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Interactive fixed effects in a panel threshold model with latent group structures

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Abstract

This paper examines panel threshold regression models with latent group structures. The first part consists of a replication of the panel structure threshold regression (PSTR) model from Miao et al. (2020), where additive fixed effects are used to capture some heterogeneity. Then, the PSTR model is extended with interactive fixed effects (IFEs), such that the heterogeneity can vary both cross-sectionally and over time. The multiplicative form of IFEs ensure that the heterogeneity can be captured more flexibly. Through the dependence of common factors, IFEs allow for strong correlation across time and individuals of the regressors. Monte Carlo simulations show that the estimators perform well in finite samples. The method is applied to the relationship between economic growth and financial development, where the results show evidence to support the threshold model with IFEs and latent group structures.

Contents

1 Introduction

This study investigates several features to model unobserved heterogeneity in a panel data setup. The first part uses a threshold model with latent group structures, which is similar to the panel structure threshold regression (PSTR) model from Miao et al. (2020). In the PSTR model, both the slope coefficients and threshold parameters exhibit latent group structures. Miao et al. (2020) show that their estimators perform reasonably well in finite samples and they capture a large degree of heterogeneity that cannot be captured by traditional panel threshold regressions.

To capture even more heterogeneity, the second part of this research extends the PSTR model by incorporating interactive fixed effects (IFEs). The literature on threshold models, interactive fixed effects, and latent group structures are fastly developing, but no study combines all those concepts into one model.

Capturing unobserved heterogeneity in panel data is commonly done via individual-specific time-invariant fixed effects, but this is still restrictive. When IFEs are used, the unobservable heterogeneity can vary across individuals and time. The multiplicative form of the IFEs captures heterogeneity more flexibly, as it allows for common time-varying shocks (factors) to affect the cross-sectional individuals with individual-specific sensitivities (factor loadings).

IFEs allow serial and cross-sectional correlation of regressors through the dependence on common factors. Traditional models without IFEs do not allow for this correlation. However, in many macro or financial data, this dependence is present. As IFEs allow and control for unobserved heterogeneity, which is one of the most attractive features of panel data models, models with IFEs are becoming very appealing for empirical studies.

This paper considers the diminishing threshold effects framework from Hansen (2000), and uses an iterative least squares (LS) estimation procedure for both the PSTR model and the with IFEs extended model. Miao et al. (2020) also use LS estimation iteratively for their PSTR model. They use an algorithm, similar to the one from Bonhomme and Manresa (2015), which updates the group allocation at the end of the estimation procedure. Bai (2009) shows how to use LS estimation for a model with IFEs. I combine the estimation methods from Miao et al. (2020) and Bai (2009) into one estimation algorithm to be able to capture all the concepts discussed above into one complex model. In most studies, the number of groups and the number of factors that need to be included in the model are estimated using some criteria. I assume those numbers to be known, as this estimation is beyond the scope of research. IFEs are incorporated in the form of one single factor.

To examine the finite sample performance of the estimators, I perform Monte Carlo simulations. The estimators of both the PSTR model and the extended model with IFEs perform well in terms of bias, root mean squared error, and coverage probability of the 95% confidence interval. The practicality of the model is shown by an empirical application on the relationship between economic growth and financial development. This relationship already contains both threshold effects and IFEs, as is shown by Miao et al. (2019). I use similar data from the World Development Indicators (WDI) to investigate the presence of group structures next to the existing threshold effects and IFEs. The estimation results show strong evidence to support the inclusion of latent group structures next to the threshold effect and IFEs.

The rest of this paper is outlined as follows. In Section 2, I discuss relevant literature on threshold models, group structures and IFEs in a panel framework. Section 3 shows the PSTR model and estimation procedure where next to threshold effects and latent group structures, additive fixed effects are incorporated to capture some additional heterogeneity. In Section 4, I introduce IFEs to the model and adjust the estimation method where needed. Section 5 reports the Monte Carlo simulation findings and Section 6 provides the application on the relationship between economic growth and financial development. Finally, Section 7 concludes.

2 Literature review

There is a widespread range of literature on threshold models with lots of applications in economics (Hansen (2011)), and still, threshold regression models is a fast developing concept. Hansen (1999) evaluated static panel threshold models with exogenous regressors and threshold variables. Seo and Shin (2016) developed a GMM estimation method for the analysis of dynamic threshold models with additive fixed effects, where either the regressors or the threshold variable could be endogenous. The estimation and inference in a dynamic panel threshold model with IFEs are investigated by Miao et al. (2019). Similar to the threshold model literature, the literature covering panel data models with IFEs is also rapidly growing. A far from exclusive list of papers studying IFEs is Bai (2009), Bai and Liao (2016), Li et al. (2016) and Moon and Weidner (2017). Incorporating latent group structures is another way to capture unobserved heterogeneity. Studies discuss a big variety of methods for group classification. For instance, Su et al. (2016) and Su and Ju (2018) use a variant of the Lasso procedure, or Lin and Ng (2012) and Bonhomme and Manresa (2015) who achieve group classification via K-Means. This research will focus on the concept of latent group structures in panel threshold models (such as the PSTR model from Miao et al. (2020)) and extend this by incorporating IFEs.

Threshold models can be traced back to Tong (1978). Chan (1993) developed a threshold estimator asymptotic theory with fixed threshold effects. The concept of shrinking threshold effects, which are the most common in the current literature, was introduced by Hansen (2000). He developed a full statistical theory of the LS estimator for a linear cross-sectional model with threshold effects. Dang et al. (2012) and Seo and Shin (2016) propose a GMM type of estimation method for dynamic panel threshold models, while Ramírez-Rondán (2015) advocates for maximum likelihood estimation (MLE). All these studies assume either mutual or exclusive exogeneity for regressors and threshold variables. This is restrictive in a lot of empirical applications. Therefore, recent studies (Caner and Hansen (2004) and Yu and Phillips (2018), among others) consider endogeneity.

