Master Thesis Economics and Business- Economics of Markets and Organizations

The cost of anonymity: Data driven price discrimination with repeated purchases

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Abstract:

This thesis examines how a monopolist acts in a market in which it can identify consumers and price discriminate them based on personal data from consumers who previously bought his product due to a identifying technology. To protect themselves from these consumers can anonymize their data at a cost through hiding technology. The ability of a firm to price discriminate predicts an increase in producer surplus and a decrease in consumer surplus, whereas the possibility of anonymizing their data is expected to partly restore the lost consumer surplus. I find that it is always beneficial for the monopolist to utilize the identifying technology, as this increases his profits. However, I find that the hiding technology is harmful for consumers, as individual consumers who use this technology exert an externality on unidentified consumers in the market. Due to this externality consumers are collectively better off if they decide not to anonymize their data. Finally I find that the inflow of new consumers in the second period exerts a dampening effect on the externality.

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1. Introduction

In a recent hearing in the U.S. Senate the leaders of Amazon, Apple, Facebook and Google were confronted for using their market power to abuse their dominant position in order to crush competitors and receive huge profits, as a result of the data they hoard from their consumers. Few can dispute that such companies nowadays have more big data at their disposal than ever. Tech giants like Amazon, Apple, Facebook, Google and Microsoft amass a lot of data from their customers, and for most of these companies it constitutes a large portion of their business model, as such firms have found many ways of using big data, like targeting advertisements, product improvements or price discrimination. Recent technological developments have enabled companies to do so with increased precision. However, similar technological developments have enabled consumers to negate such practices by firms. Tools like Addblock specifically prevent firms from targeting adds to consumers, and by using a Virtual Private Network (VPN) consumers can encrypt their personal data, which disables firms from linking the data back to them.

When specifically looking at price discrimination, the ability of a firm to price discriminate with increased precision predicts a decrease in consumer surplus and increase in profits. In a previous study it was found that when a monopolist can identify a consumer her valuation from their data, it always has an incentive to use such technology (Belleflamme and Vergote, 2016). However, the same technology which allows firms to harness and analyze big data, allows consumers to protect their privacy. This option is expected to restore at least a part of that lost surplus. Yet, Belleflamme and Vergote (2016) find that the usage of privacy-protecting technologies in a setting where a monopolist can price discriminate may hurt consumers even more.

However, this model does not completely fit the story I presented earlier. As stated previously, a lot of data firms have is on existing customers. Yet, in their paper Belleflamme and Vergote simply assume a monopolist has access to the data of all consumers in the market. Therefore I would like to analyze this model in a setting with repeated purchases, where a firm has to collect data from previous sales before it can be used to price discriminate. In such a setting I expect the monopolist to lower his price in the previous period in order to build up his database. If this is the case, this could potentially have some consequences for the results of Belleflamme and Vergote (2016). Firstly, a reduced price in the first period means the firm has to sacrifice profits. It could be that a firm is unwilling to use identifying technology to price discriminate, as increased profits in the exploitation period are not high enough to warrant the sacrifice in profits needed to build up a database. Secondly, if the monopolist drops his price in the first period, consumers gain an increase in surplus for this period. It could be that in a setting with repeated purchases it is beneficial for consumer to use the tracking technology due to this increase.

I find that a monopolist always has an incentive to use the identifying technology when he has the opportunity to do so, and therefore he benefits from this technology. Furthermore, I find that the hiding technology is only beneficial for consumers when they collectively choose to either use or ignore the technology. This happens because individual consumers who use the technology exert an externality on consumers who do not. Furthermore, I find that this externality is dampened if new consumers enter the market in the second period.

The remainder of this paper will be structured as follows. First I will briefly discuss the related literature. Secondly, the model will be presented, after which I will present and analyze the results of the base model. Afterwards I will present an extension of the model and will analyze the changed results this extension causes. Subsequently I will briefly discuss an alternate variant of the identifying technology. Finally, the paper ends with a conclusion.

2. Related Literature

Over the years the ability of firms to price discriminate due to data has been an ongoing research topic. Earlier research includes Taylor (2004), Acquisti and Varian (2005) and Calzori and Pavan (2006). In Taylor (2004) a firm can sell the data from its previous consumers to another firm, which can then price discriminate based on this data. In their model the welfare depends on whether consumers anticipate the sale of the list or not. If enough consumers do not anticipate the sale, the firm will price discriminate. However, if enough consumers do anticipate the sale of the list, the monopolist is better of protecting its consumers privacy, which is also beneficial for the consumers. Aquisti and Varian (2005) find similar results in a two-period model in which firms have tracking technologies and consumers have access to hiding technologies. They find that when a large enough portion of consumers do not protect their privacy firms are willing to condition their pricing on purchase history, but only with enhanced personalized services. Otherwise conditioning their prices does not raise the profits of firms. Complementary to the previously mentioned papers, Villas-Boas (2004) shows how strategic consumers may harm a firm when the firm can dynamically target its prices. The reason is that consumers can strategically wait to purchase today so that they remain unidentified tomorrow, and benefit from the lower uniform prices targeted at new consumers. This behaviour hurts firms both through a reduction in sales and a smaller effect of price discrimination, and can lead to the firm preferring a voluntary privacy-protecting policy. Calzori and Pavan (2006) analyse a setting between two firms who share their data in order to uncover their customers willingness to pay. Interestingly, within their setting data sharing may reduce information distortions and enhance social welfare. These earlier works mainly show that consumer data does not have to be protected as long as consumers are actively aware of the consequences of sharing their data with firms.

More recent research into data driven price discriminaton includes Belleflamme and Vergote (2016), Chen et al. (2020) and Conitzer et al. (2012). Belleflamme and Vergote (2016) examine a situation in which a monopolist can perfectly identify any customer in the market due to an identifying technology, but allows consumers to anonymize their data through hiding technology. They find that a monopolist will always use such technology, but that consumers are collectively better off not using it. This is the case because by anonymizing themselves they exert an externality on other consumers, due to the monopolist raising his uniform price. Chen et al. (2018) study a duopoly were each firm chooses personalized prices for its targeted consumers, who can be active or passive in identity management. They find that when consumers are active , firms choose to not serve the entire market when the non-targeted segment is too small, which can lead to lower social welfare. Furthermore, Conitzer et al. (2012) study price discrimination in a market where a monopolist can distinguish past consumers from new consumers, with past consumers having the option of maintaining their anonymity and avoiding identification. They find that when consumers can costlessly maintain their anonymity they are the worst off. Increasing the cost can benefit consumers, but only up to a certain point. Ichihashi (2020) examines a situation with a multi-product seller and a consumer who can choose how much information about herself she would like to reveal. This information is consequently used for product recommendations. Although the consumer benefits from good recommendations based on her preferences, the firm can also use this information to price discriminate. He shows that the firm has an incentive to commit to not use the information for pricing in order to encourage information disclosure. Yet, this commitment is harmful for the consumer, who would be better of withholding some of her information.

