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Forecasting spare part demand: installed base concepts, failure rate function and varied expected lifetimes

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Abstract

Stopping production and marketing of a product, does not remove the need for spare parts. Therefore, this end-of-life decision requires the manufacturer to determine how many components have to be produced in the final production run. However, this size decision is often found to be a complex problem. The demand for spare parts is closely related to the amount of products in use by consumers, the installed base, but retailers of consumer products do not keep track of the installed base of their products. This paper uses four installed base concepts in order to replace the functionality of the real measurement. Forecasting with these concepts indeed improved the forecast performance in majority of the cases. In addition, the failure rate functions of the spare parts are estimated and the assumption of a fixed lifetime is relaxed. However, when dealing with larger amount of spare parts, estimating failure rates is not a suggested approach. The same advice holds for gaining different expected lifetimes through five adaptations of lifetime length. Sampling life expectancy from uniquely specified Weibull distributions could, however, be beneficial for forecasting spare part demand.

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1 Introduction

When products are in use by consumers, essential parts might break down or fail to fulfill their role in making the product usable. The owners might want to replace these failing components with spare parts, in order to repair the product and extend its lifetime. However, the production of these spare parts is a complicated process. The components differ in demand size and their availability seems of great importance when the production of the product itself has stopped. Logically, the demand of the spare parts and thus the production depends on the amount of products in use at that moment, the so called *Installed Base* (IB). However, in business-to-consumer situations specifically, the sellers often do not keep track of the IB of their products, as selling the item is their main goal. Therefore, including this measurement when forecasting spare part demand is only a recent method and is at the early stages of development. Previous published papers tend to only address the use of historical data. However, letting the forecast depend only on sales and returns might not be accurate, as demand may depend on more factors.

Kim, Dekker, and Heij (2017) have developed four installed base concepts as a replacement for the lack of information on the real installed base. The research concludes that in majority of the cases these concepts seem to produce better forecasts than only using historical data. However, not one specific model is suitable for every spare part. This leads to the research question of this paper;

> *"Which installed base concept produces the best forecast performance for what type of spare part?"*

To give an answer to this question, this paper will deal with three sub questions;

1. *Do all forecasts of spare part demand improve when adding installed base information as explanatory factor?*
2. *Are forecasts being improved when adding estimated failure rate information of the spare part to the installed base models?*
3. *Are forecasts being improved by letting the product units that are sold vary in expected lifetime length?*

To answer these questions the research done by Kim, Dekker, and Heij (2017) is replicated first. The dataset that is used is provided by the Western European warehouse of Samsung Electronics. It contains demand data for spare parts of different types of products, which is smoothed by means of the exponential weighted moving average method. These data are used to specify the four different IB concepts, which are being modelled to forecast the demand of spare parts in the products end of life phase. Based on three forecast-error measures, the forecasts are compared with each other to confirm or deny the hypotheses of which installed base fits which spare part best. Besides this replication part, I examine two potential ways to increase the forecasting performance. First, the failure rates of the spare parts are estimated through the Weibull distribution and are added as an explanatory factor for their demand. Secondly, the assumption of a fixed lifespan of the products is relaxed by giving each unit sold a different expected lifetime. This is done in two ways; (i) by five predefined lifetime alternatives and (ii) by sampling from a Weibull distribution. Replicating the research, however, did not give the same results. Moreover, applying the new methods did not give a clear suggestion for when these approaches might be beneficial.

The continuation of the paper is organized as follows. In Section 2, an overview is given of previous research directions regarding spare part demand. In Section 3, the methodology that is used in this research is discussed and elaborated. Section 4 illustrates the given dataset by Samsung Electronics and the results of applying the methods on this information are given in Section 5. The research finishes with concluding remarks in Section 6.

2 Research background

The life cycle of a product consists of three time phases; (i) the initial phase where the sales of the product increase, (ii) the mature phase where the sales decline towards zero and (iii) the end-of-life (EOL) phase where the product is no longer produced nor sold. The demand for spare parts in this last phase is of great importance. In the first phase the amount of consumers requiring spare parts is expected to be relatively low, as most products are at the early stages of consumption. In the second phase the demand is expected to rise, but in the EOL phase the demand is more erratic. Demand could possibly still increase in the beginning of this phase, but that is not always the case. Besides, demand will eventually reduce to zero because the products all reach the end of their lifetimes. That is why this paper focuses on forecasting the demand for spare parts in this last period, the end-of-life phase.

2.1 Historical sales data

Forecasting demand in general has been a relevant problem for stores and manufacturers. However, the demand of spare parts is intermittent; periods of zero demand interchange with periods of very high demand. The methods that are used the most for these cases are moving average modelling and exponential smoothing of the data, Axsäter (2015). Because the demand of spare parts fluctuates heavily over time, exponential smoothing turns out to be a strong and often used method, as it reduces the high changes in demand. However, both these methods tend to overestimate the mean level of intermittent demand, Boylan and Syntetos (2010). Johnston and Boylan (1996) proposed an adjusted exponentially weighted moving average (EWMA) method for forecasting the intermittent demand. The research concludes that this adjusted method performs better, in terms of the mean square prediction errors, than a traditional EMWA.

In business-to-business (B2B) relations, an often discussed forecasting method for intermittent demand was written by Croston (1972). Instead of focusing on the mean demand per period, he divided the demand into the time of occurrence and its size. By dividing these components he made two separate estimates: one of the inter-demand interval and one of the size of the demand when it occurs. This approach in forecasting is called the Croston's method (CR) and is very popular in previous literature regarding spare part demand.

Besides Croston's method various other approaches for forecasting this demand have been developed. Bootstrapping, for example, is a non-parametric approach that does not require to make an assumption on the distribution of the demand, Efron (1992). Another non-parametric tool for forecasting intermittent demand is neural networks, Gutierrez, Solis, and Mukhopadhyay (2008). It captures the relation between points with non-zero demand and the inter-arrival rate of demands. Thirdly, judgmental forecasting is often used to adjust quantitatively derived forecasts (Goodwin (2002)) and can result in an improved forecast accuracy, Syntetos et al. (2009). However, this is not a practical application as it requires managing time and in practice one is easily dealing with thousands of spare parts.

2.2 Installed base

The amount of iPhones that Apple has sold, the amount of cars that Volkswagen has manufactured, the amount of solar panels that JinkoSolar has placed, et cetera. These are all examples of installed base. In general, the installed base is defined as the set of products or systems that have been sold by an organization and are still in use (Summers (2007)). However, these products and systems require maintenance service or supply for new spare parts when they break down. As these services are required in this research, I define the installed base as the set of systems or products for which an organization provides after-sales services (Dekker et al. (2013)).

As previously mentioned, forecasting spare part demand using installed base methods is a recent point of interest. In the startup phase of this research direction, installed base was assumed to be constant over time. This means that the amount of products in use is not changing in the initial, mature and end-of-life phase. Soon after, several researchers acknowledge the change of the installed base over the products life cycle. The amount of products in use increases during the initial phase, reaches a maximum level in the mature phase, and decreases during the end-of-life phase, Inderfurth and Mukherjee (2008).

A lot of papers investigate the life cycle of products which are not sold after their production stop, which indicates a decrease in installs in the EOL phase. This decline in installed base values is of great importance concludes Chou, Hsu, and Lu (2016). They argue that the spare part production costs may be much larger during the EOL phase than in the mature phase. This decrease in spare part demand can also be a cause of non-replacement decisions by the consumers. They might not want to replace the component when the product is relatively old. The probability of this non-replacement decision is introduced by Ritchie and Wilcox (1977). Chou, Hsu, and Lu (2016) conclude that regressing on this probability produces more accurate forecasts than using only historical sales data. Lu and Wang (2015) discuss another reason for the decrease in this willingness; the cumulative number of breakdowns that the product has experienced. A combination of the two reasons is examined by Lu and Hjelle (2016), they depend the probability of repairing on both; the number of failures and the use time.

Kim, Dekker, and Heij (2017) also deal with products that are not sold after their production stop. They propose four different installed base concepts to forecast the future demand of spare parts. However, there are some limitations regarding this research. A long estimation period during the first two phases is necessary and a short EOL phase is preferred. Besides this, the deterioration of the spare parts depends on the age of the product (Van der Auweraer, Boute, and Syntetos (2019)). Additionally, Van der Auweraer and Boute (2019) state another weakness; the part age and product age are not distinguished. Moreover, only the age of the products are considered. The part age, however, can be of great importance too, as it contains information about the failure frequency of the spare parts. Using this information could possibly lead to better explanation of the occurrence and frequency of spare part demand, which is the goal of the research. The paper evaluates the acquired forecasts by means of the mean absolute prediction error (MAPE) and root mean squared prediction error (RMSPE). However, these measures are not necessarily the best choice when intermittent data is studied (Kolassa (2016)). The MAPE is indeed suitable for comparing multiple forecasts, but it is not optimal for evaluating the overall performance of the results. Moreover, Murray, Agard, and Barajas (2018) state that the RMSPE is not as useful for comparing performance between clusters because error size is skewed by data magnitude. The forecast evaluation should therefore focus more on the differences between the predicted and the actual time-series.

Another research direction when examining spare parts is the impact of the environment.

Environmental factors like air pressure, temperature, humidity, etc., can have an effect on the reliability characteristics of products. Research done by Ghodrati (2011) suggests a method using system reliability and environmental data to calculate the expected number of spare parts needed. Forecasting spare part demand with environment factors is of great importance according to Ghodrati, Akersten, and Kumar (2007). Their research concludes that ignoring these factors results in potential losses. Due to not considering the operating environment, the risk of shortage of spare parts could lead to unnecessary downtime of machines. This results in a temporary production stop, which affects companies both production wise and financially.

Finally, we look at the very first attempt in including installed base in forecasting spare part demand. Shaunty and Hare Jr (1960) investigate the connection between spare part demand and product usage based on the number of landings of airplanes. They estimate the failure rate of a spare part per landing. Another approach for estimating the failure rate of spare parts can be found in the continuation of this paper. However, contrary to the research by Shaunty and Hare Jr (1960), I propose this method in a business-to-consumer (B2C) setting.

3 Methodology

3.1 The installed base concepts

Manufacturers and sellers of machines for B2B trading, like airplanes and cars, do not only earn their money from selling their products, but also from repairing them. In contrary to B2C trading, where the goals of the seller is accomplished when the retailer sells the product. This is one of the reasons why B2B sellers keep track of their installed base and B2C sellers do not. In this last case, four installed base concepts can be defined and used to replicate the functionality of the real installed base. First, the *lifetime installed base* (IBL) is the amount of units in use by the consumers. It keeps track of the amount of products leaving and returning to the warehouse and decreases with the amount of products that exceed their expected lifetime. The definition for the IBL at the end of week t, formulated by Kim, Dekker, and Heij (2017), is given by

$$IBL(t) = \sum_{i=t-L+1}^{t} (S(i) - R(i)). \tag{1}$$

With $S(t)$ being the product sales in time period t, $R(t)$ is the amount of returned products from customers in that period, and L denotes the expected average lifetime of the product. Where $S(i) = R(i) = 0$ for $i < 1$.

Second, we define the *warranty installed base* (IBW). Consumers might let the choice of replacing a component depend on the warranty regulations. In some cases, they might only repair the broken component if it is covered by the products warranty. Afterwards, customers have to invest in the reparation themselves, but this might lead to them buying a new product instead. When a product comes with a warranty of W periods of time, the IBW is equal to

$$IBW(t) = \sum_{i=t-W+1}^{t} (S(i) - R(i)) \tag{2}$$

(with $S(i) = R(i) = 0$ for $i < 1$) (Kim, Dekker, and Heij (2017)). The warranty period is part of the lifetime of the product, hereby the IBL and IBW will be the same in the warranty period, but will diverge afterwards.

