The performance of the enhanced dual strength momentum strategy in the US stock market

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Abstract

This research paper studies the enhanced dual strength momentum strategy and its robustness towards momentum crashes. This research paper does this by means of the Kolmogorov-Smirnov Distance measure. Furthermore, the performance of the enhanced dual strength momentum strategy is based on the US stock market, and the strategy is to be compared against the original dual strength momentum strategy. Results overall show that the enhanced dual strength momentum strategy is more robust towards momentum crashes relative to the original dual strength momentum strategy. Moreover, the enhanced dual strength momentum strategy is a profitable trading strategy, including transaction cost. Finally, the enhanced dual strength momentum strategy performs differently based on different sector structures. Results overall are robust by means of including transaction cost, different sample sizes, and sample splits.

1 Introduction

This research paper studies the dual strength momentum strategy and its robustness towards momentum crashes. The dual strength momentum strategy combines both absolute strength momentum and relative strength momentum together. The relative strength momentum strategy, also known as the momentum strategy, buys past winners' stocks and sells past losers' stocks. The momentum strategy is also known for its proneness towards momentum crashes. Moreover, the strategy has been around for decades, yet there is limited literature available on the strategy and its application towards momentum crashes. Momentum crash is when past winners' stocks suddenly lose and past losers' stocks suddenly win. Furthermore, this dual strength momentum strategy is an enhanced version of the original dual strength momentum strategy by Antonacci (2016). For the original dual strength momentum strategy, the author (2016) too simply combines both absolute strength momentum and relative strength momentum together, without incorporating a robust solution towards momentum crashes. Therefore, this research paper will construct a dual strength momentum strategy, one that is robust towards momentum crashes. Furthermore, the original dual strength momentum strategy excludes transaction cost. Antonacci (2016) states that the transaction cost for the original dual strength momentum strategy would be minimum as the strategy is based on a formation period of twelve months, which comes with lower transaction cost. This statement however is questionable as this very same dual strength momentum strategy is also based on a holding period of one month, which comes with higher transaction cost. Carhart (1997) and Novy-Marx & Velikov (2014) state that transaction cost alone could diminish an entire portfolio's return. Finally, the performance of the enhanced dual strength momentum strategy will be assessed, not only via the US stock market, but also via different US sector structures. This research paper will therefore answer the following research question:

"How does the enhanced dual strength momentum strategy perform in the US stock market?"

To answer this research question, this research paper will assess the performance of the enhanced dual strength momentum strategy with the original dual strength momentum strategy serving as a bench mark. This research paper will do so by means of the following hypothesis (1):

"The enhanced dual strength momentum strategy is more robust towards momentum crashes relative to the original dual strength momentum strategy"

Another way to answer this research question is to assess the profitability of the enhanced dual strength momentum strategy, including transaction cost. This research paper will do so by means of the following hypothesis (2):

"The enhanced dual strength momentum strategy is a profitable trading strategy, including transaction cost".

A third way to answer this research question is to assess whether the performance of the enhanced dual strength momentum strategy would differ based on different sector structures. This research paper will do so by means of the following hypothesis (3):

"The enhanced dual momentum strategy performs differently based on different sector structures".

This research paper proceeds in the following manner. Section two is the theoretical background, which is to bring the reader up to speed on the most current theories regarding the applications that are relevant for this research paper. Section three highlights the data that was required for this research paper. Section four explains the methods that have been used for conducting this research. Section five presents the empirical results. Section six analyzes these empirical results. Section seven provides robustness testing. Section eight concludes based on the results, and section nine provides recommendations for future research papers.

2 Theoretical background

This chapter will focus on past research papers that have been written which are relevant for this research paper. The goal is to bring the reader up to speed on the most current theories regarding the applications that are relevant for this research paper. The following sub-headers will provide more information on the topic and how this topic relates to this research paper.

Relative strength momentum

The origin of momentum investing can be traced back before the time of Donchian. It was Jegadeesh & Titman (1993) however, who made momentum investing mainstream. The authors (1993) define momentum investing when past winners' stocks continue to win and past losers' stocks continue to lose. After their (1993) momentum strategy was introduced, which already came with various formation and holding periods, countless of authors introduced their "own" momentum strategies. To name a few, there is momentum with lag between the formation and holding period to account for stock price overreactions [Carhart (1997)], there is momentum based on industries [Moskowitz and Grinblatt (2002)], there is momentum based on business cycles [Bacmann, Dubois & Isakov (2001)], there is momentum based on reversal [Jegadeesh & Titman (2001)], there is momentum which takes into account stocks' fundamental values [Barberis, Greenwood, Jin & Shleifer (2018)], there is momentum based on absolute strength rather than relative strength [Antonacci (2013)], and there is momentum which combines both absolute strength and relative strength [Antonacci (2016)]. To further touch upon momentum strength, Antonacci (2016) defines the momentum strategy by Jegadeesh & Titman (1993) as a relative strength momentum strategy as a stock is compared against other stocks in the cross-section. According to Cooper, Gutierrez & Hameed (2005), since stocks are compared in the cross-section, it is imperative for the market to be in an upward state as stocks are highly regime dependent.

Absolute strength momentum

Antonacci (2013) states that there is a considerable body of research on relative strength momentum, but not so much on absolute strength momentum. The author (2013) defines absolute strength momentum when a stock is compared to its past performances. Similar to that of Antonacci (2013) is Moskowitz, Ooi & Pedersen (2012), who define absolute strength momentum as time-series momentum as a stock is compared to its performances over time. The authors (2012) further state that with time-series momentum, a stock's future return can be predicted by means of its past return. Regardless, both Moskowitz et al (2012) and Antonacci (2013) define a stock an absolute strength winner (loser) when that stock has outperformed itself over time. This definition is different from that of Gulen & Petkova (2018) however. The authors (2018) define a stock an absolute strength winner (loser) only when that stock has reached the top (bottom) 10% of its historical value. Yang & Zhang (2019) on the other hand, define these types of stocks as extreme absolute strength winners (losers). Though all four research papers (2012) (2013) (2018) (2019) do state that absolute strength momentum is much more stable throughout time as compared to relative strength momentum. According to Gulen & Petkova (2018), this is because with absolute strength momentum, stocks are compared on a time-series basis and will therefore not move as significantly as compared to relative strength momentum, which compares stocks on a cross-sectional basis.

Dual strength momentum

Antonacci (2016) constructed the dual strength momentum strategy by combining absolute strength momentum [Antonacci (2013)] with relative strength momentum [Carhart (1997)]. Antonacci (2016) further states that for a stock to be classified as a dual strength winner (loser), the stock must be first classified as both absolute strength winner (loser) and relative strength winner (loser), as the author (2016) describes both components to be highly regime dependent. According to Antonacci (2016), by combining both absolute strength momentum and relative strength momentum, the dual strength momentum strategy yields significantly higher returns, while bearing lower risks. This hypothesis is realized by the author (2016) by incorporating Treasury bills into the strategy. Bessembinder (2018) states that Treasury bills are considered a low risk investment, and by incorporating this asset class into a strategy imposes diversification. However, Mishkin & White (2002) state that stocks and bonds are only well diversified (during down markets) if the economy is either predicted to be weaker or faces greater uncertainty. The authors (2002) further state that anything else (e.g. the economy is facing tighter monetary policies) results in both stocks and bonds to drop in value.

Momentum reversals

De Bondt & Thaler (1985) define momentum reversals when past winners' stocks in the long term lose and past losers' stocks in the long term win (3+ years). Both de Bondt & Thaler (1985) and Jegadeesh & Titman (2001) motivate that these momentum reversals are due to the fact that investors at some point recognize that winning (losing) stocks are overbought (oversold), which results in these stocks becoming overvalued (undervalued). The authors (1985) (2001) further state that at this stage, investors buy the undervalued stocks and sell the overvalued stocks. In addition to the conventional momentum strategy, Jegadeesh & Titman (2001) also constructed the reversal momentum strategy which buys longer term losers' stocks and sells longer term winners' stocks. Next to long term momentum reversals, Daniel & Moskowitz (2016) document on short momentum reversals, also known as momentum crashes. The authors (2016) state that unlike long term momentum reversals, short term momentum reversals are more sudden and more severe. These reversal indicators are characterized by Daniel & Moskowitz (2016) when markets are down. Furthermore, both Cooper et al (2005) and Antonacci (2016) motivate that momentum crashes are more severe with relative strength momentum compared to absolute strength momentum. According to the authors (2005) (2016), this is due to the fact that with relative strength momentum, stocks are compared on a cross-sectional basis, and are highly regime dependent, with each other and with the market. Daniel & Moskowitz (2016) document a recent momentum crash in which a losers' portfolio rose by 163% against a corresponding winners' portfolio which only rose by 8% (p. 1).

Identifying momentum crashes

Several authors have documented possible insights on identifying momentum crashes. For instance, Stivers & Sun (2010) find that a momentum crash is due when a stock shows low momentum premium, combined with high market volatility. Gulen & Petkova's (2018) on the other hand, find that a momentum crash is due when a stock's recent (ranking) value deviates significantly from its long-run value. Finally, Barberis et al (2018) find that a momentum crash is due when a stock's recent (ranking) value consistently increases (decreases) without corresponding good (bad) news on a company's fundamental values. According to the authors

(2010) (2018) (2018), it is important for momentum investors to recognize when momentum continues and when momentum is due for crash.

Optimal portfolio and transaction cost

Jegadeesh & Titman (1993) introduced the relative strength momentum strategy which comes with various formation and holding periods. According to the authors (1993), a formation period is defined as the look-back period for which the stock is assessed, while a holding period is defined as the forward period for which the stock is held. Furthermore, Assogbavi & Leonard (2008) define an optimal portfolio as a portfolio which generates the highest return, while bearing the lowest risk. The authors (2008) state that portfolios with a minimum formation and holding period are optimal. Their (2008) research paper however, excluded transaction cost. Carhart (1997) and Novy-Marx & Velikov (2014) state that transaction cost alone could diminish an entire portfolio's return. The authors (1997) (2014) therefore speak of a tradeoff between minimizing (maximizing) time lag on the one hand, and incurring higher (lower) transaction cost on the other hand.

3 Data

This chapter will focus on the data that have been collected for conducting this research. The following sub-headers will provide more information on the types of data collected, its source, and its purpose for this research.

Stock prices

The first type of data that were collected are daily and monthly stock prices for all US listed companies. Daily prices are required for deriving the stocks' daily idiosyncratic volatility, which will be converted into monthly idiosyncratic volatility as the main analysis uses monthly intervals. For the monthly analysis, monthly stock prices are required for the period from January 1970 to December 2019. These US stock prices were collected from the *Center of Research in Security Prices* (CRSP) which is part of *Wharton Research Data Services* (WRDS). This research paper focuses on the US equity market only as CRSP is the only data source at the moment that provides stock prices for all US listed companies as one extractable package.

Treasury bills

The second type of data that were collected are daily and monthly US Treasury bills. As with stock prices, the same case is applicable for daily Treasury bills, which is that these daily Treasury bills are required for deriving the stocks' daily idiosyncratic volatility. For the monthly analysis, monthly Treasury bills are required for the period from January 1970 to December 2019. These Treasury bills were also collected from CRSP WRDS. Monthly US Treasury bills serve as a complementary component for the dual strength momentum strategy as will be explained during the methodology section.

Stock market index

The third type of data that were collected are daily and monthly stock market indexes, in particular the *Standard & Poor 500* index (S&P500). As with stock prices, the same case is applicable for daily index, which is that this daily index is required for deriving the stocks' daily idiosyncratic volatility. For the monthly analysis, monthly S&P500 data is required for the period from January 1970 to December 2019. This index was also collected from CRSP WRDS. The S&P500 will mainly be used for relative purposes. More details regarding this concept will be explained during the methodology section.

Company ratios

The fourth type of data that were collected are company ratios, which include price-to-book, return-on-asset, and return-on-equity. These ratios, together with stock prices, were used to construct the dual strength momentum strategy. Note that WRDS only offers company ratios starting from January 1990 and onwards. Furthermore, these ratios were collected from the *Financial Ratios Suite by WRDS* subscription. Company ratios serve as additional metrics in assessing the behavior of momentum, which again will be explained more in-depth during the methodology section.

Risk factors

The fifth and final type of data that were collected are the daily and monthly Fama & French–Carhart factors, which include market, size, value, and momentum. As with stock prices, the same case is applicable for the daily Fama & French market factor. This daily market factor is required for deriving the stocks' daily idiosyncratic volatility. For the monthly analysis, Fama & French–Carhart factors are required for the period from January 1970 up to December 2019. All four factors were collected from the Kenneth R. French Library. These four factors were required for testing the dual strength momentum strategy on its significance.

4 Research Methodology

This chapter will focus on the methodology that have been used for conducting this research. The following sub-headers will provide more information on the particular method, its history with past research papers, and its application.

Dual strength momentum strategy

The dual strength momentum strategy combines both the absolute strength momentum strategy with the relative strength momentum strategy. Moreover, this research paper has constructed two different dual strength momentum strategies. The enhanced dual strength momentum strategy serves as the lead strategy for this research paper. The original dual strength momentum strategy by Antonacci (2016) serves as a complementary. The goal is to have both strategies compared against each other in terms of performances. Furthermore, for the sake of easier reading, the enhanced dual strength momentum strategy will be referred to as dual strength one and the original dual strength momentum strategy will be referred to as dual strength two. The following sections explain how both strategies were constructed.

Dual strength one

Dual strength one is based on a multi-step selection process. On the first step, the absolute strength momentum strategy is constructed. On the second step, the Kolmogorov-Smirnov Distance measure is implemented for identifying momentum crashes. On the third step, the relative strength momentum strategy is constructed. On the fourth step, several key ratios related to a stock's fundamental value are implemented for assessing stock momentum. On the fifth step, a measure is implemented for measuring stock momentum. On the sixth and final step, steps one to five are combined to construct dual strength one. The following sub-headers will explain step-by-step on how this strategy was constructed.

Step one: Absolute strength in dual strength one

For the absolute strength component, this research paper has adopted a similar approach as that of Gulen & Petkova (2018). This includes a formation period of 12 months, which comes with a lag period of 1 month as Carhart (1997) found that returns during the formation period would be higher the following year, but not thereafter. The formation period is followed by a holding period of 1 month. Gulen & Petkova (2018) explain this via the following setup which is also the same setup that this research paper has adopted for dual strength one:

"At the beginning of January of year t, we record cumulative returns for all stocks over the period January of year t-1 to November of year t-1. These returns are ranked on the basis of the historical distribution of all January to November cumulative returns. If a stock's cumulative return over January of year t-1 to November of year t-1 falls in the top (bottom) 10% of the historical distribution of January-November returns, we classify that stock as an absolute strength winner (loser) at the beginning of January of year t. This process is to be repeated every month as the historical distribution of 11-month is updated every month" (pp. 10, 11).

Step two: Identifying momentum crashes

For identifying momentum crashes, this research paper measured the distance between the historical distribution of cumulative returns and the ranking distribution of cumulative returns. This research paper did so by means of a Kolmogorov-Smirnov Distance (KS-D) measure. The KS-D measure is a non-parametric test; meaning, it can be calculated without assuming the type of the underlying distributions of cumulative stock returns. Furthermore, the KS-D measure is defined as the maximum of the absolute difference between F1 (the historical distribution of cumulative returns up to month t-2) and F2 (the ranking distribution of cumulative returns up to month t-2), and it is stated as follows:

$$\mathrm{D} = \max_{\mathrm{x}}(|F1(x) - F2(x)|)$$

where a larger D value indicates a larger absolute deviation between the historical distribution of cumulative returns and the ranking distribution of cumulative returns. A KS-D value was then calculated for every month for the period January 1970 through December 2019. These months were then classified into five quintiles which is based on the respective KS-D value. Portfolios with quintiles 1 to 4 will proceed to step three. Portfolios with quintile 5 are excluded and monthly Treasury bills will be bought instead. Furthermore, to avoid look-ahead bias, these KS-D quintiles are continuously updated on a monthly rolling basis starting from January 1970, and progressing its way up until December 2019.

