ERASMUS UNIVERSITY ROTTERDAM

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Master Thesis Quantitative Finance

Modeling extreme European Market Risk

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Abstract

This research explores whether the risk of having extreme losses in the European stock market can be predicted by incorporating macro-economic variables. To answer this question, we investigated the tail distribution of weekly stock losses on six different European indices (AEX, DAX, CAC40, PSI-20, IBEX35 and FTSE MIB). We set up an Extreme Value Theory (EVT) machine learning framework using shrinkage regression techniques, such as LASSO. Our results show that when adding a limited amount of macro-economic covariates to the tail distribution of weekly losses, the prediction for the VaR is improved for five of the six indices. Inflation, short-term interest rate, industrial production and the USD/EUR exchange rate appear to have predicting power for the tail risk. No predicting power is found when using unemployment and the long-term interest rate. This study implies the importance of macro-economic information when estimating financial risks of investments in the European stock market, which can be of added value for investors and regulators.

Keywords— Extreme Value Theory; Peaks-Over-Threshold; Generalized Pareto distribution; Poisson point process; Macro-economic covariates; Value-at-Risk; European stock market; Shrinkage regression

Contents

1 Introduction

During the last two decades, Europe has suffered with economic crises that had a severe impact on the economic landscape. Bigger than predicted financial risks materialized during these crises. It sparked the search of methods that are able to model extreme events related to macro-economic variables.

One method is the implementation of covariates when describing the tail of financial return series, as done by for example Chavez-Demoulin and Embrechts (2004). Next to that Chavez-Demoulin, Embrechts, and Hofert (2016) develops a methodology for modelling the tail of financial loss data depending on covariates. In this research we apply this method to investigate European stock market risk related to macro-economic covariates.

Macro-economic variables have predictive power for future stock market returns, see as an early example Asprem (1989). Asprem (1989) finds for France, Germany and the Netherlands that changes in stock prices are positively correlated to macro-economic variables. Moreover, Cheung and Lai (1999) states that there is a role for macro-economic variables in accounting for stock market movements in the countries France, Germany and Italy. This result is confirmed by the more recent findings in Peiró (2016) for stock prices in France, Germany and the United Kingdom.

The relation between macro-economic variables and stock market returns does not necessarily imply that these macro-economic variables can predict extreme losses or in other words future market risk. This research aims to explore if we can predict risk on extreme losses in the European stock market by incorporating macro-economic variables as covariates. Moreover, our research takes more Eurozone countries into account, with using more frequent and longer periods of data.

We use the Peaks-over-Threshold (POT) method (from Extreme Value Theory (EVT)) to focus on extreme market losses. The POT method uses the Generalized Pareto Distribution (GPD) to model the tail distribution of losses. The parameters of the GPD are related to a limited number of macro-economic covariates. After determining the conditional GPD parameters, we calculate two relevant risk measures: the Value-at-Risk and Expected Shortfall.

To select proper covariates in the model we use machine learning techniques. The amount of covariates selected in the final model are limited, while the collection of potential covariates are large. We use machine learning techniques such as shrinkage methods (LASSO or Ridge Regression). We apply these methods to a model selection procedure. This results in an EVT machine learning framework which can be used for further research.

Investigating tail risks by means of POT has been studied in previous literature. An early example is Davison and Smith (1990), which analyses the extremes in environmental time series data in a POT context by modelling the size and occurrence of exceedances. Next to that, their research investigates the point process of exceedances and apply the POT framework to investigate nuclear siting policies. They conclude that the GPD and POT framework is sufficient in modelling tail risk of high level exceedances. Also estimating financial tail risks using the GPD have been investigated. For example, Jondeau and Rockinger (1999) uses EVT to estimate the tail distribution of stock market returns in 27 countries. These countries are divided in emerging and mature markets. They find a good fit of the GPD on mature markets, suggesting it can be an interesting tool for risk measurement. Next to that, Gençay and Selçuk (2004) studied extreme values of daily returns of stock indices in nine emerging markets and found that GPD fit the tail distributions well. Moreover, when testing each distribution on their Value-at-Risk forecasting performance, the GPD clearly outperforms other distributions, signalling that GPD is an important distribution in describing financial tail risks.

Macro-economic covariates can help to predict stock market returns, as we have discussed earlier in this section. In addition to that, Nasseh and Strauss (2000) finds evidence of a long-run relationship between stock prices and macro-economic variables in Europe, when using domestic as well as international macro variables. Another example can be found on the stock markets in the United States. Chen, Roll, and Ross (1986) show that the predictive power in the US stock market increases when adding macro-economic variables to their models. Moreover, macro-economic variables can also enhance the prediction of stock volatility, as is shown by Errunza and Hogan (1998), who investigate the then largest seven European equity markets. This research explores whether macro-economic covariates also have predictive power concerning the risk on extreme European stock market losses.

One source on incorporating covariates in describing financial tail distributions is Chavez-Demoulin and Embrechts (2004), who set out nonparametric or semiparametric techniques to describe extremes of nonstationary data, such as financial returns. They do this by smoothing generalized additive models and incorporating covariates in these models. Recent research of Chavez-Demoulin et al. (2016) builds forward onto these techniques and set out a framework for modelling simulated and insurance loss data depending on covariates. We use this research when incorporating macro-economic covariates to model the tail distributions of our extreme losses.

The empirical results show that the optimal models often select one or no covariate in describing the tail distribution parameters. Implementing the shrinkage regression approach does not provide better results, only two out of twelve models selected were formed by using shrinkage regression. However, adding macro-economic covariates do improve the estimation of tail risk for five of the six indices considered, with the only exception being the Portuguese Stock Index (PSI-20). Inflation, short-term interest rate, production and the USD/EUR exchange rate all have predicting power on the tail risk of losses. For unemployment and the long-term interest rate we find no predicting power when these are included for any of the six indices.

This thesis is organized as follows. Section 2 provides a description of the methodology that is followed in this research. Next to that, Section 3 studies the data of the financial losses as well as the covariates that we use in the analysis in depth. The results of our analysis are presented in Section 4. Lastly Section 5 provides a discussion and concluding remarks on the presented results and methodology.

2 Methodology

2.1 The general set-up for i.i.d. losses

Serial dependence is a known issue when using stock return data. To overcome this problem, we follow the two-step conditional EVT filtration approach as described in McNeil and Frey (2000). First we fit a suitable type of GARCH process to the financial losses L_t and extract the residuals as X_t. We choose to fit an AR(1) process with GARCH(1,1) errors to the losses L_t which can be written as

$$\begin{aligned} L_t &= \mu_t + \sigma_t X_t, \\ \mu_t &= \mu + \phi(L_{t-1} - \mu), \\ \sigma_t^2 &= \omega_0 + \omega_1(L_{t-1} - \mu_{t-1})^2 + \psi_1 \sigma_{t-1}^2, \end{aligned} \tag{1}$$

where $t \in \{1, \ldots, n\}$, $\omega_0 > 0$, $\omega_1 \geqslant 0$, $\psi_1 \geqslant 0$, $\omega_1 + \psi_1 < 1$ and the underlying distribution of X_t is scaled to have unit variance and follows a t-distribution with κ degrees of freedom, which we estimate as a parameter in the model. After estimating this model, we filter out the estimated residual series $\{X_t\}$. We follow the Peaks-Over-Threshold approaches as in McNeil, Frey, and Embrechts (2015, Chapter 5) to analyse large observations (peaks) that exceed a certain level (threshold) u. The amount of losses exceeding the chosen threshold is denoted as N_u. Denote the exceedances as $\tilde{X}_1, \ldots, \tilde{X}_{N_u}$ which is a subset from all losses $X_1, \ldots, X_n$. Each exceedance is then used to calculate the excesses $Y_j = \tilde{X}_j - u$ for $j = \{1, \ldots, N_u\}$.

Our goal is to model the distribution of our excesses Y_j. Following Embrechts, Klüppelberg, and Mikosch (1997, Chapter 3), the main excess distribution in POT analysis is the Generalized Pareto Distribution (GPD), of which the distribution function can be written as:

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \frac{\xi x}{\beta})^{-1/\xi} & \xi \neq 0 \\ 1 - \exp(-\frac{x}{\beta}) & \xi = 0 \end{cases} \tag{2}$$

for $x \geqslant 0$ if $\xi > 0$ and $x \in [0, -\beta/\xi]$ if $\xi \leqslant 0$.

If the losses $X_1, \ldots, X_n$ are independent and identically distributed, the arrivals of exceedances follow a Poisson process with an intensity parameter λ. Moreover, the excesses $Y_1, \ldots, Y_{N_u}$ are asymptotically independent of the number of exceedances N_u.

2.2 Implementing Covariates

In Section 2.1 we discussed the case when the residuals $\{X_t\}$ are assumed to be independent and identically distributed over time. However, we are interested in whether the extreme behaviour of the losses is related to one or more covariates. For example, our financial market risk losses might depend on macro-economic variables that could predict the distribution of these losses.

We will thus extend the classical EVT with a dynamic approach by modelling the parameters as dependent of one or more covariates. These covariates are all macro-economic covariates, summarized in vector $m = (m_1, m_2, \ldots, m_c)$. The mathematical notation below is comparable to the covariate models in Chavez-Demoulin et al. (2016).

For the POT model we have in total three parameters, $\lambda(t)$, $\xi(t)$ and $\beta(t)$, at time t. However, as seen in Chavez-Demoulin et al. (2016), we cannot model the parameters $\xi(t)$ and $\beta(t)$ directly linear on the covariates, because of the simultaneous fitting procedure. We thus reparameterize $\beta(t)$ as

$$\beta(t) = \frac{\exp\left(\nu(t)\right)}{1+\xi(t)}, \quad \text{with} \quad \nu(t) = \log\left[\beta(t)\cdot(1+\xi(t))\right],$$

where we assume that $\xi(t) > -1$. We then let $\nu(t)$ linearly depend on covariates.

We model the parameters $\lambda(t), \xi(t)$ and $\nu(t)$ to be dependent on the macro economic covariates as follows:

$$\begin{aligned}
\lambda(t) &= \exp\left(m_1(t)\gamma_{\lambda,1} + m_2(t)\gamma_{\lambda,2} + \cdots + m_c(t)\gamma_{\lambda,c}\right) \\
\xi(t) &= m_1(t)\gamma_{\xi,1} + m_2(t)\gamma_{\xi,2} + \cdots + m_c(t)\gamma_{\xi,c} \\
\nu(t) &= m_1(t)\gamma_{\nu,1} + m_2(t)\gamma_{\nu,2} + \cdots + m_c(t)\gamma_{\nu,c},
\end{aligned} \tag{3}$$

where $\gamma_{p,1}, \ldots, \gamma_{p,c}$ are the regression coefficients for our temporal parameters $p \in \{\lambda, \xi, \nu\}$. Furthermore, $m_1(t), \ldots, m_c(t)$ are all c macro-economic covariates at time $t \in [0, T]$.

To determine the parameters $\gamma_{p,q}$ for $p \in \{\lambda, \xi, \nu\}$ and $q \in \{1, 2, \ldots, c\}$ we will make use of Maximum Likelihood estimation over our excesses. We focus on the time periods $\{t_1, \ldots, t_{N_u}\}$, where t_j corresponds to exceedance $\tilde{X}_j$ occuring, for $j = \{1, \ldots, N_u\}$. The covariate values on these exceedance time periods, combined with the log-likelihood maximization will then be used to determine the parameters.

We assume that the weekly returns are independent. We write the likelihood function as the product of marginal likelihoods for each excess as follows:

$$\begin{aligned}\mathcal{L}(\lambda(t),\xi(t),\beta(t);Y_1,\dots,Y_{N_u}) &= L_{\lambda,\xi,\beta}(\lambda(t),\xi(t),\beta(t);Y_1,\dots,Y_{N_u}) \\ &= L_\lambda(\lambda(t))\cdot L_{\xi,\beta}(\xi(t),\beta(t);Y_1,\dots,Y_{N_u}),\end{aligned} \tag{4}$$

where we can split the joint likelihood because of the asymptotic independence of our excesses and number of exceedances, as discussed in Section 2.1.

The likelihood for the arrival of exceedances over a time interval $[0,T]$ is

$$L_\lambda(\lambda(t)) = \exp\left(-\int_0^T \lambda(t)dt\right)\prod_{j=1}^{N_u}\lambda(t_j), \tag{5}$$

and the likelihood for our excesses $Y_1,\dots,Y_{N_u}$ can be formulated as

$$L_{\xi,\beta}(\xi(t),\beta(t);Y_1,\dots,Y_{N_u}) = \prod_{j=1}^{N_u} g_{\xi(t_j),\beta(t_j)}(Y_{t_j}), \tag{6}$$

with $g_{\xi(t_j),\beta(t_j)}(Y_{t_j})$ being the marginal likelihood function for each excess Y_{t_j}.

Following the expression in Equation (4), (5) and (6), we can formulate the log-likelihood function as

$$\log\mathcal{L}(\lambda(t),\xi(t),\beta(t);Y_1,\dots,Y_{N_u}) = \ell_\lambda(\lambda(t)) + \ell_{\xi,\beta}(\xi(t),\beta(t);Y_{t_1},\dots,Y_{t_{N_u}}), \tag{7}$$

with

$$\ell_\lambda(\lambda(t)) = \sum_{j=1}^{N_u}\log\lambda(t_j) - \sum_{t=1}^{T}\lambda(t),$$

where we replaced the integral term from Equation (5) by the sum since we assume that the intensity $\lambda(t)$ is constant in each time unit, and

$$\ell_{\xi,\beta}(\xi(t),\beta(t);Y_{t_1},\dots,Y_{t_{N_u}}) = \sum_{j=1}^{N_u}\log g_{\xi(t_j),\beta(t_j)}(Y_{t_j}),$$

where

$$\log g_{\xi(t_j),\beta(t_j)}(y) = \begin{cases} -\log(\beta(t_j)) - (1+1/\xi(t_j))\log(1+\frac{\xi(t_j)y}{\beta(t_j)}), & \text{if } \xi(t_j)>0, y\geqslant 0 \text{ or } \xi(t_j)<0, y\in[0,-\frac{\beta(t_j)}{\xi(t_j)}), \\ -\log(\beta(t_j)) - \frac{y}{\beta(t_j)} & \text{if } \xi(t_j)=0, \\ -\infty, & \text{otherwise.}\end{cases}$$

We can maximize the two likelihood functions seperately to estimate the parameters $\gamma_{\lambda,q}$ in $\lambda(t)$, $\gamma_{\xi,q}$ in $\xi(t)$ and $\gamma_{\nu,q}$ in $\beta(t)$ for $q \in \{1, \ldots, c\}$.

