ERLR})^2$,
$g^*_R = (p_R - p_{ERMR})^2 - (p_R - p_{ERLR})^2$ if $w_{R+ER} < 0.5$, and $C^* = \{ER, M, R\}$

21. If $w_{ER} > w_M > w_L > w_R$, then

a. $y^* = p_{ERR}, g^*_{ER} = G - g^*_R, g^*_R = (p_R - p_{ERR})^2 - (p_R - p_{MER})^2$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$

b. $y^* = p_{ERLR}, g^*_{ER} = G - g^*_L - g^*_R, g^*_L = (p_L - p_{ERLR})^2 - (p_L - p_M)^2$,

$g_R^* = (p_L - p_{ERLR})^2 - (p_L - p_R)^2$ if $w_{R+ER} < 0.5$, and $C^* = \{ER, L, R\}$.

22. If $w_{ER} > w_M > w_L > w_R$, then
    a. $y^* = p_{ERR}, g_{ER}^* = G - g_R^*, g_R^* = (p_R - p_{ERR})^2 - (p_R - p_{MER})^2$ if $w_{R+ER} > 0.5$, and $C^* = \{R, ER\}$
    b. $y^* = p_{ERL}, g_{ER}^* = G - g_L^*, g_L^* = (p_L - p_{ERL})^2 - (p_L - p_M)^2$ if $w_{R+ER} < 0.5$, and $C^* = \{ER, L\}$.
23. If $w_{ER} > w_R > w_L > w_M$, then
    a_1. $y^* = p_R, g_{ER}^* = G, g_M^* = 0$, if $w_{R+M} > 0.5$, and $\Delta(ER - R) \leq \Delta(R - M)$ and $C^* = \{ER, M\}$
    a_2. $y^* = p_{ERM}, g_{ER}^* = G - g_M^*, g_M^* = (p_R - p_{ERM})^2$, if $w_{R+M} > 0.5$, and $\Delta(ER - R) > \Delta(R - M)$ and $C^* = \{ER, R\}$
    b. $y^* = p_R, g_{ER}^* = G, g_L^* = 0$ if $w_{R+M} < 0.5$, and $C^* = \{ER, L\}$.
24. If $w_{ER} > w_R > w_L > w_M$, then
    a. $y^* = p_R, g_{ER}^* = G, g_M^* = 0$, if $\Delta(ER - R) \leq \Delta(R - M)$ and $C^* = \{ER, M\}$
    b. $y^* = p_{ERM}, g_{ER}^* = G - g_M^*, g_M^* = (p_R - p_{ERM})^2$, if $\Delta(ER - R) > \Delta(R - M)$ and $C^* = \{ER, R\}$

Proof If $w_d > 0.5$, d will not need any assistance and can implement its first best proposal $y^* = p_d,\ g_d^* = G,\ C^* = d$

If $w_{d+g} > 0.5$, then $C^* = d, g$ and d will maximise $U_d = g_d^* - (p_d - y^*)^2$ subject to $g_g^* + (p_g - y^*)^2 = u_g^t$. Filling in the required terms and maximising yields that $y^* = \frac{p_d + p_g}{2},\ g_g^* = (p_g - y^*) + u_g^t$. However, if the indifference condition of the supporting party ($g_g^* + (p_g - y^*)^2 = u_g^t$) is binding, party d will not have to propose the maximised y^*, but can offer something more beneficial to itself. This occurs if $(p_g - y^*)^2 > u_g^t$. It follows that only if $g_g^* = 0$, that y^* can be closer to p_d than to p_g, as otherwise party d would be better off proposing the middle point.

If $w_{d+g} < 0.5$, d will have to form a coalition with three parties, so $C^* = d, g, h$ and d will maximise $U_d = g_d^* - (p_d - y^*)^2$ subject to $g_g^* + (p_g - y^*)^2 = u_g^t$ and $g_h^* + (p_h - y^*)^2 = u_h^t$. It follows that $y^* = \frac{p_d + p_g + p_h}{3},\ g_g^* = (p_g - y^*)^2 + u_g^t,\ g_h^* = (p_h - y^*)^2 + u_h^t$. Once more, if one of the two constraints is binding, it follows that $g_k^* = 0$ and that y^* will be closer to the preferred policy platforms of the other collaborating partners.

