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Testing different GARCH models for bull and bear markets

Author: M. K. Jankovic
Student number: 456560mj
Thesis supervisor: Dr. S. van den Hauwe
Second assessor: Dr. J. Kil
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ABSTRACT

Bull and bear phases were detected for three US stock market indices (S&P 500, Dow Jones Industrial Average, Nasdaq) in the period of 1990/01/01-2020/01/01 and their asymmetric characteristics described and measured. The findings show long steady bull phases interrupted by short and significant bear phases and we noted that the longest bull phase was followed by the longest bear phase. A comparison of the ability of symmetric (GARCH) and asymmetric (EGARCH and GJR-GARCH) models to describe the volatility of the detected phases was performed by using the Akaike Information Criterion and the Bayesian Information Criterion. The asymmetric models performed better than the symmetric GARCH model, in particular the EGARCH model was the best model. These results were in line with the findings in the forecast procedure, where the EGARCH model was the best model for the volatility forecast over the entire period whereas the GJR-GARCH model was better for forecasting single phases. In summary, we show evidence of the superiority of asymmetric models when compared to the symmetric GARCH model.

Keywords:
GARCH models, bull/bear, asymmetry, leverage effect, forecast accuracy

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CHAPTER 1 Introduction

There is extensive literature about the estimation and testing procedure of different GARCH models, applied on a broad range of financial data (Christoffersen & Jacobs, 2004; Lamoureux & Lastrapes, 1990; Nelson & Cao, 1992). It is the aim of this paper to test different GARCH models for bull and bear market regimes and to evaluate their forecasting accuracy. Bull (bear) markets are commonly described in the financial literature as phases where market prices are rising (falling) (Chauvet & Potter, 2000). The detection of bull and bear phases does not follow a strict procedure what leaves options open for researchers. In our work we follow the approach of Pagan and Sossounov (2003), but we investigate a more current time period. Furthermore, we add an extension for the censoring procedure of phases, following the methodology of Gil-Alana et al. (2014), in order to remove short and insignificant phases.

The accurate detection of bull and bear phases is important for investors and risk managers. Firstly, as stated by Kole and Van Dijk (2017), adequate information of the past, current and future state of the economy is crucial as market regimes have an influence on asset prices. Secondly, the modelling and forecasting of the time-varying volatility of financial time-series is important to obtain information for accurate pricing and risk management (Hansen & Lunde, 2001). Therefore, a broad range of different models were developed by researchers and practitioners to describe and forecast volatility, where the first model was the ARCH model of Engle (1982). The GARCH model is a generalization of the ARCH model, introduced by Bollerslev (1986).

We can distinguish between symmetric and asymmetric GARCH models: in the symmetric models positive and negative returns are treated equally, in asymmetric models the value of the volatility depends on the sign of the return (Christoffersen, 2012). There is no consensus in the literature if asymmetric GARCH models are able to overperform symmetric GARCH models to describe volatility in the markets. To my best understanding there is no literature that explicitly studies the performance of the selected symmetric and asymmetric GARCH models for bull and bear market phases. Gil-Alana et al. (2014) present a similar research approach but just for the simple GARCH model and present bull and bear phases for a shorter time horizon. Ding and Meade (2010) measured forecast accuracy but not for asymmetric GARCH models. This thesis is adding value to the existing literature, in particular for investors and risk managers, through connecting characteristics of bull and bear phases with a testing procedure of GARCH model extensions for different volatility regimes. This leads to the following main research question for this thesis:

How do bull and bear phases differ and which GARCH model (we consider GARCH, EGARCH, GJR-GARCH) performs best during bull and bear phases?

Therefore, we aim in the first part of this thesis to provide the reader with the right measures to detect the specific nature of bull and bear market phases and to take into consideration these measures for the model selection. In the second part, after detection and analysis of the phases in our dataset, we will answer the main research question by testing the models in two ways: first, we will estimate the model parameters and assess which model best describes the data. Secondly, we will perform a forecast procedure to compare the ability of the models to predict volatility.

The remainder of this paper is structured as follows. In Section 2 the relevant literature is discussed, Section 3 describes the data and in Section 4 the methodology is outlined. In Section 5 results will be presented with concluding remarks in Section 6.

CHAPTER 2 Literature Review

In the first subsection of this chapter, the literature is systematically reviewed for methods how bull and bear markets can be detected and how the specific nature of bull and bear phases is described in the literature. In the second subsection of this chapter the literature for GARCH models is presented. In the third subsection further literature is provided that supports the development of the main hypothesis.

2.1 Bull and bear markets in the literature

It has been discussed by a great number of authors that there is not a single method how to detect bull and bear market regimes (Pagan & Sossounov, 2003). One of the commonly used methods is to apply algorithms consisting of a set of rules for the analysis of time series. Bry and Boschan (1971) presented in their ground-breaking work of the cyclical analysis of time series a procedure for the programmed determination of turning points of business cycles. The algorithm is a pattern-recognition program consisting of a sequence of steps. The algorithm was applied in academia by many other researchers including Cashin et al., (2002) or Harding and Pagan (2002). Given that the Bry and Boschan algorithm does not specifically address asset prices, Pagan and Sossounov (2003) stated that for the analysis of asset prices a modified version is required. Therefore, Pagan and Sossounov (2003) presented a similar method where they developed a simple framework for the detection of bull and bear markets with a sequence of steps including requirements for the duration of the phases and for the price changes. One of the deviations from the Bry and Boschan algorithm is that the data is not smoothed. This decision was motivated by the fact that excluding outliers in asset prices could prevent the detection of relevant stock market events.

Lunde and Timmermann (2004) developed an alternative algorithm which only considers requirements for the price changes by introducing a binary variable that takes the value 1 during a bull market and 0 for a bear market. One of the shortcomings of this algorithm is that the analyst must be informed about the initial state of the market.

Hamilton (1989) presented a two-state Markov-switching model that he applied to U.S.GNP growth rates for the detection of business cycles and the transition probabilities between the two states, corresponding to bull and bear markets.

The various methods to detect bull and bear market regimes were broadly applied by researchers in the financial literature. Gil-Alana et al. (2014) detected bull and bear phases with a method similar to the framework of Pagan and Sossounov (2003). An important finding of Gil-Alana et al. (2014) was that many short and irrelevant phases were detected where the percentual increase (for bull) or decrease (for bear) during the phase was smaller than a specific threshold. The authors addressed this problem by eliminating all cycles with insignificant peaks and troughs and achieved consistent results for the detected bull and bear periods for different stock market indices.

There exists a considerable body of literature on the nature and specific characteristics of bull and bear market phases (Angelidis et al., 2015; Gonzalez et al., 2005). Pagan and Sossounov (2003) summarized the characteristics of the phases with specific measures such as amplitude, duration or the shape of the phases. Some of the widespread findings in the literature are that bull phases are longer than bear phases (Pagan & Sossounov, 2003) and that in bear periods the volatility of stock returns is higher than in bull periods (Kole & Van Dijk, 2017). There is also extensive research about the impact of news on volatility (Engle & Ng, 1993) and how the reaction to good and bad news differs with respect to bull and bear phases (Below & Johnson, 1996).

2.2 GARCH models in the literature

A feature of financial time series data is, that the volatility is time-varying, also described in the literature by the term heteroskedasticity (Engle, 2001). The first model able to capture this feature was the autoregressive conditional heteroskedasticity (ARCH) model, introduced by Engle (1982), in which the volatility at time t+1 is estimated using past information. The GARCH model is a generalization of the ARCH model and was developed by Bollerslev (1986). The GARCH model describes the conditional variance as a function of lagged squared logarithmic returns and past values of the conditional variance. The GARCH model of Bollerslev (1986) has been discussed and applied by a great number of authors in literature (Engle & Patton, 2007; Katsiampa, 2017; Oberholzer & Venter, 2015) and many new models were developed as listed in the glossary of Bollerslev (2008).

The vast number of modifications of the traditional GARCH model were developed with the aim to overperform the traditional GARCH model. One shortcoming of the GARCH model is that it cannot capture asymmetric effects or the leverage effect in financial time series. The leverage effect was introduced to the literature by Black (1976) and states that there is negative correlation between volatility and returns, meaning that there will be an increase in volatility after bad news and a decrease in volatility after good news.

Seminal contributions for asymmetric GARCH models have been made in the literature with the development of the EGARCH model by Nelson (1991) and the GJR-GARCH model by Glosten et al. (1993). Both models are extensions of the traditional GARCH model and are able to capture the leverage effect in financial time series data. Hansen and Lunde (2001) compared the performance of a large number of volatility models, including the EGARCH and GJR-GARCH model, with the objective to analyse whether more sophisticated models can improve the forecast accuracy of volatility in comparison to the simple GARCH model. It was surprising for the authors, that they could not find evidence that the simple GARCH model was outperformed by other models, given that they estimated over 300 different models and that the simple GARCH model is not able to capture the previously described leverage effect.

2.3 Further literature with subsequent hypothesis development

As previously stated, many papers found evidence that stock market volatility during bear markets is higher than during bull markets (Maheu & McCurdy, 2000) and bull markets tend to last longer than bear markets (Pagan & Sossounov, 2003). In the literature, stylized facts of asset returns are widely discussed with regards to GARCH models and with regards to the question how these models are able to describe these stylized facts of time series (Malmsten & Teräsvirta, 2004). As stated by Christoffersen (2012) one of these stylized facts is the leverage effect for equity and equity indices. The EGARCH and the GJR-GARCH model are both able to capture this effect through assigning weights to the returns dependent if the returns are positive or negative.

There is evidence in research that the impact of news is perceived differently during bull and bear markets. Kurov (2010) analysed the impact of monetary policy shocks on investor sentiment separately for bull and bear markets. He states that the reaction to new information is related to the market conditions and that for specific stocks the reaction to news is stronger during bear markets than during bull markets. Lim and Sek (2013) investigated different periods of the financial crisis and found evidence that asymmetric models are in comparison to the simple GARCH model better in capturing stock market volatility during periods with high fluctuation (crisis period and recovery period). This finding is supported by a study of El Jebari and Hakmaoui (2018), that find evidence that an asymmetric model overperformed other models, including the simple GARCH model, in the analysis of the Moroccan stock-market index.

As previously stated, other researchers show that more evolved models are not able to overperform the simple GARCH model (Hanse & Lunde, 2001). We conclude that there is no consensus in the literature which GARCH model performs best and which model to use for bull and bear phases. Therefore, we can formulate the following hypothesis that we aim to confirm or reject.

Hypothesis: The symmetric GARCH model is able to describe well the market volatility for bull and bear phases and more evolved asymmetric models do not deliver better results.

CHAPTER 3 Data

3.1 Data selection

The focus in this paper lies on the US market with a dataset consisting of major US stock indices, namely the S&P500 (^GSPC), the Dow Jones Industrial Average (^DJI) and the Nasdaq Composite (^IXIC). The choice of these stock market indices is motivated by the fact that all three indices are leading indicators for the US economy. With 30, 500 and over 2500 constituents for the DJI, S&P500 and the Nasdaq, respectively, each index represents a different range of constituents. From a historical perspective it is interesting to note that the DJI was founded over 100 years ago by Charles Dow, who invented the Dow Theory that aimed to divide the market into different phases (Dagnino, 2013; Hamilton, 1919; Pagan & Sossounov, 2003). The research data in this thesis is retrieved from YAHOO Finance for the period of 1990/01/01 – 2020/01/01. For the manual detection of bull and bear phases monthly open prices of the three indices are used. For the detection of bull and bear phases, that is performed in R, monthly close prices are used. For the remaining calculations, including descriptive statistics, estimation of GARCH model parameters, and testing of the forecasting accuracy, the dataset consists of daily close prices. The raw data series is transformed into a series of logarithmic returns. This step is explained in more detail in the methodology section.

As bull and bear phases are normally periods of several months (Pagan & Sossounov, 2003), we decided to select a time span of 30 years to ensure that there are enough data observations to detect bull and bear phases that we consider as significant, especially because we do also apply censoring operations where insignificant periods are deleted. Furthermore, it was an important selection criterion to include recent data, as bull and bear phase characteristics change over time (Pagan & Sossounov, 2003), in particular in the past the phases were shorter and alternated with a higher frequency. An advantage of the selected time period is that the financial crisis of the years 2007/08, that produced high levels of volatility in the stock market, as described by Schwert (2011), is entirely covered by the data. Charles and Darné (2014) investigated large volatility shocks for the DJI in the period of 1928-2013, where they also detected the terrorist attack in September as major shock, and found evidence that the time of the financial crisis and the following recession showed similar characteristics as the recession from 1929-1934 with high volatility levels for a long period of time. A further important event that motivates us to select this time period is the inclusion of the dot-com bubble in the data, where the Nasdaq reached a bear-market low in 2002 after a fall of over 70 percent from the peak in 2000.