Eventually we estimate risk measures on our loss sequence to model our financial risk. Using the estimates of $\hat{\lambda(t)}$, $\hat{\xi(t)}$ and $\hat{\beta(t)}$, we can estimate two risk measures, the Value-at-Risk (VaR) and the Expected Shortfall (ES), conditionally on the sequence X_t, as follows

$$\widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{X}}(t) = u + \frac{\hat{\beta}(t)}{\hat{\xi}(t)} \left(\left(\frac{1-\alpha}{1-e^{-\hat{\lambda}(t)}} \right)^{-\hat{\xi}(t)} - 1 \right), \tag{8}$$

where we approximate the tail probability of exceedance $\bar{F}(u) = P\left(X > u\right)$ by $1 - e^{\hat{\lambda}(t)}$, and

$$\widehat{\mathrm{ES}}_{\alpha}^{\mathrm{X}}(t) = \begin{cases} \dfrac{\widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{X}}(t) + \hat{\beta}(t) - \hat{\xi}(t)u}{1-\hat{\xi}(t)}, & \text{if} \quad \hat{\xi}(t) \in (0,1), \\ \infty, & \text{if} \quad \hat{\xi}(t) \geqslant 1. \end{cases} \tag{9}$$

After estimating the conditional VaR and ES for X_t as $\widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{X}}(t)$, we estimate the conditional risk measures for our losses L_t, $\widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{L}}(t)$ and $\widehat{\mathrm{ES}}_{\alpha}^{\mathrm{L}}(t)$, by:

$$\begin{aligned} \widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{L}}(t) &= \hat{\mu}_t + \hat{\sigma}_t \cdot \widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{X}}(t) \\ \widehat{\mathrm{ES}}_{\alpha}^{\mathrm{L}}(t) &= \hat{\mu}_t + \hat{\sigma}_t \cdot \widehat{\mathrm{ES}}_{\alpha}^{\mathrm{X}}(t) \end{aligned} \tag{10}$$

where $\widehat{\mathrm{VaR}}_{\alpha}^{\mathrm{X}}(t)$ and $\widehat{\mathrm{ES}}_{\alpha}^{\mathrm{X}}(t)$ are the conditional VaR and ES from respectively Equation (8) and (9), and $\hat{\mu}_t$ and $\hat{\sigma}_t$ are respectively the estimated mean and standard deviation from the GARCH process in Equation (1).

2.3 Model Selection

Maximizing the likelihood in Equation (7) will give us estimates for the non-stationary parameters $\hat{\lambda}(t)$, $\hat{\xi}(t)$ and $\hat{\beta}(t)$ which we can use to estimate our risk measures. Since we have a large list of covariates, we have a large catalogue of potential models to choose from when estimating our parameters. We thus need to narrow down our model choices and pick the better performing model.

Before discussing the approaches let us first set out the model evaluation criterium. Our models will always be nested, thus we can use a likelihood ratio test to choose the covariate model which has the highest maximum likelihood score. We will use the deviance statistic $\mathcal{D}$, defined by Coles, Bawa, Trenner, and Dorazio (2001) as

$$\mathcal{D} = 2\left(\ell_1(\mathcal{M}_1) - \ell_0(\mathcal{M}_0)\right). \tag{11}$$

Here $\mathcal{M}_0$ and $\mathcal{M}_1$ represent two covariate models where $\mathcal{M}_0 \subset \mathcal{M}_1$. Also $\ell_0(\mathcal{M}_0)$ and $\ell_1(\mathcal{M}_1)$ are the maximized log-likelihoods under model $\mathcal{M}_0$ and $\mathcal{M}_1$ respectively. When $\mathcal{D}$ is large, this indicates that model $\mathcal{M}_1$ is preferred to model $\mathcal{M}_0$. This decision is made with α-level significance. In other words, we reject model $\mathcal{M}_0$ in favour of $\mathcal{M}_1$ if $\mathcal{D} > c_\alpha$, where c_α is the $(1-\alpha)$ quantile of the χ^2_p-distribution, with p the difference of amount of parameters in model $\mathcal{M}_0$ and $\mathcal{M}_1$.

2.3.1 Stepwise Regression Approach

The first approach we will use is the Stepwise Regression approach. Using this approach we iteratively add or remove covariates to our parameter models in Equation (3), ending up with a subset of covariates that best describe the non-stationary parameter of our interest. The Stepwise Regression approach is carried out in three ways: (i) Forward Selection, (ii) Backward Selection or (iii) Stepwise Selection.

With Forward Selection, we start off with a model with no covariates, iteratively adding the variable that increases the likelihood score the most, and stopping when we do not find a significant increase in maximum likelihood score. Backward Selection starts off with a model with all covariates implemented, iteratively removing the variable that has the most negative impact on the likelihood score, and stopping when no significant increase in maximum likelihood score can be achieved. Stepwise Selection combines Forward and Backward Selection: starting off with a model with no covariates, we first add a new variable to the model which increases the likelihood score the most and then remove any variables that have negative impact on the likelihood score. This procedure will be carried out iteratively until our maximum likelihood score does not significantly increase.

2.3.2 Penalized Regression Approach

One of the most common issues when fitting variables in a regression model is the risk of overfitting. To avoid overfitting we can simplify our model by reducing the amount of variables it holds. However, we want to take the bias-variance regression trade-off in account when doing this, which means we can not simplify our model too much without having a significant increase in its variance. The Penalized Regression approach takes this into account by using regularized models.

Regularized models contain a regularization term that penalizes our parameter models for each covariate it adds. In other words, by using the Penalized Regression approach, we only add variables that significantly contribute to our parameter model. To start off we need to rewrite our log-likelihood formula in Equation (7) as follows:

$$\log \mathcal{L}(\lambda(t), \xi(t), \beta(t); Y_1, \ldots, Y_{N_u}) = \ell_\lambda(\lambda(t)) + \ell_{\xi,\beta}(\xi(t), \beta(t); Y_{t_1}, \ldots, Y_{t_{N_u}}) - \sum_p \pi_p \|\gamma_p\|_v^v, \quad (12)$$

where we call the last term our penalization term. In this term $\|\cdot\|_v$ is the standard l_v-norm, thus $\|\gamma_p\|_v^v = \left(\sum_{q=1}^c |\gamma_{p,q}|^v\right)$, and π_p is the regularization term for parameter $p \in \{\lambda, \xi, \beta\}$. When maximizing the likelihood in Equation (12), we will penalize the likelihood score by the amount of variables that have been added to the parameter models. We will implement the Penalized Regression approach in three ways: (i) Ridge Regression, (ii) Least Absolute Shrinkage and Selection Operator (LASSO) Regression and (iii) ElasticNet Regression.

Ridge Regression, according to Hoerl and Kennard (1970), is a technique that takes multicollinearity into account. It penalizes the maximum likelihood score by implementing the squared l_2-norm for every variable added. In other words, we take $v = 2$, and thus penalize every variable added by its squared value. LASSO Regression, according to Tibshirani (1996), implements the absolute l_1-norm as penalization, thus with $v = 1$, penalizing the likelihood by every variable's absolute value. This makes that some coefficients are shrinked to exactly zero, in contrast to Ridge Regression, which increases the interpretability and simplicity of the model.

Finally, ElasticNet Regression, as described by Zou and Hastie (2005), combines the penalizations of Ridge and LASSO Regression, by implementing both an l_1-norm as well an l_2-norm in the penalization term. To implement this we need to rewrite our penalization term, such that the target likelihood becomes

$$\log \mathcal{L}(\lambda(t), \xi(t), \beta(t); Y_1, \ldots, Y_{N_u}) = \ell(\lambda(t)) + \ell(\xi(t), \beta(t); Y_{t_1}, \ldots, Y_{t_{N_u}}) - \sum_{v=1}^{2} \sum_p \pi_p \|\gamma_p\|_v^v. \quad (13)$$

3 Data

3.1 Stock Data description

In this thesis we investigate the market risk of six European countries that are part of the European Monetary Union (EMU): France, Germany, Italy, the Netherlands, Portugal and Spain.

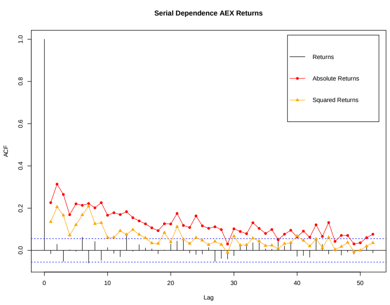
All data are collected from Bloomberg Finance or Thomas DataStream. For each country we pick the major stock index, described in Appendix A.1. We download all available end-of-day stock index prices from 1 January 1995 to 31 December 2018. We transform these prices to weekly return indices. This frequency is chosen over monthly return indices to have enough observations in our data sample to analyse by means of Extreme Value Theory. The descriptive statistics of these weekly returns are depicted in Table 1.

Table 1: Descriptive Statistics Indices Weekly Returns

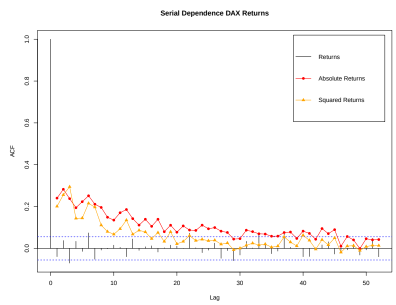
Index	**# Losses**	μ (%)	σ (%)	**Min**	**Q1**	**Median**	**Q3**	**Max**
AEX	527	0.038	2.890	-15.246	-1.266	0.192	1.543	14.171
DAX	522	0.100	3.000	-14.822	-1.267	0.333	1.778	16.116
CAC40	538	0.044	2.898	-13.754	-1.315	0.227	1.650	15.481
PSI-20	553	0.009	2.631	-17.241	-1.284	0.115	1.441	14.568
IBEX35	530	0.074	2.952	-13.209	-1.487	0.191	1.761	13.126
FTSE MIB	468	-0.024	3.195	-13.726	-1.572	0.214	1.672	12.750

μ (%) = Sample mean (in %) σ (%) = Sample standard deviation (in %) Min = Minimum sample return Q1 = 1^{st} sample quantile
Q3 = 3^{rd} sample quantile Max = Maximum sample return
Note: Sample period is from January 1995 to December 2018.

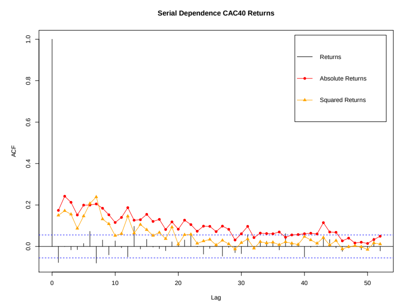
Moreover, we investigate whether our weekly returns suffer from a high serial dependence, such that we can still assume independence in our data sample. Figure 1 depicts the autocorrelation function of our weekly returns, absolute weekly returns and squared weekly returns for the AEX Index. When looking at a horizon of 48 lags (being approximately one trading year of weekly returns), we see very little autocorrelation. The absolute and squared weekly returns obviously show some autocorrelation for the first lags, meaning that we are dealing with serial dependent data. We thus apply the methodology in Section 2 to the filtered residuals series $\{X_t\}$.



