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Nonlinearity in Fiscal Policy Response: A Quantile Regression Approach

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Abstract

The effects of fiscal policy on the economy continues to be studied extensively by both academics and practitioners. Recently the field has been the subject of some debate with regard to whether or not fiscal policy effects are characterised by nonlinearities rooted in the business cycle. This paper aims to evaluate this by help of Quantile Vector Autoregression (QVAR) models, a method developed in Cecchetti and Li (2008) and first applied to this context in Linnemann and Winkler (2016). Besides using Quantile Regression to evaluate nonlinearity through QVAR, this paper also researches the benefits possessed by Quantile Regression in a forecast setting and how best to apply it. For this, forecasts are calculated both by means of QVAR and by Quantile Regression Forests (QRF) (Meinshausen, 2006). These represent two separate but promising ways to apply Quantile Regression and thus their performance is evaluated against one another. I find considerable differences in fiscal policy effects across quantiles, thereby giving a strong indication of nonlinear responses to government spending both for GDP and for unemployment. In terms of forecasting, I find that Quantile Regression methods provide additional value to conditional mean forecasts and that QRF provides more accurate forecasts, however, not significantly so.

The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, Erasmus School of Economics or Erasmus University Rotterdam.

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1 Introduction

As one of the main tools of central banks to combat downturns in economic activity, fiscal policy has been and continues to be the subject of considerable economic research. Superior understanding of how spending shocks affect key economic indicators such as GDP and unemployment rate allows for improved stimulus package design, characterized by more efficient means of sheltering the economy from major economic downturns. Early research into spending effects, such as Blanchard and Perotti (2002), consisted of simple Vector Autoregressive (VAR) analyses and derivative Impulse Response Functions (IRFs). VAR models were a natural starting point for research into the relationship between fiscal policy and output/unemployment due to its ability to represent dynamic processes. This ability allows VAR to generally outperform univariate models in terms of forecasting performance. In the past two decades a number of more involved methods have been applied to the context of fiscal policy effects, see e.g. Mountford and Uhlig (2009), Auerbach and Gorodnichenko (2012), Owyang, Ramey, and Zubairy (2013). Lately, focus within the field has shifted towards nonlinearities arising from economic conditions such as, for example, economic slack. Some examples of methods dealing with this nonlinearity include Smooth Transition Vector Autoregression (STVAR) as in Auerbach and Gorodnichenko (2012), local projection as in Auerbach and Gorodnichenko (2013) as well as Owyang et al. (2013) and most relevant to this thesis Quantile Vector Autoregressions (QVAR) as in Linnemann and Winkler (2016). The correct modeling of nonlinear aspects would allow for a better understanding of how fiscal policy affects the economy and it would provide a better toolset for practitioners to more accurately forecast how the economy will respond to economic stimulus. Whilst VAR, STVAR & local projection methods allow for forecasting at the conditional mean, QVAR and other Quantile Regression based methods, such as Quantile Regression Forests (QRF) (Meinshausen, 2006), allow for further understanding of the conditional distribution through quantile dependent model specifications. How this translates exactly to the methodology is dependent on the model in which it is utilized but for QVARs, it allows one to estimate the effect of explanatory variables with different coefficients at different ends of the distribution and for QRF it allows for different estimates within leaf nodes.

This thesis adds to the existing literature on how economic indicators are affected by government spending and how QVAR and other Quantile Regression methods can be leveraged to provide more accurate and informative forecasts. Specifically, this paper addresses the following research questions: To which extent do economic measures such as GDP and the unemployment rate exhibit nonlinear responses to government spending and how can the Quantile Regression methodology best be applied to forecast responses to fiscal policy at various quantiles?

As mentioned there have been a number of different approaches used to account for nonlinearity in response to government spending, for example the inclusion of regime switching in VAR models in Auerbach and Gorodnichenko (2012) and the use of QVARs in Linnemann and Winkler (2016). In this paper I use the QVAR method applied in Linnemann and Winkler (2016), both for an analysis of fiscal policy multipliers and for forecasting parts of the conditional distribution of output. QVAR works in a similar way as traditional VAR methods, however, it relies on an asymmetric absolute value loss function as opposed to a traditional squared loss function. This loss function is dependent on the quantile the QVAR is estimated at and this is what gives it the quantile dependent specification. In addition to the QVAR, I will also consider the application of Quantile Regression in a tree based method to see which of the two provide more accurate forecasts of the response distribution. This Quantile Regression Forest (QRF) method is new in the context of fiscal policy effects, and it will be interesting to see how it performs against the proven QVAR method. QRFs rely on growing a large amount of regression trees as in Random Forests, however, whereas Random Forests

average observations in leaf nodes, QRF keeps all observations to get a conditional distribution estimate. Based on Quantile Regressions and Random Forests, QRF inherits some of their respective advantages. Specifically, QRF like QVAR allows for quantile dependent specification, however, where the QVAR is restricted to a linear form within each equation for a given quantile, QRF is much more flexible and as such can incorporate far more nonlinearities. This characteristic lends to the intuition that it will likely perform better than QVAR. The accuracy of the two quantile forecasting methods needs to be evaluated by means of a non-trivial loss function. This comes down to the fact that when forecasting at quantiles other than $q = 0.5$, the true value will not be representative of what the method wants to forecast. Rather, one would like to forecast a value that will over/under-estimate the true value with a certain probability and therefore traditional evaluation methods such as, for example, the Root Mean Squared Forecast Error (RMSFE) do not suffice. This paper makes use of the same asymmetric absolute value loss function used to estimate QVAR to evaluate the forecasts.

I find that government spending effects vary considerably across quantiles when analysing the impulse responses for $q = 0.1$, $q = 0.5$ and $q = 0.9$. Namely, output (unemployment) increases (decreases) considerably more (less) at $q = 0.1$ than at $q = 0.5$ and $q = 0.9$, in line with Keynesian economics. This finding is tested to see if it also holds up when correcting for the persistence of government spending shocks, this fact is evident when considering the maximum (minimum) point-to-point and cumulative ratio. When analysing the quantile forecasts it is clear that they follow and give a good indication of the true distribution of a conditional forecast. When I compare the QVAR and QRF forecasts I find that the QRF performs better, however, not enough to say they are significantly different at 5%.

The paper is structured such that it first examines and discusses previous literature investigating fiscal policy effects. After this, a description of the raw data and its sources is given together with an explanation of all the transformations applied. Following the data section, the methodology for the QVAR and QRF methods is provided together with a subsection on Impulse Response Functions. Also found in the methodology are subsections discussing the pseudo out-of-sample forecasting procedure and the forecast evaluation. After the methodology, I present the results of the thesis in two parts. First the results related to estimation of the fiscal policy effects on output and unemployment are presented, then the results relevant for the quantile forecasts for output are discussed. The paper is then concluded with a discussion of the findings and suggestions for further research.

2 Literature Review

Owing to the cyclical and inherently unstable nature of the economy, central banks play a key role in maintaining stable growth by mitigating swings in demand. One of the key tools available to central banks for stabilisation purposes is fiscal policy and as such there is a great deal of both empirical and theoretical literature focusing on its effects on the economy and how best to implement it. Early research into fiscal multipliers and the effects of fiscal policy shocks was led by Blanchard and Perotti (2002). In their 2002 paper they adopted the structural Vector Autoregression (VAR) framework to investigate the dynamics of GDP responses to shocks in both taxes and government spending. Blanchard and Perotti find results in line with standard Keynesian economics in the sense that when government spending increases, output increases and when taxes increase, output falls. Besides confirming the Keynesian model, Blanchard and Perotti find peak fiscal and tax shock multipliers to be small and close to 1.

As time has progressed more sophisticated methods have been incorporated to gain a better understanding

of the intricacies of fiscal policy and multipliers of government spending shocks. Specifically, Auerbach and Gorodnichenko (2012) extended the VAR framework of Blanchard and Perotti (2002) to incorporate regime switching through Smooth Transition Vector Autoregressive models (STVAR). These models allow for the possibility to be either in a recession or an expansionary setting and weights the two based on a moving average of GDP growth. This was one of the first specifications that allowed for the possibility of nonlinear multipliers and intended to investigate the variation in effects with respect to economic conditions. The economic intuition in testing this hypothesis lies in the Keynesian thought that government spending has a larger effect in times of recession as government spending is less likely to crowd out private spending in times of economic slack. As intuition suggested this specification held promise as Auerbach and Gorodnichenko found that fiscal multipliers were considerably greater in times of recession as opposed to expansion. They found fiscal multipliers to be around 1 in the linear model, similar to Blanchard and Perotti (2002), between 0 and 0.5 in times of expansion and between 1 and 1.5 in times of recession. Besides looking at peak multiplier values like Blanchard and Perotti (2002), Auerbach and Gorodnichenko also look into the ratio of the sum of output responses to the sum of government spending response. This cumulative ratio normalizes the output response with respect to the spending response and gives a better indication of the fiscal policy effectiveness under the two regimes. Here too, Auerbach and Gorodnichenko find fiscal policy to be considerably more effective in recession as opposed to expansion.

Since the introduction of nonlinearity by the STVAR model framework there have been a number of other methods used to model possible variations in the response to government spending. Auerbach and Gorodnichenko (2013) and Owyang et al. (2013) for example rely on local projection (Jorda, 2005) as opposed to STVAR. These have the advantage of relaxing the STVAR assumption that all shocks are perfectly unanticipated, which has been put off as an unrealistic assumption in Ramey (2011). Besides this, local projection models make the incorporation of nonlinearities in the model simple and easy to interpret. Using local projection, Auerbach and Gorodnichenko (2013) find similar results to their previous paper in the sense that fiscal multipliers are considerably higher in times of recession than in times of expansion. With their new methodology they find multipliers of around 1 in the linear model around 2 in recessions and around 0 in expansions with peak multipliers of 0.97, 0.2, and 3.48 in the linear model, expansionary and recessionary periods respectively. When applying the same technique of Jorda (2005), Owyang et al. (2013) find not only considerably lower multipliers but also that they do not significantly differ over business cycles. Specifically, their analysis results in peak multipliers of 0.88 in the linear model, 0.83 in times of high unemployment (recession) and 0.93 in times of low unemployment (expansion). There are a number of potential reasons for the differing results found in Auerbach and Gorodnichenko (2013) and Owyang et al. (2013), the most plausible of which is their distinct definitions for economic slack. Namely, where Auerbach and Gorodnichenko (2013) define economic slack as a considerable deviation from trend in output growth, Owyang et al. (2013) define slack to take place when the unemployment rate is above 6.5%. Which assumption is correct is far from a settled dispute and its discussion is beyond the scope of this thesis, however, it leads us nicely into the next step in this section.

In their 2016 paper, Linnemann and Winkler set out to improve upon the work by Auerbach and Gorodnichenko (2012) and Owyang et al. (2013) by applying the ideas of Quantile Regression as put forth by Koenker and Bassett (1978) in a VAR framework. The resulting QVARs hold significant advantages, as mentioned earlier in the introduction. They improve considerably on the linear specification by allowing for quantile dependent coefficients and they improve on strict regime switching models as they do not require a classification of the business cycle. As was discussed earlier this labeling of recessions or times of economic

slack can have a large impact on the results and arguably, both the measure based on the unemployment rate (Owyang et al., 2013) as well as the one based on the GDP growth (Auerbach & Gorodnichenko, 2013) provide a poor approximation of the level of slack in the economy. While they could be improved upon to comprise of more sophisticated recession prediction models in a forecasting setting this would not necessarily help as even these have difficulties in predicting recessions (Fernald & Trehan, 2006). These two arguments form the appeal of the Quantile Regression approach which allows for nonlinear investigation without having to specify regimes.

Despite the advantages, Linnemann and Winkler (2016) also find some drawbacks of Quantile Regressions in the context of fiscal policy multipliers. Specifically, whilst more correct in a misspecified setting the Quantile Regressions provide inference of a lower quality than the STVAR and local projection models in the case of correct model specification (good estimate for economic slack). This comes down to the lack of an exact functional form for the nonlinearity in the response of GDP and unemployment rate to spending shocks. Besides this, Linnemann and Winkler mention that quantile specific effects are difficult to incorporate into stabilisation policy owing to the theoretical need to know in which section of the conditional distribution the economy is in at a given time. This criticism, however, is becoming less warranted as forecasting techniques have advanced and conditional GDP distribution estimates have become more widely available. Specifically, there are a number of sources for estimates of the distribution such as, among others, the Blue Chip Economic Survey and the Federal Reserve Bank of Philadelphia’s Survey of Professional Forecasters. Additionally, there has been an increasing literary focus on forecasting the entire GDP distribution as seen in Adrian, Boyarchenko, and Giannone (2019). In addition to the concerns listed by Linnemann and Winkler, a point of weakness in the model lies in its failure to capture nonlinearities other than through varying the quantile the model is estimated at. This is likely not mentioned as further forms of nonlinearity have not been researched as in depth as those caused by business cycles.

While intuition suggests that QVARs can provide good forecasts it is important to test their accuracy in a real world setting and up against other candidates. A method that holds considerable promise is Quantile Regression Forests (QRF), as developed by Meinshausen (2006). This methodology extends the random forest estimation developed in Breiman (2001) to be useful in estimation at points other than the conditional mean. QRFs, like QVARs, have the appeal of being able to incorporate nonlinearity without relying on a regime classification. However, it sets itself apart from QVAR in its ability to address the QVARs weakness in modeling for other potential nonlinearities. Specifically, the QRF’s specification allows it to consider a wide range of nonlinearity only limited by the explanatory variables it is built upon. While this is a considerable benefit it also comes with the drawback of more difficult interpretation, as no parameter estimates are retrieved. This however, does not take away from its potential in a forecast setting and this is the main reason for its consideration in answering the research question of how best to apply Quantile Regression for forecasting purposes.

3 Data

The main goal of this paper is to investigate the effects of fiscal policy on the economic measures of GDP and the unemployment rate at different quantiles of their conditional distribution. In order to conduct this analysis there are a number of data series that are made use of. This section discusses where raw data is obtained from, as well as its derivations and transformations.