3.2 Descriptive statistics

In Table 1 we present summary statistics of the daily log-returns of the three stock indices for the entire sample period of 1990/01/01-2020/01/01, and for the three longest bull and bear phases of each index (detected in R). When looking at the entire period, we have a total number of 7558 observations per index. The number of daily observations for the three longest bull phases are 1535, 1367, and 1409, for the S&P 500, the DJI, and the Nasdaq, respectively. Given the number of observations, we note that the three longest bear phases are shorter than the three longest bull phases, with 498, 667, and 627 daily observations for the S&P500, the DJI, and the Nasdaq, respectively. The mean of the log-returns is around zero, but we can observe small positive means for bull phases and small negative means for bear phases. Additionally, one sees that the standard deviations are higher in bear phases compared to bull phases. In the data we can observe that the longest bear phase of the S&P500 has the lowest range value, compared to all other range values. We do also see that for the Nasdaq the range value in the longest bear phase (2000/03/01-2002/09/01) coincides with the range value of the entire period. This particular phase is related to the previously mentioned dot-com bubble, that peaked in 2000. From the mostly negative skewness in the data, we can see that we have long left tails and asymmetric returns.

Table 1: Summary statistics for the S&P 500, DJI and Nasdaq for 1990/01/01-2020/01/01 and tests for normality and stationarity

Index	n Obs.	Mean	St. Dev.	Min.	Max.	Range	Skew.	Kurt.	JB[1] Value	JB p-value	DF[2] Value	ADF[3] p-value
S&P	7558	0.0003	0.0110	-0.0947	0.1096	0.2043	-0.2672	8.8534	24791	<0.01	-19.7550	<0.01
DJI	7558	0.0003	0.0105	-0.0820	0.1051	0.1871	-0.1950	8.2067	21273	<0.01	-20.1070	<0.01
Nasdaq	7558	0.0004	0.0143	-0.1017	0.1325	0.2342	-0.1158	6.4794	13248	<0.01	-18.6020	<0.01
S&P (Bear $_{longest}$) [4]	498	-0.0010	0.0143	-0.0505	0.0557	0.1062	0.3126	1.2539	42	<0.01	-8.0557	<0.01
S&P (Bull $_{longest}$) [5]	1535	0.0008	0.0103	-0.0711	0.0499	0.1210	-0.4506	5.1270	1740	<0.01	-11.9240	<0.01
DJI (Bear $_{longest}$) [6]	667	-0.0004	0.0137	-0.0740	0.0615	0.1355	-0.0985	2.6389	197	<0.01	-8.5345	<0.01
DJI (Bull $_{longest}$) [7]	1367	0.0008	0.0096	-0.0745	0.0486	0.1231	-0.5936	5.8621	2047	<0.01	-10.8280	<0.01
Nasdaq (Bear $_{longest}$) [8]	627	-0.0021	0.0275	-0.1017	0.1325	0.2342	0.3434	1.3510	61	<0.01	-8.5405	<0.01
Nasdaq (Bull $_{longest}$) [9]	1409	0.0012	0.0131	-0.0895	0.0585	0.1480	-0.5976	3.9766	1017	<0.01	-10.8750	<0.01

Note: 1) JB: Jarque-Bera, 2) Dickey-Fuller, 3) Augmented Dickey-Fuller, 4) 2000/09/01-2002/09/01, 5) 1994/07/01-2000/08/01, 6) 2000/01/01-2002/09/01, 7) 1994/07/01-1999/12/01, 8) 2000/03/01-2002/09/01, 9) 1994/07/01-2000/02/01.

The results of the Jarque-Bera test (Jarque & Bera, 1980) in Table 1 show us for all analyzed data subsamples that we can reject the null hypothesis of normality on the level of 1 percent. Testing the data for stationarity, the Augmented Dickey-Fuller test (Dickey & Fuller, 1979) shows us for all data subsamples that we can reject the null hypothesis of unit root on the level of 1 percent, meaning that the data is stationary.

When we look at the plotted log return series of the three indices in Figure 1, we can see that periods of high volatility are followed by periods of low volatility, what is described by financial analysts as volatility clustering (Engle, 2001).

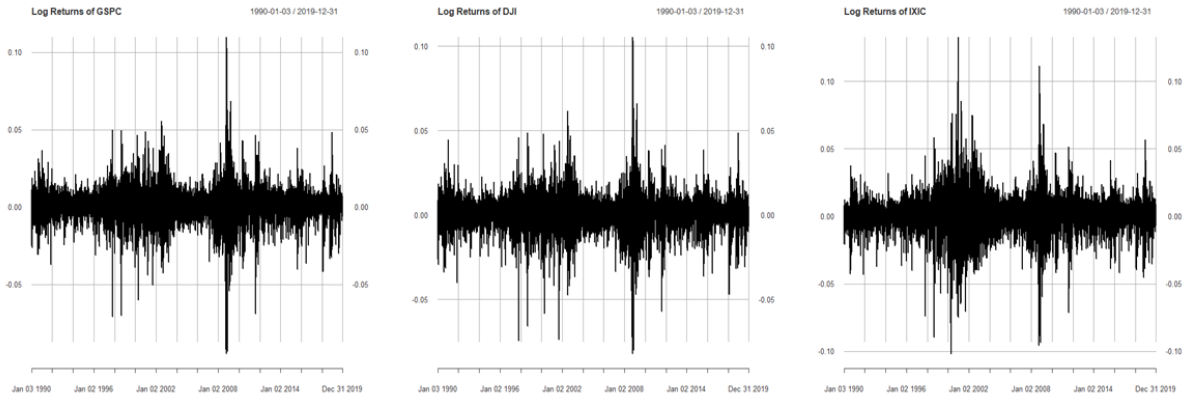

Figure 1: Daily observations of log returns from 1990/01/01-2020/01/01 for the S&P 500, the DJI and the Nasdaq

We can also see in Figure 1 random fluctuation around zero for all three indices. Therefore, it appears that there is little or no autocorrelation, what can be shown with the autocorrelation functions (ACF) of the log returns in Figure 2, plotted for the three indices up to 35 lags. The plots show little autocorrelation and we do not see a systematic pattern. Autocorrelation is defined as the correlation between the data set and the same data set lagged by t days. The null hypothesis of an autocorrelation coefficient equal to zero is rejected if the sample autocorrelations are outside the dashed bounds (lower and upper bound for the 95% confidence interval) in the figure. In the ACF plots of the squared log returns in Figure 3, we observe positive autocorrelations that are just slowly decaying towards zero. This last observation gives us further evidence of volatility clustering and is therefore important for our work with GARCH models.

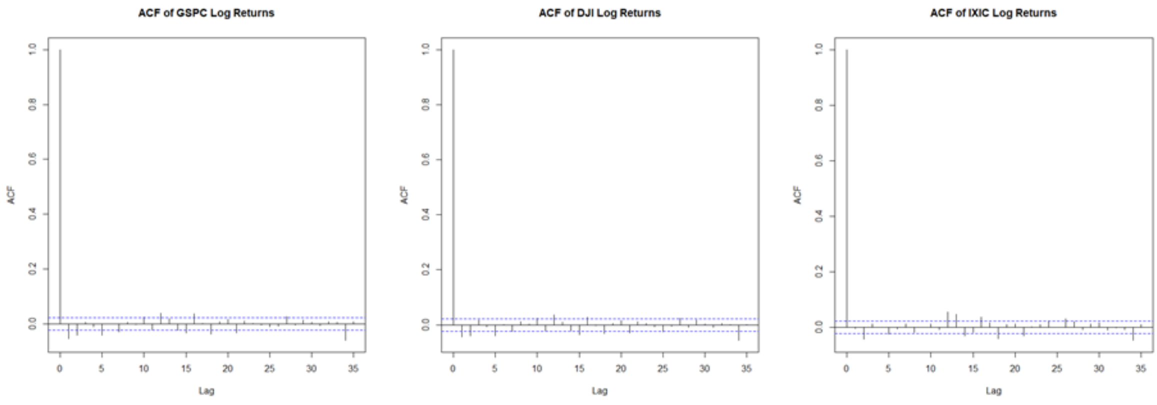

Figure 2: ACF plots for log returns from 1990/01/01-2020/01/01 for the S&P 500, the DJI, and the Nasdaq
Note: The dotted lines are the limits of the 95% confidence interval. Lags between 0 and 35 days.

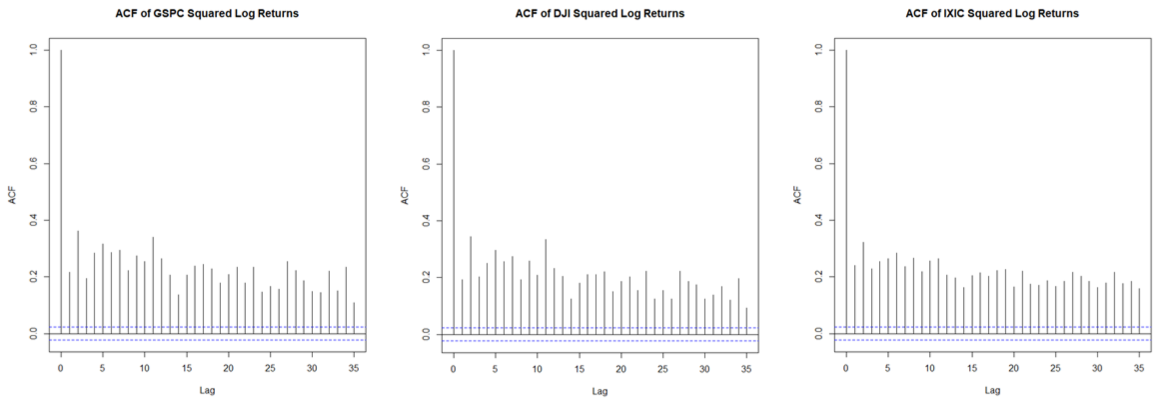

Figure 3: ACF plots for squared log-returns from 1990/01/01-2020/01/01 for the S&P 500, the DJI, and the Nasdaq

Note: The dotted lines are the limits of the 95% confidence interval. Lags between 0 and 35 days.

CHAPTER 4 Methodology

This section outlines the methods applied in this thesis to detect and describe bull and bear markets and provides the reader with an introduction to the GARCH models used for the prediction of volatility.

4.1 Bull and bear markets

To follow the definition of Chauvet and Potter (2000) there are extensive periods of time when equity prices are rising and falling. These two distinct phases can be described by following two market states: bull and bear. Furthermore, the authors state that bear markets are connected with negative excess return, what is a relevant finding for investors and financial risk managers. Other definitions of bull (bear) markets require that the rise (fall) during a market period is higher than a specific threshold, in general 20-25 percent, in order to obtain the label bull (bear) market or lasts for a certain period of time (Gil Alana et al., 2014; Pagan & Sossounov, 2003). The resulting problem is, that there is no consensus amongst researchers and practitioners how to date bull and bear markets. As stated in the literature review, Bry and Boschan (1971) developed an algorithm for the detection of turning points in the business cycle. The algorithm consists of several steps, where the detection of initial turning points is followed by censoring operations. Beside a duration of at least 15 months for cycles (starting at the peak, passing the trough and ending at the next peak) the algorithm sets the duration of a phase to a minimum of 5 months. Pagan and Sossounov (2003) and Gonzalez et al. (2005) adapted this algorithm for dating bull and bear markets.

4.1.1 Bull and bear market detection (manually)

In this thesis the methodology of Pagan and Sossounov (2003) is applied for the detection of bull and bear markets. Following the procedure of Gil-Alana et al. (2014) we add a further constraint (step 7) to prevent the detection of not significant short phases. As we aim to perform GARCH modelling of these bull and bear phases, the sample size should be large enough: the rule of thumb given by Christofferson (2012) is to use around 1000 daily observations for the estimation process. Nevertheless, given the specific nature of bear phases, to be of short duration, we are not able to meet this condition and are therefore detecting the three longest bear phases in R for the entire period beside the manual detection procedure.