Lemma 3, 4. Party equilibrium vote shares

The length of the policy spectrum is 2 (and we thus need to scale all obtained shares by this length), and the vote tipping point between party k and j is $\frac{p_k + p_j}{2}$. Thus, party k obtains all votes between the tipping point and its own placement p_k. Then, the vote share of party k is such that $w_k = 0.5$(left hand side votes + right hand side votes). If there is no other party situated at a side of party k, then it obtains all votes between the end of the spectrum on that side and p_k. A vote share on, say, the right hand side of party k, if party j is situated right from party k then yields: $\frac{p_k + p_j}{2} - p_k = \frac{p_j - p_k}{2}$. If no party is situated on the left hand side of party k, then it obtains $p_k - (-1) = 1 + p_k$ on that side. Bringing these together and scaling by the size of the spectrum,

we get that:
If $p_k < p_j < p_z$ (no competition on the left of k): $0.5(1 + p_k + \frac{p_j - p_k}{2})$
If $p_j < p_k < p_z$ (competition on both ends): $0.5(\frac{p_k - p_j}{2} + \frac{p_z - p_k}{2})$
If $p_j < p_z < p_k$ (no competition on the right of k): $0.5(1 - p_k + \frac{p_k - p_z}{2})$

Lemma 5. Proof of uniqueness of three-party equilibrium

We will start off with party L and R, as the incentives are completely symmetrical and can hence be analysed at once. Hence, I will only go over L's position and invoke symmetry at the end. In this setting, the following feasible outcomes are possible[10]:
a) $w_L > w_R > w_M$, b) $w_L > w_M > w_R$, c) $w_R > w_L > w_M$, d) $w_R > w_M > w_L$.

If $\mathbf{d_L = d_R}$: a) $U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2$ **b)** $U_L = G - (p_L - p_M)^2$ **c)** $U_L = -(p_L - p_{RM})^2$ **d)** $U_L = -(p_L - p_M)^2$
If $\mathbf{d_L < d_R}$: a) $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{LR})^2 - (p_L - p_{LM})^2$ **b)** $U_L = G - (p_L - p_M)^2$
If $\mathbf{d_L > d_R}$: c) $U_L = -(p_L - p_{RM})^2$ **d)** $U_L = -(p_L - p_M)^2$

L wants to maximise its payoff, which means it will have to optimise the outcomes above to compute its best response strategy. In all cases, the maximisation (subject to $p_L < p_M < p_R$) yielded that $p_L = p_M - \epsilon$. Of course, one alternative is the case in which L can obtain an absolute majority. In that case, it will position itself such that $p_L = 0$ and will achieve its first-best result with certainty. In the case where R achieves its first-best proposal as a result of an outright majority, L maximises $U_L = -(p_L - p_R)^2$ subject to $p_L < p_M < p_R$, which also yields $p_L = p_M - \epsilon$. This leads to the following electoral strategy:

$$p_L = \begin{cases} 0 & \text{if } p_M > 0 \\ p_M - \epsilon & \text{if } p_M \leq 0 \end{cases}$$

Invoking symmetry will yield R's strategy:

$$p_R = \begin{cases} 0 & \text{if } p_M < 0 \\ p_M + \epsilon & \text{if } p_M \geq 0 \end{cases}$$

Let us move on to party M. Party M is faced by two options: position in the middle, i.e. $p_M = 0$, or facilitate an absolute majority of a competitor party. Naturally, M would prefer to become the largest party, as this is the only case in which it can gain a positive utility outcome. However, these incentives are the same for all parties, meaning that R and L will also attempt to become

[10] Some payoffs are omitted, as these outcomes are by definition impossible.

the biggest parties. By doing so, they will reduce party M's vote share. Hence, a scenario in which M is largest is impossible and will not occur. Moreover, the highest vote share M can achieve is 0.5, which is insufficient for an outright majority. The other possible outcomes, and M's associated payoffs are:

The four following cases apply:[11]
a) $w_L > w_R > w_M$, b) $w_L > w_M > w_R$, c) $w_R > w_L > w_M$, d) $w_R > w_M > w_L$.

