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MASTER THESIS

Comparative and non-comparative aspects of desert-adjusted axiologies

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Contents

1 Introduction

In a passage of *Killing, Letting Die, and the Trolley Problem*, Judith Jarvis Thomson proposes the following "Transplant" case example [1]. Imagine that Ann is a transplant surgeon and that she has five patients who each needs an organ in order to survive. They do not all need the same organ. One of them needs a heart, another one a liver, a third a kidney, and so on. Unfortunately, Ann does not have all these organs readily available for transplant, but since they differ from each other, she faces the following choice. Ann can choose to either kill an innocent visitor to the hospital and distribute his organs to them, or to let her five patients die. In the first case, five people live at the expense of one, while in the second five people die but an innocent person lives. What should she choose?

According to utilitarianism, the surgeon should kill the innocent and take his organs to save the five people. This theory holds that a certain action is morally right if it produces the greatest sum of benefits. Since saving five lives produces more good than saving a single life, then the overall good in the world is increased by killing the innocent. Thus, utilitarianism concludes that this is the morally right choice to make.

Many authors in the literature are not inclined to accept the utilitarian conclusion, for they take it to be in rather sharp contrast with our common-sense intuitions [1],[2],[3]. Indeed, it seems intuitively plausible that killing an innocent is not a morally permissible action, even if it resulted in a net benefit. That action is wrong in itself and should be simply forbidden.

In the literature, this intuition is accounted for by *deontological ethics* [4]. This theory comes with a list of moral principles that define when an action is right, as for example "you shall not kill innocent people", and so it determines the rightness of an action independently of what happens in the world. Its rival theory is called *consequentialism*, and it contains utilitarianism as one of its branches [5]. Consequentialism holds that to assess the rightness of an action we need to look at the good it brings about in the world, i.e. at its consequences.

As the Transplant case shows, these two theories can be in sharp contrast with each other, and the contrast seems to be unfavorable to consequentialism but not to deontology. In fact, it is one of its branches, utilitarianism, that does not correctly account for some of our moral intuitions. To illustrate, utilitarianism concludes that killing the innocent in Transplant is the morally right action, for it saves five lives instead of one and so by performing it one realizes more good in the world than by doing otherwise. However, our moral intuitions are strongly against this conclusion. We take that action as simply unjust. It thus seems that we should not understand the rightness of an action as dependent on the goodness of its consequences, for that leads us to derive the wrong conclusion that we should kill the innocent. Instead, the deontological principle "you shall not kill innocent people" correctly prescribes not to do what is wrong. Hence, it seems that utilitarianism prescribes unjust actions, whereas deontology does not.

The criticism according to which utilitarianism allows for injustice as killing innocents is an *objection from justice* [6]. As we have seen, it depends on the definition of right as conditional on the goodness of the consequences. Since this definition of right is the basic tenet of consequentialism, the criticism seems to be directed not only to utilitarianism, but rather to consequentialism as such. Deontological accounts instead seem to be immune to it, and so many were motivated to give up consequentialism and adopt a deontological perspective.

More so, the literature has produced several kinds of objection from justice, which can differ greatly from the Transplant case. In *The Right and the Good*, William David Ross illustrates one of them by means of the following example, which we call "Wicked and Virtuous" case [2]. Imagine to have the choice between giving 10 units of value to a virtuous person, or giving 50 units of value to a wicked one. You can choose only one of the two options. What should you choose?

According to utilitarianism, you should benefit the wicked person. By choosing that action, the overall good increases more than if you benefited the virtuous person, so utilitarianism concludes that that is the right action. However, I believe that also in this case not many moral philosophers would agree that benefiting wicked people is right. Probably, they would rather say that the virtuous should be benefited, for they deserve it. Indeed, the deontological principle "you should distribute good according to what people deserve", which dates back to Aristotle, seems intuitively unobjectionable.[1] Hence, it seems again to be the case that deontological ethics is on the right track, while consequentialism prescribes unjust actions.

W.D. Ross was among the ones who thought consequentialism was to be rejected one time and for all [2]. However, not every scholar in the field agrees. Fred Feldman, for example, submits that the problem does not lie in consequentialism itself, but rather in utilitarianism and in the way this theory defines the good [6]. In particular, utilitarianism fails to account for our intuitions because it fails to consider the distinctions between people, and instead defines the good as the overall sum of individual benefit (independently of who receives it). Feldman then proposes a solution. His strategy is to let the definition of good be determined not only by individual benefit, but also by individual *desert*, and then define the right as dependent on this definition of good. In this way, it is possible to address the objection from justice without having to giving up the consequentialist framework, and instead by remaining in it [6]. The theory that he proposes goes under the name of *desert-adjusted consequentialism* (DAC).

According to DAC, the good is that people are treated according to what they deserve, and the right actions are the ones that realize this good. Importantly, Feldman's theory only considers whether each person *individually* is treated according her desert, and so it determines the good without making comparisons between people. It is then an *individualistic* or *non-comparative* moral theory.

Thomas Hurka doubts that a moral theory defined on merely individualistic grounds is sufficient to do justice to all our moral intuitions and claims that we need to account for comparative considerations as well [7]. To illustrate his doubts, consider the following "Equal Treatment" example. Imagine to have to distribute 10 units of value between two people who are equally virtuous, i.e. who have equal desert. The choice is between giving all the 10 units to one of them, or giving 5 units each. What should you choose?

Intuitively, we would say that equally deserving people should be treated equally, and so that the right action is to give 5 units of value each. Indeed, it seems that we have no reason to benefit only one of them, and that the deontological principle "equally deserving people should be treated equally" should

[1] In the Nichomachean Ethics, Book V, Aristotle writes that "all [people] agree that what is just in distribution must be according to merit".

be respected. This principle has a comparative nature, for it refers to the relationships between persons, their desert levels and their modes of treatment. We have seen that Feldman's theory is defined in non-comparative terms, for it takes into account how individuals are treated only with respect to their own desert level. Can a non-comparative definition of DAC account for moral intuitions of comparative nature? If not, the objection from justice strikes back once again, as the consequentialist theory does not account for a deontological principle representing one of our strong moral intuitions.[2]

The goal of this thesis is to work towards an answer to such question. In particular, we are interested in understanding the relation between comparative and non-comparative justice, and whether the two can coexist in such a way that our moral intuitions are respected. As we will see, this is not a trivial problem, for the comparative and non-comparative dimension often contradict each other. Moreover, finding an answer to this question is highly relevant in the debate between consequentialist and deontological ethics, as we have seen that if DAC does not account for comparative requirements of justice, then it falls prey of another objection. This might mean that DAC is insufficient to redeem consequentialism from the objections from justice, and that this theory needs thus to be complemented with deontological elements.

In the thesis, I will study these issues in a formal framework. To briefly illustrate, both utilitarianism and DAC use suitably defined functions to represent the value of the total consequence of an action. Utilitarianism defines it as the sum of the individual good, while DAC as the sum of desert-adjusted individual good. I will examine the latter kind of functions, discuss what properties the literature claimed they should have and asses whether these properties, defined in non-comparative terms, are compatible with comparative aspects of justice mentioned above. This is the main goal of the thesis.

Note that the formal representation I will use is meant to abstract away the many specificities of the problem and to capture its more fundamental and general structure. Perhaps then, the formal framework I will be studying can be used to account for other issues as well.

[2]Note that the Equal Treatment example is also applicable to utilitarianism, and not only to DAC specifically, as utilitarianism too fails to account for the comparative consideration that the example discusses. However, comparative and non-comparative considerations are more relevant for DAC, for this theory explicitly involves consideration of desert in individualistic terms.

In fact, a debate about comparative and non-comparative aspects also exists in the literature about fairness. Broome has developed an influential theory of fairness, according to which fairness is the proportional satisfaction of claims [8]. This theory is strictly comparative in nature, for proportionality essentially requires that the extent to which each claim is satisfied takes into account the other agents' claims as well. However, some authors have argued for the importance of a notion of individual fairness, and have objected that the strictly comparative notion cannot account for it [9],[10]. In that literature, nobody has yet come up with a theory that accounts for the relation between individual and comparative fairness. Since the structure between fairness claims and desert claims seem to be similar, for one might take individual desert to give rise to both kinds of claims, my research might also illuminate the relation between individual and comparative (Broomean) fairness.

Finally, let us see the structure of the thesis. In the **first chapter**, we will introduce the theory of desert-adjusted consequentialism. We will claim that any non-comparatively defined axiology that does not satisfy comparative requirements is subject to an objection from justice. Since the same holds for comparatively defined axiologies that do not satisfy non-comparative requirements, we will conclude that the two are complementary aspects of any desert-adjusted axiology, and so any such theory should comprise both. The literature though has proposed only non-comparatively defined desert-adjusted axiologies, and so in the **second chapter** we will present two of them, namely Feldman's and Arrhenius' ones. We will mainly discuss the formal aspects of these theories, as for example what axioms the functions should satisfy so to comply to our intuitions. This implies that the chapter will mainly have a technical nature, but the discussion it contains will give us the necessary understanding of non-comparative theories to be able to move to their assessment with respect to comparative justice matters, which is the goal of the thesis. Towards that objective, in the **third chapter**, we will introduce the definition of this kind of justice, which in the literature is widely believed to be equivalent to the *proportional* division of goods. However, scholars as Kagan disregard it as such, for it cannot handle cases of zero and negative numbers. We will then propose an argument according to which the adoption of a strictly positive scale is justified, thereby ruling out Kagan's objection and rehabilitating proportionality principle as equivalent to comparative justice. Having gained a more precise understanding of comparative justice, in the **fourth chapter**, we will go back to Feldman's and Arrhenius' formal proposals and see how they fare with respect to comparative requirements so understood. We will see that they fail to satisfy it, and the reasons why this is the case can be made formally precise. The formalization will illustrate how to find a remedy to this failure and thereby also suggest how to induce these theories to satisfy proportionality as well. We will conclude the chapter showing that there exists a kind of non-comparatively defined functions that satisfy proportionality, so our result will also show that comparative and non-comparative aspects of justice can be satisfied by a single function, i.e. they are compatible. The final remarks hold what possible future ramification the present work has.

2 Desert-adjusted consequentialism

As we said in the introduction, there are broadly two theories that provide the definition of rightness of an action: *deontological* [4] and *consequentialist* ethics [5]. The theories we will discuss and analyse in the present thesis are of the consequentialist kind, and thus they abide to the principle:

(**C**) An act is morally right just in case, when compared to other available acts, it realizes a state of affairs of maximal intrinsic value.

Clearly, this principle is not yet enough to provide a full definition of rightness and thus of consequentialism, as it does not specify *when* a state of affairs has maximal intrinsic value, or *what* intrinsic value is. For this reason, consequentialism needs to augment the framework that (**C**) sets forth with a *theory of value*, also called *axiology*.

An axiology is a philosophical theory that accounts for what is good or what can be said to have value. There are many ways in which something is good and many perspectives under which one can make a value claim. For example, one can say that something is good *for someone* or *to do something*, and attribute an instrumental value to the good [5]. A consequentialist theory, being a moral theory, is interested in axiologies that account for a specific kind of good: the non-instrumental good, also called *intrinsic good*, *intrinsic value*, or good *simpliciter*.

The intrinsic good is a property that many different things can be endowed with. In this work, we study the intrinsic value that a given *distribution of good* among a certain population has. In particular, we are interested in examining how the intrinsic value of a state of affairs varies at the variation of the distribution of good. This will direct our focus on the actions that distribute goods, where the right actions will be the ones realizing the distribution of good with greatest intrinsic value.

Note that in a consequentialist framework, the descriptive claim that a certain distribution has the greatest value, is equivalent, by principle (**C**), to the normative claim that the right choice of action is the one that realizes that distribution. This means that sentences as "states of affairs x is more valuable than states of affairs y" will be taken to mean that we should distribute or redistribute welfare so that x obtains, and not y. As another terminological note, the word "state of affairs" will be used interchangeably with the term "possible world". They both refer to the total consequence of an action, so in this case, to a given distribution of welfare among a certain set of agents.

A prominent consequentialist axiology is provided by utilitarianism. It holds that the intrinsic value (IV) of a possible world is equal to the total sum of welfare[1][2] that individuals in the possible world receive [12]. More formally, IV is given by the outcome of the function $IV : W^* \to \mathbb{R}$, defined as $IV(W) = \Sigma_{i \in N}(w_i)$, where W^* is the set of possible worlds, W is a possible world, $\mathbb{R}$ is the real line, N is the set of agents, and w_i is the numerical representation of the amount of individual welfare that each agent i receives at W. A possible world is a certain distribution of welfare, i.e. a set of welfare levels $W = \{w_1, w_2, \ldots, w_n\}$.

Utilitarianism then characterizes the intrinsic value of a possible world only by appealing to information about how much welfare the agents in it receive. This means that it abides to the following axiological principle, and it is thus a species of *welfarism*:

(**W**) The intrinsic value of a possible world is a function of, and only of, the individual welfare of the persons that inhabit that world.

By stating that individual welfare is the *only* variable that matters in the definition of what is good, this principle and thus utilitarianism submit that morality is ultimately about individual welfare [13].

This claim is problematic, and it leads utilitarianism into accepting cases of "distributional improprieties" [6]. Indeed, because it assumes principle (**W**), the theory fails to take into account information about other features of agents, as for example their moral worth, their basic needs, basic rights, or their legitimate claims. In other words, the theory fails to consider what individuals *deserve* to receive. As we have seen in the introduction, utilitarianism can then

[1]We will discuss the welfare notion in the next section. For now we can take it as equivalent to individual good. We will use the terms "welfare" and "well-being" interchangeably.

[2]Note that the IV-function is equivalent to what in welfare economics or social choice theory is called *social welfare function* [11].

end up prescribing to benefit a less deserving person over a more deserving one, if this implies the maximization of total welfare. This is clearly improper, and the literature has called this the *objection from justice*. The objection shows utilitarianism to be an inadequate moral theory.

To answer the objection, Fred Feldman proposes to modify principle (**W**) in such a way that it includes *all* the relevant information and that it correctly attributes an higher IV to those possible worlds in which agents receive the welfare they deserve [6]. Then, instead of (**W**), he proposes to use the following principle:

(**DW**) The intrinsic value of a possible world is a function of, and only of, the well-being and desert levels of the agents that inhabit that world.

We call this new framework *desert-adjusted consequentialism*,[3] and the new axiology it is complemented with *desert-adjusted axiology*. Here, the IV of a possible world is defined as a function of two variables: (1) the amount of welfare the recipient *receives*, and (2) the amount of welfare the recipient *deserves* [6]. Essentially, DAC distinguishes between the good that should be given to a person for she deserves it, and the good that the person actually receives, and then says that justice is done when the amount of received and deserved welfare coincide.

This new principle requires a new definition of the IV-function. Feldman aims at minimally changing the utilitarian function, by retaining as many features as possible while introducing considerations of desert. Consequently, he proposes to define the desert-adjusted IV-function too as the *sum* of (desert-adjusted) intrinsic values of the lives of agents living in that world. More formally, the function IV is now defined as $IV(W) = \Sigma_{i \in N} C_i(w_i, d_i)$, where W and N are again a possible world and the set of agents, $C_i : (w_i, d_i) \rightarrow \mathbb{R}$, for every $i \in N$, are the *contribution functions*, stating for each agent how much her life (w_i, d_i) contributes to the value of the possible world W, with w_i again as the numerical representation of the amount of welfare the agent i receives (received welfare, or receipt), and d_i as the numerical representation of the amount of welfare agent i deserves (deserved welfare, or desert).

The contribution function C describes how valuable it is that an agent receives a certain amount of welfare given her desert level. Proponents of DAC call this the *intrinsic value of a life*. The notion of intrinsic value is the same we have

[3] Feldman calls this proposal *justice-adjusted consequentialism* [6]. We change the name to render it even more intuitive.

seen above, but applied to individual lives and not to states of affairs. This means that the *C*-function aims at representing the non-instrumental value that a life has. In Feldman's proposal, an agent's life is maximally valuable when, other things being equal, the welfare she receives *fits* the welfare she deserves, and the better the fit between the two levels, the higher the intrinsic value of the agent's life. This is called the *fit-idea* (*F-idea*).

(*F-idea*) The intrinsic value of an agent's life i is a function of, and only of, i's welfare and desert levels, and it is maximized when the two levels are equal.

This idea dates back to Aristotele, and the principle of just distribution according to desert. It represents our intuitions about retribution and compensation, as we intuitively say that justice requires us to reward people for their efforts and compensate them for their bad luck, i.e. to give to each what they deserve. Note that this implies that the fit-idea has an optimality structure, in the sense that, for any level of desert, it says that there is a quantity of well-being that it is best in desert terms for the agent to have [7].

The fit-idea is not the only way in which one might define justice in relation to desert. Another option is to define it according to the *merit-idea*, which holds that the higher the agent's desert is, the higher the intrinsic value of her life is, no matter the amount of welfare she receives. It represents the intuition that, other things being equal, it is better if an agent is more deserving than less. The merit-idea works well when desert is equated with moral worth, as the worthier a population is, the better. However, as we will soon see, Feldman and the literature that has spawned from his work, use a broader definition of desert, one that includes not only moral worth, but also need and bad luck as desert bases [14]. Note that this definition of desert is incompatible with the merit-idea and with any C-axiom that realizes it, contrary to what Feldman and other authors suggest [6][14]. Indeed, this combining this definition of desert with the merit idea would amount to say that the intrinsic value of an agent's life increases when her needs increase, which is clearly implausible.[4]

Nevertheless, that literature is still debating about whether the fit- or the

[4] A similar remark is made by Ingmar Persson [15]. He claims that the merit-idea has implausible implications if taken in conjunction with the assumption that legal ownership can be a desert basis. This is because then by this idea we can conclude that the life of an agent is intrinsically more valuable if the agent owns more, which is dubious at best.

merit-idea, or perhaps a combination of the two, should be the guiding principle of an appropriate *C*-function, and if any of the three, in what way that should happen. In fact, the whole discussion about desert-adjusted axiologies revolves around capturing our intuitions about the IV of a possible world in an appropriate way, and so it substantially revolves around the definition of the contribution function and its behavior. The present thesis is devoted to contributing to this debate.

Before moving to the discussion of what assumptions Feldman and the other proponents of DAC must make in order to provide a definition of contribution function, let us mention that besides Feldman's proposal, several other authors have put theirs on the market [16],[14],[17],[18]. As we said, these differ with respect to the definition of the *C*-function, and the properties that the authors think this function should have. We will see some of these proposals in full in the next chapter.