When the warranty has expired, the demand for spare parts might still exist. This is due to the effect that the remaining economic value exceeds the costs of repairing the product. This occurrence leads to the third installed base concept; the *economic installed base* (IBE). The following notation by Kim, Dekker, and Heij (2017) is used. Let $c(t)$ be the repair costs in period t and let $v_i(t)$ be the remaining economic value in period t of a product bought in period i. If $v_i(t) > c(t)$, then the decision for repair is defined as $E_i(t) = 1$. When $v_i(t) \leq c(t)$, the user will not replace the component: $E_i(t) = 0$. The IBE is then defined as the part of the IBL where repair is economically beneficial, given by,

$$IBE(t) = \sum_{i=t-L+1}^{t} E_i(t) \times (S(i) - R(i)) \tag{3}$$

(with $S(i) = R(i) = 0$ for $i < 1$). As reparation is free in the warranty period, the economic value always exceeds the repair costs in this time period.

The IBE assumes that the evaluation of economic value is the same for all customers, but this can differ from person to person, depending on their preferences and tastes. Some people might want to have the newest product or want to follow the latest trends. For these cases the *mixed economic installed base* (IBM) is defined. It is identical to the IBE, but with heterogeneous decay rates for the consumers. As previously done by Rogers (2010) and Kim, Dekker, and Heij (2017), the consumers are divided in five groups. The groups differ in quickness of willing to replace the product, resulting in a different expected life cycle for each group. Identical to Rogers (2010), the size of the segments are as follows: 2.5% innovators (0.6), 13.5% early adopters (0.7), 34% early majority (1.0), 34% late majority (1.05), and 16% laggards (1.3). With in parenthesis the concerned life cycle as a fraction of the average product lifetime.

3.2 Installed base modelling & estimation

As the demand of spare parts is intermittent the demand data is smoothed by taking an exponentially weighted moving average (EWMA) with smoothing factor α. The model specification that is used follows from Kim, Dekker, and Heij (2017) and is given by

$$\ln(1 + D_s(t)) = b_0 + b_1 \times \ln(1 + IB(t)) + b_2 \times AGE(t) + \varepsilon(t). \tag{4}$$

With $D_s(t)$ equal to the size of smoothed demand in period t. $AGE(t)$ denotes the mean age of the products included in the installed base at the end of period t and is specified for all installed base concepts. The unobserved error term $\varepsilon(t)$ is assumed to follow an autoregressive (AR) process. By making this assumption, ordinary least squares can be used to estimate the model coefficients b_0, b_1, and b_2 in (4). This model is estimated for all four installed base concepts. As the demand must be positively related to installed base, the installed base term $ln(1 + IB(t))$ is removed when its coefficient is negative ($b_1 < 0$).

To generate black box forecasts, a pure AR model is examined. This model is obtained when $b_1 = b_2 = 0$ in (4). The error term is hereby modelled as $\varepsilon_t = c_1\varepsilon_{t-1} + \cdots + c_p\varepsilon_{t-p} + \omega_t$. The amount of lags p is determined by forward selection; the order is increased until the extra lag term becomes insignificant at a 5% significance level. This AR order p is specified for each product and is also used in the installed base models of the product.

In summary, the models are estimated in the initial and mature phases as follows. First, the average lifetime, sales and the spare part demand data is determined. With the smoothed demand data, an AR model is estimated for the initial and mature phases of the product and a suitable AR order p is selected. With this order p, four types of installed base models are estimated for the first two phases and the installed base variable is removed when it has a negative coefficient. With these models, the spare part demand in the EOL phase of all products is forecasted. These forecasts are, just like the dependent variable, smoothed demand values.

3.3 Forecast measures

Afterwards, these forecasts are compared with each other based on their predictive power, which is measured based on five criteria. These prediction measures are calculated by comparing the forecast of $D_s(t)$ directly with the actual demand values $D(t)$. The first measure being the sum of the forecast errors. Suppose the EOL phase runs from t_2 to t_3 and let $F(t)$ be the smoothed demand forecasts, with $t_2 \leq t \leq t_3$. Then the sum of forecast errors is given by

$$\text{SUM} = \sum_{t=t_2}^{t_3} (F(t) - D(t)) / \sum_{t=t_2}^{t_3} D(t). \tag{5}$$

The closer to zero, the better the accuracy of the forecasts. A negative value indicates under-estimation of the demand of spare parts and a positive value corresponds with over-estimation. The other two measure criteria are the mean absolute prediction error (MAPE) in (6) and the root mean squared prediction error (RMSPE) in (7).

$$MAPE = \frac{\sum_{t=t_2}^{t_3} |F(t) - D(t)|}{\sum_{t=t_2}^{t_3} D(t)}, \tag{6}$$

$$RMSPE = \frac{\sqrt{\frac{1}{t_3-t_2+1}\sum_{t=t_2}^{t_3}(F(t) - D(t))^2}}{\frac{1}{t_3-t_2+1}\sum_{t=t_2}^{t_3} D(t)} = \frac{\sqrt{\sum_{t=t_2}^{t_3}(F(t) - D(t))^2}}{\sum_{t=t_2}^{t_3} D(t)/\sqrt{t_3 - t_2 + 1}}. \tag{7}$$

Finally, to test if the forecasts of two models are significantly different from each other, I use the Diebold-Mariano test statistic

$$DM = \frac{\overline{d}}{\sqrt{V(\hat{d}_{t+1})/P}} \overset{a}{\sim} N(0, 1), \tag{8}$$

where P is the amount of weeks in the EOL phase and d_{t+1} is the loss function. The loss function is the difference between the squared forecast errors of the first model (a) minus the squared forecast errors of second model (b): $d_{t+1} = e^2_{a,t+1} - e^2_{b,t+1}$. Furthermore, $\overline{d}$ is the mean difference of the squared prediction errors produced by the two models and the variance is approximated by the loss function $V(\hat{d}_{t+1}) = \frac{1}{P-1}\sum_{t=T}^{T+P-1}(d_{t+1} - \overline{d})^2$. Under the null hypothesis, the forecast errors are the same. Thus, when the resulting p-value exceeds 0.05, the null-hypothesis can be rejected with a significance of 5%; the forecast errors of model a and model b differ significantly.

3.4 Failure rate estimation & modelling

According to Van der Auweraer, Boute, and Syntetos (2019), information on the failure rate is something which captures a large part of the demand generating process. As this rate is not available, I estimate the failure rate function of each spare part and include these values into the existing model to possibly further improve the forecasts. This is done as follows. Let Y be the time of failure of the product requiring a spare part for repair. Then Y is assumed to follow a 2-parameter Weibull distribution, $Y \sim \text{WEI}(\beta, \eta)$ with probability density function given by

$$f(y_i|\beta, \eta) = \frac{\beta t^{\beta-1}}{\eta^\beta} \times \exp\left\{-\left(\frac{y_i}{\eta}\right)^\beta\right\}, \tag{9}$$

with $i = 0 \ldots n$ and n the amount of demand occurrences. The shape parameter β and scale parameter η are being estimated by means of Maximum Likelihood Estimation (MLE) with ML function denoted as

$$\begin{aligned} \text{L}(\beta, \eta|\mathbf{y}) &= \prod_{i=1}^{n} f\left(y_i|\beta, \eta\right) \\ &= \prod_{i=1}^{n}\left[\frac{\beta}{\eta^\beta} y_i^{(\beta-1)} \exp\left\{-\left(\frac{y_i}{\eta}\right)^\beta\right\}\right] \\ &= \left(\frac{\beta}{\eta^\beta}\right)^n \prod_{i=1}^{n} y_i^{(\beta-1)} \times \exp\left\{-\left(\frac{\sum y_i}{\eta}\right)^\beta\right\}. \end{aligned} \tag{10}$$

When taking the logarithm and simplifying the equation, we get the final optimization function

$$\ell(\beta, \eta|\mathbf{y}) = n\log(\beta) - \beta n\log(\eta) + (\beta - 1)\sum_{i=1}^{n}\log(y_i) - \sum_{i=1}^{n}\left(\frac{y_i}{\eta}\right)^{\beta}. \tag{11}$$

Minimizing this function gives an estimation for the parameters β and η. When $\beta < 1$ the failure rate decreases over time, $\beta > 1$ indicates an increasing failure rate and $\beta = 1$ states that the failure rate is constant over time. With these estimated parameters, the failure rate on a given time t is the failure rate over the last t time periods and can be calculated through the Weibull failure rate function ($FR(t)$), that is,

$$\mathrm{FR}(t) = \frac{\hat{\beta}_{ML}}{\hat{\eta}_{ML}}\left(\frac{t}{\hat{\eta}_{ML}}\right)^{\hat{\beta}_{ML}-1}. \tag{12}$$

In the model specification process done by Kim, Dekker, and Heij (2017), Y was assumed to follow an exponential distribution. Which means that the failure rate is assumed to be constant over time and the installed base model (4) is based on this assumption. However, this is unlikely to be the case with products that experience a production stop. As rewriting the demand probability $p_d(t)$ with Weibull survival functions is too complex, the failure rate function (12) is simply added to (4) and results in the following model specification

$$\ln(1 + D_s(t)) = b_0 + b_1 \times \ln(1 + IB(t)) + b_2 \times AGE(t) + b_3 \times FR(t) + \varepsilon(t). \tag{13}$$

Which is again estimated with least squares estimation and follows the forecast procedure stated in Section 3.2.

3.5 Varying lifetime

In all methodology aforementioned, the lifetime of the product is assumed to be fixed. However, in practice the lifetime of a product is different for every unit produced. This is due to external factors that the product experiences, like influences of the environment. Just as IBM assumes five different lifetimes for every product, I apply this assumption while redetermining the lifetime installed base and the corresponding average age. Every unit sold will be associated by one of the following five types of expected lifetimes (L): $0.6L$ (2.5%), $0.7L$ (13.5%), $1.0L$ (34%), $1.05L$ (34%) and $1.3L$ (16%). These percentages can be seen as probabilities that a product will be unusable by one of these points in time. With these probabilities, s lifetimes are generated, with s being the amount of sales in the initial and mature product phases. For example, the amount of products that are expected to be discarded after $0.6L$ time units is given by $0.025 \times s$. When the lifetimes are generated, every unit that is sold is given a randomly sampled lifetime. This requires the number of units that are exceeding their expected lifetime to be corrected for the newly associated lifespans. When a certain end-of-life is given, the time where the products life is expected to end, is registered. When the IBL and its average age are recalculated, forecasts are made with the new data to determine what effect a varied lifetime has on the forecast performance of the IBL model. Implementing this part of the IBM in the IBL, still make the two non-identical. Where the IBM includes the exponential decay rate of the product, the new IBL only takes the expected life ending of products into account. Which results in IBL only altered slightly and IBE and IBM still being more accommodating installed base concepts.

To further investigate the effect of different lifetimes for a specific kind of product, another method is applied. In this method, the time until a product reaches the end of its lifetime is assumed to follow a Weibull distribution: $L \sim \text{WEI}(\beta, \eta)$. To be clear, in Section 3.4 the time until a product fails and requiring repair was assumed to follow this distribution, whereas here the time until the product is unusable is assumed to be Weibull distributed. Let l_i be the expected lifetime of the i'th sold product unit, then the pdf for this lifetime is given by

$$f(l_i|\beta, \eta) = \frac{\beta t^{\beta-1}}{\eta^{\beta}} \times \exp\left\{-\left(\frac{l_i}{\eta}\right)^{\beta}\right\}, \tag{14}$$

with $i = 1, \ldots, s$. In order to acquire the lifetimes from a Weibull distribution that matches the reliability characteristics of the product, the shape (β) and scale (η) parameters are predefined for every product category. Just like in the five lifetimes method, s lifetimes are sampled from the Weibull pdf specified for the concerned product category. Hereafter, every unit sold gets an expected end-of-life date and the IBL and average age values are recalculated and used to make new forecasts. In this way of sampling the lifetimes, I allow them to be chosen continuously instead out of five predefined alternatives.

