ERASMUS UNIVERSITY ROTTERDAM Erasmus School of Economics

Bachelor Thesis Economie en Bedrijfseconomie

Overconfidence and over-optimism: a theoretical approach to optimal bidding in auctioning

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Abstract

This research covers the topic of optimal bidding strategies in first-price sealed bid private valuation auctions. It aims to answer the research question: *How do overconfidence and over-optimism affect the optimal bidding strategy in first-price sealed bid auctions?* Literature will be used to define these behavioural biases and describe the literature on the topic so far. The answer to the research question will then be formed by using the model of McMillan (1992) and extending this model to take into account these biases. Over-optimism and overconfidence increase the perceived chance of winning the auction and leave the optimal bid unchanged if the believed chance of winning the auction is increased due to a multiplicative parameter. If the number of bidders is believed to be lower or the chance of winning is additively believed to be higher, bidders in optimum decrease their bid.

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Introduction

Auctions are important for the economy because they involve enormous amounts of money. Many products are standardly sold through auctioning and some markets display similar characteristics as auctions without formally being one (Klemperer, 1999). Theory distinguishes between four basic types of auctions: the ascending-bid auction, the descending-bid auction, the first-price sealed bid auction and the second-price sealed bid auction. The four types have different optimal bidding strategies. The valuation of the object for sale can be *common value,* meaning that the value is the same for everyone but private information about the valuation can differ. The value can also be a pure *private valuation,* in which case the buyer has a private valuation only known by himself. It could also be the case that there is a combination of both types (Klemperer, 1999).

It is becoming increasingly clear that individuals do not always act in a way that maximizes their utility. In behavioural economics, many biases are found that influence the way people behave and make choices. One of these biases is over-optimism. Sharot (2011) states that over-optimism is the tendency to overestimate the likelihood of positive events. Capraa, Laniera, and Meerb (2010) found that through mood, over-optimism can influence the valuation of the object, the willingness to pay, and the bidding behaviour. Another behavioural bias that occurs in auctioning is overconfidence. The difference between over-optimism and overconfidence is that overconfidence often results in people tending to believe the precision of their information is higher (Herz, Schunk, & Zehnder, 2014). In auctioning, it could thus very well be that overconfident people believe probabilities of winning to be higher just as over-optimistic people would. Because this distinction is relatively small and not yet exactly determined for auctioning, over-optimism and overestimation will be examined in the same sub question. Overconfident people may overestimate how certain they are of the valuation. This is also believed to be the reason for the occurrence of the winner's curse, a situation in which the winner of an auction overbids the value of the object because his estimate of the value was the most optimistic one. There are three types of overconfidence according to Moore and Healy (2008), namely overestimation, overprecision and overplacement.

Behavioural biases can have a big impact on the outcome of auctions, potentially leading to losses for the bidders. To my knowledge, most of the research regarding optimal bidding is done focussing on first-price or second-price auctions, but not on sealed bid auctions. Also, there has not been a lot of research about over-optimism in auctioning. How these types of biases exactly affect the optimal bidding strategy has not yet been determined by researchers. Therefore, it can be very insightful to do more research on this subject, as it could help people understand bidding behaviour better. This paper will thus aim to expand the basic model of optimal bidding under first-price sealed bid auctions to account for over-optimism and overconfidence. The research question is:

How do overconfidence and over-optimism affect the optimal bidding strategy in first-price sealed bid auctions?

To answer this question, a few sub questions will be answered. These sub questions are:

(1) What is the effect of over-optimism or overestimation on the bidding strategy?
(2) What is the effect of overprecision on the bidding strategy?

The social relevance is that a lot of money is at stake in total in these types of auctions. Also, understanding the effect of behavioural biases is important because of the enormous effect it could have on bidders. They can be helped to understand their behaviour and limitations better, potentially saving them a lot of money. This does not affect overall surplus, because the bidders loss is the sellers gain. In general, the bidder is the weaker party. If society values the interests of the bidder more than that of the auctioneer, society as a whole is better off. Also, potentially the results from this research can provide auctioneers with information they can use to better organize their auctions. Perhaps behavioural biases make people bid more, meaning it is in the auctioneers best interest to let biased bidders bid biasedly. On the other hand, if bidders bid less due to biases, the auctioneer would do well to reduce bias in the hopes of increasing bids. The scientific relevance is the contribution to auction theory and the expansion of theory regarding optimal bidding strategy and behavioural biases to first-price sealed bid auctions. This research aims to explain how the behavioural biases overconfidence and over-optimism can affect bidders and what this does to their optimal bidding strategy. This is a contribution to existing theory, as theory regarding this subject is scarce.

Next, the existing theory will be discussed and the sub questions will be introduced in the theoretical framework. After this the model by McMillan (1992), that will be the basis for the models that follow, will be explained. It will also be discussed in what ways this model can be altered to account for the behavioural biases that are of interest in this research. Following the methodology, is the results section. In this section the results of the models will be discussed extensively. After this, a conclusion and discussion will follow.

Theoretical framework

The existing literature on the subject of the optimal bidding strategy for first-price sealed bid private valuation auctions for bidders that are subjected to over-optimism and overconfidence is limited. To my knowledge, there has not yet been explicitly tested how these biases alter the bidding behaviour and the bids people make. Also, theoretically there is little known on how these biases change the bidding function. Therefore, the literature is not sufficient to convincingly predict in what way overconfidence and over-optimism affect the bidding function. This research will thus look at the different ways these biases can affect the bidding function and how that would affect the optimal bidding strategy. Multiple possible ways in which over-optimism and overconfidence affect bidders beliefs will be explored to see how the bidding function would be affected.

Even though the literature does not explicitly state how over-optimism and overconfidence alter the optimal bid for first-price sealed bid auctions, it can help predict the potential effects on the bidding function. Capraa, Laniera, and Meerb (2010) did an experiment and found that over-optimism can have multiple effects on bidding behaviour. This over-optimism was induced through mood. The type of auction in this research is not a first-price sealed bid private valuation auction. Nevertheless, general lessons about the effect of over-optimism could be used to determine the effect it could have on other types of auctions. They found that mood can increase the willingness to pay and can also alter peoples valuations and change their bidding behaviour. This research will not go into the origin of the over-optimism, but will focus on the effect it has on the bidding strategy. Johnson and Tversky (1983) write that over-optimism makes people misperceive probabilities when the outcome of an event is desirable or undesirable. People tend to attach higher probabilities to desirable events and lower probabilities to undesirable events. Because people willingly join auctions and want to win the object, winning the auction can be seen as a desirable event. Over-optimism can thus affect bidders in such a way that they consistently believe the chance of winning to be higher and the chance of losing to be lower. It can also alter the valuation of a bidder.

In auctions that sell an object that is purely a private value object, it seems unlikely that rational bidders would overbid, as it is known to them what the exact valuation is. However, Goeree, Holt and Palfrey (2002) show with their model that in first-price private value auctions there can also be overbidding if the cost of overbidding is relatively low. The optimal bidding strategy for first price private value auctions would be to stop bidding when the price reaches the bidders private valuation. It is thus in practice not always the case that individuals act according to what would be their optimal strategy. Perhaps people do not always know what their exact private valuation is. Overconfidence could be an explanation for this phenomenon. Theory distinguishes between tree different types of overconfidence, as described by Moore and Healy (2008). The first one is the overestimation of one's actual performance, also known as overestimation. Overestimation can vary from overestimating the chance of success to overestimating the level of control. If overestimation in auctioning leads to bidders overestimating the chance of winning the auction, the effect is similar to that of over-optimism. Formally, the distinction between over-optimism and overconfidence in the existing literature is that over-optimistic people overestimate their chance of success and overconfident people tend to overestimate the accuracy with which they estimate this chance of success (Herz, Schunk, & Zehnder, 2014). In auctioning however, this distinction is not yet clearly found and the effects of these biases can be very similar. Therefore the first sub question is:

(1) What is the effect of over-optimism or overestimation on the bidding strategy?

The second type of overconfidence is the overplacement of performance relative to others. This is also called the better-than-the-average effect or overplacement. An example of this is asking a certain number of people if they believe themselves to be a better driver than the average. In expectation, 50% of people are. However, if more than 50% of people believe themselves to be better, this is due to overplacement. The last type is excessive precision in one's beliefs, or overprecision. Usually, this is investigated by asking people their 90% confidence intervals for numerical answers to questions. If 10 questions are answered, in expectation 9 of those should fall within the confidence interval. In general, the results of these experiments show that people are correct significantly less than they would be were they actually 90% confident.

