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Using different kinds of installed base models to forecast consumers' spare part demand over the end-of-life phase in the B2C case

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Abstract

By stopping the production of consumer goods, a manufacturer has to decide the number of parts to produce to cover the demand of consumers for spare parts over the end-of-life phase. Using different kinds of installed base variables, it is possible to forecast the end-of-life phase based on the initial and mature sales periods. Consumer decisions on whether to repair a product depend on the specific product and the characteristics of the spare part. Furthermore, all consumers differ in their behaviour and some want to innovate faster than others. To forecast the demand for spare parts over the end-of-life phase multiple models are defined by using different kinds of installed base information. The consumers' behaviour can be accounted to a different installed base variable. This paper provides forecasts results over the end-of-life phase for eighteen different spare parts in the B2C case. The results of these forecasts are often better than the standard black-box method. Using installed base variables therefore supports final production decisions to cover the spare part demand of consumers over the end-of-life phase.

Contents

1 Introduction

In the current society people rely on a lot of complex products, when a component breaks products will be deemed useless. Spare parts are necessary to function the product again. Therefore, the availability of these spare parts is an important issue for users. However, providing the spare parts can be a challenging task for Original Equipment Manufacturers (OEMs). Spare parts management is difficult because of the highly erratic and intermittent demand of spare parts. The OEMs, or other service logistics companies, have difficulties with the interaction between decreasing the holding costs, and having enough spare parts in stock to guarantee spare parts for their customers. Uncertainty could be reduced for the OEMs by providing more accurate forecasts for the spare part demand. Especially because spare part forecasts are important by determining the total number of spare parts produced in the final production run that guarantees spare parts availability over the *end-of-life* phase (EOL) (see e.g. Van der Heijden and Iskandar (2013)).

To forecast this erratic demand, proper variables are needed, for example the *installed base* variable (IB). The IB variable keeps track of the amount of products in circulation during at a certain time, which is interesting, because the products in use generate the spare part demand. Unfortunately, to obtain the actual size of the IB is rather difficult, especially in the Business-to-Consumer (B2C) case, where no contracts between OEMs and consumers are provided. Regarding the small amount of information OEMs have, Wagner and Lindemann (2008) concluded that companies only have a 'cloudy view' of their current installed base. OEMs try to keep track of their installed base, by using the number of sales and returned products.
This paper is an extension of the paper of Kim et al. (2017), and tries to give more accurate forecast performances over the EOL phase. Kim et al. (2017) use different kinds of installed base concepts which can be used to create different kinds of models. Each model gives different forecast performances of the spare parts demand over the EOL phase. Four different installed bases are provided by Kim et al. (2017): lifetime IB, warranty IB, economic IB, and mixed IB. In this paper a new installed base is introduced, the *older installed base* (IBO), which takes only the more older products into account, because only older products may generate spare parts demand, which gives more accurate forecast performances. Furthermore, assumptions of Kim et al. (2017) will be discussed to improve the forecast performance. The research question is: *which model gives the most accurate forecast performance of spare parts demand over the end-of-life phase of a production period, a standard forecasting model or a model including the installed base information?* An empirically validation is made to compare between the forecasts performances.

The remainder of this paper is structured as follows. Sect. 2 discusses relevant information that forms the background for this research. In Sect. 3 the forecast model and evaluation methods are provided. In Sect 4 the available data is described. In Sect 5 the results are given of different forecasts results. Sect. 6 gives an overall conclusion and Sect. 7 gives ideas of future work.

2 Related Works

2.1 Background literature

This paper reproduces and expands on the paper of Kim et al. (2017). The main topic of this paper is spare part demand forecasting for consumer products over their end-of-life phase, using the concept of installed base(IB). Firstly, background literature is given to introduce important concepts in our research. Secondly, we review some papers which uses Kim et al. (2017) as background literature.

Cohen et al. (1990) introduce the installed base information, and mention it as a way of updating forecasts. Auramo and Ala-Risku (2005) focus on obtaining the IB information, and discuss installed base information for service logistics. Wagner and Lindemann (2008) use seven engineering companies to perform a case-study on spare parts management. They observe that companies have problems in keeping track of their own installed base, which makes forecasting difficult. Dekker et al. (2013) introduce an installed base definition: the installed base is the whole set of systems or products for which an organisation provides after sales services. Jin and Liao (2009) assume that the IB is known and use simulation to control inventory to satisfy maintenance demand for spare parts. Jalil et al. (2011) highlight the value of the IB concept and describe further experience with IBM. Dekker et al. (2013) review the concept introduced by Jalil et al. (2011) and use several applications. Bacchetti and Saccani (2012) investigate the gap between research and practice in spare parts management. They give an overview of spare part demand forecasting.

This paper contributes to this discourse by proposing IB concepts that can be applied in practice for Business-to-Consumer (B2C) supply management of spare parts, by forecasting spare part demand in the end-of-life phase of consumer products. Keeping track of the number of products in use, which are stored in the installed base, is much harder for the B2C case than for the B2B case, according to Dekker et al. (2013). In the B2B, users have service contracts with manufacturers to guarantee spare part supply, which the B2C case does not have. Furthermore, keeping track of the IB is hard for the OEM, due to the presence of between sellers, e.g. Media Markt or big supermarket companies. These companies regularly buy big amounts of products at the OEM, which makes it hard for the OEM to follow the actual sales data of the consumer. Van der Auweraer et al. (2019) describes that the installed base information consists of three main sources of information that drive spare part demand: (1) the size and status of the installed base and the status of the spare part itself; (2) the maintenance policy; and (3) the environmental factors which affects the reliability of products and their spare parts. Kim et al. (2017) only use the first source of information, the status and size of the installed base. The maintenance policy is too hard to handle in a B2C case (see e.g. Dekker et al. (2013)). Kim et al. (2017) do not use environmental factors in the data. This paper uses the same data files as Kim et al. (2017) use.

