This paper studies the effect of non-uniform distributions on quality competition in regulated markets. Our model finds firms opting for minimal differentiated outcomes, regardless of the type of distribution. In symmetric models, regulated prices only affect quality levels. An unrestricted non-symmetric setting allows the regulator to alter both quality levels and hospital locations. In addition, distributions with diffuse demand see 40% higher transport costs compared to distributions with centralized demand. The former profits more from socially planned locations than the latter. Furthermore, in contrast to the symmetric distributions, welfare is maximized when hospitals are not located at the 25th and 75th percentile, but are shift towards less concentrated areas. Finally, depending on the size of (relative) transport and fixed hospital location costs, the regulator will be inclined to either choose for a central location or an infinite number of locations.

The views stated in this thesis are those of the author and not necessarily those of Erasmus School of Economics or Erasmus University Rotterdam.
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1. Introduction

In general, welfare states face similar challenges for their healthcare systems. Firstly, due to increasing urbanization, population growth and an ageing population, these systems experience an increased demand for healthcare in urbanised centres whereas a decrease in demand for healthcare in rural areas. Secondly, due to technological progress, patients today have more options in the treatment they desire compared to patients 30 years ago. To keep up with the pace of technological progress, systems costs have increased considerably, in turn becoming too expensive for lower and middle income groups. Both trends lead to increased pressure on hospitals to improve both accessibility and quality levels, whilst not cutting back on quality of care. The purpose of this thesis is to examine the interaction between location and quality choices hospitals make in light of different distributions of demand. We are interested in finding how this affects the optimal strategy for hospitals and the regulator.

One way to overcome demand shortages is for hospitals to centralize or to merge. Decreasing the number of locations implies fewer overhead costs, thereby resulting in decreased costs for administrative tasks. In addition, merging hospital management leads to the opportunity for a hospital-collective to rotate medical-specialists between the different locations. Nevertheless, centralizing hospitals does come at a cost. Besides leading to longer travel times for households consuming higher degrees of healthcare, it also changes the structure of the healthcare market. Fewer firms supplying goods may lead to increased market power for those firms that remain supplying the good. More abstractly,

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Motta (2004) argues that firm market power often results in lower incentives for firms to increase their productivity. Put differently, hospitals are less inclined to specialize in the good, or level of quality they provide. Given this argument, it is arguable whether in the long run such mergers increase utility of the public.

Another solution often discussed by policymakers is boosting competition in the healthcare market. Empirical studies find increased competition leading to more efficient allocation of resources within the hospital (Burgess et al., 2005; Propper, 2018). Nevertheless, increased competition is contradictory to centralizing hospitals. More competition analogically implies more suppliers of healthcare, whereas centralizing or merging healthcare implies fewer suppliers. According to the theoretical literature – discussed in the next paragraph – this does not seem to apply to the market for healthcare.

1.1. General Characteristics of the market for hospitalized care

Broadly speaking, the market for healthcare can be summarized as follows: Hospitals are unable to compete on prices, but are able to compete horizontally, on location, and vertically, on the quality of care provided. Healthcare systems are funded differently per country. This thesis limits itself to welfare states where consumers, generally, pay a lump sum fee for access to the system. For example: Britain’s NHS is funded by public expenses and in mainland (western-) Europe, insurance is mandatory. After insurance is bought, the consumer gains full access, but that in some systems additional payments are required to cover the patients’ own risk.

Insurance firms and hospitals negotiate the unit price, resulting in administrated or regulated prices. All hospitals face the same regulated price. Hospitals maximize their own utility, thereby maximizing profits. Furthermore, a hospital determines the amount of quality it provides and where it locates. Consumers maximize utility when maximizing received quality levels and minimizing travel costs.

Brekke et al. (2006) derive a model in a Hotelling setting where demand is uniformly distributed and prices are administrated. Notably, firms are only able to compete on location and quality. They set up two games. In the first game, firms choose their location and quality first, where after the regulator sets prices. In the second game, a regulator commits to a price first, and

thereafter hospitals choose quality and location outputs. The first game encounters suboptimal outcomes in relation to quality and location. Locations are too far away for most consumers and quality is too high when compared to the social optimum. The second game provides more optimal outcomes for the level of quality provided by the firm, but the horizontal mix is too extensive.

In 1929 Harold Hotelling’s paper on spatial competition was published. Hotelling argued that horizontal competition leads to minimal differentiation. Since then additional research has been conducted in a similar setting. Moreover, his model has been extended. Some extensions also take quality competition into account.

Brekke e.a. build upon research previously conducted by D’Aspremont e.a. (1979), amongst others. D’Aspremont e.a. introduce the possibility of maximal differentiation. This opposes the standard Hotelling (1929) outcome of minimal differentiation, but does not account for quality competition. In the D’Aspremont setting firms locate away from each other, thereby achieving a higher degree of market power and dampening competition. Spence (1975) also studies quality competition and compares the objective of the monopolist to that of the social planner. He finds that the social planner optimizes the utility of the average consumer, whereas the monopolist optimizes utility of the marginal consumer. The quality supplied by the monopolist is either too little or too large when compared to the social optimum. The amount of quality delivered is optimal only when the marginal consumer is equal to the average consumer.

Calem and Rizzo (1995) take a similar approach, but place the model in a Hotelling-duopoly setting, whereby firms compete on the quality delivered and their specialty mix. In addition, the paper assumes that hospitals pay part of the consumers’ travel expenses. In the equilibrium firms either supplies too much quality or delivers too little levels of quality when compared to the social optimum output. Quality levels can be too large as the costs involved with maintaining these levels oppresses the welfare outcome. Firm mergers however do result in healthcare offered which is closely linked to the social optimum.

Gaynor discusses regulated prices in his book on competition on healthcare markets (2007). When prices are fixed, firms are incentivized to increase competition on the characteristics of their supplied products. Applied to the healthcare market, this results in increasing levels of quality that are inefficiently high.

Besides extending the Hotelling model with competition on quality, some scholars have also introduced non-uniform distribution of demand to the Hotelling model.
Neven (1986) models an oligopolistic game within a Hotelling setup where demand is characterized in triangular distribution. Firms first choose location and then set their price. He finds that under more concentrated distribution of the population, duopolistic competitors centralize locations.

Tabuchi and Thisse (1995) study the effect of market concentration on quality levels in a Hotelling model. By taking triangular, symmetric distributions they find that hospitals centralize their location when demand is more concentrated. In addition, when market boundaries are relaxed they find hospitals locating outside of the model to decrease quality level competition.

Anderson, Goerree and Ramer (1997) study the location-then-price-model in a non-uniform distribution setting. They do so with a log concave distribution, whereby they study the effect the slope has on the location of hospitals. They find that in symmetric models in an optimal welfare setting, hospitals locate at relative similar locations, whereas in non-symmetric distributions, locations partially diverge towards the flanks of the model.

Other research conducted on competition in healthcare markets but beyond the scope of quality and location competition focusses, inter alia, on asymmetric information and labour markets. For example; Klein and Leffer (1981) are study asymmetric-information between hospitals and consumers. They find a Nash-equilibrium when a game is played repeatedly and whereby quality is inelastic and prices are elastic. Genevieve and Chifang (2016) consider an approach whereby the supply of qualified labour determines the supply of healthcare and thereby the healthcare received by the household. They argue that tight labour markets result in market power for hospitals attracting best qualified staff. Both approaches are not considered for this thesis.

### 1.2. Research question and hypothesis

Generally, theoretical literature argues policymakers should centralize hospital management. However, empirical research suggests that more hospitals results in higher hospital productivity, whereas fewer hospitals deter the productivity of hospitals\(^9\). These findings contradict each other. A thorough understanding of the market is necessary to tackle the current challenge regulators face to supply healthcare to areas where population is less concentrated.

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We argue that current research has insufficiently studied location-quality competition in the realm of different spatial distributions. Extending current models by introducing different spatial distributions of population contributes towards more realistic models\(^{10}\). The purpose of this thesis is to examine the interaction between location and quality choices hospitals make and how population distribution changes optimal location strategy for hospitals when prices are regulated\(^{11}\).

We apply the setup of Brekke e.a. (2006) but alter the spatial distribution of demand and, for comparison, assume transport costs to be linear. Additionally, we address the possibility of the regulator introducing additional hospitals to the model in a welfare setting.

Our findings are similar to those of Brekke e.a. (2006). In competitive settings consumers face higher transport expenses when compared to the optimal welfare outcome. Our model however finds hospitals opting for minimal differentiation rather than maximal differentiation. We argue that these outcomes better grasp the hospital location choice in reality, but do not reflect vertical competition. Adding non-uniform distributions to the model does allow for strategic locations effects. The strategic effect however is always trumped by the direct effect. In symmetric models, the regulator can therefore not influence hospital location decisions. Countries where demand is diffuse, for example the United States, see 40% higher transport costs when compared to countries with centralized demand such as France. The former profits more from socially planned locations than the latter. In addition we find optimal regulated price to be dependent on the density of the population. The less dense the population is at the location of the indifferent consumer, the higher the regulated price becomes. Uncertainty with regards to regulated prices does not affect the derived outcomes. It however does allow for more certainty in setting optimal quality levels.

Non-symmetric distribution with demand concentrated at the left side of the distribution form an exception. In countries such as Norway, the regulator can slightly influence the location of both hospitals, as long as the desired location change from the population median is not too large. Furthermore, welfare is maximized when hospitals are not located at the 25\(^{th}\) and 75\(^{th}\)

\(^{10}\) In reality, demand is not uniformly distributed: rural areas see a lower concentrated population as compared to concentrated populations in cities. A uniform distribution is highly stylized when compared to reality.

\(^{11}\) Theoretical research previously conducted in a Hotelling setting has either taken quality-location competition in uniformly distributed market into account or have modelled non-uniform demand. No research has previous been conducted by combining location-quality competition under regulated priced and with demand being non-uniformly distributed.
percentile, but are shifted towards the 28th and 79th percentile of the cumulative population. This shifts the location of the indifferent consumer towards the 58th percentile. Finally we show that the regulator will choose for one central hospital when relative transport costs and fixed hospital costs are low enough. Whenever transport costs and fixed hospital costs become too large the regulator will choose an infinite number of hospitals. This provides an argument in favour of centralizing and decentralizing hospital locations, depending on the of sort of healthcare provided and the on the tightness of labour markets.

The paper is structured in seven sections. Section one provides an introduction to the problem, an overview of the literature and addresses the main research questions. Section two sets the outline for the baseline model and provides insights into the different distributions used for the analysis. Section three computes the optimal quality level, location and regulated price outcomes in a competitive setting, endogenously and exogenously. In the fourth section the setup of the model is altered. Instead of regulators setting the price first, the research assumes they the price is set after hospitals have chosen their location. Section five addresses the implications for total welfare when the regulator gets to choose optimal quality levels and the optimal location. Section six discusses the choice the regulator makes when it can choose the optimal number of firms in a uniformly distributed market. Section seven draws a conclusion. Based on four distributions, this paper tries to find a general line. Any mistakes encountered in the paper are the sole responsibility of the author.

2. The Model

Consider four different distributions of demand of which one is uniform, whereas the other three are not. All demand distributions are incorporated into a similar setup. The setup sees two hospitals competing in a Hotelling setting. They are only able to compete with each other on quality and location. Prices are regulated by the administrator. Consumers gain full access to the healthcare system once they have paid a mandatory fee to the insurance company. The game is played in three stages: the administrator first sets a price, hospitals then choose their location and lastly choose the optimal amount of quality supplied. Summarized, this means;

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12 We stylizes that consumer face no costs when receiving treatment from the hospital. Modelling the relationship between consumer and insurance company and insurance company and hospital is beyond the scope of this thesis.
A. The administrator sets a price;
B. Hospitals A and B choose their location;
C. Both hospitals set the amount of quality they will supply to the market.

2.1. **Spatial distribution functions**

The four distributions represent stylized versions of countries. The uniform distribution, distribution \( f(x) \) pictured in graph 1, can be considered a stylized version of the Tokaido corridor in Japan. Stretching for over 1000 kilometres, the Tokaido corridor captures all major Japanese cities. Approximately throughout the whole corridor, excluding the uninhabitable parts, the population density is comparable. Function \( g(x) \) in Graph 1 compares to a country with a central city located in the middle of the country. Examples are Hungary with its capital Budapest, France with Paris and the state of Colorado in the United States with its state capital Denver being the densely-populated centre of the state. Function \( h(x) \) on the other hand captures a population distribution that is diverging from the centre. The population is most concentrated at the borders of the model, so at \( x=0 \) and \( x=1 \). With some imagination, this can be seen as a stylized version of the United States spanning from coast to coast. Most people live in the large urban areas near the ocean. Fewer people live in the more centrally located parts of the country. Norway on the other hand has a denser population density in the south where cities as Oslo and Bergen are situated. The more north travelled, the less concentrated the population density becomes. Function \( k(x) \) provides a stylized representation of Norway, where \( x=0 \) represents south of the country and \( x=1 \) represents north. The interested reader is referred to appendix A.1. for visual substantiation of the cited distributions. The distributions can also be used to gain insight into the spatial spreading of certain population groups. Younger generations are often more urban
based, whereas retired people often reside in outlying areas\textsuperscript{13}. By gathering insights into the effect of the spatial distribution of demand, target-group-specific-care can be better supplied.

