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Predicting The Returns Of The Worst Financial Day Of The Corona Crisis Using The Measure Of Systematic Risk During The 2008 Financial Crisis

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Abstract

This paper analyzes the prediction of the returns of the worst financial day of the corona crisis so far. This paper concludes that the proposed estimator by Van Oordt and Zhou (2019) for the measure of systematic risk under extreme market conditions outperforms the conditional regression approach in predicting the returns of the worst financial day of the corona crisis so far, using data from the 2008 financial crisis. This superior performance is due to the higher persistence of the measure of systematic risk under extreme market conditions between the 2008 financial crisis and the corona crisis, when one uses the proposed estimator. Finally, this paper shows that, using hedging portfolios formed during the 2008 financial crisis, one can achieve higher economic gain during the worst financial days of the corona crisis when one uses estimates of the measure of systematic risk under extreme market conditions instead of the estimated regular betas.

1 Introduction

On the 17th of November 2019 is the world introduced to the COVID-19 (corona) virus. With its origin in the Chinese city of Wuhan, the spread of the virus in China and the rest of the world is unprecedented. Lucchese and Pianta (2020) argue that the corona crisis will have a huge impact on the economy, saying: "The slow-down of the economy could simply make the mass of private debt unpayable", which implies huge consequences for every branch in the economy. The virus has the world in a stranglehold and the IMF even argues that the corona crisis will lead to the biggest financial crisis ever on record since the 1930s depression (IMF (2020)). Kelly and Jiang (2014) analyze, in other economic crises of the past, the event that the value of a portfolio of stocks moves more than 3 standard deviations from the current price, the tail risk. Analyzing the tail risk of U.S. stocks between 1963 and 2010, Kelly and Jiang (2014) show that high risk for extreme conditions in the 1980s recession also occurs at around the same magnitude during the 2008 financial crisis, for the pooled stocks they use in their research. To analyze whether the returns during the corona crisis of stocks listed in the United States can be predicted using past crises, in line with the similarities of different financial crises explained by Kelly and Jiang (2014), the research question this paper will investigate is: "In what extend are returns of U.S. stocks predictable during the corona crisis using their systematic risk measure during the 2008 financial crisis?"

In this paper, I will test the hypothesis that the measure of systematic risk during the 2008 financial crisis for a group of U.S. listed companies is a good predictor for returns on the worst financial day during the corona crisis. First, I use the proposed estimator for the measure of systematic risk under extreme market conditions as described by Van Oordt and Zhou (2019) to estimate the measure of systematic risk under extreme conditions during 2007-2012, which covers the 2008 financial crisis. I make use of the conditional regression model to estimate the same measure of systematic risk during 2007-2012, to make a comparison to a 'benchmark' prediction method. Then, I predict the returns of the worst financial day of the corona crisis so far using the estimated measures of systematic risk under extreme market conditions, using both estimation approaches. Second, in order to explain the difference in prediction performance, I analyze the persistence of the measure of systematic risk between the 2008 financial crisis and the corona crisis. I use three statistical tests to test the persistence between the estimated measures of systematic risk under extreme market conditions between the 2008 financial crisis and the corona crisis. Lastly, I show what economic gain can be achieved using hedging portfolios. I follow Van Oordt and Zhou (2016) to select companies based on their measure of systematic risk during the 2008 financial crisis and invest in a long position for companies with a low measure of systematic risk and invest in a short position for companies with a high measure of systematic risk.

Van Oordt and Zhou (2019) analyze systematic risk under extreme market conditions. The proposed estimator for the measure of systematic risk under extreme market conditions outperforms the conditional linear regression approach, demonstrated by a simulation study. Van Oordt and Zhou (2019) show an empirical application, for the more accurate estimator for the measure of systematic risk under extreme market conditions, which concludes that the proposed estimator can outperform the conditional regression approach for predicting the returns of different industry portfolios on the worst financial day within multiple subperiods between 1931 and 2010. This research makes use of the proposed estimator to estimate the measure of systematic risk under extreme market conditions of U.S. stocks during the 2008 financial crisis and the corona crisis. Kelly and Jiang (2014) perform an analyses on time-varying tail risk of individual and aggregated U.S. stock returns. One of their conclusions, among others, is that tail risk has high predictive power for aggregated market returns. In contrast, this research compares two different financial crises, rather than a continuous time period. Fahlenbrach, Prilmeier, and Stulz (2012) concludes that the returns of banks in the 2008 financial crisis can be predicted using their returns during the 1998 crisis. Similarly, this research compares the 2008 financial crisis and the corona crisis to analyze in what extend the returns of the worst financial day of the corona crisis so far can be predicted using the estimated measure of systematic risk under extreme market conditions during the 2008 financial crisis. Van Oordt and Zhou (2016) construct a zero investment portfolio by sorting on the spread between the measure of systematic risk under extreme market conditions and the market beta. Similarly, this paper selects companies for each zero investment portfolio (hedging portfolio) by sorting the companies with respect to their measure of systematic risk under extreme market conditions during the 2008 financial crisis, to form a hedging portfolio during 2015-2020, which covers the corona crisis.

In this paper, I first show that the proposed estimator by Van Oordt and Zhou (2019) leads to a lower root mean squared error (RMSE) for predicting the worst day of the corona crisis so far than the conditional regression approach, by using the measure of systematic risk under extreme market conditions during the 2008 financial crisis for 48 SP500 companies. Second, to investigate the superior prediction performance, I analyze the persistence of estimated measures of systematic risk under extreme market conditions between the 2008 financial crisis and the corona crisis. When one uses the proposed estimator by Van Oordt and Zhou (2019), one can conclude that the measure of systematic risk under extreme market conditions is persistent, while using the conditional regression approach one can not conclude this. Lastly, I practically show what economic gain can be achieved using hedging portfolios formed during the 2008 financial crisis. One can form a hedging portfolio with higher returns during the corona crisis when one uses the proposed estimator for systematic risk under extreme market conditions by Van Oordt and Zhou (2019) to estimate tail betas, instead of using estimated regular betas.

2 Methodology

2.1 Definition tail beta and estimator

For a linear model between two heavy-tailed variables, where the explanatory variable has an extreme high or low value, Van Oordt and Zhou (2019) propose an estimator for the systematic risk measure. The model is defined as $Y = \beta^T X + \epsilon$, for $X < Q_X(\bar{p})$, where Y and X are the returns of a stock portfolio and market portfolio, respectively. Here, β^T is the tail beta and regarded as a measure of systematic risk under extreme market conditions. Furthermore, $\bar{p}$ is a very small probability and ϵ is the error term, which is assumed to be independent of the conditional $X < Q_X(\bar{p})$. Here, $Q_x(\bar{p})$ is the quantile function of X defined as: $Q_x(\bar{p}) = sup\ \{c : Pr(X) \leq \bar{p}\}$.

