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BSc International Bachelor Economics and Business Economics (IBEB)

TESTING THE FISHER HYPOTHESIS FOR A RECENT PERIOD:

Evidence from Survey of Inflation Expectations

ABSTRACT:

The objective of this paper is to examine the Fisher hypothesis focusing on the recent period and a selection of four markets; the US, the Euro Area, Switzerland and Sweden. The relationship between interest rates and expected inflation is modeled using a survey of inflation expectations and estimated by the Johansen method of cointegration. Contrary to most of the recent literature, we find coefficients above one and the estimates are not significantly different from the hypothesized tax adjusted level for the US, the Euro Area and Sweden. We show that the test results on the Fisher hypothesis may be sensitive to the choice of inflation expectation proxy. We also find that as of 1990, the German and Swiss long term bond rates have included a significant predictive content about future inflation rates.

Keywords:
the Fisher hypothesis, the Fisher effect, survey expectations, inflation expectations, inflation forecasting,

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TABLE OF CONTENTS

1. Introduction

The Fisher effect is one of the oldest paradigms of Financial Economics, with origins dating back to the 19th century. Over the past decades, the Fisher equation has been subject to scrutiny by many authors. Throughout the years, the theoretical representations and empirical models have been gradually extended to account for additional complexities and more realistic assumptions. Behind the thread of an academic literature lies a simple and very intuitive idea, a proposition that nominal interest rates should adjust to the changes of expected inflation, thereby leaving the real rate unaffected. The concept itself refers to the notion of money illusion, and an assumption that if investors are rational, they demand a compensation for the expected loss in the purchasing power.

In this paper we review the theoretical background of the Fisher equation, and reexamine the hypothesis using the Johansen method. The objective is to review the effect on four markets; the US, the Euro Area, Switzerland and Sweden. We focus on two main sub-questions. First we estimate the long run cointegrating relationship between interest rates and inflation and investigate whether the estimates are sensitive to the common assumption that agents form expectations rationally. For this purpose, we employ recent survey data of expected inflation. Second, we examine whether in the recent period, the bond rate included a significant predictive content about inflation.

The paper follows a simple structure. The next chapter reviews the early history of the Fisher effect. Chapter 3 discusses details on the Fisher equation and some theoretical and empirical issues. In that chapter we also review some of the recent results and their interpretation. Chapter 4 is empirical and presents the design of our methodology, our findings and the interpretation. Concluding remarks are found in chapter 5.

2. History of Fisher Hypothesis

2.1 Origins of the Fisher Relation and Early History of the Paradigm (Humphrey 1993) [1]

Although the Fisher effect is frequently credited to *The Theory of Interest (1930)*, a well known work by Irving Fisher, the formal representation as an equation does not appear in the book. *The Theory of Interest* does contain a definition of the relationship and number of empirical tests, but the equation appeared much earlier in Fisher's monograph *Appreciation and Interest (1896).* Although Fisher was the first to define the relationship as an equation and statistically test its validity, he was not the first to formulate inflation as a difference between real and nominal rates of interest. (Dimand 1999)

[1] For more elaborate review on the early history of Fisher effect refer to Humphrey (1993), as this section offers only a reduced summary of his article.

Before Irving Fisher, several authors have attempted to describe the relation between interest rates and inflation using anecdotal evidence. As early as in 1738, William Douglass noted how extensive issue of colonial currency resulted in depreciation of money, while the loans denominated in silver coins had lower rates of interest. Douglas suggested a presence of a premium in the rate of nominal interest, which reflected the loss of purchasing power. He reasoned that if the paper money depreciated at the rate of 7%, while the interest on silver was 6%, lenders would demand 13% rate in the paper money loan. Douglas summarized;

> "The quantity of paper credit sinks the value of the principal, and the lender to save himself, is obliged to lay the growing loss of the principal, upon the interest." Douglas (1940)

In 1811, Henry Thornton gave a more precise definition of the relation. He specified that it was the expected rate of inflation that determined the interest rate premium. Thornton implied that it was unnecessary for anyone to predict the inflation rate, and as long as lenders predicted their rates of returns, inflation expectations would be formed indirectly. Thornton used this logic to explain why interest rates in Britain have gone up after Bank of England has lifted its obligation to convert paper money into gold.

Among the first contributors was also John Stuart Mill, who elaborated on the hypothesis in the sixth edition of his *Principles of Political Economy (1865)*. Mill's primary innovation was to acknowledge that inflation had also affected the value of interest received - a factor which should be incorporated into the premium. This was a new insight, as earlier writers were concerned merely with the reduction in the value of the principal.

In the late 19th century, the relation between inflation and interest rates received a growing interest from scholars. Among them was a Dutch economist Jacob de Hass, who developed a framework, whereby he decomposed the interest rate premium into three elements. He described the first one of them as a "remuneration for the abstinence, i.e., the hire of capital," and the second as an insurance against credit risk. De Hass proposed that the third factor which determined interest rate premium was the rate of inflation.

The discussion on the hypothesis was further extended by Alfred Marshall in the two editions of his *Principles of Economics (1890)*. Marshall outlined two key propositions. First, he suggested that inflation expectations are derived from realized inflation, meaning that expectations lag behind the actual values. Second, Marshal argued that lenders and borrowers hold different inflation expectations, and as such, changes in expectations cause disproportional adjustment of the supply and demand for the loans. These two effects combined, Marshall concluded, cause economic disturbances, sufficient enough to create trade cycles.

2.2 The Empirical Foundation by Irving Fisher

Irving Fisher has initially addressed the real/nominal rate analysis in his *Appreciation and Interest (1896)*. In this book, he modelled the relation between real and nominal interest rates.

Fisher derived his equation arguing the efficiency between the loan markets. This argument rests on two assumptions. First, Fisher assumed a perfect foresight of inflation. Under perfect foresight, the price of one basket of goods, which costs one dollar at the beginning of the year, will rise precisely at the rate of the expected inflation Π^e, and will cost $(1+\Pi^e)$ at the end of the year[2]. Second, he assumed that any loan contract can be denominated in both the currency and commodity. In such case, a borrower has two options for taking a loan. One option is to take a one dollar loan in paper currency, whereby an interest has to be repaid on top of the principal, and the borrower has to pay back $(1+I)$ when the loan is due. Alternatively, the borrower may take a loan denominated in commodities, and borrow a basket of goods at the price of one dollar. Then, he will be liable to repay $(1+R)$ baskets of goods at the maturity of the loan. This payment can be liquidated at the price $(1+R)(1+\Pi^e)$, which is composed of the expected inflation and the real rate. In the absence of arbitrage opportunities, the prices of the two loans are equal so that equation (1) holds.

$$(1) \qquad (1+I) = (1+R)(1+\Pi^e)$$

If the above equation does not hold, two identical loans have different prices, and arbitrage opportunities arise. For instance, if the dollar loan is cheaper, it may be possible to extract a profit by borrowing at the dollar rate $(1+I)$, using proceeds to buy a basket of commodities and lending it at the equal maturity at the rate of $(1+R)$. At maturity the proceeds from the commodity loan can be liquidated, and the dollar loan is repaid with profit. Thus, if the dollar loans are cheaper, investors will exploit arbitrage by buying commodity loans and selling dollar loans until supply and demand adjust and equilibrium prices are restored. Fisher formulated his conclusion using equation (2),

$$(2) \qquad I = R + \Pi^e + R\Pi^e$$

[2] The capitalized terms I, R, Π^e refer to annually compounded variables. Continuously compounded variables (in logs) are depicted by small letters i ,π and r.

where the nominal rate of interest I is determined by the real rate of interest R, inflation Π^e and the product of the two terms.

2.3 The Fisher's Empirical Findings and Conclusions

Fisher used equation (2) to verify the hypothesized relationship empirically and found that nominal rates reacted poorly to changes of future inflation. Specifically, he showed that real rates responded inversely to movements in nominal rates. Moreover, the real rates were shown to be negative in some periods and had much higher variability compared to nominal rates.

When the hypothesis about perfect foresight of inflation failed to be supported by facts, Fisher presented an alternative; a model of imperfect foresight. He suggested that nominal rates are slow to adjust to new levels when inflation rises, and real rates temporarily fall as a result. Following Alfred Marshal, Fisher argued that lenders and borrowers derive the inflation expectations differently. Borrowers, he reasoned, are typically entrepreneurs, who borrow capital to fund their investments and forecast their profits from the current prices and cost of capital. Entrepreneurs interpret the rise in real rates as a potential increase in profits, which encourages them to increase their investment activity. As a result, the demand for the loan capital rises.

Lenders, according to Fisher, forecast the returns from realized profits and thus they are slower to respond to price increases. Slow response of lenders means that while the supply curve of loans is initially fixed, the demand curve shifts to the left, which results in falling real rates. Eventually, after their realized profits have fallen, lenders adjust to the new conditions and nominal rate rises.

Fisher used the discrepancy in the way expectations are formed as a basis for his trade cycle model. He illustrated how inflation creates expectations of higher profits, which in turn generates more business investment. On the contrary, deflation, which triggers a fall in real rates, may discourage business investment and cause an economic downturn.