Step three: Relative strength in dual strength one

For the relative strength component, this research paper has adopted a similar approach as that of Carhart (1997). This includes a formation period of 12 months, which comes with a lag period of 1 month as the author (1997) found that returns during the formation period would be higher the following year, but not thereafter. The formation period is followed by a holding period of 1 month. Furthermore, only portfolios in the top (bottom) 10% of a particular month (of a particular year) will be bought (sold) as they are considered relative strength winners (losers). The reason why this research paper has opted for the setup of Carhart (1997), and not that of Jegadeesh & Titman (1993), is for the sake of comparing dual strength one with dual strength two which also comes with a holding period of one month. One difference though is that this relative strength strategy includes two criteria for defining relative strength winners and losers instead of one according to Jegadeesh & Titman (1993). The first criterion is the portfolio's ranking performance relative to the other portfolio's ranking performances as documented by the authors (1993). The second criterion is that only portfolios with Kolmogorov-Smirnov Distance (KS-D) quintiles 1 to 4 are considered as described in step two.

Step four: Implementing fundamental ratios

For implementing fundamental ratios, this research paper has adopted a similar approach as that of Barberis et al (2018). This includes the return-on-asset (ROA) and return-on-equity (ROE) ratios. ROA is defined as earnings before tax over assets. ROE is defined as earnings before tax over equity. Furthermore, both ROA and ROE are recorded as a growth rate which takes the value at month t over its value at month t-1. The reason for this step is because of the following scenario, which was brought to the attention by Gulen & Petkova (2018). The authors (2018) presented two states of the world in which a stock yields a higher (lower) ranking cumulative return relative to its historical cumulative return. State of the world one: the stock yields a higher (lower) ranking cumulative return relative to its historical cumulative return due to the stock receiving good (bad) news over the ranking period. State of the world two: the stock yields a higher (lower) ranking cumulative return relative to its historical cumulative return due to the stock undergoing a continuous period of extrapolation. To put this scenario in context, its Kolmogorov-Smirnov Distance (KS-D) quintile has increased for the better (worse) in terms of the ranking cumulative return as a winner (loser), but so has the probability of a momentum crash. Therefore, a momentum crash does not necessarily have to take place at KS-D quintile 5, but it can also take place at any KS-D quintiles other than 5. According to Barberis et al (2018), momentum investors should not only pay attention to a stock value, but also to its fundamental value when engaging in a momentum strategy. The authors (2018) state that by doing so, a momentum strategy becomes much more sustainable and thereby less prone to momentum crashes.

Step five: Stock momentum

For assessing stock momentum, this research paper identifies two types of momentum. The first type of momentum is a stock value at month t-2 relative to its long-run value which runs up to month t-2. This research paper has recorded the mean value of the stock's historical cumulative returns as its long-run value. Furthermore, in order for a winners' (losers') stock to gain momentum, the ranking distribution of its cumulative return at month t-2 ought to be larger (smaller) than its historical distribution of cumulative return at month t-2.

The second type of momentum is a stock value at month t relative to the market value also at month t. This research paper has adopted a similar approach as that of Gulen & Petkova (2018), which is by means of the price-to-book (PTB) ratio. PTB is defined as stock value over book value. This research paper has recorded the mean value of all stocks up to month t as the market value. Furthermore, in order for a winners' (losers') stock to gain momentum, the stock's PTB at month t ought to be larger (smaller) than the market's PTB at month t.

Step six: Combining steps one to five

For a portfolio to be classified as a winner (loser), all five aforementioned steps ought to be satisfied in given order. Finally, dual strength one is defined as the winners' portfolios *minus* the losers' portfolios.

$$\begin{aligned} Winners'\ portfolio = {} & absolute\ strength\ winner\ \&\ KS-D\ measure \\ & < 5\ \&\ relative\ strength\ winner\ \&\ ROA\ growth\ rate > 0\ \&\ ROE\ growth\ rate \\ & > 0\ \&\ ranking\ distribution > historical\ distribution\ \&\ ptb > mean(ptb) \end{aligned}$$

$$\begin{aligned} Losers'\ portfolio = {} & absolute\ strength\ loser\ \&\ KS-D\ measure \\ & < 5\ \&\ relative\ strength\ loser\ \&\ ROA\ growth\ rate < 0\ \&\ ROE\ growth\ rate \\ & < 0\ \&\ ranking\ distribution < historical\ distribution\ \&\ ptb < mean(ptb) \end{aligned}$$

Dual strength two

Antonacci (2016) based dual strength two on a three-step selection process. On the first step, the relative strength momentum strategy is constructed. On the second step, the absolute strength momentum strategy is constructed. On the third and final step, steps one and two are combined to construct dual strength two. The following sub-headers will explain step-by-step on how this strategy was constructed.

Step one: Relative strength in dual strength two

For the relative strength component, Antonacci (2016) has adopted an identical approach to that of Carhart (1997). This includes a formation period of 12 months, which comes with a lag period of 1 month as the author (1997) found that returns during the formation period would be higher the following year, but not thereafter. The formation period is followed by a holding period of 1 month. Furthermore, only portfolios in the top (bottom) 10% of a particular month (of a particular year) will be bought (sold) as they are considered relative strength winners (losers).

Step two: Absolute strength in dual strength two

All the relative winners' and losers' portfolios that were selected from step one are assessed whether they also carry absolute momentum. Antonacci (2016) does this by means of their performances relative to Treasury bills at month t-12. The author (2016) motivates that if a stock has outperformed Treasury bills over time, then it too is likely to continue to show a positive excess return at month t+1 by virtue of the transitive property since Treasury bill returns should remain positive over time (p. 5). Antonacci (2016) bases this theory on 1) Moskowitz et al (2012), who state that in absolute momentum, there is significant positive auto-covariance between a stock's excess return at month t+1 and its excess return at t-12; and on 2) Case, Yang & Yildirim (2010), who state that there is a positive correlation between a stock's performance and Treasury bills. The author (2016) further states that if the stock does not show positive momentum relative to Treasury bills (meaning it does not have positive absolute momentum), Treasury bills are selected as an alternative proxy investment until the selected asset is stronger than Treasury bills (p. 5).

Step three: Combining steps one and two

For a portfolio to be classified as a winner (loser), both steps one and two ought to be satisfied in given order. Finally, dual strength two is defined as the winners' portfolios *minus* the losers' portfolios.

$$Winners'portfolio = relative\ strength\ winner\ \&\ absolute\ strength\ winner$$

$$Losers'portfolio = relative\ strength\ lower\ \&\ absolute\ strength\ loser$$

Transaction cost

Estimating transaction cost

For estimating transaction cost, this research paper has adopted a similar approach as that of Roll (1984). The author (1984) motivates that in essence the cost of trading equals one-half the (posted) effective bid-ask spread. Moreover, this research paper has adjusted the estimated transaction cost by the appropriate formation and holding period frequency. This approach might seem outdated, considering the fact that more recent research papers exist in calculating transaction cost, including that of Hasbrouck (2009) and Novy-Marx & Velikov (2014). However, Novy-Marx & Velikov (2014) has adopted the same measure as that of Hasbrouck (2009) for estimating transaction cost, which is by means of using the Bayesian Gibbs sampler for estimating the effective bid-ask spread. Hasbrouck (2009), in turn, based this estimation on the original Roll (1984) model. Regardless, the more recent theory on estimating transaction cost can be traced back to the original Roll (1984) model. Roll (1984) defines the model as follows:

$$Vt = Vt - 1 + \ \varepsilon t$$

$$Pt = Vt + cQt$$

where Vt is the underlying "efficient value", which is denoted as the log quote midpoint price prior to trading, Pt is the observed trading price, Qt is a random indicator for the direction of the trade which either takes the value of +1 (-1) depending on whether the trading took place at ask (bid), εt is a random disturbance which reflects public information about the stock, and c is the effective cost of trading.

Bid-ask spread

This research paper has estimated transaction cost by means of the Roll (1984) model, which equals the weighted average of one-half the (posted) effective bid-ask spread, adjusted for the appropriate formation and holding frequency. According to Gryglewics & Eisert (2019), the formula for estimating the effective bid-ask spread is as follows:

$$s = \frac{Offer\ price - Bid\ price}{Mid - market\ price}$$

where $Offer\ price - Bid\ price$ represents the bid-offer spread, which equals the difference between the highest price that a buyer is willing to pay for an asset and the lowest price that a seller is willing to accept. The $Mid - market\ price$ equals the average of the bid and ask price.

Euclidean distance

Upon extracting the stock prices from CRSP, a significant proportion of observations came with missing bid and ask prices. Instead of excluding these observations from the dataset, this research paper has opted for an approach as suggested by Novy-Marx & Velikov (2014). The authors (2014) motivate that certain variables explain the cross-sectional variation in estimating transaction cost. Therefore, for each month, this research paper ranked all companies on market equity and estimated idiosyncratic volatility. For observations where transaction cost could not be estimated directly, these missing cost of transactions were replaced by the nearest match stock for which transaction cost could be estimated. The closest match is defined by the shortest Euclidean distance in rank space in market equity and estimated idiosyncratic volatility. The shortest Euclidean distance in rank space between companies i and j equals as follows:

$$\sqrt{(rankMEi - rankMEj)^2 + (rankIVOLi - rankIVOLj)^2}$$

where “ME” and “IVOL” stand for market equity and idiosyncratic volatility, respectively. Market equity is measured as the stock price multiplied by the number of shares outstanding. Idiosyncratic volatility is measured as the standard deviation of residuals of past three months’ daily returns on the daily excess market return.

Optimal portfolio construction

The enhanced dual strength momentum strategy (dual strength one) comes with a formation (J) and holding (K) period of J=12 and K=1, respectively. In addition to this setup, this research paper has constructed dual strength one based on other Js and Ks. Assogbavi & Leonard (2008) did so for formation periods {12, 9, 6, and 3} and holding periods {12, 9, 6, and 3}. The authors (2008) find that a momentum strategy would achieve higher returns by shortening the J and K periods. Their (2008) finding however excluded transaction cost. Antonacci (2016) has constructed the original dual strength momentum strategy (dual strength two), also excluding transaction cost. The author (2016) states that the transaction cost for dual strength two would be minimum as the strategy is based on a formation period of 12 months which comes with lower transaction cost. This statement however is questionable as this very same dual strength momentum strategy is also based on a holding period of one month which comes with higher transaction cost. Carhart (1997) and Novy-Marx & Velikov (2014) state that transaction cost alone could diminish an entire portfolio's return. Therefore, this research paper has extended on the work of Assogbavi & Leonard (2008) and Antonacci (2016) by constructing dual strength one through different Js and Ks, including transaction cost.

Portfolio performance on sector structures

This research paper has also constructed the enhanced dual strength momentum strategy (dual strength one) based on different sector structures. This is done by means of the sector classification system which is produced by Morningstar[1]. This sector classification system presents three sector structures: cyclical, defensive and sensitive. Sectors in the cyclical structure are highly sensitive to business cycle peaks and troughs. Moreover, sectors in the defensive structure are anti-cyclical. Finally, sectors in the sensitive structure are moderately correlated with business cycles. Furthermore, Bacmann et al (2001) motivate that momentum strategies perform differently based on different business cycles. For instance, Antonacci (2016) has constructed the dual strength momentum strategy based on the "real estate investment trust" industry. This research paper however has constructed the dual strength momentum strategy based on sector structures, instead of sectors (or industries for that matter). This is due

[1] https://www.morningstar.com/content/dam/marketing/apac/au/pdfs/Legal/StockSectorStructure_Factsheet.pdf?

to the fact that with some sectors (industries), an insufficient number of observations were available upon classification [not to mention the six-step criteria it needs to go through for it to be marked as a potential dual strength winner (loser)]. Furthermore, it should be noted that upon extracting the data from CRSP, a small proportion of observations came with missing sectors. These missing sectors were excluded from the data set. Finally, this research paper has compared and contrasted the portfolio performances of the three sector structures against each other and against the S&P 500 index.

Asset pricing model

This research paper has employed the Fama & French–Carhart 4-factor model as its asset pricing model. The Fama & French–Carhart asset pricing model is presented as follows:

$$EXr = \alpha + \beta mkt * EXmkt + \beta hml * HML + \beta smb * SMB + \beta umd * UMD + \epsilon t$$

where $EXmkt$ stands for monthly excess return of the CRSP value-weighted index, HML stands for monthly premium of the book-to-market factor, SMB stands for monthly premium of the size factor, UMD stands for monthly premium on winners minus losers, α stands for monthly excess risk adjusted return, and ϵt stands for some random error term.

The monthly excess returns, EXr, produced by the dual strength momentum strategy are to be regressed against the $EXmkt$, HML, SMB and UMD factors to assess to what extent these factors explain the significance of the strategy. The reason why this research paper has opted for the Fama & French–Carhart 4-factor model, and not the Fama & French–Carhart 6-factor model, which include the profitability and investment factors, was because these two factors are still relatively recent discoveries and the research of these factors in different markets and time periods is still limited[2].

Other data preparation

Upon extracting the data from CRSP, a significant proportion of this data came with missing values. In order to maintain as much of the original dataset as possible, various estimates were used instead. For instance, of those observations for which their prices were missing, an

[2] https://www.robeco.com/en/insights/2015/10/fama-french-5-factor-model-why-more-is-not-always-better.html

average between the bid price and ask price were used (if available). Furthermore, these prices, including the number of shares outstanding, have been correctly adjusted by the appropriate cumulative factor adjustment. Only those observations for which all prices were missing, were excluded. In addition, all prices for which the average, by company, were below $1, were excluded as well.

5 Empirical Results

This chapter will focus on the results that have been produced for this research. The following sub-headers will provide the specifics of what each result mean, and its relevance towards the analysis.

The Kolmogorov-Smirnov Distance measure

The Kolmogorov-Smirnov Distance (KS-D) measure measures the distance between the ranking distributions of cumulative returns relative to its historical distributions of cumulative returns. A more significant distance indicates a larger momentum crash being on the way. Moreover, the KS-D measure provides insights on the probability of implementing a successful momentum strategy as it measures the number of winners' stocks which is required to hedge against the number of losers' stocks, and vice versa. Moreover, to further elaborate on this concept, consider the following three portfolios, which all have been taken from the sample.

Portfolio 1

Portfolio 1 covers the ranking distribution of 11-month cumulative returns from the period April 2003 up to February 2004. This portfolio is to be compared against the historical distribution of 11-month cumulative returns which covers the period from January 1970 to February 2004.