Algorithms are presented in the business cycle literature as a set of rules (Bry and Boschan, 1971) and we will also present the steps in the same manner for our manual detection procedure of bull and bear phases. The set is divided in two parts, A and B. In part A the initial turning points are detected, whereas part B is a censoring procedure, in which not significant phases are eliminated. In this thesis the manual detection of bull and bear phases was performed across the three indices, S&P500, DJI and Nasdaq, for the period from January 1990 till December 2019.

The applied set of rules for the detection of bull and bear phases, retrieved from Pagan and Sossounov (2003) except for step 7, that is based on Gil-Alana et al. (2014), is as following:

A. Determination of initial turning points

1. Determination of initial turning points by going through every data point S_t (monthly open index prices) with a window of eight months on each side (from S_{t-8} to S_{t+8}) and defining a local peak (trough) in case S_t is the highest (lowest) value in the window
2. If with the previous rule consecutive peaks (troughs) are detected, selection of the highest peak (lowest trough) amongst them

B. Censoring operations

3. Elimination of turns in the first and last six months of the data
4. Elimination of peaks (troughs) at both tails of the data set that are lower (higher) than the values in the tails
5. Elimination of cycles that are shorter than 16 months
6. Elimination of phases that are shorter than 4 months except the rise/fall over the phase exceeds a threshold of 20%
7. Elimination of two successive troughs and peaks if the fall/rise between them is below 20%
8. Listing of final turning points.

4.1.2 Bull and bear market detection (in R)

The statistical software R was also used to detect bear and bull phases with the aim to find the three longest bull and bear phases during the investigated period. As input values for the program (package: bbdetection) following parameters have to be specified: window (τ_{window}), elimination of peaks and troughs (τ_{censor}), minimal cycle length (τ_{cycle}), minimal phase duration (τ_{phase}), threshold rise/fall (θ) for short phases. All measures are expressed in months except for the threshold that is a percentage value. To be able to compare the manually detected phases and the phases detected in R, we use for the statistical program the same values as proposed in the study of Pagan and Sossounov (2003) and in the manual set of rules, meaning: $\tau\_{window} = 8$; $\tau\_{censor} = 6$; $\tau\_{cycle} = 16$; $\tau\_{phase} = 4$; $\theta = 20$. In contrast to the manual detection procedure, the statistical software does not censor phases with an amplitude smaller than 20% (rule 7 in our set of rules). We note, if we slightly change the values of the censoring parameters (e.g. τ_{window}), the censoring procedure delivers similar final turning points.

4.1.3 Nature of bull and bear markets

Measures from the study of Pagan and Sossounov (2003) are presented below and will be used to test if there is asymmetry in the nature of bull and bear phases. Furthermore, a new measure is introduced that describes the steepness of bull and bear phases. The following formulas are defined for bear phases but are also valid for bull phases, by simply replacing the word bear with bull (if not differently stated).

D_{bear}: Average duration of bear phases, in months. The sum of the duration of all bear phases is divided by the number of bear phases as stated by following expression

$$D_{bear} = \frac{\sum_{i=1}^{n_{bear}} d_i}{n_{bear}} \tag{1}$$

where n_{bear} is the number of bear phases and d_i is the duration of the i-th bear phase (in months).

A_{bear}: Average amplitude of bear phases, in percentage. The sum of the amplitudes of all bear phases is divided by the number of bear phases, as illustrated by following formula

$$A_{bear} = \frac{\sum_{i=1}^{n_{bear}} a_i}{n_{bear}} \tag{2}$$

where a_i is the amplitude (in percentage) of the i-th bear phase, that is $a_i = \ln(S_{end}) - \ln(S_{start})$ for each phase and $S_{end(start)}$ denotes the index price at the end (beginning) of the phase.

G_{bear}: Average slope during bear phases, that is the average change in amplitude during 1 month, defined as the average amplitude divided by the average duration. Thus,

$$G_{bear} = \frac{A_{bear}}{D_{bear}} \tag{3}$$

where G_{bear} is measured in percent per month and describes the steepness of the phases, that is their gradient.

$B_{bear\ (bull)}$: The percentage of bear (bull) phases with an amplitude smaller (larger) or equal to -0.2 (0.2), corresponding to a decrease (increase) of at least 20 percent.

C_{bear}: Average cumulated amplitude during bear phases, defined as the average over all bear phases of the cumulated difference between the monthly $\log(S_t)$ and its value at the beginning of the phase.

Thus,

$$C_{bear} = \frac{\sum_{i=1}^{n_{bear}} (\sum_{k=1}^{d_i} log(S_k^i) - \log(S_1^i))}{n_{bear}} \quad (4)$$

where S_k^i is the index price in the k-th month of the i-th bear phase (analogously S_1^i is the index price at the beginning of the i-th bear phase). We can interpret C_{bear} as the average of the black shaded areas in Figure 4 (and C_{bull} as the average of the grey shaded areas).

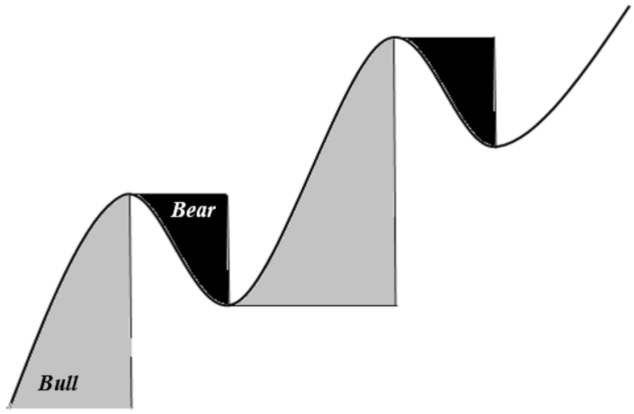

Figure 4: Schematic illustration of the C_{bear} and C_{bull} measures
Note: Own illustration.

4.2 GARCH models

As shown in the data analysis financial time series exhibit volatility clustering, meaning that periods of high volatility (high market uncertainty) are followed by periods of low volatility (low market uncertainty) (Christoffersen, 2012). Furthermore, an asymmetry between bull and bear phases is observed, in particular in our data set bull phases are much longer than bear phases but bear phases are steeper. ARCH and GARCH models are developed to capture time-changing volatility. Estimations and forecasts of conditional variance can be performed by many different GARCH models. In our thesis we consider first the standard GARCH model of Bollerslev (1986), that does not include asymmetry in the conditional volatility equation, and secondly we consider two models that both are able to describe asymmetry, namely the EGARCH model by Nelson (1991) and the GJR-GARCH model by Glosten et al. (1993). We aim to test these models in particular to examine if the asymmetric models (and which of them) significantly enhance the volatility estimation as they are possibly able to capture the bull and bear features, as both models include the leverage effect.

In this thesis we will test models that allow only for one lag, meaning the GARCH (1,1), the EGARCH (1,1) and the GJR-GARCH (1,1) models. This decision is supported by a paper of Angabini and Wasiuzzaman (2011), that found no evidence, that models with higher lags overperform models with one lag.

The following five subsections (4.2.1-4.2.5) follow the notation and theoretical explanation outlined in Christoffersen (2012).

4.2.1 General assumptions

As financial time series often exhibit non-stationarity, we can use the relative changes in the prices for statistical analysis. In this thesis we will use the daily log return of the asset which is defined by

$$R_{t+1} = \ln(S_{t+1}) - \ln(S_t) = \ln\left(\frac{S_{t+1}}{S_t}\right) \tag{5}$$

where S_t is the index price at time t. We note that for small relative changes in the prices, the log return is approximately equal to the percentual change of the prices.

The generic form of our model of the daily log returns is

$$R_{t+1} = \mu_{t+1} + \sigma_{t+1} z_{t+1}, with\ z_{t+1} \sim i.i.d. D(0,1) \tag{6}$$

where μ_{t+1} is the conditional mean (in this thesis assumed to be zero as it is statistically not possible to reject the assumption that the conditional mean is equal to zero), σ_{t+1} is the conditional standard deviation and z_{t+1} is an independently and identically distributed (i.i.d.) innovation term and follows a distribution with mean equal to zero and variance equal to one. In this thesis we assume that z_{t+1} is normally distributed with mean equal to zero and variance equal to one. Based on the previously mentioned assumptions we can rewrite the equation for our model of the daily log returns as

$$R_{t+1} = \sigma_{t+1} z_{t+1}, with\ z_{t+1} \sim i.i.d. N(0,1). \tag{7}$$

We then need to build a model for the time-varying variance, σ_{t+1}^2. We consider three different models, GARCH, GJR-GARCH and EGARCH.

4.2.2 Basic GARCH model

The GARCH (Generalized Auto Regressive Conditional Heteroskedasticity) model of Bollerslev (1986) is defined as

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta \sigma_t^2. \tag{8}$$

The parameters alpha (α), beta (β) and omega (ω) need to fulfill the conditions $\alpha + \beta < 1$ and $\omega > 0$ in order to assure that the long-run average variance (unconditional variance) is well defined in the model, as the unconditional variance σ^2 in this model is given by

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta}. \tag{9}$$

Furthermore, the model is well defined if the conditions $\alpha \geq 0$ and $\beta \geq 0$ hold, since the right-hand side of the model equation has to be positive. Note that this form of the model is often referred to as GARCH (1,1). The first value in parenthesis describes the number of autoregressive lags and the second value the number of moving average lags (Engle, 2001). The sum of the parameters $\alpha + \beta$ is the persistence of the GARCH model. A large persistence means that when the variance deviates from its long run average, this deviation will last for a long time.

Given a time series of returns R_t, we take as starting value σ_1^2 the variance of the whole returns data set. We can then calculate the variance σ_2^2 inserting the first return observation R_1 and the calculated variance σ_1^2 in the model equation. With this procedure, the variance estimates σ_t^2 for the whole data set can be determined. To find the best values for the parameters (α, β, ω), maximum likelihood estimation is used.

We note that the basic GARCH model is a symmetric model, where positive and negative logarithmic returns R_t are treated in the exact same way, as only R_t^2 enters the formula, meaning that the variance estimate σ_{t+1}^2 does not depend on the sign of R_t. Therefore, this model is not capable to describe the leverage effect, where negative logarithmic returns increase the variance more than positive logarithmic returns of the same magnitude. To include this effect, we consider other models, where the value of σ_{t+1}^2 depends on the sign of the logarithmic return R_t.

4.2.3 GJR-GARCH model

In the Glosten-Jagannathan-Runkle-GARCH (GJR-GARCH) model, introduced by Glosten et al. (1993), the value of the conditional variance is sensitive to the sign of the past returns. The GJR-GARCH model is defined by

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \gamma I_t R_t^2 + \beta \sigma_t^2 \tag{10}$$

where I_t is an indicator variable that is equal to 1 if the logarithmic return R_t is negative, otherwise I_t is 0. The term with I_t introduces asymmetry and the model captures the leverage effect when the estimated value for the parameter gamma (γ) is larger than zero ($\gamma > 0$). This implies that negative returns cause a larger variance.

If the conditions $\omega > 0$, $\frac{\alpha+\gamma}{2} > 0$ and $\beta > 0$ hold, the variance σ_{t+1}^2 is positive. Furthermore, the condition $\frac{\alpha+\gamma}{2} + \beta < 1$ assures that the long-run variance σ^2 in this model

$$\sigma^2 = \frac{\omega}{1 - \frac{\alpha+\gamma}{2} - \beta} \tag{11}$$

is well defined. As for the GARCH model, the parameter estimates $(\alpha, \beta, \gamma, \omega)$ are obtained by the maximum likelihood method.

4.2.4 EGARCH model

The Exponential GARCH (EGARCH) model, developed by Nelson (1991), is another model able to capture asymmetry in the relation between return and volatility. The model is defined by following equation

$$\log(\sigma_{t+1}^2) = \omega + \alpha\left(\left|\frac{R_t}{\sigma_t}\right| - E\left[\left|\frac{R_t}{\sigma_t}\right|\right]\right) + \gamma\frac{R_t}{\sigma_t} + \beta\log\left(\sigma_t^2\right) \tag{12}$$

where $E\left[\left|\frac{R_t}{\sigma_t}\right|\right]$ is the expected value of the absolute value of $\frac{R_t}{\sigma_t}$. The parameters do not have to be positive, since the dependent variable $\log(\sigma_{t+1}^2)$ can also be negative due to the logarithmic function. The parameter γ describes the asymmetry of the model. The leverage effect is captured if $\gamma < 0$, resulting in a large variance when the returns are negative. On the other hand, if $\gamma > 0$, the variance is large when the returns are positive. As for the previous models, estimations for the parameters $(\alpha, \beta, \gamma, \omega)$ are obtained by using the maximum likelihood method.