The spatial distributions of the population ($y_i(\hat{x})$) and its corresponding undefined integral ($Y_i(\hat{x})$) – capturing the population from point zero up to point $\hat{x}$ - are represented by the functions in Table 1. The table also captures the total population for the interval $x \in [0,1]$ in column N.

<table>
<thead>
<tr>
<th>Spatial distribution – $y_i(\hat{x})$</th>
<th>Undefined integral - $Y_i(\hat{x})$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i.$ $f(\hat{x}) = 1$</td>
<td>$F(\hat{x}) = \hat{x}$</td>
<td>1</td>
</tr>
<tr>
<td>$ii.$ $g(\hat{x}) = -3\hat{x}^2 + 3\hat{x}$</td>
<td>$G(\hat{x}) = -\hat{x}^3 + \frac{3}{2}\hat{x}^2$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$iii.$ $h(\hat{x}) = \hat{x}^2 - \hat{x} + 1$</td>
<td>$H(\hat{x}) = \frac{1}{3}\hat{x}^3 - \frac{1}{2}\hat{x}^2 + \hat{x}$</td>
<td>$\frac{5}{6}$</td>
</tr>
<tr>
<td>$iv.$ $k(\hat{x}) = 1 - \hat{x}$</td>
<td>$K(\hat{x}) = \hat{x} - \frac{\hat{x}^2}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Table 1

\textbf{2.2. Characteristics of the model}

Besides the distribution of the population, the model has four elements, respectively;

a. The regulated per-patient-price hospitals receive, parameter $p$;

b. The amount of quality delivered, parameter $\theta$;

c. The location firms choose, parameters $a$ and $b$.

d. The travel costs patients pay to visit the hospital, parameter $t$.

Hospitals are unable to compete on prices as both hospitals face the same regulated price. Hospitals however are able to compete on location and the quality of care they provide. Both parameters are endogenous and determined by the model. Given the interval of the model, we assume that hospital A locates within the interval $[0, \text{population median}]$ whereas hospital B locates within the interval $[\text{population median}, 1]$. The location assumption implies that hospitals only locate at the same location when they are located at the population median. Whenever hospitals locate at the same location both share the entire market. On average both supply 50% of the market.

Function 1. is the consumers’ utility function and is determined by the consumers’ valuation of care ($v$), the maximum level of quality of healthcare ($\theta$) delivered on the market, the cost for

the distance travelled to the hospital \((t(x - a))\) and the price of healthcare \((p)\). The objective of the consumer is to optimize its utility. The level of quality is determined by the hospital.

The consumer does take price into account, but as prices are similar at both hospitals, the price paid for healthcare does not influence the consumers’ decision to choose for a particular hospital. The consumer is focused on maximizing the degree of quality it receives and minimizing the costs it pays for travel. In point \(\hat{x}\) the consumer is indifferent between choosing hospital A or B. Functions 1. to 3. characterize the consumers’ behaviour.

\[
U = v + \theta - t(x - a) - p
\]  

(1.)

We measure the location of hospital A from 0, whereas the location of hospital B is measured from 1. The location of hospital A (B) is defined by parameter \(a\) (\(b\)). The larger parameter \(a\) (\(b\)), the larger parameter \(a\) (\(b\)), the closer hospital A (B) is located towards the population median. Parameter \(\hat{x}\) represents the location of the indifferent consumer. The consumer is indifferent between both hospitals whenever function 2. holds.

\[
v + \theta_a - t|\hat{x} - a| - p = v + \theta_b - t|1 - b - \hat{x}| - p
\]  

(2.)

Rearranging terms of function 2. finds function 3. This function yields the location of the indifferent consumer as a function of the locations of hospital A and B and quality levels of both hospitals.

\[
\hat{x} = \frac{1}{2} \left(1 - b + a + \frac{\theta_a - \theta_b}{t}\right)
\]  

(3.)

Function 5. represents the hospital profit function. Hospital revenue is determined by administered prices multiplied by the number of patients treated. The number of patients treated for hospital A equals all patients left of the location of the indifferent consumer. Hospital B treats all patients right of the indifferent consumer\(^{15}\).

Hospitals encounter two types of costs. Firstly, hospitals encounter per patient costs which are dependent on exogenous parameter \(c\) multiplied by the number of patients treated. Secondly hospitals encounter fixed quality level costs which are endogenously determined by quality levels. Quality levels equal parameter \(\theta_i, i = a, b\). We assume that quality levels costs to be

\(^{14}\text{The consumer pays } t \text{ times the distance travelled.}\)

\(^{15}\text{The number of patients treated by hospital A is equal to the cumulative of the population density (the integral of } y_i(\hat{x}) : Y_i(\hat{x}) \text{) from 0 to the location of the indifferent consumer. The number of patients treated by hospital B equals the total population (N) minus the number of patients treated by hospital A} = (N - Y_i(\hat{x}))\)
found by taking quadratic quality levels and dividing these by two. Function 4. gives the cost function. Combining revenues and costs obtains function 5.

\[ C_i = c * Y_i(\bar{x}) + \frac{(\theta_i)^2}{2} \]  (4.)

\[ \pi_i = (p - c)Y_i(\bar{x}) - \frac{\theta_i^2}{2} \]  (5.)

By assuming that prices are set by the regulator, the model provides a tool for policymakers to alter supplied quality and hospital locations. Profits are made as long as regulated prices are higher than per patient treatment costs and fixed quality level costs do not exceed this income. Analogically, with increased markups or by increasing the number of patients treated, the hospital gains larger resources which it can employ the increase quality levels. Increasing mark-ups provides financial room for hospitals to increase quality levels. Decreasing mark-ups incentivizes fiercer competition for patients and may consequently also see quality levels increase. Regulated prices therefore provide a tool to policymakers to either in- or decrease quality levels by adjusting regulated prices.

3. Quality and location choices with certainty on prices

This section derives the Nash equilibrium for prices by first deriving the subgame equilibria for quality levels and locations. Section 3.1 does so for quality, section 3.2 considers the effect of exogenous locations on quality level outcomes. Section 3.3 determines endogenous locations and discusses the effect of non-uniform distributions. Section 3.4 derives the price Nash-equilibrium and welfare outcomes in a competitive setting.

3.1. Optimal quality levels

The quality subgame finds \( \theta_a^* \) and \( \theta_b^* \) to be equal. Optimal quality levels for both hospitals can be characterized by function 6. which is a function of the mark-up hospitals encounter, transport costs and hospital locations. For mathematical proof, please consult appendix A.3.1.

\[ \theta_a^* = \theta_b^* = \frac{p-c}{2t} y_i(\bar{x}) = \frac{p-c}{2t} y_i(a, b) \]  (6.)

The optimal level of quality is dependent on the distribution function and the location of the indifferent consumer - \( y_i(\bar{x}) \). By totally differentiating function 6. over the locations of hospitals A and B, transport costs (t) and price (p) the comparative statistics are yielded. The results are displayed in Table 2.
The location of hospital A nor the location of hospital B have an effect on the level of quality in distribution $i$. As the population is evenly spread, optimal quality levels are equal for every location within the interval.

For distributions $ii$ and $iii$, whenever hospital A and hospital B are located equally far from the population median the location of both hospitals does not affect quality levels. Allowing for differences in the (relative) location of both hospitals does see hospital locations affecting quality levels. In distribution $ii$ for example, whenever the location of hospital A (B) is closer to the population median than the location of hospital B (A), hospital A is incentivized to decrease (increase) quality levels. In distribution $iii$ the effect is opposite. In this setting hospital A is incentivized to decrease (increase) quality levels whenever it is located closer to (further away from) the population median than hospital B. The effects in both distributions can be attributed to the spatial spreading of the model.

Moving the location of hospital A towards the population median incentivizes A to decrease quality levels in distribution $iv$. Moving the location of hospitals B towards the population median on the other hand incentivizes and increase in hospital A’s optimal quality levels. Hospitals can dampen quality competition by jointly moving their location towards less concentrated areas.

For all distributions an increase in transport costs results in lower levels of quality supplied whereas an increase in regulated prices increases quality levels. When facing higher transport costs, consumers will consequently settle for lower quality levels. Higher reimbursements (by means of higher regulated prices) allow hospitals to pay for higher quality levels.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$\frac{d\theta_a(a, b, p)}{da}$</th>
<th>$\frac{d\theta_a(a, b, p)}{db}$</th>
<th>$\frac{d\theta_a(a, b, p)}{dt}$</th>
<th>$\frac{d\theta_a(a, b, p)}{dp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>i.</strong></td>
<td>0</td>
<td>0</td>
<td>$-\frac{(p-c)}{2t^2} &lt; 0$</td>
<td>$\frac{1}{2t} &gt; 0$</td>
</tr>
<tr>
<td><strong>ii.</strong></td>
<td>$\frac{3(p-c)}{4t} (b - a) \leq 0$</td>
<td>$\frac{3(p-c)}{4t} (a - b) \leq 0$</td>
<td>$-\frac{(p-c)}{2t^2} (g(x)) \leq 0$</td>
<td>$\frac{(-3x^2+3x)}{2t} \geq 0$</td>
</tr>
<tr>
<td><strong>iii.</strong></td>
<td>$\frac{(p-c)}{4t} (a - b) \geq 0$</td>
<td>$\frac{(p-c)}{4t} (b - a) \leq 0$</td>
<td>$-\frac{(p-c)}{2t^2} (k(x)) &lt; 0$</td>
<td>$\frac{(x^2-x+1)}{2t} &gt; 0$</td>
</tr>
<tr>
<td><strong>iv.</strong></td>
<td>$-\frac{(p-c)}{4t} &lt; 0$</td>
<td>$\frac{(p-c)}{4t} &gt; 0$</td>
<td>$-\frac{(p-c)}{(2t)^2} (1 + b - a) &lt; 0$</td>
<td>$\frac{(1+b-a)}{4t} \geq 0$</td>
</tr>
</tbody>
</table>

Table 2

---

$16$ The population density is never negative.
3.2. Exogenous location

As determined in the previous section both hospitals have similar optimal quality levels as best response function for both hospitals equal function $6$. Exogenous locations that mirror each other – $a = b$ – in symmetric models see no change in optimal quality levels as the location of the indifferent consumer stays similar in all models. When exogenous hospital locations are assumed to be non-symmetric, the location of the indifferent consumer changes and consequently so do optimal quality levels$^{17}$.

For distribution $i$ this does not affect optimal quality levels. As demand is uniformly distributed, population density is equal for all possible locations. Hence changes in the location of the indifferent consumer does not affect optimal quality levels.

For both $a > b$ and $b > a$, the optimal quality levels both hospitals provide to the market in distribution $ii$ decrease and increase in distribution $iii$.

With regards to distribution $iv$, whenever the location of the indifferent consumer moves to a location where population is more concentrated optimal quality levels increase. Whenever the indifferent consumer is located at a less concentrated areas optimal quality levels decrease. The location of hospital A has a positive effect on the location of the indifferent consumer whereas the effect of the location of hospital B is negative.

Considering that prices are regulated and optimal quality levels (and hence quality level costs) are similar for both hospitals, the exogenous locations of both hospitals might result in losses for one of the hospitals. As long as hospitals are able to take a sufficient part of the market and prices are not set too high hospitals two hospitals will remain active. Appendix A.3.2. addresses under what conditions both hospitals remain active on the market in an exogenous setting.

The regulator consequently has a delicate task in determining the regulated price when locations are exogenously set. The financial position of hospitals with a small marketshare must be strengthened for these hospitals to maintain or increase quality levels. To provide such additional financial means, the regulator is inclined to set a higher regulated price. The results may be contrary: Setting higher regulated prices allows for more fierce quality level competition and increases the likelihood of the hospital with lower marketshare leaving the market. The regulator runs the risk of a sole competitor remaining. Competition may be

$^{17}$ In other words, $a \neq b$ but rather $a > b$ or $b > a$.  

12
dampened by setting lower prices. This however results in lower quality levels which may not be in the interest of society.

### 3.3. Endogenous location

Hospitals simultaneously choose their location, thereby anticipating the quality levels that both will set in the subsequent stage. In Table 3 we have summarized hospital profit functions (third column) and first order conditions of hospital profit functions over location (fourth column). Column $j$ indicates the distribution. Column $i$ refers to the respective firm.

