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Time Series Decomposition for Studying Persistence Heterogeneity in Macroeconomic Variables

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Abstract

This paper investigates a time series decomposition method with the use of Discreet Wavelet Transforms. I revisit the research of Ortu, Tamoni, and Tebaldi (2013) by identifying persistent components concealed in consumption growth that exhibit high correlations with certain economic proxies. I show that the components of consumption growth demand a significant annual risk premium investors should be receiving for holding cash flows. Additionally, by means of the decomposition method, I establish substantial comovements between fluctuations in consumption growth, GDP growth and investment growth in the long-run related to the business cycle.

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1 Introduction

When studying the dynamics of consumption growth, persistence properties play a significant role in the discovery of long-run risk. Ortu, Tamoni, and Tebaldi (2013) highlight the problem that arises while trying to detect the long-run risk in practice; namely that standard statistical tests fail to distinguish a time series from white noise and thus are not able to estimate the highly persistent components embedded in these series. The reason behind this is that these persistent components only account for a slight fraction of the total volatility.

Any time series can be considered as a 1-Dimensional (henceforth, 1-D) signal, this is where wavelet transforms can make a difference. Tangirala, Mukhopadhyay, and Tiwari (2013) call attention to the immense relevance wavelet transforms have had in numerous fields, econometrics among others, over the past three decades. This reason for this, it being a useful tool for the analysis of signals (Schimmack, Nguyen, and Mercorelli 2016), a multiresolution analysis in particular in this paper. Wavelet transforms facilitate the use of a multi-scale framework in which one is able to decompose any 1-D signal into components at a number of different resolutions, labeled by a certain scale parameter.

This paper makes use of a certain kind of wavelet transforms, namely Discreet Wavelet Transform (henceforth, DWT). In a DWT, the wavelets are discretely sampled. Given a time series, DWT can be used at each level of persistence, up to the largest desirable level, which results in a full decomposition of the original time series. DWT has a fundamental advantage over the generally used Fourier Transforms, temporal resolution. This means that its functions are able to capture information both in the time and the frequency domain, i.e. the location in time (Holschneider et al. 1990). Therefore, DWT is preferred as it can accurately provide information about frequencies at certain times because of its better resolution (Schimmack, Nguyen, and Mercorelli 2016). In addition, it is more suitable for capturing signals with sharp spikes, which are frequent in growth time series.

Ortu, Tamoni, and Tebaldi (2013) propose a method to decompose a time series based on the level of aggregation, in other words, the persistence of the components which make up the time series. This method relies on a specific DWT, namely the Haar wavelet as introduced by Haar (1909). Different spaces of shock segregate layers generated by the time series based on their half-life. Therefore, economic events that take place at distinct time scales can be captured. This persistence based decomposition has essential implications in practice for the predictability of consumption growth, the intertemporal elasticity of substitution (henceforth, IES), and the equity premia generated by holding cash flows.

Ortu, Tamoni, and Tebaldi (2013) bring forward the empirical pricing implications the high-persistence components induce in practice. Financial agents who have recursive preferences wish to receive a long-run risk premium for maintaining cash flows whose future shocks are positively correlated with the fluctuations in consumption growth. In order to price the predictable components of consumption growth, a comprehensive analysis has to be performed on a long-run risk economy in which these components are the exogenous driving variables of the economy. For more a extensive explanation I refer to their Ortu, Tamoni, and Tebaldi (2013), since this is not the main topic of this paper.

The approach of Ortu, Tamoni, and Tebaldi (2013) differs on this aspect in terms of pricing, in comparison to the previous literature by Bansal and Yaron (2004). Ortu, Tamoni, and Tebaldi (2013) price the different components of consumption growth with each one corresponding to a specific level of persistence. The focus is thus on the entire term structure of consumption risk and not as much on the long-run risk itself.

In this paper, I attempt to replicate and discuss the following findings of Ortu, Tamoni, and Tebaldi (2013). Firstly, with regard to distinguishing a time series from white noise by means of Monte Carlo simulations based on a multi-scale autoregressive process. Secondly, concerning consumption growth and its predictability by determining the optimal number of components through a variance ratio test, identifying proxies that exhibit comovements and testing the predictability of consumption and dividend components by the price-dividend ratio and the price-consumption ratio. Thirdly, I reproduce the IES estimates and the multi-scale autoregressive process estimates which I use to reconstruct the equity premia and its term structure at different levels of persistence. Finally, I test the robustness of the findings with the use of annual data.

In addition, I extend their research by deploying the decomposition on other macroeconomic variables, together with the use of more recent data. I concentrate my analysis on the three respective growth series of income, measured by the GDP, consumption and investment. I discuss the findings in a macroeconomic context.

In conclusion, the research question of this paper is stated as follows: how can the persistent components be uncovered and what is the impact of persistence properties of consumption growth and other macroeconomic variables on their predictability and the term structure of risk premia?

In the continuation of this paper I discuss the necessary methodology related to my research. Furthermore, I describe the data set in detail for the replication and the extension part. In the next step I provide and explain the produced results. Finally, I interpret the main findings in the conclusion. Results I do not find key to my research are shown in the Appendix.

2 Methodology

2.1 Time series decomposition

The decomposition of a given time series $\{g_t\}_{t\in Z}$ into different components, each corresponding to a specific level of aggregation or time scale, start with the following. I first consider, following the notation of Ortu, Tamoni, and Tebaldi (2013), sample means. I construct these sample means, i.e. moving averages, over a certain interval by looking at a window of observations in the past of size 2^j:

$$\pi_t^{(j)} = \frac{1}{2^j}\sum_{p=0}^{2^j-1} g_{t-p} \tag{1}$$

where j $\geq$ 1 and $\pi_t^{(0)} \equiv g_t$ (indicating that the first observation is equivalent to the 'first' sample mean of size $2^0 = 1$). Hereafter, I formulate $g_t^{(j)}$ which make up j different, newly derived time series $\{g_t^{(j)}\}_{t\in Z}$, the components with aggregation level j, by taking the difference of the moving averages over an interval of sizes 2^{j-1} and 2^j:

$$g_t^{(j)} = \pi_t^{(j-1)} - \pi_t^{(j)} \tag{2}$$

where the sample mean over a 2^j-long period, $\pi_t^{(j)}$, serves as a low band pass filter that attenuates all the components, i.e. signals, of which the frequency exceeds 2^j. This entails that when a higher order filter (in j), $\pi_t^{(j)}$, is applied, only the low frequency components remain and therefore shifting the focus to the longer term. Thus, $g_t^{(j)}$ contains the shocks of which the frequency lies on the interval $[2^{j-1}$,$2^j)$. In other words, $g_t^{(j)}$ now only incorporates the fluctuations that have a half-life between 2^{j-1} and 2^j periods.

To illustrate, I assume the original time series $\{g_t\}_{t\in Z}$ is observed at quarterly intervals and has a length of 1 year, i.e. 4 periods. Consequently, the $\{g_t^{(1)}\}_{t\in Z}$ component captures shocks with a quarterly frequency resolution lying on the interval 1 - 2 quarters after applying the two period '$\pi_t^{(1)}$-filter'. Subsequently, $\{g_t^{(2)}\}$ contains the shocks with a 2 - 4 quarter frequency and $\pi_t^{(2)}$ symbolizes a long-run average as it captures shocks that exceed a half-life of $2^J = 4$ periods, where J represents the largest possible level of aggregation given the original time series. The maximum number for J given $T = 2^J$ observations is equal to $\log_2(T)$.

In conclusion, the original time series g_t can be expressed as the sum over j-different, $1 \leq j \leq J$, components and a long-run average:

$$g_t = \sum_{j=1}^{J} g_t^{(j)} + \pi_t^{(J)} \quad \forall\, J \geq 1 \tag{3}$$

The components are constructed with the use of overlapping moving averages which may lead to a serial correlation in the components that is not actually present in the original time series. This can cause a biased analysis of the time series and should thus be avoided. The solution to this problem is to decimate the different components, hereby eliminating all serial correlation. The process of decimation maintains only the observations that are relevant for reconstructing the original time series g_t and can be understood as follows:

$$\{g_t^{(j)}, t = k2^j, k \in Z\} \tag{4}$$

$$\{\pi_t^{(j)}, t = k2^j, k \in Z\} \tag{5}$$

where for the j-th component a 2^j 'jump' is taken between observations to ensure no false serial correlation is present in the component. This process is characterized by the use of a scaled Haar matrix, the $(2^j \mathrm{x} 2^j)$ operator $\tau^{(j)}$, that produces the decimated components. For illustration I take the J = 3 case:

$$\begin{pmatrix} 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 \\ 1/8 & 1/8 & 1/8 & 1/8 & -1/8 & -1/8 & -1/8 & -1/8 \\ 1/4 & 1/4 & -1/4 & -1/4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 & -1/4 & -1/4 \\ 1/2 & -1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \\ g_{t-4} \\ g_{t-5} \\ g_{t-6} \\ g_{t-7} \end{pmatrix} = \begin{pmatrix} \pi_t^{(3)} \\ g_t^{(3)} \\ g_t^{(2)} \\ g_{t-4}^{(2)} \\ g_t^{(1)} \\ g_{t-2}^{(1)} \\ g_{t-4}^{(1)} \\ g_{t-6}^{(1)} \end{pmatrix} \tag{6}$$

where the (8x8) matrix is $\tau^{(3)}$. By pre-multiplying the vector of the decimated components with $(\tau^{(3)})^{(-1)}$, the original time series can be reconstructed. If the number of observations for the original time series exceeds 2^j, this process is simply repeated $\frac{T}{2^j}$ times. The appendix contains a simulation example regarding the benefits of decomposing a time series.