The three types of overconfidence could have different effects on the bidding strategy. Overestimation could lead to bidders believing they are more likely to win the auction than they actually are. The bidder makes a wrong estimation of the chance of winning, by structurally thinking the chance of winning is higher. Bazerman and Samuelson (1983) found that in common value auctions overestimation increased in the number of bidders and in the uncertainty of the valuation of the object. However, to my knowledge, little research has been done for private value auctions and how behavioural biases affect the optimal bidding strategy. Perhaps the chance of winning the auction for first-price sealed bid private value auctions would be structurally overestimated dependent on the number of bidders, as has been found for common value auctions.

Overplacement of performance seems less likely to influence the bidding strategy of first-price sealed bid private valuation auctions. Overplacement might make bidders believe they performed better than the average bidder, but this is after the auction took place. Therefore, this specific type of overconfidence will not be investigated in this research. The third type of overconfidence, overprecision, could potentially influence the bidding strategy. If a bidder is more confident in his private valuation, he believes potential overbidding to be less common. For example, if a bidder values an object with a 10% margin of error, but believes this margin to be 5%, he fails to take into account some range of potential losses. It could also make the bidder believe others won't value the object as highly as himself. To examine the effect this can have on the bidding strategy, the second sub question is:

(2) What is the effect of overprecision on the bidding strategy?

Methodology

The basic model that will be used is the one described by McMillan (1992). It assumes that bidders know their own valuation, but not those of other bidders. The valuations lie somewhere between 0 and 1 and are uniformly distributed. Every bidder has his own private valuation and bidders are homogenous. In the model there are i bidders, in which i=1,2,3…n. The bid of bidder i is denoted as B_i. The valuation of bidder i is denoted by V_i. Valuations are uniformly distributed between 0 and 1, meaning that any valuation ≥0 or ≤1 is equally likely. Bidders assume other bidders shade their bid, so that the bid of other bidders B_j is their valuation V_j multiplied with a factor k. If k=1 a bidder bids his total valuation. Because of the uniform distribution, the probability that B_i is higher than someone else's bid is: Pr(B_i ≥ k*V_j) = Pr(k*V_j < B_i) = Pr($V_j < \frac{B_i}{k}$) = $\frac{B_i}{k}$.

There are n bidders, of which one is bidder i himself. The probability of winning the auction with n bidders therefore is: $(\frac{B_i}{k})^{n-1}$. The probability of winning thus depends on the number of competitors (n-1), the bid of bidder i and the shading of other bidders. Assumed is that k is a parameter with values above 0 and below or equal to 1. In general, if B_i is lower than k, $\frac{B_i}{k}$ is smaller than 1. A function smaller than one to the power of n-1, decreases if n increases. The more competitors there are, the smaller the chance of winning the auction.

If bidder i wins the auction, he earns his valuation minus his bid. This can be represented by (V_i - B_i). The profit for bidder i when winning the auction thus only depends on his own valuation V_i and what the bidder has to pay for the object, his bid B_i. Bidder B_i maximizes his probability of winning multiplied with the payoff of winning. This can be seen as the expected value of bid B_i. The maximization problem can be represented as:

(1) $\max_{B_i}(\frac{B_i}{k})^{n-1}(V_i - B_i)$.

The shape of this function depends on the values of the parameters, but in general the shape of the function is comparable to the figure shown below. In Figure 1, the expected value of this function is plotted. This has been done for values of $V_i = 0.9$, $k = 0.9$ and n = 9. The x-axis is B_i and the y-axis can be seen as the expected value of a certain bid B_i, labelled as EV(B_i). For these specific values of the parameters, the bid that maximizes the expected value is at $B_i = 0.8$. There are two forces when increasing a bid. The higher the bid, the higher the chance of winning the auction. This increases the expected profit of the auction. On the other hand, increasing the bid also increases the cost of winning the auction. This lowers the expected profit of the auction. Bidders thus have to consider both forces when determining what to bid.

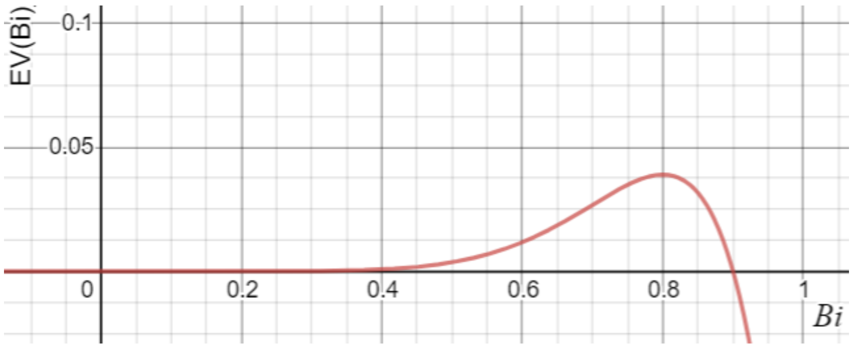

Figure 1. Plot of $(\frac{B_i}{k})^{n-1}(V_i$ - $B_i)$ with $V_i = 0.9$, $k = 0.9$ and n = 9

The maximization problem in expression (1) can be derived more extensively to find the bid that maximizes the expected value. This bid can be seen as the optimal bid for bidder i. Graphically, the expected value maximizing bid is the peak of the function, which is at $B_i = 0.8$ in Figure 1. Mathematically, the optimal bid can be found by deriving expression (1) as has been done below:

$$\left(\frac{B_i}{k}\right)^{n-2}(V_i - B_i)(n-1)\frac{1}{k} - \left(\frac{B_i}{k}\right)^{n-1} = 0$$

$$(\frac{1}{k})^{n-2}(B_i)^{n-2}(V_i - B_i)(n-1)\frac{1}{k} - (B_i)^{n-1}(\frac{1}{k})^{n-1} = 0$$

$$(B_i)^{n-2}(V_i - B_i)(n-1) - (B_i)^{n-1} = 0$$

$$(V_i - B_i)(n-1) = \frac{{B_i}^{n-1}}{{B_i}^{n-2}}$$

$$(V_i - B_i)(n-1) = B_i$$

$$V_i(n-1) = B_i + B_i(n-1)$$

$$B_i^* = V_i(1 - \frac{1}{n})$$

The general results from this model is that bidders shade their valuation V_i with fraction k = $(1 - \frac{1}{n})$. The more competitors there are, the closer k will get to 1. Bidder i thus shades less if the number of competitors increases. This model will be the basis for expansion to over-optimism and overconfidence.

Accounting for over-optimims and overconfidence

Because the existing literature on the topic of overconfidence and over-optimism does not convincingly predict how the bidding function is affected by these biases, multiple possibilities will be explored. This will be done by translating what theory suggests is the affect of these biases on the beliefs of bidders to the models or by trying other possible alterrations to the parameters of the models. Both multiplicative or additive parameters will be examined. Moreover, the biases can have character traits in common. For example, both over-optimism and overconfidence can change a bidders perception of the probability of winning the auction. Therefore, some models could potentially be a representation for both biases.

Over-optimism and overconfidence can make bidders overestimate their chances of success. The chance of success, $(\frac{B_i}{k})^{n-1}$, is dependent on B_i, k and n. These parameters could thus all be misperceived by bidders because they are over-optimistic or overconfident. It could also be the case that the biases do not affect these specific parameters, but instead the total probability of winning the auction. This could be perceived to be multiplicatively or additively higher. Because the exact way in which the bidding function is affected is not known yet, these options will be examined.