The two closest papers to this paper are Belleflamme and Vergote (2016), as this paper is an extension of their model and Conitzer et al. (2012), as I use a similar repeated purchase setting. Regardless this paper has some distinguishing features. Compared to Belleflamme and Vergote, my model adds data collection through repeated purchases. In addition to this the timing when people choose to hide changes. In Belleflamme and Vergote, consumers can decide whether to use hiding technology or not during their purchase, whereas in my model they have to do this before they know the prices they will pay. Compared to Conitzer et al. (2012), I use a different identification technology. In my model the potential identification chance can range from no to full identification chance of consumers, whereas in Conitzer et al. (2012) the only information the monopolist learns is whether a consumer previously purchased. Therefore their monopolist can only engage in third degree price discrimination, in contrary to the monopolist is this model, who can engage in personal pricing. These things make my model distinct from previous related literature.

3. The model

Each period a monopolist produces a certain product for a total of two periods at a constant marginal cost, which is set to zero for the sake of simplicity. A unit mass of consumers have unit demand for the product the monopolist produces. A consumer her valuation, noted by v, will be drawn at the beginning of the first period and is uniformly distributed between $[0,1]$. A consumer draws this valuation for both periods.

The monopolist can choose to use an identification technology which enables him to potentially identify consumers from who it has personalized data. However, at the beginning of the first period the monopolist has an empty dataset, which he can fill up through sales. Therefore the monopolist can collect personalized data from a consumer if she has previously bought a product at p_1 from the monopolist. Thus, the monopolist builds up his database trough repeated purchases from consumers. Then, through usage of his identification technology the monopolist may be able to derive the valuation from each individual consumer from the personalized data with probability λ (with $0 \leq \lambda \leq 1$). Thus, with probability λ a monopolist learns the valuation of a particular consumer, which enables him to charge her a personalized price $p_2(v) = v$ which fully extracts this consumer's surplus. However, with probability $(1 - \lambda)$ the monopolist is unable to extract anything from this consumer her personalized data and is only able to charge a consumer a uniform price p_2. Therefore λ can be interpreted as the precision of the monopolist his identification technology, which may for instance be a better algorithm. In the literature we also see a common variant of this with imperfect identification, where with probability λ a firm can identify whether a customer has previously bought in the first period, in which case the firm can use third degree price discrimination. I opt to use the perfect identification variant, as this way my model will remain close to the paper of Belleflamme and Vergote (2016), and will make the results better comparable. As in almost all price discrimination models, arbitrage is supposed to be impossible or prohibitively costly.

To counteract the identification technology of the monopolist consumers have access to hiding technology, which prevents the monopolist of learning a consumers valuation by anonymizing their data before making their first purchase. Therefore consumers only now the current price p_1 when making a decision about the hiding technology, in contrary to Belleflamme and Vergote (2016), where they know all of the prices when making this decision. Consumers can access the hiding technology at cost c with the benefit of never having to pay their valuation for the monopolist product, regardless of the precision of the monopolists identification technology. An example of such technology may be a VPN, which encrypts it's users data, making it unable for websites to track and subsequently identify its users.

I analyze the following game. First the monopolist decides whether to use the tracking technology before making any sales. Afterwards the monopolist sets the first period price p_1. Upon observing this price, consumers make their purchasing decisions, and simultaneously decide whether they want to use the hiding technology or not. During the first period the monopolist collects personalized data from all purchases on which it is possible. At the start of the second period the monopolist sets two different prices: A schedule of personalized prices for consumers the monopolist will identify, and a regular price p_2 for consumers the monopolist will be unable to identify. After setting these prices consumers will either be identified or remain anonymous, and will again make their purchasing decisions. The equilibrium concept used will be Perfect Bayesian Equilibrium (PBE), as the monopolist will have beliefs about how consumers will act based on their expectation of p_2. Afterwards I have to check if this belief can be sustained in equilibrium.

4. Equilibria

Just as in Belleflamme and Vergote (2016) I will consider two benchmarks before introducing the hiding technology. The first benchmark is the case were neither the identifying nor the hiding technology is available. The second benchmark will be the case were the monopolist has access to the identifying technology, but consumers do not have access to the hiding technology. Finally I will examine the case were both technologies are available. All derivations can be found in the appendix.

a. No identifying technology

When neither of the technologies are available, we end up in a situation with business as usual. It can then be easily derived that the optimal price in the first period is $p_1 = 1/2$, and the price in the second period is $p_2 = 1/2$. These prices then yields a total profit of $\pi = 1/2$ and a total consumer surplus of $CS = 1/4$.

b. Tracking technology without hiding technology

Suppose the monopolist uses the identifying technology. In that case consumers must pay the uniform price p_2 in the second period if unidentified, and their valuation v otherwise. Consumers will buy if $p_2 \leq v$, thus if identified, a consumer will always buy, as they then pay $p(v) = v$. Therefore the optimization problem for the second period will be:

$$\max_{p_2} \lambda \int_{p_1}^{1} v dv + (1 - \lambda) \int_{p_2}^{1} p_2 dv$$

Through redeveloping this can be simplified as:

$$\max_{p_2} \lambda \left(\frac{1}{2} - \frac{1}{2} p_1^2\right) + (1 - \lambda)(1 - p_2)p_2$$

After optimization the first order condition shows the monopolist will charge a second period price $p_2 = 1/2$ to consumers he could not identify and will extract their full valuation v upon identification. For the first period the optimization problem for the monopolist is:

$$\max_{p_1}(1 - p_1)p_1 + \lambda \left(\frac{1}{2} - \frac{1}{2} p_1^2\right) + (1 - \lambda)(1 - p_2)p_2$$

