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Bachelor Thesis [Econometrics and Operations Research]

The Relationship between Demographic Structures and Macroeconomic Variables:
A study on the cointegration relationship and time-varying coefficients

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Abstract

This paper gives insights into the relationship between demographic structures and macroeconomic variables, which is emphasized to be important. Mainly because of the fact that the population is aging and that the contribution of different age groups to these variables is considered to be different. At first, I use the vector autoregressive model as suggested in Aksoy et al. (2019) to determine the long-run relationships between the macroeconomic variables and three different age groups (0-19; young dependents, 20-59; working population, and 60+; old dependents). The effects of the working population and the old dependents on the macroeconomic variables turn out to be opposite. All in all, there is a clear existence of the life-cycle pattern in the coefficients. Secondly, I determine whether the macroeconomic variables are stationary by means of the augmented Dickey-Fuller test and the Philips-Perron test. Thereafter, the cointegration relationships are examined by means of Johansen's methodology. The trace test and maximum eigenvalue test suggest six linearly independent cointegartion relationships between the endogenous macroecnomic variables. At last, I use a time dummy approach to examine whether the coefficients differ over time. It turns out that there is fluctuation in the coefficients, especially for the dependent groups.

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1 Introduction

The biological phenomenon of birth and death is a continuous process in our contemporary life, without us actually realizing it. Every day the population structure changes, not only by means of birth and death, but also due to aging. Consider the baby boomers - born between 1942 and 1962 - who are close to, or already in their retirement. Figure 1 shows the ratio of three age groups over the period of 1970-2014. During this period, the ratio of the 60+ age group has strongly increased with approximately ten percentage points. The opposite happened with the 0-19 age group. The 20-59 age group seems fairly stable over time, containing a small drop from the 21st century.

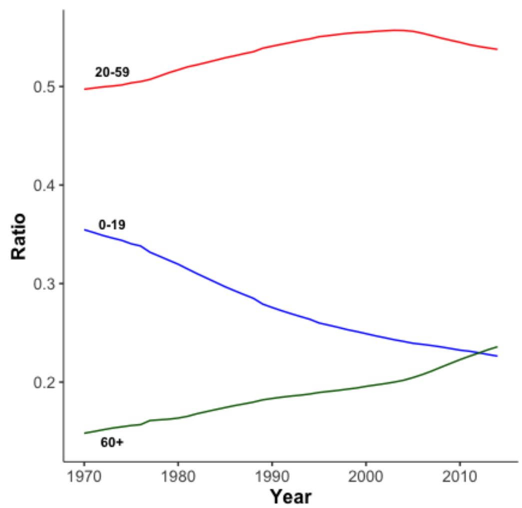


Figure 1: Average share of the three age groups by year

When dividing the population in these three age groups, namely, the young dependents (0-19), working population (20-59) and the old dependents (60+), it is plausible to think that their contribution to macroeconomic variables is different. Someone aged 30 is more likely to work than someone aged 15 or 73, due to educational and retirement purposes. This is actually one aspect of the life-cycle hypothesis, which says that age structures matters because of their different saving behaviours. In addition to the aforementioned baby boomers, the lower fertility rate and increased longevity also ensure different population structures.

In this paper, I capture the consequences of a changing demographic structure on different macroeconomic variables, which are elaborated in more detail in Section 3. This is done by a vector autoregressive (VAR) model (Aksoy, Basso, Smith, & Grasl, 2019). Furthermore, I examine the cointegration relationships by means of a vector error correction model (VECM) which is mostly based on the methodology of Johansen (1988, 1991). The error correction coefficients give an idea about which variables are error correcting when the system is out of equilibrium. At last, I incorporate a time dummy approach that allows coefficients to differ over time, in contrast to the VAR model.

In practical, the given insights can be used widely. For example, it helps to understand and determine monetary policies. The policies can be adjusted, accommodating or contradicting, in such a way that it is found optimal for a country's economic society, by the policymakers.

The paper is structured as follows: in Section 2 I will discuss the existing literature about cointegration relationships and time-varying coefficient models. Section 3 describes the data and gives some characterisations of the variables. In section 4, I will explain the used methodology in detail. The corresponding results will follow in Section 5. The paper is concluded in Section 6.

2 Literature

In this section, the literature of the various topics is discussed. At first, in Section 2.1, I will briefly discuss the literature on modeling the macroeconomic effects of demography. Thereafter, the literature on cointegration relationships is discussed in Section 2.2. At last, I will discuss the literature on admitting time-varying coefficients in Section 2.3.

2.1 Modeling macroeconomic effects of demography

In the past studies on the macroeconomic effects of demography, Higgins and Williamson (1997), and Bloom et al. (2007) mostly focused on a single macroeconomic variable of interest. They use average data from the past five years and summarize the demography by a single statistic, or using a low-order polynomials in the parameters as is done by Fair and Dominguez (1991), and Higgins (1998). A single equation model is also used by Favero and Galasso (2015), however they use annual data and population weights for estimation. Feyrer (2007) examined the relation between the working population and aggregate productivity. Again, a single macroeconomic variable is considered. However, he used a panel data setting and strictly focused on the age structure of the workforce. He found that changes in this structure is significantly correlated with with changes in the aggregate productivity.

Furthermore, there are multiple papers that provides insights in the relationship between demographic structures and stock prices, or asset returns. Poterba (2001) considered the historical relation between population structure, particularly the prime saving population (aged 40 to 64), and several financial instruments. He shows that the theoretical models suggest a change in the equilibrium returns on financial assets due to changes in the demographic structure. However, it is difficult to provide robust evidence of such relationships due to the limited power of statistical tests. Jamal and Quayes (2004) focused on the same age group as Poterba did. Although, they studied the influence on the price-dividend ratio and stock market activities. They concluded an expected drop in the demand for financial assets caused by a decline in the proportion of this age group.

In contrast to the single equation models, Aksoy, Basso, Smith and Grasl (2019) decided to estimate a panel vector autoregressive model (VAR) using annual data, a more detailed representation of the demographic structure and a larger sample. Therefore, they allowed the macroeconomic variables to interact and potentially capture the general equilibrium effects.

2.2 Cointegration relationships

Hondroyiannis and Papapetrou (2005) studied the relationship of macroeconomic variables (per capita output, real wage) and demographic variables (fertility rate, age dependency ratio) in eight European countries over the period 1960-1998. They perform stationarity and (non)-cointegration tests for each country individually, as well as panel tests. At first, they examine the order of integration of the variables by means of panel unit root tests as proposed by Hadri (2000). Thereafter, they test for cointegration in the individual country and in the panel data. Here, they use the Johansen maximum likelihood for testing the multivariate model for each individual country (Johansen, 1988). For panel cointegration tests, they use several tests presented by Kao (1999) and Pedroni (1997, 1999, 2000), to determine the existence of cointegration in a multivariate framework. They found a long-run relationship between the four variables.