Portfolio 2

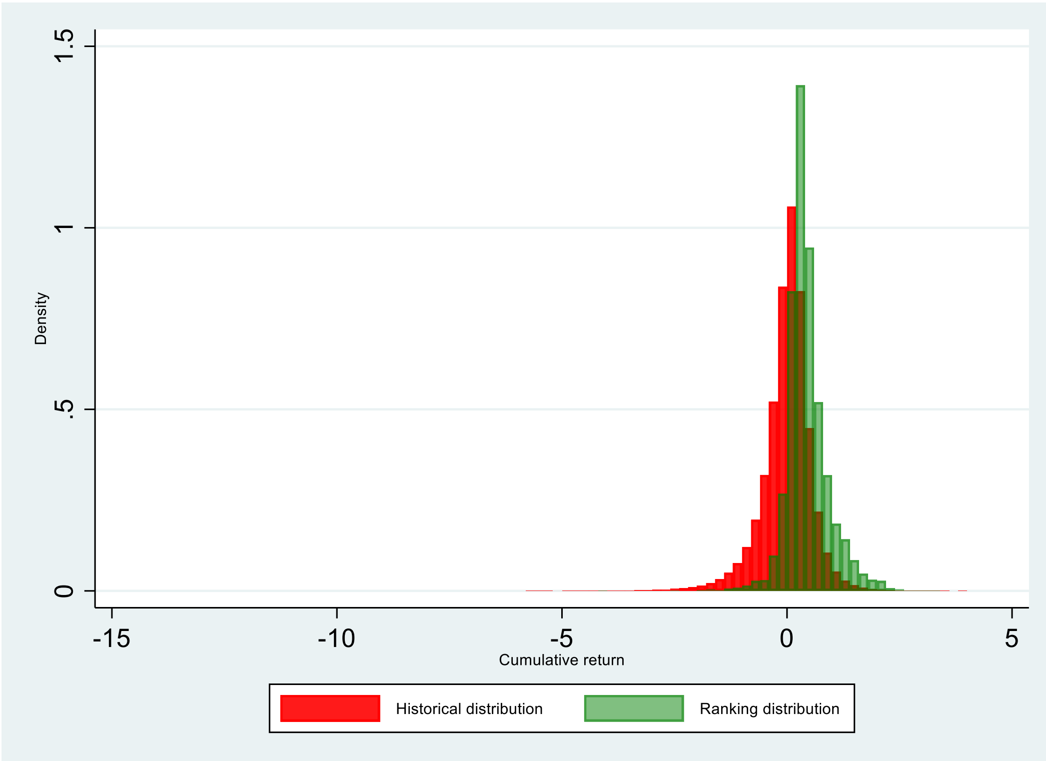
Portfolio 2 covers the ranking distribution of 11-month cumulative returns from the period May 2008 up to March 2009. This portfolio is to be compared against the historical distribution of 11-month cumulative returns which covers the period from January 1970 to March 2009.

Portfolio 3

Portfolio 3 covers the ranking distribution of 11-month cumulative returns from the period December 2014 up to October 2015. This portfolio is to be compared against the historical distribution of 11-month cumulative returns which covers the period from January 1970 to October 2015.

Figures 1, 2, and 3 visualize these portfolios and how they would stand against their respective historical distributions. For the sake of saving space, only these three portfolios were provided. Figure 4 provides a more abstract overview of the KS-D measure, which covers all the portfolios that were bought and sold throughout the sample period.

Figure 1: Ranking distribution of cumulative returns for portfolio 1



Summary statistics for historical (h) and ranking (r) distributions

Date: February 2004

Number of observations: 4721

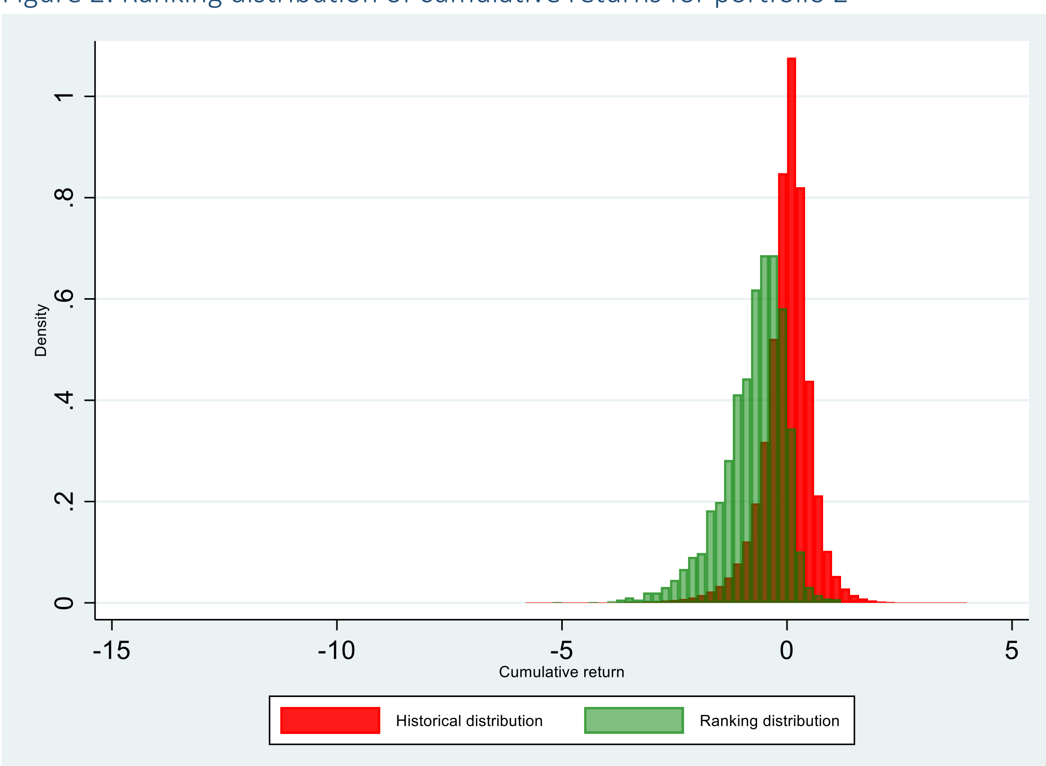
Mean: 0.1463 (h), 0.7806 (r)

Standard deviation: 0.0073 (h), 0.0126 (r)

Skewness: 8.35 (h), 6.34 (r)

KS-D quintile: 5

Figure 2: Ranking distribution of cumulative returns for portfolio 2



Summary statistics for historical (h) and ranking (r) distributions

Date: March 2009

Number of observations: 4188

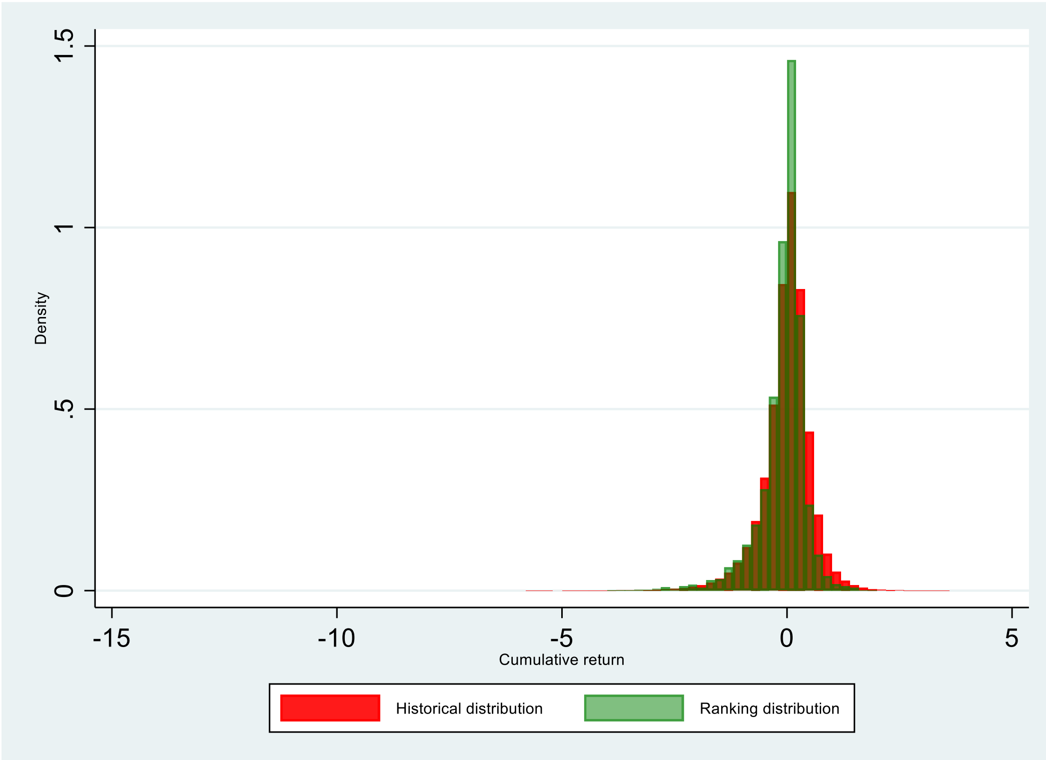
Mean: 0.1421 (h), -0.3951 (r)

Standard deviation: 0.0072 (h), 0.0036 (r)

Skewness: 8.22 (h), 1.22 (r)

KS-D quintile: 1

Figure 3: Ranking distribution of cumulative returns for portfolio 3



Summary statistics for historical (h) and ranking (r) distributions

Date: October 2015

Number of observations: 3595

Mean: 0.1483 (h), 0.0234 (r)

Standard deviation: 0.0000 (h), 0.0046 (r)

Skewness: -1.37 (h), 2.25 (r)

KS-D quintile: 1

Portfolio 1 (continued)

The summary statistics for this portfolio concerns February 2004. The summary statistics shows that February 2004 comes with 4721 absolute strength candidates. Based on the shape of both distributions, approximately 30% or 1416 are potential absolute strength losers and 70% or 3305 are potential absolute strength winners. In addition, the historical distribution of 11-month cumulative returns comes with an average return of 14.63%, a standard deviation of 0.73, and skewness of 8.35. This while the ranking distribution of 11-month cumulative returns comes with an average return of 78.06%, a standard deviation of 1.26, and skewness of 6.34. Furthermore, the ranking distribution is allocated a Kolmogorov-Smirnov Distance (KS-D) quintile of 5. As a result, 1416 stocks are qualified to be an absolute strength loser, while 3305 stocks are qualified to be an absolute strength winner[3].

[3] For scaling purposes, both the historical and ranking distributions of cumulative returns in figures 1, 2, and 3 were converted into log-values.

Portfolio 2 (continued)

The summary statistics for this portfolio concerns March 2009. The summary statistics shows that March 2009 comes with 4188 absolute strength candidates. Based on the shape of both distributions, approximately 20% or 838 are potential absolute strength winners and 80% or 3350 are potential absolute strength losers. In addition, the historical distribution of 11-month cumulative returns comes with an average return of 14.21%, a standard deviation of 0.72, and skewness of 8.22. This while the ranking distribution of 11-month cumulative returns comes with an average return of -39.51%, a standard deviation of 0.36, and skewness of 1.22. Furthermore, the ranking distribution is allocated a Kolmogorov-Smirnov Distance (KS-D) quintile of 1. As a result, 838 stocks are qualified to be an absolute strength winner, while 3350 stocks are qualified to be an absolute strength loser.

Portfolio 3 (continued)

The summary statistics for this portfolio concerns October 2015. The summary statistics shows that October 2015 comes with 3595 absolute strength candidates. Based on the shape of both distributions, approximately 49% or 1762 are potential absolute strength winners and 51% or 1833 are potential absolute strength losers. In addition, the historical distribution of 11-month cumulative returns comes with an average return of 14.83%, a standard deviation of 0.00, and skewness of -1.37. This while the ranking distribution of 11-month cumulative returns comes with an average return of 2.34%, a standard deviation of 0.46, and skewness of 2.25. Furthermore, the ranking distribution is allocated a Kolmogorov-Smirnov Distance (KS-D) quintile of 1. As a result, 1762 stocks are qualified to be an absolute strength winner, while 1833 stocks are qualified to be an absolute strength loser. Clearly portfolio 3 is the only optimal portfolio amongst the three portfolios for implementing a successful momentum strategy; one that is based on a sufficient number of winners' stocks which is required to hedge against a number of losers' stocks, and vice versa. Figure 4 provides a more abstract overview of the KS-D measure; one that covers all the portfolios that were bought and sold throughout the sample period.

Figure 4: Ranking distribution of cumulative returns for all portfolios

This figure presents all the portfolios that were bought and sold throughout the sample period. The vertical axis highlights the ranking distributions of cumulative return, which are expressed as a fraction. The horizontal axis highlights the respective dates. The sample period is January 1990 to December 2019.

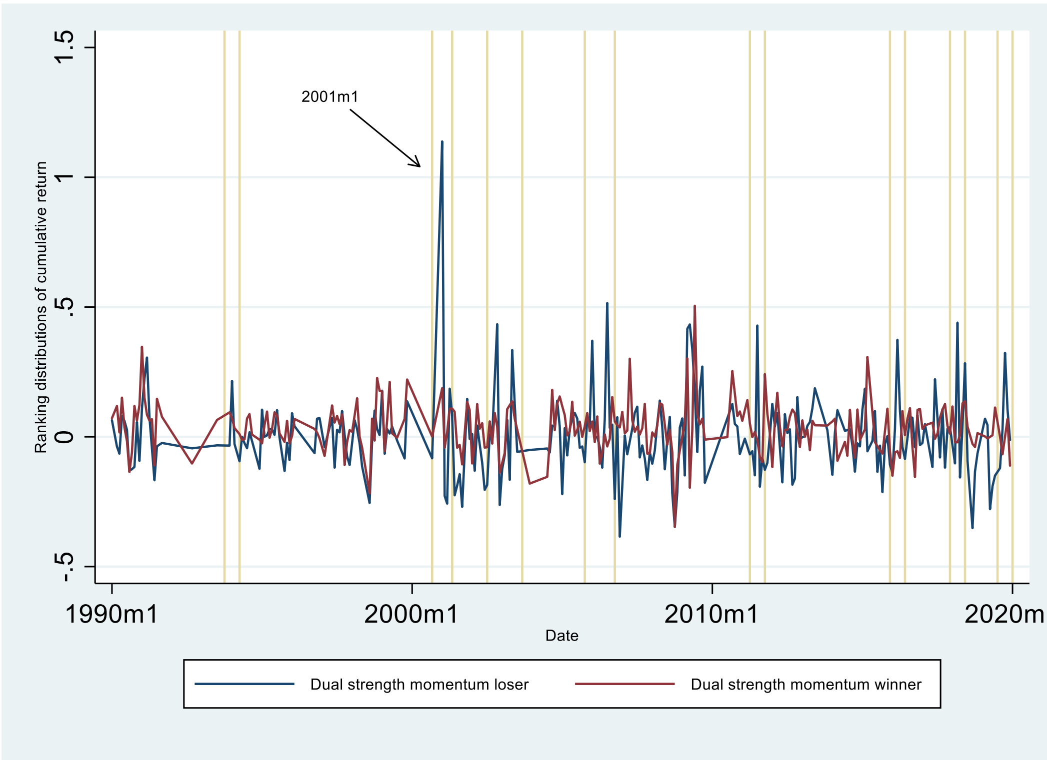
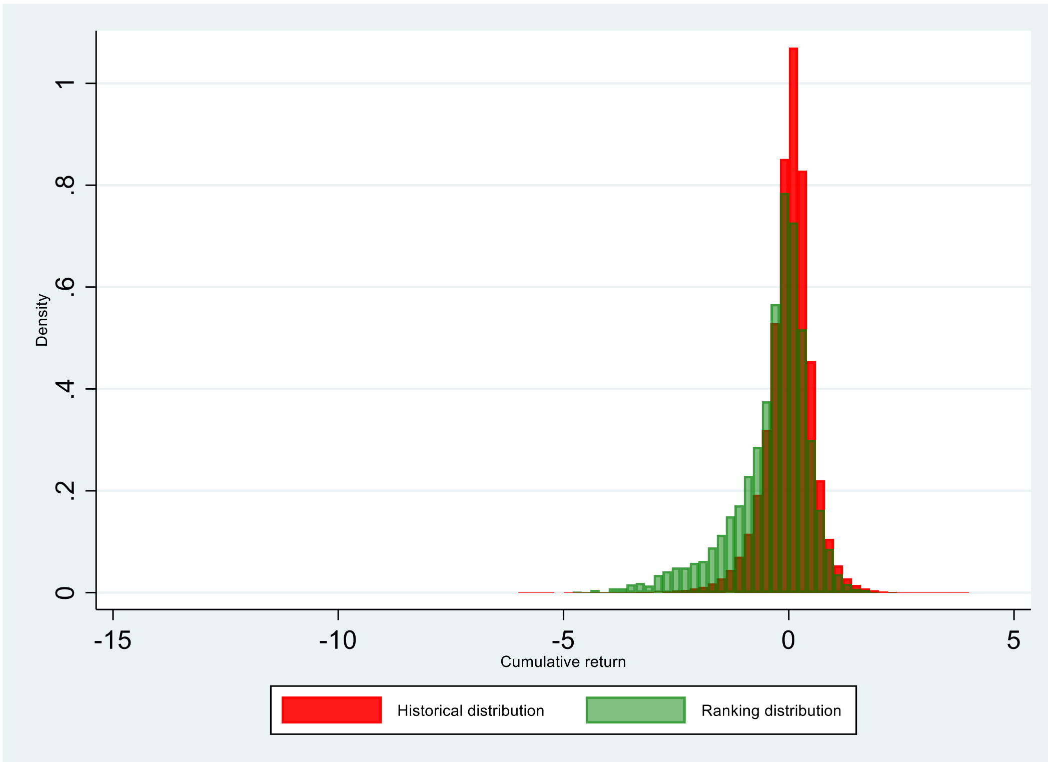


Figure 4 shows that throughout the sample period, losers' portfolios have outperformed the winners' portfolios. These losers' portfolios have been marked with golden stripes. Take for instance the portfolio of January 2001. This portfolio could not have been allocated a Kolmogorov-Smirnov Distance (KS-D) quintile of 5 (otherwise Treasury bills would have been bought instead), yet it showed a very crash alike behavior. Scoping deeper into the matter, figure 5 presents the portfolio of January 2001 (hereafter referred to as portfolio 4) more in-depth. As can be observed from figure 5, the portfolio's ranking distribution is not deviated as significantly from its historical distribution compared to for instance portfolio 2[4]. What also can be observed is that both portfolios 2 and 4 come with KS-D quintiles 1 and 3, respectively. Based on the KS-D measure, portfolio 2 should have been allocated a larger KS-D quintile

[4] See figure 2

relative to portfolio 4. This is due to the ranking distribution of portfolio 2 being more deviated from its historical distribution relative to the ranking distribution of portfolio 4 from its historical distribution.