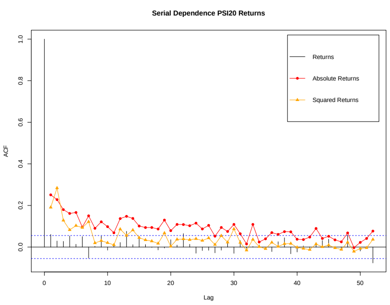
(a) AEX Index



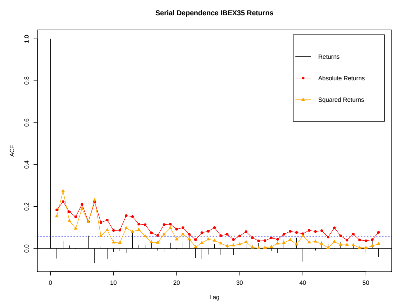
(b) DAX Index



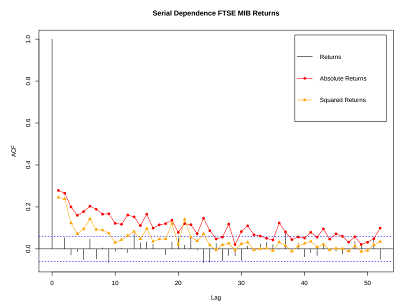
(c) CAC40 Index



(d) PSI-20 Index



(e) IBEX35 Index



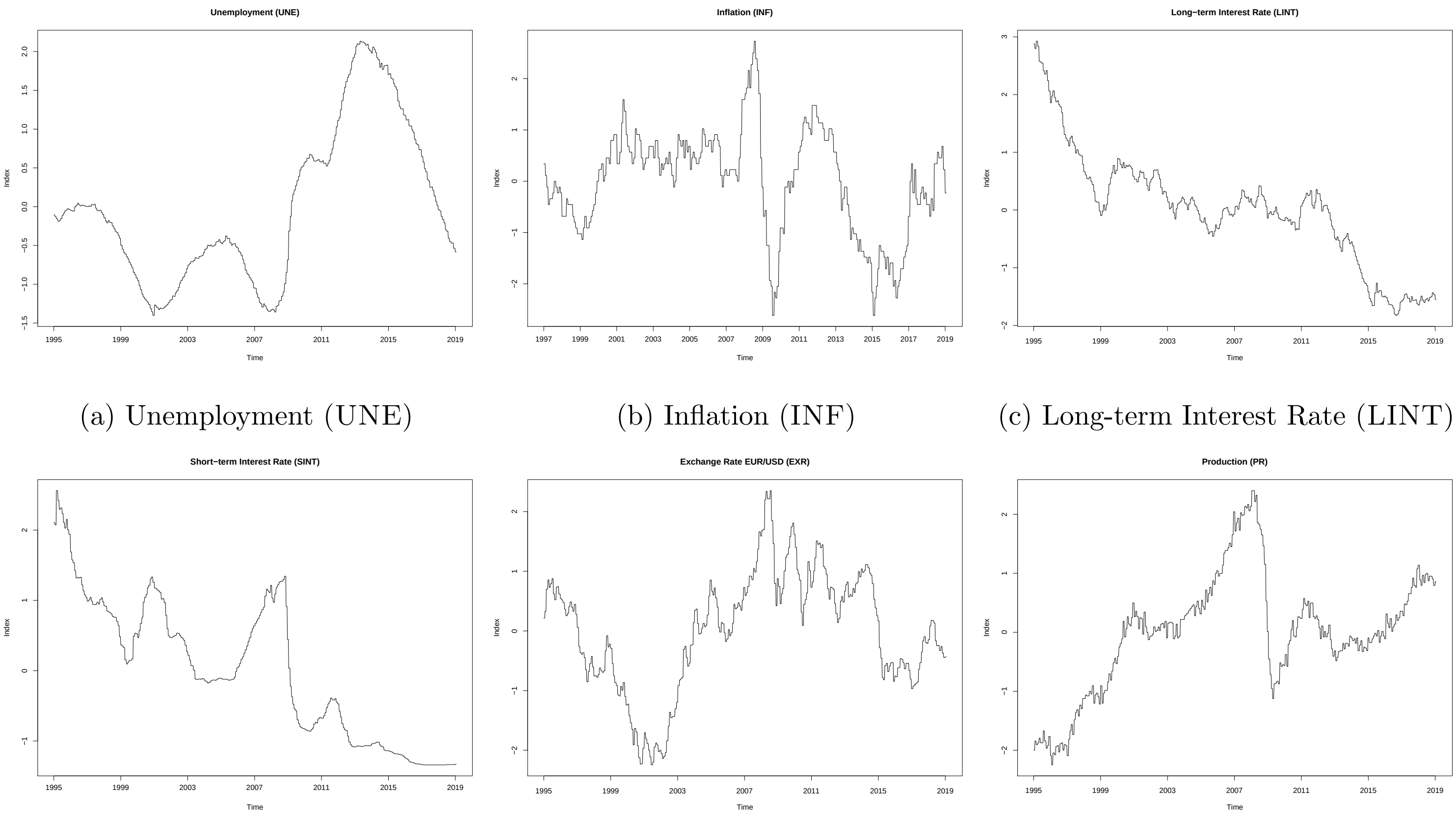
(f) FTSE MIB Index

Figure 1: Serial Dependence of Indices: Plotted Autocorrelation of the Returns (solid black lines), Absolute Returns (red lines) and Squared Returns (orange lines) over 48 lags

3.2 Covariates

Next to stock data we have a limited selection of macro-economic covariates. We choose the following Eurozone macro-economic variables to be used as a covariate in our analysis: unemployment (UNE); inflation (INF); long-term interest rate (LINT); short-term interest rate (SINT); USD/EUR exchange rate (EXR) and industrial production (PR). We collect the covariates from the statistical office of the European Union Eurostat, if possible, or from Bloomberg Finance. A detailed description of the chosen covariates can be found in Appendix A.2.

The covariate sample exists of monthly covariates. Given that the serial dependence is low enough to not violate the independence assumption, and that our returns are described as weekly losses, we convert the chosen covariates to a weekly frequency by interpolation. We interpolate the weekly covariates from the monthly covariates by assuming them to be the same during each week. In other words, considering that each month consists of four or occasionally five weeks, we take the same weekly covariate value for each four or five weeks of every month. Next to that, each covariate has a different size/measurement unity. We standardize these by demeaning each series and dividing them by their standard deviation. Figure 2 depicts all standardized weekly covariates over their available time periods.



(a) Unemployment (UNE) (b) Inflation (INF) (c) Long-term Interest Rate (LINT)

(d) Short-term Interest Rate (SINT) (e) USD/EUR Exchange Rate (EXR) (f) Production (PR)

Figure 2: Standardized Macro-economic Covariates plotted over Sample Period from 1st of January 1995 to 31st of December 2018

4 Empirical Results

In this section we apply the methodology set up in Section 2 to the financial losses. We start off by choosing the threshold per index by means of a mean excess plot. Next we set up a benchmark model, estimating parameters with no covariates, and examine our selected models by analyzing the chosen covariate parameters for the GPD parameters ξ, β and intensity parameter λ. We compare the parameters and likelihood score of each model, and choose the best performing models using the Akaike Information Criterion (AIC). For these models we then estimate the VaR and ES risk measures and compare these with the benchmark model per index by using backtesting.

4.1 Threshold Choice

Statistical inference on the POT models starts with fitting the GPD to the observed threshold exceedances, determined by the choice of threshold u. When we choose the threshold too low, we will violate the asymptotic properties of the underlying distribution, leading to a high bias. However, choosing the threshold too high will lead to very few exceedances resulting in a high estimation uncertainty.

One method to choose the threshold is based on the mean excess plot. This approach is based on the linearity of the mean excess function $e(u)$, as described by (McNeil et al., 2015, p. 150). We use the sample mean excess function

$$e_{N_u} = \frac{1}{N_u} \sum_{j=1}^{N_u} Y_j,$$

where N_u are the number of exceedances above threshold u and Y_j are the excesses. When plotting this together with the ordered sequence of data one gets a mean excess plot, defined as $\{(u, e_{N_u}) : 2 \leqslant i \leqslant N_u\}$, which effectively leads to evaluating the mean excess at higher order statistics of X_i. The linear property of the mean excess function then tells us that this plot should be approximately linear above a proper threshold.

Figure 3 shows the mean excess plot for the weekly losses of the DAX index. We observe a clear downward-sloping linear pattern of the plotted exceedances up until threshold level 1.06. From there we observe a clear break in the linear pattern, where it changes to a linear pattern with a different slope. This is until the threshold level of 2.3, where we again detect a linear pattern with different slope above this threshold level. Since we are searching a linear pattern above the threshold, we would normally choose the threshold level to be 2.3. However, at the right-hand side of a mean excess plot we are plotting the mean of a small number of large excesses. Adopting this threshold leads to very few exceedances on which we can perform the model estimation, leading to a high uncertainty and thus variance. Hence, to be certain we do not choose the threshold too high, we appoint it to be the level above which we observe a linear pattern and have a sufficient amount of excesses. We assume an amount as sufficient when it consists of approximately 10-20% of the full sample. In other words, we thus choose our threshold level u to be 1.06 for the DAX index, which is marked with the red dotted line in Figure 3. The mean excess plots for all other indices can be found in Appendix B.1.

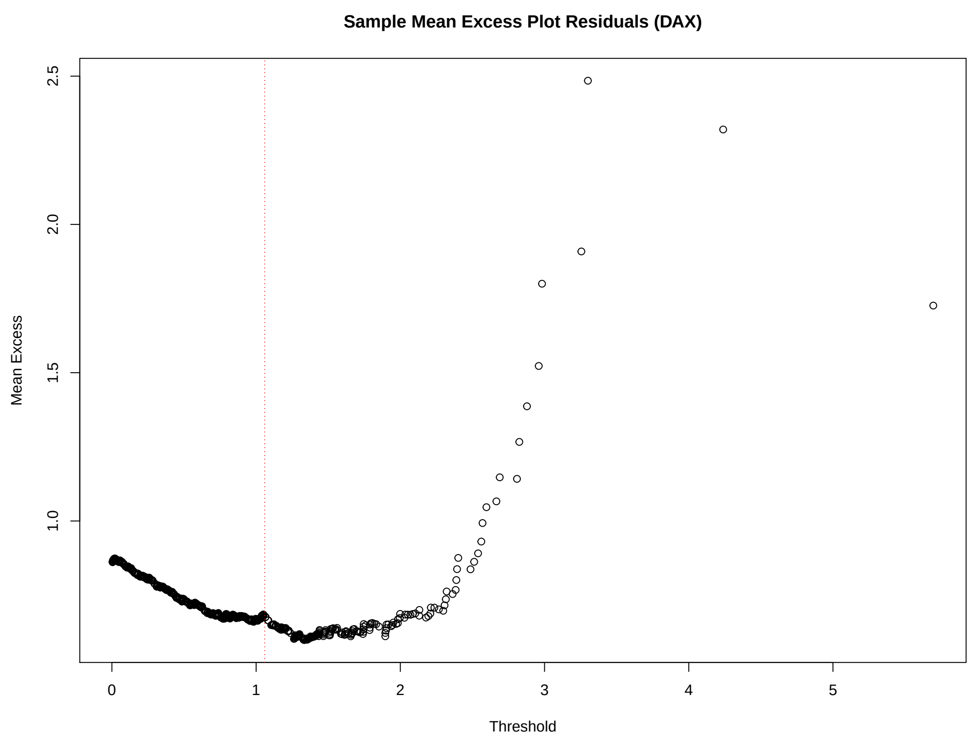


Figure 3: Mean Excess Plot of Residuals X_t of DAX Index

Table 2 shows us the chosen threshold levels for the indices, together with the total number of exceedances above the chosen threshold and in which percentile this threshold falls. We see that all chosen thresholds lead to exceedances of around 17% to 14% of the full residuals sample, with the exemption being the FTSE MIB index with exceedances of 20% of the full sample.

Table 2: Threshold Level and Percentile of Indices

Index	**AEX**	**DAX**	**CAC40**	**PSI-20**	**IBEX35**	**FTSE MIB**
Threshold u	1.23	1.06	1.00	0.81	1.13	0.86
# Exceedances	168	186	206	230	170	218
Percentile	86%	85%	83%	81%	86%	80%

Note: This table shows the number of exceedances above each threshold selected per index, as well as the percentile of total residuals corresponding to the selected threshold

4.2 Model Selection

4.2.1 Benchmark Model

Before adding covariates to the parameters of the GPD, we first estimate a 'benchmark' model, where the parameters ξ, β and λ are not dependent on covariates. Table 3 shows the benchmark models for all indices. We see that the value of the shape parameter ξ and scale parameter β is somewhat equal for the DAX, CAC40 and IBEX35 index. This means that the benchmark model, without covariates implemented, estimated the financial tail distribution of losses to be similar in these indices. The intensity parameter λ is mostly different among the indices. We only observe a resemblance for the AEX and IBEX35 index, meaning that the occurence of losses above their threshold follows a similar Poisson point process.

4.2.2 Covariate Models

Next, we add the covariates to our GPD model parameters. The results of estimation are shown in Table 4-11. In each table, we show the parameter estimated, with the standard error in brackets below. When a model does not use a covariate in describing the parameters for any of the indices, it is left out of the table. The last lines of every table show the log-likelihood and Akaike Information Criterion (AIC) for each model. The AIC, as formulated in Akaike (1998), is an approach to compare non-nested models with a possible different amount of parameters. It is calculated as $\text{AIC}_m = -2\ell_m(\mathcal{M}_m) + 2p_m$, where $\ell_m(\mathcal{M}_m)$ is the maximized log-likelihood of model $\mathcal{M}_m$, with m being one of the six model selection approaches, and p_m are the amount of variables used in estimating model $\mathcal{M}_m$. We favour the model for which the AIC criterion is the smallest.

Table 3: Estimated Benchmark Model without Covariate Dependence

Index	**AEX**	**DAX**	**CAC40**	**PSI-20**	**IBEX35**	**FTSE MIB**
ξ	0.138	0.033	0.038	0.015	0.036	-0.022
	(0.090)	(0.055)	(0.055)	(0.058)	(0.051)	(0.045)
β	0.507	0.653	0.596	0.680	0.592	0.696
	(0.060)	(0.060)	(0.053)	(0.060)	(0.055)	(0.057)
Likelihood	-76.918	-112.830	-107.084	-144.697	-87.156	-134.238
AIC	157.836	229.660	218.168	293.393	178.313	272.476
λ	0.134	0.149	0.165	0.184	0.136	0.199
	(0.010)	(0.011)	(0.011)	(0.012)	(0.010)	(0.013)
Likelihood	-505.434	-540.656	-577.752	-619.716	-509.439	-570.054
AIC	1012.867	1083.311	1157.504	1243.432	1020.878	1142.108

Note: This table shows the estimated parameters with standard errors between brackets below.
The sample period is from January 1995 to December 2018.

Table 4-6 show the estimated covariate parameters for the ξ and β parameters for all indices using the Stepwise Regression approaches. Using Forward Selection we observe in Table 4 that the estimation of the scale parameter β is not dependent on covariates using this model. The shape parameter ξ depends on one covariate however, different for almost all indices. Moreover, no constant is added to the models of the different shape parameters for all indices. In terms of AIC score this model outperforms the benchmark model by having a lower AIC score for all indices.