4 Data

To test the performance expectations of the installed base concepts and to give an answer to the research question, data provided by the Western European warehouse of Samsung Electronics is investigated. My supervisor Dekker provided me with the dataset. It contains six products; three product categories consisting of two product types. The range and amount of weeks included in the data is different for each product, as can be seen in Table 1. This is due to the difference in expected lifetime of the product categories. Besides this information, the amount of sales, life expectancy and sensitivity for technological trends is included. Emphasizing the diversity in characteristics of the refrigerators, televisions and smartphones.

For the amount of weeks of data, information on the amount of sales and returns is available, as well as the amount of units that reach the end of their warranty period and lifetime. The

Table 1: Product characteristics and available data

	Type	Life cycle	Trendy	Sales	From	Till	Estimation	Forecast	Lifetime
Refrigerator	1	Long	Low	538,386	08.12	14.13	279	36	676
	2	Long	Low	166,782	08.32	14.13	229	66	676
Television	1	Short	Low	36,766	09.23	14.13	100	152	360
	2	Short	Low	50,986	10.12	14.13	108	102	360
Smartphone	1	Short	High	348,153	10.24	14.13	109	89	160
	2	Short	High	694,816	11.19	14.13	90	61	160

Sales shows the total product sales.
From and Till indicate the data range in the format year.week (e.g., 14.13 is week 13 of 2014).
Estimation and Forecast are the number of weeks of data available for estimation and forecast analysis, respectively.
Lifetime is average lifetime in weeks.
Source: Kim, Dekker, and Heij (2017).

dataset includes actual and smoothed demand data for three spare parts of each product, resulting in eighteen spare parts in total. Besides this, the installed base values for each of the four IB specifications and the mean age of the products are available for each week in the data range. To illustrate the behaviour of the four installed bases over time, Figure 1 shows the size of the four installed base concepts for television type 1, with IBE and IBM specified for its LCD panel. These IB values were not available before the research of Kim, Dekker, and Heij (2017). They computed these values themselves and made them available to me for further research. To check their results, I replicated the data by performing the calculations myself. The procedure and used code can be found in Section A.1 included in the Appendix. However, the calculated smoothed demand of the refrigerator types is lagged by one time unit in the dataset. Corresponding with $D_s(t)$ actually being equal to $D_s(t-1)$. I recalculated these values and corrected them, but is it unclear whether this error was made before or after the research done in 2017.

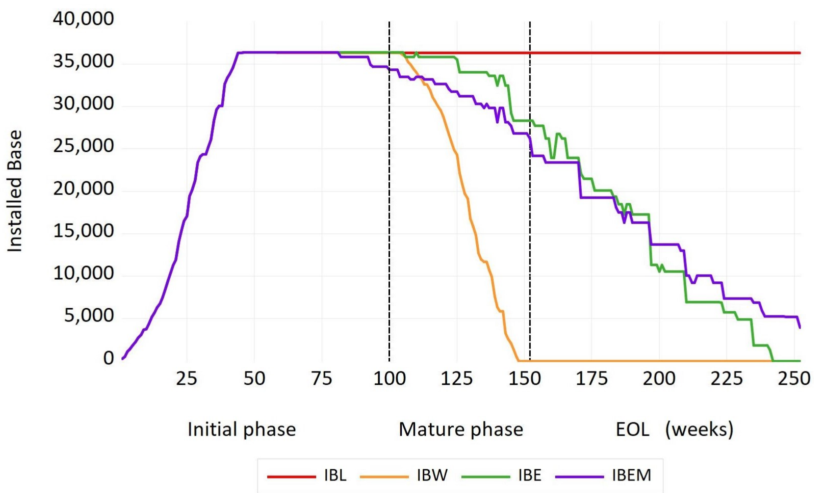

Figure 1: Size of installed base concepts for television type 1 (regarding LCD panel)

The available spare parts have their own value to the consumers of the products. In Table 2 is visible that spare parts differ from each other in necessity and price, resulting in differences in importance to the customers.

Table 2: Spare part characteristics and data information

		Essential	Expensive	Demand		Price (%)		Hypothesis
				1	2	1	2	
Refrigerator	Compressor	Yes	Yes	5,678	6,090	46.7	39.8	L
	Circuit board	No	Yes	9,596	3,518	7.6	5.6	E
	Door gasket	No	No	4,581	698	3.5	3.4	W
Television	LCD panel	Yes	Yes	868	889	47.0	39.4	W
	Circuit board	No	Yes	562	774	9.6	8.2	W
	Cover	No	No	152	230	3.2	4.5	W
Smartphone	Touch screen	Yes	Yes	21,499	58,413	19.8	25.8	1W, 2M
	Circuit board	No	Yes	6,325	14,492	28.6	40.0	1W, 2M
	Back cover	No	No	5,259	11,033	1.6	1.2	L

Essential indicates whether spare part is essential for product, and Expensive describes relative price of spare part compared to product price.
Demand is amount of spare parts demanded by customers for products of type 1 and type 2.
Price (%) is price of spare part as percentage of product price.
Hypothesis is the installed base concept that is expected to forecast best (L for IBL, W for IBW, E for IBE, M for IBM).
Source: Kim, Dekker, and Heij (2017).

Due to expected unequal demand behaviour, the four installed base concepts are expected to fit best for certain spare parts and thus to forecast best in specific situations. The four hypotheses given by Kim, Dekker, and Heij (2017) are as follows. (i) "The IBL, is expected to give the best forecasts for essential and expensive spare parts of non-trendy products with long lifetime." This is due to the decision whether the consumer will repair the product after it is covered by warranty, by trading off the potentially extended lifetime against the costs. When a product has a long lifetime, the benefits are larger. (ii) "The IBW is expected to better forecast the spare part demand for products with a short life cycle." This is due to the assumption that the consumer might only want to replace the failing component when it is covered by the warranty, as a repair does not extend its lifetime considerably. (iii) "Non-essential spare parts of products with a long lifetime are expected to get the best forecasts from IBE." When a non-essential part fails, the consumer has to make the decision whether the remaining lifetime compensates for the repair costs. If the failing spare part is expensive but not necessary, the product can still be used without replacing the component. Finally, (iv) "the IBM is expected to forecast the best if consumers differ much in their acceptance of new products." Based on these expectations, Table 2 also includes these hypotheses applied for every spare part. As refrigerators have a long expected lifetime, the essential compressor is expected to forecast best with the IBL. However, just like the circuit board, it could also be explained best by IBE, as a wide lifespan indicates that the remaining economic value will be greater than the repair cost for a considerable amount of time. Television spare parts are expected to be forecasted best by the IBW, as their utility declines rather fast over time. The estimation period of smartphone type 1 is not sufficiently long enough as that of type 2. Therefore, the hypotheses of the essential and expensive parts differ for both types. These hypotheses are being tested and discussed in the next section.

5 Results

5.1 Installed base models

To illustrate the modelling and forecasting procedure from Section 3.2, I examine in Section A.2 one specific spare part and follow through the roadmap of the code that is used. I have fully written the program myself in R and requires the following input; the spare part in question, the last week of the mature phase (t_2) and the last week of the EOL phase of the product (t_3). Running the code gives the forecast results of the black box model and the four installed base models for that specific spare part. This includes the five produced forecasts, its prediction errors and the three forecast measures for every model. I have written the program such that it automatically follows the steps elaborated in Section A.2, without interruption. The full script, including code for correcting the smoothed demand values of the refrigerators, can be found in Section A.5. The results of all spare parts regarding the television types can be found in Table 3. The forecast measures for the refrigerator and smartphone types can be found in Table 8 and 9 in Section A.3.

If we compare the results with those obtained in the research of Kim, Dekker, and Heij (2017), then 17 out of 90 forecasts give substantially different error measures. Especially for the LCD panel case detailed in Section A.2. The forecast for this spare part give measures between 2.00-3.00 for the lifetime installed base, whereas these values were found to be between 22.00-25.00 in the paper published in 2017. This seems like a significant improvement on the research of 2017, but some measures are higher than acquired before. This specific case is, however, the most extreme difference of the seventeen anomalies. The other sixteen cases state differences between 1.00 and 4.50 approximately. If we compare the television results in Table 3 with our hypotheses given in Table 2, then the best choice indeed seems using the warranty installed base to predict demand for the last three spare parts. On the other hand, the forecast results of the first three components do not show a clear suggestion for the warranty installed base.

Table 3: Forecast results for television spare parts

				Installed base			
Spare part	**Type**		**AR**	**L**	**W**	**E**	**M**
LCD panel	1	SUM	3.93	2.40	0.33	-1.00	-1.00
		MAPE	4.02	2.51	1.08	1.00	1.00
		RMSPE	4.25	2.67	1.72	1.75	1.75
	2	SUM	3.30	1.47	0.59	-1.00	-1.00
		MAPE	3.30	1.51	1.08	1.00	1.00
		RMSPE	3.58	1.69	1.51	1.43	1.43
Circuit board	1	SUM	2.20	-0.12	-0.37	-0.12	0.21
		MAPE	2.43	0.67	0.66	0.69	0.76
		RMSPE	2.64	0.92	0.97	0.97	0.98
	2	SUM	6.07	8.80	1.07	4.27	9.26
		MAPE	6.07	8.80	1.68	4.40	9.26
		RMSPE	7.15	10.83	2.33	5.82	11.36
Cover	1	SUM	13.83	6.92	2.80	6.99	7.47
		MAPE	14.44	7.59	3.45	7.65	8.12
		RMSPE	14.67	8.24	6.45	8.41	8.66
	2	SUM	4.38	1.53	0.96	1.61	1.56
		MAPE	4.76	2.24	1.76	2.30	2.27
		RMSPE	5.11	2.86	2.85	2.90	2.87

Underlined values are found to be the best results.

Overall, the results of six out of the eighteen spare parts seem to confirm their hypothesis, but twelve do not. However, in several cases the differences are not of large magnitude. Therefore, the results of the hypothesis installed base are tested against the seemingly best performing installed base according to the outcomes. These two installed base concepts can also be given as input in the estimation and forecast program given in Section A.5. This produces beside the forecasts results, also the p-values of the corresponding t-test and DM-test. These p-values regarding every spare parts one-sided null-hypothesis can be found in Table 4. As can be concluded from this table, few hypotheses are confirmed by the findings of this research. This is in contradiction with the paper by Kim, Dekker, and Heij (2017), which confirmed the stated hypothesis in about 50% of the tests. Whereas here, only about 25% is confirmed by the results.