2.2 Variance ratio test

Ortu, Tamoni, and Tebaldi (2013) propose a test to distinguish a white noise process from a time series that has serially correlated, decimated components. This variance ratio test is based on a test for serial correlation introduced by Gencay and Signori (2015) (their working paper is used). The test statistic resembles the ratio between the sample variance of a decimated component which has $\frac{T}{2^j}$ observations, $\frac{(g_t^{(j)})' g_t^{(j)}}{\frac{T}{2^j}}$, and the sample variance of the original time series:

$$\widehat{\xi_j} = \frac{2^j * (g_t^{(j)})' g_t^{(j)}}{(X_t^{(J)})' X_t^{(J)}} \tag{7}$$

where $X_t^{(J)}$ is the vector containing the time series g_t. The null hypothesis of this test is no serial correlation. In order to be able to reject or not reject the null hypothesis, $\widehat{\xi_j}$ is rescaled so it converges to a standard normal distribution:

$$\sqrt{\frac{T}{a_j}}(\widehat{\xi_j} - \frac{1}{2^j}) \xrightarrow{d} N(0,1) \tag{8}$$

where $a_j = \frac{binom(2^j,2)}{2^j(2^{2(j-1)})}$. This rescaled test statistic can be interpreted as the rescaled deviation of $\widehat{\xi_j}$ from the theoretical sample mean $\frac{1}{2^j}$ of the j-th decimated component.

Ortu, Tamoni, and Tebaldi (2013) examine the behaviour of their variance ratio test through Monte Carlo simulations. This exercise demonstrates that the test displays appropriate power at the desired level j for which the component has persistence. The replication of this exercise is added to the appendix.

2.3 Predictive regressions

In order to investigate the predictability of consumption growth (and dividend growth), I run a series of predictive regressions following Ortu, Tamoni, and Tebaldi (2013):

$$g_{t+2^j}^{(j)} = \beta_{0,j} + \beta_{1,j} x_t^{(j)} + \epsilon_{t+2^j}^{(j)} \tag{9}$$

where $x_t^{(j)}$ is the j-th component of the price-consumption ratio or the price-dividend ratio and $g_{t+2^j}^{(j)}$ is either the j-th component of consumption growth or dividend growth. Notice that the explanatory variable is from 2^j period earlier in comparison to the regressor. For these regressions I use the redundant, i.e. non-decimated components. Taking the lag of 2^j between the regressor and the corresponding regressand ensures that the OLS estimates are unbiased. However, the residuals $\epsilon_{t+2^j}^{(j)}$ might be correlated. To account for this issue I use Hansen and Hodrick corrected t-statistics and standard errors that are heteroskedastic-serial consistent. Hansen and Hodrick require a specification for the lag. In this case the lag is set equal to $2^j - 1$, stemming from the corresponding number of overlapping data points.

2.4 Wavelet-Generalized Least Squares for estimating the IES

The persistence based decomposition has essential implications for the estimation of the IES. Historically, the approach to estimate the IES, ψ, is done in the following way (Hansen and Singleton 1983):

$$r_{f,t} = \alpha_f + \frac{1}{\psi} g_t \tag{10}$$

where $r_{f,t}$ is the real risk-free rate and g_t the consumption growth series. The utilization of equation (10) often provides an inaccurate estimation of ψ (Ortu, Tamoni, and Tebaldi 2013).

Therefore, the decomposition of a time series into different components with a certain level of persistence offers an opportunity to be applied to the IES estimation. Applying the decomposition to both sides of (10) generates the following system of equations:

$$r_{f,t}^{(j)} = \alpha_f + \frac{1}{\psi} g_t^{(j)} \tag{11}$$

where $r_{f,t}^{(j)}$ and $g_t^{(j)}$ are the j-th component of the real risk free rate and consumption growth respectively. Following Fadili and Bullmore (2002), the most efficient way to estimate this set of restricted equations is using a form of GLS, Wavelet-GLS (henceforth, WLS). They show that under these conditions, the WLS-estimator is theoretically the closest to the BLUE. Fadili and Bullmore (2002) propose the following algorithm to estimate such a regression with a restricted coefficient:

1. Initialize an OLS fit with the use of the regression in (10)
2. Decompose the design matrix X and the y vector using the DWT, obtaining:
$$X_w \ (2^J\text{x}2) = \begin{pmatrix} 1 & g_t^{(J+1)} \\ 1 & g_t^{(J)} \\ \vdots & \vdots \\ 1 & g_t^{(1)} \end{pmatrix} \text{ and } y_w \ (2^J\text{x}1) = \begin{pmatrix} r_{f,t}^{(J+1)} \\ r_{f,t}^{(J)} \\ \vdots \\ r_{f,t}^{(1)} \end{pmatrix}$$
3. Calculate the detail residuals vector $(2^J\text{x}1)$ for each persistence level j: $d_j = y_j - x_j\widehat{\beta}$ for j = 1,...,J+1 where J+1 is associated with the long term average $\pi_t^{(J)}$. y_j and x_j are $r_{f,t}^{(j)}$ and $g_t^{(j)}$ from equation (11) respectfully.
4. Specify the estimated $(2^J\text{x}2^J)$ diagonal variance-covariance matrix of the noise (the residuals):
$$\widehat{\Sigma_j} = \begin{pmatrix} S_{d_{J+1}} & & & & \\ & S_{d_J} & & & \\ & & \begin{pmatrix} S_{d_{J-1}} & \\ & S_{d_{J-1}} \end{pmatrix} & & \\ & & & \ddots & \\ & & & & \begin{pmatrix} S_{d_1} & & \\ & \ddots & \\ & & S_{d_1} \end{pmatrix} \end{pmatrix}$$
where S_{d_j} is the variance of d_j for j = 1,...,J+1.
5. Estimate the parameter vector $\widehat{\beta} = (X_w'\widehat{\Sigma_j}^{-1}X_w)^{-1}(X_w'\widehat{\Sigma_j}^{-1}y_w)$
6. Go to step 3 and iterate until the change in successive $\widehat{\beta}$ estimates is smaller than 10^{-2}.

2.5 Multi-scale autoregressive process

Following Ortu, Tamoni, and Tebaldi (2013), I assume that the components of consumption growth follow an autoregressive process at different scales. Taken together, the j decimated components comprise a multi-scale autoregressives system. The process for the decimated components $g_t^{(j)}$ can be expressed as follows:

$$g_{t+2^j}^{(j)} = \rho_j g_t^{(j)} + \epsilon_{t+2^j}^{(j)} \tag{12}$$

I highlight the lag of 2^j between the regressor and the explanatory variable that ensures he OLS estimates are unbiased. This can be understood as the j-th lagged decimated component trying to predict the next 2^j periods.

2.6 Term structure of risk premia

The empirical pricing implications the persistent components induce in practice can be measured by a long-run risk, or equity premium that financial agents who have recursive preferences wish to receive for holding cash flows whose future shocks are positively correlated with the fluctuations in consumption growth. In order to price the predictable components of consumption growth, Ortu, Tamoni, and Tebaldi (2013) perform a comprehensive analysis on a Bansal and Yaron (2004), long-run risk economy in which these components are the exogenous driving variables of the economy. Hence, for more a comprehensive explanation I refer to their research since this is not the main topic of this paper. I highlight only the relevant final estimation equations.

The annualized risk prices are collected in the vector $\widehat{\lambda}_\epsilon$ with the elements:

$$\widehat{\lambda}_{\epsilon,j} = k_1(1-\theta)\widehat{A}_j * \frac{4}{2^j} \tag{13}$$

where $k_1 = 0.988$, as specified by Ortu, Tamoni, and Tebaldi (2013). $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, with γ the risk aversion parameter and ψ the IES. $\widehat{A}_j = \frac{\widehat{\rho}_j}{\widehat{\beta}_{1,j}}$, with $\widehat{\rho}_j$ the estimate from (12) and $\widehat{\beta}_{1,j}$ the estimate from (9) with regard to the price-dividend ratio.

The annualized risk exposure for the component at time-scale j is captured by:

$$\widehat{\mathbf{Q}}_{jj}\widehat{A}_j^m * \frac{4}{2^j} \tag{14}$$

where $\widehat{\mathbf{Q}}$ is the estimated variance-covariance matrix from the residuals $\epsilon_{t+2^j}^{(j)}$ in (12). $\widehat{A}_j^m = \frac{\widehat{\rho}_j}{\widehat{\beta}_{1,j}}$, with $\widehat{\rho}_j$ the estimate from (12) and $\widehat{\beta}_{1,j}$ the estimate from (9) with regard to the price-consumption ratio.

