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Capturing crash sensitivity with copulas

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Abstract

This paper revisits the 3-step tail dependence estimation method of Chabi-Yo et al. (2018), which measures the crash sensitivity of individual stocks with the market return. With portfolio sorts analysis on basis of Lower Tail Dependence we find that investors receive compensation for holding crash-sensitive stocks, but on the other hand that holding weak LTD stocks protects against a market crash. We observe difference in impact of LTD on future returns during a post market crash period and the remaining years. We find that reducing the sample period on which we estimate tail dependence does not improve the persistence of the model. Additionally using kernels for estimating marginal distributions of return samples does not improve the model, but we propose a heuristic that has a better fit in the presence of a substantial amount of duplicate observations.

Contents

1 Introduction

During a market crash investors are at risk to make big losses with the stocks in their portfolios. Therefore crash-aversion prevails amongst investors, who want to hedge against this risk by holding crash-insensitive stocks. Accordingly crash-sensitive stocks seem unattractive, as these stocks have bad returns during a market crash. However, Chabi-Yo et al. (2018) show that holding crash-sensitive stocks leads to compensation for the risk. We capture the crash sensitivity of assets with a flexible copula framework that fits the dependence structure of the returns and measure it with lower-tail dependence coefficient (LTD) that defines the probability that individual stocks will decrease in value given that the market returns are bad.

Although the groundwork for copula methods was done by Sklar (1959), the technique has become increasingly popular since the nineties. According to Embrechts (2009) this can be contributed to one sector: Finance, and in particular quantitative risk management. As investors discovered that more accurate measurements of stock sensitivity to the market can prevent big losses. Nowadays there is more awareness for market crash, since a market crash is again in our recent memory.

In this paper we investigate whether we can find the equivalent results as Chabi-Yo et al. (2018) for our dataset: that stocks are persistent in lower tail dependence, holding strong LTD stocks yields a return spread and that the impact of LTD differs from being in a post market crash period or not. In addition to this, we research the impact of shortening the estimation period for the tail dependence coefficient in persistence as well as the fit of alternative marginal distribution estimation methods that could lead to a better fit. We apply our methods on 20 stocks from CRSP trading from Jan 1, 1994 to Dec 31, 2019.

For estimating tail dependence we use the 3-step procedure as proposed by Chabi-Yo et al. (2018) on a rolling window sample 12-months. This 3-step procedure uses a flexible copula framework that is able to capture both lower tail dependence and upper tail dependence. We fit 64 different copula combinations to the empirical marginal distributions of the stock and market returns with a canonical-maximum likelihood procedure of Genest et al. (1995). For each month we select the copula combination that has the smallest Integrated Anderson-Darling distance to its empirical copula, as proposed by Ane & Kharoubi (2003). Then on basis of the tail dependence coefficients, we analyze the future returns of univariate portfolio sorts. The difference between strong and weak LTD portfolios gives us insight in the compensation that is received for holding crash-sensitive stocks.

Next we analyze the persistence of stocks to observe if tail dependence at a given time indicates that the stock will have similar tail dependence in the near future. We use a Fama-MacBeth regression to investigate the impact of LTD on future returns during a post market crash period and in the remaining years. We use Gaussian kernels and a heuristic to investigate whether we can improve the non-parametrical estimation of marginal distributions, that was introduced by Charpentier et al. (2007).

In this research paper we research the impact of using alternative methods for estimating the marginal distribution, which could result in a better fit of the flexible copula framework to the data. As our heuristic results in a strictly uniform $U(0,1)$ distributed marginal distributions for every rolling window sample. Also we observe the impact of shortening the rolling window, to see whether this is a valid alternative for the model.

We report that for a 6-month rolling window model there is persistence in the extreme portfolios that consist of stocks with very high LTD or very low LTD. However, for the portfolios in between there is no certainty in where the stock is going to be located in the near future. Therefore we prefer the 12-month rolling window model, that shows more persistence in the extreme portfolios and also to a smaller extent in the portfolios in between.

Holding the strong LTD (crash-sensitive) stocks in our data yields return premium of 3.78% per year over holding crash-insensitive stocks. In addition, we find that the impact of LTD differs over time when it is measured in a post market crash period or remaining years. During a post market crash period the enhanced awareness of investors results in a return premium of 7.80% per year for holding strong LTD stocks, as investors are willing to pay and yield lower returns for crash-insensitive stocks.

We find that the kernel method model does not improve fit of marginal distributions. On the other hand the heuristic does lead to better fitting copula combinations when there are many duplicate return observations in the rolling window sample. However, it is difficult to state that this is completely due to the 'cleaning' effect of the heuristic.

The 3-step procedure of Chabi-Yo et al. (2018) that we pursue is unique in the sense that it uses the best of 64 combinations of 16 unique copulas to estimate both lower, neutral and upper tail dependence at a firm-month level. Hu (2006) used mixture copulas that combined two copulas to capture dependence structures across stock markets and Zimmer (2012) used similar mixture copulas on housing prices. It also corresponds with the theory of Gennaioli et al. (2012), that shows that the awareness for a market crash differs over time.

In Section 2 we report the dataset for our research. In Section 3, we clarify the general framework of copulas and explain the methods used for our research. The results of our research are presented in Section 4. Lastly, the concluding remarks can be encountered in Section 5.

2 Data

The data used in this research consists of stocks from CRSP trading with share code 10 and 11 on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotations (NASDAQ) between Jan 1, 1994 and Dec 31, 2019. With a rolling window of 12 months, this leads to 300 estimations per stock. It is a computationally heavy burden to replicate the results for all stocks. Therefore, we estimate our results on a small representative sample.

To have stock sample that is as representative as possible, the stocks that were not on the market throughout the whole sample period are filtered out. We do not want our results to be driven by very small stocks. Therefore we do not consider stocks that belong to the lowest 1% market capitalization at the beginning of our sample period (the first quarter of 1994). Hereafter, we split all stocks into 3 groups based on market capitalization levels on the last trading day of the sample: large cap (10 billion or more), medium cap (2-10 billion) and small cap stocks (300 million - 2 billion). To have a representative sample, we want to include small, medium and large sized stocks pro ratio. These groups each account for around a third of stocks that suffice the filter condition.

Therefore, we randomly picked 7 large cap, 7 medium cap and 6 small cap stocks from a pre-filtered selection of stocks. (See Appendix.A-Table 11 for definitive selection of stocks). This gives us a sample with $6,000$ firm-month observations in total.

In Table 1, the summary statistics of the returns of the stocks in the considered sample, the total market return (equal-weighted, excluding dividend) and the one month T-bill Rate are shown.

Table 1: Summary statistics of the stock returns, market return, one month T-bill Rate.

	Mean	25% Quantile	Median	75% Quantile	Standard Deviation
Stock Returns	0.06%	-1.04%	0.00%	1.06%	0.028
Total Market Return	0.06%	-0.35%	0.13%	0.53%	0.010
T-bill rate	0.21%	0.04%	0.17%	0.40%	0.002

3 Methodology

In this section we explain what kind of econometric methods and statistical techniques we use to research the problem. In Section 3.1, we explain the theoretical framework of copulas, based on the papers Patton (2012) and Durante & Sempi (2010). Then, in Section 3.2, we discuss the estimation method for tail dependence from Chabi-Yo et al. (2018). Subsequently, the alternative methods for estimating the marginal distributions are considered in Section 3.3. At last, we discuss the performance measures that we use in Section 3.4.