In addition, Huynh, Mallik and Hettihewa (2006) studied the impact of the middle-aged working population, aged 40-64, on the share prices of Australia for the period 1965-2002. The used methods are similar to the ones used by Hondroyiannis and Papapetrou, except the additional use of the Phillips-Perron (PP) test. This test has an advantage over, for example, the augmented Dickey-Fuller (ADF) test since it produces robust estimation results, even when the series contains serial correlation and time-dependent heteroskedasticity, or when there exists a structural break. Their study is very focused, in the fact that they only consider one particular age group (40-64) and one particular country (Australia). Possible incorporation of these aforementioned methods in panel data and the use of a number of age groups, allows for more comparable outcomes. Not just between different countries, but also between different age groups within the country.

In contrast to Hondroyiannis and Papapetrou, Gupta and Guidi (2012) do not necessarily explore cointegration relations in macroeconomic and demographic variables, but provide some very useful cointegration methodologies in order to explore interdependence. To determine the links between the Indian market and three developed Asian markets, they use, at first, the Engle-Granger cointegration methodology (Engle & Granger, 1987). This methodology analyses the stationarity of the error terms series obtained from an equation with level values of time series that are non-stationary, but eventually become stationary when differences are taken. The estimated error terms are then examined by the augmented Dickey-Fuller (ADF) test. The next technique they use, similar to Hondroyiannis and Papapetrou, is the Johansen's methodology, in which the vector autoregressive model (VAR) is the starting point. The dependent variables are integrated of order one - $I(1)$ - and the error term is a zero mean white noise process. Then, the VAR model can be written as a vector error correction model and different likelihood ratio tests can be computed, such as the trace and maximum eigenvalue tests. At last, they consider three alternative models, proposed by Gregory and Hansen (1996), that take into account possible breaks in the cointegration relationship. Again, the estimated error terms of the so-called C model, C/T model and C/S model are then investigated for stationarity by the ADF test.

Again, Kwon and Shin (1999) do not focus on the demographic part, but do consider cointegration and causality between macroeconomic variables. By means of the unit-root test and Granger causality test, their vector error correction model shows that stock indices are cointegrated with a set of macroecomic variables, for example exchange rates and money supply.

2.3 Time-varying coefficients

Models that assume the underlying data-generating process to be stable, tend to suffer in terms of parameter instability. Therefore, Brown, Song and McGillivray (1997) used the time-varying coefficient methodology (TVC), in which the generated process is treated as unstable. In comparison with the error correction model, the vector autoregressive model and an autoregressive time series regression, the TVC outperforms the aforementioned models based on their forecasting performance.

Another method, introduced by Nakajima, Kasuya and Watanabe (2011), is to use a time-varying parameter vector autoregressive (TVP-VAR) model. In here, the parameters follow a random walk process and are estimated by means of a Markov chain Monte Carlo method (MCMC). Eventually, the TVP-VAR is compared to fixed parameter VAR models by means of marginal likelihood, which indicates that the TVP-VAR model is the best fit for Japanese economic data.

In addition, Koop and Korobilis (2013) also used a TVP-VAR model. They mention that the MCMC algorithm works well for small TVP-VAR models, but becomes computationally very demanding for large models. This is mostly due to the fact that it is a posterior simulation algorithm, where thousands of draws must be taken to make the algorithm converge. For this reason, they consider a forgetting factor λ, which is restricted to be greater than zero and less or equal to one. It is, in fact, a scaling factor that computes a conditional variance-covariance matrix at time t from the same matrix at time $t-1$.

Christopoulos and León-Ledesma (2008) propose Granger (non)-causality tests based on a VAR model. Their logistic smooth transition autoregressive (LSTAR) model uses time as a transition variable. The Granger causality test allows for smooth breaks in the causal variables. They conclude that the LSTAR model has better forecasting performance compared to a VAR model.

Another approach which has mainly been applied in the academic literature is the time dummy method. Haan (2004) and Diewert et. al (2009) considered this approach mainly in the use of a hedonic regression. Such regressions estimate the effect of of multiple factors on the price of a good, or the demand. Haan concluded that the time dummy approach can be justified, because it might lower the standard error of the index by an increased number of observations. Although, an imperfectly specified model could cause a possible bias.

Originality comes in when an aforementioned time-varying coefficient model is used for panel data, deriving the effect of a changing demographic structure on macroeconomic variables.

3 Data

The data consists of several macroeconomic variables, Y_{it}, and population shares, W_{it}, of 21 OECD countries over the period 1970-2014 (Aksoy et al., 2019). The vector of macroeconomic variables contains six endogenous variables: the growth rate of the real GDP, g_{it}; the share of investment in GDP, I_{it}; the share of personal savings in GDP, S_{it}; the logarithms of hours worked per capita, H_{it}; the real short-term interest rate, rr_{it}; and the rate of inflation, π_{it}. The vector of these six variables is denoted as $Y_{it} = (g_{it}, I_{it}, S_{it}, H_{it}, rr_{it}, \pi_{it})'$. This means that the variables are present for every country i given time period t. Figure 2 shows the distribution histograms for each of the six variables.

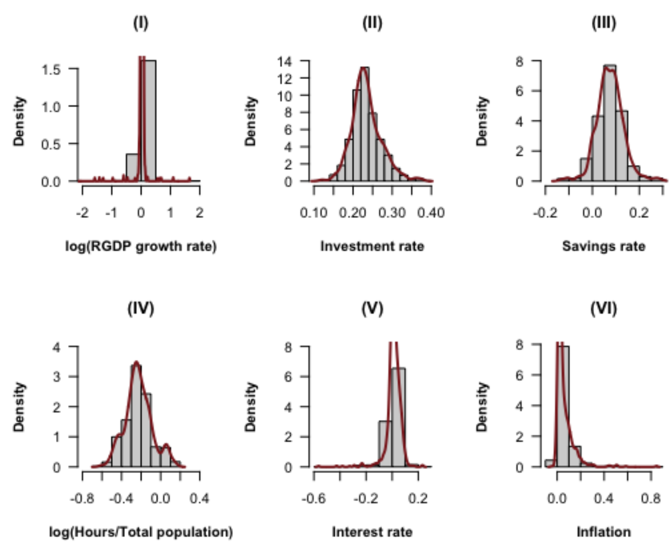


Figure 2: Distributions of: (I) the growth rate of the real GDP,g_{it}; (II) the share of investment in GDP, I_{it}; (III) the share of personal savings in GDP, S_{it}; (IV) the logarithm of hours worked per capita, H_{it}; (V) the real short-term interest rate, rr_{it}; (VI) the rate of inflation, π_{it}

Actually, none of the six endogenous variables is normally distributed. Especially the growth rate of the real GDP, interest rate and the inflation do not seem to be normally distributed. On the other hand, the investment rate, the savings rate and the logarithm of hours worked per capita appear to be close to a normal distributed variable. However, Table 1 shows that also these variables are, in fact, not normally distributed.

	g	I	S	H	rr	π
Mean	0.0048	0.2343	0.0755	-0.2260	0.0113	0.0571
Median	0.0298	0.2299	0.0735	-0.2348	0.0130	0.0320
Maximum	1.6485	0.3802	0.3062	0.1632	0.2342	0.8439
Minimum	-4.4332	0.1159	-0.1353	-0.6172	-0.5639	-0.0448
Std. dev	0.2900	0.0382	0.0576	0.1393	0.0569	0.0726
Skewness	-9.4375	0.6148	0.1901	0.2589	-3.0889	3.9202
Kurtosis	114.0018	1.0808	1.7053	0.1391	22.6979	25.8570
Jarque-Bera test	487978	98.523	112.14	10.578	20254	26719
	$(2.2e^{-16})$	$(2.2e^{-16})$	$(2.2e^{-16})$	(0.006)	$(2.2e^{-16})$	$(2.2e^{-16})$

Note: The p-values for the Jarque-Bera test are shown in brackets.