4.2.5 Parameter estimation with maximum likelihood

As previously stated, we assumed that the innovation terms z_{t+1} are standard normally distributed and therefore the returns R_t follow a normal distribution with variance σ_t^2 (we assumed zero mean, resulting in the equation $R_{t+1} = \sigma_{t+1}z_{t+1}$).

The likelihood (l_t) of R_t is then given by the normal distribution

$$l_t = \frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(-\frac{R_t^2}{2\sigma_t^2}\right) \tag{13}$$

and the total likelihood (L) for the whole data set is the product

$$L = \prod_{t=1}^{T} l_t = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{R_t^2}{2\sigma_t^2}\right). \quad (14)$$

The likelihood is a function of the parameters of our models, since the estimated variances σ_t^2 are calculated using the model equation. We estimate the optimal values of the parameters by maximizing the total likelihood. For simplicity, we take the logarithm of the likelihood function log(L) to calculate with sums instead of products. Note that since the logarithm function is monotonically increasing, the set of parameters which maximize log(L) coincides with the set of parameters maximizing L. Our task is thus to maximize

$$Max \ln L = Max \sum_{t=1}^{T} \ln(l_t) = Max \sum_{t=1}^{T} \left[-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{R_t^2}{\sigma_t^2}\right]. \quad (15)$$

Since the first term $-\frac{1}{2}\ln(2\pi)$ is constant, we can neglect it for the maximization process and only consider

$$Max \sum_{t=1}^{T} \left[-\frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{R_t^2}{\sigma_t^2}\right] = Max \left[-\frac{1}{2}\left(\sum_{t=1}^{T} \ln(\sigma_t^2) + \frac{R_t^2}{\sigma_t^2}\right)\right]. \quad (16)$$

We maximize this function using numerical methods. With an increasing number of data ($T \rightarrow \infty$), the estimated parameters converge towards their true values and their variance decreases to zero (Kiefer & Wolfowitz, 1956). We note that these numerical methods are not always unproblematic, depending on the number of parameters. The numerical methods require initial values for the parameters and the maximization process is dependent on these initial values, since the algorithm is searching for the maximum of the function starting from there. Therefore, the obtained set of optimal parameters can depend on the initial values. In a worst-case scenario, the numerical method would only find a local maximum of the function and not the global maximum. This problem is getting worse when the number of parameters in the model increases.

4.2.6 Model comparison with AIC and BIC

In order to find the model that best fit our data we need a criterium to assess model fit, which allows to compare models with different number of parameters (GARCH has three parameters, EGARCH and GJR-GARCH four parameters). With "best fit" we mean a model that describes the key features of the data with the smallest possible number of parameters. In particular we want to avoid overfitting, in which the fit improves only because of an increasing number of unjustified parameters.

Following Burnham and Anderson (2004), we use for the model comparison the Akaike Information Criterion (AIC), that is defined by

$$AIC = 2k - 2\log(L) \tag{17}$$

and the Bayesian Information Criterion (BIC), that is defined by

$$BIC = \log(T)\, k - 2\log(L) \tag{18}$$

where k is the number of parameters, L is the maximal total likelihood of the data set, and T is the sample size.

We note that when the number of parameters k is too small to fully describe the data set, the likelihood L is small and thus AIC/BIC is dominated by the logarithmic term, resulting in a large value of AIC/BIC. On the other hand, if the number of parameters is large, AIC/BIC is dominated by the linear term 2k, also leading to a large value for AIC/BIC. Between these two limits AIC/BIC has a minimum, corresponding to a model with an optimal number of parameters. With these criteria we can make a comparison between our models and select the best model by looking at which of them has the lowest AIC/BIC values. As stated by Burnham and Anderson (2004), we are aware that with these criteria we cannot make any statement about the absolute quality of the models, we can only do a comparison between them. Therefore, the best model detected by us is not necessarily the best of all possible models.

4.2.7 Forecasting procedure

We perform a forecast procedure in order to investigate, for how long in the future the models are able to predict volatility, in particular we are interested in how many months ahead and how accurate a model with a fixed set of parameters is able to produce useful volatility estimates. More precisely, we perform an out-of-sample-forecast without re-estimation of the model parameters, meaning that we estimate the parameters a single time using the entire data set without the last 100 (80, 60, 40, 20) days. Then the obtained set of parameters is used to estimate all volatilities, σ_t, for the last 100 (80, 60, 40, 20) days. Some parameter sets and estimated volatilities are presented in Appendix 2.

In the forecast procedure, we follow the methodology of Ding and Meade (2010) and use the root-mean-square error (RMSE) to measure the differences between the values predicted by our models and the observed data. Given that volatility is not observable, we compare the standard deviation of the observed logarithmic returns R_t with the square root of the mean of the estimated variances σ_t^2 over a month. We remember that a month is defined as period of 20 consecutive weekdays.

We compare the RMSE values for the forecasts for 1-4 months ahead for each phase separately (three longest bull and bear phases). The RMSE, as defined by Ding and Meade (2010), for h months ahead (h=1, 2, 3, 4) is calculated for the out-of-sample forecast data as

$$RMSE_{h\,months\,ahead} = \sqrt{\frac{1}{6-h}\sum_{m=1}^{6-h}\left[SD(r_t|t\epsilon A,B) - \sqrt{\text{Mean}(\hat{\sigma}_t^2|t\epsilon A,B)}\right]^2} \quad (19)$$

where m=1 (2, 3, 4, 5) denotes the 1st (2nd, 3rd, 4th, 5th) forecast procedure over 100 (80, 60, 40, 20) days and $A = T_m + 20(h-1) + 1$ and $B = T_m + 20h$ are the beginning and end of the h-th month after the forecast starting day T_m. Thus, $T_m = T - (100 - 20(m-1) - 1)$ where T is the total number of observations in the data set. For further clarification, we provide following examples: for h=2, the focus lies on the second month of the out-of-sample forecast for 100, 80, 60 and 40 days, respectively (the forecast for 20 days has no second month); for h=4 the RMSE can be only calculated over 2 months, meaning the fourth month of the out-of-sample forecast for 100 and 80 days.

CHAPTER 5 Results

5.1 Bull and bear markets results

In the following two subsection we will present the results (bull and bear phases) that we obtained from the manual detection process and from the detection process with the statistical software.

5.1.1 Manually detected phases

In Table 2 we see an overview of all manually detected phases for the period of 1990/01/01-2020/01/01. For the S&P500 and the DJI five phases were detected. For the Nasdaq six phases were detected, but the first phase is very short and possibly the end of a longer bear phase.

Table 2: Manually detected bull and bear phases for all indices in the period of 1990/01/01-2020/01/01

Index	#	Phase	Start Date	End Date	D[1)] (months)
S&P	1	Bull	Jan 1990	Aug 2000	128
	2	Bear	Sep 2000	Sep 2002	25
	3	Bull	Oct 2002	Oct 2007	61
	4	Bear	Nov 2007	Feb 2009	16
	5	Bull	Mar 2009	Dec 2019	130
DJI	1	Bull	Jan 1990	Dec 1999	120
	2	Bear	Jan 2000	Sep 2002	33
	3	Bull	Oct 2002	Oct 2007	61
	4	Bear	Nov 2007	Feb 2009	16
	5	Bull	Mar 2009	Dec 2019	130
Nasdaq	1	Bear	Jan 1990	Oct 1990	10
	2	Bull	Nov 1990	Feb 2000	112
	3	Bear	Mar 2000	Sep 2002	31
	4	Bull	Oct 2002	Oct 2007	61
	5	Bear	Nov 2007	Feb 2009	16
	6	Bull	Mar 2009	Dec 2019	130

Note: 1) D: Duration of the phase in months.

We note that phases of the three indices mostly coincide, what is an indication that the defined framework delivers valid results. Only the end date of the first bull phase is different for the three indices, and varies in the period between December 1999 for the DJI and August 2000 for the S&P500.

The values of the measures outlined in Chapter 4 to describe the nature of bull and bear phases are presented in Table 3. The amplitude A (in percentage) in bull phases is larger than in bear phases (on average about 2 times larger), meaning that the increase in the market during bull phases is larger than the fall during bear phases. For the duration D (in months) we observe that on average bull phases are much longer than bear phases, in particular the average bull phase is approximately 5 times longer than the average bear phase. For the slope G (in percentage/month) we find evidence that bear phases are on average steeper than bull phases, more precisely in average 2-3 times steeper. In particular, Nasdaq had the largest value for G during bear phases. In summary, bull phases are longer and have a larger amplitude than bear phases, but the bear phases are much steeper, meaning that the market steadily increases over long periods, interrupted by short but massive decreases that could be interpreted as recessions.

Given this asymmetry in the behaviour during bear and bull phases, we expect that the models capturing the leverage effect (EGARCH, GJR-GARCH) are better able to describe the data than symmetric models (GARCH). In particular, remembering that the logarithmic returns R_t are approximately equal to the percentual change in the asset prices: during a bear phase, R_t is on average negative, since the asset prices are falling. Because of the leverage effect, the negative value of R_t causes a large variance σ_{t+1} and is therefore also permitting a large R_{t+1} (since $R_{t+1} = \sigma_{t+1} z_{t+1}$): this is equivalent to a large change of the asset price during one time unit from t to t+1, corresponding to a large (negative) slope. During a bull phase this change is smaller, corresponding to a smaller (positive) slope.

Table 3: Measures describing the nature of bull and bear phases for the three considered stock indices for the period of 1990/01/01-2020/01/01

Measure	S&P 500	DJI	Nasdaq
A bear	-0.6861	-0.5475	-0.8157
A bull	1.1965	1.1462	1.8136
D bear	20.5000	24.5000	19.0000
D bull	106.3333	103.6667	101.0000
C bear	-5.7778	-4.1512	-9.8215
C bull	70.6382	64.6855	102.3346
B bear	1	1	1
B bull	1	1	1
G bear	-0.0335	-0.0223	-0.0429
G bull	0.0113	0.0111	0.0180

Note: A (in %), D (in months), C (in %), B (no units), G (%/months).

Both B_{bear} and B_{bull} are equal to 1, since all phases have an amplitude larger than 0.2 (for bull) or smaller than -0.2 (for bear), because of step 7 in the censoring procedure, where all phases with a smaller amplitude are deleted. We interpreted the measure C as an average area. Since the bull phases are much longer and have a larger amplitude, C_{bull} is larger than C_{bear}. Given that $\frac{D_{bull}}{D_{bear}} \cong 5$ and $\frac{A_{bull}}{|A_{bear}|} \cong 2$ it is expected that $\frac{C_{bull}}{|C_{bear}|} \cong 10$ which is consistent with our data.

5.1.2 Detected phases in R

Figure 5 shows the evolution of the monthly log prices of the S&P500, the DJI, and the Nasdaq with all bull and bear phases, detected in R. The grey (white) shaded areas are bear (bull) phases. We observe that in general bear phases are shorter than bull phases. We also note that our manually detected turning points are also identified by R. In addition, R detected short bear phases which correspond to censored phases in the manual detection. We note that the manually detected phases can also be identified by just looking at the movements in the figure, what should be always applied as a control mechanism after the detection with a framework. The Nasdaq index shows the most prominent decrease following the dot-com bubble.