Figure 5: Ranking distribution of cumulative returns for portfolio 4



Stock price and fundamental value

The following section presents the regression outputs of the Kolmogorov-Smirnov Distance (KS-D) measure for the ranking distribution of stock returns on price-to-book (PTB), return-on-asset (ROA), and return-on-equity (ROE) ratios. PTB is defined as stock value over book value. ROA is defined as earnings before tax over assets. ROE is defined as earnings before tax over equity. Furthermore, both ROA and ROE are recorded as a growth rate which takes the value at month t over its value at month t-1. PTB is recorded at month t. The goal is to examine the behavior of the KS-D ranking distribution relative to the historical distribution by means of these ratios. Table 1 presents the results.

Table 1: The effect of fundamental value on stock prices

This table presents the regression outputs of the KS-D measure for the ranking distribution of stock returns on PTB, ROA, and ROE ratios. The goal is to examine the behavior of the KS-D ranking distribution relative to the historical distribution by means of these ratios. For scaling purposes, log values were used for PTB, ROA growth rate, and ROE growth rate. Furthermore, R-squared is reported in percentage. Robust t-statistics are reported in parenthesis. The sample period is January 1990 to December 2019.

	Kolmogorov-Smirnov distance measure for the ranking distribution of stock returns		
PTB	0.32		0.32
	(215.73)		(211.24)
ROA growth rate		0.07	0.06
		(20.89)	(20.76)
ROE growth rate		0.01	0.01
		(8.68)	(4.59)
R-squared	7.33	0.05	7.30

Table 1 shows that as PTB, ROA, and ROE increase, the ranking distribution of a stock increases; meaning, it deviates further away from its historical distribution. Furthermore, PTB seems to show a stronger effect relative to either ROA or ROE; meaning, an increase of PTB causes the ranking distribution of a stock to deviate further away from its historical distribution relative to an increase of either ROA or ROE. Regardless, an increase of either of these ratios results in a stronger demand relative to supply for a particular stock. To visualize this concept, figure 6 {a, b, c, and d} illustrates the case when there is an increase of demand relative to supply for a particular stock. Similarly, figure 7 {a, b, c, and d} illustrates the case when there is a decrease of demand relative to supply for a particular stock. Do note that the visualizations that were presented in figures 6 and 7 are only meant for visualization purposes and that they do not represent a particular time period.

Regardless, figures 6 and 7 do show the progressive deviation of the ranking distribution of cumulative returns relative to its historical distribution. Furthermore, figure 6 seems to show a lesser deviation of the ranking distribution relative to its historical distribution as compared to the ranking and historical distributions presented in figure 7.