Table 5 depicts the estimated covariate parameters for the Backwards Selection model. We notice that, in contrast to the Forward Selection model, almost all covariates are included in describing the shape and scale parameters for all indices. However, the standard errors depicted in brackets below each covariate parameter estimate are high for most covariates. The AIC score for this model is also higher than for the benchmark model. We conclude that this model is overfitted.

Table 6 shows similar results for the Stepwise Selection model to the Forward Selection model. In fact, for the DAX, CAC40 and PSI-20 index the Stepwise Selection model chooses exactly the same covariates. When investigating the AEX, IBEX35 and FTSE MIB index, we notice that the AIC score is higher compared to the AIC score when using the Forward Selection model for these indices.

Table 4: Covariate Parameters and Likelihood describing GPD parameters $\hat{\xi}$ and $\hat{\beta}$ using Forward Selection

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
Parameter	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β
Inflation	0.097						0.048					
	(0.045)						(0.040)					
Short-Term									0.095		0.105	
Interest Rate									(0.048)		(0.053)	
Exchange Rate			0.103									
			(0.044)									
Production					0.163							
					(0.067)							
Constant		0.576		0.679		0.600		0.686		0.609		0.689
		(0.044)		(0.051)		(0.042)		(0.045)		(0.047)		(0.048)
Likelihood	-76.396		-110.542		-104.093		-143.957		-84.940		-132.224	
AIC	156.793		225.084		212.186		291.915		173.880		268.449	

Note: This table shows the estimated covariate parameters with standard errors between brackets below.

Unemployment and long-term interest rate have no covariate parameters included for any of the indices and are thus left out of this table.

The sample period is from January 1995 to December 2018.

Table 5: Covariate Parameters and Likelihood describing GPD parameters $\hat{\xi}$ and $\hat{\beta}$ using Backward Selection

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
Parameter	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β
Unemployment	-0.096	0.158	-0.076	0.048	0.217		0.137	0.048		0.104	0.167	0.082
	(0.229)	(0.107)	(0.530)	(0.203)	(0.352)		(0.174)	(0.115)		(0.086)	(0.180)	(0.120)
Inflation	0.216	-0.122	0.120	-0.118	0.302	-0.132	0.233	-0.098	0.336	-0.243	0.117	-0.159
	(0.212)	(0.096)	(0.134)	(0.089)	(0.541)	(0.403)	(0.135)	(0.114)	(0.124)	(0.094)	(0.127)	(0.126)
Long-Term	-0.127		-0.172	-0.143	-0.288	0.070	-0.400	0.105	-0.132		0.101	0.034
Interest Rate	(0.248)		(0.251)	(0.190)	(0.363)	(0.251)	(0.254)	(0.149)	(0.176)		(0.215)	(0.198)
Short-Term		0.183	0.094	0.232	0.142	0.036	0.274		-0.011	0.232	0.217	
Interest Rate		(0.100)	(0.408)	(0.270)	(0.265)	(0.160)	(0.147)		(0.096)	(0.107)	(0.122)	
Exchange Rate	-0.047	-0.050	0.201	-0.077	-0.156	0.006	-0.115	0.035	0.138	-0.061	-0.035	-0.099
	(0.267)	(0.087)	(0.193)	(0.086)	(0.552)	(0.149)	(0.114)	(0.078)	(0.079)	(0.078)	(0.173)	(0.095)
Production	-0.079	0.193				0.104	-0.270	0.238	-0.250	0.243	-0.131	0.254
	(0.237)	(0.094)				(0.191)	(0.169)	(0.151)	(0.109)	(0.129)	(0.179)	(0.190)
C	0.078	0.521	-0.151	0.767	0.002	0.671		0.678	-0.125	0.689		0.699
	(0.095)	(0.063)	(0.065)	(0.079)	(0.080)	(0.095)		(0.047)	(0.064)	(0.073)		(0.054)
Likelihood	-72.367		-103.413		-103.124		-139.042		-78.612		-127.895	
AIC	168.734		230.827		230.248		302.084		181.224		279.790	

Note: This table shows the estimated covariate parameters with standard errors between brackets below.

The sample period is from January 1995 to December 2018.

Table 6: Covariate Parameters and Likelihood describing GPD parameters $\hat{\xi}$ and $\hat{\beta}$ using Stepwise Selection

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
Parameter	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β
Unemployment											-0.088	
											(0.057)	
Inflation							0.048		0.086			
							(0.040)		(0.048)			
Short-Term	0.090											
Interest Rate	(0.047)											
Exchange Rate			0.103									
			(0.044)									
Production					0.163							
					(0.067)							
C		0.580		0.679		0.600		0.686		0.608		0.680
		(0.045)		(0.051)		(0.042)		(0.045)		(0.047)		(0.046)
Likelihood	-76.595		-110.542		-104.093		-143.957		-85.088		-133.024	
AIC	157.189		225.084		212.186		291.915		174.176		270.048	

Note: This table shows the estimated covariate parameters with standard errors between brackets below.

long-term interest rate has no covariate parameters included for any of the indices and is thus left out of this table.

The sample period is from January 1995 to December 2018.

Table 7: Covariate Parameters and Likelihood describing GPD parameters $\hat{\xi}$ and $\hat{\beta}$ using Ridge Regression

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
Parameter	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β
Unemployment		0.066					0.183			0.050		
		(0.072)					(0.107)			(0.071)		
Inflation			0.051	-0.143	0.101	-0.063	0.144		0.241	-0.235	0.143	-0.109
			(0.179)	(0.084)	(0.048)	(0.058)	(0.082)		(0.092)	(0.086)	(0.046)	(0.057)
Long-Term			0.135				-0.275					
Interest Rate			(0.206)				(0.153)					
Short-Term		0.119					0.264			0.159		
Interest Rate		(0.076)					(0.115)			(0.085)		
Exchange Rate			0.178	-0.104			-0.089		0.107			-0.035
			(0.082)	(0.070)			(0.059)		(0.060)			(0.044)
Production		0.132	-0.011	0.114			-0.157	0.132	-0.163	0.190		
		(0.071)	(0.195)	(0.070)			(0.121)	(0.066)	(0.117)	(0.118)		
C	0.587	0.574	-0.088	0.732	-0.037	0.639	-0.069	0.723	-0.108	0.684	-0.140	0.782
	(0.072)	(0.045)	(0.066)	(0.074)	(0.063)	(0.061)	(0.064)	(0.066)	(0.060)	(0.070)	(0.053)	(0.069)
Likelihood	-75.632		-106.554		-105.188		-139.584		-79.553		-129.602	
AIC	161.264		231.108		218.376		297.168		177.106		269.205	

Note: This table shows the estimated covariate parameters with standard errors between brackets below.

The sample period is from January 1995 to December 2018.

Table 8: Covariate Parameters and Likelihood describing GPD parameters $\hat{\xi}$ and $\hat{\beta}$ using LASSO Regression

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
Parameter	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β
Unemployment							0.183		-0.446	0.251		
							(0.107)		(0.230)	(0.125)		
Inflation			0.157	-0.137	0.101	-0.063	0.144		0.341	-0.238	0.143	-0.109
			(0.070)	(0.085)	(0.048)	(0.058)	(0.082)		(0.118)	(0.093)	(0.046)	(0.057)
Long-Term							-0.275		-0.383			
Interest Rate							(0.153)		(0.173)			
Short-Term		0.061					0.264		-0.259	0.380		
Interest Rate		(0.044)					(0.115)		(0.149)	(0.129)		
Exchange Rate			0.161	-0.097			-0.089		0.264	-0.106		-0.035
			(0.073)	(0.069)			(0.059)		(0.118)	(0.094)		(0.044)
Production		0.095	-0.120	0.111			-0.157	0.132	-0.451	0.274		
		(0.056)	(0.078)	(0.069)			(0.121)	(0.066)	(0.174)	(0.156)		
C	0.587	0.573	-0.093	0.736	-0.037	0.639	-0.069	0.723	-0.137	0.712	-0.140	0.782
	(0.044)	(0.044)	(0.066)	(0.074)	(0.063)	(0.061)	(0.064)	(0.066)	(0.073)	(0.085)	(0.053)	(0.069)
Likelihood	-76.048		-106.826		-105.188		-139.584		-77.364		-129.602	
AIC	160.095		229.652		218.376		297.168		180.729		269.205	

Note: This table shows the estimated covariate parameters with standard errors between brackets below.

The sample period is from January 1995 to December 2018.

Table 9: Covariate Parameters and Likelihood describing GPD parameters $\hat{\xi}$ and $\hat{\beta}$ using ElasticNet Regression

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
Parameter	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β	ξ	β
Unemployment		-0.027					0.234					
		(0.041)					(0.104)					
Inflation				-0.024	0.101	-0.063	0.065		0.154	-0.153		-0.006
				(0.057)	(0.048)	(0.058)	(0.067)		(0.086)	(0.067)		(0.042)
Long-Term							-0.160					
Interest Rate							(0.124)					
Short-Term							0.266			0.114		
Interest Rate							(0.113)			(0.062)		
Exchange Rate			0.181	-0.113			-0.109		0.111			-0.029
			(0.058)	(0.066)			(0.059)		(0.059)			(0.044)
Production		0.073		0.047				0.099	-0.020			
		(0.049)		(0.053)				(0.062)	(0.082)			
C	0.587	0.574	-0.058	0.707	-0.037	0.639	-0.065	0.714	-0.118	0.697	-0.017	0.695
	(0.041)	(0.044)	(0.065)	(0.069)	(0.063)	(0.061)	(0.064)	(0.066)	(0.059)	(0.072)	(0.046)	(0.056)
Likelihood	-76.818		-108.685		-105.188		-140.171		-80.918		-133.960	
AIC	161.636		229.369		218.376		296.342		175.835		275.920	

Note: This table shows the estimated covariate parameters with standard errors between brackets below.

The sample period is from January 1995 to December 2018.

Table 7-9 show the estimated covariate parameters for the ξ and β parameters using the Penalized Regression approach, for all six indices. When applying the Penalized Regression approaches, different covariates describe the shape and scale parameters for different indices. However, none of the Penalized Regression approaches seem to improve the estimation of the shape and scale parameters of any of the indices, when examining the AIC score.

We conclude that estimating the shape and scale parameters for the indices by the different approaches presented lead to different models selected. However, not all models result in selecting the most relevant covariates. When we observe the AIC score per model for each index, we find that this score is minimized for all indices when we use the Forward Selection model. We thus select the covariate parameters from Table 4 to describe the dynamic shape and scale parameters for all indices.

Next, Table 10 & 11 depict the covariate parameter estimates for respectively the Stepwise Regression and Penalized Regression approaches for the intensity parameter λ.

In Table 10 we observe that the Forward Selection model does not pick any of the covariates as a parameter to describe λ. However, the Backward Selection model chooses to include almost all covariates again. Nonetheless, the standard errors of these covariate estimates are again high when comparing them with the estimated parameters, instigating the assumption of a model that is overfitted. The Stepwise Selection model results in exactly the same covariate parameter estimations as the Forward Selection model and is thus left out of the table.

Table 11 displays that when choosing covariates by means of the Penalized Regression approaches, we obtain different covariates for different indices. In case of the DAX and IBEX35 index, this leads to a higher log-likelihood and lower AIC score than the Forward Selection model. The inflation and long-term interest rate covariate are not chosen in describing the intensity parameter λ and are thus left out of the table.

Table 10: Covariate Parameters and Likelihood describing parameter $\hat{\lambda}$ using Stepwise Regression models

Index	**AEX**	**DAX**	**CAC40**	**PSI20**	**IBEX35**	**FTSE MIB**
			Forward Selection			
Constant	-1.925	-1.824	-1.721	-1.611	-1.913	-1.531
	(0.077)	(0.073)	(0.070)	(0.066)	(0.077)	(0.068)
Likelihood	-491.449	-525.173	-560.604	-600.570	-495.288	-551.808
AIC	984.898	1052.345	1123.208	1203.141	992.575	1105.616
			Backward Selection			
Unemployment	0.084	0.143	0.265	0.189	0.289	0.077
	(0.162)	(0.156)	(0.149)	(0.123)	(0.160)	(0.135)
Inflation	0.066	0.080	0.066	0.163		0.089
	(0.105)	(0.100)	(0.094)	(0.088)		(0.088)
Long-Term				-0.392	0.120	
Interest Rate				(0.171)	(0.223)	
Short-Term	0.043	0.106	0.245	0.408	0.097	0.079
Interest Rate	(0.155)	(0.149)	(0.143)	(0.205)	(0.235)	(0.148)
Exchange Rate	-0.116	-0.160	-0.120	-0.046	-0.145	0.030
	(0.095)	(0.090)	(0.087)	(0.073)	(0.101)	(0.084)
Production	0.192	0.200	0.253		0.394	0.079
	(0.128)	(0.123)	(0.116)		(0.144)	(0.127)
Constant	-1.943	-1.842	-1.740	-1.626	-1.944	-1.544
	(0.079)	(0.075)	(0.071)	(0.067)	(0.079)	(0.078)
Likelihood	-488.523	-521.670	-556.627	-597.032	-490.325	-549.336
AIC	989.046	1055.340	1125.254	1206.065	992.651	1110.671

Note: This table shows the estimated covariate parameters with standard errors between brackets below.
Inflation and long-term interest rate have no covariate parameters included for any of the indices and any of the models and are thus left out of this table.
The sample period is from January 1995 to December 2018.

We conclude that estimating the intensity parameter λ by the regression approaches again lead to different model choices, albeit not for all approaches. Comparing the AIC scores we select the covariate parameters from the Forward Selection approach in Table 10 to describe the dynamic intensity parameter for the AEX, CAC40, PSI-20 and FTSE MIB index. For the DAX and IBEX35 index we find the lowest AIC score in Table 11. We thus use the covariate parameters using the Ridge Regression approach for the DAX index and the ElasticNet Regression approach for the IBEX35 index.