In this thesis I make use of quarterly US data from the 1st of January 1970 to the 1st of October

2019, which comprises the longest period for which all the relevant variables are available. The study will estimate fiscal policy effects on the GDP and the unemployment rate and as such the first point of order is obtaining these two data sets. The GDP, in billions of chained 2012 USD, is obtained from the U.S. Bureau of Economic Analysis (BEA) and the unemployment rate is obtained from the U.S. Bureau of Labor Statistics (BLS). Secondly, I obtain the main explanatory variable, government spending, as real government consumption expenditures and gross investment also from the BEA. In addition to this a number of control variables will also be included in the analysis. These are, as in Linnemann and Winkler (2016), real net taxes, government debt held by the public over GDP and the short run real interest rate. Government debt held by the public over GDP is directly retrievable from the Federal Reserve Bank of St. Louis, while the other two are derived from series provided either by BEA or from the Board of Governors of the Federal Reserve (BFED). Specifically, real net taxes are calculated as the government current tax receipts plus contributions for government social insurance minus government current transfer payments all over the GDP deflator, which is indexed to 2012. Short run real interest rate is calculated as the federal funds rate minus the annualized log change in the GDP deflator. Now that the required data series have been obtained, I apply log quadratic detrending to all the variables except the real interest rate and the unemployment rate. This transformation seems to be a question of preference as there is both research that applies quadratic detrending (see for example Blanchard and Perotti (2002)) and research that makes use of a simple log transformation (see for example Auerbach and Gorodnichenko (2012)). In this thesis I make use of it in order to obtain results as comparable as possible to Linnemann and Winkler (2016).

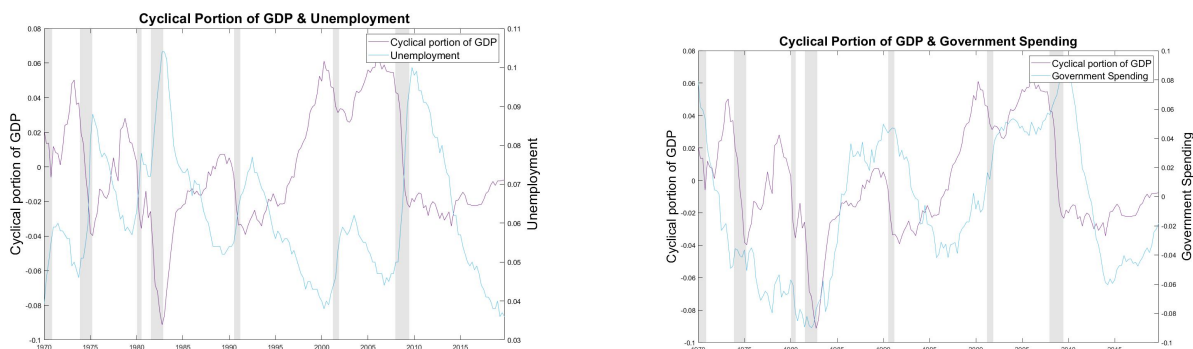


Figure 1: Plots of the cyclical component of the log of Real GDP and the unemployment rate (left) as well as of the cyclical components of the log of real GDP and log of government spending (right). Grey bars represent National Bureau of Economic Research (NBER) recessions.

In figure 1, in the graph on the left hand side, the two response variables of interest are plotted, namely, the GDP and the unemployment rate. Their negative correlation is clear to see as when the GDP falls, it is accompanied with a rise in unemployment. The grey bars represent the NBER classified recessions and one can see that they are accompanied with considerable GDP falls and large increases in unemployment. In the graph on the right plotting the GDP and the government spending it is clear that they move together in some sense with government spending generally following output, however, there are also some clear cases of active fiscal policy likely aimed at stimulating the economy. Two such examples are found in high spending during both the early 90s recession as well as the 2008 great financial crisis.

4 Methodology

For an analysis into the nonlinearity of fiscal policy effects and forecasting quantiles of the conditional output distribution, this paper makes use of two methods based off of Quantile Regression as developed in Koenker and Bassett (1978). To be specific I make use of Quantile Vector Autoregressions (QVAR) (Cecchetti & Li, 2008) and Quantile Regression Forests (QRF) (Meinshausen, 2006). To make the methodology as clear as possible to the reader, I first introduce basic Quantile Regressions, then explain how these are extended to get a QVAR setting, before moving off on a tangent to formulate Quantile Regression Forests. After the QVAR and QRF estimation procedure has been covered, the methodology behind Impulse Response Functions and pseudo out-of-sample forecasting is presented. Finally, an explanation of the methodology used to compare the quantile forecasts generated by QVAR and QRF is provided.

4.1 Quantile Regressions

Quantile Regressions make up the generalized form of the Least Absolute Error (LAE) estimator, which has historically been used as a more robust alternative to the standard Least Squares (LS) regression in terms of dealing with outliers. However, where the standard LS and LAE estimators are occupied with modeling the relationship between y_t and a $k \times 1$ vector of explanatory variable x_t , at the conditional mean of y_t , Quantile Regressions allow for a more thorough analysis by considering effects at points throughout the conditional distribution. Thus as opposed to having $E(y_t|x_t) = x_t'\beta$, where the parameter estimates given by β are constant throughout the y_t distribution, one can make use of the quantile function below (Bassett & Koenker, 1982):

$$Q_q(y_t|x_t) = x_t'\beta(q) \quad (1)$$

The quantile function gives an estimate for y_t dependent on a vector x_t given a quantile q . This notation allows for the parameters $\beta(q)$ to differ depending on the quantile of the distribution the estimator is calculated for. In order to see how these $\beta(q)$ s are obtained consider first a general estimator given by minimizing an arbitrary loss function:

$$\hat{\beta} = \arg \min_{\beta} \sum_{t=1}^T \rho(\epsilon_t(\beta)) \quad (2)$$

in this specification ρ_q denotes a weighting function and $\epsilon_t(\beta)$ is the error term or the deviation of the fitted value with parameter β from the true value. The Ordinary Least Squares (OLS) estimator is now given simply by using a quadratic weighting function, that is $\rho(\epsilon_t) = \epsilon_t^2$. However, if one were to instead use a tilted or asymmetric absolute value function to weight the errors, then one obtains the Quantile Regression estimator. As such the weighting function ρ_q is specified by:

$$\rho_q(\epsilon_t) = \begin{cases} q|\epsilon_t| & \text{for } \epsilon_t > 0 \\ (1-q)|\epsilon_t| & \text{for } \epsilon_t \leq 0 \end{cases} \quad (3)$$

This weighting function results in a penalty of q for positive errors and $(1-q)$ for negative errors. This means that for different values of q the estimated $\hat{\beta}(q)$ changes and the probability of y_t being greater than or smaller to $x_t'\hat{\beta}(q)$ changes with it. For the special case of $q = 0.5$, we obtain the LAE estimator that was discussed

earlier. The LAE estimator is mean centered as the negative deviations are weighted the same as the positive ones and thereby the probability of y_t being less than $x'_t \hat{\beta}(q)$ is equivalent to the reverse. However, if, for example, $q = 0.9$ then the positive deviations are weighted 9 times what the negative deviations are and the probability of y_t being less than $x'_t \hat{\beta}(q)$ is 9 times as big as the reverse.

By specifying different quantiles q in the minimization problem above, one can obtain information about how the explanatory variables affect the dependent variable y_t , at parts of the conditional distribution other than the mean. In the context of the research question, this allows us to check the effects of fiscal policy along different parts of the conditional distributions of both GDP and the unemployment rate, thereby telling us if the effects vary a lot or are relatively constant.

With the loss function in equation 2 and the weighting function in equation 3 we can specify the quantile specific parameter estimate as:

$$\hat{\beta}(q) = \arg \min_{\beta(q)} \sum_t \rho_q[y_t - x'_t(\beta(q))] \quad (4)$$

Equation 4 above may be used to provide a simple analysis of the nonlinearity of a factor such as government spending, however, it is much more interesting to look at this in a multiple series environment such as VAR. This is due to the dynamic relationship between spending shocks and GDP/unemployment rate and a number of other variables. This brings us on to the Quantile Vector Autoregression (QVAR).

4.2 Quantile Vector Autoregression

In their paper investigating the impact of asset price booms, Cecchetti and Li (2008) extend Koenker and Bassett's Quantile Regression framework to allow them to work with vector autoregressions. This melange of methodology is an excellent fit for fiscal policy as it allows for an analysis of interactions over a longer period of time as well as of effects at different parts of the conditional distribution.

For the general QVAR case we apply Quantile Regression to a p^{th} order VAR model encompassing k relevant variables. That is we have a vector y_t of k relevant variables given by $y_t = (y_{1t}, y_{2t}, \dots, y_{kt})'$ and a vector of quantiles given by $q = (q_1, q_2, \dots, q_k)'$ postulating the quantile at which each of the k variables will be evaluated at. Hence, the general QVAR model is given by:

$$Q_q(y_t | y_{t-1}, \dots, y_{t-p}) = c(q) + \sum_{i=1}^p B_i(q) y_{t-i} + \epsilon_t(q) \quad (5)$$

In this setting $B_i(q)$ is a $k \times k$ matrix containing the effects of the i^{th} lag of the k variables on the quantiles of the conditional distribution of y_t , where the quantiles at which the variables are evaluated at are specified by the vector q . $c(q)$ is a $k \times 1$ vector of constants and $\epsilon_t(q)$ is a $k \times 1$ vector of errors. Specifically, they contain the following:

$$B_i(q) = \begin{bmatrix} \beta_{i,11}(q_1) & \dots & \beta_{i,1k}(q_1) \\ \beta_{i,21}(q_2) & \dots & \beta_{i,2k}(q_2) \\ \vdots & & \vdots \\ \beta_{i,k1}(q_k) & \dots & \beta_{i,kk}(q_k) \end{bmatrix}, \quad c(q) = \begin{bmatrix} c_1(q_1) \\ c_2(q_2) \\ \vdots \\ c_k(q_k) \end{bmatrix}, \quad \epsilon_t(q) = \begin{bmatrix} \epsilon_{1t}(q_1) \\ \epsilon_{2t}(q_2) \\ \vdots \\ \epsilon_{kt}(q_k) \end{bmatrix}$$

As can be seen from this specification the parameter estimates for each variable are allowed to vary with the quantile that is evaluated, thereby, fulfilling the goal of utilizing this methodology.

With the general QVAR given by equation 5 there are a number of decisions needed to make the frame-

work specific to our case of analysing the nonlinear effects of government spending. In this thesis I follow Linnemann and Winkler (2016) in terms of both variable and lag selection. As such, I perform the QVAR for GDP, re-estimating for the different q vectors, and for the unemployment rate separately. That is when estimating the model I only include either of the economic indicators at any given time. Besides the variable of interest, I make use of the same four other control variables as Linnemann and Winkler (2016) bringing the number of variables that enter the QVAR to $k = 5$. In addition to the variables included in the QVAR the lag length also needs to be specified. This is set to $p = 4$ which is in line with previous literature such as Blanchard and Perotti (2002) as well as Linnemann and Winkler (2016). Finally, the quantiles at which the QVAR will be evaluated at, needs to be specified. For this any arbitrary list of quantiles could be decided upon, however, for ease of interpretation for the reader and ease of comparison with Linnemann and Winkler (2016), I make use of 0.1, 0.5 and 0.9. Hence, the three 5×1 q vectors used in this paper are

$$q_{0.1} = \begin{bmatrix} 0.1 \\ 0.5 \\ \vdots \\ 0.5 \end{bmatrix}, \quad q_{0.5} = \begin{bmatrix} 0.5 \\ 0.5 \\ \vdots \\ 0.5 \end{bmatrix}, \quad q_{0.9} = \begin{bmatrix} 0.9 \\ 0.5 \\ \vdots \\ 0.5 \end{bmatrix}$$

where the first term in the vector refers to the economic indicator which is either GDP or the unemployment rate and the other four terms are the control variables which are all evaluated at their mean.

4.3 Quantile Regression Forests

The second quantile approach that I pursue in this paper is that of Quantile Regression Forests (QRF), as put forth in Meinshausen (2006). The methodology behind this approach is presented in three parts. Firstly, I demonstrate the founding methodology, namely, regression trees. Then I show how these are used in the setting of Random Forests before I extend this concept further by looking at Quantile Regression Forests. QRF can be applied generally and not necessarily in a time series setting. In the below formulations I denote the methodology for forecasting purposes. Specifically, it is written to provide a one step ahead forecast given a set of T previously observed points. That is we wish to obtain \hat{y}_{T+1} relying on explanatory variable vector x_{T+1} , in this case as we work with lagged variables x_{T+1} can be considered as information all known at time T . Further information regarding the out of sample forecasting can be found in subsection 4.5.

4.3.1 Regression Trees

Regression trees are a type of decision tree that splits the predictor space into subsets of the possible x . These subsets are then used to provide a prediction for a given observation vector x_{T+1} by evaluating logical expressions at multiple nodes or split points in the tree. Once, x_{T+1} has passed through the entire tree, it necessarily ends at a so called leaf node where the prediction for y_{T+1} is provided. This prediction is generally provided by the mean or average of the training data that also reached that particular leaf node. There are a number of ways or rules one can follow to construct a regression tree, however, the basics involve minimizing the squared residuals given a split in one of the predictor variables. Specifically, a tree is created by choosing a number of predictor variables to treat as candidates for a split. For each candidate, all possible split points will be evaluated by use of the training data and the chosen split or decision is that which leads to the greatest improvement in the sum of squared residuals. Once a split has been made the process is repeated in order to create more splits until a stopping criterion is met. Some typical decisions left to be

made include, whether to include all predictor variables as candidates at every split point or whether one should instead use a subset, and potentially how that subset is decided upon. Furthermore, a decision has to be made when to stop splitting or in other words what is an acceptable minimum number of training observations that fall into a given node. This is an important step as it will help avoid overfitting the trees.

4.3.2 Random Forests

Random forests as developed by Breiman (2001) is a regression tree based prediction method. It relies on growing a large number of regression trees from bootstrapped or 'bagged' samples taken from a set of size T containing previously observed observations (y_t, x_t) for $t \in T$. The trademark of the random forest approach is the fact that the node split decision rule is randomized. Specifically, at every node in the tree only a subset of randomly chosen predictor variables are selected as possible candidates to split over. This subset size is a so called hyperparameter that the researcher can fine tune by making use of 'out-of-bag' samples, which will be covered in more detail later in the methodology. Once this forest, built out of a large number of individual regression trees, has been grown it is used to develop a weighted average of the observed response variables that will serve as the prediction for a given x_{T+1} .