Table 4: Results of hypothesis tests

	Type	Hypothesis	Outcome		Test			Conclusion
			2020	2017	H_0	SUM	MAPE	
Refrigerator								
Compressor	1	L	L	L	L >W	0.718	0.017	Confirmed (1x)
	2	L	M	L	M >L	0.000	0.014	Denied (2x)
Circuit board	1	E	M	E	M >E	0.000	0.104	Denied (1x)
	2	E	AR	AR	AR >E	0.000	0.746	Denied (1x)
Door gasket	1	W	AR	M	AR >W	0.999	0.999	Weakly denied (2x)
	2	W	AR	W	AR >W	0.000	0.000	Denied (1x)
Television								
LCD panel	1	W	W	W	W >E	-	0.433	Weakly confirmed (1x)
	2	W	W	W	W >M	-	0.697	Weakly confirmed (1x)
Circuit board	1	W	L	E	L >W	0.000	0.094	Denied (1x)
	2	W	W	W	W >E	0.000	0.000	Confirmed (2x)
Cover	1	W	W	W	W >L	0.000	0.000	Confirmed (2x)
	2	W	W	W	W >L	0.000	0.463	Confirmed (1x)
Smartphone								
Circuit board	1	W	E/M	W	M >W	-	0.000	Denied (1x)
	2	M	E	M	E >M	-	0.324	Weakly denied (1x)
Touch screen	1	W	E	W	E >W	0.000	0.000	Denied (2x)
	2	M	AR	W	AR >M	0.000	0.000	Denied (2x)
Back cover	1	L	W	W	W >L	0.000	0.000	Denied (2x)
	2	L	AR	W	AR >L	0.000	0.000	Denied (2x)

Outcome shows the installed base that provides the best forecasts in this research and results of Kim, Dekker, and Heij (2017). If best method is not clear, then method with best SUM is taken as outcome.

Test H_0, given by A > B, tests null-hypothesis that method A provides better forecasts than method B; if the outcome confirms the hypothesis, then A is the hypothesis and B is second-best method; if the outcome differs from the hypothesis, then A is the outcome and B is the hypothesis.

Test SUM and MAPE show the one-sided p-value (H_0 : A > B) of the t-test for mean error and Diebold-Mariano test for absolute errors, respectively. "-" corresponds with two forecast means with opposites signs, which makes it unable to preform a one-sided t-test.

Conclusion "Confirmed" states that the outcome equals the hypothesis and that it is significantly better than the second best outcome. "Weakly confirmed" states that the hypothesis is the best outcome, but not significantly better than the second best. "Denied" states that the outcome is significantly better than the hypothesis. "Weakly denied" states that the outcome is not significantly better than the hypothesis.

5.2 Adding estimated failure rates

As mentioned in Section 3.4, the failure rate function is estimated through MLE. Using RStudio, I estimate the loglikelihood function and generate the failure rate for every point in time with the code given in Section A.6. For the previously used example, the LCD panel component of television type 1, this results in the parameter estimations $\hat{\beta} = 0.013$ and $\hat{\eta} \approx 0.000$, which indicate a decreasing failure rate over time for this spare part. Moreover, small shape parameters are found for all eighteen spare parts, indicating high variability and a decrease in need for spare parts over time. The failure rate is calculated for every time period t with (12). The results of including this failure rate function into the models can be found in Table 5.

Table 5: Forecast results for television spare parts with estimated failure rates

						Installed base			
Spare part	**Type**	**Shape $\hat{\beta}$**	**Scale $\hat{\eta}$**		**AR**	**L**	**W**	**E**	**M**
LCD panel	1	0.013	0.000	SUM	3.93	2.37	0.81	-1.00	-1.00
				MAPE	4.02	2.42	1.49	1.00	1.00
				RMSPE	4.25	3.61	2.44	1.75	1.75
	2	0.021	0.000	SUM	3.30	1.40	0.64	-1.00	-1.00
				MAPE	3.30	1.44	1.12	1.00	1.00
				RMSPE	3.58	1.64	1.57	1.43	1.43
Circuit board	1	0.014	0.000	SUM	2.20	-0.10	-0.34	-0.08	0.26
				MAPE	2.43	0.67	0.66	0.71	0.77
				RMSPE	2.64	0.92	0.96	0.97	0.99
	2	0.017	0.000	SUM	6.07	8.66	1.08	4.23	9.26
				MAPE	6.07	8.66	1.69	4.37	9.26
				RMSPE	7.15	10.63	2.34	5.76	11.36
Cover	1	0.009	0.000	SUM	13.83	9.43	3.81	9.51	10.05
				MAPE	14.44	9.98	4.35	10.05	10.59
				RMSPE	14.67	10.73	8.17	10.96	11.21
	2	0.010	0.000	SUM	4.38	1.64	1.08	1.72	1.68
				MAPE	4.76	2.32	1.84	2.39	2.35
				RMSPE	5.11	2.93	2.95	2.97	2.94

Green indicates a decreased error measure and red indicates an increased error measure.
The AR results do not change as these do not depend on the failure rate.

As can be seen in the table, only 5 out of these 24 forecasts improved by adding estimated failure rates. The demand forecasts for the cover of the televisions even got worse in all cases. The results for the spare parts of the refrigerators and smartphones can be found in Tables 10 and 11 in Section A.4. For these two product categories, this approach is in general not beneficial. Except for the door gasket of the refrigerators, these forecasts tend to improve based on the change in forecast measures. In conclusion, it depends on the ageing of the spare part whether it is beneficial to also model the failure rates. Where the LCD panel and circuit board of the televisions give mixed results, the demand for door gaskets of the refrigerators are predicted better. Therefore, it seems that this refrigerator component ages the fastest of all parts, resulting in better demand explanation when estimating and including its failure rate function.

5.3 Varying lifetime

The code that implements the methods discussed in this section can be found in Section A.7.

5.3.1 Predefined alternatives

First, the lifetimes are varied by means of the five predefined lifespan alterations by Rogers (2010); $0.6L$ (2.5%), $0.7L$ (13.5%), $1.0L$ (34%), $1.05L$ (34%) and $1.3L$ (16%). The earliest a product could reach the end of its lifespan is 0.6 times the average. The average lifetime of the refrigerators is 676 weeks, 0.6 times 676 is 406, but there are only 315 and 295 weeks of data available for the refrigerator types. This is also a problem for television type 2. In these three cases, this variation of expected lifetime thus does not have an influence on the installed base concepts. The results of forecasting with varied expected lifetimes for the smartphone types and television type 1 can be found in Table 6.

Table 6: Changes in IBL forecast measures with predefined lifetime alternatives

	Type	**SUM**	Difference	**MAPE**	Difference	**RMSPE**	Difference
Television							
LCD panel	1	2.39	-0.01	2.51	-0.01	2.67	=
Circuit board	1	-0.13	-0.01*	0.67	=	0.93	+0.01
Cover	1	6.90	-0.02	7.56	-0.03	8.23	-0.01
Smartphone							
Touch screen	1	5.79	-0.82	5.80	-0.81	6.11	-0.84
	2	1.22	-0.01	1.27	=	1.47	=
Circuit board	1	3.38	-1.16	3.52	-1.24	3.77	-1.06
	2	5.44	+0.37	5.44	+0.37	6.18	+0.50
Back cover	1	9.63	+4.72	9.93	+4.72	14.86	+9.09
	2	0.73	=	1.13	-0.01	1.37	+0.01

Difference indicates the subtraction of old result from new result: a negative sign indicates that forecast measure has decreased and thus, new forecast is better based on this specific forecast measure.
* as the measure is already of negative size, the decrease here is not an improvement.

It can be concluded that out of the nine spare parts, only two forecasts are worse when using flexible lifetimes. These being the circuit board of smartphone type 2 and the back cover of smartphones of type 1. On the other hand, there are also only two cases for which the method improves the forecasts substantially; the touch screen and circuit board of smartphone type 1.

5.3.2 Weibull distribution

Instead of letting the lifetimes be one out of five options, I now sample the lifetimes from a Weibull distribution. The parameters that are used for the refrigerators are $\beta = 2.15$ and $\eta = 18.76$. These parameters change the average lifetime from 13 years to 16.6 years, but are suggested by Welch and Rogers (2010) for refrigerators specifically. The parameters used for the televisions are found by Kalmykova et al. (2015), where the parameters were estimated for lifetime data of LCD televisions with an average lifespan of six years. They find the shape parameter $\beta = 3.75$ and scale parameter $\eta = 6.45$. The parameters used for the smartphones originate from the research done by He, Wang, and Zuo (2018), where they investigated the lifespan of several mobile phones. The resulting parameters for smartphones are given by $\beta =$ 2.45 and $\eta = 2.83$. These three Weibull specifications are used to redetermine when a product reaches its end-of-life after it is sold to the customer. The results from forecasting with the newly obtained IBL and average age values can be found in Table 7 for every product.

Table 7: Changes in IBL forecast measures with Weibull distributed lifetimes

	Type	**SUM**	Difference	**MAPE**	Difference	**RMSPE**	Difference
Refrigerator							
Compressor	1	0.80	-0.01	0.90	-0.01	1.04	-0.01
	2	0.32	-0.01	0.62	=	0.74	-0.01
Circuit board	1	0.09	=	0.26	=	0.34	=
	2	0.62	+0.01	0.69	+0.01	0.84	+0.01
Door gasket	1	0.19	=	0.54	=	0.68	=
	2	2.20	-0.01	2.27	-0.01	2.48	-0.01
Television							
LCD panel	1	2.30	-0.10	2.42	-0.09	2.58	-0.09
	2	1.41	-0.06	1.45	-0.06	1.64	-0.05
Circuit board	1	-0.19	-0.07*	0.68	+0.01	0.93	+0.01
	2	9.11	+0.31	9.11	+0.31	11.30	+0.47
Cover	1	6.52	-0.40	7.18	-0.41	7.98	-0.26
	2	1.45	-0.08	2.18	-0.06	2.83	-0.03
Smartphone							
Touch screen	1	6.50	-0.11	6.51	-0.10	6.83	-0.12
	2	1.23	=	1.27	=	1.47	=
Circuit board	1	4.37	-0.17	4.51	-0.25	4.65	-0.18
	2	5.08	+0.01	5.08	+0.01	5.69	+0.01
Back cover	1	5.10	+0.19	5.40	+0.19	6.07	+0.30
	2	0.73	=	1.12	=	1.36	=

Difference indicates the subtraction of old result form new result: a negative sign indicates that measure has decreased and thus, new forecast is better based on this specific forecast measure.

* as the measure is already of negative size, the decrease here is not an improvement.

The first thing that can be concluded from the table is that varying the lifetime does little to the refrigerator forecasts. Secondly, the forecast measures of the televisions' LCD panels and covers seem to decrease. Which indicate better forecasts for these spare parts, but the circuit board forecasts tend to get worse. For smartphone type 1, only the forecasts of demand for a new back cover get worse. The demand for the other two components is predicted better with a varied lifetime. Similar to the refrigerators, the forecasts for smartphone type 2 seem to differ very little to none. In short, the demand for six spare parts are predicted better, those of nine do not change significantly and three spare part predictions got worse.

6 Discussion & Conclusion

Stopping the production of a product brings the decision on how many spare parts the manufacturer has to produce in the final production run. This depends heavily on the size expectation of spare part demand in the end-of-life phase. The installed base of products is a measure which correlates heavily with the demand for its spare parts. Which brings me to the research question of this paper: *Which installed base concept produces the best forecast performance for what type of spare part?*. This paper proposed several methods to use installed base concepts to predict the size of spare part demand for consumer products. In addition to a replication of the research done by Kim, Dekker, and Heij (2017), three standalone extensions of the original methods have been performed. First, by estimating the failure rate of the spare parts and adding these to the models. Second, by relaxing the assumption of a fixed lifespan by attaining a different lifetime to every unit that is sold over the initial and mature phases. This is done by (i) attaining one of the five lifetime alternatives defined by Rogers (2010) or (ii) by sampling a lifetime from uniquely specified Weibull distributions. These methods were applied to eighteen spare parts belonging to six products from three product categories to give an answer to the three sub questions.

Do all forecasts of spare part demand improve when adding installed base information as explanatory factor? Unfortunately, replicating the research of Kim, Dekker, and Heij (2017) did not give the previously obtained results. 44% of the outcomes regarding the preferred installed base type were equal to those obtained in 2017. However, forecasting with installed base information is in 72% of the cases better than forecasting with a simple black box model. The hypothesis that the warranty installed base would forecast best for the television spare parts can be confirmed for the most part, suggesting that the warranty installed base is indeed expected to better forecast the spare part demand for products that experience a rather fast decline in utility. On the other hand, the remainder of the hypotheses cannot be confirmed by the results of this research.

