ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics
Department of Econometrics

Estimating linear and nonlinear effects of fiscal policy using vector autoregression, vector error correction and quantile regression models

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Supervisor
dr. Annika AM SCHNUCKER

Author
Rafaël SOMAIL

Second assessor
dr. Yutao SUN

Student ID
473118

Abstract

This research estimates the linear and nonlinear effects of fiscal policy on output and government spending. Using quarterly US data obtained from the Federal Reserve Bank of St. Louis, we obtain impulse responses and expanding window forecasts for different estimation procedures, namely a VAR, a VECM which considers cointegration relations and a quantile VAR. We find that higher government spending overall increases output and decreases unemployment. Furthermore, nonlinear estimation performed with quantile regression shows that effects are notably different at different quantiles. Lastly, evaluating the forecasting performance of different models leads to the conclusion that the quantile vector autoregression is most suitable for estimating effects of fiscal policy, even though a linear VAR model is still very adequate. Considering cointegration relations does not bring improvements in forecasting performance.

Contents

1 Introduction

Fiscal policy refers to government spending, but also the government's policy towards levying taxes, which has measurable effects on both microeconomy and macroeconomy. As an example, economic theory states that increasing government spending will likely lead to increases in economic growth and could also possibly reduce the unemployment rate. We draw our attention to the macroeconomical effects of adjustments in the fiscal policy, where most focus is on government spending. Namely, this paper mainly focuses on the effects of fiscal policy on output, measured with GDP, and unemployment.

Earlier contribution to this type of research comes for instance from Blanchard and Perotti (2002), who partly use a mixed structural VAR to show that an increase in government spending has a positive effect on output, and that an increase in taxes has a negative effect. More recently, research has been performed that does not simply analyse the effects of fiscal policy in a linear fashion, but also looks at the effects in a nonlinear fashion. Take for example Auerbach and Gorodnichenko (2012) who partly use regime-switching structural vector autoregression models to find that effects are dependent on the business cycle. They observe that fiscal policy is more effective during a recession than during an expansion. Even Owyang et al. (2013) find that fiscal multipliers are larger during recessions, which they found using state-dependent local projections. So, one can look at the difference between the effects dependent on what the state of the output or unemployment is and compare the resulting outcomes.

This research thus implements multiple different estimation procedures to estimate linear and nonlinear effects of fiscal policy. The first estimation is a vector autoregression with first differences (VAR), the second estimation is a vector error correction model (VECM) and the third is a quantile vector autoregression (quantile VAR), which is all estimated using macroeconomic variables from the US. Most of these variables are integrated. The vector autoregression uses first differenced data and the vector error correction model considers the data in levels to subsequently transform into first differenced data, whereas the quantile regressions are performed using detrended data and first differenced data. The main goal of performing both these linear and nonlinear estimations is to explore whether there are indications that the outcomes depend on how the data is used, namely by implementing detrending or the first difference operators. Furthermore, the vector error correction model can be compared to the vector autoregression with first differences and the quantile vector autoregression with first differences as it considers the cointegration relation. The interesting aspect of the cointegration relation is that one can transform two or more integrated variables into a linear combination that has a lower order of integration. This represents a long run equilibrium among the variables. So, the three models are evaluated according to their forecasting performance.

Firstly, the quantile regression procedures are based on the research performed by Linnemann and Winkler (2016). Traditional ordinary least squares regression describes only the effects of changes in the independent variables on the conditional mean of the dependent variable. It assumes that changes in the explanatory variables shift the whole conditional distribution of the dependent variables. Contrarily, quantile regression allows for different effects at different tails of the conditional distribution of the dependent variable, which makes this an adequate model to investigate whether fiscal policy effects differ among different quantiles.

We perform equation-by-equation quantile regressions, with the structure of a VAR(p), to obtain impulse responses that are quantile-specific. This allows for comparison between results across different quantiles, which captures whether there are quantile-specific parameters leading to different responses to the dependent variable. For example, when output is booming and thus in a higher quantile of its conditional distribution one can observe different responses to a shock in government spending compared to when output is depressed, so it is in a lower quantile of its conditional distribution.

The vector autoregression with first differences and vector error correction model (VECM) again use the same macroeconomic variables from the US. To my knowledge, the relevant existing literature has not used the VECM to estimate the effects of fiscal policy. These models are implemented in a linear fashion, thus analysing the average relation between the dependent variables which is decided to be the first difference of output and unemployment. An advantage of this model is that it represents the central relation, which allows for a general conclusion on the effects of fiscal policy. A second advantage is that the VECM captures the cointegration between the variables, which is a linear combination of the non-stationary variables which will be stationary that represents a long-run relationship among the macroeconomic variables. A disadvantage of both models is that they do not include information on the state of the economy, but this is nevertheless accounted for in this research by performing the third estimation procedure, the quantile VAR.

The following main research questions is answered in this research:

Which linear or nonlinear model is most suitable for estimating the effects of fiscal policy, and is there an improvement in estimating the effects by considering cointegration relations?

The first part of this question refers to comparing the three estimation procedures mentioned above, in which there is a comparison based on forecasting performances of all three models, based on first differenced data. Secondly, we explore whether accounting for possible cointegration relations among the variables improves the forecasting performance, which is done by comparing all three models as they all have similar structures but the vector error correction model adds the error correction term derived from the long term equilibrium. This can be considered relevant for the existing literature as we try to find whether a non-linear or linear model would be more suitable, but we also want to see whether considering long-run relations in the estimation brings improvements.

This research can be considered very relevant for governments, and especially the second part, because if for example this paper proves that the response of output to an increase in government spending is more positive during a recession compared to an upturn, the government can alter its policies depending on the state of the economy. It can increase government spending more during a recession, but it will need to so less during an upturn. This can furthermore be reflected in the so-called fiscal multiplier, which measures the change of the output relative to the change of the government spending. One is simply asking: How much of the government spending is reflected in the GDP? An alternative version of the fiscal multiplier will be considered in this research, namely the maximum point-to-point ratio, minimum point-to-point ratio and the cumulative ratio that will be based on response values of output and unemployment on government spending (for more information see section 3.5).

In this research we find that the best performing model in forecasting output is the VAR model implemented with first differences, whereas the unemployment rate is forecasted the best using the quantile VAR model with first differences. We notice a lot of non-linearities among the different quantiles of the conditional distribution of the dependent variables output and unemployment, which leads to the notion that the quantile VAR is most prominent for estimating effects of fiscal policy.

The rest of this research has the following structure: Section 2 informs on the used data, and Section 3 gives information about the used econometric methods applied. Then, the main results of this research are in Section 4, and we conclude in Section 5. To consider the limitations of the research we refer to Section 6.

2 Data

Two sets of data are considered in this research, where both consider the same macroeconomic data with two different types of transformations. The first set of data contains several log-quadratic detrended variables in order to represent only the cyclical component of the considered time series. The Appendix contains more on how this is performed in Section 7.1 to clarify how the log-quadratic detrending is performed.

The data consists of quarterly data from the United States considering the sample 1955Q1 to 2013Q4. The variables considered are government spending g_t, real net taxes t_t, output o_t, unemployment U_t and the short-run real interest rate R_t. The following variables are log-quadratically detrended as described above: output, government spending and real net taxes, and are thus denoted with lower case. The exact construction of these variables can be found in the Appendix in section 7.1. The decision has been made to not include the debt variable in the estimation as in the original paper of Linnemann and Winkler (2016).[1] The reason for this was that it was unavailable for the same sample as their paper.

Some of the variables seem to have a unit root according to the Augmented Dickey Fuller test, meaning they are assumed to be non-stationary, as can be confirmed from Table 8 in the Appendix. Performing the Augmented Dickey Fuller test tells that the non-stationary variables are assumed to be output, government spending and interest (see Appendix 8). Also, for more information about what the Augmented Dickey Fuller test entails, see Section 3.1. The variables output O_t,

[1] In the Appendix in Figure 12, 13, 14 some of the unused impulse response results can be found in which two different implementations of the debt variable were considered, namely using a different sample starting from 1966Q1 and a yearly series that was interpolated to quarterly frequency.

government spending G_t and taxes T_t have an upward trend, which is observed in the data. The tax variable seems to be trend stationary. This all can be confirmed from Figure 5 in the Appendix.

The second set of data considers the original variables without log-quadratic detrending. This is performed in this way in order to capture the cointegration relation among the non-stationary I(1) variables. The variables considered here are government spending G_t, real net taxes T_t, output O_t, unemployment U_t, and the short-run real interest rate R_t. These variables are all first differenced before they are used to compute forecasts, even when they do not exhibit non-stationary behaviour. The reason this is done is in order to make a proper comparison among the different regression models because the VECM first differences each series, and only in this way it would be possible to see whether adding cointegration relations to the estimation brings improvement or not. Again, the exact construction of these variables can be found in the Appendix in section 7.1. Important to note is thus that variables that are in capital are in their original form whereas variables in lower case are log-quadratically detrended throughout the rest of this research.

3 Methodology

This paper replicates part of the paper Linnemann and Winkler (2016), which mainly analyses nonlinear effects of government spending on the output and unemployment, and we further extend on this research by also looking at the effects on output and unemployment in a linear fashion. First, we implement two VAR models with first differences, then we implement two VEC models, both for linear estimation, and then we implement two quantile VAR models for nonlinear estimation. The reason there are two estimations for each model is because the variables output and unemployment are interchanged as dependent variables. This is also based on how Linnemann and Winkler (2016) decided to perform their estimation. Each model is analysed with respect to the estimated impulse response. However, before we perform any estimation we first determine whether the variables contain any unit roots.

3.1 Unit Root Testing

To test the stationarity of the variables, the Augmented Dickey-Fuller test is performed. This tests the null hypothesis that the series contains a unit root against the alternative that the series does not contain unit root. Alternatively, in the following regression we test whether $H_0 : \rho = 0$, against $H_a : \rho < 0$ holds:

$$y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{h} \Delta y_{t-i} + \epsilon_t$$

Hence, tests with a P-value smaller than 0.05 lead to rejection of the null hypothesis, which is that the variable is integrated. The assumption is made in this case that there is a constant and a deterministic trend driving the data, which is evaluated by plotting the data. The t-statistic of ρ determines whether the null hypothesis is rejected or not. The decision is made to let the optimal lag order h to be determined using the Schwarz Information Criterion.

3.2 Vector Autoregression with First Differences

This research extends on Linnemann and Winkler (2016) by considering a second set of variables, where there is no detrending, and use it for implementing a VAR model with first differences and VECM. One of the goals in this research is to compare the forecasting performance of the vector error correction model which considers cointegration relations with a model that does not consider this. A suitable model would be a vector autoregression where the first differences are considered. The model is of the following form:

$$\Delta y_t = c + A_1 \Delta y_{t-i} + \cdots + A_{p-1} \Delta y_{t-p+1} + \epsilon_t$$

Here we have $y_t = (G_t, \zeta_t, T_t, R_t)'$, where $\zeta_t \in \{O_t, U_t\}$. So, we regress the first difference of government spending, either output or unemployment, taxes and interest on p of their lags. By first differencing all series, we are sure all of them are stationary even when they already were stationary. The lag order of this system is determined with the Akaike Informaiton Criterion (AIC). Namely, first the optimal lag order p^* is determined for a simple VAR model which leads to the optimal lag order of $p^* - 1$. The decision is made to include in the model a constant c as some of the variables do not have zero mean in their first difference. Furthermore, this assumes there is a trend in the data, which is deduced from the plots of the data in levels which can be found in the Appendix in Figure 5. The parameter estimates are estimated for each equation with Ordinary Least Squares.