The first order condition then shows that the monopolist will charge a price $p_1 = 1/(2 + \lambda)$ in the first period. The intuition behind this result is that when the monopolist lowers his price in the first period, he has more consumers he can potentially identify in the second period. Thus, when consumers are unable to hide, the monopolist is willing to lower his price in the first period in order to build up a database to extract consumers surplus in the second period. This is also the reason why p_1 is smaller than p_2, as the monopolist lowers his price for exploitative purposes. The corresponding total profits are equal to:

$$\pi = \int_{p_1}^{1} p_1 dv + \lambda \int_{p_1}^{1} v dv + (1-\lambda) \int_{p_2}^{1} p_2 dv = \frac{4 + 3\lambda + \lambda^2}{4(2+\lambda)}$$

The corresponding consumer surplus is computed as:

$$CS = \int_{p_1}^{1} (v - p_1) dv + \lambda \times 0 + (1-\lambda) \int_{p_2}^{1} (v - p_2) dv = \frac{1 + \lambda^2}{2(2+\lambda)^2} + \frac{1-\lambda}{8}$$

Similar to Belleflamme and Vergote (2016), I also find that when consumer are unable to hide their identity, the monopolist his profit increases and the consumer surplus decreases when the precision of tracking technology λ increases. I observe that even though the monopolist has to sacrifice some of his profits in period one, his total profits are still increasing in λ. Similarly, even though consumers gain some welfare due to the lower price in the first period, it can be seen that the total consumer surplus is decreasing in λ. Thus, the lower price in the first period does not adequality compensate consumers in comparison to the exploitation they might experience in the second period due to the use of their personalized data.

c. Identifying technology with hiding technology

Now suppose the monopolist uses the identifying technology, with consumers having access to the hiding technology. In equilibrium consumers correctly anticipate that they will pay a price p_2 if they are unidentified in the second period, but have their entire surplus extracted in the case they are identified. Consumers with a valuations $v \geq p_1$and $v \geq p_2$ gain an expected surplus of $v - p_1 + (1 - \lambda)(v - E[p_2])$ if they do not use the hiding technology, whereas consumers gain a surplus of $2v - p_1 - E[p_2] - c$ if they do use the hiding technology. After comparing these two options I can derive a threshold after which some consumers will choose to use the hiding technology, which is:

$$2v - p_1 - E[p_2] - c \geq v - p_1 + (1-\lambda)(v - E[p_2]) \quad => \quad v \geq E[p_2] + \frac{c}{\lambda} \equiv \bar{v}$$

This expression shows that the price setting in the first period is irrelevant in the decision to use the hiding technology. Whether a consumer buys a product in the first period while using the hiding technology, or she buys the product without this technology, either way she pays p_1. Furthermore, the expression shows that there exist customers who hide as long as $\bar{v} < 1$, whom I will refer to as active consumers from this point onwards. Finally, the expression also shows that no consumer will use the tracking technology when $\bar{v} \geq 1$. I will refer to these consumers as passive consumers. This derivation essentially shows there are two types of consumer behavior, depending on their expectation of the second period price p_2. Either consumers are passive and expect $E[p_2] \geq 1 - c/\lambda$, after which no consumer will use the hiding technology, or some consumers are active and expect $E[p_2] < 1 - c/\lambda$, which induces some of them to hide. Logically, active consumers are of higher valuation than passive consumers. This is the case because even though both groups obtain no surplus when identified, active consumers obtain a higher surplus when they can avoid identification than passive consumers.

Consider the first option where all consumers are passive. In this case, obtaining the hiding technology is prohibitively expensive for all of them. As no consumer hides her identity, almost the same analysis as in the second benchmark can be applied. Thus, optimal price setting requires that in the second period the monopolist sets a price $p_2 = 1/2$ and in the first period a price $p_1 = 1/(2+\lambda)$. Where the analysis differs is that this optimal price setting can only be applied if in equilibrium the belief of consumers is *right*. Since in equilibrium expectation is equal to the optimal price, it must be the case that $p_2 \geq 1 - c/\lambda$. By substituting $p_2 = 1/2$ I find that in equilibrium this pricing strategy is only possible when $c \geq \lambda/2$.

Now consider the second option in which some consumers are active. In this case the monopolist can potentially identify any customer who previously bought the product and did not use the hiding technology with probability λ, which are the customers with valuations ranging from p_1 to $\bar{v}$. These consumers are charged their valuation v_2. Consumers who did not use the hiding technology can remain unidentified with probability $(1-\lambda)$ and pay a uniform price p_2 when $p_2 \leq v_2$. These consumers have valuations ranging from p_2 to $\bar{v}$. Finally there are consumer who use the hiding technology with valuations ranging from $\bar{v}$ to 1, and these consumers also pay the uniform price p_2, as they are certain to remain unidentified. Therefore the monopolist's maximization problem for the second period becomes:

$$\max_{p_2} \lambda \int_{p_1}^{\bar{v}} v dv + (1-\lambda) \int_{p_2}^{\bar{v}} p_2 dv + \int_{\bar{v}}^{1} p_2 dv$$

Trough the substitution of $\bar{v}$ and redeveloping this can be simplified as:

$$\max_{p_2} \frac{1}{2}\lambda\left(\left(E[p_2] + \frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1 - \lambda\left(E[p_2] + \frac{c}{\lambda}\right) - (1-\lambda)p_2\right)p_2$$

From the first order condition we can then derive that the optimal price in the second period equals $p_2 = (1-c)/(2-\lambda)$. Within this price are two effects. First, for $\lambda > 0$ the monopolist finds it profitable to charge a higher price to unidentified customers in the second period. This can be explained by one of two reasons provided by Belleflamme and Vergote (2016), which is that the monopolist knows that the customers who hide are of high valuation, therefore it is profitable for him to charge those customers a higher regular price. The other reason which Belleflamme and Vergote explain, that in equilibrium the monopolist raises the threshold $\bar{v}$ which discourages people to hide does not apply in this model, as the monopolist does not have direct control over the threshold within my model. Secondly, it can be seen that the optimal second period price is decreasing as the cost of the hiding technology c increases. An explanation for this is related to the previous point. When consumers hide, the monopolist knows they are of high valuation, and has an incentive to charge a higher price in general. However, when the technology is expensive, less people hide, therefore the monopolist learns of less consumers that they are of high ability, and thus has less incentive to increase his price. This effect continues until $c \geq \lambda/2$, after which the equilibrium examined previously occurs, in which no consumer hides. Furthermore, it might be worthwhile to highlight that an equilibrium in which all relevant consumers (with valuation $v \geq p_1$) use the hiding technology can never exist, due to the consumers with valuations between p_1 and p_2. These consumers will never use the hiding technology, as in the second period they will never buy if they are not identified. Thus, these consumers want to be identified, and will never use the hiding technology. Contrary to the model of Belleflamme and Vergote (2016), there is an effect of c in my model. This is because in Belleflamme and Vergote the monopolist can directly influence consumer behavior through p_2, as the consumer knows the exact value of p_2, and thus the monopolist can negate the value of c. In my model this is not possible, as the consumer must activate the hiding technology before knowing the exact value of p_2. Therefore the monopolist has to account for the value of c when deciding on his price. I find that in equilibrium the belief is only correct if $c < \lambda/2$. In the first period the maximization problem is:

$$\max_{p_1}(1-p_1)p_1 + \frac{1}{2}\lambda\left(\left(E[p_2]+\frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1-\lambda\left(E[p_2]+\frac{c}{\lambda}\right) - (1-\lambda)p_2\right)$$

After optimization I find that the optimal price in the first period when consumers are allowed to hide equals $p_1 = 1/(2+\lambda)$. What is interesting is that even though people now hide their identity, the monopolist is still willing to lower his price to the same degree as when people are unable to hide. There could be several explanations for this. Firstly, the monopolist lowers his price in order to try identify customers with a lower valuation, as high valuation customers are already very likely to purchase the monopolist's product. Secondly, it anticipates that higher valuation consumers will hide, and therefore asks a higher regular price, which still allows the monopolist to keep the first period price low even though high valuation customers hide. With the combination of these prices, the total profits when $c < \lambda/2$ is computed as:

$$\pi = (1-p_1)p_1 + \frac{1}{2}\lambda\left(\left(E[p_2]+\frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1-\lambda\left(E[p_2]+\frac{c}{\lambda}\right) - (1-\lambda)p_2\right)$$

As in equilibrium $E[p_2] = p_2$, through substitution and redeveloping this can be simplified to:

$$\pi = (1-p_1)p_1 + \frac{1}{2}\lambda\left(\left(p_2+\frac{c}{\lambda}\right)^2 - p_1^2\right) + p_2(1-p_2-c) = \frac{2\lambda + c^2(2+\lambda)(1-\lambda)}{\lambda(2+\lambda)(2-\lambda)}$$

Summarizing, these results yields the following equilibria: if $c \geq \lambda/2$, the firm has the belief that $E[p_2] \geq 1 - c/\lambda$, indicating that no consumer will utilize the hiding technology. Consequently the monopolist sets a price $p_1 = 1/(2+\lambda)$ in the first period, and in the second periods he sets a price $p_2 = 1/2$ for unidentified consumers and a price $p(v) = v$ for identified consumers. In the first period consumers will buy when $p_1 \leq v$. When unidentified a consumer will only buy in the second period when $p_2 \leq v$, whereas upon identification a consumer will always buy the monopolist his product. If

$c < \lambda/2$, the monopolist has the belief that $E[p_2] < 1 - c/\lambda$, indicating that some consumers will use the hiding technology. In such cases, the first period price is $p_1 = 1/(2+\lambda)$, the second period price for unidentified consumers is $p_2 = (1-c)/(2-\lambda)$ and identified consumers pay $p(v) = v$ in the second period. The strategy of a consumer remains the same in this equilibrium: She buys in the first period when $p_1 \leq v$, and if she remains anonymous she buys in the second period if $p_2 \leq v$. Upon identification she will always buy in the second period.

Now I can check if the tracking technology is actually profitable for the monopolist. I find that for every value of λ (thus both cases), the monopolist has an incentive to use the identifying technology, as it yields him higher profits. Furthermore, for all values of c the monopolist has also an incentive to use the technology, due to the same reasons as with λ. Therefore the monopolist will always choose to use utilize such technology when available.

5. Effect on consumer welfare

In the previous section I have already established that the consumer surplus is decreasing in λ when hiding technology is unavailable. Now I can check whether the addition of the hiding technology has consequences for the welfare of consumers. In Belleflamme and Vergote's (2016) paper they found that consumers are better off when they refrain from using the hiding technology, as by using such technology they exert a negative externality on other consumers, because the monopolist reacts by raising his uniform price. It will be interesting to explore whether the self-collection of data changes this result.

In order to do so I must first determine the consumer surpluses for the two cases examined previously. First, for $c \geq \lambda/2$ we end up a similar case as examined in the second benchmark, where hiding technology is unavailable. However, in this case the hiding technology is prohibitively expensive instead of unavailable. The consumer surplus equals:

$$CS = \int_{p_1}^{1} (v - p_1)dv + \lambda \times 0 + (1-\lambda)\int_{p_2}^{1} (v - p_2)dv = \frac{1+\lambda^2}{2(2+\lambda)^2} + \frac{1-\lambda}{8}$$

Secondly, for $c < \lambda/2$ we find that some consumers utilize the hiding technology. In this instance the consumer surplus equals:

$$CS = \int_{p_1}^{1} (v - p_1)dv + \lambda \times 0 + (1-\lambda)\int_{p_2}^{v_c(p_2)} (v - p_2)dv + \int_{v_c(p_2)}^{1} (v - p_2 - c)dv$$

$$= \frac{1+\lambda^2}{2(2+\lambda)^2} + \frac{(2-\lambda)^2c^2 - 2\lambda(1-\lambda+c)(2-\lambda)c + \lambda(1-\lambda+c)^2}{2\lambda(2-\lambda)^2}$$

All these consumer surpluses share their relations to λ and c (where available). In both cases consumer welfare is decreasing in the likelihood to be identified λ and increasing in the cost of using the hiding technology c.

From this point the result from Belleflamme and Vergote (2016) can quickly be replicated. Even when the usage of the hiding technology would be costless ($c = 0$), consumers would still be better of if not one of them used the hiding technology. This is the case because even though it may be individually rational to use the hiding technology, collectively consumers suffer because using the technology raises the uniform price. Therefore within this model it is better for consumers to decline them access to such technology, either by making it prohibitively expensive ($c \geq \lambda/2$) or simply blockading such technology from the market. Thus, the self-collection of data does not change the main result obtained from Belleflamme and Vergote (2016).