In standard quality-location models (D’Aspremont e.a., 1979) transport costs are assumed convex. Applying such a setup results in two possible outcomes, both corner solutions. One solution is minimal differentiation whereby firms choose to centralize their location and set equal quality levels. Another solution is maximal differentiation: here firms experience a centrifugal force that dampens quality competition and increases market power for the firms operating on the market. The literature describes this outcome as a strategic (competition) effect dominating a direct (competition) effect. Both minimal and maximal differentiation imply relatively similar location outcomes for hospitals A and B. Our model assumes transport costs to be linear.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$i$</th>
<th>Profit function - $\pi_{ji}$</th>
<th>FOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.</td>
<td>a</td>
<td>$\frac{1}{2} (p-c)(1-b+a) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2$</td>
<td>$\left( \frac{p-c}{2} \right) &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>$(p-c) \left( 1 - \frac{1}{2} (1-b+a) \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2$</td>
<td>$\left( \frac{p-c}{2} \right) &gt; 0$</td>
</tr>
<tr>
<td>ii.</td>
<td>a</td>
<td>$(p-c) \left( -\hat{x}^2 + \frac{1}{2} \hat{x}^2 \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2 (h(\hat{x}))^2$</td>
<td>$g(a,b) \left( \frac{p-c}{2} \right) \left( 1 + \frac{3(p-c)}{(2t)^2} (a-b) \right) &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>$(p-c) \left( \frac{1}{2} \hat{x}^2 + \frac{3}{2} \hat{x}^2 \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2 (h(\hat{x}))^2$</td>
<td>$g(a,b) \left( \frac{p-c}{2} \right) \left( 1 + \frac{3(p-c)}{(2t)^2} (b-a) \right) &gt; 0$</td>
</tr>
<tr>
<td>iii.</td>
<td>a</td>
<td>$(p-c) \left( \frac{1}{3} \hat{x}^3 - \frac{1}{2} \hat{x}^2 + \hat{x} \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2 (k(\hat{x}))^2$</td>
<td>$h(a,b) \left( \frac{p-c}{2} \right) \left( 1 + \frac{p-c}{(2t)^2} (a-b) \right) &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>$(p-c) \left( \frac{1}{2} \hat{x}^2 + \frac{1}{2} \hat{x}^2 - \hat{x} \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2 (k(\hat{x}))^2$</td>
<td>$h(a,b) \left( \frac{p-c}{2} \right) \left( 1 + \frac{p-c}{(2t)^2} (b-a) \right) &gt; 0$</td>
</tr>
<tr>
<td>iv.</td>
<td>a</td>
<td>$(p-c) \left( \hat{x} - \frac{\hat{x}^2}{2} \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2 (g(\hat{x}))^2$</td>
<td>$k(a,b) \left( \frac{p-c}{2} \right) \left( 1 + \frac{p-c}{(2t)^2} \right) \geq 0$</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>$(p-c) \left( \frac{1}{2} \hat{x} - \frac{\hat{x}^2}{2} \right) - \frac{1}{2} \left( \frac{p-c}{2t} \right)^2 (g(\hat{x}))^2$</td>
<td>$k(a,b) \left( \frac{p-c}{2} \right) \left( 1 + \frac{p-c}{(2t)^2} \right)$</td>
</tr>
</tbody>
</table>

Table 3

In principal, only the direct effect occurs or the direct effect dominates the strategic effect. Only in distributions iii does the strategic effect trump the direct effect. Since the outcome of the strategic effect equals the outcome of the direct effect in this distribution we only encounter
one type of outcome, namely **minimally differentiated** firms. In the symmetric models both firms locate at the population median as deviating from this location will decrease firm profits. In non-symmetric models hospitals also locate adjacent to each other, but the regulator is able to alter combined hospital locations by setting a higher regulated price. Consequentially, the regulated price only affects quality levels in symmetric models, whereas it does influence location choice in non-symmetric models. Appendices A.3.3.2.– A.3.3.4. provide the derivations of the results of Table 3. Section 3.3.1. discusses hospital locations in non-symmetric settings.

### 3.3.1. Distribution iv | Norway

Given our location restriction, the optimal strategy of hospitals A is to always locate at the population median. The location of hospital B however is dependent on the relationship between the regulated price and (exogenous) transport costs.

Whenever transport costs are sufficiently large the best response of hospital B is to locate at the population median. The outcome implies **minimal differentiation** as hospitals locate at the same location and expect to set similar optimal quality levels at the third stage of the game.

Whenever transport costs are too small, the strategic effect dominates the direct effect and the best response of hospital B is to leave the market. Any form of competition with hospital A increases quality level costs more than it increases revenue gains. Hospital A remains as sole supplier. In the final stage of the model, hospital A will have no incentive to set any level of quality as this would deteriorate its profits\(^{18}\).

**Proof**

\[
\frac{d\pi_b}{db} = \frac{(p-c)}{4} \left(1 + b - a\right) \left(1 - \frac{p-c}{2t}\right)^2
\]

if \( t \geq \sqrt{\frac{p-c}{4}} \), \( \frac{d\pi_b}{db} \geq 0; \) hence hospital B locates at population median

if \( t < \sqrt{\frac{p-c}{4}} \), \( \frac{d\pi_b}{db} \leq 0; \) hospital B leaves the market

Assuming \( a = \frac{2-\sqrt{2}}{2} ; 1 - b = 1; \ \hat{x} = \frac{4-\sqrt{2}}{4}; t < \sqrt{\frac{p-c}{4}} \)

\[
\pi_b = (p-c) \left( \frac{1}{2} - \frac{4-\sqrt{2}}{4} + \left(\frac{4-\sqrt{2}}{4}\right)^2\right) - \frac{1}{2} (p-c) \left(\frac{4-\sqrt{2}}{4}\right)^2 = (p-c) \left(\frac{\sqrt{2}}{4} - \frac{1}{2}\right) < 0
\]

\(^{18}\theta_a(p,c,t,\hat{x}) = \frac{p-c}{2t} \gamma_t(\hat{x}) = \theta_a(p,c,t,1) = 0\)
We allow for additional outcomes by relaxing the location restriction. Hospitals become allowed to locate at any location within interval $a, 1 - b \in [0,1]$. Assuming quality levels are equal at both hospitals, consumers will choose the hospital located closest to them. Whenever both hospitals locate at exactly the same location we assume both hospitals to supply healthcare to the entire market. On average each hospital supplies healthcare to half of the market.

When transport costs are equal to or larger than $\frac{1}{2} \sqrt{p-c}$, hospitals remain located at the population median. Deviating from the population median results in a lower market share. Additionally, whenever hospitals deviate to the left, quality level costs increase. Deviating to the right sees quality level costs decreasing. Nonetheless, the reduction in quality level costs does not outweigh the decrease in revenue.

In this unrestricted setting, hospitals make a loss when transport costs fall below $\frac{1}{2} \sqrt{p-c}$ and both remain located at the population median. By jointly relocating to a less concentrated area, both hospitals could lower optimal quality levels, whilst at the same time provide healthcare to on average half of the consumers each. In our Norway model, this implies hospitals relocating to the right of the population median. Further deviation of one hospital to an even less concentrated area is detrimental to the profits of the deviating hospital. Deviation of one of the hospitals to the left is however advantageous. Both hospitals consequently have an incentive to locate to the left. Hospitals will continue to locate left of the other hospital up to the point where deviating to the left is as bad as not deviating. In the long run both will relocate to the population median. At first sight, hospitals mutually locating in less concentrated areas appears to be an unstable equilibrium.

Proof

Both hospitals supply exactly half of the market. This means each hospital supplies to $\frac{1}{4}$ consumers. The hospital profit functions becomes:

$$\pi_i = (p - c) \frac{1}{4} - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} (1 - x)^2$$ for $i = a, b$.

To find the location where both share the market and make no profits nor losses the profit function should equal 0. This location is $x = a = 1 - b$

$$\pi_i = (p - c) \frac{1}{4} - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} (1 - x)^2 \geq 0 \rightarrow (1 - x)^2 \leq \frac{4t^2}{2(p-c)}$$

Consequently, consumers located left of hospital A will always prefer hospital A, whereas consumers located right of hospital B will always prefer hospital B.
The location of hospital A and hospital B equals
\[ x = a = 1 - b = 1 - \frac{\sqrt{2t^2}}{p-c} \]
To control whether this equilibrium is stable imagine a small deviation of one of the
hospital to the right.
- **Given the location of the indifferent consumer quality level cost** = \( \frac{p-c}{4} \),
- \( \pi_i = (p-c)(\text{patients}) - \frac{p-c}{4} < 0 \); as patients < \( \frac{1}{4} \),
- Deviating is unprofitable.

To check whether this equilibrium is stable imagine a small deviation of one of the hospital
to the left.
- **Given the location of the indifferent consumer quality level cost** = \( \frac{p-c}{4} \),
- \( \pi_i = (p-c)(\text{patients}) - \frac{p-c}{4} > 0 \); as patients > \( \frac{1}{4} \)
- Deviation is profitable.

**Cost in the population median**
\[
\pi_i = \frac{p-c}{4} - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} \left( 1 - \frac{2-\sqrt{2}}{2} \right)^2 = \frac{p-c}{4} - \frac{1}{4} \frac{(p-c)^2}{(2t)^2}
\]
\[
\pi_i = \frac{1}{4} - \frac{1}{16} \frac{(p-c)}{t^2} < 0 \text{ if } t < \frac{1}{2} \sqrt{p-c} \text{ or if } p > 4t^2 + c
\]

We find the joint location outcome to be dependent on the depreciation rate of the hospitals. If
hospitals value future profits more than current profits, they will be inclined to remain in less
densely concentrated areas as it will result in no losses. If the regulator sets a high regulated
price to force hospitals towards less concentrated areas, the depreciation rate at which hospitals
will deviate will decrease. In other words, the higher the regulated price, the more unstable the
joint outcome becomes and the more inclined hospitals are to deviate and return to the
population median. The regulator therefore does have a tool to shift hospitals to the right of the
population median, but the effectiveness of the tool decreases with every additional increase in
regulated prices: every price increase drives down the depreciation rate of hospitals and
increases the chance of hospitals relocating to the population median.

**Proof**

For firms, the decisions becomes to deviate in period 1, and thereafter relocate to the
population median for infinite periods, or to make zero profits in the joint location
infinitely.
\[
(p-c)\left(\text{patients} - \frac{1}{4}\right) + \frac{d}{1-d} \left( \frac{1}{4} - \frac{(p-c)}{(4t)^2} \right) > 0
\]
\[ \frac{d}{1-d} \left( \frac{1}{4} - \frac{(p-c)}{(4t)^2} \right) > (p - c) \left( \frac{1}{4} - \text{(patients)} \right) \]

\[ d \left( \frac{1}{4} - \frac{(p-c)}{(4t)^2} \right) + d(p - c) \left( \frac{1}{4} - \text{(patients)} \right) > (p - c) \left( \frac{1}{4} - \text{(patients)} \right) \]

\[ d > \frac{(p-c) \left( \frac{1}{4} - \text{(patients)} \right)}{(p-c) \left( \frac{1}{4} - \text{(patients)} \right) + \left( \frac{1}{4} - \frac{(p-c)}{(4t)^2} \right)} \]

\[ \frac{(p-c)}{(4t)^2} = q \to d > \frac{ms}{ms + \left( \frac{1}{4} - q \right)} \]

When \( p > 4t^2 + c \to q > \frac{1}{4} \) and \( d < \)

1. Graph 3.1. depicts the change in the depreciation rate when \( q \) increases. The larger \( p \), the larger \( q \) becomes and the smaller \( d \) becomes. This increases the willingness for hospitals to deviate and return to the population median.

On a different note, higher regulated prices are also dependent on the institutional design of the regulator and its objective. If the regulator is politically appointed and cannot operate independently, it is arguable whether a majority of the electorate agrees to an objective that increases healthcare prices in order to make healthcare less accessible to the majority.

**3.3.2. Discussion and Conclusion on endogenous location choices**

Our model allows for strategic competition when demand is not uniformly distributed whereas a standard location model in a uniform setting does not. In distributions \( ii \) however, the strategic effect is eclipsed by the direct effect. Furthermore we encounter *minimally differentiated* outcomes in all distributions.

The outcome is opposite that of a standard quality-location model (d’Aspremont e.a., 1979). In this model quality competition in dampened by firms locating away from each other. The greater the absolute distance between both hospitals, the larger the incentive is for hospitals to lower optimal quality levels (and hence decrease quality level costs).

In our setup only relative locations, thereby meaning the relative difference of hospital locations to the location of the indifferent consumer, affect quality levels (cost). The absolute difference in hospital locations does not affect quality levels (cost). For the location outcome of the model this implies that hospitals deviating away from one another do not affect quality level...
competition as long as the change both hospitals are mutual. Any changes in location that are not mutual do lead altered quality levels. In symmetrical models, such non-mutual changes are unlikely in an endogenous setting. In a non-symmetric setting these changes can occur. By committing to an increase of regulated prices the regulator can slightly shift hospital locations to less populated areas. Hospitals will relocate at the population median if regulated prices are set too high.

Our outcome seems to correspond to the results empirical research (Schneider, 1967; Luft e.a., 1984; Lang e.a., 2016) finds. The research finds hospitals to minimally differentiate. Location-wise, hospitals pool together. With regards to vertical differentiation the research finds hospitals to, in general, not to differ from a standard mix.

In actual markets the regulator can use these insights in determining optimal regulated prices. In countries such as Japan, France and the United States, the regulator can raise quality levels without influencing the hospitals’ location decision. In a country such as Norway, price increases which see upon an increase in quality levels can lead to hospitals relocation to less populated areas without increasing quality levels. By setting too high prices, hospitals become inclined to relocate back to the population median and set higher quality levels. Note that this allows the regulator to, slightly, increase access to healthcare in less populous area.

3.4. Optimal prices and welfare outcomes

Given the previous subsections, and not allowing for credible threats or infinite periods, we conclude that increases in regulated prices only affect quality levels. This implies that all hospitals locate at the population median.

Before obtaining the optimal prices and welfare outcomes, we want to control for the profitability of the market. If hospitals are unable to make any profits, they will not enter the market. To control for non-negative profits, regardless of the location of the hospital and the location of the indifferent consumer, the first and second order derivative of the hospital profit function, incorporating optimal quality in the profit function, are derived over parameter $p$.