Table 1: Summary descriptive statistics

The variables deviate from normal characteristic skewness and kurtosis, respectively 0 and 3. This is also confirmed by the Jarque-Bera test, where the null hypothesis states that the variable is normally distributed. Given the p-values of this test, the null hypothesis can be rejected for each variable.

In addition to the six endogenous variables, there are two more control variables used. At first, the lagged oil price is considered to allow for global shocks. Both the first and second lag of this variable are used. Second, the population growth is considered to capture the effect of demographic structure. For this variable it holds that the actual value and the first lag are used. Distribution plots and summary descriptive statistics for these variables are provided in Appendix A.

Furthermore, for each country i the population is split into three age groups, representing the age structure: the young dependents, aged below 20; the working population, aged 20 to 60; and the old dependents, aged 60 and above. The contribution of age group $j = 1, 2, 3$ (0-19, 20-59, 60+) to the total population is denoted by w_{jit}. Obviously, there is exact collinearity since $\sum_{j=1}^{3} w_{jit} = 1$ represents the entire population. Therefore, a restriction has to be included. The coefficients have to sum to zero, using $(w_{jit} - w_{3it})$ as explanatory variables, after which the coefficient for the old dependents is recovered from $\delta_3 = -\sum_{j=1}^{2} \delta_j$.

4 Methodology

4.1 VARX(1) model

In order to capture the consequences of a changing demographic structure on different macroeconomic variables, Aksoy, Basso, Smith and Grasl (2019) used an augmented panel VARX(1) model as shown in equation 1.

$$Y_{it} = a_i + AY_{i,t-1} + DW_{it} + u_{it} \tag{1}$$

Here, the macroeconomic variables are denoted by the vector Y_{it} and the population shares by W_{it} for each of the countries $i = 1, 2, ..., N$. The model assumes slope homogeneity, because they found that heterogeneous slopes in combination with relatively low degrees of freedom resulted in poor parameters estimations, which is in line with the findings of Baltagi, Griffin, and Xiong (2000). They say that homogeneous estimators tend to have better forecasting results. Although slope homogeneity is assumed, they allow for intercept heterogeneity by means of a_i. Eventually, two additional controls are added: the lagged oil price, to allow for global shocks, and the population growth, to comprehend the effects of the demographic structure instead of the population effect.

After estimation of a_i, A and D from equation 1, it is possible to represent the system's equilibrium for the long-run, as is done in equation 2.

$$Y_{it}^{*} = (I - A)^{-1} a_i + (I - A)^{-1} D W_{it} \tag{2}$$

Immediately, it is possible to see that the equation does not contain the macroeconomic variables at the previous time period ($Y_{i,t-1}$) as regressors anymore, instead this is now incorporated in the dependent variable Y_{it}^{*}. The effect of the demographic variables is denoted by $D_{LR} = (I - A)^{-1} D$, which exhibits the relation between the endogenous variables in addition to the direct effect of the demographics on each variable. Now, by acquiring the demographic attractor for the economic variables at any moment in time, it is possible to isolate the long-run contribution of demography to each variable in each country. This is shown in equation 3.

$$Y_{it}^{D} = (I - A)^{-1} D W_{it} = D_{LR} W_{it} \tag{3}$$

Each element in matrix D_{LR}, denoted by $d_{ij}(A, D)$, is a function of the matrices A and D. The nonlinear Wald test is used to test whether each element or a combination of elements in D_{LR} are significantly different from zero. The null hypothesis that arises is indicated as follows: $H_0 : D_{LR}(i, j) = 0$ or $H_0 : d_{ij}(A, D) = 0$. The nonlinear Wald test statistic is calculated as in equation 4.

$$d_{ij}(\hat{A}, \hat{D})^{T} [d'_{ij}(\hat{A}, \hat{D})(\hat{V}(A, D)) d'_{ij}(\hat{A}, \hat{D})^{T}]^{-1} d_{ij}(\hat{A}, \hat{D}) \xrightarrow{D} \chi^2_Q \tag{4}$$

Here, $\hat{V}(A, D)$ denotes the estimated variance-covariance matrix and $d'_{ij}(\hat{A}, \hat{D})$ is the gradient of the function $d_{ij}(\hat{A}, \hat{D})$. The nonlinear Wald test has an asymptotic chi-square distribution with Q degrees of freedom.

4.2 Cointegration

4.2.1 Augmented Dickey-Fuller test

Aksoy, Basso, Smith and Grasl (2019) assumed that all the variables are stationary. When considering cointegration relationships, the stationarity of the variables has to be tested. If time series variables are integrated of order one - $I(1)$, which means that the time series contains a unit root - then the difference within the same time series are considered to be stationary - $I(0)$, integrated of order zero.

Many studies, therefore, consider a unit root test when examining cointegration relationships, for example (Huynh, Mallik, & Hettihewa, 2006) and (Gupta & Guidi, 2012). They both use the augmented Dickey-Fuller test provided by Dickey & Fuller (1979). Here, the null hypothesis of the time series containing a unit root is tested against the alternative hypothesis of the time series being stationary. The mathematical representation of the hypotheses is given in equation 5.

$$H_0 : \gamma = 0, \qquad H_a : \gamma < 0 \tag{5}$$

The hypothesis is applied to the following time series model (eq. 6).

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + ... + \delta_{p-1} \Delta yt - p + 1 + \epsilon_t \tag{6}$$

Here, a random walk model is denoted if the restrictions $\alpha = 0$ and $\beta = 0$ are imposed. For a random walk with a drift, only the restriction $\beta = 0$ must hold. The corresponding test statistic is given by:

$$DF = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \tag{7}$$

Here, $\hat{\gamma}$ is the estimated parameter for y_{t-1} and SE($\hat{\gamma}$) is the standard error for this parameter.