Goldberger (1972) and Joreskog and Goldberger (1975) were one of the first to incorporate factor models in econometrics. IFEs models with one factor are studied by Holtz-Eakin et al. (1988), who proposed a quasi differencing method, and Ahn et al. (2001), among others. A basic approach to control the unobserved heterogeneity is to treat λ_i and F_t as fixed-effects parameters to be estimated (as is done by Chamberlain (1984) and Arellano and Honore (2001)). Controlling fixed effects by directly estimating them results in the incidental parameter problem (Neyman and Scott (1948) and Nickell (1981) among others). The fixed T framework is studied by Kiefer (1980). Ahn et al. (2001) found the LS method to be inconsistent under fixed T and proposed a GMM estimate. For large T, an asymptotic bias can persist in a dynamic panel with fixed effects (Nickell (1981)) or in a nonlinear panel with fixed effects (Hahn and Newey (2004)). Hahn and Newey (2004) and Bai (2009), among others, propose bias-corrected estimators. Bai (2009) uses a large N and large T panel setup and assumes strictly exogenous regressors. A different approach is to use predetermined regressors, as is done by Moon and Weidner (2017).

As mentioned earlier, Miao et al. (2019) propose a panel threshold model with IFEs. In their research, they confirm the necessity of incorporating both the threshold effect and the interactive fixed effects into the model. Together with the latent group structures, these features should be able to capture a lot of heterogeneity in the panel and provide right estimates, which the results of this paper will confirm.

3 Panel structure threshold regression

In this section, first the PSTR model with additive fixed effects is derived (*Section 3.1*). Subsequently, the estimation procedure of this model is explained (*Section 3.2*).

3.1 PSTR model

The model with additive fixed effects is obtained from Miao et al. (2020). They define their PSTR model as

$$y_{it} = x'_{it}\beta^0_{g_i} + x'_{it}\delta^0_{g_i} d_{it}(\gamma^0_{g_i}) + \mu^0_i + \epsilon_{it}, \qquad i = 1, ..., N, \quad t = 1, ..., T, \tag{1}$$

where N is the number of cross-sectional units and T the number of periods. Moreover, let x_{it} be a $K \times 1$ vector of observable regressors, β a $K \times 1$ vector of slope coefficients, δ a $K \times 1$ vector of slope coefficients representing the threshold-effects, γ a scalar threshold coefficient, $d_{it}(\gamma) = 1\{q_{it} \leq \gamma\}$, q_{it} is a scalar threshold variable and ϵ_{it} is the idiosyncratic error term. μ_i is the additive fixed effects parameter that captures heterogeneity across individuals and is time-invariant. The superscript zero indicates the true parameter value.

The subscript g_i denotes the group membership variable which indicates to which group individual i belongs, such that $g_i \in \mathcal{G} = \{1, ..., G\}$, where G is the known number of groups. In contrary to the number of groups, the group allocation $\mathbf{G} = (g_1, ..., g_N)' \in \mathcal{G}^N$ is not known and has to be estimated from the data. The slope and threshold parameters can vary across groups, i.e. each group $g \in \mathcal{G}$ has its own set of parameters $(\beta^{0\prime}_g, \delta^{0\prime}_g, \gamma^{0\prime}_g)'$.

The slope and threshold-effect coefficients are summarized in $\Theta = (\theta'_1, ..., \theta'_G)' \in \mathcal{B}^G$, where $\theta_g = (\beta'_g, \delta'_g)'$. I will use a shrinking-threshold-effect, as is proposed by Hansen (2000), such that the threshold-effect converges to zero when N and T approach infinity simultaneously (that is $\delta^0_g \to 0$ as $(N, T) \to \infty$ for each $g \in \mathcal{G}$). I define the scalar threshold parameters as $\mathbf{D} = (\gamma_1, ..., \gamma_G)' \in \Gamma^G$, where I assume that $\gamma^0_g \in \Gamma = [\underline{\gamma}, \overline{\gamma}]$ for all $g \in \mathcal{G}$, where $\underline{\gamma}$ and $\overline{\gamma}$ are two fixed constants.

3.2 Estimation PSTR model

For the estimation of Model (1), I first reconstruct the model, such that it becomes

$$y_{it} = z_{it}(\gamma^0_{g^0_i})'\theta^0_{g^0_i} + \mu^0_i + \epsilon_{it}, \qquad i = 1, ..., N, \quad t = 1, ..., T, \tag{2}$$

where $z_{it}(\gamma) = (x_{it}', x_{it}' d_{it}(\gamma))'$. To get rid of the individual time-invariant fixed effects μ_i, which is convenient for the estimation, I perform a within-transformation. This leads to the model

$$\tilde{y}_{it} = \tilde{z}_{it}(\gamma^0_{g^0_i})'\theta^0_{g^0_i} + \tilde{\epsilon}_{it}, \qquad i = 1, ..., N, \quad t = 1, ..., T, \tag{3}$$

where $\tilde{y}_{it} = y_{it} - \frac{1}{T}\sum_{s=1}^{T} y_{is}$, and $\tilde{z}_{it}$ and $\tilde{\epsilon}_{it}$ are defined analogously.

The search for the least squares estimator of $(\Theta, \mathbf{D}, \mathbf{G})$ can start, which is the triplet that minimizes the sum of squared residuals (SSR). Hence, the estimators are defined as

$$(\hat{\Theta}, \hat{\mathbf{D}}, \hat{\mathbf{G}}) = \operatorname{argmin}_{(\Theta, \mathbf{D}, \mathbf{G}) \in \, \mathcal{B}^G \times \Gamma^G \times \mathcal{G}^N} \mathrm{SSR}(\Theta, \mathbf{D}, \mathbf{G}), \tag{4}$$

where the objective function is defined as

$$\mathrm{SSR}(\Theta, \mathbf{D}, \mathbf{G}) = \sum_{i=1}^{N}\sum_{t=1}^{T} (\tilde{y}_{it} - \tilde{z}_{it}(\gamma_{g_i})'\theta_{g_i})^2. \tag{5}$$

When the threshold $\mathbf{D}$ and the group allocation $\mathbf{G}$ are known, the estimators of the slope coefficients θ_g for all $g \in \mathcal{G}$ are defined as

$$\hat{\theta}_g(\mathbf{D}, \mathbf{G}) = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} 1\{g_i = g\}\tilde{z}_{it}(\gamma_g)'\tilde{z}_{it}(\gamma_g)\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T} 1\{g_i = g\}\tilde{z}_{it}(\gamma_g)'\tilde{y}_{it}\right). \tag{6}$$

Note that Θ is concentrated out of its estimator. I assume the group allocation $\mathbf{G}$ to be known. The estimator of the threshold $\mathbf{D}$ can now be computed as

$$\hat{\mathbf{D}} = \operatorname{argmin}_{\mathbf{D} \in \Gamma^G} \mathrm{SSR}(\hat{\Theta}(\mathbf{D}, \mathbf{G}), \mathbf{D}, \mathbf{G}). \tag{7}$$

Let $\hat{\Theta}(\mathbf{D}, \mathbf{G}) = (\hat{\theta}_1(\mathbf{D}, \mathbf{G})', ..., \hat{\theta}_G(\mathbf{D}, \mathbf{G})')'$ which are defined in Equation (6). Since I assume a known group allocation $\mathbf{G}$, I can compute $\hat{\mathbf{D}}$ and subsequently $\hat{\Theta}$ which is defined as $\hat{\Theta} = \hat{\Theta}(\hat{\mathbf{D}}, \mathbf{G})$.