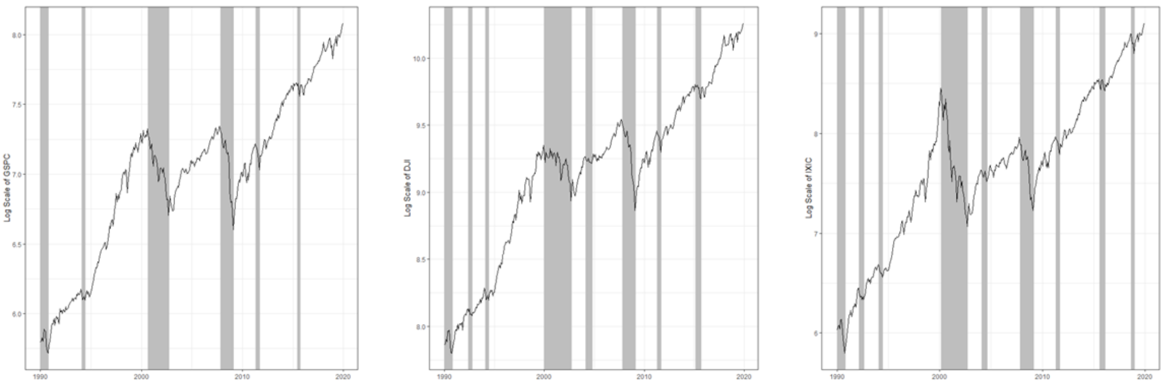

Figure 5: Evolution of the monthly log prices of the S&P 500, DJI and Nasdaq, respectively and all detected phases by R

Note: Grey shaded areas are bear phases.

Table 4 lists the three longest phases for each index for bull and bear phases, respectively. We note that because of additional bear phases, the duration of the bull phases is shorter when compared to the manual detection. We observe that the longest bull phase across all three indices is followed by the longest bear phase.

Table 4: Three longest detected bull and bear phases in R for all indices in the period of 1990/01/01-2020/01/01

Index	Longest	Phase	StartDate	EndDate	D[1] (months)
S&P	1	Bear	Sep 2000	Sep 2002	25
	2	Bear	Nov 2007	Feb 2009	16
	3	Bear	Jan 1990	Oct 1990	10
	1	Bull	Jul 1994	Aug 2000	74
	2	Bull	Oct 2002	Oct 2007	61
	3	Bull	Oct 2015	Dec 2019	51
DJI	1	Bear	Jan 2000	Sep 2002	33
	2	Bear	Nov 2007	Feb 2009	16
	3	Bear	Jan 1990	Oct 1990	10
	1	Bull	Jul 1994	Dec 1999	66
	2	Bull	Oct 2015	Dec 2019	51
	3	Bull	Oct 2011	Feb 2015	41
Nasdaq	1	Bear	Mar 2000	Sep 2002	31
	2	Bear	Nov 2007	Feb 2009	16
	3	Bear	Jan 1990	Oct 1990	10
	1	Bull	Jul 1994	Feb 2000	68
	2	Bull	Oct 2011	Jul 2015	46
	3	Bull	Sep 2004	Oct 2007	38

Note: 1) D: Duration of the phase in months.

5.2 GARCH models results

In the following two subsections we will present the results for the optimal model selection based on the AIC and BIC values (subsection 5.2.1) and the results of the forecasting procedure, where the forecasting accuracy of the different GARCH models is evaluated with the RMSE (subsection 5.2.2).

5.2.1 Optimal model selection

Table 5 shows the AIC and BIC values that were received by estimating the three considered GARCH models for each manually detected phase. Table 6 shows the same for the phases detected by R but only for the three longest bull and bear phases.

Table 5: AIC / BIC values for all models and indices for the manually detected phases

			AIC			BIC		
Index	#	Phase	GARCH	EGARCH	GJR-GARCH	GARCH	EGARCH	GJR-GARCH
S&P	1	Bull	*-6.7290*	**-6.7540**	-6.7460	*-6.7230*	**-6.7450**	-6.7370
	2	Bear	*-5.7430*	**-5.8230**	-5.7950	*-5.7180*	**-5.7900**	-5.7610
	3	Bull	*-6.8440*	-6.8650	**-6.8700**	*-6.8310*	-6.8490	**-6.8530**
	4	Bear	*-5.0470*	-5.0650	**-5.0750**	*-5.0110*	-5.0170	**-5.0270**
	5	Bull	*-6.7060*	**-6.7650**	-6.7550	*-6.6990*	**-6.7570**	-6.7460
DJI	1	Bull	*-6.7510*	**-6.7750**	-6.7670	*-6.7440*	**-6.7650**	-6.7570
	2	Bear	*-5.8360*	**-5.9030**	-5.8760	*-5.8160*	**-5.8760**	-5.8490
	3	Bull	*-6.9050*	-6.9240	**-6.9280**	*-6.8930*	-6.9080	**-6.9120**
	4	Bear	*-5.1970*	-5.2130	**-5.2200**	*-5.1610*	-5.1650	**-5.1720**
	5	Bull	*-6.8000*	**-6.8570**	-6.8510	*-6.7930*	**-6.8480**	-6.8420
Nasdaq	1	Bear	*-6.6650*	**-6.7340**	-6.7320	*-6.6130*	**-6.6650**	-6.6630
	2	Bull	*-6.3590*	**-6.3760**	-6.3690	*-6.3510*	**-6.3670**	-6.3590
	3	Bear	*-4.4560*	**-4.5210**	-4.5010	*-4.4350*	**-4.4930**	-4.4730
	4	Bull	*-6.2770*	**-6.2830**	-6.2820	*-6.2650*	**-6.2670**	-6.2660
	5	Bear	*-4.8930*	-4.9120	**-4.9280**	*-4.8570*	-4.8640	**-4.8800**
	6	Bull	*-6.3650*	**-6.4210**	-6.4120	*-6.3580*	**-6.4130**	-6.4030

Note: Bold (italic) values indicate best (worst) model.

Table 6: AIC / BIC values for all models and indices for the three longest bear and bull phases detected by R

			AIC			BIC		
Index	Longest	Phase	GARCH	EGARCH	GJR-GARCH	GARCH	EGARCH	GJR-GARCH
S&P	1	Bear	*-5.7430*	**-5.8230**	-5.7950	*-5.7180*	**-5.7900**	-5.7610
	2	Bear	*-5.0470*	-5.0650	**-5.0750**	*-5.0110*	-5.0170	**-5.0270**
	3	Bear	*-6.4620*	**-6.5390**	-6.5030	*-6.4100*	**-6.4700**	-6.4340
	1	Bull	*-6.5300*	**-6.5790**	-6.5620	*-6.5190*	**-6.5660**	-6.5490
	2	Bull	*-6.8440*	-6.8650	**-6.8700**	*-6.8310*	-6.8490	**-6.8530**
	3	Bull	*-7.0330*	**-7.1030**	-7.0820	*-7.0190*	**-7.0840**	-7.0630
DJI	1	Bear	*-5.8360*	**-5.9030**	-5.8760	*-5.8160*	**-5.8760**	-5.8490
	2	Bear	*-5.1970*	-5.2130	**-5.2200**	*-5.1610*	-5.1650	**-5.1720**
	3	Bear	*-6.4080*	**-6.4980**	-6.4430	*-6.3560*	**-6.4300**	-6.3740
	1	Bull	*-6.6170*	**-6.6540**	-6.6400	*-6.6060*	**-6.6390**	-6.6250
	2	Bull	*-7.0220*	**-7.0830**	-7.0690	*-7.0070*	**-7.0640**	-7.0500
	3	Bull	*-6.9890*	**-7.0360**	-7.0300	*-6.9720*	**-7.0130**	-7.0080
Nasdaq	1	Bear	*-4.4560*	**-4.5210**	-4.5010	*-4.4350*	**-4.4930**	-4.4730
	2	Bear	*-4.8930*	-4.9120	**-4.9280**	*-4.8570*	-4.8640	**-4.8800**
	3	Bear	*-6.6650*	**-6.7340**	-6.7320	*-6.6130*	**-6.6650**	-6.6630
	1	Bull	*-6.1100*	**-6.1340**	-6.1250	*-6.0990*	**-6.1190**	-6.1100
	2	Bull	*-6.5600*	**-6.6050**	-6.5930	*-6.5450*	**-6.5840**	-6.5720
	3	Bull	*-6.6520*	**-6.6790**	-6.6700	*-6.6340*	**-6.6550**	-6.6460

Note: Bold (italic) values indicate best (worst) model.

We observe that the GARCH model is in all considered time periods the worst model, signalling that its three parameters are not good enough to capture all characteristics of our time series. We would like to emphasize that the comparison of the EGARCH and GJR-GARCH models using AIC/BIC only depends on the likelihood (L), since both models have four parameters. In particular AIC = 8-log(L), meaning that the lowest value of AIC is obtained by the model with the highest likelihood, as can be seen for example in Table 8, where the likelihood is listed. We note that for the phases in which the GJR-GARCH model is the best, the likelihoods of EGARCH and GJR-GARCH are very similar, meaning that practically their fits are equally good. In summary, from our data we can conclude that the asymmetric models (EGARCH and GJR-GARCH) perform a better fit of our data and in particular the EGARCH model is the overall best performing model over all phases.

Table 7 shows the AIC and BIC values for modelling of the entire period. Again, the EGARCH model is the best performing model.

Table 7: AIC / BIC values for all models and indices for the entire period (1990/01/01-2020/01/01)

		AIC			BIC		
Index	Period	GARCH	EGARCH	GJR-GARCH	GARCH	EGARCH	GJR-GARCH
S&P	1990-2020 (entire period)	*-6.5910*	**-6.6350**	-6.6300	*-6.5890*	**-6.6310**	-6.6260
DJI	1990-2020 (entire period)	*-6.6330*	**-6.6730**	-6.6680	*-6.6310*	**-6.6700**	-6.6640
Nasdaq	1990-2020 (entire period)	*-6.1210*	**-6.1450**	-6.1440	*-6.1180*	**-6.1420**	-6.1410

Note: Bold (italic) values indicate best (worst) model.

We can further state that the EGARCH parameters in the model evaluation over the entire period are similar to the parameters in the evaluation of the longest phases, in our data set bull phases, as shown in Table 8. This means that the longest phase has a major influence on the parameter estimate for the entire period and the other phases only have a weak impact. This could explain the positive value of gamma we found, corresponding to the inverse leverage effect behaviour in which good news increases the future volatility more than bad news. A negative value of gamma is only found in bear phases (not for all), corresponding to the leverage effect, in which bad news increases the volatility more than good news. We did expect a negative value of gamma in the analysis of the entire period, because of the different steepness of bear and bull phases: the average slope is larger for bear phases than for bull phases. But, as previously noted, the longest bull phase could dominate the parameter estimation, leading to a positive gamma value.

Table 8 also shows the p-values and the statistical significance of the parameter estimates indicated by stars. We note that the EGARCH model is always significant (this is also valid across all phases) at least on the 5 % level, what is not given for the other models. The significance of the model parameters is a good indicator that the given model is able to describe the data.

In Appendix A the complete tables for the parameter estimation of all manually detected phases are given.

Table 8: Estimated model parameters, AIC / BIC values and Maximum Likelihood for model comparison of the entire period (1990/01/01-2020/01/01) for all three indices and for the longest bull phase of the S&P 500 (Jul 1994-Aug 2000)

S&P 500, 1990-2020 (entire period)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0925	0.8940		-6.5910	-6.5890	24911.7567		GARCH
p-value	0.0375 *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.2523	-0.1317	0.9724	0.1458	-6.6350	-6.6310	25076.4953	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0032	0.8949	0.1669	-6.6300	-6.6260	25058.0815		
p-value	0.0000 * * *	0.0015 * *	0.0000 * * *	0.0000 * * *					
DJI, 1990-2020 (entire period)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0926	0.8920		-6.6330	-6.6310	25070.3924		GARCH
p-value	0.0162 *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.2693	-0.1229	0.9706	0.1536	-6.6730	-6.6700	25222.1876	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0076	0.8948	0.1567	-6.6680	-6.6640	25202.4871		
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
Nasdaq, 1990-2020 (entire period)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0953	0.8917		-6.1210	-6.1180	23133.5421		GARCH
p-value	0.0060 * *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.1963	-0.0911	0.9772	0.1704	-6.1450	-6.1420	23226.8991	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0303	0.8890	0.1268	-6.1440	-6.1410	23222.9499		
p-value	0.0487 *	0.0001 * * *	0.0000 * * *	0.0000 * * *					
S&P 500, 1994-2000 (longest bull phase detected in R)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0687	0.9303		-6.5300	-6.5190	5014.3981		GARCH
p-value	0.4750	0.0000 * * *	0.0000 * * *						
EGARCH	-0.2671	-0.1342	0.9699	0.1466	-6.5790	-6.5660	5053.7517	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0011	0.9090	0.1703	-6.5620	-6.5490	5040.6780		
p-value	0.0000 * * *	0.6958	0.0000 * * *	0.0000 * * *					

Note: The stars denote the level of significance, ***($p \leq 0.001$), **($p \leq 0.01$), *($p \leq 0.05$). The presented results are based on conventional standard errors.