Figure 6: An increase of demand relative to supply

Figure 6a

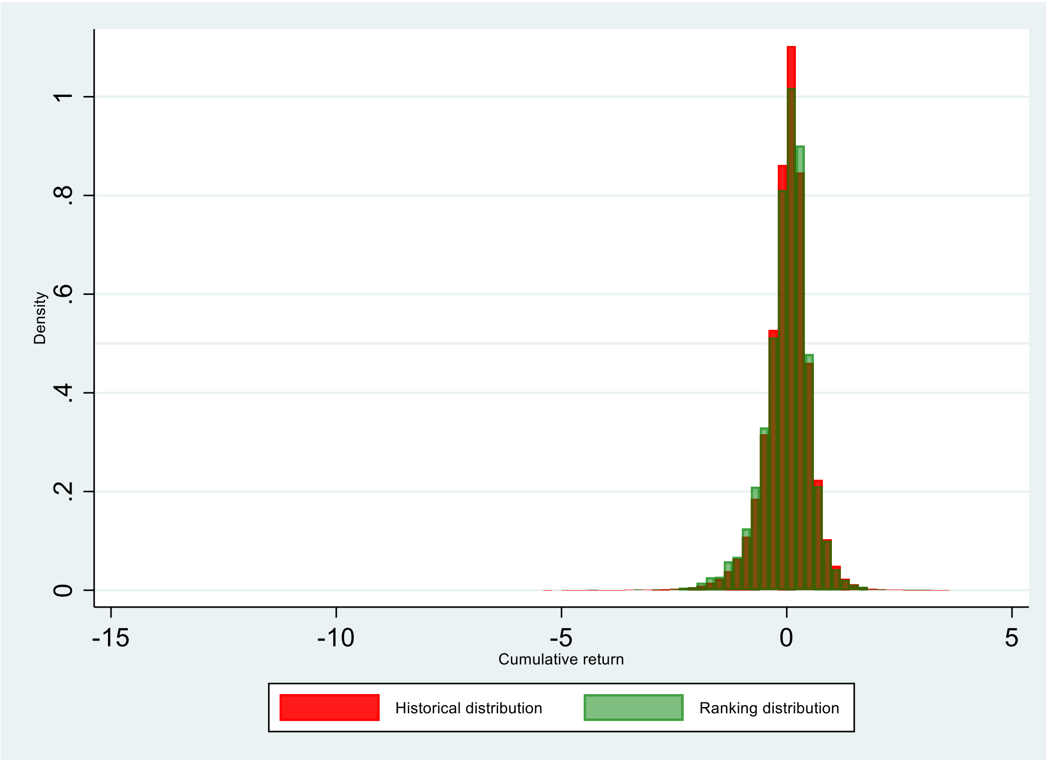


Figure 6b

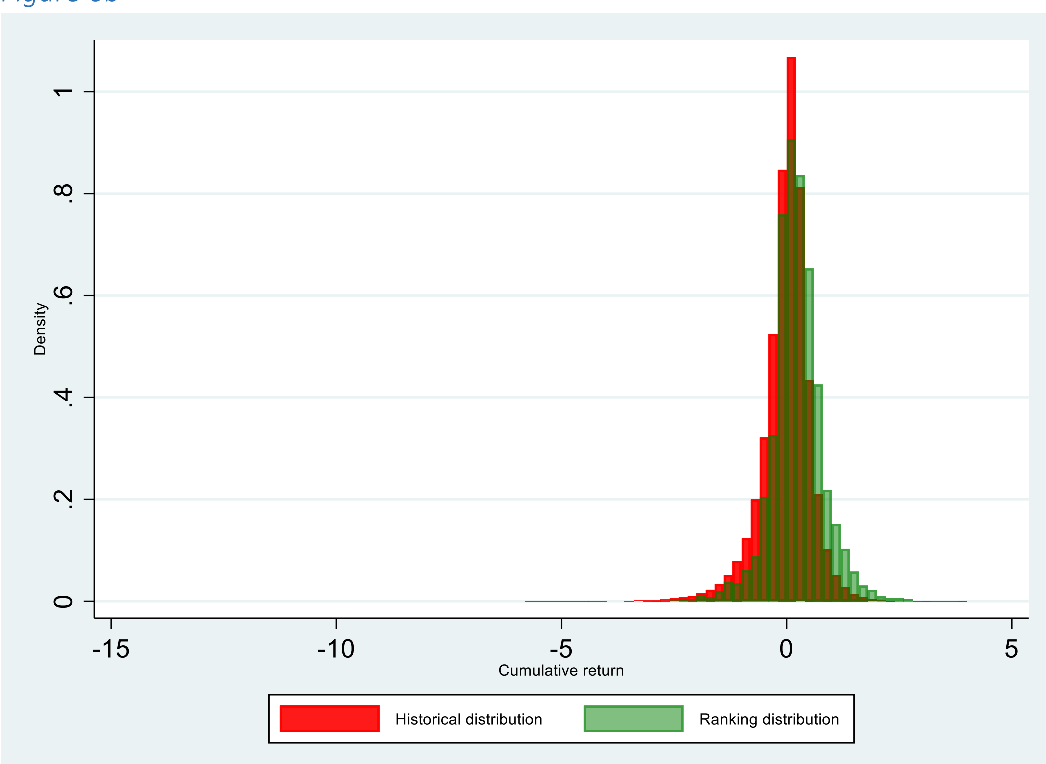


Figure 6c

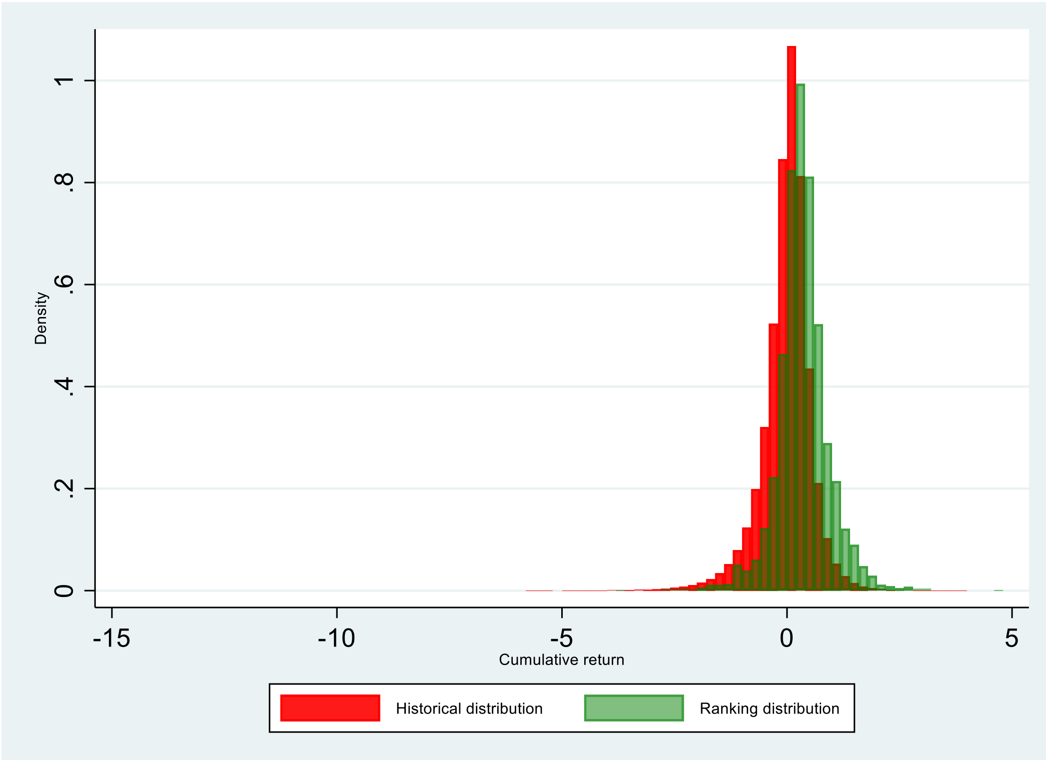


Figure 6d

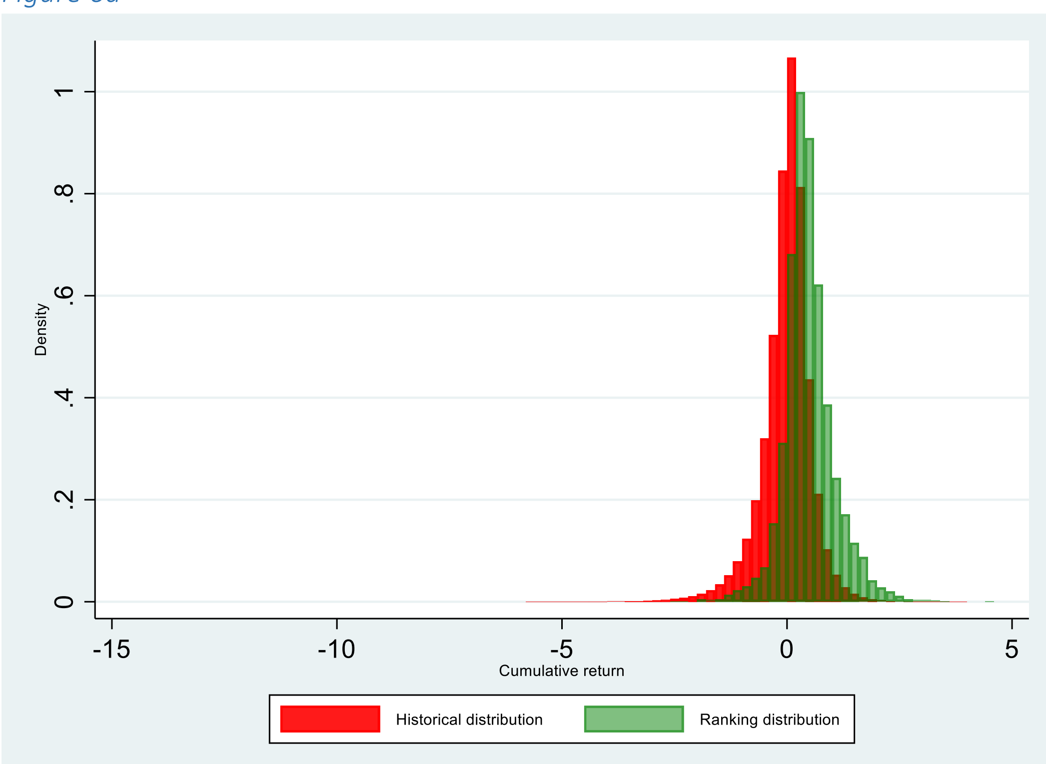


Figure 7: A decrease of demand relative to supply

Figure 7a

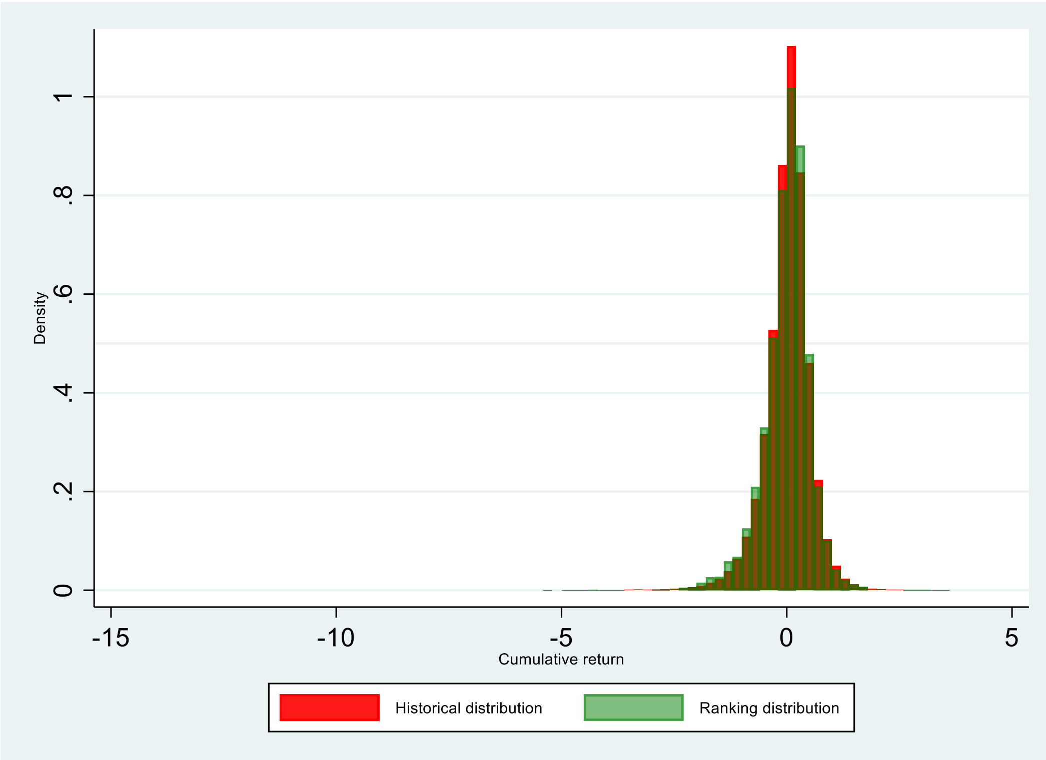


Figure 7b

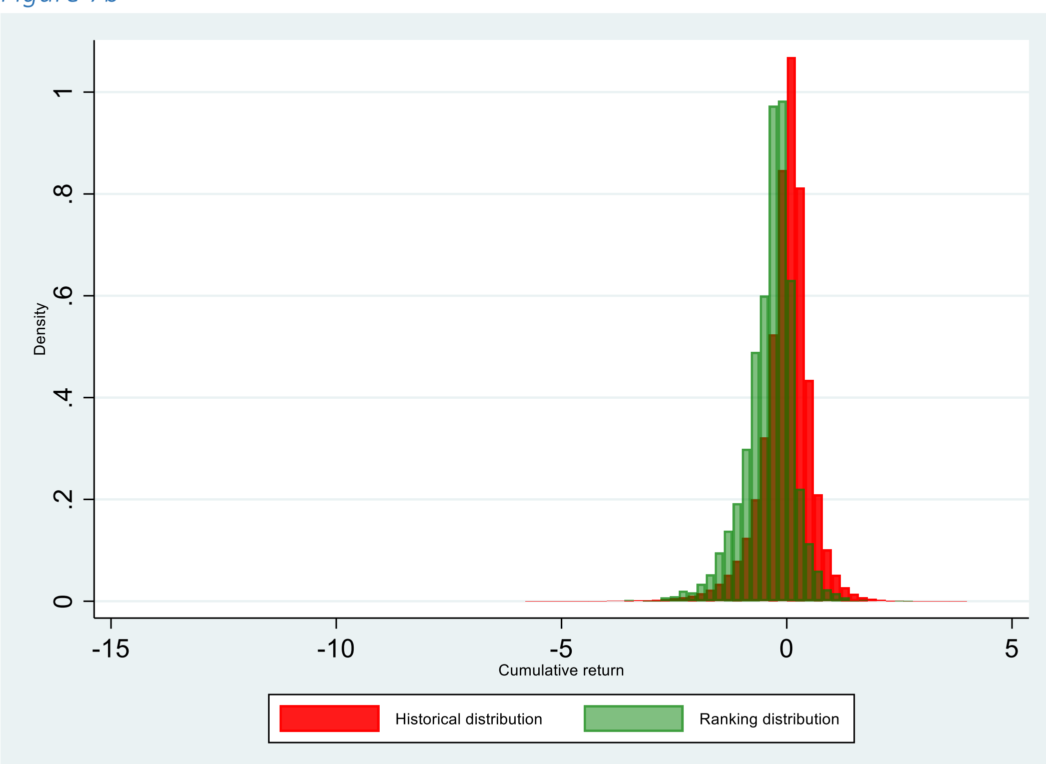


Figure 7c

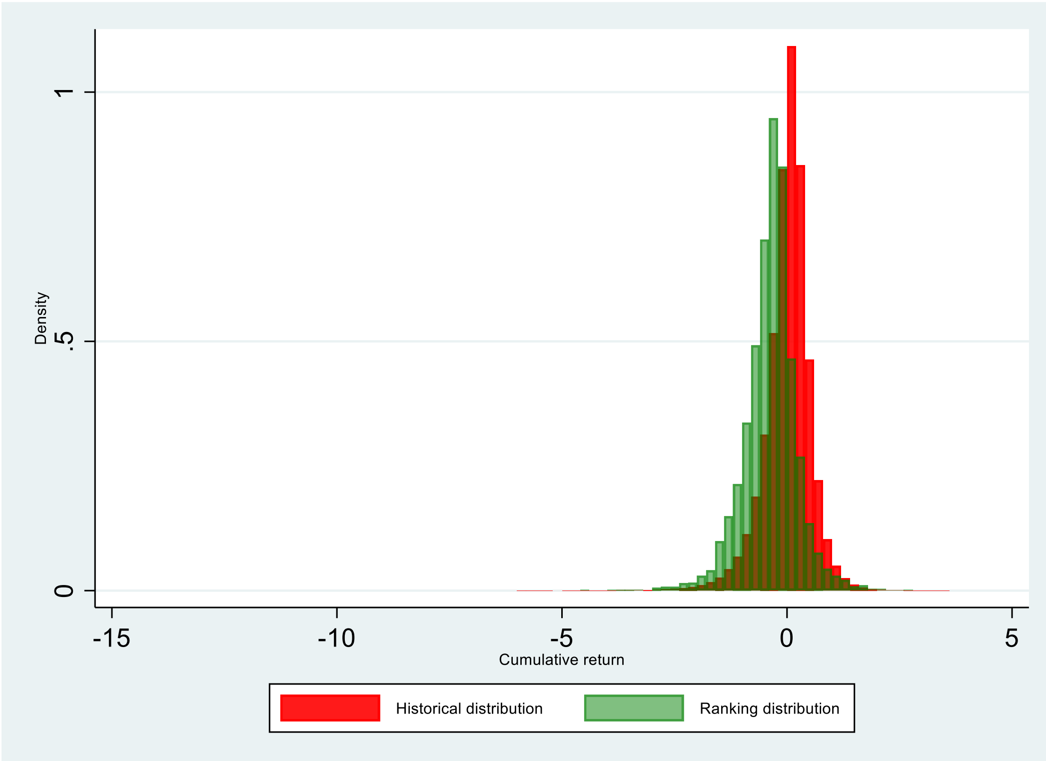
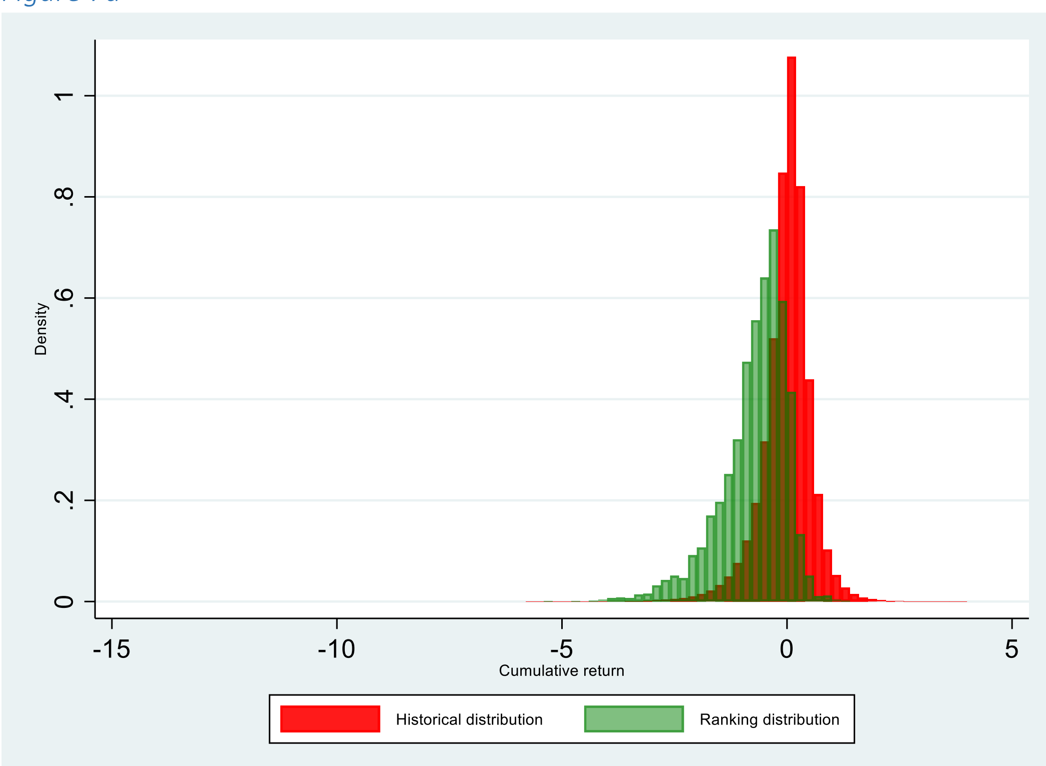


Figure 7d

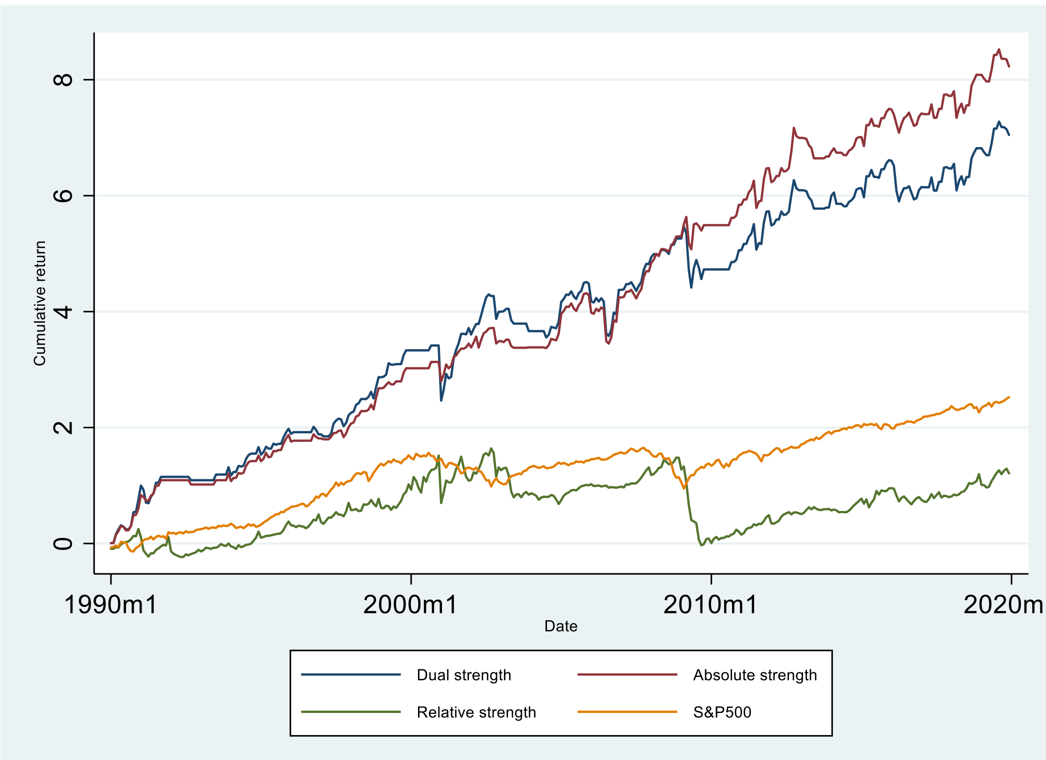


Portfolio performances: Dual versus Absolute versus Relative

Figure 8 provides the cumulative return of the dual, absolute, relative strength momentum strategies as constructed by this research paper, Gulen & Petkova (2018), and Carhart (1997), respectively. The S&P500 index has also been included which serves as a market benchmark.

Figure 8: Cumulative return amongst momentum strategies

This figure presents the gross performances of the dual, absolute, relative strength momentum strategies. In addition, the S&P500 index has been included which serves as a market benchmark. Furthermore, the vertical axis highlights the cumulative returns, expressed as a fraction. The horizontal axis highlights the respective dates. The sample period is January 1990 to December 2019.



Both the dual and absolute strength momentum strategies remain relatively stable throughout time. The relative strength momentum strategy on the other hand, performs poorly relative to the other momentum strategies, but also relative to the S&P500. Moreover, dual strength performs slightly worse relative to absolute strength. The performance gap between the two strategies were hardly noticeable until the period 2008-2009 came along. After that period, the gap had become more significant with dual strength performing worse and absolute strength performing better. Similar results hold true for relative strength and the S&P500. Table 2 presents the monthly performances for all three momentum strategies.

Table 2: Monthly performances for dual, absolute, and relative

This table presents the monthly performances for the dual, absolute, and relative strength momentum strategies. Average return, average excess return, standard deviation, and alpha are reported in percentages. Both the average excess return and alpha are regressed on the Fama French–Carhart model, which include a market risk factor, size factor, value factor and momentum factor. “P1” represents the losers’ portfolio, while “P10” represents the winners’ portfolio. The respective momentum strategy is constructed by the winners’ portfolio *minus* the losers’ portfolio. Furthermore, all three momentum strategies are reported excluding Treasury bills (if applicable), and excluding transaction cost. Robust t-statistics are reported as *** p < 0.01, ** p < 0.05, and * p < 0.10. The sample period is January 1990 to December 2019.