Results can be found in Table 5. Notice that in all distributions, for all $\lim_{p \to \infty} \pi_a < 0$ the cost for providing the level of quality becomes so large, that the hospital cannot profitably provide that level. In addition, when $\lim_{p \to c} \pi_a = 0$ hospitals make no revenues and are unable to provide any quality. However, taking $\lim_{p \to c} \left( \frac{d\pi_a}{dp} \right) > 0$. This implies the regulator can set a price, larger than
As determined in section 3.1, hospitals are anticipated to set equal quality levels. It therefore
areas see relatively higher regulated prices than countries where the indifferent consumer is
quality levels equal \( \frac{1}{2} \). Countries where the indifferent consumer is located in less concentrated
are the location and the total welfare outcomes for all distributions. In all distributions optimal
set.

The second derivative being smaller than zero implies the profit function is concave. In other
words, decreasing returns to revenue. This implies there is an optimal price the regulator can
set.

<table>
<thead>
<tr>
<th>( \pi_a )</th>
<th>( \frac{d\pi_a}{dp} )</th>
<th>( \frac{d^2\pi_a}{d^2p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. ( (p - c)\bar{x} - \frac{1}{2} \frac{(p-c)}{2t}^2 )</td>
<td>( \frac{(1-b+a)}{2} )</td>
<td>( \frac{p-c}{(2t)^2} = 0 )</td>
</tr>
<tr>
<td>ii. ( (p - c)G(\bar{x}) - \frac{1}{2} \frac{(p-c)^2}{2t^2} g(\bar{x})^2 )</td>
<td>( G(x) - \frac{(p-c)^2}{(2t)^2} (g(a,b))^2 = 0 )</td>
<td>( -\frac{(g(a,b))^2}{(2t)^2} &lt; 0 )</td>
</tr>
<tr>
<td>iii. ( (p - c)H(\bar{x}) - \frac{1}{2} \frac{(p-c)^2}{2t^2} h(\bar{x})^2 )</td>
<td>( H(x) - \frac{(p-c)^2}{(2t)^2} (h(a,b))^2 = 0 )</td>
<td>( -\frac{(h(a,b))^2}{(2t)^2} &lt; 0 )</td>
</tr>
<tr>
<td>iv. ( (p - c)K(\bar{x}) - \frac{1}{2} \frac{(p-c)^2}{2t^2} k(\bar{x})^2 )</td>
<td>( K(x) - \frac{p-c}{(2t)^2} (k(a,b))^2 = 0 )</td>
<td>( -\frac{(k(a,b))^2}{(2t)^2} &lt; 0 )</td>
</tr>
</tbody>
</table>

Table 4

Substituting the parameters with derived optimal outcomes rewrites function 7. into function 8.

\[
W = U(v, \theta, t, a, b, p) + \pi_a(\theta, t, a, b, p) + \pi_b(\theta, t, a, b, p) \quad (7.)
\]

\[
W = vN + \theta - t \left( \int_0^a (a-x) y_i dx + \int_a^\bar{x} (x-a) y_i dx + \int_{\bar{x}}^{1-b} ((1-b)-x) y_i dx + \int_{1-b}^1 (x-(1-b)) y_i dx \right) - cN - \theta^2 \quad (8.)
\]

As determined in section 3.1, hospitals are anticipated to set equal quality levels. It therefore
suffices to only take the derivative of function 9. rather than function 8. The regulators’ optimal
price is found when the condition under function 10. holds.

\[
\theta^*(p, y_i(a, b), t) - \theta^2 (p, y_i(a, b), t) \quad (9.)
\]

\[
\frac{dW}{dp} = \frac{d\theta}{dp} = p^* = \frac{t}{y_i(a, b)} + c \quad (10.)
\]

Table 6 provides the results for the optimal price and the outcomes for quality levels. The
regulated price smoothen quality levels and is depicted in the first row of the table. Also added
are the location and the total welfare outcomes for all distributions. In all distributions optimal
quality levels equal \( \frac{1}{2} \). Countries where the indifferent consumer is located in less concentrated
areas see relatively higher regulated prices than countries where the indifferent consumer is
located in concentrated areas.

| \( \theta_l \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
| Location | \( a = 1 - b = \frac{1}{2} \) | \( a = 1 - b = \frac{1}{2} \) | \( a = 1 - b = \frac{1}{2} \) | \( a = 1 - b = \frac{2-\sqrt{2}}{2} \) |

\| \( p \) | \( t + c \) | \( \frac{4}{3} t + c \) | \( \frac{4}{3} t + c \) | \( \sqrt{2}t + c \) |
\| \( \theta \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
Nevertheless, optimal regulated price outcomes should be interpreted with care as the derived outcomes are anticipated optimal outcomes. For example, mis-interpretation of the transport costs by the regulator might see the regulator set a non-optimal price. Under such circumstances quality levels will either be set too high or too low. Setting an optimal price is therefore no guarantee that actual quality levels will attain expected quality levels. In addition the regulator could run the risk of hospitals entering into a cartel to lower quality level costs. Moreover, it is inefficient not to merge both hospitals when both are located at the same location when contemplating the results from a welfare perspective. As hospitals in all settings locate at the same location it may prove beneficial for the regulator to incentivize mergers. A monopolist however does not have any incentive to provide quality levels. To combat this the regulator would have to introduce a monitoring agency. Doing so will also involve costs and possibly decrease welfare even more. Only when monitoring costs and increased quality levels supplied by a monopolist exceed the quality levels of two competing hospital will it prove beneficial to merge both hospitals into a sole supplier. This analysis however is beyond the scope of the thesis.

Also beyond the scope of this thesis, but an interesting topic for additional research is how minimal differentiation incentivizes hospital to increase efficiency. Profit maximizing hospitals cannot independently decrease quality levels as this would shift consumers to their competitor. Hospital profitability can therefore only be increased by producing more efficiently. In turn this means reducing per patient costs. The regulator can incentivize hospitals to lower per patient costs by setting a price that is (slightly) lower than the optimal price. This might actually be in the interest of society.

Moreover, we have modelled consumers to have homogenous preferences whereas heterogeneity of preferences would perhaps be more realistic. For example, consumers in the rural parts of distribution spend on average an additional 50% on travel expenses than

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20 Or finding ways to bring down fixed quality level costs. In reality this for example implies lower costs for drug development and lower costs for medical equipment that yields similar or more advanced capabilities as more expensive previous medical equipment.
consumers in the populous areas. Consumers in less populous areas may therefore value minimizing transport costs (much) more than optimizing quality levels whereas consumers in more populous areas value increased quality levels more than minimizing transport cost. Accounting for such differences does decrease the operability of the model.

Note that transport costs in the welfare equation are proportionate to the total population. Bringing up all populations to N=1 makes it better possible to compare welfare outcomes. The results in Table 7 have all been brought up to N=1. Comparing welfare outcomes does not change optimal quality level outcomes. These levels are independent of the number of consumers and stay equal for all distribution. Transport costs however are proportionate to the total population. To compare outcomes distribution i is taken as benchmark. Countries such as France and Norway profit more from the setup of the model when compared to Japan and the United States. Hospitals in France and Norway locate closer to the most populous areas, whereas in the United States hospitals will locate away from the populous areas. When the population is equal in all distributions travel costs in the United states are 40% higher than the travel costs of consumers in France.

<table>
<thead>
<tr>
<th>Welfare (N=1) i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v + \frac{1}{4} - \frac{t}{4} - c$</td>
<td>$v + \frac{1}{4} - \frac{3t}{16} - c$</td>
<td>$v + \frac{1}{4} - \frac{21t}{80} - c$</td>
<td>$v + \frac{1}{4} - \frac{t}{3} (2 - \sqrt{2}) - c$</td>
</tr>
</tbody>
</table>

Table 6

4. Uncertainty about regulatory prices

Now consider a setup where the hospital first chooses location, the regulator then sets the price, and finally, the hospital set quality levels. The setup of the model is summarized by;

B. Hospitals choosing location;
   A. Regulator setting the price;
   C. Hospitals choosing optimal level of quality.

21 travel expenses left of population median = $\int_{0}^{\frac{3 - \sqrt{2}}{2}} \frac{(3 - \sqrt{2} - x)(1 - x)}{2} \, dx = \frac{1}{24} (8 - 5 \sqrt{2})$.

22 The populations of distributions ii and iv are half of that of distribution i, whereas distribution iii’s population is 5/6.

23 $1 - \left( \frac{29}{20} \cdot \frac{16}{27} \right) = \left( 1 - \frac{2}{5} \right) = 0.4 = 40\%$
Optimal quality levels have already been derived in section 3.1. These best response functions remain the same as this step remains unchanged. In the second stage the regulator sets a regulated price by anticipating optimal quality levels in the final stage. Substituting the derived levels of quality into the welfare function finds the functions in column W in Table 8. The regulator sets a welfare optimizing price whenever the regulated price pays for the per patient costs and provides a markup which smoothens quality levels.

\[ j \]
\[ W \]
\[ \frac{dW}{dp} \]
\[ p = t + c \]

\[ i. \] \[ v + \frac{1}{2t}(p - c) - t(distance) - c - \frac{(p-c)^2}{(2t)^2} \]
\[ \frac{dW}{dp} \]
\[ p = \frac{t}{(-3\hat{x}^2 + 3\hat{x})} + c \]

\[ ii. \] \[ \frac{v}{2} + \frac{(p-c)(-3\hat{x}^2 + 3\hat{x})}{(2t)} - \frac{c}{2} \frac{(p-c)^2}{(2t)^2} \]
\[ \frac{dW}{dp} \]
\[ p = \frac{t}{(\hat{x}^2 - \hat{x} + 1)} + c \]

\[ iii. \] \[ \frac{5v}{6} + \frac{p-c}{2t}(\hat{x}^2 - \hat{x} + 1) - \frac{5c}{6} \frac{(p-c)^2}{(2t)^2} (\hat{x}^2 - \hat{x} + 1) \]
\[ \frac{dW}{dp} \]
\[ p = \frac{t}{1-x} + c \]

\[ iv. \] \[ \frac{v}{2} + \frac{(p-c)(1 - \hat{x})}{2t} - \frac{c}{2} \frac{(p-c)^2}{(2t)^2} (1 - \hat{x})^2 \]
\[ \frac{dW}{dp} \]
\[ p = \frac{t}{1-x} + c \]

Table 7

In the first stage hospitals anticipate the optimal regulated price. Column \( \pi_{ji} \) of Table 9 presents the hospital profit functions when anticipated optimal prices are substituted. The fourth column yields the FOC results for hospital profits over hospital location. Derivations and steps are provided in appendix A.4. All FOC’s in column \( \frac{d\pi_{ji}}{di} \) of Table 9 are found to be larger than zero. Firms with a positive FOC maximize their profits by locating at the population median. Deviating from the population median will result in lower profits. Under this setup the direct effect always dominates the strategic effect. The outcome implies minimal differentiation.

Furthermore the price the regulator sets only affects the anticipated optimal quality levels. This result does not differ from the result in section 3., but does provide more certainty as hospital location choice is no longer anticipated but known. Via the regulated price the regulator is capable of smoothing quality levels to the social optimal outcome.

\[ j \]
\[ i \]
\[ \pi_{ji} \]
\[ \text{substituted firm profits} \]
\[ \frac{d\pi_{ji}}{di} \]
\[ \text{FOC firm profit over firm location} \]

\[ a \]
\[ \frac{t}{2}(1 - b + a) - \frac{1}{8} \]
\[ \frac{t}{2} > 0 \]
\[ b \]
\[ \frac{t}{2}(1 + b - a) - \frac{1}{8} \]
\[ \frac{t}{2} > 0 \]
The derived results for welfare level, hospital location, regulated price and quality levels are rendered in Table 10. Altering stages in the game does not result in different outcomes as compared to the game played in section 3. This also is evident when comparing the outcomes of Table 6 to those of Table 10: there is no difference in outcomes. The regulator cannot threaten hospitals by changing the moment at which it determines the optimal price. It cannot, endogenously, change location outcomes. Optimal quality levels remain equal to the optimal outcomes of the game played in the previous section. In contrast to what we found for Norway in the previous section, the Norwegian regulator can now no longer credibly threaten hospitals to relocate to less concentrated areas. To increase its influence, the Norwegian regulator will prefer the setup of section 3 over the setup of this section. The other three countries are indifferent as optimal outcomes do not differ between setups.