Assume that we observe the identically and independent distributed observations of (X, Y) as the following sequence: (X_1, Y_1),...,(X_n, Y_n). Rank the observations $\{X_1, ..., X_n\}$ as $\{X_{n,1} \leq X_{n,2} \leq ... \leq X_{n,n}\}$, then, $X_{n,i}$ is an order statistic. To estimate the tail beta, Van Oordt and Zhou (2019) propose an estimator, using the tail dependence measure and the quantiles of X and Y. Here, k denotes the amount of observations included in the tail. Van Oordt and Zhou (2019) show that the proposed estimator is consistent and asymptotically normally distributed under certain conditions. The definition of the proposed estimator is as follows:

$$\hat{\beta}^T := \hat{\tau}(k/n)^{1/\hat{\alpha}_x} \frac{\hat{Q}_y(k/n)}{\hat{Q}_x(k/n)} \tag{1}$$

Here, $\hat{\alpha}_x$ is the Hill estimator for the tail index of X: $\frac{1}{\hat{\alpha}_x} := \frac{1}{k_1} \sum_{i=1}^{k_1} log\left(\frac{X_{n,i}}{X_{n,k_1+1}}\right)$, where k_1 is an intermediate sequence with $k_1 := k_1(n) \to \infty$ and $k_1/n \to 0$ as $n \to \infty$. To estimate the τ-measure, one may use the following estimator, where $Y_{n,k+1}$ is the $(k+1)$-th lowest order statistic of Y_t:

$$\hat{\tau}(k/n) := \frac{1}{k} \sum_{t=1}^{n} \mathbf{1}_{\{Y_t < Y_{n,k+1}, X_t < X_{n,k+1}\}} \tag{2}$$

The quantiles of X and Y at the probability level k/n are: $\hat{Q}_x(k/n)$ and $\hat{Q}_y(k/n)$, estimated by their $(k+1)$-th lowest order statistics: $X_{n,k+1}$ and $Y_{n,k+1}$ respectively.

2.2 Simulation procedure

To analyze the performance of the proposed estimator, I perform a simulation study as follows. I compare the performance of the proposed estimator to the performance of a conditional regression using tail observations. The simulations evaluate the performance of both estimators if the data generating process is in line with the linear tail model in equation (1). For the data generating process, I apply both global linear models and segmented linear models to generate the values of X and ϵ, leading to the aggregated values of Y. A global linear model is defined as: $Y = \beta^T X + \epsilon$. A segmented linear model is defined as: $Y = \beta^T X + \epsilon,\ X < BP(Breakpoint)$ and as $Y = \beta X + \epsilon,\ X \geq BP(Breakpoint)$, where the breakpoint is yet to be defined.

Van Oordt and Zhou (2019) propose three global linear models and two segmented linear models. For the global linear models, the relation is unaffected by the observation, implied by using $\beta = \beta^T = 0.5, 1, 1.5$. For the segmented linear models, if the observed value of X is higher than the third percentile of X, I use $\beta = 1$ to generate the value of Y. In other cases, I use a linear model with $\beta^T = 0.5$ and $\beta^T = 1.5$ to generate the value of Y.

Van Oordt and Zhou (2019) consider different data generating processes for X and ϵ. First, they consider random draws of the Student's t-distribution with three, four and five degrees to generate values of X and ϵ. The Student's t-distribution is heavy-tailed, where the degrees of freedom are equal to the tail index. In addition, they consider to generate models for temporal independence of X and ϵ using a GARCH(1,1) model. The GARCH(1,1) model is defined as: $Z_t = \sigma_{Z,t}\eta_t$, where $\sigma^2_{Z,t} = \psi_0 + \psi_1 Z^2_{t-1} + \psi_2\sigma^2_{t-1}$, for $Z = X, \epsilon$. Additionally, they use two different parameter choices for (ψ_0, ψ_1, ψ_2): (ψ_0, ψ_1, ψ_2) = (0.5, 0.11, 0.88), with normally distributed innovations η_t and (ψ_0, ψ_1, ψ_2) = (0.5, 0.08, 0.91), with innovations η_t distributed around a Student's t-distribution with eight degrees of freedom. These two different parameter choices imply that X and ϵ are heavy-tailed with tail indices for the first set of parameter values 3.68 and 3.82 for the second set of parameter values. Due to the similarity across all simulations, I only consider random draws of the Student's t-distribution with four degrees of freedom to generate values of X and ϵ.

For the data generating processes, I generate m = 10,000 samples. Each simulated sample consists of 1250 random observations of (X_t, Y_t). β^T is estimated from both the conditional regression approach and the proposed estimator. To compare the performance of the two methods, I compare the true value of β^T to the estimated value $\hat{\beta}^T$. Then, I calculate the mean squared error (MSE), the estimation bias and the estimation variance for the two estimation methods and use them to analyze the estimation performance. I calculate the MSE as $m^{-1}\sum_{i=1}^m(\beta^T - \hat{\beta}^T)^2$, where i = 1,...,m indicates which sample is used in the simulation. Moreover, I calculate the squared bias as $(\beta^T - \bar{\hat{\beta}}{}^T_i)^2$ and the variance as $m^{-1}\sum_{i=1}^m(\bar{\beta}^T - \hat{\beta}^T_i)^2$, where $\bar{\beta}^T = m^{-1}\sum_{i=1}^m \hat{\beta}^T_i$. The amount of observations in the tail for each simulation, denoted by k, range from 5 to 100.

2.3 Empirical application

Following the empirical application by Van Oordt and Zhou (2019), I compare the performance of the proposed estimator and the conditional regression approach. Using the data from the website of Kenneth French (explained in more detail in the data section), I divide the returns series from 1931 until 2010 into 16 five-year subperiods. For both prediction methods, I estimate the losses per industry portfolio on the worst market day within each subperiod. Using the returns of portfolio j as the dependent variable and the excess market returns as the independent variable in the linear tail model, I estimate the coefficient β^T_j. I estimate β^T_j using k = 25 observations out of approximately 1315 observations in each subperiod, where I exclude the worst financial day in each subperiod, corresponding to the day with the highest market loss. I denote the estimates of β^T_j as $\hat{\beta}^T_{OLS,j}$ and $\hat{\beta}^T_{EVT,j}$ for the conditional regression approach and the proposed estimator, respectively. Following the estimates of β^T_j, I predict the losses on each portfolio j on the day where the market realized the largest loss. I denote $L_m = -\min\{R^e_{m,1}, ..., R^e_{m,t}\}$ as the largest loss on the market portfolio. Additionally, I denote $L_j = -R^e_{j,t^*}$ as the actual loss on a specific industry portfolio on the day t^* where the market portfolio suffered the worst loss. Then, following the two prediction approaches, the projected losses for each portfolio are $\hat{L}_{EVT,j} = L_m\hat{\beta}_{EVT,j^T}$ and $\hat{L}_{OLS,j} = L_m\hat{\beta}^T_{OLS,j}$.