$(\hat{\Theta}, \hat{\mathbf{D}})$ is the pair that minimizes the objective function for a given group allocation $\mathbf{G}$. However, it is possible to reduce the SSR even further, namely by reallocating the groups. I check for each observation i whether it would reach a lower squared residual with the estimates of a different group, i.e. whether it would be beneficial for the objective value if observation i would switch to a different group. When the groups are reassigned, the estimation process recommences. This procedure iterates until no observation switches groups anymore and the coefficients are converged. The algorithm that describes this procedure (which is proposed by Miao et al. (2020)) is the following:

Algorithm 1

Choose an initial group allocation $\mathbf{G}^{(0)}$. Let $s = 0$.

1. For given $\mathbf{G}^{(s)}$, compute

$$\mathbf{D}^{(s)} = \text{argmin}_{\mathbf{D}\in\Gamma^G} \text{SSR}(\hat{\Theta}(\mathbf{D}, \mathbf{G}^{(s)}), \mathbf{D}, \mathbf{G}^{(s)}).$$

2. For given $\mathbf{G}^{(s)}$ and $\mathbf{D}^{(s)}$, compute

$$\Theta^{(s)} = \hat{\Theta}(\mathbf{D}^{(s)}, \mathbf{G}^{(s)}).$$

3. For given $\mathbf{D}^{(s)}$ and $\Theta^{(s)}$, compute

$$\mathbf{G}^{(s+1)} = \text{argmin}_{\mathbf{G}\in\mathcal{G}^N} \text{SSR}(\Theta^{(s)}, \mathbf{D}^{(s)}, \mathbf{G}).$$

4. Set $s = s + 1$ and go to step 1. Repeat until convergence of the coefficients.

4 Interactive fixed effects

In the previous section, the PSTR model was considered. This section first describes a model where the additive fixed effects are interchanged with IFEs (*Section 4.1*). Thereafter, the estimation procedure of this extended model is elaborated (*Section 4.2*).

4.1 IFEs model

The model with interactive fixed effects is similar to the model from Section 3.1. Only the additive fixed effects parameter μ_i is removed and interactive fixed effects are included in a multiplicative form of factors and their loadings. The model becomes

$$y_{it} = x_{it}'\beta_{g_i}^0 + x_{it}'\delta_{g_i}^0 d_{it}(\gamma_{g_i}^0) + \lambda_i^{0\prime} f_t^0 + \epsilon_{it}, \qquad i = 1, ..., N, \quad t = 1, ..., T, \tag{8}$$

where, again, N is the number of cross-sectional units and T the number of time periods. x_{it} is a $K \times 1$ vector of observable regressors, β a $K \times 1$ vector of slope coefficients, δ a $K \times 1$ vector of slope coefficients representing the threshold-effects, γ a scalar threshold coefficient, $d_{it}(\gamma) = 1\{q_{it} \leq \gamma\}$, q_{it} is a scalar threshold variable and ϵ_{it} is the idiosyncratic error term.

λ_i is an $R^0 \times 1$ vector of unobserved factor loadings, f_t is an $R^0 \times 1$ vector of unobserved common factors. R^0 is the true number of factors. Note that the latent group structure is only applicable for the slope and threshold parameters.

4.2 Estimation IFEs model

For the estimation procedure, it is convenient to introduce the matrix notation of the model, that is

$$Y = X \cdot \beta^0 + X(\gamma^0) \cdot \delta^0 + \Lambda^0 F^{0\prime} + \epsilon, \tag{9}$$

where the product $X \cdot \beta$ takes into account the group membership, such that $[X \cdot \beta]_{it} = x'_{it}\beta_{g_i}$. Similarly, $[X(\gamma) \cdot \delta]_{it} = x'_{it}d_{it}(\gamma_{g_i})\delta_{g_i}$. Let $\Lambda = (\lambda_1, ..., \lambda_N)'$, an $N \times R$ matrix that contains N loadings for each factor. Let $F = (f_1, ..., f_T)'$, a $T \times R$ matrix that contains T time periods for each factor.

In contrast to Section 3, where the mere interests layed in the estimation of $(\theta_g^{0\prime}, \gamma_g^{0\prime})$ for all $g \in \mathcal{G}$, I am now also interested in the estimation of λ_i^0 for all $i \in \{1, ..., N\}$ and f_t^0 for all $t \in \{1, ..., T\}$. Following Bai and Ng (2002) and Bai (2003), I consider the following identification restrictions:

- $F'F/T = I_R$
- $\Lambda'\Lambda$ is a diagonal matrix with diagonal elements ordered in descending order

Those restrictions are necessary because $\Lambda F' = \Lambda A A^{-1} F'$ for any $R \times R$ invertible matrix A. An $R \times R$ invertible matrix has R^2 free elements. Hence, we need R^2 restrictions. The normalization $F'F/T = I_R$ is commonly used and implies $R(R+1)/2$ restrictions. However, this normalization still leaves rotation indeterminacy. To get rid of this, I define $\Lambda'\Lambda$ as a diagonal matrix. This fulfills the additional $R(R-1)/2$ restrictions that were needed for identification. Thus Λ and F are uniquely determined with these two sets of restrictions.