3.1 Theoretical framework of copulas

A copula C is a function that describes the dependence structure between random variables $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n)$. Sklar (1959) introduced the term *copula* by stating that an n-dimensional joint distribution could be traced back to n univariate marginal distributions and a multi-dimensional copula. As is presented in Equation 1, where F_i is the marginal distribution of $\mathbf{x}_i$ for $i \in \{1, 2, ..., n\}$ and observations over time t $(x_{i,1}, ..., x_{i,t})$ are in $\mathbb{R} = [-\infty, \infty]$.

$$F(\mathbf{x}) = C\{F_1(\mathbf{x}_1), ..., F_n(\mathbf{x}_n)\}. \tag{1}$$

In short, a copula C can be defined as C: $[0,1]^n \to [0,1]$. Where n univariate marginal cumulative distributions $U(0,1)$ define a multivariate cumulative distribution. When the univariate marginal distribution F_i is continuous for every $i \in \{1, ..., n\}$, the formula for copula C is unique and can be defined as

$$C(u_1, ..., u_n) = F(F_1^{-1}(\mathbf{u}_1), ..., F_n^{-1}(\mathbf{u}_n)), \tag{2}$$

where F_i^{-1} is the inverse of F_i and $\mathbf{u}_i$ are random variables distributed $U(0,1)$.

In this paper we are interested in measuring the crash sensitivity of an individual stock during extreme events as a market crash (Lower-left Tail Dependence) and a market boom (Upper-right Tail Dependence). More theoretically defined, when we have two random variables (X_1, X_2), with marginal cumulative distributions (F_{x_1}, F_{x_2}), respectively. Then we can define Equation 3 as a tail dependence measure for the left tail and Equation 4 for the right tail, with q being a random quantile.

$$P_l(q) = Pr[X_1 < F_{x_1}^{-1}(q) | X_2 < F_{x_2}^{-1}(q)] \tag{3}$$

$$P_r(q) = Pr[X_1 > F_{x_1}^{-1}(q) | X_2 > F_{x_2}^{-1}(q)] \tag{4}$$

We define LTD and UTD as,

$$LTD = \lim_{q \to 0^+} P_l(q) \quad \text{and} \quad UTD = \lim_{q \to 1^-} P_r(q) \tag{5}$$

In simple terms, LTD (UTD) measures the probability that an extreme low (high) value for one random variable coincides with an extreme low (high) value for the other.

Sensitivity to the market is most commonly measured with the β_s in CAPM, where it represents the linear correlation coefficient. This model does not suffice for our research problem as it can not measure the sensitivity in the tails. Figure 1 shows that although the linear correlation is roughly the same, the dependence structure can differ substantially. The advantage of a copula is that it can capture tail dependence, because a copula is flexible and a valid tool for modelling the varying behaviours of marginal distributions.

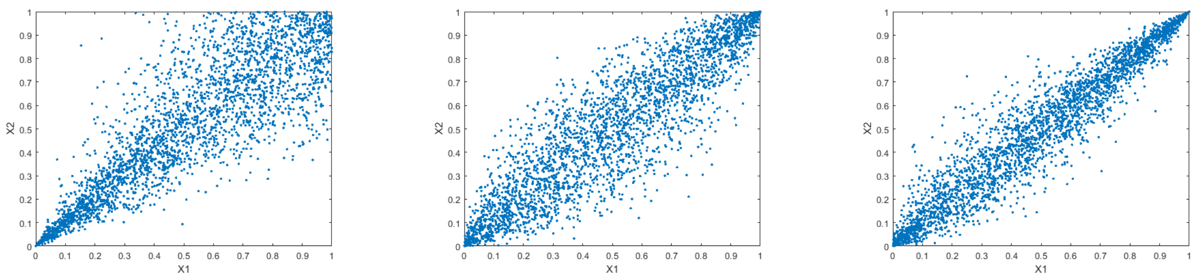

Figure 1: Three scatter plots with different measures of tail dependence where copula defines the dependence structure on 3,000 random observations on bivariate copulas. All scatter plots have a linear correlation of roughly 0.9. From left to right a.) Clayton copula that exhibits lower tail dependence b.) Gaussian copula that exhibits no tail dependence c.) Gumbel copula that exhibits upper tail dependence.

In this paper we consider two random variables $X_{1,s}$ and X_2 (the returns of individual stock s and the market returns, respectively, in the sample period). A copula defines its dependence structure from the univariate marginal distributions $F_1(x_{1,s})$ and $F_2(x_2)$ of the bivariate distribution $F(x_{1,s}, x_2)$. Each copula has a dependence parameter θ that parameterizes the copula and thus defines the bivariate distribution $F(x_{1,s}, x_2)$,

$$F(x_{1,s}, x_2) = C(F_1(x_{1,s}), F_2(x_2); \theta). \tag{6}$$

With the dependence parameter θ, it is possible to compute the tail dependence coefficient.

3.2 3-step Tail Dependence Estimation Procedure

The main focus of the paper Chabi-Yo et al. (2018) is to research whether the stocks with strong LTD have significantly higher return premia than weak LTD stocks. Therefore we need to compute the tail dependence at firm-month level. The estimation of the LTD and UTD-coefficients in Chabi-Yo et al. (2018) is done via a 3-step approach.

First, we estimate the marginal distributions $\hat{F}_i(x)$ and $\hat{F}_m(x)$. The marginal distributions of stock return r_i and market return r_m are estimated non-parametrically by their scaled empirical distribution functions, the formulas are provided in Equation 7 and 8. In these equations $\{r_{i,k}, r_{m,k}\}_{k=1}^{n}$

is the 12-month rolling window sample, that includes all daily individual stock returns and daily market returns that have been registered in the prior year. This way the method accounts for the time-varying dependence of a stock and the market, as the dependence measures will vary from month to month.

$$\hat{F}_i(x)^{(\mathbf{1})} = \frac{1}{n+1}\sum_{k=1}^{n} \mathbf{1}_{r_{i,k} \leq x} \tag{7}$$

$$\hat{F}_m(x)^{(\mathbf{1})} = \frac{1}{n+1}\sum_{k=1}^{n} \mathbf{1}_{r_{m,k} \leq x} \tag{8}$$

Hereafter, we estimate the dependence parameters θ_i (also referred to as copula parameters) with the canonical maximum-likelihood method, as proposed by Genest et al. (1995). Most copulas can not measure both LTD and UTD, therefore we use a copula framework that allows for flexibility, as is shown in Equation 9.

$$C(u_1, u_2, \Theta) = w_1 \times C_{LTD}(u_1, u_2, \theta_1) + w_2 \times C_{NTD}(u_1, u_2, \theta_2) + (1 - w_1 - w_2) \times C_{UTD}(u_1, u_2, \theta_3) \tag{9}$$

For each month we estimate 64 (=4x4x4) different convex copula combinations by considering four copulas that allow for LTD (C_{LTD}), four copulas that allow for UTD (C_{UTD}) and four copulas that exhibit no tail dependence (C_{NTD}). The parametric forms of the copulas can be found in the Internet Appendix Table IA.I of Chabi-Yo et al. (2018). This copula framework provides that we can select the best copula combination by using weights w_1 and w_2, as shown in Equation 9. Therefore we estimate five copula parameters Θ_j ($\theta_1, \theta_2, \theta_3, w_1$ and w_2) simultaneously via the canonical maximum-likelihood procedure of Genest et al. (1995), as shown in Equation 10. In this step we fit the copula framework as good as possible to the dependence structure of the marginal distributions $\hat{F}_i(x)$ and $\hat{F}_m(x)$ that are estimated in Equation 7 and 8.

$$\hat{\Theta}_j = \arg\max_{\Theta_j} L_j(\Theta_j) \quad \text{with} \quad L_j(\Theta_j) = \sum_{k=1}^{n} \ln(c_j(\hat{F}_{i,r_{i,k}}, \hat{F}_{m,r_{m,k}}; \Theta_j)) \tag{10}$$

Where $L_j(\Theta_j)$ is the log-likelihood function of copula combination $j \in \{1, ..., 64\}$ and copula density function $c_j(\cdot, \cdot; \Theta_j)$ is the *pdf* of the copula framework in Equation 9 for copula combination j.

