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Bachelor Thesis Econometrics & Operational Research

Persistence Heterogeneity and the Long-Term valuation of Equity and Bonds under Consumption Risk

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Abstract

In this paper we replicated the methods Ortu et al. (2013) for the decomposition of a time series. We use their decomposition to find persistent components in log consumption growth. Also, we replicate their results that expected consumption growth predicts the financial ratios at a certain persistence level. With predictive relation, we replicate the results of Ortu et al. (2013) for the determination of the risk premia on equity under consumption risk. We find similar evidence that the equity risk premia under consumption risk are significantly determined at levels of persistence 2, 3 and 6. As an extension, we use the methods of Ortu et al. (2013) to determine the risk premia under consumption risk for bonds. To do so, we first provide evidence that a similar assumption of expected consumption growth predicting the log bond price holds. Then we extend the long-run valuation framework to bonds and compute the risk premia for the Fama-Bliss bond portfolio with yearly maturities from 1 to 5 years. For bonds with yearly maturities from 1 to 5 years, we find that the risk premia under consumption risk can significantly determined for all maturities at levels of persistence 1 and 4.

Contents

1 Introduction

The decomposition of the consumption growth rate from Ortu et al. (2013) has disentangled the time series and unraveled some persistence components that determine the consumption growth rates. They proposed a solution to a major problem in the empirical detection of long-run risk. The implications of the decomposition has fundamental consequences for asset pricing. Agents behaving in accordance to the utility function of Epstein and Zin (1989), require a compensation for holding cash flows. The fluctuations of these cash flows are positively correlated with expected changes in consumption grows, which in equilibrium generates a long-run risk premium. Through the methods of Ortu et al. (2013) a time serie can be decomposed into layers with different levels of persistence. These different layers can be seen as a classification of the half-life of a shock, so as to capture economic phenomena occurring at different time-scales.

Ortu et al. (2013) investigate different economic proxies for the different components. To do so, they rely on time series that are economically significant and characterised by a half-life that is similar to the component of interest[1]. They document that there is a particular strong correlation between the cyclical consumption growth variations captured by the below-business-cycle frequency and long-run productivity growth. This is similar to the findings of Pastor and Veronesi (2009), Kaltenbrunner and Lochstoer (2010) and Croce (2014) who find that shifts in the long-run rate of productivity growth are a key factor in driving slow-moving consumption components. Furthermore, Ortu et al. (2013) point out that these components with business-cycle frequencies are correlated with well-known economic indicators of economic activity. The high-frequency components taken together have a yearly half-life which is closely related to the half-life of the shocks in Bansal and Yaron (2004).

Furthermore, the paper of Ortu et al. (2013) uses the paper of Bansal and Yaron (2004), from which the latter explains many inconsistencies in the long-run that affect predictions of dynamic asset pricing models. Ortu et al. (2013) slightly adapt the long-run framework of Bansal and Yaron (2004). The standard long-run framework prices the latent persistent conditional mean of consumption growth, whereas the framework used by Ortu et al. (2013) prices the different components of consumption growth. Additionally, the standard long-run framework uses a AR-process for the growth variables, however this process is indirectly done by a unobserved time series. The paper of Ortu et al. (2013) defines a more direct AR-process based on consumption growth.

[1]In appendix A, we further elaborate on the relations Ortu et al. (2013) found.

Ortu et al. (2013) therefore argue that they are more focused on the entire term of consumption risk.

By using the decomposition of Ortu et al. (2013), we will extend the literature on long-run valuation framework to determine the risk premia of the discounted zero coupon bonds from the Fama-Bliss bond portfolio under consumption risk. For our extension, we will use the long-run framework of Hasseltoft (2012) for pricing bonds, which can be seen as an extension on the standard framework of Bansal and Yaron (2004). We will follow Ortu et al. (2013) by moving the attention of pricing the latent persistent conditional mean of consumption growth by pricing the different components of consumption growth. Moreover, we will directly make use of the AR-process of different components of consumption growth as is done in Ortu et al. (2013). Similar work on the return of bonds has been done by Cochrane and Piazzesi (2005), and Viceira (2012). Cochrane and Piazzesi (2005) study time variation in expected excess bond returns through a factor model. Viceira (2012) study the volatility bond return and the risk of bonds measured through the covariance of bonds with stock return and consumption growth. Different than they, we will study the risk premia that is created through consumption risk rather than the time-variation of the excess returns.

The paper is organised as follows: we will first introduce the data we have used in this paper. In the next section, we will introduce the decomposition method of Ortu et al. (2013). We will investigate the implications of this decomposition in for testing on persistency and determining the number of components. We will then determine whether consumption growth consists of persistent components. In the section thereafter, we will first replicate the findings of Ortu et al. (2013) on the IES. Then, we will investigate the basic assumption of our long-run valuation framework by looking in to the predictability of consumption growth by the financial ratios and the log discounted bond price. The two subsections thereafter determines the risk premia of equity and bonds respectively. The final section concludes.

2 Data

The data used for this paper is the data used by Ortu et al. (2013)[2]. To replicate the analyses of Ortu et al. (2013) we use the quarterly data of consumption growth, dividend growth, log price-dividend ratio, log price-consumption ratio, and the real interest rate. The the growth rates are obtained by taking the first difference between two consecutive observations in the corresponding

[2]Obtained from the website of Andreas Tamoni: https://andreatamoni.meltinbit.com

log series. For the consumption growth, Ortu et al. (2013) followed Bansal and Yaron (2004) and Beeler and Campbell (2009) by using as a proxy for the total consumption, the U.S. consumption of durable goods and services. The data for the consumption on durable goods and services are obtained from the U.S. Bureau of Economics analysis. For the consumption on durable goods and services we make the end-of-period assumption. This implies that all consumption happens at the end of quarter t. The dividend growth and the log price-dividend ratio in the file of Ortu et al. (2013) are obtained from the CRSP files. The nominal quantities are deflated by using the personal consumption deflator. The price index used for this deflation is also covered by the CRSP files. To proxy for price-consumption, Ortu et al. (2013) follow Duffee (2005) by using the ratio of the market capitalization of publicly traded stocks to total consumption on nondurables and services. Stock market wealth is measured through the end of the month market capitalization of the CRSP value-weighted index. For the comparability to the consumption data, the stock market wealth is measured by in real terms per capita. For the real risk-free rate, Ortu et al. (2013) follow Bansal et al. (2016) by constructing the ex ante real risk-free rate as a fitted value from a projection of the ex-post real rate on the current nominal yield and inflation over the previous year. Ortu et al. (2013) use monthly observations on three-month nominal yield from the CRSP Fama Risk-Free Rate tapes and CPI series to run the predictive regression. For all observations, Ortu et al. (2013) obtain a postwar quarterly U.S. series over the period 1947Q2-2011Q4. In their robustness analyses they also use a long-run annual series over the period 1930-2010.

To extend our analysis to bonds, we obtained data of Fama-Bliss discounted bond portfolio from CRSP which is the same data Cochrane and Piazzesi (2005) use in their determination of bond risk premia. This data contains monthly observations for the discounted bond price for bonds with a yearly maturity of 1 up to 5 years. The sample of our bond data is monthly and stretches the period June 1952 up to December 2011. We converted the monthly data to quarterly by using only the discounted bond price at the last month of each quarter.

3 Persistence Heterogeneity

In this section we will be covering the persistence decomposition from Ortu et al. (2013). We will first show how to decompose a time series along its persistence component. Secondly, we will be using a specific Data Generating Process to show the difference in analysis on persistence between decomposed and aggregated time series. We will redo the specific test for serial correlation used by Ortu et al. (2013), and use this test on consumption growth to detect if any correlation can be found. At last, we will also check how many persistent components should be considered. All our results will be compared to the original findings of Ortu et al. (2013).

3.1 Decomposing time series along the persistence dimension

Before we will introduce the decomposition of a certain time series, we will first look into the definition of a moving average and a component. Consider a time series $\{g_t\}_{t \in Z}$, where Z is the defined as the set of observations. The moving average is defined as

$$\pi_t^{(j)} = \frac{1}{2^j} \sum_{p=0}^{2^j-1} g_{t-p} \tag{1}$$

where $\pi_t^{(0)} = g_t$. For each increase in j with one, the sample used for the moving average will double. From this fact, a recursive relation in j for the moving average is given by

$$\pi_t^{(j)} = \frac{\pi_t^{(j-1)} - \pi_{t-2^{j-1}}^{(j-1)}}{2} \tag{2}$$

Note how $\pi_t^{(j-1)}$ and $\pi_{t-2^{j-1}}^{(j-1)}$ divide the sample space of $\pi_t^{(j)}$ in to two disjoint sub-spaces which jointly span the sample space of $\pi_t^{(j)}$. We will define the difference between sub-samples on time t with the total sample used for $\pi_t^{(j)}$ as a component. In other words, a component is the difference of the mean based on the previous $2^{(j-1)}$ observations in period t and the mean based on the previous 2^j observations in period t. A component $g_t^{(j)}$ is mathematically defined as

$$g_t^{(j)} = \pi_t^{(j-1)} - \pi_t^{(j)} \tag{3}$$

An intuitive explanation of $g_t^{(j)}$ is given by Ortu et al. (2013). They explain that $g_t^{(j)}$ captures the fluctuations that survive by averaging over 2^{j-1} observations and disappear by averaging over 2^j observations. In other words, $g_t^{(j)}$ captures the fluctuations with a half-life in the interval $[2^{j-1}, 2^j)$. $\pi_t^{(j)}$ captures the fluctuations whose half-life exceed 2^j.[3] By using $\pi_t^{(0)} = g_t$ and summing over J components, it holds that

$$g_t = \sum_{j=1}^{J} g_t^{(j)} + \pi_t^{(J)} \tag{4}$$

for any integer $J \geq 1$. Explained in words, equation (4) decomposes the series g_t into a sum of components whose half-life belong to a specific interval and a residual term that represents a long-term average.

[3] Ortu et al. (2013) relate in their Technical Appendix $g_t^{(j)}$ and $\pi_t^{(j)}$ to their Fourier Spectra.

Using the components $g_t^{(j)}$ and moving averages $\pi_t^{(J)}$ for each observation t, will lead to a certain significance of correlation due to the overlap of the used samples. In order to prevent such spurious correlation from happening, we will follow Ortu et al. (2013) by using only a sub-series of the components and the moving average. More specific, we will only use components and moving averages such that their samples do not overlap. The sub-series that will be used is then defined by

$$\begin{aligned}&\{g_t^{(j)}, t = k2^j, k \in Z\}\\&\{\pi_t^{(j)}, t = k2^j, k \in Z\}\end{aligned} \tag{5}$$

Just as in Ortu et al. (2013), we will refer to these sub-series as decimated components.

In fact, Ortu et al. (2013) show that these decimated components contains all and only the information needed to reconstruct the original time series g_t. These decimated components can therefore be seen as the decomposition of g_t, where the irrelevant information has been deleted. Transforming the original series g_t to the decimated components can be done by the inverse of the scaled Haarmatrix $H^{(J)}$. The fact that the scaled $H^{(J)}$ is invertible can be seen by the property of orthogonality of $H^{(J)}$, which means that there exists a diagonal matrix $\Lambda^{(J)}$ such that $\Lambda^{(J)} = H^{(J)}(H^{(J)})^T$. The elements of this diagonal matrix $\Lambda^{(J)}$ are nonzero, which means that $\Lambda^{(J)}$ has an inverse. The inverse of $H^{(J)}$ is given by $(H^{(J)})^{-1} = (H^{(J)})^T(\Lambda^{(J)})^{-1}$. For a general way to construct $H^{(J)}$, Ortu et al. (2013) refer to Mallat (1989). From Mallat (1989), it can in general be seen that the diagonal elements of $\Lambda^{(J)}$ are given by $\lambda_1 = \lambda_2 = \frac{1}{2}$, $\lambda_k = \frac{1}{2^{J-j+1}}$ for $k = 2^{j-1}, ..., 2^j$ and $j = 2, ..., J$. For an illustration, see the paper of Ortu et al. (2013).

3.2 Example of a data generating process of a time series with a persistent dimension

We will use the framework of Ortu et al. (2013) for exploring a data generating process for a time series with a persistent dimension. In this framework, the decimated components are simulated with or without a correlation between the consecutive components. These decimated components can then be used to construct a time series g_t. The construction of g_t can be done by means of the scaled Haarmatrix $H^{(J)}$ as defined in the previous section. Formally, Ortu et al. (2013) define for $t = k2^j$, $k \in Z$, and let

$$\begin{aligned}g_t^{(j)} &= \epsilon_t^{(j)}, \quad \forall j < J^*\\ g_{t+2^{J^*}}^{(J^*)} &= \rho_{J^*} g_t^{(J^*)} + \epsilon_{t+2^{J^*}}^{(J^*)}\\ \pi_t^{(J^*)} &= \eta_t^{(J^*)}\end{aligned} \tag{6}$$

with $\epsilon_t^{(j)} \sim N(0, 2^{-j}\sigma^2)$, $\epsilon_t^{(J^*)} \sim N(0, (1-\rho_{J^*}^2)2^{-J^*}\sigma^2)$, and $\eta_t^{(J^*)} \sim N(0, 2^{-J^*}\sigma^2)$. Furthermore, we assume that the innovations $\epsilon_t^{(j)}$, $\epsilon_t^{(j')}$ are uncorrelated for $j \neq j'$, and that $\epsilon_t^{(j)}$ is uncorrelated with $\eta_t^{(J^*)}$ for all j. In other words, the innovations are uncorrelated across levels of persistence.

By this method of construction, $Var(g_t) = \sum_{j=1}^{J^*} Var(g_t^{(j)}) + Var(\pi_t^{(J^*)}) = \sum_{j=1}^{J^*} 2^{-j}\sigma^2 + 2^{-J^*}\sigma^2 = \sigma^2$. So that means that the persistent component explains a fraction 2^{-J^*} of the total variance. In Appendix B, we have done a simulation exercise just as in Ortu et al. (2013) where we construct a time series according to equation (6).

3.3 Persistence test

To test for serial correlation in the decimated components dimension, Ortu et al. (2013) have defined a new type of serial correlation test based on the family of serial correlation tests introduced by Gencay and Signori (2015).

Let g_t be a weakly stationary time series, and assume that $E[g_t] = 0$ and $Var(g_t) = \sigma^2$. Define $(X_T^{(J)})^T = [g_T, g_{T-1}, ..., g_1]$ as the vector containing the observations of $\{g_t\}$. As we have seen in the section (3.1), we can use the scaled Haarmatrix $H^{(J)}$ to create the decimated components from the series $\{g_t\}$ and vice versa. As a consequence, we can express the variance of the series g_t as the sum of variances of the decimated components. Define $\mathbf{g^{(j)}} = [g_{2^j}^{(j)}, ..., g_{k2^j}^{(j)}, ..., g_T^{(j)}]^T$ so that we can write the sample variance of g_t as

$$\frac{(X_T^{(J)})^T X_T^{(J)}}{T} = \frac{((\Lambda^{(J)})^{-1/2} H^{(J)} X_T(J))^T ((\Lambda^{(J)})^{-1/2} H^{(J)} X_T^{(J)})}{T} = \frac{\sum_{j=1}^{J} \mathbf{g^{(j)}}^T \mathbf{g^{(j)}}}{T} \tag{7}$$

The first equality can be seen by the fact that $(H^{(J)})^T (\Lambda^{(J)})^{-1} H^{(J)} = I$. The second equality can be seen by the definition of the diagonal elements of $\Lambda^{(J)}$ as mentioned in section (3.1). The factor 2^j can also be understood by the fact that the decimated component has in fact $T/2^j$ observations. This means that the sample variance in that case is equal to $\frac{\mathbf{g^{(j)}}^T \mathbf{g^{(j)}}}{T/2^j}$.