2.1 Assumptions underlying DAC

2.1.1 The concept of well-being

The first assumption we discuss regards well-being. Proponents of DAC must assume that a w_i number can be obtained, i.e. that individual well-being can be measured. In addition, to be able to measure it one must assume that there exists a unique and uncontroversial way to define it, which would then be used in the measurements. However, the ongoing debate on what constitutes human well-being testifies that scholars deeply disagree about what that is, and this implies that both assumptions are very problematic. Let us briefly review the debate.

In *Reasons and Persons*, Derek Parfit distinguishes the two very different ways in which the welfare notion can be understood: as a subjective and as an objective notion [19]. According to the subjective understanding, the truth about well-being is to be found in one's personal desires or preferences. The more these are satisfied, the higher the well-being is. The theory that define welfare as preference satisfaction, or pleasure achievement, is called *Hedonism*; the one that defines it as the realization of one's desires, is called *Desire-Fulfillment theory*.

According to the objective understanding, the notion of well-being is illustrated by an objective-list, which enumerates things that are good for the individual-self, independently of whether the subject actually desires any of them. The more elements of the list are satisfied, the higher her well-being is [20]. Utilitarians often adopt the subjective perspective and define well-being as preference satisfaction. However, this adoption does not manage to remove the ambiguity of the well-being notion, as the concept of preference has proven elusive, or difficult to measure too [21] [22].

Besides the disagreement about the definition of welfare and the possibility to measure it, and even assuming these away, scholars still disagree about what is *the right scale* to measure it [14]. The authors we discuss in this paper, however, assume that a ratio scale is adequate to make sense of our intuitions regarding welfare. To illustrate why, a ratio scale is defined as unique up to similarity transformation, which means that it admits all transformations that have the form $f(x) = \alpha x$, with $\alpha > 0$. Then, through such a scale one can represent claims as "Alice has two times higher welfare than Bob", which are ubiquitous in axiological discussions. Moreover, ratio scales have a natural zero point, and utilitarians use it to represent zero welfare levels [14]. This level represents the neutral welfare level, and it is understood in absolute terms, i.e. equal for everyone. The assumption of the existence of a neutral level of well-being that is equal for everyone is also problematic, for it is unclear what that amounts to, if anything, and in the literature it is rarely discussed.

Lastly, the term "welfare level" is used here to indicate that an agent has or has received a certain quantity of well-being. The welfare level is in fact the numerical representation of individual welfare corresponding to w_i, where i is an agent. The variable w_i is usually allowed to vary from positive to negative numbers.

2.1.2 The concept of desert

The second assumption we discuss concerns the desert notion. Since DAC defines the value-function on two distinct variables, one of which is the agents' desert level, DAC proponents must assume that a unique and precise definition of desert, which they can use to measure the desert level of the agents in the population.

However, besides discussing only very briefly what welfare is, Feldman and the rest of the literature on desert-adjusted axiologies discuss very briefly the desert concept too. They take the notion as a primitive[5] of their theory, and they do not delve much into it. Mostly, it is Feldman who discusses how DAC defines and uses the concept. He claims that it is not equal to individual merit, or merely paired with considerations about moral virtue or vice. Rather, the concept is taken to encompass more general considerations about distributive justice, so that "to say that a person deserves some good is to say that it would be “distributionally appropriate" [from the perspective of justice] for him to get it" [6].

Clearly, there can be hundreds of reasons according to which it is distributionally appropriate to give someone some good. In more technical terms, there can be multiple *desert bases*. Feldman mentions five categories, leaving open the possibility of there being more than that: (i) excessive or deficient past receipt of welfare, where "excessive past receipt lowers your desert level for a good; deficient past receipt increases it"; (ii) the agents' moral worthiness; (iii) the agents' basic rights or (legitimate) claims; (iv) bad luck; (v) basic needs, e.g. medical treatments [6] [25].

The assumption of these five as classes of desert bases makes clear that Feldman and proponents of desert-adjusted axiologies use the desert concept in a broader (but still intuitive) way than how another portion of literature uses it [26]. Indeed, there the desert concept has been tied together with the concept of responsibility, i.e. something is a desert basis only if an agent can be said responsible for bringing it about [27]. This has been called "The Responsibility Principle" [28]. Feldman instead explicitly claims that a desert basis can be independent from what can be responsibly brought about [25]. To see why this is the case, consider the first class of desert basis: (i) excessive or deficient past receipt of welfare. An agent past receipt of welfare can be, and often is, independent from what is under the agent's control. So if past receipt is a basis for desert, then an agent's desert level does not depend on her voluntary actions, or those under her responsibility. In the case of basic human rights (or bad luck or basic needs) this is even clearer. An agent does not have a right because she did some voluntary action through which she achieved the right. She simply has it.

It seems that there is a rather stringent reason why proponents of DAC need a broad and general conception of desert, even though they never make such

[5] A primitive concept is a concept that is not further defined in terms of other, more basic concepts. It is itself a basic one [23]. Some have advanced the hypothesis that primitive concepts are in principle unlearnable [24]. One might object that this is the case for the desert concept.

reason explicit. To see what it is, recall that this new kind of consequentialism is introduced to answer the objection from justice. In the objection, the injustice is caused by the fact that utilitarianism does not account for distinctions between agents' features, i.e. agents' needs, bad luck, legitimate claims, etc. But each of these things must be taken into account in a just distribution of welfare, as otherwise they become bases for distributional improprieties and give rise to alternative instances of the objection from justice. The reason why proponents of DAC then seem to need a broad conception of desert is that they aim at answering *all* these versions of the objection, and they aim at doing it by constructing their axiology on principle (**DW**) instead of (**W**). According to that principle, the only additional element that matters in the definition of what is good is individual desert. This means that, to answer all versions of the objection, DAC proponents must redefine the desert concept in such a way that is sufficiently broad to meet them. The only way to do it is by letting desert comprise all the bases for distributional improprieties that give rise to an objection from justice. In other words, the set of desert bases is now taken to be equal to the set of bases for distribution improprieties. Only if that is the case, the (**DW**) principle alone is sufficient to answer every instance of the objection.

Importantly, note that the way in which the set of desert bases is defined has implications for the behavior that the IV- and the C-functions have. For example, we mentioned above that if need is a desert basis then the C-function cannot represent the merit-idea, for that would mean that the value of a life increases when its needs increase, which is implausible. Then, if we want the IV-function to be guided by the (**DW**) principle alone, as well as DAC to answer all the objections from justice, then the function must not have the property of increasing on the sole basis that the desert level increases. This shows that the properties of the C-function, and therefore also of the IV-function, substantially depend on the definition of desert.

In the following, we will use the term "desert level" to represent the amount of desert an agent has. An agent's desert level does not merely represent what we could call "situational desert", namely the merit that she gains, given that she performed a certain action in a certain situation. Rather, it represents the more comprehensive amount of desert that her life, taken as a whole, has. Intuitively, this amount of desert increases or decreases owing to the actions she performs or the particular situations of bad/good luck in which she finds herself, i.e. owing to the situational desert. However, note that these variations are assumed away in the discussion of desert-adjusted axiologies, which simply considers desert values as given and the desert concept as primitive.

2.1.3 Individualism: additive separability

As we have seen above, DAC defines the intrinsic value of a possible world W as $IV(W) = \Sigma_{i \in N} C(w_i, d_i)$. This means that it takes such value to be an *additive* sum of individual contributions that are assumed to be *separated* from one another. This kind of consequentialism then holds the *additive separability* of the contributions that the agents make to the IV of the world they inhabit.

This assumption is endorsed by utilitarians and DAC theorists alike, and it is a strong one. It amounts to say that the theory abides to *individualism*, in two interrelated senses. Firstly, it holds that we can specify, for each agent i individually, what i's contribution C_i to the intrinsic value of a possible world is, and secondly, it holds that the contribution of an agent i to the intrinsic value of a world is defined without appealing to characteristics of agents other than i.

Prima facie, the assumption of additive separability may seem to be a natural one. It is also a convenient one, as it allows one to restrict one's attention to the individual contribution function. However, some authors have contested it. Thomas Hurka, for example, doubts that an individualistic or non-comparative contribution function is the appropriate ground for a theory of value designed to determine the morally right distribution of resources. Distributive justice, he claims, is "essentially holistic" [7].

2.2 The complementarity of comparative and individualistic principles

To illustrate Hurka's doubts, imagine the following situation. Consider two agents, Alice and Bob, and suppose that they are equally virtuous. Imagine further that they both receive more welfare than they deserve, but Bob receives considerably more welfare than Alice. Intuitively, we deem this situation as unjust, because equally deserving agents should receive an equal amount of welfare. By the same intuition, if we wanted to improve this situation and redistribute welfare so to reestablish justice, or, in other words, if we wanted to increase the intrinsic value of that possible world, we would say that the best option is either to level Alice's welfare level up, or to level Bob's one down,[6] so to make them even. Indeed, it seems that only if they were even, justice would be obtained.

Interestingly, the reasoning we proposed in the evaluation of the situation as unjust, as well as the one we used in the suggestion of a solution, was not guided by the non-comparative fit-idea. Indeed, if we had used that principle to judge how valuable that possible world is, we would still have concluded that it is low, and so to some extent unjust, but we would have concluded it on different grounds than the ones we provided. We would have said that since none of the agents received an amount of welfare that matches her desert level, the intrinsic value of their lives is less than optimal. In addition, if we had used the same principle to improve the situation and solve the injustice, we would have concluded that levelling up Alice's welfare level would have lowered the intrinsic value of the outcome even more, as the degree of fit between her welfare and desert levels would have decreased even more. In a similar way, we would have concluded that levelling down only Bob's welfare level would have improved the situation, but would not have let it reach its optimal intrinsic value, because both agents' degree of fit would not have been perfect yet. What this principle suggests as best option is to level both agents' welfare levels down, so that they fit their desert levels.

The levelling down option has traditionally received numerous criticisms. Mainly, these depend on a widespread agreement that it is always better to have more welfare than less, implying that the conclusion drawn by using the non-comparative principle is a rather counterintuitive or implausible one. Hurka too agrees that levelling down is not the right choice, and instead claims that, "at least in one sense", levelling up Alice's welfare would improve the intrinsic value of the outcome [7]. To justify this claim, he points at the equality between the desert levels of the agents involved, and submits that it is unjust that equally deserving agents receive unequal amounts of welfare, independently of whether this is above or below the amounts of welfare they individually deserve. Then, reducing these discrepancies is an improvement in the IV of the possible world in which they happen.

Let us further try to understand the comparative injustice involved above with a more concrete example. Imagine that Alice and Bob live in the same country and they do the same job, say that they are financial specialists. They both have studied many years at the university to obtain their current position,

[6] For simplicity, we do not consider the case in which we make their welfare level even by slightly leveling up Alice's and slightly leveling down Bob's welfare level, so to let them meet in the middle. However, analogous reasoning applies to that case too.

and they are both highly qualified to do it. Because of this, they both sell a comparatively high amount of financial products, i.e. they perform similarly, and extremely well. Unfortunately though, in the country in which they live, the wage gap is still substantially wide, despite the fact that the country itself is wealthy and generally advanced. Then, even though both Alice and Bob receive a generous salary for their work, even higher than average, Bob still receives a way higher salary than Alice. Clearly, the situation is unjust, for Alice and Bob do the same job, are equally qualified to do it, and perform equally well. Clearly, equally meritorious people deserve to be treated equally.[7]

With this concrete example, it becomes evident that reducing the discrepancies between the welfare that equally meritorious agents receive improves the IV of the possible world in which it happens. It also becomes evident that this should not be done by levelling both agents' received welfare down, so to meet their desert level, for that would lower both agents' salaries, which is unnecessary. What it seems we should do is to either level Alice's welfare up, or level Bob's one down, or do a little of both. Only in either of these cases the comparative perspective would judge the IV of the possible world increased. At the same time, we have seen that none of these cases is what the non-comparative fit-idea prescribes, and that if any of them obtained, the non-comparative perspective would judge the IV of the possible world reduced. Thus, there seems to be a tension between comparative and non-comparative perspectives, in the sense that if one of the two is satisfied, sometimes the other one is not. However, it seems that the IV of a possible world should be evaluated using both perspectives, on pain of letting our axiology be blind to some kind of injustice.

Indeed, each of the two perspectives alone lets some injustice go unnoticed. The comparative perspective only considers comparisons between agents' welfare and desert levels, and disregards completely the extent to which the first matches with the second in individual assessments. Then, this perspective alone is unable to recognize that the state of affairs in which the welfare an agent receives is insufficient to meet the welfare the agent deserves has lower value than the state of affairs in which it does not. Importantly, by failing to

[7]Note that the desert level involved in this example is of the kind we called *situational*, i.e. it is the amount of desert that an agent has on the basis of a given situation. Even though we claimed that we are interested in desert levels representing only the second option, the intuition about the injustice involved in differential treatments of equally deserving people, that should result from the example, can be generalized to the case we are interested in, too.

acknowledge this difference in value claims, the axiology ends up describing as morally just a distribution that is actually (non-comparatively) unjust. This means that the objection from justice strikes back, this time against a strictly comparative axiology (i.e. one that does not satisfy individualistic properties).

Analogously, the non-comparative perspective alone is blind to the comparative kind of injustice. Since to evaluate the life of an agent this perspective only considers the degree of fit between received and deserved welfare, it does not make any comparison between agents' features. Therefore, the sort of injustice that is based on such comparisons goes unnoticed, and the axiology grounded merely on a non-comparative perspective fails to acknowledge a difference in value claims. Once again, if that comparative dimension is not accounted for, the objection from justice strikes back.

Hence, it seems that the comparative and non-comparative perspectives are complementary elements of any desert-adjusted axiology that aims at correctly representing the IV of possible worlds. Indeed, realizing only one of them without the other makes the axiology incomplete and thus inadequate. As we have seen, the literature has proposed non-comparative axiologies, as Feldman's one. However, it has not discussed whether they satisfy comparative properties, or when it is the case that they do. In the rest of the thesis, we will work towards providing an answer to these questions, from a formal perspective. To that goal, in the next quite technical chapter, we will introduce and examine two of the formal non-comparative desert-adjusted axiologies that have already been proposed in the literature: Feldman's and Arrhenius' ones.

3 Individualistic Desert-Adjusted Axiologies: Formal Properties

In the preceding chapter, we have seen that one of the options to answer the objection from justice is to introduce considerations of desert in the axiology of a consequentialist theory. We have also presented some informal non-comparative principles that state how to introduce such considerations, and so how desert should interact with welfare and increase or decrease the contribution of each agent to the IV of a possible world. We called these principles the fit- and merit-ideas. However, we have not yet explored how exactly to implement these principles in the contribution function and we have not yet decided the overall behavior that the C- and IV-functions should have so that these principles as well as our moral intuitions are correctly represented. What properties should these functions satisfy?

In this chapter, we will focus on the answers that two proponents of DAC have given to this question. Firstly, we will examine Feldman's account and the six intuitive principles for the desert-adjusted axiology that he suggests [6]. Subsequently, we will introduce and discuss part of Gustaf Arrhenius' proposal for the properties of the C-function [14]. Since some of Feldman's remarks have been proposed in an informal way, then previous to their analysis, we will introduce a formal framework into which we will translate them. Besides increasing precision, the formalization has the advantage of offering a uniform ground to discuss and compare Feldman's and Arrhenius' proposals for the properties of the functions. Secondly, it will help us understanding what these properties are or should be, by making clearer what implications they have. Importantly then, we are not starting our discussion with a set of desirable properties for the IV- and the C-functions that we merely aim at formalizing. Instead, we will use the formalization to gain insights on such properties, testing them by means of our judgments about their concrete instantiations.

The structure of the chapter is the following. In the first section, we will introduce a specific definition and formalization of the contribution function and a specific formal framework that we will use throughout the thesis. In the second section, we will discuss Feldman's proposal, and turn the informal properties that he suggests in formal axioms of the contribution function, according to the framework we chose to work with. In the third section, we will discuss Arrhenius' proposals for the properties of the contribution function and the formalization of the fit-idea. We will see that he distinguishes two parts in the fit-idea and formalizes each of them separately.

As we said, the present is a technical chapter, and is entirely focused on presenting two of the individualistic desert-adjusted axiologies that have been proposed in the literature. The theories we present here are the ones that we will later assess with respect to comparative requirements of justice. Besides presenting the theories themselves, in this chapter we will also discuss what alternatives there could be to the properties that they have been endowed with. The discussion about such properties is still open, but in the fourth Chapter we will see that some of them are particularly relevant to comparative justice.

3.1 First step to formalize the C-function

In the previous chapter, we have introduced Feldman's formalization of the intrinsic value of a possible world as the function $IV(W) = \Sigma_{i \in N} C_i(w_i, d_i)$. We did not further specify the definition of the C-function itself, only mentioning that Feldman proposes to let it range over received and deserved welfare and to define it by means of the fit-idea. Since this proposal is in rather vague and informal terms, there are many ways to make it more precise. In this thesis, we will adopt the one that has been firstly suggested by Ingmar Persson [15]. Discussing Feldman's theory, he proposes to define the C-function as a function of the following values:

(a) the value of received welfare;

(b) the value of the fit between received and deserved welfare.

Note that while this definition specifies the C-function as given by those two values, it does not depart from Feldman's initial proposal. This, because such a function still ranges over the two variables he initially suggested, namely received welfare w and deserved welfare d, and is defined according to the fit-idea.[1] Assuming this definition then allows us to retain the essence of Feldman's proposal while giving us a more precise characterization of the function. In addition, this definition is particularly advantageous to us. Since it combines the two variables in such a way that isolates the value of received welfare from the value of the fit, it allows us to study the properties of each component in separation from the other.

Note, however, that by adopting it we are making two substantial assumptions. Firstly, we assume that the basic carriers of intrinsic value of an agent's life are two: received welfare and the fit between received and deserved welfare. Secondly, we assume the possibility of separating the values of these two carriers from each other. In other words, we are assuming that there exist two things in an agent's life that are good as such, and that these two things are good in two distinct ways. Indeed, the value of received welfare coincides with what is intrinsically good *for the agent herself*, whereas the value of the fit captures what is good *from the perspective of justice*. The latter value does not necessarily coincide with what is good for the agent. For example, one might say that from the perspective of justice it is good to keep a vicious agent in prison, and yet this is not good from the perspective of the agent herself. Persson uses the terms *personal* and *impersonal values* to name the two values,[2] respectively [15]. These are particularly illustrative terms, as it seems that the value of received welfare is a more personal kind of value than the value of the fit. The former is about what makes the life of an agent go best, whereas the latter refers to the justice of that agent's life going best, and so it has an impersonal nature.