With the estimated copula parameters $\hat{\Theta}_j$ for copula combinations $j = \{1, ..., 64\}$, we can select the best copula combination on basis of the Integrated Anderson-Darling test (IAD), as is proposed in Ane & Kharoubi (2003). The IAD-test allows us to measure how well the estimated copula combination $\hat{C}_j(\cdot, \cdot; \hat{\Theta}_j)$ distribution fits the empirical distribution, particularly in the tails. To measure the goodness of fit of each copula combination, we use the empirical copula $\hat{C}_{(n)}$ introduced in Deheuvels (1980), as shown in Equation 11.

$$\hat{C}_{(n)}\left(\frac{t_i}{n}, \frac{t_m}{n}\right) = \frac{1}{n}\sum_{k=1}^{n} \mathbf{1}_{R_{i,k} \leq t_i} \times \mathbf{1}_{R_{m,k} \leq t_m} \tag{11}$$

Where $R_{i,k}$ ($R_{m,k}$) stands for the rank statistic of stock (market) return observation $r_{i,k}$ ($r_{m,k}$). This means that largest individual stock return observation in the rolling window sample $R_{i,k} = n$ and for the smallest individual return $R_{i,k} = 1$. Hereafter, t_i and t_m are defined on the lattice provided in Equation 12.

$$L = \left[\left(\frac{t_i}{n}, \frac{t_m}{n}\right), t_i = 0, 1, ..., n, \quad t_m = 0, 1, ..., n\right] \tag{12}$$

Ultimately, we select the copula combination $C_j(\cdot,\cdot;\hat{\Theta}_j)$ that has the smallest IAD distance $D_{j,IAD}$ to the empirical copula $\hat{C}_{(n)}$. The formula for the distance is provided in Equation 13.

$$D_{j,IAD} = \sum_{t_i=1}^{n}\sum_{t_m=1}^{n} \frac{\left(\hat{C}_{(n)}\left(\frac{t_i}{n},\frac{t_m}{n}\right) - C_j\left(\frac{t_i}{n},\frac{t_m}{n};\hat{\Theta}_j\right)\right)^2}{C_j\left(\frac{t_i}{n},\frac{t_m}{n};\hat{\Theta}_j\right) \times \left(1 - C_j\left(\frac{t_i}{n},\frac{t_m}{n};\hat{\Theta}_j\right)\right)} \tag{13}$$

With the knowledge of which copula combination C_j fits best for month l and the corresponding copula parameters $\hat{\Theta}_j$, we can calculate the tail dependence coefficients LTD ($w_1 \times \text{LTD}(\theta_1)$) and UTD ($(1 - w_1 - w_2) \times \text{UTD}(\theta_3)$). The formulas for LTD (UTD) of C_{LTD} (C_{UTD}), are displayed in the Appendix.A-Table 12.

3.3 Alternative Estimation Methods for Marginals

In this section we explain the alternative methods that we use instead of Equation 7 and 8. First, we propose the Duplicates Removing-Heuristic. Then we discuss the Kernel Method.

3.3.1 Duplicates Removing-Heuristic

From the scaled empirical marginal distribution in Equation 7 and 8, we expect the marginal distribution functions $\hat{F}_i(x)$ and $\hat{F}_m(x)$ to be distributed uniformly on the interval $(0,1)$ ($U(0,1)$). This is a direct consequence if the condition that all $\{r_{i,k}, r_{m,k}\}_{k=1}^{n}$ in the rolling window sample have unique values is suffice. Then stocks will be assigned to their quantile-'rank' on the interval $(0,1)$. However, we discovered that in the beginning of the sample period most stocks have a substantial amount of duplicate observations. Between $1994 - 1997$ the rolling window samples of the stocks contain on average 75 duplicate values. Whereas the total market return in the sample period has on average one duplicate value. Subsequently, there are extreme cases of 12-month rolling window samples that consist of 100 unique return observations out of a total of 250 observations. This is due to the less accurate storing of stock prices (less decimals) and less active trading in the beginning of our sample period ($1994 - 2000$). Chordia et al. (2011) demonstrated that number of trades have increased over the years $1993 - 2008$ on the NYSE.

The duplicate observations are an obstruction for the marginal distribution function $\hat{F}_i(x)$ of Equation 7 and 8 to estimate a strictly uniform marginal distribution, as in Figure 2b. The duplicate observations are ranked with upward bias. For example, for observation $r_{i,k} = 0$, the marginal distribution function $\hat{F}_i(r_{i,k})$ includes all other duplicate zero observations as smaller than $r_{i,k}$ in Equation 7. Therefore all zero return observations get a higher quantile-rank in the marginal distribution than they would get if it was a unique value. This results in a marginal distribution that is not uniformly distributed $U(0,1)$ (with distortion), as is shown in Figure 2a.

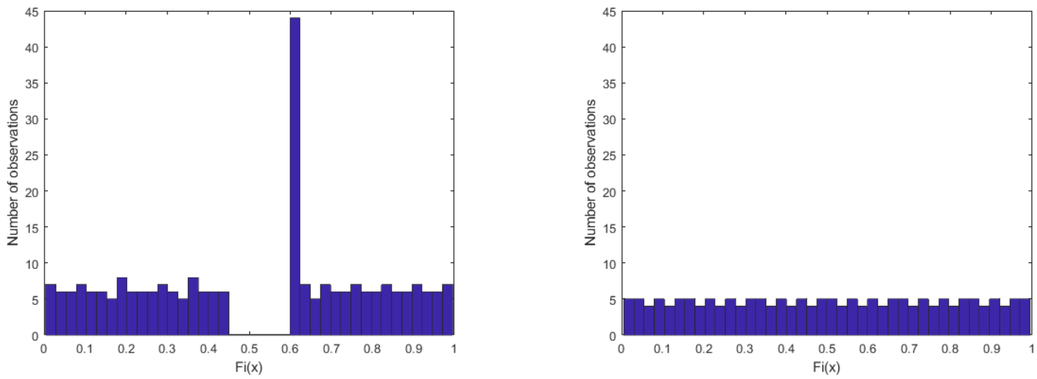

Figure 2: From left to right a.) marginal distribution of stock returns for a rolling window sample as proposed by Chabi-Yo et al. (2018) b.) the marginal distribution for the same rolling window computed with the proposed duplicates-heuristic

To eliminate the distortion of the duplicate return observations we propose a heuristic, such that the marginal distributions are strictly $U(0,1)$ distributed. First, we identify all days that have a duplicate return r_{i,d_j} for stock i in the rolling window, where $d_j \in \{1,...,n\}$ represents the duplicate observations for each return value j. Next we collect the market returns $r_{m,d}$ for all days with duplicate individual stock returns. Subsequently, we compute the average market return $\overline{r_m}$ for these days such that we can construct a unique pair $(r_{i,d_j}, \overline{r_{m,d_j}})$ that represents the duplicate observations. Finally, we delete all duplicate return days from the rolling window sample $\{r_{i,k}, r_{m,k}\}_{k=1}^{n}$ and replace them with the unique pairs $(r_{i,d}, \overline{r_{m,d}})$. Hereafter, we continue with the 3-step tail dependence estimation procedure at Equation 7. One should note that the rolling window sample with returns after this duplicates removing-procedure is also used for constructing the empirical copula in Equation 11.

This heuristic results in a smaller rolling window sample, but the output of the duplicates removing-heuristic is a strictly uniformly marginal distribution, as is exhibited in Figure 2b. If there are no duplicate values in the rolling window sample, then the heuristic procedure is exactly equal to the 3-step procedure in Chabi-Yo et al. (2018). From 2003 on we detect on average five duplicate observations per rolling window sample, therefore we estimate tail dependence with this method over the period $1995 - 2005$.

3.3.2 Kernel Method

Besides handling the distortion of duplicate values, we are also interested whether we can improve the estimation of tail dependence by estimating the marginal distribution more accurately using the Kernel Method. Charpentier et al. (2007) state that a well-specified marginal function can improve the results, but also that misspecification causes deterioration. Ultimately, without valuable prior information, non-parametric estimation should be used for the estimation of the marginal distribution, such as the empirical distribution function in Equation 7 and 8 that is also introduced by Charpentier et al. (2007).