To compare both approaches, I calculate the root mean-squared error (RMSE) as $\sqrt{N^{-1}\sum_j e^2_{EVT,j}}$ and

$\sqrt{N^{-1}\sum_j e^2_{OLS,j}}$, where $e_{EVT,j} = L_j - \hat{L}_{EVT,j}$ and $e_{OLS,j} = L_j - \hat{L}_{OLS,j}$. Consequentally, the method with the lowest RMSE performs best in predicting the portfolio losses. Finally, to conclude whether the difference in RMSE is significant, I perform a Diebold and Mariano (1995) type of test. Using the paired differences $d_j = e^2_{EVT,j} - e^2_{OLS,j}$ and the relations: $\bar{d} = N^{-1}\sum_j d_j$ and $\hat{V}(\bar{d}) = \sum_j (d_j - \bar{d})^2/N$, I calculate the test statistic as follows:

$$t - stat = \frac{\bar{d}}{\sqrt{\hat{V}(\bar{d})/(N-1)}} \tag{3}$$

Following the results of the test statistic for each period and a significance level of 5%, one can conclude whether the difference in RMSE is significant or not.

2.4 Empirical extension

2.4.1 Predicting the returns of the worst financial day during the corona crisis

In this research, I analyze the prediction performance of using estimated tail betas during the 2008 financial crisis for predicting the returns of the worst financial day of the corona crisis so far. Using the same 48 industry portfolios as Van Oordt and Zhou (2019), I estimate the tail betas for each industry portfolio during nine subperiods of five years between 1971 and 2020, including the 2008 financial crisis (2007-2012) and the corona crisis (2015-2020). Then, I calculate the RMSE for predicting the returns of the worst financial day of the corona crisis so far, by using the estimated tail betas for each company during all nine subperiods, for both the conditional regression approach and the proposed estimator. I test the difference of the RMSE's of both estimation approaches within each subperiod with the same Diebold and Mariano (1995) type of test as described in the methodology of the empirical application section at a 5% significance level. I compare the RMSE's of the 2008 financial crisis subperiod (2007-2012) to all other subperiods for predicting the returns of the worst financial day of the corona crisis so far, to analyze the relative prediction performance of using estimated tail betas during the 2008 financial crisis (2007-2012). Moreover, I perform multiple Diebold and Mariano (1995) type of tests, as explained in the methodology section of the empirical application, at a 5% significance level to test whether the difference in RMSE's between the 2008 financial crisis and all other periods, when one uses the proposed estimator, are significant.

2.4.2 Statistical tests for the persistence of tail betas

After concluding to what extend the worst financial day of the corona crisis so far can be predicted using estimated tail betas during the 2008 financial crisis, I analyze the persistence of the estimated tail betas between the 2008 financial crisis and the corona crisis. To test the persistence of the estimated tail betas between the two periods, I perform three statistical tests.

To test whether the difference between the paired sets of tail betas between the two periods follow a symmetric distribution around zero, I perform a Wilcoxon signed-rank test (Wilcoxon, Katti, and Wilcox (1970)) at a 5% significance level. It implies, if the null hypothesis can not be rejected, that there is not a significant indication that the distribution of the difference of the two sets of tail betas is not symmetric around a zero mean, giving an argument for persistence of the two sets of tail betas.

To test whether the values of the predicted tail betas differ over time across companies, I regress the estimated tail betas corresponding to the corona crisis on the estimated tail beta of the 2008 financial crisis. $\beta_{2006-2012}$ is a vector containing all estimated tail betas during 2007-2012 and $\beta_{2015-2020}$ is a vector containing all estimated tail betas during 2015-2020. The linear regression model, which I estimate using OLS, is as follows: $\beta_{2016-2020} = \alpha\beta_{2008-2012} + \epsilon$. The null hypothesis is $\alpha = 1$, because $\alpha = 1$ implies that the linear relation between the two sets of tail betas differs only in terms of a random noise, meaning that there is no significant indication that the values of the two sets of tail betas differ. Then, I perform a Student's t-test at a 5% significance level to test the null hypothesis.

For the last test, I first form two vectors for both estimation approaches separately. For the proposed estimator, I form a vector containing the differences between the estimated tail betas during the 2008 financial crisis (2007-2012) and the corresponding estimated tail betas during the corona crisis (2015-2020). Similarly, I form a vector for the conditional regression approach, containing the differences between the estimated tail betas during the 2008 financial crisis (2007-2012) and the corona crisis (2015-2020). Then, I calculate the absolute mean of both vectors. I use a pair wised t-test at a 5% significance level, to test whether the difference in absolute means is significant. Since I can observe which estimation approach leads to a lower absolute mean of the vector containing the differences between the two sets of tail betas, I can conclude whether one of the two 'difference means' is lower than the other. The lower the differences between the two sets of tail betas, the more persistent the tail beta is. I will refer to this test as the 'difference mean test'.

2.4.3 Constructing hedge portfolios

We can profit using this information, if the estimated tail betas during the 2008 financial crisis can help to predict the returns of the worst financial day of the corona crisis so far. To show what economic gain is achievable using the proposed estimator, I use 48 individual companies, which are explained in more detail in the data section.

To show what economic gain is achievable using the proposed estimator, I construct multiple hedging portfolios. After estimating the tail betas during the 2008 financial crisis (2007-2012), I sort them from high to low as in Van Oordt and Zhou (2016). Then, I divide the companies (with corresponding tail beta) into two groups: one group contains the top 24 companies with the highest tail beta values and the other group contains the top 24 companies with the lowest tail beta values. Using the two groups, I create 24 different portfolios where the amount of companies in each group ranges from 1 to 24. The first portfolio consists of the company with the highest tail beta and the company with the lowest tail beta. The second portfolio consists of the two companies with the highest tail beta and the two companies with the lowest tail beta. The last portfolio consists of all companies included in both groups. Then, the investment position for each company is short for the companies in the 'high tail beta' group and long for the companies in the 'low tail beta' group. Equal weights are assigned to each company within the same investment position, depending on the amount of used companies in each investment position.

To compare the performance of the hedging portfolios, I also construct hedging portfolios based on the regular betas of each company, using the same hedging portfolio construction method as for the tail betas, by applying OLS on all observations during the 2008 financial crisis (2007-2012). I analyze the performance of all 24 portfolios using the regular betas and tail betas to 'hedge' out all risk during the period 2015-2020, meaning that the ideal hedging portfolio is the one with the highest return. Then, I compare the realized returns of all 24 portfolios using the regular betas and tail betas during the 20 worst and best financial days of 2015-2020. I perform eight Student's t-tests at a 5% significance level to test the null hypothesis that for each of the three selections of trading days during 2015-2020 and for the two different betas, the returns are equal to zero. Furthermore, I test whether the difference in returns using both betas is significant within each period.

3 Data

I follow Van Oordt and Zhou (2019) to use 48 industry-specific stock portfolios with their value-weighted returns and the general market index in the United States from 1931 until 2010. I divide the data into 16 five-year subperiods with 1315 observations on average per subperiod. The data source comes from the personal website of Kenneth French, containing the returns of companies listed on NYSE, AMEX and NASDAQ in the CRSP database. If any portfolio contains no returns in a certain subperiod, I exclude them from the analysis of the subperiod in question. After 1969 are all 48 portfolios available, whereas before, five portfolios have missing returns until July 1963 and the portfolio 'Healthcare' has missing returns between July 1963 and July 1969. Following Van Oordt and Zhou (2019), portfolios are excluded under the condition that $\hat{\alpha}_j \leq \frac{1}{2}\hat{\alpha}_m$. In most subperiods, I exclude no portfolio from the analysis.