If $\mathbf{d_L = d_R}$: $U_M = 0$ at all times.
If $\mathbf{d_L > d_R}$: c) $U_M = -(p_M - p_{RL})^2$, **d)** $U_M = 0$
If $\mathbf{d_L < d_R}$: a) $U_M = -(p_M - p_{RL})^2$, **b)** $U_M = 0$

Now we are in a position to verify whether there exist feasible cases in which it is not optimal to stay in the middle. This is the case in two out of the eight cases as portrayed above, and I will explain one of them (the other follows by symmetry) here. Two conditions need to be realised:
1) $d_L < d_R$ and 2) $w_L > w_R > w_M$. We know that 2) happens if $\frac{d_R}{2} + (1 - d_R) > \frac{d_L + d_R}{2}$. Rewriting yields that $d_R < 1 - 0.5d_L$. Adding the condition $d_L < d_R$ gives us that M will receive a negative utility if situated at $p_M = 0$ and $d_L < d_R < 1 - 0.5d_L$. Now we need to check if, within this spectrum, the payoff would be less than the payoff in a case of absolute majority. We know from the strategy of L and R that they would situate in the centre-point of the spectrum in a case where an absolute majority is possible. Because of damage limitation, M would position itself as close to the flank-party as possible. Thus, in this case, its payoff would be $U_M = -\epsilon^2$. Hence, in order for deviation to be a viable strategy, the disutility if M would remain in $p_M = 0$ would have to be at least $-\epsilon^2$, which is the case if $|p_{LR} > \epsilon|$. Thus, M's optimal strategy can be formulated as follows:

$$p_M = \begin{cases} 0 \pm \epsilon & \text{if } d_R < d_L < 1 - 0.5d_R,\ |p_{RL}| > \epsilon \\ 0 \pm \epsilon & \text{if } d_L < d_R < 1 - 0.5d_L,\ |p_{RL}| > \epsilon \\ p_k \pm \epsilon & \text{if } p_k = 0,\ k \in (L, R) \\ 0 & \text{otherwise} \end{cases}$$

Taking the strategies together, we find the unique equilibrium as described in Lemma 5.

Lemma 6. Proof of uniqueness of four party electoral equilibrium

Let me start with party ER, since this position is extremely straight-forward. As mentioned before, ER is restricted to the position $p_{ER} = 1$. Hence, any incentive to deviate originating from any part of the game has to be ignored as a result of before mentioned considerations.

[11] omitted cases are by definition impossible

Moving on, we will continue with the middle party, M. In the four-party game, it is no longer the case at all times that a deviation from the middle point will grant a party an absolute majority, as this now only holds for party L, due to increased competition on the right flank. The possible outcomes for M can be summarised as follows:
a) $w_L > w_M > w_R > w_{ER}$, b) $w_L > w_R > w_M > w_{ER}$, c) $w_L > w_R > w_{ER} > w_M$.
The following utility outcomes would result:
if $\mathbf{d_L = d_R}$: **a)** $U_M = 0$ **b)** $U_M = 0$ **c)** $U_M = 0$
if $\mathbf{d_L < d_R}$: **a)** $U_M = 0$ **b)** $U_M = 0$ **c)** $U_M = -(p_M - p_{RL})^2$
if $\mathbf{d_L < d_R}$: **a)** $U_M = -(p_M - p_{LM})^2$ **b)** $U_M = -(p_M - p_{LM})^2$ **c)** $U_M = -(p_M - p_{RL})^2$