Firstly, the case for a multiplicative parameter will be examined. This multiplicative parameter is a constant that is multiplied with the total probability of winning the auction. To represent this in the model, a parameter α will be added. The parameter increases if a bidder believes to have a higher chance of winning the auction. In other words, the bidder believes the chance of winning to be multiplied by α. If α is 1, the function represents the function of an unbiased bidder. Any value larger than 1 means that the bidder believes his chance of winning the auction to be higher. This only influences the perceived probability of winning the auction, and not the profit for the bidder if he turns out to win the auction. The bidder's valuation does not change, so the only difference is the beliefs of the bidder that he has a higher chance of winning the auction. The probability of winning the auction thus becomes $\alpha(\frac{B_i}{k})^{n-1}$ and the profit for the bidder if he wins the auction remains $(V_i - B_i)$. The optimization problem for this setting is defined as expression (A), of which the extensive derivation can be found in Appendix A:

(A) $\underset{B_i}{Max}\,\alpha(\frac{B_i}{k})^{n-1}(V_i - B_i)$

Another way the probability of winning the auction could be perceived to be higher, is if the total probability of winning the auction is increased with an additive parameter. To represent this in the model, a parameter c will be added as a constant to the probability of winning the auction. If c = 0 the model is the same as for an unbiased bidder. c Can take any value ≥ 0. However, the sum of

$(\frac{B_i}{k})^{n-1} + c \leq 1$. This is because probabilities cannot exceed 1. Values smaller than 0 would imply the bidder thinks he has less chance to win, which is unlikely for an over-optimistic or overconfident bidder. The bidder will thus believe his probability to be c higher than it in reality is. The extensive derivation of this setting can be found in Appendix B. The optimization problem is denoted by (B):

(B) $\underset{B_i}{Max}((\frac{B_i}{k})^{n-1} + c)(V_i - B_i)$

One more way in which the bidder can think he has a higher chance of winning the auction, is if he believes the chance of bidding more than someone else is additively higher. For every additional competitor, the bidder thinks his chance of winning the auction is higher. This can be represented by adding a constant d to the chance of bidding higher than someone else. Assumed will be that $\frac{B_i}{k} + d \leq 1$, because the probability of winning the auction cannot become higher than 1. The optimization problem that follows is denoted by expression (C) and the extensive derivation of this maximization problem can be found in Appendix C:

(C) $\underset{B_i}{Max}\left(\frac{B_i}{k} + d\right)^{n-1}(V_i - B_i)$

Next, the cases in which the total probability of winning the auction is perceived to be higher will be examined by altering the parameters that are an input for this probability. Firstly, the setting for which a bidder believes there to be less competitors is investigated. The less competitors there are, the more likely the bidder is to win the auction. The number of competitors could be believed to be lower by a fixed number of bidders. This will be represented in the model by subtracting an additive parameter e. If e = 0, the model becomes the model by McMillan (1992) and represents the case for unbiased bidders. Assumed will be that e is never larger than $n - 1$, because otherwise it would mean that the bidder can also believe there to be a negative number of competitors. This is not possible. The probability of winning the auction therefore becomes: $(\frac{B_i}{k})^{n-1-e}$. The optimization problem for this setting is denoted below in equation (D). The derivation of this model is shown in Appendix D.

(D) $\max_{B_i}(\frac{B_i}{k})^{n-1-e}(V_i - B_i)$.

A similar setting as in equation (D) will be instigated next. However, the parameter will not be additive but multiplicative. The bidder will believe his chances of winning the auction to be higher, because he believes there to be less competitors by a fraction p. In other words, if the number of competitors increases by a certain amount, the bidder does not take all of them into account but just a fraction of them. This will be represented in the model by subtracting pn from the number of competitors, making the probability of winning the auction $(\frac{B_i}{k})^{n-1-pn}$. If p = 0, the model represents bidders that are not subjected to biases. Assumed will be that the value of p can vary between 0 and 1. In addition, $n - 1 - pn \geq 0$. Otherwise the bidder could potentially believe there to be a negative number of competitors, which is not possible. The extensive derivation of this model is found in Appendix E. The maximization problem for this setting is:

(E) $\max_{B_i}(\frac{B_i}{k})^{n-1-pn}(V_i - B_i)$.

Another potential effect these biases could have on the bidding function, is that the bidder thinks others shade their bid more. This changes the bidders beliefs about the chance he has to win the auction, by changing the beliefs about the probability of bidding higher than someone else. To represent this in the model, competitor bidders are believed to bid (k-z)*V_j. They are thus believed to bid less than in the model for unbiases bidders, as (k-z)*V_j < k*V_j. The believed probability of bidding higher than someone else therefore becomes: Pr(B_i ≥ (k-z)*V_j) = Pr((k-z)*V_j < B_i) = Pr($V_j < \frac{B_i}{k-z}$). Assumed will be that 0 ≤ z < k, because otherwise it would mean that bidders shade with a negative fraction, which is not possible. If this restriction on z is not assumed, it can also increase the fraction of shading to values above 1. This would mean that bidders bid over their private valuation, which would result in potential losses. This can happen if z < 0, because $(k - -z) = (k + z)$. The optimization problem for this way of representing biases in the model is derived in Appendix F. The new maximization problem denoted by expression (F) becomes:

(F) $\max_{B_i}(\frac{B_i}{k-z})^{n-1}(V_i - B_i)$.

Increasing the believed probability of winning the auction for the bidder, could also be because the bidder believes to be more precise in his estimates than his competitors. This can be represented in multiple ways. Again, theory does not convincingly dictate how overconfidence affects the bidding function for first-price sealed bid private valuation auctions. Therefore, multiple possibilities will be examined. Firstly, the bidder can believe the valuations to be distributed differently. For example he does not believe the valuations to be between 0 and 1, but between 0 and a parameter γ. This can be represented in the model by changing the range of valuations with parameter γ. This parameter can take any value above 0. If γ is 1, the model represents the model for unbiased bidders and is thus the same as the basic model. If γ is 1.2, the bidder believes the valuations can be 20% higher than they are in reality. Therefore, the valuations of competitors will now be believed to be uniformly distributed between 0 an γ. This alters the input of the model, because the introduced parameter changes the beliefs the bidder has about the chance of bidding higher than someone else. The probability of bidding higher than a competitor becomes Pr($V_j < \frac{B_i}{k}$) = γ$\frac{B_i}{k}$. This changes the probability of winning the auction to $(\gamma \frac{B_i}{k})^{n-1}$. The maximization problem for this setting is denoted in equation (G) and the extensive derivation for this model is found in Appendix G.

(G) $\underset{B_i}{Max} (\gamma \frac{B_i}{k})^{n-1}(V_i - B_i)$

So far, all the models are based on the model by McMillan (1992). This model is symmetrical, and thus assumes all bidders are homogenous. It therefore does not allow for examining situations in which bidders are heterogenous. In practice, it can very well be the case that bidders are biased in different ways or that some are more biased than others. Examining heterogeneity in bidders that are over-optimistic or overconfident could thus be very insightful. A setting in which heterogeneity occurs, is when a bidder believes others to not bid at the extremes. This can be a potential effect of overconfidence. For example, the bidder believes the valuation of his competitors to have a smaller distribution than his own. For simplicity, this setting will be looked at for two bidders. Bidder i is the biased bidder and bidder j is the unbiased bidder. In this setting, bidder i believes $0 \leq V_j \leq 0.8$. His own valuation V_i can then be either between 0 and 0.8 or above 0.8. This is a heterogenous model, as bidder i is different from the other bidder, because of his bias. If V_i ≥ 0.8, the bidder believes no one will bid more than his valuation. Bidding higher than 0.8 would then only result in more costs, without increasing the probability of winning. Assumed will be that the unbiased bidder j follows the general optimal bidding strategy that follows from the basic model by McMillan (1992), which is to bid B_j^* = $V_j(1 - \frac{1}{2}) = \frac{1}{2}V_j$. Bidder i knows bidder j follows this bidding strategy. Because this setting is not for symmetrical bidders, the optimal bid of the biases bidder i will be examined from a strategic point of view. Model H can be found more extensively in Appendix H.

Below in Table 1 an overview of the models is provided. It shortly describes how the models are different from the model by McMillan (1992) and what assumptions are made on the parameters:

Table 1

Overview of the models describing the difference from the basic model and the assumptions on the parameters.