The annualized equity premium called for by the j-th component can be estimated as a (scaled) product of the risk price and the risk exposure:

$$E_{t,j}[r_{m,t+1} - r_{f,t}] = \widehat{\lambda}_{\epsilon,j}\widehat{\mathbf{Q}}_{jj}\widehat{A}_j^m \tag{15}$$

3 Data

3.1 Replication data

The data for replication are made available by one of the co-authors on his website[1]. In addition, in appendix A, Ortu, Tamoni, and Tebaldi (2013) elaborate on the used data and on the construction of the variables that they use in their empirical analysis. Five variables are most relevant to study the persistence heterogeneity in long-run risk:

1. The growth rates of the log series of consumption (real nondurables and services per capita in chained (2005) dollars)
2. The growth rates of the log series of dividend
3. The log of the price-dividend ratio
4. The log of the price-consumption ratio
5. The real risk-free rate

Ortu, Tamoni, and Tebaldi (2013) state that the data derive from the Bureau of Economic Analysis (henceforth, BEA) and the Center for Research in Security Prices (henceforth, CRSP). All the data are related to the United States.

However, to construct these variables used in the research, transformations or conversions have to be made from the 'raw' data. The appendix A does not clearly mention how this is done. If one seeks to understand how the used variables are exactly constructed, it requires thorough work.

To test the ability of the components of financial ratios to predict the components of consumption growth and cash flows under persistence heterogeneity, Ortu, Tamoni, and Tebaldi (2013) use US postwar quarterly data over the period 1947Q2-2011Q4 (259 data points) with regard to three-month nominal yield, derived from CRSP. To check the robustness for this, they use an annual series over the period 1930-2010, obtained from CRSP as well.

For the initialization of the filtering procedure used in the detection of the persistent components, both quarterly and annual data is used. The quarterly data range over the period 1927Q1-1946Q4 and is collected from the CRSP database. The use of annual data requires the same amount of data points and thus, observations from further back in time are needed. For this purpose, Ortu, Tamoni, and Tebaldi (2013) use the annual dataset obtained from Robert Shiller's website[2].

Lastly, a time series of the annual productivity is used for comparison of the persistent components. The time series over the period 1948-2011 is from Bureau of Labor Statistics.

[1] https://andreatamoni.meltinbit.com/events/publications

[2] http://www.econ.yale.edu/ shiller/data.htm

In conclusion, the data set provided by the co-author is not structured well and contains a lot of unused variables. I construct a separate data file, including only the data relevant for my research. The appendix A lacks on clearly describing what data are exactly used. It requires detailed research on what has to be done to be able to replicate the results.

3.2 Extension Data

In addition to the file containing the data for the reproduction, I construct a new data set that includes the data from the original sources. This way, I can include only relevant data for my research and perform the cleaning myself. This makes it easy to compare with the provided data set and include more recent data. To study persistence heterogeneity in the long-run in a macroeconomic context, three variables are relevant for my research:

1. The growth rates of the log series of consumption (real nondurables and services per capita in chained (2012) dollars)
2. The growth rates of the log series of GDP (real, per capita in chained (2012) dollars)
3. The growth rates of the log series of investment (private residential- and private nonresidential fixed investment in billions of dollars)

The data derive from BEA and are retrieved from the Federal Reserve Bank of St. Louis (FRED). The effective sample is 1956Q1-2019Q4.

4 Results

4.1 Consumption growth

In this section, I focus on the consequences of applying the time series decomposition on consumption growth with regard to the findings of Ortu, Tamoni, and Tebaldi (2013).

4.1.1 Variance ratio test for consumption growth

The results of applying the variance ratio test to the quarterly consumption growth series over 1948Q1-2011Q4 are reported in Table 1. The shown values are not precisely the same as Ortu, Tamoni, and Tebaldi (2013) report, however, they are in the same order of magnitude. With regard to the significance, only the rescaled statistic for the third component is not significant in my analysis, whereas Ortu, Tamoni, and Tebaldi (2013) do not reject the null hypothesis for both the third and seventh component.

Table 1: This table reports the values of the rescaled test statistic for the quarterly consumption growth series over 1948Q1-2011Q4. Significant values that reject the null hypothesis of no serial correlation at $\alpha = 0.05$ are denoted in bold.

Persistence level j =	1	2	3	4	5	6	7
$\sqrt{\frac{T}{a_j}}(\widehat{\xi}_j - \frac{1}{2^j})$	**-4.106**	**-1.978**	0.791	**5.422**	**2.027**	**3.258**	**2.780**

The rejection of a white noise process at several levels of persistence indicates that consumption growth consists of multiple persistent components.

Following Ortu, Tamoni, and Tebaldi (2013), I determine the optimal number of components to be extracted from the consumption growth series by sequentially applying the variance ratio test to $\{\pi_t^{(j)}, t = k2^j, k \in Z\}$ for j = 1,...,7. Since $\pi_t^{(J=8)}$ only consist out of one observation that represents the long run average over the period 1948Q1-2011Q4, I do not take into account this component. $\pi_t^{(j)}$ includes the possible shocks with a persistence that exceeds 2^j periods. Thus, the first value of j, for which $\pi_t^{(j)}$ cannot be distinguished from white noise, corresponds to maximum level of persistence in the original time series g_t. The results of this sequential exercise are shown in Table 2.

Table 2: This table reports the values of the rescaled test statistic for the quarterly consumption growth series over 1948Q1-2011Q4. The component at scale k is extracted from $\pi^{(J)}$ for J = 1,...,7. Significant values that reject the null hypothesis of no serial correlation at $\alpha = 0.05$ are denoted in bold.

Scale k =	1	2	3	4	5	6	7
$\pi^{(1)}$	**-3.907**	-0.603	**3.313**	0.961	**2.027**	1.779	**4.268**
$\pi^{(2)}$	**-2.353**	1.332	-0.033	0.905	0.872	**2.645**	
$\pi^{(3)}$	-0.022	-0.703	0.218	0.322	1.672		
$\pi^{(4)}$	-0.873	0.225	0.326	1.686			
$\pi^{(5)}$	-0.261	-0.003	1.155				
$\pi^{(6)}$	-0.167	1.009					
$\pi^{(7)}$	1						

Unfortunately, the results I produce are substantially different from the values reported by Ortu, Tamoni, and Tebaldi (2013). The first value of j for which (a component of) $\pi^{(j)}$ does not reject the null hypothesis, is 3 in my analysis as one can observe from Table 2, contrary to $\pi^{(7)}$ that Ortu, Tamoni, and Tebaldi (2013) find. I do not have a clear explanation for this significant dissimilarity. It is possible Ortu, Tamoni, and Tebaldi (2013) perform some additional steps in the testing process that they do not state or, that they handle the data in a peculiar way that is not explained.

4.1.2 Comovements components of consumption growth

Following Ortu, Tamoni, and Tebaldi (2013), I identify certain economic proxies that exhibit a significant correlation with the different components of consumption growth for j = 2,...,6. Firstly, the shocks that are incorporated in $g_t^{(2)}$ and $g_t^{(3)}$ that last between $2^{(2-1)} = 2$ and $2^{(3)} = 8$ periods, i.e. between one half and two years for a quarterly sample, show considerable correlations with the fourth-quarter growth rate of consumption. This comovement is shown in Figure 4 in the appendix. This figure is in line with Ortu, Tamoni, and Tebaldi (2013).

Secondly, the fourth and fifth component added together capture fluctuations with a half-life between $2^{(4-1)} = 8$ and $2^5 = 32$ periods, i.e. between two and eight years for a quarterly sample. The sum of $g_t^{(4)}$ and $g_t^{(5)}$ exhibits a notable (negative) correlation with economic variables related to the business cycle, namely, the term spread and the default yield spread. The comovement of the sum of the fourth and fifth component with these business cycle indicators is plotted in Figure 5 in the appendix which is in compliance with Ortu, Tamoni, and Tebaldi (2013).

Lastly, regarding the identification of an economic proxy for the sixth component of consumption growth that includes shocks that last between $2^{(6-1)} = 32$ and $2^6 = 64$ periods, i.e. between eight and sixteen years for a quarterly sample, Ortu, Tamoni, and Tebaldi (2013) highlight a considerable correlation with Total Factor Productivity (henceforth, TFP). In particular, the sixth component filtered out of the TFP growth series. Figure 6 in the appendix shows the comovement of $g_t^{(6)}$ and the sixth component of TFP growth. This figure is consistent with Ortu, Tamoni, and Tebaldi (2013).

4.1.3 Predictive regressions for consumption- and dividend growth

To start, I run OLS regressions of the components of the consumption growth series on the components of the log price-dividend ratio. Figure 7 in the appendix shows the log price-dividend ratio and the time series of consumption growth. Figure 7 is in line with Ortu, Tamoni, and Tebaldi (2013). The results of the regressions are displayed in Table 3.

Table 3: This table reports the OLS estimates, multiplied by -100, of the regression of the components of the consumption growth, $g^{(j)}_{t+2^j}$, on the components of the log price-dividend ratio $pd^{(j)}_t$. Significant values at $\alpha = 0.05$ are denoted in bold. Hansen and Hodrick corrected t-statistics are shown in parenthesis and the adjusted R^2 in brackets. The sample period is 1947Q2-2011Q4.