Are forecasts being improved when adding estimated failure rate information of the spare part to the installed base models? Estimating the failure rate function of a spare part and adding this to the installed base models results in different effects. There is only one spare part that is predicted noticeably better and for two components adding their failure rates has its pros and cons. In general, executing this method is not in favour of the forecast performance of the installed base models.

Are forecasts being improved by letting the product units that are sold vary in expected lifetime length? The first method of varying the lifespans only gave in 22% of the cases better forecasts, but also 22% of the predictions showed a decreased forecasting performance. However, sampling the lifetimes from a Weibull distribution gave more promising results. 17% of the forecasts got worse and 33% got better. The only clear pattern in these results is that varying the lifetime of refrigerators does little to the forecast performance. Which can be argued by the fact that refrigerators have a very long lifetime. Variations in this average lifetime of sixteen years did not impact the lifetime installed base substantially as the available data only captured six years.

In conclusion, *Which installed base concept produces the best forecast performance for what type of spare part?*. The previously obtained results of 2017 are only partly confirmed by the research in this paper. As I have applied the methodology that is stated in the paper by Kim, Dekker, and Heij (2017), I question the full reproducability of the estimation and forecast procedure. Presumably, the used methodology contains several details that have been left out in writing the report. Concluding which installed base is best for which type of spare part is thus not possible based on the results of this research. Furthermore, the newly introduced methods improved the forecasts in some cases, but did not change which installed base performed best. Therefore, it seems that the installed base concepts indeed perform best for certain spare parts and that extending the methodology only impacts the size of the predictive power. The forecast performance mostly decreased when estimating the failure rate function. This method is not suggested when dealing with larger amount of spare parts, as it depends on the ageing of the spare part whether this approach would be beneficial. Moreover, the failure rate is estimated with demand occurrences for the spare part, not with actual failure data. There could also be consumers who do not want to replace the component when it fails. The same advice holds for attaining different lifetimes by means of the five alternatives, where no clear suggestion could be concluded from the results. Sampling lifetimes from the Weibull distributions, however, could be beneficial for forecasting spare part demand. For further research I suggest examining these newly proposed methods in combination with the correct methodology used by Kim, Dekker, and Heij (2017), as this could possibly result in different conclusions.

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A Appendix

A.1 Data replication

In this section, I try to replicate the given data for the compressor spare part of the first type of refrigerator. Originally, the following data are available to the previous researchers:

1. *S(t)*; the amount of units sold in period t.
2. *R(t)*; the amount of units returned to the store in period t.
3. *D(t)*; the real demand for the spare part in period t.
4. *'out of life'(t)*; the amount of units exceeding their expected lifetime in period t.
5. *out of warranty(t)*; the amount of units exceeding their warranty period in period t.

The following variables are also given, but computed by Kim, Dekker, and Heij (2017):

1. $D_s(t)$; the smoothed demand for the spare part in period t.
2. *IBL(t), IBW(t), IBE(t)* & *IBM(t)*; the size of the installed base concepts in period t.
3. $IBL_{nr}(t)$ & $IBW_{nr}(t)$; IBL and IBW in period t without taking the returns into account.
4. *AGEP(t), AGEW(t), AGEE(t)* & *AGEM(t)*; the mean age of the installed base concepts in period t.

Smoothing the demand

In order to smooth the demand, the exponentially weighted moving average method is used, with smoothing factor $\alpha = 0.06$. This should lead to the following recursive relation:

$$D_s(t) = \alpha \times D(t) + (1-\alpha) \times D_s(t-1) \tag{15}$$

$$= 0.06 \times D(t) + 0.94 \times D_s(t-1). \tag{16}$$

Running this recursive calculation with the R code below, gives indeed the smoothed demand given in the dataset. However, the smoothed demand of the refrigerators lags one time unit. Code for correcting this error is included in the model estimation code of Section A.5.

```
DSMO <- rep(0, 315)
for (i in 1:315){
  if (i != 1){
 DSMO[i] <- data$DACT[i]*0.06 + DSMO[i-1]*0.94
  }
}
```

Installed base concepts

The IBL and IBW are calculated through (1) and (2). When running the R code for calculating these vectors, as can be seen below, the values are indeed equal to those given in the dataset.

```
IBL <- rep(0, 315)
IBW <- rep(0, 315)
IBL[1] <- data$sales[1] - data$return[1]
IBW[1] <- data$sales[1] - data$return[1]
for (t in 2:315){
  IBL[t] <- IBL[t-1] + data$sales[t] - data$return[t] - data$`out of life`[t]
  IBW[t] <- IBW[t-1] + data$sales[t] - data$return[t] - data$`out of warranty`[t]
}
IBW <- ifelse(IBW < 0, 0, IBW)
```

For the IBE and IBM the repair cost and remaining economic value have to be known in order to construct $E_i(t)$ in (3). As this information is not available, the repair cost is set equal to the price of the spare part and the remaining economic value of the product is determined by assuming exponential value decay and final unity value at the end of the expected lifetime (Kim, Dekker, and Heij (2017)). As the price of the spare part is only available in a percentage of the product price, I assume the price of the spare part on time t ($c(t)$) is equal to the price of the product on time t (p_t) times this percentage. This leads to the following equation

$$v_i(t) = p_i \times \exp\{a_i \times (t - i)\}. \tag{17}$$

With p_i the price of the product bought at time i and a_i the decay rate for products sold in this period: $a_i = -ln(p_i)/L$. Obtained from the condition that $1 = p_i \times \exp\{a_i \times L\}$. The R code given below should produce the economic installed base values for the refrigerator type 1, unfortunately the outcome is different.

```
IBE <- rep(0, 315)
for (t in 1:315){
  for (i in 1:min(t, 279)){
    a <- -log(p[i])/L
    v <- p[i]*exp(a*(t-i))
    if (v>(p[min(t, 279)]*0.467)){
      IBE[t] <- IBE[t] + data$sales[i] - data$return[i]
    }
  }
}
IBE <- ifelse(IBE < 0, 0, IBE)
```

Due to this circumstance, replicating the mixed economic installed base is not possible. The explanation of the computations made by Kim, Dekker, and Heij (2017) are not detailed enough to reproduce. Moreover, the example elaborated in the paper of 2017, does not equal the information given in the paper and the data that is available to me. For example, the price of the refrigerator type 1 is said to be €550 instead of an average price of €220 in the given dataset. The price of the spare part is said to equal €100, which is 18.3% of the product price. However, the table which is given in the paper states that the price of the spare part is 46.7%.

Mean age

The mean age of the installed base concepts is defined for the end of each time period, that is, the products bought in period t are in this period treated as already 1 time period old. This results in the following calculation for the mean age of the lifetime and warranty installed bases

$$AGEP(t) = \frac{(AGEP(t-1)+1) \times IBL_{nr}(t-1) + S(t) - (L+2) \times \mathit{out\ of\ life}(t)}{IBL_{nr}(t)} \tag{18}$$

$$AGEW(t) = \frac{(AGEW(t-1)+1) \times IBW_{nr}(t-1) + S(t) - (W+1) \times \mathit{out\ of\ warranty}(t)}{IBW_{nr}(t)} \tag{19}$$

With the R code below, the values given in the dataset are acquired.

```
AGEL <- rep(0, 315)
AGEW <- rep(0, 315)
AGEL[1] = 1
AGEW[1] = 1
for (t in 2:315){
  AGEP[t] <- ((AGEP[t-1]+1)*data$`IBL(no return)`[t-1]+data$sales[t])/data$`IBL(no
      return)`[t]
  AGEW[t] <- ((AGEW[t-1]+1)*data$`IBW(no return)`[t-1]+data$sales[t]-(105)*data$`
    out of warranty`[t])/data$`IBW(no return)`[t]
}
```

A.2 Estimation and forecast procedure

To illustrate the modelling and forecasting procedure from Section 3.2, I examine one spare part and follow through the roadmap of the code that is used. The spare part that will be examined in specific is the LCD panel of television type 1. First, the AR order p has to be determined. This is done by regressing the dependent variable $ln(1+D_s(t))$ on a constant and forming an AR model with the produced residuals. As Section 3.2 describes, the order p is increased until the next added lag of residuals is found to be insignificant. For this LCD panel, significant lags for the residuals are found for $p = 2$, resulting in the following relation: $\varepsilon_t = 1.19\varepsilon_{t-1} - 0.22\varepsilon_{t-2} + \omega_t$.

With this equation, the model in (4) is rewritten as follows. Let Y_t be equal to $ln(1 + D_s(t))$, $X_{1,t}$ equal to $ln(1 + IB(t))$ and let $X_{2,t}$ be $AGE(t)$, then (4) is equivalent to

$$\begin{aligned} Y_t &= b_0 + b_1 \times X_{1,t} + b_2 \times X_{2,t} + \varepsilon_t && (20) \\ &= b_0 + b_1 \times X_{1,t} + b_2 \times X_{2,t} + 1.19\varepsilon_{t-1} - 0.22\varepsilon_{t-2} + \omega_t && (21) \\ &= b_0 + b_1 \times X_{1,t} + b_2 \times X_{2,t} + 1.19 \times (Y_{t-1} - (b_0 + b_1 \times X_{1,t-1} + b_2 \times X_{2,t-1})) \\ &\quad - 0.22 \times (Y_{t-2} - (b_0 + b_1 \times X_{1,t-2} + b_2 \times X_{2,t-2})) + \omega_t. && (22) \end{aligned}$$

Which simplifies to

$$\begin{aligned} Y_t - 1.19Y_{t-1} + 0.22Y_{t-2} &= (b_0 - 1.19b_0 + 0.22b_0) \\ &\quad + (b_1X_{1,t} - 1.19b_1X_{1,t-1} + 0.22b_1X_{1,t-2}) && (23) \\ &\quad + (b_2X_{2,t} - 1.19b_2X_{2,t-1} + 0.22b_2X_{2,t-2}) + \omega_t \\ Y_t - 1.19Y_{t-1} + 0.22Y_{t-2} &= b_0(1 - 1.19 + 0.22) + b_1(X_{1,t} - 1.19X_{1,t-1} + 0.22X_{1,t-2}) \\ &\quad + b_2(X_{2,t} - 1.19X_{2,t-1} + 0.22X_{2,t-2}) + \omega_t && (24) \\ \frac{Y_t^*}{c_0} &= b_0 + b_1 \times \frac{X_{1,t}^*}{c_0} + b_2 \times \frac{X_{2,t}^*}{c_0} + \omega_t && (25) \\ Y_t^{**} &= b_0 + b_1 \times X_{1,t}^{**} + b_2 \times X_{2,t}^{**} + \omega_t. && (26) \end{aligned}$$

With $c_0 = (1 - 1.19 + 0.22) = 0.03$, $Y_t^{**} = (Y_t - 1.19Y_{t-1} + 0.22Y_{t-2})/c_0$ and $X_{i,t}^{**} = (X_{i,t} - 1.19X_{i,t-1} + 0.22X_{i,t-2})/c_0$. This model is estimated with least squares and gives the following coefficients for the lifetime installed base; $\hat{b_0} = 2.50$, $\hat{b_1} = -0.01$ and $\hat{b_2} = -0.005$. This indicates a negative relation between installed base and the demand for the LCD panel. Hereby, the installed base values are set equal to zero while forecasting. To obtain the black box model, b_1 and b_2 are set to equal zero before performing least squares, resulting in a regression of Y_t^{**} on a constant. This gives a b_0 estimate of 2.11.