	Dual strength			Absolute strength			Relative strength		
	P1	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0.52	3.41	2.89	0.24	3.58	3.34	1.16	1.50	0.34
$\overline{R}$ - $\overline{R}$f	0.29	3.18***	2.89***	0.01	3.36***	3.34***	0.94	1.28***	0.34
t($\overline{R}$ - $\overline{R}$f)	(0.29)	(5.26)	(2.86)	(0.02)	(5.70)	(3.97)	(1.58)	(3.49)	(0.71)
SD	15.81	9.44	15.79	12.68	9.23	13.21	11.30	6.93	8.95
SR	0.02	0.34	0.18	0.00	0.36	0.25	0.08	0.18	0.04
α	0.41	2.45***	2.04**	-0.25	2.58***	2.83***	0.73*	0.18	-0.55
t(α)	(0.45)	(4.85)	(2.23)	(0.36)	(5.11)	(3.63)	(1.83)	(1.62)	(1.55)

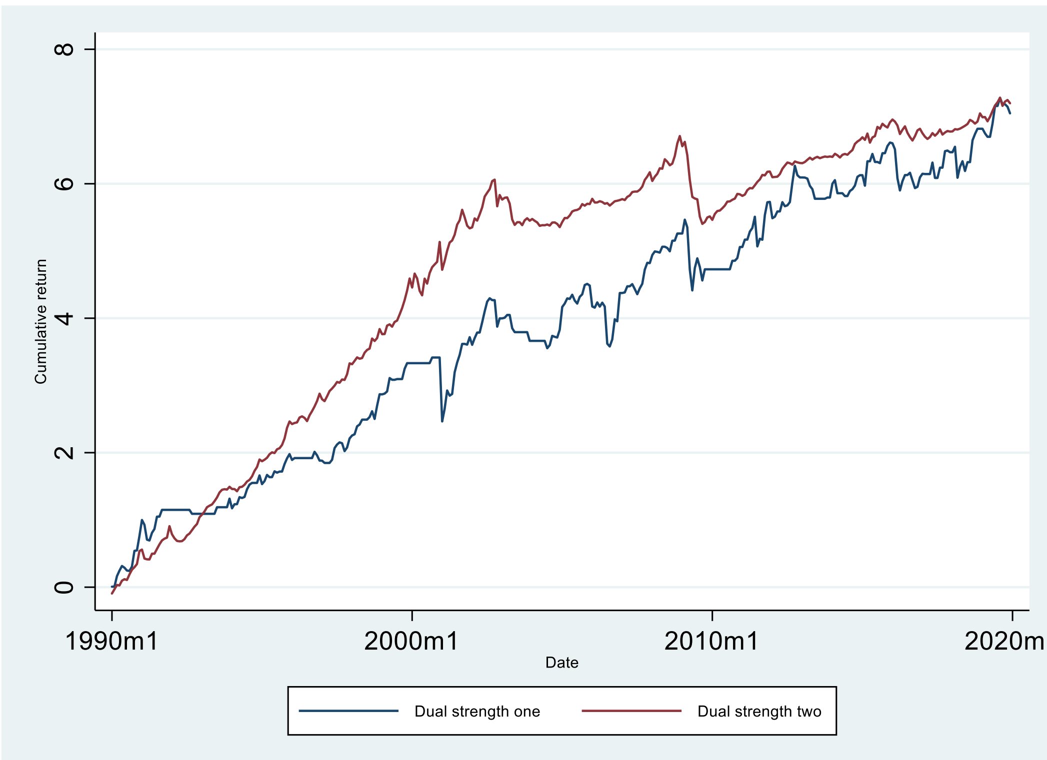
Table 2 reconfirms what was already presented in figure 8. Namely, the absolute strength momentum strategy outperforms both the dual and relative strength momentum strategies, not only in terms of average excess returns but also in terms of risk return. In addition, only the absolute and dual strength momentum strategies produce a monthly significant alpha. The absolute strength produces a significant alpha of 2.83% at a significance level of 1%, while the dual strength produces a significant alpha of 2.04% at a significance level of 5%.

Portfolio performances: Dual strength one versus Dual strength two

Focusing now on the comparison of both dual strength momentum strategies as constructed by this research paper and Antonacci (2016). Figure 9 provides the cumulative return of both dual strength momentum strategies performing throughout time.

Figure 9: Cumulative returns amongst dual strength momentum strategies

This figure presents the gross performances of dual strength one and dual strength two. The vertical axis highlights the cumulative returns, expressed as a fraction. The horizontal axis highlights the respective dates. Moreover, both momentum strategies are reported excluding Treasury bills, and excluding transaction cost. The sample period is January 1990 to December 2019.



Dual strength one performs relatively stable throughout time. The same cannot be said about dual strength two. This strategy significantly outperforms dual strength one until the period 2008-2009 came along. After this period, the performance gap between both dual strength momentum strategies had narrowed down, with dual strength one still showing signs of a strong upward momentum, while dual strength two showing signs of a "concave" alike upward momentum. Overall it seems that dual strength two is more volatile towards specific market conditions relative to dual strength one. This also explains the outperformance of dual strength two relative to dual strength one at the initial stage, which is then followed by comparable performances. Table 3 presents the monthly performances for dual strength one once more, but this time side by side with dual strength two for comparison purposes.

Table 3: Monthly performances for dual strength one and dual strength two

This table presents the monthly characteristics for dual strength one and dual strength two. Average return, average excess return, standard deviation, alpha, max drawdown and fraction Treasury bills are reported in percentages. Both the average excess return and alpha are regressed on the Fama French–Carhart model, which include a market risk factor, size factor, value factor and momentum factor. “P1” represents the losers’ portfolio, while “P10” represents the winners’ portfolio. The respective momentum strategy is constructed by the winners’ portfolio *minus* the losers’ portfolio. Furthermore, both dual strength momentum strategies are reported excluding Treasury bills (if applicable), and excluding transaction cost. Robust t-statistics are reported as *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.10$. The sample period is January 1990 to December 2019.

	Dual strength one			Dual strength two		
	P1	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0,52	3,41	2,89	-0,51	1,49	2,00
$\overline{R}$ - $\overline{R}$f	0,29	3.18***	2.89***	-0,73	1.27***	2.00***
t($\overline{R}$ - $\overline{R}$f)	(0,29)	(5,26)	(2,86)	(1,44)	(3,45)	(4,78)
SD	15,81	9,44	15,79	9,60	6,97	7,93
SR	0,02	0,34	0,18	-0,08	0,18	0,25
α	0,41	2.45***	2.04**	-1.07***	0,18	1.25***
t(α)	(0,45)	(4,85)	(2,23)	(3,56)	(1,42)	(4,26)
Max drawdown			-6,46			-11,60
Fraction Treasury bills			32,22			56,11

Both dual momentum strategies produce a monthly significant alpha. Dual strength one produces a significant alpha of 2.04% at a significance level of 5%, while dual strength two produces a significant alpha of 1.25% at a significance level of 1%. Furthermore, table 3, combined with figure 9, provides valuable insights on both portfolio performances during certain market conditions. For example, compared to dual strength two, dual strength one holds on average a lesser proportion of Treasury bills relative to its total portfolio holding. Treasury bills are considered a low risk investment. This also explains the higher standard deviation and therefore lower Sharpe ratio of dual strength one relative to dual strength two as there is less diversification. Regardless, dual strength one still shows a higher Max drawdown relative to dual strength two. Max drawdown is defined as the maximum difference between a stock’s trough and peak value, expressed in terms of peak value. Table 4 presents the monthly drawdowns amongst the two momentum strategies during the both the dot-com and sub-prime mortgage crises.

Table 4: Monthly drawdowns

This table presents the drawdowns at month t for both dual strength momentum strategies. These drawdowns were calculated relative to their respective cumulative portfolio value up to month t-1. Both cumulative portfolio values cover the period from January 1970 and onwards. Furthermore, drawdowns are reported in percentages. KS-D Q is a dummy variable which takes the value 1 if Kolmogorov-Smirnov Distance (KS-D) is in its highest quintile {5}, and 0 otherwise {1, 2, 3, and 4}. Red KS-D Q values indicate additional criteria other than KS-D which were not met (e.g. return-on-asset and return-on-equity growth rates > 0, while price-to-book stock < price-to-market market, or vice versa).

Period	Dual strength two	Dual strength one	KS-D Q
2000m01	(1,11)	-	1
2000m03	(0,59)	-	1
2000m04	(1,51)	-	1
2000m05	(0,56)	-	1
2000m07	(0,61)	-	1
2001m01	(3,25)	(7,21)	0
2001m10	(0,85)	-	1
2001m11	(0,95)	(0,08)	0
2001m12	(0,31)	0,83	0
Sub-total	**(9,74)**	**(6,46)**	
2008m01	(0,95)	0,83	0
2008m05	(0,06)	0,59	0
2008m07	(0,27)	(0,12)	0
2008m08	(0,38)	(0,34)	0
2009m01	(1,06)	-	0
2009m03	(1,40)	0,75	0
2009m04	(2,61)	(4,16)	0
2009m05	(1,86)	(2,13)	0
2009m06	(0,17)	2,35	0
2009m07	(0,09)	1,00	0
2009m08	(1,93)	(0,88)	0
2009m09	(0,82)	(1,38)	0
Sub-total	**(11,60)**	**(3,49)**	

Table 4 shows that dual strength two performs worse relative to dual strength one during both the dot-com and sub-prime mortgage crises. Focusing now on dual strength one and its Kolmogorov-Smirnov Distance (KS-D) measure. During the dot-com crisis, the strategy shows solid performance at the initial stage, which was then followed by poor performance as the strategy suffered a one-month drawdown of up to 7.21%. During the sub-prime mortgage crisis however, the strategy shows solid performance, but not necessarily due to its KS-D measure, but rather due to its implemented price-to-book (PTB), return-on-asset (ROA) and return-on-equity (ROE) ratios. Incorporating the KS-D measure, combined with these (fundamental) ratios, makes dual strength one thus less prone to market crashes relative to dual strength two.

Dual strength one with transaction cost

The following section presents the regression outputs of transaction cost estimates on transaction cost determinants. Moreover, daily data were used for determining transaction cost. Note that due to the large sample size, the regressions had to be split into multiple 10-year time intervals. Table 5 presents the most important features. Table 5 however only represents an abstract version of the complete output. Refer to appendix A for the complete and detailed overview.

Table 5: Determinants of transaction costs

This table presents the regression outputs of transaction cost estimates on lagged transaction cost, market equity, and idiosyncratic volatility. Transaction costs that could be directly estimated by means of the (posted) effective bid-ask spread equals one-half the (posted) effective bid-ask spread as suggested by Roll (1984). Furthermore, "lag_T_cost", "ME" and "IVOL" stand for lagged transaction cost, market equity and idiosyncratic volatility, respectively. Transaction cost were lagged by 21 days. Market equity was measured as the stock price multiplied by the number of shares outstanding. Idiosyncratic volatility was measured as the standard deviation of residuals of past three months' daily returns on the daily excess market return. Moreover, both "ME1" and "ME2" are log converted in order to account for skewness towards larger values. Finally, the difference between "ME1" and "ME2" is that "ME1" is non-squared whereas "ME2" is squared, as done by Novy-Marx & Velikov (2014). Both market equity and idiosyncratic volatility use end of July values. R-squared is reported in percentage. Robust t-statistics are reported in parenthesis. Furthermore, the estimated regressions are on an annual frequency and cover sub-periods ranging from January 1990 to December 2019.

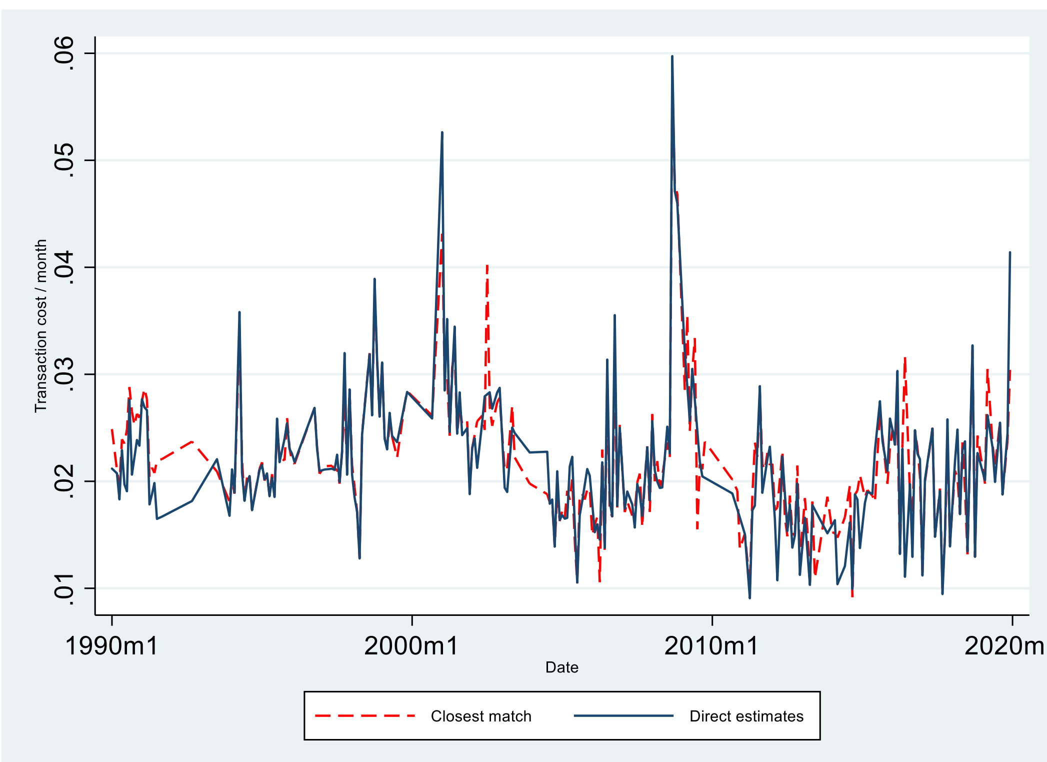
	January 1990 - December 1999						January 2000 - December 2009						January 2010 - December 2019					
lag_T_cost	0.44					0.53	0.21					0.09	0.38					0.28
	(35.98)					(37.26)	(17.06)					(7.68)	(26.23)					(16.77)
log_ME1		-0.97	-3.65		-3.08	-1.53		-0.36	-0.99		-0.22	-0.22		-0.32	-0.78		-0.71	-0.47
		(74.09)	(57.55)		(38.52)	(29.16)		(47.80)	(21.75)		(4.79)	(4.86)		(56.06)	(21.49)		(18.89)	(13.79)
log_ME2			0.28		0.24	0.12			0.06		0.01	0.01			0.04		0.03	0.02
			(49.67)		(36.22)	(27.22)			(16.08)		(2.75)	(3.04)			(14.41)		(12.50)	(8.80)
IVOL				0.45	0.20	0.10				0.53	0.50	0.46				0.00	0.00	0.00
				(17.58)	(9.93)	(6.52)				(32.00)	(27.78)	(23.65)				(2,216.29)	(201.56)	(178.13)
R-squared	25.39	17.47	28.83	13.84	29.85	49.34	6.98	6.73	8.01	29.56	30.15	30.97	12.47	11.51	12.59	0.36	12.19	17.73

Based on table 5, all the determinants that were included for determining transaction cost are highly significant. Furthermore, lagged transaction cost seems to be positively correlated with transaction cost. This while market equity is negatively correlated with transaction cost. Similar to lagged transaction cost, is idiosyncratic volatility, which seems to be positively correlated with transaction cost as well. Figure 10 presents the transaction cost estimates by sorting stocks on market equity and idiosyncratic volatility. The sorting process is done by means of the shortest Euclidean distance in rank space between companies i and j which is depicted as follows:

$$\sqrt{(rankMEi - rankMEj)^2 + (rankIVOLi - rankIVOLj)^2}$$

Figure 10: Estimated transaction cost

This figure presents the difference between the direct estimated transaction cost and the closest match. The transaction cost that were directly estimated / matched for this figure is based on the dual strength momentum strategy. This strategy comes with a formation period of 12 month, which includes a lag period of 1 month, followed by a holding period of 1 month. The vertical axis highlights the average transaction cost per month, expressed as a fraction. The horizontal axis highlights the respective dates in which the transaction cost occur.



Apart from a few deviations, the closest match transaction cost comes fairly close with its direct estimates. Moreover, the average transaction cost seems to be approximately two percent per month. What also can be observed from figure 10 is that transaction cost seems to be higher during times when markets are down. Two dates within figure 10 that refer to this market condition are the periods 2001-2002 and 2008-2009, which represents the dot-com and sub-prime mortgage crises, respectively. Overall the Euclidean distance sorting process does solid work in estimating transaction cost that otherwise would not have been possible via direct estimates. Table 6 presents the dual strength momentum strategy by means of various formation (J) and holding (K) periods (J-K), including transaction cost. Do note that only 12-K has K = 1. This was done for the sake of comparing dual strength one with dual strength two. The other Js come with K = 3, 6, 9, and 12.

Commencing with 12-K. The strategy produces significant net average excess return and alpha at a 10% significance level at the minimum, except for K=1 and K=12. The 3-K strategy produces comparable results with only K=12 being insignificant. The opposite is true for 9-K. This strategy produces insignificant net average excess return or alpha for that matter. The 6-K strategy produces comparable results with only K=3 being significant at a 1% significance level. In addition, (net) average excess return and alpha all seem to be decreasing as the holding period increases. The same holds true for their associated statistical significance, with the exception of 12-1, which was already insignificant to begin with. Furthermore, the statistical significance also seems to be decreasing as transaction cost are included. Only 12-K and 3-K appear to be robust for the most part. The same holds true for J-3.