Table 8

<table>
<thead>
<tr>
<th></th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>t + c</td>
<td>(\frac{4}{3}t + c)</td>
<td>(\frac{4}{3}t + c)</td>
<td>(t\sqrt{2} + c)</td>
</tr>
<tr>
<td>(\theta_i)</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>Location</td>
<td>(a = 1 - b = \frac{1}{2})</td>
<td>(a = 1 - b = \frac{1}{2})</td>
<td>(a = 1 - b = \frac{1}{2})</td>
<td>(a = 1 - b = \frac{2}{\sqrt{2}})</td>
</tr>
<tr>
<td>Welfare</td>
<td>(v + \frac{t}{4} - \frac{c}{4})</td>
<td>(v + \frac{1}{4} + \frac{3t}{32} - \frac{c}{2})</td>
<td>(\frac{5v}{6} + \frac{1}{4} + \frac{7t}{32} - \frac{5c}{6})</td>
<td>(v + \frac{1}{4} + \frac{1}{6}(2 - \sqrt{2}) - \frac{c}{2})</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>(\frac{5}{6})</td>
<td>(\frac{5}{6})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(\pi_{t_i}, i = a, b)</td>
<td>(\frac{t}{2} - \frac{1}{8})</td>
<td>(\frac{1}{3}t - \frac{1}{8})</td>
<td>(\frac{5}{9}t - \frac{1}{8})</td>
<td>(\frac{\sqrt{\pi}}{4}t - \frac{1}{8})</td>
</tr>
</tbody>
</table>

Table 9
5. Optimal welfare outcomes

This section analyses the actions a regulator can take to optimize delivered quality and minimize consumer transport costs. To optimize welfare the regulator sets optimal quality levels and optimal hospital locations. In contrast to the previous settings the regulator chooses the optimal location and the optimal quality level simultaneously. To solve the model backward induction is not required. Furthermore note that welfare maximization by means of setting an optimal regulated price is no longer necessary when optimizing function 8.

5.1. Optimal welfare quality levels

In the social optimum the regulator sets equal quality levels for both hospitals. Quality levels do not depend on the distribution of demand. Taking the first order condition finds optimal quality levels to equal \( \frac{1}{2} \).

Proof

\[
\frac{dw}{d\theta} = 1 - 2\theta_w^* = 0
\]

Optimal quality level is set when \( \theta_w^* = \frac{1}{2} \)

5.2. Hospital locations in a welfare setting for symmetric distributions

We optimize the locations of hospitals A and B for distribution \( i, ii \) and \( iii \) to maximize welfare levels. For mathematical proof of the optimal locations for all three distributions, please consult appendix A.5.1., A.5.1.2. and A.5.1.3. respectively. Welfare is maximized when hospitals locate at the 25th and 75th percentile of the cumulative population distribution. The distribution has no effect on relative location outcome in symmetric models. Location and welfare outcomes can be found in Table 11. Note that transport costs in the welfare equation are proportionate to the total population. Bringing the population levels up to \( N=1 \) finds the outcomes of the final row.

<table>
<thead>
<tr>
<th></th>
<th>( i, Japan )</th>
<th>( ii, France )</th>
<th>( iii, United States )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( a )</td>
<td>( \frac{1}{4} )</td>
<td>0.32635</td>
<td>0.23088</td>
</tr>
<tr>
<td>1-b</td>
<td>( \frac{3}{4} )</td>
<td>0.67365</td>
<td>0.76912</td>
</tr>
<tr>
<td>Welfare</td>
<td>( v + \frac{1}{4} - \frac{t}{8} - c )</td>
<td>( v + \frac{1}{4} - 0.0513t - \frac{c}{2} )</td>
<td>( \frac{5v}{6} + \frac{1}{4} - 0.1049t - \frac{5c}{6} )</td>
</tr>
<tr>
<td>Welfare</td>
<td>( v + \frac{1}{4} - 0.125t - c )</td>
<td>( v + \frac{1}{4} - 0.1026t - \frac{c}{2} )</td>
<td>( v + \frac{1}{4} - 0.1259t - c )</td>
</tr>
</tbody>
</table>

Table 10
Figure 5.1 compares absolute welfare outcomes of planned setting to the competitive setting. We assume parameters $v$ and $c$ to be fixed. Thereby only comparing the effect transport costs have on the total welfare outcome. The dashed lines represent the planned outcome for distributions $i$, $ii$ and $iii$. The continuous lines represent the competitive outcome for distributions $i$, $ii$ and $iii$. Note that the competitive outcomes only equal the planned outcomes when transport costs are assumed zero. In all other cases the dashed lines trump the continuous lines. We therefore conclude a planned setting to be more efficient than a competitive setting.

The results allow for a discussion on the desirability of a socially planned setting versus a competitive setting. Regardless of the distribution of demand introducing a social planner increases the welfare outcome. As discussed in section 3. a sole supplier introduces a monitoring issue to the model to control whether optimal quality levels are reached. In a competitive model these monitoring costs are not necessary. Altering the model to a competitive model with exogenous locations allows for lower monitoring costs due to competition and a higher overall welfare outcome as exogenous locations are set to maximize welfare.

With regards to the distribution of demand, we learn that introduction of a social planner affects transport costs specifically. Distribution $iii$ relatively sees the largest effects. Transport costs decrease with more than 52% in a planned setting, whereas distributions $i$ and $ii$ see a decrease of 50% and 45,28% respectively. We contribute the difference to the spatial spreading of the population. Countries as the United States profit more from a socially planned healthcare locations than countries such as France and Japan. Regulators in developing countries deciding on the structure of their hospital system and location of their hospitals have a larger incentive to choose for a socially planned setting when the population is evenly spread over the country.

\[ i = \left( \frac{1}{6} - \frac{1}{4} \right) / \frac{1}{4} = -\frac{1}{2}; \ ii = (0.0513 - \frac{3}{32}) / \frac{3}{32} \approx -0.4528; \ iii = (0.1049 - \frac{7}{32}) / \frac{7}{32} \approx -0.52 \]
or distributed centrifugally. States where demand is centralized can gain relatively more by introducing competition.

5.3. Hospital locations in a welfare setting in a non-symmetric distribution

Hospital locations in distribution iv are found to lie at approximately the 28th and 79th percentile of the cumulative population respectively. The results of Table 12 are derived in appendix A.5.2. on page A-xiii. Furthermore, when compared to symmetric distributions where both hospitals supply exactly 50% of the consumer, hospital A supplies healthcare to approximately 57.3% of consumers, where hospital B does so for the remaining 42.3% of consumers. In a competitive distribution iv setting both hospitals optimally supply exactly half of the market. Optimal strategy for the social planner becomes to slightly increase the travel expenses of the population in the more densely populated area. Therewith allowing a decrease in travel expenses for the rural population. The decrease in travel expenses of the rural population trumps the increase in travel expenses by the urban population. In total travel expenses in comparison to the competitive setting are decreased by almost 47%25. The social planned setting is preferable to the competitive setting26.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\theta_i)</th>
<th>(a)</th>
<th>(1-b)</th>
<th>(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>iv</td>
<td>(\frac{1}{2})</td>
<td>(\frac{4-\sqrt{(10+\sqrt{2})}}{4})</td>
<td>(\frac{4-\sqrt{2+\sqrt{2}}}{4})</td>
<td>(2-\frac{1}{\sqrt{2}})</td>
</tr>
</tbody>
</table>

Welfare \(\frac{V}{2} + \frac{1}{4} - \frac{t}{12} (4 - \sqrt{10 + \sqrt{2}}) - \frac{c}{2}\)

Table 11

6. N-firms in a welfare setting

The regulators’ aim is to maximize welfare. Up to now we have considered models with two locations in both a competitive and non-competitive setting. These computations have resulted in four welfare outcomes, all of which are a function of parameters \(v\), \(t\) and \(c\).

\[\frac{1}{\sqrt{2}} (\sqrt{\gamma} - \sqrt{\gamma - \sqrt{2}}) - \frac{1}{3} (\sqrt{\gamma - \gamma}) \approx -0.4695\]

\[\frac{1}{\sqrt{2}} (\sqrt{\gamma} - \sqrt{\gamma - \sqrt{2}}) - \frac{1}{3} (\sqrt{\gamma - \gamma}) \approx -0.4695\]

26 We do not control for deterioration of perceived valuation of care when the hospital in the urbanized area becomes over utilized. Over time a majority of the electorate will be inclined to shift additional resources from the lesser used rural hospital to the over used urban hospital. This in turn might decrease the amount of resources allocated to the rural hospital and deteriorate quality levels there.
Given the value of parameter $t$, welfare outcomes can change drastically when the value of this parameter changes. In addition, this ignores the possibility the regulator has to introduce an additional competitor to the game. To enhance operability, demand in the model in this section is distributed uniformly. Furthermore, we assume the regulator can only introduce an additional competitor in a welfare setting\textsuperscript{27}. Finally, to model reality better, we also add additional fixed hospital location costs to the model (parameter $k$). These costs are undefined but represent a convex function that is dependent on the number of locations.

Therefore the regulator does not only face the task of setting the optimal price, but also has to choose the optimal market structure. This setup can be described as similar to adding an additional step to the models. Given the previously employed steps, we now introduce step $Z$ to the model. We assume every hospital to treat exactly the same number of patients as the other hospitals. Hence, every hospital delivers healthcare to $\frac{1}{n}$ of the market. The regulator first chooses the number of competitors, represented by parameter $n$. In the second stage it sets quality levels and locations simultaneously. The game is solved by backward induction.

Y. Regulator chooses number of hospitals;
Z. Regulator sets location and sets quality level.

Setting optimal levels of quality and determining optimal locations from a welfare perspective finds optimal quality levels to be equal to $\frac{1}{n}$ in a setting with $n$ number of hospitals. Deriving the optimal location finds that hospitals are located at $\frac{1}{2n}$ of the market.

**Proof:**

\[
W = v + \theta_w - t \left( \sum_{i=1}^{N} \int_{0}^{i}(i-x)dx + \int_{i}^{\frac{1}{n}}(x-i)dx \right) - p - \sum_{n=1}^{N} \pi_n(\theta_w, k(n))
\]

(11.)

\[
W = v + \theta_w - nt \left( \int_{0}^{i}(i-x)dx + \int_{i}^{\frac{1}{n}}(x-i)dx \right) - c - \frac{n}{2} \theta_w^2 - nk(n)
\]

\[
\frac{dW}{d\theta_w} = 1 - n\theta_w^* \quad \text{implies optimal quality level:} \quad \theta_w^* = \frac{1}{n}
\]

\[
\frac{dW}{di} = 2ni^* - 1 \quad \text{implies optimal location:} \quad i^* = \frac{1}{2n}
\]

\textsuperscript{27} A competitive Hotelling model with an uneven number of competitors, with a monopolist being the exception, does not result in a Nash-equilibrium. Therefore only welfare settings are studied.
Interpreting the results finds quality levels decreasing and overall travel expenses decreasing for every additional location. To maximize the welfare outcome the regulator has to decide how many locations it will allow on the market. The answer is derived by substituting $\theta^*_w$ and $t^*$ into function 11. (outcome is function 12.) and taking the FOC of this function over the number of hospital locations. Depending on the value of transport costs the regulator either chooses 1 location or infinite locations.

Proof:

$$W = v + \frac{1}{n} - \frac{nt}{4n^2} - c - \frac{n}{2n^2} - nk(n) = v + \frac{1}{2n} - \frac{t}{4n} - c - nk(n)$$

$$\frac{dW}{dn} = \frac{t-2}{4n^2} - k(n)(1 + n \frac{dk}{dn})$$

if $t < 2$ and $k(n) > 0 \rightarrow \frac{dW}{dn} < 0$

if $t > 2$ and $k(n) \frac{t-2}{4n^2} \rightarrow \frac{dW}{dn} < 0$

The regulator chooses for minimal number of locations.

if $t > 2$ and $k(n) < \frac{t-2}{4n^2} \rightarrow \frac{dW}{dn} > 0$

The regulator chooses for as many locations as possible.

The regulator wants to maximize total welfare outcomes. In this setting, given that transport costs are lower than 2 and fixed hospital location costs are high, the lesser the number of hospitals active on the market, the higher the welfare outcome will be. Once transport costs become larger than 2, the regulator will still not be inclined to introduce a maximal number of locations if fixed hospital location cost are sufficiently high. Only when transport costs are high and fixed hospital location costs are low will the regulator be incentivized to introduce as many location as possible.

Once again the regulator has a delicate task, this time in setting the number of locations. The more hospitals it introduces the lower transport expenses and quality levels will be and the higher fixed hospital location costs will become. In settings where the transport costs are low and fixed location costs high the regulator will be inclined to minimize the number of locations. This can be explained by the following examples; Take two diseases. One of the diseases requires specialist care, the other does not. The willingness to pay to combat the former disease is higher than the willingness to pay for the latter. To put this in perspective of transport costs we should therefore not view these at their absolute value. Transport costs must be viewed in proportion to the value of care they relate to. If the value of care is high, read care of a high quality, it is desirable from a social perspective to centralize healthcare. Decentralization is
beneficial if the value of care is relatively low. A real life example are centralized clinics providing high quality cancer treatment versus the decentralized location of General Practitioners. In addition, General Practitioners provide a good example of situations where the relative costs of travel can be high – think of elderly who frequently need a shot against the flu, babies that need a monthly checkup after delivery, or, more in general, diseases that need a frequent checkup. Due to the frequency of visits relative transport costs become high and fixed location costs are relatively low. The regulator will want to maximize the number of locations as long as the number of locations does not increase hospital fixed location costs too much. Over time the size of the fixed hospital location costs will prove debatable. This may prove in line with the argument made by Genevieve and Chifang (2016). For example, when labour markets are tight, hospital location costs will lie substantially higher when compared to loose labour markets. Regulators might therefore be inclined to choose for a central location in tight labour markets and for decentral locations in a loose labour market. However, labour markets are not static, but tend to change over time. Choosing for a central location might prove beneficial during a period of tight labour markets, but may prove to be a non-optimal decision over time when labour markets become more loose.

Above all, the questions the regulator has to deal with when setting the optimal number of locations is deriving the value of parameter $t$ and deciding on supervision of the sole hospital delivering the optimal quality. Answering these questions is however beyond the scope of this thesis.