Table 11: Covariate Parameters and Likelihood describing parameter $\hat{\lambda}$ using Penalized Regression models

Index	**AEX**	**DAX**	**CAC40**	**PSI20**	**IBEX35**	**FTSE MIB**
			Ridge Regression			
Unemployment			0.261	-0.015	0.112	
			(0.147)	(0.088)	(0.085)	
Short-Term			0.275	0.002		
Interest Rate			(0.135)	(0.087)		
Exchange Rate	-0.052	-0.109	-0.114		-0.069	
	(0.077)	(0.073)	(0.086)		(0.085)	
Production		0.155	0.279		0.252	
		(0.078)	(0.107)		(0.094)	
Constant	-1.927	-1.838	-1.739	-1.611	-1.938	-1.531
	(0.077)	(0.074)	(0.071)	(0.066)	(0.079)	(0.068)
Likelihood	-491.223	-522.525	-556.874	-600.538	-491.337	-551.808
AIC	986.446	1051.049	1123.748	1207.076	990.674	1105.616
			LASSO Regression			
Unemployment			0.006	-0.015	0.112	
			(0.079)	(0.088)	(0.085)	
Short-Term				0.002		
Interest Rate				(0.087)		
Exchange Rate		-0.083	-0.029		-0.069	
		(0.073)	(0.076)		(0.085)	
Production			0.131		0.252	
			(0.080)		(0.094)	
Constant	-1.925	-1.827	-1.729	-1.611	-1.938	-1.531
	(0.077)	(0.074)	(0.070)	(0.066)	(0.079)	(0.068)
Likelihood	-491.449	-524.527	-558.999	-600.538	-491.337	-551.808
AIC	984.898	1053.054	1125.998	1207.076	990.674	1105.616
			ElasticNet Regression			
Unemployment			0.261			
			(0.147)			
Short-Term			0.275			
Interest Rate			(0.135)			
Exchange Rate		-0.083	-0.114			
		(0.073)	(0.086)			
Production			0.279		0.190	
			(0.107)		(0.077)	
Constant	-1.925	-1.827	-1.739	-1.611	-1.931	-1.531
	(0.077)	(0.074)	(0.071)	(0.066)	(0.078)	(0.068)
Likelihood	-491.449	-524.527	-556.874	-600.570	-492.250	-551.808
AIC	984.898	1053.054	1123.748	1203.141	988.499	1105.616

Note: This table shows the estimated covariate parameters with standard errors between brackets below. None of the covariates have covariate parameters included for any of the indices when using the 'Forward Selection' model and are thus left out of this table. The sample period is from January 1995 to December 2018.

4.2.3 Economic Relevance

We investigate the selected covariate parameters in the economic context. Since the covariates are standardized monthly growth rates interpolated to a weekly frequency, we can interpret the parameters as follows. The shape parameter ξ is estimated linearly dependent on covariates in Equation (3). This means that the constant represents the arithmetic mean of ξ, being insignificantly different from zero. The covariate parameter estimates for ξ can then directly be seen as an increase or decrease of the shape parameter when that covariate increases or decreases over time. For example, if in Table 4 for the AEX the monthly inflation growth rate increases by one standard deviation, the shape parameter will increase by around 0.097 if all other covariates are held constant. The interpretation for the scale parameter β is different, because of the reparameterization with the ν parameter. This means that the logarithmic value of β, scaled by a factor of $1 + \xi$, is linearly dependent on covariates. The same is the case for λ, of which its log value is linearly dependent on covariates. This means that the constant now represents the geometric mean of the parameter β or λ respectively, guaranteeing a positive β and λ. The covariate parameter estimates now need to be interpreted as an increase or decrease of the log value of the scale or intensity parameter. This in practice means for example for the β parameter in Table 5 for the AEX that an increase of the monthly unemployment growth rate by one standard deviation leads to an increase of our scale parameter of around 17.1% ($e^{0.158} \approx 1.1712$) when holding all other variables constant.

We examine the impact of every index covariate parameter on the 95%-significant VaR of the residuals in Table 12. We calculate the VaR when all covariates are assumed to be zero or, in other words, at the mean of the covariates. Then we examine the VaR level when we observe a shock. This corresponds with an increase of one standard deviation for one covariate, whilst the other variables are held constant. We also denote the percentage change or increment of the VaR level.

We observe that for four out of six indices only the shape parameter is influenced by macro-economic covariates. This is inflation for the AEX and PSI-20 index, production for the CAC40 index and the short-term interest rate for the FTSE MIB index. Next to that, for the DAX and IBEX35 index, also the intensity parameter is influenced by macro-economic covariates. For the DAX index this is the USD/EUR exchange rate, which is the same covariate as for the shape parameter. For the IBEX35 index we notice that the short-term interest rate influences the shape parameter, whereas production affects the intensity parameter. The effect of a shock in these parameters is thus shown separately in Table 12.

Table 12: Impact of estimated covariates in GPD parameters on 95%-significant VaR of Residuals X_t for all indices

Index	**AEX**	**DAX**	**CAC40**	**PSI-20**	**IBEX35**		**FTSE MIB**
Parameter	ξ	ξ & λ	ξ	ξ	ξ	λ	ξ
VaR at mean	1.805	1.793	1.712	1.693	1.814	1.814	1.796
VaR after shock	1.834	1.759	1.785	1.720	1.848	1.935	1.866
Increment	1.6%	-1.9%	4.3%	1.6%	1.9%	6.7%	3.9%

Note: This table depicts the impact of the estimated covariates in the parameters shown on the 95%-significant Value-at-Risk for all indices. VaR at mean is the VaR level when all covariates are held zero, or in other words, the VaR level at the mean of the covariates. VaR after shock is the VaR level after an increase of the covariates with one standard deviation. Increment is the percentage change of the VaR level from the VaR at mean to the VaR after shock.

All in all we conclude that the covariates of inflation, short-term interest rate, exchange rate and production all influence the VaR of the six indices. When inflation, short-term interest rate or production increases, the VaR also increases. A higher inflation means higher price growth and thus there is more uncertainty of the possible costs or investments of large companies on the market. These can become more expensive and increase the VaR. When this risk namely materializes, this can lead to higher losses. Higher short-term interest rate leads to money on the money market becoming more expensive. Because of this, the uncertainty of the costs of company investments increases, leading to potentially higher costs in the future and thus an increase of the VaR. Namely, when this risk materializes this again leads to higher losses. Higher production in essence are a signal of a growing economy and leads to more company profit. The VaR is thus not expected to increase when the industrial production increases, which contradicts our empirical findings. A potential reason is as follows: a growing economy may lead to other uncertainties emerging. For example, investors might assume that a high production growth will lead to less production growth in the future, when e.g. a shock or recession occurs. This can lead to a higher VaR, even when the economy is growing. Then, in times of recession, this risk materializes and leads to higher losses. Lastly, we also noticed that when the USD/EUR exchange rate increases, the VaR decreases. Since a higher USD/EUR exchange rate means that the euro is stronger with regards to the US dollar, this signals a strong state of the economy of the Eurozone, leading to more investor confidence in the (European) stock markets and less high losses.

4.3 Risk Measures

For all six indices we selected the best performing models in previous section using their AIC scores. The scale and shape parameters are described by the Forward Selection models for all six indices, and the intensity parameters by the Forward Selection models for the AEX, CAC40, PSI-20 and FTSE MIB index, the Ridge Regression model for the DAX index and the ElasticNet Regression model for the IBEX35 index. We use these models to calculate the Value-at-Risk (VaR) and Expected Shortfall (ES) as risk measures over time. Figure 4 shows the VaR for the DAX index plotted over time. We plot the dynamic VaR at 95% significance level (red line) against the VaR at 95% significance level of the benchmark (blue dashed line) with the weekly losses for the DAX in the background (grey). The plots of the dynamic VaR for other indices, as well as the ES for all indices, can be found in Appendix B.3.

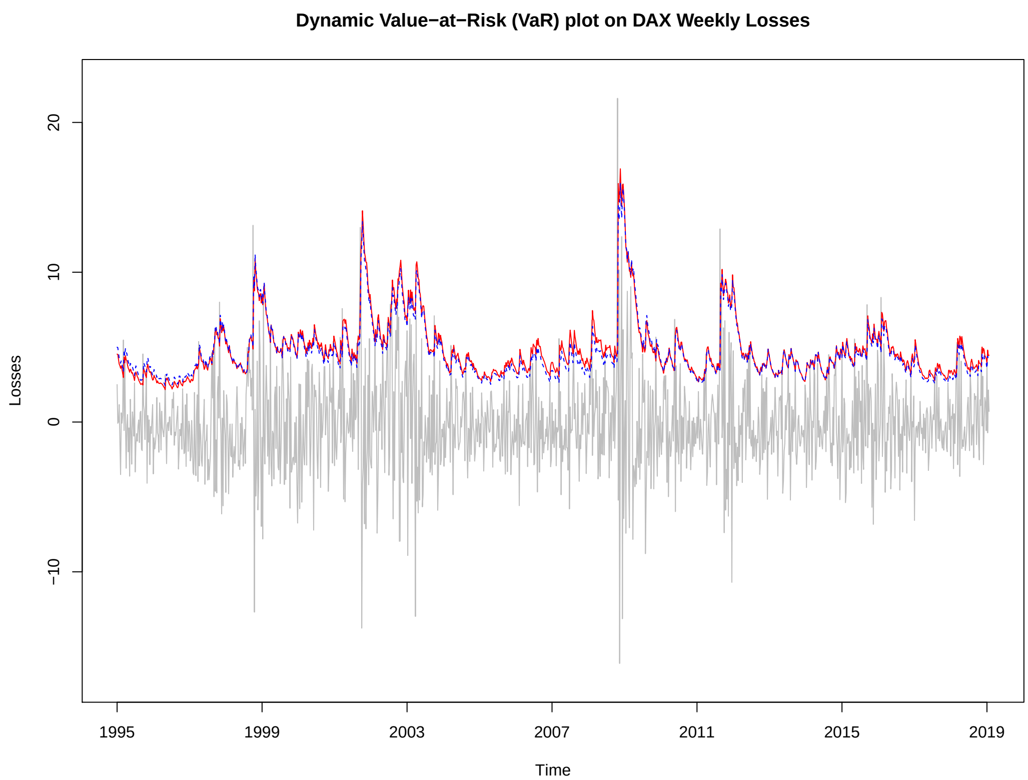


Figure 4: Value-at-Risk plot for the DAX index: The VaR of the Dynamic Model (red line) and VaR of the Benchmark Model (blue dashed line) are plotted against the Losses (grey in background)

We observe that the dynamic VaR model follows a similar pattern to the benchmark model, both reacting rapidly to the losses that appear. The covariates may have little improvement on estimating the risk measures for the DAX index. However, in periods of large losses (especially around the crisis in 2008) that the dynamic VaR is slightly higher than the VaR of the benchmark model, showing that the covariates improve the VaR estimation in times of financial turmoil. In other words, including covariates in modelling risk measures seems to be effective.

To verify if adding covariates aid in measuring risk we perform a backtest of the VaR estimates on the financial losses. Per index we compare the estimated VaR for the dynamic model and the benchmark model with the actual observed weekly loss. We keep track of the violations of these estimates. A violation occurs when the actual weekly loss on a date is higher than the estimated VaR. By counting these violations, we can compare the models. This is done by a binomial distribution test, which shows systematically under- or overestimation of the VaR.

Table 13 depict the backtesting results for the VaR measure of all indices on three probability levels (α = 95%, 99% and 99.5%). At the 95%-significant VaR we observe that for most indices the dynamic model performs better than the benchmark model, since the number of violations are closer to the expected number of violations at this probability level. Only the dynamic model for the PSI-20 index seems to perform about just as well as the benchmark model when looking at the amount of under- or overestimation. A similar conclusion can be drawn for the 99%-significant VaR, except for the FTSE MIB index, AEX index and again the PSI-20 index. The dynamic model performs similar in case of the PSI-20 and FTSE MIB index and slightly worse in case of the AEX index compared to the benchmark model. Finally the 99.5%-significant VaR shows an underestimation of the VaR by the dynamic model in cases of the AEX and CAC40 index and similar results to the benchmark model for the DAX, PSI-20, IBEX35 and FTSE MIB index. However, this probability level is very high, resulting in a low amount of expected violations. The last row for the 99.5%-significant VaR thus needs to be interpreted with caution.

When comparing both VaR models for all indices we can conclude that the dynamic model improves on the benchmark model. In other words, we observe improvement when adding covariates to our risk measures. However, this is not as strong for all indices. In particular for the PSI-20 index we detect less improvement in VaR estimation than the other indices. This matches our findings that the estimated covariate models for the shape and scale parameter of the PSI-20 index have little improvement in log-likelihood compared to the model with no covariates implemented.

Table 13: Binomial two-sided Backtest of Estimates of the Dynamic Model (D) and Benchmark Model (BM) for the 95%-, 99%- and 99.5%-significant Value-at-Risk for all Indices

Index	**AEX**		**DAX**		**CAC40**		**PSI20**		**IBEX35**		**FTSE MIB**	
	D	BM	D	BM	D	BM	D	BM	D	BM	D	BM
95% significant VaR												
Expected Violations	63	63	63	63	63	63	63	63	63	63	55	55
Violations	63	70	63	75	69	75	60	66	62	71	58	60
p-value	(0.948)	(0.330)	(0.948)	(0.119)	(0.399)	(0.119)	(0.795)	(0.650)	(0.938)	(0.270)	(0.627)	(0.446)
99% significant VaR												
Expected Violations	13	13	13	13	13	13	13	13	13	13	11	11
Violations	17	15	12	9	11	9	10	10	10	9	9	9
p-value	(0.199)	(0.475)	(0.883)	(0.393)	(0.777)	(0.393)	(0.570)	(0.570)	(0.570)	(0.393)	(0.650)	(0.650)
99.5% significant VaR												
Expected Violations	6	6	6	6	6	6	6	6	6	6	5	5
Violations	7	7	8	5	10	7	7	7	4	5	4	4
p-value	(0.153)	(0.435)	(0.917)	(0.840)	(0.421)	(0.687)	(0.687)	(0.687)	(0.543)	(0.840)	(0.671)	(0.671)

D = Dynamic model BM = Benchmark Model

Note: This table shows the expected violations and real violations of the Value-at-Risk at different probability levels with the p-value for the two-sided Binomial test in brackets below. The sample period is from January 1995 to December 2018.