The test statistics of Ortu et al. (2013) is built on the above variance decomposition and rely on the comparison to the contribution to the total variance of the different components of a white noise process on one side and a process with serially correlated decimated components on the other side. The test statistic is defined by the proportion of the contribution of a decimated component to the total sample variance of the time series. Mathematically, the proportion is defined as

$$\hat{\xi}_j = \frac{2^j \mathbf{g^{(j)}}^T \mathbf{g^{(j)}}}{(X_T^{(J)})^T X_T^{(J)}} \tag{8}$$

Under no serial correlation the proportion of the variances should be equal to $1/2^j$. To see this, remember that the decimated components are equal to the differences of two sums based on 2^j and $2^{(j-1)}$ observations of g_t. Since $2^{(j-1)}$ observations of g_t overlap, we can rewrite the decimated component as $g_t^{(j)} = \frac{1}{2^j}\sum_{p=0}^{2^j-1} g_{t-p} - \frac{1}{2^{j-1}}\sum_{p=0}^{2^{j-1}-1} g_{t-p} = \frac{1}{2^j}(\sum_{p=0}^{2^{j-1}-1} g_{t-p} - \sum_{p=2^{j-1}}^{2^j-1} g_{t-p})$. The decimated component $g_t^{(j)}$ is therefore constructed out of 2^j random variables with either a negative of a positive sign. Using then the fact that under no serial correlation $Var(X+Y) = Var(X) + Var(Y)$ and $Var(aX) = a^2Var(X)$, it can easily be seen that the variance of $g_t^{(j)}$ equals $\frac{1}{2^j}$.

Letting $Cov(g_t, g_{t-k})/Var(g_t) = \rho_k$ we can test for the null hypothesis $\rho_k = 0$ for all $k \geq 1$ against $\rho \neq 0$ for some $k \geq 1$. Under the proper re-scaling factor a_j, the test statistic converges to a standard normal distribution

$$\sqrt{Ta_j}(\hat{\xi}_j - \frac{1}{2^j}) \xrightarrow{d} N(0,1) \tag{9}$$

The proper re-scaling factor is defined as $a_j = \frac{2^j 2^{2(j-1)}}{binom(2^j,2)}$ where $binom(2^j, 2)$ is the binomial coefficient.

3.4 Simulating experiment with the persistence test

Following Ortu et al. (2013), we will redo their Monte Carlo Simulation for the performance of the test in finite samples. We will simulate the series according to equation (6) mentioned in section (3.2). The simulated time series will be simulated for 256 observations and for 2048 observations. The smaller sample will be compatible to the sample of consumption growth. Just as Ortu et al. (2013), we will choose $J = 6$ as the component containing the persistent dimension. The series will be simulated with ρ being equal to 0.2 or 0.4. Furthermore, the proportion of the total variance that can be explained by the persistent component will be equal to 0.03, 0.05 and 0.07.

In each simulation, we compute the re-scaled test statistic $\hat{\xi}_j$ and carry out a two-tailed test. We repeat this simulation 5000 times and obtain a probability of rejecting the null hypothesis. Table (1) shows the results of the rejecting probabilities for a time series containing a persistent component. As can be seen from table (1), in small samples the probability of rejecting the null hypothesis for component with level of persistence 6 varies between 30% and 70% as the proportion increases, which is +10% higher compared to Ortu et al. (2013). In the bigger sample, the probability of rejecting the null hypothesis in that case varies between 80% and 100%, which is again a +10% higher compared to Ortu et al. (2013). Similar to Ortu et al. (2013), we do not over-reject the null hypothesis in the small sample when the components are uncorrelated. In the big sample however, we do find a over-rejections when the proportion is 0.05 and 0.07 for $j = 1$ and $j = 2$.[4]

Table 1: simulated rejection probabilities of the persistence test

			T=256							T=2048						
ρ	prop.	j =..	1	2	3	4	5	6	7	1	2	3	4	j=5	6	7
0.2	0.03		0.054	0.049	0.043	0.041	0.040	0.283	0.045	0.081	0.056	0.051	0.051	0.047	0.807	0.039
0.2	0.05		0.070	0.051	0.040	0.037	0.043	0.572	0.045	0.202	0.098	0.063	0.051	0.045	0.996	0.040
0.2	0.07		0.114	0.058	0.047	0.033	0.034	0.727	0.044	0.434	0.171	0.086	0.061	0.041	1.000	0.035
0.4	0.03		0.060	0.048	0.040	0.039	0.041	0.288	0.045	0.079	0.054	0.049	0.049	0.043	0.772	0.045
0.4	0.05		0.075	0.055	0.042	0.032	0.033	0.544	0.043	0.220	0.098	0.062	0.049	0.042	0.991	0.035
0.4	0.07		0.112	0.060	0.046	0.036	0.030	0.700	0.044	0.417	0.178	0.092	0.059	0.038	1.000	0.026

Table (1) shows the results of the simulated probabilities when repeating the simulation 5000 times with a different parametric choices; prop. stands for proportion

3.5 Determining number of persistent components

Crucial when decomposing a time series in components is to know how many components to choose. Ortu et al. (2013) solve this issue by testing the series of decimated series $\pi^{(J)}$ for J=1,2,.. until for a certain J^* we do not reject hypothesis of white noise . Remember that we mentioned in equation (4) how $\pi^{(J)}$ is referred to as the residual who captures the long-term fluctuations that can not be captured by the components. If we consider $\pi^{(J)}$ as a residual in a time series, we can conclude that the amount of components to choose should be the amount such that the decimated series of $\pi^{(J)}$ could be considered white noise. In other words, the components should capture the fluctuations in g_t sufficiently such that the residual fluctuations captured by $\pi^{(J)}$ could be considered white noise.

Testing the decimated series $\pi^{(J)}$ on serial correlation could be done by considering the decimated series of $\pi^{(J)}$ as a time series on itself. In that case, we could consider the persistence test given by formula (9) for $\pi^{(J)}$. We could consider $\pi^{(J)}$ as white noise, if the serial correlation test is insignificant. This implies that we do not find a significant re-scaled test statistic for $\pi^{(J)}$ for any of the components of $\pi^{(J)}$.

If we would like to rewrite the test for $\pi^{(J)}$, consider that the decimated time series is given by

$$\{\pi_t^{(J)}, t = k2^J, k \in Z\} \tag{10}$$

Where for each $j \geq 1$, the decimated series of $\pi^{(J)}$ contains $T/2^J$ observations. If we consider that

[4]In appendix B, we will also perform this test for a white noise series. In that case there is no over-rejection.

formally $\pi^{(J)} = \sum_{q=1}^{Q} g_t^{(J+q)}$ where Q is the maximum number of components before the sample is saturated, then we could see that the components of $\pi_t^{(J)}$ are similar to $g_t^{(J+q)}$. If we define $\Pi_T^{(J)} = [\pi_T^{(J)}, ..., \pi_{t-k2^J}^{(J)}, ..., \pi_{T-T/2^J}^{(J)}]^T$, we could rewrite the test statistic given by formula (9). The serial correlation test for $\pi_t^{(J)}$ in terms of the decimated component $g_t^{(J+q)}$ is given by

$$\hat{\xi}_q = \frac{\mathbf{g^{(J+q)}}^T \mathbf{g^{(J+q)}}/(T/2^{J+q})}{(\Pi_T^{(J)})^T \Pi_T^{(J)}/(T/2^J)} = \frac{2^q \mathbf{g^{(J+q)}}^T \mathbf{g^{(J+q)}}}{(\Pi_T^{(J)})^T \Pi_T^{(J)}} \tag{11}$$

Under the proper re-scaling factor a_q, this test statistic converges to a standard normal distribution

$$\sqrt{\frac{Ta_q}{2^J}}(\hat{\xi}_q - \frac{1}{2^q}) \xrightarrow{d} N(0,1) \tag{12}$$

The proper re-scaling factor is defined as $a_q = \frac{2^q 2^{2(q-1)}}{binom(2^q,2)}$ where $binom(2^q, 2)$ is the binomial coefficient.

3.6 Variance ratio test for consumption growth and determining optimal components

For the valuation framework used by Ortu et al. (2013), we will test whether there are persistence components in consumption growth. For the tests on persistence, we will use the quarterly data of consumption growth for the period 1948Q2-2011Q4. In order for the assumption $E[g_t] = 0$ to hold, we will de-mean the consumption growth series g_t before we compute our test statistics. As the sample mean is 0.005 with a standard deviation of 0.005, it might also be assumed that the mean of consumption growth is equal to zero.

Table (2), shows the results of the re-scaled test statistics $\hat{\xi}_j$. Bold values in table (2) are significant against a 5% level. The only re-scaled test statistic that is not significant in our case is $\hat{\xi}_3$, which is similar to the findings of Ortu et al. (2013). The main difference between both results is, that in our case the re-scaled test statistic $\hat{\xi}_7$ is not significant.

Table (3), shows the result of the re-scaled test statistics $\hat{\xi}_q$ to test for serial correlation in the decimated series π_t^J. Again, the bold values are significant against a 5% significance level. Interesting to see is that in contrary to Ortu et al. (2013), we do not find as many significant persistent components as Ortu et al. (2013). Ortu et al. (2013) rejected π_t^J to be white noise up to the scale level of J, whereas we do not reject the hypothesis of π_t^J for $J \geq 3$.

In appendix D, we add the test statistics without de-meaning the series of consumption growth. In that case, the findings are in general similar. We again reject the hypothesis of consumption growth to have no serial in the components. In this case, we find that the re-scaled test statistics $\hat{\xi}_j$

are significant for $j = 1, 2$. The serial correlated test for the decimated series π_t^J are then significant for levels 1 up to 4.

Table 2: persistence test on consumption growth

level	j=1	2	3	4	5	6	7
zvalues	**-4.12**	**-1.99**	0.76	**5.42**	**2.02**	**3.19**	**2.78**

Table (2) shows the re-scaled test statistic of ξ_j for consumption growth using the sample 1948Q2-2011Q4. Significant values against a 5% significance level are represented in bold.

Table 3: determining the optimal amount of components for consumption growth

level	scale q=1	2	3	4	5	6	7
J=1	**-3.91**	-0.60	**3.31**	0.96	**2.02**	1.78	**4.27**
J=2	**-2.35**	1.33	-0.03	0.90	0.88	**2.64**	
J=3	-0.02	-0.70	0.21	0.33	1.67		
J=4	-0.87	0.22	0.33	1.69			
J=5	-0.27	0.00	1.16				
J=6	-0.17	1.01					
J=7	1.00						

Table (3) shows the re-scaled test statistic of ξ_q for $\pi_T^{(J)}$ using the decimated sample 1948Q2-2011Q4. Significant values against a 5% significance level are represented in bold.

4 Persistence Heterogeneity, Predictability, and Long-Run Valuation

In this section we will first follow Ortu et al. (2013) by following their steps for computing the long-run risk premia. First we use the decomposition to the estimation of the IES. Then we will use the decomposition to verify the assumption that consumption growth can be predicted by financial ratios and the log discounted bond price. Finally, we will determine the market risk premium of equity and the Fama-Bliss discounted bonds under consumption risk.

The composition method of Ortu et al. (2013) reconciles the long-run risk framework with the empirical evidence on two main aspects. First, it provides a different way of determining the IES. Second, it reconciles the literature on a requirement in the long run risk literature that price-dividend ratio predicts consumption growth (for example Bansal and Yaron (2004) and Beeler and Campbell (2009)). This requirement has been rejected empirically. The price-dividend ratio, on one hand, is (close to) a unit process (Ortu et al. (2013) mention to see, for example, Torous et al. (2004); Campbell and Yogo (2006); Lettau and Van Nieuwerburgh (2008))[5]. Consumption growth on the other hand closely resembles a white noise and therefore does not share the high persistence of the dividend ratio. By decomposing consumption growth in to components with different layers of persistence, Ortu et al. (2013) show that highly persistent components are indeed predictable by the financial ratios.

4.1 Estimating the IES under persistence heterogeneity

With the decomposition method of Ortu et al. (2013), they find a suitable approach to estimate the IES in the literature. The standard approach to estimate the IES (see Hansen and Singleton (1983)) is to estimate

$$r_{f,t} = \alpha_f + \frac{1}{\psi} g_t + \sigma_f u_t \tag{13}$$

Empirical estimates of equation (13) typically find an estimate of ψ lower than one (Ortu et al. (2013) mention to see, for example, Hall (1988); Campbell and Mankiw (1990)). An estimate of ψ contradicts a basic paradigm in the long-run risk literature. For the small-in-volatility component to contribute to the equity premium, one requires an IES greater than one. Ortu et al. (2013) resolves the issue of estimating an IES smaller than one, by using their decomposition in equation (13). Ortu et al. (2013) use the operator $H^{(J)}$ for both $r_{f,t}$ and g_t. The decimated components of $r_{f,t}$ are therefore regressed on the decimated components of g_t with the corresponding level of persistence. The IES is constrained to be equal across levels of persistence. Formally, Ortu et al. (2013) use the regression

$$r_{f,t}^{(j)} = \frac{1}{\psi} g_t^{(j)} + \sigma_{f,j} u_t^{(j)} \tag{14}$$

We will estimate equation (14) using the GLS technique introduced by Fadili and Bullmore (2002), where we weigh each observation through an estimate of $\sigma_{f,j}$. We regress equation (14) with a nonlinear regression method and a weighting matrix Ω which is based on the residuals through the proposed method by Fadili and Bullmore (2002). We repeat the estimation until the estimated ψ converges. The computed standard errors are estimated through the covariance matrix based on the gradients, where we do not take in to an account that the observations are weighted to maintain

[5] For a plot of both time series, see appendix E

robustness. Just as Ortu et al. (2013), we estimate ψ over the sample 1948Q1-2011Q4 and over the sub-samples 1948Q1-1979Q4 and 1980Q1-2011Q4.

Table (4) shows the estimation results. The significant estimates (against a 5% significance level) are represented in bold. Compared to Ortu et al. (2013) we find higher estimates over the sample 1948Q1-2011Q4 and the sample 1948Q1-1979Q4. The estimate of the sample 1948Q1-1979Q4 is in our case not significant, whereas in Ortu et al. (2013) it was. Also, over the full sample we estimate the IES to be roughly 1.5 higher than Ortu et al. (2013). The estimate for the sample 1980Q1-2011Q4 is almost identical. The fact that IES of the sample 1948Q1-1979Q4 is estimated to be insignificant can be due to the fact that the gradient method we used for our the computation of our standard errors can be considered as a rough method. This causes our estimate over this sub-sample to be insignificant, however it is not far from being significant against a 10% significance level. Overall, our estimates seem to support the key hypothesis of the long-run valuation approach. based on magnitude, our estimates can therefore also be placed in line with Ortu et al. (2013), Attanasio and Weber (1993), Beaudry and Van Wincoop (1996) and Vissing-Jørgensen (2002).

Table 4: Estimation of IES over different samples

sample	ψ	t-value	R^2
1948Q1-2011Q4	**7.13**	3.24	0.04
1948Q1-1979Q4	11.74	1.55	0.02
1980Q1-2011Q4	**4.05**	3.35	0.09

Significant estimates against a significance level of 10% are represented in bold.