The least squares estimator of $(\Theta, \mathbf{D}, \mathbf{G}, \Lambda, F)$ is defined as the quintet that minimizes the sum of squared residuals. It can be obtained by solving

$$(\hat{\Theta}, \hat{\mathbf{D}}, \hat{\mathbf{G}}, \hat{\Lambda}, \hat{F}) = \text{argmin}_{(\Theta, \mathbf{D}, \mathbf{G}, \Lambda, F) \in \mathcal{B}^G \times \Gamma^G \times \mathcal{G}^N \times \mathrm{R}^N \times \mathrm{R}^T} \text{SSR}(\Theta, \mathbf{D}, \mathbf{G}, \Lambda, F), \tag{10}$$

where the objective function is defined as

$$
\begin{aligned}
\mathrm{SSR}(\Theta,\mathbf{D},\mathbf{G},\Lambda,F) &= \sum_{i=1}^{N}\sum_{t=1}^{T}\Big(y_{it}-x_{it}'\beta_{g_i}-x_{it}(\gamma_{g_i})'\delta_{g_i}-\lambda_i' f_t\Big)^2 \\
&= \big\|Y-X\cdot\beta-X(\gamma)\cdot\delta-\Lambda F'\big\|_F^2 \\
&= \mathrm{Tr}\Bigg[\Big(Y-X\cdot\beta-X(\gamma)\cdot\delta-\Lambda F'\Big)'\Big(Y-X\cdot\beta-X(\gamma)\cdot\delta-\Lambda F'\Big)\Bigg]
\end{aligned}
\tag{11}
$$

Again, I assume that the group allocation is known. Using an equivalent approach as the one from Moon and Weidner (2017), I can formulate the expression for $\mathcal{Q}(\Theta,\mathbf{D})$ which denotes the minimum value of $\mathrm{SSR}(\Theta,\mathbf{D},\mathbf{G},\Lambda,F)$ over Λ and F, given that $\mathbf{G}$ is known. Therefore, it holds that

$$
(\hat{\Theta},\hat{\mathbf{D}}) = \mathrm{argmin}_{(\Theta,\mathbf{D})\in\, \mathcal{B}^G\times\Gamma^G}\ \mathcal{Q}(\Theta,\mathbf{D}) \tag{12}
$$

where $\mathcal{Q}(\Theta,\mathbf{D})$ is defined as

$$
\begin{aligned}
\mathcal{Q}(\Theta,\mathbf{D}) &= \min_{\Lambda,F}\ \mathrm{SSR}(\Theta,\mathbf{D},\mathbf{G},\Lambda,F) \\
&= \min_F\ \mathrm{Tr}\Bigg[\Big(Y-X\cdot\beta-X(\gamma)\cdot\delta\Big)M_F\Big(Y-X\cdot\beta-X(\gamma)\cdot\delta\Big)'\Bigg] \\
&= \sum_{r=R+1}^{T}\mu_r\Bigg[\Big(Y-X\cdot\beta-X(\gamma)\cdot\delta\Big)'\Big(Y-X\cdot\beta-X(\gamma)\cdot\delta\Big)\Bigg]
\end{aligned}
\tag{13}
$$

The second expression for $\mathcal{Q}(\Theta,\mathbf{D})$ in Equation (13) is obtained by concentrating out Λ (see Bai (2009)). The optimal F is given by the R eigenvectors that correspond to the R largest eigenvalues of the $T\times T$ matrix $\big(Y-X\cdot\beta-X(\gamma)\cdot\delta\big)'\big(Y-X\cdot\beta-X(\gamma)\cdot\delta\big)$. Bai (2009) confirms this by stating that $\hat{F}$ can be found by solving $P\hat{F}=\hat{F}V$, where P is the $T\times T$ matrix mentioned earlier, and V a diagonal matrix consisting of the R largest eigenvalues of P in decreasing order. Therefore, $\mathcal{Q}$ is equal to the sum over the $T-R$ smallest eigenvalues of this $T\times T$ matrix. Equivalence between the lines of Equation (13) is shown by Moon and Weidner (2017).

The estimates $(\hat{\Theta},\hat{\mathbf{D}})$ are obtained by performing a grid search over the possible value of $\mathbf{D}$ and find the corresponding value of Θ that minimizes $\mathcal{Q}(\Theta,\mathbf{D})$. $(\hat{\Theta},\hat{\mathbf{D}})$ are the values of $\mathbf{D}$ and Θ that generate the lowest value for $\mathcal{Q}(\Theta,\mathbf{D})$. According to Hansen (1999), it suffices to search over the values of the threshold variable q_{it}, which are at most NT values. I eliminate the smallest and largest 5% to reduce the number of possible values for the threshold parameter to $0.9NT$.

The estimate $\hat{F}$ can simply be constructed with the R eigenvectors that correspond to the R

largest eigenvalues of the matrix mentioned above, using $(\hat{\Theta}, \hat{\mathbf{D}})$ for the values of the parameters.

Using the steps in Bai (2009), I can write $\hat{\Lambda}$ as a function of Θ, $\mathbf{D}$ and F. The estimator of Λ can be computed using the following formula:

$$\hat{\Lambda}' = (\hat{\lambda}_1, ..., \hat{\lambda}_N) = T^{-1}\Big[\hat{F}'\big(Y_1 - X_1 \cdot \hat{\beta} - X(\hat{\gamma})_1 \cdot \hat{\delta}\big), ..., \hat{F}'\big(Y_N - X_N \cdot \hat{\beta} - X(\hat{\gamma})_N \cdot \hat{\delta}\big)\Big]. \quad (14)$$

Earlier, I assumed that the group allocation was known for the purpose of estimating the other parameters. Now that these are all estimated, the group allocation is being reconsidered in order to decrease the SSR even further. After reassigning the groups, the estimation procedure can start again. This results in an algorithm that is somewhat similar to Algorithm 1 in Section 3.2. The algorithm for the estimation of a threshold model with interactive fixed effects and latent group structures is shown in Algorithm 2.

Algorithm 2
Choose an initial group allocation $\mathbf{G}^{(0)}$. Let $s = 0$.

1. For given $\mathbf{G}^{(s)}$, compute

$$(\Theta^{(s)}, \mathbf{D}^{(s)}) = \text{argmin}_{(\Theta,\mathbf{D})\in \mathcal{B}^G\times\Gamma^G} \mathcal{Q}(\Theta, \mathbf{D}).$$

2. For given $\mathbf{G}^{(s)}$, $\Theta^{(s)}$ and $\mathbf{D}^{(s)}$, construct $F^{(s)}$ using the R eigenvectors that correspond to the R largest eigenvalues of the $T \times T$ matrix $\big(Y - X \cdot \beta^{(s)} - X(\gamma^{(s)}) \cdot \delta^{(s)}\big)'\big(Y - X \cdot \beta^{(s)} - X(\gamma^{(s)}) \cdot \delta^{(s)}\big)$.