2.2 Installed base concepts

Forecasting is often based on the so-called black-box models, which are popular in business because of their simplicity. Black-box models use only the historical demand data. The black-box models are introduced by Box et al. (2015). To get more accurate forecast results for the spare part demand, historical sales data are important. These sales data are handy to create the installed base variables. In this research it is more useful to use the installed base information than historical sales data, because the installed base shows the number of products in use per time unit, which can lead to upcoming spare parts demand. We explore different installed bases, which differs on given criteria. The most general installed base is the *lifetime installed base*(IBL). Per time unit (week, month, quarter, or year) the lifetime installed base is updated with the number of sales and the number of returned items. Let L denote the average lifetime of the product. Products which have a higher lifetime than the average lifetime (L) of the specific product are called *out- of-life*, and are kept out of the IBL variable. Here, S(t) is the number of sales in time t, and R(t) the number of returned products in time t. For all upcoming installed base variables applies: S(i) = R(i) = 0 for i< 1). The IBL at the end of week t is defined in Eq. 1 as follows

$$IBL(t) = \sum_{i=t-L+1}^{t} (S(i) - R(i)). \tag{1}$$

Beside the IBL we define the *warranty installed base* (IBW) as follows. This IBW counts the number of products or systems which are still in use, and are still in their warranty period. Consumers may determine their decision for repair based on product warranty regulations. The warranty installed base formulation is important, because there could be consumers that will only consider repairment for products with a valid warranty. The warranty period (W) is always strictly smaller than or equal to the lifetime period (L) of each product. If W is larger than L, W is manually set equal to L. After the warranty period, customers have to pay for the repair of their goods themselves, which may result into purchasing a new product instead of repairing their old ones. Each product has its own warranty period, and is determined by the EOMs. Eq. 2 gives the IBW at the end of week t after a warranty period of W periods

$$IBW(t) = \sum_{i=t-W+1}^{t} (S(i) - R(i)). \tag{2}$$

Fig. 2.2.1 shows an example of the IBL and the IBW curve introduced by Inderfurth and Mukherjee (2008) and updated by Kim et al. (2017). In the initial phase the sales per time unit grows, in the mature phase the sales gradually fall back. In the EOL phase the sales data is equal to zero, because the production has been stopped. Obviously, if the warranty period is almost as long as the average lifetime of a product, the IBW would become more and more similar to the IBL. In the first period, the initial phase, the IBL is equal to the IBW because all the newly produced products are still in their warranty period.

In Eq. 3 the *economic installed base* (IBE) is defined. This kind of installed base takes the economical value of the product into account. If the remaining economic value exceeds the repair costs, consumers may still generate demand for spare parts, because the consumers may want to repair their product. For period t, let $E_i(t) = 1$ if $v_i(t) > c(t)$ and $E_i(t) = 0$ if $v_i(t) \leq$ c(t), where c(t) is the repair costs in period t, and $v_i(t)$ is the remaining economic value of the product bought in week i. If $E_i(t) = 1$, there are economical reasons to proceed to repair of their products.

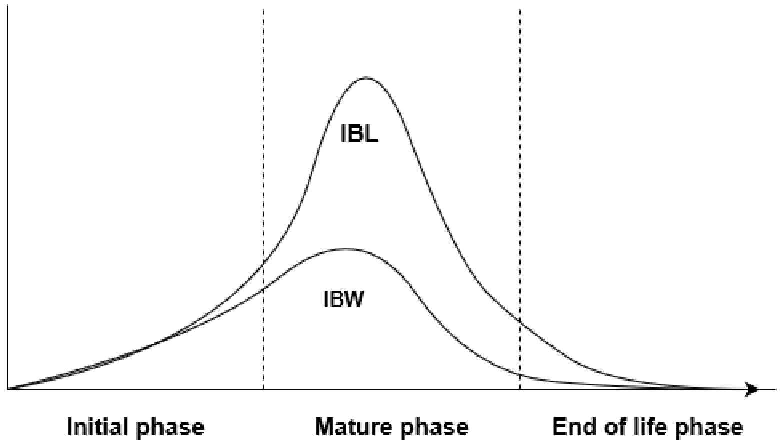

Figure 2.2.1: Sketch of installed base (IBL and IBW) against time on the horizontal axis, updated from Kim et al. (2017).

$$IBE(t) = \sum_{i=t-L+1}^{t} E_i(t) \times (S(i) - R(i)). \tag{3}$$

In the construction of IBE, it is assumed that all consumers apply the same decay rate for the remaining value of the product. The remaining value $v_i(t)$ is determined by assuming exponential value decay. Let p_i be the price of the product sold in period i, then the decay rate a_i is obtained from the condition that $p_i \times exp(a_i \times L) = 1$. So, $a_i = -ln(p_i)/L$. The remaining value in period t is $v_i(t) = p_i \times exp(a_i \times (t-1))$. The difference between the IBE and the *mixed economic installed base* (IBM) is the subjective evaluation of the remaining value of the product. In the IBE all the consumers apply the same decay rate for the remaining value of the product. In the IBM, the remaining value of a product depends on heterogeneous tastes. Consumers who are more sensitive for social trends and technological innovations, the subjective lifetime is shorter than for consumers who are less sensitive for trends or innovations. Consumers are divided in five adopter segments. In line with Rogers (2003), the consumers are distributes as follows: 2.5% innovators (0.6), 13.5% early adopters (0.7), 34% normal adopters (1.0), 34% late adopters (1.05), and 16% laggers (1.3). In parenthesis is the fraction given of the lifecycle compared to the overall average within each segment.

Finally, the *older installed base* (IBO) is defined, as an extension of the paper of Kim et al. (2017). Each IB variable shows the number of products in use per time t. For all t, the average age of the products stored in IB(t), is calculated per installed base variable. The IBO takes the average age of the IBL method into account by constructing the IBO variable. If a product is older than the mean age of the IBL at time t, the product is kept in the IBO(t). The age of a product in week t bought in week i is formulated as $d_i(t)$. The mean age of the IBL information is denoted by AGEL(t). So, $E_i(t) = 1$ if $d_i(t) > AGEL(t)$ and $E_i(t) = 0$ if $d_i(t) \leq AGEL(t)$. Eq. 4 gives a mathematical representation of the older installed base.

$$IBO(t) = \sum_{i=t-L+1}^{t} E_i(t) \times (S(i) - R(i)). \tag{4}$$

3 Methodology

This section consists of two different parts. Firstly, a model with different explanatory variables is presented. Secondly, three different criteria is introduced to test the accuracy of the forecast results.