4.2 Predictability of consumption and dividend growth under persistence heterogeneity

A key assumption for the determination of the long-run term structure of equity premia is the ability of expected consumption growth to determine the financial ratios. Beeler and Campbell (2009) test this by checking the ability of the financial ratios to predict future consumption growth. Ortu et al. (2013) test the ability of the components of financial ratios to predict the components of consumption growth with the same level of persistence by running the regressions. Similarly, a key assumption for the determination of the long-run term structure of the bonds is the ability of expected consumption growth to determine the log bond price. We will follow the reasoning of

Beeler and Campbell (2009) and Ortu et al. (2013) to test the ability by using the log bond price to predict future consumption growth on the same level of persistence[6].

The model to test the ability of financial ratios to predict financial ratios is formally given as

$$\begin{aligned} g^{(j)}_{t+2^j} &= \beta_{0,j} + \beta^{g}_{1,j} z^{(j)}_{m,t} + \epsilon^{(j)}_{t+2^j} \\ g^{(j)}_{t+2^j} &= \tilde{\beta_{0,j}} + \tilde{\beta_{1,j}}^{g} z^{(j)}_{a,t} + \epsilon^{(j)}_{t+2^j} \end{aligned} \tag{15}$$

where $z^{(j)}_{m,t}$ and $z^{(j)}_{a,t}$ are the components with level of persistence j of respectively the price-consumption and price-dividend series. Differently to Beeler and Campbell (2009) who analyse these predictability's at an aggregate level, Ortu et al. (2013) analyse the predict abilities at different level of persistence. To increase the sample size, we follow Ortu et al. (2013) by using all components and not the decimated components. The coefficients resulting from this regression will still be consistent, but not efficient. To correct for serial correlation, we will use a variant of HAC standard errors introduced by Hansen and Hodrick (1980). Table (5) and (6), show the estimation results of the regression defined in equation (15). Significant estimates against a 10% significance level are represented in bold. The estimates in table (5) and (6) are multiplied by -100 to align the results that we found to the results of Ortu et al. (2013). Ortu et al. (2013) mention that the estimates are multiplied by a 100, but not that the sign has also changed in their output.

Table (5) and (6) shows that under our multiplication, the results are similar to that of Ortu et al. (2013). We do however, find some differences in the significance of our estimates in table (6). Ortu et al. (2013) find significant estimates for the level of persistence 3 and 6 at a significance level of 5% when predicting consumption growth by price-consumption. In our regression, we find significant estimates for the level of persistence 2 and 3 at a significance level of 10%. These differences do not change the overall conclusion that components of the financial ratio of persistence are able to predict the components consumption growth with the same level of persistence[7].

To extend the long-run risk framework to bonds, it is key that the log bond price is able to predict consumption growth. The extension of Hasseltoft (2012) in the Bansal et al. (2016) suggest that expected consumption growth determines the log bond price with a maturity of n periods. Similarly to the way Beeler and Campbell (2009) and Ortu et al. (2013) investigated this relation,

[6] In addition Ortu et al. (2013) argue that a similar relation must hold between the financial ratios and dividend growth. The results are shown in appendix G. We also tested whether the log discounted bond price is able to predict dividend growth. These results are also shown in appendix G.

[7] We check the robustness of this conclusion by re-doing our analysis with the yearly data of log market returns and the log price-dividend ratio. These results are shown in appendix F

Table 5: log price-dividend

level	j=1	j=2	j=3	j=4	j=5	j=6	j=7
pd	0.25	**0.61**	0.48	-0.24	0.24	**0.37**	0.01
t-value	0.58	2.06	1.45	-0.79	0.94	1.88	0.08
R^2	0.00	0.02	0.02	0.01	0.03	0.19	0.00

Table 6: log price-consumption

level	j=1	j=2	j=3	j=4	j=5	j=6	j=7
pc	0.31	**0.39**	**0.40**	-0.05	0.25	0.15	0.05
t-value	0.99	1.80	1.89	-0.24	1.57	0.80	0.49
R^2	0.00	0.02	0.04	0.00	0.08	0.04	0.02

Table (5) and table (6) estimate the consumption growth on either the log price-dividend ratio or the log price-consumption ratio. Significant estimates against a significance level of 10% are represented in bold.

we will investigate whether the log bond price with a maturity of n periods can predict consumption growth. Formally we will test this through

$$g_{t+2^j}^{(j)} = \beta_{0,j}^n + \beta_{1,j}^n q_{t,n}^{(j)} + \nu_{t+2^j}^n \tag{16}$$

where $q_{t,n}^{(j)}$ is the component with level of persistence j from the discounted bond price with a maturity of n-periods. To estimate this equation we will again use the full set of data available for bonds. Using the Redundant Haarmatrix, this means that we have a effective sample for the period 1952Q3-2011Q4. Due to the overlap in the construction of the components we again encounter the problem of serial correlation. Therefore, in line with the methods above, we will again use the standard errors defined by Hansen and Hodrick (1980).

Table (7) shows the results of the estimation of $\beta_{1,j}^n$ for different levels of maturity in the Fama-Bliss discounted bond portfolio. In brackets in the column to the right of the estimates are the associated t-values. To keep the table readable, we left the R^2 out of the table. Estimates that can significantly determine consumption growth against a 10% level of significance are represented in bold. As can be seen from table (7), the log bond prices for all used maturity periods can predict the log consumption growth at level of persistence 1 and 4.

4.3 Persistence heterogeneity and the term structure of equity premia

Ortu et al. (2013) show that the layers of consumption growth with heterogeneous persistence generate a term structure of equity risk premia. To show this, they consider a Bansal and Yaron (2004) economy in which a representative agent with recursive preferences having a Epstein and Zin (1989) faces a consumption stream g_t whose decimated components $g_t^{(j)}$ follow multiscale autoregressive processes, that is,

$$g_{t+2^j}^{(j)} = \rho_j g_t^{(j)} + \epsilon_{t+2^j}^{(j)} \tag{17}$$

Table 7: Shows the coefficients of the consumption growth on the log bond price scaled by a 100

	Maturity=1		Maturity=2		Maturity=3		Maturity=4		Maturity=5	
level	Estimate	[t-value]	Estimate	[t-value]	Estimate	[t-value]	Estimate	[t-value]	Estimate	[t-value]
j=1	**0.16**	**[4.17]**	**0.10**	**[4.22]**	**0.07**	**[3.70]**	**0.05**	**[3.33]**	**0.04**	**[3.12]**
j=2	0.02	[0.72]	0.01	[0.31]	0.00	[0.28]	0.00	[0.31]	0.00	[0.13]
j=3	0.03	[0.83]	0.02	[0.82]	0.01	[0.75]	0.01	[0.50]	0.01	[0.51]
j=4	**-0.07**	**[-2.22]**	**-0.04**	**[-2.00]**	**-0.03**	**[-1.93]**	**-0.02**	**[-1.85]**	**-0.02**	**[-1.77]**
j=5	0.00	[0.09]	0.00	[0.07]	0.00	[0.05]	0.00	[-0.04]	0.00	[-0.06]
j=6	-0.03	[-0.89]	-0.02	[-0.89]	-0.01	[-0.87]	-0.01	[-0.81]	-0.01	[-0.82]
j=7	-0.01	[-1.07]	-0.01	[-1.02]	0.00	[-1.05]	0.00	[-1.07]	0.00	[-1.05]

Consistent to this, Ortu et al. (2013) allow the leverage effect to cash flows to be different across levels of persistence, that is, the decimated components $g^{(j)}_{t+2^j}$ of the log dividend growth series satisfy

$$gd^{(j)}_{t+2^j} = \phi_j g^{(j)}_t + \epsilon^{(j)}_{t+2^j} \tag{18}$$

To determine the term structure of risk premia, Ortu et al. (2013) log-linearize the return on the consumption claim, $r_{a,t+1}$ and the return on the market portfolio $r_{m,t+1}$, similar to Campbell and Shiller (1988), to express them in terms of the log price-consumption and log price-dividend ratio $z_{a,t}, z_{m,t}$ as follows [8]:

$$\begin{aligned} r_{a,t+1} &= \kappa_0 + \kappa_1 z_{a,t+1} - z_{a,t} + g_{t+1} \\ r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + gd_{t+1} \end{aligned} \tag{19}$$

Furthermore, Ortu et al. (2013) conjecture a linear relation between the persistent components $z^{(j)}_{m,t+1}$, $z^{(j)}_{a,t+1}$ of the financial ratios and the consumption components $g^{(j)}_t$ [9]

$$\begin{aligned} z^{(j)}_{a,t+1} &= A_{0,j} + A_j g^{(j)}_{t+1} \\ z^{(j)}_{m,t+1} &= A^m_{0,j} + A^m_j g^{(j)}_{t+1} \end{aligned} \tag{20}$$

From the Campbell and Shiller (1988) equations, Ortu et al. (2013) show in their paper that with persistence heterogeneity, the equity premia for consumption claim asset, $r_{a,t+1}$, and for the market portfolio, $r_{m,t+1}$, take the form

$$E_t[r_{a,t+1} - r_f] + 0.5\sigma^2_{r_{a,t}} = \gamma \iota^T Q \iota + \kappa_1 \lambda^T_\epsilon Q A \tag{21}$$

$$E_t[r_{m,t+1} - r_f] + 0.5\sigma^2_{r_{m,t}} = \kappa_{1,m} \lambda^T_\epsilon Q A_m \tag{22}$$

[8] Similar to Ortu et al. (2013), we set $\kappa_1 = \kappa_{1,m} = 0.988$ in the computation of the risk premia

[9] Bansal et al. (2016) conjectures the same relation on a aggregate level

where $\lambda_\epsilon = \kappa_1(1-\theta)A$ and $\theta = (1-\gamma)/(1-1/\psi)$. The parameters γ and ψ are respectively the risk aversion and the intertemporal elasticity of substitution. The matrix Q is the covariance matrix from the innovations of equation (17). We believe however, that his representation of the equity risk premium is not correct. In our computation the correct risk premium is [10]

$$E_t[r_{m,t+1} - r_f] + 0.5\sigma^2_{r_{m,t}} = \kappa_{1,m}(\gamma \mathbf{1}^T + \lambda_\epsilon)QA^m_1 \tag{23}$$

For the entries of A and A^m (both vectors) there are different ways to compute them. By solving the Euler Equation in the Bansal and Yaron (2004) economy, Ortu et al. (2013) show that the entries can be determined in the state variables by

$$\begin{aligned} A &= (1 - \tfrac{1}{\psi})(I_J - \kappa_1 M)^{-1} M\mathbf{1} \\ A_m &= (I_J - \kappa_{1,m} M)^{-1} M(\mathbf{\Phi 1} - \tfrac{1}{\psi}\mathbf{1}) \end{aligned} \tag{24}$$

Where $M = -diag(\rho_1, ..., \rho_J)$ and $(\phi) = diag(\phi_1, ..., \phi_J)$. However, we believe that in the determination of A_m Ortu et al. (2013) made a small mistake.[11] The correct specification of A_m is given by

$$A_m = (I_J - \kappa_{1,m} M)^{-1}(\mathbf{\Phi 1} - \tfrac{1}{\psi} M\mathbf{1}) \tag{25}$$

An alternative way of Ortu et al. (2013) to estimate the entries of $\hat{A}_j$ and $\hat{A}^m_j$ is by using equation (15) and (17). It can be shown that through these formulas $\hat{A}_j = \hat{\rho}_j/\hat{\tilde{\beta}}^g_{1,j}$ and $\hat{A}^m_j = \hat{\rho}_j/\hat{\beta}^g_{1,j}$ where $\hat{\beta}^g_{1,j}$ and $\hat{\tilde{\beta}}^g_{1,j}$ correspond to the coefficients $\hat{\beta}^g_{1,j}$ and $\tilde{\beta_{1,j}}^g$ estimated in section 4.2. This way of estimating the entries remains correct, even after our correction. The market return's exposure to shocks are represented by the vector QA^m, whereas the risk compensations for these shocks are given by λ_ϵ in the risk premium of Ortu et al. (2013). In our corrected risk premium, the risk compensation for shocks are given by $(\gamma \mathbf{1}^T + \kappa_{1,m}\lambda^T_\epsilon)$.

Table (8) shows the estimated coefficients of the multiscale autoregressive process for the components of consumption growth. These estimates are based on the full sample of components (not the decimated). We therefore use the standard errors of Hansen and Hodrick (1980) again to obtain a robust inference about the estimated coefficients. Compared to Ortu et al. (2013), we only find 2 significant autoregressive processes, namely the processes at levels 2 and 7. Ortu et al. (2013) also finds the significant autoregressive processes for the levels 3, 5, and 6. We compute the annualized half-life based on $\frac{-2^j}{4}\frac{log(2)}{log(|\rho_j|)}$. As can be seen from table (8) we estimate the annualized half-lives to

[10] For prove, see appendix L

[11] For proof, see appendix M

be roughly between 0.4 and 16.9. These estimates are comparable with the half-lives given by Ortu et al. (2013) who estimate the annualized half-lives to be between 0.4 and 10.8.

Table (9) shows the estimated diagonal elements of the matrix Q (the off-diagonal elements are assumed to be small or zero), the risk price, the risk exposure and the risk premium under the parameters choices of $\gamma = 5$ and $\psi = 5$. The value of γ is chosen to be equal to the estimate of Ortu et al. (2013) and the risk aversion is chosen equal to the estimate of Ghysels et al. (2004). We also estimated the risk premia with $\psi = 2.5$ (estimated by Bansal et al. (2016)) and with $\gamma = 7.5$ (estimated by Bansal and Yaron (2004)). These results are shown in appendix N. The risk price, the risk exposure and the risk premium are annualised.

Table 8: Estimation of AR(1) process for g_t^j over different samples

persistence level	ρ	t-value	HL (in Years)	R^2
j=1	-0.01	-0.14	0.80	0.00
j=2	**-0.15**	-2.14	0.37	0.03
j=3	-0.11	-0.96	0.62	0.01
j=4	-0.08	-0.51	1.10	0.00
j=5	-0.14	-0.70	2.84	0.02
j=6	-0.23	-1.36	7.62	0.09
j=7	**0.26**	3.01	16.49	0.11

Significant estimates against a significance level of 10% are represented in bold.