For the empirical extension, I use the returns of 48 different companies between 04/01/2007 and 04/01/2012, in which the 2008 financial crisis occurred. To conclude whether the predicted tail betas during the 2008 financial crisis of each company performs well in predicting the returns of the worst financial day of the corona crisis so far, I use the returns of each company between 01/05/2015 and 30/04/2020. To be precise, the 48 companies are part of the SP100 (subset of the constituents of the SP500) and contain companies in all industries. In the appendix section is a list included for all used companies. The data source is Yahoo Finance for the companies and market index and the personal website of Kenneth French for the risk free rate (to calculate the excess market returns). So far, the worst financial day of the corona crisis for the SP500 is the 16th of March, with a realized excess market loss of 12.58%. From now on, I will denote the 'worst day of the corona crisis so far' as the 'corona crash'.

4 Results

In this section, I will first show the simulation results, followed by the results of the empirical application and finally the results of the empirical extension.

4.1 Simulation results

Following the simulation study as explained in the methodology section, Figures 1 and 2 show the simulation results. Figure 1 shows the results for the simulations of the global linear model. We conclude for small values of k, the number of used observations in estimation, that the mean squared error (MSE) of the proposed estimator lower is than the conditional regression model. For high values of k is the MSE of the conditional regression approach lower than the proposed estimator. Furthermore, the second and third column show the decomposition of the MSE into squared bias and variance. We conclude that the squared bias of the proposed estimator is higher than the conditional regression approach and the variance of the conditional regression approach is higher than the proposed estimator.

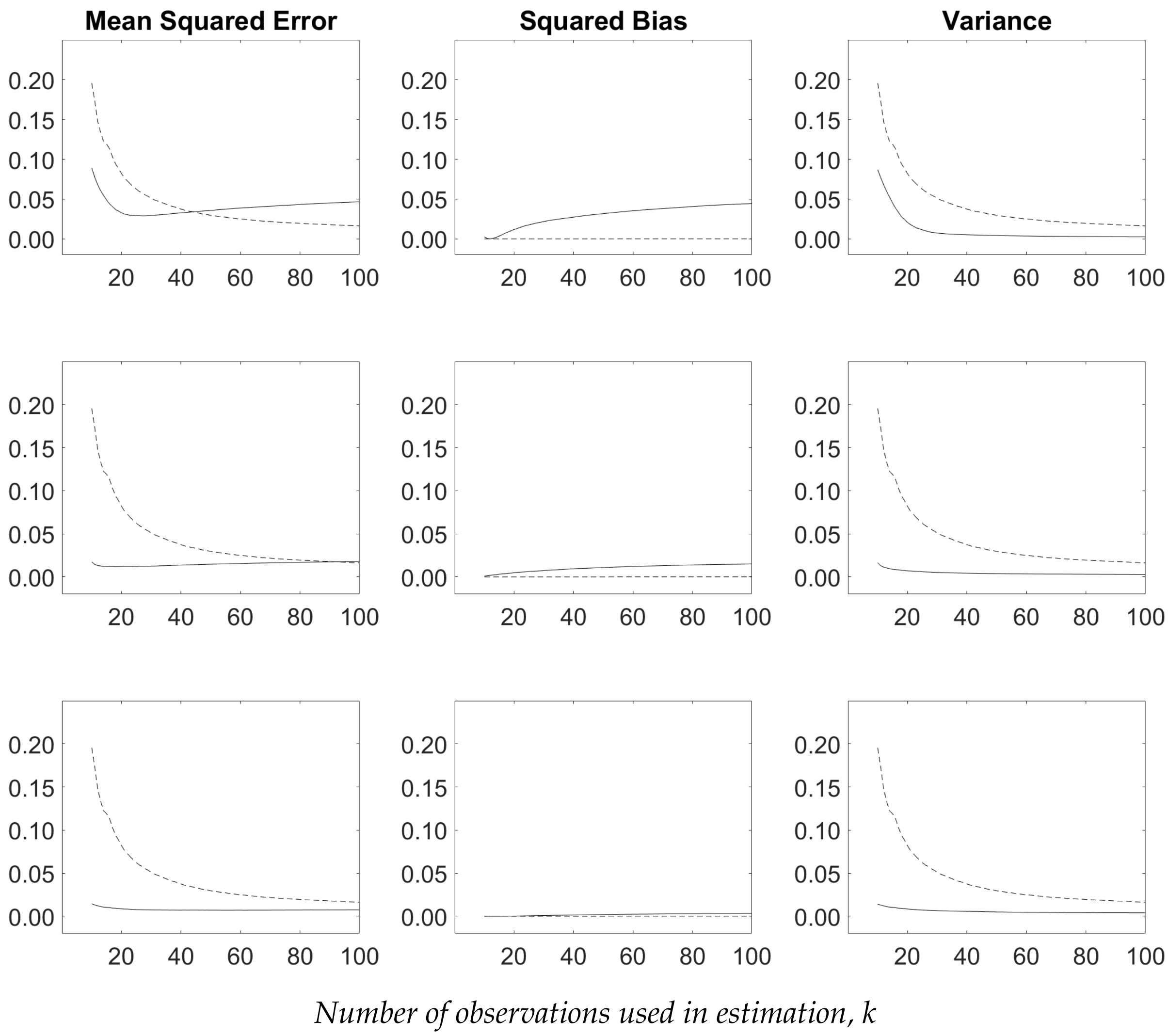

Figure 1: Simulations with a global linear model. The solid lines show the simulation results for the proposed estimator and the dashed lines report the simulation results for the conditional regression model. For both methods, the mean squared error, squared bias and variance are shown. The first row of figures corresponds to $\beta^T = \beta = 0.5$, the second row to $\beta^T = \beta = 1$ and the third row to $\beta^T = \beta = 1.5$

Additionally, Figure 2 shows the simulations using a segmented linear model. The MSE is lower for the proposed estimator than the conditional regression approach, for the model where $\beta^T = 0.5$, when we use low values of k and higher for high values of k. Similar to the global linear model, the squared bias for the proposed estimator is higher than the conditional regression approach and the variance of the proposed estimator is lower than the conditional regression approach. In contrast, the MSE of the proposed estimator is always lower than the conditional regression approach, for the model where $\beta^T = 1.5$.