Even if more precise than Feldman's, Persson's definition of the C-function is still informal. Then, there are still many ways to formalize it, as there are many ways to combine (a) and (b). We will assume the following one, which formalizes it as the *sum* of (a) and (b):

$$C_i(w_i, d_i) = WV(w_i, d_i) + FV(w_i, d_i) \tag{3.1}$$

where i is an agent, w_i is the welfare agent i receives, d_i is the welfare agent i deserves, $WV(w_i, d_i)$ is the representation of (a), which we will call *welfare value*,

[1] We will see below that there is a sense in which instead it departs from Feldman's proposal.

[2] Broome uses the terms *personal* and *communal goods* to name the same concepts, respectively [29].

and $FV(w_i, d_i)$ is the representation of (b), which we will call *fit-value*.

This formalization of the contribution function was firstly suggested by Gustaf Arrhenius [14]. Importantly, it proposes to *add* the two *separate* components of the function, i.e. it assumes their *additive separability*. Recall that we have encountered the additive separability assumption in the last chapter, when discussing desert-adjusted consequentialism. There, it was applied to the contribution functions of the agents in a possible world, which were indeed defined as additively separable from one another. Here, it is still applied to the individual contribution functions, and in addition it is applied also to the welfare and fit-value components of each contribution function.

Clearly, these assumptions have consequences for the definition of the IV-function, which is now given by the following additively separable function:

$$\begin{aligned} IV(W) = & C_1(w_1, d_1) + C_2(w_2, d_2) + \cdots + C_n(w_n, d_n) = \\ & (WV(w_1, d_1) + FV(w_1, d_1)) + (WV(w_2, d_2) + \\ & FV(w_2, d_2)) + \cdots + (WV(w_n, d_n) + FV(w_n, d_n)) \end{aligned} \tag{3.2}$$

where W is a possible world and the other symbols are interpreted as above.

Recall that the assumption of additive separability of the intrinsic value of agents lives is a *substantial* assumption, as it amounts to assume individualism, as we have seen above. However, it is also a *simplicity* assumptions, as it makes the analysis of the intrinsic value of a possible world easier to handle or manipulate. Indeed, even though those components could have been combined in other ways, for example by multiplying rather than summing them, addition is a simpler way to combine them, and since we do not have any reason to adopt a more complicated form, we stick to that.[3]

Another simplicity assumption that we make is the following:

$$WV(w_i, d_i) = w_i \tag{3.3}$$

This means that for each agent i, the value of the welfare i receives is equal to i's welfare level. In other words, we do not distinguish between the welfare an agent receives and the value that this welfare has. Note that this is not always the case. If welfare levels are interpreted as amounts of material goods that agents receive, then their value might differ from the concrete amount of

[3]Note that the same comment holds also for assumption (3.1).

good received and thus the two could be represented with different numbers. This means that assumption (3.3) is not the only way to relate welfare values and welfare levels. Another way could be to take $WV(w_i, d_i) = 2w_i$, for every agent i. However, we again have no reason to choose this more elaborated option, so we stick with the simplest $WV(w_i, d_i) = w_i$.

Assumptions (3.1), (3.2), and (3.3) together compose the formal framework that we will use to analyze Feldman's and the other proposals. Since this framework was firstly proposed by Arrhenius, let us call it the "Arrheniusean framework", and turn to discuss the proposals by means of it.

3.2 Feldman's proposal

As we saw above, Feldman is the first to put forward a desert-adjusted consequentialist theory. This theory, originally spelled out in hedonistic terms, is a first account of how the intrinsic value of an agent's life depends on the interaction between individual desert levels and individual welfare levels. In a nutshell, this theory holds that positive and negative desert levels interact with episodes of pleasure or pain by either increasing or decreasing their value. A neutral desert level instead leaves the value of pleasure and pain unmodified.

This theory is presented by means of six principles and six graphs. Each principle states what is the IV of an agent's life, given her welfare and desert levels, while each graph pictures a possible interpretation of a principle. In the following, we will discuss only the first three principles, i.e. the ones about the interaction of desert with episodes of pleasure, and we will leave the analogous discussion of the ones about pain in the Appendix. After having introduced them, we will translate the principles in formal axioms for the contribution function, by adopting the Arrheniusean framework presented above. Note that Feldman did not propose any formalization of the principles, and so in particular he did not assume the additive separability of welfare and fit values as in the framework we adopt. The conclusions we will draw are thus dependent on this or the other assumptions of the Arrheniusean framework.

Lastly, note that even if he did not suggest any particular formalization, Feldman did suggest some graphic interpretations of the principles and the behavior of the contribution function. The graphs thus contain a considerable part of the theory, which is the reason why in what follows it will be important to also inspect them carefully, and formulate axioms that are congenial to them too.

3.2.1 Principles

The first principles that we analyze are the ones about the goodness of pleasure:

P1: Positive desert enhances the intrinsic goodness of pleasure.

P2: Negative desert mitigates the intrinsic goodness of pleasure.

P3. Neutral desert neither enhances nor mitigates the intrinsic goodness of pleasure.

These principles present the relations between desert and welfare levels. The intuition behind the first is that when a person who deserves it experiences pleasure, something good and just has happened. This goodness is created by the match between the positive nature of pleasure and positive desert. Similarly, the intuition behind the second is that when a person who does not deserve it experiences pleasure, something bad and unjust has happened. This badness is instead created by the *mis*match between pleasure and negative desert. Lastly, the intuition behind the third is that if a person has neutral desert, it is neither good nor bad that she experiences pleasure.[4]

As they are formulated, these principles seem to be based on essentially three elements: a desert level, a positive welfare level (pleasure) and the intrinsic value of this welfare (goodness of pleasure). The principles then claim that the intrinsic value of welfare might change depending on the desert level of the person who receives the welfare. The total intrinsic value of her life ultimately resides in this final modified value.

The principles do not explicitly mention the fit-value of a life. How do they talk about it then? As we saw above, Feldman takes the increment or decrement in welfare value induced by individual desert to represent how just or how good it is that an agent receives that amount of welfare. This change in the welfare value then represents the justice component of the intrinsic value of the agent's life, i.e. the fit-value.

[4]Clearly, one could say that there is instead a mismatch here too. This is one of the criticism that has been directed to Feldman's proposal and we will see it in the fourth chapter, as it relates to comparative justice as well.

That being said, let us now consider each of the principles more closely and translate them in our formal framework.

P1 states that the value of positive welfare is increased by positive desert. Translated in our framework, P1 says that the IV of the life of an agent with positive welfare is higher than the value of its welfare, when the agent has positive desert. In formal terms, P1 states that if d and w are positive, then $C = w + FV$ is greater than $WV = w$. Since the latter is positive, then also $C = w + FV$ must be positive, which implies that FV must be positive too.
According to P2, the value of positive welfare is decreased by negative desert. In our framework, P2 says that the IV of the life of an agent with positive welfare is lower than the value of its welfare when the agent has negative desert. This means that when d is negative but w is positive, P2 states that $C = WV + FV$ has lower value than WV. For this to happen, FV must in this case have negative value.
Lastly, P3 states that neutral desert levels leave the goodness of pleasure unmodified. This principle can be represented in our framework by assuming that when $d = 0$ and w is positive, then P3 demands $C = WV$. For this to happen, FV must be equal to zero.

We summarize the import of axioms P1-P3 through the table below.

	P1: $\mathbf{d > 0}$	**P2: $\mathbf{d < 0}$**	**P3: $\mathbf{d = 0}$**
$\mathbf{w > 0}$	FV > 0	FV < 0	FV $= 0$

TABLE 3.1: Positive Welfare

To illustrate these principles, Feldman proposes three distinct graphs, each of which represents one of the possible interpretations of one of the principles. We merged the three graphs in the single one below, where the red line represents a possible interpretation of P1, in the case an agent S has positive desert level +10, while the blue and green line represent possible interpretation of P2 for negative desert level -10.[5] The horizontal axis represents the hedonic level, so the amounts of pleasure that the agent experiences, while the vertical axis represents the intrinsic value of such pleasure.

Let us first discuss the red function. Since Feldman explicitly mentions the

[5]Note that the graphs are silent on what happens for other desert levels.

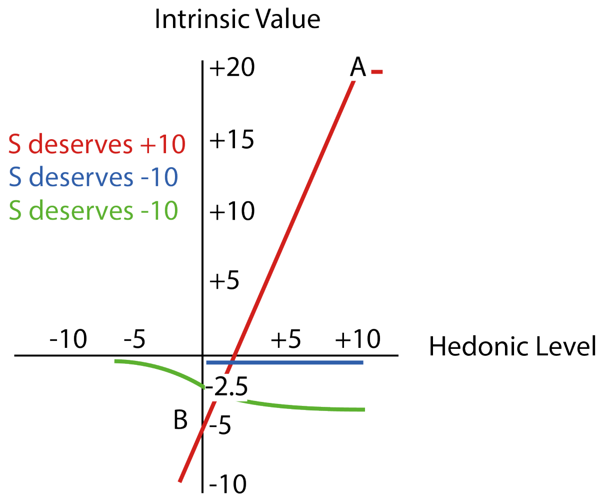

FIGURE 3.1: Graphs Positive Welfare

values of two points, namely A and B, it is possible to deduce which contribution function the red line represents. The function is the following:

$$C(w, 10) = 2.5w - 5 \tag{3.4}$$

Given this function, let us discuss the three points of interest mentioned by Feldman. The first is point A, representing the pair composed by $WV = 10$ and $IV = 20$. This point is meant to show the content of P1, namely that the value of 10 units of pleasure is enhanced if it is received by an agent S such that $d_S = 10$. Since in our framework, $WV = w$ then we have $w = 10$ and if we plug in the C-function $w = 10$, we get $C(10, 10) = (2.5 \cdot 10) - 5 = 20$, consistently with Feldman's take. Since we assumed that $C = WV + FV$, then from the value of C we can also derive the value of FV, i.e. $FV = C - WV5 = 10$. This is consistent with what Table 3.1 states.

The second point of interest is the one that flattens the curve on the right of A, indicatively representing a pair composed by $WV > 10$ and 3.5 an $IV \leq 20$. Feldman takes this point as showing that as an agent starts receiving more than she deserves, "additional increments of pleasure have decreasing marginal value"[6] [6]. Let us call *undeserved welfare*, a welfare level that is higher than the agent's desert level, *deserved welfare*, a welfare level that is equal to it, and *over-deserved welfare*, a welfare level that is lower than it. Then, the flattening point represents the idea, not explicit in any of the principles, that undeserved welfare is less valuable than deserved welfare. Note that we

[6] This is not clear from the picture in Feldman's paper, where it rather seems that the IV stays equal.

cannot derive the existence of points on the right of A by using function (3.4) and, e.g., $w = 11$, as this is a linear function and thus we would get a value higher than 20, i.e. $C(11, 10) = (2.5 \cdot 11) - 5 = 22.5$. Moreover, if the function was instead linear, the fit-value of $(11, 10)$ would be equal to 11.5, which is higher than the fit-value of $(10, 10)$. This would mean that the fit-value does not have its maximum at $w = d$, which is the contrary of what the fit-idea states.

Because of this reason, it seems that the only two values of the function that Feldman explicitly mentions are not actually sufficient to construct the function he pictures in the graph. Then, we propose to redefine the contribution function that the red line represents by intervals, and let it be equal to 20 whenever $w \geq 10$. The resulting function is the following, which we will call "Feldman's Function":

$$C(w, 10) = \begin{cases} 2.5w - 5 & \text{for } w < 10 \\ 20 & \text{for } w \geq 10 \end{cases} \tag{3.5}$$

This function is consistent with Feldman's informal suggestion about the flattening point on the right of A, as we have $C(11, 10) = 20$, by 3.5 and $11 \geq 10$. Moreover, the fit-value now has its maximal value (also) at $d = w$, as we have $FV(w, 10) = 10$ whenever $w \geq 10$. This is also consistent with what Table 3.1 says.

The third point of interest is B, representing the pair composed by $WV = 0$ and $IV = -5$. Point B shows that if an agent S, such that $d_S = 10$, experiences neither pleasure nor pain, this is intrinsically bad, and thus the value of S's life is negative. Interestingly, also this fact is not captured by any of the principles, as none of them talks about neutral welfare levels. Note that equation (3.6) agrees that $C(0, 10) = (2.5 \cdot 0) - 5$ and $FV(0, 10) = -5 - 0 = -5$. However, we cannot compare this result with Table 3.1, as it is silent on welfare levels equal to zero.

Let us now get back to illustrate the other two functions represented in 3.1, the blue and green lines. For these two functions, Feldman does not explicitly mention some of their points, so we cannot deduce what function underlies them. We will proceed by discussing them informally. The lines are different but possible representations of P2. The blue line shows that pleasure is mitigated by negative desert level in the sense that its intrinsic value becomes null. In this case then, Feldman takes $FV = -WV$, so that $C = WV + FV = 0$.

Since $C = 0$, $WV = w$ and w is positive by assumption, then $C < w$, as P2 states. The green line instead shows that pleasure is mitigated by negative desert in the sense that its value becomes negative. Feldman calls the idea that negative desert can sometimes invert the intrinsic value of pleasure, the "transvaluation" of the goodness of pleasure into badness. We can represent this interpretation of P2 by taking $FV < -WV$. Clearly then we have $C < w$, consistently with P2.

Note that Feldman does not add in the graph the behavior of the contribution function when $d = 0$. This is probably because it is straightforward what that is. Indeed, by principle P3 we have

$$C(w, 0) = w \quad \text{for } w > 0 \tag{3.6}$$

So much for what concerns episodes of pleasure. Feldman then proposes other three principles to capture the interaction between individual desert and episodes of pain.

They are the following:

P4: Positive desert aggravates the intrinsic badness of pain.

P5: Negative desert mitigates the intrinsic badness of pain.

P6: Neutral desert neither enhances nor mitigates the intrinsic badness of pain.

By adopting an analogous reasoning to the one used for principles P1-P3, the reader can understand principles P4-P6 too. Anyway, in the Appendix we discuss the three graphs that Feldman uses to illustrate the above principles, merged again in a single graph in Figure A.1.

Note that none of the principles talks about how desert interacts with the neutral amount of pleasure and pain, i.e. the welfare level equal to zero. Indeed, each of the principles is either about the interaction of desert with pleasure or about the interaction of desert with pain. The theory is in general silent about how desert relates with neutral well-being levels, except for what it says by means of the graphs that we have seen above, i.e. for agents that have desert level equal to +/- 10.

Despite this, in the fourth chapter we will see that some authors have still discussed Feldman's proposal with respect to such neutral welfare level, and proposed strong criticisms to it that points at its failure to satisfy comparative justice. However, for now, let us move to the discussion of another non-comparative theory that the literature has proposed: Arrhenius' one.

3.3 Arrhenius' proposal

Gustaf Arrhenius proposes a formal analysis of the contribution function and, in particular, of the axioms that it should satisfy [14]. As we have seen above, he adopts a specific framework, the Arrheniusean, which introduces a specific representation of the function, namely $C(w,d) = w + FV(w,d)$. His work focuses on the second component of the C-function, the fit-value function FV, and comprises several proposals for how to capture the fit-idea and its properties.

The discussion that follows is centered on the implications and formal details that possible formalizations of such properties have. Some implications will be more preferable than others, although in the end Arrhenius does not settle on a correct or final formalization of the fit-value. He rather claims that his is just a preliminary proposal, or a "default theory", which can then be used to build more elaborated version of desert-adjusted axiologies [14].

For the sake of the analysis, Arrhenius suggests to split the fit-idea in two components and to analyse each of them separately.[7] The two parts are the following:

> *The First Central Fit-Idea:* The better the fit between receipt and desert, the higher the fit value.
>
> *The Second Central Fit-Idea*: The contributive value of a given increase in fit by a change in the receipt decreases the closer to the desert level one gets.

The first part captures the fit-idea as such, namely that a receipt is more valuable if it is closer to individual desert level, and that justice is done when a perfect fit occurs. The second idea instead represents the intuition that "the greater the mismatch between receipt and merit, the greater the urgency to increase the fit between receipt and merit by adjusting the receipt" [14]. Even though Arrhenius leaves unspecified the relationships between the two ideas, they seem to be strictly related. Indeed, by the first fit-idea we can derive that

[7] We have modified the formulation of the two fit-ideas as well as of the properties of the function, to make them consistent with the terminology used here.

if an agent receives way less welfare than she deserves, her life has lower value than the life of an agent that receives almost as much welfare as she deserves. And if this is the case, we would say that there is more urgency in improving the life of the former rather than the latter agent, which is what the second idea says.[8]

Note that the two parts are distinct and represent different properties of the fit-idea. Indeed, a fit-value function might realize the first but not the second. Consider for example the following graph we designed, where on the horizontal axis there is the welfare level w and on the vertial axis there is the fit-value FV:

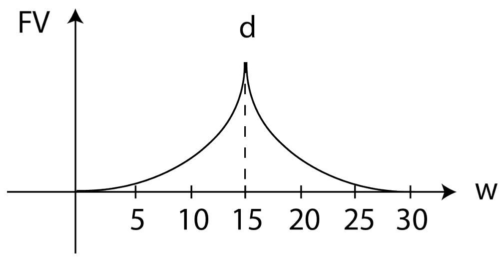

FIGURE 3.2: Function satisfying the first fit-idea but not the second.

In the graph, we represented the fit-value function for agents with desert level equal to 15. The function satisfies the first fit-idea, because the closer to desert d one gets, the higher the fit value FV. However, it does not satisfy the second fit-idea, because the closer to desert d one gets, the greater the contributive value of the increase in fit.

Given the informal definition of the fit-idea, it is not clear whether there exists a function that realizes the second but not the first fit-idea.[9] However, the graph in figure 3.2 above and its related argument are sufficient to show that the two parts are distinct and thus represent different properties of the fit-idea.

[8]Note that the second fit-idea captures a particular perspective about justice, namely that the worse off (with respect to their own desert level) should be benefited first. This recalls prioritarian perspectives on what is just (benefiting the worse-off first), and one might say that it is a special version of them. If this is true, then saying that the second idea is part of the fit-idea as such amounts to saying that the fit-idea represents a very specific perspective about what is just, one that is not necessarily shared by everybody. Some libertarians for example would not agree with these prioritarian views, because it is not necessarily true that they respect individual freedom [30], [31].