In his later book, *Theory of Interest (1930),* Fisher used the distributed lag structure to examine the relationship between nominal interest rates and inflation. In support of the hypothesis, the results showed that the nominal rate did respond to changes in price level, only with very considerable time lags. He found a correlation coefficient of 0.86 for US and 0.98 for UK when the price changes were spread as much as 20 and 28 years for the two countries respectively. Much of his attention was therefore focused on setting the theoretical ground to explain the lagged adjustment. Fisher (1930, p. 451) concluded:

"We have found evidence general and specific… that price changes do, generally and perceptibly affect the interest rate in the direction indicated by a priori theory. But since forethought is imperfect, the effects are smaller than the theory requires and lag behind price movements, in some periods very greatly. When the effects of price changes upon interest rates are distributed over several years, we have found remarkably high coefficients of correlation, thus indication that interest rates follow price changes closely in degree, though rather distantly in time."

These results inspired many to re-examine Fisher's finding, and the subject has received a lot of attention in the empirical literature. (Cooray 2003) The following section reviews the standard testing procedures applied on the Fisher hypothesis and some of the key findings.

3. The Test Variants of the Fisher Hypothesises: Theoretical and Statistical Issues

The literature that has been written on the subject since 1930 is marked by a gradual development. Two milestones have significantly shaped the Fisher effect. First, a revised definition of the hypothesis has been suggested upon the development of the theory of the Efficient Markets. Second, innovation in methods of time-series data analysis such as methods of co-integration, have had a profound influence on empirical hypothesis testing. Other important issues, both theoretical and technical, have been addressed in the literature. This section reviews Fisher's original equation, and discusses its later variants which were developed to incorporate new definitions and address statistical problems. We first discuss the influence of the rational expectations hypothesis along with several other theoretical issues. Second, we concentrate on unit root processes, along with some additional statistical issues and the effects of monetary policy. Last, we review some of the standard empirical results.

3.1 Defining the Fisher Hypothesis: Rational Expectations and Other Issues

Fisher's Original Proposal: Lagged adjustment mechanism

One of the major theoretical problems Fisher faced was determining how inflation expectations are formed. A related technical problem was finding an appropriate measure of inflation expectations, to test the relation. Fisher developed a hypothesis that inflation is formed from past realized value, and is therefore backward looking in nature. Using this assumption, he implemented the distributed lag mechanism on inflation as a proxy for expectations. The traditional way of testing this relationship involved estimating the following equation:

(3) $$i_t = \beta_0 + \beta_1 \pi_{t-1} + \varepsilon_t$$

where β_0 is the estimate of the constant real rate, and ε is the error term. Empirical equation (3) is obtained by taking natural logarithms of Fisher's theoretical relationship in eq. (1) and rewriting it in an empirical form.

The subsequent works of Cagan (1956), Meiselman (1962), Sergeant (1969), Yohe and Karnosky (1969) and Lahiri (1976) followed this approach and were mostly focused on verifying Fisher's empirical results. These studies generally confirmed Fisher's findings about significant lag effects in the formation of expectations. However, Yohe and Karnosky (1969), Lahiri (1976) and particularly Gibson (1970) found that a significant improvement in the formation speed of expectations is supported by data from the 60s. (Cooray 2003)

Rational Expectations Hypothesis

With the advent of the rational expectation theory developed by Muth (1961) and the efficient market hypothesis put forward by Fama (1970), the Fisher hypothesis was reformulated. Muth proposed that market participants hold rational expectations about future price changes, meaning they use all available information to form expectations. Such rational expectations are one component of the future realized inflation. The other is the random shock. If we assume that the random shock is on average zero, expectations should on average equal realizations.

Fama (1975) criticised Fisher's traditional equation on the grounds of rational expectations hypothesis. The problem, he pointed out, was that traditional methodology which relies on past values of inflation, implicitly assumes market inefficiency. Consider equation (3) which represents Fisher's assumption that economic agents derive inflation expectations from past realized information. This assumption can be described using (4),

(4) $$\pi_t^e = \pi_{t-1} + \varepsilon_t$$

where π_t^e are inflation expectations and π_{t-1} is the past realized inflation. This formulation creates two quite undesirable restrictions. First, it restricts the possibility that inflation is not predictable from any information (other then π_{t-1}) available at the time t. Second, if we dismiss the first restriction, then it is necessary to assume that market agents, in forming their expectations, will unintentionally or deliberately ignore the information that is available to them. The second statement is no more than a definition of market inefficiency, while the first may be difficult to square with the theory, since potentially useful information about inflation is theoretically available.

Fama (1975) proposed to describe the relationship alternatively. Let us first define the change in purchasing power from time *t-1* to *t* as Δ_t, which can be defined as follows:

(5) $$\Delta_t = \frac{p_{t-1}}{p_t - p_{t-1}} = \frac{1}{1+\Pi_t} - 1$$

where p_t is the price level at time *t*. The right hand side of the equation implies that Δ_t refers to the inverse of the inflation rate $(1+ \Pi_t)$. Fama argues that economic agents use all information from the informational set Φ_{t-1} available to them at time *t-1* to form expectations about future inflation, so the following holds:

(6) $$E(\Delta_t | \Phi_{t-1}) = E(r) - I_t$$

where *E(r)* is the constant real rate of interest. Using (6), he implies that all the variation in nominal rate I_t is a "direct reflection of the variation in the market's assessment of the expected value of Δ_t". Therefore, the information Φ_{t-1} available at *t-1* is fully summarized in the value of I_t. If markets are efficient, as Fama argued, nominal rates should absorb all the variance in inflationary expectations, leaving the real rates constant. This formulation stipulates that a test on the Fisher effect revolves around the hypothesis about constant real rates. The following empirical equation was tested by Fama (1975):

(7) $$\Delta_t = r + \beta_1 I_t + \varepsilon_t$$

where Δ_t is the inverse of the future realized inflation, serving as a proxy for the inflation expectations. The hypothesis is rejected if the coefficient estimates are not consistent with the condition,

(8) $$\beta_0 = E(r),\ \beta_1 = -1$$

This variant of the equation has been subsequently referred to as "inflation forecast test", as it measures the informational content of future inflation in nominal rates of interest. The following equivalent of (7) has been customarily adopted by later studies:

(9) $$\pi_{t+m} = r + \beta_1 i_t + \varepsilon_t$$

where π_{t+m} is the continuously compounded future inflation from time t to m, while m is the maturity of the loan contract[3]. Under (9), the Fisher hypothesis is rejected if β_1 significantly differs from one. A popular reverse representation, frequently used in literature, is formulated as:

(10) $$i_t = r + \beta_1 \pi_{t+m} + \varepsilon_t$$

Fama found supportive evidence of the Fisher hypothesis and concluded that the equilibrium real interest rate on six month US Treasury bills is constant and therefore unaffected by expected inflation rate. However, subsequent studies of Hess and Bicksler (1975), Carlson (1977), Joines (1977), and Nelson and Schwert (1977), which conducted Fama's inflation forecast test, did not confirm Fama's findings. (Cooray 2002)

The Issue of the Simultaneous Hypothesis

Fama's model based on the REH offered a solution to a problem which stems from the traditional model (3), whereby this equation provides a premise for market inefficiency. In an ideal situation, where perfect foresight of inflation exists and the informational set Φ_{t-1} contains information useful in predicting inflation, Fama's innovated version (6) is a good general representation of Fisher's Hypothesis. Consider the basic assumption of the REH in (10), from which follows that:

(11) $$E(\Delta_t | \Phi_{t-1}) = \Delta_t + \varepsilon_t$$

where ε is the unpredictable random shock, which is on average assumed to be zero. If the information set Φ_{t-1} is limited and the predictability of inflation is low, (11) may not hold and the empirical test as described by (7) will be rejected. Obviously, the result of the Fisher test is depended on the formulation of the REH at question and whether such formulation is consistent with reality. Inevitably, the test on the Fisher hypothesis is a simultaneous test on a given formulation of the rational expectation hypothesis. If the REH formulation at hand is rejected, so is the Fisher hypothesis, regardless of its validity. Thus the inflation forecast test suggested by Fama, like Fisher's original one, both suffer from the fact that the hypothesis about real/nominal rates is inseparable from the additional hypothesis about formation of expectations.

[3] Fama (1975) ignores the cross-term $R\Pi^e$ from equation (2), suggesting that this term is very small when monthly data is employed. More precise definition is attained, when continuously compounded data (natural logarithm of inflation and interest rates) are employed, as in (9).

Use of Survey Data

A solution to this problem of a simultaneous hypothesis was suggested even prior to the implementation of the REH. Gibson (1972) criticized the use of past realizations as a proxy, pronouncing its disadvantage of double hypothesis testing. Concerned this issue, Gibson wrote,

> These studies [of supportive evidence] relied, however, on the hypothesis that the expected rate of inflation is dependent upon past rates of price change, so that this hypothesis was being tested along with Fisher's.

To avoid this, he proposed the use of a more direct measure of inflation expectations, namely the survey data on expectations. Specifically, Gibson proposed a model based on the oldest survey of economists' expectations, the well known Livingston survey.

The idea behind this solution is simple, yet appealing. The opinions of economists, particularly if they are responsible for managing considerable amounts of assets, are contained in the general expectations of the market. There certainly may be a discussion about the weights of such opinions within the general expectation of a full market. If, however, survey participants include large corporations, banks or other institutional investors whose assets form a considerable proportion of the total market, their expectations may weigh heavily.