Now that the intuition behind the Random Forests procedure has been covered the process is made more rigorous by introducing some notation. This notation will serve as the framework for the mathematical formulation of predictions from single trees and from forests of trees. Let a tree be defined by $T(\theta)$ where θ represents a random vector that specifies the splits at the different nodes of the tree. With the tree built it is useful to define the possible x . If x is made up of k predictor variables then the possible x s is made up of some subspace of the vector space \mathbb{R}^k . Specifically, I define this subspace containing all possible x as $B \subseteq \mathbb{R}^k$. Once the space of possible x s has been defined it is easy to see that each leaf l of the tree $T(\theta)$ needs to be a subspace of B as any x should be attributable to a leaf. Define this leaf specific subspace of B as R_l . At this point one can define the tree specific leaf of a given x value as $l(x, \theta)$ which is the leaf that is obtained when evaluating x in a tree $T(\theta)$.

Now one can see the weight function used in single tree predictions as:

$$w_t(x_{T+1}, \theta) = \frac{I_{x_t \in R_{l(x_{T+1}, \theta)}}}{\sum_{j=1}^T I_{x_j \in R_{l(x_{T+1}, \theta)}}} \quad (6)$$

where, I represents the indicator function which takes value 1 when an x_t or x_j , from the T in-sample observations, falls in the same leaf as x_{T+1} for a tree $T(\theta)$. $w_t(x_{T+1}, \theta)$ can be seen as the weight given to each observation in the in-sample data set, where its vector of explanatory variables x_t fell in the same leaf node as x_{T+1} .

The prediction of a single tree $T(\theta)$ is a weighted average of the original observations y_t for $t \in T$ given by:

$$\hat{u}(x_{T+1}) = \sum_{t \in T} w_t(x_{T+1}, \theta) y_t \quad (7)$$

The single tree estimation is simply extended to the random forest estimation by averaging the prediction of the n trees $(T(\theta_1), \dots, T(\theta_n))$ in the forest. As such the weights can be interpreted as the average weight a given observation had over all the trees in a forest for a given x_{T+1} . Specifically, they can be written as:

$$\bar{w}_t(x_{T+1}) = \frac{1}{n} \sum_{j=1}^n w_t(x_{T+1}, \theta_j) \quad (8)$$

to define the random forest estimation.

$$\hat{u}(x_{T+1}) = \sum_{t \in T} \bar{w}_t(x_{T+1}) y_t \quad (9)$$

It is now possible to see that the random forest prediction is a weighted average of the T observations. This is shown as the $\sum_{t \in T} w_t(x_{T+1}, \theta_j) = 1$ owing to the specification in 6, which then leads to $\sum_{t \in T} \bar{w}_t(x_{T+1}) = 1$. The weights give an indication of how much an observation is used in the individual tree predictions relative to other observations for a given x_{T+1} . This means that the value y_t has a relatively high weight in determining y_{T+1} when the conditional distribution of y_t is similar for x_t and x_{T+1} .

4.3.3 Quantile Regression Forests

Now that the theory behind random forests has been introduced I generalize the procedure's estimation of the conditional mean by using an algorithm developed by Meinshausen (2006). In order to do this it is good to first define what we wish to obtain using the random forest framework from the previous section. Meinshausen (2006) makes use of the conditional distribution to obtain estimates for the conditional quantiles and as such we need to adapt the Random Forest method above to estimate the conditional distribution function. The conditional distribution function can be written as:

$$F(Y|X = x) = P(Y \leq y|X = x) = E(I_{Y \leq y}|X = x) \quad (10)$$

Notice that when written as a conditional expectation the conditional distribution function takes a similar form to the conditional mean as found by standard Random Forest specification. To be specific, they are $E(I_{Y \leq y}|X = x)$ and $E(Y|X = x)$ respectively. Now the Random Forest methodology needs to be transformed to fit the expectation in equation 10. This is a fairly simple task as one can simply replace the y_t in equation 9 with the indicator function $I_{y \leq Y}$. Thereby giving an estimator for the conditional distribution function:

$$\hat{F}(y|x_{T+1}) = \sum_{t=1}^T \bar{w}_t(x) I_{Y_t \leq y} \quad (11)$$

The algorithm used to obtain the above estimator for the conditional distribution is given in Appendix A.

Once the conditional distribution has been estimated make use of it to obtain estimates of the conditional quantiles $Q_q(y|x_{t+1})$. There are numerous ways in which to do this, however, in this paper I follow the framework of Meinshausen (2006) and make use of:

$$Q_q(y|x) = \inf\{y : \hat{F}(y|x) \geq q\} \quad (12)$$

In the above equation \inf represents the infimum of the set of y that satisfies $F(y|x) \geq q$, hence giving the largest value that is less than or equal to all the values in the set.

For estimation of the Quantile Regression Forest I make use of the package `quantregForest` (Meinshausen, 2017) in R (Team, 2017) and tune the hyperparameter `mtry` by help of the package `caret` (Kuhn, 2020).

4.4 Impulse Response Functions

VAR models are largely superior to univariate time series models, owing to their ability to incorporate more information on variables closely related to the research interest. However, the interpretation of VAR

models is considerably more challenging owing to the endogeneity problem arising from the interrelationship of the variables in the system. As the system is modeled in a dynamic manner one cannot interpret the coefficients as one would with a simple linear regression and as such a causal analysis is considerably more difficult in a VAR setting. Impulse response functions (IRFs) provide a good way to conduct causal analysis, by tracking how variables react to an exogenous shock in the system and thereby giving an indication of how one variable 'responds' to an 'impulse' in another. In the context of the research question this allows us to investigate how a spending shock is set to affect either GDP or the unemployment rate over time, while taking into consideration the dynamic relationship with the control variables. There are a number of steps involved in tracing out the IRFS and these will be outlined below with a specific focus on QVAR. Firstly, one needs to obtain the QVAR coefficients and residuals from the model by applying the methodology in section 4.2. The residuals are used to find how shocks relate across equations, information crucial to shock the system with an exogenous or uncorrelated innovation. As the covariance matrix of the residuals is most often not diagonal there is evidence of correlation between the error terms. This suggests that variable shocks often accompany one another and as such if one introduces a unit shock to a variable it is unrealistic to set all the other residuals or shocks to zero as intuition would suggest. This means that we need some form of transformation, and this is given by the Cholesky Decomposition. In performing the Cholesky decomposition it is important to consider the order of the variables as it imposes a recursive causal structure. Generally, economic intuition is used to obtain an ordering of most exogenous to most endogenous. In this thesis I make use of the order $(g_t, y_t, \tau_t, r_t, d_t)$ for the output analysis and $(g_t, u_t, \tau_t, r_t, d_t)$ when looking into effects on unemployment. These orderings are the same as those utilized in Linnemann and Winkler (2016). The ordering is based on the assumption that the effects of change in government spending are not felt before the quarter after they take place. This assumption has been used widely since its introduction in Blanchard and Perotti (2002), but it has also seen some resistance. Most notably in Ramey (2011), which argues that treating unexpected and expected changes in fiscal policy equally will taint the results. However, Linnemann and Winkler (2016) find that correcting for fiscal foresight does not affect their results and as such this thesis does not test the consequences of this assumption.

4.5 Pseudo Out-of-Sample Forecasting

For the out-of-sample forecast analysis I made the decision to use a 60-40 split in the data, where 60% or 30 years of data is used to train the models and 40% or 20 years is used to evaluate the models. This allows us to compare the models under similar circumstances to a realistic scenario, namely using past data to forecast future, yet to be observed effects of fiscal policy. Besides the 60-40 split decision I also settled on using a rolling window approach. This means that the training data is updated for each new quarter and includes all the 30 years of data leading up to the point we wish to forecast. This has the advantage of allowing for structural changes over time which in turn helps improve forecasting accuracy. In this thesis the estimation sample is denoted by T and changes depending on what forecast is being made. Specifically, for the first forecast, namely the first quarter of 2000, T consists of $t = 1, \dots, 120$ and this moves by one as the forecast we wish to make increases in index. That is T consists of $t = 2, \dots, 121$ for the second quarter and so on. Besides T we also have S which represents the out-of-sample forecasts that is used for forecast evaluation. Unlike T this does not change and consists of $t = 121, \dots, 200$.

4.6 Evaluation of Forecasts

In order to answer the second part of my research question, namely how to best apply Quantile Regression methodology for forecasting purposes, I evaluate the forecasts of the two methods by means of accuracy. The evaluation of conditional quantile forecasts is less trivial than forecasts of the conditional mean. This is due to the fact that in essence one hopes to overestimate or underestimate the true value with a certain probability depending on the quantile used. For this reason it does not make sense to use, for example, mean squared forecast errors. Instead, one can follow in the steps of Meinshausen (2006) and make use of the loss function specified in equation 3. Specifically, I will compare the mean of these weighted absolute deviations over a forecast sample for a number of q values. This mean loss can be calculated as follows:

$$\bar{L}_q = \frac{1}{S} \sum_{t \in S} \rho_q(\epsilon_{qt}) \quad (13)$$

Where S represents the set of observations in the pseudo out-of-sample, $\rho_q(\epsilon_t)$ is as defined in equation 3 and ϵ_{qt} is the forecast error at time t estimated with quantile q . This is a much better measure to evaluate the forecasts as the loss is minimal for the actual conditional quantile function.

In order to evaluate the significance of a potential difference in mean loss of the QVAR forecasts and the QRF forecasts I will, as in Meinshausen (2006), rely on bootstrapping. Specifically, I make use of the forecast sample to calculate the mean loss for both the QVAR and the QRF. Then, I bootstrap a 95% confidence interval for the QVAR mean loss. This is done by resampling and calculating the mean loss of the bootstrapped sample 5000 times. Thereafter, the means are ordered and the 125th and 4875th means in the ordered list are provided, representing the lower and upper bounds of the 95% confidence interval respectively. I then check to see if there is a significant difference in the performance of QRF by analysing whether its mean loss falls within or outside the confidence interval. In order to bootstrap the 95% confidence interval for the mean loss of the QVAR forecasts I make use of the boot package in R (Ripley, 2020).

5 Results

In this section I will discuss the results of the investigation and answer the research questions from the introduction. The section is structured such that the investigation into the nonlinear effects is presented before the section looking into the application of Quantile Regression for forecasting purposes. The subsections tackling the nonlinear effects of fiscal policy are structured in such a way that they first present the results of this paper and then compare them to the findings of Linnemann and Winkler (2016), which provided the inspiration for this paper.

5.1 Fiscal Policy Effects on Output

For the analysis of fiscal policy effects I begin by analysing how output responds to changes in government spending at different parts of its distribution. Specifically, the effects are studied at $q = (0.1, 0.5, 0.9)$ and as such represent how output is set to change at three different parts of its distribution. Namely, when $q = 0.1$ output is in its lowest 10%, when $q = 0.5$ output is at its mean and when $q = 0.9$ output is in its highest 10%. These different quantiles give an indication of the business cycle as high levels of output are generally associated with expansions and the opposite with recessions. Figure 2 plots the fitted values of the QVAR at the 10th and 90th percentiles together with the actual values. Given that the QVAR makes use of only

lagged variables these can be seen almost as one-step-ahead forecasts, however, the QVARs are estimated for the entire sample and as such the forecasts make use of data not available at that time. Nevertheless, it gives a good indication of what the 10th and 90th QVARs represent. In analysing figure 2, one of the main takeaways is that when output increases considerably the actual values are near the quantile estimate for the 90th percentile and when output decreases drastically it is very near the quantile estimate for the 10th percentile. This is particularly clear when considering the 1991-2001 expansion and the 2007-2009 Great Recession. This shows the additional value of conditional quantile estimates as they give a better estimate than would an unconditional method at these times of big increase or decrease in output. When looking at how the QVAR fitted values shown in figure 2 compare to those obtained in Linnemann and Winkler (2016) it is easy to see that they resemble considerably. Whilst, Linnemann and Winkler evaluate a different time period from this thesis their findings are very similar as they make the observation that true values fall short of median forecasts in times of low growth and exceed them in times of high growth. For aid in visualizing this exact observation I include figure 10 in appendix A.

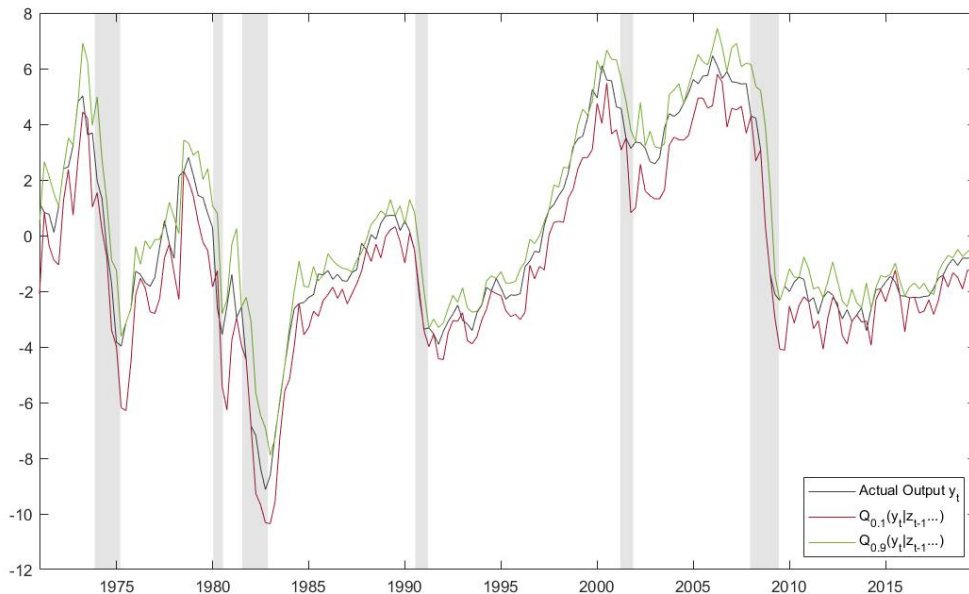


Figure 2: Actual output versus QVAR fitted values at 10th and 90th percentiles.