Charpentier et al. (2007) also propose another method in their paper to estimate smooth marginal CDF distributions non-parametrically. This is the Kernel Method that uses a univariate Gaussian kernel function $K : \mathbb{R} \rightarrow \mathbb{R}$, $\int K = 1$ as shown in Equation 14.

$$\hat{F}_k(x)^{(\mathbf{2})} = \frac{1}{n} \sum_{k=1}^{n} \mathbf{K} \left(\frac{x - r_{i,k}}{h} \right) \tag{14}$$

Parameter h stands for the bandwidth of the kernel. This parameter is set by rule of thumb of Scott (1992), which is optimal for normal distributions.

3.4 Performance Measures

In this section we discuss the performance measures, persistence in LTD, Fama-MacBeth regression and IAD-statistics, that we have use to research whether we have upgraded our model in persistence and fit, and for investigating if we find the same phenomena for our dataset as Chabi-Yo et al. (2018).

3.4.1 Persistence in LTD

Risk-averse investors want to be protected against market crashes. Therefore, they might be interested in holding stocks with weak LTD in their portfolio. However, we first need to determine if a stock that has weak LTD coefficients in month l will have weak LTD again in the future, before investors are able to profit from holding weak LTD stocks when the market crashes.

We evaluate this persistence of stocks in LTD by constructing a Markov-chain-like matrix. This matrix measures the relative frequency that a stock transitions from portfolio i in year $t-1$ (month $l-12$) to portfolio j year t (month l). We study the transitions over a year, so the 12-month rolling windows are disjoint. As the estimation of month l and $l-1$ share eleven months of the same observations. Consequently, the results would naturally show persistence in LTD. The model has good persistence when there is a plausible likelihood that a stock will return in (or near) the same portfolio and thus has equivalent LTD measurements.

With respect to the persistence in LTD we investigate the effect of shortening the 12-month rolling window to a 6-month rolling window. This way only the return observations, r_i and r_m, in the half-year prior influence the estimation of tail dependence coefficients. Therefore a 6-month rolling window can be an advantage, as the considered sample consists of observations that are highly relevant for the state of the stock. Whereas a 12-month rolling window can consist of observations from a year prior that do not represent the current state of the stock. On the other hand, the tail dependence measurements can be more volatile, because extreme observations will have more influence on the average estimation.

3.4.2 Fama-MacBeth Regression

We use the Fama-MacBeth regression as introduced by Fama & MacBeth (1973) to explain the influence of risk factors, such as LTD, on stock returns. First, we regress the stock returns against factors LTD and UTD, see Equation 15. This measures to what extent each stock is exposed to a factor for every period in T.

$$\begin{aligned} R_{1,t} &= \alpha_1 + \beta_{1,F_{LTD}} F_{LTD,t} + \beta_{1,F_{UTD}} F_{UTD,t} + \epsilon_{1,t} \\ R_{2,t} &= \alpha_2 + \beta_{2,F_{LTD}} F_{LTD,t} + \beta_{2,F_{UTD}} F_{UTD,t} + \epsilon_{2,t} \\ &\vdots \\ R_{n,t} &= \alpha_n + \beta_{n,F_{LTD}} F_{LTD,t} + \beta_{n,F_{UTD}} F_{UTD,t} + \epsilon_{n,t} \end{aligned} \tag{15}$$

Where $R_{i,t}$ is the return of stock i at time t. $F_{j,t}$ is factor j at time t and β_{i,F_j} is the factor exposure of stock i to factor j. Then we regress the cross-section of stock returns $R_{i,t}$ against the factor exposures $\hat{\beta}_{i,F}$, that we computed in the previous step, see Equation 16. In this step we essentially measure if a larger factor exposure leads to higher returns.

$$\begin{aligned} R_{i,1} &= \gamma_{1,0} + \gamma_{1,1}\hat{\beta}_{i,F_{LTD}} + \gamma_{1,2}\hat{\beta}_{i,F_{UTD}} + \epsilon_{i,1} \\ R_{i,2} &= \gamma_{2,0} + \gamma_{2,1}\hat{\beta}_{i,F_{LTD}} + \gamma_{2,2}\hat{\beta}_{i,F_{UTD}} + \epsilon_{i,2} \\ &\vdots \\ R_{i,1} &= \gamma_{T,0} + \gamma_{T,1}\hat{\beta}_{i,F_{LTD}} + \gamma_{T,2}\hat{\beta}_{i,F_{UTD}} + \epsilon_{i,T} \end{aligned} \tag{16}$$

The regression coefficients γ are then used to calculate the risk premium, by averaging γ over T. The t-statistics are computed using the Newey & West (1986) standard errors with four monthly lags.

3.4.3 Integrated Anderson-Darling Statistic

In Section 3.2 we demonstrated that we can measure the fit for each copula combination with the IAD-test. We can use this performance measure to compare the Kernel Method with the Empirical Distribution Method as proposed in Chabi-Yo et al. (2018), because we use the same rolling window sample and construct the empirical copula on the same returns.

For the Duplicates Removing-Heuristic this is more complicated. It does consider the same rolling window sample at the start of the 3-step procedure. However, the heuristic summarizes the duplicate observations into one observation. This means that if the 12-month rolling window sample consists of d duplicates, the IAD-statistic is computed over approximately d^2 less observations, such that we can expect beforehand that the IAD-statistic will be smaller. Therefore we can not state with certainty whether the Duplicate Removing-Heuristic has a better fit to the marginal distributions, because of the smaller amount of observations or because of a better fit. However it remains interesting to observe if the IAD-statistic decrease is proportional to the decline in observations used for computing the IAD-statistic.

4 Results

In this section we research if the phenomena that are demonstrated in Chabi-Yo et al. (2018) also can be found for our data. Firstly, we present the statistics found for our data in Section 4.1. Then we investigate persistence in LTD and the impact of shortening the rolling window on the persistence in Section 4.2. In Section 4.3 we observe the difference in return premia for strong and weak LTD stocks. Hereafter, we examine this difference after a market crash in Section 4.4. At last, we consider the performance of the alternative methods for estimation the marginal distributions in Section 4.5.

4.1 Sample results

We find that there is more LTD than UTD present in our data sample see Figure 3 (for underlying statistics see Appendix.A-Table 13). We can also deduct from this table that the LTD coefficient fluctuates more than UTD coefficient. Both coefficients do not show a trend. For LTD we observe a crash just before 2000 and high LTD from 2006 − 2012 which coincides with the financial crisis of 2008. LTD and UTD seem to move in the same direction as they have a correlation of 0.4519.

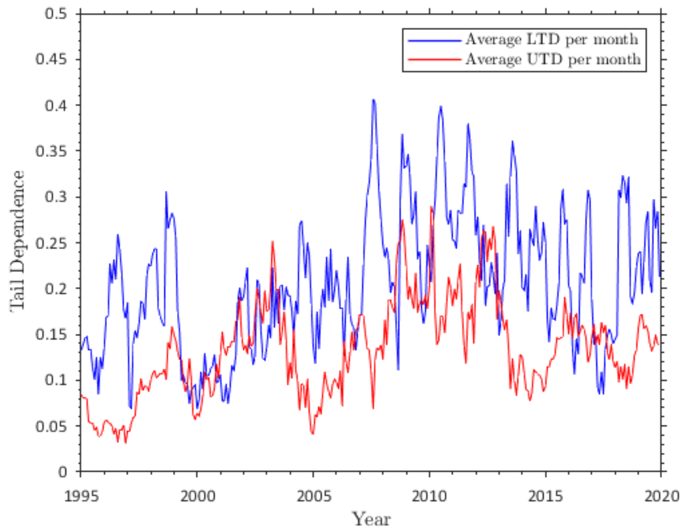

Figure 3: Average Tail Dependence measured per month in sample period from January 1994 - December 2019.

Furthermore, the copula combinations are selected with roughly the same frequency, as there is no dominant copula combination. In Appendix.A-Table 14 the percentages of selection frequency per copula combination can be found, with the Rotated Galambos-Frank-Joe combination being selected the most 4.22% and the Rotated Gumbel-FGM-Rotated Clayton combination the least with 0.48%.