[9]However, as of now, it seems false that there exists a function that realizes the second fit-idea but not the first. Indeed, as it is stated, the second fit-idea seems to require that the derivative of the function decreases the closer to desert one gets. It does not seem to be possible to satisfy this requirement without having increases in value the closer to desert one gets.

For each part, Arrhenius proposes multiple formalizations, which are one the refinement of the other. We will now introduce and discuss most of them in turn. We will first state each of them in formal terms and then present a possible instantiation of each property through a graph. The picture will give the reader an immediate idea of what the proposal amounts to.

3.3.1 The First Central Fit-Idea

This part captures the fit-idea as we presented it in the first chapter, for it implies that the fit-value is maximized when receipt equals desert, and that it is anyway better that the two values are close to each other. Let us see the options that Arrhenius proposes to formalize it.

The first option.

F1-1: If $d \geq w_1 > w_2$ then $FV(w_1, d) > FV(w_2, d)$; and
If $w_1 > w_2 \geq d$ then $FV(w_1, d) < FV(w_2, d)$;

In words this says that "if two lives with the same desert both have less welfare than they deserve, or both have more welfare than they deserve, then the life with the least difference between desert and receipt of welfare has the highest fit value. Moreover, if one life receives exactly what she deserves, and another life receives more or less than she deserves, then the former life has the highest fit value" [14].[10]

We can represent axiom F1-1 in the following way:

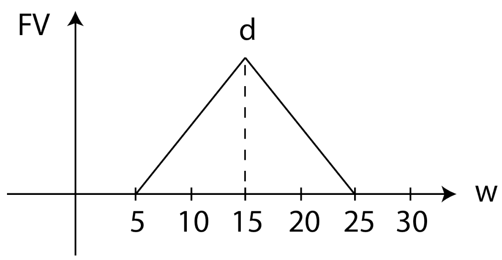

FIGURE 3.3: Function satisfying F1-1

This is the graph of the fit-value function of an agent S such that $d_S = 15$ and it represents axiom F1-1. To illustrate, consider two receipt levels w_1, w_2 such that $d \geq w_1 > w_2$, e.g. $w_1 = 10, w_2 = 5$. In the graph we have $FV(w_1, d) > FV(w_2, d)$, so the first clause of F1-1 is satisfied. Consider now w_1, w_2 such

[10]Here we reported a modified version of Arrhenius' verbal formulation of the axiom, namely such that it matches our terminology. We will do the same every time we report the axioms in words.

that $w_1 > w_2 \geq d$, e.g. $w_1 = 20, w_2 = 25$. In the graph we have $FV(w_1, d) < FV(w_2, d)$, so also the second clause is satisfied.

As also the graph illustrates, F1-1 represents the optimality structure that the first fit-idea involves, for it directly implies that if $w_1 = d$ and $w_2 \neq d$ then $FV(w_1, d) > FV(w_2, d)$. However, it represents the idea in a rather weak way. Firstly, it applies only to agents that have the same desert level d. Secondly, it only compares welfare levels that either are both higher than or equal to the desert level, or that are both lower than or equal to it, but it does not compare two welfare levels that are one above and one below the desert level. Then, consider for example $FV(4,3)$ and $FV(2,3)$. From F1-1 we can derive that $FV(4,3) < FV(3,3)$ and that $FV(2,3) < FV(3,3)$. However, F1-1 does not say anything about whether $FV(4,3) > FV(2,3)$ or $FV(4,3) < FV(2,3)$. Arrhenius then claims that the axiom is compatible with either option, i.e. F1-1 is compatible with the *asymmetry* between the value of under- and over-deserved welfare. We represent the options in the following graphs.

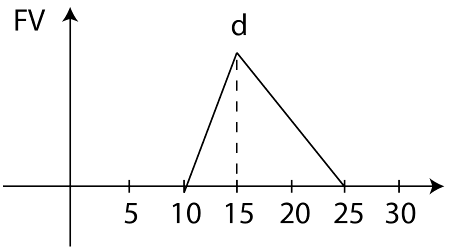

FIGURE 3.4: Steeper on the Left

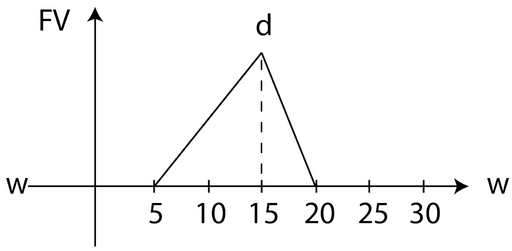

FIGURE 3.5: Steeper on the Right

Note that there are two kinds of asymmetry, and while the asymmetry of the graph on the left seems plausible, the asymmetry on the right is harder to justify. Indeed, it seems more valuable to have more welfare than less, but Arrhenius thinks we would not say that it is better to have less welfare than more.

There are two ways in which he proposes to adjust F1-1 so that it only implies the asymmetry on the left. One is by introducing another property of the fit-value that states that undeserved welfare induces a higher fit-value than the over-deserved one:

A1: If $|w_1 - d| = |w_2 - d|$ and $w_1 > w_2$, then $FV(w_1, d) > FV(w_2, d)$.[11]

By A1, we get that $FV(4,3) > FV(2,3)$ and not the opposite, and so it imposes the asymmetry that the graph on the right shows, as desired. The other option to represent that receiving more welfare than less is better, is to recall that we are assuming $WV(w,d) = w$ and that $C = w + FV$. Then, in this case we have $WV(4,3) > WV(2,3)$, so we obtain $C(4,3) > C(2,3)$. The difference between the two options lies in that the first one takes the asymmetry between under- and over-deserved welfare to induce an asymmetry in the fit-value, while the second one let the asymmetry lie in the individual welfare that the agents receive. Since the fit-value is the component that captures how just a distribution of welfare is from the perspective of individual desert, then the first option takes the asymmetry to be part of such matters, while the second one does not. Depending on the perspective that one takes, it is possible to adopt either option.

In addition to the above, Arrhenius says that under some interpretations F1-1 might be too strong. Take for example the need-base interpretation of desert. One might think that if all needs are satisfied, and so either $w_i = d$ or $w_i > d$ for both agents i, then all the demands of justice are satisfied too. Then we should have $FV(w_1, d) = FV(w_2, d)$. However, this is not what F1-1 states. Consider two lives such that $w > d$, e.g. $(5,3)$ and $(6,3)$. By F1-1 we have $FV(5,3) > FV(6,3)$ and not $FV(5,3) = FV(6,3)$ as the need-base interpretation of desert seem to require.

As Arrhenius suggests, this problem can be easily remedied by adjusting the second clause of F1-1 in the following way:

F1-1′: If $d \geq w_1 > w_2$ then $FV(w_1, d) > FV(w_2, d)$; and
If $w_1 > w_2 \geq d$ then $FV(w_1, d) = FV(w_2, d)$;

F1-1′ represents the idea that "there are no distributional concerns left when everybody has received at least as much as they merit" [14].[12] A possible interpretation of axiom F1-1′ is displayed in the following graph:

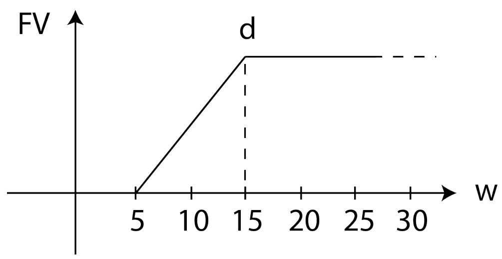

FIGURE 3.6: Function satisfying F1-1′

[12]In the following chapter, we will see that it is not true that there are no such concerns.

In the graph, for every w_1, w_2 such that $w_1 > w_2 \geq d$ we have $FV(w_1, d) = FV(w_2, d)$, as the new clause in F1-1′ requires. As it is clear from the graph, F1-1′ implies the asymmetry of under- and over-deserved welfare. The formal proof goes like this. Consider for example two lives $(11, 10)$ and $(9, 10)$. By the first clause of F1-1′ we have $FV(10, 10) > FV(9, 10)$, and by the second clause we have that $FV(11, 10) = FV(10, 10)$. Applying transitivity, we get $FV(11, 10) > FV(9, 10)$.

Note that the first fit-idea, stated informally, is silent on whether the fit-value function should be symmetric or asymmetric in the value of under- and over-deserved welfare is not mentioned there. Indeed, the idea simply states that the fit-value should increase as the receipt gets closer to the desert.

The second option.

F1-2: If $|w_1 - d| < |w_2 - d|$, then $FV(w_1, d) > FV(w_2, d)$; and
If $|w_1 - d| = |w_2 - d|$, then $FV(w_1, d) = FV(w_2, d)$;

In words this option says that "if two agents have the same desert level, and the difference between the first agent's receipt and desert is less than (equal to) the difference between the second agent's receipt and desert, then the fit value of the first agent's life is greater than (equal to) the fit value of the second agent's life" [14].

A representation of axiom F1-2 is given by the graph in Figure 3.3 above, where $d = 15$.[13] To illustrate why the graph represents F1-2, consider two receipt levels w_1, w_2 such that $|w_1 - d| < |w_2 - d|$, e.g. $w_1 = 10, w_2 = 5$. In the graph, we have $FV(10, 15) > FV(5, 15)$, so the first clause of F1-2 is satisfied. Consider now some w_3, w_4 such that $|w_3 - d| = |w_4 - d|$,, e.g. $w_3 = 20, w_4 = 10$. In the graph we have $FV(10, 15) = FV(20, 15)$, so also the second clause is satisfied.

Because of this second clause, F1-2 implies the *symmetry* between values of under- and over-deserved welfare, as the graph clearly shows. As we have seen in the discussion of F1-1, this might be taken as an undesirable feature. Clearly then one might decide to replace the second clause of F1-2 with A1 above, which solves the problem.

A positive implication of F1-2 is that the fit-value function has a maximum when perfect fit $w = d$ occurs, as the fit-idea seems to require. To illustrate,

[13] Note that also the graphs in Figure 3.2 satisfies axiom F1-2, but the ones in Figures 3.4 and 3.5 clearly do not.

by its first clause we have that if $d = w_1$, then for any other w_2, it holds that $FV(w_1, d) > FV(w_2, d)$.

Importantly, the following proposition holds:

Proposition 3.3.1. *In the Arrheniusean framework, F1-2 implies that if n units of welfare are to be distributed among a certain number of people with a total desert level of n, then the distribution where each person gets exactly what she deserves is better than any alternative distribution.*

Proof. The proof of this proposition can be found in the Appendix. □

Proposition 3.3.1 is interesting because it shows that even though F1-2 does not apply to agents with different desert levels, it does satisfy properties about such agents.[14] However, even though F1-2 does not apply to distinct d, it is clearly possible to strengthen it so that it applies to that too, as the following option shows.

The third option.

F1-3: If $|w_1 - d_1| < |w_2 - d_2|$, then $FV(w_1, d_1) > FV(w_2, d_2)$; and
If $|w_1 - d_1| = |w_2 - d_2|$, then $FV(w_1, d_1) = FV(w_2, d_2)$;

In words this option says that "if the difference between receipt and desert in a given life is smaller than (equal to) the difference between receipt and desert in another life, then the fit value of the former life is greater than (equal to) the fit value of the latter life" [14].

This axiom can be represented by the graph in Figure 3.7:

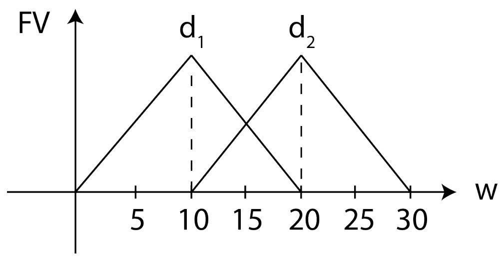

FIGURE 3.7: Function satisfying F1-3

The graph represents F1-3: for every d_1, d_2, the two functions have the same slope on both sides of d, and so they satisfy the first clause. Moreover, they

[14] The proposition shows that Arrhenius' theory satisfies one of the principles that Carlson claims to be essential for any desert-adjusted axiology. The principle is called **J1** and we will see it in the fourth chapter.

preserve the symmetry between values of over- and un-deserved welfare, and so they satisfy the second clause too.

F1-3 has interesting implications. For example, in the Arrheniusean framework, F1-3 implies, for any fixed welfare level w, the following two principles:

D1: If $|w - d_1| < |w - d_2|$, then $C(w, d_1) > C(w, d_2)$
D2: If $d_1 = w$ and $d_2 \neq w$, then $C(w, d_1) > C(w, d_2)$

For any welfare level w, D1 states that the intrinsic value of a life is higher if the fit between welfare and merit is closer, while D2 states that if a life has perfect fit between welfare and desert levels, then the value of that life is higher than any other in which that fit is not perfect.

Note that in the graph above we have depicted the fit-value functions as having the same peaks. This is because when $w_1 = d_1$ and $w_2 = d_2$, F1-3 directly implies that $FV(w_1, d_1) = FV(w_2, d_2)$, for every w_1, w_2. Arrhenius finds this implication interesting as it captures the plausible idea that justice has the same value in every situation. However, one might disagree with this implication and instead think that the fit-value of a very virtuous person getting exactly what she deserves should be higher than the value of a very vicious person getting what she deserves. Then, one might require the fit-value function to satisfy the following axiom:

A2: If $w_2 > w_1$, then $FV(w_2, w_2) > FV(w_2, w_1)$

In words this says that the greater the receipt, the highest the value of the perfect fit. The function in Figure 3.8 satisfies this axiom:

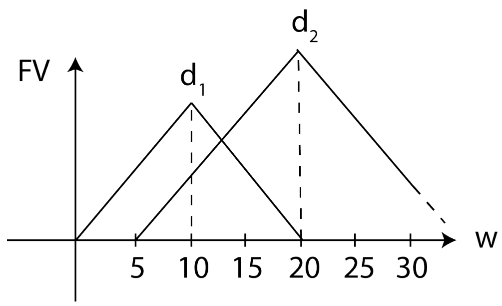

FIGURE 3.8: Function satisfying A2

Clearly this graph represents A2, because the two peaks have different heights, as the axiom requires.

Arrhenius suggests a second reason in favor of A2 and against F1-3, and so another reason why we should think of the fit-value as depending on the magnitude of the receipt and desert. The reason is illustrated by the following argument. F1-3 directly implies that $FV(1,2) = FV(99,100)$ and $FV(2,2) = FV(100,100)$. It follows that $FV(2,2) - FV(1,2) = FV(100,100) - FV(99,100)$. This means that if we were to distribute one unit of welfare to either $(1,2)$ or $(99,100)$, the increase in fit-value would be the same. By $WV = w$, this is equivalent to saying that the increase in *intrinsic* value of the life in either case would be the same, i.e. we would do equally good by giving one unit of welfare to one agent or to the other. This clearly is objectionable from the perspective of *proportional justice*, as $(1,2)$ is much worse-off than $(99,100)$, and so we should benefit the former and not the latter. Importantly, if it was the case that $FV(2,2) \neq FV(100,100)$ or $FV(1,2) \neq FV(99,100)$, and so if A2 obtained, we could not have derived the same conclusion. In the next chapters of the thesis, we will discuss proportionality in greater detail.

Arrhenius suggests yet another criticism to F1-3: by its first clause, this axiom directly implies that $FV(10,10) > FV(10,20)$. Since the receipt of the two lives is equal, then in the Arrheniusean framework this implies $C(10,10) > C(10,20)$. Arrhenius takes this to meaning that the value of a possible world can be improved by reducing people's moral worth, which is clearly implausible. Then, he suggests to reject the axiom and adopt F1-2 instead, which does not have the implication because it only considers lives with the same desert.

Note that Arrhenius reads this as a bad implication of F1-3 only because he adopts the moral interpretation of desert. However, as we have seen above, DAC proponents also consider the need-base interpretation of desert, i.e. the one according to which need can be a basis of desert claims. Adopting this latter interpretation, it does not seem anymore that the implication that $C(10,10) > C(10,20)$ is so wrong. In fact, one might say that a life is intrinsically better than another if it has less needs and all of them are satisfied. Then, under this perspective F1-3 is not subject of the latter criticism.

Anyway, Arrhenius concludes that F1-3 should be rejected as a representation of the first fit-idea, in favor of F1-2.

3.3.2 The Second Central Fit-Idea

> *The Second Central Fit-Idea*: The contributive value of a given increase in fit by a change in the receipt decreases the closer to the

desert level one gets.

As we said above, this idea says that the ones who are further from their desert level should be benefited first. As an example, consider three lives with the same desert level, e.g. $(5, 10), (4, 10), (3, 10)$. This idea says that "the increase in fit value from a change in pleasure from 4 to 5 is less than the increase in fit value from a change in pleasure from 3 to 4. In other words, $FV(4, 10) - FV(3, 10) > FV(5, 10) - FV(4, 10)$" [14]. Let us see how Arrhenius proposes to formalize it.

The first option

F2-1: If $|e_1| = |e_2|$, $|w_1 - d| > |w_1 + e_1 - d|$, $|w_2 - d| > |w_2 + e_2 - d|$, and $|w_1 - d| > |w_2 - d|$ then $FV(w_2 + e_2, d) - FV(w_2, d) < FV(w_1 + e_1, d) - FV(w_1, d)$

In words this axiom says that "if we can increase the fit between desert and receipt in two lives with the same desert level by adjusting their welfare level up or down by a fixed amount, then the life with the greater difference between receipt and desert will get the greater increase in fit value from the adjustment of its welfare level" [14].

This axiom can be represented as the graph in Figure 3.9.

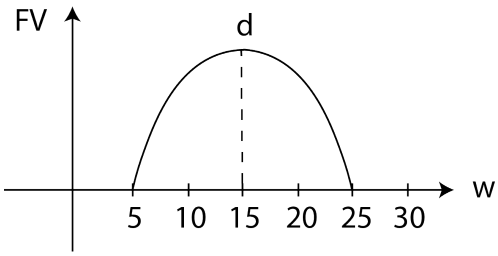

FIGURE 3.9: Function satisfying F2-1

This graph represents F2-1: consider $e_1 = e_2 = 5$ and $w_1 = 5, w_2 = 10$. Then, $|5 - 15| > |5 + 5 - 15|$, $|10 - 15| > |10 + 5 - 15|$, and $|5 - 15| > |10 - 15|$ and so e_1, e_2, w_1, w_2 satisfy the assumptions of F2-1. Given that graph, for these values we clearly have $FV(15, 15) - FV(10, 15) < FV(10, 15) - (5, 15)$, as F2-1 wants.