I now move on to interpret the meaning of the QVAR estimates in an effect context. Interpretation of VAR coefficients is difficult due to the interrelation between the variables and as such, as explained in the methodology, I make use of impulse response functions (IRFs). In the figure below (figure 3) are the impulse responses of output and government spending when faced with a 1% shock in government spending. From the graphs in the first row depicting the reaction of output, it is clear to see that the response is quantile dependent. Specifically, we see a peak increase of 0.65%, 0.24% and 0.32% in output at the quantiles $q = 0.1$, $q = 0.5$ and $q = 0.9$ respectively. Showing that fiscal policy is considerably more effective at lower points in the conditional distribution of output. It is clear that for all three of the investigated quantiles there is an initial impact shock of between 0.1% for $q = 0.1$ and 0.3% for $q = 0.9$, however, the responses begin to diverge as time passes. Specifically, the graphs show that fiscal policy responses are positive throughout the

5 years for $q = 0.1$ and $q = 0.5$, albeit at $q = 0.5$ the response tends to 0 fairly quickly. Furthermore, one can see that responses to government spending at the top 10% turn negative after the impact shock, showing that the increase in spending is not strong enough to prevent the mean recursion tendency of the output.

Besides showing the responses of output, figure 3 also shows the response of government spending, and thus gives an indication of how persistent government spending shocks are. For example at $q = 0.1$ there is a brief increasing trend before the response slowly falls toward 0 reaching 0.62% after 5 years. This shows that an increase in government spending is generally persistent over a number of periods. This is also the case for $q = 0.5$ and $q = 0.9$, however, they tend to approach 0 faster. This observation begs the question of whether the larger increase in output at $q = 0.1$ is due to the first shock taking place in a lower employment environment or if it is due to the fact that the likelihood of additional shocks is greater at this quantile.

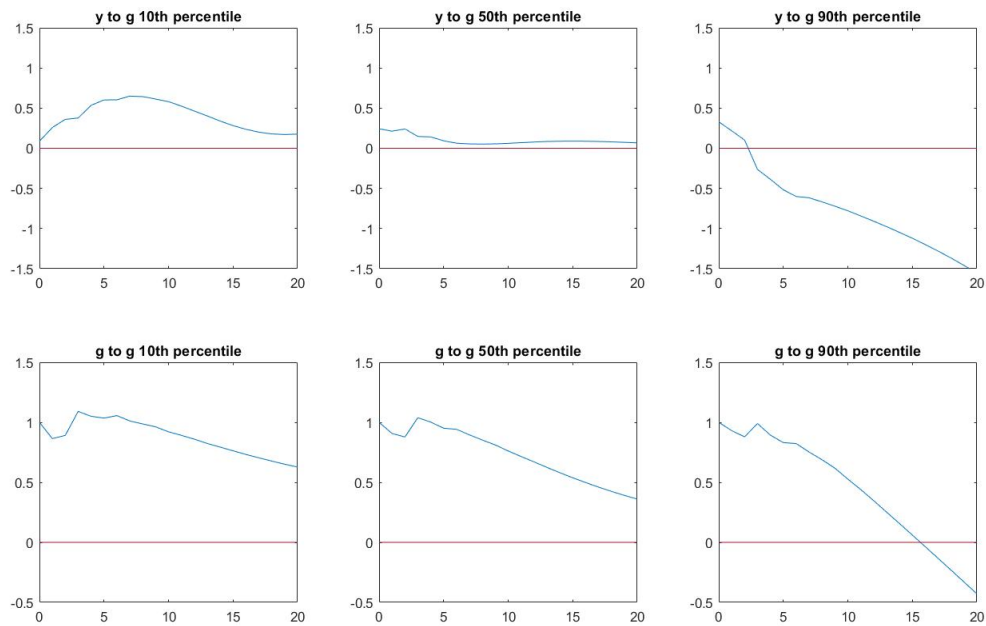


Figure 3: Impulse responses at $q = 0.1$, $q = 0.5$, $q = 0.9$ quantiles of the output distribution.

In order to investigate this hypothesis I make use of the maximum point-to-point ratio and the cumulative ratio as defined in Linnemann and Winkler (2016). These give an indication of the response of output after having normalized it for government spending. The maximum point-to-point ratio is the maximum value of the ratio of output response to government spending response over the 3 year period following the initial shock. As such it is calculated as $MR(q) = \max_{h \in \{0, \dots, 12\}} \hat{y}_h(q) / \hat{g}_h(q)$. The cumulative ratio takes a similar approach, however, it takes the ratio between the summed responses and is thus calculated as $CR(q) = \sum_{h=0, \dots, 12} \hat{y}_h(q) / \sum_{h=0, \dots, 12} \hat{g}_h(q)$. The maximum point-to-point ratio and the cumulative ratio are plotted below for quantiles ranging from 0.025 and 0.975 at intervals of 0.025. The decreasing trend in the two ratios as the quantile is increased is obvious to see and suggests that the effectiveness of fiscal policy decreases with higher levels of conditional output even when taking into consideration persistence of government spending shocks. The max point-to-point ratio suggests that fiscal policy effectiveness decreases fast in quantiles up until $q = 0.4$ before it levels off. However, the cumulative ratio shows that this fall off in effectiveness continues past $q = 0.4$ and one can see that the cumulative ratio even turns negative around

$q = 0.6$. The reason in the discrepancy between the showings of these two ratios is that the impact effect of the government spending shock is fairly consistent across quantiles. What these ratios do agree upon though is that there is considerable nonlinearity in the responses over the quantiles and that in general, responses are much larger in times of recession or depressed output.

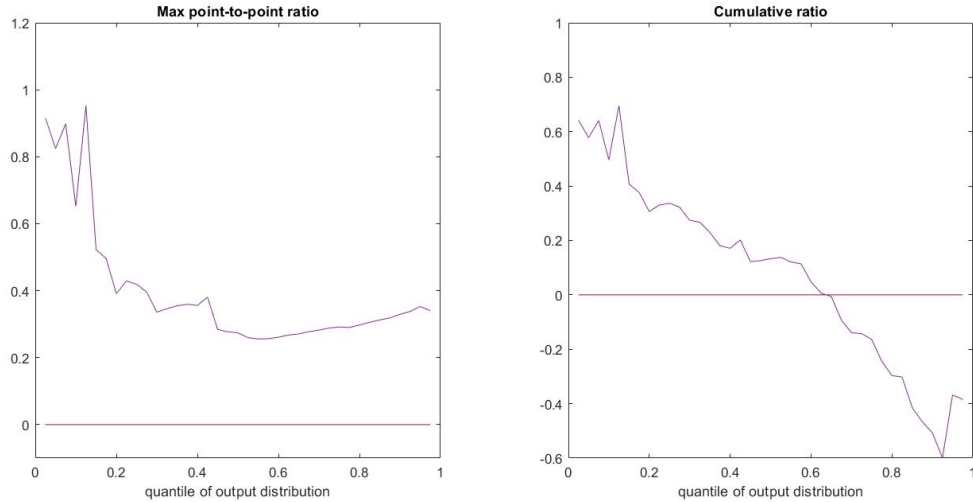


Figure 4: Maximum point-to-point and cumulative ratio over different quantiles of the output distribution.

The results obtained above looking at the nonlinearity of fiscal policy effects are for the most part in line with previous research. The main benchmark for the study is as previously explained Linnemann and Winkler (2016) and as such, it is natural to explain similarities and contrasts in our findings. When looking at the impulse response functions for the lowest decile or $q = 0.1$ one can see that the output effect peaks between seven and eight quarters after the shock, very close to Linnemann and Winkler who find the peak to be between seven and nine quarters after the shock. In this paper the peak output response is obtained as 0.65% which is lower than what Linnemann and Winkler find, however, it is only marginally smaller and still falls within their bootstrapped 90% confidence interval. In terms of the government spending response there is more contrast as where Linnemann and Winkler find that government spending response increases fairly consistently over the five years I find that it increases only for the first six quarters before tending towards zero. Hence, their results suggest that government spending shocks are more persistent than what we find. At the median or $q = 0.5$ the findings are very similar for both the output and the government spending responses both in terms of level and pattern in the response over time. At the highest 10% of conditional output, that is $q = 0.9$, there is once more considerable differences between my findings and those of Linnemann and Winkler (2016). While the findings are consistent for the first three quarters they diverge thereafter. Specifically, the impulse responses I obtain become increasingly more negative whilst in Linnemann and Winkler (2016) they simply tend to zero and hover there for five years. In terms of the government spending response, the pattern is more similar as the response is seen tending to zero over time. However, in my findings this approach to zero is faster than in Linnemann and Winkler (2016) and after four years my response falls below zero while in Linnemann and Winkler (2016) the response stays positive throughout the five year period. Overall, the nonlinearity patterns are similar as multipliers and government spending persistence are greater at lower levels of output than at high levels. However, at high levels of output I find both output and government spending to drop faster and further after the impact effect. These

contrasts in findings are most likely due to different sample periods as where (Linnemann & Winkler, 2016) base their analysis on a sample spanning from 1955Q1 to 2013Q4 this thesis makes use of a more up to date sample from 1970Q1 to 2019Q4. As such the the second longest expansion in NBER history running from 1961 to 1969, fueled by expansive fiscal policy resulting from the Vietnam War effort, is not included in my analysis. This exclusion is by no means a hindrance, however it is important to note.

5.2 Fiscal Policy Effects on Unemployment

In the previous section the nonlinearity embedded in output response to government spending is analysed. In this section this will be extended to the unemployment rate so as to investigate if the nonlinearity is specific to output or more general to characteristics of the business cycle. The structure of the analysis is similar to the previous section as the behaviour of the QVARs are examined first, before the effects are studied through the IRFs and the minimum point-to-point and cumulative ratios.

In figure 5 below, results are similar to those in the case of output. Namely, in times of increasing (decreasing) unemployment the true value is considerably closer to the QVAR estimated at the 90th (10th) percentile. This is very evident during and right at the back of recessions and provides more information on the relationship between government spending and the unemployment rate than an unconditional estimation method.

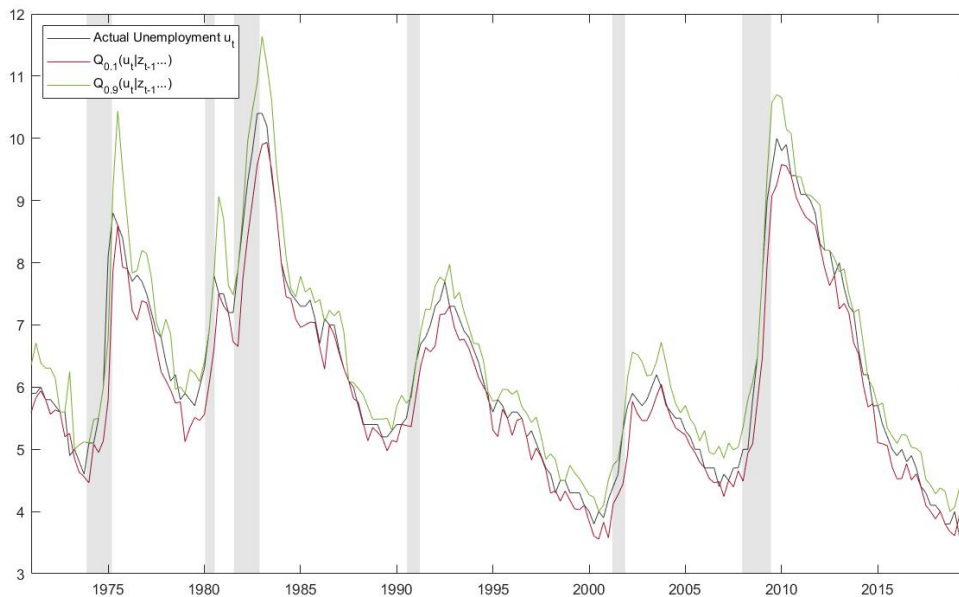


Figure 5: Actual unemployment versus QVAR fitted values at 10th and 90th percentiles.

Now I consider the Impulse Response Functions (IRFs) of unemployment and government spending, assuming a 1% orthogonal shock in government spending. The resulting IRFs are plotted in the graphs in figure 6. In general the resulting IRFs are in line with Keynesian economics in the sense that an increase in government spending leads to a decrease in unemployment. However, the magnitude of this decrease is considerably different across quantiles of the conditional unemployment distribution. When unemployment is in its lowest 10%, there is an initial decrease of unemployment which bottoms at approximately -0.15%

around the 3rd or 4th quarter after the spending shock, before slowly increasing to approximately 0. When unemployment is at its mean, or $q = 0.5$, there is a similar pattern as in the lowest decile, however, in this case the lower unemployment is persistent and approaches -0.05% towards the end of the five year observation period. In the cases where unemployment is at its highest 10%, the impact of expansionary fiscal policy is considerably more pronounced. Specifically, the unemployment rate continues to fall up to between twelve and fourteen quarters after the shock, at which point the unemployment rate is around 0.41% lower than prior to the shock. This is a considerable amount and really showcases the increased efficacy of fiscal policy in times of market distress.

The aforementioned results are generally in line with those of Linnemann and Winkler, as the effect of government spending becomes more pronounced at higher quantiles of the conditional distribution of unemployment. However, across the distribution this paper finds that unemployment decreases by a greater magnitude. Specifically, whereas Linnemann and Winkler (2016) find a max decrease of approximately -0.05% , -0.12% and -0.25% at $q = 0.1$, $q = 0.5$ and $q = 0.9$ respectively, this paper finds an effect almost twice as big at the two extreme quantiles ($q = 0.1$ and $q = 0.9$). The reason for this discrepancy is once again likely data related, however, specific event driven cause is less obvious.

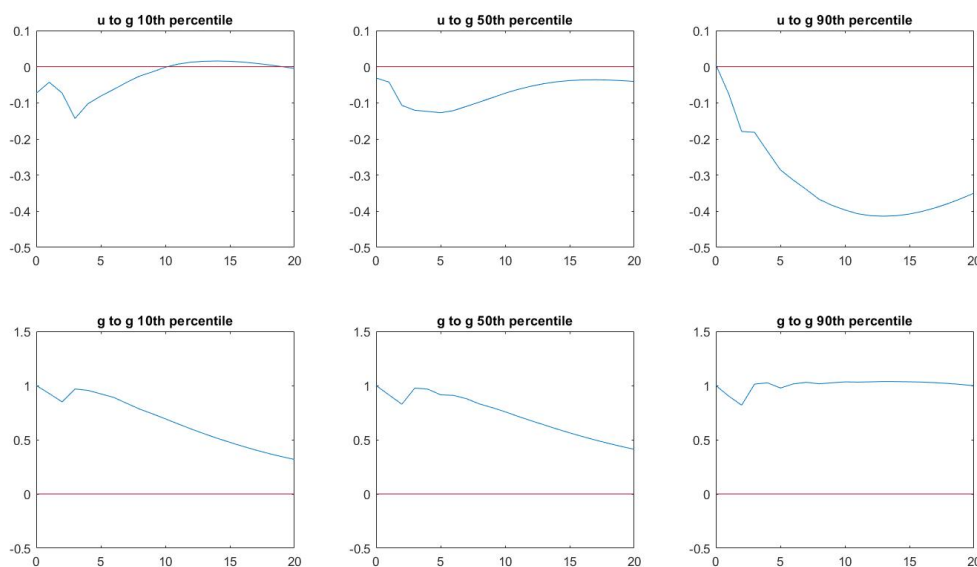


Figure 6: Impulse responses at $q = 0.1$, $q = 0.5$, $q = 0.9$ quantiles of the unemployment distribution.