At any rate, the use of survey data as an expectation proxy does not generate a dependence on the secondary hypothesis. If the survey is representative of the market, the REH and its formulation - whether weak or strong, can be determined separately from the test of the Fisher hypothesis, for instance by applying standard methods of rationality testing of expectations. Rejection of a given definition of the REH does not automatically rule out the proper adjustment of nominal rates to inflation, as this can be tested separately. The only condition which hereby remains is that the expectations contained in a given survey publication are representative of the market.

Gibson's approach yielded more support to the Fisher effect than many of the earlier studies. His results show a very short distributed lag of inflation in nominal rates and a significant full adjustment in nominal rates is found with a time lag of only 6 months.

One to One Adjustment and the Issue of Taxes

Empirical models (9) and (10) require coefficient estimates of 1 for β_1, under the null hypothesis. For, if they are not, shocks to the expected inflation are not fully reflected in the nominal rate. However, this reasoning is theoretically sound only in a tax free environment. Darby (1975) and Feldstein (1976) showed that the relation is more complicated in a world with taxes. In a market where interest proceeds are taxed, investors should care about their tax adjusted returns. If τ is the tax rate, the correct tax adjusted formula is $i(1 - \tau) = \beta_1 + \beta_1\pi^e_t + \varepsilon$. Alternatively, without specifying the tax rate in the equation, the coefficient of β_1 should be higher than 1. Crowder and Hofmann (1996) argue the correct tax adjusted range of β_1 is 1.20-1.50.

The Issue of Constant Real Rates

Irving Fisher's original hypothesis was that the real rate of interest is not affected by inflation. Fisher believed in the absence of money illusion, whereby borrowers are rational to the extent that they demand compensation for expected reduction in the purchasing power of currency. For practical or other purposes, some authors who wrote on the subject later redefined this hypothesis, as a condition of constant real rates. The two formulations are far from equivalent and the latter places additional burden of proof on the hypothesis. In reality, real rates, whether they are affected by business cycles or demand for capital, need not be constant, while simultaneously they may be uncorrelated with expected inflation. Evidence of constant real rates is a sufficient but not a necessary condition. However, up until recently, the assumption of constant real rates had been regarded necessary, as is implied by equations (9) and (10). Only relatively recent innovations in time series techniques and the development cointegration models, have led to an opportunity to relax the assumption of constant real rates. These methods are discussed in the following section.

3.2 Unit Root and other Technical Issues

Unit Root in Inflation and Interest rates

The line of testing described above, as formulated by (9) and (10), has been identified to cause a few other problems. Some authors[4] suggested a possibility that tested variables follow unit root processes. Unit root, or alternatively, integrated order one I(1), is a linear stochastic process which is non-stationary in levels and stationary in first differences. The case of the inflation being a unit root process can be illustrated as follows:

(12) $$\pi_t = m\pi_{t-1} + \varepsilon_t$$

where m relates to the order of the process, and m is 1 under unit root. Granger and Newbold (1974) showed that if traditional regression methods are applied to unit root variables, a risk arises that the regression will be spurious and coefficients test biased, particularly if variables share no long run relationship.

[4] Hess and Bicksler (1975), Carlson (1977) and Fama and Gibbons (1982)

This problem was generally ignored in previous literature. A number of studies, including Hess and Bicksler (1975), Carlson (1977) and Fama and Gibbons (1982), suggested a possibility that inflation and nominal rates are unit roots, but no formal tests have been carried out. Mishkin (1992) was among the first to conduct a thorough unit root analysis for both inflation and nominal rates. He applied the Dickey–Fuller test and the Phillips modified test, and with respect to the former he used a Monte Carlo simulation in order to obtain critical values for small sample distribution. The results showed that it was impossible to reject the null hypothesis of unit root for neither inflation nor nominal rates. Mishkin concluded that conventional methods of regression analysis are not applicable to the case of the Fisher effect as they may be associated with spurious regressions.

These findings quite rightly aroused new questions about the validity of the previous literature. At the very least, these conclusions meant that the Fisher hypothesis had to be reviewed in a new light, using new methods to account for statistical issues. Mishkin (1992) applied new methods and his example was followed by many others, including Evans and Lewis (1995), Mehra (1998) Crowder and Hoffman (1996), Peng (1995). In section 4 we review the results of these works, but first we discuss some other methodological issues in greater detail.

Long run Fisher Equation

An optimal case of the two non-stationary variables is when they move together in the long run. In such case there may exist a linear relationship that makes a combination of these two non-stationary variables stationary. If such a linear relationship exists, it can be found by testing the stationarity of the error term ε, which comes from an estimate of the simple OLS regression, and can be represented as:

(13) $$\varepsilon_t = i_t - \beta_1 \pi_t - \beta_0$$

If the error term is confirmed stationary, then variables are said to be cointegrated, meaning the model has a long run solution. This method was proposed by Engle and Granger (1987). Note that ε_t in (13) equals to the real rate spread, which determines the long run real rate *r*. Thus, equation (13) is simply a test on the stationarity of the real rate. Apart from the apparent benefits of the statistical viability, this method offers to relax the assumption that real rate is constant. Under the long run solution of the Fisher hypothesis, the assumption about the real rates is reduced to stationarity. The right side of equation (13) is referred to as a vector of the long run relationship.

It may also be of interest to identify the short run dynamics of the relationship. In early literature, this typically involved differencing I(1) variables to achieve stationarity, before applying OLS (as illustrated by 14).

(14) $$\Delta(i_t) = \alpha_0 + \alpha_1 \Delta(\pi_t) + \varepsilon_t .$$

The above representation is a statistically feasible solution as it is free from the effects of spurious regression. The problem is, however, that (14) does not include the relevant information about a long run solution of the model. Engle and Granger (1987) therefore proposed to extend the model to incorporated the long run vector as follows:

(15) $$\Delta i_t = \alpha_{10} + \alpha_{11}(i_{t-1} - \beta_1 \pi_{t-1} - \beta_0) + \alpha_{12} \Delta i_{t-1} + \alpha_{13} \Delta \pi_{t-1} + \varepsilon_t$$

(16) $$\Delta \pi_t = \alpha_{20} + \alpha_{21}(i_{t-1} - \beta_1 \pi_{t-1} - \beta_0) + \alpha_{22} \Delta \pi_{t-1} + \alpha_{13} \Delta i_{t-1} + \varepsilon_t$$

This system consists of a combination of lagged first differences and the cointegrating vector. The expression $(i_{t-1} - \beta\pi_{t-1} + \beta_0)$ is referred to as the *Vector Error Correction* term or VEC. The coefficients α_{11} and α_{21} can be referred to as speed of adjustment coefficients because they measure the tendency of the model to correct for deviations from the equilibrium. Specifically, they show the proportion of the equilibrium error from the last period that is corrected. (Brooks 2002)

The coefficient signs of α_{11} and α_{21} provide information about the short term dynamics of the relationship. Consider an example where a positive shock to the expected inflation at time *t-1* triggers a positive adjustment of the nominal rate at time *t*. Consequently, α_{11} assumes a negative value. This suggests that deviations from the long term relationships are corrected by a subsequent adjustment of the interest rate. If the coefficient α_{12} is significantly positive, the equilibrium errors are also corrected by a subsequent adjustment in the expected inflation. If only one of the coefficients significantly assumes the correct sign, such an effect can be described as an evidence of weak causality.

The remaining alphas further illustrate the short run dynamics of the relationship. The coefficients α_{12} and α_{22} measure the presence of autocorrelation whereas α_{13} and α_{23} measure serial correlation. The estimates of α_{13} and α_{23} provide information about the so called Granger causality between the variables. The coefficients α are part of what is referred to as a *Vector Auto Regressive* (VAR) component of the model. Note that in (15 -16) both VEC and VAR components contain constants, although it is also possible to estimate the relationship without them. (Brooks 2002)

The Issue of Stochastic Environment

The works of Lucas (1978), Benniga Protopapadakis (1983) and Shome Smith Pinkerton(1988) have shown that solution to the Fisher equation is more complex in a stochastic environment. A generalized form of the Fisher equation as suggested by Benniga/Protopapadakis (1983) follows as:

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(17) $$\frac{1}{1+i_1} = E\left(\frac{1}{r}\right)E\left(\frac{p_0}{p_1}\right) + \beta \operatorname{cov}\left(\frac{u'(C_{t+1})}{u'(C_t)}, \frac{p_0}{p_1}\right)$$

where *u'* is a first order condition of a given utility function to consumption C_t. This definition under uncertainty implies that risk-averse investors demand a risk premium, which is given by the covariance between the rate of marginal utility to consumption and the expected change in price level p_0/p_1. The risk premium is positive if inflation negatively co-varies with the business cycle (high nominal prices in low consumption states). This means that agents require additional premium when inflation makes their returns more volatile.