Variable	Persistence level j =	1	2	3	4	5	6	7
		0.24	**0.61**	0.48	-0.24	0.24	**0.37**	**0.02**
$pd^{(j)}_t$		(-0.82)	(-2.13)	(-1.40)	(0.65)	(-0.90)	(-1.85)	(3.02)
		[0.00]	[0.02]	[0.03]	[0.01]	[0.03]	[0.19]	[0.00]

The results from my regressions are similar to ones Ortu, Tamoni, and Tebaldi (2013) produce. The values are in the same order of magnitude, however, not exactly the same and the sign of the estimates is turned around. I suspect their values are multiplied by '-100' instead of the stated '100'. The corresponding coefficients, with Ortu, Tamoni, and Tebaldi (2013), are labeled significant, except for the significant coefficient at j = 7 in my analysis. This is most likely due to an error in my calculations since it is remarkable that a coefficient so close to zero is significant. I highlight the significant coefficient for j = 6 that is able to explain a substantial part of future consumption growth with the adjusted R^2 equal to 0.19.

Secondly, I run OLS regressions of the components of the consumption growth series on the components of the log price-consumption ratio. The results are shown in Table 4.

Table 4: This table reports the OLS estimates, multiplied by -100, of the regression of the components of the consumption growth, $g^{(j)}_{t+2^j}$, on the components of the log price-consumption ratio $pc^{(j)}_t$. Significant values at $\alpha = 0.05$ are denoted in bold. Hansen and Hodrick corrected t-statistics are shown in parenthesis and the adjusted R^2 in brackets. The sample period is 1947Q2-2011Q4.

Variable	Persistence level j =	1	2	3	4	5	6	7
		0.31	**0.40**	**0.40**	-0.050	**0.26**	0.15	**0.06**
$pc^{(j)}_t$		(-1.53)	(-2.37)	(-2.46)	(0.26)	(5.4)	(-1.33)	(47.64)
		[0.00]	[0.02]	[0.04]	[0.00]	[0.08]	[0.04]	[0.03]

Most of the conclusions from the previous regressions apply to this case as well. The results from my regressions are in agreement with the ones Ortu, Tamoni, and Tebaldi (2013) compute. The values are in the same order of magnitude, but not exactly the same and the sign of the estimates is turned around repeatedly. In my analysis, the coefficients at j = 2, 3, 5 and 7 are labeled significant as opposed to only the estimates for j = 3 and 6 in their regressions. The significance of the seventh component is likely to be caused by the reason discussed above, namely that my calculation of the Hansen-Hodrick corrected standard errors contain an error. I am unable to explain the difference in significant estimates for the other values of j.

To summarize, regarding the predictive power of the financial ratios for future shocks in consumption growth, I identify a significant second component when using both the price-consumption and price-dividend ratio. For the third component, only the price-consumption ratio shows predictive power. Fluctuations with lower frequencies are captured by a significant sixth component for the price-dividend ratio whereas the fifth components shows predictive power with the use of the price-consumption ratio.

Finally, I run OLS regressions of the components of the dividend growth series on the components of the log price-dividend ratio. The estimated coefficients of these regressions are reported in Table 5.

Table 5: This table reports the OLS estimates, multiplied by 100, of the regression of the components of the log dividend growth, $g^{(j)}_{t+2^j}$, on the components of the log price-dividend ratio $pd^{(j)}_t$. Significant values at $\alpha = 0.05$ are denoted in bold. Hansen and Hodrick corrected t-statistics are shown in parenthesis and the adjusted R^2 in brackets. The sample period is 1947Q2-2011Q4.

Variable	Persistence level j =	1	2	3	4	5	6	7
		12.04	**-2.83**	**-4.15**	2.61	0.026	0.97	**1.43**
$pd^{(j)}_t$		(0.87)	(-1.92)	(-2.23)	(1.67)	(0.03)	(1.32)	(-54.30)
		[0.00]	[0.00]	[0.04]	[0.04]	[0.00]	[0.08]	[0.28]

My estimates are unfortunately not in line with the coefficients Ortu, Tamoni, and Tebaldi (2013) generate. The values differ substantially in both order of magnitude and sign. The components for j = 2 and 3 are significant in my case, whereas the third and sixth component show predictability in their analysis. Again, the significance of the seventh component is likely to be caused by the reason discussed earlier.

In conclusion, my results are partially in agreement with Ortu, Tamoni, and Tebaldi (2013). The different regressions show that consumption growth does contain predictable components by the corresponding components of the financial ratios.

4.1.4 IES estimation

Using the WLS-algorithm specified in the methodology section, I estimate the IES for three different sample periods. The outcome of exercise is reported in Table 6.

Table 6: This table reports the WLS-estimates of ψ, the IES, with the use of the real risk free rate. The top row uses the full sample 1948Q1-2011Q4 with T = 256 to estimate ψ where consequently the maximum number of components is eight. The bottom two rows use the 1948Q1-1979Q4 and 1980Q1-2011Q4 sample respectively, both with T = 128 to estimate ψ where consequently the maximum number of components is seven.

Variable	Sample	$\widehat{\psi}$
$r_{f,t}$	1948Q1 - 2011Q4	8.23
$r_{f,t}$	1948Q1 - 1979Q4	9.97
$r_{f,t}$	1980Q1 - 2011Q4	4.94

My IES-estimates are in line with the ψ-values Ortu, Tamoni, and Tebaldi (2013) compute. Despite having the same order of magnitude, the values are not precisely similar as my $\widehat{\psi}$'s deviate approximately three and one units for the first two rows and the last row respectively. I do not have a clear explanation for these deviations. I write my own code for the adaption of the WLS-estimation Gencay and Signori (2015) propose, and thus suspect it can be due to a difference in the estimation method. Ortu, Tamoni, and Tebaldi (2013) do not distinctly describe the steps they take or the exact data that is used related to the risk-free rate.

4.1.5 Multi-scale autoregressive process

I estimate ρ_j in the multi-scale autoregressive system for j = 1,..,7. I report the results of these regressions in Table 7.

My estimated values for ρ_j are very close to the ones Ortu, Tamoni, and Tebaldi (2013) report. My $\widehat{\rho}_j$'s are not more than a hundredth off, except for $\widehat{\rho}_7$, that has a deviation of approximately 0.1. However, in my analysis, only $\widehat{\rho}_2$ and $\widehat{\rho}_7$ can be labeled as significant, whereas Ortu, Tamoni, and Tebaldi (2013) find that for j = 2, 5, 6 and 7 the estimates are significant.

I point out that the half-lives I obtain are calculated by a different formula Ortu, Tamoni, and Tebaldi (2013) use. Their stated formula for this does not provide the correct values, even in their own tables. Consequentially, I suspect they use the formula I provide in Table 7. In Table 7, I report only the half-lives of the significant coefficients in my analysis, which deviate from Ortu, Tamoni, and Tebaldi (2013) for the straightforward reason that my $\widehat{\rho}_j$'s are different. Tables 8 and 14 contain all the estimated half-lives.

Table 7: This table reports the estimates of ρ_j for the regressions of $g^{(j)}_{t+2^j}$ on its own lagged component $g^{(j)}_t$, for j = 1,..,7. Significant values at $\alpha = 0.05$ are denoted in bold. Hansen and Hodrick corrected t-statistics are shown in parenthesis and the adjusted R^2 in brackets. Half-lives in annual units are calculated by: $-\frac{ln(2)}{ln(|\rho_j|)} * \frac{2^j}{4}$. The sample period is 1947Q2-2010Q4.

Variable	$\widehat{\rho}_j$		Half-life (years)	
$g^{(1)}_{t+2^1}$	-0.01	(-0.08)	-	[0.00]
$g^{(2)}_{t+2^2}$	**-0.16**	(-2.74)	0.4	[0.03]
$g^{(3)}_{t+2^3}$	-0.11	(-0.88)	-	[0.01]
$g^{(4)}_{t+2^4}$	-0.08	(-0.47)	-	[0.00]
$g^{(5)}_{t+2^5}$	-0.14	(-0.94)	-	[0.02]
$g^{(6)}_{t+2^6}$	-0.23	(-1.07)	-	[0.09]
$g^{(7)}_{t+2^7}$	**0.26**	(-4.51)	16.4	[0.12]

4.1.6 Term structure of risk premia

With the use of the equations (13) - (15) discussed in the methodology section, I estimate the annualized equity premium (in %) demanded by the j-th component of consumption growth. I report the results in Tables 8 and 14. Table 14 is added to the appendix.

The risk premia I estimate are unfortunately not in line with the values Ortu, Tamoni, and Tebaldi (2013) produce and neither are the risk exposure and risk price estimates. I point out that for some of the components, the risk premium differs up to a factor of five. The estimated risk premia for the first component lie between 0.17 and 0.34 percent, where Ortu, Tamoni, and Tebaldi (2013) estimate a maximum of 0.01 percent commanded by the first component of consumption growth. Additionally, although my estimated risk premium for the seventh component is only a maximum of two hundredths off in all four combinations of γ and ψ, the estimated risk exposures and risk prices are by no means similar to Ortu, Tamoni, and Tebaldi (2013).