In terms of the government spending IRFs the results are generally more similar to those of Linnemann and Winkler (2016). Overall, there is a higher level of persistence for higher quantiles. Furthermore, it becomes clear that the IRFs for $q = 0.1$, $q = 0.5$, and $q = 0.9$ in the unemployment QVAR are very similar to the IRFs for $q = 0.9$, $q = 0.5$, and $q = 0.1$ respectively in the output QVAR. This is due to the fact that the lowest decile of unemployment gives a good indication of a healthy and expansionary market, which generally coincides with high levels of output. In the IRFs it is clear that for higher unemployment the government spending is more persistent. This, once again brings up the concern from earlier that potentially the greater effect, seen in the IRF for unemployment at $q = 0.1$, could be down to the higher likelihood of further increases in government spending after the initial shock.

In order to analyse this I make use of the minimum point-to-point and cumulative ratio. These, as mentioned in the previous section, normalize the unemployment rate response for potential persistence in government spending. These ratios are defined very similarly to those used in the output analysis, however for unemployment it becomes a minimum point-to-point ratio as the effect of government spending is generally negative for unemployment. The resulting graphs can be seen below in figure 7. Here it becomes clear that even after normalizing for the persistence of government spending, there is considerable nonlinearity in the effect on unemployment. This can be seen by both the minimum point-to-point ratio and the cumulative ratio which steadily decrease, albeit slowly at first, over the value of q . At $q = 0.8$ and up, the slope sharpens considerably suggesting greater efficacy of government spending shocks at these levels. This sharp drop-off helps explain the relatively large difference between the IRF at $q = 0.5$ and $q = 0.9$ in figure 6. When comparing my findings to those of Linnemann and Winkler (2016) there are some clear similarities and differences. Firstly, the slow negative trend observed in this paper's results up to $q = 0.8$ is very similar to that of Linnemann and Winkler (2016) both for the minimum point-to-point and cumulative ratio. However, for $q > 0.8$ the results diverge, namely, the large drop-off is not observed in Linnemann and Winkler (2016). This is once again likely related to the fact this thesis makes use of a different sample range.

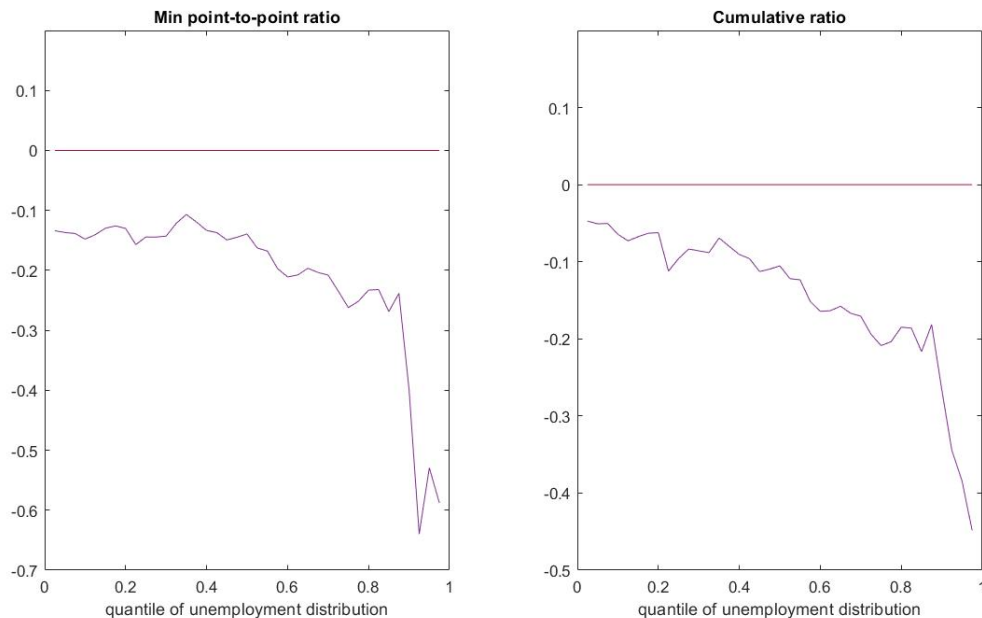


Figure 7: Minimum point-to-point and cumulative ratio over different quantiles of the unemployment distribution.

5.3 Quantile forecasts by QVAR & QRF

In this section I present the out-of-sample forecasts for the 10th, 50th and 90th percentiles of output by using the QVAR and QRF, as specified in the methodology. I then provide an analysis of their accuracy by use of the tailored loss function derived from equation 3, in order to answer the second part of the research question.

5.3.1 Output Forecasts

In figure 9, the QVAR forecasts are plotted together with the true output values. From this graph it is clear to see that the forecasts track the true value of output well and that the different quantiles are biased in the desired direction. Specifically, we see the forecast for $q = 0.1$, depicted by the red line, generally underestimates while the forecast for $q = 0.9$, shown in green, predominantly overestimates the true output. In the pseudo out-of-sample forecasts graph (figure 8), some of the features from the fitted values graph (figure 2) reemerge. Namely, in times of extended downturn or recession the true value tends to be nearer the lower quantile forecast whilst in times of expansion it is nearer the higher quantile forecast. It is clear from the graph that the forecasting method is responsive and exhibits delay. As a result it is not able to predict large downturns or following recoveries as evidenced by the 2007-2010 financial crisis period, which contains first a large decrease in output and then a subsequent stabilization. The QVAR model needs approximately four quarters or one year to recover from its heavy underestimation of output following the drop caused by the Great Recession.

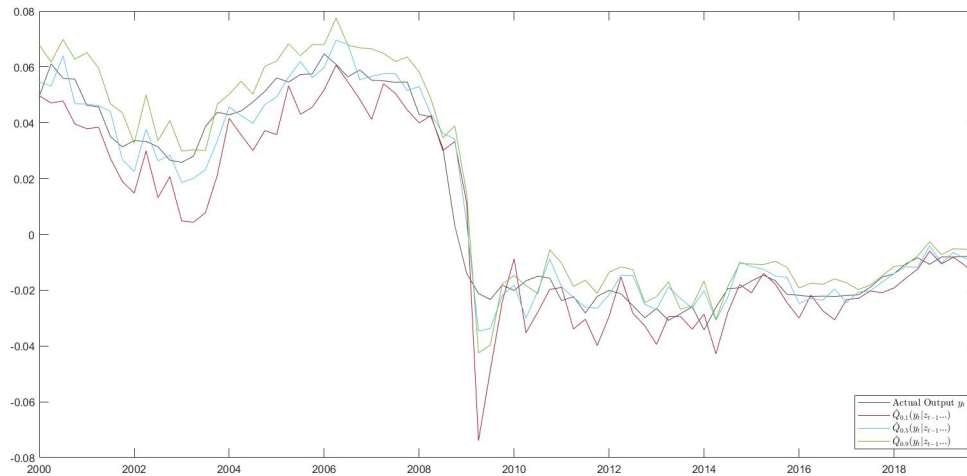


Figure 8: QVAR forecasts for 10th, 50th and 90th percentile plotted with true output.

Now that the QVAR forecasts have been considered it is interesting to see how the QRF method might differ. The forecasts for the $q = 0.1$, $q = 0.5$, and $q = 0.9$ quantiles can be seen together with the true output in figure 9. The most notable part of this graph is the considerable improvement over the QVAR model in the period during and shortly after the Great Recession. Specifically, the QRF model's lower quantile output forecast is considerably closer to the true value of output during the crash of the Great Recession. In addition to this the model stabilizes much faster in the period afterwards, using only about one quarter as opposed to four to do so. The better performance during the crisis could, however, come at a cost to forecasting performance under more normal circumstances. In particular the QVAR provides a tighter and seemingly more accurate forecast in the period leading up to the Great Recession, namely, from 2000 to 2006.

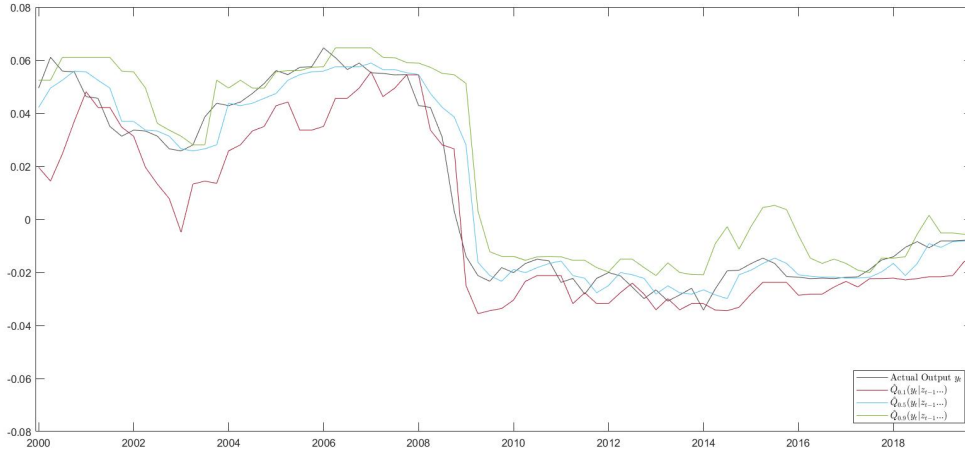


Figure 9: QRF forecasts for 10th, 50th and 90th percentile plotted with true output.

While eyeballing the forecasts does provide some insight into which method performs superior, it is much more conclusive to analyze it analytically. For this, as mentioned in section 4.6, I make use of the Quantile Regression loss function as specified in equation 3. The results of this analysis can be seen in table 1. In this table results are reported for the $q = 0.1$, $q = 0.5$ and $q = 0.9$ quantiles. Immediately it is evident that the QRF mean loss is smaller than that of the QVAR at all three quantiles considered by between 5 and 10%. However, as seen in 1 it still falls within the interval and thus there is not enough evidence to say that the QRF mean loss is smaller than that of the QVAR at a 5% significance level.

Table 1: QVAR vs QRF Forecast Results

Quantile	QVAR Mean Loss	QVAR Mean Loss 95% CI	QRF Mean Loss
Q = 0.1	1.843	1.094 - 2.829	1.695
Q = 0.5	2.677	2.175 - 3.287	2.478
Q = 0.9	1.386	0.875 - 2.093	1.324

Notes: This table indicates the evaluation results of the out of sample forecasts. The mean loss is made using the quantile loss function. It is provided for the QVAR and the QRF forecasts. Besides this a 95% confidence interval is provided by using resampling methods. All the loss values are multiplied by a thousand for ease of visualization.

There are a number improvements that can be made to the QRF model that could possibly improve forecast accuracy. Specifically, more regressors could be included in the form of more lags or different variables. This is due to QRF's excellent performance for high dimensional data. Besides this one could make use of a different function to tune the mtry hyperparameter. In this paper Caret is used, which optimizes the mtry based on the RMSFE at the 50th percentile. One could look into minimizing the mean loss described by equation 3 for each of the three quantiles such that the mtry parameter can vary with the quantile that is being forecasted. These changes would all likely improve the forecasting performance of the model.

It could be interesting to look into the forecasting performance of the two Quantile Regression methods for additional time horizons. One could, for example, evaluate forecasting performance for 2-steps, 3-steps and 4-steps ahead, which as a result would encompass predictions for anywhere between three months and

a year. For the QVAR such a prediction is still feasible computationally as the baseline computation time is fairly low, however, as the forecasting horizon is increased for the QRF the computation time becomes impractically long owing in part to its already long 1-step-ahead computation time. As such this thesis does not provide comparisons for more than 1-step ahead in time. I remain humble to the fact that there could be ways to increase efficiency for the QRF forecasting and therefore encourage research into this.

6 Conclusion

The main goal of this paper is to investigate the presence of nonlinear tendencies in responses of output and unemployment to government spending. In addition to this the thesis also examines how best to apply the concept of Quantile Regression for purposes of obtaining superior forecasts of the different quantiles of the conditional distribution of output. For this analysis I make use of data from the United States from the first quarter of 1970 until the fourth quarter of 2019. This period offers variation in output, unemployment and government spending and as such provides an excellent case for an investigation into fiscal policy effects. From the QVAR analysis it is clear that nonlinearity plays a prevalent part in responses to fiscal policy both for output and for unemployment. Comparing the peaks and troughs of the IRFs the difference between the output effect at $q = 0.1$ and $q = 0.5$ amounted to a magnitude of two and for unemployment the difference between the effect at $q = 0.5$ and $q = 0.9$ amounted to almost a magnitude of four, when considering a 1% increase in government spending. There also exists differences across quantiles in the persistence of government spending, namely for low levels of output and high levels of unemployment, government spending is considerably more sustained than otherwise. This makes intuitive sense as in downturns governments are more likely to engage in expansionary policy. As discussed this brought about the conundrum of whether the differences across quantiles is due to slack in the economy or if it is because of greater persistence in government spending. The maximum (minimum) point-to-point and cumulative ratio, as measures normalizing for the government spending persistence, show that it is in fact, as predicted by Keynesian economics, due to greater slack in the economy.

For the second part of the research question, relating to the forecasting of conditional quantiles of the output distribution, it is clear that both the QVAR and the QRF perform fairly well overall. However, there are some clear differences with regards to when the methods perform relatively well and relatively poorly. It is clear that the QRF is more sensitive and adaptable and as such is better at forecasting during periods of large changes in output. This is very clear when examining the data during the financial crisis from 2007 to 2009. However, in time of more stable production it is clear to see that the QVAR provides tighter and more accurate forecasts. Analytically, the QRF method has lower mean loss when calculated by the quantile loss function and thus better approximates the true conditional quantile functions, however, the difference is not significant at a 5% level and therefore we do not have enough evidence to disprove equal forecasting ability.