3. For given $\mathbf{G}^{(s)}$, $\Theta^{(s)}$, $\mathbf{D}^{(s)}$ and $F^{(s)}$, compute

$$\Lambda^{(s)\prime} = T^{-1}F^{(s)\prime}\big(Y - X \cdot \beta^{(s)} - X(\gamma^{(s)}) \cdot \delta^{(s)}\big).$$

4. For given $\Theta^{(s)}$, $\mathbf{D}^{(s)}$, $F^{(s)}$ and $\Theta^{(s)}$, compute

$$\mathbf{G}^{(s+1)} = \text{argmin}_{\mathbf{G}\in\mathcal{G}^N} \text{SSR}(\Theta^{(s)}, \mathbf{D}^{(s)}, \mathbf{G}, \Lambda^{(s)}, F^{(s)}).$$

5. Set $s = s + 1$ and go to step 1. Repeat until convergence of the coefficients.

5 Monte Carlo simulations

Monte Carlo (MC) simulations are used to evaluate the performance of the estimates in finite samples for both models in this section. First, I evaluate the performances of the estimators of the PSTR model (*Section 5.1*). Subsequently, the data generating process (DGP) is adapted so that I can investigate the, with IFEs extended, model (*Section 5.2*).

5.1 MC with PSTR model

In this subsection, first a DGP is introduced that is consistent with the PSTR model from Miao et al. (2020) (*Section 5.1.1*). Then, the results of the Monte Carlo simulation are provided and discussed (*Section 5.1.2*).

5.1.1 Data generating process PSTR model

The DGP for the PSTR model that I use, is a static panel structure model, which is obtained from Miao et al. (2020). From now on, I call this data generating process DGP 1. The case were both the slope coefficients and the threshold parameters are heterogeneous (and hence vary across groups) is considered. DGP 1 looks as follows:

$$y_{it} = \mu_i + \beta_{g_i} x_{it} + \delta_{g_i} x_{it} 1\{q_{it} > \gamma_{g_i}\} + \epsilon_{it}, \tag{15}$$

where $\mu_i = T^{-1} \sum_{t=1}^{T} x_{it}$, and x_{it} follows i.i.d.N(0,1). The slope coefficients $\theta_{g_i} = (\beta'_{g_i}, \delta'_{g_i})'$ are heterogeneous across the three groups that I specify, and are defined as

$$(\beta_1, \beta_2, \beta_3) = (1, 1.75, 2.5)$$

$$(\delta_1, \delta_2, \delta_3) = (1, 1, 1) \times c(NT)^{-0.1},$$

where c controls the size of the threshold effect and is set to 1. The groups are divided with the proportions 0.3 : 0.3 : 0.4, which are fixed ratios. I generate the threshold variable q_{it} from an i.i.d.N(1,1). ϵ_{it} is a heteroskedastic error term, which is computed as $\epsilon_{it} = \sigma_{it} e_{it}$, where $\sigma_{it} = (s + 0.1x_{it}^2)^{1/2}$, where s (which I equal to 0.5) controls the signal-to-noise ratio, and e_{it} is drawn from an i.i.d.N(0,1). The heterogeneous thresholds are defined as $\mathbf{D} = (\gamma_1, \gamma_2, \gamma_3)' = (0.5, 1, 1.5)'$.

I use two cross-sectional sample sizes ($N \in \{50, 100\}$) and two different time periods ($T \in \{30, 60\}$), thus in total there are four combinations of dimensions. I replicate the estimation procedure 100 times.

5.1.2 Estimation results PSTR model

The accuracy of group classification is evaluated using the average misclassification rate across replications. For the slope coefficients (β and δ), I provide the bias, the root mean squared error (RMSE), and the coverage probability (CP) of the two-sided nominal 95% confidence interval. I evaluate the threshold parameter estimates (γ) based on the bias and the coverage probability.

Table 1 shows the average misclassification rate across replications. As can be seen, the misclassification rate does not change drastically when N increases. However, it significantly decreases when T increases.

Table 1: Misclassification rate DGP 1

	T = 30	T = 60
N = 50	0.0226	0.0024
N = 100	0.0257	0.0013

Table 2 presents the results for the slope coefficients and threshold parameters of DGP 1. The slope coefficients are estimated precisely, as the biases are small and the coverage probabilities are generally close to the nominal 95% level. DGP 1 also provides a precise estimation of the threshold parameter, as the coverage probabilities, which are all close to the nominal level, suggest. As could be expected, the performance measures generally improve as N or T increase.

Table 2: Finite sample performances of the slope coefficients and threshold values of DGP 1

		β			δ			γ	
		Bias	RMSE	CP	Bias	RMSE	CP	Bias	CP
N=50	Group 1	-0.007	0.089	0.86	0.010	0.099	0.90	-0.005	0.94
T=30	Group 2	-0.011	0.073	0.88	0.000	0.082	0.91	-0.010	0.92
	Group 3	0.002	0.037	0.94	-0.008	0.078	0.90	-0.019	0.91
N=50	Group 1	0.003	0.056	0.90	0.002	0.061	0.94	-0.006	0.90
T=60	Group 2	-0.002	0.048	0.89	-0.008	0.061	0.90	-0.001	0.91
	Group 3	-0.002	0.033	0.91	-0.003	0.061	0.90	-0.004	0.94
N=100	Group 1	0.003	0.053	0.96	-0.003	0.062	0.94	0.006	0.88
T=30	Group 2	0.006	0.044	0.90	-0.015	0.057	0.92	-0.003	0.94
	Group 3	0.002	0.033	0.86	-0.003	0.053	0.95	-0.003	0.89
N=100	Group 1	0.000	0.040	0.88	0.001	0.047	0.89	0.006	0.93
T=60	Group 2	0.001	0.029	0.92	-0.005	0.040	0.93	-0.003	0.93
	Group 3	0.003	0.020	0.94	-0.004	0.046	0.90	0.000	0.95

5.2 MC with IFEs model

In this subsection, I start by providing a DGP that includes IFEs (*Section 5.2.1*). Next, the simulation results of this DGP are stated (*Section 5.2.2*).