4.2.2 Phillips-Perron test

Next, the Phillips-Perron test (Phillips & Perron, 1988) is considered. As stated in (Huynh et al., 2006), this test has an advantage over the traditional augmented Dickey-Fuller test. It produces robust estimates in case the time series has serial correlation and time-dependent heteroskedasticity, or in case it contains a structural break.

Again, the null hypothesis of the time series containing a unit root is tested against the alternative hypothesis of the time series being stationary. The mathematical representation is given in equation 8.

$$H_0 : \rho = 1, \qquad H_a : \rho < 1 \tag{8}$$

Compared to the ADF test, the hypotheses of the PP test is applied to a smilar time series model:

$$\Delta y_t = (\rho - 1) y_{t-1} + \mu_t \tag{9}$$

Again, similar to the ADF test it is possible to include a linear time trend and a constant term. In contrast to the ADF test, the PP test uses a correction in the t-test for ρ which is non-parametric. Therefore, the estimation is robust for serial correlation and heteroskedasticity.

4.2.3 Johansen's methodology

After investigating the stationarity properties of the data, it is possible to test for cointegration in individual countries as well as in the panel data, as is done by Gupta and Guidi (2012), and Hondroyiannis and

Papapetrou (2005). They use the methodology provided by Johansen (1988, 1991). His starting point is the vector autoregression (VAR) model of order p (as is shown in section 4.1, p equals one).

$$z_t = c + A_1 z_{t-1} + ... + A_p z_{t-p} + \mu_t \tag{10}$$

Here, z_t is a vector of variables, and μ_t is a white noise zero mean error term.

The VAR model can be rewritten as a vector error correction model (VECM):

$$\Delta z_t = c + \Pi z_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + \mu_t \tag{11}$$

here, coefficient matrix $\Pi = \sum_{i=1}^{p} A_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$. When it holds that the coefficient matrix has a reduced rank, $r < n$, then n x r matrices α and β exist such that $\Pi = \alpha\beta'$ and $\beta' z_t$ is stationary. Now, r denotes the amount of cointegration relationships, α contains the error correction coefficients and β is the cointegration vector. Matrix α tells us which of the variables is error correction when the system is out of equilibrium.

Furthermore, Johansen (1988, 1991) provides two different likelihood ratio tests for the significance of the cointegration relations, in other words, for the significance of the reduced rank of the matrix. The trace test and maximum eigenvalue test are provided in equations 12 & 13 .

$$H_0 : r = r_0, \quad H_a : r_0 < r < K, \qquad \lambda_{trace} = -T \sum_{i=r_0+1}^{K} ln(1 - \lambda_i) \tag{12}$$

$$H_0 : r = r_0, \quad H_a : r = r_0 + 1, \qquad \lambda_{max} = -T ln(1 - \lambda_{r_0+1}) \tag{13}$$

Here, T is the sample size, and λ_i and λ_{r_0+1} are the estimated characteristic roots from the matrix. The trace test investigates the null hypothesis of r cointegration vectors versus the alternative hypothesis of more than r cointegration vectors. The maximum eigenvalue test examines the null hypothesis of r cointegration vectors versus the alternative hypothesis of $r - 1$ cointegration vectors.

4.3 The time dummy approach

In order to examine whether the coefficients for the demographic variables vary over time, it is possible to include a time-varying dummy matrix into equation 1. The equation is represented as follows:

$$Y_{it} = a_i + AY_{i,t-1} + DU_i W_{it} + u_{it} \tag{14}$$

Where U_i is the n x t dummy matrix for country i. Here, n is equal to the amount of observations and t represents the years (1970-2014). Therefore, it holds that $u_{nt} = 1$ when there is an observation n during year t. For each of the countries it will result in an identity matrix, provided that all years have an observation. The interaction between the dummy variables and the demographic variables results in coefficient estimates for each year. For implementation of the regressions in R, see Appendix B Table 9.

To test whether the interaction between the dummy variables and the demographic variables is significant, the F-test for joint significance is used. The null hypothesis states that the coefficients for these variables are all equal to zero, and is tested against the alternative hypothesis that they are not equal to zero. The F-statistic is then calculated as in equation 15.

$$F_{q,n-k-1} = \frac{(R_U^2 - R_R^2)/(df_R - df_U)}{(1 - R_U^2)/(n - k - 1)} \tag{15}$$

Here, q is equal to the amount of restrictions, n is the amount of observations, k is the amount of variables in the unrestricted model, R_U^2 is the R-squared of the unrestricted model, R_R^2 is the R-squared of the restricted model, df_R is the degrees of freedom in the restricted model and df_U is the degrees of freedom in the unrestricted model.

5 Results

5.1 VARX(1) model

The long-run demographic parameter effects D_{LR} for the three age groups β_i, as estimated in equation 3, are shown in Table 2. Clearly, the effects of the workers (β_2) and the old dependents (β_3) are opposite. Workers contribute positively to growth, investment, savings, hours worked per capita and the real interest rate, whereas the old dependents contribute negatively to each of these variables. Furthermore, the effect directions - either positive or negative - of the young dependents (β_1) and the old dependents are equal for 3 out of 6 variables. The negative effect for both the young and old dependents on the worked hours seems reasonable, considering educational purposes for the young dependents and retirement for old dependents. The opposite happens for the savings rate variable, in which the young dependents contribute positively whereas the old dependents contribute negatively. A possible explanation for this is that young dependents are still very reliant on their parents on that age and therefore do not have the need to spend a lot of money. In addition, many parents already save money for the young dependents for their future life. The negative effect for the old dependents, on the other hand, is due to the fact that they have saved money prior to their retirement. Considering their relatively shorter life expectancy, they do not feel the need to save money anymore. All in all, it is reasonable to say that the long-term demographic effects show life-cycle patterns.

The corresponding p-values of the parameter estimates show that the null hypothesis is rejected for most of the estimates, considering a significance level of ten percent. In fact, 11 out of 18 parameters are estimated precisely. It is noticeable that, especially among the workers, there are many parameters for which the null hypothesis is not rejected.

	β_1	β_2	β_3
g	0.04	0.06	-0.10
	(0.15)	(0.28)	(0.04)
I	0.13	0.08	-0.22
	(0.01)	(0.48)	(0.02)
S	0.24	0.16	-0.40
	(0.00)	(0.29)	(0.00)
H	-0.54	1.08	-0.54
	(0.00)	(0.00)	(0.05)
rr	-0.05	0.46	-0.42
	(0.72)	(0.15)	(0.10)
π	0.70	-0.75	0.05
	(0.00)	(0.00)	(0.68)

Note: The p-values are shown in brackets, where the $H_0 : D_{LR}(i,j) = 0$ is tested.