4.2 Persistence

The results of the LTD transition analysis are visualized in Figure 4 and the underlying numbers are reported in Table 2 and 3. The matrices in Figure 4 show the relative frequency that a stock in portfolio i in year $t-1$ can be found in portfolio j in year t, with $i, j \in \{1, ...5\}$ (where 1 (5) represents the weakest (strongest) LTD portfolio). We can deduct that the LTD transition matrix for the 6-month rolling window is flatter than the matrix for the 12-month rolling window.

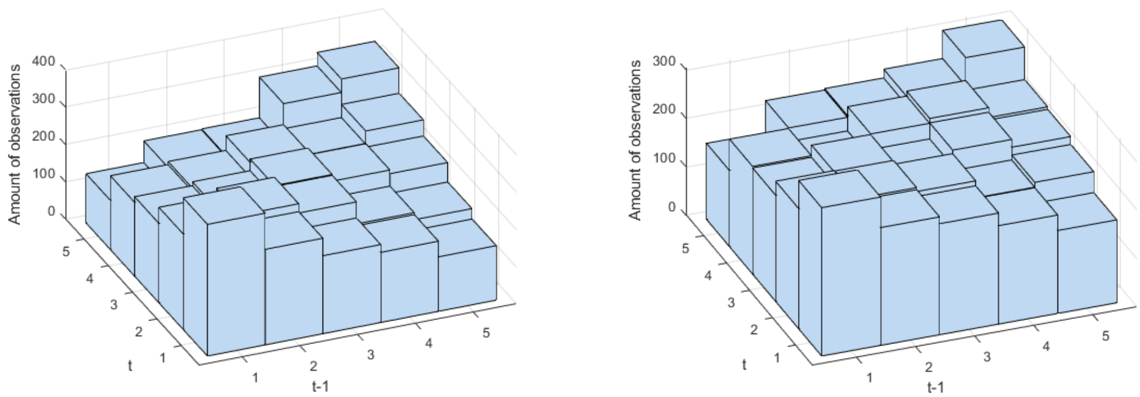

Figure 4: Visualizations of the LTD transition matrices, where the left matrix a.) exhibits the frequency that stocks transition from portfolios on basis of LTD estimated on a 12-month rolling window. The right matrix b.) exhibits the same transition-frequency based on LTD estimated on a 6-month rolling window.

Table 2 provides the relative frequency that stocks transition from portfolios, which are the underlying percentages of Figure 4a. Without persistence in LTD the transition probabilities would be around 20% for all stocks and all years, because then the stock transitions randomly from portfolio. Table 2 implies that there is persistence in LTD, because if a stock is in either the strongest or the weakest LTD portfolio in year $t-1$, it has a probability of 29.25% and 30.64%, respectively, to appear in the same portfolio in year t. We also regressed the LTD in year t on the LTD in year $t-1$, with Newey & West (1986) standard errors (on four monthly lags). This resulted in a coefficient 0.441 with t-statistic (3.81) (significant at 1% level), which confirms that the model exhibits substantial persistence.

Table 2: The empirical probabilities that a stock will be portfolio j in year t given that the stock was sorted in portfolio i in year $t-1$, where the LTD is estimated on basis of a rolling window of 12 months.

		t				
		1	2	3	4	5
	1	30.64%	22.22%	18.49%	17.19%	11.46%
	2	22.22%	24.05%	19.36%	17.88%	16.49%
t-1	3	18.14%	21.88%	22.05%	21.27%	16.67%
	4	16.32%	16.75%	20.49%	20.31%	26.13%
	5	12.67%	15.10%	19.62%	23.35%	29.25%

The extreme portfolios show distinctly more LTD persistence, but in portfolios 2 and 4 we observe a clear difference. When a stock is in a weak portfolio 2 in year $t-1$ it has bigger probability to be found in a weak/weaker portfolio ((1+2) = 46.27%) than in a strong/stronger portfolio ((4+5) = 34.37%). The same holds for portfolio 4 in opposite reasoning. Nonetheless, this result also displays that the LTD of stocks changes over time.

The results in Table 3 show that the extreme portfolios (the weakest (1) and strongest (5) LTD portfolios) exhibit persistence in LTD, as the stocks in these portfolios have a higher likelihood (26.56% and 26.22%) to be found in the same portfolio a year later. However, these probabilities are lower than for the 12-month rolling window model. When we regress the LTD of year t on the LTD of year $t-1$, this gives a coefficient of 0.326 with t-statistic (2.69), which is significant at the

10% level. This indicates persistence, but to a lesser extent. Furthermore, Table 3 demonstrates that there is convincing lower probability for a stock in portfolio 1 (5) in year $t-1$ to be found in the other portfolio 5 (1) in year t (14.84% (13.63%)).

Table 3: The empirical probabilities that a stock will be portfolio j in year t given that the stock was sorted in portfolio i in year $t-1$, where the LTD is estimated on basis of a rolling window of 6 months.

		t				
		1	2	3	4	5
	1	26.56%	21.53%	19.01%	19.27%	13.63%
	2	21.09%	21.44%	21.18%	16.93%	19.36%
t-1	3	19.88%	20.75%	18.49%	21.27%	19.62%
	4	17.62%	17.71%	21.35%	22.14%	21.18%
	5	14.84%	18.58%	19.97%	20.40%	26.22%

Besides the lower probabilities for the extreme portfolios, it is noticeable that the probabilities for the portfolios with less tail dependence (2,3,4) differ less. For the 6-month rolling window model the difference is around 3% for these portfolios. This means that when a stock is found in portfolio $i \in \{2,3,4\}$ in year $t-1$, it appears in portfolio $j \in \{1,2,3,4,5\}$ with approximately the same probability. Therefore we prefer the 12-month rolling window model, as there is more persistence present for portfolios 2 and 4.

The results for persistence in LTD encourage holding weak LTD stocks, as it indicates that the stock will have weak LTD in the future. Table 4 shows that having strong LTD stocks pays off, as it prevents investors from big losses during the Asian Crisis and the Dot-Com Bubble Burst. During the Lehman Crisis there is no clear prevention of big losses on basis of LTD.

Table 4: Daily portfolio returns on crisis days (Asian Crisis (October 27th, 1997), Dot-Com Bubble Burst (April 14th, 2000), Lehman Crisis (October 15th, 2008)), where portfolios are sorted on basis of the most recent rolling window.

	Asian Crisis	Dot-Com Bubble Burst	Lehman Crisis
5 Strongest LTD Portfolio	-6.77%	-6.79%	-10.67%
4 Strong LTD Portfolio	-8.60%	-7.26%	-11.48%
3 Neutral LTD Portfolio	-6.39%	-2.80%	-7.99%
2 Weak LTD Portfolio	-5.13%	-5.09%	-12.18%
1 Weakest LTD Portfolio	-2.52%	-1.81%	-10.93%
Strongest - Weakest	-4.25%	-4.99%	0.27%

4.3 Weak vs. Strong LTD Stocks

We are interested in the difference in future return premia of individual stocks with strong tail dependence and those with weak tail dependence. To measure this difference, we sort all the stocks in our sample into five portfolios sorted on tail dependence (Strongest, Strong, Neutral, Weak and Weakest Tail Dependence portfolio). As our sample consists of 20 stocks, this results in four stocks per portfolio. Every month the stocks are redistributed based on their LTD and UTD-coefficient. We assign the stocks with the highest (lowest) tail dependence coefficient to the strongest (weakest) LTD/UTD portfolio. Then we collect the monthly returns of the stocks in next month $l+1$, such that we mimic a trading situation, where we know the tail dependence coefficients of all stocks at the end of month l. With these future monthly returns we compute the equal-weighted portfolio returns. Hereafter, we subtract the monthly T-bill rate (the risk free rate) that can be found in the Kenneth R. French Data Library, such that the results represent excess returns.

Table 5: Results of univariate equal-weighted portfolio sorts based on the LTD coefficients estimated on a 12-month rolling window for the 20 individual stocks in our sample in the sample period Jan. 1995 to Dec. 2019. All results in the second and third column are significant, unless the t-statistic is provided. We test the significance with t-statistics (in parentheses), that are computed with Newey & West (1986) standard errors with 4 monthly lags. *, ** and *** represent 1%, 5% and 10% significance respectively.