Thus, what we have here is that if the distance to M is equal for L and R, i.e. if $p_M = \frac{p_L + p_R}{2}$, M will have a payoff of $U_M = 0$ with 100% certainty. If this is the case, M will not want to deviate to the right hand side and yield L an outright majority, since it would always be worse off. Furthermore, it has nothing to gain from moving to the left, as it cannot improve its utility outcome from doing so. This is the case, because M could only induce a positive utility outcome by becoming the winner of the election. In a uniform distribution game, however, M cannot alter its own vote share by announcing a different policy position. Things change if $d_L \neq d_R$. In this case, M will want to move, such that the equality is restored, i.e. by moving to $p_M = \frac{p_L + p_R}{2}$. Even though optimising the other case would yield that $p_M = p_L + \epsilon$, we know that $p_{LM} < p_{LR}$ at all times. Thus, moving to the middle point between L and R is sufficient. If M finds itself in a position where L obtains an absolute majority, it will maximise $U_M = -(p_M - p_L)^2$, yielding $p_M = p_L + \epsilon$

Thus: $p_M = \begin{cases} \text{p}_L + \epsilon & \text{if p}_L = 0 \\ \frac{p_L + p_R}{2} & \text{otherwise} \end{cases}$

Moving on to party R. For the right-wing party, much like the centrist party, it is impossible to alter its own vote share. R can expect the following payoffs:
a) $w_L > w_M > w_R > w_{ER}$, b) $w_L > w_R > w_M > w_{ER}$, c) $w_L > w_R > w_{ER} > w_M$.
a) $U_R = -(p_R - p_M)^2$, **b)** $U_R = -(p_R - p_{LM})^2$, **c)** $U_R = (p_R - p_{LM})^2$.
In cases b and c, R would optimise $U_R = (p_R - p_{LM})^2$, constrained to $p_R > p_M$, yielding that $p_R = p_M + \epsilon$). However, R would prefer option a over option b and c, and so it will want to make sure that its vote share is smaller than M's vote share. R can achieve this by moving to the right, which leaves its own vote share unaltered but increases M's vote share. M will have no incentive to deviate, as R's movement to the right ensures that d_R is increasing, which is beneficial to M too. Thus, in this case, R will want to move a minimum distance to the right to ensure that option a occurs. The amount it has to move is a simple exercise: $w_R < w_M$. We know from the voting equilibrium that we can write this in the following manner: $0.5(\frac{p_{ER} - p_R}{2} + \frac{p_R - p_M}{2}) < 0.5(\frac{p_R - p_M}{2} + \frac{p_M - p_L}{2})$. Simplifying yields: $(p_{ER} - p_R) < (p_M - p_L)$. Maximising R's payoff subject to the constraint above

yields that $p_R = p_L + 1 + \epsilon$. Knowing this, we can conclude that the distance R has to travel is dependent on the positioning of L. However, R will not move to the right indefinitely. At some point, the distance between R and M (remember, the payoff R wants to induce is $U_R = (p_R - p_M)^2$) becomes too large, and it is no longer beneficial to force the move towards the right. Solving $(0 + \epsilon - p_{ML})^2 < (p_R - p_M)^2$, where $p_R = 1 + l + \epsilon$, yields that the move is only beneficial if $p_L < p_M - \frac{2}{3}$. We know, however, that L will never position itself such that $p_L < -0.5$. Thus, this strategy is not feasible and hence, R has a dominant strategy:

$p_R = p_M + \epsilon$

Lastly, let us analyse the possible positions that L would be able to credibly and feasibly consider. L knows that, if it takes a position such that $p_L > -0.5$, it will win the election with certainty. It also knows that if the opportunity occurs that L can position itself at $p_L = 0$, it will be able to capture an absolute majority and implement its first-best package. However, its own vote share is not all that L cares about. Once more, three options out of the 24 possible legislative equilibria are possible:

$a) w_L > w_M > w_R > w_{ER}$, b) $w_L > w_R > w_M > w_{ER}, c) w_L > w_R > w_{ER} > w_M$.