Model:	*Difference from basic model:*	*Assumptions parameters:*
A	A multiplicative parameter α increases the believed total probability of winning the auction to: $\alpha(\frac{B_i}{k})^{n-1}$.	α ≥ 1
B	An additive parameter c increases the believed total probability of winning the auction to: $((\frac{B_i}{k})^{n-1} + c)$.	$(\frac{B_i}{k})^{n-1} - 1 \geq c \geq 0$

C	An additive parameter increases the believed chance of bidding higher than someone else to: $\frac{B_i}{k} + d$.	$0 \geq d \geq 1 - \frac{B_i}{k}$
D	An additive parameter e is subtracted from the number of competitors, making the believed probability of winning the auction: $(\frac{B_i}{k})^{n-1-e}$.	$n - 1 > e \geq 0$
E	A multiplicative parameter pn is subtracted from the number of competitors, making the believed probability of winning the auction: $(\frac{B_i}{k})^{n-1-pn}$.	$0 \leq p < 1$ $n - pn \geq 1$
F	An additive parameter z is subtracted from the fraction k with witch competitors shade. The believed probability of winning the auction becomes: $(\frac{B_i}{k-z})^{n-1}$.	$k > z \geq 0$
G	Parameter γ is multiplied with the potential valuations of competitors changing the believed probability of winning the auction to: $(\gamma \frac{B_i}{k})^{n-1}$.	$\gamma > 0$
H	The bidder believes his competitors won't bid above 0.8, this model will be looked at from a strategic point of view.	

Results

Model A in Appendix A contains the model in which a multiplicative parameter α was multiplied with the total probability of winning the auction. The optimal bidding strategy is unchanged compared to the basic model by McMillan (1992). The optimal bid for a bidder is B_i^* = V_i(1 - $\frac{1}{n}$). This thus means that if the number of competitors n increases, the bidder maximizes his expected value by increasing his bid. The optimal shading fraction is k = (1 - $\frac{1}{n}$). In Figure 2 a plot is made to compare the expected value function of a bidder that is not biased with a bidder that is biased in the way that is represented by model A. The function for the unbiased bidder is in green versus the biased bidder in red. The values of the parameters are $V_i = 0.9$, $k = 0.9$, n = 9 and α = 2. It can be seen that the parameter α amplifies the peak of the function, but does not shift the optimal bid.

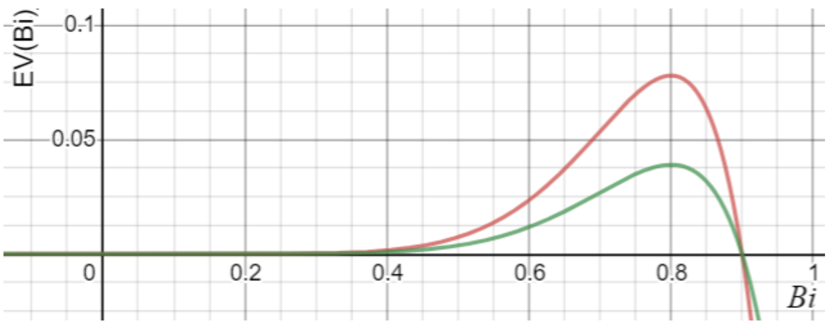

Figure 2. Plot of model A (red line) and the basic model by McMillan (1992) with $V_i = 0.9$, $k = 0.9$, n = 9 and α = 2

The multiplicative parameter increases the expected value with factor α, but does not change the bid at which this expected value is maximized. This is the reason the optimal bidding function remains unchanged compared to the original model. This is contradictory with the findings of Capraa, Laniera, and Meerb (2010). They wrote that over-optimism increases the perceived chance of winning and the willingness to pay.

Model B in Appendix B contains the model in which an additive parameter c is added to the total probability of winning the auction. Algebraically, this model is difficult to optimize and solve. Therefore a plot of this model is made in comparison to the basic model for different values of c. This plot can be found in Figure 8 in Appendix B. For very low values of c there is an optimal bid. This optimal bid decreases in c. For higher values of c there is no optimal bid, because the peak is no longer present. This is because there are corner solutions when c is not relatively small. It would then be optimal for bidders to bid 0. For different values of V_i these corner solutions also remain, as is shown in Figure 9 in Appendix B. This model seems less realistic than the other models, because it implies that even if a bidder bids 0, he perceives a positive chance of winning the auction.

In Appendix C model C is shown. In model C an additive parameter d is added to the probability of bidding higher than another bidder. The optimal bid for bidder i in this model is $B_i^* = V_i\left(1 - \frac{1}{n}\right) - \frac{d}{n}\left(1 - \frac{1}{n}\right)$. Because both d and n are positive parameters, $V_i\left(1 - \frac{1}{n}\right) - \frac{d}{n}\left(1 - \frac{1}{n}\right) < V_i(1 - \frac{1}{n})$. The expected value maximizing bid is thus lower than it is in the basic model. Figure 3 contains a plot of model C in red and the basic model in green. It can be seen that the peak is amplified in model C and shifted to the left. The parameters in this figure have values $V_i = 0.9$, $k = 0.9$, n = 9 and d = 0.1. The effect of an increase in the parameter d is $B_i^{*\prime}(d) = -\frac{1}{n}(1 - \frac{1}{n})$. Because n is a positive parameter, this expression is always negative. The higher d, the lower the optimal bid becomes.

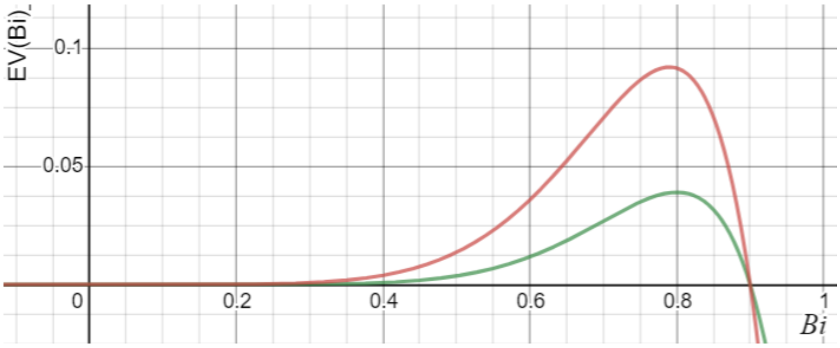

Figure 3. Plot of model C in red and the basic model in green with $V_i = 0.9$, $k = 0.9$, n = 9 and d = 0.1

This model corresponds to what Bazerman and Samuelson (1983) found for common value auctions. They wrote that overestimation increased in the number of bidders.

Model D is shown in Appendix D. In this model an additive parameter e is subtracted from the total number of bidders. The optimal bid for this model is B_i^* = $V_i(1 - \frac{1}{n-e})$. The fraction k with which the bidder shades his bid is thus $(1 - \frac{1}{n-e})$. Because $\frac{1}{n} < \frac{1}{n-e}$, $V_i(1 - \frac{1}{n-e}) < V_i(1 - \frac{1}{n})$. Compared to the basic model, biased bidders in model D bid less in optimum. The effect of an increase in e on the optimal bid is $B_i^{*\prime}(e)$ = $V_i(-\frac{1}{(n-e)^2})$. This expression is always negative, meaning that the more biased the bidder is, the lower his bid will become. This can also be shown graphically, as can be seen in Figure 4. The expression $B_i^{*\prime}(e)$ = $V_i(-\frac{1}{(n-e)^2})$ becomes very small for large values of n, meaning that if there are more competitors, the effect of the bias on the optimal bid becomes smaller. This is also intuitively logical, as e represents the number of competitors that are not taken into account by the bidder. If n is large, the effect of e becomes relatively small because there are a lot of competitors already.

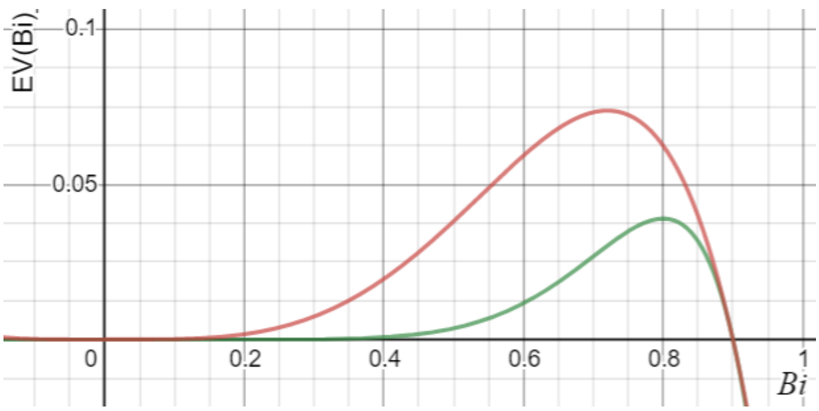

Figure 4. Plot of the basic model by McMillan (1992) in green and model D in red with $V_i = 0.9$, $k = 0.9$, n = 9 and e = 4

In Appendix E model E is derived extensively. In this model a multiplicative parameter pn is subtracted from the total number of bidders. The profit maximizing bid for this model is: B_i^* = $V_i(1 - \frac{1}{n(1-p)})$. Compared to the optimal bid in the model for unbiased bidders, this is lower. Because $\frac{1}{n(1-p)}$ > $\frac{1}{n}$, $V_i(1 - \frac{1}{n(1-p)})$ < $V_i(1 - \frac{1}{n})$. The higher p, the more biased the bidder is. The effect of p on the optimal bid is: $B_i^{*\prime}(p)$ = $V_i(-\frac{n}{p(p-2)})$. This expression is always negative. An increase in p thus means a decrease in the profit maximizing bid. This can also be shown graphically, as follows from Figure 5.