3.3 Vector Error Correction Model

In economics there is often a long-run equilibrium between variables. Given that some variables are non-stationary, a simple VAR model would not be suitable in this case because there can be a spurious regression with I(1) variables, which is why we consider a vector error correction model.

The implemented vector error correction model has the following form:

$$\Delta x_t = c + \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_{p-1} \Delta x_{t-p+1} + u_t$$

where $x_t = (G_t, \zeta_t, T_t, R_t)'$, $\alpha = (\alpha_G, \alpha_\zeta, \alpha_T, \alpha_r)$, $\beta_t' = (1, -\beta_\zeta, -\beta_T, -\beta_R)$ and

$$\mu_t = (\mu_{G,t}, \mu_{\zeta,t}, \mu_{T,t}, \mu_{R,t})', \zeta \in \{O_t, U_t\}$$

We interpret $\beta' x_{t-1}$ as the cointegrating relationship which is a long-run relation of the variables in equilibrium. Then, α can be interpreted as the loadings vector which represents the sensitivity of the changes in the dependent variable to the cointegration equation $\beta' x_{t-1}$. It can also be interpreted as a speed adjustment parameter, namely when the equilibrium does not hold the parameters in α will pull the corresponding variable back towards it's equilibrium relationship. Both left and right hand side of the equations are stationary as the regressors are all first differenced. Next, there is a restriction which normalizes some of the coefficients in β'. Namely if a cointegrating vector is found, a multiplication with any non-zero number can still be stationary. Therefore, the following normalized form holds: $\beta' = [I_r, \beta_{(K-r)\times r}]$, where r is the cointegration rank and

K the number of variables. All parameter estimates are computed with the Johansen estimation strategy, which utilizes maximum likelihood, so the parameters for α and β' can be computed simultaneously.

Again, the decision is made to include in the model a constant c as some of the variables do not have zero mean in their first difference. The lag order of this system is also determined with the Akaike Informaiton Criterion (AIC). Also, the Johansen Cointegration Test tests what might be an approriate number of cointegration relations. After estimating this model the impulse responses are considered in order to see the effect of a unit shock in government spending on output and unemployment.

3.4 Johansen Cointegration Test

Determination of the cointegration rank is performed with the Trace test and Maximum Eigenvalue test, which is shortly discussed below.

3.4.1 Trace Test and Maximum Eigenvalue Test

This test has the following hypotheses, $H_0 : r = r_0$ against $H_1 : r < r_0 < K$.

The Trace statistic is defined as follows:

$$Trace(K, r_0) = -T \sum_{i=r_0+1}^{K} \log(1 - \lambda_i)$$

Here, the λ_i represents the i-th eigenvalue of a special matrix of which the determinant is taken in computing the maximum likelihood of the VECM. This test statistic should be small under null hypothesis $r = r_0$ and this hypothesis is rejected once it exceeds some critical value which is determined by a nonstandard distribution that is simulated. So, a sequence of tests is performed until we accept the hypothesis that $r = r^*$.

Furthermore, the Maximum Eigenvalue test has the following hypotheses: $H_0 : r = r_0$ against $H_1 : r = r_0 + 1$ and this is tested with the following test statistic:

$$\lambda_{max} = -T \log(1 - \lambda_{r_0+1})$$

Similarly to the Trace test, this test is performed sequentially until the null hypothesis gets rejected and the optimal number of cointegration relations, the rank of $\alpha\beta'$, is found.

3.5 Quantile Vector Autoregressive Model

The quantile regression model originates from Koenker and Bassett Jr (1978), and it can be seen as an enhanced form of the well-known general ordinary least squares model. Ordinary least squares estimates a β, which measures the marginal effects of explanatory variables on the dependent variable, that will minimize the sum of squared residuals, in order to have model that predicts the expected value of y_t given x_t. Quantile regression will focus more on the distribution of y_t which depends on the quantiles q ∈ (0,1). If $F(y_t)$ is the probability distribution of y_t, then the qth quantile is represented by the quantile function $Q_q(\cdot)$, with the property that $Q_q(y_t) = F^{-1}(q)$. Essentially, the quantile of a distribution represents the value for which a certain provided fraction of the observations is less than or equal to that value. The general form of a quantile regression is:

$$Q_q(y_t|x_t) = x_t'\beta(q),$$

where x_t is a K by 1 vector with the explanatory variables and $\beta(q)$ is the marginal effect of the explanatory variables on the provided quantile of y_t.

The regression parameter $\beta(q)$ for a q-th quantile regression can be found by solving the following optimization problem:

$$\min_{\beta(q)\in R^K} \sum_{t\in\{t:y_t\geq x_t'\beta(q)\}} q|y_t - x_t'\beta(q)| + \sum_{t\in\{t:y_t\leq x_t'\beta(q)\}} (1-q)|y_t - x_t'\beta(q)|$$

This minimization problem solves for the marginal effect parameters in such a way that a fraction q of the dependent variable data lies below the estimated fit and a fraction $1-q$ lies above it. This mathematical problem can be interpreted in the following way: If the residual is positive then a weight of q is given to the loss function, whereas if the residual is negative a weight of 1-q is given to the loss function. For example, if $q = 0.8$ a positive residual will lead to a higher loss function value which leads the minimization problem to set the $\beta(q)$ in such a way that there will be a smaller residual by giving more weight to observations that will lie above the estimated fit. This solves for the parameter estimate $\beta(q)$ in such a way that the estimated fit of the model, which is $x_t'\beta(q)$, will be as close as possible to the datapoints of the dependent variable as there is a unique solution to the minimization problem.

Now, the first model that is estimated in this research is a quantile VAR model. We have $y_t = (y_{1,t}, ..., y_{K,t})$, a vector of k explanatory variables at each point in time, and $q = (q_1, ..., q_K)$, a vector of K values for the quantiles at which the conditional distribution of the variables in y_t will be assessed. The quantile VAR model has p lags that are based on previous literature, which suggested to use four lags. The model is of the following form:

$$Q_q(y_t|y_{t-1}, ..., y_{t-p}) = c(q) + \sum_{i=1}^{p} B_i(q)y_{t-i},$$

where

$$c(q) = \begin{bmatrix} c_1(q_1) \\ \vdots \\ c_K(q_K) \end{bmatrix} \text{ and } B_(q) = \begin{bmatrix} \beta_{i,11}(q_1) & \dots & \beta_{i,1k}(q_1) \\ \vdots & & \vdots \\ \beta_{i,K1}(q_K) & \dots & \beta_{i,kk}(q_K) \end{bmatrix}$$

This model is estimated with an equation-by-equation approach in *EViews*. The variables that are used are also the following two sets $y_t = (g_t, \zeta_t, t_t, R_t)'$, where $\zeta_t \in \{o_t, U_t\}$. So, we either add output or unemployment to the system, as has been proposed by Linnemann and Winkler (2016). Also, the variables government spending g_t, real net taxes t_t and the real interest rate R_t are only evaluated at $q = 0.5$, while output o_t and unemployment U_t are evaluated at multiple quantiles, namely $q \in \{0.1, 0.5, 0.9\}$. This way we evaluate whether effects of government spending are dependent on the quantile of the distribution of output and unemployment.

The main goal of setting up this model is to look at the impulse response function which represents the change of each of the dependent variables caused by a unit shock in one of the equations, measured over multiple periods $h = 0, \ldots, H$. More on this is discussed next in section 3.6.

3.6 Computing Impulse Response Functions

After estimation of a vector autoregression the impulse response functions can be estimated. There is only focus on the responses of output and unemployment after an impulse shock in government spending. Also, the impulse responses are based on a Cholesky decomposed covariance matrix of the residuals to ensure there is no correlation between the residuals.

We consider the following VAR(p) model: $Y_t = AY_{t-1} + U_t$, where

$$Y_t \equiv \begin{pmatrix} y_t \\ \vdots \\ y_{t-p+1} \end{pmatrix}, A \equiv \begin{bmatrix} A_1 & A_2 & \ldots & A_{p-1} & A_p \\ I_K & 0 & & 0 & 0 \\ 0 & I_K & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & I_K & 0 \end{bmatrix}, \text{ and } U_t \equiv \begin{pmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

We have $J \equiv [I_K, 0_{K \times K(p-1)}]$ and the response matrix of all variables at period i is calculated by:

$$\underset{K \times K}{\Phi_i} = JA^i J'$$

To ensure the orthogonality of the errors the response is post-multiplied with the Cholesky decomposition of the covariance matrix of the residual terms. The covariance matrix $E(u_t u_t')$ is estimated as follows: $\hat{\Sigma} = \frac{\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'}{(T-(K \times p+1))}$, where we adjust for the degrees of freedom. Namely, we use T observations to estimate the regression coefficients for K variables, all evaluated at p lags and also one constant is involved which means we estimate $K \times p + 1$ coefficients. The covariance matrix can be represented by its Cholesky decomposition, namely: $\Sigma = PP'$, where P is a lower triangular matrix. Post-multiplication with P ensures there is no correlation among the residuals anymore, which gives more reliable estimates for the impulse response functions.

The computation of the impulse response functions for the quantile VAR with the detrended data is performed with a self-written code in *MATLAB (R2020a)*, whereas for the other models: the VAR in first difference and the VECM, it is performed in *EViews* using existing functions also based on a Cholesky decomposition of the residual covariance matrix.

Assessing fiscal multipliers is not feasible in this research so we assess another alternative for the quantile VAR. The point-to-point ratio $R_h(q)$ is defined as $\frac{\hat{o}_h(q)}{\hat{g}_h(q)}$ or $\frac{\hat{u}_h(q)}{\hat{g}_h(q)}$, where $\hat{o}_h(q)$ represents the response of output h periods after a unit shock in government spending, $\hat{u}_h(q)$ represents the response of unemployment h periods after a unit shock in government spending and $\hat{g}_h(q)$ represents the response of government spending h periods after a unit shock in government spending. We then calculate the maximum point-to-point ratio for output, minimum point-to-point ratio for unemployment and the cumulative ratio, defined as:

$$MaxR(q) = \max_{h\in\{0,\ldots,12\}} R_h(q),\ MinR(q) = \min_{h\in\{0,\ldots,12\}} R_h(q),\ CR(q) = \frac{\sum_{h=0}^{12} \hat{o}_h(q)}{\sum_{h=0}^{12} \hat{g}_h(q)}$$

$$\text{and } CR(q) = \frac{\sum_{h=0}^{12} \hat{u}_h(q)}{\sum_{h=0}^{12} \hat{g}_h(q)}$$

This calculation measures the maximum, minimum or cumulative responses of output or unemployment relative to simultaneous government spending response for each quantile of the distribution of output or unemployment. They cannot be interpreted as dollar-for-dollar, but they measure the percentage change of output or unemployment relative to the percentage change of government spending.