The above formulation of the Fisher theorem was tested empirically by Shome, Smith and Pinkerton (1988). They derive and test the following equation:

(18) $$i_t = \beta_0 + \beta_1 E(\pi_{t+1}) + \beta_2 E(r_{t+1}) + \beta_3 \operatorname{var}(r_{t+1}) + \beta_4 \operatorname{cov}(r_{t+1}, \pi_{t+1}) + \beta_5 \operatorname{var}(\pi_{t+1}) + \varepsilon_t$$

where under the null hypothesis of a Fisher effect, β_1 and β_5 are expected to be 1 and -0.5 respectively. The coefficient β_5 relates to the so called Jensen's inequality term, which enters the equation when the expected inflation is decomposed under the assumption of log-normality. The authors employ the real changes in the national accounts to substitute for the real consumption rate *r*. Note that in (18) we expect a negative β_4, indicating that risk averse agents require a positive risk premium for negative covariance between inflation and consumption rate.

Shome, Smith and Pinkerton (1988) find that inflation variance can not explain changes in nominal rates and the Jensen inequality term is insignificant. Furthermore, they find β_4 significantly positive and conclude that investors demand a positive premium for covariance between inflation and the real rates, meaning that investors are risk averse.

With respect to the last two terms of representation (18), Evans and Lewis (1995) indicate that they are inconsequential in the long run model (equation 14). They argue that these terms are in theory stationary and therefore should be inconsequential in the cointegrating equation. Nevertheless, the terms may still be of concern in the VAR representation of the model.

The Issue of Structural Breaks in Inflation

Some studies, including King and Watson (1992) and Crowder and Hoffman (1996), suggested that nominal rates do not move one-to-one with expected inflation in the long run. Such findings can be interpreted as evidence against the Fisher hypothesis as the real rates appear to suffer from permanent shocks to inflation. Evans and Lewis (1995) confirm this evidence, finding long run coefficients below one. They, however, attempt to provide an alternative interpretation. Evans and Lewis (1995) suggest that inflation processes are not stable, but rather contain structural shifts. This hypothesis is confirmed using the Markov switching model of inflation, which shows that structural shifts are present in inflation processes. When Evans and Lewis (1995) incorporate the inflation shifts into the Fisher equation, they find support for the one-to-one long run Fisher relation.

The Fisher Hypothesis and Monetary Policy

At the time when Irving Fisher developed his hypothesis, the market environment of commodity prices and interest rates was arguably less complex. Today, among other things, these markets are typically often influenced by active monetary policy. Under active policy, central banks purposefully adjust nominal rates, often to contain inflation or to smooth the business cycle, depending on their objective. The basic premise of the Fisher hypothesis is that the nominal rates on loan capital are determined by the market, on the basis of supply and demand for capital. It rests on an assumption of efficiency between loan markets of commodities and currencies. Central banks, via their policies, may directly influence the prices of currency loans and indirectly the prices of commodities. Consequently, if the price efficiency between these two markets is not attained, it is not clear who is to blame, whether the markets themselves, for not being able to determine the prices right, or central banks, which use their policy tools to influence the prices.

We can re-consider the Fisher theorem in the light the three types of monetary policies; counter-cyclical, pro-cyclical and constant money supply. Under the basic definition of the Fisher hypothesis, real rates share no correlation with inflation. This, according to Benniga Protopapadakis (1983), is only likely under pro-cyclical monetary policy.

Consider a case where monetary policy is state independent and the supply of money is fixed. Under this policy mode, states with low consumption are characteristic by relatively high nominal prices, due to the lower purchasing power of money. Inflation thus correlates negatively with the real rates. If the monetary policy is countercyclical (low consumption states are countered by increases in money supply), then nominal prices are even higher and covariance yet more negative under this policy. Under the pro-cyclical policy of “real bills” doctrine, low consumption states have low nominal prices, as the purchasing power of money is not greatly affected by the cycle. Under this regime, as Benniga and Protopapadakis argue, the covariance is slightly negative or positive. Hypothetical low covariance of real rates and inflation under this regime offers room for the Fisher hypothesis. With other monetary regimes, the analysis of the Fisher effect is more complex, particularly in the cases of active policy, whereby nominal rates are systematically controlled and inflation is targeted (Smant 1996).

A supportive evidence for this view is found by Mehra (1998), who tests whether expectations about future inflation in United States are contained in the long term bond rates. Mehra finds that the bond rates contained significant information about future inflation prior to 1979, but after Federal Reserve adopted a policy of inflation stability, the information content in bond rates has deteriorated. More detailed discussion on this and other relevant studies is offered in the next section.

3.3 The Standard Empirical Results

The above two sections identified seven issues which typically accompany empirical studies of the Fisher hypothesis. Due to the existence of these issues, it is important that empirical results are interpreted with caution. A number of studies has examined the Fisher hypothesis incorporating the long run (cointegration) model. In this section we briefly review four of these studies: Mishkin (1992), Peng (1995), Crowder Hoffman (1996), Mehra (1998).

Mishkin (1992)

As discussed above, Mishkin (1992) was among the first studies to present evidence of unit root processes in nominal rates and inflation. He uses monthly data of inflation and twelve month U.S Treasury bills from February 1964 to December 1986. Mishkin employs a Monte Carlo simulation to obtain small sample critical values for the ADF test. The dataset is split into four subsamples according to different periods and the evidence shows that unit root can not be rejected for any one of them. Unit root is found in both nominal rates and inflation. Subsequently, the study concentrates on the co-integration model of the Fisher effect.

Mishkin uses Engle and Granger's (1987) methodology, estimating a version of equation (13) (with coefficient on i_t). The unit root test is applied to the error term from this equation, in two versions; $(\pi_t - \beta i_t)$ and $(\pi_t - i_t)$, the second of which implies that coefficient β equals 1. Supportive evidence is found under both versions and for all subsamples. Mishkin also conducts a tests of the short run Fisher effect employing inflation forecast equation. He finds no evidence of the short run effect, as most coefficient estimates are non-significant or even negative. However, according to Mishkin, the lack of short run relationship can not be interpreted as a rejection of the hypothesis. He concludes:

> "[Irving Fisher] viewed the positive relationship between inflation and interest rates as long-run phenomenon. The evidence in this paper thus supports a return to Irving Fisher's original characterization of the inflation-interest rate relationship."

Peng (1995)

Peng (1995) extends the analysis of the long run Fisher effect by including additional markets, namely France, UK, Germany and Japan. Three month interest rates and consumer price indices are utilized for a sample period of 1957 to 1994. Peng (1995) makes use of the ADF test for unit root analysis. The evidence of unit root in inflation and nominal rates is found for France, UK and the US, but not for Germany and Japan, where the presence of unit root in both variables is rejected at 1% level. Subsequently, different methods are used for the two groups. The first group with unit root processes is analyzed by Engle-Granger and Johansen methods. Such long run analysis is not applicable to the datasets of Germany and Japan, given the variables are I(0). Consequently, a traditional OLS equation is applied to these two markets.

The results provide support for strong cointegration in the US, UK and France. Utilizing Johansen methodology, Peng shows that cointegrating coefficients are very close to one for all three countries, suggesting evidence of one to one adjustment between expected inflation and interest rates. Somewhat weaker evidence for the Fisher effect in Germany and Japan is provided by the OLS regression as β coefficients are lower when compared to the US, UK and France.

Peng attributes the cross-country differences in the results, to different regimes of monetary policy. As he argues, the results for Germany relate to the function of the Bundesbank, which has been "formally independent and legally obliged to achieve and maintain price stability" and which conducts a very active anti-inflationary policy. Such policy has helped to successfully maintain persistently low inflation in Germany. The Bank of Japan, although not formally independent, is also identified with high anti-inflationary sentiment. Such monetary regimes have, according to the author, contributed to the relatively weaker Fisher effect.

Crowder and Hoffmann (1996)

Crowder and Hoffmann (1996) argue that considerably limited evidence has been found for the tax adjusted Fisher equation, whereby the inflation coefficient should hypothetically range between values of 1.2 to 1.5. This is partly a response to the conclusion of Evan and Lewis (1995), who found an inflation coefficient of less than one, which they attributed to the existence of structural breaks in inflation. Another study of reference, Mishkin (1992), has in fact observed coefficients insignificantly different from their tax adjusted range of 1.2 - 1.5.

Crowder and Hoffmann apply three month T-bill rates and price deflator data for the period 1952 to 1991. They estimate the long run equation using the Johansen reduced rank method, considering two sets of inflation data, one of which is tax adjusted. The estimates obtained for the non-adjusted and adjusted sets are 1.34 and 0.97 respectively, which yields support for the traditional tax adjusted relationship. These results are in contrast to previous studies (Mishkin 1992, Evans and Lewis 1995) which, as Crowder and Hoffman argue, is due to the application of the Johansen reduced rank approach in their own study. Finally, they conduct variance decomposition and innovations analysis. They find evidence that nominal rates fully accommodate an inflation shock to a tax adjusted level, although the adjustment period is quite long, ranging from 6 to 8 years. Crowder and Hoffmann conclude that such a slow adjustment creates significant downward pressure on real rates during the adjustment period.

Mehra (1998)

Mehra (1998) concentrates on the short term Fisher effect, studying predictive content of the bond rate with respect to future inflation. The author finds favorable evidence from cointegration tests, indicating the presence of the long run relationship. Subsequently, he employs a vector error correction model, with the correction term consisting of the bond-inflation spread. The model is also designed to control for monetary policy and real growth, by incorporating a federal funds rate spread and output gap.