As we will see in the fourth chapter, axiom F2-1 succeeds in inducing the IV-function to satisfy some (not all) desirable comparative properties. With respect to what we will see in that chapter, it is now important to note that functions that satisfy axiom F2-1, as the one in Figure 3.9, also satisfy the property of *strict concavity*, i.e.:

Definition 3.3.2 (Strict Concavity)**.** For any real valued function f on an interval, f is said to be concave iff for any x and y in the interval and for any $\alpha \in [0,1]$,

$$f(\alpha x + (1-\alpha)y) > \alpha f(x) + (1-\alpha)f(y)$$

Indeed, even if Arrhenius does not mention it, axiom F2-1 is satisfied by functions satisfying this property. The concavity property will be particularly interesting for us with respect to comparative justice, and we will see it again in the fourth chapter. We show that axiom F2-1 satisfy it in three steps: (1) we define another axiom that any concave function satisfies, (2) we show that any function satisfying the axiom also satisfies F2-1, (3) by transitivity we conclude that F2-1 is satisfied by concave functions. The axiom we introduce is the following:

G1: For any $e < w$, if $|w + e - d| < |w - d|$,
then $FV(w+e,d) - FV(w,d) < FV(w,d) - FV(w-e,d)$.

In words this axiom says that the gain in contribution value obtained by adding a certain amount of welfare to a life is smaller than the loss in contribution value obtained by subtracting the same amount of welfare to it.

Proposition 3.3.3. *In the Arrheniusean framework, any function that is strictly concave satisfies G1.*

Proof. The proof of this proposition can be found in the Appendix. □

Proposition 3.3.4. *In the Arrheniusean framework, any function that satisfies G1 satisfies F2-1 as well.*

Proof. The proof of this proposition can be found in the Appendix. □

Proposition 3.3.5. *In the Arrheniusean framework, any function that is strictly concave satisfies F2-1.*

Proof. The conclusion follows by applying transitivity to Proposition 3.3.3 and Proposition 3.3.4. □

This concludes our formal discussion of the non-comparative properties of Feldman's and Arrhenius' desert adjusted axiologies. To briefly summarise it, we have seen that Feldman proposes six intuitive principles for a desert adjusted axiology, which are not fully formalized, and we have introduced and discussed the several ways in which Arrhenius suggests to formalize both parts of the fit-idea. Besides showing how a non-comparative desert-adjusted axiology is in general defined, and how Feldman's and Arrhenius' ones are in particular constructed, the upshot of the chapter is also to introduce us to how the theories can be manipulated to ensure that they satisfy the desired properties, even though we did not settle on what these properties are. We will adopt a similar way to manipulate them to reach the goal of the thesis, namely to work towards the understanding of whether a non-comparatively defined axiology can satisfy comparative properties. To that goal, in the next chapter, we will move to introduce the concept of comparative justice and define what that exactly is. Having gained such understanding, in the fourth Chapter we will then be able to go back to the formal discussion of the theories presented here and try to formally implement the precise definition of comparative justice.

4 Comparative Justice and Proportionality

In the last chapter, we have seen Feldman's and Arrhenius' proposals for the properties of the contribution function. The theories they put forward are defined in noncomparative terms. Indeed, they define the intrinsic value of a possible world as the *sum* of the individual contribution functions and these are specified by means of the fit-idea and thus only by using features of individual lives separately from each other, as their welfare and desert levels. This means that there is nothing in their theories that explicitly assigns value to comparisons between desert and welfare levels of different agents, i.e. to patterns of distribution of welfare among agents relative to given desert levels. As we have seen in the introduction, some authors objected that such theories are insufficient to account for *all* the demands of distributive justice. This kind of justice is holistic rather than individualistic, as it also involves comparisons between agents, for example in that it demands that equally deserving agents are treated equally. If this is the case, then our ethical theory should represent the distribution of welfare that satisfies this requirement as having higher value than one that does not, thereby representing that we should strive to obtain the former rather than the latter. But this requirement directly involves comparisons between desert and welfare levels of different agents, and so it is not clear that a non-comparatively defined theory satisfies it.

In the remainder of the thesis, we will work towards a clarification of this. To that goal, in this chapter we will firstly introduce the concept of comparative justice and its two components (section 1). After having acknowledged that comparative justice is usually understood in terms of proportionality, we will discuss a famous problem that proportionality has, which relates to the treatment of negative and zero welfare levels (subsection 1.1). We will consider Shally Kagan's argument to reject the proportionality principle, and then propose an argument to restore the principle, which essentially amounts to the proposal of a strictly positive scale to measure welfare and desert (section 2). We will then move to discussing the multiple ways in which comparative justice can be introduced in a non-comparatively defined theory (section 3).

4.1 What comparative justice is

In the literature, it is uniformly assumed that comparative justice is the union of the following two normative precepts:[1]

(1) Treat agents with the same desert in the same way; and

(2) Treat agents with different desert in appropriately different ways.

These principles define a *comparative* kind of justice, because they define what is just as dependent on *comparisons* between desert and welfare levels of different agents.

In our discussion, the treatment agents receive is in terms of welfare they have, receive or experience, and so the two principles amount to say that when two agents have the same desert they should receive the same amount of welfare, whereas when one of them has greater (lower) desert than the other, she should receive a higher (lower) amount of welfare.

Clearly, not *any* higher (lower) amount of welfare would do. If one has to distribute 60 units of welfare to among two agents, A and B, whose desert levels are 10 and 5 respectively, it won't do to give 55 units of welfare to A and 5 to B. This distribution gives way more than she deserves to one agent but not the other, which is comparatively unjust. The second principle in fact says that agents with different desert levels should be treated in *appropriately* different ways.

There are many ways to make this formally precise, but since Aristotle, a distribution that is *proportionate* to what agents deserve has been taken to be such an appropriate way [33], [7], [35], [36].[2]

[1] It is often said that Joel Feinberg drew attention on this distinction [32]. To the best of my knowledge, this distinction is unquestioned, and there is wide range of authors that re-cites it, e.g. [33], [7], [34].

[2] Other ways would be for example to take the absolute distance between the agents' desert and receipt, and then claim that justice requires this to be equal among the agents. However, this sometimes gives counterintuitive results. Consider two lives (1,2) and (99,100). According to this principle, the distribution of welfare is comparatively just as the difference is the same. However, the first agent has half the welfare she deserves, while the second almost all of it, and so this distribution does not seem so just, after all.

To illustrate, the proportional view thinks that "comparative desert is satisfied when my level of well-being stands to your level of well-being as my level of [desert] stands to your level of [desert]" [35]. In more formal terms, according to this view, comparative justice is obtained when we have $w_A : w_B = d_A : d_B$, where A is me and B is you. In the example above, the comparatively just distribution is then $\frac{2}{3}$ of the total to A and $\frac{1}{3}$ to B.

Importantly, notice that not only this distribution seems intuitively just, but it also respects the second comparative precept, as the more deserving agent is given more welfare than the less deserving one. In addition, notice that the proportional distribution would satisfy the first precept too, in the case in which two agents have the same desert level, because then it prescribe to give them an equal amount of welfare.

Thus, the two comparative precepts can be seen as immediate implications of the distributive principle according to which the best distribution of welfare is the one that is *proportional* to what the agents deserve. Comparative justice then boils down to requiring the satisfaction of **PP**:

PP Treat agents proportionately to what they deserve.

More formally, this amounts to give $w_i = \frac{d_i}{\Sigma_{j \in N} d_j} \cdot E$ to each agent i, where d_i, w_i are again agent i's desert and welfare levels, and E is the estate, i.e. the total amount of welfare to distribute. **PP** is a distributive principle, in the sense that it states how a given estate is best distributed. Also the fit-idea states what is the best distribution of some welfare, as it requires that each agents' receipt fits their desert level. Then, in an additively separable axiology as the one we are discussing, these two principles both indicate how to maximize the sum of the contribution functions. However, note that the fit-idea and the proportionality principle profoundly differ. Firstly, the fit-idea applies to individual agents, and it is thus an individualistic principle, guiding the behavior of the contribution function. The proportionality principle instead relates different agents' desert and welfare levels to derive what the best distribution is, and so it is not an individualistic principle or a property of the contribution function. Instead, it is a property of the entire distribution of welfare. It is then a property of the intrinsic value function, rather than of the contribution function.

Interestingly, despite having been taken as the comparative principle *par excellence*, the proportionality principle has the famous problem of being inapplicable to negative and zero numbers. In those cases, it gives completely wrong results. This problem has been taken as a substantial one and has led scholars such as Shelly Kagan to make the strong claim that proportionality is *incorrect* as a principle for comparative desert [35]. In the next section, we will see what the problem is and propose an argument to circumvent it.

4.2 Problem with the proportionality principle

Let us first see what is the problem when the zero number is involved. Consider two agents A and B and suppose that A has desert level equal to 0 and B has desert level n, for some number n. According to the proportionality principle, we measure the amount of welfare that the agents should receive by means of the following calculations: $w_A = \frac{d_A}{\Sigma_{j\in\{A,B\}} d_j} \cdot E = \frac{0}{0+n} \cdot E = 0$, and $w_B = \frac{d_B}{\Sigma_{j\in\{A,B\}} d_j} \cdot E = \frac{n}{0+n} \cdot E = E$. The proportionality rule then allocates zero welfare to A and the total amount of the estate to B, irrespectively of B's desert level. Now suppose, more concretely, that B deserves 1 unit of welfare more than A, e.g. $d_A = 0$ and $d_B = 1$, and suppose that $E = 50$. Then A receives zero and B receives the whole 50 units of welfare, even though she is just one unit more deserving than A. And the same goes if B is 20 units more deserving than A or hundreds of units more deserving than A.

These conclusions are clearly problematic. The principle gives agent A zero welfare *independently* of the amount of welfare and desert level agent B has. When zero levels are involved the proportionality principle does not give results that are proportional to each other and thus comparatively just. The comparative dimension is completely lost.

Similarly problematic conclusions are reached for negative welfare or desert levels. To illustrate, suppose that A's desert level is -5, and B's desert level is 1. By the proportionality principle, the welfare that they should receive is $w_A = \frac{-5}{-4} \cdot E$, which is a positive number, and $w_B = \frac{1}{-4} \cdot E$, which is a negative number. Then, the principle concludes that A should receive a positive amount of welfare, while B a negative one, even if B is the one having positive desert. This is clearly unjust.

Because of these reasons, in *The Geometry of Desert*, Kagan rejects proportionality as a principle for comparative desert, and goes on to propose an alternative one [35]. However, he notices that all the problems this principle has depend on the theory allowing zero and negative levels of welfare and desert. Therefore, he claims that by using a strictly positive scale for both values one would escape the counterexamples and redeem the proportionality principle. This is because such a scale excludes zero and negative numbers, for example by having zero as its lower bound that could only be approached but never be reached.

Consequently, in the definition of a comparative ethical theory, it seems that one has the choice to either keep negative and zero numbers and reject proportionality, or to keep proportionality and reject negative and zero numbers. Kagan chooses the first option, claiming that the idea of using a strictly positive scale (with zero as its lower bound that can never be reached) to represent welfare and desert is implausible and counterintuitive. In support of his choice, he makes the following claims [35]:

1. *Counterintuitive:* It is counterintuitive to represent the welfare of lives that are not worth living or the desert of very vicious people with positive amounts of welfare or desert;

2. *Lower bound for vice:* It is implausible to claim that there exists a "lower bound for vice, a level of perfect vice below which it is simply impossible to go". It rather seems that people can always be more vicious;

3. *Lower bound for well-being:* It is implausible to claim that there exists a lower bound for well-being too. Indeed, also in this case it seems that well-being can get worse and worse with no limit. Moreover, it is not clear why this lower bound of well being would be something that could never be reached.

4. *Appropriateness of the scale:* It is unclear whether a strictly positive scale is the appropriate scale to reason about comparative desert. The scale one uses to elaborate one's theory can completely shape the conclusions one obtains.[3] Then, when proposing a strictly positive scale, one should check that it has the desired implications.

These are Kagan's objections to a strictly positive scale that has zero as a lower bound that can never be reached. Because of them, Kagan rejects proportionality as the comparative principle of an appropriate ethical theory.

But are his objections really compelling? As we said above, proportionality has a really long history, dating back to Aristotle, and we should only reject a well established principle if we have strongly compelling reasons against it. We think this is not the case for most of Kagan's objections, as they

[3] Kagan gives several elaborate example of why this is the case. The interested reader can refer to *The Geometry of Desert*, section "absolute zero" [35].

can be straightforwardly overcome. In the next section, we will suggest an argument to do so, by constructing a strictly positive scale from some assumptions that the literature we have been considering in this thesis makes. We will see that adopting such a scale and representing individual welfare and desert with positive numbers is not so counterintuitive after all, and thus that proportionality principle does not necessarily have to be rejected.

4.3 Argument for a strictly positive scale for welfare

The assumptions we will move from are the ones made by the proponents of DAC about the notions of well-being and desert. In order to illustrate them, we will make use of terminology deriving from Feinberg's seminal paper *Justice and Personal Desert*, where he proposes the formula characterizing desert judgments [37]. The formula is "A deserves X by virtue of P", where A is an agent, X is a mode of treatment, and P is a desert bases. Feinberg claims that the desert basis P can be either a fact about A or a property that A satisfies, whereas the mode of treatment X is the way in which that fact or property about A is to be compensated.

In the DAC literature, X is taken to represent an amount of welfare, the amount that agent A should receive (deserves), given that she satisfies property P. Note that only in the case in which P is the property "being agent A" we have that X represents the amount of welfare that A's life, taken as a whole, deserves. In every other case, P represents a "situational" desert basis, i.e. a specific single fact or property about A, as the fact that A behaved well in a certain situation. Importantly, proponents of DAC are mainly interested in the former kind of desert bases, thus to the overall desert level of agents, for they take their axiologies to refer to agents' lives taken as a whole. Indeed, d_A symbolizes the mode of treatment that A deserves, given what she is and what she has done overall in her life [6],[16],[14]. They thus take P to be always representing the property of being a certain agent.

Clearly, this overall mode of treatment d_A is obtained by aggregating all the situational modes of treatment X that agent A deserves, given that she satisfies several situational desert bases P. The DAC literature usually remains agnostic about how to aggregate them [6].[4] In fact, note that in the DAC theories we have seen in Chapter 2, the desert basis P does not enter the scene, i.e. there

[4] Kagan discusses such matter in detail [35].

is no element that directly represents them. The theory starts by assuming an aggregated d for every agent.

Nevertheless, as we have seen in Chapter 1, desert bases do play a role in the theory, and proponents of DAC make explicit assumptions about them. These are among the assumptions that we will use to argue for the strictly positive scale and answer Kagan's objections, which are the following: (i) the set of desert bases comprises needs and basic rights; (ii) needs and basic rights are desert bases that correspond to a positive mode of treatment, and thus raise the agent's overall desert level; (iii) the desert level of each agent corresponds to a certain amount of welfare that the agent should receive.

Since every human has basic rights and needs, then by these assumptions we have that every agent i, *in principle*, has an overall positive mode of treatment, i.e. an overall positive desert level d_i. We say "in principle", because this mode of treatment is obtained by abstracting away all the actions that agents have performed, i.e. by making a move similar to what Rawls does, and placing ourselves under a veil of ignorance with respect to these actions. Under such veil, we see that every agent, in virtue of being human, satisfies such desert bases and thus deserves a positive mode of treatment. Let us call τ the mode of treatment every agent deserves in virtue of being human. By assumption (iii), τ corresponds to an amount of welfare. Then, we take τ to be the threshold representing the *minimal amount of welfare* that an agent should receive in order to have her basic rights and needs covered.

This τ level has a subjective nature, as it is thought to comprise agents' needs and these might differ from one another. For example, an agent might need certain medical treatments while another might need none. Then, for every agent, τ might be placed at a different level in the welfare scale. Moreover, even if the τ threshold is in principle independent of the agent's actions or responsibilities, in practice an agent can still increase or decrease that amount by satisfying additional positive or negative desert bases. By satisfying a particularly negative one, the agent's overall desert level decreases and might go below τ. We take this as equivalent to saying that the agent is vicious and deserves not to have some of her basic rights or needs satisfied. For example, it could be that she deserves to go to prison, i.e. she deserves her freedom (basic right) to be taken away from her.

In this way, it seems that the τ level allows us to represent both positive and negative well-being states and desert levels on a positive scale: the levels lying above τ represent positive amounts of welfare or desert, and the ones lying below τ represent negative ones. Furthermore, contrarily to Kagan's suggestion, we do not need to assume that in our scale there exists a lower bound for welfare, i.e. an absolute zero. We can simply exclude it from our scale, while allowing the scale to have an infinity of numbers below 1. We have thus no need of positing a lower bound. Simply think of the positive part of the real numbers for an example of such a scale.

The scale for welfare and desert that results from the argument above is a *strictly positive scale*, because it excludes the existence of both negative and zero welfare and desert levels, which a *standard scale* would instead include, while retaining only the positive ones. In the next section, we will see how the adoption of this scale can answer Kagan's objection.

Answers to Kagan's objections

As Kagan submits, a strictly positive scale avoids the problem of the proportionality rule. What about the other objections to using such a scale in a DAC theory that Kagan puts forward? Let us see how our proposal answers to most of them in turn.

1. *Counterintuitive*: Firstly, it does not seem counterintuitive any more to represent negative situations with positive numbers. This is because of the τ level, whose existence follows from the same assumptions that the DAC literature makes. Thanks to such a level, there is an intuitive representation of lives not worth living and of the desert of vicious people through positive numbers: these situations correspond to levels or numbers preceding τ.

2. *Lower bound for vice*: First of all, note that we assumed that in our strictly positive scale there is no lower bound for welfare. Let us take the real line as an example of such a scale, where between any two points in the scale there is an infinity of more numbers. Since we assumed that the numbers below τ represents negative welfare levels, then we have an infinite amount of negative welfare levels. Proponents of DAC take desert levels to be amount of welfare that agents should receive, as assumption (iii) states. Since in our theory we have no lower bound for negative welfare, then we have no lower bound for negative desert either, and one can take as small of a number one wishes to represent the desert levels of highly vicious people (the problem then arises of what number would be the appropriate one to represent the agents' vice. We consider this problem in 4, below).

3. *Lower bound for well-being*: Analogous to 2.