6. Inflow of new consumers

Even though I have now found my main results, I would like to make an extension to the model. Unlike this model, in real markets there continually is an inflow of new consumers who might be interested in buying the monopolists product. If we add this to the model, this would probably dampen the incentive of the monopolist to raise his price in the second period. This could occur because new consumers enter the market in the second period, they are automatically unidentified, as identification can only occur in the first period. If the monopolist would then raise his uniform price too high, a new consumer consequently is less likely to buy the monopolist his product. For the existing consumers nothing changes: If identified they pay $p(v) = v$ and if unidentified they have to pay p_2.

Therefore I want to add a mass of consumers m in the second period. As stated before, these consumers cannot be identified by the monopolist, and are charged the uniform price p_2. As the regular consumers, these new consumer their valuation is drawn from the same uniform distribution between $[0,1]$. Just like an unidentified consumer a new consumer will buy the monopolist his product when $p_2 \leq v$. Therefore the chance a new consumer buys the product given p_2 is:

$$\Pr(v > p_2) = 1 - \Pr(\mathrm{v} \leq \mathrm{p}_2) = 1 - p_2$$

Since each consumer faces the same decision, the additional profits mass m brings is:

$$\pi_m = m(1 - p_2)p_2$$

Let us check the changes that this additional demand potentially causes for the cases with both technologies. So first, I will check how the mass m influences the results when all consumers act passive, and afterwards I will check how mass m influences the monopolist his price setting when some consumers are active. The cutoff rule derived in the earlier section does not change. So for $\bar{v} \geq 1$ all consumers will be passive, and for $\bar{v} < 1$ there exist some active consumers. Again this yields the following types of consumer behaviour based on the expected price: either consumers are passive and expect $E[p_2] \geq 1 - c/\lambda$, after which no consumer will use the hiding technology, or some consumers are active and expect $E[p_2] < 1 - c/\lambda$, which induces some of them to hide. I find that when all consumers are passive the monopolist does not have an incentive to change his price. This occurs because if the monopolist could differentiate between the new consumers and unidentified old consumers he would both charge them $p_2 = 1/2$. However, the monopolist cannot distinguish such consumers, which is no problem, as he wants to charge them the same price. When some consumers are active, I find that the monopolist has an incentive to slightly lower his price compared to when no new consumers enter the market. Contrary to the case with only passive consumers, when some consumers are active the will charge different prices to unidentified returning consumers and new consumers. The monopolist the charge returning consumers a price $p_2 = (1-c)/(2-\lambda)$ and new consumers a price of $p_2 = 1/2$. However, he cannot distinguish these consumer groups and therefore he will lower his uniform price p_2 such that he sells to more of the new consumers and consequently gain more profits from them.

First, I will discuss the case when all consumers are passive in their identity management. In equilibrium the price must than satisfy the belief that $E[p_2] \geq 1 - c/\lambda$. When all consumers are passive the monopolist his maximization problem in the second period changes the following way:

$$\max_{p_2} \lambda\left(\frac{1}{2} - \frac{1}{2}\,p_1^2\right) + (1-\lambda)(1-p_2)p_2 + m(1-p_2)p_2$$

After optimization I find that the optimal price is still $p_2 = 1/2$, so here I observe no different behaviour by the monopolist compared to the case were no new consumers enter the market. However, this price must satisfy the belief that $E[p_2] \geq 1 - c/\lambda$. Just like the case without new consumers this belief can only be correct when $c \geq \lambda/2$. The first period price also remains the same, as the new consumers have no influence on this price, because they only afterwards exist.

Now I will discuss the case when some consumers are active. In equilibrium this behaviour will only occur when the belief that $E[p_2] < 1 - c/\lambda$ is satisfied. The additional mass of consumers changes the maximization problem of the monopolist in the second period in the following way:

$$\max_{p_2} \frac{1}{2}\lambda\left(\left(E[p_2] + \frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1 - \lambda\left(E[p_2] + \frac{c}{\lambda}\right) - (1-\lambda)p_2\right)p_2 + m(1-p_2)p_2$$

After optimization I find that the additional consumers in the second period do have an effect on the pricing strategy of the monopolist, unlike the case with only passive customers. In equilibrium the monopolist sets a price of $p_2 = (1+m-c)/(2+2m-\lambda)$. As mentioned previously, this price can only hold in equilibrium when $E[p_2] < 1 - c/\lambda$. Similar to the case without new consumers, I find that this belief is satisfied when $c < \lambda/2$. In equilibrium p_2 reacts similarly to λ and c *as* when no new consumers enter the market, thus p_2 is increasing and λ and decreasing in c, for the same reasons as in the previous section. With respect to the new mass of consumers m, I find that p_2 decreases as a larger mass of consumers enter the market. This can be seen by deriving p_2 with respect to m:

$$\frac{dp_2}{dm} = \frac{c-\lambda}{(2+2m-\lambda)^2}$$

By examining the derivative it can be quickly seen that p_2 is decreasing in m when $c < \lambda$. Since this equilibrium can only exist when $c < \lambda/2$, within this equilibrium p_2 is always decreasing in m. Thus the additional mass of consumers contains a dampening effect on p_2. An explanation for this is quite simple. The monopolist has no incentive to raise his price too high when new consumers enter the market, as he loses out on potential sales. Thus, by lowering his second period price the monopolist can increase his sales and consequently his profits.

Summarizing, when a mass of consumers enter the market in the second period the following equilibria can occur: if $c \geq \lambda/2$ the monopolist has the belief $E[p_2] \geq 1 - c/\lambda$, which indicates that no consumers uses the hiding technology. Under these circumstances the monopolist will set a first period price of $p_1 = 1/(2+\lambda)$, a second period price for unidentified consumers of $p_2 = 1/2$ and a price for identified consumers of $p(v) = v$. A consumer will only buy in the first period if $p_1 \leq v$. When she remains unidentified in the second period she will only buy if $p_2 \leq v$, whereas she will always buy upon identification. If $c < \lambda/2$ the monopolist has the belief $E[p_2] < 1 - c/\lambda$, indicating that some consumers will use the hiding technology. In this case the firm will set a first period price of $p_1 = 1/(2+\lambda)$, and second period prices of $p_2 = (1+m-c)/(2+2m-\lambda)$ for unidentified consumers and $p(v) = v$ for identified consumers. A consumer her strategy remains the same as the previous equilibrium: she will buy in the first period if $p_1 \leq v$, she will buy if $p_2 \leq v$ if she remains unidentified and the second period she will always buy upon identification.