Mehra finds that predictive content of the bond rate was significant prior to 1979, while the component which controls for monetary policy was not. Conversely, the bond rate is insignificant in the post 1979 period and the term that controls for the federal funds spread correlates negatively with future inflation, which implies that the Fed had a policy of containing inflation. Mehra concludes that deterioration in forecastability of inflation using the bond rate is due to the change in the Fed's policy stance, which was directed towards a more aggressive inflation reduction in the post 1979 period. The results confirm the theoretical proposition that active monetary policy weakens the Fisher effect.

4. Empirical Analysis

In this chapter we empirically examine the Fisher hypothesis over the recent period for four different markets; the US, Sweden, Germany and Switzerland. We analyze the hypothesis in three steps. First, we examine whether the long run adjustment between variables is in line with the tax adjusted hypothesis. Second, we relax the assumption of the REH and reexamine the conclusion. Last, we examine whether future inflation is forecastable from the current bond rate. The chapter is structured accordingly. The following section introduces the general framework of the methodology. Subsequent sections present the results of the three tests.

4.1 The Model and Methods of Hypothesis Testing

Johansen's VEC Model

To test the long run Fisher hypothesis, we apply the Johansen test of cointegration developed by Johansen (1988) and Johansen and Juselius (1990). The advantages of this approach over the Engel and Granger method is that it provides a more generalized framework, allowing for a number of the cointegrating relationships to be examined simultaneously and also facilitates statistical testing of the cointegrating vectors. The method is based on a Vector Error Correction system which can be represented by (19).

(19) $$\Delta y_t = \Pi X_{t-1} + \sum_{k=1}^{j-1} \Gamma_k \Delta y_{t-k} + \varepsilon_t$$

In our case, y_t is a 2x1 vector of two variables, nominal bond rates and inflation. Π is the coefficient matrix such that $\Pi = \alpha\beta$, whereby β is a matrix of cointegrating vectors and α is a matrix of adjustment parameters entering each equation. In this bivariate case, α and β are 2 x 1 vectors of coefficients. The second term represents the lag dynamics of the model, where Γ is a matrix of lag coefficients, and k is a number of lags. (Brooks 2002)

The test of cointegration is computed from the matrix Π, by ordering its eigenvalues λ. Two test statistics are available under the Johansen method, the *Trace* test (eq. 20) and the *Maximum Eigenvalue* test (eq 21).

(20) $$\lambda_{trace}(r) = -T \sum_{i=r+1}^{g} \ln(1-\lambda_i)$$

(21) $$\lambda_{\max}(r, r+1) = -T \ln(1-\lambda_{r+1})$$

The *Trace* test is designed to examine the hypothesis of r cointegrating vectors against the alternative of n vectors, whereby n is the number of variables. Under the *Maximum Eigenvalue* test, the alternative tested is $r+1$ vectors versus the null hypothesis of r vectors. Johansen and Juselius (1990) provide the critical values for these tests. (Hjalmarssonand and Österholm, 2007 and Brooks, 2002)

Additional Terms: Covariance Risk and Jensen's Alpha

Consider again the Fisher equation (17 and 18) derived under stochastic environment. It was argued by Evans and Lewis (1995) that two terms of equation (18) are inconsequential in the long run, as they are assumed stationary. Under this assumption, these terms do not appear in the cointegrating vector of equation (19). Nevertheless, these terms should remain in the VAR section of the system, the section which illustrates the short run dynamics of the model. Including these terms we may rewrite (19) as:

$$\Delta y_t = \Pi X_{t-1} + \sum_{k=1}^{j-1} \Gamma_k \Delta y_{t-k} + \Phi\Delta \operatorname{cov}[\Delta c_{t+m} +, \pi_{t+m}] + \Xi\Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t \tag{22}$$

where the third term represents the covariance between expected consumption growth and inflation and the last term represents Jensen's inequality. The representations Φ and Ξ are 2 x 1 matrices of coefficients which appear at the terms 3 and 4.

Bivariate Model Specification

In line with most of the previous literature, we start out with the assumption of the REH. As discussed above, this assumption is well represented in equation (10), whereby expected inflation is substituted by the realized future inflation. To fully illustrate all specifications of our model, we formulate:

$$i_t^m = \beta_0 + \beta_1 \pi_{t+m} + U_t \tag{23}$$

$$\Delta i_t^m = \alpha_0 + \alpha_1(\hat{U}_{t-1}) + \sum_{k=1}^{j-1} \alpha_{2k} \Delta \pi_{t+m-k} + \sum_{k=1}^{j-1} \alpha_{3k} \Delta i_{t-k}^m + \alpha_{4k} \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \alpha_{5k} \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t \tag{24}$$

$$\Delta \pi_{t+m} = \gamma_0 + \gamma_1(\hat{U}_{t-1}) + \sum_{k=1}^{j-1} \gamma_{2k} \Delta \pi_{t+m-k} + \sum_{k=1}^{j-1} \gamma_{3k} \Delta i_t^m + \gamma_{4k} \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \gamma_{5k} \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t \tag{25}$$

where (23) is the cointegrating equation and the coefficients β define the cointegrating vector. For each method, we test whether the cointegrating coefficients β_1 are significantly different from the tax adjusted level[5] of 1.35. Equations (24) and (25) describe the VAR section of the model, where the coefficients α_1 and γ_1 are the adjustment parameters. The lag coefficients α_{2k}, γ_{2k} measure the autocorrelation, while the lag coefficients α_{3k}, γ_{3k} provide information about causality. The realized future inflation[6] π_{t+m} is computed as:

$$\pi_{t+m} = \ln\left(\frac{p_{t+m}}{p_t}\right) \tag{26}$$

where p_{t+m} is the price level at time $t+m$. Note that m refers to the maturity of the bond, which is issued at time t. To correctly model the system, it is necessary that the bond maturity corresponds to the price index change from time t to $t+m$. The model is estimated at the monthly frequency of data and we

[5] It has been argued by Evans and Lewis (1995) that the hypothesized tax adjusted long run coefficient should be approximately 1.35.

[6] (25) is a simplified alternative to the equation (5)

begin the test by setting *m* equal to twelve months. The last two terms in equations (24-25) are computed as follows:

$$\text{(27)} \qquad \mathrm{var}[\pi_{t+m}] = \frac{1}{m}\sum_{i=t}^{t+m}(\pi_i - \overline{\pi}_i)^2$$

$$\text{(28)} \qquad \mathrm{cov}[\Delta c_{t+m}, \pi_{t+m}] = \frac{1}{m}\sum_{i=t}^{t+m}(gdp_i - \overline{gdp_i})(\pi_i - \overline{\pi}_i)$$

where the Jensen's alpha (27) is the twelve month variance of future inflation. For the covariance term in (28), we substitute the consumption growth with the real output growth and compute the twelve month covariance with inflation. National accounts data is only available on quarterly basis and does not correspond to the monthly frequency of inflation. In order to avoid any bias in the covariance term, we compute equation (28) such that inflation is averaged within each quarter.

Lag length Specifications

To estimate the above model, a specification regarding the lag length is necessary. Because it has been suggested that this test may be sensitive to the number of lags chosen, we take the following approach. Initially, we set a maximum lag length for the test to 15. This number is equivalent to 5 quarters, which is slightly lower compared with Mehra (1998), who uses 8 quarters. However, in our dataset we need to account for the monthly frequency of the data and the fact that beyond 15 lags the model may become highly inflated even with two variables. As a next step, we estimate the cointegration test starting with 15 lags and then dropping one lag at a time so that 15 models are estimated each time. Subsequently, we analyze whether the results are sensitive to the choice of the lag length. Finally, we select the VAR model with the lowest Schwarz information criterion[7].

Data

Concerning the choice of the interest rates, we employ yields on one year government bonds for all four countries. The yields on one year US Treasury bills and Swedish government bonds are collected from DataStream. Swiss Confederation bonds rates are obtained from Swiss National Bank. The government bonds in the Euro Area are issued only by the national governments and to the best of our knowledge, the Euro Area composite yields are not available for bonds with one year maturity. Therefore, we use 12 month Euribor money market rates instead, which are obtained from the European Central Bank. The price indexes for all countries are obtained from IMF International Financial Statistics.

[7] If the Schwarz information criterion is nearly the same for two nested models, we also consider the Akaike information criterion of the two models.

The test is conducted at the monthly frequency for the period from 07-2001 to 07-2007. This period is chosen to correspond to the test in the subsequent section 3.3, whereby the period is limited by the availability of survey data. The following section 3.2 presents the cointegration test results for the system (23-25), where the rational expectation hypothesis is assumed.

4.2 The Cointegration Test Assuming the REH: Test 1

Unit Root Analysis

We conduct a root analysis on both variables i and π_{t+m} which are used in the model (23-25). Two different tests are used for this purpose, the Augmented Dickey Fuller test and the KPSS test. The Augmented Dickey Fuller test (ADF) takes a unit root as the null hypothesis. This has lead to some criticism, because as some argue this test suffers from poor precision when variables are near unit root processes. Kwaitkowski et. al. (1992) have developed an alternative test (KPSS), which assumes stationarity under the null hypothesis, the so called stationarity test. Applying the combination of the stationarity and unit root tests allows for confirmatory analysis and higher robustness of the test results (Brooks 2002). We follow this approach and analyze the data using both tests. The first three rows of *Table 1* show critical values of the two tests under 1, 5 and 10 % significance levels. The diagram below displays the test results for bond rates and inflation of the four countries.