5 Conclusion

In this thesis we investigated whether we could better predict market risk by adding covariates to the tail distribution of weekly stock losses on six different European indices (AEX, DAX, CAC40, PSI-20, IBEX35 and FTSE MIB). We set up an EVT machine learning framework that has aided in selecting the right amount of covariates per model, and thus can be used for further research.

We observe that when adding a limited amount of macro-economic covariates to the tail distribution of weekly losses, the prediction for the VaR is improved for five of the six indices. The inflation, short-term interest rate, and industrial production in the Eurozone appear to have positive predicting power for the tail risk. Next to that, the USD/EUR exchange rate appears to have a negative effect on the tail risk. Moreover there is no predicting power in any of the six indices when using unemployment and the long-term interest rate.

5.1 Discussion

We conclude that incorporating macro-economic variables as covariates in describing the tail distribution of European stock market returns does aid in predicting the risk of extreme losses. Macro-economic information is thus of added value when estimating the financial risks of the European stock market. For investors this finding supports that not only business-specific information, such as the gross profit or book-to-market ratio of companies, but also macro-economic data such as inflation or short-term interest rate are important when investing on the European stock market. Moreover, this finding can also be of value in the policy debate among regulators. Regulators may use their information together with their macro-economic policy to better predict the financial risks that are or might be taken on the European stock market by institutional investors and banks.

5.2 Potential Improvements

When carrying out this research on the weekly tail distribution losses of European stock market returns we encountered different disadvantages, as well as had to make assumptions and concessions. Some of these require a discussion, which is presented below.

First of all let us discuss the data used in this research. We chose to select macro-economic covariates with monthly frequency, but consider weekly returns of the indices. This resulted in the weekly interpolation of these macro-economic covariates. This frequency mismatch can lead to a less transparent interpretation of the effect of macro-economic covariates on the tail distributions of these six indices. However, we have chosen to do so to make up for the data scarcity of index returns over the given time period. Considering monthly returns over a period of 24 years (from 1995-2018) would lead to too little observations to be able to use EVT models on the tail distributions. In other words, we would have too little tail observations to say anything meaningful about them. Since the European Monetary Union started to form around the end of the 20th century it would be meaningless to increase the considered time period of our data sample until we would have enough monthly returns. This not only would mean that we had to increase the start of the time period to around the beginning of the 1960's, but also a lot of data such as European macro-economic covariates or even indices would not be relevant or simply not be available. The choice of weekly interpolation can next to this be justified in the sense that the monthly covariates selected were not only not available in a weekly frequency, but can also be assumed to change very little during a month. In other words, even if it was possible to select weekly covariates, the assumption can ben made that this would have very little to no effect on the results of explanatory power of these covariates.

Next to the frequency of the covariates, it is also important to discuss the implementation of these covariates in our model. In line with Chavez-Demoulin et al. (2016) we chose to let our parameters ξ, ν and $\log(\lambda)$ linearly depend on the covariates. However, one can argue that the effect of these macro-economic variables are not only linear. Next to that, we selected a limited amount of covariates in our models. It of course can be argued that more important covariates were not selected and could potentially have a better impact on estimating the VaR or ES. By introducing an EVT machine learning framework in this research we give room to this concern for further research with more macro-economic covariates.

Appendices

A Data Description

A.1 Stock Indices Description

Table A1: Stock Indices Description

Index	**Country**	**Date**	
		From	Until
CAC40	France	1-1-1995	31-12-2018
DAX	Germany	1-1-1995	31-12-2018
FTSE MIB	Italy	1-1-1998	31-12-2018
AEX	The Netherlands	1-1-1995	31-12-2018
PSI-20	Portugal	1-1-1995	31-12-2018
IBEX35	Spain	1-1-1995	31-12-2018

Note: This table shows the stock indices considered in this research. Index is the abbreviated index name. Country is the country of origin of the index. Date is the date from and until when the index returns are available.
Source: Compustat Global through Wharton Research Data Services & Thomas Reuters DataStream

A.2 Covariates Description

Table A2: Covariates Description

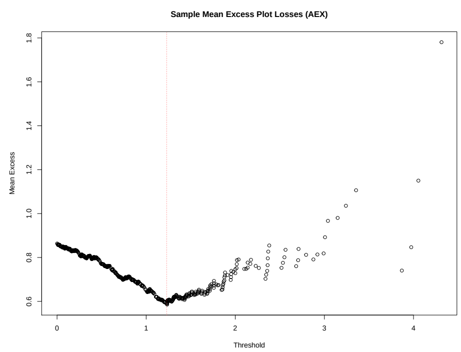
Covariate		Date		Observations	Measurement
		From	Until		
UNE	Unemployment	01-1995	12-2018	288	% of active population
INF	Inflation	01-1997	12-2018	264	Annual rate of change
LINT	Long-Term Interest Rate	01-1995	12-2018	288	% per annum
SINT	Short-Term Interest Rate	01-1995	12-2018	288	3-month rate % per annum
EXR	Exchange Rate	01-1995	12-2018	288	Monthly average rate against USD
PR	Industrial Production	01-1995	12-2018	288	Volume index (2010 = 100)

Note: This table shows the monthly covariates used in this research. Covariate is the name of the covariate (with abbreviation in front). Date is the date from and until when the covariate is available. Observations shows the amount of observations there are available. Measurement depicts the type of measurement of the original monthly covariate

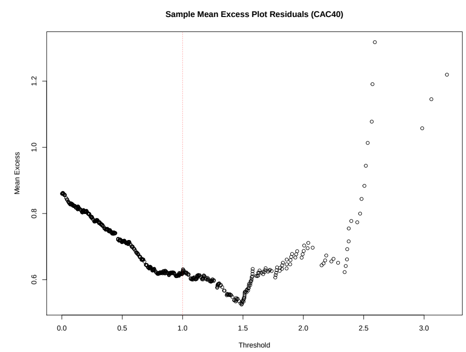
Source: Eurostat, the statistical office of the European Union

B Affixed Results

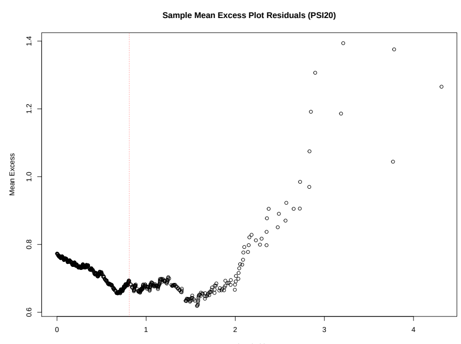
B.1 Mean Excess Plots



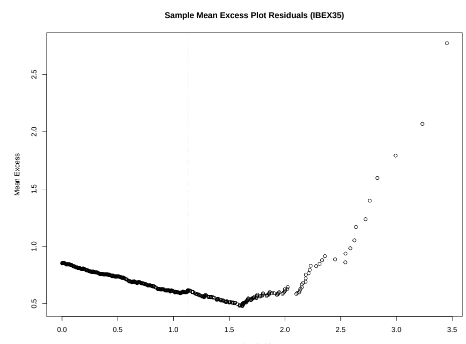
(a) AEX Index



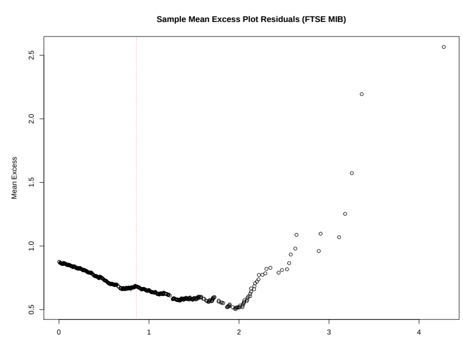
(b) CAC40 Index



(c) PSI20 Index



(d) IBEX35 Index



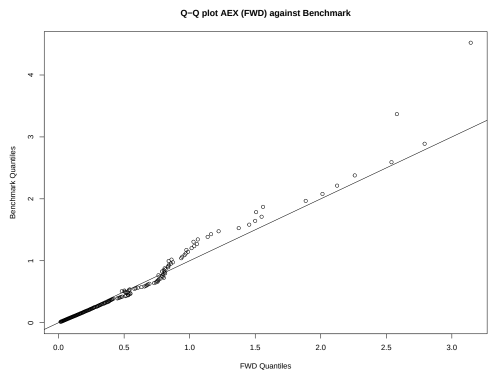
(e) FTSE MIB Index

Figure B1: Mean Excess Plots of Residuals X_t for the AEX, CAC40, PSI20, IBEX35 and FTSE MIB index

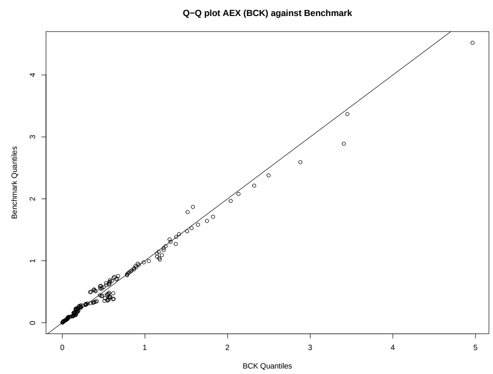
B.2 Model Selection: Q-Q plot

B.2.1 AEX Index - Netherlands

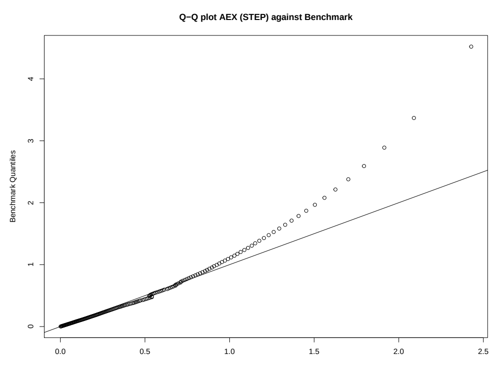
Figure B2: QQ-Plot of Residuals X_t of selected models for the GPD parameters against the quantiles of the GPD distribution estimated without using covariates (Benchmark)



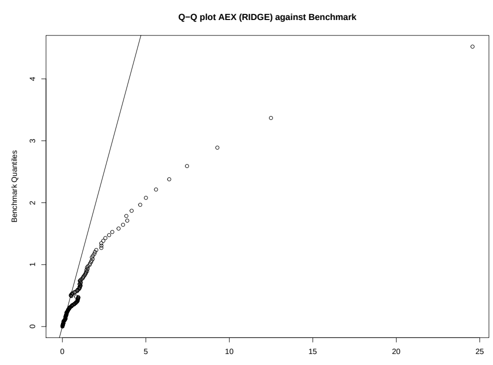
(a) Forward Selection (FWD)



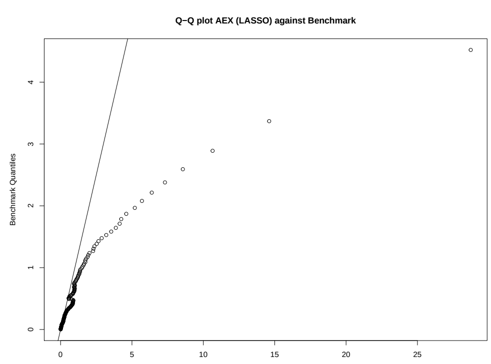
(b) Backward Selection (BCK)



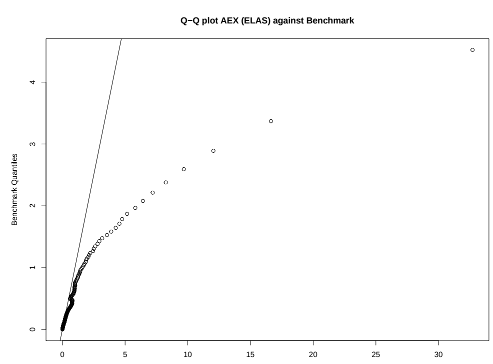
(c) Stepwise Selection (STEP)



(d) Ridge Regression (RIDGE)



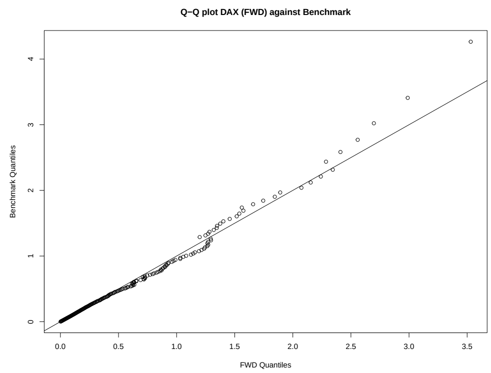
(e) LASSO Regression (LASSO)



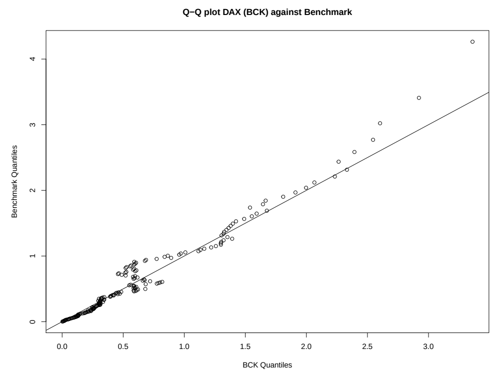
(f) ElasticNet Regression (ELAS)