I highlight the fact that for the fourth and seventh component, both the risk exposure and the risk price are negative, which is different from Ortu, Tamoni, and Tebaldi (2013). Since both values are negative, the resulting risk premia are not affected by this changed sign. My values for $\widehat{\mathbf{Q}}_{jj}$ however, are very similar to the estimates of Ortu, Tamoni, and Tebaldi (2013).

Table 8: This table reports the annualized equity premium, $E_{t,j}[r_{m,t+1} - r_{f,t}]$, for j = 1,...,7 (in %). $\widehat{\mathbf{Q}}_{jj}$-estimates are shown in the second column. $\gamma = 5$ and $\psi = 5$ in top panel. $\gamma = 7.5$ and $\psi = 5$ in the bottom panel. The risk exposure and the risk price are annualized as well.

$\gamma = 5$ & $\psi = 5$					
Scale j =	Half-life (years)	$\widehat{\mathbf{Q}}_{jj}$ (1 x 10^{-5})	Risk exposure (1 x 10^{-6})	Risk price	Risk premium (%)
1	0.08	0.90	64.51	53.82	0.17
2	0.4	0.49	192.68	150.24	2.90
3	0.6	0.42	57.11	66.68	0.76
4	1.1	0.30	-118.51	-49.33	2.34
5	2.8	0.16	11.11	43.60	0.39
6	7.6	0.08	8.20	23.55	0.31
7	16.4	0.02	-2.27	-225.08	1.63
$\gamma = 7.5$ & $\psi = 5$					
Scale j =	Half-life (years)	$\widehat{\mathbf{Q}}_{jj}$ (1 x 10^{-5})	Risk exposure (1 x 10^{-6})	Risk price	Risk premium (%)
1	0.08	0.90	64.51	81.86	0.26
2	0.4	0.49	192.68	228.49	4.40
3	0.6	0.42	57.11	101.41	1.16
4	1.1	0.30	-118.51	-75.03	3.56
5	2.8	0.16	11.11	66.30	0.59
6	7.6	0.08	8.20	35.82	0.47
7	16.4	0.02	-2.27	-342.31	2.48

In conclusion, despite my estimated risk premia differing substantially from Ortu, Tamoni, and Tebaldi (2013), this exercise shows that the different components of consumption growth call for a significant annual risk premium investors should be receiving.

4.2 Macroeconomic analysis

In this section, I focus on the consequences of applying the time series decomposition on other macroeconomic variables, next to consumption growth, and discuss the findings in a macroeconomic context.

There exist numerous economic theories on the covariability, or comovement, of economic variables in the long-run (Müller and Watson 2018). The long-run relationship between consumption and income is considered to be one of the most studied ones while they appear to move proportionally throughout history. Klein and Kosobud (1961) even label the consumption-income ratio as one of the 'great ratios' in economics. The investment-income ratio is another one of those celebrated ratios. All variations in income, consumption and investment seem to arise from variations in TFP growth. In section 4.1.2, I identify a significant correlation between TFP growth and consumption growth in the long-run, for the sixth component of both respective series. Hence, in this analysis, I concentrate on the three variables consumption, income and investment. In order to extend my analysis to more recent times, I consider the growth series of the three respective variables over the period 1959Q1-2019Q4.

Figure 1: This figure displays the comovement of the series of consumption growth, GDP growth and investment growth over the period 1959Q1-2019Q4.

Figure 1 plots the respective growth series of consumption, GDP and investment. At a short glance, one can notice that the three variables appear to move closely together in the long-run, although investment growth shows to be more volatile in comparison to consumption- and GDP growth. Taking a look at the correlations between the three variables, the consumption growth series and the GDP growth series display a correlation of 0.54 in the relevant sample period. The correlation between consumption- and investment growth is 0.48. Finally, GDP growth and investment growth exhibit a sizeable correlation of 67%.

4.2.1 Variance ratio tests

In order to determine if the three series consist of any persistence components and if so, on which time-scales, I apply the variance ratio test. I report the results in Tables 9, 10 and 11.

Table 9: This table reports the values of the rescaled test statistic for the quarterly consumption growth series over 1956Q1-2019Q4. Significant values that reject the null hypothesis of no serial correlation at $\alpha = 0.05$ are denoted in bold.

Persistence level j =	1	2	3	4	5	6	7
$\sqrt{\frac{T}{a_j}}(\widehat{\xi}_j - \frac{1}{2^j})$	**-5.102**	**-2.284**	1.308	**6.110**	**2.824**	**3.754**	1.898

I observe from Table 9 that the test rejects a white noise process at exactly the same levels of persistence as in the older sample, see Table 1, except at j = 7 for which the test just fails to reject the null hypothesis. This indicates that in this different, but overlapping, sample of consumption growth, multiple persistent components are still present.

Table 10: This table reports the values of the rescaled test statistic for the quarterly income growth series over 1956Q1-2019Q4. Significant values that reject the null hypothesis of no serial correlation at $\alpha = 0.05$ are denoted in bold.

Persistence level j =	1	2	3	4	5	6	7
$\sqrt{\frac{T}{a_j}}(\widehat{\xi}_j - \frac{1}{2^j})$	**-2.691**	-0.389	1.702	1.927	**2.549**	0.261	-0.152

Table 10 shows that the components of GDP growth at level j = 1 and 5 reject the null hypothesis of white noise. I point out that at time-scale j = 4, the variance ratio test is almost able to classify the fourth component as persistent.

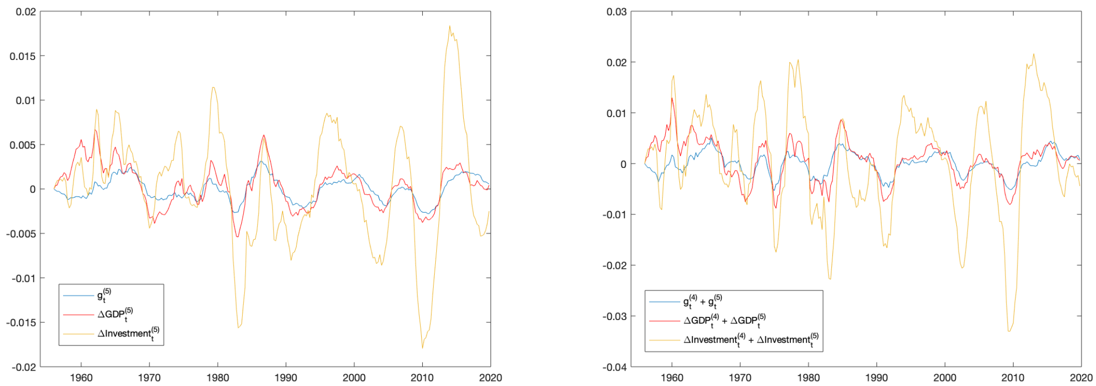
Table 11: This table reports the values of the rescaled test statistic for the quarterly investment growth series over 1956Q1-2019Q4. Significant values that reject the null hypothesis of no serial correlation at $\alpha = 0.05$ are denoted in bold.

Persistence level j =	1	2	3	4	5	6	7
$\sqrt{\frac{T}{a_j}}(\widehat{\xi}_j - \frac{1}{2^j})$	**-5.971**	0.054	1.110	**4.081**	**3.804**	**3.756**	1.583

The rejection of a white noise process at several levels of persistence, namely for j = 1, 4, 5 and 6 in Table 11 indicates that four persistent components are present in the investment growth series.

4.2.2 Long-run analysis

I notice that for all three variables, the first and the fifth component display persistent properties. A rejection of the null hypothesis at j = 1 does not reveal much because the half-lives of the shocks captured by this component are by construction very volatile. Since at j = 4, the component for income is almost persistent and the values of the test statistic at this level is the largest for the other two variables, I decide to take this component into account as well. Thus, I focus on investigating the scale-interactions at the fourth and fifth level of persistence. The fourth and fifth component capture fluctuations with a half-life between between two and four, and four and eight years respectively. I plot the fifth components and the sum of the fourth and fifth components of the respective series in Figure 2.



(a)

(b)

Figure 2: This figure displays the comovement of g_t^5, together with the fifth component of GDP growth and investment growth in (a), and it displays the comovement of $g_t^4 + g_t^5$, together with the respective sums of the fourth and fifth component of GDP growth and investment growth in (b). The sample is 1959Q1-2019Q4.

Looking at Figure 2, it is evident that all three series display clear cyclical patterns in line with the corresponding half-lives related to the business cycle (Burns and Mitchell 1946). The substantial long-run comovement is apparent between consumption and income, but less so for investment growth. This is in line with the long-run projections of the three growth series identified by Müller and Watson (2018) who estimate a correlation close to one between consumption and income, and a smaller, but still substantial correlation with investment. Nonetheless, the correlation between $g_t^{(5)}$ and $\Delta Investment_t^{(5)}$ is 0.67 and 0.56 between $\Delta GDP_t^{(5)}$ and $\Delta Investment_t^{(5)}$. The correlation between the $g_t^{(5)}$ series and $\Delta GDP_t^{(5)}$ is a sound 71%. When taking the respective sums of the fourth and fifth components, the correlations increase to 0.73, 0.60 and 0.71 respectively.