The utility payoffs for L are:

If $d_L = d_R$: **a)** $U_L = G - (p_L - p_M)^2$ **b)**$_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2$

c) $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{RL})^2 - (p_L - p_{LM})^2$

If $d_L < d_R$: **a)** $U_L = G - (p_L - p_M)^2$ **b)** $U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2$

c) $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{RL})^2 - (p_L - p_{LM})^2$

If $d_L > d_R$: **a)** $U_L = G - (p_R - p_M)^2 - (p_L - p_{LR})^2$ **b)** $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{RL})^2 - (p_L - p_{LM})^2$ **c)** $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{RL})^2 - (p_L - p_{LM})^2$

It becomes clear that there are two options that party L prefers, as they yield the exact same utility outcome: either option b where $d_L > d_R$ or option c, regardless of the relative distances. However, option b is very difficult to enforce, as it relies on the condition $d_L > d_R$, whereas to enforce outcome b, L would have to win over voters from the centrist party, i.e. move more towards M. Thus, by doing so, it is actually decreasing d_L. Moreover, L has another motive to move towards the middle. By doing so, L minimises the distance between the policy outcome y^* **regardless of which of the three outcomes ensues:** $y^* \in (p_{LM}, p_M, p_{LR})$ is always situated to the right of party L. Thus, by minimising the distance, L maximises its utility. Technically, maximising $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{RL})^2 - (p_L - p_{LM})^2$ subject to $p_L < p_M$ yields $p_L = p_M - \epsilon$. Maximising any other potential outcome as presented above subject to $p_L < p_M < p_R$ yield the same outcome, as a matter of fact. Hence, party L will position itself at $p_M - \epsilon$. This is the ultimate position it chooses, because party L knows that if it chooses to move so far to the centre, R can no longer make sure that $d_L > \Delta(R - ER)$. In the case in which an opportunity presents itself to obtain absolute majority, L will position itself such that $p_L = 0$, as this ensures absolute majority at all times. Thus:

$$p_L = \begin{cases} p_M - \epsilon & \text{if } p_M \leq 0 \\ 0 & \text{if } p_M > 0 \end{cases}$$

Taking the strategies together, we find the unique equilibrium as described in Lemma 6.

Lemma 7. Proof of uniqueness of bliss point equilibrium[12]

Let us consider the possible cases and their associated equilibrium outcomes for party M:
a) $w_L > w_M > w_R > w_{ER}$, b) $w_L > w_R > w_M > w_{ER}$, c) $w_L > w_R > w_{ER} > w_M$

If $\mathbf{d_L = d_R}$: a) $U_M = 0$ **b)** $U_M = 0$ **c)** $U_M = 0$,
If $\mathbf{d_L < d_R}$: a) $U_M = 0$ **b)** $U_M = 0$ **c)** $U_M = -(p_M - p_{LR})^2$
If $\mathbf{d_L > d_R}$: a) $U_M = -(p_M - p_{LM})^2$ **b)** $U_M = -(p_M - p_{LM})^2$ **c)** $U_M = -(p_M - p_{RL})^2$

If $d_L > d_R$, then $p_{LM} < p_{RL}$. M will achieve a better outcome if $p_M = p_{RL}$, as this ensures that $d_L = d_R$. Hence, its objective if $d_L > d_R$ is to achieve this. Maximising $U_M = -(p_M - p_{RL})^2 - (p_M - 0)^2$ yields $p_M = \frac{p_L + p_R}{4}$, whereas maximising $U_M = -(p_M - p_{LM})^2 - (p_M - 0)^2$ yields $p_M = \frac{p_L}{5}$. However, if $d_L \leq d_R$, M will want to pursue the other option, $p_M = \frac{p_L + p_R}{4}$. If $p_L = 0$, then M will want to maximise $-(p_M - p_L)^2 - (p_M - 0)^2$, yielding $p_L + \epsilon$, i.e. locate as close to L as possible.

$$p_M = \begin{cases} \mathrm{p}_L + \epsilon & \text{if } \mathrm{p}_L = 0 \\ \frac{p_L + p_R}{4} & \text{otherwise} \end{cases}$$

Moving on to party R:
a) $w_L > w_M > w_R > w_{ER}$, b) $w_L > w_R > w_M > w_{ER}$, c) $w_L > w_R > w_{ER} > w_M$

a) $U_R = -(p_R - p_M)^2$, **b)** $U_R = -(p_R - p_{LM})^2$, **c)** $U_R = (p_R - p_{LM})^2$.