3.7 Forecasting Evaluation

One-step ahead forecasts are computed for the following three models: the VAR model, the quantile VAR model where the dependent variable is evaluated at its median and the VECM, where each model has first differenced data as regressors. For each model there are expanding window estimations which computes one-step ahead out of sample forecasts. The first 60 observations are used to estimate the first model in order to forecast the 61st observation, which was at 1970Q1. Then a new model is estimated in order to forecast the 62nd observation using the previous 61 observations, and it goes on in this recursive fashion.

Also, four-step ahead forecasts are computed in the following manner: The first 60 observations are used to estimate the first model in order to forecast the 61st, 62th, 63rd and 64th observations using forecasted values to estimate the next forecast. The coefficients of the model stay the same. Then we recompute the coefficients of the model when we set up the forecast for the 65th, 66th, 67th and 68th observations using all previous data. Again, this goes on in this recursive way. To implement this estimation programs in *EViews* are written, which is found in the Appendix under Section 7.3.

In order to evaluate the accuracy of the forecasts we look at two criteria: the RMSE and the MAE.

The Root Mean Squared Error (RMSE) for each model are analysed. The RMSE is in this case mathematically defined as follows:

$$RMSE = \sqrt{\frac{1}{T - T_0} \sum_{t=T_0+1}^{T} (\hat{y}_t - y_t)^2},$$

where $\hat{y}_t$ denotes the predicted value of y_t, T is the total number of observations and T_0 is the number of observations used to estimate the first model.

Furthermore, the Mean Absolute Error (MAE) for each model are analysed, which has the following mathematical definition in this case:

$$MAE = \frac{1}{T - T_0} \sum_{t=T_0+1}^{T} |\hat{y}_t - y_t|,$$

In order to examine whether for example forecasts from model 1 are significantly better compared to the forecasts of model 2 the Diebold-Mariano test is performed. The test statistic is defined to be:

$$DM = \frac{\bar{d}}{s_d} \sim N(0,1),$$

where we set up the following two definitions of the differential d: $d_{sq,t} = (\hat{y}_{1,t} - y_t)^2 - (\hat{y}_{2,t} - y_t)^2$ or $d_{abs,t} = |\hat{y}_{1,t} - y_t| - |\hat{y}_{2,t} - y_t|$. $\bar{d}$ represents its average and s_d represents the standard deviation of d. One-sided P-values are provided as the RMSE and MAE indicate which model has less forecasting error which already creates a hypothesis which model performs better than the other.

Furthermore, to illustrate whether adding all these models is useful, simple random walk one-step ahead forecasts are also performed. Namely, the forecast for variable $\hat{y}_{t+1}$ is simply the observed value one quarter before, namely y_t. We can then also clearly see whether adding more sophisticated models leads to better outcomes compared to the random walk forecasts.

Most importantly, the value for T_0 is varied when analyzing the errors. The reason this is done because we assume that it might lead to unproper conclusions when we just choose one value for T_0 and conclude that there is one winning model that forecasts the best. Namely, over different considered samples different conclusions could possibly be drawn. So, in order to get a more accurate view of which model performs good we assess the evaluation for different values of T_0. According to the RMSE and MAE we decide which models is to be compared with the Diebold-Mariano test, as we choose the two best performing models. To implement this all, a code has been written in MATLAB (2020) which can be found in the Appendix in section 7.3.

4 Results

4.1 Vector Autoregression with First Differences

The vector autoregressions with first differences has been estimated with four lags of the first differences, based on the Akaike Information Criterion. This model has also been used to set impulse response functions to observe its behaviour. There is no clear conclusion derived from the impulse response function in Figure 16a, which is in the Appendix. The effect alternates around zero and dies out after roughly 10 periods which is as is expected since the analysed series are both stable I(0) processes. Because the data is first differenced it is not possible to really find economical interpretations to these impulse responses.

4.2 Vector Error Correction Model

Estimating different VAR(p) models gave the outcome that an optimal lag order would be 5 lags, given the Akaike Information Criterion. Thus, the VECM, where output is added, and the VECM, where unemployment is added instead, have been implemented with 4 lags of the first difference. The cointegration rank for the estimation with the VEC model with output is equal to 2 after performing the Johanson cointegration test, whereas it was equal to 1 for the VEC model with unemployment. More in-depth results on these test can be found in the Appendix in Tables 9, 10, 11, 12, 13 and 14. Furthermore, the assumption was made that overall there is a linear trend in the data, which added a constant to the cointegration relations and to the regression with first differences as regressors. Assessing the subfigures in Figure 5 in the Appendix confirms this is a proper assumption to make for most of the variables, especially government spending, output and real net taxes.

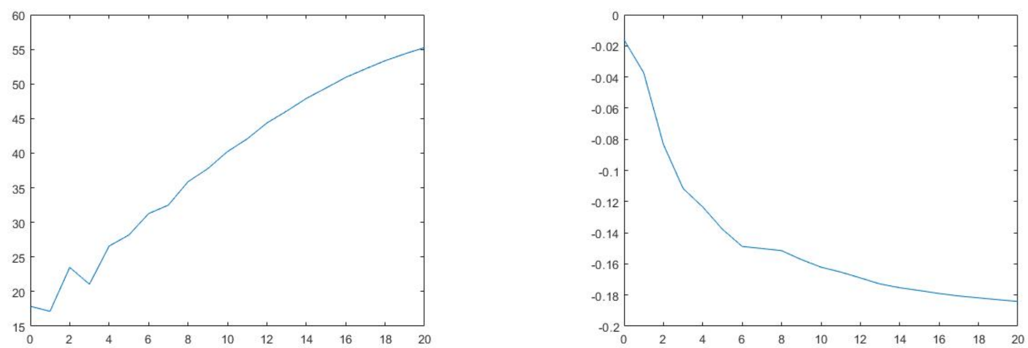
Analyzing the cointegration equations we find multiple relations among the variables. Firstly, considering the first cointegration relation which represents the long-run equilibrium in the VECM with output, we observe the government spending, taxes and interest are positively related. Secondly, considering the second cointegration relation, we observe that output and taxes are positively related whereas output and interest are negatively related. The exact coefficients of β can be found in the Appendix in Table 15. This aligns with economic theory to some extent because it is expected that higher government spending will be possible when more taxes are levied, as both go paired with economic growth. However, one can question whether taxes and output are always to be considered positively related. McNabb (2018) brings this to question as he finds that increases in taxes can even lead to a reduction of GDP growth. The fact that interest and output are negatively related is somewhat counter intuitive as economic theory states that they are supposed to be positively related, as the well-known Taylor rule states. Nevertheless, the t-statistic for the coefficient for the interest rate was not statistically significant at the 5% level which might explain the outcome.

Also, the cointegration relationships derived from the VECM with unemployment show that government spending, taxes and interest are positively related, and that government spending and unemployment are negatively related. This is all aligned with the economic theory, as for example more government spending is expected to reduce unemployment. Again, the exact coefficients of β can be found in the Appendix in Table 16.

What we observe in Figure 1a is that a one time shock will result in a permanent effect due to the non-stationarity of the series. This is also in line with what is observed in the book of Lütkepohl

(2005), where a non-stationary series is also observed to have a permanent effect caused by a shock. Namely, in this case a shock occurring will not die out due to the fact that in the autoregressive data generating process the sum of the autoregressive coefficients of the lags is near 1 or exceeds it. We clearly see that in this impulse response function an increase in government spending with one billion Dollars will lead to an increase of 55 billion Dollars after 20 quarters.

Furthermore, in Figure 1b we observe that an increase in government spending leads to a reduction in unemployment. After 20 quarters there is a reduction of roughly -0.18% which is not very considerable. Nevertheless, this is also aligned with the results that are observed next in the quantile VAR models, where there is also not a very large decrease for the unemployment measured at its median.



(a) (b)

Figure 1: a): Response of output on a unit shock in government spending (measured in billions of Dollars). b): Response of unemployment to a shock in government spending (measured in percentages).

4.3 Quantile VAR

A quantile VAR(4) model with detrended data has been estimated, where the lag order has been based on previous literature from Linnemann and Winkler (2016). Namely, for each variable a quantile regression has been performed containing four lags of output o_t, government spending g_t, tax t_t and interest R_t and a constant using the sample 1955Q1 to 2013Q4. We obtain an in-sample fit where we look at the log quadratically detrended GDP, namely output o_t, which can be seen in Figure 2. The 10% and 90% quantiles of output were considered. The blue line represents the actual output o_t, the orange line the 10% quantile fitted value and the green line the 90% quantile fitted value. We observe in the figure that the actual output coincides with the 10% quantile forecast mostly during downturns and coincides with the 90% quantile forecast mostly during upturns. This finding is also in line with the findings of Linnemann and Winkler (2016).

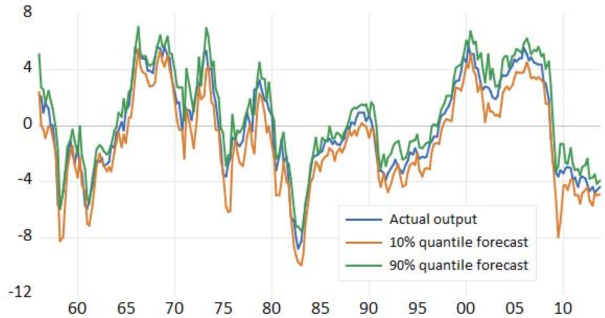


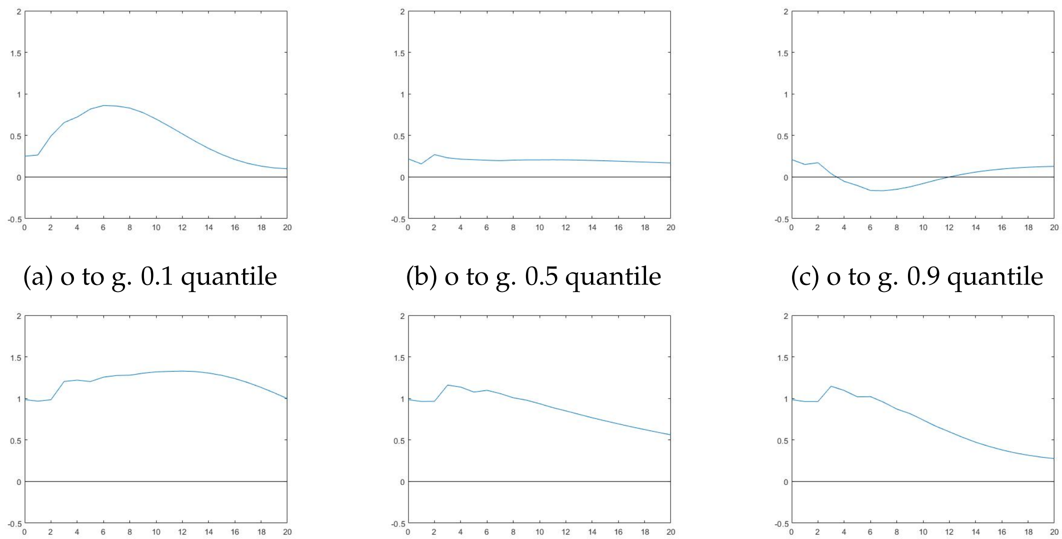
Figure 2: In-sample fit of output obtained with a quantile VAR(4) model for 10% and 90% quantiles.