5 Conclusion

The goal of this paper is to replicate the findings of Ortu, Tamoni, and Tebaldi (2013) and to extend their research by applying the decomposition to other macroeconomic variables, next to consumption growth, and discuss these results in a macroeconomic context. The research question is stated as follows: how can the persistent components be uncovered and what is the impact of persistence properties of consumption growth and other macroeconomic variables on their predictability and the term structure of risk premia?

In line with Ortu, Tamoni, and Tebaldi (2013), I identify persistent components concealed in consumption growth, by means of a DWT and a variance ratio test, that exhibit high correlations with certain economic proxies. The financial ratios of price-dividend and price-consumption show to have predictive power for consumption growth at multiple levels of persistence. Moreover, using the WLS-algorithm, I estimate an IES greater than one consistent with Ortu, Tamoni, and Tebaldi (2013). Finally, revisiting the term structure of risk premia, I show that the different components of consumption growth demand a significant annual risk premium investors should be receiving. Despite my estimates and results rarely being exactly in compliance with Ortu, Tamoni, and Tebaldi (2013), but most often not differing substantially, I am able to draw reasonably similar conclusions.

Additionally, decomposing a time series along the persistence dimension allows for one to compare economic variables at specific time-scales and identify proxies or large correlations between certain macroeconomic series. I establish substantial comovements between fluctuations in consumption growth, GDP growth and investment growth in the long-run related to the business cycle which can be relevant to predict future shocks.

My research is limited in a number of ways. Firstly, the framework of Ortu, Tamoni, and Tebaldi (2013) related to consumption growth is not easily applicable to other economic variables and thus does not allow for straightforward and accessible extensions. Furthermore, the absence of code to replicate the majority of the results forced me to thoroughly check into all details of the methodology, leaving less room to perform a more in-depth extension of the topic.

Future research can be devoted to investigating scale-interactions between different economic variables that is allowed through decomposing a time series into a number of components with specific half-lives. This could be relevant for identifying proxies for certain variables and establishing high correlations in order to predict future shocks. In addition, one can apply the decomposition method to financial asset data, e.g. bonds, and formulate an accompanying framework to determine if such assets demand any equity premia.

6 Appendix

6.1 Persistence versus white noise example

The benefits of the decomposition method can be shown by means of a simple simulation exercise that highlights the importance of detecting a persistent, decimated component that otherwise would not have been observed and consequently have labeled the time series as white noise. I simulate decimated components for J = 4: $g_t^{(1)}$, $g_t^{(2)}$, $g_t^{(3)}$, $g_t^{(4)}$ and $\pi_t^{(4)}$ where only the persistent component $g_t^{(4)}$ follows an autoregressive process and the others are independent normal innovations. Figure 3 shows the reconstructed time series and the simulated components. The right panel shows the autocorrelation functions corresponding to the series in the left panel. I observe that the autocorrelation function of the reconstructed time series does not show any significant lags and thus looks like white noise while one of the components does display persistence.

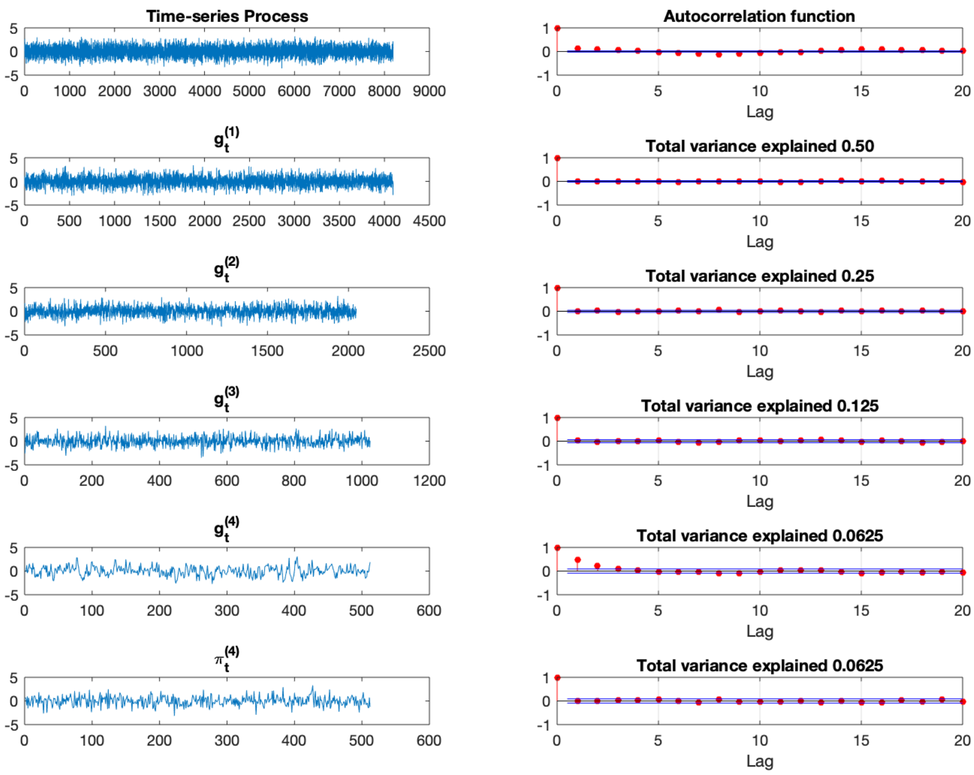


Figure 3: This figure displays the reconstructed time series on the top left with below the simulated components, $g_t^{(1)}$, $g_t^{(2)}$, $g_t^{(3)}$, $g_t^{(4)}$ and $\pi_t^{(4)}$ where only the persistent component $g_t^{(4)}$ follows an autoregressive process and the others are independent normal innovations. The right panel shows the autocorrelation functions corresponding to the series in the left panel.

6.2 Performance variance ratio test

I test the behaviour of the variance ratio proposed by Ortu, Tamoni, and Tebaldi (2013) through Monte Carlo simulations with N = 5000. I simulate decimated components for J = 6: $g_t^{(1)}$, $g_t^{(2)}$, $g_t^{(3)}$, $g_t^{(4)}$, $g_t^{(5)}$, $g_t^{(6)}$ and $\pi_t^{(6)}$ where only the persistent component $g_t^{(6)}$ follows an autoregressive process and the others are independent normal innovations. Hereafter, I reconstruct the original time series using the inverse of $\tau^{(j)}$. I set ρ_J^2 equal to 0.2 or 0.4 and the proportion of the total variance explained by $g_t^{(J)}$, $\frac{Var(g_t^{(J)})}{Var(g_t)}$, equal to 0.03, 0.05 or 0.07. I run the simulation for T = 256 and T = 2048. The results of this exercise are shown in Table 12.

Table 12: This table shows the probabilities of the variance ratio test for rejecting the null hypothesis of no serial correlation, i.e. white noise, against an autoregressive process for α = 0.05. I simulate decimated components for J = 6: $g_t^{(1)}$, $g_t^{(2)}$, $g_t^{(3)}$, $g_t^{(4)}$, $g_t^{(5)}$, $g_t^{(6)}$ and $\pi_t^{(6)}$ where only the persistent component $g_t^{(6)}$ follows an autoregressive process and the others are independent normal innovations. The outcome is based on N = 5000 replications.

T = 256	Persistence level j =	1	2	3	4	5	6	7
ρ_J	$\frac{Var(g_t^{(J)})}{Var(g_t)}$							
0.2	0.03	0.058	0.048	0.047	0.040	0.040	0.277	0.048
0.2	0.05	0.082	0.049	0.042	0.037	0.036	0.557	0.045
0.2	0.07	0.116	0.056	0.039	0.034	0.033	0.718	0.042
0.4	0.03	0.060	0.048	0.047	0.040	0.040	0.252	0.048
0.4	0.05	0.080	0.048	0.042	0.037	0.037	0.517	0.045
0.4	0.07	0.115	0.057	0.040	0.034	0.033	0.680	0.042
T = 2048	Persistence level j =	1	2	3	4	5	6	7
ρ_J	$\frac{Var(g_t^{(J)})}{Var(g_t)}$							
0.2	0.03	0.075	0.055	0.052	0.046	0.041	0.816	0.039
0.2	0.05	0.210	0.092	0.063	0.047	0.039	0.997	0.035
0.2	0.07	0.427	0.174	0.094	0.055	0.043	1.000	0.032
0.4	0.03	0.076	0.054	0.052	0.046	0.041	0.776	0.039
0.4	0.05	0.207	0.093	0.064	0.047	0.039	0.993	0.035
0.4	0.07	0.425	0.173	0.095	0.057	0.043	1.000	0.032

I observe that for the T = 256 case, my results are very similar to Ortu, Tamoni, and Tebaldi (2013), except for a slight over-rejection of the null hypothesis for the first component when $\frac{Var(g_t^{(J)})}{Var(g_t)}$ becomes larger. This outcome shows the high power of the variance ratio test only at the 'correct' time scale of J = 6 for which the component is persistent. Unfortunately, in my Monte Carlo analysis for T = 2048, a large over-rejection of the null hypothesis of white noise is present for the first two components, compared to Ortu, Tamoni, and Tebaldi (2013). This is particularly true for $\frac{Var(g_t^{(J)})}{Var(g_t)}$ equal to 0.05 and 0.07. I suspect this is due to a mistake in my code but I am not able to find it. I point out that the test still is able to correctly reject the null hypothesis for the persistent component at J = 6.