3.1 Spare part demand formulations

Using the installed base variables, spare part demand can be estimated. The presented model uses two different explanatory variables: IB(t), and AGE(t) which is the mean age of the products stored in the installed base in period t. To create the forecast model, two different times are defined, T_1 and T_2. Both can be seen as random variables, with survival distributions $S_i(t) = P_i(T_i > t)$, i=1, 2. Here T_1 is the (continuously measured) time of failure of the product requiring a spare part for repair. T_2 is the time where the customer ends the use of the product.
The demand for spare parts has a probability $p_d(t)$ in period t, that runs continuously from t-1 to t, and is equal to the distribution $P(t-1 < T_1 < t, T_2 > t)$. This probability distribution takes the rule $T_1 < T_2$ into account. The probability of the demand is calculated without this rule, so $P(t-1 < T_1 < t | T_2 > t) = P(t-1 < T_1 < t)$. The probability's demand derivation can be seen in Eq. 5.

$$\begin{aligned} p_d(t) &= P(t-1 < T_1 < t, T_2 > t) = P(t-1 < T_1 < t | T_2 > t) \times P(T_2 > t) \\ &= P(t-1 < T_1 < t) \times P(T_2 > t) \\ &= (S_1(t-1) - S_1(t)) \times S_2(t). \end{aligned} \tag{5}$$

By assumption of constant hazard rates over time, the product and spare part do no age, then the survival functions are exponential, that is $S_i = exp(-a_i t)$ with $a_i > 0$. Eq. 5 becomes

$$p_d(t) = (exp(a_1) - 1) \times exp(-(a_1 + a_2)t). \tag{6}$$

The total demand for spare parts D(t) is calculated using the expression of $p_d(t)$ and the obtained installed bases. $D(t) = p_d(t) \times IB(t)$. By taking the natural logarithm, D(t) becomes equal to

$$ln(D(t)) = b_0 + ln(IB(t)) + b_2 \times t. \tag{7}$$

The demand for spare parts can be estimated more accurate by using an extra variable, AGE(t). This variable stores the mean age of the installed base in period t, and has more power by forecasting the demand of spare parts. So each installed base variable has its own AGE variable, which stores the mean age of the products stored in the specific IB variable per time t. In addition, the coefficient b_1 is added to the model. Now, the model accounts for the fact that not all the products generate spare part demand. Some products can be disused and only a portion of all break-downs will be repaired. Using the variable AGE(t), Eq. 7 changes into

$$ln(D(t)) = b_0 + b_1 \times ln(IB(t)) + b_2 \times AGE(t). \tag{8}$$

3.2 Model selection and forecast performance

Not all details of the data generating process are known, due to our limited available demand data. Forecasts will be made by changing Eq. 8 into a regression model, by adding unobserved error terms $\varepsilon(t)$. Furthermore, a 1 is added to all the demand data and installed bases, to avert zero values. The demand data can be estimated using the regression model

$$ln(1 + D(t)) = b_0 + b_1 \times ln(1 + IB(t)) + b_2 \times AGE(t) + \varepsilon(t). \tag{9}$$

In Eq. 9, the $\varepsilon(t)$ follows an AR process. Which means the unknown coefficients b_0,b_1, and b_2, can easily be obtained using ordinary least squares.
The demand for spare parts could be very erratic, which is why the exponentially weighted moving average method (EWMA) is used, to convert the actual demand data into a less erratic version. In Eq. 10 a formulation of the EMWA model is given

$$S_t = \begin{cases} Y_t, & t = 1 \\ \alpha \times Y_t + (1 - \alpha) \times S_{t-1}, & t > 1 \end{cases} \tag{10}$$

For this EWMA method, a value of α is necessary to optimise the smoothed version of the demand data compared with the actual demand data.Kim et al. (2017) use a value for α of 0.06, determined by Inc (1996). This research tries to optimise this α factor. The optimal α, or a more optimal one than 0.06, can be found by using three different criteria. These criteria gain insight in the forecasts' performance over the EOL phase. The first criterion is the summed error, which is the difference between the summed forecast and the summed demand over the EOL phase. This summed demand is the actual demand and not the less erattic EMWA demand. By changing the value for α, the forecast performance changes. Suppose that actual demand data D(t) are available for the EOL phase for periods from $t_1 \leq t \leq t_2$. F(t) is the forecast for these periods. In Eq. 11 the function for the first criterion is given.

$$SUM = \frac{\sum_{t=t_1}^{t_2} (F(t) - D(t))}{\sum_{t=t_1}^{t_2} D(t)} \tag{11}$$

A positive value for the SUM criterion correspondents with an over-estimation of the demand for spare parts. A negative value for the SUM criterion belongs to an under-estimation. One other criteria, MAPE, takes the absolute value of the SUM criteria. The RMSPE criterion is the root mean squared prediction error.

$$MAPE = \frac{\sum_{t=t_1}^{t_2} |F(t) - D(t)|}{\sum_{t=t_1}^{t_2} D(t)} \tag{12}$$

$$RMSPE = \frac{\sqrt{\sum_{t=t_1}^{t_2} (F(t) - D(t))^2}}{\sum_{t=t_1}^{t_2} D(t)/\sqrt{t_2 - t_1 + 1}} \tag{13}$$

For all the three different criteria, the value of α has influence on the forecast performance. By choosing the correct value of α for the different kind of products, lower values of the different criteria can be obtained.

For every product we forecast with six different models. The first one is using the black-box models. These are obtained using pure AR models, so $b_1 = b_2 = 0$. The value of b_0 is obtained during the initial and mature phase. The error term is modeled as $\varepsilon(t) = c_1 \times \varepsilon(t-1) + ... + c_p \times \varepsilon(t-p) + \omega(t)$, where $\omega(t)$ is a white noise process. The amount of lags in the AR model is obtained using forward selection and using a significance level of 5%. Extensions with the AR models do not improve the forecasts, according to Kim et al. (2017).

The other four models are all provided with a different installed base variable. If $b_1 < 0$, than the IB(t) variable is removed from the model. There are no restrictions on the b_2 coefficient. Insignificant coefficients are not removed from the model, because insignificance may be due to a short estimation period. The residuals for each of the five IB models follow the same AR order as the AR model has for each product. To forecast the demand for spare parts in the end-of-life phase, we estimate the coefficients for all the six different models over the initial and mature phase. If the forecasted demand in the EOL phase is negative, the forecasted demand is manually set equal to zero, because negative demand does not exist. In addition, if the installed base variable is equal to zero on time t, the forecasted demand is manually set equal to zero, because no products needs a repair.

4 Data

4.1 Overview of different spare parts

The spare parts data in this case study is provided by the Western European warehouse of Samsung Electronics. The data consists of three different kinds of consumer products: refrigerators, televisions, and mobile phones. Each type of product has two different kinds of its own type, so so there is data for six different products. Each product consists of (at least) three different parts. So the data consists of eighteen different spare parts. In Table 4.1.1 an overview of the different kinds of products is provided.

Each kind of product has its own lifecycle, price and sales period. The product‘s average lifecycle is the expected amount of times the products fulfills its function. The lifecycle of the refrigerators are determined to be 676 weeks, according to Seiders et al. (2007). Search (2012) determines the lifecycle of a television on 360 weeks. Mobile phones have an average lifecycle of 160 weeks, according to Entner (2011).