With these two models, forecasts of smoothed demand are produced by inserting the data of the explanatory variables into an alteration of (26). For (i) the black box model this is given as

$$Y_{t+1}^{**} = 2.11 \quad (27)$$

$$Y_{t+1}^{*} = 2.11 \times 0.03 \quad (28)$$

$$Y_{t+1} = 0.08 + 1.19Y_t - 0.22Y_{t-1}. \quad (29)$$

and for (ii) the (lifetime) installed base given as

$$Y^{**}_{t+1} = 2.50 - 0.01 \times X^{**}_{1,t+1} - 0.005 \times X^{**}_{2,t+1} \tag{30}$$

$$Y^{*}_{t+1} = 2.50 \times 0.03 - 0.01 \times X^{*}_{1,t+1} - 0.005 \times X^{*}_{2,t+1} \tag{31}$$

$$Y_{t+1} = 0.09 - 0.01 \times X^{*}_{1,t+1} - 0.005 \times X^{*}_{2,t+1} + 1.19Y_t - 0.22Y_{t-1}. \tag{32}$$

Where the lifetime installed base $X^*_{1,t+1}$ is equal to zero, as argued before. The forecasts of smoothed demand calculated with (29) and (32) are compared with the real demand values in order to produce the forecast measures and test statistics stated in Section 3.3.

A.3 Forecast results

Table 8: Forecast results for refrigerator spare parts

				Installed base			
Spare part	**Type**		**AR**	**L**	**W**	**E**	**M**
Compressor	1	SUM	1.22	0.81	<u>0.73</u>	1.27	1.27
		MAPE	1.32	<u>0.91</u>	1.07	1.36	1.36
		RMSPE	1.47	<u>1.05</u>	1.24	1.52	1.52
	2	SUM	0.79	0.33	0.57	0.43	<u>0.26</u>
		MAPE	0.96	0.62	0.80	0.70	<u>0.58</u>
		RMSPE	1.07	0.75	0.90	0.80	<u>0.73</u>
Circuit board	1	SUM	-0.10	0.09	-0.21	0.06	<u>0.04</u>
		MAPE	0.27	0.26	0.38	<u>0.25</u>	<u>0.25</u>
		RMSPE	0.33	0.34	0.54	0.33	<u>0.32</u>
	2	SUM	<u>-0.09</u>	0.61	0.11	0.64	0.55
		MAPE	<u>0.27</u>	0.68	0.31	0.72	0.63
		RMSPE	<u>0.36</u>	0.83	0.38	0.87	0.75
Door gasket	1	SUM	<u>-0.09</u>	0.19	-0.12	0.17	-0.18
		MAPE	<u>0.38</u>	0.54	0.53	0.52	0.53
		RMSPE	<u>0.60</u>	0.68	0.69	0.67	0.67
	2	SUM	<u>1.43</u>	2.21	2.27	2.21	2.22
		MAPE	<u>1.61</u>	2.28	2.33	2.28	2.28
		RMSPE	<u>1.80</u>	2.49	2.54	2.49	2.50

Underlined values are found to be the best results.

Table 9: Forecast results for smartphone spare parts

				Installed base			
Spare part	**Type**		**AR**	**L**	**W**	**E**	**M**
Touch screen	1	SUM	5.82	6.61	3.11	-1.00	-1.00
		MAPE	5.84	6.61	3.17	1.00	1.00
		RMSPE	6.16	6.95	4.63	2.07	2.07
	2	SUM	0.64	1.23	1.25	-0.08	0.29
		MAPE	0.80	1.27	1.29	0.84	0.87
		RMSPE	0.95	1.47	1.49	1.03	1.07
Circuit board	1	SUM	4.80	4.54	2.32	1.34	3.03
		MAPE	4.96	4.76	2.60	1.71	3.25
		RMSPE	5.15	4.83	3.64	2.62	3.97
	2	SUM	1.45	5.07	4.15	1.57	2.39
		MAPE	1.50	5.07	4.15	1.68	2.41
		RMSPE	1.63	5.68	4.51	1.88	2.59
Back cover	1	SUM	3.30	4.91	1.01	5.41	5.38
		MAPE	3.64	5.21	1.76	5.70	5.68
		RMSPE	3.94	5.77	2.31	6.43	6.46
	2	SUM	0.38	0.73	0.72	0.67	1.05
		MAPE	0.83	1.12	1.11	1.06	1.42
		RMSPE	1.11	1.36	1.35	1.30	1.67

Underlined values are found to be the best results.

A.4 Forecast results with estimated failure rates

Table 10: Forecast results for refrigerator spare parts with estimated failure rates

						Installed base			
Spare part	**Type**	$\hat{\beta}$	$\hat{\eta}$		**AR**	**L**	**W**	**E**	**M**
Compressor	1	0.034	0.445	SUM	1.22	0.83	0.75	1.29	1.30
				MAPE	1.32	0.93	1.09	1.39	1.39
				RMSPE	1.47	1.07	1.26	1.55	1.55
	2	0.060	4.728	SUM	0.79	0.70	1.01	0.84	0.62
				MAPE	0.96	0.89	1.14	1.00	0.83
				RMSPE	1.07	0.97	1.26	1.10	0.92
Circuit board	1	0.167	23.133	SUM	-0.10	0.11	-0.19	0.09	0.07
				MAPE	0.27	0.26	0.38	0.26	0.25
				RMSPE	0.33	0.35	0.54	0.34	0.33
	2	0.044	0.903	SUM	-0.09	0.66	0.17	0.70	0.61
				MAPE	0.27	0.74	0.34	0.77	0.68
				RMSPE	0.36	0.89	0.41	0.93	0.81
Door gasket	1	0.042	0.954	SUM	-0.09	0.18	-0.11	0.17	-0.18
				MAPE	0.38	0.53	0.53	0.52	0.53
				RMSPE	0.60	0.67	0.70	0.67	0.67
	2	0.016	1.089	SUM	1.43	2.01	2.11	2.02	2.03
				MAPE	1.61	2.09	2.18	2.10	2.11
				RMSPE	1.80	2.31	2.40	2.32	2.32

Green indicates a decreased error measure and red indicates an increased error measure.
The AR results do not change as these do not depend on the failure rate.

Table 11: Forecast results for smartphone spare parts with estimated failure rates

						Installed base			
Spare part	**Type**	$\hat{\beta}$	$\hat{\eta}$		**AR**	**L**	**W**	**E**	**M**
Touch screen	1	0.015	0.000	SUM	5.82	9.28	4.72	-1.00	-1.00
				MAPE	5.84	9.28	4.77	1.00	1.00
				RMSPE	6.16	9.69	6.95	2.07	2.07
	2	0.080	36.498	SUM	0.64	2.20	2.85	0.51	1.43
				MAPE	0.80	2.22	2.87	1.41	1.99
				RMSPE	0.95	2.40	3.18	1.77	2.53
Circuit board	1	0.056	13.745	SUM	4.80	7.12	4.00	1.96	4.47
				MAPE	4.96	7.25	4.26	2.32	4.67
				RMSPE	5.15	7.48	6.09	3.59	5.72
	2	0.156	227.693	SUM	1.45	14.22	14.15	8.60	11.02
				MAPE	1.50	14.22	14.15	8.67	11.02
				RMSPE	1.63	15.79	15.70	10.16	12.07
Back cover	1	0.024	0.007	SUM	3.30	4.66	1.13	5.48	5.50
				MAPE	3.64	4.96	1.88	5.78	5.79
				RMSPE	3.94	5.47	2.48	6.50	6.55
	2	0.123	35.764	SUM	0.38	0.90	1.22	0.96	1.30
				MAPE	0.83	1.26	1.57	1.32	1.64
				RMSPE	1.11	1.45	1.79	1.51	1.84

Green indicates a decreased error measure and red indicates an increased error measure.

The AR results do not change as these do not depend on the failure rate.