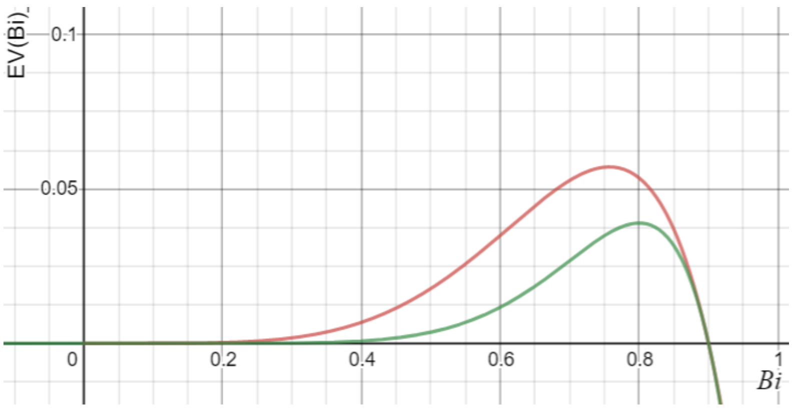

Figure 5. Plot of the basic model in green and model E in red with $V_i = 0.9$, $k = 0.9$, n = 9 and p = 0.3

Both model D and E show that the more biased the bidder is, the less he will bid. This is again contradictory to what Capraa, Laniera, and Meerb (2010) wrote.

Model F is derived extensively in Appendix F. In this model an additive parameter z was subtracted from the fraction with which competitors shade. The optimal bid for this model remained unchanged compared to the model by McMillan (1992), namely B_i^* = $V_i(1 - \frac{1}{n})$. The more competitors there are, the less the bidder shades his bid. In Figure 6 the function of this model is plotted in red compared to the basic model in green. The peak of model F is amplified, meaning the expected value is higher. The optimal bid does not change.

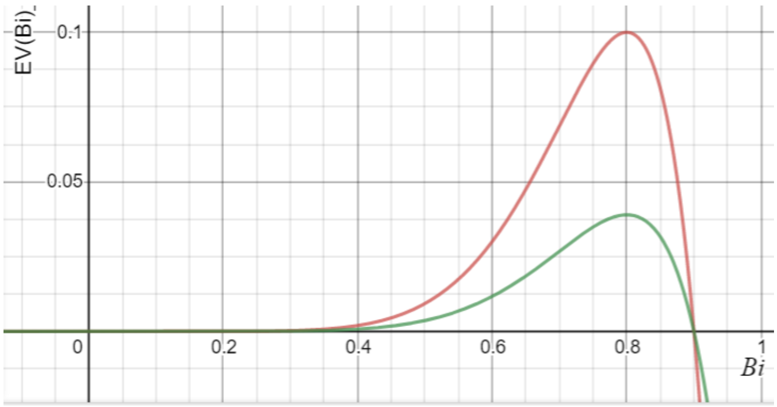

Figure 6. Plot of basic model in green and model F in blue with $V_i = 0.9$, $k = 0.9$, n = 9 and z = 0.1

In Appendix G model G is shown. In this model a multiplicative parameter γ is multiplied with the valuations of the competitor bidders. For this model, the optimal bid also remains unchanged compared to the basic model. The optimal bid is thus B_i^* = V_i(1 - $\frac{1}{n}$). This can also be shown graphically, as has been done in Figure 7. Again, the peak of the function had been amplified due to the beliefs of the bidder that he has a higher chance of winning. The bid at which this peak is obtained remains the same.

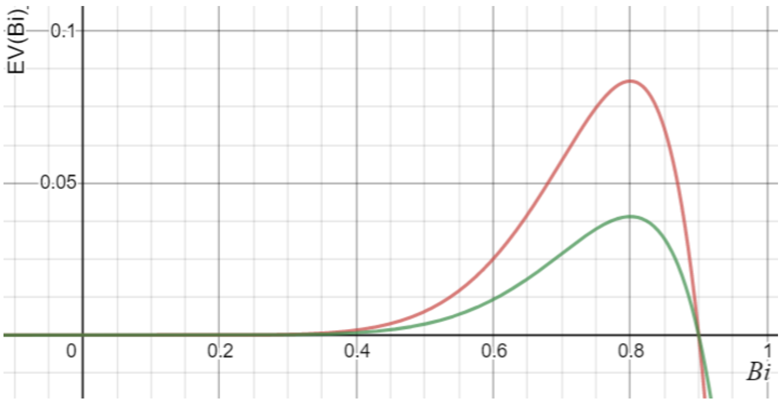

Figure 7. Plot of the basic model in green and model G in red with $V_i = 0.9$, $k = 0.9$, n = 9 and γ = 1.1

In Appendix H model H can be found more extensively. In this model, the bidder believes his competitors to never bid above 0.8. Bidder j follows the optimal bidding function B_j^* = V_j(1 - $\frac{1}{n}$). Bidder i knows this, and thus believes that bidder j will never bid above 0.4. Strategically, bidder i would never bid above 0.4, because this does not increase the perceived probability of winning and only decreases the believed expected value. However, in reality bidder j can bid up to 0.5. It can therefore be the case that bidder i loses an otherwise profitable auction, if his competitor bids above 0.4. Table 2 contains values for B_i at which bidder i with a certain value of V_i believes his expected value to be highest and also the bid that in reality has the highest expected value.[1] It can be seen that for valuations above 0.85 bidder i could have a higher expected value by increasing his bid. Bidder i will thus for some valuations make bids that are suboptimal because of his bias.

Table 2

The bid with the highest believed expected value and expected value for some values of V_i for a strategic bidder with only one competitor that believes his competitor to have a valuation below 0.8.

V_i	B_i with the highest believed expected value*	B_i with the highest expected value**
0.8	0.4	0.4
0.85	0.4	0.4/0.45
0.9	0.4	0.45
0.95	0.4	0.45/0.5
1	0.4	0.5

*found by choosing the bid with the highest believed expected value for the level of V_i from Table 4 in appendix H

** found by choosing the bid with the highest expected value for the level of V_i from Table 4 in appendix H

If bidder i were to follow the bidding function B_i^* = $V_i(1 - \frac{1}{n})$, and there would be more competitors, at some point increasing his bid would no longer result in higher chances of winning. For values of $V_i \leq 0.8 - \frac{4}{5n}$, the optimal bidding function according to bidder i would be B_i^* = $V_i(1 - \frac{1}{n})$. For values of $V_i > 0.8 - \frac{4}{5n}$ it is to bid $B_i^* = 0.8$. For some values of V_i and n this is shown in Table 2. For $V_i = 0.85$ and n = 17, $B_i^* = 0.8$. From this point onwards an increase in the number of competitors would not increase the optimal bid anymore, as the believed probability no longer increases due to a higher bid.

Table 3

The number of competitors at which increasing B_i no longer increases the probability of winning the auction for model H.

[1] Table 2 is based on Table 4 in appendix H, which uses increments of 0.05. It could thus be the case that the (believed) expected value maximizing bid differs slightly from the values shown in table 2.

V_i	n	B_i^*
0.85	17	0.8
0.9	9	0.8
0.95	6	0.79
	7	0.81

*found by inserting the values of the parameters into $V_i(1 - \frac{1}{n})$

Conclusion and discussion

This research tried to find an answer to the research question by using a theoretical model and trying to account for behavioural biases to see how the optimal bid would be affected. This research question is:

How do overconfidence and over-optimism affect the optimal bidding strategy in first-price sealed bid auctions?