Refrigerator 1 and 2 are two separated products. The sales periods for both refrigerators are long enough to make sufficient forecasts for the full EOL phase. Refrigerator 1 has an estimation period of 279 weeks and a forecast period of 36 weeks, while refrigerator 2 has an estimation period of 229 weeks and a forecast period of 66 weeks.

Giachetti and Marchi (2010) find that the market for mobile phones is highly competitive. In this case study, smartphone 1 is an early version of smartphone 2, so smartphone 2 is an upgrade of smartphone 1. They have both a warranty period of two years. The sales period of smartphone 2 already starts during the sales period of smartphone 1. The sales period of smartphone 1 is around a year, which is too short to make proper forecasts over the full EOL phase. We decide to forecast remaining EOL demand one year after the end of product sales.

Smartphone 2 has a sales period of nearly two years, which is sufficient to forecast the EOL

Table 4.1.1: Overview of six products with features

Product	Consumer sentiments		Sales	Period	Estimate	Forecast	Lifecycle
	Life cycle	Tech trendy					
Refrigerator 1	Long	Low	538.386	08.12 - 13.29	279	36	676
Refrigerator 2	Long	Low	166.782	08.32 - 12.51	229	66	676
Television 1	Short	Low	36.766	09.23 - 10.17	100	152	360
Television 2	Short	Low	50.986	10.12 - 11.15	108	102	360
Smartphone 1	Short	High	348.153	10.24 - 11.28	109	89	160
Smartphone 2	Short	High	694.816	11.19 - 13.04	90	6	160

Table notes

Tech trendy is the amount of innovation possible

Sales are the total product sales, indicated in the format year.week(e.g., 04.37 is week 37 of 2004).

Estimate and Forecast show the number of weeks of data available respectively for estimation and for forecast analysis.

Lifecycle is the average lifetime in weeks.

phase. For televisions, we make the same choices as for the smartphones. The short sales period makes forecasting the EOL phase a to challenging task. All methods, including the black-box, are far off the mark. We consider to forecast remaining EOL demand one year after the end of product sales. The estimation period then becomes 100 weeks with a forecast period of 152 weeks for television 1. Television 2 has about two years of forecast evaluation, as type 2 was introduced about one year after type 1.

Each product has three different kinds of parts which, if broken, may contribute to spare part demand. In Table 4.1.2 is an overview of the different spare parts provided. We derived per spare part our forecast hypothesis, which installed base model gives the most accurate forecast performance of spare part demand over the EOL phase. The hypothesis is derived from other spare part features, especially the " "Essential" and "Expensive" feature. If a spare part is essential, the necessity to repair is high. The product loses its function if the essential part is broken. The "Expensive" feature shows if the spare part is an expensive repair or not. An assumption is that expensive spare parts are less likely to get a repair than less expensive parts.

4.2 Hypothesis of best forecast method

The hypothesis is that the best way to forecast the refrigerator compressors by using the IBO information. The refrigerator has a long lifecycle, which supports consumers to repair their product instead of by buying a new refrigerator. Moreover, the compressor is an essential part and are necessary to operate the product. Compressors may break by using the refrigerator often and for a long time, not essentially by using the refrigerator wrong by human behaviour. It is imaginable that only the more older refrigerators gives more demand for compressors that younger refrigerators. Although the circuit board is expensive, it is not essential, meaning that the refrigerator is still operational without circuit board. The lifecycle of the refrigerator is rather long, way longer than the warranty period. Consumers may repair the circuit board only if this economical beneficial, due to the high utility of the refrigerator until end of life. So, it is expected that IBE provides the best forecast results. The door gasket is not essential and not expensive.

Table 4.1.2: Overview of six products with features

Spare part	Essential	Expensive	Demand		EOL%		Price%		Hypothesis
			1	2	1	2	1	2	
For refrigerator									
Compressor	Yes	Yes	5,678	6,090	4.4	13.6	18.3	39.8	O
Circuit board	No	Yes	9,596	3,518	17.7	36.5	7.6	5.6	E
Door gasket	No	No	4,581	698	17.7	10.0	3.5	3.4	E
For television									
LCD panel	Yes	Yes	868	889	24.9	20.6	47.0	39.4	E
Circuit board	No	Yes	562	774	37.7	39.1	9.6	8.2	W
Cover	No	No	152	230	9.2	16.1	3.2	4.5	W
For smartphone									
Touch screen	Yes	Yes	21,299	58,413	16.5	23.8	19.8	25.8	1W,2M
Circuit board	No	Yes	6,325	14,492	14.2	37.6	28.6	40.0	1W,2M
Back cover	No	No	5,259	11,033	42.4	45.8	1.6	1.2	1W,2M

Table notes

"Essential" describes the essence of a spare part for the product. "Expensive" represent the costs of the spare part compared to the product price.

"Demand" indicates the demand for spare parts over the analysis data.

EOL% is the percentage of the demand during the EOL-phase compared with the spare part demand over the initial and mature phase.

Price% is the price of the spare part as a percentage of the product price, for type 1 and type 2.

"Hypothesis" shows the hypothesis of the best forecast method (L for IBL, W for IBW, E for IBE, O for IBO, and M for IBM)

We expect a forecast method using the IBE information, as for the same reasoning for the circuit board. Due to long lifecycle, consumers may repair this part only if it is economical beneficial.
The television consists of a LCD panel which is essential, without it the television is useless. Televisions has a relatively short lifecycle. The repair of the LCD panel is expensive. The IBE information is the best option for the forecast method of televisions. It is possible that consumers will only opt for repair if the remaining value of the television is higher than the replacement costs. Otherwise, consumers may buy a new television, because of their relatively short lifecycle. An expensive but not essential spare part is the circuit board of a television. Consumers may only opt for repair of this spare product if the television is still in its warranty period. The urge to repair the circuit board is less after the warranty period, due to the televisions' short lifecycle. Consumers may wait with repairing the circuit board by buying a new television. The televisions' cover is not essential and not expensive. Due to the televisions' short lifecycle, the IBW is expected to provide the bets forecast.

The smartphones' touch screen is essential and expensive. The IBW may result in the best forecast for smartphone 1, as smartphone 1 is an upgraded version of smartphone 2. The lifecycle of smartphone 1 is relatively short, so consumers may repair its touch screen only if the smartphone is still in its warranty period. If it is out of its warranty, consumers may buy the new smartphone 2, to upgrade their smartphone use. IBM is expected to provide the best forecast for smartphone 2 because this smartphone is the most enhanced smartphone on the market. Only if the economic value of the remaining smartphone is higher than the repair costs, taking different consumers preferences into account, the product is repaired. The circuit board is not essential but expensive. In order to come to the best forecast for smartphone 1

IBW information is expected to be the most accurate. While the forecast for smartphone 2 is expected to be most accurate using IBM. The back cover of a smartphone is non-essential and not expensive. It is expected that the demand for the back covers for smartphone 1 best predicted by IBW. If smartphone 1 is out of its warranty period, consumers may opt to buy the more enhanced smartphone 2 instead of repairing the product. It is expected that the demand for the back covers for smartphone 2 is best predicted by IBM, due to economical reasons.