4.3.1 Response of Output to Government Spending

After estimating the quantile VAR equation-by-equation with the variables (g_t, o_t, t_t, R_t), in this order with four lags, we obtain impulse response functions depending on the quantile at which output has been estimated. This ordering ensures that government spending is assumed to be exogenous. Namely, the response of output depends on government spending, but the opposite does not hold, because we used the Cholesky decomposition of the residual covariance matrix. This implementation has also been advised by Blanchard and Perotti (2002).

In Figure 3 we find the orthogonalized impulse responses of output and government spending, based on percentages as they are log quadratically detrended. For example, Figure 3a shows the response of output to government spending, which is hump shaped. This shows that when output is at the lowest decile, a 1% shock in government spending results in a very positive increase in output, up to roughly 0.85%. When output is estimated at its median we find only a very small increase in output shortly after a 1% shock in government spending. Lastly, when output is booming we find that a 1% shock in government spending even unexpectedly results for some period in negative response of output, which is nevertheless not far away from zero. These results are much in line with the findings of Linnemann and Winkler (2016), but are still different looking at for example Figure 3c. This most likely is the case due to the fact that the debt variable they used in the estimation has been omitted in this research.

Also, looking at Table 1 we find from the maximum point-to-point ratio and cumulative ratio that when output is depressed government spending has a higher multiplier compared to when it is at the median of its conditional distribution or even booming at its highest decile. The interpretation of the multipliers can be seen as for $q = 0.1$ that for a 1% increase in government spending, output increases with 0.69% at maximum, and for the cumulative ratio we take into account the overall effect over the first 12 quarters, which is 0.55% for $q = 0.1$. We even again observe that the cumulative ratio is negative for booming output, but not very different from zero.



(a) o to g. 0.1 quantile (b) o to g. 0.5 quantile (c) o to g. 0.9 quantile

(d) g to g. 0.1 quantile (e) g to g. 0.5 quantile (f) g to g. 0.9 quantile

Figure 3: Impulse responses at different quantiles of the output distribution. The lines show the responses of output and government spending to a 1% government spending shock. These orthogonalized IRFs are based on a quantile VAR model with the variables government spending, output, real net taxes and real interest (on a quarterly basis).

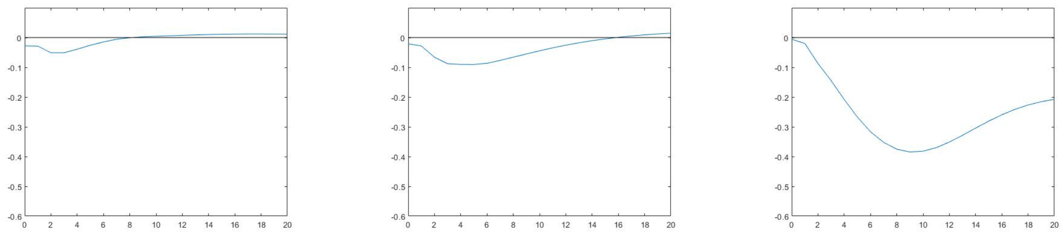
	$q = 0.1$	$q = 0.5$	$q = 0.9$
MaxR	0.69	0.28	0.21
CR	0.55	0.21	-0.03

Table 1: Maximum point-to-point ratio and Cumulative ratio for all three quantiles at which output is evaluated in the quantile VAR.

4.3.2 Response of Unemployment to Government Spending

Next, we analyse the response of the unemployment rate to shocks in government spending. Looking at Figure 4 we again find different responses to government spending shocks across the different quantiles of the distribution of the unemployment rate. When the labor market has little unemployment, at $q = 0.1$, the response is a slight reduction in unemployment and afterwards it turns positive, but not very different from zero. Then, for the median of the unemployment the decreasing effect caused by the 1% shock in government spending is slightly larger than for $q = 0.10$, but still not very different from zero. Lastly, when the labor market is depressed, a 1% government spending shock brings a strong decrease to unemployment which almost reaches -0.4 percentage points after more than eight quarters.

To continue, the Table 2 shows the minimum point-to-point ratios and cumulative ratios for the three different quantiles of the distribution of unemployment. We see that as the quantile increases one can overall conclude that the effects are stronger. For example, when unemployment is predicted to be high in a depressed labor market the multiplier effect of a government spending shock can reach up to -0.34 percentage points for $q = 0.9$, which is much stronger compared to for example the -0.05 percentage points for when unemployment is at its lowest decile.



(a) u to g. 0.1 quantile (b) u to g. 0.5 quantile (c) u to g. 0.9 quantile

Figure 4: Impulse responses at different quantiles of the output distribution. The lines show the responses of unemployment to a 1% government spending shock. These orthogonalized IRFs are based on a quantile VAR model with the variables government spending, unemployment, real net taxes and real interest (on a quarterly basis).

	$q = 0.1$	$q = 0.5$	$q = 0.9$
MinR	-0.05	-0.10	-0.34
CR	-0.02	-0.07	-0.23

Table 2: Minimum point-to-point ratio and Cumulative ratio for three quantiles at which unemployent is evaluated in the quantile VAR.

4.4 Robustness of Quantile VAR

Figure 3 and Figure 4 were computed in multiple different ways during the performance of the research. Upcoming results can be confirmed by looking at the Appendix in Figures 11, 12, 13, 14, 15. Firstly, the interest rate was differently defined because monthly series at the beginning of each quarter were used instead of average quarterly series. This gave very similar outcomes for the sample 1955Q1 till 2013Q4 except for the response of output to government spending at $q = 0.9$. Furthermore, two different implementations were performed in which a debt variable were included aswell. The first implementation was changing the sample to 1966Q1 to 2013Q4 and include a debt variable that was available from 1966Q1, which gave very different outcomes compared to Linnemann and Winkler (2016) who considered the same variables but the sample 1955Q1 to 2013Q4. Furthermore, an implementation was performed with an interpolated version of the debt variable (which had a yearly frequency), with the sample 1955Q1 to 2013Q4. This also gave very different outcomes. Lastly, for the impulse responses considering unemployment we find that adding the interest variable which considered monthly series instead of average quarterly frequency gave similar but still different outcomes, especially for $q = 0.5$.

What this learns us is that impulse response functions are very sensitive to which regressors are added, but also that different samples with the same variables can lead to totally different figures.

4.5 Forecast Evaluation

4.5.1 Forecast Evaluation for Output

We have set up expanding window forecasts for the VECM, VAR in first differences, the quantile VAR for $q = 0.5$ in first differences and the simple random walk model. We observe that the forecasts fit the actual values very well, which can mostly be explained from the fact that the window models have been re-estimated before performing each forecast, instead of having just one model with the same coefficients which forecasts an entire sample. Also, we observe that in the period of 2009 the forecasts are clearly different from the actual observed output. This can mainly be elucidated by the fact that during the financial crisis in this period a lot of external shocks occured in the economy, causing the forecasts to be less accurate. In the Appendix the one-step ahead forecasts and the four-step ahead forecasts graph for each of the models can be found in Figures 7 and 8. It is also observed that forecasting one year ahead leads to less accurate forecasts compared to one quarter ahead forecasts, as expected.

Table 3 shows the root mean squared error and mean absolute error for all four models considering one-step ahead forecasts: the VAR in first differences, the vector error correction model, the quantile VAR in first differences and the Random Walk model. The statistics are there for different values of T_0 which is the value at which the forecast evaluation starts.

Clearly the Random Walk model performed the worst as it had highest RMSE and MAE values for all values of T_0, which shows that estimating more sophisticated models is useful. We observe that the quantile VAR in first difference at $q = 0.5$ and VECM alternate between which model is best according to both criteria over different values of T_0. Next, we find that for the values of $T_0 = 60, 90, 120, 150$ the VAR model in first difference outperformed all other models according to the criteria of the RMSE and MAE. However, for $T_0 = 210, 220$ the VECM outperformed all other models. This makes these two models candidates for performing a comparison among them to see whether the difference is significant. The fact that the quantile VAR performed well for $T_0 = 220, 230$ is not considered very important since the forecast sample evaluated is quite small there.

Now, to see whether the models differed significantly from each other we still refer to Table 3. Again for each value of T_0 we seek to see whether the VAR model compared to the VECM estimates is more accurate. The P-values are one-sided because we assume that one of the models is better than the other for each value of T_0, given the observed RMSE and MAE. The significance level is set at 10%, namely we reject when the P-value is smaller or equal to 10%. Hence, we do not reject the hypothesis that the forecasting performance is the same for most cases. We do reject for $T_0 = 60, 90$ when considering the Diebold-Mariano statistic based on the absolute error. This leads to the conclusion that the VAR model in first difference is significantly better than the

VECM. Secondly, for $T_0 = 150$ we find that the Diebold-Mariano test based on the squared errors and absolute errors both lead to the rejection of the null hypothesis, and thus again tell us that the VAR model with first differences is significantly better than the VECM.

	VAR		VECM		Quantile VAR		RW	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
$T_0 = 60$	**79.48**	**57.70***	80.47	59.50	81.43	59.35	101.57	85.53
$T_0 = 90$	**80.55**	**57.41***	81.80	59.13	82.32	58.63	106.69	90.63
$T_0 = 120$	**83.04**	**58.27**	84.11	59.72	85.18	60.48	108.68	93.63
$T_0 = 150$	**91.62***	**64.85***	93.38	67.61	93.99	68.02	117.39	101.33
$T_0 = 180$	107.56	79.42	**107.43**	**78.46**	109.21	81.16	116.83	97.75
$T_0 = 210$	135.28	98.76	**134.72**	**97.25**	137.86	100.01	121.09	99.37
$T_0 = 220$	67.80	69.75	69.69	102.20	**61.09**	**60.41**	63.75	87.01
$T_0 = 230$	65.00	66.16	69.96	96.47	**60.44**	**59.64**	66.09	77.47

Table 3: One quarter ahead forecast errors for all expanding window models where output is forecasted. Expressed in billions of Dollars. The errors in bold are the smallest. The * represents the significance of the DM-statistic, performed with the VAR and VECM, at the 10% level.

Furthermore, when we look at the errors based on the four-step ahead forecasts, which are one year ahead forecasts, we find that the VAR model with first differences overall outperforms both the two other models for all values of T_0, except for $T_0 = 230$ considering root mean squared error. The Diebold-Mariano tests in which again the VAR and VECM were compared were all significant at the 10% level, except for when $T_0 = 230$.

	VAR		VECM		Quantile VAR	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
$T_0 = 60$	**130.60***	**96.78***	143.36	101.79	145.61	109.09
$T_0 = 90$	**131.62***	**95.68**	144.70	100.60	143.68	103.97
$T_0 = 120$	**130.08***	**91.43***	149.33	102.35	149.63	106.54
$T_0 = 150$	**146.62***	**106.65***	169.58	121.08	166.40	122.38
$T_0 = 180$	**156.77***	**108.27**	166.11	116.65	183.50	134.93
$T_0 = 210$	**185.11***	**117.49***	195.39	134.85	210.22	146.59
$T_0 = 220$	**87.96***	**72.40***	117.87	101.78	113.42	93.99
$T_0 = 230$	101.54	**86.69**	**99.25**	89.65	124.77	113.46

Table 4: One year ahead forecast errors for all expanding window models where output is forecasted. Expressed in billions of Dollars. The errors in bold are the smallest. The * represents the significance of the DM-statistic, performed with the VAR and VECM, at the 10% level.