### 7. Discussion and Conclusion

Before jumping off to conclusions we want to point out that we have assumed patients to be homogenous. In real life, healthcare is valued differently by different types of patients. As are travel expenses. Think of physically disabled and elderly people who, in general, require more healthcare and in some instances have to undertake larger efforts to visit centres of healthcare\(^{28}\). Their valuation of travel expenses lie higher than those in the bloom of their lives. For those who have to undertake large travel efforts, it would make more sense to put more weight on decreasing travel expenses than to (further) increase quality levels.

\(^{28}\) E.g. Think of those with Alzheimer’s disease who are not allowed to drive themselves anymore but do require some form of healthcare.
This example immediately shows the flaws of the research. We have made many assumptions to find an answer to the research question. It is questionable whether these assumptions hold in reality. For example, the research has assumed the location of the indifferent consumer to be a leading explanation of quality levels. This for instances would imply that hospitals in scarcely populated areas require much higher prices to achieve similar quality levels as compared to those hospitals located in more densely populated areas. Analogically this would also imply that the intrinsic quality levels of doctors in scarcely populated areas cannot be better when compared to the quality levels of their counterparts in more populated areas. The model assumes such results to be impossible.

Furthermore, hospitals are also assumed to be homogenous. It is rather the question if this applies in reality. Finally, the assumed quality level cost function is set arbitrarily and might differ in reality. Then again, not making such assumption would make the thesis ambiguous. No clear results would arise and no regulators would not be able to draw any clear lessons for future policies. Nevertheless, policymakers should be aware that translating the outcomes of the thesis one-to-one to policy might affect those that are most in need of healthcare.

Overall, the research finds comparable outcomes to theoretical model of Brekke e.a. (2006), the paper on which it is based. In competitive settings, consumers face higher transport expenses when compared to the optimal welfare outcome. Our model however finds hospitals opting for minimal differentiation rather than maximal differentiation. Adding non-uniform distributions to the model does allow for strategic locations effects, despite assuming linear transport cost. The strategic effect however is always trumped by the direct effect. Furthermore, we find uncertainty with regards to the regulated price to allow for a regulated price that is better able to target optimal quality levels. The uncertainty the regulator has with regards to hospital locations in a setting with certain regulated prices is taken away as the regulator knows where both hospitals locate.

From a social welfare point of view it is beneficial to place hospitals in public hands as this decreases consumer transport expenses. For example, countries where demand is diffuse, such as the United States, Canada or Australia, see 40% higher transport costs when compared to countries with centralized demand (France) in competitive settings. The former profit more from socially planned locations than the latter, as welfare maximizing locations shift to the 25<sup>th</sup> and 75<sup>th</sup> percentile of the cumulative population. We slightly touch upon the desirability of centralized hospital management. Introducing this over a competitive market introduces monitoring costs. We argue society could see welfare maximizing outcomes when the regulator...
exogenously sets hospital locations at the welfare maximizing locations, and then allows for competition. Doing so brings down monitoring costs and gives the regulator a tool to incentivize more efficient production.

Furthermore we find optimal regulated price to be dependent on the density of the population. The less dense the population is at the location of the indifferent consumer, the higher the regulated price becomes.

Non-symmetric distribution with demand concentrated at the one side of the distribution form an exception. In countries such as Norway, the regulator can influence the location of both hospitals, but only slightly. Increasing regulated prices in this setting shifts joint hospital locations to the right, but also decreases the stability of this equilibrium. Additionally, welfare is maximized when hospitals are not located at the 25th and 75th percentile, but are shifted towards the 28th and 79th percentile of the cumulative population. This shifts the location of the indifferent consumer towards the 58th percentile, and provides better access to healthcare to the rural population. It is debatable whether quality levels will remain similar in this welfare optimizing setting. Finally we show that the regulator will choose for one central hospital when relative transport costs and fixed hospital costs are low enough. Whenever transport costs and fixed hospital costs become too large the regulator will choose an infinite number of hospitals. This provides an argument in favour of centralizing and decentralizing hospital locations, depending on the of sort of healthcare provided. However, the preferability of these decisions might change over time due to the tightness of labour markets.

When credibly committing to a price the regulator can incentives hospitals to alter their locations towards less concentrated areas in non-symmetric models and assuming linear transport costs. Future research should focus on the effects quadratic transport costs have on competition in non-symmetric distributions, and whether such settings also allow for credible commitment of the regulator to provide better access to healthcare in more rural regions.
Bibliography


Appendix

A.1.

29 https://en.populationdata.net/maps/japan-density-2010/


32 https://www.researchgate.net/figure/Map-of-Norway-The-core-regions-indicated-and-population-density-of-first-generation_fig1_311456632
A.3.1.

**Deriving optimal quality levels for all four distributions**

Note: \[ \frac{d\hat{x}}{d\theta_a} = \frac{1}{2t} \quad \text{and} \quad \frac{d\hat{x}}{d\theta_b} = -\frac{1}{2t} \]

**Distribution i**

**Profit function**

\[
\pi_a = (p - c) \left( \frac{1}{2} \left( 1 - b + a + \frac{\theta_a - \theta_b}{t} \right) \right) - \frac{1}{2} \theta_a^2
\]

\[
\pi_b = (p - c) \left( \frac{1}{2} \left( 1 + b - a + \frac{\theta_b - \theta_a}{t} \right) \right) - \frac{1}{2} \theta_b^2
\]

**FOC**

\[
\frac{d\pi_a}{d\theta_a} = \frac{(p-c)}{2t} - \theta_a = 0 \quad \Rightarrow \quad \theta_a^* = \frac{p-c}{2t}
\]

\[
\frac{d\pi_b}{d\theta_b} = \frac{(p-c)}{2t} - \theta_b = 0 \quad \Rightarrow \quad \theta_b^* = \frac{p-c}{2t}
\]

**Distribution ii**

**Profit function**

\[
\pi_a = (p - c) \left( -\hat{x}^3 + \frac{3}{2} \hat{x}^2 \right) - \frac{1}{2} \theta_a^2
\]

\[
\pi_b = (p - c) \left( \frac{1}{2} \left( 1 + b - a + \frac{\theta_b - \theta_a}{t} \right) \right) - \frac{1}{2} \theta_b^2
\]

**FOC**

\[
\frac{d\pi_a}{d\theta_a} = \frac{(p-c)}{2t} \left(-3\hat{x}^2 + 3\hat{x}\right) - \theta_a = 0
\]

\[
\frac{d\pi_b}{d\theta_b} = \frac{(p-c)}{2t} \left(-3\hat{x}^2 + 3\hat{x}\right) - \theta_b = 0
\]

**Optimal \( \theta \)**

\[
\theta_a^* = \frac{(p-c)}{2t} g(\hat{x})
\]

\[
\theta_b^* = \frac{(p-c)}{2t} g(\hat{x})
\]

**Distribution iii**

**Profit function**

\[
\pi_a = (p - c) \left( \frac{1}{3} \hat{x}^3 - \frac{1}{2} \hat{x}^2 + \hat{x} \right) - \frac{1}{2} \theta_a^2
\]

\[
\pi_b = (p - c) \left( \frac{5}{6} \hat{x}^3 + \frac{1}{2} \hat{x}^2 - \hat{x} \right) - \frac{1}{2} \theta_b^2
\]

**FOC**

\[
\frac{d\pi_a}{d\theta_a} = \frac{(p-c)}{2t} \left(\hat{x}^2 - \hat{x} + 1\right) - \theta_a = 0
\]

\[
\frac{d\pi_b}{d\theta_b} = \frac{(p-c)}{2t} \left(\hat{x}^2 - \hat{x} + 1\right) - \theta_b = 0
\]

**Optimal \( \theta \)**

\[
\theta_a^* = \frac{(p-c)}{2t} h(\hat{x})
\]

\[
\theta_b^* = \frac{(p-c)}{2t} h(\hat{x})
\]

**Distribution iv**

**Profit function**

\[
\pi_a = (p - c) \left( \hat{x} - \frac{\hat{x}^2}{2} \right) - \frac{1}{2} \theta_a^2
\]

\[
\pi_b = (p - c) \left( \frac{1}{2} \hat{x} + \frac{\hat{x}^2}{2} \right) - \frac{1}{2} \theta_b^2
\]

**FOC**

\[
\frac{d\pi_a}{d\theta_a} = \frac{(p-c)}{2t} \left(1 - \hat{x}\right) - \theta_a = 0
\]

\[
\frac{d\pi_b}{d\theta_b} = \frac{(p-c)}{2t} \left(1 - \hat{x}\right) - \theta_b = 0
\]

**Optimal \( \theta \)**

\[
\theta_a^* = \frac{(p-c)}{2t} k(\hat{x})
\]

\[
\theta_b^* = \frac{(p-c)}{2t} k(\hat{x})
\]
A.3.2.

Exogenous locations – when do hospitals at least make zero profits?

**Distribution i**

\[
\pi_a = \frac{(p-c)}{2} (1 - b + a) - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} > 0
\]

\[
\pi_a = \frac{(p-c)}{2} (1 - b + a) > \frac{1}{2} \frac{(p-c)^2}{(2t)^2}
\]

\[
a > \frac{(p-c)}{(2t)^2} - 1 + b
\]

\[
\pi_b = \frac{(p-c)}{2} (1 + b - a) - \frac{1}{2} \frac{(p-c)^2}{2t} > 0
\]

\[
b > \frac{(p-c)}{(2t)^2} + a - 1
\]

**Distribution ii**

\[
\pi_a = (p - c) \left( -\hat{x}^2 + \frac{3}{2} \hat{x}^2 \right) - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} (-3\hat{x}^2 + 3\hat{x})^2 > 0
\]

\[
\pi_a = \hat{x}^2 \left( -\hat{x} + \frac{3}{2} \right) - \frac{9}{2} \frac{(p-c)}{(2t)^2} \hat{x}^2 (1 - \hat{x})^2 > 0
\]

\[
\pi_a = \left( \frac{3}{2} - \hat{x} \right) - \frac{9}{2} \frac{(p-c)}{(2t)^2} (1 - \hat{x})^2 > 0
\]

\[
4(p-c) \sqrt{\frac{9(p-c) + 4t^2}{(p-c)^2}} + 9(p-c) - 8t^2
\]

\[
- \frac{4(p-c)}{9(p-c)} + b < a < \frac{4(p-c)}{9(p-c)} + b
\]

Analogically the outcome is similar for hospital B.

**Distribution iv**

\[
\pi_a = (p - c) \left( \hat{x} - \frac{\hat{x}^2}{2} \right) - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} (1 - \hat{x})^2 > 0
\]

\[
\pi_a = \left( \hat{x} - \frac{\hat{x}^2}{2} \right) > \frac{1}{2} \frac{(p-c)^2}{(2t)^2} (1 - \hat{x})^2
\]

Hospital A will leave the market whenever its location is smaller than following function.

\[
a < 1 + b - 4 \sqrt{\frac{t^2}{(p-c) + 4t^2}}
\]

for B

\[
\pi_b = \frac{(p-c)}{2} (1 - \hat{x})^2 \left( 1 - \frac{(p-c)}{(2t)^2} \right) > 0
\]

as long as \( p - c < 4t^2 \) all locations of B are profitable.
A.3.3.

For simplicity, given that $\theta_a = \theta_b$, $\hat{x}$ can be rewritten to; $\hat{x} = \frac{1-b+a}{2}$, $\frac{d\hat{x}}{da} = \frac{1}{2}$, $\frac{d\hat{x}}{db} = -\frac{1}{2}$

This applies to all following sections of the appendix.

A.3.3.2.