A.5 Code for modelling and forecasting

This R code gives the forecast results of the black box model and the four installed base types in the form of vectors with the name corresponding to the concerned measure or outcome. This also includes the tests for hypothesis versus outcome, several additional forecast tests and code for correcting the incorrect smoothed demand values of the refrigerators. In lines 85 through 93, the excel sheet and the other variables can be changed to examine a different spare part.

```
1 'import'
2 library(readxl)
3 library(forecast)
4 library(Hmisc)
5 library(lmtest)
6
7 REF1_E <- read_excel("REF1_AGE.xlsx", sheet = "REF1_E")
8 REF1_M <- read_excel("REF1_AGE.xlsx", sheet = "REF1_M")
9 REF1_C <- read_excel("REF1_AGE.xlsx", sheet = "REF1_C")
10 REF2_E <- read_excel("REF2_AGE.xlsx", sheet = "REF2_E")
11 REF2_M <- read_excel("REF2_AGE.xlsx", sheet = "REF2_M")
12 REF2_C <- read_excel("REF2_AGE.xlsx", sheet = "REF2_C")
13
14 CTV1_E <- read_excel("CTV1_AGE.xlsx", sheet = "CTV1_E")
15 CTV1_M <- read_excel("CTV1_AGE.xlsx", sheet = "CTV1_M")
16 CTV1_C <- read_excel("CTV1_AGE.xlsx", sheet = "CTV1_C")
17 CTV2_E <- read_excel("CTV2_AGE.xlsx", sheet = "CTV2_E")
18 CTV2_M <- read_excel("CTV2_AGE.xlsx", sheet = "CTV2_M")
19 CTV2_C <- read_excel("CTV2_AGE.xlsx", sheet = "CTV2_C")
20
21 MOB1_E <- read_excel("MOB1_AGE.xlsx", sheet = "MOB1_E")
22 MOB1_M <- read_excel("MOB1_AGE.xlsx", sheet = "MOB1_M")
23 MOB1_C <- read_excel("MOB1_AGE.xlsx", sheet = "MOB1_C")
24 MOB2_E <- read_excel("MOB2_AGE.xlsx", sheet = "MOB2_E")
25 MOB2_M <- read_excel("MOB2_AGE.xlsx", sheet = "MOB2_M")
26 MOB2_C <- read_excel("MOB2_AGE.xlsx", sheet = "MOB2_C")
27
28 'correcting for refrigerator type 1'
29 DSMOnew <- matrix(0, 315, 3)
30 data <- REF1_E
31 t2 <- 279
32 t3 <- 315
33 t4 <- t3-t2
34
```

```
for (a in 1:3){
  if (a == 2){
    data <- REF1_M
  }
  if (a == 3){
    data <- REF1_C
  }

  'expontential weighted moving average smoothing'
  DSMO <- rep(0, t3)
  DSMO <- data$DACT[1]
  for (i in 2:t3){
    DSMO[i] <- data$DACT[i]*0.06 + DSMO[i-1]*0.94
  }
  DSMO <- round(DSMO, digits = 2)
  DSMOnew[,a] <- DSMO
}
REF1_E$DSMO <- DSMOnew[,1]
REF1_M$DSMO <- DSMOnew[,2]
REF1_C$DSMO <- DSMOnew[,3]

'correcting for refrigerator type 2'
DSMOnew <- matrix(0, 295, 3)
data <- REF2_E
t2 <- 229
t3 <- 295
t4 <- t3-t2

for (a in 1:3){
  if (a == 2){
    data <- REF2_M
  }
  if (a == 3){
    data <- REF2_C
  }

  'expontential weighted moving average smoothing'
  DSMO <- rep(0, t3)
  DSMO <- data$DACT[1]
  for (i in 2:t3){
    DSMO[i] <- data$DACT[i]*0.06 + DSMO[i-1]*0.94
  }
```

```r
  DSMO <- round(DSMO, digits = 2)
  DSMOnew[,a] <- DSMO

}
REF2_E$DSMO <- DSMOnew[,1]
REF2_M$DSMO <- DSMOnew[,2]
REF2_C$DSMO <- DSMOnew[,3]

'--->INPUT<---; change for different results
(outcome and hypothesis correspond with index:
BB = 1, IBL = 2, IBW = 3, IBE = 4, IBM = 5)'
data <- CTV1_C
t2 <- 100
t3 <- 252
t4 <- t3-t2
outcome <- 3
hypothesis <- 2

'setting all non-numeric values equal to zero'
for (c in 1:t2){
  if (data$DACT[c]=="-" || is.na(data$DACT[c])){
    data$DACT[c] <- 0
  }
  if (data$DSMO[c]=="-" || is.na(data$DSMO[c])){
    data$DSMO[c] <- 0
  }
  if (data$sales[c]=="-" || is.na(data$sales[c])){
    data$sales[c] <- 0
  }
}
data$DACT <- as.numeric(data$DACT)
data$DSMO <- as.numeric(data$DSMO)
data$sales <- as.numeric(data$sales)

'initiating result matrices and arrays'
forecast <- matrix(0, t4, 5)
colnames(forecast) <- c("BB", "IBL", "IBW", "IBE", "IBM")
errorf <- matrix(0, t4, 5)
colnames(errorf) <- c("BB", "IBL", "IBW", "IBE", "IBM")
sum <- rep(0, 5)
mape <- rep(0, 5)
rmspe <- rep(0, 5)
```

```

'computational code is looping for every installed base type'
for (a in 2:5){
  IBX <- 0
  AGE <- 0
  for (b in 1:t3){
    IBX[b] <- as.numeric(data[b,(7+a)])
    AGE[b] <- as.numeric(data[b,(13+a)])
  }

  'defining variables for simplicity'
  y <- log(data$DSMO[1:t2]+1)
  ytest <- data$DACT[t2+1:t3]
  ytest <- ytest[1:t4]
  x <- cbind(log(IBX+1), AGE)

  'determining the AR order p'
  BB <- lm(log(1+data$DSMO[1:t2])~1)
  pval <- 0
  p <- 0
  while (all(pval < 0.05)) {
    p <- p+1
    reslag <- Lag(BB$residuals, 1)
    if(p>1){
      for(i in 2:p){
        reslag <- cbind(reslag, Lag(BB$residuals, i))
      }
    }
    AR <- lm(BB$residuals ~ reslag)
    for (i in 1:p) {
      pval[i] <- coeftest(AR)[i+1,4]
    }
  }
  p <- p-1
  AR <- lm(BB$residuals ~ reslag[,1:p])

  'estimating the models'
  b0 <- 1
  ynew <- y
  x1new <- x[,1]
  x2new <- x[,2]
  for (i in 1:p){
```

```
    b0 <- b0 - AR$coefficients[i+1]
    ynew <- ynew - AR$coefficients[i+1]*Lag(y, i)
    x1new <- x1new - AR$coefficients[i+1]*Lag(x[,1], i)
    x2new <- x2new - AR$coefficients[i+1]*Lag(x[,2], i)
  }
  ynew[1] <- y[1]
  x1new[1] <- x[1,1]
  x2new[1] <- x[1,2]
  if(p>1){
    ynew[2] <- y[2]-AR$coefficients[2]*y[1]
    x1new[2] <- x[2,1]-AR$coefficients[2]*x[1,1]
    x2new[2] <- x[2,2]-AR$coefficients[2]*x[1,2]
  }
  if(p>2){
    ynew[3] <- y[3]-AR$coefficients[2]*y[2]-AR$coefficients[3]*y[1]
    x1new[3] <- x[3,1]-AR$coefficients[2]*x[2,1]-AR$coefficients[3]*x[1,1]
    x2new[3] <- x[3,2]-AR$coefficients[2]*x[2,2]-AR$coefficients[3]*x[1,2]
  }
  xnew <- cbind(x1new, x2new)/b0
  xtrainnew <- cbind(x1new[1:t2], x2new[1:t2])/b0
  ynew <- ynew/b0
  IBmodel <- lm(ynew~xtrainnew)
  BBmodel <- lm(ynew~1)

  'forecasting smoothed demand with IB'
  if (IBmodel$coef[2]<0){
    xnew[,1] <- 0
  }
  for (i in (t2+1):t3){
    ylagged <- 0
    for (j in 1:p){
      ylagged <- ylagged + AR$coefficients[j+1]*y[i-j]
    }
    if (IBX[i]>0){
      y[i] <- IBmodel$coefficients[1]*b0 + IBmodel$coefficients[2]*xnew[i,1]*b0 +
    IBmodel$coefficients[3]*xnew[i,2]*b0 + ylagged
    }else{
      y[i] <- 0
    }
  }
  f <- y[t2+1:t3]
  f <- f[1:t4]
```

```
  forecast[,a] <- exp(f)-1
  errorf[,a] <- ytest - forecast[,a]

  'forecasting smoothed demand with BB'
  for (i in (t2+1):t3){
    ylagged <- 0
    for (j in 1:p){
      ylagged <- ylagged + AR$coefficients[j+1]*y[i-j]
    }
    y[i] <- BBmodel$coefficients[1]*b0 + ylagged
  }
  fBB <- y[t2+1:t3]
  fBB <- fBB[1:t4]
  forecast[,1] <- exp(fBB)-1
  errorf[,1] <- ytest - fBB

  'forecast error measures IB'
  sumnom <- 0
  mapenom <- 0
  rmspenom <- 0
  denom <- 0
  for (i in 1:t4){
    sumnom <- sumnom + forecast[i,a]-ytest[i]
    mapenom <- mapenom + abs(forecast[i,a]-ytest[i])
    rmspenom <- rmspenom + (forecast[i,a]-ytest[i])^2
    denom <- denom + ytest[i]
  }
  sum[a] <- sumnom/denom
  mape[a] <- mapenom/denom
  rmspe[a] <- sqrt(rmspenom)/(denom/sqrt(t3-t2+1))

  'forecast error measures BB'
  sumnom <- 0
  mapenom <- 0
  rmspenom <- 0
  denom <- 0
  for (i in 1:t4){
    sumnom <- sumnom + forecast[i,1]-ytest[i]
    mapenom <- mapenom + abs(forecast[i,1]-ytest[i])
    rmspenom <- rmspenom + (forecast[i,1]-ytest[i])^2
    denom <- denom + ytest[i]
  }
```

```
  sum[1] <- sumnom/denom
  mape[1] <- mapenom/denom
  rmspe[1] <- sqrt(rmspenom)/(denom/sqrt(t3-t2+1))

}


'outcome vs hypothesis: t-test'
if (mean(errorf[,outcome])<0 && mean(errorf[,hypothesis])<0){
  t <- t.test(errorf[,outcome], errorf[,hypothesis], paired = TRUE, alternative =
    "greater")
  better_tpval <- t$p.value
  if (t$p.value<0.05){
    better_t <- "yes"
  }else{
    better_t <- "no"
  }
}
if (mean(errorf[,outcome])>0 && mean(errorf[,hypothesis])>0){
  t <- t.test(errorf[,outcome], errorf[,hypothesis], paired = TRUE, alternative =
    "less")
  better_tpval <- t$p.value
  if (t$p.value<0.05){
    better_t <- "yes"
  }else{
    better_t <- "no"
  }
}
if ((mean(errorf[,outcome])>0 && mean(errorf[,hypothesis])<0)||(mean(errorf[,
    outcome])<0 && mean(errorf[,hypothesis])>0)){
  better_t <- "opposite mean error sign: unable to perform t-test"
}


'outcome vs hypothesis: DM test'
dm2 <- dm.test(errorf[,outcome], errorf[,hypothesis], alternative = "less")
better_dmpval <- dm2$p.value
if (dm2$p.value<0.05){
  better_dm <- "yes"
}else{
  better_dm <- "no"
}
```

A.6 Code for modelling and forecasting with estimated failure rates

This R script gives, just like the previous program, the forecast results of the black box model and the four installed base types. Additionally, this program estimates the failure rate function; the failure rate of the spare part for every time period. This information is then included in the estimation of the models. In lines 86 through 92, the excel sheet and the other variables can be changed to examine a different spare part.

```
1 'import'
2 library(readxl)
3 library(forecast)
4 library(Hmisc)
5 library(lmtest)
6 library(weibullness)
7 
8 REF1_E <- read_excel("REF1_AGE.xlsx", sheet = "REF1_E")
9 REF1_M <- read_excel("REF1_AGE.xlsx", sheet = "REF1_M")
10 REF1_C <- read_excel("REF1_AGE.xlsx", sheet = "REF1_C")
11 REF2_E <- read_excel("REF2_AGE.xlsx", sheet = "REF2_E")
12 REF2_M <- read_excel("REF2_AGE.xlsx", sheet = "REF2_M")
13 REF2_C <- read_excel("REF2_AGE.xlsx", sheet = "REF2_C")
14 
15 CTV1_E <- read_excel("CTV1_AGE.xlsx", sheet = "CTV1_E")
16 CTV1_M <- read_excel("CTV1_AGE.xlsx", sheet = "CTV1_M")
17 CTV1_C <- read_excel("CTV1_AGE.xlsx", sheet = "CTV1_C")
18 CTV2_E <- read_excel("CTV2_AGE.xlsx", sheet = "CTV2_E")
19 CTV2_M <- read_excel("CTV2_AGE.xlsx", sheet = "CTV2_M")
20 CTV2_C <- read_excel("CTV2_AGE.xlsx", sheet = "CTV2_C")
21 
22 MOB1_E <- read_excel("MOB1_AGE.xlsx", sheet = "MOB1_E")
23 MOB1_M <- read_excel("MOB1_AGE.xlsx", sheet = "MOB1_M")
24 MOB1_C <- read_excel("MOB1_AGE.xlsx", sheet = "MOB1_C")
25 MOB2_E <- read_excel("MOB2_AGE.xlsx", sheet = "MOB2_E")
26 MOB2_M <- read_excel("MOB2_AGE.xlsx", sheet = "MOB2_M")
27 MOB2_C <- read_excel("MOB2_AGE.xlsx", sheet = "MOB2_C")
28 
29 'correcting for refrigerator type 1'
30 DSMOnew <- matrix(0, 315, 3)
31 data <- REF1_E
32 t2 <- 279
33 t3 <- 315
34 t4 <- t3-t2
```

```

for (a in 1:3){
  if (a == 2){
    data <- REF1_M
  }
  if (a == 3){
    data <- REF1_C
  }

  'expontential weighted moving average smoothing'
  DSMO <- rep(0, t3)
  DSMO <- data$DACT[1]
  for (i in 2:t3){
    DSMO[i] <- data$DACT[i]*0.06 + DSMO[i-1]*0.94
  }
  DSMO <- round(DSMO, digits = 2)
  DSMOnew[,a] <- DSMO
}
REF1_E$DSMO <- DSMOnew[,1]
REF1_M$DSMO <- DSMOnew[,2]
REF1_C$DSMO <- DSMOnew[,3]

'correcting for refrigerator type 2'
DSMOnew <- matrix(0, 295, 3)
data <- REF2_E
t2 <- 229
t3 <- 295
t4 <- t3-t2

for (a in 1:3){
  if (a == 2){
    data <- REF2_M
  }
  if (a == 3){
    data <- REF2_C
  }

  'expontential weighted moving average smoothing'
  DSMO <- rep(0, t3)
  DSMO <- data$DACT[1]
  for (i in 2:t3){
    DSMO[i] <- data$DACT[i]*0.06 + DSMO[i-1]*0.94
```

```
  }
  DSMO <- round(DSMO, digits = 2)
  DSMOnew[,a] <- DSMO

}
REF2_E$DSMO <- DSMOnew[,1]
REF2_M$DSMO <- DSMOnew[,2]
REF2_C$DSMO <- DSMOnew[,3]

'--->INPUT<---; change for different results
(outcome and hypothesis correspond with index:
BB = 1, IBL = 2, IBW = 3, IBE = 4, IBM = 5)'
data <- CTV1_C
t2 <- 100
t3 <- 252
t4 <- t3-t2

'setting all non-numeric values equal to zero'
for (c in 1:t2){
  if (data$DACT[c]=="-" || is.na(data$DACT[c])){
    data$DACT[c] <- 0
  }
  if (data$DSMO[c]=="-" || is.na(data$DSMO[c])){
    data$DSMO[c] <- 0
  }
  if (data$sales[c]=="-" || is.na(data$sales[c])){
    data$sales[c] <- 0
  }
}
data$DACT <- as.numeric(data$DACT)
data$DSMO <- as.numeric(data$DSMO)
data$sales <- as.numeric(data$sales)

'initiating result matrices and arrays'
forecast <- matrix(0, t4, 5)
colnames(forecast) <- c("BB", "IBL", "IBW", "IBE", "IBM")
errorf <- matrix(0, t4, 5)
colnames(errorf) <- c("BB", "IBL", "IBW", "IBE", "IBM")
sum <- rep(0, 5)
mape <- rep(0, 5)
rmspe <- rep(0, 5)