Table 6: Monthly performances for dual with and without transaction cost

This table presents the monthly characteristics for the dual strength momentum strategy. Average return, average excess return, standard deviation, alpha, share turnover, and transaction cost are reported in percentages. Both the average excess returns and alphas are regressed on the Fama French–Carhart model, which include a market risk factor, size factor, value factor and momentum factor. "P1" represents the losers' portfolio, while "P10" represents the winners' portfolio. The respective momentum strategy is constructed by the winners' portfolio *minus* the losers' portfolio. The momentum strategies that are reported in this table come with a formation period of either 3, 6, 9, or 12 months, which includes a lag period of 1 month, followed by a holding period of either 1, 3, 6, 9, or 12 months. Furthermore, the dual strength momentum strategies are reported excluding Treasury bills (if applicable). Robust t-statistics are reported as *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.10$. The sample period is January 1990 to December 2019.

	12-1			12-3			12-6			12-9			12-12		
Dual strength	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0.52	3.41	2.89	1.07	2.63	1.56	1.03	2.15	1.12	1.02	1.79	0.77	1.42	1.60	0.18
$\overline{R}$ - $\overline{R}$f	0.29	3.18***	2.89***	0.84	2.40***	1.56**	0.80*	1.92***	1.12**	0.79**	1.56***	0.77**	1.19***	1.37***	0.18
t($\overline{R}$ - $\overline{R}$f)	(0.29)	(5.26)	(2.86)	(1.30)	(5.87)	(2.55)	(1.73)	(6.27)	(2.51)	(2.08)	(6.23)	(2.16)	(3.47)	(6.12)	(0.56)
SD	15.81	9.44	15.79	10.03	6.34	9.52	7.18	4.74	6.89	5.86	3.85	5.44	5.23	3.43	5.02
SR	0.02	0.34	0.18	0.08	0.38	0.16	0.11	0.41	0.16	0.13	0.41	0.14	0.23	0.4	0.04
α	0.41	2.45***	2.04**	0.60	2.06***	1.45**	0.82*	1.77***	0.96**	0.77**	1.46***	0.69*	1.16***	1.32***	0.16
t(α)	(0.45)	(4.85)	(2.23)	(0.94)	(5.60)	(2.29)	(1.73)	(6.14)	(2.06)	(1.97)	(6.11)	(1.85)	(3.28)	(6.15)	(0.46)
Share turnover			108.49			33.18			21.73			13.57			9.89
Transaction cost			2.20			0.24			0.06			0.03			0.01
$\overline{R}$ - $\overline{R}$f			0.69			1.32**			1.06**			0.74**			0.17
t($\overline{R}$ - $\overline{R}$f)			(0.68)			(2.15)			(2.37)			(2.09)			(0.52)
α			-0.19			1.21*			0.90*			0.67*			0.14
t(α)			(0.21)			(1.90)			(1.93)			(1.78)			(0.42)

	9-3			9-6			9-9			9-12		
Dual strength	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	1.47	3.11	1.65	1.27	2.06	0.78	1.29	1.68	0.40	1.40	1.39	-0.01
$\overline{R}$ - $\overline{R}$f	1.24**	2.89***	1.65**	1.05**	1.83***	0.78*	1.06***	1.46***	0.4	1.17***	1.16***	-0.01
t($\overline{R}$ - $\overline{R}$f)	(2.07)	(6.05)	(2.47)	(2.31)	(5.93)	(1.84)	(2.68)	(5.24)	(1.05)	(3.20)	(4.96)	(0.03)
SD	9.11	7.23	10.09	6.82	4.65	6.40	5.91	4.15	5.65	5.47	3.5	5.18
SR	0.14	0.40	0.16	0.15	0.39	0.12	0.18	0.35	0.07	0.21	0.33	0.00
α	1.2**	2.56***	1.36**	1.17***	1.71***	0.54	1.13***	1.40***	0.26	1.21***	1.12***	-0.09
t(α)	(2.03)	(5.71)	(2.03)	(2.64)	(5.89)	(1.24)	(2.97)	(5.23)	(0.68)	(3.40)	(5.00)	(0.27)
Share turnover			35.33			21.20			13.26			9.65
Transaction cost			0.32			0.08			0.04			0.02
$\overline{R}$ - $\overline{R}$f			1.33**			0.70*			0.36			-0.03
t($\overline{R}$ - $\overline{R}$f)			(1.99)			(1.66)			(0.95)			(0.08)
α			1.03			0.46			0.23			-0.12
t(α)			(1.55)			(1.06)			(0.59)			(0.33)

	6-3			6-6			6-9			6-12		
Dual strength	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0.43	3.04	2.60	0.61	1.65	1.05	0.79	1.27	0.48	1.11	1.10	-0.01
$\overline{R}$ - $\overline{R}$f	0.21	2.81***	2,60***	0.38	1.43***	1,05**	0,56*	1.04***	0.48	0,88***	0,87***	-0.01
t($\overline{R}$ - $\overline{R}$f)	(0.39)	(6.06)	(4.13)	(0.95)	(5.24)	(2.57)	(1.76)	(4.54)	(1.50)	(3.19)	(4.01)	(0.05)
SD	8.07	7.00	9.51	6.03	4.07	6.07	4.68	3.41	4.78	4.09	3.20	4.29
SR	0.03	0.40	0.27	0.06	0.35	0.17	0.12	0.30	0.10	0.22	0.27	0.00
α	0.26	2.64***	2,38***	0.57	1,38***	0,80*	0,73**	1.00***	0.27	1.04***	0,85***	-0.19
t(α)	(0.46)	(5.70)	(3.45)	(1.33)	(4.94)	(1.75)	(2.01)	(4.23)	(0.70)	(3.28)	(3.70)	(0.57)
Share turnover			36.24			21.74			13.60			9.90
Transaction cost			0.48			0.12			0.05			0.03
$\overline{R}$ - $\overline{R}$f			2,12***			0,93**			0.43			-0.04
t($\overline{R}$ - $\overline{R}$f)			(3.37)			(2.28)			(1.34)			(0.15)
α			1,89***			0.68			0.21			-0.22
t(α)			(2.75)			(1.49)			(0.56)			(0.65)

	3-3			3-6			3-9			3-12		
Dual strength	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0.57	3.45	2.88	0.44	1.97	1.52	0.46	1.73	1.27	0.96	1.37	0.41
$\overline{R}$ - $\overline{R}$f	0.34	3.22***	2,88***	0,22	1.74***	1,52***	0,23	1.50***	1.27***	0,73**	1.14***	0.41
t($\overline{R}$ - $\overline{R}$f)	(0.56)	(5.13)	(3.55)	(0.50)	(4.52)	(3.07)	(0.71)	(4.77)	(3.25)	(2.21)	(4.10)	(1.12)
SD	8.44	8.63	9.72	6.08	5.40	6.96	4.74	4.55	5.64	4.74	4.02	5.36
SR	0.04	0.37	0.30	0.04	0.32	0.22	0.05	0.33	0.23	0.15	0.28	0.08
α	0,04	3.04***	2,38***	0,43	1,81***	1.38**	0,25	1.50***	1.25***	0.74**	1.10***	0.36
t(α)	(0.07)	(5.08)	(3.31)	(1.01)	(4.70)	(2.55)	(0.73)	(4.65)	(2.98)	(2.22)	(3.85)	(0.91)
Share turnover			35.66			21.39			13.36			9.73
Transaction cost			1.00			0.25			0.11			0.06
$\overline{R}$ - $\overline{R}$f			1.88**			1.27***			1.16***			0.35
t($\overline{R}$ - $\overline{R}$f)			(2.06)			(2.57)			(2.97)			(0.95)
α			1.37*			1.13**			1.14***			0.30
t(α)			(1.91)			(2.09)			(2.72)			(0.75)

Portfolio performances: Cyclical versus Defensive versus Sensitive

Figure 11 presents the cumulative return of the dual strength momentum strategy based on different sector structures. The cyclical sector structure comprises sectors that are highly sensitive to business cycles, while the defensive sector structure comprises sectors that are anti-cyclical. Finally is the sensitive sector structure, which comprises sectors that have moderate correlation with business cycles. Furthermore, the S&P500 index has been included, which serves as a market benchmark.

Figure 11: Cumulative return based on different sector structures

This figure presents the gross performances of the dual strength momentum strategy based on different sector structures. In addition, the S&P500 index has been included which serves as a market benchmark. Furthermore, the vertical axis highlights the cumulative returns, expressed as a fraction. The horizontal axis highlights the respective dates. The sample period is January 1990 to December 2019.

Figure 11 shows that companies from different sector structures behave differently relative to the market. For instance, companies that are focused in the “Basic Materials”, “Real Estate”, “Consumer Cyclical”, and “Financial Services” sectors seem to be sensitive; meaning, they are moderately correlated with business cycles. Moreover, companies that are focused in the “Technology”, “Communication Services”, “Energy”, and “Industrials” sectors seem to be cyclical; meaning, they are highly sensitive to business cycle peaks and troughs. Finally, companies that are focused in the “Health Care”, “Consumer Defensive”, and “Utilities” sectors seem to be anti-cyclical; meaning, they thrive when markets are down, and vice versa.

6 Empirical Analysis

This chapter will focus on the analysis which are based on the produced results. The following sub-headers will provide the specifics of the analysis, and its relevance towards the end result.

Analyzing the Kolmogorov-Smirnov Distance measure

Figures 1 and 2 indicate that it may not always be feasible to implement the dual strength momentum strategy as the strategy is not based on well balanced portfolios. Jegadeesh & Titman (1993) state that for any momentum strategy to work, it is important to have a sufficient number of winners' portfolios such that it can be offset by a (roughly) equal number of losers' portfolios. Otherwise, Treasury bills are bought instead. Moreover, Gulen & Petkova (2018) motivate that momentum patterns in stock returns may not continue as they should during periods in which its ranking distribution of cumulative returns deviates significantly from its long-run behavior (p. 16). Referring to portfolio 1[5] as an example. Daniel & Moskowitz (2016) motivate that during these periods, investors acknowledge that losers' stocks are undervalued relative to their long-run values, while winners' stocks are overvalued relative to their long-run values. According to the authors (2016), this results in momentum crashes as suddenly there is a strong demand relative to supply for past losers' stocks and a weak demand relative to supply for past winners' stocks. During these periods, again Treasury bills are bought instead. Overall the KS-D application performs optimally during times of low and mid-market volatility as can be observed with portfolios 1[6] and 3[7]. However, the KS-D application does not seem to perform optimally during times of high market volatility as can be observed with portfolios 2[8] and 4[9]. These portfolios stem from the sub-prime mortgage and dot-com crises, respectively. Furthermore, portfolios 2 and 4 were allocated KS-D quintiles 1 and 3, respectively, and not 5 as they should have been. This is due to the application allocating quintiles based on all the months up to month t. This means that the application is relative as the produced results show that prior deviations in cumulative return between the historical and ranking distributions have ran up to 5100%.

[5] See figure 1
[6] See figure 1
[7] See figure 3
[8] See figure 2
[9] See figure 5

Furthermore, the analysis on portfolio 2[10] is consistent with that of De Bondt & Thaler (1985) and Daniel & Moskowitz (2016), who both motivate of a market overreaction. The authors (1985) (2016) find that as markets overreact, it amplifies the effect of a reversal shift, which results in a strong market rebound where there is a sudden overflow of demand (supply) relative to supply (demand) for a particular stock. Additionally, the analysis on portfolio 4[11] is consistent with that of Lim, Brooks & Kim (2008) and Tsai (2015), who both state of a post financial crisis shock. The authors (2008) (2015) find that right after the peak of such a chaotic financial environment, investors are sensitive to, not only local news, but also to that of other markets. The analysis shows that investors are more sensitive towards stock price changes relative to fundamental value changes. This is consistent with that of Barberis et al (2018), who state of stock price changes as fuelling a bubble, whereas fundamental value changes as sustaining a bubble. The analysis overall is consistent with that of Gulen & Petkova (2018), who state that a momentum crash is larger when the stocks' ranking value deviates significantly from its historical value.

Analyzing the (gross) portfolio performances

Figure 8 shows that absolute and dual strength remain relatively stable throughout time. The performance similarities between the absolute and dual strength seem to break away from the trend however as the sub-prime mortgage crisis came along. The relative component drags the dual strength momentum strategy down during market crashes. This can also be observed with the relative strength, which had already performed poorly relative to the other momentum strategies before the sub-prime mortgage crisis, and it is now performing worse, even relative to the S&P500, after the sub-prime mortgage crisis. These poor performances as set by the relative strength momentum strategy should be interpreted carefully however as the strategy failed to produce significant average excess returns or alpha for that matter. Dual and absolute strength on the other hand, did produce significant positive average excess returns and alphas. Overall the relative strength momentum strategy is more prone to momentum crashes compared to the absolute strength momentum strategy. According to Antonacci (2016), this is

[10] See figure 2
[11] See figure 5

because with absolute strength momentum, stocks are compared on a time-series basis and will therefore not move as significantly as compared to relative strength momentum, which compares stocks on a cross-sectional basis. Cooper et al (2005) and Antonacci (2016) further state that as stocks are compared on a cross-sectional basis, it makes them highly regime dependent, with each other and with the market. Overall the analysis is consistent with that of Gulen & Petkova (2018) and Daniel & Moskowitz (2016), who both state that momentum crashes are more severe with the relative strength momentum strategy compared to the absolute strength momentum strategy.

Switching the focus towards dual strength one and dual strength two. Both strategies came with a formation period of 12 months, which includes a lag period of 1 month, followed by a holding period of 1 month. Figure 9 shows that dual strength two excels beyond dual strength one when markets are thriving, but also suffers from severe losses compared to dual strength one when markets are down. According to Antonacci (2016), dual strength two is mainly based on the relative strength component, with the absolute strength component serving as a side. The author (2016) explains this concept via a two-stage selection process. During the first stage, the relative strength momentum strategy is constructed. During the second stage, Antonacci (2016) assesses whether these relative winning (losing) assets also show positive (negative) momentum towards their own past performances. Meaning, whether these relative winning (losing) assets also show positive (negative) absolute momentum. Put differently, the selection process for the second stage consist of relative winners and losers only. Not only does this make dual strength two limited in its choices, but it still results in extreme positive (negative) returns during up (down) markets.

The opposite is true for dual strength one. Dual strength one is mainly based on the absolute strength component, with the relative strength component serving as a side. The absolute strength component makes dual strength one thus more stable relative to dual strength two, which in turn, also results in less extreme positive (negative) returns during up (down) markets. And even though dual strength two is more diversified in terms of asset classes, this type of diversification does not seem to hold too well during down markets as can be observed from max drawdown, which is lower for dual strength two relative to dual strength one. Mishkin &

White (2002) state that stocks and bonds are only well diversified during down markets if the economy is either predicted to be weaker or faces greater uncertainty. Anything else results in both stocks and bonds to drop in value. Both the dot-com and sub-prime mortgage crises got worsened, not necessarily due to either of the aforementioned reasoning, but mainly due the economy facing tighter monetary policies. In addition, Eom, Park, Kim & Kaizoji (2015) find that portfolio diversification works better when it contains mean variance efficient stocks only. This idea however is clearly different from that of Jegadeesh & Titman (1993), which is buying past winners' stocks and selling past losers' stocks. The role of Treasury bills in the dual strength momentum strategy is therefore not so much for diversification purposes. Rather it serves as a compensation of what otherwise would have been opportunity cost for not investing. Table 4 shows that incorporating the KS-D measure, combined with several key ratios related to a stock's fundamental value, makes dual strength one, relative to dual strength two, less prone to momentum crashes, while boosting higher investment activity. For further optimizing the identification of momentum crashes, Yang & Zhang (2019) already expanded on this topic by classifying stock characteristics during times of low, mid, and high market volatility.