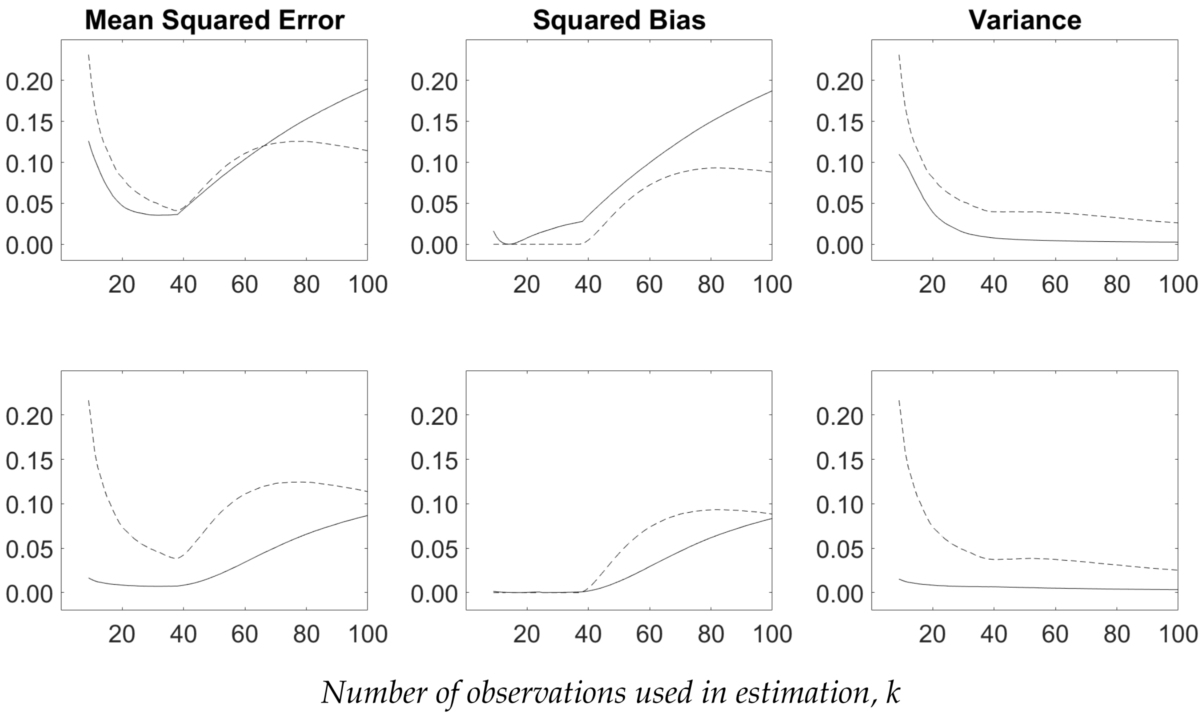

Figure 2: Simulations with a segmented linear model. The solid lines show the simulation results for the proposed estimator and the dashed lines report the simulation results for the conditional regression model. For both methods, the mean squared error, squared bias and variance are shown. The first row of figures corresponds to $\beta = 1$, $\beta^T = 0.5$ and the second row to $\beta = 1$, $\beta^T = 1.5$

4.2 Empirical Application results

Following the empirical application of the proposed estimator by Van Oordt and Zhou (2019), Table 1 shows the estimation summary for all subperiods. In Table 1, Av. $\hat{\beta}^T_{EVT,j}$ denotes the average of the estimated tail betas. Furthermore, N denotes the amount of industry portfolios used and S the amount of industry portfolios excluded by the rule $\hat{\alpha}_j \leq \hat{\alpha}_m$. Finally, Table 1 contains the minimum and maximum estimated $\hat{\beta}_{EVT,j}$ with the corresponding name of the industry portfolio for each subperiod. Following the results in Table 1, S is unequal to zero only in the periods 1931-1935 and 1976-1980. The smallest $\hat{\beta}_{EVT,j}$ is 0.35 (communication, 1946-1950) and the largest $\hat{\beta}_{EVT,j}$ is 2.51 (real estate, 1941-1945) and the average range of the $\hat{\beta}_{EVT,j}$s is 1.5. What stands out for the subperiods 1951-1955 and 1986-1990, is that these are the only subperiods with an average estimated tail beta, by using the proposed estimator, smaller than one (0.99 for both). In most subperiods are the industry portfolios with the smallest $\hat{\beta}_{EVT,j}$ the vital industries like utilities, communication and food products. In contrast, the least vital portfolio industries have the highest $\hat{\beta}_{EVT,j}$, like soda & candy, recreation and entertainment.

Table 1: Estimation summary for all subperiods

Period	Av. $\hat{\beta}^T_{EVT,j}$	N	S	Minimum $\hat{\beta}^T_{EVT,j}$		Maximum $\hat{\beta}^T_{EVT,j}$	
1931-1935	1.18	41	2	0.60	Tobacco Prdcts	1.73	Recreation
1936-1940	1.08	43	0	0.44	Tobacco Prdcts	2.04	Recreation
1941-1945	1.15	42	0	0.62	Communication	2.51	Real Estate
1946-1950	1.07	43	0	0.35	Communication	1.69	Construction
1951-1955	0.99	43	0	0.37	Communication	1.59	Aircraft
1956-1960	1.08	43	0	0.52	Food Products	1.70	Electronic Eq.
1961-1965	1.14	43	0	0.57	Utilities	1.93	Recreation
1966-1970	1.24	47	0	0.53	Utilities	1.89	Recreation
1971-1975	1.14	48	0	0.64	Utilities	1.78	Entertainment
1976-1980	1.07	45	3	0.58	Utilities	1.59	Healthcare
1981-1985	1.08	48	0	0.59	Utilities	2.28	Precious Metals
1986-1990	0.99	48	0	0.53	Utilities	1.31	Soda & Candy
1991-1995	1.15	48	0	0.62	Utilities	1.85	Shipbldng & Railrd Eq.
1996-2000	1.00	48	0	0.43	Utilities	1.85	Coal
2001-2005	1.01	48	0	0.60	Real Estate	1.70	Electronic Eq.
2006-2010	1.11	48	0	0.56	Beer & Liquor	2.13	Coal

Notes: For each subperiod, I calculate the average of the estimated tail betas, denoted by $Av.\hat{\beta}^T_{EVT,j}$. N denotes the amount of used industry portfolios and S the amount of industry portfolios excluded by the rule $\hat{\alpha}_j \leq \hat{\alpha}_m$. Finally, the lowest and highest estimated tail beta with corresponding industry portfolio are included, denoted by minimum $\hat{\beta}^T_{EVT,j}$ and maximum $\hat{\beta}^T_{EVT,j}$, respectively.

Table 2 contains the estimation performance results for all subperiods. For both the conditional regression method and the proposed estimator, I calculate the RMSE as explained in the methodology section. The proposed estimator outperforms the conditional regression approach for all subperiods. As explained in the methodology section, I perform a Diebold and Mariano (1995) type of test at a 5% significance level, to conclude whether the difference in RMSE is significant within each subperiod. Table 2 shows both the test statistic and the corresponding p-value for each subperiod. In eleven out of the sixteen subperiods, is the difference significant with a p-value smaller than 0.05.