The answer depends on how the bidding function is affected by the over-optimism and overconfidence. Because existing literature on this topic is relatively little and does not specifically address this topic, there is no certainty of how over-optimism and overconfidence will exactly influence the bidding strategy. Therefore, multiple possibilities have been explored in this research. The distinction between the biases for the specific topic of first-price sealed bid private valuation auctions is not defined clearly. The answers to the sub questions will thus be given in general, because the models in this research are not convincingly allocable to one specific bias.

Overconfidence and over-optimism either decreased the optimal bid or left it unchanged. For the cases in which the probability of winning the auction or bidding higher than someone else was multiplicatively higher, the optimal bid was not influenced. This can be explained by the shape of the function. The function represents the expected value of a bid B_i. Multiplying such a function thus only amplifies the peak, but does not shift the optimal bid. This is also intuitively logical, as the bidder believes he has the same higher chance of winning the auction for every possible B_i.

If the effect of overconfidence and over-optimism is merely an increased belief of winning the auction, but the optimal bid does not change, an auctioneer would not benefit from changing his procedures. Bidders also would not benefit from bidding differently, because if they were unbiased, their optimal bid would be the same. In practice, it may not necessarily be the case that bidders are unharmed. If bidders think the auction has a high expected value, they could be disappointed if they do not win the auction. Perhaps mentally this can have a negative effect on people.

If overconfidence or over-optimism causes bidders to believe there are less bidders, the optimal bid decreases. Because of the shape of the function, the higher n is, the higher the optimal bid becomes. Bidders will bid less if they believe there to be less competitors, shifting the peak of the function to the right. This is also intuitively logical. If bidders believe to have a higher change of winning the auction due to their beliefs about competitors, they may think they can afford to lower their bid.

In all models in which overconfidence and over-optimism changed the optimal bid, the optimal bid became lower than it would have been for the basic model. The bid is optimized from the perspective of the biased bidder. Because in the models the bidders are symmetrical, everyone equally lowers their bid. The bidders thus do not become worse off because everyone is symmetrically biased and adapts their bid accordingly. In practice it seems more likely that bidders are not equally biased. Perhaps if some bidders are more biased than others, they could be worse off. This could be the case for when a bidder that is biased relatively much makes a lower bid and loses the auction, but could have made a higher bid and won the object at a lower price than his valuation. In such a case, surplus for both the bidder and the auctioneer can be lost.

In the model where there are two heterogenous bidders and one of them believes others to never bid above a certain point, this bidder may lower his bid and will not bid above this point himself. This can potentially lead to that bidder losing an otherwise profitable auction. Again, this can happen when the biased bidder could have increased his bid to win the auction, and still bid below his private valuation.

Potentially, auctioneers can be less profitable if biases make their customers bid less. Auctioneers could take a few relatively cheap measures to lower biases bidders may have. For example, auctioneers can give their customers more information about how many competitors there are. This would help bidders correctly estimate this number, decreasing bias. Auctioneers could also inform their customers better about the probabilities of winning an auction, helping them in not overestimating this chance. If the cost of informing bidders is lower than the lower profits due to biases, these measures could be mutually beneficial for both bidders and the auctioneer.

Because the existing literature on the topic of optimal bidding strategies for first-price sealed bid auctions when bidders are over-optimistic or overconfident is limited, there is no guarantee that the models are a representation of reality. The results from the models are also contradictory to some existing literature that predicts over-optimistic bidders have a higher willingness to pay. This literature, however, focussed on other types of auctions and may not be applicable to first-price sealed bid auctions. Also, it could be that over-optimism and overconfidence affect bidders in a different way than what is represented in these models. However, they can provide useful insights on what would happen if bidders are biased in the way that is represented in the models. Also, many assumptions are made in these models, that in reality may not hold. For example, the assumption that valuations are uniformly distributed could be unrealistic. Perhaps valuations are normally distributed or skewed. The findings of this research are mostly based on one basic model. Therefore it could be useful to investigate if similar results are found when it is based on other types of models.

More research is needed to find out if in practice bidders follow these bidding functions. If empirical data can be collected on this topic, it could shed more light on how these biases affect the bidding function. Also, more research is needed for the other types of auctions, as there are many more auctions in which a lot of money is involved. For example in ascending bid common value auctions the bids of others can be seen as signals, which can affect the beliefs of the bidders as well. Also, second-price sealed bid auctions could have very different results. In these auctions, the highest bidder pays the second highest bid and not his own. Moreover, there can exist many more behavioural biases that influence people when joining an auction. Learning more about the biases people have and how they affect their behaviour can help them safe money. For the auctioneer, more profit could be made if bidding strategies and functions are better understood.

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Appendix A: model A

For this model, the same assumptions as in the model by McMillan (1992) are used. The probability of having a higher bid than someone else is thus: Pr(B_i ≥ k*V_j) = Pr(k*V_j < B_i) = Pr(V_j < $\frac{B_i}{k}$) = $\frac{B_i}{k}$. The probability of winning the auction is: $(\frac{B_i}{k})^{n-1}$. The bidder however will believe the probability of winning to be α times higher than it in reality is. Thus the maximization problem becomes:

(1) $\underset{B_i}{Max}\,\alpha(\frac{B_i}{k})^{n-1}(V_i - B_i)$

Maximizing this expression and solving it gives the optimal bid for this setting, as will be shown below:

$\alpha(\frac{B_i}{k})^{n-2}(V_i - B_i)(n-1)\frac{1}{k} - \alpha(\frac{B_i}{k})^{n-1} = 0$

$\alpha(\frac{1}{k})^{n-2} (B_i)^{n-2}(V_i - B_i)(n-1)\frac{1}{k} - \alpha(B_i)^{n-1}(\frac{1}{k})^{n-1} = 0$

$(B_i)^{n-2}(V_i - B_i)(n-1) - (B_i)^{n-1} = 0$

$(V_i - B_i)(n-1) = \frac{{B_i}^{n-1}}{{B_i}^{n-2}}$

$(V_i - B_i)(n-1) = B_i$

$V_i (n-1) = B_i + B_i(n-1)$

$\frac{V_i(n-1)}{n} = B_i$

$B_i^* = V_i(1 - \frac{1}{n})$

The optimal bidding strategy thus remains the same. The bidder shades his bid with fraction k = (1 - $\frac{1}{n}$). This fraction k increases in the number of competitor bidders, increasing the bid.

Appendix B: model B

The same assumptions as in the model by McMillan (19992) are used. Over-optimism will now be represented by a constant that is added to the total probability of winning the auction. The probability of winning the auction in reality is: $\left(\frac{B_i}{k}\right)^{n-1}$. The bidder however believes this probability to be $(\left(\frac{B_i}{k}\right)^{n-1} + c)$. This results in the following optimization problem:

(1) $\underset{B_i}{Max}(\left(\frac{B_i}{k}\right)^{n-1} + c)(V_i - B_i)$

Maximizing this expression results in the following equation:

$$\underset{B_i}{Max}\left(\frac{B_i}{k}\right)^{n-1}(V_i - B_i) + c(V_i - B_i)$$

$$\left(\frac{B_i}{k}\right)^{n-2}(V_i - B_i)(n-1)\frac{1}{k} - \left(\frac{B_i}{k}\right)^{n-1} - c = 0$$

To derive this further, complicated math would be required. This is because for values of c that are not relatively low, there is no peak in the function that represents the optimal bid. For higher values of c there are corner solutions, meaning it would be optimal for the bidder to bid 0. This can also be seen in Figure 8 and Figure 9.

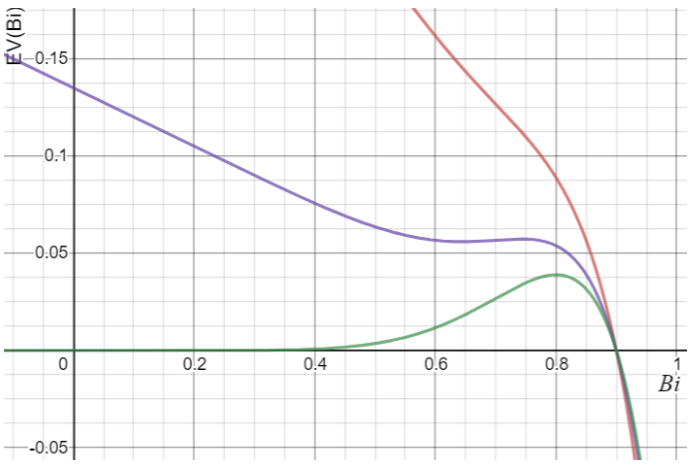

Figure 8. Plot of the basic model by McMillan (1992) in green and of model B for values c = 0.15 and c = 0.5 in blue and red respectively with $V_i = 0.9$, $k = 0.9$ and n = 9.