Deriving the optimal location for hospitals A and B in distribution ii

For hospital A

\[
\frac{d\pi_a}{da} = (p - c) \left( -3\hat{x}^2 \cdot \frac{d\hat{x}}{da} + 3\hat{x} \cdot \frac{d\hat{x}}{da} \right) - \frac{(p - c)^2}{(2t)^2} \left( -3\hat{x}^2 + 3\hat{x} \right) \left( -6\hat{x} \cdot \frac{d\hat{x}}{da} + 3 \cdot \frac{d\hat{x}}{da} \right)
\]

\[
\frac{d\pi_a}{da} = \frac{(p - c)}{2} \left( -3\hat{x}^2 + 3\hat{x} \right) \left( 1 - \left( -6\hat{x} + 3 \right) \frac{(p - c)}{(2t)^2} \right)
\]

\[
= \frac{p - c}{2} \left( -3\hat{x}^2 + 3\hat{x} \right) \left( 1 + \frac{3(p - c)}{(2t)^2} (a - b) \right)
\]

if $a > b - \frac{(2t)^2}{3(p-c)}$; $\frac{d\pi_a}{da} \geq 0$; hospital A locates at population median

if $a \leq b - \frac{(2t)^2}{3(p-c)}$; $\frac{d\pi_a}{da} < 0$; hospital A locates at 0

if $\lim_{t \to 0} a = b \rightarrow \frac{d\pi_a}{da} < 0$; hospital A locates at 0

For hospital B

$\pi_b = (p - c) \left( \frac{1}{2} + \hat{x}^3 - \frac{3}{2} \hat{x}^2 \right) - \frac{1}{2} \frac{(p-c)^2}{(2t)^2} \left( -3\hat{x}^2 + 3\hat{x} \right)^2$

\[
\frac{d\pi_b}{db} = (p - c) \left( 3\hat{x}^2 \cdot \frac{d\hat{x}}{db} - 3\hat{x} \cdot \frac{d\hat{x}}{db} \right) - \frac{(p - c)^2}{(2t)^2} \left( -3\hat{x}^2 + 3\hat{x} \right) \left( -6\hat{x} \cdot \frac{d\hat{x}}{db} + 3 \cdot \frac{d\hat{x}}{db} \right)
\]

\[
\frac{d\pi_b}{db} = \frac{(p - c)}{2} \left( -3\hat{x}^2 + 3\hat{x} \right) + \frac{(p - c)^2}{2(2t)^2} \left( -3\hat{x}^2 + 3\hat{x} \right)(-6\hat{x} + 3)
\]

\[
= \frac{(p - c)}{2} \left( -3\hat{x}^2 + 3\hat{x} \right) \left( 1 + \frac{3(p - c)}{(2t)^2} (b - a) \right) = 0
\]

if $b > a - \frac{(2t)^2}{3(p-c)}$; $\frac{d\pi_a}{da} \geq 0$; hospital B locates at population median

if $b \leq a - \frac{(2t)^2}{3(p-c)}$; $\frac{d\pi_a}{da} < 0$; hospital B locates at 1

if $\lim_{t \to 0} b = a \rightarrow \frac{d\pi_a}{da} < 0$; hospital B locates at 1
A.3.3.3.

deriving the optimal location for hospitals A and B in distribution iii

For hospital A

\[
\frac{d\pi_a}{da} = (p - c) \left( \hat{x}^2 \frac{d\hat{x}}{da} - \hat{x} \cdot \frac{dx}{da} + 1 \cdot \frac{dx}{da} \right) - \frac{(p-c)^2}{(2t)^2} \left( \hat{x}^2 - \hat{x} + 1 \right) \left( 2\hat{x} \cdot \frac{d\hat{x}}{da} - 1 \cdot \frac{dx}{da} \right) = 0
\]

\[
\frac{d\pi_a}{da} = \frac{(p - c)}{2} \left( \hat{x}^2 - \hat{x} + 1 \right) - \frac{(p - c)^2}{2(2t)^2} \left( \hat{x}^2 - \hat{x} + 1 \right)(2\hat{x} - 1)
\]

\[
= \frac{(p - c)}{2} \left( \hat{x}^2 - \hat{x} + 1 \right) \left( 1 - \frac{p - c}{(2t)^2} (a - b) \right) = 0
\]

if \( a > b + \frac{(2t)^2}{p-c} \); \( \frac{d\pi_a}{da} < 0 \); hospital A locates at 0; however impossible outcome

if \( a \leq b + \frac{(2t)^2}{p-c} \); \( \frac{d\pi_a}{da} \geq 0 \); hospital A locates at population median

if \( \lim_{t \to 0} a = b \rightarrow \) hospital A locates at population median

For hospital B

\[
\frac{d\pi_b}{db} = (p - c)(-x^2 + x - 1) \frac{d\hat{x}}{db} - \frac{(p - c)^2}{(2t)^2} \left( \hat{x}^2 - \hat{x} + 1 \right) \left( 2\hat{x} - 1 \right) \frac{d\hat{x}}{db} = 0
\]

\[
\frac{d\pi_b}{db} = \frac{(p - c)}{2} \left( \hat{x}^2 - \hat{x} + 1 \right) + \frac{(p - c)^2}{2(2t)^2} \left( \hat{x}^2 - \hat{x} + 1 \right)(2\hat{x} - 1)
\]

\[
= \frac{(p - c)}{2} \left( \hat{x}^2 - \hat{x} + 1 \right) \left( 1 - \frac{p - c}{(2t)^2} (b - a) \right) = 0
\]

if \( b > a + \frac{2t^2}{p-c} \); \( \frac{d\pi_b}{db} < 0 \); hospital B locates at 1; however impossible outcome

if \( b \leq a + \frac{2t^2}{p-c} \); \( \frac{d\pi_b}{db} \geq 0 \); hospital B locates at population median

if \( \lim_{t \to 0} b = a \rightarrow \) hospital B locates at population median

\( b > a + \frac{2t^2}{p-c} \) and \( a > b + \frac{(2t)^2}{p-c} \) are impossible solutions. Whenever \( \frac{d\pi_a}{da} < 0 \) hospital A will set its location at zero. Hospital B’s FOC = \( \frac{d\pi_b}{db} \geq 0 \). Therefore it will set its location at the population median. By doing so \( \frac{d\pi_a}{da} \) becomes \( \geq 0 \). Hence hospital A sets its location at the population median.
A.3.3.4.

**Deriving the optimal location for hospitals A and B in distribution iv**

**Optimal location choice for hospital A:**

\[
\pi_a = (p - c)\left(\hat{x} - \frac{\hat{x}^2}{2}\right) - \frac{1}{8}\left(\frac{p-c}{2t}\right)^2 (1 + b - a)^2
\]

\[
\frac{d\pi_a}{da} = (p - c)\left(\frac{d\hat{x}}{da} - \hat{x} \frac{d\hat{x}}{da}\right) - \frac{1}{4}\left(\frac{p-c}{2t}\right)^2 (1 + b - a) \ast -1
\]

\[
\frac{d\pi_a}{da} = \frac{p-c}{2} (1 - \hat{x}) - \frac{1}{4}\left(\frac{p-c}{2t}\right)^2 (1 + b - a) = \frac{p-c}{4} (1 + b - a) + \frac{(p-c)^2}{4 + 4t^2} (1 + b - a)
\]

\[
= \frac{p-c}{4} (1 + b - a) \left(1 + \frac{p-c}{(2t)^2}\right) > 0
\]

Given the possible locations of hospitals A and B, hospital A sets its location at the population median.

**Optimal location choice for hospital B:**

\[
\pi_b = (p - c)\left(\frac{1}{2} - \hat{x} + \frac{\hat{x}^2}{2}\right) - \frac{1}{8}\left(\frac{p-c}{2t}\right)^2 (1 + b - a)^2 = 0
\]

\[
\frac{d\pi_b}{db} = (p - c)\left(- \frac{d\hat{x}}{db} + \hat{x} \frac{d\hat{x}}{db}\right) - \frac{1}{4}\left(\frac{p-c}{2t}\right)^2 (1 + b - a)
\]

\[
= \frac{(p-c)}{2} (1 - \hat{x}) - \frac{1}{4}\left(\frac{p-c}{2t}\right)^2 (1 + b - a)
\]

\[
= \frac{(p-c)}{4} (1 + b - a) - \frac{1}{4}\left(\frac{p-c}{2t}\right)^2 (1 + b - a)
\]

\[
= \frac{(p-c)}{4} (1 + b - a) \left(1 - \frac{p-c}{(2t)^2}\right)
\]

Location of hospital B depends on the size of \(\frac{p-c}{(2t)^2}\)

A.3.4.

**Setting optimal prices and welfare solution in a competitive setting**

For all distributions the following function applies.

\[
W = Nv + \theta - t \left(\int_0^{\text{pop.med}} (\text{pop.med} - x) dx + \int_{\text{pop.med}}^1 (x - \text{pop.med}) dx\right) - Nc - \theta^2
\]

Welfare solution distribution i
\[ a = 1 - b = \hat{x} = \frac{1}{2} \]

\[ W = v + \frac{p-c}{2t} - t \left( \int_{0}^{\frac{1}{2}} (x - \frac{1}{2}) - x \right) dx + \int_{\frac{1}{2}}^{1} (x - \frac{1}{2}) dx - c - \frac{(p-c)^2}{(2t)^2} \]

\[ \frac{dW}{dp} = \frac{1}{2t} - \frac{2(p-c)}{4t^2} = 0 \rightarrow p = t + c \]

Quality levels:
\[ \theta = \frac{t+c-c}{2t} = \frac{1}{2} \]

Welfare outcome:
\[ W = v + \frac{1}{4} - \frac{t}{4} - c \]

Hospital profits:
\[ \pi_i = (p-c) \frac{1}{2} - \frac{1}{8} = \frac{t}{2} - \frac{1}{8} \quad i = a, b \]

**Welfare solution distribution ii**

\[ a = 1 - b = \hat{x} = \frac{1}{2} \]

\[ W = v + \frac{p-c}{2t} \left( -3\hat{x}^2 + 3\hat{x} \right) - t \left( \int_{0}^{\frac{1}{2}} (x - \frac{1}{2}) \left( -3x^2 + 3x \right) dx + \int_{\frac{1}{2}}^{1} (x - \frac{1}{2}) \left( -3x^2 + 3x \right) dx \right) \]

\[ \frac{dW}{dp} = \frac{3}{8t} - \frac{(p-c)^2}{2t^2} \left( \frac{9}{16} \right) = 0 \]

\[ \frac{3}{8t} = (p-c) \frac{1}{2t^2} \left( \frac{9}{16} \right) \rightarrow \frac{3}{8t} = (p-c) \frac{1}{2t^2} \left( \frac{3 \times 3}{2 \times 8} \right) \rightarrow p = \frac{4}{3} t + c \]

Quality levels:
\[ \theta_i = \frac{\frac{4}{3} t + c - c}{2t} = \frac{1}{2} \quad i = a, b \]

Welfare outcome:
\[ W = v + \frac{t}{4} - t \left( \frac{3}{32} \right) - \frac{c}{2} \]

Hospital profits:
\[ \pi_i = \frac{1}{3} t - \frac{1}{8} \quad i = a, b \]

**Welfare solution to distribution iii**
\( a = 1 - b = \hat{x} = \frac{1}{2} \)

\[
W = \frac{5}{6}v + \frac{p - c}{2t} \left( \frac{3}{4} \right) - t \left( \int_{0}^{1} \left( \frac{1}{2} - x \right)(x^2 - x + 1)dx + \int_{\frac{1}{2}}^{1} \left( x - \frac{1}{2} \right)(x^2 - x + 1)dx \right) - \frac{5}{6}c
- (p - c)^2 \frac{2}{(2t)^2} \left( \frac{3}{4} \right)^2
\]

\[
\frac{dW}{dp} = \frac{1}{2t} \left( \frac{3}{4} \right) - (p - c)^2 \frac{2}{(2t)^2} \left( \frac{3}{4} \right)^2 = 0 \Rightarrow \frac{2(p-c)}{(2t)^2} = \frac{9(p-c)}{8(2t)^2} = \frac{3}{8t} \Rightarrow p = \frac{4}{3}t + c
\]

Quality levels:

\( \theta_i = \frac{\sqrt{2}}{2t} \left( \sqrt{2} \right) = \left( \frac{\sqrt{2}}{2} \right)^2 = \frac{1}{2} \) \( i = a, b \)

Welfare setting outcome:

\[
W = \frac{5}{6}v + \frac{1}{4} - t \left( \frac{7}{32} \right) - \frac{5}{6}c
\]

Hospital profits:

\[
\pi_i = \frac{5}{9}t - \frac{1}{8} \quad i = a, b
\]

Welfare solution distribution \( iv \)

\( a = 1 - b = \hat{x} = \frac{2-\sqrt{2}}{2} \)

\[
W = \frac{v}{2} + \frac{(p-c)}{2t} \left( 1 - (1 - \frac{\sqrt{2}}{2}) \right) - t \left( \int_{0}^{\frac{\sqrt{2}}{2}} \left( \frac{2-\sqrt{2}}{2} - x \right)(1-x)dx + \int_{\frac{\sqrt{2}}{2}}^{1} \left( x - \frac{2-\sqrt{2}}{2} \right)(1-x)dx \right) - \frac{c}{2} - \frac{(p-c)^2}{(2t)^2} \left( 1 - (1 - \frac{\sqrt{2}}{2}) \right)^2
\]

\[
W = \frac{v}{2} + \frac{(p-c)}{2t} \left( \frac{1}{2} \sqrt{2} \right) - t(distance) - \frac{c}{2} - \frac{(p-c)^2}{(2t)^2} \left( \frac{1}{2} \sqrt{2} \right)^2
\]

\[
\frac{dW}{dp} = \frac{\sqrt{2}t}{4t} - \frac{2(p-c)}{8t^2} = 0 \Rightarrow \frac{\sqrt{2}t}{4t}p = \frac{8t}{8} \sqrt{2} + c = t \sqrt{2} + c
\]

Quality levels:

\( \theta_i = \frac{\sqrt{2}t}{2t} \left( \frac{\sqrt{2}}{2} \right) = \left( \frac{\sqrt{2}}{2} \right)^2 = \frac{1}{2} \) \( i = a, b \)

Welfare outcome:

\[
W = \frac{v}{2} + \frac{1}{4} - \frac{t}{6} \left( 2 - \sqrt{2} \right) - \frac{c}{2}
\]

Hospital profits:

\[
\pi_i = \left( t \sqrt{2} \right) \ast \frac{1}{4} - \frac{1}{8} \quad i = a, b
\]

A.4.
What happens to hospital locations if the setup of the model is altered?