Overall, we can conclude from these findings that the VAR model with first differences performs well for one-step ahead forecasts and is even much better when considering the four-step ahead forecasts.

4.5.2 Forecast Evaluation for Unemployment

We performed one-step ahead and four-step ahead forecasts for unemployment based on the expanding window as before. Based on the outcomes one could say the forecasts are quite close to the actual values when considering the one-step ahead forecasts, again this likely holds due to the fact that an expanding window has been performed, which incorporates all previous observations. However, the four-step ahead forecasts are observed to be very inaccurate. Again, this can be confirmed in the Appendix in Figure 9 and Figure 10.

Table 5 again show the root mean squared error and mean absolute error, for all different models over different starting values for the forecast evaluation. This time we observe that for $T_0 = 60, 90, 120, 150, 180$ the VAR model is the best based on the RMSE, whereas based on the MAE the quantile VAR is the best for each value of MAE, except for $T_0 = 230$. At some values of T_0 we observe that the MAE values were equal for several models. Nevertheless, the two interesting models to compare in this case turn out to be the VAR in first differences and the quantile VAR in first differences, at $q = 0.5$. Based on the squared errors the VAR model was better whereas based on the absolute error the quantile VAR model was better, which already leads to difficulty in really concluding which model can be seen as better performing. It holds that only for $T_0 = 220$ the quantile VAR at $q = 0.5$ is significantly better than the VAR model, at the 10% level, concluding from both the DM-statistics based on squared errors and absolute errors.

	VAR		VECM		Quantile VAR		RW	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
$T_0 = 60$	**0.312**	0.229	0.320	0.242	0.313	**0.226**	0.353	0.244
$T_0 = 90$	**0.253**	0.192	0.263	0.206	0.263	**0.194**	0.322	0.226
$T_0 = 120$	**0.227**	**0.173**	0.244	0.190	0.233	**0.173**	0.279	0.194
$T_0 = 150$	**0.242**	0.184	0.260	0.202	0.243	**0.180**	0.300	0.203
$T_0 = 180$	**0.271**	0.207	0.284	0.214	0.272	**0.197**	0.348	0.233
$T_0 = 210$	0.325	0.241	**0.321**	**0.236**	0.340	**0.236**	0.460	0.329
$T_0 = 220$	0.241	0.196	0.283	0.214	**0.201***	**0.165***	0.235	0.196
$T_0 = 230$	0.114	0.104	**0.100**	**0.078**	**0.100**	0.089	0.226	0.211

Table 5: One quarter ahead forecast errors for all expanding window models where unemployment is forecasted. Expressed in percentages. The errors in bold are the smallest. The * represents the significance of the DM-statistic, performed with the quantile VAR and VAR, at the 10% significance level.

Now, considering the Table 6 we find the four-step ahead forecast errors. We observe that the VECM is performing seemingly better than before, namely starting from the value of $T_0 = 180$ to $T_0 = 230$ the mean absolute errors are all lower compared to the VAR model. This shows that increasing the sample size with which the VECM is estimated can possibly improve the accuracy of the estimation, as then the long-run equilibrium is better defined. Nevertheless, the best performing model is again clearly the quantile VAR model, which significantly outperforms the VAR model for almost all values of T_0 as can be seen in the table.

	VAR		VECM		Quantile VAR	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
$T_0 = 60$	0.742	0.527	0.800	0.567	**0.667***	**0.489***
$T_0 = 90$	0.647	0.451	0.703	0.489	**0.576***	**0.425**
$T_0 = 120$	0.542	0.371	0.706	0.475	**0.450***	**0.333***
$T_0 = 150$	0.610	0.430	0.785	0.543	**0.470***	**0.345***
$T_0 = 180$	0.722	0.534	0.798	0.511	**0.530***	**0.406***
$T_0 = 210$	0.911	0.693	0.953	0.578	**0.517***	**0.394***
$T_0 = 220$	0.577	0.503	0.381	**0.290**	**0.352***	0.303*
$T_0 = 230$	0.414	0.328	**0.203**	0.199	0.322*	0.279

Table 6: One year ahead forecast errors for all expanding window models where unemployment is forecasted. Expressed in percentages. The errors in bold are the smallest. The * represents the significance of the DM-statistic, performed with the quantile VAR and VAR, at the 10% significance level.

Thus, when considering unemployment we clearly observe that the quantile VAR model is the best performing model. This is mostly based on the results of the four-step ahead forecasts since there it had overall the lowest RMSE and MAE values.

5 Conclusion

This research conducts estimation of linear and nonlinear effects of government spending using US data. In essence, the response of output and unemployment are analysed, in linear and nonlinear ways. For the linear estimation a vector error correction model and a vector autoregression with first differences are performed, to see whether adding a long-run equilibrium does improve the estimation of output and unemployment. The approach for the nonlinear estimation implemented is a quantile VAR using detrended data and first differenced data. We thus implement multiple models in this research with the aim of managing to find which model forecasts best, considering output and unemployment.

The findings of this research are that the VAR with first differences overall performs the best regarding its forecasting performance. This is mostly concluded from the fact that at some results there is a significant improvement in forecasting performance compared to the VECM, regarding the output variable as the endogenous variable. Nevertheless, the quantile VAR model with first differences, evaluated at its median can be considered also as a well performing model as it has significantly better forecasts compared to the VAR model with first differences again for some results, regarding the unemployment as the dependent variable.

The nonlinear quantile VAR is still considered to be more relevant in estimating the effects of fiscal policy. Firstly, we have confirmed that nonlinearities exist as different quantiles of the distribution of output and unemployment showed to have different impulse response functions, which is why a nonlinear approach is much better. Furthermore, when this model was performed using first differenced data it had adequate accuracy regarding the forecasting of unemployment. Thus, we conclude the quantile VAR model is much more appropriate for estimating the effects of fiscal policy.

Also, it turns out to be that there is not a significant improvement when considering the cointegration relations. Adding a long-run equilibrium to the estimation leads to overfitting apparently.

Overall, this research has thus confirmed earlier findings of Linnemann and Winkler 2016, but has also found that linear estimation with vector error correction model leads to the same conclusions drawn, even when data is not detrended. However, the truth is that nonlinear estimation is much more suitable for estimating the effects as we have seen that there are differences in them among different quantiles of the distribution. Developing more nonlinear different estimation procedures in order to estimate the effects of fiscal policy to assess its effectiveness is recommended for future research.

6 Discussion

Sample size could be considered an issue of this research, as for example performing the Diebold-Mariano test assumes normality of the test statistic and with little observations this is not accurate. Take for example the fact that we evaluate only six forecasts at $T_0 = 230$ which does not give an accurate estimate. For lower values of T_0 the Diebold-Mariano test was more accurate. Increasing the sample size might possibly lead to different conclusions which is an interesting aspect for future research.

Furthermore, the VECM has been performed with variables that are not concluded to be non-stationary, even though cointegration requires non-stationary variables in order to transform into stationairy series. Nevertheless, adding stationary variables such as unemployment and taxes to the estimation likely has been reason that the Johansen tests gave the outcome that the number of cointegration relations is nonzero sometimes (at different assumptions), thus increasing the number cointegration relations. However, it was necessary to include all the stationary and non-stationary variables due to the fact that then a proper comparison among the three models would be possible as they all have the same variables.

Lastly, taking first differences of all the data is not very economically meaningful. For example, to take the first difference of an interest rate does not have a proper interpretation for the research. Nevertheless, performing this was necessary in order to manage to compare the forecasts among the models. Also, the Augmented Dickey-Fuller test hinted at performing the estimation with first differences for the interest rate, likely due to the assumptions made.

Overall, this research is sensitive to many assumptions that are necessary for producing results.

7 Appendix

7.1 Data 1 and Data 2

Variable	Construction	Series Title & ID
Output	lqd(GDPC1)	Real Gross Domestic Product (GDPC1)
Government Spending	lqd(GCEC96)	Real Government Consumption Expenditures and Gross Investment (GCEC96)
Real interest rate	$\frac{FEDFUNDS}{100} - log(\frac{GDPDEF}{GDPDEF(-1)}) * 4$	1) Effective Federal Funds Rate (FEDFUNDS), 2) Gross Domestic Product: Implicit Price Deflator (GDPDEF)
Unemployment	$\frac{UNRATE}{100}$	Unemployment Rate (UNRATE)
Net Taxes	lqd($\frac{W054R...+W782R...-A084R...}{GDPDEF}$)	1) Government Current Tax Receipts (W054RC1Q027SBEA), 2) Government Current Receipts: Contributions for Government Social Insurance (W782RC1Q027SBEA), 3) Government Current Transfer Payments (A084RC1Q027SBEA), 4) Gross Domestic Product: Implicit Price Deflator (GDPDEF)
Debt (not used)	lqd(interpolate(FYP...))	Gross Federal Debt Held by the Public as Percent of Gross Domestic Product (FYPUGDA188S)
	lqd((GFD...))	Federal Debt: Total Public Debt as Percent of Gross Domestic Product (GFDEGDQ188S) (available from 1966Q1)

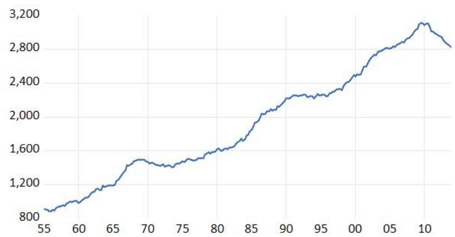
Table 7: Every series is taken from the Federal Reserve Bank of St. Louis database on the following website: https://fred.stlouisfed.org/. Each used variable has a quarterly frequency. The lqd($\cdot$) represents the log-quadratic detrending operator and the interpolate($\cdot$) represents that the annual data has been converted into quarterly data by means of interpolation. The above mentioned data is used as dataset 1, whereas dataset 2 considers the exact same variables but without the lqd($\cdot$) operator, so there is no form of detrending. These variables that are not detrended, but in their original level form, are denoted by capital letters in the text. In the research these variables in dataset 2 are first differenced.

For log-quadratic detrending we consider the same approach as implemented by Mendoza (1991) to detrend the variables: Let $y_t = log(Y_t)$, where $log(\cdot)$ denotes the natural logarithm and Y_t is the originally considered variable measured over time. We assume y_t consists of a trend component and a cyclical component, namely $y_t = y_t^t + y_t^c$. We regress the following equation using ordinary least squares:

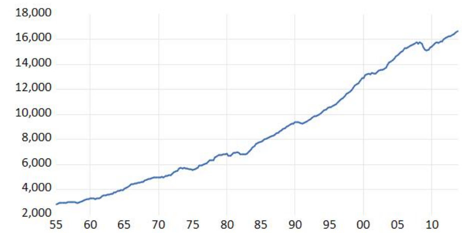
$$y_t = a + bt + ct^2 + \epsilon_t,$$
where we have trend: $y_t^t = a + bt + ct^2$, and cycle: $y_t^c = \epsilon_t$

Thus we can retrieve the cyclical component $y_t^c = \epsilon_t$. This way we likely also turn a variable which possibly has a unit root into a variable that likely has no unit root, since it is detrended.