4. *Appropriateness of the scale:* First of all, note that this is not an objection, strictly speaking. Rather, Kagan *doubts* that a strictly positive scale would be appropriate to reason about desert and proposes an example to show that there exists a strictly positive scale that is inappropriate. Clearly, it does not follow that *every* strictly positive scale would be inappropriate. The issue is rather the technical one of defining a concrete scale that represents welfare and desert appropriately. However, the discussion of such a scale would bring us too far from our main topic, i.e. how to combine non-comparative and comparative justice, and at the same time it would end up occupying most of this chapter, so we omit it. Nevertheless, we point at some relevant constraints that anyone who tried to find such a scale, starting from our proposals above, would have to consider.

   One thing that one would need to consider is that the τ level seems to correspond to the zero in the standard scale. This is because we take welfare levels above τ to be positive and below τ to be negative, which implies that at τ they are either (i) both positive and negative, or (ii) neither positive nor negative. For simplicity, let us say that they are *neutral*. On the one hand, assuming them to be neutral would be helpful in the construction of a scale, for it would set a unifying starting point from which one could work to construct the scale. In other words, it would provide the level corresponding to the zero in the standard scale. On the other hand though, we take τ to comprise *subjective* basic needs, as specific medical treatments, and so this level cannot actually correspond to the objective zero level in the standard scale.[5]

   One might try to circumvent this problem by excluding such subjective needs from the τ amount of welfare, and by positing instead a unique and objective τ level that covers only basic rights and objective needs that every agent has. However, this still might not be sufficient to imply that the τ level is what we intuitively think of a *neutral* level, namely a level at which nothing good and nothing bad happens to the agent. Indeed, some people might find themselves at τ without having all their basic medical treatments covered, and would thus suffer.

[5]The zero has an objective nature in the sense that it is defined *irrespectively* of the agent who finds herself at that level and *previous* to considering agents' states of being.

> In the strictly positive scale that one would construct from our considerations then, one would ideally have keep the subjectivity of the τ level, while at the same time find a way to make the scale objective, so that agents' welfare levels could still be compared. In any case, the rest of the chapter will be developed under the assumption that such a scale exists, and the following discussion will be based on it.

Now that we have seen how one might go to save the proportionality principle, let us move to see what is needed to actually obtain an axiology that prescribes the proportional distribution of welfare as the most valuable one.

4.4 How to introduce proportionality in a noncomparative theory

As we have seen in the previous section, the **PP** principle is a property of the IV-function, and not of the C-function. This is because it describes how a distribution of welfare among different agents should be, and not what a single agent should receive. However, the literature we have discussed above defines the IV-function as the addition of separate contributions functions, one for each agent, which means that the IV-function too is built on noncomparative elements, i.e. it is a noncomparative function. Because of this reason, we said in the introduction of this chapter that it is not clear whether it satisfies comparative properties. If it does satisfy them, then it is only because they are induced by the additively separable C-functions, i.e. they are induced "from below".

Importantly, there are several ways in which one might induce it from below. For example, one might decide to introduce them by dispersing "the value of comparative desert over individual lives". Arrhenius suggests to do so by adding a comparative component to the individual contribution function [14]:

$$C(w,d) = WV(w,d) + FV(w,d) + CW(w,d) \tag{4.1}$$

Function (4.1) is a contribution function, in which $CW(w,d)$ represents "the individual's share of comparative (..) value" [14]. We can illustrate the reasoning behind such a representation by considering two agents, Ann and Bob, who have the same desert level. They both receive less welfare than they deserve, but Ann receives more welfare than Bob. In this situation, one might say that the comparative injustice is a bad feature of Bob's life, and locate this badness in the comparative element *CW* of Bob's contribution function. The result is that Bob's life has lower intrinsic value than Ann's, for he has lower welfare, lower fit-value, *and* lower comparative value.

Clearly, the IV-function induced by Function (4.1) satisfies comparative properties, as we have forced the comparative dimension in the contribution function by explicitly adding an element that represents it. Moreover, Function (4.1) is individualistic, because it introduces the comparative element by using only features of single agents considered separately (their welfare and desert). In other words, it introduces comparative properties as a non-comparative feature of agents. In addition, since we are not modifying its additive separable definition, this implies that the IV-function induced by such a C-function is individualistic too.

There are three things that should be noticed about this proposal. First of all, since it captures comparative justice as if it was a feature of individual agents' lives considered separately, one might think that the representation that Function (4.1) proposes is unsatisfactory. After all, comparative justice is comparative and not individualistic, and so it is not about agents' features considered in isolation. Rather, it is concerned with *patterns* of distribution of welfare, i.e. with *relations* between agents and their features. The relational information is not captured by Function (4.1). This function instead represents the impact that comparative (in)justice has on the agent's life, i.e. how bad it is for the agent himself that he receives less than an equally deserving other agent, which is far from what comparative justice is about. Then, even if an axiology defined on Function (4.1) succeeds in representing a noncomparative function with comparative properties, it still seems to fail in the representation of the conceptual import of comparative considerations, i.e. the relational nature of justice. The mere addition of a comparative component in each contribution function is then unsatisfactory, at least on the methodological and conceptual level.

In addition to being unsatisfactory, the strategy that Function (4.1) proposes might also be unnecessary, for two reasons. The first is that Bob's life would anyway result having lower value than Ann's one, as it would anyway have lower welfare and lower fit-value than Ann. What is the point of adding a comparative component then? If our new axiology gives exactly the same results as the old one, the addition of the comparative element is unnecessary. Especially if, as we have just said, the comparative component does not really capture comparative properties, but just the non-comparative reflection that

these have on individual lives.
The second reason is that the same comparatively just results might be obtainable by determining which, among the properties that the C-function can be endowed with, are sufficient to induce a comparatively just IV-function. By adopting this perhaps more parsimonious strategy and by defining the C-function according to such properties, one would obtain comparatively just results from a genuinely non-comparative basis, rather than by adding a comparative component to the C-function.

As good as this might sound, one might not be convinced by this strategy either. After all, we are now proposing to use a non-comparative function to capture comparative properties, which method we just claimed being unsatisfactory. In other words, it might seem that the first of our criticisms above applies here as well. But in fact it does not. The reason is that while Function (4.1) represents the comparative element as a non-comparative feature of individual agents, thereby resulting conceptually odd, this second method seeks to obtain the comparatively just output by simply deciding on what set of properties the C-function should have. In that way, instead of forcing the comparative dimension into a non-comparative form, it endows the non-comparative part of the function with those properties that will then induce a comparatively just IV-function. The comparative element will then still be captured by this latter function, i.e. by the aggregative element of our formal axiology, which seems correct.

This is a promising strategy to capture comparative justice. In fact, some authors in the literature have already shown that, in some cases, this kind of justice can be successfully obtained in a non-comparative environment. The following chapter will be devoted to this discussion.

5 Comparative Justice and Proportionality: Formal Properties

In the last chapter, we have seen that the literature on comparative justice considers this kind of justice to be realized by a distribution of welfare that is proportional to the agents' desert levels (this is what we called **PP**). At the same time, because of its failure to deal with zero or negative numbers, we have seen that the literature often disregards proportionality as an appropriate distributive principle. But this needs not be: we proposed an argument which justifies the adoption of a strictly positive scale, at least given the assumptions about desert that DAC theorists make. With such a scale, the problems of **PP** can be easily circumvented, implying that now it makes sense again to try and achieve a desert-adjusted axiology that satisfies the proportionality principle. As we said at the end of the previous chapter, our next step is then to understand whether and how it is possible to implement the principle in a individualistically defined contribution function. This concretely means defining properties for the non-comparative C-function so that it successfully induces an IV-function that satisfies proportionality. Then, the resulting axiology would not only be individualistically just, but also comparatively so, thereby also showing that non-comparatively defined desert-adjusted axiologies can be made compatible with comparative requirements, which was the goal of the present thesis.

We will start our analysis by going back to Feldman's theory, to discuss how that theory fares with respect to the satisfaction of comparative requirements. Scholars as Carlson, Hurka or Persson have already raised interesting counterexamples against his theory, that all point at its failure to satisfy some comparative requirements. After having seen what principles they think Feldman's theory fails to satisfy, we will move to discuss Arrhenius' theory and its relations with the proportionality principle. Arrhenius himself has put forward the issue of comparative justice. We will go through his analysis and we will see that the theory he proposes does not achieve the full satisfaction of **PP**. We will see that there is a formally precise reason why it does not, and, building on this finding, we will make our own proposal. We will introduce an axiom for the C-function that allows the satisfaction of comparative requirements and, if appropriately complemented, also of the proportionality principle.

5.1 Comparative justice in Feldman's theory

As we said above, Feldman's theory has been subject to strong criticisms, most of which point at the fact that it does not satisfy comparative requirements. Erik Carlson, for example, proposed some such criticisms, and claimed that owing to them the theory fails to answer the objection from justice [16] [15].

The criticisms Carlson proposes seem to depend on two interrelated features of the theory:

(a) The claim that neutral desert levels leave positive and negative welfare values unmodified, i.e. principles P3 and P6 for positive and negative welfare levels, respectively;

(b) It fails to treat equally deserving agents equally.

Note that (a) concerns the non-comparative dimension of the theory, as it is about the interaction of individual desert and welfare levels, while (b) concerns its comparative dimension, as it looks at whether different agents with equal desert levels are treated equally, i.e. it compares them. However, we will soon see that not only (b) but also (a) has implications that make the theory *comparatively unjust*.

Carlson proposes several counterexamples to show why (a) and (b) are problematic features of Feldman's theory. One of them captures both aspects (a) and (b) at the same time and so it is the one we will now consider [16].[1] We will call it Carlson Neutral and represent it through the table below. This example

[1] Note that strictly speaking it is unclear whether Carlson's counterexample really goes through. In fact, Carlson uses receipt levels equal to zero, i.e. equal to the "receipt of neither pleasure nor pain", and then applies Feldman's principles to reason about them [16]. But we have seen above that Feldman's theory is silent about such receipts, except for what he says through the graphs. In such graphs, he does not say anything about the interaction between zero desert levels and 0, 100 or -100 welfare levels. Hence, we cannot be sure that the conclusions that Carlson draws about such interactions are correct.

involves two agents A and B, and two different distributions of welfare, i.e. option (i) and option (ii) in Table 5.1 below. As in the previous chapters, w_i and d_i represent received welfare (pleasure or pain) and deserved welfare that agent i receives, respectively, where i can be either A or B. In both options, it is assumed that A and B have desert level equal to 0, which is why in the table we have $FV_A(w_A, 0)$ and $FV_B(w_B, 0)$.

Before examining the table, let us briefly elaborate on why we still use the zero number after having argued for the adoption of a strictly positive scale in the previous chapter. We choose to still use zero, as well as negative numbers, because all the counterexamples that Carlson proposes in [16] involve them, and none of them can be sensibly translated using a strictly positive scale.[2] The adoption of this scale would simply blow them away. One might say that this is an advantage, however such an adoption would also imply that Feldman's principles P1-P6 do not make sense any more, as they are constructed using positive and negative numbers. Then, to be able to analyse Feldman's original proposal and Carlson's examples, we need to momentarily bracket our assumption of a strictly positive scale and, only for what concerns Feldman's discussion, adopt a standard one instead. We will see that it is important for us to do this and go through Carlson's analysis, as at the end of this discussion, we will have gained a useful insight into how to introduce **PP** in a non-comparatively axiology. Then, let us turn to the Carlson's example, which the following table illustrates.

Options	w_A	$FV_A(w_A, 0)$	w_B	$FV_B(w_B, 0)$	Total Value
(i)	100	0	-100	0	0
(ii)	0	0	0	0	0

TABLE 5.1: Carlson Neutral

As we can see from Table 5.1, according to Feldman's theory, option (i) and (ii) result in equal total value. The fit-value does not influence the result, i.e. it is zero in both cases. This is because of principles P3 and P6 of Feldman's theory, which hold that neutral desert levels do not have any effect on the agent's positive and negative welfare level, respectively.

The equal total value of the two options means that the theory is indifferent with respect to which is the right option to choose, i.e. that the following proposition holds.

[2]We omit the proof of this claim here because it would lead us too far from the present discussion.

Proposition 5.1.1. *Given two agents A and B, and two options (i) and (ii) as described in Table 5.1, Feldman's theory concludes that the options have equal total intrinsic value.*

Proof. The proof of this proposition can be found in the Appendix. □

Carlson points out that this conclusion is incorrect for two reasons. First of all, A and B have the same desert levels, namely zero, and so justice demands that they are treated equally. However, among the two options, only option (ii) treats the agents equally. Option (i) does not: it gives a high amount of (undeserved) pleasure to agent A and a high amount of (undeserved) pain to agent B. Yet, option (i) is considered as having the same intrinsic value as option (ii), which must then be incorrect. The second reason why Feldman's conclusion is incorrect is that option (ii) satisfies the agents' desert perfectly, while option (i) does not. This again means that option (ii) should have higher intrinsic value than option (i), which is not the case. These two issues show that Feldman's theory fails to satisfy the comparative as well as the non-comparative aspects of justice, respectively.

In his paper [16], Carlson suggests that the incorrectness resides in principles P3 and P6 and in how the theory treats neutral desert levels. As we have seen in Chapter 2, these principles state that a neutral desert level has no effect on the value of pleasure and pain. This has the undesirable consequence that, when desert levels are neutral, the desert-adjusted axiology is not sensitive to considerations of desert and instead seems to prescribe exactly what utilitarianism does. Therefore, when such desert levels are involved, the theory can still incur in the objection from justice, which implies that it is still an inappropriate ethical theory.

To repeat, Carlson's example also shows that Feldman's theory fails with respect to comparative considerations of distributive justice. A very similar criticism is raised also by Ingmar Persson and Thomas Hurka [15], [7]. They both claim that there is a principle of justice that demands the equal treatment of equals. Clearly, this principle comes in operation only when there is more than one agent involved. However, Feldman defines the contribution function and the intrinsic value of possible worlds using the fit-idea only, and this does not take into account the presence of more than one individual. Such presence is taken into account by Feldman's theory merely in the equation of intrinsic value of a possible world with the sum of the agents' contributions. This implies that Feldman's theory "presents justice as an *intra*-personal matter". However, distributive justice involves an *inter*-personal or *holistic* dimension, according to which the value of a possible world considers also *comparisons* between agents' receipt and desert, as well as between agents' fit-values. Indeed, in Chapter 3 we saw that a distribution that satisfies the proportionality principle is unanimously considered just, and that principle is essentially based on comparisons between agents.

Carlson's principles

To further capture the properties that an axiology should satisfy and that seem to be neglected by Feldman's theory, Carlson puts forward two principles. The first one is the following:

> **J1** If n units of welfare are to be distributed among a certain number of people with a total desert level of n, then the distribution where each person gets exactly what she deserves is better than any alternative distribution.[3]

More formally, this principle amounts to say that, given a total amount of welfare E and a set of agents N such that $\Sigma_{i \in N} d_i = E$, the IV-function is maximized when for every agent $i \in N$, we have $w_i = d_i$.

Carlson Neutral shows that this principle is not satisfied by Feldman's theory. In fact, in that example we have that $\Sigma_{i \in N} d_i = E$, but we do not have that, in every option, for every agent $i \in N$, $w_i = d_i$. Yet, according to Feldman's theory, the options are equally valuable.

As Carlson says, if an axiology does not satisfy this principle, then it "sometimes allows that some person gets more than she deserves and another person gets less, even though there is no reason in terms of maximizing net pleasure to allow this. Such a theory, it seems, permits us to depart from the requirements of justice for no good reason" [16].

The other principle Carlson proposes is the following:

> **J2** If a given amount of welfare is to be distributed among a number of agents with the same desert level, the equal distribution is intrinsically better than any other distribution.[4]

[3] We use the reformulation of the principle that Arrhenius proposes [14].

[4] We reformulated the principle so to make it consistent with our terminology.

More formally, this principle amounts to say that, given a total amount of welfare E and a set of agents N such that for every $i, j \in N$ we have $d_i = d_j$, the IV-function is maximized when we have $\frac{E}{N} = w_i$ for every $i \in N$.

As the reader can immediately verify, the counterexample Carlson Neutral goes through also because Feldman's theory does not satisfy **J2**. Note that **J2** is a *comparative* principle, as it makes comparisons between agents in order to assess what is the distribution of welfare that maximizes intrinsic value. If the agents have equal desert, then the principle says that there is no better distribution than the one that gives to each an even amount of welfare, no matter whether this is above or below the agents' desert levels.[5]

Because of its comparative nature, J2 is a principle that regulates the behavior of the IV-function, and not of the C-function. Indeed, the latter only considers individual agents, while the former takes as input the entire possible world, and thus sets of agents' lives i, defined as the pair (w_i, d_i).

Interestingly, note that if we now go back assuming the strictly positive scale we argued for in the last chapter, then we have that principles **J1** and **J2** capture special cases of the **PP** principle. **J1** captures the case in which there is enough welfare to fully satisfy everybody's desert, no matter what agents' desert levels are (i.e. how positive they are). In this case, an axiology satisfying **J1** would prescribe to give to each what they deserve, which is exactly what an axiology satisfying **PP** would do.

Proposition 5.1.2. *Given a strictly positive scale, for any distribution of an estate E such that $E = \Sigma_{i \in N} d_i$ according to an axiology, if the axiology satisfies **J1**, then it satisfies **PP**.*

Proof. Immediate. □

Instead, **J2** captures the case in which the agents have the same desert level, no matter how much estate we have to distribute to them. In this case, an axiology satisfying **J2** would prescribe to give to every agent the same amount of welfare, which is exactly what an axiology satisfying **PP** would do.

Proposition 5.1.3. *Given a strictly positive scale, for any distribution of welfare according to an axiology, if the axiology satisfies **J2**, then it is proportional to the agent's desert level.*

[5] Note that this means that **J2** does not require us to distribute the entire amount of welfare.

Proof. Immediate. □

This means that, if we assume a strictly positive scale, then Carlson's criticisms to Feldman's theory can actually be boiled down to the claim that the theory does not satisfy the proportionality principle. Once again, the literature is reclaiming (aspects of) the principle as a fundamental requirement of justice.

5.1.1 What went wrong?

We will now move to the discussion of what is the problem of Feldman's theory, that does not allow it to be comparatively just, i.e. to satisfy the proportionality principle. Besides principles P3 and P6, which Carlson already pointed out as leading to severe problems, we submit that the reason why Feldman's function fails to satisfy **J2**, and thus comparative requirements of justice, resides in that it is a *linear* function.[6] Because of this, whenever we redistribute welfare from one agent to the other, we increase the first agent's contribution of an extent that is equal to the extent to which we decrease the other agent's contribution. Therefore, redistributing welfare from one agent to another, as happens in Carlson Neutral where an agent receives 100 units of welfare that are taken away from the other, does not affect the total intrinsic value of the distribution. In other words, in a linear function f, for any amounts of welfare a, x_1, x_2, such that $x_1 = x_2$, we have $f(x_1) + f(x_2) = f(x_1 + a) + f(x_2 - a)$.