It is clear that party R would prefer case a over b and c. R would be able to induce case a if $w_M > w_R : d_L > \Delta(R - ER)$ R would obtain $U_R = -(p_R - p_M)^2 - (p_R - 0.5)^2$. Maximising subject to $d_L > \Delta(R - ER)$, it appears that the optimal strategy would be $p_R = 1 + p_L + \epsilon$. However, this is only viable if it is a better option than the next best, which would be a case in which $U_R = -(p_R - p_{LM})^2 - (p_R - 0.5)^2$ is optimised, which occurs at $p_R = \frac{p_{LM}+1}{4}$. Equating

[12] The bliss point argument is omitted in all cases as it is equal in all outcomes and is only dependent on a party's position, not on the legislative outcome.

the two projected outcomes results in the restriction that R will only attempt to pursue option a if $p_L < -0.46$. Taking this into consideration, we find:

$$p_R = \begin{cases} 1 + p_L + \epsilon & \text{if } d_L > \Delta(R - ER),\ p_L < -0.46 \\ \frac{p_{LM}+0.5}{2} & \text{otherwise} \end{cases}$$

Moving on to party L:

a) $w_L > w_M > w_R > w_{ER}$, b) $w_L > w_R > w_M > w_{ER}$, c) $w_L > w_R > w_{ER} > w_M$

If $\mathbf{d_L = d_R}$: a) $U_L = G - (p_L - p_M)^2 - (p_L + 0.5)^2$ **b)** $U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$ **c)** $U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$
If $\mathbf{d_L < d_R}$: a) $U_L = G - (p_L - p_M)^2 - (p_L + 0.5)^2$ **b)** $U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$ **c)** $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{LR})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$
If $\mathbf{d_L > d_R}$: a) $U_L = G - (p_L - p_M)^2 - (p_L + 0.5)^2$ **b)** $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{LR})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$ **c)** $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{LR})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$

L will want to achieve either case b) or c) if $d_L > d_R$ or case c) if $d_L < d_R$, as in these cases it achieves the most favourable combination of equilibrium policy and portfolio distribution. L, then, maximises $U_L = G - (p_M - p_{LM})^2 + (p_M - p_{LR})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$, resulting in $p_L = \frac{p_R-2}{5}$ as its best strategy. Its second best would be the case where $U_L = G - (p_M - p_{LM})^2 - (p_L - p_{LM})^2 - (p_L + 0.5)^2$. Maximising here yields $p_L = \frac{p_M+1}{3}$. However, L would only try to induce this outcome if $d_L = d_R$, in which the two aforementioned outcomes are equal. Alternatively, if the opportunity arises, L would be able to achieve absolute majority if $w_L > 0.5$ which happens if $p_L > -p_M,\ p_M > 0$. Maximising $U_L = G - (p_L - 0.5)^2$ subject to the aforementioned constraint, we receive that $p_L = -p_M + \epsilon$. L is not always willing to do this, as, alternatively, L could just collaborate with p_M and not move as far from its bliss point. If L would do this, it would situate at $p_L = \frac{m-7}{19}$. Solving for the two options yields that from $p_M = 0.18$ onwards, it is a viable strategy to pursue the absolute majority. Before that, the move to the centre is too costly for L.

$$p_L = \begin{cases} \frac{p_R-2}{5} & \text{if } p_M \leq 0 \\ \frac{m-7}{19} & \text{if } 0 < p_M < 0.18 \\ -p_M + \epsilon & \text{if } p_M > 0.18 \end{cases}$$

Solving the following strategies as a system, we can conclude that the equilibrium as described in Lemma 7 is indeed a unique equilibrium in this game.