The aforementioned conclusions help answer the main research questions of this thesis. However, I am humble to the fact that this research merely scratches the surface of potential analysis of nonlinearities in fiscal policy and how best to forecast the effects of government spending. As such I wish to outline some limitations, provide some suggestions on potential improvements to my methodology and give possible directions for further study. First of all, this paper is limited to investigating nonlinear responses in fiscal policy from a production level perspective. That is I solely investigate how fiscal policy effects vary with the level of slack in the economy, proxied either by GDP or by unemployment. After conducting this research it seems reasonable to expect other forms of nonlinearity to exist in fiscal policy effects as well. One example

of this is monetary policy stance, during expansionary fiscal policy. Whilst this thesis does make use of the interest rate as a control variable one could for example look into how responses in output and unemployment differ when rates are near the zero lower bound. Besides further analysis of nonlinearity, future research could focus on improving forecasting performance. This could be done in a number of ways. Firstly, if one considers using the QRF method from this paper one could look to increase the number of variables used in the model. This is due to the ability of QRF to handle high dimensional data. In addition to this one could look to further optimize the m try hyperparameter to this specific use case. Specifically, one could make use of the loss function in equation 3, which is also used for its performance evaluation. This would allow the m try to differ across quantiles and would likely improve the minimization of the mean loss when compared to using the RMSFE at only the mean. Furthermore, as there are periods of relatively good and relatively bad performance between the QVAR and QRF method, the forecasts lend themselves to be excellent candidates for a forecast combination approach. Possible methodology could be based on Gibbs and Vasnev (2017) which makes use of bias forecasts to determine a conditionally optimal weighting scheme for the different forecasts. Last but not least, researchers could look into which of the two Quantile Regression methods provide more accurate forecasts at longer time horizons such as periods between half a year to a year ahead in time.

With results showing definitive nonlinearity in fiscal policy and a methodology that opens the dialogue on how to most accurately forecast the effects, this thesis provides an encouraging baseline for future study. As explained above this research can take on several directions, a large portion of which will come with considerable social benefit owing to the great importance of effective and efficient economic stimulus.

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Appendix A

Algorithm to obtain Conditional Distribution Estimator

1. Grow a forest of n trees by using n independent and identically distributed random parameter vectors θ_j just as in random forest estimation. However, as opposed to just keeping the average of the observations that fall into a given leaf, keep a note of all the observations that end up at that leaf. Do this for all the leaves in all trees.
2. Now obtain the weights $w_t(x_{T+1}, \theta_j)$ and $\bar{w}_t(x_{T+1})$ by using equations 6 and 8.
3. Finally, calculate $\hat{F}(y|x_{T+1})$ with equation 11 for all y in range of the observations.

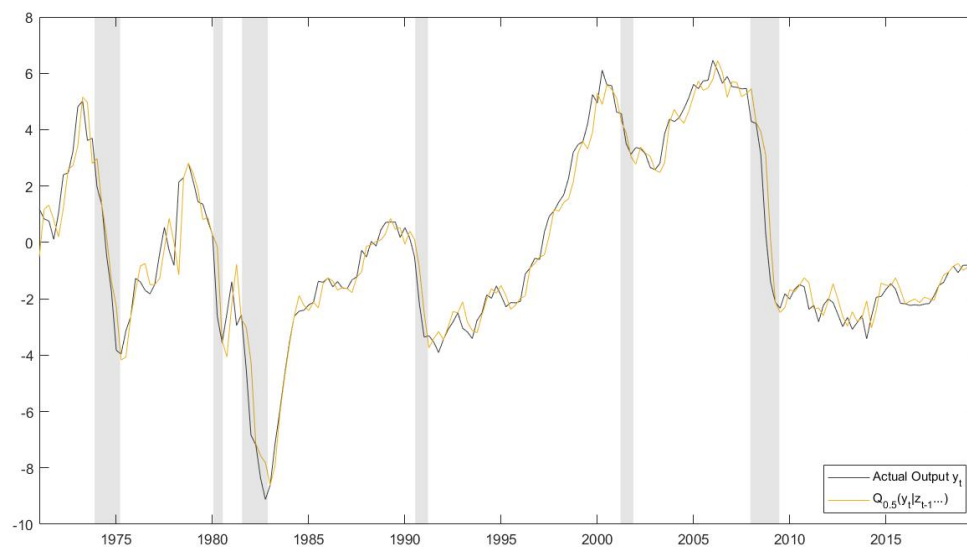


Figure 10: Actual output versus fitted values 50th percentile.

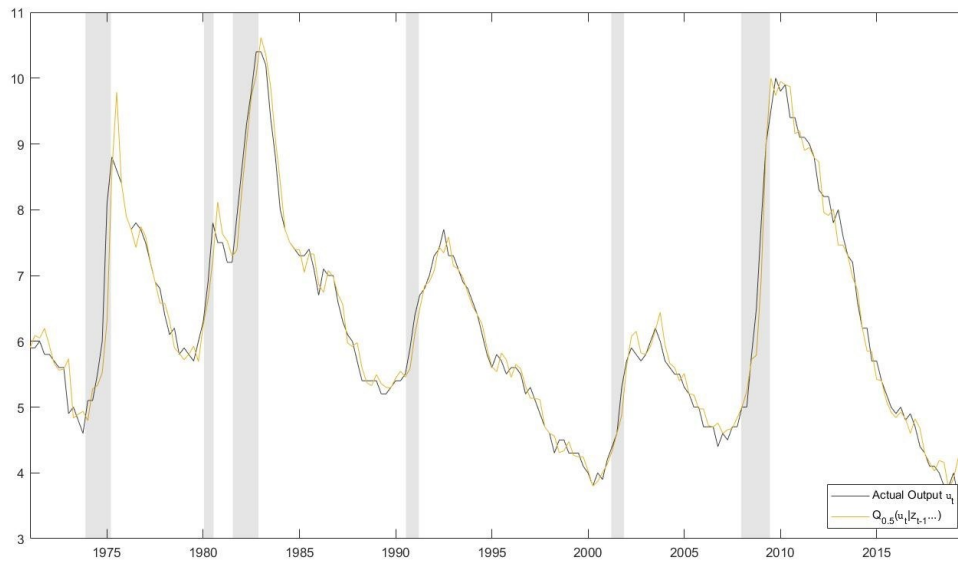


Figure 11: Actual unemployment versus fitted values 50th percentile.

Impulse Response Functions

The IRFs are calculated in a similar manner for both output and unemployment. The R script below is for calculating IRFs for output. We follow the methods in Lütkepohl (2007) to calculate the IRFs from the VAR estimates.

```

library(vars)
library(dyn)
library(quantreg)
library(gdata)
library(MASS)
library(expm)
library("xlsx")

#Load Data
fiscal_data <- read.xls("cleanThesisData.xlsx", perl = "C:/Strawberry/perl/bin/perl.exe")
output = ts(fiscal_data$outputTransformed)
govSpending = ts(fiscal_data$govSpendingTransformed)
unemployment = ts(fiscal_data$unemployment)
netTaxes = ts(fiscal_data$netTaxesTransformed)
debtRatio = ts(fiscal_data$debtToGDPTransformed)
realInterest = ts(fiscal_data$realInterest)

#----- Replication Impulse Response Figure 2 (Output)-----#

#Perform Quantile Vector Autoregression Estimation (Equation by Equation)

```

```

#10th percentile
tau = 0.1 # Select quantile
quantRegGovSpending <- dynrq(govSpending ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4)
  + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegOutput <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = tau)
quantRegNetTaxes <- dynrq(netTaxes ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegRealInterest <- dynrq(realInterest ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4
) + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegDebtRatio <- dynrq(debtRatio ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)

#Save estimation output
model_summaryGovSpending <- summary(quantRegGovSpending)
model_summaryOutput <- summary(quantRegOutput)
model_summaryNetTaxes <- summary(quantRegNetTaxes)
model_summaryRealInterest <- summary(quantRegRealInterest)
model_summaryDebtRatio <- summary(quantRegDebtRatio)

#Get VAR matrices (same order)
coefficients <- t(cbind(model_summaryGovSpending$coefficients[,1],model_summaryOutput$coefficients
  [,1],model_summaryNetTaxes$coefficients[,1],model_summaryRealInterest$coefficients[,1],model_
  summaryDebtRatio$coefficients[,1]))
intercept <- coefficients[,1]
A1 <- matrix(nrow = 5, ncol = 5)
A2 <- matrix(nrow = 5, ncol = 5)
A3 <- matrix(nrow = 5, ncol = 5)
A4 <- matrix(nrow = 5, ncol = 5)

for (i in 0:4)
{
  A1[, (i+1)] <- coefficients[, i*4+2]
  A2[, (i+1)] <- coefficients[, i*4+3]
  A3[, (i+1)] <- coefficients[, i*4+4]
  A4[, (i+1)] <- coefficients[, i*4+5]
}

#Impulse Response Function
resids <- cbind(model_summaryGovSpending$residuals,model_summaryOutput$residuals,model_
  summaryNetTaxes$residuals,model_summaryRealInterest$residuals,model_summaryDebtRatio$residuals)
#obtain residuals in order to find covariance matrix
covMatrix <- (1/(196-5))*(t(resids))%*%resids #sample covariance matrix
Pprime <- chol(covMatrix) #Make use of same notation as New Introduction to Multiple Time Series
  Analysis (Ltkepohl, 2007)
P <- t(Pprime)

```

```

D <- diag(diag(P))
Dinv <- ginv(D)
W <- P%*%Dinv
A <- ginv(W)

A1star <- A%*%A1
A2star <- A%*%A2
A3star <- A%*%A3
A4star <- A%*%A4

govSpendingShock <- rbind(0.01,0,0,0,0)
correctedShock <- W%*%govSpendingShock
prevObs <- cbind(matrix(0L, nrow = 5, ncol = 3),correctedShock)

for (i in 1:20)
{
  newestIndex <- ncol(prevObs)
  newObs <- W%*%A1star%*%prevObs[,newestIndex] + W%*%A2star%*%prevObs[,newestIndex-1] + W%*%A3star
    %*%prevObs[,newestIndex-2] + W%*%A4star%*%prevObs[,newestIndex-3]
  prevObs <- cbind(prevObs,newObs)
}

impulseResponse10th <- prevObs[, (ncol(prevObs)-20):ncol(prevObs)]*100

#Plot Impulse Responses
plot(impulseResponse10th[2,],
      main = "Impulse_Response_Function_GovSpending_on_Output_10th_Percentile",
      ylab = "",
      xlab = "",
      type = "l",
      ylim = c(-1.5, 1.5),
      xlim = c(2,20),
      col = "blue")
abline(h = 0, col = "red")

plot(impulseResponse10th[1,],
      main = "Impulse_Response_Function_GovSpending_on_GovSpending_10th_Percentile",
      ylab = "",
      xlab = "",
      type = "l",
      ylim = c(-0.5, 2),
      xlim = c(2,20),
      col = "blue")
abline(h = 0, col = "red")

#50th percentile

```

```

tau = 0.5 # Select quantile
quantRegGovSpending <- dynrq(govSpending ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4)
  + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegOutput <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = tau)
quantRegNetTaxes <- dynrq(netTaxes ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegRealInterest <- dynrq(realInterest ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4
) + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegDebtRatio <- dynrq(debtRatio ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)

#Save estimation output
model_summaryGovSpending <- summary(quantRegGovSpending)
model_summaryOutput <- summary(quantRegOutput)
model_summaryNetTaxes <- summary(quantRegNetTaxes)
model_summaryRealInterest <- summary(quantRegRealInterest)
model_summaryDebtRatio <- summary(quantRegDebtRatio)

#Get VAR matrices (same order)
coefficients <- t(cbind(model_summaryGovSpending$coefficients[,1],model_summaryOutput$coefficients
  [,1],model_summaryNetTaxes$coefficients[,1],model_summaryRealInterest$coefficients[,1],model_
  summaryDebtRatio$coefficients[,1]))
intercept <- coefficients[,1]
A1 <- matrix(nrow = 5, ncol = 5)
A2 <- matrix(nrow = 5, ncol = 5)
A3 <- matrix(nrow = 5, ncol = 5)
A4 <- matrix(nrow = 5, ncol = 5)

for (i in 0:4)
{
  A1[, (i+1)] <- coefficients[, i*4+2]
  A2[, (i+1)] <- coefficients[, i*4+3]
  A3[, (i+1)] <- coefficients[, i*4+4]
  A4[, (i+1)] <- coefficients[, i*4+5]
}

#Impulse Response Function
resids <- cbind(model_summaryGovSpending$residuals,model_summaryOutput$residuals,model_
  summaryNetTaxes$residuals,model_summaryRealInterest$residuals,model_summaryDebtRatio$residuals)
#obtain residuals in order to find covariance matrix
covMatrix <- (1/(196-5))*(t(resids))%*%resids #sample covariance matrix
Pprime <- chol(covMatrix) #Make use of same notation as New Introduction to Multiple Time Series
  Analysis (Ltkkepohl, 2007)
P <- t(Pprime)
D <- diag(diag(P))

```

```

Dinv <- ginv(D)
W <- P%*%Dinv
A <- ginv(W)

A1star <- A%*%A1
A2star <- A%*%A2
A3star <- A%*%A3
A4star <- A%*%A4

govSpendingShock <- rbind(0.01,0,0,0,0)
correctedShock <- W%*%govSpendingShock
prevObs <- cbind(matrix(0L, nrow = 5, ncol = 3),correctedShock)

for (i in 1:20)
{
  newestIndex <- ncol(prevObs)
  newObs <- W%*%A1star%*%prevObs[,newestIndex] + W%*%A2star%*%prevObs[,newestIndex-1] + W%*%A3star
    %*%prevObs[,newestIndex-2] + W%*%A4star%*%prevObs[,newestIndex-3]
  prevObs <- cbind(prevObs,newObs)
}

impulseResponse50th <- prevObs[, (ncol(prevObs)-20):ncol(prevObs)]*100

#Plot Impulse Responses
plot(impulseResponse50th[2,],
      main = "Impulse_Response_Function_GovSpending_on_Output_50th_Percentile",
      ylab = "",
      xlab = "",
      type = "l",
      ylim = c(-1.5, 1.5),
      xlim = c(2,20),
      col = "blue")
abline(h = 0, col = "red")

plot(impulseResponse50th[1,],
      main = "Impulse_Response_Function_GovSpending_on_GovSpending_50th_Percentile",
      ylab = "",
      xlab = "",
      type = "l",
      ylim = c(-0.5, 2),
      xlim = c(2,20),
      col = "blue")
abline(h = 0, col = "red")

#90th Percentile

tau = 0.9 # Select quantile