B.2.2 DAX Index - Germany

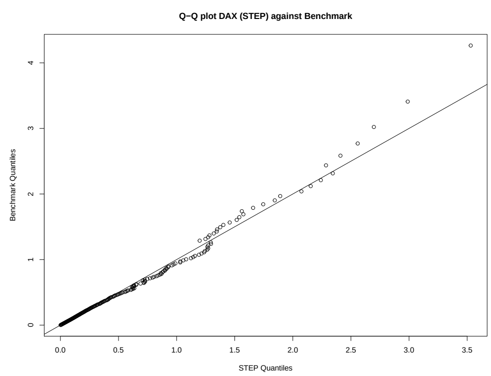
Figure B3: QQ-Plot of Residuals X_t of selected models for the GPD parameters against the quantiles of the GPD distribution estimated without using covariates (Benchmark)



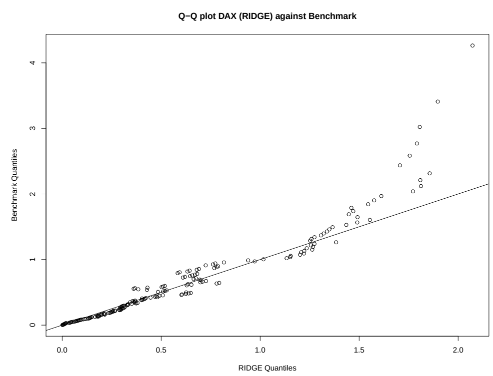
(a) Forward Selection (FWD)



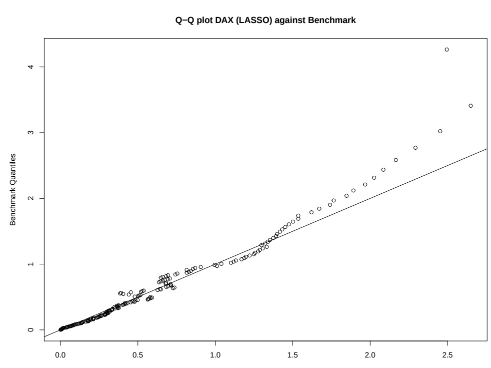
(b) Backward Selection (BCK)



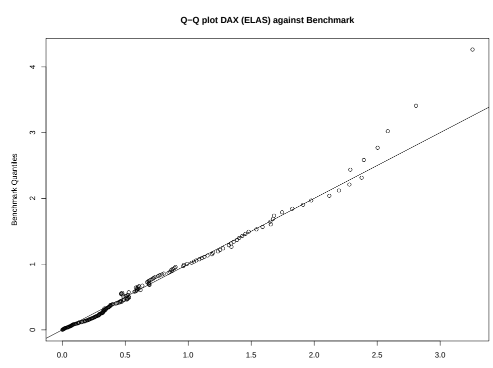
(c) Stepwise Selection (STEP)



(d) Ridge Regression (RIDGE)



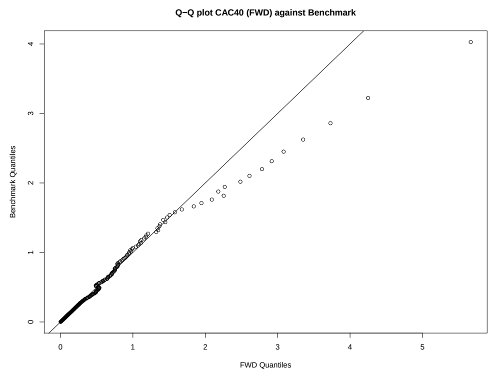
(e) LASSO Regression (LASSO)



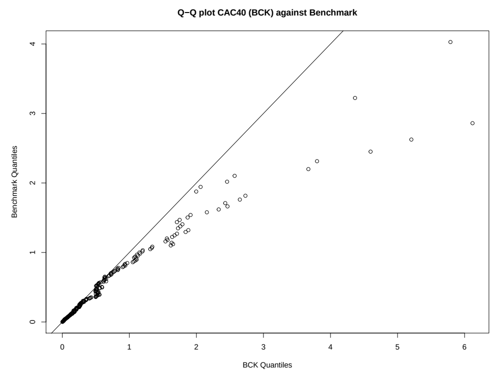
(f) ElasticNet Regression (ELAS)

B.2.3 CAC40 Index - France

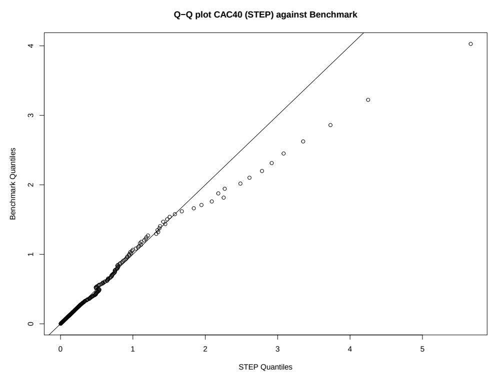
Figure B4: QQ-Plot of Residuals X_t of selected models for the GPD parameters against the quantiles of the GPD distribution estimated without using covariates (Benchmark)



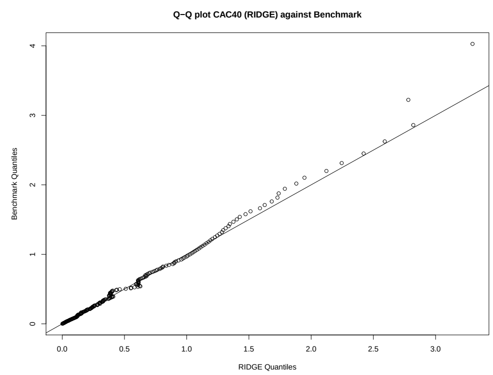
(a) Forward Selection (FWD)



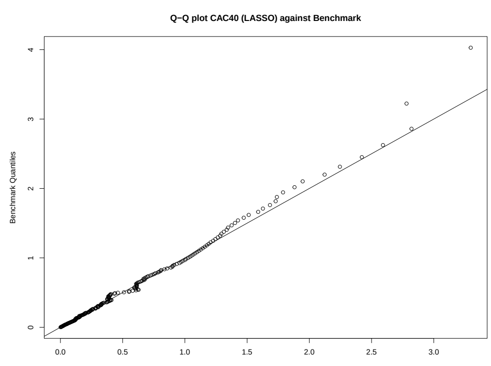
(b) Backward Selection (BCK)



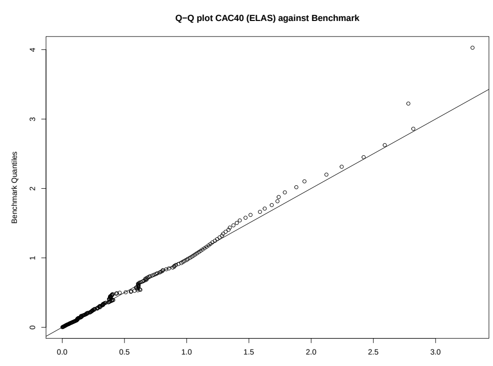
(c) Stepwise Selection (STEP)



(d) Ridge Regression (RIDGE)



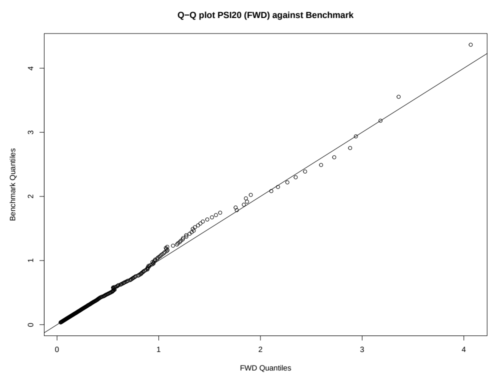
(e) LASSO Regression (LASSO)



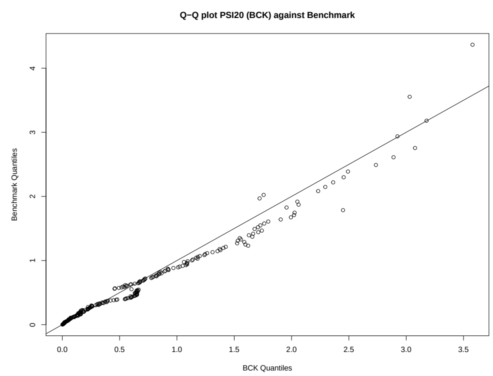
(f) ElasticNet Regression (ELAS)

B.2.4 PSI-20 Index - Portugal

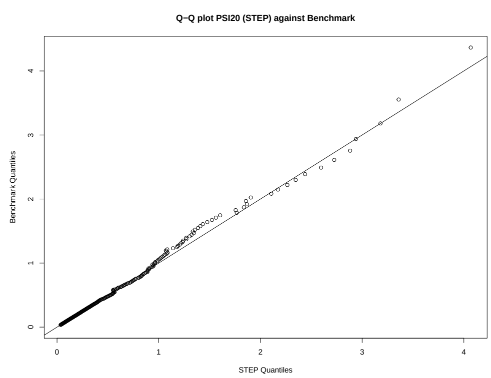
Figure B5: QQ-Plot of Residuals X_t of selected models for the GPD parameters against the quantiles of the GPD distribution estimated without using covariates (Benchmark)



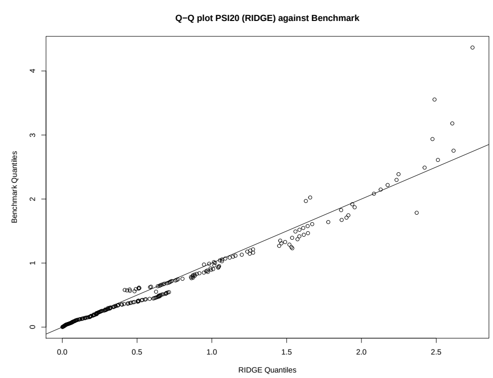
(a) Forward Selection (FWD)



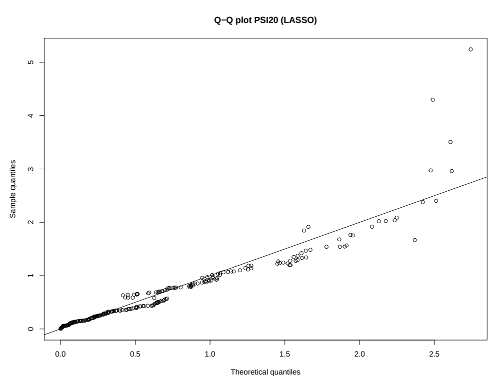
(b) Backward Selection (BCK)



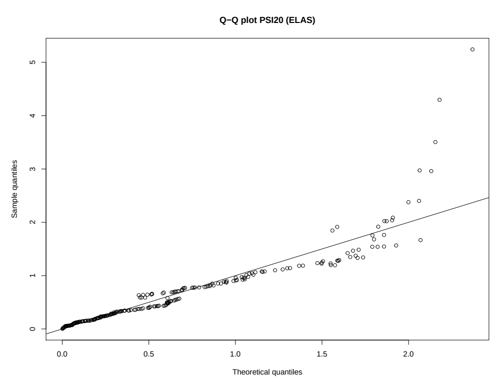
(c) Stepwise Selection (STEP)



(d) Ridge Regression (RIDGE)



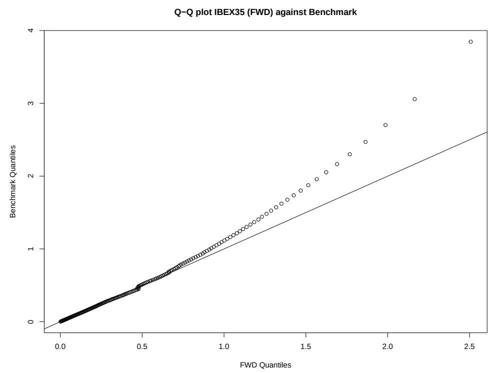
(e) LASSO Regression (LASSO)



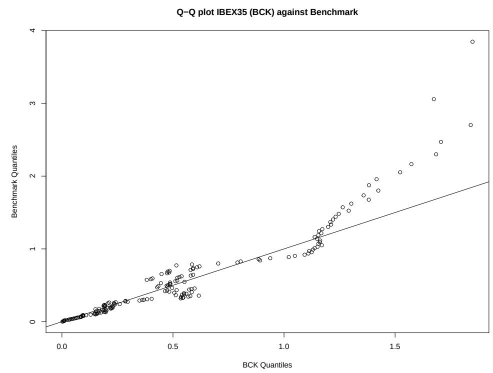
(f) ElasticNet Regression (ELAS)

B.2.5 IBEX35 Index - Spain

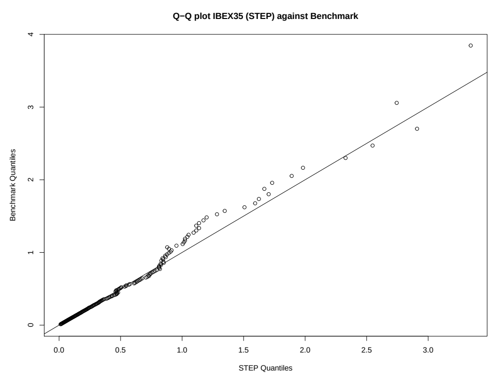
Figure B6: QQ-Plot of Residuals X_t of selected models for the GPD parameters against the quantiles of the GPD distribution estimated without using covariates (Benchmark)



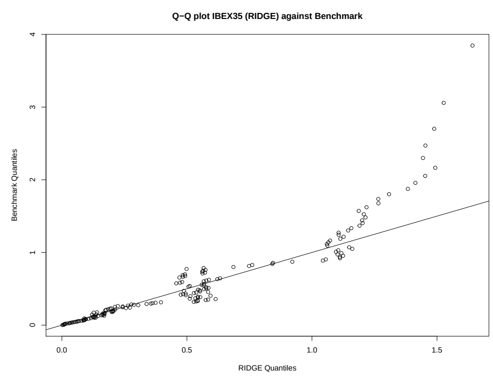
(a) Forward Selection (FWD)