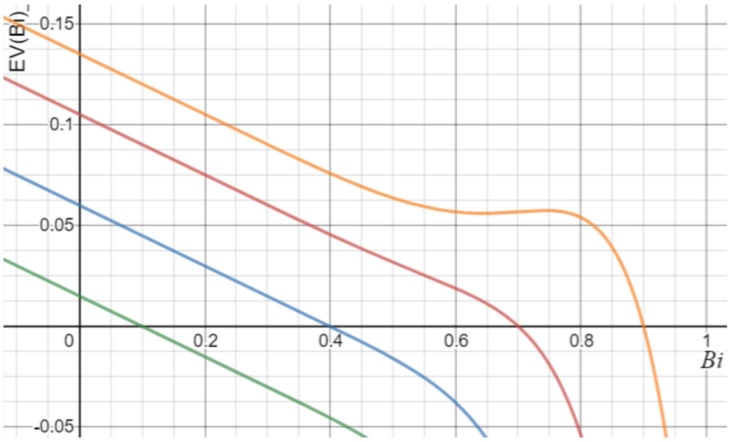

Figure 9. Plot of model B with $V_i = 0.9, 0.7, 0.4$ and 0.1 in orange, red, blue and green respectively and with c = 0.15, k = 0.9 and n = 9.

Appendix C: model C

The assumptions used for this model are the same as in the model by McMillan (1992). In this model however, the bidder believes that the probability of bidding higher than someone else is d higher than it in reality is. This probability thus is $\frac{B_i}{k} + d$. The total probability of winning the auction the bidder believes to be is therefore $\left(\frac{B_i}{k} + d\right)^{n-1}$. If the bidder wins the auction, he earns: $(V_i - B_i)$. The maximization problem becomes:

(1) $\underset{B_i}{Max} \left(\frac{B_i}{k} + d\right)^{n-1} (V_i - B_i)$

Maximizing this expression and solving it gives the optimal bid for this setting, as will be shown below:

$$\left(\frac{B_i}{k} + d\right)^{n-2} (V_i - B_i)(n-1)\frac{1}{k} - (\frac{B_i}{k} + d)^{n-1} = 0$$

$$(V_i - B_i)(n-1)\frac{1}{k} = \frac{(\frac{B_i}{k}+d)^{n-1}}{(\frac{B_i}{k}+d)^{n-2}}$$

$$(V_i - B_i)(n-1)\frac{1}{k} = \frac{B_i}{k} + d$$

$$V_i(n-1) - dk = B_i + B_i(n-1)$$

$$B_i^* = \frac{(V_i)(n-1)}{n} - \frac{dk}{n} = V_i\left(1 - \frac{1}{n}\right) - \frac{dk}{n}$$

This expression for the bid that maximizes the expected value still contains parameter k. The optimal bid is thus the bidders valuation shaded with fraction k subtracted with a function that depends on the number of bidders, parameter d and this fraction k. Because k equals the fraction with which bidders shade their valuation, k can be substituted with $(1 - \frac{1}{n})$ to allow further simplifying of the expression into a function that is dependent on les parameters. Substituting k = $\left(1 - \frac{1}{n}\right)$ into $B_i^* = V_i\left(1 - \frac{1}{n}\right) - \frac{dk}{n}$ yields:

$$B_i^* = V_i\left(1 - \frac{1}{n}\right) - \frac{d}{n}\left(1 - \frac{1}{n}\right)$$

Differentiating the optimal bidding strategy, the effect of an increase of the following parameters are found:

$$B_i^{*\prime}(d) = -\frac{1}{n}(1 - \frac{1}{n})$$

$$B_i^{*\prime}(n) = \frac{1}{n}(V_i - d - \frac{2d}{n})$$

$B_i^{*\prime}(V_i) = 1 - \frac{1}{n}$

Appendix D: model D

Once again, the same assumption from the model by McMillan (1992) are used. In this setting, however, bidders believe there to be e less competitor bidders in the auction. Because the probability of winning the auction can never be larger than 1, assumed will be that e ≤ n-1. The chance of winning the auction thus changes to $(\frac{B_i}{k})^{n-1-e}$. The maximization problem is given in equation (1):

(1) $\max_{B_i}(\frac{B_i}{k})^{n-1-e}(V_i - B_i)$.

Maximizing this expression and solving it gives the optimal bid for this setting, as will be shown below:

$(\frac{B_i}{k})^{n-2-e}(V_i - B_i)(n-1-e)\frac{1}{k} - (\frac{B_i}{k})^{n-1-e} = 0$

$(V_i - B_i)(n-1-e)\frac{1}{k} = \frac{(\frac{B_i}{k})^{n-1-e}}{(\frac{B_i}{k})^{n-2-e}}$

$(V_i - B_i)(n-1-e)\frac{1}{k} = \frac{B_i}{k}$

$V_i(n-1-e) = B_i + B_i(n-1-e)$

$V_i(n-1-e) = B_i(n-e)$

$B_i^* = \frac{V_i(n-1-e)}{n-e} = V_i(1 - \frac{1}{n-e})$

The bidder will now shade with k = $1 - \frac{1}{n-e}$.

Differentiation gives the effect of the parameters on the optimal bid:

$B_i^{*\prime}(n) = V_i(\frac{1}{(n-e)^2})$

$B_i^{*\prime}(e) = V_i(-\frac{1}{(n-e)^2})$

$B_i^{*\prime}(V_i) = 1 - \frac{1}{n-e}$

Appendix E: model E

The same assumptions as are used in the model by McMillan (1992) are used for this model. However, now the bidder will believe there to be less competition by fraction (1-p). The probability of winning the auction therefore becomes $(\frac{B_i}{k})^{n-1-pn}$. Assumed will be that: 0≤p<1. If p = 0, the bidder correctly estimates the number of competitors. This yields the following maximization problem:

(1) $\max_{B_i}(\frac{B_i}{k})^{n-1-pn}(V_i - B_i)$.

Maximizing this expression and solving it gives the optimal bid for this setting, as will be shown below:

$$\left(\frac{B_i}{k}\right)^{n-2-pn}\left(\frac{1}{k}\right)(n-1-np)(V_i - B_i) - (\frac{B_i}{k})^{n-1-np} = 0$$

$$(V_i - B_i)(n-1-np)\frac{1}{k} = \frac{(\frac{B_i}{k})^{n-1-np}}{(\frac{B_i}{k})^{n-2-np}}$$

$$(V_i - B_i)(n-1-np)\frac{1}{k} = \frac{B_i}{k}$$

$$V_i(n-1-np) = B_i + B_i(n-1-np)$$

$$V_i(n-1-np) = B_i(n-np)$$

$$B_i^* = \frac{V_i(n-1-np)}{n-np} = V_i(1-\frac{1}{n(1-p)})$$

Bidders thus shade with k = $(1-\frac{1}{n(1-p)})$.

Differentiating B_i^* to the different parameters gives the effect of those parameters on the optimal bid:

$$B_i^{*\prime}(n) = V_i(\frac{1}{(n-e)^2})$$

$$B_i^{*\prime}(p) = V_i(-\frac{n}{p(p-2)})$$

$$B_i^{*\prime}(V_i) = 1 - \frac{1}{n(1-p)}$$

Appendix F: model F

The assumptions from the model by McMillan (1992) are a basis for this model. In addition, bidders believe others to shade more than themselves. To represent this, competitor bidders in the model will bid (k-z)*V_j. Also, 0 ≤ z < k, so that bids remain positive and the total fraction of shading should won't exceed 1. The probability of bidding higher than someone else is: Pr(B_i ≥ (k-z)*V_j) = Pr((k-z)*V_j < B_i) = Pr($V_j < \frac{B_i}{k-z}$). This results in the following bidding function:

(1) $\max_{B_i}(\frac{B_i}{k-z})^{n-1}(V_i - B_i)$.