Portfolio	LTD	Return	CAPM Alpha	FF5 Alpha
5 Strongest LTD Portfolio	0.37	1.171%	0.490%*	0.301%
4 Strong LTD Portfolio	0.27	1.332%	0.674%*	0.623%
3 Neutral LTD Portfolio	0.20	1.060%	0.442%	0.157%
2 Weak LTD Portfolio	0.13	0.913%	0.407%	0.037%
1 Weakest LTD Portfolio	0.05	0.856%	0.363%	-0.045%
Strongest - Weakest	0.32***	0.315%	0.127%	0.347%
	(19.20)	(0.87)	(0.36)	(0.86)

In Chabi-Yo et al. (2018) the main focus lies on the results of value-weighted portfolios. This means that the authors assign more weight to stocks with higher tail dependence coefficients within each portfolio. We choose not to focus on these results, because our portfolios consist of four stocks. When using value-weighted portfolios, this results in portfolios that are quickly driven by one or two stocks with slightly higher tail dependence coefficients than the other stocks in that portfolio.

Table 5 shows that the portfolio with strongest LTD has 0.32 more LTD than the portfolio with the weakest lower tail dependence. In the third column, the excess returns of the portfolios are exhibited. The return spread between the strongest LTD portfolio and the weakest LTD portfolio is 0.315% per month (3.78% per year), which is not significant. This is due to the high standard errors for the portfolio returns, which is a consequence of having small portfolios consisting of four stocks. However, this result confirms the intuition that investors receive a compensation for holding stocks with strong LTD.

One should note that the returns do not decrease monotonically as the strong LTD portfolio (4) has higher monthly return than the strongest (5) LTD portfolio. In the fourth and fifth column the average monthly alpha in the CAPM of Sharpe (1964) and the Fama & French (2015) 5-factor model, respectively, are presented. These results are coherent with the findings of the 'Return' column. The strongest LTD portfolio has higher returns than the weakest LTD portfolio in 0.127% and 0.347% per month (1.52% and 4.16% per annum), but that these return spreads are also not significant.

Table 6: Results of univariate equal-weighted portfolio sorts based on the UTD coefficients estimated a 12-month rolling window for the 20 individual stocks in our sample in the sample period Jan. 1995 to Dec. 2019. All results in the second and third column are significant, unless the t-statistic is provided. We test the significance with t-statistics (in parentheses), that are computed with Newey & West (1986) standard errors with 4 monthly lags. *, ** and *** represent 1%, 5% and 10% significance respectively

Portfolio	UTD	Return	CAPM Alpha	FF5 Alpha
5 Strongest UTD Portfolio	0.28	0.976%	0.321%	0.134%
4 Strong UTD Portfolio	0.19	1.046%	0.471%	0.199%
3 Neutral UTD Portfolio	0.12	0.892%	0.305%	0.122%
2 Weak UTD Portfolio	0.06	1.126%	0.603%*	0.263%
1 Weakest UTD Portfolio	0.01	1.295%	0.676%	0.356%
Strongest - Weakest	0.27***	-0.319%	-0.355%	-0.221%
	(11.24)	(-1.15)	(-1.24)	(-0.75)

The results of Table 6 suggest that investors pay a premium for holding the strongest UTD stocks, as these stocks are more attractive due to the fact that they are more likely to increase in value during a market boom. The difference between the strongest UTD Portfolio return and Alphas and the weakest LTD Portfolio return and Alphas are substantial at -0.319%, -0.355 and -0.221% per month (-3.83%, -4.25% and -2.66% per annum) for the excess returns, CAPM Alpha model and FF5 Alpha model, respectively. However, alike the LTD results in Table 5 the return and Alpha spreads are not significant.

4.4 Time Varying Fear of Investors

In this section we investigate the phenomenon that during a post market crash, investors are more risk-averse. Gennaioli et al. (2012) show that investors structurally underestimate the risk of a market crash, when there has not been a big crash in recent memory. On the other hand, it is also found that investors change their perspective and enhance their awareness of bad news after a crisis.

We conduct two Fama-MacBeth regressions that measure the impact of LTD on returns in a post market crash period and the remaining years (see Table 7). For the post market crash period we consider 5 years after the three most relevant crises in our sample on October 27th, 1997 (Asian Crisis), April 14th, 2000 (Dot-Com Bubble Burst) and October 15th, 2008 (Lehman Crisis). This divides the sample in two halves: 150 month observations per stock in the Post Market Crash periods and 150 observations per stock in the remaining years.

Table 7: Fama-MacBeth regression of monthly future excess returns over the risk-free rate on LTD and UTD. Post Market Crash period is October 1997-March 2005 and October 2008-September 2013. In parentheses the t-statistics are presented computed on Newey & West (1986) standard errors with four monthly lags. *,**, and *** stand for 10%, 5% and 1% level of significance, respectively.

	Post Market Crash	Remaining Years
Constant	0.0111***	0.0079***
	(3.298)	(3.042)
LTD	0.0248	-0.0882***
	(0.748)	(-2.815)
UTD	-0.0352	-0.0277
	(1.306)	(-1.193)

Table 7 shows that during a post market crash period the impact of LTD is positive on returns, whereas for the remaining years we observe a significant negative impact. This suggests that the occurrence of a market crash does increase the LTD premium and that the fading of crash awareness amongst investors in the remaining years decreases the LTD premium. It does imply that the impact of LTD differs between the a post market crash period and the remaining years.

The results in Table 8 show that there is a significant return spread during the post market crash between the Strong LTD portfolio and the Weak LTD portfolio of 0.66% per month (7.80% per annum). This indicates that investors are willing to pay a risk premium for weak LTD stocks in the aftermath of a market crash. In the remaining years there is no clear distinction in portfolio returns based on LTD sorts. This could indicate that the awareness for market crashes decreased in the remaining years.

Table 8: Results of univariate equal-weighted portfolio sorts based on the LTD coefficients estimated on a 12-month rolling window for the 20 individual stocks in our sample, where we divide the sample period in the Post Market Crash period is October 1997 - March 2005 and October 2008 - September 2013 and the Remaining Years. The t-statistics are based on Newey & West (1986) standard errors with four monthly lags.

Portfolio	Post Market Crash		Remaining Years	
5 Strongest LTD Portfolio	0.36	1.26%	0.37	1.08%
4 Strong LTD Portfolio	0.27	1.18%	0.28	1.49%
3 Neutral LTD Portfolio	0.20	0.67%	0.21	1.46%
2 Weak LTD Portfolio	0.11	1.06%	0.14	0.77%
1 Weakest LTD Portfolio	0.03	0.60%	0.06	1.11%
Strongest - Weakest	0.31***	0.66%*	0.33***	-0.03%
	(11.99)	(1.72)	(15.12)	(-0.06)

4.5 Alternative Methods for Empirical Marginal Distribution Functions

In this section we examine the performance of alternative marginal distribution estimation methods. First we present the results of the Duplicates Removing-Heuristic. Lastly we discuss the results of the Kernel Method.

4.5.1 Duplicates Removing-Heuristic

For the application of this heuristic it is most interesting to focus on the period between 1995 and 2000, because in that period the rolling windows consider a substantial amount of duplicate return days, in particular zero return observations ($r_{i,k} = 0$). This means that for that period the heuristic differs convincingly from 12-month rolling window model.

In Figure 5 we observe that the average IAD-statistic for the Duplicates Removing-Heuristic is constant around 25 and substantially smaller than that of the 12-month rolling window model. One should note that the heuristic model also has a better fit than the 'distorted' 6-month rolling window model, although the heuristic model considers roughly 50 observations more in its rolling window sample (170 vs. 125 of 6-month rolling window model). This indicates that also for the 6-month rolling window model the distortion disturbs the fitting of the copula framework on the observations and that the improvement of fit by the heuristic model can not solely be contributed to the decrease in the amount of observations over which the IAD-statistic is estimated.