Analyzing the (net) portfolio performances

Table 5 shows that lagged transaction cost and idiosyncratic volatility result in higher transaction cost, while market capitalization results in lower transaction cost. The analysis is consistent with that of Novy-Marx & Velikov (2014) and Hasbrouck (2009). The authors (2014) (2009) motivate that as a company is valued more, its stock is said to be more liquid, and therefore easier to trade. Bozhkov, Lee, Sivarajah, Despoudi & Nandy (2018) motivate similarly for idiosyncratic volatility. As a stock carries more idiosyncratic risk, it is considered less popular to trade, thus less liquid, and it therefore incurs higher transaction cost. Furthermore, based on figure 10, it seems that market crashes are positively correlated with transaction cost. This is due to traders not revealing private information upon trading. And when they do, they do so in a large scaled fashion, causing short-term high frequency trading. The overall analysis is consistent with that of Lee (1998), who motivates that as private information becomes public, it results in failed information aggregation, which upon revealing may cause the stock market to overreact, yielding a high volatility in trading.

Shifting the focus to the dual strength momentum strategy (including transaction cost). Table 6 shows that the (net) average excess return and alpha decrease as the holding period increases. This is due to the market reversing in the long-term with past winners becoming losers, and vice versa. This analysis is consistent with that of De Bondt & Thaler (1985) and Jegadeesh & Titman (2001), who both motivate that investors at some point recognize that winning (losing) stocks are overbought (oversold), which results in these stocks becoming overvalued (undervalued). At this stage, investors buy the undervalued stocks and sell the overvalued stocks. Furthermore, it seems that the shorter the formation and holding period, the better the strategy will perform as portfolio rebalancing is more frequent. This holds true not only for gross performances, as already stated by Assogbavi & Leonard (2008), but also for net performances. This analysis should be carefully interpreted however as each research paper might use a different methodology for calculating transaction cost. Take for instance Carhart (1997), who find that transaction cost could wipe out most of the portfolio return due to high rebalancing frequencies. Furthermore, for all Js-Ks, it holds that the gross average excess return, alpha and its significance power decrease as transaction cost are included. This analysis is consistent with that of Novy-Marx & Velikov (2014), who state of a decreasing profitability and its associated statistical significance as a result of including transaction cost.

Analyzing the sector structures

Figure 11 shows that the market has changed. Companies that are focused in the "Basic Materials", "Real Estate", "Consumer Cyclical", and "Financial Services" industries, which is marked by Morningstar[12] as cyclical seem to be rather sensitive; meaning, they are moderately correlated with business cycles. Similarly, companies that are focused in the "Technology", "Communication Services", "Energy", and "Industrials" industries, which is marked by Morningstar as sensitive seem to be rather cyclical; meaning, they are highly sensitive to business cycle peaks and troughs. Companies that are focused in the "Health Care", "Consumer Defensive", and "Utilities" industries, which is marked by Morningstar as anti-cyclical seem to remain consistent; meaning, they thrive when markets are down, and vice versa.

[12] https://www.morningstar.com/content/dam/marketing/apac/au/pdfs/Legal/StockSectorStructure_Factsheet.pdf?

7 Robustness testing

For robustness testing, this research paper has already compared the dual strength momentum strategy by means of different formation (J) and holding (K) periods (J-K), including transaction cost. The results overall seem to be robust as average gross excess return and alpha remain significant when including transaction cost, except for the 12-1 setup. For the following section, this research paper presents two more variations for robustness testing.

Different sample

The original sample covers the period from January 1990 to December 2019. The sample used for this robustness testing covers the period from January 1974 to December 2011. The reason for this particular time period is because Antonacci (2016) used this time period for the original dual strength momentum strategy. Table 7 presents the results.

Sample split

The original sample covers the period from January 1990 to December 2019. For this robustness testing, this research paper split the original sample into two sub-samples. The first sub-sample covers the period from January 1990 to December 2004. The second sub-sample covers the period from January 2005 to December 2019. Table 8 presents the results.

Table 7: Robustness testing #1

This table presents the monthly characteristics for the dual strength momentum strategy. Average return, average excess return, standard deviation, alpha, share turnover, and transaction cost are reported in percentages. Both the average excess returns and alphas are regressed on the Fama French–Carhart model, which include a market risk factor, size factor, value factor and momentum factor. “P1” represents the losers’ portfolio, while “P10” represents the winners’ portfolio. The respective momentum strategy is constructed by the winners’ portfolio *minus* the losers’ portfolio. The momentum strategies that are reported in this table come with a formation period of 12 months, which includes a lag period of 1 month, followed by a holding period of either 1, 3, 6, 9, or 12 months. Furthermore, the dual strength momentum strategies are reported excluding Treasury bills (if applicable). Robust t-statistics are reported as *** p < 0.01, ** p < 0.05, and * p < 0.10. The sample period is January 1974 to December 2011.

	12-1			12-3			12-6			12-9			12-12		
Dual strength	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0.01	5.27	5.25	1.04	3.81	2.77	1.21	2.69	1.48	1.22	2.43	1.21	1.58	2.23	0.65
$\overline{R}$ - $\overline{R}$f	-0.43	4.82***	5.25***	0.60	3.37***	2.77***	0.77**	2.25***	1.48***	0.78**	1.99***	1.21***	1.14***	1.79***	0.65**
t($\overline{R}$ - $\overline{R}$f)	(0.52)	(8.11)	(6.21)	(1.18)	(8.59)	(5.48)	(1.97)	(7.90)	(3.94)	(2.30)	(8.46)	(3.91)	(3.69)	(8.25)	(2.09)
SD	14.11	10.22	14.53	8.68	6.69	8.62	6.68	4.87	6.42	5.81	4.03	5.30	5.29	3.70	5.27
SR	-0.03	0.47	0.36	0.07	0.50	0.32	0.12	0.46	0.23	0.13	0.49	0.23	0.22	0.48	0.12
α	-0.14	3.64***	3.78***	0.46	2.95***	2.49***	0.93**	2.08***	1.16***	0.84**	1.89***	1.06***	1.22***	1.75***	0.53
t(α)	(0.16)	(7.25)	(4.31)	(0.88)	(7.94)	(4.47)	(2.27)	(7.49)	(2.73)	(2.37)	(7.66)	(2.93)	(3.71)	(7.51)	(1.50)
Share turnover			110.00			36.54			21.93			13.70			9.97
Transaction cost			2.05			0.23			0.06			0.03			0.01
$\overline{R}$ - $\overline{R}$f			3.20***			2.54***			1.42***			1.18***			0.64**
t($\overline{R}$ - $\overline{R}$f)			(3.77)			(5.03)			(3.79)			(3.89)			(2.04)
α			1.66*			2.25***			1.10***			1.03***			0.52
t(α)			(1.88)			(4.05)			(2.59)			(2.86)			(1.46)

The results overall seem to be robust as average gross excess return and alpha remain significant when including transaction cost. A slight difference relative to the original sample is that the 12-1 setup remains significant, even with transaction cost included, albeit its significance power drops greatly. Furthermore, the average excess return, alpha and their respective significance power all seem to be decreasing as the holding period increases, which is again consistent with the original sample.

Table 8: Robustness testing #2

This table presents the monthly characteristics for the dual strength momentum strategy. Average return, average excess return, standard deviation, and alpha are reported in percentages. Both the average excess returns and alphas are regressed on the Fama French–Carhart model, which include a market risk factor, size factor, value factor and momentum factor. “P1” represents the losers’ portfolio, while “P10” represents the winners’ portfolio. The respective momentum strategy is constructed by the winners’ portfolio *minus* the losers’ portfolio. The momentum strategies that are reported in this table come with a formation period of 12 months, which includes a lag period of 1 month, followed by a holding period of 1 month. Furthermore, the dual strength momentum strategies are reported excluding Treasury bills (if applicable), and excluding transaction cost. Robust t-statistics are reported as *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.10$. The first sub-sample covers the period from January 1990 to December 2004. The second sub-sample covers the period from January 2005 to December 2019.

	January 1990 - December 2004			January 2005 - December 2019		
Dual strength	**P1**	**P10**	**P10 - P1**	**P1**	**P10**	**P10 - P1**
$\overline{R}$	0.42	3.99	3.58	0.60	2.95	2.35
$\overline{R}$ - $\overline{R}$f	0.05	3.63***	3.58***	0.48	2.83***	2.35
t($\overline{R}$ - $\overline{R}$f)	(0.03)	(4.07)	(2.57)	(0.36)	(3.44)	(1.63)
SD	16.32	9.22	14.42	15.46	9.62	16.82
SR	0.00	0.39	0.25	0.03	0.29	0.14
α	1.21	3.06***	1.86	-0.11	2.04***	2.15*
t(α)	(0.83)	(4.77)	(1.32)	(0.10)	(2.88)	(1.70)

The results overall seem to be less robust as average gross excess return and alpha either do not remain significant or become weakly significant as the original sample is split into sub-samples.

8 Conclusion

This research paper tried to answer the following research question:

"How does the enhanced dual strength momentum strategy perform in the US stock market?"

Hypothesis 1

To answer this research question, this research paper assessed the performance of the enhanced dual strength momentum strategy (dual strength one) with the original dual strength momentum strategy (dual strength two) serving as a bench mark. This research paper did so by means of the following hypothesis:

"The enhanced dual strength momentum strategy is more robust towards momentum crashes relative to the original dual strength momentum strategy"

Dual strength one is more robust towards momentum crashes relative to dual strength two. Dual strength one does this by means of the Kolmogorov-Smirnov Distance (KS-D) measure, which measures the distance between the ranking distributions of cumulative stock returns relative to its historical distributions of cumulative stock returns. A more significant distance indicates a larger momentum crash being on the way. Furthermore, the KS-D measure provides insights on the probability of implementing a successful momentum strategy as it measures the number of winners' stocks which is required to hedge against the number of losers' stocks, and vice versa. In addition to the KS-D measure, dual strength one incorporates the price-to-book (PTB), return-on-asset (ROA), and return-on-equity (ROE) ratios. These ratios serve as additional metrics for assessing whether an increase (decrease) of demand relative to supply for a particular stock is due to extrapolation or due to the stock receiving good (bad) news over the ranking period. Furthermore, an increase (decrease) of PTB results in a stronger (weaker) demand relative to supply for a particular stock compared to an increase (decrease) of ROA and ROE. The effect of an increasing (decreasing) PTB is said to fuel up the momentum, whereas with ROA and ROE, momentum is more sustainable. Overall dual strength one suffers less from momentum crashes when markets are down as the max drawdown for dual strength one is higher relative to that of dual strength two.

Hypothesis 2

Another way this research paper tried to answer this research question was to assess the profitability of the enhanced dual strength momentum strategy (dual strength one), including transaction cost. This research paper did so by means of the following hypothesis:

"The enhanced dual strength momentum strategy is a profitable trading strategy, including transaction cost".

Dual strength one came with various formation and holding periods, with each combination resulting in different profitability. Overall the shorter the formation and holding period, the higher the profitability, with the 6-month formation and 3-month holding period achieving the highest and most significant alpha.

Hypothesis 3

A third way this research paper tried to answer this research question was to assess whether the performance of the enhanced dual strength momentum strategy (dual strength one) would differ based on different sector structures. This research paper did so by means of the following hypothesis:

"The enhanced dual momentum strategy performs differently based on different sector structures".

Dual strength one was constructed based on companies that are either marked as cyclical, anti-cyclical, or sensitive. Companies that are focused in the "Basic Materials", "Real Estate", "Consumer Cyclical", and "Financial Services" industries are sensitive; meaning, they are moderately correlated with business cycles. Similarly, companies that are focused in the "Technology", "Communication Services", "Energy", and "Industrials" industries are cyclical; meaning, they are highly sensitive to business cycle peaks and troughs. Companies that are focused in the "Health Care", "Consumer Defensive", and "Utilities" industries are anti-cyclical; meaning, they thrive when markets are down, and vice versa.

9 Recommendation

For those who wish to construct a momentum strategy with the Kolmogorov-Smirnov Distance (KS-D) measure included, this research paper recommends to further optimize the KS-D application. Throughout this research, this research paper has taken a single sample for its main empirical results and analysis. This resulted in the KS-D measure forecasting less-than-perfect results. Therefore, future research papers should split the sample into sub-samples based on particular market characteristics. One possibility would be to split the sample in small-, mid-, and large-cap stocks. Another possibility would be to split the sample in value and growth stocks. A final possibility would be to split the sample in low, mid, and high volatility stocks.

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Appendix A: Determinants of transaction cost

This appendix presents the regression outputs of transaction cost estimates on lagged transaction cost, market equity, and idiosyncratic volatility. Transaction costs that could be directly estimated by means of the (posted) effective bid-ask spread equals one-half the (posted) effective bid-ask spread as suggested by Roll (1984). Furthermore, “lag_T_cost”, “ME” and “IVOL” stand for lagged transaction cost, market equity and idiosyncratic volatility, respectively. Transaction cost were lagged by 21 days. Market equity was measured as the stock price multiplied by the number of shares outstanding. Idiosyncratic volatility was measured as the standard deviation of residuals of past three months’ daily returns on the daily excess market return. Moreover, both “ME1” and “ME2” are log converted in order to account for skewness towards larger values. Finally, the difference between “ME1” and “ME2” is that “ME1” is non-squared whereas “ME2” is squared, as done by Novy-Marx & Velikov (2014). Both market equity and idiosyncratic volatility use end of July values. R-squared is reported in percentage. Robust t-statistics are reported in parenthesis. Furthermore, the estimated regressions are on an annual frequency and cover sub-periods ranging from January 1970 to December 2019.

	January 1970 - December 1979						**January 1980 - December 1989**					
lag_T_cost	0.82					0.89	0.76					0.76
	(80.47)					(121.66)	(87.85)					(91.46)
log_ME1		-1.37	-4.49		-4.52	-0.63		-1.59	-5.63		-5.59	-1.35
		(83.06)	(64.76)		(63.64)	(19.68)		(94.43)	(84.27)		(77.44)	(27.08)
log_ME2			0.43		0.43	0.06			0.50		0.50	0.12
			(52.44)		(51.38)	(19.17)			(71.27)		(67.13)	(25.92)
IVOL				0.50	0.02	-0.03				0.56	0.16	0.06
				(22.47)	(1.17)	(2.52)				(28.26)	(8.74)	(4.00)
R-squared	74.14	28.36	43.75	3.70	44.24	87.52	62.44	26.36	43.13	4.53	43.41	75.76

	January 1990 - December 1999						**January 2000 - December 2009**						**January 2010 - December 2019**					
lag_T_cost	0.44					0.53	0.21					0.09	0.38					0.28
	(35.98)					(37.26)	(17.06)					(7.68)	(26.23)					(16.77)
log_ME1		-0.97	-3.65		-3.08	-1.53		-0.36	-0.99		-0.22	-0.22		-0.32	-0.78		-0.71	-0.47
		(74.09)	(57.55)		(38.52)	(29.16)		(47.80)	(21.75)		(4.79)	(4.86)		(56.06)	(21.49)		(18.89)	(13.79)
log_ME2			0.28		0.24	0.12			0.06		0.01	0.01			0.04		0.03	0.02
			(49.67)		(36.22)	(27.22)			(16.08)		(2.75)	(3.04)			(14.41)		(12.50)	(8.80)
IVOL				0.45	0.20	0.10				0.53	0.50	0.46				0.00	0.00	0.00
				(17.58)	(9.93)	(6.52)				(32.00)	(27.78)	(23.65)				(2,216.29)	(201.56)	(178.13)
R-squared	25.39	17.47	28.83	13.84	29.85	49.34	6.98	6.73	8.01	29.56	30.15	30.97	12.47	11.51	12.59	0.36	12.19	17.73