4.4 Persistence heterogeneity and the term structure of the Fama-Bliss discounted bond portfolio

In this section, we will construct a risk premia for the Fama-Bliss bond portfolio with the decomposition of Ortu et al. (2013) and the extension of Hasseltoft (2012) to the Bansal and Yaron (2004) economy. As Bansal and Yaron (2004) conjectures a linear relation between the financial ratios and the expected consumption growth, so does Hasseltoft (2012) conjecture a similar linear relation to the bond price with a maturity of n periods and the expected growth component.[12] Now for the determination of the risk premia on bonds, we will conjecture a linear relation to components of the bond price with a maturity of n periods and the components of consumption growth. This is similar to the relation conjectures by Ortu et al. (2013) on the components consumption growth

[12]In appendix H, we show the basic equations of the Bansal and Yaron (2004) with the Hasseltoft (2012) extension

Table 9: Risk premia for Equity

			Ortu et al.		Corrected	
Level	$Q_(j,j)$ ($x10^-5$)	Risk Exposure ($x10^-6$)	Risk Price	Risk Premium	Risk Price	Risk Premium
j=1	0.89	73.98	51.71	0.19	41.71	0.15
j=2	**0.48**	**123.58**	**312.45**	**3.82**	**307.45**	**3.76**
j=3	**0.42**	**48.91**	**111.42**	**1.08**	**108.92**	**1.05**
j=4	0.31	29.28	339.26	3.93	340.51	3.94
j=5	0.17	10.84	47.22	0.40	46.59	0.40
j=6	**0.09**	**3.34**	**72.31**	**0.38**	**72.00**	**0.38**
j=7	0.02	14.32	130.07	5.89	130.22	5.90

Table (9) shows the estimation ouput when when $\gamma = 5$ and $\psi = 5$. In the second and third column the variance of the innovation shocks (x10^{-5}) and the exposure of return (x10^{-6}) to those shocks are shown; in the columns thereafter the price and the risk premium (in %) for equity for both methods are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

and the components of the financial ratios. Formally, our linear relation is given by

$$q_{t,n}^{(j)} = D_{0,n}^{(j)} + D_{1,n}^{(j)} g_t^{(j)} \tag{26}$$

In the framework of Hasseltoft (2012), the one-period holding return for a bond with a maturity of n periods is defined as

$$h_{t,n} = q_{t+1,n-1} - q_{t,n} \tag{27}$$

With the definition an asset return, we can express the risk premium on a bond with a maturity of n periods as[13]

$$E_t[h_{n,t+1} - r_f] + 0.5\sigma_{r_{m,t}}^2 = (\gamma \mathbf{1}^T + \lambda_\epsilon^T) Q D_{1,n-1} \tag{28}$$

with $D_{1,n-1} = [D_{1,n-1}^1, ..., D_{1,n-1}^J]^T]$ and the other variables defined as in the previous section. The exposure of innovation of consumption growth at different persistence levels is given by the vector $QD_{1,n-1}$, and $(\gamma \mathbf{1}^T + \lambda_\epsilon^T)$ is a vector with the compensation of the risk exposure at different levels of persistence. Note that with $\kappa_{1,m} \approx 1$ the price of the risk exposure for bonds and equity are almost identical. Again, just as is done by Ortu et al. (2013) we could express the vector $D_{n,1}$ in to the state variables by exploiting the Euler equation, the multiscale AR-process of consumption growth

[13] For proof, see appendix K

and the decomposition of Ortu et al. (2013). Doing so, we find the following recursive expression [14]

$$D_{1,n} = M(D_{1,n-1} + \frac{1}{\psi}\iota) \tag{29}$$

with $D_{1,0} = 0$. To estimate the entries of $D_{1,n}$, we could use a similar method as Ortu et al. (2013). It can be shown through the linear relation of equation (16) and the multiscale AR-process of equation (21) that the estimates for the entries of $D_{1,n}$ are given by $\hat{D}^{(j)}_{1,n} = \hat{\rho}_j/\hat{\beta}^n_{1,j}$. However, due to the fact that our estimates of $\hat{\beta}^n_{1,j}$ which are not significant tend to be close to 0, we would not use such relation to prevent to inflate our estimates. Therefore, we will estimate the entries of $D_{1,n}$ directly through an OLS regression on equation (26) with the full sample of components.[15] Because our data is quarterly and our maturities are yearly we will assume that $D_{1,n} \approx D_{1,n-1}$. [16] In other words, the premium risk would normally be determined by the estimated coefficient $D_{1,n-1}$ which has a period less to its maturity (in our case a quarter). Since we don't have data that contains bonds with maturities of 3, 7, 11, 15 or 19 quarters, we will assume that these coefficients are close to the maturities we do have.

Table (10) shows the results of the computed diagonal elements of Q, the compensation to the exposure of log consumption growth shocks, and the exposure and risk premium for each discounted bond with a maturity of 1 up to 5 years. Again we only show the results of the risk premia for bonds under the parametric choices of $\gamma = 5$ and $\psi = 5$. The results of the risk premia of bonds under other parametric choices are shown in appendix M. In table (10), the variables are in bold when consumption growth with a persistence level of j could be determined by the component of the log bond price with the same level of persistence. For the other parametric choices of Ortu et al. (2013) we refer to the appendix. From the table we can see that the risk premium increases as the maturity increases up to a maturity of 4 for the significant components. The risk premium of a bond with a maturity of 5 years is slightly lower than the risk premium of a bond with a maturity of 4 years.

5 Conclusion and Further Research

In summary, we have used the techniques of Ortu et al. (2013) to detect persistence heterogeneity in log consumption growth. As Ortu et al. (2013) we did find some persistent components, but we

[14]For proof, see appendix J

[15]In appendix O, the results from this regression is shown

[16]One could evade this problem by using only yearly data in the model, but we did not do this since it would lead to a loss of data points

Table 10: Risk premia for the Fama-French bond portfolio

			Maturity=1		Maturity=2		Maturity=3		Maturity=4		Maturity=5	
level	Q(j,j)	Price	Exposure	Premium	Exposure	Premium	Exposure	Premium	Exposure	Premium	Exposure	Premium
j=1	0.89	28.89	**279.97**	**0.40**	**364.35**	**0.53**	**420.72**	**0.61**	**587.05**	**0.85**	**575.82**	**0.83**
j=2	0.48	229.97	108.81	2.50	202.62	4.66	224.41	5.16	262.58	6.04	285.13	6.56
j=3	0.42	81.29	15.36	0.25	49.47	0.80	46.49	0.76	49.29	0.80	54.39	0.88
j=4	0.31	256.38	**24.17**	**2.48**	**30.87**	**3.17**	**43.72**	**4.48**	**63.55**	**6.52**	**63.24**	**6.49**
j=5	0.17	34.88	8.42	-0.24	9.05	-0.25	6.95	-0.19	6.08	-0.17	5.54	-0.15
j=6	0.09	54.07	11.30	0.98	20.71	1.79	30.33	2.62	38.70	3.35	43.09	3.73
j=7	0.02	97.97	1.67	-0.52	2.94	-0.92	4.21	-1.32	5.58	-1.75	6.34	-1.99

Table (10) shows the estimation ouput when $\gamma = 5$ and $\psi = 5$. In the second and third column the variance of the innovation shocks (scaled by 10^{-5}) and the price of the exposure to those shocks are shown; in the columns thereafter the exposure (scaled by 10^{-6}) and the risk premium (in %) for bonds with different maturities are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

did not find as many significant persistent components as they did. We also replicated the results on the IES. The estimates we found were of similar size, but were not as consistent as in Ortu et al. (2013). For the predictive ability of expected consumption growth on the financial ratios, we found similar evidence as Ortu et al. (2013). With these results, we estimated the risk premium for equity under consumption risk on a persistence level. As an extension, we used the decomposition to show that some components expected consumption growth predicts the corresponding components of the log discounted bond price. We used the long-run valuation framework to estimate the risk premia for bonds under consumption risk on a persistence level.

Further research could be done on this topic by looking at the implications of our model for bonds, by investigating its implications to yields. Or, one could further extend our risk premia for bonds by considering the extension of Hasseltoft (2012) of log bonds price determined by expected consumption growth and expected inflation growth. Finally, we think that a similar method could be derived for the dynamics of long-run risk in money markets derived by d'Addona (2017) for the economy of Bansal and Yaron (2004).

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A Identification of the consumption components

In this section, we will show the identification of some of the consumption growth components that Ortu et al. (2013) found. Ortu et al. (2013) concentrate their analysis on the components with level of persistence 2 to 7. The first component resembles a clear statistical (random) noise such that they identify it with a contemporaneous i.i.d. consumption[17].

The second and third component capture fluctuations with a half-life between one-half and two years. Ortu et al. (2013) argue that one way of identifying these two components with observable economic factors is to follow the lead of Jagannathan and Wang (2007) and Møller and Rangvid (2012), who analyse the ability of the fourth quarter consumption growth rate to predict expected excess returns on stocks. The intuition behind this ability is that due to cultural and institutional features consumption and investment decisions are aligned to the fourth quarter. Their used variable in these analyses aims to capture economic and financial choices happening with a yearly frequency which is similar to our second and third component. The correlation to their variable with the second component is estimated by Ortu et al. (2013) to be 0.60. When adding the third component to the second component, the correlation rises to 0.72. Figure (1) shows the second component with the variable of the fourth quarter effect.

The fourth and fifth component taken together capture fluctuations lasting from two to eight years. Ortu et al. (2013) argue that it is widely accepted that this interval is similar to that of business-cycle fluctuations in economic activity (Ortu et al. (2013) refer to Burns and Mitchell (1946)). Therefore, they look at indicators of the business-cycle, such as the term spread, to identify the fourth and fifth component of consumption growth. The term spread is defined as the slope of the Treasury yield curve. The Treasury yield curve is the difference between the ten-year constant-maturity yield and the three-month constant-maturity Treasury yield. Another indicator Ortu et al. (2013) use is the Baa-Aaa credit spread. Figure (2) shows the sum of the fourth and fifth component with the mentioned term spread and the mentioned default spread. The correlation Ortu et al. (2013) found between the components and these indicators are -30% for the term spread and -45% for the default spread.

The sixth component of consumption growth is a slow-moving series with a half-life of about eight years. To search for a valid proxy, Ortu et al. (2013) follow the literature on the long-run risk with

[17]For a replication of the figures and the data, we refer to Ortu et al. (2013) and the website of Andreas Tamoni mentioned in the data section

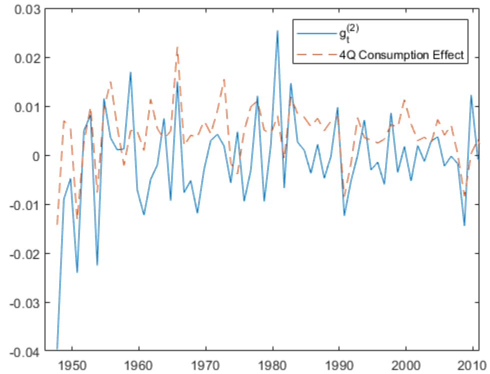

Figure 1: this figure shows the second component of consumption growth and the fourth quarter effect

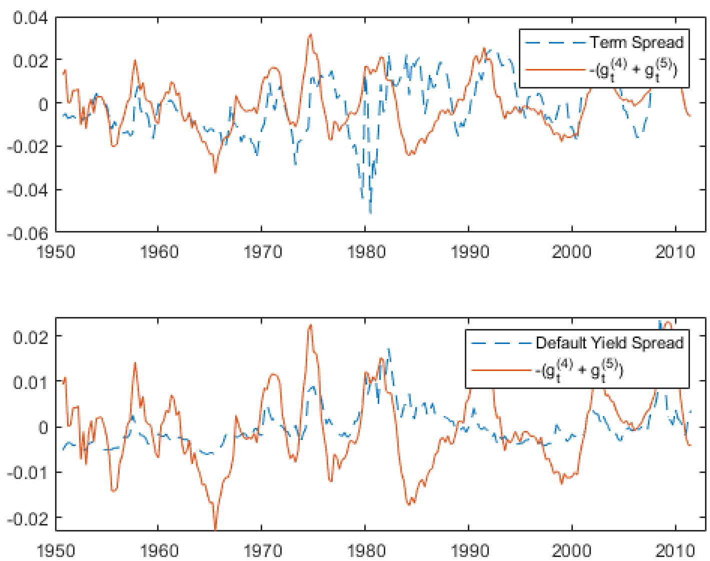

Figure 2: This figure shows the negative co-movement of the fourth and fith component of consumption growth and the bussiness-cycle indicators

production (Ortu et al. (2013) mention to see, for example, Kaltenbrunner and Lochstoer (2010) and Croce (2014)). Moreover, Ortu et al. (2013) investigate whether shocks to productivity growth can explain these persistent fluctuations in consumption. Figure (3) plots the sixth component of consumption growth together with the sixth component filtered out of the multifactor productivity growth index (TFP). Ortu et al. (2013) estimate the correlation between the two components at 0.61.

Finally, for the identification of the seventh component, Ortu et al. (2013) look at demographic trends as a possible economic proxy. Ortu et al. (2013) look at the live births in the United States. These life birds have featured alternating twenty-year periods of boom and busts and are therefore consistent with the half-life of our seventh component (Ortu et al. (2013) refer to Geanakoplos et al. (2004) for an example). Ortu et al. (2013) estimate the correlation at 30% between the ratio of middle-aged population to population of young adults and the seventh component, which still leaves place for unexplained variability om the slow-moving component of consumption growth. Furthermore, Ortu et al. (2013) argue that the sparsity of data for the seventh component is less strong than the previous ones.

In extend, To comprehend the long-run variations in the data Ortu et al. (2013) give an interpretation for the components with the yearly data. With the equations (1) - (4), we can show that taking the sum over the components with level of persistence higher than 6 is equal to the log consumption growth over 32 quarters. This smooth consumption growth series that corresponds to frequencies of 32 quarters or above is strictly analogue with the medium-frequency component of Comin and Gertler (2006). In figure (4) we plot the truncated sum of components for the consumption growth series and TFP series. The correlation between this two truncated sums is remarkable with 0.47.

The vast majority of action in terms of long-run fluctuations in consumption and TFP growth rates comes from the sixth component. In other words, components with higher half-life than between eight and sixteen years seem to be of lesser importance. In figure (5), we plot these sixth components with the truncated sum of higher frequency components. The correlation between the sixth component and the truncated sum are equal to 85% and 54% respectively for the consumption growth and TFP growth.