5.2.1 Data generating process IFEs model

The process for generating the data in the with IFEs extended model (DGP 2) is very similar to DGP 1. Only here, I remove the time-invariant additive fixed effects (μ_i), and instead I use IFEs. DGP 2 looks as follows:

$$y_{it} = \beta_{g_i} x_{it} + \delta_{g_i} x_{it} 1\{q_{it} > \gamma_{g_i}\} + \lambda_i f_t + \epsilon_{it}, \tag{16}$$

where x_{it}, q_{it} and ϵ_{it} are generated the same way as in GDP 1, and the slope coefficients and threshold parameters have the same values as in GDP 1. λ_i follows from an i.i.d.N(0,1) and f_t from an i.i.d. $0.7 \times N(0,1)$.

The combinations of dimensions and the number of iterations are the same as for DGP 1.

5.2.2 Estimation results IFEs model

For DGP 2, the same accuracy measures are computed as earlier for DGP 1. According to Table 3, which presents the average misclassification rate across replications for DGP 2, the same finding as earlier can be concluded; the accuracy of classification strongly increases when T increases, while the cross-sectional sample size N does not seem to influence the misclassification rate too much.

Table 3: Misclassification rate DGP 2

	T = 30	T = 60
N = 50	0.0242	0.0018
N = 100	0.0253	0.0026

Table 4 shows the simulation results for DGP 2. The performances are comparable to the ones from DGP 1. They are generally not way better or worse. This implies that the estimation preciseness of the coefficients is more or less similar. However, this does not mean that the the model with IFEs performs equivalently to the PSTR model. In the next section, I show that IFEs increase the fit of the model.

An interesting finding is that the average biases of λ and F are rather small (-0.008 and 0.018 respectively), and the multiplication of the two, which forms the interactive fixed effect, has an average bias of -0.001.

Table 4: Finite sample performances of the slope coefficients and threshold values of DGP 2

		β			δ			γ	
		Bias	RMSE	CP	Bias	RMSE	CP	Bias	CP
N=50	Group 1	-0.016	0.076	0.91	0.022	0.093	0.91	0.000	0.93
T=30	Group 2	-0.015	0.072	0.85	0.017	0.094	0.86	-0.020	0.95
	Group 3	-0.010	0.061	0.91	0.022	0.086	0.90	-0.042	0.98
N=50	Group 1	-0.005	0.059	0.86	0.009	0.070	0.90	0.001	0.93
T=60	Group 2	-0.009	0.046	0.88	0.009	0.063	0.89	-0.008	0.92
	Group 3	-0.006	0.032	0.89	0.018	0.056	0.91	-0.004	0.92
N=100	Group 1	-0.012	0.058	0.87	0.009	0.065	0.91	0.001	0.94
T=30	Group 2	-0.008	0.046	0.88	0.020	0.062	0.95	0.005	0.93
	Group 3	0.002	0.032	0.92	0.001	0.055	0.92	0.006	0.93
N=100	Group 1	-0.008	0.036	0.93	0.008	0.047	0.91	0.000	0.96
T=60	Group 2	-0.005	0.030	0.92	0.002	0.042	0.93	-0.001	0.97
	Group 3	0.000	0.020	0.92	0.007	0.041	0.94	0.003	0.95

6 Empirical application

Now that I have evaluated the finite properties of the estimators, I apply the estimation approach form Section 4 to the relationship between financial development and economic growth. This relationship has been studied by many already. First, I provide a short literature overview (*Section 6.1*), after which the model is introduced (*Section 6.2*) and the results are presented and discussed (*Section 6.3*).

6.1 Literature review on finance-growth relationship

Economists are not always on the same line when it comes to the role of the financial sector in economic growth. Nobel prize winner Lucas (1988) dismisses finance as an "over-stressed" determinant of economic growth, while recent studies (Levine (2004) and Law and Singh (2014) among others) have demonstrated that there is a positive long-run association between economic growth and indicators of financial development. They suggest that a well-developed financial market is growth-enhancing. However, researchers at the Bank for International Settlement (BIS) and International Monetary Fund (IMF) have argued that the level of financial development is good only up to a certain point, after which it counteracts on economic growth. This implies that the relationship between finance and growth has a turning point.

Law and Singh (2014) use a panel data set of 87 countries covering 1980 through 2010, on several macroeconomic series. They confirm the threshold effect in the finance-growth relationship. However, as Miao et al. (2019) argue, the cross-sectional correlation that originates from the unobserved common factors is generally ignored. With their empirical study, they confirm the necessity of both threshold effects and interactive fixed effects. Even more heterogeneity can be captured by clustering the countries. Therefore, I revisit this relation and include all the concepts that are discussed in this research.

6.2 Finance-growth model

After adapting the model from Law and Singh (2014) and from Miao et al. (2019) to incorporate all the features discussed, the model becomes

$$GROWTH_{it} = \beta_{g_i} FIN_{it} + \delta_{g_i} FIN_{it} 1\{FIN_{it} > \gamma_{g_i}\} + \alpha'_{g_i} X_{it} + \lambda'_i f_t + \epsilon_{it}, \tag{17}$$

where $GROWTH_{it}$ denotes the economic growth of country i in year t, FIN_{it} the level of financial development of country i in year t and X_{it} a vector of control variables. Similar to Miao et al. (2019), I do not consider a threshold effect in the slope of the control variables.

Similar to Law and Singh (2014), I collect annual data from the World Development Indicators database between 1970 and 2018. This results in a balanced panel with $N = T = 49$. The measure of financial development are the financial resources that a country provides to the private sector (domestic credit to private sector (CPS)) as a percentage of the GDP. The control variables are initial per capita GDP, population growth and a constant.

In Table 5, the descriptive statistics of the used variables are shown. As can be seen, the economic growth has a relative high deviation compared to the domestic credit to private sector.

Table 5: Descriptive statistics

	Unit of measurement	mean	std dev	median	min	max
Growth	%	3.693	4.320	3.901	-24.049	39.487
CPS	log(% of GDP)	3.353	0.893	3.288	0.433	5.733
Lag GDP Per Capita	log(US$) 2010 constant price	8.246	1.426	8.057	5.609	11.431
Population Growth	%	1.885	1.001	2.010	-1.475	6.017

Countries: Algeria, Australia, Bolivia, Cameroon, Chile, Congo Rep., Costa Rica, Cote d'Ivoire, Denmark, Ecuador, Egypt, El Salvador, Gabon, Ghana, Guatemala, Guyana, Haiti, Honduras, Iceland, India, Israel, Jamaica, Japan, Kenya, Malaysia, Mali, Mexico, Niger, Norway, Pakistan, Panama, Paraguay, Peru, Philippines, Senegal, Sierra Leone, Singapore, Sri Lanka, Sudan, Sweden, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, United Kingdom, United States, Uruguay, Zambia.
N = 49 cross-country, T = 1970-2018.