5 Results

5.1 Illustrative case: back cover of a smartphone

We analyze the demand for the back cover of smartphone 1 over the EOL. The back cover is a dispensable part, the smartphone may still work if the back cover is broken. As given in Table 5.2.3 the spare part is 1.6 percent of the product price, which is 6.4 euros, excluding labour and handling costs. The warranty period is 2 years. The expected average lifetime is, according to Entner (2011), over three years, which is relatively short for consumer electronic products.
The smartphone market has expanded rapidly in recent years. Giachetti and Marchi (2010) show insight in the smartphone market, which is highly competitive, not only between brands but also between products of the same brand. Consumers feel the urge to upgrade their smartphone with a newer mobile phone which has better properties. These replacements between mobile phones reduces the spare part demand, because the user time is relatively short. Therefore, we expect a best forecast method using the IBW information.
The sales period for smartphones is slightly more than a year. To forecast the spare part demand, we forecast remaing EOL demand one year after the end of product sales. Fig. 5.1.1 shows time plots over the end-of-life phase of the actual demand, the EWMA smoothed demand and the five different forecasts results with different installed base information.

IBW is clearly doing best and is rather successful in tracking the EMWA smoothed demand, and the actual demand. The IBO is following the AR model, and the IBL, IBE, and IBM are clearly not the most accurate forecasts.

5.1.1 Forecast results

The number of autoregressive lags is determined by the black box model. The demand for spare sparts follows an AR(2) model: $ln(1+D_s(t)) = 5.52+\varepsilon(t)$, where $\varepsilon(t) = 1.28\times\varepsilon_{t-1}-0.28\times\varepsilon_{t-2}+\omega(t)$. The model has an R^2 equal to 0.997. Each installed base model has the same AR order as the AR model, with two autoregressive lags. Table 5.1.1 shows the different error values. Here the forecast error is equal to the difference between the forecast values and the non-smoothed demand values. The IBL and the IBW variable have a negative coefficient and has been kept out of the model. The IBE, IBO and IBM variables have positive coefficients.

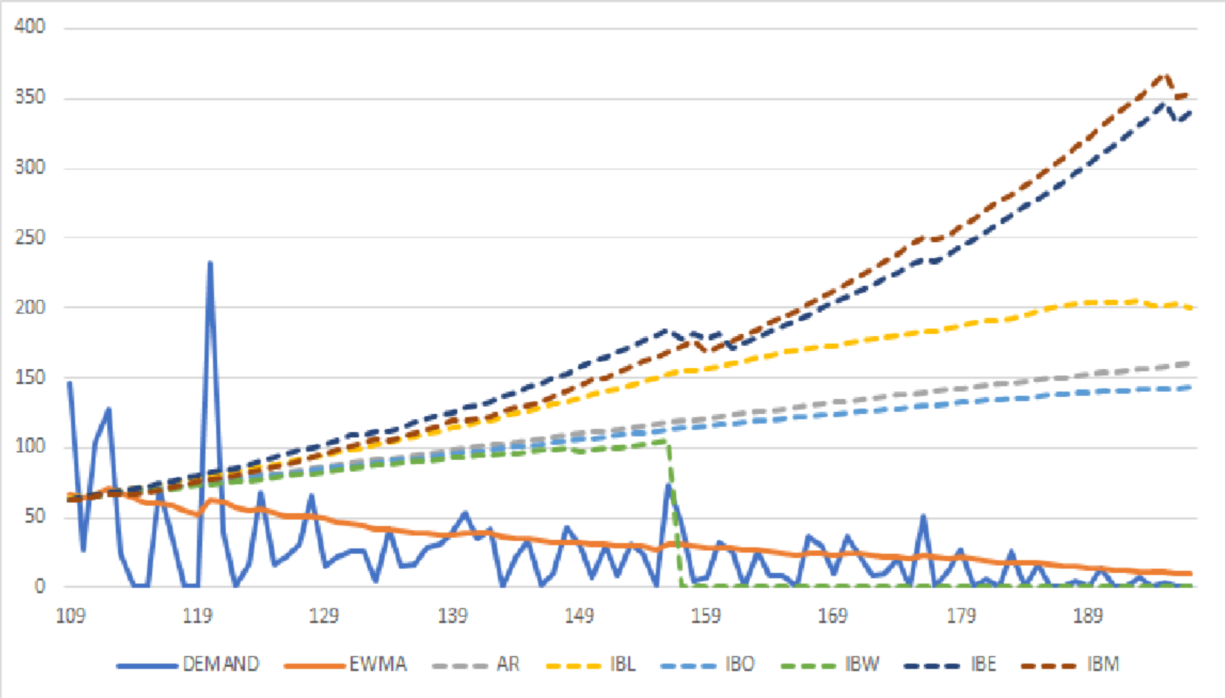

Figure 5.1.1: Sketch of actual demand and EWMA smoothed demand for the back cover of smartphone 1 during EOL-phase and black-box(AR) forecast, five different installed base forecasts (IBL, IBW, IBE, IBO, and IBM), all measured in units on vertical axis against week number of EOL-phase on horizontal axis.

Table 5.1.1: Forecast results for the back cover of smartphone 1

	Forecast error	SUM	MAPE	RMSPE
AR(2)	7861.60	3.52	3.82	4.12
IBL	10270.97	4.60	4.90	5.35
IBW	1856.00	0.83	1.59	2.06
IBE	13305.55	5.96	6.26	7.15
IBO	7335.68	3.29	3.59	3.85
IBM	13400.85	6.00	6.31	7.39

Each criteria shows that the IBW model has the best forecast performance, and that the IBO model has the second-best forecast performance.The conclusion here is that the warranty installed base contains the best information to forecast the demand for the back covers of the smartphone 1. Back covers are non-essential and not expensive, which is in line with the features of the warranty installed base. Consumers may wait with repairing there back cover, by buying the newer smartphone 2.

5.2 Spare part demand smoothing

The EWMA model, presented in Eq. 10, uses α factor. Inc (1996) uses a value of 0.06. First, a argumentative insight is given why using the EWMA formula is useful, which is completed with a mathematical diversion. Secondly, an optimal value for the α is derived.