**Distribution i**

\[
\pi_a = \frac{1}{2} (1 - b + a) - \frac{1}{8} \\
\pi_b = \frac{1}{2} (1 + b - a) - \frac{1}{8}
\]

\[
\frac{d\pi_a}{da} = \frac{t}{2} > 0 \\
\frac{d\pi_b}{db} = \frac{t}{2} > 0
\]

**Distribution ii**

For hospital A:

\[
\pi_a = \frac{t}{(-3x^2 + 3x)} \left( -\hat{x}^3 + \frac{3}{2} \hat{x}^2 \right) - \frac{1}{8}
\]

\[
\frac{d\pi_a}{da} = -\frac{t}{2} \left( -\hat{x}^3 (a, b) + \frac{3}{2} \hat{x}^2 (a, b) \right) \left( -6 \hat{x} (a, b) + 3 \right) + \frac{t}{2} > 0
\]

Rewriting and rearranging terms finds

\[
\frac{d\pi_a}{da} = -\frac{t \hat{x}^2 (-\hat{x} + \frac{3}{2} (\frac{1}{2} (1 - b + a)) + 3)}{(3\hat{x})^2 (1 - \hat{x})^2} + \frac{t}{2} = \frac{t}{2} \left( 1 - \frac{2(2 + b - a)(b - a)}{3(1 + b - a)^2} \right) > 0
\]

For hospital B

\[
\pi_b = \frac{t}{(-3x^2 + 3x)} \left( \frac{1}{2} + \hat{x}^3 - \frac{3}{2} \hat{x}^2 \right) - \frac{1}{8}
\]

\[
\frac{d\pi_b}{db} = \frac{t}{2} \left( \frac{1}{2} \hat{x}^3 (a, b) - \frac{3}{2} \hat{x}^2 (a, b) \right) \left( -6 \hat{x} (a, b) + 3 \right) + \frac{t}{2} = \frac{t}{2} \left( \frac{1}{2} \hat{x}^3 (a, b) - \frac{3}{2} \hat{x}^2 (a, b) \right)^2 \left( \frac{3(1 - b + a)}{2} \right)^2 \left( \frac{1}{2} \hat{x}^3 (a, b) + 1 \right)^2 + 1
\]

Computer analysis finds Figure 2 and Figure 3. The figures show that within market boundaries the FOC does not equal zero. Hence the optimal location for both hospitals is to location at the population median.

**Distribution iii**

For hospital A:
For hospital A:

\[
\pi_a = \left( \frac{t}{x^2 + 1} \right) \left( \frac{1}{3} x^3 - \frac{1}{2} x^2 + \hat{x} \right) - \frac{1}{8}
\]

\[
\frac{d\pi_a}{da} = \frac{t}{2} \left( \frac{1}{x^2 + 1} \right) \left( \frac{1}{3} x^3 - \frac{1}{2} x^2 + \hat{x} \right) - \frac{1}{8}
\]

For hospital B:

\[
\pi_b = \left( \frac{t}{x^2 + 1} \right) \left( \frac{5}{6} x^3 + \frac{1}{2} x^2 - \hat{x} \right) - \frac{1}{8}
\]

\[
\frac{d\pi_b}{db} = \frac{t}{2} \left( \frac{5}{6} x^3 + \frac{1}{2} x^2 - \hat{x} \right) - \frac{1}{8}
\]

Computer analysis finds Figure 4 and Figure 5. The figures show that within market boundaries the FOC does not equal zero. Hence the optimal location for both hospitals is to location at the population median.

**Hospital A! distribution iv | optimal location decision**

**Hospital B! Distribution iv | optimal location**

**Figure 4** | The gradient equals zero.

**Figure 5** | The gradient equals zero

**Distribution iv**

**For hospital A:**

\[
\pi_a = \left( \frac{t}{x^2 + 1} \right) \left( \hat{x} - \frac{x^2}{2} \right) - \frac{1}{8} = t(1 - \hat{x})^{-1} \left( \hat{x} - \frac{x^2}{2} \right) - \frac{1}{8}
\]

\[
\frac{d\pi_a}{da} = \frac{t}{2} \left( \frac{1}{(1-x^2)^2} \right) \left( \hat{x} - \frac{x^2}{2} \right) + 1 = 0
\]

\[
\frac{d\pi_a}{da} = \left( \hat{x} - \frac{x^2}{2} \right) = -1 + 2\hat{x} - \hat{x}^2
\]

\[
\frac{x^2}{2} - \hat{x} + 1 > 0
\]

**For hospital B:**
\[
\pi_b = \left( \frac{t}{1-x} \right) \left( \frac{1}{2} - \hat{x} + \frac{x^2}{2} \right) - \frac{1}{8} = t(1-\hat{x})^{-1} \left( \frac{1}{2} - \hat{x} + \frac{x^2}{2} \right) - \frac{1}{8}
\]
\[
\frac{d\pi_b}{db} = t \cdot (1-x)^{-2} \cdot \frac{dx}{db} \left( \frac{1}{2} - \hat{x} + \frac{x^2}{2} \right) + t(1-x)^{-1}(-1+x) \cdot \frac{dx}{db}
\]
\[
\frac{d\pi_b}{db} = -\frac{t(1-x)^2}{4(1-x)^2} + \frac{t(1-x)}{2(1-x)} = -\frac{t}{4} + \frac{t}{2} = \frac{t}{4} > 0
\]

Using the previously derived outcomes yields the location results of table 12.

<table>
<thead>
<tr>
<th>j</th>
<th>i.</th>
<th>ii.</th>
<th>iii.</th>
<th>iv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>(\frac{d\pi_j}{di})</td>
<td>(\frac{p-c}{2}) &gt; 0</td>
<td>(\frac{p-c}{2}) &gt; 0</td>
<td>(\frac{d\pi_a}{da}) &gt; 0</td>
<td>(\frac{d\pi_b}{db}) &gt; 0</td>
</tr>
</tbody>
</table>

Table 12

A.5.1.

For all distributions the following function applies:

\[
W = vN + \theta - t \left( \int_0^a (a-x)y_i(x)dx + \int_a^\hat{x} (x-a)y_i(x)dx + \int_{\hat{x}}^{1-b} (1-b-x)y_i(x)dx \right) + \int_{1-b}^1 (1-x)y_i(x)dx - pN + \pi_a + \pi_b
\]

Finding optimal locations for hospitals in an optimal welfare setting in distribution i

\[
W = v + \theta - t \left( a^2 + x^2 - a\hat{x} - (1-b)\hat{x} + \frac{1}{2} + (1-b)^2 - (1-b) \right) - c - \theta^2
\]

\[
\frac{dW}{da} = -t \left( 2a + 2\hat{x} \frac{dx}{da} - \hat{x} - a \frac{dx}{da} - (1-b) \frac{dx}{da} \right) = 0 \implies \frac{3}{2}a - \frac{1-b}{2} = 0 \implies a = \frac{1+b}{3}
\]

\[
\frac{dW}{db} = -t \left( 2\hat{x} \frac{dx}{db} - a \frac{dx}{db} + \hat{x} - (1-b) \frac{dx}{db} - 2(1-b) + 1 \right) = \frac{a}{2} - \frac{3}{2}(1-b) + 1 = 0
\]

\[
a = \frac{1}{4}
\]

\[
b = \frac{1}{4}; x = 1/2
\]

\[
W = v + \frac{1}{4} - \frac{t}{8} - c
\]

A.5.1.2.
Optimal location from a welfare perspective for distribution $ii$

\[ W = v + \theta - t \left( \int_0^a (a - x)(-3x^2 + 3x)dx + \int_a^x (x - a)(-3x^2 + 3x)dx \right. \]
\[ + \int_0^{1-b} ((1 - b) - x)(-3x^2 + 3x)dx + \int_0^{1-b} (x - (1 - b))(3x^2 + 3x)dx \right) - \frac{c}{2} \]
\[ - \theta^2 \]

Using computer analysis finds as rewritten integral

\[ W = v + \theta - t \left(-\frac{a^4 + a^3 - a}{2} + (1 - b)^4 \right) - \frac{c}{2} \]

Using computer analysis finds:

\[ a \approx 0.32635 \quad x = 1/2 \]
\[ b \approx 0.32635; \quad 1 - b \approx 0.6736 \]

\[ W = \frac{v}{2} + \frac{1}{4} - t \left(-\frac{(0.32635)^4}{2} + (0.32635)^3 - \frac{0.32635}{4} - \frac{(0.67365)^4}{2} + (0.67365)^3 - \frac{3(0.67365)^3}{4} + \frac{13}{32} \right) - \frac{c}{2} = \frac{v}{2} + \frac{1}{4} - 0.0513t - \frac{c}{2} \]

A.5.1.3.

Optimal location from a welfare perspective for distribution $iii$

\[ W = v + \theta - t \left( \int_0^a (a - x)(x^2 - x + 1)dx + \int_a^x (x - a)(x^2 - x + 1)dx \right. \]
\[ + \int_0^{1-b} ((1 - b) - x)(x^2 - x + 1)dx + \int_0^{1-b} (x - (1 - b))(x^2 - x + 1)dx \right) - \frac{5c}{6} \]
\[ - \theta^2 \]

Using computer analysis finds as rewritten integral

\[ dW \over da = -t \left(\frac{2a^3 - a^2 + 2a - 5}{12} \right) = 0 \quad \quad dW \over db = -t \left(\frac{-5}{4} + 2(1 - b) - (1 - b)^2 + \frac{2(1 - b)^3}{3} \right) = 0 \]

Using computer analysis finds:

\[ a \approx 0.23088 \quad x = 1/2 \]
\[ b \approx 0.23088 \quad 1 - b \approx 0.76912 \]

\[ W = v + \theta - t \left(\frac{a^4}{6} - \frac{a^3}{3} + a^2 - \frac{5a}{12} + \frac{(1 - b)^4}{6} - \frac{(1 - b)^3}{3} + \frac{(1 - b)^2}{4} - \frac{5(1 - b)}{4} + \frac{59}{96} \right) - \frac{5c}{6} \]

\[ - \theta^2 \]
\[ W = \frac{5v}{6} + \frac{1}{4} - 0.1049t - \frac{5c}{6} \]

A.5.2. Determining optimal location for distribution iv: taking first order derivatives of \( W \) over \( a \) and \( b \)

\[
W = -t \left( -\frac{1}{6} (a - 3)a^2 - \frac{1}{6} (a - x)^2 (a + 2x - 3) + \frac{1}{6} (b - 2x + 2)(b + x - 1)^2 + \frac{1}{6} b^3 \right)
\]

\[
W = -\frac{1}{6} (a - 3)a^2 \pm \frac{1}{6} \left( a - \left(\frac{1}{2} (1 - b + a)\right) \right)^2 \left( a + 2 \left(\frac{1}{2} (1 - b + a)\right) - 3 \right) + \frac{1}{6} \left( b - 2 \left(\frac{1}{2} (1 - b + a)\right) + 2 \right) \left( b + \left(\frac{1}{2} (1 - b + a)\right) - 1 \right)^2 + \frac{1}{6} b^3
\]

\[
W = -\frac{1}{6} (a - 3)a^2 - \frac{1}{6} \left( a - \frac{1}{2} (1 - b + a) \right)^2 \left( a + (1 - b + a) - 3 \right) + \frac{1}{6} (b - (1 - b + a) + 2) \left( b + \frac{1}{2} (1 - b + a) - 1 \right)^2 + \frac{b^3}{6}
\]

**Solution for hospital A**

\[
\frac{dW}{da} = \frac{1}{8} \left( -7a^2 - 2a(b - 7) + b^2 + 2b - 3 \right) = 0
\]

\[
a = \frac{1}{7} (-2\sqrt{2}b^2 + \sqrt{7} - b + 7) \quad \text{(A.1)}
\]

\[
a = \frac{1}{7} (2\sqrt{2}b^2 + \sqrt{7} - b + 7) \quad \text{(A.2)}
\]

**Solution for hospital B**

\[
\frac{dW}{db} = \frac{1}{6} \left( -a^2 + 2a(b + 1) + 7b^2 - 2b - 1 \right) = 0
\]

\[
b = \frac{1}{7} (-2\sqrt{2}\sqrt{(a^2 - 2a + 1) - a + 1}) \quad \text{B.1}
\]

\[
b = \frac{1}{7} (2\sqrt{2}\sqrt{(a^2 - 2a + 1) - a + 1}) \quad \text{B.2}
\]

**Combining outcomes A.1 and B.2 finds a solution:**

\[
a = \frac{4 - \sqrt{10} + \sqrt{2}}{4}
\]

\[
b = \frac{\sqrt{2} + \sqrt{2}}{4}
\]

\[
x = \frac{2 - \sqrt{1 + \frac{1}{16}}}{2}
\]

\[
1 - b = \frac{4 - \sqrt{2} + \sqrt{2}}{4}
\]

Optimal solutions are slightly altered to the right of where \( N=25\% \) and \( N=75\% \)

\[
W = \frac{v}{2} + \frac{1}{4} - t \left( \int_0^a (a - x)(1 - x) \, dx + \int_a^x (x - a)(1 - x) \, dx + \int_x^b (B - x)(1 - x) \, dx + \int_b^1 (x - B) \, dx \right) - \frac{c}{2}
\]

\[
W = \frac{v}{2} + \frac{1}{4} - t \left( 0.01145 + 0.01313 + 0.01079 + 0.01643 \right) - \frac{c}{2} = \frac{1}{4} - \frac{t}{12} \left( 4 - \sqrt{10} + \sqrt{2} \right) - \frac{c}{2}
\]

A-xiii