The results of the ADF tests show a general support for a unit root in both variables. In none of the cases, we are able to reject unit root at 5% level. The KPSS confirms the evidence of ADF only for the US and Sweden, where the test rejects stationarity for both variables. For Euro Area and Switzerland, the ADF test results are not confirmed by KPSS. Only for Swiss inflation is the stationarity rejected at 10% level.

To summarize, the results provide a strong evidence of a unit root in the variables from the US and Sweden. Conflicting evidence on both variables is found for the Euro Area and Switzerland. This result suggests a possibility that the variables are either both stationary or unit root or of different orders of integration. The last property would be particularly undesirable because it would imply the existence of no relationship between bond rates and inflation. On the other hand, if both variables are stationary, the cointegration analysis is not applicable.

Even though no clear evidence of unit root is found for the Euro Area and Switzerland, we still proceed with the cointegration test for all countries. It may be argued for instance that a failure to reject stationary is a result of the low number of observations, or the limited length of the period. At any rate, the unit root test results for the Euro Area and Switzerland must be interpreted with caution.

Table 1 Unit Root Analysis on i and π_{t+m}

	i - interest rates		π_{t+m} - realized future inflation	
Critical values	ADF	KPSS	ADF	KPSS
1%	-3.524	0.739	-3.527	0.739
5%	-2.902	0.463	-2.904	0.463
10%	-2.589	0.347	-2.589	0.347
US	0.050	0.802***	-1.379	0.767***
	(0.960)		(0.588)	
Euro Area	-0.652	0.290	-0.348	0.326
	(0.852)		(0.912)	
Switzerland	-1.358	0.300	-1.101	0.458*
	(0.598)		(0.712)	
Sweden	-1.496	0.552**	-0.567	0.354*
	(0.530)		(0.871)	

*,**,*** indicate the rejection of the null hypothesis at 10,5, and 1% level respectively. The null hypothesis of ADF is unit root. The null hypothesis of KPSS test is stationarity. The table displays test values for the respective test with p-values below in brackets.

Trace and Maximum Eigenvalue tests

To proceed further with the cointegration analysis, additional choice has to be made with respect to the trend specification. Most previous authors, including Mishkin (1992), Evans and Lewis (1995), Peng (1995) and Mehra (1998), typically assume that the real rate is mean stationary. Crowder and Hoffman (1996) also allow for a possibility that the real rate contains a linear trend. This assumption can be facilitated by allowing for deterministic trend in the vector of the cointegrating relationship.

To analyze whether the results are sensitive to such specification, we conduct the following exercise. The Trace and Maximum Eigenvalue test is performed using both specifications. First, the trend in the cointegrating equation is restricted to zero. Subsequently, we allow for a linear trend in the cointegrating vector. The test is carried out with a maximum lag length of 15 months and then re-estimated for lower numbers of lags, dropping one lag at a time. We analyze whether the results are sensitive to either the lag length or trend specification.

The summary of test results is available in *Table 2*. The table displays a cointegrating rank - the number of cointegrating vectors found significant at 5% level. Each cell refers to a different specification on the trend and lag length. The number before the slash shows the rank as estimated

Table 2: Trend and Lag Length specification summary by rank of the cointegrating relationship

System 1 (i, π_{t+m}),

	Restriction of No Trend				Linear Trend			
Lag length	US	Euro Area	Switzerland	Sweden	US	Euro Area	Switzerland	Sweden
1	0/0	0/0	0/0	0/0	0/0	0/0	1/1	0/0
2	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
3	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
4	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
5	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
6	0/0	0/0	0/0	0/0	0/0	0/0	1/0	0/0
7	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0
8	0/0	0/0	0/0	2/0	0/0	0/0	1/1	1/1
9	1/1	0/0	0/0	0/0	2/2	1/1	0/0	1/1
10	1/1	1/1	1/1	0/0	2/2	2/0	1/0	1/1
11	1/1	1/1	1/1	1/1	2/2	2/2	2/2	2/2
12	1/1	1/0	0/0	0/0	1/1	1/0	1/1	1/0
13	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1
14	0/0	0/0	0/0	1/0	1/1	1/1	1/1	1/1
15	0/0	0/0	1/1	0/0	1/1	1/1	2/2	1/1

The table presents the summary of the Trace and Maximum Eigenvalue tests. Each cell displays the rank of the cointegrating relationships at 5% level. The number before the slash refers to the rank according to the Trace test, followed by rank according to the Maximum Eigenvalue test.

by the Trace test, followed by the rank according to the Maximum Eigenvalue test[8].

The evidence suggests some sensitivity to both the trend specification and the lag length. There appears to be less evidence of the cointegration under the restriction of no trend. The test results for all countries are highly sensitive to the lag length and cointegration is only found for models between 10 to 12 lags. More supportive evidence is found under the specification of a linear trend. The support for the cointegration typically deteriorates only for the lag lengths below 7 months. For models with higher lag lengths, consistent support for cointegration is found.

In summary, the results show evidence in favor of a trend stationarity real rate. It appears that trending real rate provides a somewhat more flexible setting, compared to the restriction of no trend. We believe that this is because our sample consists of a fairly short period, which encompasses only one business cycle, wherein the development of the real rate may be well approximated by a linear trend.

To proceed, we allow for a linear trend in the cointegrating vector, and select the model with the optimal leg length according to the lowest Schwarz criteria. The minimum lag length for any model is set according to results from *Table 2* and only the models with lag levels supported by the Trace test are considered. The Trace and Maximum Eigenvalue test results for the chosen specifications are

[8] Finding at least one cointegrating vector is sufficient because there are only 2 variables in the system.

Table 3: Trace and Maximum Eigenvalue tests;

	US		Euro Area		Switzerland		Sweden	
	Trace	Max. Eigenv.	Trace	Max. Eigenv.	Trace	Max. Eigenv.	Trace	Max. Eigenv.
5% Critical Value	25.87	19.39	25.87	19.39	25.87	19.39	25.87	19.39
System (i, π_{t+m})								
eigenvalue	0.44***	0.44***	0.27**	0.27*	0.24**	0.24*	0.29***	0.29**
test statistics	44.89	34.35	29.07	18.95	28.49	17.37	40.84	20.49
p-value	(0.000)	(0.000)	(0.019)	(0.057)	(0.023)	(0.096)	(0.000)	(0.035)
k	12	12	12	12	8	8	11	11

*,**,*** indicate the rejection of the null hypothesis at 10,5, and 1%. Trace test has the null hypothesis of zero cointegrating vectors. The Maximum Eigenvalue test has the null hypothesis of zero cointegrating vectors against the alternative of one vector. The first row displays eigenvalues with the test statistics below and p-values in brackets. Last row reports the VAR lag length k, which was chosen using the Schwarz criterion.

presented in *Table 3*. The tests results for US and Sweden indicate cointegration at 1% significance level.

Cointegrating Vector: Long Run Test of the Fisher Hypothesis

We estimate the VEC system (23-25) for all four countries using the specifications described above. We impose two restrictions; first, $\beta_1=1$ and subsequently $\beta_1= 1.35$, the latter being the hypothesized tax adjusted level[9] of the inflation coefficient. The coefficient estimates of the cointegrating vector are found in *Table 4.*

The first row shows the unrestricted long run coefficients β_1, which represent the inflation term[10]. For the US and Sweden, the results suggest a fairly solid evidence of a long run Fisher effect. In line with most of the previous literature[11], which assumes REH, the coefficient estimates are below 1. The first restrictions $\beta_1=1$ is evidently supported for both US and Sweden. The results on the tax adjusted restrictions are not as clear-cut. We record that the restrictions for both countries are nearly rejected at 10% level.

The β_1 coefficients for the Euro Area and Sweden are estimated highly imprecisely. We observe that even the values of the standard t-tests indicate that coefficients are insignificant. Furthermore, the coefficient estimates are highly sensitive to the choice of the lag length. Estimates at other lag levels often lead to very high or even nonsensical results. The β_1 coefficient for the Euro Area is far from the hypothesized level, and even negative for Switzerland. Still, none of the restrictions are rejected for

[9] 1.35 is the rate hypothesized by Evans and Lewis (1996). Although some cross country differences in tax rates may exists, we believe this to be a reasonable approximation.

[10] According to standard procedure, the nominal rate term is normalized to 1.