To be able to compare the model, I introduce five benchmark models. From now on, I will refer to Equation (17) as Model 1. The other models use the same variables as Model 1, but features such as threshold effects, IFEs and group structures are being left out. Table 6 provides an overview of the features that are included in the models. Note that all models use the exact same data. The models differ in the way they are structured.

Table 6: Summary of the models

	Threshold effect	IFEs	latent group structure
Model 1	✓	✓	✓
Model 2	✓	✗	✓
Model 3	✓	✗	✗
Model 4	✗	✓	✓
Model 5	✗	✗	✓
Model 6	✗	✗	✗

6.3 Finance-growth estimation results

Table 7 displays the estimation results of the six different models. In this subsection, I will mainly focus on the results from Model 1, which is the model where the concepts of threshold effects, IFEs, and latent group structures are incorporated. In the end, I compare the different models by their coefficient of determination.

After several iterations with different initial allocations, the coefficients are converged and the countries are divided into two groups of similar size (23 and 26 for group 1 and 2 respectively). Group 1 mainly consists of rich countries such as Australia, Denmark, Norway, Sweden, the UK and the USA. The poorer countries, such as Ghana, Mali, Philippines, Sierra Leone, Sudan and Zambia, form group 2.

We can see that the threshold coefficient γ for the two groups are 2.385 (group 1) and 4.565 (group 2) which correspond to a credit to the private sector as large as 10.86% and 96.06% of the GDP respectively. In the first group, only 6.46% of the observations falls below the corresponding threshold level, which means the vast majority of the subsample obtains the threshold effect. In contrary to the group 1, group 2 has a vast majority (93.55%) of the observations that is smaller than their threshold level.

The estimates of β and δ for the group 1 suggest that CPS is a negative and statistically significant determinant of economic growth if it is less than the threshold level, and the threshold effect that arises for observations larger than the threshold, is positive and statistically significant. However, the total effect of financial indicator on growth remains negative for observations above the threshold level (as $-2.373 + 1.353 < 0$).

The estimates of group 2 show that financial development positively and significantly explains economic growth for observations below the threshold, while the threshold effect is negative and statistically significant. We see that the total effect of CPS remains positive for observations above the threshold (as $1.149 - 0.496 > 0$).

In both groups, the effect of financial development on economic growth does not change sign when observations exceed the estimated threshold. This is in contrast with the results from Miao et al. (2019), who show that the effect of financial development on growth is positive until the financial development exceeds a certain threshold. Their findings are in line with the literature from Section 6.1.

Nevertheless, in my findings, the threshold effect is present, as the threshold-effect parameters (δ) are significant at a 1% significance level. The reason why the effect of financial development does not switch sign as soon as it exceeds a certain threshold, could be because the countries are divided into two groups. One group consists of countries where financial development on average has a negative impact on growth (group 1), and a second group where financial development generally stimulates economic growth (group 2). However, for both groups, the finance-effect of the observations that exceed the estimated threshold changes towards the other direction, which is to some extend in line with the literature.

Although the effects of the control variables seem to be similar across the two groups, the threshold coefficients and the effect of financial development on growth seem to differ substantially. It would not be good for the model to combine the two groups into one big sample, thus the segmentation of the countries helped in explaining economic growth.

Table 7: Finance-growth relationship: estimation results

	Model 1				Model 2				Model 3	
	Group 1		Group 2		Group 1		Group 2			
Threshold (γ):										
Threshold level	2.385 (log10.86)		4.565 (log96.06)		2.398 log(11.00)		4.569 log(96.45)		4.680 log(107.77)	
Sample Quantile	6.46%		93.55%		7.19%		90.11%		90.88%	
β (CPS)	-2.373***	(0.449)	1.149***	(0.188)	-2.437***	(0.457)	1.151***	(0.188)	0.693***	(0.147)
δ (CPS)	1.353***	(0.300)	-0.496***	(0.117)	1.351***	(0.292)	-0.492***	(0.103)	-0.365***	(0.075)
Lag GDP Per Capita	0.704***	(0.126)	0.048	(0.149)	0.696***	(0.127)	0.032	(0.148)	0.095	(0.091)
Population Growth	0.875***	(0.139)	0.710***	(0.173)	0.810***	(0.140)	0.717***	(0.170)	0.821***	(0.109)
Intercept	-0.926	(1.034)	-0.955	(1.178)	-0.541	(1.044)	-0.882	(1.176)	-0.798	(0.809)
Sample size	23		26		23		26		49	
	Model 4				Model 5				Model 6	
	Group 1		Group 2		Group 1		Group 2			
CPS	-1.016***	(0.202)	1.017***	(0.182)	-0.760***	(0.197)	1.037***	(0.185)	0.411***	(0.135)
Lag GDP Per Capita	0.761***	(0.124)	0.010	(0.149)	0.605***	(0.122)	-0.042	(0.151)	0.039	(0.091)
Population Growth	0.717***	(0.139)	0.675***	(0.175)	0.664***	(0.138)	0.737***	(0.178)	0.812***	(0.109)
Intercept	-1.349*	(1.027)	-0.352	(1.153)	-0.794	(1.027)	-0.106	(1.159)	0.466	(0.770)
Sample size	25		24		25		24		49	

The values without parentheses (the left column) are the least squares estimates and the values in parentheses (the right column) are the corresponding standard errors.
*, ** and *** imply significance at a 10%, 5% and 1% significance level repectively.

The R-squared measures of the models are displayed in Table 8. We can see that the model that includes the threshold effect, interactive fixed effects and latent group structures explains economic growth the best. It explains the dependent variable even three times as much as the model that does not include any of the features (Model 6). Therefore, including those effects and structures results in a better regression model.