Table 2: The long-run demographic impact: D_{LR}

Table 3 shows the conditional forecasts for a changing demographic structure on the average annual GDP growth. To measure this, demographic predictions for each country are used ($W_{i,t+h}$). Then, together with the long-run demographic impact matrix, the expected demographic changes on the long-run are estimated as follows: $Y_{i,t+h} = D_{LR}(W_{i,t+h} - W_{i,t}) + Y_{i,t}$. For each country in the sample it holds that the average annual GDP growth will drop in 2025, compared to the sample average of 1970-2010 and the projected estimates at 2015. The highest absolute decrease will occur in Spain (0.90 percentage points), whereas the absolute smallest decrease will occur in Sweden (0.31 percentage points). The last column shows the probability that the change in annual GDP growth is actually positive, as a result of a changing demography. Only for Sweden it holds that this chance is higher than 10 percent.

	Sample average (1970-2010)	Projected at 2015	Projected at 2025	Change (Δg) (2015-2025)	$\Pr(\Delta g > 0)$
Australia	3.33	3.11	2.60	-0.51	0.089
Austria	2.51	2.37	1.53	-0.84	0.080
Belgium	2.57	2.45	1.88	-0.57	0.087
Canada	3.05	2.69	1.81	-0.88	0.080
Denmark	2.18	1.97	1.50	-0.46	0.060
Finland	2.80	2.44	1.93	-0.51	0.084
France	2.57	2.26	1.72	-0.54	0.068
Germany	2.11	1.91	1.15	-0.77	0.081
Greece	3.77	3.49	2.82	-0.67	0.061
Iceland	3.46	3.13	2.39	-0.74	0.065
Ireland	5.00	4.59	3.99	-0.61	0.068
Italy	2.91	2.64	1.84	-0.79	0.073
Japan	2.95	2.56	2.14	-0.42	0.069
Netherlands	3.01	2.68	1.90	-0.78	0.066
New Zealand	2.72	2.47	1.78	-0.69	0.066
Norway	3.63	3.53	3.10	-0.43	0.063
Portugal	3.69	3.33	2.58	-0.75	0.056
Spain	3.73	3.41	2.51	-0.90	0.073
Sweden	2.20	2.17	1.86	-0.31	0.119
Switzerland	2.22	2.13	1.40	-0.74	0.084
United Kingdom	2.60	2.47	1.96	-0.51	0.087
United States	2.85	2.53	1.87	-0.65	0.066

Table 3: Average predicted impact on GDP growth by country

Furthermore, the variables output growth, real interest rate, investment rate and savings rate, projected for the countries United States, Japan, Italy and France, are shown in Figure 3. Again, the sample average for the period 1970-2010 is the initial point. The projection in 2030 for each country and each variable is expected to decrease over time, compared to the starting point. These projections are fairly precise considering the 80 and 90 percent one-sided tests, respectively the dark gray and light gray areas. Interesting is the drop in the real interest rate, which starts slightly above 0 percent for each of the given countries, but becomes negative when we move towards 2030. It means that banks will charge interest instead of pay interest, when you keep cash with them. This provides an incentive to spend or invest your money instead of saving it, which is in line with the projection of the savings rate for the given countries.

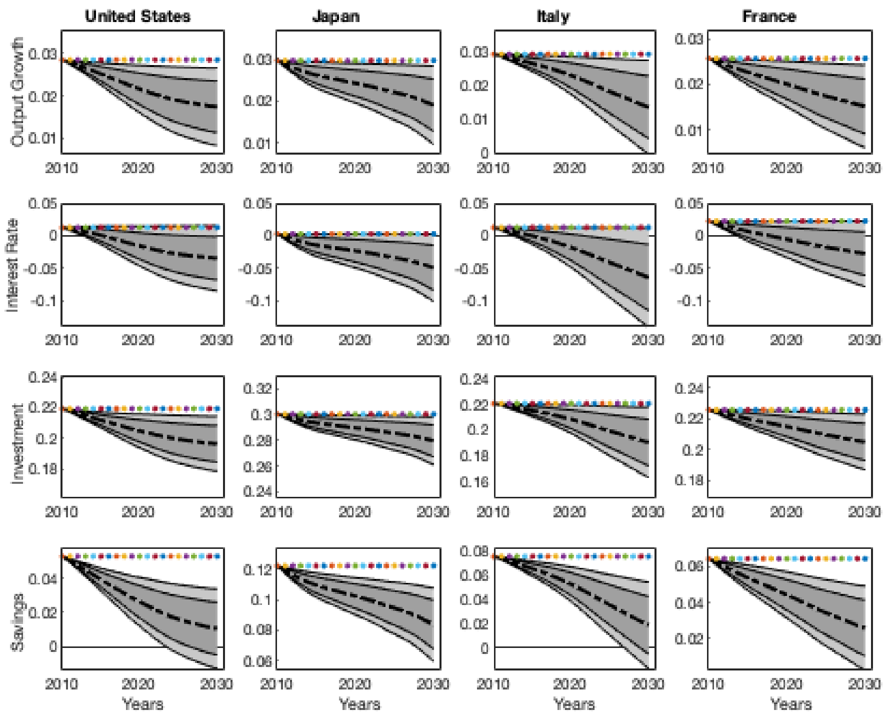


Figure 3: Impact of predicted demographic structure

5.2 Stationarity and cointegration

To test if the time series variables contain a unit root, the augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test are used. Table 4 shows the results of these tests. Considering the null hypothesis of both tests, which stated that the time series variable contains a unit root, and the small p-values that are linked with this, it is reasonable to reject the null hypothesis of a unit root. Therefore each of the time series variables is stationary - $I(0)$.

	ADF	P	PP	P
g	-17.825	0.01	-531.34	0.01
I	-9.337	0.01	-125.21	0.01
S	-7.713	0.01	-106.87	0.01
H	-4.588	0.01	-36.53	0.01
rr	-7.761	0.01	-191.84	0.01
π	-6.779	0.01	-111.25	0.01

Table 4: Test statistics for the Augmented Dickey-Fuller test and Phillips-Perron test.

Since all the variables are integrated of the same order, the Johansen's methodology can be applied. The one-to-one relationship between the vector autoregressive model and the vector error correction model ensures that one model can be rewritten to another model. The A_1 matrix gives the coefficient estimates for the endogenous variables in the VARX(1) model (Appendix B, Table 8). Here, the rows are the dependent variables $y_t = (g_t, I_t, S_t, H_t, rr_t, \pi_t)'$ and the columns correspond to the first lag of these variables $y_{t-1} = (g_{t-1}, I_{t-1}, S_{t-1}, H_{t-1}, rr_{t-1}, \pi_{t-1})'$. Therefore, the endogenous part of the VARX(1) model is represented as: $y_t = A_1 y_{t-1}$. It is interesting to see that the the lagged real GDP growth (g_{t-1}) positively affects all the dependent variables except for the savings rate (-0.0166). Furthermore, the lagged variables of hours worked per capita (H_{t-1}) and real interest rate (rr_{t-1}) negatively affect all the dependent variables except for the corresponding lead dependent variable, respectively 0.8906 and 0.8472. In fact, all the lagged variables positively affect their corresponding lead variable, as shown by the diagonal of the matrix.