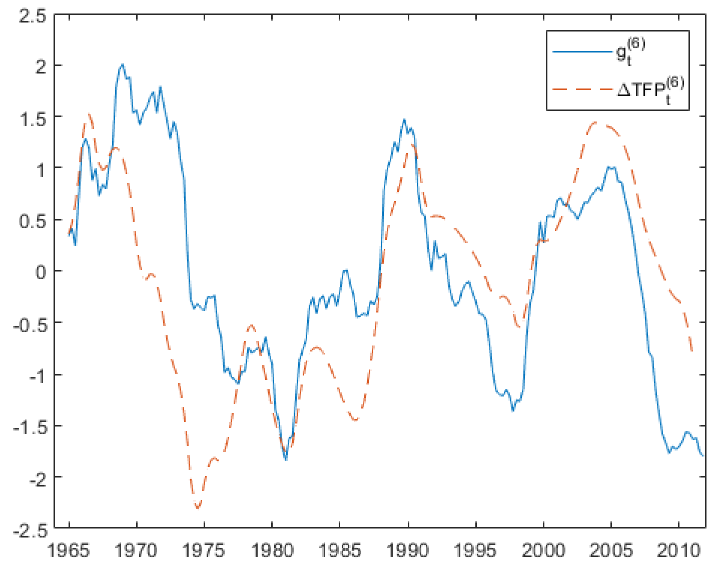

Figure 3: This figure shows the sixth consumption component and the total factor of productivity

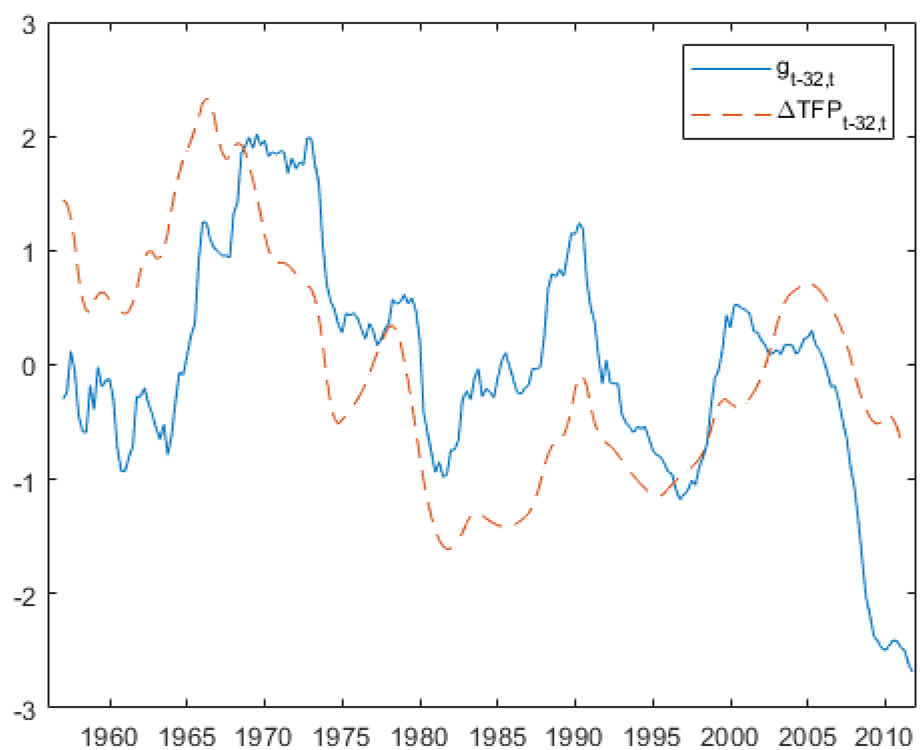

Figure 4: Plot of the truncated sums of higher frequency components for consumption growth and TFP growth

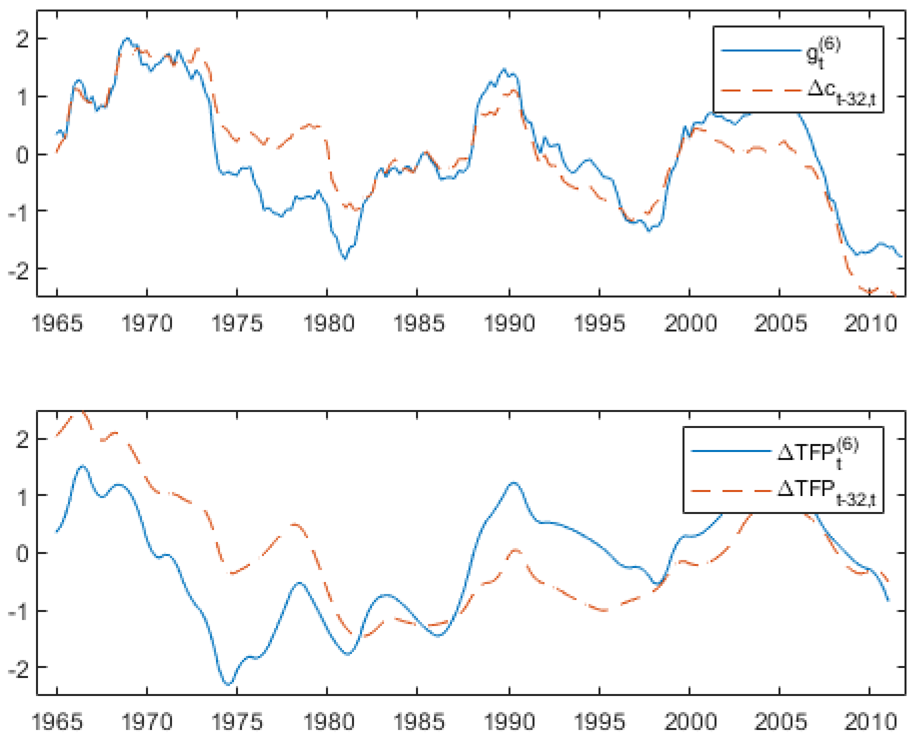

Figure 5: Plot of the truncated sums of higher frequency components for consumption growth and TFP growth with the corresponding sixth component

B Simulating a time series from its components

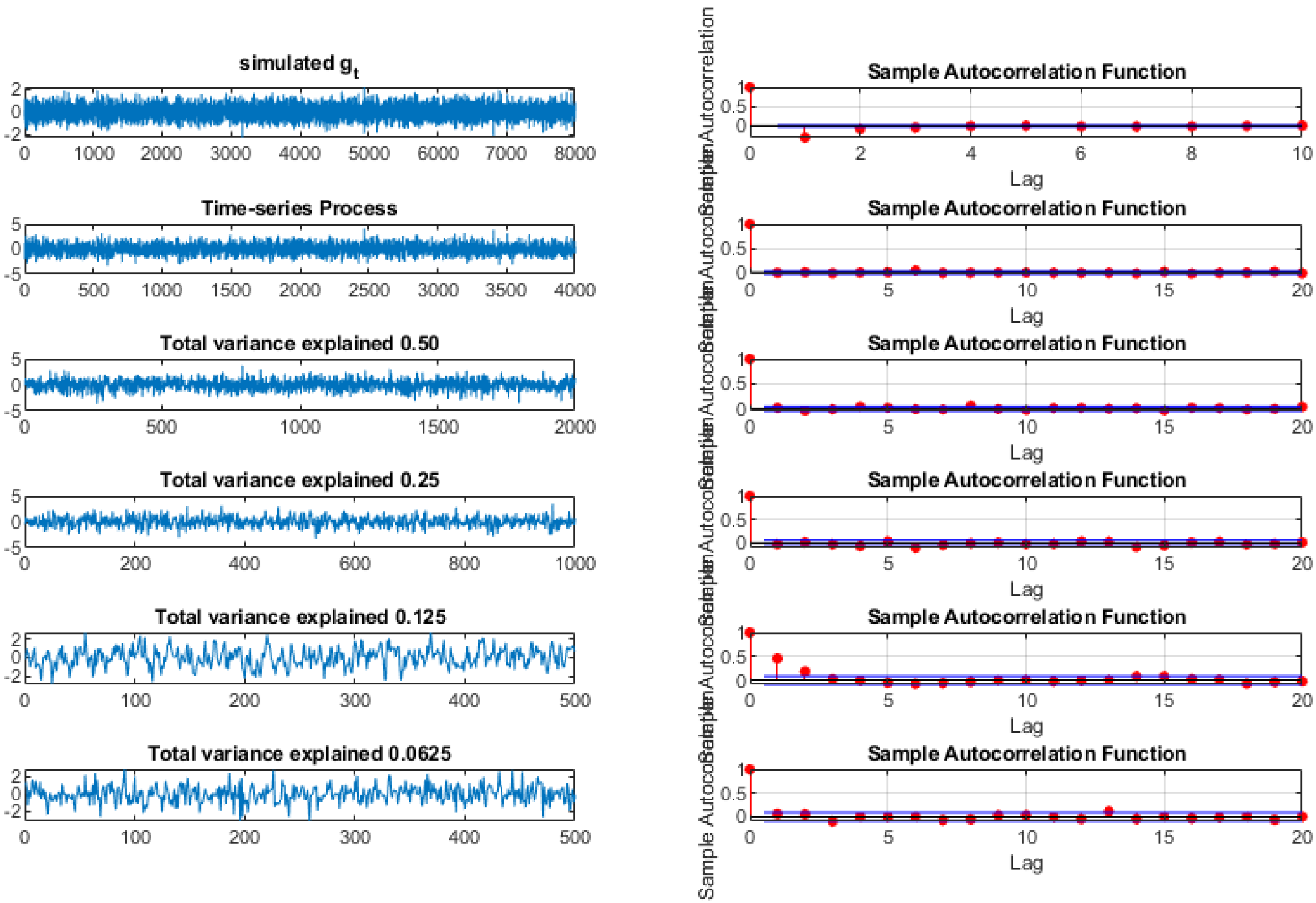

Figure 6: Simulated time series from its components, where component $J = 4$ contains a persistent element

C Results of the persistence heterogeneity test with white noise

In table (11) we show the rejection probabilities when the simulated series is white noise. This is in addition to the test that has been performed in section (3.4). For a given sample size, we simulate the series according to a standard normal distribution and compute the test statistic given by equation (9) for levels of persistence 1 up to 7.

Table 11: Simulated rejection probabilities of the persistence test with a white noise series

	T=256							T=2048						
j =..	1	2	3	4	5	6	7	1	2	3	4	j=5	6	7
	0.049	0.051	0.049	0.048	0.040	0.049	0.053	0.051	0.048	0.047	0.049	0.049	0.047	0.046

Table (11) shows the Simulated Probabilities of Rejecting the Null Hypothesis when the simulated series is White Noise, simulation repeated 5000 times.

D Serial correlation test results for consumption growth without de-meaning

This section shows the test results of section 3.6 when we do not de-mean the log consumption growth series. Table (12) shows the results of the serial correlation test on the log consumption growth. As it can be seen; in case we do not de-mean our time series, we find significant disproportions of the variance explained in levels of persistence 1 and 2 meaning that we can reject the hypothesis of no serial correlation.

Table (13) shows the results of the serial correlation test on the moving averages of different levels of persistence in case we do not de-mean log consumption growth. It follows from the table that in this case we find significant evidence that the moving averages at levels 1 up to 4 contain serial correlation.

Table 12: Rescaled test statistic of ξ_j for consumption growth without de-meaning

level	j=1	2	3	4	5	6	7
zvalues	**-7.55**	**-4.16**	-1.63	1.42	0.08	1.01	0.97

In table (12) significant values against a significance level of 5% are represented in bold.

Table 13: Rescaled test statistic of ξ_q for consumption growth without de-meaning

	q=1	2	3	4	5	6	7
j=1	**-6.26**	**-2.91**	-0.32	-0.78	0.03	0.18	1.41
j=2	**-4.48**	-1.63	-1.39	-0.62	-0.34	0.48	
j=3	**-2.88**	-1.86	-1.03	-0.65	-0.05		
j=4	**-2.51**	-1.33	-0.84	-0.33			
j=5	-1.80	-1.02	-0.54				
j=6	-1.33	-0.69					
j=7	-0.92						

In table (13) significant values against a significance level of 5% are represented in bold.

E Price-dividend ratio with consumption growth

Figure (7) plots the log consumption growth with the log price-dividend ratio. The graph gives a visual representation in the mismatch between both series. On one hand, log consumption growth is close to a white noise series. On the other hand, the log price-dividend ratio resembles almost a unit process.

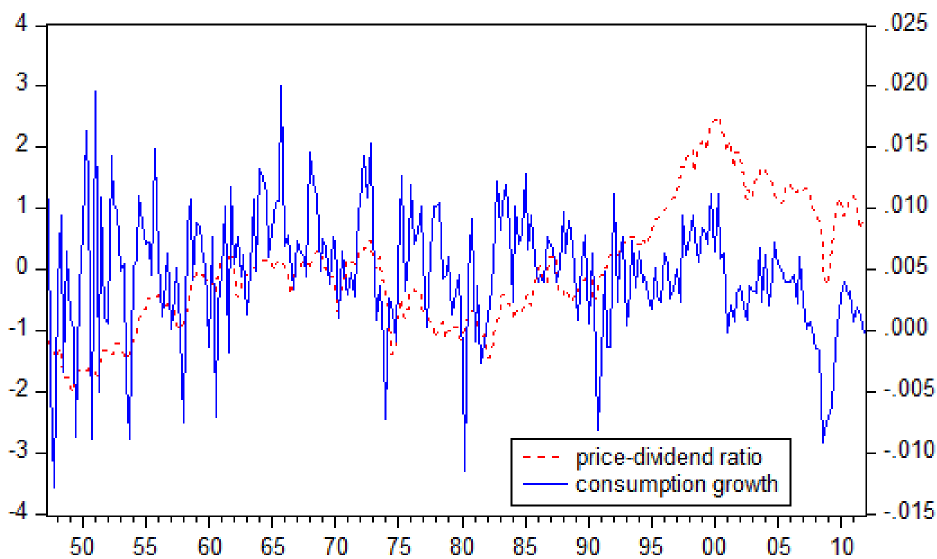

Figure 7: This figure plots log consumption growth with the log price-dividend ratio. Both series have been normalized.

F Robustness check on the predictability of consumption growth by the financial ratios

In order to check the robustness of our inference on the ability of the financial ratios to predict consumption growth, we follow Ortu et al. (2013) and estimate equation (15) in the main text with the yearly data. We constrain ourselves to the robustness of the log price-dividend ratio, since we do not have much yearly data on the log price-consumption ratio and the log discounted bond price.

Table (14) shows the estimation results when estimating the ability of log price-dividend to predict future consumption growth. Significant estimates against a level of 10% are shown in bold. For the smaller estimation sample of 1948-2011, we find the components at level 1 and 5 of persistence to be significant. For the bigger estimation sample of 1930-2011, we only estimate the coefficient with level of persistence 3 to be insignificant against a significance level of 10%. Against a level of 5%, we see that the estimate of the bigger sample size becomes insignificant at level of persistence j=2. For the other estimates, the conclusion remains the same. The estimated coefficients we found are in comparable size to those of Ortu et al. (2013). The significance, however, does differ a bit for some coefficients. Ortu et al. (2013) found the first and fourth estimate to be significant for both sample sizes.

Table 14: Testing the ability of price-dividend ratio to predict consumption growth

	1948-2011			1930-2011		
level	pd	t-value	R^2	pd	t-value	R^2
j=1	**3.46**	4.06	0.21	**2.70**	2.70	0.09
j=2	-0.27	-0.26	0.00	**1.80**	1.81	0.06
j=3	0.47	0.49	0.01	-0.80	-0.93	0.03
j=4	1.17	1.42	0.12	**1.53**	2.47	0.24
j=5	**0.88**	4.46	0.46	**0.66**	2.44	0.34

Significant estimates against a significance level of 10% in Table (14) are represented in bold. The standard errors are the standard errors defined by Hansen and Hodrick (1980).

G Predicting dividend growth by financial ratios and the log bond prices

In addition to the predictive ability of the components financial ratios to predict the components consumption growth, Ortu et al. (2013) also examine the ability of financial ratios to predict cash flows. They argue that in extending the standard long-run risk model to their persistence heterogeneity framework, one would expect that those components of financial ratios that are useful in forecasting the components of consumption growth are also useful in predicting variations at the same level of persistence in cash flows. To test this, Ortu et al. (2013) run the regression

$$\begin{aligned} gd_{t+2^j}^{(j)} &= \beta_{0,j}^{gd} + \beta_{0,j}^{gd} z_{m,t}^{(j)} + \nu_{t+2^j}^{(j)} \\ gd_{t+2^j}^{(j)} &= \tilde{\beta_{0,j}^{gd}} + \tilde{\beta_{0,j}^{gd}} z_{a,t}^{(j)} + \eta_{t+2^j}^{(j)} \end{aligned} \tag{I.30}$$

where again $z_{m,t}^{(j)}$ and $z_{a,t}^{(j)}$ are the components with level of persistence j of respectively the price-consumption and price-dividend series. Ortu et al. (2013) use for the left-hand side the cash dividend following much of the earlier literature (see, for example, Cochrane (1992)). Again, in this regression we use the standard errors defined by Hansen and Hodrick (1980). Significant estimates against a 10% significance level are represented in bold. The estimates in table (15) and (16) are again multiplied by -100 to align with the results that we found to the results of Ortu et al. (2013), where Ortu et al. (2013) again only mentioned the multiplication of the estimates by a 100.