5.2.2 Forecasting accuracy

Table 9 shows the RMSE values for the model forecast for 1-4 months ahead for the longest phases detected by R.

Table 9: RMSE values for 1-4 months volatility forecasts for bull and bear phases

Index	Longest	Phase	Model	4 (80 days)	3 (60 days)	2 (40 days)	1 (20 days)	RMSE*smallest*	RMSE*largest*
S&P	1	Bear	GARCH	0.00397	0.00373	0.00342	0.00318		
			EGARCH	*0.00789*	*0.00598*	*0.00517*	*0.00416*		*EGARCH*
			GJR-GARCH	**0.00373**	**0.00318**	**0.00275**	**0.00255**	**GJR-GARCH**	
	2	Bear	GARCH	**0.00723**	0.02226	0.02143	0.01256		
			EGARCH	*7246*	*2995774*	*Inf*	*0.01438*		*EGARCH*
			GJR-GARCH	0.00744	**0.01228**	**0.01165**	**0.01013**	**GJR-GARCH**	
	3	Bear	GARCH	0.00792	0.00594	**0.00489**	**0.00418**		
			EGARCH	*0.01280*	*0.01145*	*0.01075*	*0.00679*		*EGARCH*
			GJR-GARCH	**0.00461**	**0.00559**	0.00502	0.00426	**GJR-GARCH**[1)]	
	1	Bull	GARCH	*0.00368*	*0.00303*	*0.00305*	0.00272		*GARCH*
			EGARCH	0.00258	0.00212	0.00288	*0.00285*		
			GJR-GARCH	**0.00222**	**0.00190**	**0.00190**	**0.00197**	**GJR-GARCH**	
	2	Bull	GARCH	*0.00294*	*0.00287*	*0.00255*	*0.00226*		*GARCH*
			EGARCH	0.00241	0.00244	0.00217	0.00190		
			GJR-GARCH	**0.00184**	**0.00183**	**0.00163**	**0.00143**	**GJR-GARCH**	
	3	Bull	GARCH	*0.00159*	*0.00137*	*0.00128*	*0.00138*		*GARCH*
			EGARCH	**0.00131**	**0.00116**	0.00114	0.00113	**EGARCH**[1)]	
			GJR-GARCH	0.00137	0.00122	**0.00109**	**0.00109**		
DJI	1	Bear	GARCH	0.00409	*0.00351*	*0.00327*	*0.00317*		*GARCH*
			EGARCH	**0.00342**	**0.00293**	**0.00266**	**0.00248**	**EGARCH**	
			GJR-GARCH	*0.00425*	0.00351	0.00309	0.00288		
	2	Bear	GARCH	**0.00512**	0.01934	0.01996	0.01043		
			EGARCH	*0.01513*	*0.02764*	*0.02574*	*0.01165*		*EGARCH*
			GJR-GARCH	0.00970	**0.01142**	**0.01169**	**0.00913**	**GJR-GARCH**	
	3	Bear	GARCH	0.00840	0.00633	**0.00515**	**0.00449**		
			EGARCH	*0.01308*	*0.01180*	*0.01061*	*0.00698*		*EGARCH*
			GJR-GARCH	**0.00620**	**0.00621**	0.00553	0.00470	**GJR-GARCH**[1)]	
	1	Bull	GARCH	*0.00242*	0.00218	0.00206	0.00192		
			EGARCH	0.00173	*0.00232*	*0.00210*	*0.00205*		*EGARCH*
			GJR-GARCH	**0.00162**	**0.00186**	**0.00184**	**0.00179**	**GJR-GARCH**	
	2	Bull	GARCH	*0.00149*	*0.00133*	*0.00127*	*0.00139*		*GARCH*
			EGARCH	0.00138	0.00129	0.00123	0.00131		
			GJR-GARCH	**0.00138**	**0.00129**	**0.00116**	**0.00121**	**GJR-GARCH**	
	3	Bull	GARCH	*0.00158*	*0.00254*	*0.00229*	*0.00205*		*GARCH*
			EGARCH	0.00144	**0.00178**	**0.00156**	**0.00138**	**EGARCH**	
			GJR-GARCH	**0.00122**	0.00187	0.00163	0.00145		
Nasdaq	1	Bear	GARCH	0.00212	0.00194	0.00311	*0.00362*		
			EGARCH	*0.00323*	*0.00429*	*0.00374*	0.00299		*EGARCH*
			GJR-GARCH	**0.00093**	**0.00066**	**0.00078**	**0.00215**	**GJR-GARCH**	
	2	Bear	GARCH	0.00826	0.01979	0.01914	0.01176		
			EGARCH	*0.01857*	*0.02721*	*0.02582*	*0.01453*		*EGARCH*
			GJR-GARCH	**0.00126**	**0.01176**	**0.01056**	**0.00891**	**GJR-GARCH**	
	3	Bear	GARCH	0.01087	0.00873	0.00729	0.00634		
			EGARCH	*0.01432*	*0.00955*	*0.00901*	*0.00775*		*EGARCH*
			GJR-GARCH	**0.00411**	**0.00386**	**0.00623**	**0.00625**	**GJR-GARCH**	
	1	Bull	GARCH	*0.00391*	*0.00322*	0.00281	0.00255		*GARCH*[2)]
			EGARCH	0.00356	0.00321	*0.00291*	*0.00262*		
			GJR-GARCH	**0.00238**	**0.00232**	**0.00204**	**0.00187**	**GJR-GARCH**	
	2	Bull	GARCH	**0.00092**	**0.00076**	**0.00067**	*0.00112*	**GARCH**	
			EGARCH	*0.00100*	*0.00089*	0.00084	**0.00095**		*EGARCH*
			GJR-GARCH	0.00098	0.00089	*0.00089*	0.00096		
	3	Bull	GARCH	0.00364	0.00320	0.00281	0.00252		
			EGARCH	*0.00390*	*0.00349*	*0.00304*	*0.00262*		*EGARCH*
			GJR-GARCH	**0.00291**	**0.00262**	**0.00229**	**0.00202**	**GJR-GARCH**	
Final model evaluation								**GJR-GARCH**	*EGARCH*

Note: Bold (italic) values indicate best (worst) model. 1) Selected here because of the smaller mean value over all 4 periods compared to the other model. 2) Selected here because of the larger mean value over all 4 periods compared to the other model.

Table 10 shows the same but for the forecast procedure for the entire period. As expected, the RMSE values become in most cases larger with increasing forecast horizon: in general, the estimated parameters perform better in describing the data of the first month than the data of the fourth month. This behaviour is consistent with the general fact that the influence of past events decreases over time.

The observed RMSE increase is gradually and slow. This means that the studied models are capable to deliver a useful forecast of the future volatility development for a certain number of months ahead.

The three considered models perform almost equally well as the differences in the RMSE values are small, but in general the GJR-GARCH is mostly the better model for forecast in a single phase (bull or bear). On the other hand, we observe that for forecast procedures using the whole data set (and not only single phases), the EGARCH model performs generally slightly better. It seems that for the forecast using large data sets it is best to use the EGARCH model, for shorter data sets the GJR-GARCH model.

Table 10: RMSE values for 1-4 months volatility forecasts for the entire period (1990/01/01-2020/01/01)

Index	Period	Model	4 (80 days)	3 (60 days)	2 (40 days)	1 (20 days)	RMSE*smallest*	RMSE*largest*
S&P	1990-2020	GARCH	*0.00180*	*0.00191*	*0.00177*	0.00158		*GARCH*
		EGARCH	**0.00119**	**0.00171**	**0.00157**	**0.00140**	**EGARCH**	
		GJR-GARCH	0.00149	0.00181	0.00175	*0.00164*		
DJI	1990-2020	GARCH	*0.00163*	0.00173	0.00164	0.00149		
		EGARCH	**0.00113**	**0.00162**	**0.00154**	**0.00138**	**EGARCH**	
		GJR-GARCH	0.00142	*0.00175*	*0.00174*	*0.00161*		*GJR-GARCH*
Nasdaq	1990-2020	GARCH	*0.00231*	*0.00220*	0.00209	0.00187		*GARCH* 2)
		EGARCH	**0.00179**	0.00218	*0.00215*	*0.00195*		
		GJR-GARCH	0.00194	**0.00193**	**0.00190**	**0.00176**	**GJR-GARCH**	
Final model evaluation							**EGARCH**	*GARCH*

Note: Bold (italic) values indicate best (worst) model. 2) Selected here because of the larger mean value over all 4 periods compared to the other model.

In summary, we suggest based on our findings to perform forecast analysis with both EGARCH and GJR-GARCH models. Ideally, the two models deliver consistent results. In this case the forecast can be regarded as valuable, also given that the magnitude of the RMSE is small. In the worst case, the two models give different results and the forecast has to be treated with caution.

We note that the EGARCH model is sensitive to outlier return values: because of them, in some cases, the parameter estimation procedure breaks down delivering useless results. For example, the unusually large return of 13th October 2008 during the financial crisis causes such a break down, as can be seen in the second bear phase for the S&P500 in Table 9 and in Appendix C. On the other hand, GARCH and GJR-GARCH do not seem to have difficulties with this event.

CHAPTER 6 Conclusion

We studied the characteristics of bull and bear phases for three leading US stock market indices (S&P500, DJI, Nasdaq) in the period between January 1990 and January 2020. The market evolution of the prices during this period was characterized by long steady bull phases interrupted by short and significant bear phases. We detected bull and bear phases with a manual procedure where we defined a set of rules and also using statistical software, and found consistent results for both methods. We then described the nature of the detected phases using measures that are able to describe the characteristics of the phases. In particular we introduced a measure describing the average slope of the phases finding an asymmetry between bull and bear phases: bear phases were 2-3 times steeper than bull phases. This pronounced asymmetry was found consistently across all measures that we did apply.

We fitted these phases using the GARCH, EGARCH and GJR-GARCH model. We used the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to select the optimal model. We observe that our data are best described by the asymmetric models, in particular the EGARCH model is overall the best model for both types of phases (bull and bear). It is important to note that the parameter estimation for the EGARCH model always delivered statistically significant values whereas the other model parameters were less or not consistently significant. We did the same analysis also for the entire period where our findings were confirmed with the EGARCH being the best model. In this last case, the parameter γ, describing the asymmetry in the EGARCH model, was found to be positive, corresponding to an inverse leverage effect, in which good news increases the future volatility more than bad news. This finding is not in line with the results of Angabini and Wasiuzzaman (2011), that applied a similar methodology to analyse the Malaysian stock market during the financial crisis. They did find a negative value for gamma corresponding to the leverage effect. We explained our finding by observing that the bull phases are much longer than the bear phases and therefore they could dominate the parameter estimation over the entire period. Since all bull phases have consistently a positive value of γ, one can expect that also the parameter estimation over the entire period delivers a positive value of γ.

A forecast procedure was performed with out-of-sample data where the last observations were withheld and parameters estimated on the remaining data. With these parameters, volatility estimates were produced for the withheld observations. The forecasting accuracy was measured by the root-mean-squared-error (RMSE), which is a further method to compare the models and their ability to be used for volatility forecasts. We noted, that the magnitude of the RMSE values were consistently small and only slowly and gradually increasing with an increasing forecast horizon up to 4 months ahead. We found that also in this investigation the asymmetric models performed better than the symmetric GARCH model. In particular it seems that for longer estimation periods the EGARCH model is the better choice, for shorter estimations periods GJR-GARCH is the better model. These findings are in

line with findings in the literature, where the GJR-GARCH model is found to be the best model for volatility forecasts, closely followed by the EGARCH model (Liu & Hung, 2010).
With these results we fully addressed the underlying research question in this thesis for bull and bear markets and for the model selection process.

Nevertheless, we are aware that in the past the market evolution displayed other characteristics compared to the considered time period in this study, with shorter phases and a higher frequency of bull-bear phases interchange (Pagan & Sossounov, 2003). For this reason, we cannot assume that our results are also applicable to past periods.