7.2 Additional Results



(a) Government spending (billions of Dollars)



(b) Output (billions of Dollars)

(c) Real net taxes (billions of Dollars)

(d) Interest rate (percentage)

(e) Unemployment (percentage)

Figure 5: Plots of all considered variables in their original level form.

H_0: series contains unit root	Dickey Fuller statistic	P-value	Specification
Output	-1.92	0.64	Trend and intercept
Government spending	-2.45	0.35	Trend and intercept
Net Taxes	-4.16	0.01	Trend and intercept
Real Interest rate	-2.74	0.22	Trend and intercept
Unemployment rate	-3.89	0.00	Intercept

Table 8: Augmented Dickey-Fuller tests performed for the variables in levels.

Data Trend:	None	None	Linear	Linear	Quadratic
Test Type:	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend	No Trend	No Trend	Trend	Trend
Trace	3	2	2	2	1
Max-Eig	2	2	2	2	1

Table 9: Cointegration rank with output: Under most assumptions it holds that 2 cointegration relations hold following from both the Trace test and Maximum Eigenvalue test.

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.14	76.49	47.86	0.00
At most 1 *	0.13	40.55	29.80	0.00
At most 2	0.04	8.83	15.49	0.38
At most 3	0.00	0.01	3.84	0.93

Table 10: Cointegration rank with output: Trace test performed for the variables in levels. 4 lags of the first differences are used and it allows for a linear deterministic trend in the data and constant.

Hypothesized No. of CE(s)	Eigenvalue	Max. Eigenvalue Statistic	0.05 Critical Value	Prob.
None *	0.14	35.93	27.58	0.00
At most 1 *	0.13	31.72	21.13	0.00
At most 2	0.04	8.82	14.26	0.30
At most 3	0.00	0.01	3.84	0.93

Table 11: Cointegration rank with output: Maximum Eigenvalue test performed for the variables in levels. 4 lags of the first differences are used and it allows for a linear deterministic trend in the data and constant.

Data Trend:	None	None	Linear	Linear	Quadratic
Test Type	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend	No Trend	No Trend	Trend	Trend
Trace	1	1	1	1	1
Max-Eig	1	1	1	1	1

Table 12: Cointegration rank with unemployment: Under most assumptions it holds that 1 cointegration relation holds following from both the Trace test and Maximum Eigenvalue test.

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.14	56.36	47.86	0.01
At most 1	0.05	20.63	29.80	0.38
At most 2	0.03	9.01	15.49	0.36
At most 3	0.01	2.18	3.84	0.14

Table 13: Cointegration rank with unemployment: Trace test performed for the variables in levels. 4 lags of the first differences are used and it allows for a linear deterministic trend in the data and constant.

Hypothesized No. of CE(s)	Eigenvalue	Max. Eigenvalue Statistic	0.05 Critical Value	Prob.**
None *	0.14	35.73	27.58	0.00
At most 1	0.05	11.62	21.13	0.59
At most 2	0.03	6.83	14.26	0.51
At most 3	0.01	2.18	3.84	0.14

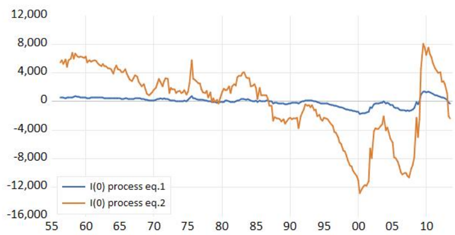
Table 14: Cointegration rank with unemployment: Maximum Eigenvalue test performed for the variables in levels. 4 lags of the first differences are used and it allows for a linear deterministic trend in the data and constant.

Cointegrating Eq:	CointEq1	CointEq2
Government Spending(-1)	1.00	0.00
Output(-1)	0.00	1.00
Tax(-1)	-211.252 (-9.93)	-1632.27 (-9.42)
Interest(-1)	-593.69 (-0.14)	22755.67 (0.67)
C	781.23	11814.41

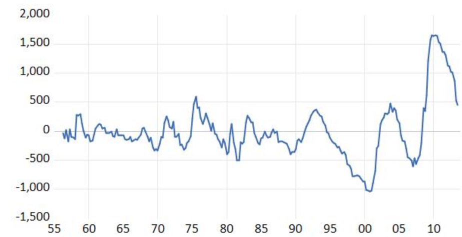
Table 15: Estimated cointegration relations derived from VECM where output is added. T-statistics are in brackets.

Cointegrating Eq:	CointEq1
Government Spending(-1)	1.00
Unemployment(-1)	6333.29 (1.39)
Tax(-1)	-129.18 (-10.39)
Interest(-1)	-5542.05 (-1.80)
C	-553.50

Table 16: Estimated cointegration relation derived from VECM where unemployment is added. T-statistics are in brackets.



(a) I(0) Error correction terms based on VECM with output.



(b) I(0) Error correction terms based on VECM with unemployment.

Figure 6: Error correction terms resulting from the cointegration relations for the VECM with output and the VECM with unemployment.

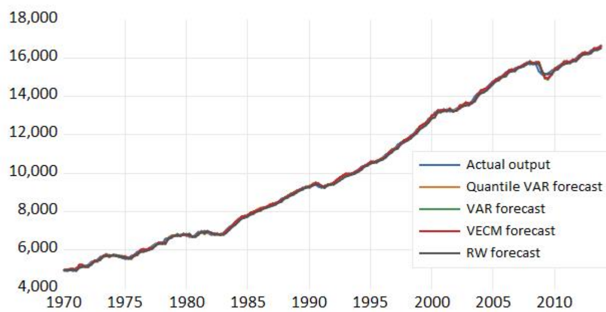


Figure 7: Actuals and one-step ahead forecasts for output obtained from an expanding window for each of the four models, namely VAR with first differences, VECM, quantile VAR with first differences and the random walk model.

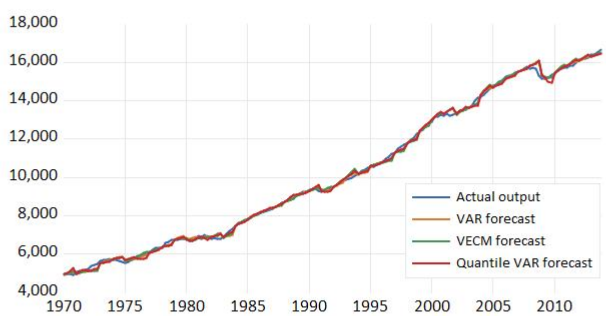


Figure 8: Actuals and four-step ahead forecasts for output obtained from an expanding window for each of the three models, namely VAR with first differences, VECM and quantile VAR with first differences.

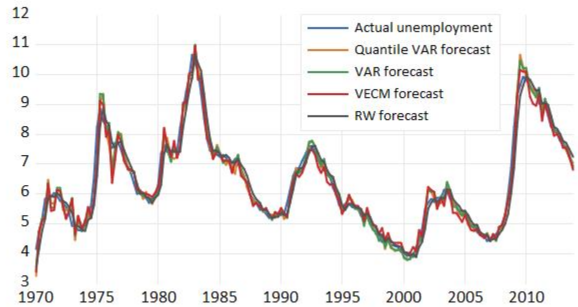


Figure 9: Actuals and one-step ahead forecasts for the unemployment rate obtained from an expanding window for each of the four models, namely VAR with first differences, VECM, quantile VAR with first differences and the random walk model.

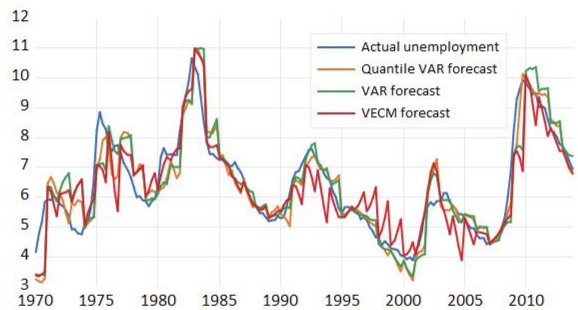
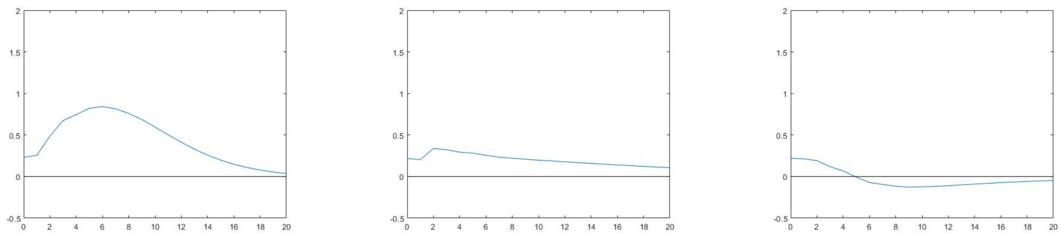
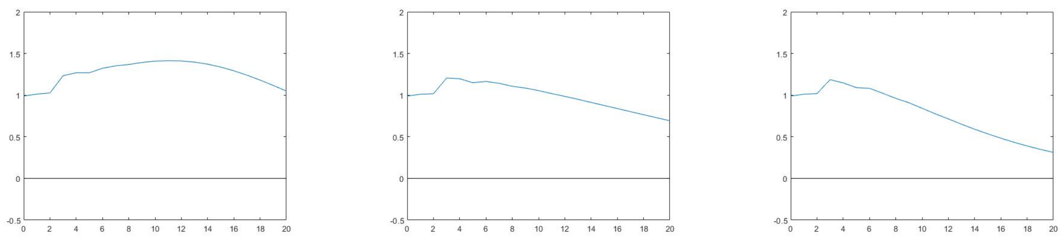


Figure 10: Actuals and four-step ahead forecasts for the unemployment rate obtained from an expanding windows for each of the three models, namely VAR with first differences, VECM and quantile VAR with first differences.