$$A_1 = \begin{pmatrix} 0.2269 & -0.0390 & 0.0569 & -0.0584 & -0.0632 & -0.1410 \\ 0.1004 & 0.8200 & 0.0817 & -0.0092 & -0.0438 & -0.0340 \\ -0.0166 & -0.0900 & 0.8689 & -0.0308 & -0.0705 & -0.0769 \\ 0.1930 & 0.0083 & 0.0779 & 0.8906 & -0.0674 & -0.0241 \\ 0.0919 & -0.1632 & -0.1046 & -0.0031 & 0.8472 & 0.2422 \\ 0.0076 & 0.2475 & 0.0752 & -0.0158 & -0.1379 & 0.5630 \end{pmatrix}$$

Now, the endogenous coefficient matrix (Π) is considered for the vector error correction model (Appendix B, Table 10). The matrix is estimated as shown in equation 11. Again, the rows correspond to the dependent variables which are now represented as $\Delta y_t = (\Delta g_t, \Delta I_t, \Delta S_t, \Delta H_t, \Delta rr_t, \Delta \pi_t)'$. The representation of the columns remains the same. The representation of the endogenous part is as follows: $\Delta y_t = \Pi y_{t-1}$. In fact, it follows that the coefficients of the VECM model can be obtained from the VAR model, since it holds that $\Pi = A_1 - I$, where I corresponds to the identity matrix.

$$\Pi = \begin{pmatrix} -0.7731 & -0.0390 & 0.0569 & -0.0584 & -0.0632 & -0.1410 \\ 0.1004 & -0.1800 & 0.0817 & -0.0092 & -0.0438 & -0.0340 \\ -0.0166 & -0.0900 & -0.1310 & -0.0308 & -0.0705 & -0.0769 \\ 0.1930 & 0.0083 & 0.0779 & -0.1094 & -0.0674 & -0.0241 \\ 0.0919 & -0.1632 & -0.1046 & -0.0031 & -0.1528 & 0.2422 \\ 0.0076 & 0.2475 & 0.0752 & -0.0158 & -0.1379 & -0.4370 \end{pmatrix}$$

Johansen considered two different tests for investigating cointegration relationships. The results for the trace test as well as the maximum eigenvalue test are presented in Table 5. The tests are performed on a model with two lags (instead of one) for the endogenous variables. Therefore, the short term parameters concentration is not neglected. The corresponding A_1 and A_2 matrices for this VAR model, and the Π and Γ_1 matrices for this VECM model are provided in Appendix C. In both tables, the first column represents the null hypothesis for these tests, the second column gives the specific test statistic and from the third column the critical values are given for the indicated significance level. The test statistics slightly differ among the tests. The null hypothesis on top of the tables is the most interesting one, since the previous ones have already been rejected at a 1% significance level. It states the null hypothesis of at most five cointegration relationships which, again, is rejected since the test statistic exceeds the critical values ($12.83 > 11.65$). Both tests indicate that the amount of cointegration relationships (r) among the endogenous variables is greater than five. Combining this with the use of six endogenous variables, results in six linearly independent cointegration relations. Therefore, there is no reduced rank in matrix Π (Appendix C).

	Test	10%	5%	1%
r $\leq$ 5	12.83	6.50	8.18	11.65
r $\leq$ 4	72.74	15.66	17.95	23.52
r $\leq$ 3	140.54	28.71	31.52	37.22
r $\leq$ 2	241.15	45.23	48.28	55.43
r $\leq$ 1	377.59	66.49	70.60	78.87
r = 0	760.13	85.18	90.39	104.20

	Test	10%	5%	1%
r $\leq$ 5	12.83	6.50	8.18	11.65
r $\leq$ 4	59.92	12.91	14.90	19.19
r $\leq$ 3	67.79	18.90	21.07	25.75
r $\leq$ 2	100.61	24.78	27.14	32.14
r $\leq$ 1	136.45	30.84	33.32	38.78
r = 0	382.54	36.25	39.43	44.59

Table 5: Test statistics and critical values for the trace test (left) and the maximum eigenvalue test (right).

Since there is no reduced rank, the matrices α and β are 6 x 6 matrices. The rows and columns can be interpreted as in the A_1 matrix, with the y_t and y_{t-1} vectors. Matrix α corresponds to the loading matrix containing the error correction coefficients. Intuitively, it tells which of the variables is error correcting when the system is out of equilibrium. All of the values in the matrix are rounded to three decimals, which does not necessarily mean that the value of 0.000 is meaningless. However, these values are very small. The matrix shows that especially the growth in real GDP and the savings rate are error correction when the system is out of equilibrium, considering columns one and three. On the other hand, the investment rate, the hours worked per capita, the real interest rate and the rate of inflation do not error correct at all, or do error correct to a small extend.

$$\alpha = \begin{pmatrix} -1.006 & 0.000 & 0.007 & 0.000 & 0.002 & 0.000 \\ 0.004 & 0.000 & -0.001 & 0.000 & 0.000 & 0.000 \\ -0.005 & 0.000 & -0.002 & 0.000 & 0.000 & 0.000 \\ 0.017 & 0.000 & -0.002 & 0.000 & 0.000 & 0.000 \\ 0.017 & 0.000 & -0.003 & 0.000 & 0.000 & 0.000 \\ -0.009 & 0.000 & 0.001 & 0.000 & 0.000 & 0.000 \end{pmatrix}$$

Matrix β is called the cointegration matrix, which is a combination of several eigenvectors. It shows the cointegration relationships between the endogenous variables. The matrix is standardized to the first row.

$$\beta = \begin{pmatrix} 1.000 & 1.000 & 1.000 & 1.000 & 1.000 & 1.000 \\ 0.283 & 570.713 & 47.028 & -309.187 & -72.075 & -47.141 \\ -0.275 & -66.890 & 10.683 & 281.540 & -29.045 & -25.478 \\ -0.025 & -45.166 & 2.200 & 41.066 & -0.346 & 61.770 \\ -0.327 & -159.339 & 50.483 & 28.115 & 157.749 & -25.187 \\ -0.144 & 52.381 & 1.935 & 92.553 & 184.041 & -14.891 \end{pmatrix}$$

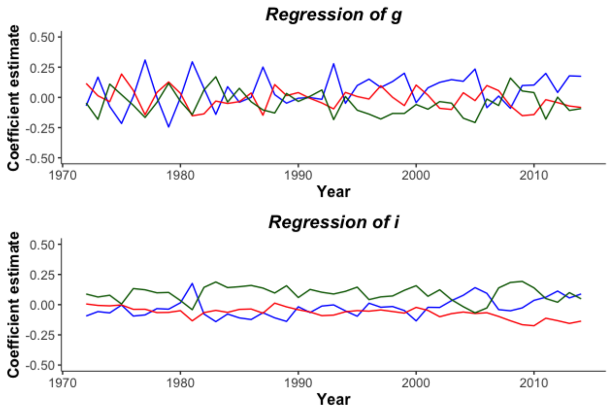
Since the condition for reduced rank $r < n$ is not satisfied, it is possible that $\alpha\beta' = \Pi$ will not hold. Although the tests state that there are six linearly independent cointegration relationships, deriving correct statistical conclusions about the exact cointegration relationships is hard.