```

```
'estimating the failure rate'
fails <- data$DACT[1:t3]
for (x in 1:t3){
  if (fails[x]==0 || is.na(fails[x])){
    fails[x] <- 1e-100
  }
}
weib <- weibull.mle(fails, threshold = 0)
beta <- weib$shape
eta <- weib$scale
failrate <- seq(1, t3)
failrate <- (beta/eta)*((failrate/eta)^(beta-1))

'computational code is looping for every installed base type'
for (a in 2:5){
  IBX <- 0
  AGE <- 0
  for (b in 1:t3){
    IBX[b] <- as.numeric(data[b,(7+a)])
    AGE[b] <- as.numeric(data[b,(13+a)])
  }

  'defining variables for simplicity'
  y <- log(data$DSMO[1:t2]+1)
  ytest <- data$DACT[t2+1:t3]
  ytest <- ytest[1:t4]
  x <- cbind(log(IBX+1), AGE, failrate)

  'determining the AR order p'
  BB <- lm(log(1+data$DSMO[1:t2])~1)
  pval <- 0
  p <- 0
  while (all(pval < 0.05)) {
    p <- p+1
    reslag <- Lag(BB$residuals, 1)
    if(p>1){
      for(i in 2:p){
        reslag <- cbind(reslag, Lag(BB$residuals, i))
      }
    }
    AR <- lm(BB$residuals ~ reslag)
    for (i in 1:p) {
```

```
      pval[i] <- coeftest(AR)[i+1,4]
    }
  }
  p <- p-1
  AR <- lm(BB$residuals ~ reslag[,1:p])

  'estimating the models'
  b0 <- 1
  ynew <- y
  x1new <- x[,1]
  x2new <- x[,2]
  x3new <- x[,3]
  for (i in 1:p){
    b0 <- b0 - AR$coefficients[i+1]
    ynew <- ynew - AR$coefficients[i+1]*Lag(y, i)
    x1new <- x1new - AR$coefficients[i+1]*Lag(x[,1], i)
    x2new <- x2new - AR$coefficients[i+1]*Lag(x[,2], i)
    x3new <- x3new - AR$coefficients[i+1]*Lag(x[,3], i)
  }
  ynew[1] <- y[1]
  x1new[1] <- x[1,1]
  x2new[1] <- x[1,2]
  x3new[1] <- x[1,3]
  if(p>1){
    ynew[2] <- y[2]-AR$coefficients[2]*y[1]
    x1new[2] <- x[2,1]-AR$coefficients[2]*x[1,1]
    x2new[2] <- x[2,2]-AR$coefficients[2]*x[1,2]
    x3new[2] <- x[2,3]-AR$coefficients[2]*x[1,3]
  }
  if(p>2){
    ynew[3] <- y[3]-AR$coefficients[2]*y[2]-AR$coefficients[3]*y[1]
    x1new[3] <- x[3,1]-AR$coefficients[2]*x[2,1]-AR$coefficients[3]*x[1,1]
    x2new[3] <- x[3,2]-AR$coefficients[2]*x[2,2]-AR$coefficients[3]*x[1,2]
    x3new[3] <- x[3,3]-AR$coefficients[2]*x[2,3]-AR$coefficients[3]*x[1,3]
  }
  xnew <- cbind(x1new, x2new, x3new)/b0
  xtrainnew <- cbind(x1new[1:t2], x2new[1:t2], x3new[1:t2])/b0
  ynew <- ynew/b0
  IBmodel <- lm(ynew~xtrainnew)
  BBmodel <- lm(ynew~1)

  'forecasting smoothed demand with IB'
```

```
if (IBmodel$coef[2]<0){
  xnew[,1] <- 0
}
for (i in (t2+1):t3){
  ylagged <- 0
  for (j in 1:p){
    ylagged <- ylagged + AR$coefficients[j+1]*y[i-j]
  }
  if (!is.na(IBX[i]) && IBX[i]>0){
    y[i] <- IBmodel$coefficients[1]*b0 + IBmodel$coefficients[2]*xnew[i,1]*b0 +
   IBmodel$coefficients[3]*xnew[i,2]*b0 + IBmodel$coefficients[4]*xnew[i,3]*b0 +
   ylagged
  }else{
    y[i] <- 0
  }
}
f <- y[t2+1:t3]
f <- f[1:t4]
forecast[,a] <- exp(f)-1
errorf[,a] <- ytest - forecast[,a]

'forecasting smoothed demand with BB'
for (i in (t2+1):t3){
  ylagged <- 0
  for (j in 1:p){
    ylagged <- ylagged + AR$coefficients[j+1]*y[i-j]
  }
  y[i] <- BBmodel$coefficients[1]*b0 + ylagged
}
fBB <- y[t2+1:t3]
fBB <- fBB[1:t4]
forecast[,1] <- exp(fBB)-1
errorf[,1] <- ytest - fBB

'forecast error measures IB'
sumnom <- 0
mapenom <- 0
rmspenom <- 0
denom <- 0
for (i in 1:t4){
  sumnom <- sumnom + forecast[i,a]-ytest[i]
  mapenom <- mapenom + abs(forecast[i,a]-ytest[i])
```

```
      rmspenom <- rmspenom + (forecast[i,a]-ytest[i])^2
      denom <- denom + ytest[i]
    }
    sum[a] <- sumnom/denom
    mape[a] <- mapenom/denom
    rmspe[a] <- sqrt(rmspenom)/(denom/sqrt(t3-t2+1))

    'forecast error measures BB'
    sumnom <- 0
    mapenom <- 0
    rmspenom <- 0
    denom <- 0
    for (i in 1:t4){
      sumnom <- sumnom + forecast[i,1]-ytest[i]
      mapenom <- mapenom + abs(forecast[i,1]-ytest[i])
      rmspenom <- rmspenom + (forecast[i,1]-ytest[i])^2
      denom <- denom + ytest[i]
    }
    sum[1] <- sumnom/denom
    mape[1] <- mapenom/denom
    rmspe[1] <- sqrt(rmspenom)/(denom/sqrt(t3-t2+1))

}
```

A.7 Code for varying the lifetime

The following code snippet is added before the loop at line 124 in the program of Section A.5. It varies the lifetime according to the five predefined lifetime alterations explained in Section 3.5 and recalculates the 'out of life', IBL and AGEP values. The input variables in lines 2 and 3 can be added to the input section of the original script of Section A.5.

```
1 '---> INPUT <---'
2 sales <- 36766
3 L <- 252
4
5 'attaining an expected lifetime to all units sold'
6 lifetime <- c(0.6, 0.7, 1, 1.05, 1.3)
7 count <- c(ceiling(sales*0.025), ceiling(sales*0.135),
8            ceiling(sales*0.34), ceiling(sales*0.34),
9            ceiling(sales*0.16))
10 consumers <- c(rep(1, count[1]), rep(2, count[2]),
11                rep(3, count[3]), rep(4, count[4]),
```

```
                  rep(5, count[5]))
outoflife <- rep(0, t3)
for (i in 1:t2){
  buyers <- sample(consumers, data$sales[i])
  sort(buyers)
  for (j in 1:5){
    if (round(i+L*lifetime[j])<(t3+1)){
      outoflife[ceiling(i+L*lifetime[j])] <- outoflife[ceiling(i+L*lifetime[j])]+
      length(which(buyers==j))
    }
    count[j] <- count[j] - length(which(buyers==j))
  }
  consumers <- c(rep(1, count[1]), rep(2, count[2]),
                  rep(3, count[3]), rep(4, count[4]),
                  rep(5, count[5]))
}
data$`out of life` <- outoflife

'recalculating IBL and AGEP'
IBL <- rep(0, t3)
noreturn <- rep(0, t3)
IBL[1] <- data$sales[1] - data$return[1]
noreturn[1] <- data$sales[1]
for (t in 2:t3){
  noreturn[t] <- noreturn[t-1] + data$sales[t] - data$`out of life`[t]
  IBL[t] <- IBL[t-1] + data$sales[t] - data$return[t] - data$`out of life`[t]
}
data$IBL <- IBL
data$`IBL(no return)` <- noreturn
AGEP <- rep(0, t3)
AGEP[1] = 1
for (t in 2:t3){
  AGEP[t] <- ((AGEP[t-1]+1)*data$`IBL(no return)`[t-1]+data$sales[t])/data$`IBL(no
      return)`[t]
}
data$AGEP <- AGEP
```

The following code snippet can also be added before the loop at line 124 in the program of Section A.5. It varies the lifetime by means of the Weibull distribution explained in Section 3.5 and recalculates the 'out of life' vector. The 'recalculating IBL and AGEP' fragment here is the same as that of the previous lifetime script. The input variables in lines 2, 3 and 4 can be added to the input section of the original script of Section A.5.

```
'---> INPUT <----'
sales <- 36766
shape <- 3.75014
scale <- 6.45085

'attaining an expected lifetime to all units sold'
set.seed(1)
lifetimes <- rweibull(sales, shape=shape, scale=scale)
lifetimes <- ceiling(lifetimes*52)
outoflife <- rep(0, 1000)
for (i in 1:t2){
  if (data$sales[i]!= 0){
    buyers <- sample(lifetimes, data$sales[i])
    buyers <- sort(buyers)
    outoflife[buyers+i] <- outoflife[buyers+i] + 1
    for (j in 1:length(buyers)){
      index <- match(buyers[j], lifetimes)
      lifetimes[index[1]] <- NA
    }
    lifetimes <- lifetimes[!is.na(lifetimes)]
  }
}
outoflife <- outoflife[1:t3]
data$`out of life` <- outoflife

'recalculating IBL and AGEP'
IBL <- rep(0, t3)
noreturn <- rep(0, t3)
IBL[1] <- data$sales[1] - data$return[1]
noreturn[1] <- data$sales[1]
for (t in 2:t3){
  noreturn[t] <- noreturn[t-1] + data$sales[t] - data$`out of life`[t]
  IBL[t] <- IBL[t-1] + data$sales[t] - data$return[t] - data$`out of life`[t]
}
data$IBL <- IBL
data$`IBL(no return)` <- noreturn
AGEP <- rep(0, t3)
AGEP[1] = 1
for (t in 2:t3){
  AGEP[t] <- ((AGEP[t-1]+1)*data$`IBL(no return)`[t-1]+data$sales[t])/data$`IBL(no return)`[t] }
data$AGEP <- AGEP
```