One shortcoming of this research is that the data sample for bear phases was short and therefore GARCH estimation could be influenced, meaning that our results for bear phases could be less significant than for bull phases. In further research, to improve the modelling of bear phases with GARCH models, one could simulate longer data samples with similar characteristics as bear markets. A similar approach was used by (Ding & Meade, 2010) to investigate the forecasting accuracy of stochastic volatility, GARCH and EWMA models for different volatility scenarios.

For simplicity we assumed in our study that the returns follow a normal distribution but this assumption is not consistent with real data (Christoffersen, 2012) and other distributions could improve the estimation accuracy.

The hypothesis stating that the symmetric GARCH model is accurate to describe bull and bear phases, is not consistent with our findings. The asymmetric models EGARCH and GJR-GARCH deliver overall better results. The results of the symmetric GARCH model were in most cases the worst what is a further indication to reject the underlying hypothesis of this paper.

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APPENDIX A

Table A1: Estimated model parameters, AIC / BIC values and Maximum Likelihood for model comparison for the S&P 500 (manually detected phases)

S&P: Bull (Jan 1990-Aug 2000)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0444	0.9523		-6.7290	-6.7230	8996.9409		GARCH
p-value	0.5986	0.0000 * * *	0.0000 * * *						
EGARCH	-0.1697	-0.0815	0.9813	0.1245	-6.7540	-6.7450	9030.6286	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0130	0.9352	0.0881	-6.7460	-6.7370	9020.3159		
p-value	0.7194	0.3646	0.0000 * * *	0.0506					
S&P: Bear (Sep 2000-Sep 2002)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1219	0.8096		-5.7430	-5.7180	1433.1308		GARCH
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.1272	-0.1518	0.9870	0.0075	-5.8230	-5.7900	1454.0229	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0000	0.8708	0.1840	-5.7950	-5.7610	1446.9952		
p-value	0.0000 * * *	1.0000	0.0000 * * *	0.0000 * * *					
S&P: Bull (Oct 2002-Oct 2007)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0564	0.9233		-6.8440	-6.8310	4304.2762		GARCH
p-value	0.2467	0.0000 * * *	0.0000 * * *						
EGARCH	-0.1902	-0.1005	0.9798	0.0983	-6.8650	-6.8490	4318.6147		
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0000	0.9302	0.1101	-6.8700	-6.8530	4321.4813	GJR-GARCH	
p-value	0.2895	1.0000	0.0000 * * *	0.0000 * * *					
S&P: Bear (Nov 2007-Feb 2009)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1265	0.8640		-5.0470	-5.0110	792.8432		GARCH
p-value	0.2984	0.0001 * * *	0.0000 * * *						
EGARCH	-0.1594	-0.1724	0.9818	0.1407	-5.0650	-5.0170	796.6242		
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0000	0.8883	0.1770	-5.0750	-5.0270	798.2225	GJR-GARCH	
p-value	0.1616	1.0000	0.0000 * * *	0.0000 * * *					
S&P: Bull (Mar 2009-Dec 2019)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1508	0.8175		-6.7060	-6.6990	9079.0298		GARCH
p-value	0.0361 *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.4809	-0.2066	0.9479	0.2097	-6.7650	-6.7570	9161.0398	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0087	0.8269	0.2814	-6.7550	-6.7460	9146.4417		
p-value	0.0000 * * *	0.0087 * *	0.0000 * * *	0.0000 * * *					

Note: The stars denote the level of significance, ***($p \leq 0.001$), **($p \leq 0.01$), *($p \leq 0.05$). The presented results are based on conventional standard errors.

Table A2: Estimated model parameters, AIC / BIC values and Maximum Likelihood for model comparison for the DJI (manually detected phases)

DJI: Bull (Jan 1990-Dec 1990)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0471	0.9459		-6.7510	-6.7440	8458.6720		GARCH
p-value	0.1348	0.0000 * * *	0.0000 * * *						
EGARCH	-0.2024	-0.0734	0.9780	0.1190	-6.7750	-6.7650	8489.1900	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0155	0.9278	0.0812	-6.7670	-6.7570	8479.2081		
p-value	0.1270	0.0460 *	0.0000 * * *	0.0000 * * *					
DJI: Bear (Jan 2000-Sep 2002)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1083	0.8491		-5.8360	-5.8160	1949.2792		GARCH
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.1561	-0.1566	0.9831	0.0448	-5.9030	-5.8760	1972.7340	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0179 *					
GJR-GARCH	0.0000	0.0000	0.8807	0.1800	-5.8760	-5.8490	1963.6525		
p-value	0.0024 * *	1.0000	0.0000 * * *	0.0000 * * *					
DJI: Bull (Oct 2002-Oct 2007)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0597	0.9178		-6.9050	-6.8930	4342.6583		GARCH
p-value	0.2198	0.0000 * * *	0.0000 * * *						
EGARCH	-0.2260	-0.1002	0.9761	0.1223	-6.9240	-6.9080	4355.7752		
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0060	0.9158	0.1207	-6.9280	-6.9120	4358.1855	GJR-GARCH	
p-value	0.2878	0.5847	0.0000 * * *	0.0002 * * *					
DJI: Bear (Nov 2007-Feb 2009)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1193	0.8668		-5.1970	-5.1610	816.2876		GARCH
p-value	0.3292	0.0001 * * *	0.0000 * * *						
EGARCH	-0.1610	-0.1542	0.9817	0.1411	-5.2130	-5.1650	819.8294		
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0000	0.8915	0.1725	-5.2200	-5.1720	820.9085	GJR-GARCH	
p-value	0.6054	1.0000	0.0000 * * *	0.1119					
DJI: Bull (Mar 2009-Dec 2019)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1534	0.8130		-6.8000	-6.7930	9206.6093		GARCH
p-value	0.1148	0.0000 * * *	0.0000 * * *						
EGARCH	-0.5345	-0.1928	0.9429	0.2130	-6.8570	-6.8480	9284.6841	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0055	0.8303	0.2742	-6.8510	-6.8420	9276.5194		
p-value	0.0000 * * *	0.0052 * *	0.0000 * * *	0.0000 * * *					

Note: The stars denote the level of significance, ***($p \leq 0.001$), **($p \leq 0.01$), *($p \leq 0.05$). The presented results are based on conventional standard errors.

Table A3: Estimated model parameters, AIC / BIC values and Maximum Likelihood for model comparison for the Nasdaq (manually detected phases)

Nasdaq: Bear (Jan 1990-Oct 1990)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1701	0.8122		-6.6650	-6.6130	629.5080		GARCH
p-value	0.6413	0.0374 *	0.0000 * * *						
EGARCH	-1.6160	-0.2227	0.8336	0.2007	-6.7340	-6.6650	636.9979	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0001 * * *					
GJR-GARCH	0.0000	0.0000	0.6503	0.3401	-6.7320	-6.6630	636.7639		
p-value	0.0000 * * *	1.0000	0.0000 * * *	0.0005 * * *					
Nasdaq: Bull (Nov 1990-Feb 2000)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1052	0.8752		-6.3590	-6.3510	7426.9413		GARCH
p-value	0.0819	0.0000 * * *	0.0000 * * *						
EGARCH	-0.2606	-0.0664	0.9704	0.2177	-6.3760	-6.3670	7448.4267	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0723	0.8563	0.0963	-6.3690	-6.3590	7439.8491		
p-value	0.4155	0.0000 * * *	0.0000 * * *	0.0002 * * *					
Nasdaq: Bear (Mar 2000-Sep 2002)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1048	0.8660		-4.4560	-4.4350	1400.0222		GARCH
p-value	0.0446 *	0.0003 * * *	0.0000 * * *						
EGARCH	-0.0702	-0.1282	0.9923	-0.0161	-4.5210	-4.4930	1421.3952	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0000	0.8991	0.1582	-4.5010	-4.4730	1415.0202		
p-value	0.0190 *	1.0000	0.0000 * * *	0.0000 * * *					
Nasdaq: Bull (Oct 2002-Oct 2007)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.0387	0.9517		-6.2770	-6.2650	3948.1487		GARCH
p-value	0.6351	0.0495 *	0.0000 * * *						
EGARCH	-0.0664	-0.0433	0.9924	0.0825	-6.2830	-6.2670	3953.0467	EGARCH	
p-value	0.0000 * * *	0.0002 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0169	0.9542	0.0454	-6.2820	-6.2660	3952.5433		
p-value	0.7883	0.3649	0.0000 * * *	0.0561					
Nasdaq: Bear (Nov 2007-Feb 2009)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1116	0.8768		-4.8930	-4.8570	768.6996		GARCH
p-value	0.4002	0.0010 * * *	0.0000 * * *						
EGARCH	-0.1893	-0.1659	0.9775	0.1344	-4.9120	-4.8640	772.7779		
p-value	0.0001 * * *	0.0000 * * *	0.0000 * * *	0.0122 *					
GJR-GARCH	0.0000	0.0000	0.8881	0.1756	-4.9280	-4.8800	775.1586	GJR-GARCH	
p-value	0.0001 * * *	1.0000	0.0000 * * *	0.0002 * * *					
Nasdaq: Bull (Mar 2009-Dec 2019)									
Model	Omega	Alpha	Beta	Gamma	AIC	BIC	Likelihood	$Model_{first}$	$Model_{last}$
GARCH	0.0000	0.1227	0.8380		-6.3650	-6.3580	8617.4149		GARCH
p-value	0.0002 * * *	0.0000 * * *	0.0000 * * *						
EGARCH	-0.5376	-0.1974	0.9402	0.1663	-6.4210	-6.4130	8695.2358	EGARCH	
p-value	0.0000 * * *	0.0000 * * *	0.0000 * * *	0.0000 * * *					
GJR-GARCH	0.0000	0.0000	0.8369	0.2512	-6.4120	-6.4030	8682.1643		
p-value	0.0000 * * *	0.9981	0.0000 * * *	0.0000 * * *					

Note: The stars denote the level of significance, ***($p \leq 0.001$), **($p \leq 0.01$), *($p \leq 0.05$). The presented results are based on conventional standard errors.

APPENDIX B

Table B1: Realized returns and sigma forecast estimates for GARCH, EGARCH and GJR-GARCH for the last 100 days of the longest bear phase of the S&P500