Table 2: Estimation performance for all subperiods

Period	Worst day	Market loss	RMSE OLS	RMSE EVT	t-Stat	p-Value
1931-1935	July 21, 1933	9.21	6.90	4.57	2.03	0.025
1936-1940	October 18, 1937	8.20	4.58	3.03	2.11	0.02
1941-1945	December 8, 1941	4.15	2.50	1.76	3.20	0.001
1946-1950	September 3, 1946	6.90	2.43	1.46	1.63	0.055
1951-1955	September 26, 1955	6.52	3.48	1.49	2.31	0.013
1956-1960	October 21, 1957	3.04	2.00	1.20	3.12	0.002
1961-1965	May 28, 1962	7.00	3.26	2.73	1.44	0.079
1966-1970	May 25, 1970	3.21	2.37	1.49	2.19	0.017
1971-1975	November 18, 1974	3.57	2.50	1.01	3.65	0.000
1976-1980	October 9, 1979	3.44	3.14	0.86	5.52	0.000
1981-1985	October 25, 1982	3.62	2.24	1.03	4.02	0.000
1986-1990	October 19, 1987	17.44	6.87	4.15	1.39	0.086
1991-1995	November 15, 1991	3.55	2.85	2.01	1.80	0.039
1996-2000	April 14, 2000	6.72	4.21	3.34	0.84	0.20
2001-2005	September 17, 2001	5.03	5.82	5.19	1.07	0.15
2006-2010	December 1, 2008	8.95	3.35	1.43	3.65	0.000

Notes: For each subperiod, I show the worst financial day with corresponding market loss. Furthermore, I calculate the RMSE for both the proposed estimator (EVT) and the conditional regression approach (OLS). The test statistic (t-Stat) and p-Value correspond to the Diebold and Mariano (1995) type of test, as explained in the methodology section, for testing the difference between the RMSE's of the two estimation approaches.

4.3 Extension results

4.3.1 Predicting the corona crash using estimated tail betas during different periods

To analyze in what extend the corona crash can be predicted using the estimated tail betas during the 2008 financial crisis (2007-2012), I use multiple subperiods to show the relative prediction performance of the estimated tail betas during the 2008 financial crisis (2007-2012) for predicting the corona crash. I use the same 48 different industry portfolios used by Van Oordt and Zhou (2019) between 1971 and 2020. In Table 3, the RMSE of both the conditional regression approach and the proposed estimator are shown for predicting the corona crash, using the estimated tail betas from a given subperiod. Additionally, the Diebold and Mariano (1995) type of test at a 5% significance level, as explained in the methodology section, is shown with corresponding p-value, to conclude whether the difference between the two RMSE's is significant. The subperiods 1986-1990 and 1996-2000 are the only subperiods where the p-value is larger than 0.05, meaning there is no significant difference between the two estimation approaches. There is also no significant difference within these two periods between the two estimation methods for predicting the worst financial day, see Table 2.

Note that the RMSE for predicting the corona crash is the smallest when using the tail betas estimated from data in the most recent period 2015-2020. In Table 3, I test the difference in RMSE of the proposed estimator for predicting the corona crash using estimated tail betas during the 2008 financial crisis (2007-2012) and any other subperiod, using a Diebold and Mariano (1995) type of test at a 5% significance level, as explained in the methodology section. The difference is only significant when one compares the 2008 financial crisis (2007-2012) with the corona crisis itself (2015-2020) and the subperiod 1986-1990. This means that across all subperiods, using data from the current period (2015-2020) leads to the best performance in estimating the corona crash, nevertheless, this is not an authentic out-of-sample analysis. One can conclude that tail betas estimated from data in 1986-1990 outperforms tail betas estimated from data in the 2008 financial crisis (2007-2012) for predicting the corona crash, since the RMSE is significantly smaller. The reason for this superior prediction performance is due to the nature of the financial crisis in the 1980s. Similar to the corona crisis, the HIV/AIDS crisis is also a healthcare crisis, leading to an economic crisis across the world. The 2008 financial crisis on the contrast, has a completely different nature, which is more banking related.

Table 3: Estimation summary for all subperiods for predicting the corona crash using estimated tail betas.

Period	N	RMSE OLS	RMSE EVT	t-Stat	p-Value	t-Stat A.P.	p-Value A.P.
1971-1975	48	8.61	4.83	2.87	0.003	1.28	0.10
1976-1980	47	7.89	4.36	5.53	0.000	0.26	0.40
1981-1985	48	7.29	5.71	4.21	0.000	0.06	0.48
1986-1990	48	4.63	4.99	1.14	0.13	1.74	0.044
1991-1995	48	6.24	4.39	2.74	0.004	1.21	0.12
1996-2000	48	6.08	5.69	0.62	0.27	1.58	0.06
2001-2005	48	10.36	4.94	2.17	0.018	1.19	0.12
2007-2012	48	5.96	5.17	2.54	0.007		
2015-2020	48	5.59	4.09	1.96	0.028	3.79	0.000

Notes: For each five year subperiod between 1971-2020, including the 2008 financial crisis (2007-2012) and the corona crisis (2015-2020), I calculate the RMSE when one uses the estimated tail betas to predict the returns of the corona crash using both estimation approaches. Furthermore, I perform a Diebold and Mariano (1995) type of test at a 5% significance level, as explained in the methodology section, to test the difference between the RMSE's of both the proposed estimator (EVT) and the conditional regression model (OLS). Finally, I perform a Diebold and Mariano (1995) type of test to test the difference between the RMSE of the proposed estimator of the financial crisis (2007-2012) between all other subperiods, denoted by t-Stat A.P. (across periods).

For estimating the corona crash using different subperiods, the proposed estimator outperforms the conditional regression approach, except for the period 1986-1990. Because the difference between RMSE for the two methods is insignificant in this period, we conclude that overall the proposed estimator is better than the conditional regression approach.

Knowing what the estimation performance is of the two estimation approaches to predict the corona crash by using the estimated tail betas during the 2008 financial crisis (2007-2012), I make an analysis of the persistence of the estimated tail betas between the 2008 financial crisis (2007-2012) and the corona crisis (2015-2020). Table 4 shows the results for the Wilcoxon signed rank test, the linear regression test and the difference mean test, following the three explained statistical tests in the methodology section. For the two sets of estimated tail betas using the conditional regression approach, we can reject the null hypothesis corresponding to the Wilcoxon signed rank test at a 5% significance level, but we can not reject the null hypothesis of $\alpha = 1$, by using the linear regression test at a 5% significance level. For the two sets of tail betas estimated using the proposed estimator on the other hand, we can not reject the null hypothesis corresponding to the Wilcoxon signed rank test at a 5% significance level, but we can reject the null hypothesis of α=1, using the linear regression test at a 5% significance level. To Decide which of the two estimation approaches is more persistent, I now analyze the difference mean test. The absolute mean of the set of differences in estimated tail betas using the proposed estimator (0.066) is lower than the absolute mean of the set of differences in estimated tail betas using the conditional regression approach (0.11). Since the null hypothesis of equal means is rejected, using a pair wised Student's t test at a 5% significance level, we conclude that the estimated tail betas using the proposed estimator are more persistent than the conditional regression approach, confirming the superior prediction performance.

Table 4: Statistical tests for persistence tail betas

	OLS	EVT
Wilcoxon stat.	384 (0.018)	442 (0.067)
$\hat{\alpha}$	1.00	0.90
t-Stat	0.14 (0.44)	3.9 (0.00)
Mean Diff. test	2.56 (0.011)	

Notes: Wilcoxon signed rank test, linear regression test and mean difference test at a 5% significance level for persistence of tail betas estimated in 2007-2012 and 2015-2020 using the same 48 industry portfolios as in the empirical application. For both the conditional regression approach (OLS) and the proposed estimator (EVT), I show the test statistics with corresponding p-values in brackets.