Table (15) and (16) show the results of the regression defined in formula (I.30). Similar to Ortu et al. (2013), we find for a persistence level of 3 a significant estimate with roughly the same coefficient. Different to Ortu et al. (2013), is that we do not find a significant estimate for the coefficient according to the persistence level of 6. Instead, we find a significant estimate for the coefficient according to the persistence level of 7. In addition, our estimated coefficient is negative for both the price-dividend ratio and the price-consumption ratio. Both estimated coefficients are in Ortu et al. (2013) estimated to be positive, but are not significant.

Additional, we follow Ortu et al. (2013) by investigating whether the log bond price is also able to predict cash flows. Again, if the log bonds price is able to predict log consumption growth, one would also expect the log bond price to predict dividend growth. Similar to Ortu et al. (2013) we test this by estimating the following equations

$$gd_{t+2^j}^{(j)} = \beta_{0,j}^{gd,n} + \beta_{0,j}^{gd,n} q_{n,t}^{(j)} + \nu_{n,t+2^j}^{(j)} \tag{I.31}$$

From Table (17) it can be seen that we found 4 significant values against a 10% significance level. We

Table 15: log price-dividend

level	j=1	j=2	j=3	j=4	j=5	j=6	j=7
pd	-12.15	2.93	**4.14**	-2.62	-0.02	-0.97	**-1.41**
se	16.77	6.02	1.80	1.68	1.23	1.09	0.78
pvalue	0.47	0.63	0.02	0.12	0.99	0.37	0.07
R^2	0.00	0.00	0.04	0.04	0.00	0.07	0.27

Table 16: log price-consumption

level	j=1	j=2	j=3	j=4	j=5	j=6	j=7
pc	-16.16	0.57	**3.79**	-1.80	0.11	-0.60	**-1.39**
se	12.85	4.01	1.13	1.18	0.85	0.97	0.53
pvalue	0.21	0.89	0.00	0.13	0.89	0.54	0.01
R^2	0.01	0.00	0.07	0.04	0.00	0.04	0.37

Table (15) and Table (16) shows the estimation results of dividend growth regressed on either the log the price-dividend ratio or log price-consumption. Significant estimates against a significance level of 10% are represented in bold.

found at a persistence level of 3 that the log bond prices of maturities of 1 and 2 years significantly predicts the log dividend growth. At a persistence level of 2, we found that the log bond price with maturities of 1 and 2 years significantly predicts dividend growth.

Table 17: Testing the ability of the log discounted bond price to predict dividend growth

	Maturity=1		Maturity=2		Maturity=3		Maturity=4		Maturity=5	
level	Estimate	[t-value]	Estimate	[t-value]	Estimate	[t-value]	Estimate	[t-value]	Estimate	[t-value]
j=1	0.09	[0.05]	0.33	[0.33]	0.36	[0.45]	0.16	[0.24]	0.17	[0.29]
j=2	0.77	[1.47]	0.43	[1.39]	0.39	[1.62]	**0.39**	**[1.90]**	**0.34**	**[1.75]**
j=3	**0.38**	**[2.02]**	**0.19**	**[1.65]**	0.14	[1.54]	0.11	[1.45]	0.10	[1.33]
j=4	-0.17	[-0.94]	-0.11	[-0.98]	-0.09	[-1.04]	-0.08	[-1.11]	-0.08	[-1.22]
j=5	-0.22	[-1.09]	-0.10	[-0.91]	-0.06	[-0.72]	-0.04	[-0.58]	-0.03	[-0.47]
j=6	0.13	[0.86]	0.07	[0.88]	0.05	[0.86]	0.05	[0.90]	0.04	[0.91]
j=7	-0.11	[-1.22]	-0.06	[-1.28]	-0.05	[-1.34]	-0.04	[-1.43]	-0.03	[-1.45]

Table (17) shows the estimated coefficients (scaled by a 100) of the components of log dividend growth on the corresponding components of the log bond price with the t-value between the brackets. Significant estimates against a level of 10% are represented in bold.

H Bansal & Yaron economy with the Hasseltoft extension

Bansal and Yaron (2004) define the following framework for their economics

$$\begin{aligned} x_{t+1} &= \rho x_t + \psi_e \sigma_t \epsilon_{t+1} \\ g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} \\ g_{d,t+1} &= \mu_d + \phi x_t + \psi_d \sigma_t u_{t+1} \\ . \quad \sigma_{t+1} &= \overline{\sigma}^2 + \nu_1(\sigma_t^2 - \overline{\sigma}^2) + \sigma_\omega \omega_{t+1} \\ \epsilon_{t+1}, u_{t+1}, \eta_{t+1}, \omega_{t+1} &\sim I.I.D.N(0,1) \end{aligned} \tag{I.32}$$

In the paper of Hasseltoft (2012), he defines the log price of bonds with a maturity of n periods are defined as a linear function of the state variables in the way

$$q_{t,n} = D_{1,0} + D_{1,n} x_t + D_{2,n} \sigma_t^2 \tag{I.33}$$

The n-period continuously compounded yield is denoted as $y_{t,n} = -q_{t,n}/n$. The coefficients $D_{0,n}$, $D_{1,n}$ and $D_{2,n}$ are given by recursive relation (Hasseltoft (2012) refers to Bansal and Yaron (2004) for a proof):

$$\begin{aligned} &D_{1,n} = \rho D_{1,n-1} - \tfrac{1}{\phi} \\ &D_{2,n} = \nu_1 D_{2,n-1} + (\theta - 1) A_2 (k_1 \nu_1 - 1) + \tfrac{1}{2}(\lambda_\eta^2 + (-\lambda_\epsilon + \psi_e D_{1,n-1})^2) \end{aligned} \tag{I.34}$$

with $D_{1,0} = D_{2,0} = 0$. Furthermore, Hasseltoft (2012) defines the one period holding return for a bond with a maturity of n periods as

$$h_{t+1,n} = q_{t+1,n-1} - q_{t,n} \tag{I.35}$$

I Stochastic Discount Factor in the Long-run Risk Framework

In this section, we find an expression for the discount factor in the state variables when we use the decomposition method of Ortu et al. (2013). As a start, recall that the Euler equation in the Long-Run risk frame work can be written as

$$E[\delta^\theta G_{t+1}^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1 \tag{I.36}$$

or equivalently,

$$E[e^{m_{i,t+1} + r_{i,t+1}}] = 1 \tag{I.37}$$

where

$$m_{i,t+1} = \theta log(\delta) - \frac{\theta}{\psi} g_{t+1} - (1-\theta) r_{a,t+1} \tag{I.38}$$

where $r_{i,t+1}$ is the return on any asset i. Ortu et al. (2013) show that the discount factor can be rewritten in to variables the economic equations of the Bansal and Yaron (2004) economy. To do this, Ortu et al. (2013) use the following relation

$$g_{t+1} = -\sum_{j=1}^{J} g_{t+2^j}^{(j)} + \pi_{t+2^J}^{(J)} \approx -\sum_{j=1}^{J} g_{t+2^j}^{(j)} + \pi_g \tag{I.39}$$

The above equation holds for all variables that are decomposed and can be observed by considering some equations of the main text. Equation (2) from the main text implies

$$\pi_t^{(j-1)} - \pi_t^{(j)} = \pi_t^{(j)} - \pi_{t-2^{j-1}}^{(j-1)} \tag{I.40}$$

together with equation (3) one has the relation $g_t^{(j)} = \pi_t^{(j)} - \pi_{t-2^{j-1}}^{(j-1)}$, which can be used to rewrite

$$g_t = \sum_{j=1}^{(J)} g_t^{(j)} + \pi_t^{(J)} \approx \sum_{j=1}^{(J)} g_t^{(j)} + \pi_g$$

to equation (I.40) through some tedious algebra. Using equation (I.38), equation (I.39), equation (19), equation (20) and the multiscale AR-model of g_t, Ortu et al. (2013) rewrite m_{t+1} in terms of the state parameters and innovations.

$$\begin{aligned}
& m_{t+1} = \theta log(\delta) - \tfrac{\theta}{\psi} g_{t+1} - (1-\theta) r_{a,t+1} \\
& = \theta log(\delta) - \tfrac{\theta}{\psi} g_{t+1} - (1-\theta)(\kappa_0 + \kappa_1 z_{a,t+1} - z_{a,t} + g_{t+1}) \\
& = \theta log(\delta) - \tfrac{\theta}{\psi}(-\textstyle\sum_{j=1}^{J} g_{t+2^j}^{(j)} + \pi_g) - (1-\theta)\Bigg(\kappa_0 + \kappa_1(\textstyle\sum_{j=1}^{J} - z_{t+2^j}^{(j)}) + \kappa_1 \pi_z - (\textstyle\sum_{j=1}^{J} z_t^{(j)}) - \pi_z - (\textstyle\sum_{j=1}^{J} g_{t+2^j}^{(j)}) \\
& = \theta log(\delta) + (\theta - 1 - \tfrac{\theta}{\psi})\pi_g + (\theta - 1)(\kappa_1 - 1)\pi_z + \\
& (1 - \theta + \tfrac{\theta}{\psi}) \textstyle\sum_{j=1}^{J} g_{t+2^j}^{(j)} + (\theta - 1)\Bigg(\kappa_0 - \kappa_1 \textstyle\sum_{j=1}^{J}(A_{0,j} + A_j g_{t+2^j}^{(j)}) - \textstyle\sum_{j=1}^{J}(A_{0,j} + A_j g_t^{(j)})\Bigg)
\end{aligned} \tag{I.41}$$

Then using the auto-regressive relation of consumption growth Ortu et al. (2013) find

$$\begin{aligned}
& m_{t+1} = \theta log(\delta) + (\theta - \tfrac{\theta}{\psi} - 1)\pi_g + (\theta - 1)((\kappa_1 - 1)\pi_z + \kappa_0) \\
& + (1 - \theta + \tfrac{\theta}{\psi}) \textstyle\sum_{j=1}^{J} \rho_j g_t^{(j)} + (\theta - 1)\Bigg(\kappa_1 \textstyle\sum_{j=1}^{J}(A_{0,j} + A_j \rho_j g_t^{(j)}) + \textstyle\sum_{j=1}^{J}(A_{0,j} + A_j g_t^{(j)})\Bigg) \\
& + (1 - \theta + \tfrac{\theta}{\psi}) \textstyle\sum_{j=1}^{J} \epsilon_{t+2^j}^{(j)} + (\theta - 1)\kappa_1(\textstyle\sum_{j=1}^{J} A_j \epsilon_{t+2^j}^{(j)})
\end{aligned} \tag{I.42}$$

From which we can conclude

$$\begin{aligned}
& m_{t+1} - E_t[m_{t+1}] = (1 - \theta + \tfrac{\theta}{\psi}) \textstyle\sum_{j=1}^{J} \epsilon_{t+2^j}^{(j)} + (\theta - 1)\kappa_1(\textstyle\sum_{j=1}^{J} A_j \epsilon_{t+2^j}^{(j)}) \\
& = -\gamma \mathbf{1} \cdot \boldsymbol{\epsilon}_{\mathbf{t+1}} - \lambda_\epsilon \cdot \boldsymbol{\epsilon}_{\mathbf{t+1}}
\end{aligned} \tag{I.43}$$

where $\gamma = (\frac{\theta}{\psi} - \theta + 1)$ and $\lambda_\epsilon = \kappa_1(1-\theta)A$ with $A = [A_1, ..., A_J]^T$. Ortu et al. (2013) show that by setting $r_{a,t+1} = r_{i,t+1}$ in the Euler equation and exploiting the fact that this equation must hold true for any set of parameters, the matrix A can be rewritten such that

$$\begin{aligned} &e_j((1-\tfrac{1}{\psi})M + A(\kappa_1 M - I_J)) = 0 \\ &A = (1-\tfrac{1}{\psi})(I_J - \kappa_1 M)^{-1} M\iota \end{aligned} \tag{I.44}$$

with e_j a vector that represents the canonical relation, $M = -diag(\rho_1, ..., \rho_J)$ and $\iota = [1, ..., 1]^T$.

J Determining the coefficients $D_{0,n}$, $D_{1,n}$ under persistence heterogeneity

Similar to the linear relation Ortu et al. (2013) conjecture on the financial ratios and consumption growth, we will conjecture a linear relation on the log price bond price and consumption growth. To do so, we will use the Bansal and Yaron (2004) equations without the process for volatility. As an effect, the term $D_{2,n}$ from Hasseltoft (2012) will be omitted. However, it is quite analogue to add the term later on. The relation we will conjecture on the log bond price and consumption growth is written more formally as

$$q_{t,n}^{(j)} = D_{0,n}^{(j)} + D_{1,n}^{(j)} g_t^{(j)} \tag{I.45}$$

To find the express $D_{0,n}$ and $D_{1,n}$ in the state variables, we will use the Euler equations (note that this is the same strategy as Bansal et al. (2016), Hasseltoft (2012) and Ortu et al. (2013) use):

$$E[e^{m_{t+1}+r_{i,t+1}}] = 1$$

The Euler equation holds for any asset $r_{i,t}$, such that we can use subsitute $r_{i,t}$ for the one-period log holding return $h_{t+1,n}$. This means that we can write the Euler equation for the log bond price as

$$E[e^{m_{t+1}+h_{t+1,n}}] = 1 \tag{I.46}$$

The term $E[e^{m_{t+1}+h_{t+1,n}}]$ can be rewritten to the state variables. First, we will show how to rewrite $h_{t+1,n}$ in the state variables. Notice that (I.42) already shows how to rewrite m_{t+1} in to the state variables.