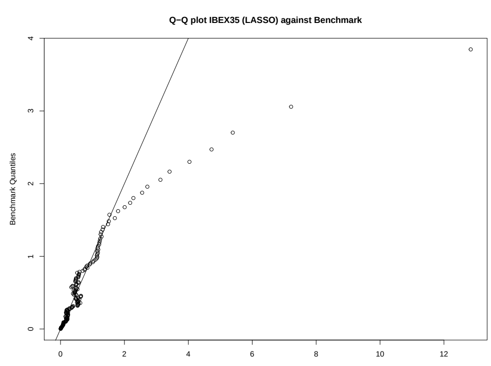
(b) Backward Selection (BCK)



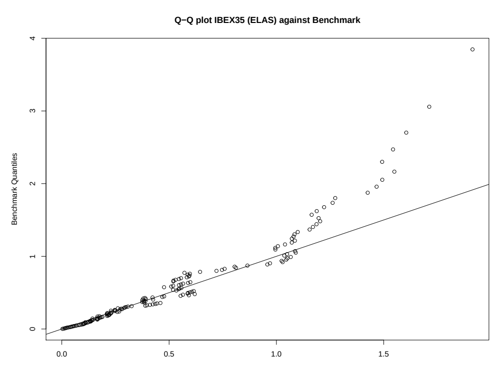
(c) Stepwise Selection (STEP)



(d) Ridge Regression (RIDGE)



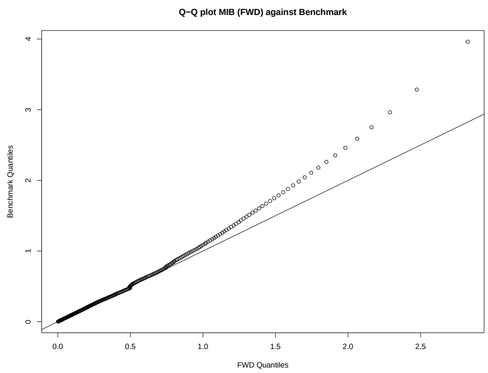
(e) LASSO Regression (LASSO)



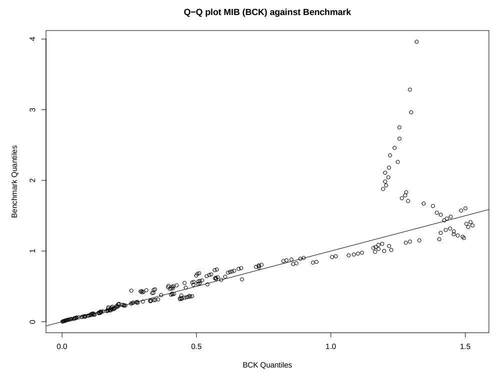
(f) ElasticNet Regression (ELAS)

B.2.6 FTSE MIB Index - Italy

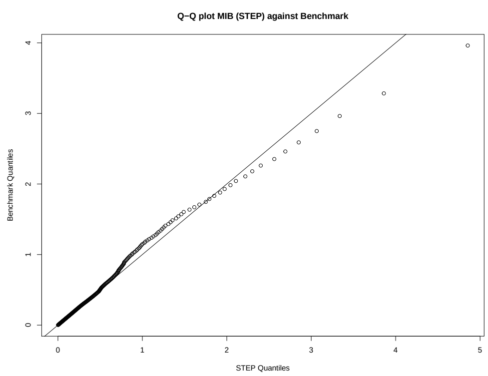
Figure B7: QQ-Plot of Residuals X_t of selected models for the GPD parameters against the quantiles of the GPD distribution estimated without using covariates (Benchmark)



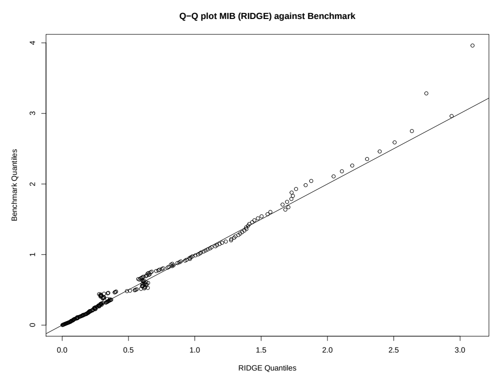
(a) Forward Selection (FWD)



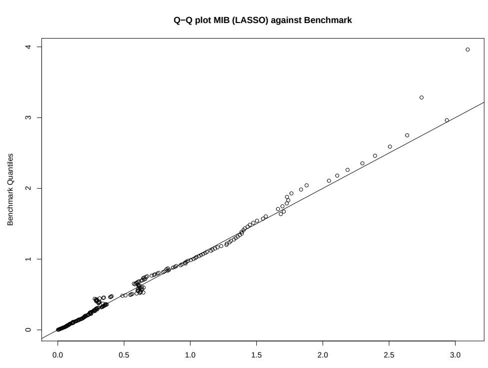
(b) Backward Selection (BCK)



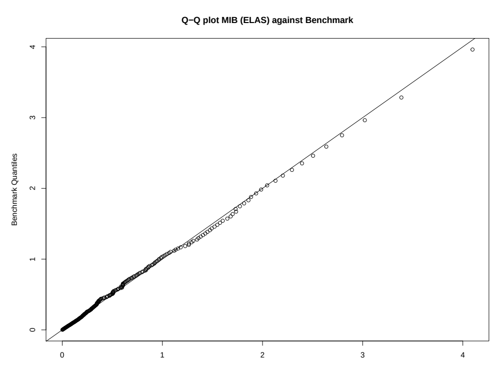
(c) Stepwise Selection (STEP)



(d) Ridge Regression (RIDGE)



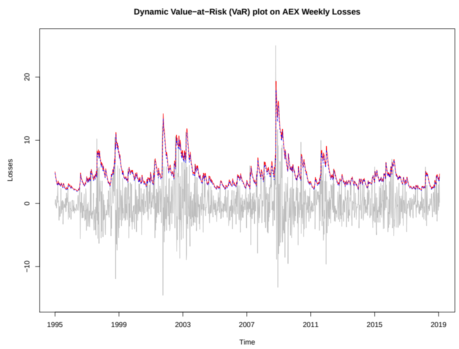
(e) LASSO Regression (LASSO)



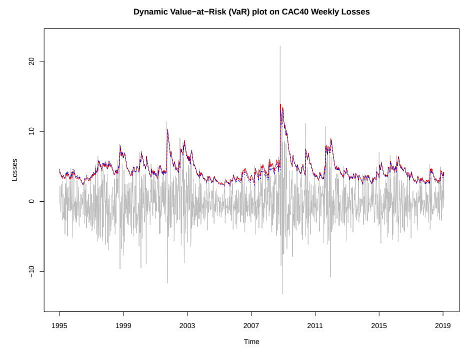
(f) ElasticNet Regression (ELAS)

B.3 Risk Measurements

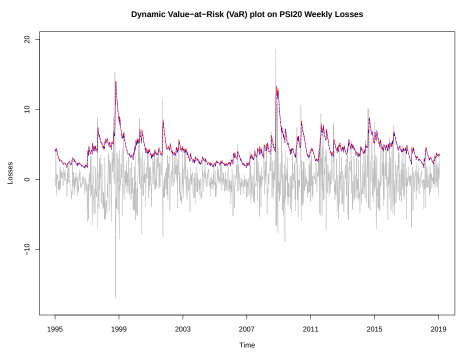
B.3.1 Value-at-Risk



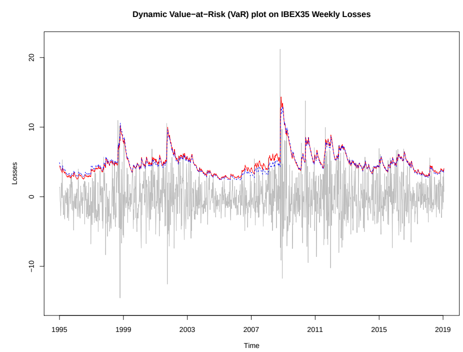
(a) AEX Index



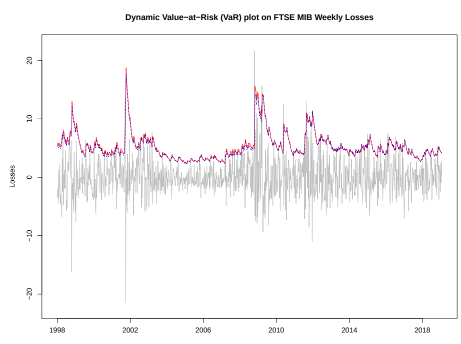
(b) CAC40 Index



(c) PSI20 Index



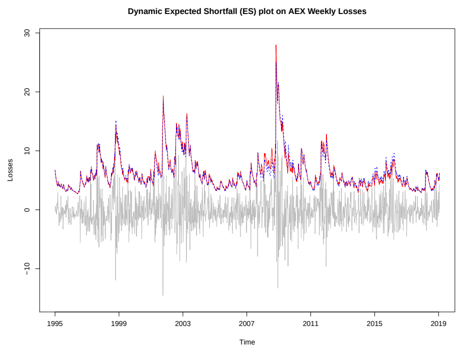
(d) IBEX35 Index



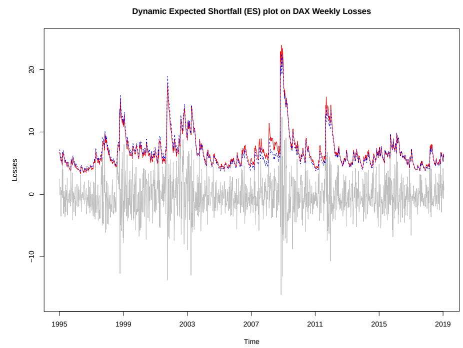
(e) FTSE MIB Index

Figure B8: Value-at-Risk plot of the Dynamic VaR Model (red line) and Benchmark VaR Model (blue dashed line) against the Losses (grey in the background) for the AEX, CAC40, PSI20, IBEX35 and FTSE MIB index

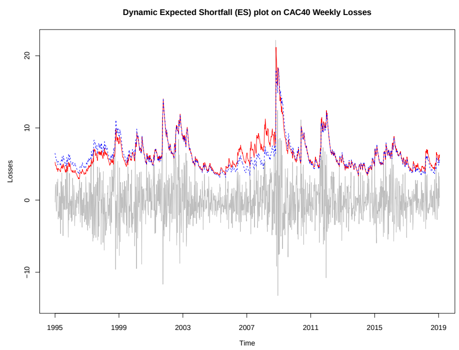
B.3.2 Expected Shortfall



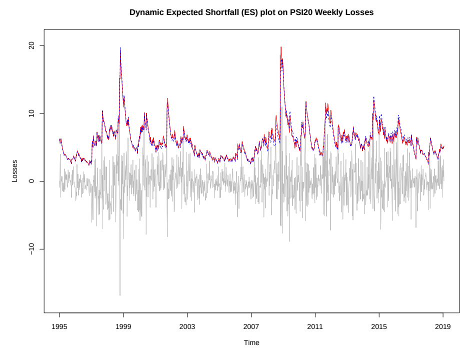
(a) DAX Index



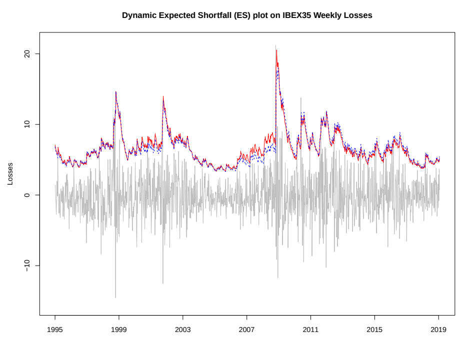
(b) DAX Index



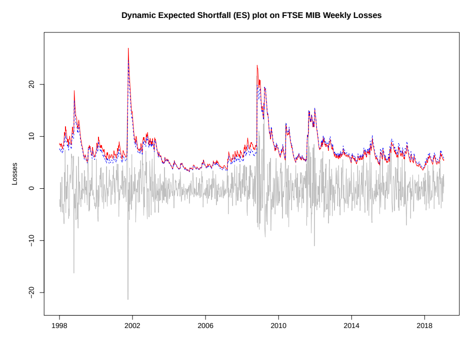
(c) CAC40 Index



(d) PSI20 Index



(e) IBEX35 Index



(f) FTSE MIB Index

Figure B9: Expected Shortfall plot of the Dynamic ES Model (red line) and Benchmark ES Model (blue dashed line) against the Losses (grey in the background) for the AEX, DAX, CAC40, PSI20, IBEX35 and FTSE MIB index

C Acronyms

AEX	Amsterdam Exchange Index
AIC	Akaike Information Criterion
CAC40	Cotation Assistée en Continu 40
DAX	Deutscher Aktienindex
EMU	European Monetary Union
ES	Expected Shortfall
EVT	Extreme Value Theory
EXR	Exchange Rate
FTSE MIB	Financial Times Stock Exchange Milano Indice di Borsa
GPD	Generalized Pareto Distribution
IBEX35	Índice Bursátil Español 35
INF	Inflation
LASSO	Least Absolute Shrinkage and Selection Operator
LINT	Long-Term Interest Rate
POT	Peaks-over-Threshold
PR	Industrial Production
PSI-20	Portugese Stock Index 20
QQ-Plot	Quantile-Quantile plot
SINT	Short-Term Interest Rate
UNE	Unemployment
USD/EUR	US Dollar/Euro
VaR	Value-at-Risk

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