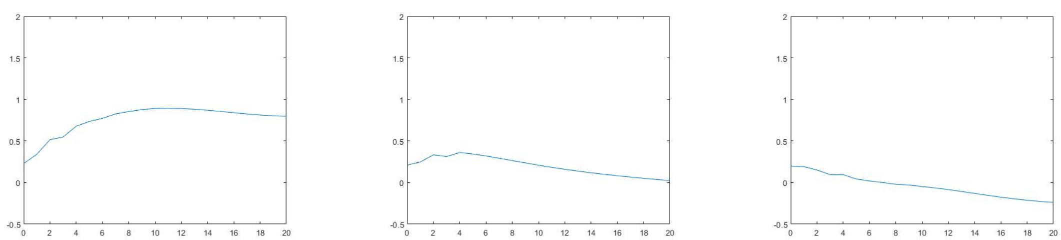


(a) o to g. 0.1 quantile (b) o to g. 0.5 quantile (c) o to g. 0.9 quantile

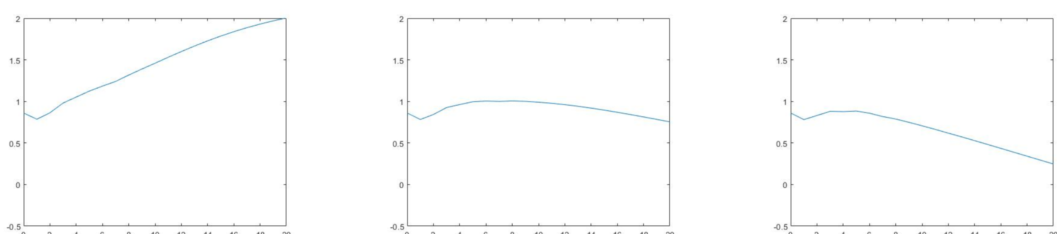


(d) g to g. 0.1 quantile (e) g to g. 0.5 quantile (f) g to g. 0.9 quantile

Figure 11: Impulse responses at different quantiles of the output distribution. The lines show the responses of output and government spending to a 1% government spending shock. The interest rate has been determined using monthly series of the fedfunds rate selected at the beginning of each quarter.

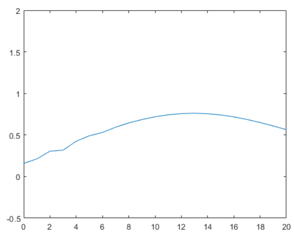


(a) o to g. 0.1 quantile (b) o to g. 0.5 quantile (c) o to g. 0.9 quantile

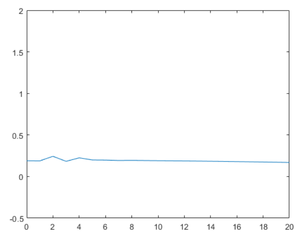


(d) g to g. 0.1 quantile (e) g to g. 0.5 quantile (f) g to g. 0.9 quantile

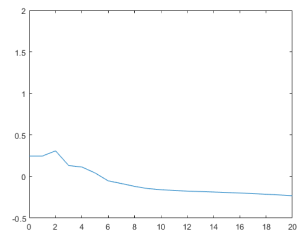
Figure 12: Impulse responses at different quantiles of the output distribution. The lines show the responses of output and government spending to a 1% government spending shock. The sample considered is 1966Q1 to 2013Q4. The interest rate has been determined using monthly series of the fedfunds rate selected at the beginning of each quarter. The quantile VAR(4) model also included the debt variable, which was available from 1966Q1.



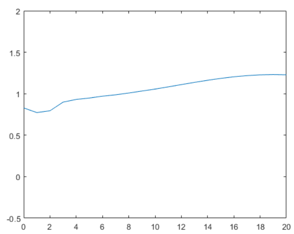
(a) o to g. 0.1 quantile



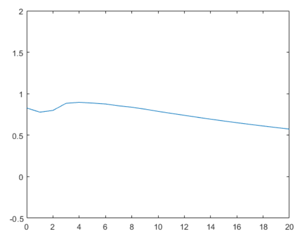
(b) o to g. 0.5 quantile



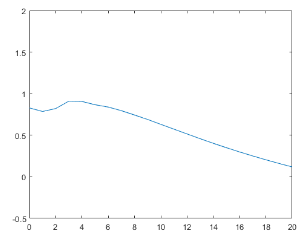
(c) o to g. 0.9 quantile



(d) g to g. 0.1 quantile

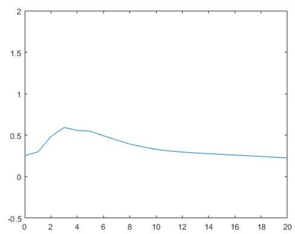


(e) g to g. 0.5 quantile

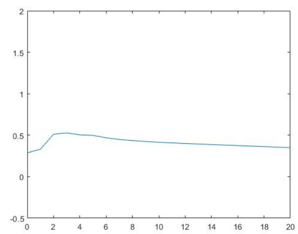


(f) g to g. 0.9 quantile

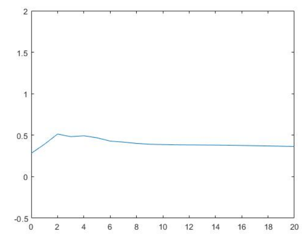
Figure 13: Impulse responses at different quantiles of the output distribution. The lines show the responses of output and government spending to a 1% government spending shock. The sample considered is 1966Q1 to 2017Q2. The interest rate has been determined using monthly series of the fedfunds rate selected at the beginning of each quarter. The quantile VAR(4) model also included the debt variable, which was available from 1966Q1.



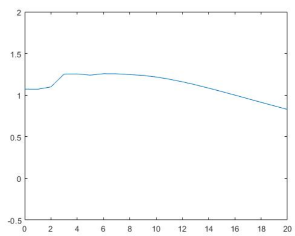
(a) o to g. 0.1 quantile



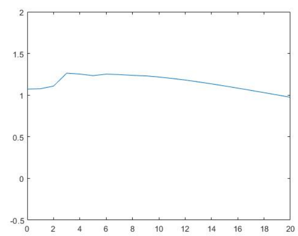
(b) o to g. 0.5 quantile



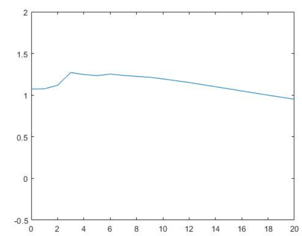
(c) o to g. 0.9 quantile



(d) g to g. 0.1 quantile

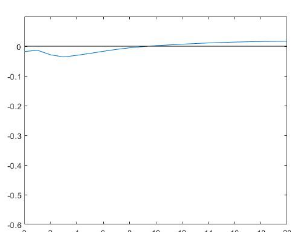


(e) g to g. 0.5 quantile

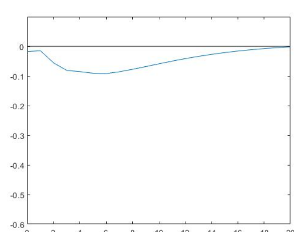


(f) g to g. 0.9 quantile

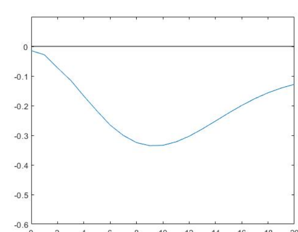
Figure 14: Impulse responses at different quantiles of the output distribution. The lines show the responses of output and government spending to a 1% government spending shock. The sample considered is 1955Q1 to 2013Q4. The interest rate has been determined using monthly series of the fedfunds rate selected at the beginning of each quarter. The quantile VAR(4) model also included the debt variable, which was available from 1955Q1 and has been interpolated as it was available in a yearly frequency.



(a) u to g. 0.1 quantile



(b) u to g. 0.5 quantile



(c) u to g. 0.9 quantile

Figure 15: Impulse responses at different quantiles of the output distribution. The lines show the responses of output and government spending to a 1% government spending shock. The interest rate has been determined using monthly series of the fedfunds rate selected at the beginning of each quarter. The unemployment rate has been determined using average quarterly series.

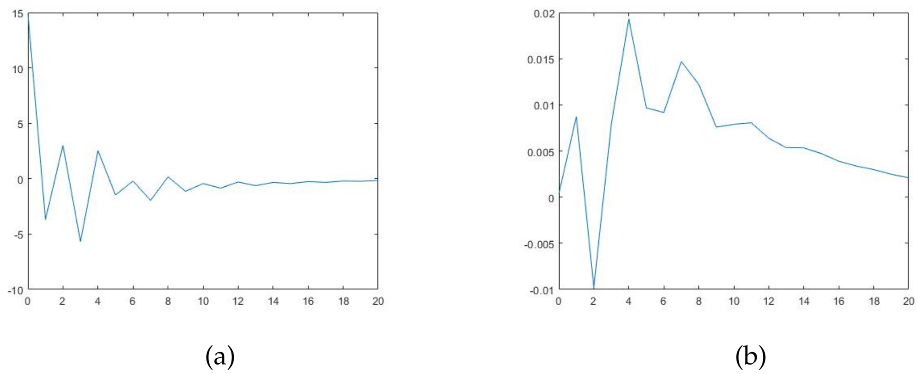


Figure 16: a): Response of ΔO_t on a shock in ΔG_t (measured in billions of Dollars). b): Response of Δu_t to a shock in ΔG_t (measured in percentages)

	DM-statistic Sq. error (one-sided P-value)	DM-statistic Abs. error (one-sided P-value)
$T_0 = 60$	0.82 (0.21)	1.26 (0.10)
$T_0 = 90$	1.11 (0.13)	1.26 (0.10)
$T_0 = 120$	1.06 (0.14)	1.11 (0.13)
$T_0 = 150$	1.54 (0.06)	1.92 (0.03)
$T_0 = 180$	-0.12 (0.55)	-0.75 (0.77)
$T_0 = 210$	-0.32 (0.63)	-0.73 (0.77)
$T_0 = 220$	0.57 (0.28)	-0.21 (0.58)
$T_0 = 230$	0.26 (0.40)	-0.19 (0.57)

Table 17: DM-statistics for comparing VAR with first differences and VECM, considering the one quarter ahead forecasts of output.

	DM-statistic Sq. error (one-sided P-value)	DM-statistic Abs. error (one-sided P-value)
$T_0 = 60$	2.70 (0.00)	1.37 (0.09)
$T_0 = 90$	2.44 (0.01)	1.15 (0.12)
$T_0 = 120$	3.09 (0.00)	2.32 (0.01)
$T_0 = 150$	3.17 (0.00)	2.42 (0.01)
$T_0 = 180$	2.15 (0.02)	1.47 (0.07)
$T_0 = 210$	1.92 (0.03)	1.92 (0.03)
$T_0 = 220$	1.98 (0.02)	2.14 (0.02)
$T_0 = 230$	-0.10 (0.54)	0.14 (0.44)

Table 18: DM-statistics for comparing VAR with first differences and VECM, considering the one year ahead forecasts of output.

	DM-statistic Sq. error (one-sided P-value)	DM-statistic Abs. error (one-sided P-value)
$T_0 = 60$	0.12 (0.45)	-0.35 (0.64)
$T_0 = 90$	0.79 (0.22)	0.21 (0.42)
$T_0 = 120$	0.56 (0.29)	0.07 (0.47)
$T_0 = 150$	0.02 (0.49)	-0.56 (0.71)
$T_0 = 180$	0.07 (0.47)	-0.94 (0.83)
$T_0 = 210$	0.56 (0.29)	-0.26 (0.60)
$T_0 = 220$	-1.45 (0.93)	-1.35 (0.91)
$T_0 = 230$	-1.17 (0.88)	-0.87 (0.81)

Table 19: DM-statistics for Quantile VAR and VAR in first differences, considering the one quarter ahead forecasts of unemployment.