[11] Evans and Lewis (1995), Mehra (1998)

Table 4: Coefficients of the Cointegrating Vectors

System 1 (i, π_{t+m}), vector $(i_{t-1} - \beta_1 \pi^e{}_{t-1} - \beta_2 v - \beta_0)$				
	US	Sweden	Euro Area	Switzeralnd
k	12	11	12	8
$\beta_1(\pi_{t+m-1})$	**0.69**	**0.88**	**3.14**	**-0.50**
st. Error	(0.37)	(0.17)	(3.53)	(0.665)
t-test	[1.86]	[5.11]	[0.89]	[0.753]
$\beta_2 v$ -trend	0.07	-0.04	0.15	0.08
β_0 - const.	-2.67	3.38	-11.34	-1.81
Restrictions:				
β_1=1, Chi-square st.	0.65	0.60	0.45	2.33
(Prob.)	(0.42)	(0.44)	(0.50)	(0.13)
β_1= 1.35, Chi-square st.	2.36	2.28	0.31	1.77
(Prob.)	**(0.12)**	**(0.13)**	**(0.58)**	**(0.18)**

The remarks *; **; *** indicate that the coefficient restrictions are rejected at 10, 5 and 1% levels, respectively.
The p-values are displayed in brackets. The results are estimated using the Johansen method.

either the Euro Area or Switzerland. We assume that imprecise coefficient estimates are the results of a low evidence of unit root in inflation and interest rates found in *Table 1,* or limited number of observations.

The trend coefficient β_2 are positive for all countries, with the mere exception of Sweden. A positive value of trend coefficient signals that the trend real rate has been positive over the sample period. This result is easy to explain as the period has been mainly dominated by a bull market. The estimates of the constant β_0 are not very informative under this specification, as they merely correspond to the intercept of the trend line.

4.3 Relaxing the Assumption of the REH: Test 2

Previous studies typically diverge in the choice of the inflation expectation proxy used for testing. Most authors of the recent literature, such as Mishkin (1992), Peng (1995), Crowder Hoffmann (1996) and Mehra (1998) have assumed the REH and employed some measure of realized future inflation, like consumer prices or GDP deflator. Few others, i.e. Evans and Lewis (1995), have employed survey expectations, typically Livingston Survey. Although the second approach has found limited application, it is possibly advantageous, as argued above, because it relaxes the implied assumption of the REH.

In this section we analyze whether the test of the Fisher hypothesis is sensitive to the assumption of the REH. To approach this question, we relax the REH assumption by employing expected inflation from survey data. As such, we simply replace the future realized inflation π_{t+m} by the inflation expectations π^e variable in (25-27) as follows:

(29) $$i_t = \beta_0 + \beta_1 \pi^e{}_t + \beta_2 v + U_t$$

(30) $$\Delta i_t = \alpha_0 + \alpha_1 (U_{t-1}) + \sum_{k=1}^{j-1} \alpha_{2k} \Delta i_{t-k} + \sum_{k=1}^{j-1} \alpha_{3k} \Delta \pi^e{}_{t-k} + \alpha_4 \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \alpha_5 \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t$$

(31) $$\Delta \pi_t^e = \gamma_0 + \gamma_1 (U_{t-1}) + \sum_{k=1}^{j-1} \gamma_{2k} \Delta \pi^e{}_{t-k} + \sum_{k=1}^{j-1} \gamma_{3k} \Delta i_{t-k} + \gamma_4 \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \gamma_5 \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t$$

All other specifications of the model and the methods are kept constant to allow for comparability with the results in the previous section. Below we address some data issues associated with the survey of expectations.

Data on Survey of Expectations

As suggested above, the use of survey expectations is one method to circumvent the REH assumption. The disadvantage is that we have to assume that survey expectations are representative of the market. While this criterion may be difficult to fulfill, few publications, notably *Livingston Survey* and *Consensus Forecast,* may be considered. *Livingston Survey* is issued twice a year by the Federal Reserve Bank of Philadelphia and contains forecasts of key US macroeconomic variables. The latter is issued by Consensus Economics Inc., a London based organization. *Consensus Forecast* is revised each month and contains forecasts on the variables of 70 countries. Both publications contain a panel of forecasts issued by some of the most important market participants, large banks and corporations. The opinions of these institutions have a significant weight, due to their own market activity and because they are often referred to by policy makers and private sector.

We select *Consensus Forecasts*, as it is the only major publication that covers a number of countries at the monthly frequency. The data is available only for limited period, from 07-2001 to 07-2007, which given the monthly frequency yields 73 observations. The forecasts are available at the horizons of 1 and 2 years. To formulate the test Fisher hypothesis correctly, the key idea is to match the forecast horizon m, formed at the time t, with the interest rate issued at the time t. To do so, a minor adjustment in the forecast dataset is needed.

To illustrate the problem, we briefly discuss how the *Consensus Forecast* panel is issued. The forecast of each variable is always issued on two different target horizons. The two forecast horizons are referred to as *current year* and *next year.* The *Current year* forecast is issued at time t and targets the inflation over the given year at time t. The *next year* is issued at the same time and it targets the inflation over the following year t+y. It is declared in the publication that the inflation forecasts target is the 12 month average of the annual inflation. We assume that all survey respondents adhere to this rule and the forecast target can be defined as follows:

$$\Pi^{T} = \frac{1}{12}\sum_{n=Jan}^{Dec}\left(\frac{CPI_{n}}{CPI_{n-12}} - 1\right) \tag{32}$$

where T is the target year and n is a calendar month of the given target year. Note that the forecast target the average inflation over the whole calendar year. The inflation value for each month is computed on an annual basis against the index level twelve months ago, CPI_{n-12}.

It is clear from equation (32) that each of the twelve publications issued within a single calendar year have the fixed forecast target. The *Consensus Forecast* issues for the months February to December contain only revisions of the first forecasts made in January. To summarize, we have a forecast with a yearly target, which is revised on monthly basis. It is easy to see that without any adjustment, this dataset is not very useful, as it can only be employed at yearly frequency, and the limited length of the dataset does not allow that.

However, we can use this data to approximately calculate the 12 month moving average of expected inflation at monthly frequency. This is done using combinations of the realized inflation, the *current year* forecast and the *next year* forecast. To begin, we convert all inflation measures into continuously compounded series using (33).

$$\pi_{\mathrm{t}} = \ln(1 + \frac{\Pi_{t}}{100}) \tag{33}$$

Next, the average expected inflation over the next 12 months can be defined as:

$$\pi^{\mathrm{e}}_{t,t+m} = \pi^{\mathrm{f}}_{t} - w_{\mathrm{t}}\overline{\pi}_{t} + w_{t}\pi^{\mathrm{f}}_{t+\mathrm{y}} \tag{34}$$

where, $\pi^{\mathrm{f}}_{\mathrm{t}}$ is the current year forecast, $\pi^{\mathrm{f}}_{\mathrm{t+y}}$ is the next year forecast and $\overline{\pi}_{t}$ is the average realized inflation of the current calendar year. There are twelve inflation realizations in any calendar year - one for each month. We define w_{t} as a simple weighting variable, which at time t signals how many realizations out of the twelve are already known. The known part of the twelve month average includes realizations of all previous months excluding the current. This is because the realized inflation for January is typically published at the beginning of February. Thus, in February *w* is 1/12 as only one of the twelve realizations is known. Equation (34) simply states that average expected inflation over next twelve months is the combination the *current year* and the *next year* forecast given by the *w* ratio, less the known proportion of the inflation average from the current calendar year.

The result of equation (34) is $\pi^e_{t,t+m}$; an inflation forecast issued at time *t*, with the target *t+m*, where *m* is 12 months. This adjusted forecast can be directly matched[12] with a yield on a one year bond issued at time *t*. Hence, this transformation allows for testing at monthly frequency. Below we report the estimated results from cointegrating relations (29-31), whereby we relax the assumption on the REH.

Test 2 Results and Interpretation

First, we conduct unit root analysis of the expected inflation $\pi^e_{t,t+m}$ and the results are reported in *Table 5.* The results are fairly consistent with the unit root result on realized inflation. The ADF test does not rejected unit root at 5% level for any country, although the result for US signals a near rejection. The KPSS test rejects stationarity for Sweden at 5%. The stationarity in the Euro Area and the US is rejected only at 10% level. No rejection occurs for Switzerland. Hence, somewhat conflicting evidence is found for Switzerland. We remind that the unit root test of interest rate above shows the conflicting evidence for both the Euro Area and Switzerland.

Table 5: Unit Root Analysis on $\pi^e_{t,t+m}$

	$\pi^e_{t,t+m}$ - expected inflation	
Critical values	ADF	KPSS
1%	-3.527	0.739
5%	-2.904	0.463
10%	-2.589	0.347
US	-2.804* (0.063)	0.440*
Euro Area	-2.346 (0.161)	0.379*
Switzerland	-2.238 (0.195)	0.154
Sweden	-1.763 (0.395)	0.492**

*,**,*** indicate the rejection of the null hypothesis at 10,5, and 1% level respectively. The null hypothesis of ADF is unit root. The null hypothesis of KPSS test is stationarity. The table displays test values for the respective test with p-values below in brackets.

We proceed with the estimation of the VEC model. We preserve all the specifications from

[12] It should be noted that the target for $\pi^e_{t,t+m}$ is one year average of the annual inflation, which for the forecast prepared at time *t* can be computed as:

$$\pi_t^T = \frac{1}{12}\sum_{n=t}^{t+m}\left(\frac{CPI_n}{CPI_{n-12}} - 1\right)$$

where *m* equals 12 months, and it is the time to maturity of the bond issued at time *t*. Note that this is not an exact equivalent to the future price level change as defined by equation (26). Nevertheless, for the purpose of the test it is safe to use it as a proxy for one year expected inflation.