5.3 Time-varying VAR model

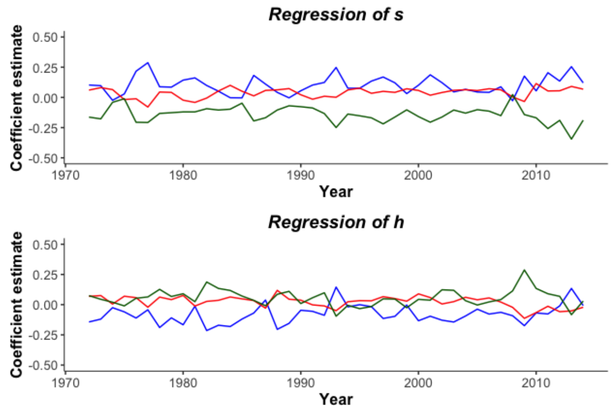
The β_i estimates over time for the dummy variable regressions are shown in Figure 4. Here, β_1 (young dependents) is denoted by the blue line, β_2 (workers) by the red line and β_3 (old dependents) by the green line. Similar to the VARX(1) model, the coefficient for the old dependent is recovered as told in Section 3.

When comparing the three lines in each regression, it is fair to say that the coefficient for the workers is relatively stable over time for each of the variables, compared to the lines of the young and old dependents. This means that the relationship between each of the endogenous variables and the workers is relatively stable over the time period of 1970-2014. A possible reason for this is the smooth transition between the demographic rates. Borderline cases may fall into the young dependent group one year after which they will be part of the working population the following year. The same holds for the exit of workers to the old dependents. The working population is dealing with process of entry and exit between the dependent groups.

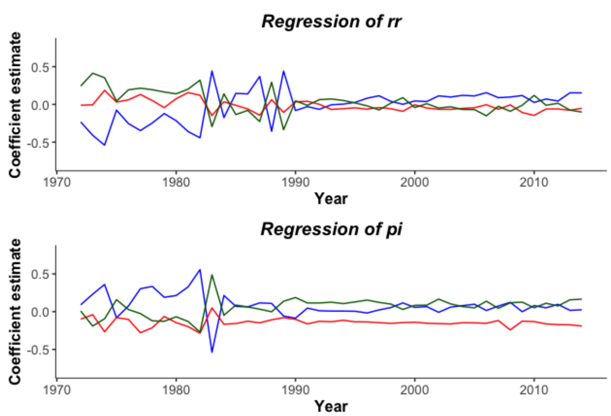
On the other hand, the coefficients of the dependent groups seem a lot less stable over time. Especially in the regressions of g, rr and π there is fluctuation in these coefficient estimates. As seen in Figure 1 in Section 1, the lines belonging to the dependent groups are fluctuating more over time than the line of the working group, which can be an indication why the coefficients also fluctuate over time. It cannot be excluded that the endogenous dependent variables also fluctuate over time and therefore cause different coefficient relationships in the regressions.



(a) Betas in regression g and i



(b) Betas in regression s and h



(c) Betas in regression rr and π

Figure 4: The β_1 (blue), β_2 (red) and β_3 (green) coefficient estimates over time period 1970-2014.

To see whether or not the coefficients are actually explaining, they are tested on their joint significance by means of a F-test. The first column describes the endogenous dependent variable in which the regression takes place (Appendix B, Table 9). The following columns give the null hypotheses for three different tests with corresponding F-statistics and p-values. The first null hypothesis means that the coefficients of the interaction between β_1 and the dummy variable for the years is equal to zero. For clarification, then β_1:factor(Year)1972 = β_1:factor(Year)1973 = .. = 0. Exactly the same is done for β_2. At last, the two tests are combined.

The table shows that the null hypothesis is rejected for every test and in every regression, when a significance level of 10 percent or lower is used. It means that the variables in the tests are joint significant, and therefore the coefficients are interpretative.

	Beta_1:factor(Year)=0		Beta_2:factor(Year)=0		Both	
	F	P	F	P	F	P
g	1.3085	$9.3e^{-2}$ *	4.6645	$< 2.2e^{-16}$ ****	4.8022	$< 2.2e^{-16}$ ****
I	2.4043	$2.4e^{-06}$ ****	3.7967	$5.9e^{-14}$ ****	3.7516	$< 2.2e^{-16}$ ****
S	1.3994	$4.9e^{-2}$ **	2.7652	$3.3e^{-08}$ ****	3.1766	$< 2.2e^{-16}$ ****
H	1.5940	$1.0e^{-2}$ **	4.5638	$< 2.2e^{-16}$ ****	4.3387	$< 2.2e^{-16}$ ****
rr	2.0421	$1.4e^{-4}$ ****	1.4966	$2.3e^{-2}$ **	2.9182	$1.06e^{-14}$ ****
π	1.5491	$1.5e^{-2}$ **	1.6779	$4.9e^{-3}$ ***	3.5192	$< 2.2e^{-16}$ ****

Note: Significance level is indicated by: * for 10%, ** for 5%, *** for 1% and **** for 0.1%

Table 6: F-statistic and p-values for the significance of dummy variables coefficient estimates.

6 Conclusion

The long-run demographic effects determined by the VARX(1) model show clear life-cycle patterns. Considering that 11 out of 18 parameters are estimated precisely, the effects of the workers and the old dependents are opposite. It indicates that the influence of the population structure should not be underestimated when estimating macroeconomic variables. Furthermore, the conditional forecasts for a changing demographic structure on the average annual GDP growth suggests a drop in each country for this macroeconomic variable when it is projected at 2015 and 2025, which indicates an aging population structure considering the negative long-run influence of the old dependents. Again, this is confirmed for the countries United States, Japan, Italy and France when considering the output growth, interest rate, investment rate and savings rate. All these variables drop when they are projected at 2020 and 2030, combined with the fact that the old dependents contribute negatively to these variables. The projections are reasonable fair, considering the relationship between the projected savings rate and interest rate. Intuitively, the savings rate will drop when the real interest rate drops, especially when the real interest rate becomes negative.

Secondly, all of the endogenous variables are stationary when the augmented Dickey-Fuller test and the Philips-Perron test are applied. The VARX(1) model can be rewritten to a vector error correction model. In here, the condition that $\Pi = A_1 - I$ is satisfied. The same holds for the corresponding VAR model with two lags. Applying the trace test and maximum eigenvalue test to the last model suggest six linearly independent cointegration relationships between the endogenous variables. The loading matrix α shows, at first, that the growth in real GDP is error correcting the most, whereas other variables do not or hardly do this. Cointegration matrix β shows the cointegration relationships between the six endogenous variables. It should be noted that it is hard to derive statistical conclusions about the exact cointegration relationships, since the tests show that there is no reduced rank $r < n$. Therefore, it is possible that $\alpha\beta = \Pi$ will not hold.