5.2.1 Usefulness of EWMA formula

Sect. 2 describes shortly the current situation between OEMs and consumers and the inconvenience in the information in the supply chain for consumer goods. Frequently, businesses as Media Markt and large supermarket chains, buy a large size of the same product at the OEMs, and keep them in stock until sold to consumers. This results in erratic sales data, because these data show the sales between the OEMs and the trade intermediary. For forecasting the demand for spare parts, it might be handy to refine these sales data. Table 5.2.1 shows the different forecast results between the actual spare part demand and the smoothed demand with $\alpha = 0.06$. We show the differences by obtaining the MAPE, and RMSPE forecast criteria for both demand data. We investigate two different parts, the door gasket of refrigerator 1 and the circuit board of refrigerator 2.

Table 5.2.1: Forecast criteria for door gasket of refrigerator 1, MAPE, RMSPE for non-smoothed demand, $MAPE^*$, $RMSPE^*$ for smoothed demand

	Door gasket refrigerator 1						Circuit board refrigerator 2					
	AR	IBL	IBW	IBE	IBO	IBM	AR	IBL	IBW	IBE	IBO	IBM
MAPE	0.57	0.60	0.75	0.59	0.57	0.59	0.64	3.59	0.84	3.59	0.44	3.11
RMSPE	0.78	0.69	0.81	0.68	0.66	0.68	0.74	4.28	0.97	4.28	0.57	3.59
MAPE*	0.38	0.47	0.43	0.45	0.43	0.44	0.27	3.09	0.37	2.99	0.94	1.43
RMSPE*	0.59	0.62	0.61	0.61	0.59	0.61	0.36	4.07	0.48	3.93	1.13	1.68

Table 5.2.1 shows that for the door gasket, smoothed demand gets better forecasts results than for a non-smoothed demand. The circuit board of refrigerator 2 gives the same conclusion overall. Only, the IBO model gives better forecast results with $\alpha = 1$. This observation does not change the opinion of using the EWMA, because by using the EWMA, lower criteria values would be obtained by the AR or IBW model than by the IBO, without EWMA. In this case study we use the EWMA formula to smooth the demand of spare parts.

5.2.2 Optimal value for alpha

We optimise the value for α to decline the forecast errors, obtained by using the SUM, MAPE, and RMSPE criteria. First, we use refrigerator 1 compressor data, given in Table 4.1.1. Several values of α has been used to smooth the demand curve over the estimation period of the data. 10 different forecasts has been executed, with all different criteria values. Table 5.2.2 shows the different outcomes of the SUM, MAPE, and RMSPE criteria.

Table 5.2.2: Criteria values for refrigerator 1 compressor with different values of alpha

	0.02	0.04	0.55	0.06	0.061	0.0615	0.062	0.065	0.08	0.10	0.12
SUM	1.09	0.67	0.607507	0.603455	0.603252	0.603132	0.603158	0.603922	0.62	0.67	0.74
MAPE	1.15	0.78	0.72704	0.72408	0.723961	0.723885	0.723925	0.724635	0.74	0.78	0.83
RMSPE	1.23	0.87	0.828866	0.8266765	0.826805	0.826795	0.826907	0.82799	0.85	0.89	0.94

It is clear from Table 5.2.2 that the value of α introduced by Inc (1996) is understandable. However, there is a indication of $\alpha = 0.0615$ by only using the refrigerator 1 compressor data.

A simultaneous sum function is defined, to declare the overall lower criteria values for smoothing the demand for spare parts with an $\alpha = 0.0615$. The sum function takes the following parts into account: the compressors of refrigerator 1 and 2, the circuit boards of televisions 1 and 2, and the back covers of smartphone 1 and 2. These products gives an overall representation of the data, because each product has one spare parts taking into account. Table 5.2.3 shows the summons of criteria values of six different AR models. Each AR model is used twice, one with an $\alpha = 0.06$ and an $\alpha = 0.0615$.

Table 5.2.3 shows the results of the sum of six different AR models, comparing the SUM, MAPE, and RMSPE values created by spare parts demand with smoothing factors of 0.06 and 0.0615.

Table 5.2.3: Criteria values for the sum function with two different values of alpha

	SUM	MAPE	RMSPE
$\alpha = 0.06$	17.138	18.361	21.501
$\alpha = 0.0615$	<u>17.129</u>	<u>18.354</u>	<u>21.491</u>

In this case study, the EWMA formula is used to smooth the spare part demand with an α equal to 0.0615.

5.3 Criteria values for different kind of forecasts

The main theoretical contribution of this case study lies in proposing installed base concepts for the B2C companies. Now we give multiple overviews of the three different products with the best forecast results. Table 5.3.1 shows the results of the different installed base forecasts for refrigerator 1 and refrigerator 2. The outcomes supports our hypothesis. The compressor for the refrigerator 1 shows accurate forecast results for the IBO model, which is in line with the hypothesis. Compressor 2 is forecasted most accurate by the IBL model, which is the standard model of the IBO. Both models give better forecast performance than the black-box method.
The results for the both circuit boards are different. The best forecast performance for the circuit board 1 is provided by using the IBM model, which is almost the same as the IBE forecast results. The outcome of the circuit board 1 is in line with the hypothesis of IBE. Circuit board 2 best forecast performance is provided by the AR(2) black-box model. The results for the door gasket are varied. The door gasket 1 has best forecast performance by using the black-box model. Door gasket 2 is forecasted most accurate by IBE, IBO, and IBM, which is in line with the hypothesis of IBE.

Table 5.3.2 gives an overview of the different forecast results for smartphones 1 and smartphone 2. Both touch screen spare parts for smartphone 1 and smartphone 2 is forecasted most accurate by using the IBE model. Touch screens are non-essential spare parts so consumers may repair the touch screen only if it is economic beneficial for the consumers. The result for touch screen of smartphone 2 is almost in line with the hypothesis of IBE. The outcomes of smartphone 1 is not in line with the hypothesis. Apparently, consumers are less influential for buying a more enhanced smartphone than expected. If the remaining value of the smartphone is larger than the repair costs, consumers may repair their smartphone 1 instead of buying smartphone 2. The circuit board 1 is forecasted most accurate by taking the IBE or the IBM