Finally I would like to discuss what the effect of this change in price is for existing consumers who previously bought in the first period. Although I did not manage to formally prove this, I can make an educated guess followed by reasoning. It is very likely that when new consumers enter the market in the second period, it makes existing consumers better off. The reasoning for this is twofold: First, as the new mass m provides a dampening effect on p_2, the deadweight loss in the second period will be smaller for existing consumers. With this lower price more existing consumers who remained unidentified will buy the product, instead of refusing to buy at the higher second period price without new consumers. Secondly, as the price is lower when new consumers enter the market, the externality posed by active consumers on unidentified consumers is lower. Consequently, these consumers will keep a larger part of their surplus.

7. Imperfect Identification

Another interesting extension would be to examine the results when the tracking technology does not perfectly identify a consumer her valuation, but instead only recognizes whether a consumer previously bought or not. In such a case a monopolist cannot engage in personalized pricing and must resort to third degree price discrimination. Upon identification the monopolist can now only charge a consumer a price p_2^i, as he recognizes her as a returning customer. Therefore he knows that this particular consumer has a valuation of at least p_1. When the monopolist fails to identify a consumer, he can only charge them a price of p_2^u. This identification mechanism could give the monopolist an incentive to not set p_1 as low as the case with perfect identification, as the information provided upon identification becomes increasingly invaluable, as with a low p_1 he only learns that a large groups of consumers have a valuation of at least the low p_1. Therefore the monopolist ideally wants to set a higher p_1 to better identify consumers of higher valuation. In other words, the monopolist could use $p_1$as a high valuation test. However, he does not want to this price too high, as then he loses out on first period sales. Unfortunately, I did not manage to fully work out this extension, however, I would still like to share some preliminary results.

Just like the case with perfect identification, without both technologies nothing changes from the regular monopoly case: In the first period the monopolist charges a price $p_1 = 1/2$ and in the second period he also charges a price $p_2^u = 1/2$. Since the monopolist cannot identify his consumers, he cannot set a price $\mathrm{p}_2^{\mathrm{i}}$. The profits and surplus over two period are respectively $\pi = 1/2$ and $CS = 1/4$.

When both technologies are added I unfortunately fail to find a price for p_2^i which is rational. However, I do find a few things which I find at least worth mentioning. First, unlike the base model, consumers in this model do have an incentive to strategically refuse to buy in the first period. In the base model this does not make sense, as $p_1 < p_2$, thus there is no strategic incentive for buying in the second period. Refusing to buy in the second period is not possible, a consumer is unable to commit to this action. If the monopolist charges this consumer a price lower than or equal to the consumer her valuation, she will always but. Consumers are willing to strategically skip a purchase in the first period when:

$$v \leq p_1 + \lambda\left(E\left[p_2^i\right] - E[p_2^u]\right)$$

This seems intuitive, as upon identification the monopolist learns that a consumer her valuation is at least $p_1$and $\lambda\left(E\left[p_2^i\right] - E[p_2^u]\right)$ is the expected rise in price upon identification. Thus consumers with a valuation lower than this threshold will never buy upon identification, and therefore they are willing to strategically not purchase in the first period. Similarly, consumers prefer skipping a purchase over buying in both periods when:

$$v \leq p_1 + c$$

So, consumers will strategically skip a purchase when they are not willing to pay the cost of the hiding technology. I expect that strategically not buying is more beneficial for consumers then buying in the first period and consequently not be willing to pay the higher price upon identification, as I expect that p_1 will be used to identify high valuation consumers. Consequently I expect that p_u is smaller than p_1 and therefore such consumers will buy in the second period instead of the first. Finally, when consumers who will purchase in both periods have to choose between the hiding technology or taking the risk of being identified, this choice is independent of their valuation. Consumer who buy in both periods prefer using the hiding technology over taking a risk when:

$$c \geq \lambda\left(E\left[p_2^i\right] - E\left[p_2^u\right]\right)$$

Thus, all consumers who purchase in two periods will anonymize when the cost of using the hiding technology is smaller than or equal to the expected price raise upon identification. So, consumers collectively choose the same action. Depending on whether such consumers collectively choose to anonymize or take a risk the threshold for people who strategically skip a purchase changes. This could have interesting consequences for my model, which I did not yet manage to uncover. In further research I could delve deeper into this.

8. Conclusion

In this paper I studied a model in which a monopolist can charge a personalized price for consumers identified through data, which he collected from previous purchases by those consumers. However, consumers can prevent identification by using hiding technology to anonymize their data at a cost. I show that the monopolist benefits from hiding technology, as individual consumers exert an externality on other consumers by using such technology. However, due to this externality consumers collectively suffer. Under such circumstances it is best for such consumers to collectively ignore the hiding technology. Finally I found that the entry of new consumers dampens the externality imposed by consumers who use the hiding technology.

Based on these results a policy recommendation would be to not promote the usage of these hiding technologies, as they harm consumers if only a part of them uses such technology. Ideally no consumer should use the technology, as this way the externality disappears. However, since it is highly unlikely to obtain full participation for any program, it would probably be better to aim for as little use as possible, such that the harm to consumers is limited. This could be done by making hiding technologies like a VPN more expensive.

From this point my model can possibly be extended in multiple directions. A logical first step could be to add competitive effects in the model. In such a case a firm will only be able to price discriminate consumers only he has identified, as otherwise price competition will push the price downwards. Another area which could be added are experience goods, such that consumers only learn their true valuation after they have consumed the product, with the firm only learning consumers a priori valuation from the data. This could make the identification technology less reliable, as it is possible for the monopolist to charge a personalized price which is too high, as a consumer could have adjusted her valuation downwards. Another interesting direction could be to investigate opt-in policies by companies, where they only obtain data from consumers when they manage to get them enrolled into a loyalty program. This could be interesting as then the price the price a customer pays in the first period will matter, contrary to my model.