		GARCH	EGARCH	GJR-GARCH
Last 100 days	R_t	σ_t	σ_t	σ_t
2002-04-11	-0.0240	0.0102	0.0094	0.0096
2002-04-12	0.0066	0.0128	0.0106	0.0135
2002-04-15	-0.0076	0.0123	0.0102	0.0129
2002-04-16	0.0231	0.0119	0.0106	0.0127
2002-04-17	-0.0020	0.0137	0.0091	0.0121
2002-04-18	-0.0014	0.0129	0.0093	0.0116
2002-04-19	0.0006	0.0122	0.0095	0.0112
2002-04-22	-0.0155	0.0116	0.0095	0.0108
2002-04-23	-0.0062	0.0123	0.0103	0.0122
2002-04-24	-0.0071	0.0118	0.0107	0.0119
2002-04-25	-0.0015	0.0115	0.0111	0.0118
2002-04-26	-0.0140	0.0111	0.0113	0.0113
2002-04-29	-0.0102	0.0116	0.0120	0.0123
2002-04-30	0.0107	0.0116	0.0126	0.0125
2002-05-01	0.0088	0.0116	0.0119	0.0119
2002-05-02	-0.0018	0.0115	0.0113	0.0114
2002-05-03	-0.0103	0.0110	0.0115	0.0110
2002-05-06	-0.0195	0.0111	0.0121	0.0114
2002-05-07	-0.0030	0.0125	0.0131	0.0135
2002-05-08	0.0368	0.0119	0.0133	0.0130
2002-05-09	-0.0147	0.0167	0.0108	0.0124
2002-05-10	-0.0169	0.0163	0.0116	0.0133
2002-05-13	0.0184	0.0161	0.0124	0.0144
2002-05-14	0.0209	0.0162	0.0112	0.0137
2002-05-15	-0.0057	0.0166	0.0097	0.0131
2002-05-16	0.0065	0.0155	0.0101	0.0127
2002-05-17	0.0076	0.0146	0.0097	0.0121
2002-05-20	-0.0134	0.0139	0.0092	0.0116
2002-05-21	-0.0111	0.0138	0.0099	0.0124
2002-05-22	0.0057	0.0135	0.0106	0.0127
2002-05-23	0.0101	0.0129	0.0102	0.0122
2002-05-24	-0.0122	0.0126	0.0096	0.0117
2002-05-28	-0.0086	0.0126	0.0102	0.0122
2002-05-29	-0.0064	0.0123	0.0107	0.0122
2002-05-30	-0.0028	0.0119	0.0111	0.0120
2002-05-31	0.0023	0.0114	0.0113	0.0116
2002-06-03	-0.0251	0.0109	0.0113	0.0111
2002-06-04	0.0000	0.0134	0.0125	0.0148
2002-06-05	0.0088	0.0127	0.0126	0.0141
2002-06-06	-0.0200	0.0123	0.0121	0.0134
2002-06-07	-0.0016	0.0135	0.0131	0.0151
2002-06-10	0.0031	0.0127	0.0132	0.0144
2002-06-11	-0.0168	0.0121	0.0131	0.0137
2002-06-12	0.0065	0.0128	0.0139	0.0147
2002-06-13	-0.0105	0.0123	0.0135	0.0140
2002-06-14	-0.0023	0.0122	0.0141	0.0140
2002-06-17	0.0283	0.0116	0.0143	0.0134
2002-06-18	0.0009	0.0146	0.0123	0.0128
2002-06-19	-0.0167	0.0136	0.0124	0.0122
2002-06-20	-0.0135	0.0140	0.0132	0.0135
2002-06-21	-0.0172	0.0139	0.0139	0.0140
2002-06-24	0.0036	0.0143	0.0148	0.0151
2002-06-25	-0.0168	0.0134	0.0146	0.0143
2002-06-26	-0.0027	0.0138	0.0154	0.0153
2002-06-27	0.0174	0.0130	0.0156	0.0145
2002-06-28	-0.0008	0.0136	0.0144	0.0138
2002-07-01	-0.0216	0.0128	0.0145	0.0132
2002-07-02	-0.0215	0.0141	0.0156	0.0153
2002-07-03	0.0062	0.0150	0.0166	0.0170
2002-07-05	0.0361	0.0142	0.0162	0.0161
2002-07-08	-0.0123	0.0179	0.0137	0.0153
2002-07-09	-0.0250	0.0171	0.0144	0.0153
2002-07-10	-0.0346	0.0179	0.0156	0.0178
2002-07-11	0.0075	0.0201	0.0173	0.0219
2002-07-12	-0.0065	0.0187	0.0168	0.0206
2002-07-15	-0.0038	0.0174	0.0171	0.0196
2002-07-16	-0.0187	0.0162	0.0174	0.0186
2002-07-17	0.0056	0.0163	0.0183	0.0191
2002-07-18	-0.0274	0.0152	0.0179	0.0181
2002-07-19	-0.0391	0.0169	0.0192	0.0204
2002-07-22	-0.0335	0.0204	0.0210	0.0250
2002-07-23	-0.0274	0.0218	0.0226	0.0272
2002-07-24	0.0557	0.0220	0.0238	0.0278
2002-07-25	-0.0056	0.0274	0.0198	0.0262
2002-07-26	0.0167	0.0250	0.0201	0.0247
2002-07-29	0.0527	0.0235	0.0189	0.0232
2002-07-30	0.0042	0.0278	0.0153	0.0219
2002-07-31	0.0097	0.0253	0.0150	0.0206
2002-08-01	-0.0300	0.0234	0.0144	0.0194
2002-08-02	-0.0234	0.0236	0.0158	0.0220
2002-08-05	-0.0349	0.0229	0.0170	0.0228
2002-08-06	0.0295	0.0240	0.0186	0.0258
2002-08-07	0.0198	0.0240	0.0165	0.0242
2002-08-08	0.0322	0.0230	0.0151	0.0228
2002-08-09	0.0035	0.0236	0.0129	0.0215
2002-08-12	-0.0053	0.0216	0.0127	0.0202
2002-08-13	-0.0219	0.0199	0.0131	0.0192
2002-08-14	0.0393	0.0197	0.0141	0.0202
2002-08-15	0.0115	0.0224	0.0115	0.0191
2002-08-16	-0.0016	0.0209	0.0107	0.0180
2002-08-19	0.0233	0.0192	0.0109	0.0170
2002-08-20	-0.0141	0.0193	0.0093	0.0161
2002-08-21	0.0126	0.0184	0.0100	0.0163
2002-08-22	0.0140	0.0175	0.0092	0.0155
2002-08-23	-0.0229	0.0168	0.0083	0.0147
2002-08-26	0.0075	0.0174	0.0095	0.0168
2002-08-27	-0.0139	0.0163	0.0090	0.0159
2002-08-28	-0.0183	0.0158	0.0097	0.0161
2002-08-29	-0.0001	0.0160	0.0107	0.0170
2002-08-30	-0.0019	0.0154	0.0159	0.0166

Table B2: Parameter estimation sets for the data set without the last 100 days of the longest bear phase of the S&P500

	Omega	Alpha	Beta	
GARCH	0.0000	0.1108	0.8124	
p-value	0.0000	0.0000	0.0000	
	Omega	Alpha	Beta	Gamma
EGARCH	-0.1178	-0.1210	0.9878	-0.0347
p-value	0.0000	0.0000	0.0000	0.0000
	Omega	Alpha	Beta	Gamma
GJR-GARCH	0.0000	0.0000	0.8740	0.1657
p-value	0.0000	1.0000	0.0000	0.0000

Table B3: Realized returns and sigma forecast estimates for GARCH, EGARCH and GJR-GARCH for the last 20 days of the longest bear phase of the S&P500

		GARCH	EGARCH	GJR-GARCH
Last 20 days	R_t	σ_t	σ_t	σ_t
2002-08-05	-0.0349	0.0240	0.0209	0.0237
2002-08-06	0.0295	0.0251	0.0233	0.0268
2002-08-07	0.0198	0.0252	0.0208	0.0252
2002-08-08	0.0322	0.0240	0.0191	0.0237
2002-08-09	0.0035	0.0247	0.0166	0.0222
2002-08-12	-0.0053	0.0226	0.0162	0.0209
2002-08-13	-0.0219	0.0208	0.0165	0.0198
2002-08-14	0.0393	0.0206	0.0179	0.0209
2002-08-15	0.0115	0.0234	0.0150	0.0197
2002-08-16	-0.0016	0.0218	0.0140	0.0186
2002-08-19	0.0233	0.0200	0.0141	0.0175
2002-08-20	-0.0141	0.0202	0.0123	0.0166
2002-08-21	0.0126	0.0192	0.0133	0.0168
2002-08-22	0.0140	0.0183	0.0123	0.0159
2002-08-23	-0.0229	0.0176	0.0112	0.0151
2002-08-26	0.0075	0.0181	0.0129	0.0173
2002-08-27	-0.0139	0.0170	0.0123	0.0164
2002-08-28	-0.0183	0.0165	0.0133	0.0166
2002-08-29	-0.0001	0.0166	0.0146	0.0176
2002-08-30	-0.0019	0.0154	0.0159	0.0166

Table B4: Parameter estimation sets for the data set without the last 20 days of the longest bear phase of the S&P500

	Omega	Alpha	Beta	
GARCH	0.0000	0.1219	0.8137	
p-value	0.0000	0.0000	0.0000	
	Omega	Alpha	Beta	Gamma
EGARCH	-0.1440	-0.1494	0.9851	-0.0078
p-value	0.0000	0.0000	0.0000	0.1429
	Omega	Alpha	Beta	Gamma
GJR-GARCH	0.0000	0.0000	0.8723	0.1828
p-value	0.0000	1.0000	0.0000	0.0000

APPENDIX C

Table C1: Realized returns and sigma forecast estimates for EGARCH for the last 100 days of the second longest bear phase of the S&P500

		EGARCH
Last 100 days	R_t	σ_t
2008-09-09	-0.0347	0.0129
2008-09-10	0.0061	0.0136
2008-09-11	0.0137	0.0134
2008-09-12	0.0021	0.0122
2008-09-15	-0.0483	0.0125
2008-09-16	0.0174	0.0132
2008-09-17	-0.0483	0.0115
2008-09-18	0.0424	0.0122
2008-09-19	0.0395	0.0077
2008-09-22	-0.0390	0.0038
2008-09-23	-0.0158	0.0042
2008-09-24	-0.0020	0.0045
2008-09-25	0.0195	0.0049
2008-09-26	0.0034	0.0029
2008-09-29	-0.0922	0.0026
2008-09-30	0.0528	0.0030
2008-10-01	-0.0046	0.0002
2008-10-02	-0.0411	0.0003
2008-10-03	-0.0136	0.0004
2008-10-06	-0.0393	0.0004
2008-10-07	-0.0591	0.0005
2008-10-08	-0.0114	0.0007
2008-10-09	-0.0792	0.0008
2008-10-10	-0.0118	0.0010
2008-10-13	0.1096	0.0011
2008-10-14	-0.0053	0.0000
2008-10-15	-0.0947	Inf
2008-10-16	0.0416	5938700000.0000
2008-10-17	-0.0062	3565505000.0000
2008-10-20	0.0466	2163665000.0000
2008-10-21	-0.0313	1326786000.0000
2008-10-22	-0.0630	821975000.0000
2008-10-23	0.0126	514363600.0000
2008-10-24	-0.0351	325045700.0000
2008-10-27	-0.0323	207392000.0000
2008-10-28	0.1025	133575100.0000
2008-10-29	-0.0111	86827980.0000
2008-10-30	0.0255	56952200.0000
2008-10-31	0.0152	37687380.0000
2008-11-03	-0.0025	25155670.0000
2008-11-04	0.0400	16933690.0000
2008-11-05	-0.0541	11493860.0000
2008-11-06	-0.0516	7865096.0000
2008-11-07	0.0284	5424900.0000
2008-11-10	-0.0127	3771003.0000
2008-11-11	-0.0223	2641368.0000
2008-11-12	-0.0533	1863968.0000
2008-11-13	0.0669	1325005.0000
2008-11-14	-0.0426	948637.3000
2008-11-17	-0.0261	683945.1000
2008-11-18	0.0098	496497.4000
2008-11-19	-0.0631	362848.5000
2008-11-20	-0.0695	266922.5000
2008-11-21	0.0613	197622.7000
2008-11-24	0.0627	147238.6000
2008-11-25	0.0065	110378.1000
2008-11-26	0.0347	83246.1300
2008-11-28	0.0096	63155.4000
2008-12-01	-0.0935	48191.2800
2008-12-02	0.0392	36981.5800
2008-12-03	0.0255	28537.1000
2008-12-04	-0.0297	22140.6900
2008-12-05	0.0358	17269.5200
2008-12-08	0.0377	13540.3200
2008-12-09	-0.0234	10670.6100
2008-12-10	0.0118	8451.1470
2008-12-11	-0.0289	6726.0860
2008-12-12	0.0070	5378.7990
2008-12-15	-0.0128	4321.5610
2008-12-16	0.0501	3488.0770
2008-12-17	-0.0096	2827.9970
2008-12-18	-0.0214	2302.9270
2008-12-19	0.0029	1883.4280
2008-12-22	-0.0185	1546.8440
2008-12-23	-0.0098	1275.6570
2008-12-24	0.0058	1056.2690
2008-12-26	0.0053	878.0727
2008-12-29	-0.0039	732.7681
2008-12-30	0.0241	613.8299
2008-12-31	0.0141	516.1042
2009-01-02	0.0311	435.5163
2009-01-05	-0.0047	368.8182
2009-01-06	0.0078	313.4270
2009-01-07	-0.0305	267.2629
2009-01-08	0.0034	228.6606
2009-01-09	-0.0215	196.2734
2009-01-12	-0.0228	169.0135
2009-01-13	0.0018	145.9961
2009-01-14	-0.0340	126.5002
2009-01-15	0.0013	109.9374
2009-01-16	0.0075	95.8242
2009-01-20	-0.0543	83.7625
2009-01-21	0.0426	73.4265
2009-01-22	-0.0153	64.5381
2009-01-23	0.0054	56.8839
2009-01-26	0.0055	50.2696
2009-01-27	0.0109	44.5394
2009-01-28	0.0330	39.5617
2009-01-29	-0.0337	35.2245
2009-01-30	-0.0231	0.0253

Table C2: Parameter estimation set for EGARCH for the data set without the last 100 days of the second longest bear phase of the S&P500

	Omega	Alpha	Beta	Gamma
EGARCH	-0.1950	-0.1487	0.9791	-0.1467
p-value	0.0000	0.0000	0.0000	0.0000