```

```

quantRegGovSpending <- dynrq(govSpending ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4)
  + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegOutput <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = tau)
quantRegNetTaxes <- dynrq(netTaxes ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegRealInterest <- dynrq(realInterest ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4
  ) + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegDebtRatio <- dynrq(debtRatio ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)

#Save estimation output
model_summaryGovSpending <- summary(quantRegGovSpending)
model_summaryOutput <- summary(quantRegOutput)
model_summaryNetTaxes <- summary(quantRegNetTaxes)
model_summaryRealInterest <- summary(quantRegRealInterest)
model_summaryDebtRatio <- summary(quantRegDebtRatio)

#Get VAR matrices (same order)
coefficients <- t(cbind(model_summaryGovSpending$coefficients[,1],model_summaryOutput$coefficients
  [,1],model_summaryNetTaxes$coefficients[,1],model_summaryRealInterest$coefficients[,1],model_
  summaryDebtRatio$coefficients[,1]))
intercept <- coefficients[,1]
A1 <- matrix(nrow = 5, ncol = 5)
A2 <- matrix(nrow = 5, ncol = 5)
A3 <- matrix(nrow = 5, ncol = 5)
A4 <- matrix(nrow = 5, ncol = 5)

for (i in 0:4)
{
  A1[, (i+1)] <- coefficients[, i*4+2]
  A2[, (i+1)] <- coefficients[, i*4+3]
  A3[, (i+1)] <- coefficients[, i*4+4]
  A4[, (i+1)] <- coefficients[, i*4+5]
}

#Impulse Response Function
resids <- cbind(model_summaryGovSpending$residuals,model_summaryOutput$residuals,model_
  summaryNetTaxes$residuals,model_summaryRealInterest$residuals,model_summaryDebtRatio$residuals)
#obtain residuals in order to find covariance matrix
covMatrix <- (1/(196-5))*(t(resids))%*%resids #sample covariance matrix
Pprime <- chol(covMatrix) #Make use of same notation as New Introduction to Multiple Time Series
  Analysis (Ltkepohl, 2007)
P <- t(Pprime)
D <- diag(diag(P))
Dinv <- ginv(D)

```



```

W <- P%*%Dinv
A <- ginv(W)

A1star <- A%*%A1
A2star <- A%*%A2
A3star <- A%*%A3
A4star <- A%*%A4

govSpendingShock <- rbind(0.01,0,0,0,0)
correctedShock <- W%*%govSpendingShock
prevObs <- cbind(matrix(0L, nrow = 5, ncol = 3),correctedShock)

for (i in 1:20)
{
  newestIndex <- ncol(prevObs)
  newObs <- W%*%A1star%*%prevObs[,newestIndex] + W%*%A2star%*%prevObs[,newestIndex-1] + W%*%A3star
    %*%prevObs[,newestIndex-2] + W%*%A4star%*%prevObs[,newestIndex-3]
  prevObs <- cbind(prevObs,newObs)
}
impulseResponse90th <- prevObs[, (ncol(prevObs)-20):ncol(prevObs)]*100

#Plot Impulse Responses
plot(impulseResponse90th[2,],
     main = "Impulse_Response_Function_GovSpending_on_Output_90th_Percentile",
     ylab = "",
     xlab = "",
     type = "l",
     ylim = c(-1.5, 1.5),
     xlim = c(2,20),
     col = "blue")
abline(h = 0, col = "red")

plot(impulseResponse90th[1,],
     main = "Impulse_Response_Function_GovSpending_on_GovSpending_90th_Percentile",
     ylab = "",
     xlab = "",
     type = "l",
     ylim = c(-0.5, 2),
     xlim = c(2,20),
     col = "blue")
abline(h = 0, col = "red")

#----- Replication Figure 1 (fitted values at 10th and 90th percentile vs actual)-----#

```

```

quantRegOutput10 <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.1)
quantRegOutput90 <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.9)
quantRegOutput50 <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)

fitted10 <- quantRegOutput10$fitted.values
fitted90 <- quantRegOutput90$fitted.values
fitted50 <- quantRegOutput50$fitted.values

plot.ts(output)
lines(fitted10, col = "red")
lines(fitted90, col = "red")

#----- Write results to excel file-----#

# #write to new workbook
# fig1data <- cbind(output,fitted10,fitted90)
# write.xlsx(fig1data, file = "replicationResults.xlsx",
# sheetName = "fig1Output", append = FALSE)
# #write to new sheet
# write.xlsx(impulseResponse10th, file = "replicationResults.xlsx",
# sheetName="impulseresponse10thOutput", append=TRUE)
# #write to new sheet
# write.xlsx(impulseResponse50th, file = "replicationResults.xlsx",
# sheetName="impulseresponse50thOutput", append=TRUE)
# #write to new sheet
# write.xlsx(impulseResponse90th, file = "replicationResults.xlsx",
# sheetName="impulseresponse90thOutput", append=TRUE)
#write to new sheet
write.xlsx(fitted50, file = "replicationResults.xlsx",
  sheetName="fitted50Output", append=TRUE)

```

Maximum (Minimum) Point-to-Point and Cumulative Ratio

Programmed similarly for output and unemployment with the exception that it is a minimum point-to-point ratio for unemployment. Below is the script for output.

```

library(vars)
library(dyn)
library(quantreg)
library(gdata)
library(MASS)
library(expm)

```

```

#Load Data
fiscal_data <- read.xls("cleanThesisData.xlsx", perl = "C:/Strawberry/perl/bin/perl.exe")
output = ts(fiscal_data$outputTransformed)
govSpending = ts(fiscal_data$govSpendingTransformed)
unemployment = ts(fiscal_data$unemployment)
netTaxes = ts(fiscal_data$netTaxesTransformed)
debtRatio = ts(fiscal_data$debtToGDPTransformed)
realInterest = ts(fiscal_data$realInterest)

#----- Extended Replication (max point-to-point and cumulative ratio)-----#

maxPointToPointRatio <- matrix(nrow = 2, ncol = 0)
cumulativeRatio <- matrix(nrow = 2, ncol =0)

#Perform estimations not relevant for loop
quantRegGovSpending <- dynrq(govSpending ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4)
  + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegNetTaxes <- dynrq(netTaxes ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegRealInterest <- dynrq(realInterest ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4
  ) + L(realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)
quantRegDebtRatio <- dynrq(debtRatio ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
  realInterest, 1:4) + L(debtRatio, 1:4), tau = 0.5)

#Save estimation output for model not relevant for loop
model_summaryGovSpending <- summary(quantRegGovSpending)
model_summaryNetTaxes <- summary(quantRegNetTaxes)
model_summaryRealInterest <- summary(quantRegRealInterest)
model_summaryDebtRatio <- summary(quantRegDebtRatio)

#prepare correction fucntion
correctionFunction <- function(x) {
  ifelse((x > 1),0,x)}

for(tau in seq(from=0.025, to=0.975, by=0.025)){
  print(tau)

  #Qauntile specific estimation
  quantRegOutput <- dynrq(output ~ L(govSpending, 1:4) + L(output, 1:4) + L(netTaxes, 1:4) + L(
    realInterest, 1:4) + L(debtRatio, 1:4), tau = tau)
  model_summaryOutput <- summary(quantRegOutput)

  #Get VAR matrices for given quantile
  coefficients <- t(cbind(model_summaryGovSpending$coefficients[,1],model_summaryOutput$
    coefficients[,1],model_summaryNetTaxes$coefficients[,1],model_summaryRealInterest$
    coefficients[,1],model_summaryDebtRatio$coefficients[,1]))
  intercept <- coefficients[,1]

```

```

A1 <- matrix(nrow = 5, ncol = 5)
A2 <- matrix(nrow = 5, ncol = 5)
A3 <- matrix(nrow = 5, ncol = 5)
A4 <- matrix(nrow = 5, ncol = 5)

for (i in 0:4)
{
  A1[, (i+1)] <- coefficients[, i*4+2]
  A2[, (i+1)] <- coefficients[, i*4+3]
  A3[, (i+1)] <- coefficients[, i*4+4]
  A4[, (i+1)] <- coefficients[, i*4+5]
}

#Calculate quantile specific impulse response function
resids <- cbind(model_summaryGovSpending$residuals, model_summaryOutput$residuals, model_
  summaryNetTaxes$residuals, model_summaryRealInterest$residuals, model_summaryDebtRatio$
  residuals) #obtain residuals in order to find covariance matrix
covMatrix <- (1/(196-5))*(t(resids))%*%resids #sample covariance matrix
Pprime <- chol(covMatrix) #Make use of same notation as New Introduction to Multiple Time Series
  Analysis (Ltkepohl, 2005)
P <- t(Pprime)
D <- diag(diag(P))
Dinv <- ginv(D)
W <- P%*%Dinv
A <- ginv(W)

A1star <- A%*%A1
A2star <- A%*%A2
A3star <- A%*%A3
A4star <- A%*%A4

govSpendingShock <- rbind(0.01, 0, 0, 0, 0)
correctedShock <- W%*%govSpendingShock
prevObs <- cbind(matrix(0L, nrow = 5, ncol = 3), correctedShock)

for (i in 1:20)
{
  newestIndex <- ncol(prevObs)
  newObs <- W%*%A1star%*%prevObs[, newestIndex] + W%*%A2star%*%prevObs[, newestIndex-1] + W%*%A3
    star%*%prevObs[, newestIndex-2] + W%*%A4star%*%prevObs[, newestIndex-3]
  prevObs <- cbind(prevObs, newObs)
}

impulseResponse <- prevObs[, (ncol(prevObs)-20):ncol(prevObs)]*100

#Max point-to-point ratio
pointToPoint <- impulseResponse[2,]/impulseResponse[1,]

```

```

correctPointToPoint <- sapply(pointToPoint, correctionFunction)
maxPointToPoint <- max(correctPointToPoint)
maxPointToPointRatio <- cbind(maxPointToPointRatio, c(tau,maxPointToPoint))

#Cumulative ratio
cumulativeOutputShock <- sum(impulseResponse[2,1:12])
cumulativeGovSpendingShock <- sum(impulseResponse[1,1:12])
cumulativeRatioScalar <- cumulativeOutputShock/cumulativeGovSpendingShock
cumulativeRatio <- cbind(cumulativeRatio, c(tau,cumulativeRatioScalar))
}
plot(maxPointToPointRatio[1,], maxPointToPointRatio[2,],
      main = "Maximum_point-to-point_ratio",
      ylab = "",
      xlab = "",
      type = "l",
      ylim = c(0, 1.2),
      xlim = c(0,1),
      col = "blue")
abline(h = 0, col = "red")
plot(cumulativeRatio[1,], cumulativeRatio[2,],
      main = "Cumulative_ratio",
      ylab = "",
      xlab = "",
      type = "l",
      ylim = c(-0.6, 1),
      xlim = c(0,1),
      col = "blue")
abline(h = 0, col = "red")

#----- Write results to excel file-----#

#write to new workbook
write.xlsx(maxPointToPointRatio, file = "extendedReplicationResults.xlsx",
           sheetName = "maxPointToPointRatioOutput", append = FALSE)
#write to new sheet
write.xlsx(cumulativeRatio, file = "extendedReplicationResults.xlsx",
           sheetName="cumulativeRatioOutput", append=TRUE)

```

Quantile Forecasts through QVAR & QRF

Forecasts are for different quantiles of output and provide a better indication of the output response to government spending than a simple mean forecast. Forecasting accuracy evaluation is done by use of the loss function specified in the methodology.

QVAR

```
library(vars)
library(dyn)
library(quantreg)
library(gdata)
library(MASS)
library(expm)
library(boot)
library("xlsx")
set.seed(1)

#Load Data
fiscal_data <- read.xls("cleanThesisData.xlsx", perl = "C:/Strawberry/perl/bin/perl.exe")
output = ts(fiscal_data$outputTransformed)
govSpending = ts(fiscal_data$govSpendingTransformed)
unemployment = ts(fiscal_data$unemployment)
netTaxes = ts(fiscal_data$netTaxesTransformed)
debtRatio = ts(fiscal_data$debtToGDPTransformed)
realInterest = ts(fiscal_data$realInterest)

#Initialize
outputForecasts10th <- vector()
outputForecasts50th <- vector()
outputForecasts90th <- vector()
trueOutput <- vector()

#Forecast for 10th percentile

tau <- 0.1 # Select quantile
for (i in 1:80) {
  print(i)
  #Set estimation sample
  inSample <- data.frame("govSpendingSample" = ts(govSpending[i:(i+119)]), "outputSample" = ts(
    output[i:(i+119)]), "netTaxesSample" = ts(netTaxes[i:(i+119)]), "realInterestSample" = ts(
    realInterest[i:(i+119)]), "debtRatioSample" = ts(debtRatio[i:(i+119)]))
  xi <- data.frame("govSpendingSample" = ts(govSpending[(i+116):(i+119)]), "outputSample" = ts(
    output[(i+116):(i+119)]), "netTaxesSample" = ts(netTaxes[(i+116):(i+119)]), "
    realInterestSample" = ts(realInterest[(i+116):(i+119)]), "debtRatioSample" = ts(debtRatio[(i+
    116):(i+119)]))
  #Perform estimation
  quantRegOutput <- dynrq(inSample$outputSample ~ L(inSample$govSpendingSample, 1:4) + L(inSample$
    outputSample, 1:4) + L(inSample$netTaxesSample, 1:4) + L(inSample$realInterestSample, 1:4) +
    L(inSample$debtRatioSample, 1:4), tau = tau)
  #Save estimation output
  model_summaryOutput <- summary(quantRegOutput)
  coefficients <- model_summaryOutput$coefficients[,1]
```

```

intercept <- coefficients[1]
A1 <- matrix(nrow = 1, ncol = 5)
A2 <- matrix(nrow = 1, ncol = 5)
A3 <- matrix(nrow = 1, ncol = 5)
A4 <- matrix(nrow = 1, ncol = 5)

for (j in 0:4)
{
  A1[1,(j+1)] <- coefficients[j*4+2]
  A2[1,(j+1)] <- coefficients[j*4+3]
  A3[1,(j+1)] <- coefficients[j*4+4]
  A4[1,(j+1)] <- coefficients[j*4+5]
}
#Perform prediction
forecast <- intercept + A1%*%as.numeric(as.vector(xi[4,])) + A2%*%as.numeric(as.vector(xi[3,])) +
  A3%*%as.numeric(as.vector(xi[2,])) + A4%*%as.numeric(as.vector(xi[1,]))
outputForecasts10th <- c(outputForecasts10th,forecast) #save forecast
trueOutput <- c(trueOutput, output[(i+120)])
}

plot(outputForecasts10th,
      main = "",
      ylab = "",
      xlab = "",
      type = "l",
      col = "blue")
lines(trueOutput)

#Generate Mean Loss

forecastError10th <- trueOutput - outputForecasts10th
plot(forecastError10th,
      type = "l")

#mean loss function
mean.loss <- function(data, index){
  lossVector <- vector()
  dt <- data[index]
  for (p in 1:length(data)) {
    if (dt[p] > 0) {
      localLoss <- tau*abs(dt[p])
    } else {
      localLoss <- (1-tau)*abs(dt[p])
    }
  }
  lossVector <- c(lossVector,localLoss)
}