Despite the over-rejection for the first (two) components, this exercise illustrates that the test reject has strong power against the null hypothesis of white noise when a persistent component is present at a time-scale j.

6.3 Figures comovements components of consumption growth

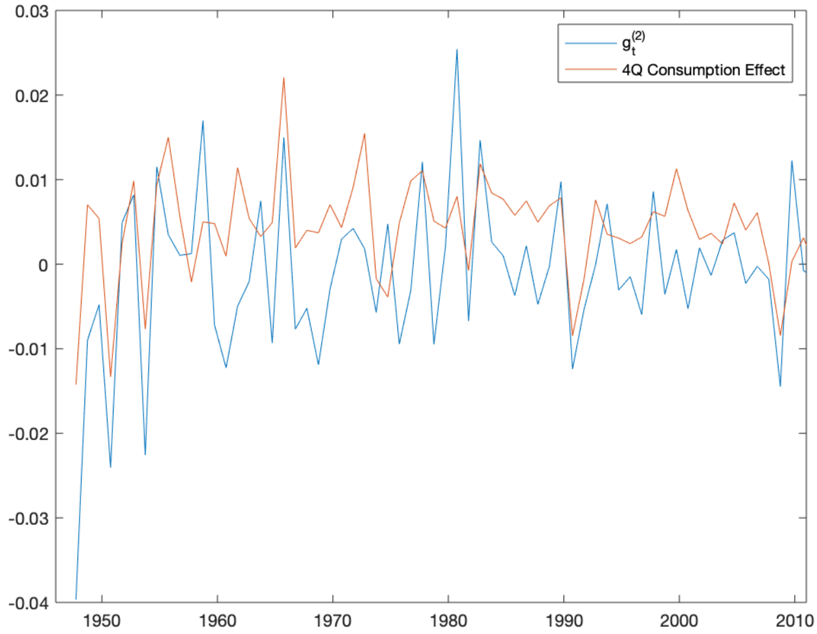


Figure 4: This figure displays the comovement of $g_t^{(2)}$ with the fourth-quarter growth rate of consumption.

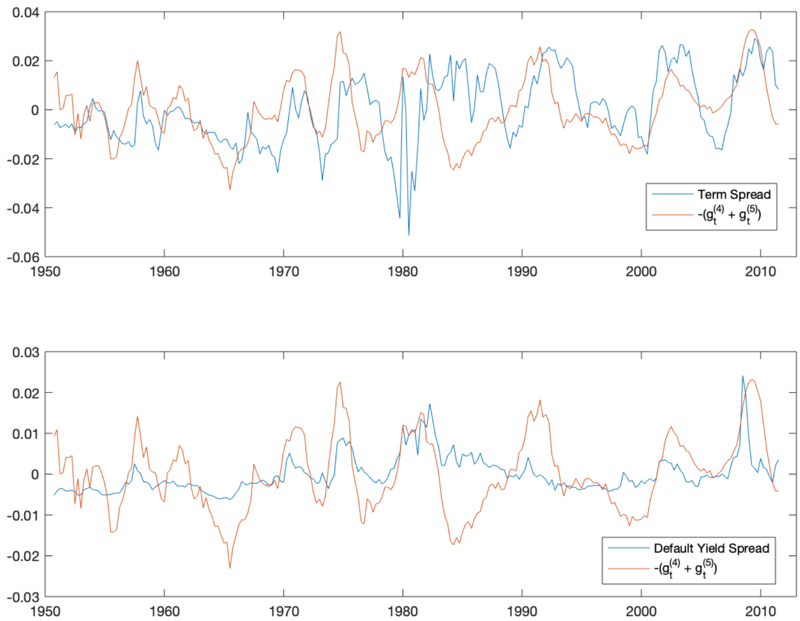


Figure 5: This figure displays the comovement of the negative of the sum of $g_t^{(4)}$ and $g_t^{(5)}$ with the term spread in the top panel and with the default yield spread in the bottom panel.

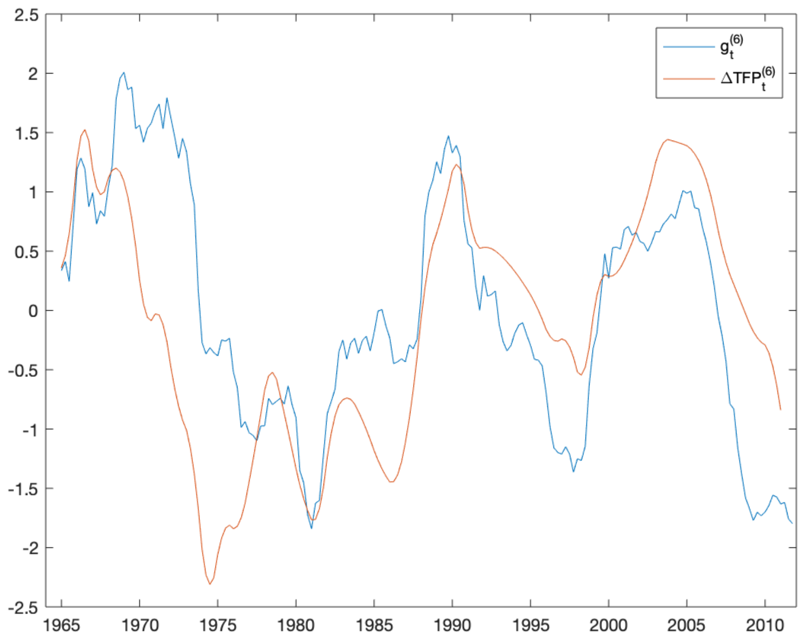


Figure 6: This figure displays the comovement of $g_t^{(6)}$ and the sixth component of TFP growth.

6.4 Figure consumption growth and the price-dividend ratio

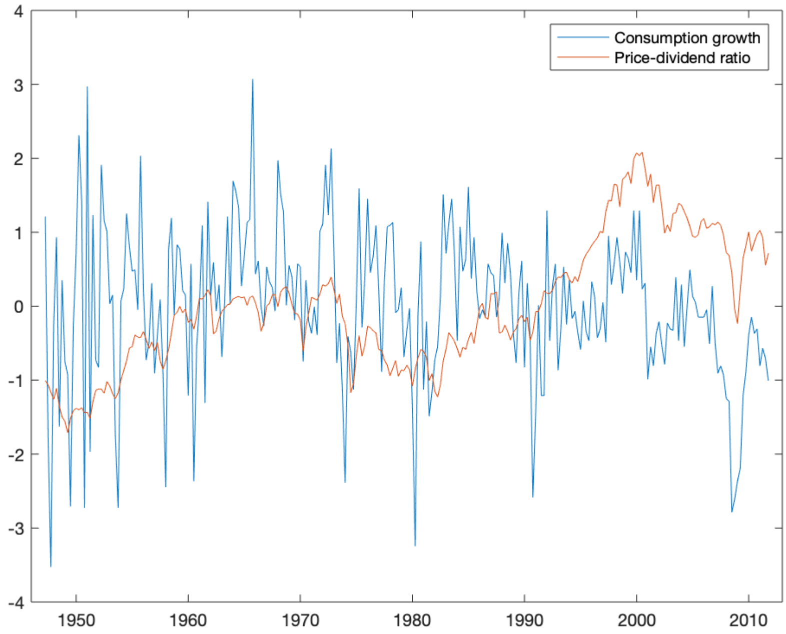


Figure 7: This figure displays the price-dividend ratio and consumption growth.

6.5 Robustness Check

I test the robustness of the findings for the predictive regressions where the explanatory variable is the price-dividend ratio with the use of annual data. I report the results of these regressions in Table 13.

Table 13: This table reports the OLS estimates, multiplied by -100, of the regression of the components of the consumption growth, $g_{t+2^j}^{(j)}$, on the components of the log price-dividend ratio $pd_t^{(j)}$. Significant values at $\alpha = 0.05$ are denoted in bold. Hansen and Hodrick corrected t-statistics are shown in parenthesis and the adjusted R^2 in brackets. The sample period is 1948-2011 and 1930-2011 in the top and lower panel respectfully.