	DM-statistic Sq. error (one-sided P-value)	DM-statistic Abs. error (one-sided P-value)
$T_0 = 60$	2.08 (0.02)	1.48 (0.07)
$T_0 = 90$	1.67 (0.05)	1.08 (0.14)
$T_0 = 120$	1.50 (0.07)	1.30 (0.10)
$T_0 = 150$	1.89 (0.03)	2.35 (0.01)
$T_0 = 180$	1.98 (0.02)	2.43 (0.01)
$T_0 = 210$	2.33 (0.01)	3.03 (0.00)
$T_0 = 220$	3.26 (0.00)	2.90 (0.00)
$T_0 = 230$	1.53 (0.06)	0.69 (0.25)

Table 20: DM-statistics for comparing VAR with first differences and Quantile VAR with first differences, considering the one year ahead forecasts of unemployment.

7.3 Programming Codes

EVIEWS ONE STEP AHEAD EXPANDING WINDOW CODE:

```
series yvar=na
for !i=60 to 235
smpl @first @first+!i-1
var var1.ls 1 4 d(gvtspend) d(gdp) d(tax) d(interest2)

smpl @first+!i @first+235
var1.forecast
yvar(!i+1)=gdp_f(!i+1)
next

```

```
series yvec1=na
for !i=60 to 235
smpl @first @first+!i-1
var vec1.ec(c,2) 1 4 gvtspend gdp tax interest2

smpl @first+!i @first+235
vec1.forecast
yvec1(!i+1)=gdp_f(!i+1)
next


series yquant=na
for !i=60 to 235
smpl @first @first+!i-1
equation quantilemodel.qreg(quant=0.5) d(gdp) C d(GVTSPEND(-1)) d(GDP
    (-1)) d(tax(-1)) d(interest2(-1)) d(GVTSPEND(-2)) d(GDP(-2)) d(tax
    (-2)) d(interest2(-2)) d(GVTSPEND(-3)) d(GDP(-3)) d(tax(-3)) d(
    interest2(-3)) d(GVTSPEND(-4)) d(GDP(-4)) d(tax(-4)) d(interest2(-4)
    )

smpl @first+!i @first+235
quantilemodel.forecast gdp_f
yquant(!i+1)=gdp_f(!i+1)
next



series uquant=na
for !i=60 to 235
smpl @first @first+!i-1
equation quantilemodel.qreg(quant=0.5) d(unemployment2) C d(GVTSPEND
    (-1)) d(unemployment2(-1)) d(tax(-1)) d(interest2(-1)) d(GVTSPEND
    (-2)) d(unemployment2(-2)) d(tax(-2)) d(interest2(-2)) d(GVTSPEND
    (-3)) d(unemployment2(-3)) d(tax(-3)) d(interest2(-3)) d(GVTSPEND
    (-4)) d(unemployment2(-4)) d(tax(-4)) d(interest2(-4))

smpl @first+!i @first+235
quantilemodel.forecast unemployment2_f
```

```
uquant(!i+1)=unemployment2_f(!i+1)
next

series uvar=na
for !i=60 to 235
smpl @first @first+!i-1
var var2.ls 1 4 d(gvtspend) d(unemployment2) d(tax) d(interest2)

smpl @first+!i @first+235
var2.forecast
uvar(!i+1)=unemployment2_f(!i+1)
next


series uvec2=na
for !i=60 to 235
smpl @first @first+!i-1
var vec2.ec(c,1) 1 4 gvtspend unemployment2 tax interest2

smpl @first+!i @first+235
vec2.forecast
uvec2(!i+1)=unemployment2_f(!i+1)
next

series rwgdp=na

for !i=60 to 235
rwgdp(!i+1)=gdp(!i)
next

series rwunemp2=na

for !i=60 to 235
rwunemp2(!i+1)=unemployment2(!i)
next


smpl @all
```

EVIEWS 4-STEP AHEAD EXPANDING WINDOW FORECAST CODE:

```
series yvar4step=na
!i=60

while !i<233
smpl @first @first+!i-1
var var14step.ls 1 4 d(gvtspend) d(gdp) d(tax) d(interest2)

smpl @first+!i @first+235
var14step.forecast
yvar4step(!i+1)=gdp_f(!i+1)
yvar4step(!i+2)=gdp_f(!i+2)
yvar4step(!i+3)=gdp_f(!i+3)
yvar4step(!i+4)=gdp_f(!i+4)
!i=!i+4
wend

series yvec14step=na
!i=60
while !i<233
smpl @first @first+!i-1
var vec14step.ec(c,2) 1 4 gvtspend gdp tax interest2

smpl @first+!i @first+235
vec14step.forecast
yvec14step(!i+1)=gdp_f(!i+1)
yvec14step(!i+2)=gdp_f(!i+2)
yvec14step(!i+3)=gdp_f(!i+3)
yvec14step(!i+4)=gdp_f(!i+4)
!i=!i+4
wend


series yquant4step=na
!i=60
while !i<233
smpl @first @first+!i-1
equation quantilemodel4step.qreg(quant=0.5) d(gdp) C d(GVTSPEND(-1)) d(
    GDP(-1)) d(tax(-1)) d(interest2(-1)) d(GVTSPEND(-2)) d(GDP(-2)) d(
    tax(-2)) d(interest2(-2)) d(GVTSPEND(-3)) d(GDP(-3)) d(tax(-3)) d(
```

```
    interest2(-3)) d(GVTSPEND(-4)) d(GDP(-4)) d(tax(-4)) d(interest2(-4)
    )

smpl @first+!i @first+235
quantilemodel4step.forecast gdp_f
yquant4step(!i+1)=gdp_f(!i+1)
yquant4step(!i+2)=gdp_f(!i+2)
yquant4step(!i+3)=gdp_f(!i+3)
yquant4step(!i+4)=gdp_f(!i+4)
!i=!i+4
wend


series uquant4step=na
!i=60
while !i<233
smpl @first @first+!i-1
equation quantilemodel.qreg(quant=0.5) d(unemployment2) C d(GVTSPEND
    (-1)) d(unemployment2(-1)) d(tax(-1)) d(interest2(-1)) d(GVTSPEND
    (-2)) d(unemployment2(-2)) d(tax(-2)) d(interest2(-2)) d(GVTSPEND
    (-3)) d(unemployment2(-3)) d(tax(-3)) d(interest2(-3)) d(GVTSPEND
    (-4)) d(unemployment2(-4)) d(tax(-4)) d(interest2(-4))

smpl @first+!i @first+235
quantilemodel.forecast unemployment2_f
uquant4step(!i+1)=unemployment2_f(!i+1)
uquant4step(!i+2)=unemployment2_f(!i+2)
uquant4step(!i+3)=unemployment2_f(!i+3)
uquant4step(!i+4)=unemployment2_f(!i+4)
!i=!i+4
wend

series uvar4step=na
!i=60
while !i<233
smpl @first @first+!i-1
var var2.ls 1 4 d(gvtspend) d(unemployment2) d(tax) d(interest2)

smpl @first+!i @first+235
```

```
var2.forecast
uvar4step(!i+1)=unemployment2_f(!i+1)
uvar4step(!i+2)=unemployment2_f(!i+2)
uvar4step(!i+3)=unemployment2_f(!i+3)
uvar4step(!i+4)=unemployment2_f(!i+4)
!i=!i+4
wend


series uvec4step=na
!i=60
while !i<233
smpl @first @first+!i-1
var vec2.ec(c,1) 1 4 gvtspend unemployment2 tax interest2

smpl @first+!i @first+235
vec2.forecast
uvec4step(!i+1)=unemployment2_f(!i+1)
uvec4step(!i+2)=unemployment2_f(!i+2)
uvec4step(!i+3)=unemployment2_f(!i+3)
uvec4step(!i+4)=unemployment2_f(!i+4)
!i=!i+4
wend


smpl @all
```

MATLAB IMPULSE RESPONSE (CHOL. DECOMP.) CODE:

```
%This code generates impulse responses based on a Cholesky composition.
%The input needed is the residuals of each regression in a matrix
%and the coefficient matrix, consisting of all coefficients except
%for the constant.

J=[eye(4) zeros(4,4*3)];
A=[P;eye(4) zeros(4,4) zeros(4,4) zeros(4,4);zeros(4,4) eye(4) zeros
    (4,4) zeros(4,4);zeros(4,4) zeros(4,4) eye(4)  zeros(4,4)];
covar=resid'*resid/(232-(4*4+1));%compute covariance of residuals
    adjusting for df

Bz=chol(covar);
```

```
impulseresp=zeros(20,16);
for i=0:20
    phi=J*A^i*J'*Bz';
    impulseresp(i+1,1)=phi(1,1);%gvtspend on gvtspend
    impulseresp(i+1,2)=phi(1,2);%gvtspend on gdp/unemp
    impulseresp(i+1,3)=phi(1,3);%gvtspend on tax
    impulseresp(i+1,4)=phi(1,4);%gvtspend on interest
    impulseresp(i+1,5)=phi(2,1);%gdp/unemp on gvtspend
    impulseresp(i+1,6)=phi(2,2);%gdp/unemp on gdp
    impulseresp(i+1,7)=phi(2,3);%gdp/unemp on tax
    impulseresp(i+1,8)=phi(2,4);%gdp/unemp on interest
    impulseresp(i+1,9)=phi(3,1);%tax on gvtspend
    impulseresp(i+1,10)=phi(3,2);%tax on gdp/unemp
    impulseresp(i+1,11)=phi(3,3);%tax on tax
    impulseresp(i+1,12)=phi(3,4);%tax on interest
    impulseresp(i+1,13)=phi(4,1);%interest on gvtspend
    impulseresp(i+1,14)=phi(4,2);%interest on gdp/unemp
    impulseresp(i+1,15)=phi(4,3);%interest on tax
    impulseresp(i+1,16)=phi(4,4);%interest on interest

end
plot( [0:20],impulseresp(:,1))

hline = refline(0, 0);
hline.Color = 'black';
axis([0 20 -0.5 2])

maximumpointopoint= max(impulseresp(1:12,5)./impulseresp(1:12,1));
cumulativeratio=sum(impulseresp(1:12,5))/sum(impulseresp(1:12,1));
```

MATLAB FORECASTING EVALUATION CODE:

```
%This code sets up RMSE, MAE, DM statistics for squared and
%absolute errors. It uses another function called 'dmtest' and
%'dmtestabs' in order to get the DM-statistics for each value of
%T0.
%T0=60,90,120,150,180,210,220,230 forecastmatrix is ordered VAR VEC QUANT RW:
rmse=zeros(6,4);
mae=zeros(6,4);
```

```
T=1;
h=1;
for i=1:4
     for j=1:8
     rmse(j,i)=sqrt(mean((forecastmatrix(T:end,i)-actual(T:end)).^2));
     mae(j,i)=mean(abs(forecastmatrix(T:end,i)-actual(T:end)));
        e1=forecastmatrix(T:end,3)-actual(T:end);%comparing errors of the two that have smallest error
        e2=forecastmatrix(T:end,1)-actual(T:end);
        DMsq(j) = dmtest(e1,e2,h);
        DMabs(j)= dmtestabs(e1,e2,h);
        Pvalsq(j)=1- normcdf(DMsq(j));
        Pvalabs(j)=1-normcdf(DMabs(j));
       if j<6
             T=T+30;
       else
            T=T+10;
       end
       end
       T=1;
end
output=[rmse mae DMsq' Pvalsq' DMabs' Pvalabs'];
```

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