```

```

    return(mean(lossVector))
}

meanLoss10th <- mean.loss(forecastError10th)

#Forecast for 50th percentile

tau <- 0.5 # Select quantile
for (i in 1:80) {
  print(i)
  #Set estimation sample
  inSample <- data.frame("govSpendingSample" = ts(govSpending[i:(i+119)]), "outputSample" = ts(
    output[i:(i+119)]), "netTaxesSample" = ts(netTaxes[i:(i+119)]), "realInterestSample" = ts(
    realInterest[i:(i+119)]), "debtRatioSample" = ts(debtRatio[i:(i+119)]))
  xi <- data.frame("govSpendingSample" = ts(govSpending[(i+116):(i+119)]), "outputSample" = ts(
    output[(i+116):(i+119)]), "netTaxesSample" = ts(netTaxes[(i+116):(i+119)]), "
    realInterestSample" = ts(realInterest[(i+116):(i+119)]), "debtRatioSample" = ts(debtRatio[(i+
    116):(i+119)]))
  #Perform estimation
  quantRegOutput <- dynrq(inSample$outputSample ~ L(inSample$govSpendingSample, 1:4) + L(inSample$
    outputSample, 1:4) + L(inSample$netTaxesSample, 1:4) + L(inSample$realInterestSample, 1:4) +
    L(inSample$debtRatioSample, 1:4), tau = tau)
  #Save estimation output
  model_summaryOutput <- summary(quantRegOutput)
  coefficients <- model_summaryOutput$coefficients[,1]

  intercept <- coefficients[1]
  A1 <- matrix(nrow = 1, ncol = 5)
  A2 <- matrix(nrow = 1, ncol = 5)
  A3 <- matrix(nrow = 1, ncol = 5)
  A4 <- matrix(nrow = 1, ncol = 5)

  for (j in 0:4)
  {
    A1[1,(j+1)] <- coefficients[j*4+2]
    A2[1,(j+1)] <- coefficients[j*4+3]
    A3[1,(j+1)] <- coefficients[j*4+4]
    A4[1,(j+1)] <- coefficients[j*4+5]
  }
  #Perform prediction
  forecast <- intercept + A1*%as.numeric(as.vector(xi[4,])) + A2*%as.numeric(as.vector(xi[3,])) +
    A3*%as.numeric(as.vector(xi[2,])) + A4*%as.numeric(as.vector(xi[1,]))
  outputForecasts50th <- c(outputForecasts50th,forecast) #save forecast
}

plot(outputForecasts50th,
      main = "",

```



```

    ylab = "",
    xlab = "",
    type = "l",
    col = "blue")
lines(trueOutput)

#Generate Mean Loss

forecastError50th <- trueOutput - outputForecasts50th
plot(forecastError50th,
     type = "l")

#mean loss function
mean.loss <- function(data, index){
  lossVector <- vector()
  dt <- data[index]
  for (p in 1:length(data)) {
    if (dt[p] > 0) {
      localLoss <- tau*abs(dt[p])
    } else {
      localLoss <- (1-tau)*abs(dt[p])
    }
    lossVector <- c(lossVector,localLoss)
  }
  return(mean(lossVector))
}

meanLoss50th <- mean.loss(forecastError50th)

#Forecast for 90th percentile

tau <- 0.9 # Select quantile
for (i in 1:80) {
  print(i)
  #Set estimation sample
  inSample <- data.frame("govSpendingSample" = ts(govSpending[i:(i+119)]), "outputSample" = ts(
    output[i:(i+119)]), "netTaxesSample" = ts(netTaxes[i:(i+119)]), "realInterestSample" = ts(
    realInterest[i:(i+119)]), "debtRatioSample" = ts(debtRatio[i:(i+119)]))
  xi <- data.frame("govSpendingSample" = ts(govSpending[(i+116):(i+119)]), "outputSample" = ts(
    output[(i+116):(i+119)]), "netTaxesSample" = ts(netTaxes[(i+116):(i+119)]), "
    realInterestSample" = ts(realInterest[(i+116):(i+119)]), "debtRatioSample" = ts(debtRatio[(i+
    116):(i+119)]))
  #Perform estimation
  quantRegOutput <- dynrq(inSample$outputSample ~ L(inSample$govSpendingSample, 1:4) + L(inSample$
    outputSample, 1:4) + L(inSample$netTaxesSample, 1:4) + L(inSample$realInterestSample, 1:4) +
    L(inSample$debtRatioSample, 1:4), tau = tau)
  #Save estimation output

```

```

model_summaryOutput <- summary(quantRegOutput)
coefficients <- model_summaryOutput$coefficients[,1]

intercept <- coefficients[1]
A1 <- matrix(nrow = 1, ncol = 5)
A2 <- matrix(nrow = 1, ncol = 5)
A3 <- matrix(nrow = 1, ncol = 5)
A4 <- matrix(nrow = 1, ncol = 5)

for (j in 0:4)
{
  A1[1,(j+1)] <- coefficients[j*4+2]
  A2[1,(j+1)] <- coefficients[j*4+3]
  A3[1,(j+1)] <- coefficients[j*4+4]
  A4[1,(j+1)] <- coefficients[j*4+5]
}
#Perform prediction
forecast <- intercept + A1%*%as.numeric(as.vector(xi[4,])) + A2%*%as.numeric(as.vector(xi[3,])) +
  A3%*%as.numeric(as.vector(xi[2,])) + A4%*%as.numeric(as.vector(xi[1,]))
outputForecasts90th <- c(outputForecasts90th,forecast) #save forecast
}

plot(outputForecasts90th,
      main = "",
      ylab = "",
      xlab = "",
      type = "l",
      col = "blue")
lines(trueOutput)

#Generate Mean Loss

forecastError90th <- trueOutput - outputForecasts90th
plot(forecastError90th,
      type = "l")

#mean loss function
mean.loss <- function(data, index){
  lossVector <- vector()
  dt <- data[index]
  for (p in 1:length(data)) {
    if (dt[p] > 0) {
      localLoss <- tau*abs(dt[p])
    } else {
      localLoss <- (1-tau)*abs(dt[p])
    }
  }
  lossVector <- c(lossVector,localLoss)
}

```

```

}
  return(mean(lossVector))
}

meanLoss90th <- mean.loss(forecastError90th)

#Perform bootstrapping to obtain 95% confidence intervals

#10th percentile
tau <- 0.1
bootResults10th <- boot(data = forecastError10th, statistic = mean.loss, R = 5000)
CI10th <- boot.ci(boot.out = bootResults10th, index = 1, type = "norm")

#50th percentile
tau <- 0.5
bootResults50th <- boot(data = forecastError50th, statistic = mean.loss, R = 5000)
CI50th <- boot.ci(boot.out = bootResults50th, index = 1, type = "norm")

#90th percentile
tau <- 0.9
bootResults90th <- boot(data = forecastError90th, statistic = mean.loss, R = 5000)
CI90th <- boot.ci(boot.out = bootResults90th, index = 1, type = "norm")

list10th <- sort(bootResults10th$t, decreasing = FALSE)
list50th <- sort(bootResults50th$t, decreasing = FALSE)
list90th <- sort(bootResults90th$t, decreasing = FALSE)
# #----- Write results to excel file-----#
#
# #write to new workbook
# outputForecasts <- cbind(outputForecasts10th,outputForecasts50th,outputForecasts90th)
# write.xlsx(outputForecasts, file = "extensionResults.xlsx",
# sheetName = "QVARForecasts", append = TRUE)

```

QRF

```

library(vars)
library(dyn)
library(quantreg)
library(gdata)
library(MASS)
library(expm)
library(boot)
library(quantregForest)
library(caret)
library("xlsx")

```

```

set.seed(1)

#Make use of lag function which places NA at start of series
lagNA <- function(x, k) {
  stopifnot(k >= 0 & k <= length(x))
  replace(rep(NA, length(x)), (k + 1):length(x), x[1:(length(x) - k)])
}

#Load Data
fiscal_data <- read.xls("cleanThesisData.xlsx", perl = "C:/Strawberry/perl/bin/perl.exe")
dataOutput <- data.frame("output" = ts(fiscal_data$outputTransformed),
  "outputL1" = lagNA(fiscal_data$outputTransformed, 1), "outputL2" = lagNA(fiscal_
    data$outputTransformed, 2), "outputL3" = lagNA(fiscal_data$outputTransformed,
    3), "outputL4" = lagNA(fiscal_data$outputTransformed, 4),
  "govSpendingL1" = lagNA(fiscal_data$govSpendingTransformed, 1), "govSpendingL2" =
    lagNA(fiscal_data$govSpendingTransformed, 2), "govSpendingL3" = lagNA(fiscal_
    data$govSpendingTransformed, 3), "govSpendingL4" = lagNA(fiscal_data$
    govSpendingTransformed, 4),
  "netTaxesL1" = lagNA(fiscal_data$netTaxesTransformed, 1), "netTaxesL2" = lagNA(
    fiscal_data$netTaxesTransformed, 2), "netTaxesL3" = lagNA(fiscal_data$
    netTaxesTransformed, 3), "netTaxesL4" = lagNA(fiscal_data$netTaxesTransformed,
    4),
  "realInterestL1" = lagNA(fiscal_data$realInterest, 1), "realInterestL2" = lagNA(
    fiscal_data$realInterest, 2), "realInterestL3" = lagNA(fiscal_data$
    realInterest, 3), "realInterestL4" = lagNA(fiscal_data$realInterest, 4),
  "debtRatioL1" = lagNA(fiscal_data$debtToGDPTransformed, 1), "debtRatioL2" = lagNA(
    fiscal_data$debtToGDPTransformed, 2), "debtRatioL3" = lagNA(fiscal_data$
    debtToGDPTransformed, 3), "debtRatioL4" = lagNA(fiscal_data$
    debtToGDPTransformed, 4))

# Remove NA rows
dataOutput <- dataOutput[-1:-4,]

#Initialize
outputForecasts <- vector()
trueOutput <- vector()
tuningParameter <- vector()

#Set number of trees
ntree <- 1000
for (i in 1:80) {
  print(i)
  #Divide into training and test data for Caret and quantregForest
  trainData <- dataOutput[i:(115+i),1:ncol(dataOutput)]
  testData <- dataOutput[(116+i),1:ncol(dataOutput)]

  xTrain <- dataOutput[i:(115+i),2:ncol(dataOutput)]

```

```

yTrain <- dataOutput[i:(115+i),1]
xTest <- dataOutput[(116+i),2:ncol(dataOutput)]

trueY <- dataOutput[(116+i),1]

#Perform hyperparameter tuning at 50th percentile using RMSFE
fitControl <- trainControl(## 10-fold CV
  method = "repeatedcv",
  number = 10,
  ## repeated ten times
  repeats = 10)

tuning <- train(output ~ ., data = trainData,
  method = "qrf",
  trControl = fitControl,
  ## This last option is actually one
  ## for gbm() that passes through
  verbose = FALSE)

mtryTuned <- tuning$finalModel$mtry
tuningParameter <- c(tuningParameter,mtryTuned)

#Compute quantile regression forests
qrf <- quantregForest(x=xTrain, y=yTrain, mtry = mtryTuned, ntree = ntree)

#Predict 0.1, 0.5 and 0.9 quantiles for test data
conditionalQuantiles <- predict(qrf, xTest)

#Save forecasts & true value
outputForecasts <- rbind(outputForecasts,conditionalQuantiles)
trueOutput <- c(trueOutput,trueY)
print("MTRY_□=□", mtryTuned)
}

plot(outputForecasts[,1],
  main = "",
  ylab = "",
  xlab = "",
  type = "l",
  col = "red")
lines(outputForecasts[,2], col = "yellow")
lines(outputForecasts[,3], col = "green")
lines(trueOutput)

#Generate Mean Loss for different quantiles

#Mean Loss Function

```

```

mean.loss <- function(data, index){
  lossVector <- vector()
  dt <- data[index]
  for (p in 1:length(data)) {
    if (dt[p] > 0) {
      localLoss <- tau*abs(dt[p])
    } else {
      localLoss <- (1-tau)*abs(dt[p])
    }
    lossVector <- c(lossVector,localLoss)
  }
  return(mean(lossVector))
}

#Generate Mean Loss 50th Percentile

forecastError50th <- trueOutput - outputForecasts[,2]
plot(forecastError50th,
     type = "l")
tau <- 0.5 # Need correct quantile
meanLoss50th <- mean.loss(forecastError50th)

#Generate Mean Loss 10th Percentile

forecastError10th <- trueOutput - outputForecasts[,1]
plot(forecastError10th,
     type = "l")
tau <- 0.1 # Need correct quantile
meanLoss10th <- mean.loss(forecastError10th)

#Generate Mean Loss 90th Percentile

forecastError90th <- trueOutput - outputForecasts[,3]
plot(forecastError90th,
     type = "l")
tau <- 0.9 # Need correct quantile
meanLoss90th <- mean.loss(forecastError90th)

#----- Write results to excel file-----#

#write to new workbook
write.xlsx(outputForecasts, file = "extensionResults.xlsx",
          sheetName = "QRFForecasts", append = FALSE)
#write to new sheet
write.xlsx(trueOutput, file = "extensionResults.xlsx",
          sheetName="actualOutput", append=TRUE)

```