We illustrate this point through the graph in Figure 5.1, which represents Feldman's contribution function. In the graph, one can see that taking some welfare from one agent (indicated in green) and giving it to the other (indicated in blue) does not change the overall intrinsic value of the distribution (the blue and the green lines are equal).

Carlson himself seems to recognize that the problem resides in the linearity property, even though he does not mention it explicitly. In fact, he proposes

[6] Recall that Feldman's function is the following:

$$C(w, 10) = \begin{cases} 2.5w - 5 & \text{for } w < 10 \\ 20 & \text{for } w \geq 10 \end{cases} \quad (5.1)$$

Note that this function is composed by two linear parts. One is clear and the other is the one that flattens out after +10. So the reasoning we proposed above holds for both parts. We illustrate the function in Figure 5.1.

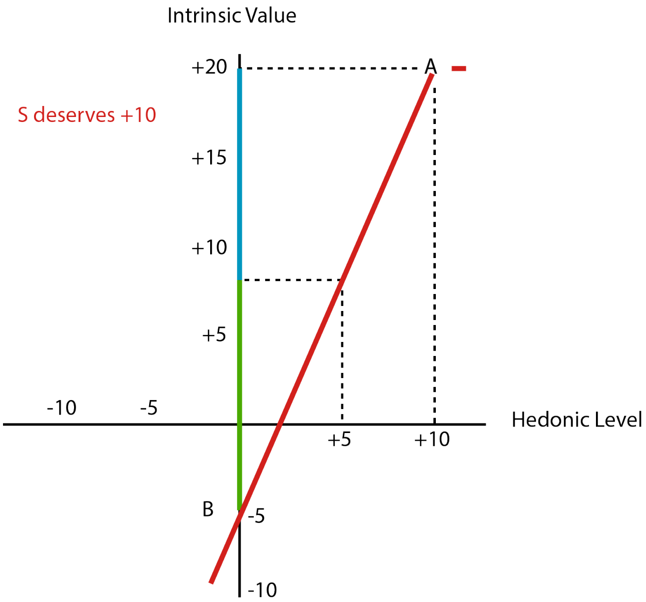

FIGURE 5.1: Linear functions are not comparatively just.

a new contribution function, that solves the problem of Feldman's and is not linear, but *concave*. Carlson's C-function is the following:[7]

$$C(w,d) = d + (w-d)^k, \text{ if } d \leq w, 0 < k < 1.$$
$$C(w,d) = d - (d-w)^k, \text{ if } d > w, k < 1.$$

This is a non-comparatively defined contribution function, because it is defined only using welfare and desert of a single agent. Carlson illustrates it through the graph in Figure 5.2:

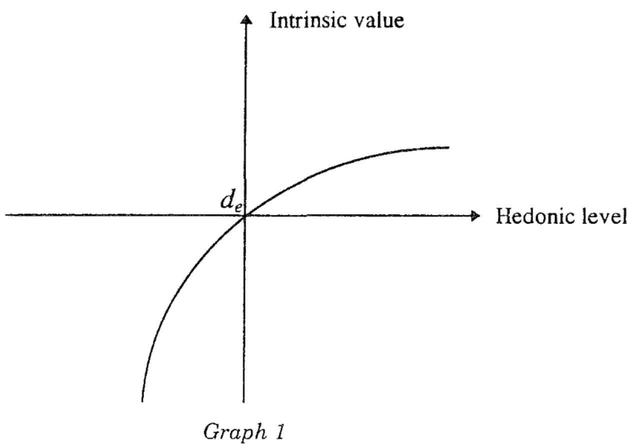

FIGURE 5.2: Carlson's function, taken from [16].

[7]The formulation of the function is taken from Arrhenius [14] and adapted to the notation used here.

He shows that this contribution function satisfies **J1** and **J2**.[8] Moreover, note that by proposition 5.1.2 and 5.1.3, we also know that, if we adopt a strictly positive scale, this function satisfies proportionality in the particular cases in which there is an amount of estate equal to the sum of the agent's desert, or in which the agents have equal desert levels.

Even if just partially satisfactory, in the sense that it only satisfies proportionality in some specific cases, the function that Carlson proposes provides us with an important first step towards the understanding of the properties that an individualistic C-function could have to satisfy comparative justice: being a concave function.[9] In fact, Carlson shows us that concave functions are such that, if two agents have the same desert level, the overall intrinsic value of their lives decreases whenever we redistribute their welfare from an even to an uneven allocation.

Importantly, we have already encountered the strict concavity property. In Chapter 2, we have seen that one of the axioms that Arrhenius proposes is satisfied by concave functions. In the next section then, we will discuss that axiom, and how Arrhenius' proposal fares with respect to comparative requirements.

5.2 Arrhenius Theory and Comparative Justice

The axiom that Arrhenius proposes and that concave functions satisfy is axiom F2-1, let us recall it.

F2-1: If $|e_1| = |e_2|$, $|w_1 - d| > |w_1 + e_1 - d|$, $|w_2 - d| > |w_2 + e_2 - d|$, and $|w_1 - d| > |w_2 - d|$ then $FV(w_2 + e_2, d) - FV(w_2, d) < FV(w_1 + e_1, d) - FV(w_1, d)$

In words this axiom says that "if we can increase the fit between desert and receipt in two lives with the same desert level by adjusting their welfare level up or down by a fixed amount, then the life with the greater difference between receipt and desert will get the greater increase in fit value from the adjustment of its welfare level" [14].

Recall that in Chapter 2 we showed that F2-1 is satisfied by concave functions, and that we showed it by means of another axiom that we introduced, namely

[8] The interested reader can find the proof in [16].

[9] Note that we do not mean to say that concave functions are the *only* kind of functions that satisfy comparative properties. Rather, we are making the existential claim that this kind of function does satisfy them, while remaining agnostic about whether other functions satisfy them as well. Clearly though, convex and linear functions do not satisfy **PP**, but its opposite.

G-1.[10] Then, the axiom can be represented by a concave function as the following.

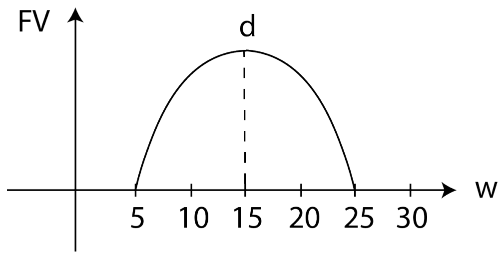

FIGURE 5.3: Function satisfying F2-1

Importantly, Arrhenius shows that, if combined with F1-2, axiom F2-1 has the following implications:[11]

D3: For any d, if $w_1 > w_2 > w_3$ and $w_1 + w_3 = 2w_2$, then $IV\langle(w_2, d), (w_2, d)\rangle > IV\langle(w_1, d), (w_3, d)\rangle$

And its generalization:

D4: For any d, and any two possible worlds $A = \langle(w_1, d), (w_2, d), \ldots, (w_n, d)\rangle$ and $B = \langle(q_1, d), (q_2, d), \ldots, (q_n, d)\rangle$, with $n \geq 2$, if $w_i \neq w_j$ for some i and $j \leq n$, and $q_i = \frac{(w_1+w_2+\cdots+w_n)}{n}$ for all $i \leq n$, then $IV(B) > IV(A)$

Implications D3 and D4 are particularly interesting for our discussion. They say that the most valuable distribution of a certain amount of welfare among a set of agents with the same desert level is an equal distribution. This is the principle that Carlson called **J2** and thus Arrhenius shows that a non-comparatively defined theory can satisfy some comparative properties, i.e. the proportionality principle in the case in which agents have *equal* desert. Note that the axiom that allows such implications to go through is an axiom that is satisfied by concave functions, as we showed above.

However, axiom F2-1 indeed only applies to agents with the same desert levels. Only if our C-function instead considers agents with *different* deserts we will be able to understand when and how the satisfaction of the full proportionality principle, and not only the part where agents have equal deserts, can be induced by a non-comparative C-function. This means that we need to generalize axiom F2-1 so that it holds for agents with different deserts. How to do it?

[10]Recall that G-1 says the following: For any $e < w$, if $|w + e - d| < |w - d|$, then $C(w + e, d) - C(w, d) < C(w, d) - C(w - e, d)$.

[11]The interested reader can find the proof in the appendix of [14].

Arrhenius proposes the following generalization (note the distinct d_1, d_2!):

F2-2: If $|e_1| = |e_2|$, $|w_1 - d_1| > |w_1 + e_1 - d_1|$, $|w_2 - d_2| > |w_2 + e_2 - d_2|$, and $|w_1 - d_1| > |w_2 - d_2|$, then $FV(w_2 + e_2, d_2) - FV(w_2, d_2) < FV(w_1 + e_1, d_1) - FV(w_1, d_1)$

In words this axiom says that "if we can increase the fit between desert and receipt in two lives by adjusting their welfare level up or down by a fixed amount, then the life with the greater difference between receipt and desert will get the greater increase in fit value from the adjustment of its welfare level" [14].

A graph that represents axiom F2-2 is the one in Figure 5.4.

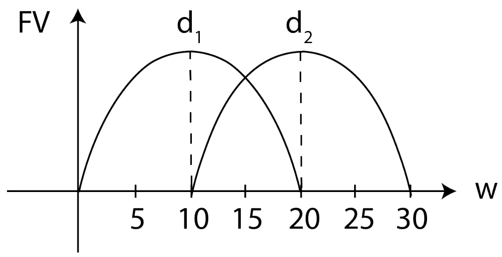

FIGURE 5.4: Function satisfying F2-2

The graph contains two times the function that we represented in Figure 3.9 and that exemplifies F2-1. This illustrates an implication of F2-2, that Arrhenius does not mention:

Proposition 5.2.1. *In the Arrheniusean framework, if the FV-function satisfies F2-2, then the function remains identical under horizontal transformation.*

Proof. The proof can be found in the Appendix. □

Note that to generalize axiom F2-1 to two distinct desert levels we do not necessarily have to impose the FV-function to remain identical for every desert level, i.e. under horizontal transformation. Instead, one could require several features of the curve to differ when the desert level differs, e.g. the slope of the curve could differ or the maximum of the function (the peaks) could increase with the increase of the desert level. The latter would be a sensible requirement, as it represents that the intrinsic value of an agent's life increases with the increase of the deservingness of the agent. This is the merit-idea, which we saw in the first chapter.

Even though Arrhenius does not mention it, it is exactly because F2-2 instead imposes that the function remains identical under horizontal transformation that he can show that the axiom implies the following principle:[12]

PD If a fixed amount of welfare is to be distributed among a certain number of people, then the distribution where each person gets the same difference between her desert and receipt is better than any alternative distribution.

PD is a comparative principle as it assigns properties to relations between receipt and desert of different agents in the possible world, i.e. to patterns of distribution. However, some of its instantiation are not so desirable. As for axiom F1-3, which we saw in Chapter 2,[13] also F2-2 implies indifference in giving one unit of welfare to $(1, 2)$ and giving it to $(99, 100)$. Moreover, the axiom also implies that the best distribution of six units of welfare among two agents such that $\langle(0, 8), (0, 4)\rangle$ is a possible world W' such that $W' = \langle(5, 8), (1, 4)\rangle$.[14] But one might object to this conclusion that the possible world with the best distribution should intuitively be $W'' = \langle(4, 8), (2, 4)\rangle$, i.e. the proportional distribution. Thus, F2-2 does not seem to capture some of our important intuitions about distributive justice. Because of these problems, Arrhenius ultimately disregards axiom F2-2 and instead chooses F2-1 for his default theory.

The reason why axiom F2-2 implies **PD** is indeed that it imposes that the FV-function remains identical under horizontal transformation. If that was not the case, e.g. if the curves had different slopes or different maxima, it would not be necessarily the case that the best distribution is the one in which everyone receives the same difference between desert and receipt. Consider for example Figure 5.5 below. In that graph, it is clearly the case that the best distribution is not always the one where the agents get the same difference: consider the distribution $w_1 = 0$ and $w_2 = 10$. Even if the difference between each agent's desert is the same, that distribution of 10 units of welfare is not the best. The distribution $w_1 = 2.5$ and $w_2 = 7.5$ for example is already better.

[12]The interested reader can find the proof in [14].

[13]Recall that axiom F1-3 is the following: F1-3: If $|w_1 - d_1| < |w_2 - d_2|$, then $FV(w_1, d_1) > FV(w_2, d_2)$, and if $|w_1 - d_1| = |w_2 - d_2|$, then $FV(w_1, d_1) = FV(w_2, d_2)$. In words this option says that "if the difference between receipt and desert in a given life is smaller than (equal to) the difference between receipt and desert in another life, then the fit value of the former life is greater than (equal to) the fit value of the latter life" [14].

[14]The interested reader can find the proof in [14]

Importantly then, since it requires the curves of FV-function to be always identical and thus induces the satisfaction of **PD**, F2-2 *prevents* **PP** to obtain.

Proposition 5.2.2. *In the Arrheniusean framework, if the FV-function satisfies F2-2, then it does not satisfy* ***PP****.*

Proof. The proof can be found in the Appendix. □

This proposition then shows that axiom F2-2 is in direct contradiction with the proportionality principle. Since we know why this happens, in the next section we will propose a modification of Arrhenius' axiom F2-2 that avoids its problem, and thereby allows for the satisfaction of proportionality.

5.3 Proposal

We propose a different way to capture comparative justice for sets of agents that have different desert levels. As we said, the axiom builds on Arrhenius' F2-2, and is in fact a minimal modification of it, sufficient to impose that the function that satisfies it is not identical across horizontal transformations. Interestingly, we will see that the axiom we propose is satisfied by strictly concave functions. The strict concavity property is particularly relevant for comparative justice, and in this section we will also elaborate on why this is the case. As we discussed above, the connection between comparative justice and the strict concavity property was already (implicitly) included both in Carlson's and in Arrhenius' proposals.

The axiom we propose, which we call G-2, states the existence of two welfare levels, relative to two (possibly distinct) desert levels, such that any redistribution of welfare from these two levels lowers the overall intrinsic value of the possible world.

The axiom is the following:

G-2: For any E, there exist w_1, w_2 such that $E = w_1 + w_2$,
and such that for any positive $e < w_1, w_2$,
if $|w_1 + e - d_1| < |w_1 - d_1|$ and $|w_2 + e - d_2| < |w_2 - d_2|$, then
(i) $FV(w_1 + e, d_1) - FV(w_1, d_1) < FV(w_2, d_2) - FV(w_2 - e, d_2)$ and
(ii) $FV(w_2 + e, d_2) - FV(w_2, d_2) < FV(w_1, d_1) - FV(w_1 - e, d_1)$.

In words it says that, for any estate, there exist two welfare levels such that if two lives with different desert levels receive it, the gain in contribution value obtained by adding a certain amount of welfare to one of the lives is smaller than the loss in contribution value obtained by subtracting the same amount of welfare to the other life, and vice versa.

Let us illustrate G-2 with the graphs in Figure 5.5 and 5.6.

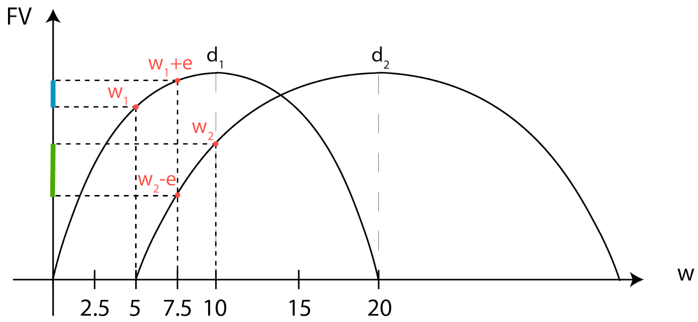

FIGURE 5.5: Graph capturing part (i) of the consequence of axiom G-2.

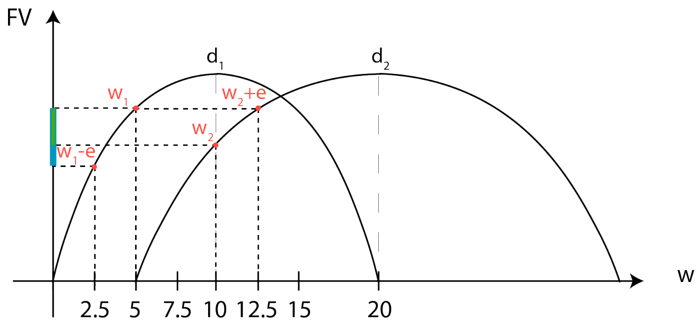

FIGURE 5.6: Graph capturing part (ii) of the consequence of axiom G-2.

As one can see from the graphs, we consider two lives, 1 and 2, with unequal desert levels, namely $d_1 = 10$ and $d_2 = 20$, respectively. Suppose that we have an estate E of 15 units of welfare to distribute among the agents and consider the distribution $w_1 = 5, w_2 = 10$, and $e = 2.5$. This distribution satisfies $e < w_1, w_2$, the antecedent of G-2. From the graph, we can see that it holds that (i) $FV(5 + 2.5, 10) - FV(5, 10) < FV(10, 20) - FV(10 - 2.5, 20)$ and (ii) $FV(10 + 2.5, 20) - FV(10, 20) < FV(5, 10) - FV(5 - 2.5, 10)$, and thus $w_1 = 5, w_2 = 10$ are welfare levels that G-2 singles out. Note that redistributing from these levels lowers the overall value of the possible world. This holds for any other redistribution of E.

The example shows that the axiom is satisfied by a function that is not identical across horizontal transformation, i.e. it has different curves for different desert levels. As we saw above, this means that G-2 allows for the satisfaction of proportionality and does not imply **PD**, as F2-2 instead does.

The example also shows that G-2 is satisfied by strictly concave functions.

Proposition 5.3.1. *For every function f, if it is strictly concave then it satisfies G-2.*

Proof. The proof can be found in the Appendix. □

Discussing both Carlson and Arrhenius, we have seen that the strict concavity property has interesting implications for comparative justice. Indeed, Carlson's function, which is concave, and Arrhenius' axiom F2-1, which we showed being satisfied by concave functions, both satisfy **J2**. Recall that this is Carlson's principle, which by Proposition 5.1.3 we know being equivalent to proportionality in the case in which the agents have equal desert levels.