$$
\begin{aligned}
h_{t+1,n} &= q_{t+1,n-1} - q_{t,n} \\
&= (-\textstyle\sum_{j=1}^J q_{n-1,t+2^j}^{(j)} + \pi_q) - (\sum_{j=1}^J q_{n,t}^{(j)} + \pi_q) \\
&= (-\textstyle\sum_{j=1}^J q_{n-1,t+2^j}^{(j)}) - \sum_{j=1}^J q_{n,t}^{(j)} \\
&= (-\textstyle\sum_{j=1}^J (D_{0,n-1}^{(j)} + D_{1,n-1}^{(j)} g_{t+2^j}^{(j)}) - \sum_{j=1}^J (D_{0,n}^{(j)} + D_{1,n}^{(j)} g_t^{(j)}) \\
&= -\textstyle\sum_{j=1}^J (D_{0,n-1}^{(j)} + D_{0,n}^{(j)}) + \sum_{j=1}^J D_{1,n-1}^{(j)} (e_j M \tilde{g}_t + e_j \epsilon_{t+1}) - \sum_{j=1}^J D_{1,n}^{(j)} g_t^{(j)} \\
&= -\iota^T (D_{0,n} + D_{0,n-1}) + D_{1,n-1}^T M \tilde{g}_t + D_{1,n-1}^T \epsilon_{t+1} - D_{1,n}^T \tilde{g}_t \\
&= -\iota^T (D_{0,n} + D_{0,n-1}) + (D_{1,n-1}^T M - D_{1,n}^T) \tilde{g}_t + D_{1,n-1}^T \epsilon_{t+1}
\end{aligned}
$$

with $M = -diag(\rho_1, ..., \rho_J)$, e_j a canonical vector that has also been used by Ortu et al. (2013), and $\tilde{g}_t = [g_t^{(1)}, ..., g_t^{(J)}]^T$. Such that we also use the relation $-g_{t+2^j} = e_j M \tilde{g}_t + e_j \epsilon_{t+1}$ which can be seen through the multiscale autoregressive relation. Similarly, we will rearrange m_{t+1}. Finally, the vector $D_{1.n-1} = [D_{1,n-1}^{(1)}, ..., D_{1,n-1}^{(J)}]^T$. The vectors $D_{0.n-1}$, $D_{0.n}$, and $D_{1.n}$ are defined analogue to $D_{1.n-1}$. To rewrite the Euler equation in a convenient way, we will also rewrite m_{t+1}. Recall that from the previous section we had

$$
\begin{aligned}
&m_{t+1} = \theta log(\delta) + (\theta - 1 - \tfrac{\theta}{\psi})\pi_g + (\theta - 1)(\kappa_1 - 1)\pi_z + \\
&(1 - \theta + \tfrac{\theta}{\psi}) \textstyle\sum_{j=1}^J g_{t+2^j}^{(j)} + (\theta - 1)\left(\kappa_0 - \kappa_1 \sum_{j=1}^J (A_{0,j} + A_j g_{t+2^j}^{(j)}) - \sum_{j=1}^J (A_{0,j} + A_j g_t^{(j)}) \right)
\end{aligned}
$$

which we can write using the solutions of A and the relation $-g_{t+2^j} = e_j M \tilde{g}_t + e_j \epsilon_{t+1}$. It follows that

$$
\begin{aligned}
&m_{t+1} = \theta log(\delta) + (1 - \theta + \tfrac{\theta}{\psi})\pi_g + (\theta - 1)(\kappa_1 - 1)\pi_z + \\
&(1 - \theta + \tfrac{\theta}{\psi}) \textstyle\sum_{j=1}^J g_{t+2^j}^{(j)} + (\theta - 1)\left(\kappa_0 - \kappa_1 \sum_{j=1}^J (A_{0,j} + A_j g_{t+2^j}^{(j)}) - \sum_{j=1}^J (A_{0,j} + A_j g_t^{(j)}) \right) \\
&= \theta log(\delta) + (1 - \theta + \tfrac{\theta}{\psi})\pi_g + (\theta - 1)(\kappa_1 - 1)\pi_z + \\
&(\theta - \tfrac{\theta}{\psi} - 1) \textstyle\sum_{j=1}^J -g_{t+2^j}^{(j)} + (\theta - 1)\left(\kappa_0 - \kappa_1 \sum_{j=1}^J (A_{0,j} + A_j g_{t+2^j}^{(j)}) - \sum_{j=1}^J (A_{0,j} + A_j g_t^{(j)}) \right) \\
&= \theta log(\delta) + (1 - \theta + \tfrac{\theta}{\psi})\pi_g + (\theta - 1)(\kappa_1 - 1)\pi_z + (\theta - \tfrac{\theta}{\psi} - 1) \textstyle\sum_{j=1}^J (e_j M \tilde{g}_t + e_j \epsilon_{t+1}) \\
&+(\theta - 1)\left(\kappa_0 - \kappa_1 \textstyle\sum_{j=1}^J (A_{0,j} + A_j(-e_j M \tilde{g}_t e_j \epsilon_{t+1})) - \sum_{j=1}^J (A_{0,j} + A_j g_t^{(j)}) \right) \\
&= \theta log(\delta) + (1 - \theta + \tfrac{\theta}{\psi})\pi_g + (\theta - 1)(\kappa_1 - 1)\pi_z + (\theta - \tfrac{\theta}{\psi} - 1)((M\iota)^T \tilde{g}_t + (\iota)^T \epsilon_{t+1}) \\
&+(\theta - 1)\kappa_0 + (1 - \theta) \textstyle\sum_{j=1}^J (1 + \kappa_1) A_{0,j} + (1 - \theta) A^T (I - \kappa_1 M) \tilde{g}_t + (\theta - 1)\kappa_1 A^T \epsilon_{t+1}
\end{aligned}
$$

Now we will focus on the terms with $\tilde{g}_t$. We will substitute A defined in (I.44) in to the equation above and rearrange. For $\tilde{g}_t$ we have

$$
(\theta - \frac{\theta}{\psi} - 1)((M\iota)^T \tilde{g}_t + (1 - \theta) A^T (I - \kappa_1 M) \tilde{g}_t = \frac{1}{\psi} (M\iota)^T \tilde{g}_t
$$

Then we will focus on the terms with ϵ_{t+1}. Rearranging gives

$$(\theta-\frac{\theta}{\psi}-1)(\iota)^T\epsilon_{t+1}+(\theta-1)\kappa_1A^T\epsilon_{t+1}=\left[(\theta-\frac{\theta}{\psi}-1)(\iota)^T+(\theta-1)\kappa_1A^T\right]\epsilon_{t+1}=\mathbf{c}_\epsilon\epsilon_{t+1}$$

where we take $\mathbf{c}_\epsilon=(\theta-\frac{\theta}{\psi}-1)(\iota)^T+(\theta-1)\kappa_1A^T$ for simplicity. Finally, for the same reasons of simplicity we will take

$$c_{m_{t+1}}=\theta log(\delta)+(1-\theta+\frac{\theta}{\psi})\pi_g+(\theta-1)(\kappa_1-1)\pi_z+(\theta-1)\kappa_0+(1-\theta)\sum_{j=1}^{J}(1+\kappa_1)A_{0,j}$$

This way, we can summarize m_{t+1} as

$$m_{t+1}=c_{m_{t+1}}+\frac{1}{\psi}(M\iota)^T\tilde{g}_t+\mathbf{c}_\epsilon\epsilon_{t+1}$$

We will use both the expressions m_{t+1} and $h_{t+1,n}$ to find solutions for $D_{0,n}$ and $D_{1,n}$. Note that the Euler Equation means the following

$$\begin{aligned}0&=h_{t+1,n}+m_{t+1}\\&=c_{m_{t+1}}+\tfrac{1}{\psi}(M\iota)^T\tilde{g}_t+\mathbf{c}_\epsilon\epsilon_{t+1}-\iota^T(D_{0,n}+D_{0,n-1})+(D_{1,n-1}^TM-D_{1,n}^T)\tilde{g}_t+D_{1,n-1}^T\epsilon_{t+1}\end{aligned}\tag{I.47}$$

Since the Euler equation must hold for any set of state variables, we have the following relation by rearranging the terms of $\tilde{g}_t$:

$$\begin{aligned}&D_{1,n-1}^TM-D_{1,n}^T+\tfrac{1}{\psi}(M\iota)^T=0\\\iff\ &D_{1,n}=M(D_{1,n-1}+\tfrac{1}{\psi}\iota)\end{aligned}\tag{I.48}$$

with $D_{1,0}=0$ (similar to Hasseltoft (2012)). By rearranging the constant terms, we find the recursive relation for $D_{0,n}$:

$$\iota^TD_{0,n}=c_{m_{t+1}}-\iota^TD_{0,n-1}\tag{I.49}$$

with again $D_{0,0}=0$.

K Risk premium for bonds

The risk premium for a bond with a maturity of n periods is given by the equation

$$E_t[h_{t+1,n}-r_{f,t}]+\frac{1}{2}Var_t[h_{t+1,n}]=-Cov_t[m_{t+1},h_{t+1,n}]\tag{I.50}$$

This means that in order to determine the risk premium for a bond with n periods we would like to write $h_{t+1,n}$ in the state variables. Recall that in the previous section we found that $h_{t+1,n}$ could be rewritten to

$$\begin{aligned} h_{t+1,n} &= q_{t+1,n-1} - q_{t,n} \\ &= -\iota^T (D_{0,n} + D_{0,n-1}) + (D_{1,n-1}^T M - D_{1,n}^T)\tilde{g}_t + D_{1,n-1}^T \epsilon_{t+1} \end{aligned} \tag{I.51}$$

This means that we can write

$$h_{t+1,n} - E[h_{t+1,n}] = D_{1,n-1}^T \epsilon_{t+1} \tag{I.52}$$

Also recall that we found in the previous section a expression for m_{t+1} as

$$m_{t+1} = c_{m_{t+1}} + \tfrac{1}{\psi}(M\iota)^T \tilde{g}_t + \mathbf{c}_\epsilon \epsilon_{t+1}$$

From which we can conclude

$$m_{t+1} - E[m_{t+1}] = \mathbf{c}_\epsilon \epsilon_{t+1}$$

Using the above expressions we can work out the risk premium as follows

$$\begin{aligned} -Cov_t[m_{t+1}, h_{t+1,n}] &= -E\Bigg[(m_{t+1} - E_t[m_{t+1}])(h_{t+1,n} - E_t[h_{t+1,n})^T\Bigg] \\ &= -E[(\mathbf{c}_\epsilon \epsilon_{t+1})(D_{1,n-1}^T \epsilon_{t+1})^T] \\ &= -\mathbf{c}_\epsilon Q D_{1,n-1} \\ &= -((\theta - \tfrac{\theta}{\psi} - 1)(\iota)^T + (\theta - 1)\kappa_1 A^T) Q D_{1,n-1} \\ &= \gamma \iota^T Q D_{1,n-1} + \lambda_\epsilon^T Q D_{1,n-1} \end{aligned} \tag{I.53}$$

where $E_t[\epsilon_{\mathbf{t+1}} \epsilon_{\mathbf{t+1}}{}^T] = Q$.

L Corrected risk premium for equity

In this section, we will present our method for determining the risk premium for equity. We will do this by rewriting $r_{m,t+1}$ and use the expression for the price kernel m_{t+1} we found earlier. We can write $r_{m,t+1}$ with the Campbell Equations as

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + gd_{t+1}$$
$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m}(\textstyle\sum_{j=1}^J (-z_{m,t+2^j}^{(j)}) + \pi_m) - \sum_{j=1}^J z_{m,t}^{(j)} - \pi_m + \sum_{j=1}^J (-gd_{m,t+2^j}^{(j)}) + \pi_{gd}$$
$$r_{m,t+1} = \kappa_{0,m} + (\kappa_{1,m} - 1)\pi_m + \pi_{gd} - \kappa_{1,m}(\textstyle\sum_{j=1}^J z_{m,t+2^j}^{(j)} - \sum_{j=1}^J z_{m,t}^{(j)} - \sum_{j=1}^J gd_{m,t+2^j}^{(j)}$$
$$r_{m,t+1} = \kappa_{0,m} + (\kappa_{1,m} - 1)\pi_m + \pi_{gd} -$$
$$\kappa_{1,m}(\textstyle\sum_{j=1}^J A_{0,j}^m + A_{1,j}^m g_{t+2^j}) - \sum_{j=1}^J (A_{0,j}^m + A_{1,j}^m g_t) - \sum_{j=1}^J (\phi_j g_t + \eta_{t+2^j})$$
$$r_{m,t+1} = \kappa_{0,m} + (\kappa_{1,m} - 1)\pi_m + \pi_{gd} - \textstyle\sum_{j=1}^J (\kappa_{1,m} + 1) A_{0,j}^m$$
$$-\kappa_{1,m}(\textstyle\sum_{j=1}^J A_{1,j}^m g_{t+2^j}) - \sum_{j=1}^J (A_{1,j}^m g_t) - \sum_{j=1}^J (\phi_j g_t + \eta_{t+2^j})$$

Now we will focus on the equation without the constant and rewrite it as

$$\begin{aligned}
&-\kappa_{1,m}(\sum_{j=1}^{J} A_{1,j}^{m} g_{t+2^j}) - \sum_{j=1}^{J}(A_{1,j}^{m} g_t) - \sum_{j=1}^{J}(\phi_j g_t + \eta_{t+2^j}) \\
&= -\kappa_{1,m}(\sum_{j=1}^{J} A_{1,j}^{m}(-e_j M\tilde{g}_t - ej\epsilon_{t+1}) - \sum_{j=1}^{J}(A_{1,j}^{m} g_t) - \sum_{j=1}^{J}(\phi_j g_t + \eta_{t+2^j}) \\
&= \kappa_{1,m}(A_1^M)^T M\tilde{g}_t + \kappa_{1,m}(A_1^M)^T \epsilon_{\mathbf{t+1}} - (A_1^M)^T \tilde{g}_t - \iota^T \Phi \tilde{g}_t - \iota^T \eta_{\mathbf{t+1}}
\end{aligned}$$

where we define Φ as a diagonal matrix, such that $\Phi = diag(\phi_1, ..., \phi_J)$. Re-plugging the above expression we find a expression for $r_{m,t+1}$

$$r_{m,t+1} = c_s + \kappa_{1,m}(A_1^M)^T M\tilde{g}_t + \kappa_{1,m}(A_1^M)^T \epsilon_{\mathbf{t+1}} - (A_1^M)^T \tilde{g}_t - \iota^T \Phi \tilde{g}_t - \iota^T \eta_{\mathbf{t+1}}$$

with $c_s = \kappa_{0,m} + (\kappa_{1,m} - 1)\pi_m + \pi_{gd} - \sum_{j=1}^{J}(\kappa_{1,m} + 1)A_{0,j}^m$. Using the above expression, we find

$$r_{m,t+1} - E[r_{m,t+1}] = \kappa_{1,m}(A_1^m)^T \epsilon_{\mathbf{t+1}} - \iota^T \eta_{\mathbf{t+1}}$$

This means that if we express the risk premium $-Cov(m_{t+1}, rm, t+1)$ as follows

$$\begin{aligned}
&-Cov(m_{t+1}, r_{m,t+1}) = -E[(m_{t+1} - E[m_{t+1}])(r_{m,t+1} - E[r_{m,t+1}])^T] \\
&= -E[(\mathbf{c}_\epsilon \epsilon_{t+1})(\kappa_{1,m}(A_1^m)^T \epsilon_{\mathbf{t+1}} - \iota^T bold\eta_{t+1})^T] \\
&= -\kappa_{1,m}\mathbf{c}_\epsilon Q A_1^m \\
&= \kappa_{1,m}(\gamma \mathbf{1}^T + \lambda_\epsilon) Q A_1^m
\end{aligned} \tag{I.54}$$

M Expressing A_1^m in the state variables

In this section, we will show how to find an expression for A_0^m and A_1^m. First we will show what kind of mistake we think Ortu et al. (2013) made when they found an expression, and then we will use our expression for m_{t+1} and $r_{m,t+1}$ to find a corrected expression. Ortu et al. (2013) use the following expression in their techniqual appendix to express A_1^m

$$\begin{aligned}
&E_t\Bigg[exp\Bigg(\theta log(\delta) + \theta(1 - \tfrac{1}{\psi})\Bigg(\sum_{j=1}^{J}(-g_{t+2^j}^{(j)}) + \pi_g\Bigg) + \\
&(\theta - 1)\Bigg(\kappa_0 + \kappa_1\Bigg(\sum_{j=1}^{J}(-z_{a,t+2^j}^{(j)} + \pi_z\Bigg) - \Bigg(\sum_{j=1}^{J} z_{a,t}^{(j)} + \pi_z\Bigg)\Bigg) - \Bigg(\sum_{j=1}^{J}(-g_{t+2^j}^{(j)}) + \pi_g\Bigg) + \\
&\kappa_{0.m} + \kappa_{1,m}\Bigg(\sum_{j=1}^{J}(-z_{m,t+2^j}^{(j)} + \pi_m\Bigg) - \Bigg(\sum_{j=1}^{J} z_{m,t}^{(j)} + \pi_z\Bigg) + \Bigg(\sum_{j=1}^{J}(-gd_{t+2^j}^{(j)} + \pi_{gd}\Bigg)\Bigg] = 1
\end{aligned} \tag{I.55}$$