Maximizing this expression and solving it gives the optimal bid for this setting, as will be shown below:

$(\frac{B_i}{k-z})^{n-2}(V_i - B_i)(n-1)\frac{1}{k-z} - (\frac{B_i}{k-z})^{n-1} = 0$

$(\frac{1}{k-z})^{n-2}(B_i)^{n-2}(V_i - B_i)(n-1)\frac{1}{k-z} - (B_i)^{n-1}(\frac{1}{k-z})^{n-1} = 0$

$(B_i)^{n-2}(V_i - B_i)(n-1) - (B_i)^{n-1} = 0$

$(V_i - B_i)(n-1) = \frac{B_i^{n-1}}{B_i^{n-2}}$

$(V_i - B_i)(n-1) = B_i$

$V_i(n-1) = B_i + B_i(n-1)$

$\frac{V_i(n-1)}{n} = B_i$

$B_i^* = V_i(1 - \frac{1}{n})$

The optimal bid thus remains unchanged. Bidders shade with k = $(1 - \frac{1}{n})$.

Appendix G: model G

The basic model by McMillan (1992) is the basis for this model, as the same assumptions are used. A parameter will be added to represent the beliefs of bidders about the distribution of the private valuations. The parameter can take values larger than or equal to 0. If γ is 1.3, the bidder believes the valuations can be 30% higher. The valuations of other bidders will now be uniformly distributed between 0 an γ. The bidder will believe the probability of bidding higher than a competitor is Pr(V_j < $\frac{B_i}{k}$) = γ$\frac{B_i}{k}$. The maximization problem becomes:

(1) $\underset{B_i}{Max}(\gamma \frac{B_i}{k})^{n-1}(V_i - B_i)$

Maximizing this expression and solving it gives the optimal bid for this setting, as will be shown below:

$(\gamma \frac{B_i}{k})^{n-2}(V_i - B_i)(n-1)\frac{\gamma}{k} - (\gamma \frac{B_i}{k})^{n-1} = 0$

$(\frac{\gamma}{k})^{n-2}\ (B_i)^{n-2}(V_i - B_i)(n-1)\ \frac{\gamma}{k} -\ (B_i)^{n-1}(\frac{\gamma}{k})^{n-1} = 0$

$(B_i)^{n-2}(V_i - B_i)(n-1) - (B_i)^{n-1} = 0$

$(V_i - B_i)(n-1) = \frac{{B_i}^{n-1}}{{B_i}^{n-2}}$

$(V_i - B_i)(n-1) = B_i$

$V_i\ (n-1) = B_i + B_i(n-1)$

$\frac{V_i(n-1)}{n} = B_i$

$B_i = V_i(1 - \frac{1}{n})$

Again, the optimal bidding strategy remains unchanged. The bidder shades his bid with fraction k = (1 - $\frac{1}{n}$), which increases in the number of competitor bidders.

Appendix H: model H

There are two bidders B_i and B_j who both compete for an object that is up for auction. Bidder i is the biased bidder, and bidder j is the unbiased bidder. Bidder i believes bidder j's valuation is between 0 and 0.8. The valuation of the competitor is thus believed to be uniformly distributed between 0 and 0.8. The unbiased bidder j bids according to the optimal bidding strategy that follows from McMillan (1992). The competitor thus bids B_j^* = $V_j(1 - \frac{1}{2}) = \frac{1}{2}V_j$. The biased bidder knows about the bidding function of his competitor, and makes a bid from a strategic point of view. Because bidder i believes bidder j will not have a valuation that exceeds 0.8, the believes that bidder j will never bid above 0.4. Bidder i will thus never bid above 0.4, because this would not increase his probability of winning and would only decrease the profit when he wins. In reality, bidder j can bid up to 0.5, because he can have a valuation up to 1. For simplicity, it is assumed that when both bidders make the same bid, bidder i wins the auction.

Table 4

Table containing both the real and believed probability of winning and expected value for different values of V_i when B_i is a certain value.

B_i	Believed probability of winning*	Believed expected value	Believed EV for a certain value of V_i		Probability of winning**	Expected value	EV for a certain value of V_i	
			V_i	Believed EV			V_i	EV
0.5	100%	$(V_i - 0.5)$	0.8	0.3	100%	$(V_i - 0.5)$	0.8	0.3
			0.85	0.35			0.85	0.35
			0.9	0.4			0.9	0.4
			0.95	0.45			0.95	0.45
			1	0.5			1	0.5
0.45	100%	$(V_i - 0.45)$	0.8	0.35	90%	$0.9(V_i - 0.45)$	0.8	0.315
			0.85	0.4			0.85	0.36
			0.9	0.45			0.9	0.405
			0.95	0.5			0.95	0.45
			1	0.55			1	0.495
0.4	100%	$(V_i - 0.4)$	0.8	0.4	80%	$0.8(V_i - 0.4)$	0.8	0.32
			0.85	0.45			0.85	0.36
			0.9	0.5			0.9	0.4

			0.95	0.55			0.95	0.44
			1	0.6			1	0.48
0.35	87.5%	$0.875(V_i - 0.35)$	0.8	0.39375	70%	$0.7(V_i - 0.35)$	0.8	0.315
			0.85	0.4375			0.85	0.35
			0.9	0.48125			0.9	0.385
			0.95	0.525			0.95	0.42
			1	0.56875			1	0.455
0.3	75%	$0.75(V_i - 0.3)$	0.8	0.375	60%	$0.6(V_i - 0.3)$	0.8	0.3
			0.85	0.4125			0.85	0.33
			0.9	0.45			0.9	0.36
			0.95	0.4875			0.95	0.39
			1	0.525			1	0.42
0.25	62.5%	$0.625(V_i - 0.25)$	0.8	0.34375	50%	$0.5(V_i - 0.25)$	0.8	0.275
			0.85	0.375			0.85	0.3
			0.9	0.40625			0.9	0.325
			0.95	0.4375			0.95	0.35
			1	0.46875			1	0.375
0.2	50%	$0.5(V_i - 0.2)$	0.8	0.3	40%	$0.4(V_i - 0.2)$	0.8	0.24
			0.85	0.325			0.85	0.26
			0.9	0.35			0.9	0.28
			0.95	0.375			0.95	0.3
			1	0.4			1	0.32
0.15	37.5%	$0.375(V_i - 0.15)$	0.8	0.24375	30%	$0.3(V_i - 0.15)$	0.8	0.195
			0.85	0.2625			0.85	0.21
			0.9	0.28125			0.9	0.225
			0.95	0.3			0.95	0.24
			1	0.31875			1	0.225
0.1	25%	$0.25(V_i - 0.1)$	0.8	0.175	20%	$0.2(V_i - 0.1)$	0.8	0.14
			0.85	0.1875			0.85	0.15
			0.9	0.2			0.9	0.16
			0.95	0.2125			0.95	0.17
			1	0.225			1	0.18
0.05	12.5%	$0.125(V_i - 0.05)$	0.8	0.09375	10%	$0.1(V_i - 0.05)$	0.8	0.075
			0.85	0.1			0.85	0.08
			0.9	0.10625			0.9	0.085
			0.95	0.1125			0.95	0.09
			1	0.11875			1	0.095

$*\left(\frac{0.4 - B_i}{0.4}\right) 100\%$

$**\left(\frac{0.5 - B_i}{0.5}\right) 100\%$

When the number of competitors increases and their bids approach their maximum valuations a bidder i that follows the optimal bidding function from the basic model, who believes competitors wont bid above 0.8, would at some point believe increasing his bid does not increase the probability of winning. The number of competitors for which the optimal bid of bidder i would no longer increase, can be calculated for different values of V_i. Using the optimal bidding function $B_i^* = V_i(1 - \frac{1}{n})$ and restraining the optimal bid to 0.8, the number of bidders for which increasing the bid does not increase the probability of winning the auction can be found.

$V_i(1 - \frac{1}{n}) \leq 0.8$

$V_i \leq 0.8 - \frac{4}{5n}$

The optimal bidding strategy from the point of view of bidder i is thus:

$B_i = V_i(1 - \frac{1}{n})$ if $V_i \leq 0.8 - \frac{4}{5n}$ and $B_i = 0.8$ if $V_i > 0.8 - \frac{4}{5n}$