4.3.2 Predicting the returns for the corona crash for 48 SP500 companies

I now switch to the 48 SP500 listed companies. For each company, I estimate the tail betas during the 2008 financial crisis (2007-2012), using both the conditional regression approach and the proposed estimator. Then, I analyze in what extend the returns of the corona crash can be predicted, using the estimated tail betas. The RMSE of the conditional regression approach is equal to 6.78 and the RMSE of the proposed estimator is equal to 5.25, for predicting the corona crash. To test the difference between the two RMSE's, I perform the same Diebold and Mariano (1995) type of test, as explained in the methodology section, at a 5% significance level. The test statistic is 2.60 and the corresponding p-value is 0.0062, meaning that the difference in RMSE is significant.

To analyze the persistence of the estimated tail betas between the 2008 financial crisis (2007-2012) and the corona crisis (2015-2020), I apply the same three statistical tests as in Table 4. Table 5 contains the three test statistics, with corresponding p-values in brackets. Again, for the conditional regression approach, we can reject the null hypothesis corresponding to the Wilcoxon signed rank test at a 5% significance level, but not the null hypothesis $\alpha = 1$, using the linear regression test at a 5% significance level. For the proposed estimator, we can not reject the null hypothesis corresponding to the Wilcoxon signed rank test but we can reject the null hypothesis $\alpha = 1$, using the linear regression test at a 5% significance level. To decide which of the two estimation approaches is more persistent, I now analyze the difference mean test. The absolute mean of the set of differences in estimated tail betas using the proposed estimator (0.054) is lower than the absolute mean of the set of differences in estimated tail betas using the conditional regression approach (0.17). Since we can reject the null hypothesis of equal means, using the pair wised Student's t-test at a 5% significance level, we conclude that the tail betas estimated using the proposed estimator are more persistent than the conditional regression approach, confirming the superior prediction performance.

Table 5: Statistical tests for persistence of tail betas using SP500 companies

	OLS	EVT
Wilcoxon stat.	423 (0.046)	576 (0.45)
$\hat{\alpha}$	0.98	0.87
t-Stat	0.20 (0.42)	3.86 (0.00)
Mean Diff. test	2.39 (0.02)	

Notes: Wilcoxon signed rank test, linear regression test and mean difference test for persistence of tail betas estimated in 2007-2012 and 2015-2020 using 48 SP500 companies at a 5% significance level. For both the conditional regression approach (OLS) and the proposed estimator (EVT), I show the test statistics with corresponding p-values in brackets.

4.3.3 Economic gain using hedging portfolios

To show the achievable economic gain, I construct hedging portfolios as explained in the methodology section. Using the estimated tail betas for each company during the 2008 financial crisis (2007-2012), I construct 24 different hedging portfolios for both the regular beta and the tail beta (estimated using the proposed estimator). In Figure 3, I show the hedging returns for all possible portfolios during the entire period 2015-2020. The constructed portfolios using the regular betas outperform the constructed portfolios using tail betas except for the portfolios containing 3, 16 and 17 companies in each investment position. The portfolio with the highest possible return is 55.26% for the constructed portfolios using the regular betas, which includes eight companies in both investment positions. The portfolio with the highest possible return is 50.09% for the constructed portfolios using tail betas, which includes three companies in both investment positions. I will analyze these two specific portfolios in more detail.

In Table 6, I show the test statistics of the Student's t-tests at a 5% significance level to test whether the returns of the two specific hedging portfolios, using three and eight companies in both investment positions, are significantly different than zero during 2015-2020 and whether the difference is significant. For the constructed portfolio using regular betas is a significant indication that the return is higher than zero, whereas for the constructed portfolio using tail betas is no significant indication that the return is higher than zero. However, the difference is not significant between the returns, meaning that there is no significant indication that either of the two portfolios performs better than the other.

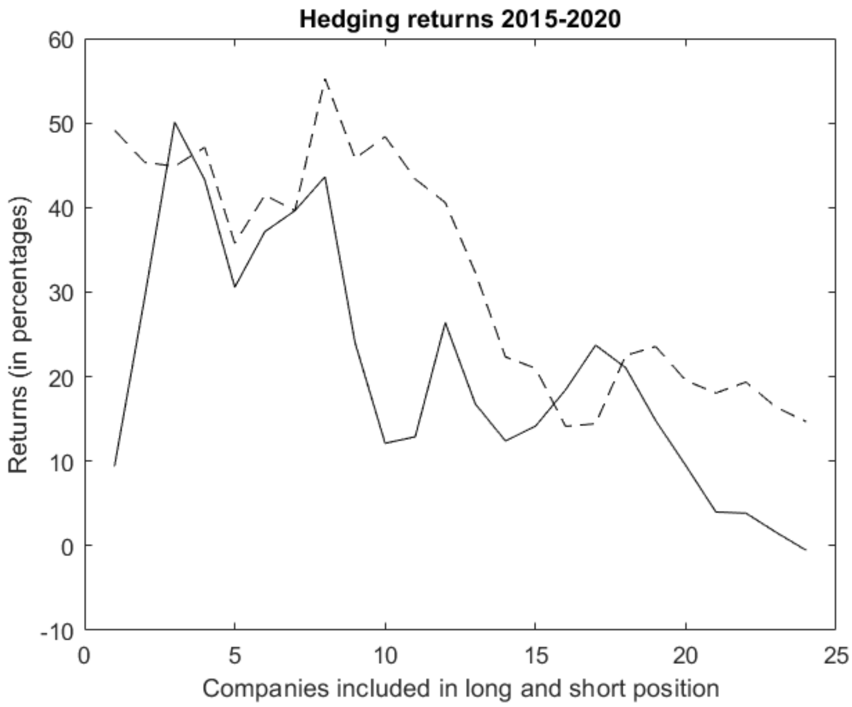

Figure 3: Hedging returns during 2015-2020 for 24 different constructed portfolios using tail betas and regular betas. The dotted lines are the returns using the regular beta, the solid lines are the returns for the tail beta.

Table 6: Testing for hedging returns.

Method	2015-2020	20 Worst Days	20 Best Days
Regular Beta	1.80 (0.036)	4.10 (0.00)	1.86 (0.032)
Tail Beta	1.00 (0.16)	2.61 (0.0051)	1.32 (0.063)
Difference	1.02 (0.15)	3.17 (0.000)	2.07 (0.019)

Notes: Student's *t*-test statistics at a 5% significance level for a portfolio with three companies in both investment positions using tail betas and a portfolio with eight companies in both investment positions using regular betas. I show the test results for testing whether the returns for each period and beta are significantly different than zero. Furthermore, I test the difference between the returns within each period for both betas. The *p*-values are shown in brackets.

To show the hedging performance of all constructed portfolios using the regular beta and the tail beta, I select both the worst and best 20 financial days during 2015-2020. Figure 4 shows the hedging returns for the 20 worst financial days during 2015-2020, for all possible 24 portfolios for each of the two different betas. The constructed portfolios using tail betas outperform almost all corresponding constructed portfolios using regular betas when one uses eight or more companies in both investment positions. Constructed portfolios using regular betas outperform almost all corresponding portfolios using tail betas when one uses seven or less companies in both investment positions. The returns for the two specific portfolios of interest, using three and eight companies in both investment positions, using tail betas and regular betas, are 53.75% and 59.57%, respectively. In Table 6, the Student's t-test statistics are shown for testing the null hypothesis that the returns are zero at a 5% significance level. For both portfolios is the null hypothesis rejected, meaning that there is a significant indication for both portfolios that a positive return can be realized during the worst 20 financial days of 2015-2020. Furthermore, there is a significant indication that the returns of the two specific portfolios are not equal during the 20 worst financial days of 2020, meaning that the constructed portfolio using tail betas outperforms the constructed portfolio using regular betas significantly during economic downturn.