I would like to end this paper on the note that I do not think this paper proves that the usage of data should be banned from markets altogether, as this paper purely focusses on the effects of data when used for the purpose of price discrimination. The increasing use of big data has brought along beneficial effects as well, for instance by creating a feedback loop which induces product improvement, or by recommending certain other products which consumers were previously unaware of. However, I do believe we need to have a discussion about how we want personal data to be used by firms moving forward. Yet far too often we observe that when data has beneficial effect for consumers, firms often have incentives to undo such beneficial effects. Therefore more research into this area is certainly needed, both in terms of beneficial and detrimental effects of data.

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10. Appendix

Tracking technology without hiding

When the monopolist uses tracking technology, and no consumers uses the hiding technology, we can solve this through backwards induction. In the second period, the monopolist sets a uniform price and a schedule of individualized prices. Consumers buy if $p_2 \leq v$, thus if identified, a consumer will always buy, as they then pay $p(v) = v$. Therefore the expected profit prior to the second period will be:

$$\pi_{n_2} = \lambda \int_{p_1}^{1} v dv + (1-\lambda) \int_{p_2}^{1} p_2 dv$$

Which can be simplified to:

$$\pi_{n_2} = \lambda \left(\frac{1}{2} - \frac{1}{2} p_1^2\right) + (1-\lambda)(1-p_2)p_2$$

Optimization with respect to p_2 then yields $p_2 = \frac{1}{2}$. Now we can determine the optimal price in the first period, which determines how many consumers are potentially identified. In order to find this we have to solve the following maximization problem:

$$\max_{p_1} \ (1-p_1)p_1 + \ \lambda \left(\frac{1}{2} - \frac{1}{2} p_1^2\right) + (1-\lambda)(1-p_2)p_2$$

Solving this yields an optimal price in the first period of $p_1 = \frac{1}{2+\lambda}$, thus the price in the first period is decreasing in λ. The corresponding per period expected profits can then be computed as follows:

$$\pi_{n_1} = \frac{1}{2+\lambda}\left(1 - \frac{1}{2+\lambda}\right) = \frac{1+\lambda}{(2+\lambda)^2}$$

$$\pi_{n_2} = \lambda\left(\frac{1}{2} - \frac{1}{2(2+\lambda)^2}\right) + (1-\lambda)\left(1 - \frac{1}{2}\right)\frac{1}{2} = \frac{\lambda}{2} - \frac{1}{2(2+\lambda)^2} + \frac{1}{4}(1-\lambda)$$

When summing these up these yield:

$$\pi_{n_1} + \pi_{n_2} = \frac{4 + 3\lambda + \lambda^2}{4(2+\lambda)} \equiv \pi_n(\lambda)$$

Tracking technology with hiding

When adding the ability for consumers to hide their identity at cost c, we must first determine who will do so. The surplus a consumer is expected to receive when hiding is equal to $2v - p_1 - E[p_2] - c$, whereas the surplus a consumer is expected to receive if he does not hide is equal to $v - p_1 + (1 - \lambda)(v - E[p_2])$. Thus, a consumer will hide when:

$$2v - p_1 - E[p_2] - c \geq v - p_1 + (1-\lambda)(v - E[p_2]) \quad => \quad v \geq E[p_2] + \frac{c}{\lambda} \equiv \bar{v}$$

From this expression we can determine that some consumers will hide when $\bar{v} < 1$, which yields a constraint of $\mathrm{E}[p_2] < 1 - \frac{c}{\lambda}$. This belief yields two types of consumer behaviours depending on their expectations of p_2. Consumer either have;

a) An expectation $E[p_2] \geq 1 - \frac{c}{\lambda}$, after which all consumers are passive

b) An expectation $E[p_2] < 1 - \frac{c}{\lambda}$ after which some consumers are active

Consider (a). In such a case the profit in the second period is equal to:

$$\pi_2 = \lambda \int_{p_1}^{1} v dv + (1-\lambda) \int_{p_2}^{1} p_2 dv$$

Which can be simplified to:

$$\pi_2 = \lambda\left(\frac{1}{2} - \frac{1}{2}p_1^2\right) + (1-\lambda)(1-p_2)p_2$$

Now we essentially have the same maximization problem as when there is no hiding technology, and optimization subsequently yields $p_2 = \frac{1}{2}$. However, this price only satisfies the belief if:

$$E[p_2] > 1 - \frac{c}{\lambda}$$

Since in equilibrium $E[p_2] = p_2$, this price satisfies the belief when:

$$p_2 > 1 - \frac{c}{\lambda} \quad => \quad c \geq \frac{\lambda}{2}$$

The corresponding profits are the same as the ones found in the case examined previously.

Now consider (b), which is the case in which the monopolist allows some consumers to hide. In that case, the corresponding profit can be computed as:

$$\pi_2 = \lambda \int_{p_1}^{\bar{v}} v dv + (1-\lambda) \int_{p_2}^{\bar{v}} p_2 dv + \int_{\bar{v}}^{1} p_2 dv$$

This expression can be simplified to:

$$\pi_2 = \frac{1}{2}\lambda\left(\left(E[p_2] + \frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1 - \lambda\left(E[p_2] + \frac{c}{\lambda}\right) - (1-\lambda)p_2\right)$$

Now the optimization problem for the second period is:

$$\max_{p_2} \frac{1}{2}\lambda\left(\left(E[p_2] + \frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1 - \lambda\left(E[p_2] + \frac{c}{\lambda}\right) - (1-\lambda)p_2\right)\ s.t. E[p_2] < 1 - \frac{c}{\lambda}$$

Optimization then yields the following first order condition:

$$1 - \lambda E[p_2] - c - 2(1-\lambda)p_2 = 0$$

Since in equilibrium $E[p_2] = p_2$, this can be simplified to:

$$1 - 2p_2 + \lambda p_2 - c = 0 \ \ => p_2 = \frac{1-c}{2-\lambda}$$

This price satisfies the belief as long as $c < \frac{\lambda}{2}$. Now we can check for the optimal price in the first period. This maximization problem is:

$$\max_{p_1}(1-p_1)p_1 + \frac{1}{2}\lambda\left(\left(E[p_2] + \frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1 - \lambda\left(E[p_2] + \frac{c}{\lambda}\right) - (1-\lambda)p_2\right)$$