At last, it can be concluded that the coefficients for the populations shares differ over time. This is to a greater extent for the young dependent and the old dependent, compared to the working population. It indicates that there is a constantly changing relationship between the population shares and the macroeconomic variables. On the one hand this can be explained by the fluctuating population shares, where, again, the young and old dependent fluctuate more over time than the workers. On the other hand, it can not be excluded that the endogenous dependent variables also fluctuate over time and therefore cause different relationships. The coefficients do fluctuate the most in the regressions of the growth in real GDP, the real interest rate and the inflation rate. This indicates that the variation of these variables could be greater than for the other variables. Furthermore, the results are interpretative because of the joint significance between the dummy variables.

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A Control Variables

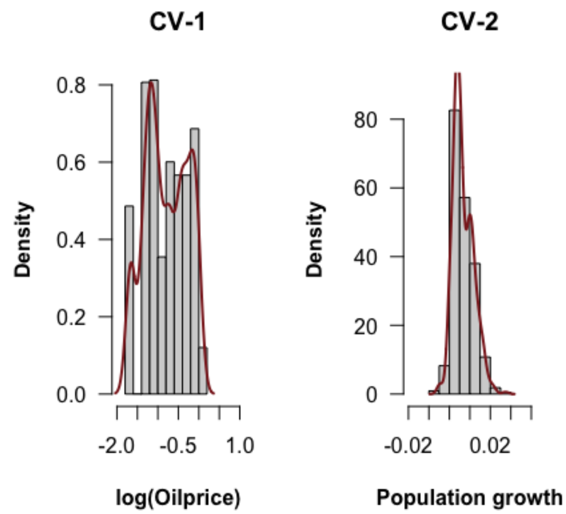

Figure 5: Distibutions of: (CV-1) the logarithm of the oil price; (CV-2) the population growth.

	CV-1	CV-2
Mean	-0.8074	0.0067
Median	-0.7941	0.0056
Maximum	0.0084	0.0283
Minimum	-1.6841	-0.0061
Std. dev	0.5062	0.0050
Skewness	-0.0235	0.6730
Kurtosis	-1.1732	0.4244
Jarque-Bera test	49.852	73.007
	($2.2e^{-16}$)	($2.2e^{-16}$)

Table 7: Summary descriptive statistics for control variables

B Regressions

Equations for the vector autoregressive model
diff(log(RGDPlevel), 1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
Investment.Rate = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
Savings.Rate = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
log(Hours/Total) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
realInterest = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
infl = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2

Table 8: Regressions of VAR model in R.

Equations for the time-varying parameter vector autoregressive model
diff(log(RGDPlevel), 1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1:factor(Year) + beta_2:factor(Year)
Investment.Rate = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1:factor(Year) + beta_2:factor(Year)
Savings.Rate = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1:factor(Year) + beta_2:factor(Year)
log(Hours/Total) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1:factor(Year) + beta_2:factor(Year)
realInterest = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1:factor(Year) + beta_2:factor(Year)
infl = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1:factor(Year) + beta_2:factor(Year)

Table 9: Regressions of TVP-VAR model in R.

Equations for the vector error correction model
diff(diff(log(RGDPlevel), 1),1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
diff(Investment.Rate,1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
diff(Savings.Rate,1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
diff(log(Hours/Total),1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
diff(realInterest,1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2
diff(infl,1) = lag(diff(log(RGDPlevel), 1), 1) + lag(Investment.Rate, 1) + lag(Savings.Rate, 1) + lag(log(Hours/Total), 1) + lag(realInterest, 1) + lag(infl, 1) + lag(log(Oil.Price)) + lag(log(Oil.Price), 2) + popGrowth + lag(popGrowth, 1) + beta_1 + beta_2

Table 10: Regressions of VECM model in R.

C VARX(2) and corresponding VECM Matrices

$$A_1 = \begin{pmatrix} 0.2212 & 0.2271 & 0.1130 & 0.0444 & -0.0884 & -0.2776 \\ 0.0300 & 1.0963 & 0.1609 & 0.0375 & -0.0649 & -0.0453 \\ -0.0829 & 0.1532 & 1.0039 & -0.0111 & -0.0136 & -0.0491 \\ 0.0678 & 0.2096 & 0.1185 & 1.2507 & -0.1031 & -0.0711 \\ 0.0166 & 0.0980 & -0.0368 & 0.1381 & 0.5747 & 0.1239 \\ 0.0387 & 0.0879 & 0.0958 & 0.0192 & -0.1258 & 0.4704 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -0.0883 & -0.2479 & -0.0758 & -0.1116 & -0.0095 & 0.1491 \\ -0.0187 & -0.2897 & -0.0979 & -0.0346 & 0.0300 & 0.0233 \\ -0.0603 & -0.2363 & -0.1542 & -0.0162 & -0.0683 & -0.0158 \\ -0.0596 & -0.2304 & -0.0655 & -0.3463 & 0.0223 & 0.0350 \\ -0.0515 & -0.2301 & -0.0703 & -0.1179 & 0.3862 & 0.1719 \\ 0.0174 & 0.1722 & -0.0286 & -0.0468 & -0.0629 & 0.0783 \end{pmatrix}$$

$$\Pi = \begin{pmatrix} -0.8671 & -0.0208 & 0.0371 & -0.0673 & -0.0979 & -0.1285 \\ 0.0113 & -0.1934 & 0.0630 & 0.0029 & -0.0349 & -0.0220 \\ -0.1431 & -0.0830 & -0.1503 & -0.0274 & -0.0820 & -0.0650 \\ 0.0082 & -0.0208 & 0.0530 & -0.0956 & -0.0808 & -0.0361 \\ -0.0349 & -0.1321 & -0.1071 & 0.0202 & -0.0391 & 0.2957 \\ 0.0561 & 0.2601 & 0.0672 & -0.0276 & -0.1887 & -0.4514 \end{pmatrix}$$

$$\Gamma_1 = \begin{pmatrix} 0.0883 & 0.2479 & 0.0758 & 0.1116 & 0.0095 & -0.1491 \\ 0.0187 & 0.2897 & 0.0979 & 0.0346 & -0.0300 & -0.0233 \\ 0.0603 & 0.2363 & 0.1542 & 0.0162 & 0.0683 & 0.0158 \\ 0.0596 & 0.2304 & 0.0655 & 0.3463 & -0.0223 & -0.0350 \\ 0.0515 & 0.2301 & 0.0703 & 0.1179 & -0.3862 & -0.1719 \\ -0.0174 & -0.1722 & 0.0286 & 0.0468 & 0.0629 & -0.0783 \end{pmatrix}$$