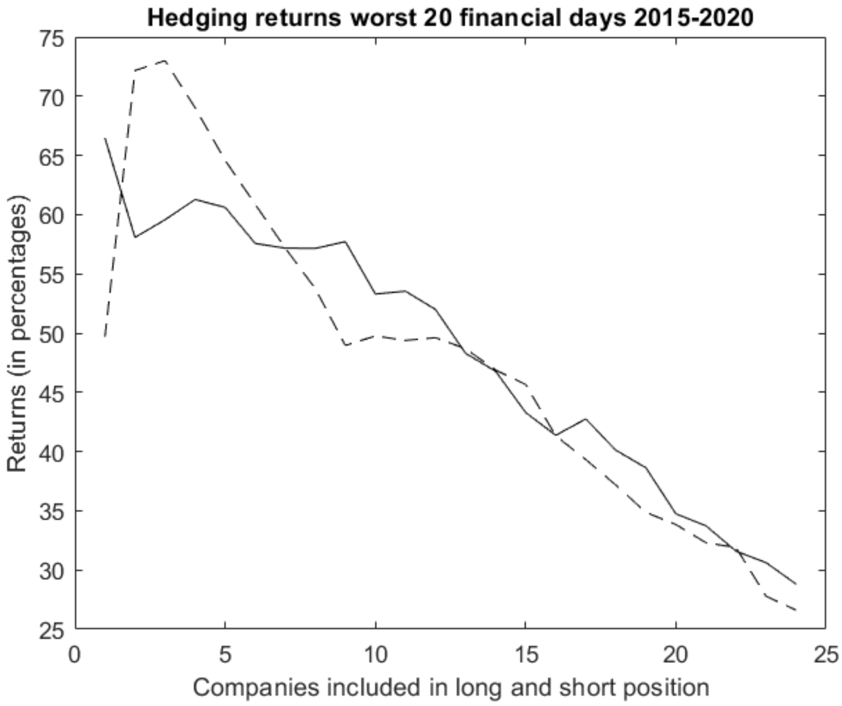

Figure 4: Hedging returns during the 20 worst financial days of 2015-2020 for 24 different constructed portfolios using tail betas and regular betas. The dotted lines are the returns for the regular beta, the solid lines for the tail beta.

Figure 5 shows the returns for the best 20 financial days of 2015-2020 for all 24 possible hedging portfolios for each of the two betas. The constructed portfolios using tail betas and the constructed portfolios using regular betas differ more for small amounts of companies used in both investment positions than for high amounts of companies used in both investment positions. The returns for the two specific portfolios of interest, using three and eight companies in both investment positions, using tail betas and regular betas, are -25.63% and -30.37%, respectively. In Table 6, the Student's *t*-test statistics are shown for testing the null hypothesis that these two returns are zero at a 5% significance level. For the constructed portfolio using tail betas is no significant indication that the return is different than zero during the 20 best financial days of 2015-2020, which is mainly due to the high standard error within this period. This implies that there is no significant indication that the portfolio costs money to hold during the 20 best performing financial days during 2015-2020. In contrast, there is a significant indication for the constructed portfolio using regular betas that the return is different than zero, implying that there is a significant indication that the portfolio costs money to hold during the 20 best financial days of 2015-2020. However, there is a significant difference between the returns of the two portfolios during the 20 best financial days of 2015-2020, meaning that the constructed portfolio using regular betas performs better during the 20 best financial days of 2015-2020, since the return of the constructed portfolio using regular betas (-25.63%) is significantly different (and therefore higher) than the return of the constructed portfolio using tail betas (-30.67%).

Figure 5: Hedging returns during the 20 best financial days of 2015-2020 for 24 different constructed portfolios using tail betas and regular betas. The dotted lines are the returns for the regular beta, the solid lines for the tail beta.

Following the results of both constructed portfolios, one can conclude that during 2015-2020, there is no significant indication that either of the two betas outperforms each other, due to the fact that the difference is not significant between the returns of both portfolios. Furthermore, by selecting the hedging portfolio with the highest possible return for both betas during 2015-2020, which is three and eight companies in both investment positions using tail betas and regular betas, respectively, one can conclude that the constructed portfolio using tail betas outperforms the constructed portfolio using regular betas during the 20 worst financial days of 2015-2020, since the return is significantly higher. However, the constructed portfolio using regular betas outperforms the constructed portfolio using tail betas during the 20 best financial days of 2015-2020, since the return is significantly higher. A side note to this conclusion is that we find no significant indication that the return for the constructed portfolio using tail betas is unequal to zero at all, during the 20 best financial days of 2015-2020.

5 Conclusion

In this paper, I first show that the proposed estimator by Van Oordt and Zhou (2019) outperforms the conditional regression approach in predicting the returns of the worst financial day of the corona crisis so far, using estimated measures of systematic risk under extreme market conditions during the 2008 financial crisis. Second, I show that the persistence of the measure of systematic risk under extreme market conditions between the 2008 financial crisis and the corona crisis is more evident using the proposed estimator, which explains the superior predicting performance. Lastly, I use the characteristics of the measure of systematic risk under extreme market conditions to achieve economic gain by constructing multiple hedging portfolios during the 2008 financial crisis using 48 SP500 companies, where the investment position for the companies with a high measure of systematic risk is short and long for the companies with a low measure of systematic risk. Using the proposed estimator to estimate tail betas, one can form a hedging portfolio during the 2008 financial crisis that outperforms a similarly constructed hedging portfolio using regular betas during the 20 worst financial days of 2015-2020, which covers the corona crisis.

Future research is needed to analyze the persistence of the measure of systematic risk under extreme economic conditions between a larger period and why certain periods are more persistent than other periods. More research can also be dedicated to analyze the weights assigned to each company in the hedging portfolios, to conclude whether or not equal weights are ideal.

6 Appendix

6.1 SP500 companies

The complete list of the SP500 companies I use is: Apple, Abott Laboraties, Adobe, American Tower, Amazon, American Express, Boeing, Biogen, Berkshire Hatheway, Citigroup, Caterpillar, Colgate Palmolive, Comcast Corp, Conoco Philips, Cisco Systems, CVS Health, Chevron, the Walt Disney Company, Duke Energy, Ford, FedEx, General Electric, Goldman Sachs, Home Depot, Honeywell, Intel, Johnsen & Johnsen, 3M, Coca-Cola, Lockheed Martin, Lowe's, Mastercard, McDonald's, MetLife, Morgan Stanley, Microsoft, NextEra Energy, Nike, Qualcomm, AT&T, Texas Instruments, United Health, Union Pacific, U.S. Bancorp, Verizon, Wells Fargo, Walmart and Exxon Mobil.

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