Table 5.3.1: Forecast results for refrigerator 1 and 2

Spare part	AR	IBL	IBW	IBE	IBO	IBM
Compressor 1						
SUM	1.16	0.60	0.67	1.10	0.56	1.11
MAPE	1.25	0.72	0.98	1.19	0.68	1.19
RMSPE	1.37	0.83	1.13	1.31	0.79	1.32
Compressor 2						
SUM	0.80	0.00	0.16	0.16	0.23	-0.05
MAPE	0.97	0.48	0.52	0.50	0.56	0.48
RMSPE	1.07	0.68	0.69	0.68	0.71	0.69
Circuit board 1						
SUM	-0.09	0.11	-0.16	0.05	0.08	0.00
MAPE	0.26	0.26	0.37	0.24	0.25	0.24
RMSPE	0.31	0.34	0.52	0.31	0.32	0.30
Circuit board 2						
SUM	-0.10	1.44	0.18	1.46	0.87	1.28
MAPE	0.25	1.50	0.34	1.51	0.93	1.34
RMSPE	0.35	1.80	0.43	1.81	1.12	1.56
Door gasket 1						
SUM	-0.09	0.12	0.08	0.10	0.23	0.13
MAPE	0.37	0.48	0.64	0.47	0.55	0.48
RMSPE	0.58	0.62	0.74	0.61	0.67	0.62
Door gasket 2						
SUM	1.44	1.09	0.38	0.26	0.26	0.29
MAPE	1.62	1.35	0.96	0.93	0.93	0.93
RMSPE	1.80	1.54	1.16	1.14	1.14	1.14

model, due to the amount of zeros in the installed base variables, which makes the demand for spare parts over the EOL phase equal to zero. The IBO model forecast the circuit board 2 most accurate, due to the assumption that only more older goods lose the circuit boards function. The IBO outcome is not in line with the IBM hypothesis. Apparently, older circuit boards are broken more often which gives the IBO a more accurate forecast result. The back covers for smartphone 1 and smartphone 2 is forecasted most accurate by using respectively the IBW and IBO model. As smartphone 2 is an improvement of smartphone 1, and was released relatively short after the sales period of smartphone 1, it is imaginable that consumers may repair their back cover for smartphone 1 only in their warranty period. So, smartphone 1's back cover is in line with the hypothesis of IBW. For smartphone 2 the IBO model gives the best forecast performance, due to that consumers use the smartphone 2 for a longer period, and only more older products may lose the back covers function. Table 5.3.3 gives the results of the forecasts of television 1 and television 2. The LCD panel is forecasted most accurate with an IBO model for television 1 and IBL for television 2. These essential parts are often repaired over the total lifetime. The hypothesis IBE is not in line with the outcomes. Apparently, the relatively long lifecycle of the televisions is a better indication than the economical benefits for the consumers. The IBL model gives the most accurate forecast performance for the circuit board 1, and IBW for circuit board 2. The circuit board's hypothesis is IBW which corresponds with the forecast outcome of circuit board 2. Circuit board 1 has five almost equal outcomes, so apparently the data of circuit board 1 was not sufficient enough to give proper forecast performances. The cover 1 is forecasted most accurate by the IBW model, equal to the hypothesis, due to the fact that the cover of a television

Table 5.3.2: Forecast results for smartphones 1 and 2

Spare part	AR	IBL	IBW	IBE	IBO	IBM
Touch Screen 1						
SUM	5.03	2.35	1.62	0.97	3.80	2.18
MAPE	5.18	2.53	1.91	1.36	3.97	2.42
RMSPE	5.35	2.65	2.58	2.07	4.13	2.94
Touch Screen 2						
SUM	1.65	4.75	3.55	1.06	3.09	1.47
MAPE	1.69	4.76	3.56	1.18	2.11	1.51
RMSPE	1.80	5.37	3.84	1.33	3.40	1.62
Circuit board 1						
SUM	6.02	4.98	2.47	-1.00	6.43	-1.00
MAPE	6.04	5.03	2.56	1.00	6.45	1.00
RMSPE	6.33	5.26	3.75	2.05	6.80	2.05
Circuit board 2						
SUM	0.71	0.76	0.75	-0.24	0.40	0.03
MAPE	0.85	0.89	0.88	0.73	0.67	0.69
RMSPE	0.99	1.04	1.02	0.93	0.76	0.86
Back cover 1						
SUM	3.52	4.60	0.83	5.96	4.35	6.00
MAPE	3.82	4.90	1.59	6.26	4.65	6.31
RMSPE	4.12	5.35	2.06	7.15	5.21	7.39
Back cover 2						
SUM	0.41	0.75	0.60	0.53	0.12	1.30
MAPE	0.86	1.14	1.02	0.96	0.65	1.66
RMSPE	1.13	1.39	1.25	1.21	0.98	1.99

is a non-essential part of the television. Cover 2 is forecasted most accurate by taking the IBL information in the model, due to the relatively long lifecycle. The difference between the two covers is due to the innovation of television 2 compared with television 1.

5.4 Lag operator into installed base information

The IBO variable is obtained by using a dynamic age constraint. The age variable, used to create the IBO variable, is corresponding with the age variable of the IBL variable. As an extension of the IBO variable, the *age installed base* (IBA) is created by considering a deterministic age constraint (ζ). It is imaginable that parts of products which are older have a higher chance of getting out of use than younger parts. A imaginable threshold for the constant age constraint is around the 70% (ζ=0.7) of the lifecycle per product. So, as an assumption, products which are older than 70% of the lifecycle has a higher change of requiring a repair. It is likely that taking only the older products into account results in a better forecast performance over the EOL-phase than other installed base variables.

This research provides a IBA forecasting for the following products: Compressor, Circuit board, and Door gasket for refrigerators 1 and 2. Furthermore, the LCD panel, Circuit board and Cover of television 2, and Touch screen, Circuit board and Back cover of smartphone 2. We take both the refrigerators into account because the large size of the lifecycle, and by the hypothesis that consumers keep their refrigerators long in use. Smartphone 2 and television 2 are taking into account because these products are an improvement of respectively smartphone 1 and television 2. Consumers may keeping these products for a longer period before switching to a newer version.