This maximization problem yields a solution in $p_1 = \frac{1}{2+\lambda}$. Thus, even though consumers may hide, the monopolist still has the same price setting as when consumers do not or are unable to hide. The corresponding profits and consumer surpluses when are:

$$\pi_1 = \frac{1}{2+\lambda}\left(1 - \frac{1}{2+\lambda}\right) = \frac{1+\lambda}{(2+\lambda)^2}$$

$$\pi_2 = \frac{1}{2}\lambda\left(\left(\frac{1-c}{2-\lambda} + \frac{c}{\lambda}\right)^2 - \frac{1}{(2+\lambda)^2}\right) + \frac{(1-\lambda)(1-c)^2}{(2-\lambda)^2}$$

Summing these up yields total profits of:

$$\pi_{1+2} = \frac{2\lambda + c^2(2+\lambda)(1-\lambda)}{\lambda(2+\lambda)(2-\lambda)} \equiv \pi(\lambda, c)$$

Consumer surplus equals:

$$CS = \int_{p_1}^{1}(v-p_1)dv + \lambda \times 0 + (1-\lambda)\int_{p_2}^{\bar{v}}(v-p_2)dv + \int_{\bar{v}}^{1}(v-p_2-c)dv$$

$$= \frac{1+\lambda^2}{2(2+\lambda)^2} + \frac{(2-\lambda)^2c^2 - 2\lambda(1-\lambda+c)(2-\lambda)c + \lambda(1-\lambda+c)^2}{2\lambda(2-\lambda)^2}$$

Profitability identifying technology

When $c \geq \frac{\lambda}{2}$, the identifying technology is profitable as long as:

$$\pi_n(\lambda) - \frac{1}{2} \geq 0 => \frac{\lambda(1-\lambda)}{4(2+\lambda)} \geq 0$$

$$\pi(\lambda, c) - \frac{1}{2} \geq 0 => \frac{2(1-\lambda)(2+\lambda)c^2 + \lambda^3}{2\lambda(2+\lambda)(2-\lambda)} \geq 0$$

All of these are larger than zero for all potential values of λ, thus the identifying technology is profitable. Furthermore, for $\pi(\lambda, c)$ it can be easily seen that this is increasing in c

Proof that consumers are better off not using hiding technology

First, I can show that the consumer surplus when hiding technology is available is increasing in c:

$$\frac{dCS}{dc} = \frac{(2-\lambda)^2 c - \lambda + \lambda c}{\lambda(2-\lambda)^2} = \frac{(4-\lambda(3-\lambda))c - \lambda}{\lambda(2-\lambda)^2}$$

This derivate is positive for all relevant values of λ between 0 and 1. As c decreases, more people use the hiding technology, which leads to the monopolist learning of more people that they have a valuation of at least $\bar{v}$. This brings me to the final part of this proof. Consumers as a whole are worse of when $c = 0$ then when c is large enough for hiding to be blockaded as:

$$CS_n(\lambda) - CS(\lambda, 0) = \frac{1-\lambda}{8} - \frac{\lambda(1-\lambda)^2}{2\lambda(2-\lambda)^2} = \frac{\lambda^2(1-\lambda)}{8(2-\lambda)^2} > 0$$

Inflow of new consumers with passive consumers

When new consumers enter the market in the second period and all consumers are passive, the monopolist his maximization problem changes as follows:

$$\max_{p_2} \lambda\left(\frac{1}{2} - \frac{1}{2}\, p_1^2\right) + (1-\lambda)(1-p_2)p_2 + m(1-p_2)p_2$$

This yields the following first order condition:

$$(1-\lambda)(1-2p_2) + m(1-2p_2) = 0$$

From the first order condition I can consequently derive that $p_2 = \frac{1}{2}$. The derivations for p_1 and the belief are the same as the base model, so for those I refer to that part of the appendix.

Inflow of new consumers with active consumers

When new consumers enter the market and some consumers are active, the monopolist his maximization problem changes as follows:

$$\max_{p_2} \frac{1}{2}\lambda\left(\left(E[p_2] + \frac{c}{\lambda}\right)^2 - p_1^2\right) + \left(1 - \lambda\left(E[p_2] + \frac{c}{\lambda}\right) - (1-\lambda)p_2\right)p_2 + m(1-p_2)p_2$$

After deriving this function I find the following first order condition:

$$1 + m - \lambda E[p_2] - c - 2p_2 - 2mp_2 + 2\lambda p_2 = 0$$

Since in equilibrium $E[p_2] = p_2$, this can be redeveloped into:

$$1 + m - c - 2p_2 - 2mp_2 + \lambda p_2 = 0$$

From this condition I derive that $p_2 = \frac{1+m-c}{2+2m-\lambda}$. However, this price can only be set in equilibrium when $E[p_2] < 1 - \frac{c}{\lambda}$. Again, since in equilibrium $E[p_2] = p_2$, this can be redeveloped into $p_2 < 1 - \frac{c}{\lambda}$. Now I can check when this price can be set in equilibrium:

$$\frac{1+m-c}{2+2m-\lambda} < 1 - \frac{c}{\lambda}$$

$$\frac{c}{\lambda} < \frac{2 + 2m - \lambda - (1 + m - c)}{2 + 2m - \lambda}$$

$$c(2 + 2m - \lambda) < (1 + m - \lambda + c)\lambda$$

$$2c + 2mc - 2c\lambda < \lambda + m\lambda - \lambda^2$$

$$2c(1 + m - \lambda) < \lambda(1 + m - \lambda)$$

From the last line it can be easily seen that that when $c < \frac{\lambda}{2}$ this belief is satisfied. For the computation for p_1 I again refer to the computations of the base model.

Thresholds imperfect identification

Consumers prefer skipping a purchase over taking a risk when:

$$v - E[p_2^u] \geq 2v - \lambda E\left[p_2^i\right] - (1 - \lambda)E[p_2^u] \quad => \quad v \leq p_1 + \lambda\left(E\left[p_2^i\right] - E[p_2^u]\right)$$

Consumers prefer skipping a purchase over using the hiding technology when:

$$v - E[p_2^u] \geq 2v - p_1 - E[p_2^u] - c \quad => \quad v \leq p_1 + c$$

Consumers prefer buying in both periods with hiding technology over buying in both periods without hiding technology when:

$$2v - p_1 - E[p_2^u] - c \geq 2v - p_1 - \lambda E\left[p_2^i\right] - (1 - \lambda)E[p_2^u] \quad => \quad c \leq \lambda(E\left[p_2^i - E[p_2^u]\right])$$