Table 8: R-squared measures Model 1-6

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
0.0821	0.0768	0.0356	0.0758	0.0704	0.0260

7 Conclusion

This research covers the least squares estimation of the PSTR model from Miao et al. (2020), and the with IFEs extended version of this model. The PSTR model combines threshold effects and latent group structures, and is proven to capture a large degree of heterogeneity that cannot be captured by traditional panel threshold regressions. IFEs allow for unobserved heterogeneity to vary cross-sectionally and serially through the dependence on common factors. This dependence is present in many macro or financial data.

Allowing and controlling for (unobserved) heterogeneity is one of the most attractive features of panel data models. Therefore, this paper aims to combine the concepts of threshold effects, IFEs, and latent group structures into one model, as no other study has done so far.

The estimators perform well in finite samples, as the Monte Carlo simulation results suggest. The preciseness of the slope and threshold estimators are similar for the PSTR model and the model with IFEs. Hence, IFEs do not increase nor decrease the accuracy of the estimators. The segmentation of the data is accurate. As the results show, the cross-sectional sample size N does not have a significant impact on the classification accuracy, in contrary to the number of time periods T, which improve the accuracy drastically when it increases.

The application of the relationship between economic growth and financial development showed the practicality of the model. The results support the model where the concepts of threshold effects, IFEs, and latent group structures are included, as it provides statistically significant estimators that give interesting insights into this finance-growth relation. Also, it outperforms several benchmark models that do not include all the discussed features.

For further research, several things can be looked at. First, it can be interesting to include the estimation of the number of groups and the number of factors, as for now, I assumed those numbers to be known. Moreover, panel regressions with endogeneity is an interesting feature to consider, as it has become a concern in recent threshold models (see Yu and Phillips (2018)). Third, I only use one threshold, while multiple thresholds might provide a better explanation in some cases.

References

Ahn, S., Lee, Y. H., and Schmidt, P. (2001). Gmm estimation of linear panel data models with time-varying individual effects. *Journal of Econometrics*, 10(2):219–255.

Arellano, M. and Honore, B. (2001). Panel data models: some recent developments. *Handbook of Econometrics*, 5:3229–3296.

Bai, J. (2003). Inferential theory for factor models of large dimensions. *Econometrica*, 71(1):135–171.

Bai, J. (2009). Panel data models with interactive fixed effects. *Econometrica*, 77(4):1229–1279.

Bai, J. and Liao, Y. (2016). Efficient estimation of approximate factor models via penalized maximum likelihood. *Journal of Econometrics*, 191(1):1–18.

Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221.

Bonhomme, S. and Manresa, E. (2015). Grouped patterns of heterogeneity in panel data. *Econometrica*, 83(3):1147–1184.

Caner, M. and Hansen, B. (2004). Instrumental variable estimation of a threshold model. *Econometric Theory*, 20(5):813–843.

Chamberlain, G. (1984). Panel data. *Handbook of Econometrics*, 2:1247–1318.

Chan, K. S. (1993). Consistency and limiting distribution of the least squares estimator of a threshold autoregressive model. *Ann. Statist.*, 195(1):520–533.

Dang, V. A., Kim, M., and Shin, Y. (2012). Asymmetric capital structure adjustments: New evidence from dynamic panel threshold models. *Journal of Empirical Finance*, 19(4):465–482.

Goldberger, A. S. (1972). Structural equation methods in the social sciences. *Econometrica*, 40(6):979–1001.

Hahn, J. and Newey, W. (2004). Jackknife and analytical bias reduction for nonlinear panel models. *Econometrica*, 72(4):1295–1319.

Hansen, B. E. (1999). Threshold effects in non-dynamic panels: Estimation, testing, and inference. *Journal of Econometrics*, 93:345–368.

Hansen, B. E. (2000). Sample splitting and threshold estimation. *Econometrica*, 68(3):575–603.

Hansen, B. E. (2011). Threshold autoregression in economics. *Statistics and Its Interface*, 195(2):123–127.

Holtz-Eakin, D., Newey, W., and Rosen, H. S. (1988). Estimating vector autoregressions with panel data. *Econometrica*, 56(6):1371–1395.

Joreskog, K. G. and Goldberger, A. S. (1975). Estimation of a model with multiple indicators and multiple causes of a single latent variable. *Journal of the American Statistical Association*, 70(351):631–639.

Kiefer, N. M. (1980). Estimation of fixed effect models for time series of cross-sections with arbitrary intertemporal covariance. *Journal of Econometrics*, 14(2):195–202.

Law, S. H. and Singh, N. (2014). Does too much finance harm economic growth? *Journal of Banking Finance*, 41(C):36–44.

Levine, R. (2004). Finance and growth: Theory and evidence. Working Paper 10766, National Bureau of Economic Research.

Li, D., Qian, J., and Su, L. (2016). Panel data models with interactive fixed effects and multiple structural breaks. *Journal of the American Statistical Association*, 111(516):1804–1819.

Lin, C.-C. and Ng, S. (2012). Estimation of panel data models with parameter heterogeneity when group membership is unknown. *Journal of Econometric Methods*, 1(1):42–55.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42.

Miao, K., Li, K., and Su, L. (2019). Panel threshold models with interactive fixed effects. *School of Economics*.

Miao, K., Su, L., and Wang, W. (2020). Panel threshold regressions with latent group structures. *Journal of Econometrics*, 214(2):451–481.

Moon, H. and Weidner, M. (2017). Dynamic linear panel regression models with interactive fixed effects. *Econometric Theory*, 33:158–195.

Neyman, J. and Scott, E. L. (1948). Consistent estimates based on partially consistent observations. *Econometrica*, 16(1):1–32.

Nickell, S. (1981). Biases in dynamic models with fixed effects. *Econometrica*, 49(6):1417–1426.

Ramírez-Rondán, N. (2015). Maximum likelihood estimation of dynamic panel threshold models. *Peruvian Economic Association*, 32.

Seo, M. H. and Shin, Y. (2016). Dynamic panels with threshold effect and endogeneity. *Journal of Econometrics*, 195(2):169–186.

Su, L. and Ju, G. (2018). Identifying latent grouped patterns in panel data models with interactive fixed effects. *Journal of Econometrics*, 206(2):554–573.

Su, L., Shi, Z., and Phillips, P. C. (2016). Identifying latent structures in panel data. *Econometrica*, 84(6):2215–2264.

Yu, P. and Phillips, P. C. (2018). Threshold regression with endogeneity. *Journal of Econometrics*, 203(1):50–68.