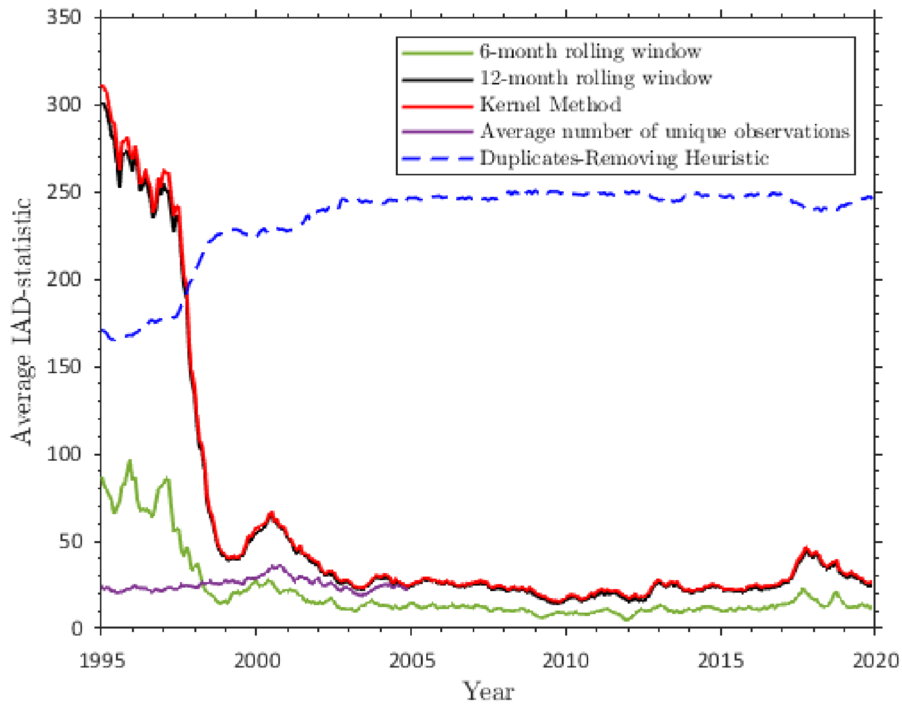

Figure 5: IAD-statistics of the different models over time, where IAD for Duplicates Removing-Heuristic observations run from Jan 1995 to Dec 2004. The other IAD graphs run over the sample period Jan 1995 to Dec 2019. The average number of unique observations per rolling window sample of the 20 stocks does correspond to the scaling of the y axis (not to the label).

In the years following 1997, we observe the movement in opposite directions as the average number of unique values in the rolling window sample increases, the difference between heuristic model and the 12-month rolling window model disappears. The IAD-statistic for the heuristic on the sample period $1995-2005$ is strictly smaller than the 12-month rolling window. We are required to do further research to state with certainty that the Duplicates-Heuristic outperforms the 12-month rolling model.

In the period $1995-2005$ that the Duplicates Removing-Heuristic differs from the 12-month rolling window, two financial crises (the Asian Crisis and the Dot-Com Bubble Burst) take place. In Table 9 we observe that the LTD levels for portfolios of both models are similar. Over the whole period we see that the heuristic model has a higher monthly return spread of 0.85% than the 12-month rolling window model with 0.41%, which suggests that the heuristic measures a more direct impact from LTD on the future returns.

We observe that for the Asian Crisis the Duplicate-Heuristic offer slightly less protection against wealth destruction (-3.79% against -4.25%). However during the Dot-Com Bubble Burst crisis the Duplicate Removing-Heuristic model prevents the weakest LTD portfolio for bigger losses than the 12-month rolling window model,as the losses for the portfolios based on LTD estimated by the heuristic model increase monotonically with the level of LTD.

Table 9: Results for univariate portfolio sorts based on LTD for the period Jan 1995 to Dec 2004. On the left the results for the model with Duplicates Removing-Heuristic and on the right the results for the 12-month rolling window. The daily returns on crisis days in the sample period: the Asian Crisis and the Dot-Com Bubble Burst (D-C Bubble).

Portfolio	LTD	Return	Asian Crisis	D-C Bubble	LTD	Return	Asian Crisis	D-C Bubble Burst
5 Strongest LTD portfolio	0.31	2.28%	-6.77%	-8.03%	0.30	2.04%	-6.77%	-6.79%
4 Strong LTD portfolio	0.21	1.61%	-9.47%	-5.34%	0.21	1.34%	-8.60%	-7.26%
3 Neutral LTD portfolio	0.15	1.00%	-4.90%	-4.72%	0.14	1.92%	-6.39%	-2.80%
2 Weak LTD portfolio	0.09	1.66%	-5.29%	-4.46%	0.08	1.04%	-5.13%	-5.09%
1 Weakest LTD portfolio	0.03	1.43%	-2.98%	-1.20%	0.03	1.64%	-2.52%	-1.81%
Strongest - Weakest	0.28	0.85%	-3.79%	-6.83%	0.27	0.41%	-4.25%	-4.98%

4.5.2 The Kernel Method

In this section, we further analyze the empirical marginal distribution function of the 12-month rolling window model with the Kernel Method. As is clear from Figure 5, the model for the Kernel Method has equivalent IAD-statistics as the 12-month rolling window model. In Table 10, it is reported that the kernel method has a higher IAD-statistic on average, that implies that the Replication Method of Chabi-Yo et al. (2018) has our preference.

Table 10: IAD-statistics comparison between Replication Method and Kernel Method over the sample period from Jan 1995 to Dec 2019.

	Mean	Median
Replication Method	56.38	45.83
Kernel Method	58.43	48.10
Difference	2.04	2.27

When analyzing the IAD-statistics per individual stock, the same case can be made, as each stock has on average a lower IAD-statistic. In total 761 of 6,000 observations had smaller IAD-statistics for the Kernel Method than for the 12-month rolling window model.

The portfolio returns of the Kernel Method are exhibited in the Appendix.A-Table 16. The returns do not differ from the portfolios sorts returns in Table 5 of the 12-month rolling window model. This implies that the LTD coefficients do not differ per individual stock per month, such that a stock is sorted to an other portfolio in any month.

5 Conclusion

The use of copulas has increased in popularity in the field of risk management since the nineties. Chabi-Yo et al. (2018) introduced a 3-step procedure that fits a flexible copula framework on a rolling window sample of returns that measures both lower tail dependence and upper tail dependence of stock returns with the market returns. They conclude that holding stocks with high LTD, that are thus crash-sensitive, yields a return premium. Therefore we question whether we can find the same results for our dataset. Additionally, we investigate the impact of shortening the rolling window on the persistence in LTD and research whether alternative methods for the empirical distribution in the 3-step procedure can improve the fit of the copulas to the returns.

After implementing the 3-step procedure of Chabi-Yo et al. (2018) we find that for our dataset strong LTD stocks in the equal-weighted strongest LTD portfolio yield an average return 3.78% per year over the weakest LTD portfolio. This is coherent with the idea that investors are willing to pay higher prices and yield lower returns for weak LTD stocks. However, we learn that this depends on the economic state, as we find that during a post market crash period LTD has a positive impact on future returns, while LTD has a negative impact in the remaining years. This suggests that the fear of a market crash is time-varying amongst investors, as we do not find a difference between strong and weak LTD stocks in returns during the remaining years.

Reducing the sample on which we estimate tail dependence to a 6-month rolling window model shows that the model still exhibits persistence. However to a lesser extent than the 12-rolling window model. For the alternative methods for empirical distribution functions we find that the Kernel Method does not improve the fit of the copula framework on the returns. However, for the proposed Duplicates Removing-Heuristic we do find better results, when the rolling window sample has a substantial amount of duplicate observations. Although we can not contribute solely to the fit of the model, as we measure the performance on a smaller summarized sample. With the rise of the amount of unique values in the rolling window sample, we find that the results of the heuristic model converge to the original 12-month rolling window model.

For further research we suggest to investigate if there is a performance measure that can measure the improvement of fit better for the heuristic, that is not biased by the difference in considered observations. Also we would like to dive in the other methods that already exist for coping with infrequent trading, which indirectly leads to duplicate observations. We could try other kernels with different bandwidths, however our results do not indicate much room for improvement.