The reason why they are able to satisfy some comparative requirements is the concavity property. Indeed, this property ensures that, for every given estate E, (1) the function induces a "comparatively just maximum". This means that according to the function, there exists some distribution of E among the agents that is comparatively just, e.g. proportional to the agents' deserts; (2) the function induces the existence of only one comparatively just maximum, i.e. the welfare levels that are comparatively just are unique.

We claim that it is the concavity property that allows (1) and (2) because if the function was instead convex or linear, then either (1) or (2) would not hold, respectively.

Proposition 5.3.2. *For every function f, if it is convex, then (1) does not hold.*

Proof. The proof can be found in the Appendix. □

Proposition 5.3.3. *For every function f, if it is linear, then (2) does not hold.*

Proof. The proof can be found in the Appendix. □

Importantly, the welfare levels that constitute the comparatively just maximum and satisfy (1) and (2) are exactly the ones that axiom G-2 singles out, i.e. the ones that are just with respect to each other and the agents' deserts, and that are such that any redistribution lowers the value of the possible world. We thus might see the axiom as capturing the property that, for every given estate, there exist two points in *comparative equilibrium*.[15]

[15]We can also see these two welfare levels as the points where the derivatives of the C-functions of each agent coincide, i.e. $\frac{\partial FV(w_1,d_1)}{\partial w} = \frac{\partial FV(w_2,d_2)}{\partial w}$.

Note that both (1) and (2) are necessary conditions for any comparatively just distribution of welfare. Indeed, in a strictly positive scale, which we are assuming, for any E there always exists a distribution that is proportional to the agents' desert levels, and it is unique. Thus, if a function did not satisfy any of the two conditions, the distribution could not be proportional to the agents' deserts. Since we have seen that proportionality is taken to be equivalent to comparative justice, this means that such a distribution would not satisfy comparative requirements. Then, G-2 and Proposition 5.3.1 show that strict concavity is itself a necessary condition for comparative justice. More precisely, our finding establishes that if we want a non-comparatively defined axiology to satisfy comparative properties as proportionality, we need to use a strictly concave function.

In addition, our finding also establishes that for any other property that a comparatively just axiology should satisfy, e.g. the properties and axioms discussed in chapter 2, should be made consistent with the property of strict concavity. Interestingly, this property excludes some of the axioms that we saw in that chapter. For example, it excludes the axioms that hold that over-deserved welfare does not increase nor decrease the intrinsic value of the agent's life, i.e. F1-1′ and F1-3.[16] Recall that this property is desirable when need is taken as the only desert basis. Indeed, once all the agent's needs are satisfied and so the agent's welfare fits her desert, it seems that the intrinsic value of her life stabilizes and that increasing her receipt even further would not increase the intrinsic value more than that. Note that in a strictly concave function it cannot be the case that after a certain point the curve flattens out, as the property imposes that it either increases or decreases.

Lastly, note that some other properties that we have seen in chapter 2 are instead compatible with the concavity property. To illustrate, both the asymmetry between over and under-deserved welfare induced by axioms F1-1 and A1 and the symmetry induced by axiom F1-2 can instead be made compatible with the concavity property and thus G-2 and proportionality. However, imposing either one of the two has substantial consequences for justice, and thus needs further discussion.

[16]Recall that axiom F1-1′ says that if $d \geq w_1 > w_2$ then $FV(w_1, d) > FV(w_2, d)$; and if $w_1 > w_2 \geq d$ then $FV(w_1, d) = FV(w_2, d)$.
Axiom F1-3 says that if $|w_1 - d_1| < |w_2 - d_2|$, then $FV(w_1, d_1) > FV(w_2, d_2)$; and if $|w_1 - d_1| = |w_2 - d_2|$, then $FV(w_1, d_1) = FV(w_2, d_2)$.

6 Conclusion and future works

In this thesis, we have discussed desert-adjusted axiologies from a formal perspective, and we have examined whether it is possible to induce comparatively just distributions of welfare in a non-comparatively defined function. In the first chapter, we have introduced the non-comparatively defined desert-adjusted consequentialism and the objection that it encounters if it does not meet comparative requirements. Since the goal of this thesis was to discuss whether non-comparatively defined desert-adjusted axiologies are formally compatible with comparative justice, in the second chapter, we have introduced the formal details of two such axiologies. Firstly, we have seen Feldman's one and principles for non-comparative justice, then we have moved to Arrhenius' proposals for the properties of the fit-value function. In the third chapter, we then moved to discussing comparative justice as distribution proportional to desert, and suggested a way to redeem proportionality principle from the failure to treat negative and zero numbers. The discussion in this chapter helped us clarifying and defining what comparative justice exactly is, so that in the fourth chapter, we could go back to the formalities with a precise idea of what to look for in non-comparative theories, namely whether they satisfy proportionality. With this in mind, we have then discussed the criticisms that have been raised to Feldman's proposals. They have been instructive, as building on them we were able to construct our own proposal. It constitutes of a modification of Arrhenius' axiom F2-2, modification that allows the function to satisfy proportionality. By incorporating two necessary conditions of proportionality and by being satisfied by strictly concave functions, our axiom G-2 shows that comparative justice itself is satisfied by concave functions. This finding shows that a non-comparatively defined theory can also satisfy comparative requirements. From a more general perspective, this finding also shows that DAC can address comparative justice as well, and thus that consequentialism is able to answers the objection from justice that we discussed in the introduction and first chapter, and does not need to be complemented with deontological aspects.

There are many paths for future work that stem from the present one. For example, in chapter 1 and in chapter 2 we have hinted at the fact that some desert bases seem to induce properties of the C-function that are incompatible with each other. Therefore, it would be very interesting to investigate whether, given the assumptions about desert that DAC proponents make, an impossibility theorem follows.[1] The theorem would show that desert cannot be defined on such different desert basis, on pain of having to give up some justice requirements.
Another interesting development of this thesis would be to examine what axiological properties are compatible with concave functions, and thus what properties are compatible with proportionality. At the end of chapter 4 we have only mentioned some, drawing from Arrhenius' proposal discussed in chapter 2. Clearly, to construct a fully fledged desert-adjusted axiology, one would have to delve deep in all such properties.
Yet another path that one could follow is to examine what concrete non-comparatively defined C-function could induce proportionality. This would mean to engage in a similar discussion to the one proposed in this thesis, but instead of the *properties* of the function, to look at the matter from a quantitative perspective, and study how to quantify the intrinsic value of a possible world with such a function.

[1]This suggestion was given to me by Dr. Constanze Binder at the EIPE seminar.

A Appendix

- **Graph of Feldman's principles P4-P6.**

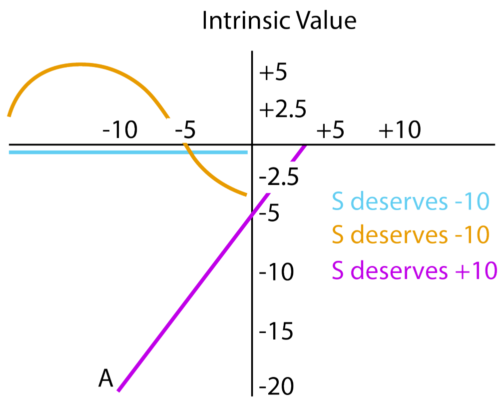

FIGURE A.1: Graph Negative Welfare

The purple line represents a possible interpretation of P4, in the case in which an agent S is such that $d_S = 10$, the light blue and the orange lines two possible interpretations of P5, in the case in which S is such that $d_S = -10$. Note that also for these lines Feldman does not explicitly mention enough of their points to deduce what the function underlying them is and so also in this case we will proceed by discussing them informally only.

Consider first the purple line. There are two points of interest in this function, but Feldman discusses only one of them, namely point A. This point represents the pair composed by $WV = -10$ and $IV = -20$. Then, Feldman is here taking $FV(-10, 10) = -10$, for then $C(-10, 10) = -10 + (-10)$ and the intrinsic value of that life is -20. This shows the content of P4, namely that the badness of pain is aggravated by positive desert. The other point we would like to highlight is B, corresponding to $WV = 0$ and $IV = -5$. This point represents the fact that it is bad that an agent with positive desert gets neither pleasure nor pain. Feldman here must be taking $FV(0, 10) = -5$, so that $C(0, 10) = -5$.

Consider now the light blue and orange lines. The intuitions that lie behind them are analogous to the ones lying behind the (dark) blue and green lines in Figure 3.1. They are different representations of the principle P5. The light blue line shows that the badness of pain is mitigated in the sense that it becomes neutral in cases of negative desert, whereas the orange line shows that it is mitigated in the sense that it gets transvaluated and may become even positive. This transvaluation represent "the retributivist axiological intuition that sometimes it is good for bad people to be punished" [6]. We omit the formal representation in Arrhenius' style, for it is simply the dual of the ones given for the dark blue and green lines above.

Again, Feldman does not add in the graph the behavior of the contribution function, in the case in which $d = 0$. However, also in this case that is straightforward, as by principle P6 we have

$$C(w, 0) = w \quad \text{for } w < 0 \tag{A.1}$$

- **Proof of counterexample** Carlson Neutral**, i.e. Proposition 5.1.1**:

Proof. Assume that the Arrheniusean framework holds. The total value of the two options is the sum of the welfare and fit-value of the two agents. The fit-value of each agent can be calculated by means of equation (3.5) above in the following way. Consider option (i). Since $d_A = 10$ and $w_A > 10$, then by equation (3.5) we have $C_A(11, 10) = 20$. Then since we are in the Arrheniusean framework, we have $FV_A(11, 10) = C_A(11, 10) - w_A = 20 - 11 = 9$. Moreover, since $d_B = 0$ and $w_B < 0$ we can use equation (A.1) to get that $C_B(-19, 0) = -19$. Then by our framework assumption, we get $FV_B(-19, 0) = C_B(-19, 0) - w_B = -19 - 19 = 0$. Hence, the total value of option (i) is ((11 + 9) + (-19 + 0))= 1.
Now consider option (ii). Since $d_A = 10$ and $w_A < 10$, then by equation (3.5) we have $C_A(0, 10) = (2.5 \cdot 0) - 5 = -5$. Then, we have $FV_A(0, 10) = C_A(0, 10) - w_A = -5 - 0 = -5$. Moreover, since $d_B = 0$ and $w_B > 0$ we can use equation (3.6) to get that $C_B(1, 0) = 1$. Then, by our framework assumption, we get $FV_B(1, 0) = C_B(1, 0) - 1 = 0$. Hence, the total value of option (ii) is $((0 - 5) + (1 + 0)) = -4$.
Since option (i) has higher total value than option (ii), then, according to Feldman's theory, option (i) is the right option. □

- **Proof of Proposition 3.3.1**:

Proof. Consider an amount of welfare E and a set of agents N. Suppose that $\Sigma_{i \in N} d_i = E$. As we saw above, F1-2 implies that the fit-value of a life is maximized when $d = w$. It follows that the sum of the agents' fit-values is highest when that happens. Since the welfare to be distributed is fixed, then the possible world with the highest IV is the one having the highest aggregate fit-value. Thus, the IV of a possible world is highest when for every agent $i \in N$ we have $d_i = w_i$, which is possible to obtain as by assumption we have $\Sigma_{i \in N} d_i = E$. We can conclude that, in the Arrheniusean framework, axiom F1-2 implies **J1**. □

- **Proof of Proposition 3.3.3**:

Proof. Consider a strictly concave function $FV(w, d)$, i.e. such that for all $\alpha \in (0, 1)$ and two points $(w + e)$ and $(w - e)$, we have that

$$FV(\alpha(w - e, d) + (1 - \alpha)(w + e, d)) > \alpha FV(w - e, d) + (1 - \alpha) FV(w + e, d).$$

Consider $\alpha = \frac{1}{2}$ and substitute it to the inequality above. Then we have

$$FV(\frac{1}{2}(w - e, d) + \frac{1}{2}(w + e, d)) > \frac{1}{2} FV(w - e, d) + \frac{1}{2} FV(w + e, d)$$

This is equivalent to $2FV(w) > FV(w - e) + FV(w + e)$ which is equivalent to G1. □

- **Proof of Proposition 3.3.4**:

Proof. Consider two points w_1, w_2, such that $|w_1 - d| > |w_2 - d|$. By G1, for every e, we have $FV(w_2 + e) - FV(w_2) < FV(w_2) - FV(w_2 - e)$. Call this (1). Without loss of generality, let us take e such that $e = w_2 - w_1$. It follows that $w_2 = w_1 + e$ and $w_1 = w_2 - e$. Thus, we can rewrite (1) as:

$$FV(w_2 + e) - FV(w_2) < FV(w_1 + e) - FV(w_1)$$

which corresponds to F2-1. □

- **Proof of Proposition 5.2.1**:

Proof. Let the FV-function satisfy F2-2 and suppose towards contradiction that there exists two desert levels d_1, d_2 such that the curves of the FV-function that each desert level induces are not identical, e.g. suppose that they are as in Figure A.2.

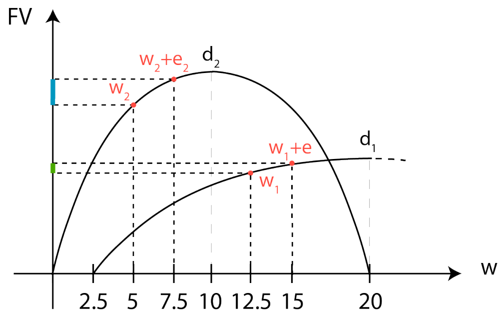

FIGURE A.2: Graph Negative Welfare

Note that in the graph there exists some w_1, w_2, e_1, e_2 that satisfy the antecedent of F2-2, i.e. $w_1 = 12.5, w_2 = 5, e_1 = e_2 = 2.5$. Clearly, for these welfare levels, we have $FV(w_2 + e_2, d_2) - FV(w_2, d_2) > FV(w_1 + e_1, d_1) - FV(w_1, d_1)$, which means that the function does not satisfy F2-2. Contradiction. Hence, if an FV-function satisfies F2-2, then for every two desert levels, the curves of the function induces are identical. □

- **Proof of Proposition 5.2.2**:

Proof. Consider a function that satisfies F2-2. Arrhenius showed that such a function satisfies **PD**. Then consider Arrhenius' example, which we introduce as a counterexample to F2-2: consider two arbitrary agents that have different desert levels, e.g. $d_1 = 8, d_2 = 4$ and suppose that we have to distribute $E = 6$ to them. Arrhenius shows that by **PD** the best distribution is $w_1 = 5$ and $w_2 = 1$. However, the proportional distribution would be $w_1 = 4$ and $w_2 = 1$. This shows that a function that satisfies **PD** contradicts **PP**. □

- **Proof of Proposition 5.3.1**:

Proof. By Proposition 3.3.3, if a function is strictly concave then it satisfies G-1. We now prove that, if a function satisfies G-1, then G-2 is also satisfied. By G-1 we have that, for any positive w, e such that $e < w$, if $|w + e - d| < |w - d|$, then we have

(1) $FV(w + e, d_1) - FV(w, d_1) < FV(w, d_1) - FV(w - e, d_1)$;

(2) $FV(w+e,d_2) - FV(w,d_2) < FV(w,d_2) - FV(w-e,d_2)$.

Take an arbitrary w_1^* and consider all w_2 such that

$$FV(w_1^* + e, d_1) - FV(w_1^*, d_1) \leq FV(w_2, d_2) - FV(w_2 - e, d_2)$$

Among those w_2 there exists a w_2^* such that

$$FV(w_2^*, d_2) - FV(w_2^* - e, d_2) = FV(w_1^*, d_1) - FV(w_1^* - e, d_1)$$

From (1), we know that for w_1^* it holds that

$$FV(w_1^*+e,d_1) - FV(w_1^*,d_1) < FV(w_1^*,d_1) - FV(w_1^*-e,d_1) = FV(w_2^*,d_2) - FV(w_2^*-e,d_2)$$

Thus, we obtain clause (i) of G-2:

$$FV(w_1^* + e, d_1) - FV(w_1^*, d_1) < FV(w_2^*, d_2) - FV(w_2^* - e, d_2)$$

By analogous reasoning, from (2) we can derive that for w_2^* we have

$$FV(w_2^*+e,d_2) - FV(w_2^*,d_2) < FV(w_2^*,d_2) - FV(w_2^*-e,d_2) = FV(w_1^*,d_1) - FV(w_1^*-e,d_1)$$

Hence, we obtain clause (ii) of G-2:

$$FV(w_2^* + e, d_2) - FV(w_2^*, d_2) < FV(w_1^*, d_1) - FV(w_1^* - e, d_1)$$

We can conclude that if a function is strictly concave, then there exists two welfare levels that satisfy clause (i) and (ii) of G-2. □

- **Proof of Proposition 5.3.2**:

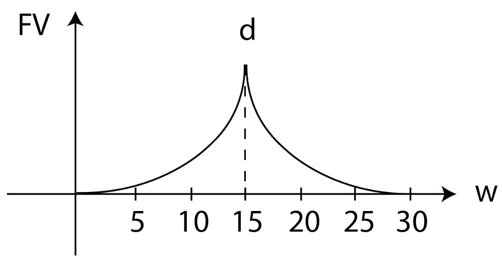

FIGURE A.3: Function satisfying the first fit-idea but not the second.

Proof. Consider a convex function, as in Figure A.3. We show that (1) does not hold. Take two agents with desert levels $d_1 = 15 = d_2$, and suppose we distribute 20 units of welfare among them such that $w_1 = 10 = w_2$. Now suppose that we want to redistribute the 20 units of welfare among them in such a way that the distribution is not proportional to their desert levels any more, e.g. we give $w_1 = 15$ and $w_2 = 5$. By simply inspecting the figure, one can note that the overall intrinsic value of the agents' life has now increased. This means that there exists two amounts of welfare that are comparatively just, i.e. proportionally so, and there exists a redistribution of welfare from these two levels that increases the overall value of the distribution. In other words, (1) does not hold. □

- **Proof of Proposition 5.3.3**:

Proof. Consider a linear function, as Feldman's function. When discussing such a function, we have already seen that (2) does not hold. Indeed, given a distribution of welfare that is proportional to the agents' desert, any redistribution of welfare from that one would not modify the intrinsic value of the distribution. This means that the function allows several distributions of welfare to be as valuable as the comparatively just one, i.e. (2) does not hold. □

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