Sample: 1948-2011						
Variable	Persistence level j =	1	2	3	4	5
		3.34	-0.57	0.72	**0.68**	0.19
$pd_t^{(j)}$		(-4.37)	(0.58)	(-1.03)	(-2.08)	(1.79)
		[0.20]	[0.00]	[0.02]	[0.05]	[0.02]
Sample: 1930-2011						
Variable	Persistence level j =	1	2	3	4	5
		3.08	**3.04**	-0.71	1.61	-0.45
$pd_t^{(j)}$		(-4.31)	(-2.07)	(0.56)	(-1.91)	(1.12)
		[0.10]	[0.10]	[0.01]	[0.14]	[0.02]

The results from my regressions are similar to ones Ortu, Tamoni, and Tebaldi (2013) produce. The values are in the same order of magnitude, however, not exactly the same and the sign of the estimates is turned around. As mentioned in the main text, I suspect their values are multiplied by '-100' instead of the stated '100'. The corresponding coefficients are labeled significant for the 1948-2011 sample. However, for the 1930-2011 sample, the component at j = 2 is significant in my analysis, whereas this is not true in their results. I highlight that for my results, the coefficient at j = 4 is almost significant as this is also the case in Ortu, Tamoni, and Tebaldi (2013).

Additionally, similar to Ortu, Tamoni, and Tebaldi (2013), I investigate the characteristics in the long-run of the shocks in log consumption growth. Firstly, I take a look at the truncated sum of the components of consumption growth greater than five together with the long run average, $\sum_{j=6}^{J} g_t^{(j)} + \pi_t^{(J)} = g_{t-2^6-1}$, that represents a smoothed series of consumption growth with shocks lasting more than 32 quarters. In Figure 8, I plot $g_{t-2^{6-1}}$, together with the corresponding truncated sum of the TFP growth series.

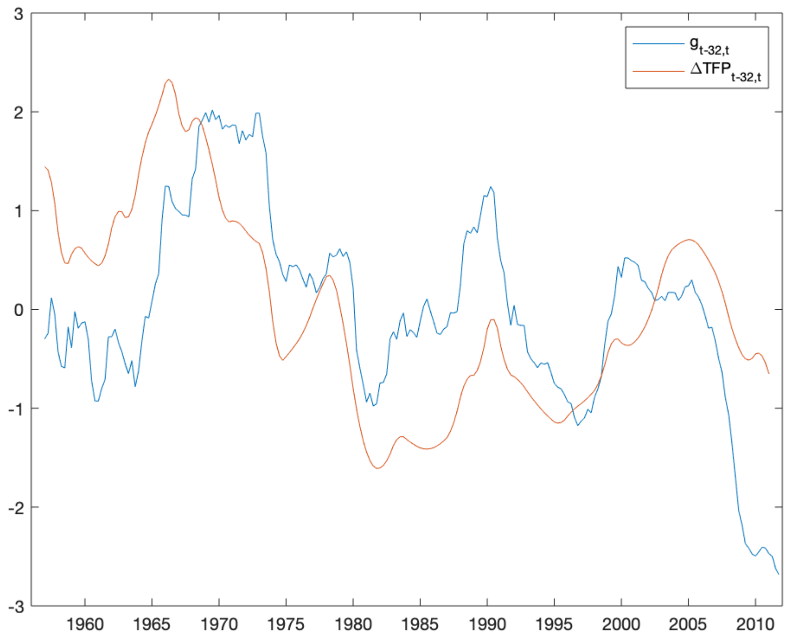


Figure 8: This figure displays the comovement of $g_{t-2^{6-1}}$ and the corresponding truncated sum of TFP growth.

Figure 8 is in line with Ortu, Tamoni, and Tebaldi (2013) and exhibits a large resemblance with Figure 6 that displays the comovement of g_t^6 and the sixth component of TFP growth. It highlights the fact that most of the long-run fluctuations in both series are captured by the sixth component of the respective series.

To further investigate this, I plot the sixth component together with the truncated sum for both the consumption growth and TFP growth series in Figure 9, which is consistent with Ortu, Tamoni, and Tebaldi (2013). I notice that both subplots indeed show substantial comovement, affirming the claim that most of the long-run shock are indeed captured by the sixth component of the corresponding series.

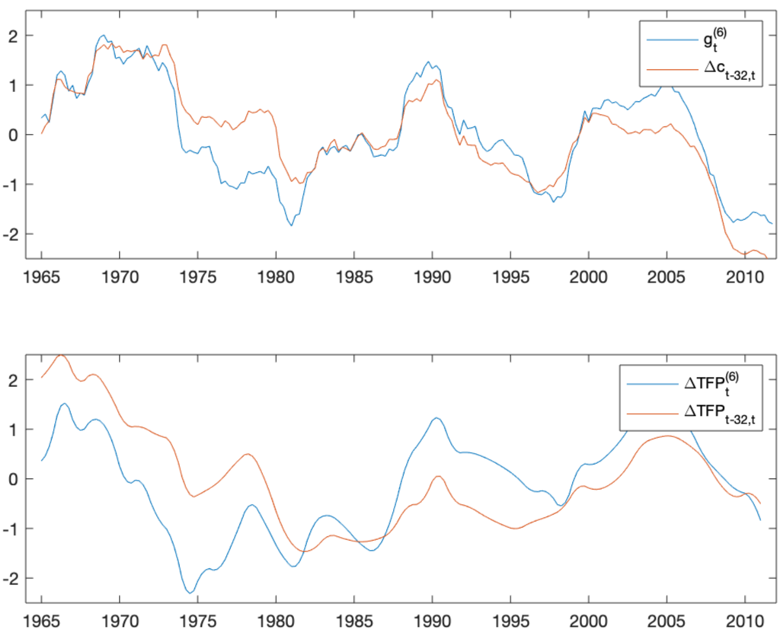


Figure 9: This figure displays the comovement of g_t^6 and $g_{t-2^{6-1}}$ for consumption growth in the top panel and the respective series of TFP growth in the bottom panel.

6.6 Table term structure of risk premia

Table 14: This table reports the annualized equity premium, $E_{t,j}[r_{m,t+1} - r_{f,t}]$, for j = 1,...,7 (in %). $\widehat{\mathbf{Q}}_{jj}$-estimates are shown in the second column. $\gamma = 5$ and $\psi = 2.5$ in top panel. $\gamma = 7.5$ and $\psi = 2.5$ in the bottom panel. The risk exposure and the risk price are annualized as well.

$\gamma = 5$ & $\psi = 2.5$					
Scale j =	Half-life (years)	$\widehat{\mathbf{Q}}_{jj}$ (1 x 10^{-5})	Risk exposure (1 x 10^{-6})	Risk price	Risk premium (%)
1	0.08	0.90	64.51	68.78	0.22
2	0.4	0.49	192.68	191.98	3.70
3	0.6	0.42	57.11	85.20	0.97
4	1.1	0.30	-118.51	-63.04	2.99
5	2.8	0.16	11.11	55.71	0.49
6	7.6	0.08	8.20	30.09	0.39
7	16.4	0.02	-2.27	-287.60	2.09
$\gamma = 7.5$ & $\psi = 2.5$					
Scale j =	Half-life (years)	$\widehat{\mathbf{Q}}_{jj}$ (1 x 10^{-5})	Risk exposure (1 x 10^{-6})	Risk price	Risk premium (%)
1	0.08	0.90	64.51	106.15	0.34
2	0.4	0.49	192.68	296.31	5.71
3	0.6	0.42	57.11	131.51	1.50
4	1.1	0.30	-118.51	-97.30	4.61
5	2.8	0.16	11.11	85.98	0.76
6	7.6	0.08	8.20	46.45	0.61
7	16.4	0.02	-2.27	-443.91	3.22

6.7 Matlab programs

6.7.1 Replication

- **simulationFigure**: This program replicates Figure 3
- **simulationWN**: This program performs one MCMC simulation in Table 12
- **simulationTable**: This program perform all the MCMC simulation in Table 12
- **VRtestLCG**: This program perform the variance ratio test for consumption growth in Table 1
- **optimalNumComp**: This program determines the optimal number of components for consumption growth in Table 2
- **replicationFigures345678**[3]: This (adjusted) program replicates Figures 4, 5, 6, 7, 8 and 9
- **IESestimationJ8**: This program estimates the IES in Table 6 for j = 8
- **IESestimationJ7**: This program estimates the IES in Table 6 for j = 7
- **MultiscaleARregressions**: This program performs the multiscale AR-regressions in Table 7
- **PredictiveRegressions**: This program performs the predictive regressions in Table 3, 4 and 5
- **TermStructureRiskPremia**: This program computes the risk exposures, risk prices and risk premia in Tables 8 and 14
- **olshacAdjusted**[4]: This (adjusted) program performs OLS with Hansen and Hodrick corrected standard errors
- **AnnualPredictiveRegressions**: This program performs the annual predictive regressions in Table 13
- **RedundantHaar**[5]: This program performs a redundant decomposition of a time series
- **ConstructTau**: This program construct the Tau matrix for a given value of j

[3]https://andreatamoni.meltinbit.com/events/publications

[4]https://www.mathworks.com/matlabcentral/fileexchange/43259-ols-with-newey-west-and-hansen-hodrick-se

[5]https://andreatamoni.meltinbit.com/events/publications

6.7.2 Extension

- **macroPlots**: This program replicates Figures 1 and 2
- **VRtestMacro**: This program perform the variance ratio test for consumption growth, GDP growth and investment growth in Tables 9, 10 and 11
- **RedundantHaar**[6]: This program performs a redundant decomposition of a time series

[6] https://andreatamoni.meltinbit.com/events/publications

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