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6 Appendix

A Additional Graphs and Tables

Table 11: List of the twenty individual stocks in our data sample. Sorted on basis of the market capitalization on the most recent day of our sample period 31 Dec. 2019.

Large Cap Stocks PERMNO	Company Name	Market Capitalization
21178	LOCKHEED MARTIN CORP	$ 110,543,622,079
72726	STATE STREET CORP	$ 29,046,205,240
22517	P P L CORP	$ 25,790,587,110
58246	NORTHERN TRUST CORP	$ 22,527,294,240
10933	MARKEL CORP	$ 15,745,680,000
46674	GENUINE PARTS CO	$ 15,403,963,860
12650	KANSAS CITY SOUTHERN	$ 15,174,280,800
Medium Cap Stocks PERMNO		
53065	INTERPUBLIC GROUP COS INC	$ 8,948,854,560
36281	SEABOARD CORP	$ 4,898,825,000
15456	FOOT LOCKER INC	$ 4,177,732,170
13507	AMERICAN NATIONAL INS CO	$ 3,148,467,700
88197	FULTON FINANCIAL CORP PA	$ 2,883,623,540
24328	E Q T CORP	$ 2,720,041,520
58334	NORTHWEST NATURAL HOLDING CO	$ 2,224,567,240
Small Cap Stocks PERMNO		
75320	UNITED STATES CELLULAR CORP	$ 1,878,002,940
76573	MUELLER INDUSTRIES	$ 1,816,768,800
77519	TRUSTCO BANK CORP NY	$ 845,116,240
13777	AMERICAN SOFTWARE INC	$ 443,175,000
11267	CATO CORP NEW	$ 399,133,850
18403	PENNEY J C CO INC	$ 355,200,000

Table 12: The copulas that are included in fitting copula combinations. The third and fourth column show the formulas for calculating the tail dependence. The last column states the domains of the dependence parameter θ.

Tail Dependence	Copula	LTD	UTD	θ domain
Lower Tail Dependence	Clayton	$2^{-1/\theta}$	-	$[0, \infty)$
	Rotated-Gumbel	$2 - 2^{-1/\theta}$	-	$[1, \infty)$
	Rotated-Joe	$2 - 2^{-1/\theta}$	-	$[1, \infty)$
	Rotated-Galambos	$2^{-1/\theta}$	-	$[0, \infty)$
Neutral Tail Dependence	Gauss	-	-	$[-1.1]$
	Frank	-	-	$(-\infty, \infty)$
	Plackett	-	-	$(0, \infty)$
	F-G-M	-	-	$[-1.1]$
Upper Tail Dependence	Joe	-	$2 - 2^{-1/\theta}$	$[1, \infty)$
	Gumbel	-	$2 - 2^{-1/\theta}$	$[1, \infty)$
	Galambos	-	$2^{-1/\theta}$	$[0, \infty)$
	Rotated-Clayton	-	$2^{-1/\theta}$	$[0, \infty)$

Table 13: Summary statistics of LTD and UTD coefficients that are estimated with the 12-month rolling window model.

	Mean	25% Quantile	Median	75% Quantile	Standard Deviation
LTD	0.20	0.09	0.19	0.30	0.140
UTD	0.13	0.03	0.12	0.21	0.116

Table 14: The frequency that each copula combination is selected with the the 12-month rolling window model.

Copula	Percentage	Copula	Percentage	Copula	Percentage	Copula	Percentage
Clayton-Gaussian-Joe	2.50%	RGumbel-Gaussian-Joe	1.23%	RJoe-Gaussian-Joe	3.92%	RGalambos-Gaussian-Joe	1.93%
Clayton-Gaussian-Gumbel	1.08%	RGumbel-Gaussian-Gumbel	0.87%	RJoe-Gaussian-Gumbel	1.48%	RGalambos-Gaussian-Gumbel	1.33%
Clayton-Gaussian-Galambos	1.43%	RGumbel-Gaussian-Galambos	0.87%	RJoe-Gaussian-Galambos	2.58%	RGalambos-Gaussian-Galambos	1.25%
Clayton-Gaussian-RClayton	2.13%	RGumbel-Gaussian-RClayton	1.00%	RJoe-Gaussian-RClayton	2.85%	RGalambos-Gaussian-RClayton	1.37%
Clayton-Frank-Joe	1.87%	RGumbel-Frank-Joe	1.32%	RJoe-Frank-Joe	3.40%	RGalambos-Frank-Joe	4.22%
Clayton-Frank-Gumbel	1.67%	RGumbel-Frank-Gumbel	0.85%	RJoe-Frank-Gumbel	2.53%	RGalambos-Frank-Gumbel	2.48%
Clayton-Frank-Galambos	2.20%	RGumbel-Frank-Galambos	1.47%	RJoe-Frank-Galambos	3.28%	RGalambos-Frank-Galambos	3.13%
Clayton-Frank-RClayton	1.40%	RGumbel-Frank-RClayton	1.17%	RJoe-Frank-RClayton	1.47%	RGalambos-Frank-RClayton	2.12%
Clayton-Plackett-Joe	1.32%	RGumbel-Plackett-Joe	0.87%	RJoe-Plackett-Joe	2.33%	RGalambos-Plackett-Joe	2.17%
Clayton-Plackett-Gumbel	0.83%	RGumbel-Plackett-Gumbel	0.57%	RJoe-Plackett-Gumbel	1.00%	RGalambos-Plackett-Gumbel	0.93%
Clayton-Plackett-Galambos	1.02%	RGumbel-Plackett-Galambos	0.77%	RJoe-Plackett-Galambos	1.77%	RGalambos-Plackett-Galambos	1.37%
Clayton-Plackett-RClayton	0.92%	RGumbel-Plackett-RClayton	0.45%	RJoe-Plackett-RClayton	0.87%	RGalambos-Plackett-RClayton	1.28%
Clayton-FGM-Joe	1.38%	RGumbel-FGM-Joe	1.25%	RJoe-FGM-Joe	2.22%	RGalambos-FGM-Joe	2.43%
Clayton-FGM-Gumbel	0.75%	RGumbel-FGM-Gumbel	0.48%	RJoe-FGM-Gumbel	1.50%	RGalambos-FGM-Gumbel	0.97%
Clayton-FGM-Galambos	1.30%	RGumbel-FGM-Galambos	0.58%	RJoe-FGM-Galambos	1.72%	RGalambos-FGM-Galambos	1.55%
Clayton-FGM-RClayton	0.73%	RGumbel-FGM-RClayton	0.58%	RJoe-FGM-RClayton	0.75%	RGalambos-FGM-RClayton	0.95%

Table 15: Returns of value-weighted LTD portfolios.

Value-Weighted Portfolio	Return
5 Strongest LTD portfolio	6.38%
4 Strong LTD portfolio	1.12%
3 Neutral LTD portfolio	-2,42%
2 Weak LTD portfolio	-0,41%
1 Weakest LTD portfolio	2,83%

Table 16: Results of univariate equal-weighted portfolio sorts based on the LTD coefficients estimated with the Kernel method for the 20 individual stocks in our sample in the sample period Jan. 1995 to Dec. 2019.

Portfolio	LTD	Return
5 Strongest LTD Portfolio	0.37	1.17%
4 Strong LTD Portfolio	0.27	1.33%
3 Neutral LTD Portfolio	0.20	1.06%
2 Weak LTD Portfolio	0.13	0.91%
1 Weakest LTD Portfolio	0.05	0.86%
Strong - Weak	0.32***	0.32%
		(0.881)

B MATLAB Code Files

B.1 CopulaMethods General.m

This code estimates all tail dependence coefficients for each stock. Within the code we explain how it runs for all different models.

B.2 Result Finalizer.m

This program computes portfolio sorted returns on basis of the tail dependence that is computed in *CopulaMethods.m*.

B.3 Rates and Market Return.data

The returns of the individual stocks and market on which we estimate the tail dependence.