In this equation they take

$$r_{m,t+1} = \kappa_{0.m} + \kappa_{1,m}\left(\sum_{j=1}^{J}(-z_{m,t+2^j}^{(j)} + \pi_m\right) - \left(\sum_{j=1}^{J} z_{m,t}^{(j)} + \pi_z\right) + \left(\sum_{j=1}^{J}(-gd_{t+2^j}^{(j)} + \pi_{gd}\right)$$

such that we can conclude that it must follow that

$$
\begin{aligned}
&m_{t+1} = \theta log(\delta) + \theta(1 - \tfrac{1}{\psi})\left(\textstyle\sum_{j=1}^{J}(-g_{t+2^j}^{(j)}) + \pi_g \right) \\
&(\theta - 1)\left(\kappa_0 + \kappa_1 \left(\textstyle\sum_{j=1}^{J}(-z_{a,t+2^j}^{(j)} + \pi_z \right) - \left(\textstyle\sum_{j=1}^{J} z_{a,t}^{(j)} + \pi_z \right)\right) - \left(\textstyle\sum_{j=1}^{J}(-g_{t+2^j}^{(j)}) + \pi_g \right)
\end{aligned}
$$

If we use our previous equations that $g_{t+1} = \sum_{j=1}^{J}(-g_{t+2^j}^{(j)}) + \pi_g$, we can further simplify m_{t+1} to its original equation.

$$
m_{t+1} = \theta log(\delta) + \theta(1 - \tfrac{1}{\psi})(g_{t+1}) + (\theta - 1)\left(\kappa_0 + \kappa_1 z_{a,t+1} - z_{a,t} + g_{t+1}\right)
$$

where we recognize the Campbell and Shiller (1988) equation for the consumption claim $r_{a,t+1} = \kappa_0 + \kappa_1 z_{a,t+1} - z_{a,t} + g_{t+1}$. With this we can finally rewrite the stochastic discount factor to

$$
m_{t+1} = \theta log(\delta) + \theta(1 - \tfrac{1}{\psi})(g_{t+1}) + (\theta - 1) r_{a,t+1}
$$

Now it is easy to see that this is not the correct stochastic discount factor that we have for an Epstein and Zin (1989) preference, since the coefficient for $\theta(1 - \frac{1}{\psi}) \neq \frac{\theta}{\psi}$. This mistake is probably made through the fact that this expression arises in the Euler equation in the case $r_{a,t+1} = r_{i,t+1}$.

To find the expression for A_0^m and A_1^m we will use the expression for the stochastic discount factor and our expression for the return on equity. By exploiting the Euler equations we know that the following relation must hold

$$
\begin{aligned}
0 &= r_{m,t+1} + m_{t+1} \\
&= c_{m_{t+1}} + \tfrac{1}{\psi}(M\iota)^T \tilde{g}_t + \mathbf{c}_\epsilon \epsilon_{t+1} + c_s + \kappa_{1,m}(A_1^M)^T M \tilde{g}_t + \kappa_{1,m}(A_1^M)^T \epsilon_{\mathbf{t+1}} - (A_{M})^{T}\tilde{g}_t - \iota^T \Phi \tilde{g}_t - \iota^T \eta_{\mathbf{t+1}}
\end{aligned}
\tag{I.56}
$$

Collecting the terms of $\tilde{g}_t$ and using the Euler equation must hold for all state variables, we find

$$
\begin{aligned}
&(\tfrac{1}{\psi}(M\iota)^T + \kappa_{1,m}(A_1^M)^T M - (A_1^M)^T - \iota^T \Phi)\tilde{g}_t = 0 \\
\iff\ & A_1^m = (I - \kappa_{1,m} M)^{-1}(\Phi\iota - \tfrac{1}{\psi} M\iota)
\end{aligned}
\tag{I.57}
$$

N Risk premia of stocks under different parametric choices

In this section we show the risk premia of equity given by equation (22) and equation (23) for different parametric choices of IES and risk aversion.

Table 18: Risk premia for equity when $\gamma = 7.5$ and $\psi = 5$

			Ortu et al.		Corrected	
Level	$Q_(j,j)$ $(x10^{-}5)$	Risk Exposure $(x10^{-}6)$	Risk Price	Risk Premium	Risk Price	Risk Premium
j=1	0.89	73.98	78.65	0.29	63.65	0.23
j=2	**0.48**	**123.58**	**475.19**	**5.81**	**467.69**	**5.71**
j=3	**0.42**	**48.91**	**169.45**	**1.64**	**165.70**	**1.60**
j=4	0.31	29.28	515.96	5.98	517.84	6.00
j=5	0.17	10.84	71.81	0.62	70.87	0.61
j=6	**0.09**	**3.34**	**109.97**	**0.58**	**109.51**	**0.58**
j=7	0.02	14.32	197.81	8.96	198.04	8.97

Table (18) shows the estimation output when when $\gamma = 7.5$ and $\psi = 5$. In the second and third column the variance of the innovation shocks (x10^{-5}) and the exposure of return (x10^{-6}) to those shocks are shown; in the columns thereafter the price and the risk premium (in %) for equity for both methods are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

Table 19: Risk premia for equity when $\gamma = 5$ and $\psi = 2.5$

			Ortu et al.		Corrected	
Level	$Q_(j,j)$ $(x10^{-}5)$	Risk Exposure $(x10^{-}6)$	Risk Price	Risk Premium	Risk Price	Risk Premium
j=1	0.89	73.98	66.08	0.24	56.08	0.21
j=2	**0.48**	**123.58**	**399.24**	**4.88**	**394.24**	**4.82**
j=3	**0.42**	**48.91**	**142.37**	**1.38**	**139.87**	**1.35**
j=4	0.31	29.28	433.50	5.02	434.75	5.03
j=5	0.17	10.84	60.33	0.52	59.71	0.51
j=6	**0.09**	**3.34**	**92.40**	**0.49**	**92.09**	**0.49**
j=7	0.02	14.32	166.20	7.53	166.35	7.53

Table (19) shows the estimation output when when $\gamma = 5$ and $\psi = 2.5$. In the second and third column the variance of the innovation shocks (x10^{-5}) and the exposure of return (x10^{-6}) to those shocks are shown; in the columns thereafter the price and the risk premium (in %) for equity for both methods are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

Table 20: Risk premia for equity when $\gamma = 7.5$ and $\psi = 2.5$

			Ortu et al.		Corrected	
Level	$Q_{(j,j)}$ ($x10^-5$)	Risk Exposure ($x10^-6$)	Risk Price	Risk Premium	Risk Price	Risk Premium
j=1	0.89	73.98	101.99	0.37	86.99	0.32
j=2	**0.48**	**123.58**	**616.23**	**7.53**	**608.73**	**7.44**
j=3	**0.42**	**48.91**	**219.74**	**2.13**	**215.99**	**2.09**
j=4	0.31	29.28	669.10	7.75	670.98	7.77
j=5	0.17	10.84	93.12	0.80	92.19	0.79
j=6	**0.09**	**3.34**	**142.61**	**0.75**	**142.15**	**0.75**
j=7	0.02	14.32	256.52	11.62	256.76	11.63

Table (20) shows the estimation output when when $\gamma = 7.5$ and $\psi = 2.5$. In the second and third column the variance of the innovation shocks (x10^{-5}) and the exposure of return (x10^{-6}) to those shocks are shown; in the columns thereafter the price and the risk premium (in %) for equity for both methods are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

O Regression of the log discounted bond price on consumption growth

To determine the estimates of $D_{1,n}^{(j)}$ for section (4.4) we estimated the coefficients of

$$q_{t,n}^{(j)} = D_{0,n}^{(j)} + D_{1,n}^{(j)} g_t^{(j)} + v_t^n \tag{I.58}$$

This regression is performed with OLS on the full sample of components that we could obtain. The effective sample we have used therefore consists of the 1952Q3 - 2011Q4. For significance we used the HAC standard errors introduced by Hansen and Hodrick (1980). Table (21) shows the estimation output of $D_{1,n}^{(j)}$ for different levels of maturity.

level	Maturity=1 Estimate	[t-value]	Maturity=2 Estimate	[t-value]	Maturity=3 Estimate	[t-value]	Maturity=4 Estimate	[t-value]	Maturity=5 Estimate	[t-value]
j=1	-15.74	[-1.36]	-20.48	[-1.07]	-23.65	[-0.98]	-33.00	[-1.18]	-32.37	[-1.05]
j=2	-22.49	[-1.34]	-41.88	[-1.44]	-46.39	[-1.23]	-54.28	[-1.21]	-58.94	[-1.19]
j=3	-7.38	[-0.21]	-23.76	[-0.39]	-22.33	[-0.29]	-23.67	[-0.26]	-26.12	[-0.27]
j=4	31.20	[0.40]	39.85	[0.31]	56.43	[0.36]	82.03	[0.46]	81.63	[0.44]
j=5	39.81	[0.30]	42.77	[0.18]	32.86	[0.11]	28.71	[0.08]	26.19	[0.07]
j=6	-202.67	[-0.71]	-371.40	[-0.69]	-543.75	[-0.73]	-693.85	[-0.76]	-772.53	[-0.73]
j=7	-297.72	[-0.53]	-523.29	[-0.50]	-749.68	[-0.52]	-993.39	[-0.56]	-1127.76	[-0.55]

Table 21: Estimates of $D_{1,n}^{(j)}$ and its t-value between brackets when regressing the components of log discounted bond price on the corresponding components consumption growth

P Risk premia of bonds under different parametric choices

In this section we show the risk premia of bonds with maturities of 1 up to 5 years given by equation (28) for different parametric choices of IES and risk aversion.

Table 22: Risk premia for bonds when $\gamma = 5$ and $\psi = 2.5$

Level	$Q_{(}j, j)$	Price	Maturity=1 Exposure	Premium	Maturity=2 Exposure	Premium	Maturity=3 Exposure	Premium	Maturity=4 Exposure	Premium	Maturity=5 Exposure	Premium
j=1	**0.89**	**39.69**	**279.97**	**0.56**	**364.35**	**0.72**	**420.72**	**0.83**	**587.05**	**1.30**	**575.82**	**1.14**
j=2	0.48	295.24	108.81	3.21	202.62	5.98	224.41	6.63	262.58	9.19	285.13	8.42
j=3	0.42	104.56	15.36	0.32	49.47	1.03	46.49	0.97	49.29	1.22	54.39	1.14
j=4	**0.31**	**327.25**	**24.17**	**3.16**	**30.87**	**4.04**	**43.72**	**5.72**	**63.55**	**9.91**	**63.24**	**8.28**
j=5	0.17	44.75	8.42	-0.30	9.05	-0.32	6.95	-0.25	6.08	-0.26	5.54	-0.20
j=6	0.09	69.17	11.30	1.25	20.71	2.29	30.33	3.36	38.70	5.09	43.09	4.77
j=7	0.02	125.14	1.67	-0.67	2.94	-1.18	4.21	-1.69	5.58	-2.66	6.34	-2.54

Table (22) shows the estimation output when $\gamma = 5$ and $\psi = 2.5$. In the second and third column the variance of the innovation shocks (scaled by 10^{-5}) and the price of the exposure to those shocks are shown; in the columns thereafter the exposure (scaled by 10^{-6}) and the risk premium (in %) for bonds with different maturities are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

Table 23: Risk premia for bonds when $\gamma = 7.5$ and $\psi = 5$

			Maturity=1		Maturity=2		Maturity=3		Maturity=4		Maturity=5	
Level	$Q_{(}j,j)$	Price	Exposure	Premium	Exposure	Premium	Exposure	Premium	Exposure	Premium	Exposure	Premium
j=1	**0.89**	**44.15**	**279.97**	**0.62**	**364.35**	**0.80**	**420.72**	**0.93**	**587.05**	**1.30**	**575.82**	**1.27**
j=2	0.48	349.85	108.81	3.81	202.62	7.09	224.41	7.85	262.58	9.19	285.13	9.98
j=3	0.42	123.68	15.36	0.38	49.47	1.22	46.49	1.15	49.29	1.22	54.39	1.35
j=4	**0.31**	**389.89**	**24.17**	**3.77**	**30.87**	**4.82**	**43.72**	**6.82**	**63.55**	**9.91**	**63.24**	**9.86**
j=5	0.17	53.07	8.42	-0.36	9.05	-0.38	6.95	-0.30	6.08	-0.26	5.54	-0.24
j=6	0.09	82.23	11.30	1.49	20.71	2.73	30.33	3.99	38.70	5.09	43.09	5.67
j=7	0.02	148.99	1.67	-0.80	2.94	-1.40	4.21	-2.01	5.58	-2.66	6.34	-3.02

Table (23) shows the estimation output when $\gamma = 7.5$ and $\psi = 5$. In the second and third column the variance of the innovation shocks (scaled by 10^{-5}) and the price of the exposure to those shocks are shown; in the columns thereafter the exposure (scaled by 10^{-6}) and the risk premium (in %) for bonds with different maturities are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

Table 24: Risk premia for bonds when $\gamma = 7.5$ and $\psi = 2.5$

			Maturity=1		Maturity=2		Maturity=3		Maturity=4		Maturity=5	
Level	$Q_{(}j,j)$	Price	Exposure	Premium	Exposure	Premium	Exposure	Premium	Exposure	Premium	Exposure	Premium
j=1	**0.89**	**61.70**	**279.97**	**0.86**	**364.35**	**1.12**	**420.72**	**1.30**	**587.05**	**1.81**	**575.82**	**1.78**
j=2	0.48	455.91	108.81	4.96	202.62	9.24	224.41	10.23	262.58	11.97	285.13	13.00
j=3	0.42	161.50	15.36	0.50	49.47	1.60	46.49	1.50	49.29	1.59	54.39	1.76
j=4	**0.31**	**505.05**	**24.17**	**4.88**	**30.87**	**6.24**	**43.72**	**8.83**	**63.55**	**12.84**	**63.24**	**12.78**
j=5	0.17	69.09	8.42	-0.47	9.05	-0.50	6.95	-0.38	6.08	-0.34	5.54	-0.31
j=6	0.09	106.78	11.30	1.93	20.71	3.54	30.33	5.18	38.70	6.61	43.09	7.36
j=7	0.02	193.14	1.67	-1.03	2.94	-1.82	4.21	-2.60	5.58	-3.45	6.34	-3.92

Table (24) shows the estimation output when when $\gamma = 7.5$ and $\psi = 2.5$. In the second and third column the variance of the innovation shocks (scaled by 10^{-5}) and the price of the exposure to those shocks are shown; in the columns thereafter the exposure (scaled by 10^{-6}) and the risk premium (in %) for bonds with different maturities are shown. Bold values shows the predictable components of consumption growth. The estimated coefficients are annualized.

Q Code

We provide the code we used in a zip file. The structure of the zip file is as follows: the zip file contains one folder for our replication of the results of Ortu et al. (2013) and one folder for our extension. The replication folder is organized as a collection of folders where each folder contains the code to replicate a certain section of the original paper of Ortu et al. (2013). In addition to this folder, we also add the codes we obtained from the website of Andreas Tamoni. In the extension folder, we have three folders which each represents a part of our extension. One folder contains the codes to investigate the predictive ability of the log discounted bond price to predict the log consumption growth. The other two folders contain codes to compute the risk premia under consumption growth for respectively equity and the Fama-Bliss bond portfolio.