Table 5.3.3: Forecast results for television 1 and 2

Spare part	AR	IBL	IBW	IBE	IBO	IBM
LCD panel 1						
SUM	4.14	0.76	0.05	-1.00	-0.03	-1.00
MAPE	4.23	1.10	0.89	1.00	0.71	1.00
RMSPE	4.44	1.36	1.41	1.74	1.17	1.74
LCD panel 2						
SUM	3.42	0.43	0.42	-1.00	0.76	-1.00
MAPE	3.43	0.76	0.93	1.00	0.91	1.00
RMSPE	3.69	0.98	1.31	1.42	1.14	1.42
Circuit board 1						
SUM	2.30	-0.18	-0.27	-0.06	0.20	0.14
MAPE	2.51	0.65	0.68	0.71	0.76	0.73
RMSPE	2.73	0.92	0.97	0.97	0.95	0.95
Circuit board 2						
SUM	6.17	7.53	0.86	4.09	6.96	8.00
MAPE	6.17	7.53	1.47	4.17	6.96	8.00
RMSPE	7.24	9.03	1.98	5.30	8.29	9.50
Cover 1						
SUM	14.04	2.99	1.89	3.23	4.24	3.52
MAPE	14.65	3.86	2.71	4.09	5.05	4.36
RMSPE	14.84	5.22	5.07	5.33	6.04	5.54
Cover 2						
SUM	4.44	1.04	2.58	1.33	3.09	1.63
MAPE	4.89	1.87	3.02	2.09	3.58	2.34
RMSPE	5.14	2.64	4.87	2.80	3.92	2.90

Instead of taking an age threshold around 70% of the lifecycle, this research takes $\zeta = 20\%$ in account, due to the short data files, according to the products' lifecycle. The IBA variable is created by changing the IBL variable. The age constraint causes a shift in the IBL variable, due to considering products which are older than a certain age, the first $\zeta\times$ lifecycle number of weeks are set equal to zero. A mathematical representation of the IBA is given in Eq. 14

$$IBA(t) = \sum_{i=t-L+1}^{t} E_i(t) \times (S(i) - R(i)) \tag{14}$$

where $E_i(t) = 1$ if $d_i(t) > \zeta\times$lifecycle. Here $d_i(t)$ is the age of a product in week t bought in week i. $E_i(t) = 0$ if $d_i(t) \leq \zeta\times$lifecycle.

Table 5.4.1 shows the criteria values for the IBL and IBA foresting for the four different products. Eight parts have better forecast results by taking an age threshold into account. Two products has almost the same forecast results. The forecast results with an 20% age threshold, are a clear indication of taking an age threshold into account. Furthermore research is necessary to optimise the value of ζ to create more accurate forecast results for the IBA model.

Table 5.4.1: Forecast results of IBL and IBA for different spare parts

	Refrigerator 1			Refrigerator 2			Smartphone 2			Television 2		
	Comp	CB	DG	Comp	CB	DG	TS	CB	BC	LCD	CB	Cover
SUM	0.60	0.11	0.12	0.00	1.44	1.09	4.75	0.76	0.75	0.43	7.53	1.04
MAPE	0.72	0.26	0.48	0.48	1.50	1.35	4.76	0.89	1.14	0.76	7.53	1.87
RMSPE	0.83	0.34	0.62	0.68	1.80	1.54	5.37	1.04	1.39	0.98	9.03	2.64
SUM*	0.35	0.14	-0.07	1.22	0.50	1.12	3.71	-0.11	-0.20	-0.67	-2.41	-0.39
MAPE*	0.53	0.27	0.37	1.34	0.60	1.37	3.74	0.51	0.54	0.85	2.41	1.37
RMSPE*	0.63	0.35	0.58	1.51	0.75	1.55	4.18	0.67	0.94	1.11	2.59	2.55

Table notes

Comp is spare part Compressor. CB is Circuit board. DG is Door gasket. TS is Touch screen. BC is Back cover. LCD is LCD panel.

SUM, MAPE, and RMSPE criteria values of the IBL forecasting.

SUM*, MAPE*, and RMSPE* criteria values of the IBA forecasting.

6 Conclusion

A production period consists of three different periods, the initial, mature, and end-of-life (EOL) phase. In the first two phases, OEMs produces a specific product. In the EOL phase the production stops, but consumers may still have demand for spare parts to repair their products. The OEMs have interest in defining the number of spare parts which are necessary to serve the consumers' spare part demand. This research focuses on consumer goods and makes different models to forecast the demand for spare parts over the EOL phase. Consumers urge to repair their products depends on different criteria, e.g. the price, the lifecycle of the product or the warranty period. Different kind of installed base models are created. The most suitable type of installed base depends on the spare part, the consumer market and the characteristics of the product. Five different installed base models are created, e.g. the lifetime installed base for spare parts with a longer lifetime. The warranty installed base focuses on products that are only repaired during the warranty period. The older installed base takes only products of old age into account. The economic and mixed installed base is useful for non-essential spare parts for products which are out of warranty. In this research we provide eighteen different spare parts and sixteen out of the eighteen spare parts have more accurate demand forecast performances over the EOL phase than a standard black box autoregressive model.

In this case study, an overview is given of the best forecast models per kind of products. OEMs may use these information to forecast the number of spare parts needed to serve the total demand of the consumers for spare parts over the EOL phase. Although the demand for spare parts over the EOL phase depends on the consumers' behaviour and preferences, it can be helpful to cluster products and spare parts in groups, depending on the characteristics and expected behaviour of the consumers. EOL demand can be forecasted per cluster by using a common type of installed base that applies for all products of that cluster.

7 Discussion

This research has some discussion points about the reliability of the paper. In addition, future work is provided to improve forecasting demand of spare parts over the EOL phase.

7.1 Disruption in the supply chain

This research provides different forecast models, created by different kinds of installed base variables. These installed base variables are created by using sales and return data. These sales and return data are provided by the OEMs. Doing research on sales data is always a bit challenging, especially if these sales data is provided by the OEM. Regularly, by selling a new consumer good, there is a trade intermediary, e.g. Media Markt or the supermarket. These companies buy at the OEM in large sizes every period. These data is hard to investigate if research is interested in sales data between the trade intermediary and the consumer. The data offered by the OEM is often erratic, because of the information disruption in the supply chain. The erratic sales data is smoothed by using the EMWA method to create a more likely sales data between the trade intermediary and the consumer.

7.2 Future work and improvements

Future work can be provided by creating new installed base variables with different characteristics, e.g. adding different kinds of costs in the IBE and IBM, e.g. handling costs or labour costs. Furthermore, improvements in this case study can be gained by exploring more information in the IBA model. In this case study, we used an age threshold of 20% to investigate if our presumption, older products affects demand for spare parts, is supposable. We chose 20% by limitations of our data. A more interesting age threshold is around 70% of the lifecycle of a product, because then the change of needing a repair of the product increases, due to the aging of the products.

If a product gets too old, approximate the total lifecycle, consumers may not spend money on the repair of the product anymore. They are more likely to buy a new product. It is conceivable to put an upper age threshold for the IBA model. A likely threshold would be around 90% of the lifecyle. More older products may be written off and consumers may buy a new improved version. This process is an example of the bathtub model.

Besides the IBA, the value for α in the EWMA formula can be optimised more, by taking more products into account for the sum function. A more general α is than declared with lower criteria values.

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