Table 6: Trend and Lag Length specification summary by rank of the cointegrating relationship Test 2

System 2 (i, $\pi^{e}_{t,t+m}$),

	Intercept and No Trend				Intercept and the Linear Trend			
Lag length	US	Euro Area	Switzerland	Sweden	US	Euro Area	Switzerland	Sweden
1	0/0	0/0	0/0	0/0	0/0	0/0	0/1	0/0
2	1/1	0/0	0/0	0/0	0/0	0/0	0/0	0/0
3	1/1	0/0	0/0	1/1	1/1	0/0	0/0	0/0
4	1/1	1/1	0/0	1/1	0/1	1/0	1/1	0/0
5	0/1	1/1	0/0	2/2	1/1	0/0	1/1	1/1
6	0/0	0/0	0/0	2/0	0/0	1/0	1/1	0/0
7	0/0	0/0	0/0	2/2	1/1	0/0	1/1	1/1
8	0/0	0/0	0/0	2/2	0/0	1/1	1/1	2/0
9	0/0	0/0	0/0	2/2	1/1	0/0	0/0	1/1
10	0/0	0/0	0/0	2/2	0/0	1/0	0/0	1/1
11	0/0	1/1	0/0	2/2	2/2	2/2	1/1	1/1
12	0/0	1/1	0/0	2/0	1/1	1/1	1/1	1/1
13	0/0	1/1	0/0	2/0	1/1	1/1	1/1	1/1
14	0/0	1/1	0/0	0/0	1/1	1/1	1/1	1/1
15	0/0	1/1	0/1	0/0	1/1	2/2	2/2	1/1

The table presents the summary of the Trace and Maximum Eigenvalue tests. Each cell displays the rank of the cointegrating relationships at 5% level. The number before the slash refers to the rank according to the Trace test, followed by rank according to the Maximum Eigenvalue test.

the previous section to ensure that the system in this section corresponds to the REH model. This approach is also supported by the Trace and Maximum Eigenvalue test summary (*Table 6*), which are in accord with the results from the REH model. Here again, the cointegration results are found to be more sensitive to the lag length under the restriction of zero trend. An interesting finding is that the results are less sensitive to the choice of lag length, when the assumption of the REH is relaxed. We find that the support for cointegration is found with as few as 3-4 monthly lags. We recall that under the assumption of the REH, cointegration was found only for models with more than 7 lags.

To return to the discussion about a trend stationarity, *Figure 1* contains a plot of the expected real rate and expected inflation. Notice that the real rate for US during this period may contain an element of a linear trend. A mere optical examination of the chart shows that such trend is not seen in the movement of the expected inflation rate, suggesting that the source of this trend is the real rate. As such, the deterministic trend in the real rate may be a useful assumption.

The Trace and Maximum Eigenvalue test results for the chosen lag length specifications are presented in *Table 7*. It appears that the results for nearly all countries show significance of the cointegrating relationship at 1% level. We also notice that the optimal leg length for all four countries is generally lower, in comparison to the REH model. In summary, the cointegration tests indicate somewhat higher support for the long run relationship when the assumption of the REH is relaxed.

Figure 1: The Expected Real Rate and the Expected Inflation

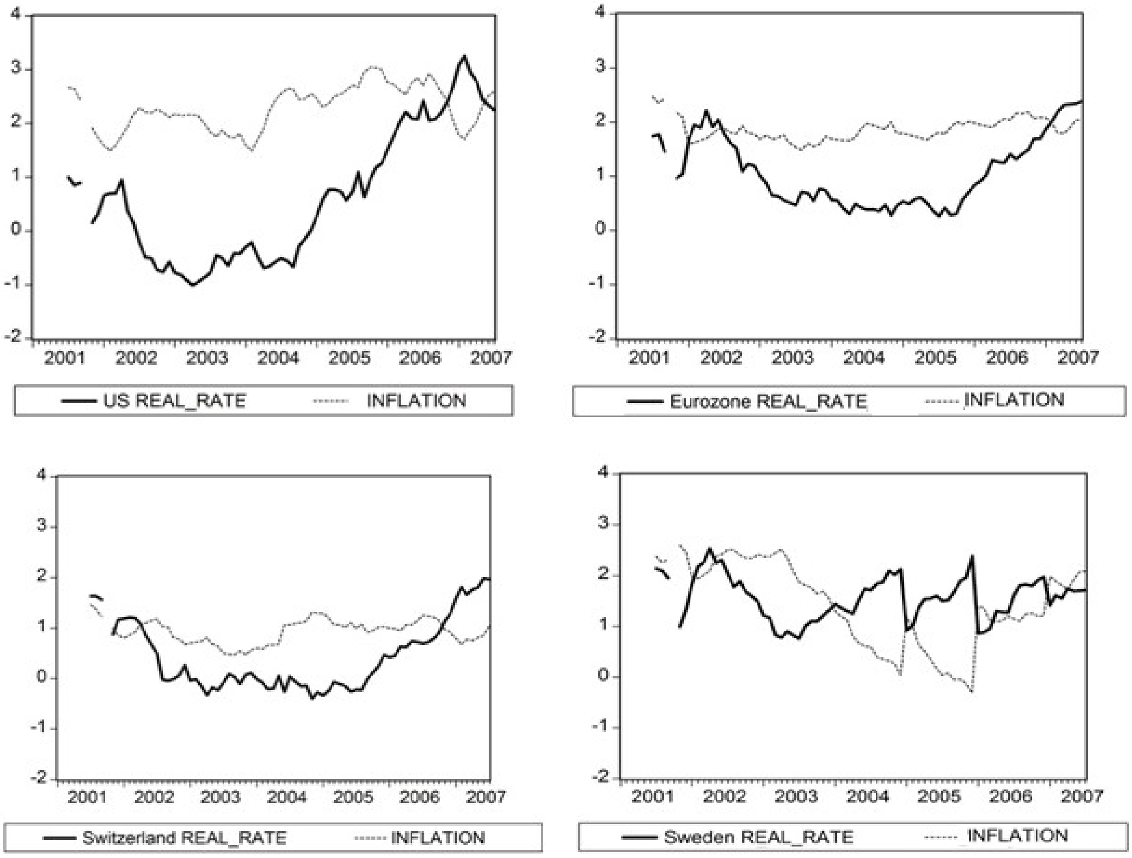


Table 7: Trace and Maximum Eigenvalue tests; System 2 (i, $\pi^e_{t,t+m}$)

	US		Euro Area		Switzerland		Sweden	
	Trace	Max. Eigenv.	Trace	Max. Eigenv.	Trace	Max. Eigenv.	Trace	Max. Eigenv.
5% Critical Value	25.87	19.39	25.87	19.39	25.87	19.39	25.87	19.39
System 2 (i, $\pi^e_{t,t+m}$)								
eigenvalue	0.37***	0.37***	0.32***	0.32***	0.30**	0.30**	0.34***	0.34***
test statistics	35.89	27.83	35.36	24.15	31.12	23.59	34.02	24.90
p-value	(0.002)	(0.002)	(0.002)	(0.009)	(0.010)	(0.012)	(0.004)	(0.007)
k	7	7	5	5	2	2	7	7

*,**,*** indicate the rejection of the null hypothesis at 10,5, and 1%. Trace test has the null hypothesis of zero cointegrating vectors. The Maximum Eigenvalue test has the null hypothesis of zero cointegrating vectors against the alternative of one vector. The first row displays eigenvalues with the test statistics below and p-values in brackets. The last row reports the VAR lag length k, which was chosen using the Schwarz criterion.

Table 8: The Coefficients of the Cointegrating Vector

System 2 (i, $\pi^e_{t,t+m}$), vector $(i_{t-1} - \beta_1\pi^e_{t-1} - \beta_2 v - \beta_0)$				
	US	Sweden	Euro Area	Switzerland
K	7	7	5	2
$\beta_1(\pi_{t+m-1})$	**1.89**	**1.80**	**3.96**	**-0.23**
st. Error	(0.42)	(0.37)	(1.39)	(1.22)
t-test	[4.46]	[4.94]	[2.85]	[0.19]
$\beta_1(\pi_{t-1})$	0.07	0.040	0.01	0.08
β_0 - *const.*	-4.60	-2.18	-4.98	-2.21
Restrictions:				
β_1=1; Chi-square st.	3.242	2.111	2.554	0.700
(Prob.)	(0.071)	(0.146)	(0.110)	(0.403)
β_1= 1.35; Chi-square st.	1.345	1.462	2.149	1.178
(Prob.)	**(0.246)**	**(0.227)**	**(0.142)**	**(0.278)**

The remarks *; **; *** indicate that the coefficient restrictions are rejected at 10, 5 and 1% levels, respectively.
The p-values are displayed in brackets. The results are estimated using the Johansen method.

Finally, the VEC model estimates are reported in *Table 8*. First we note that the results for the Euro Area and Sweden are not much affected by the change in the inflation variable. We recall that the *Test 1* estimates of the β_1 coefficient were 3.14 and -0.50 for the Euro Area and Switzerland respectively. Again, we suppose that these estimates may be affected by a conflicting evidence of unit root in the interest rate variables and possibly limited length of the dataset.

Of particular interest are the results of the cointegrating coefficient β_1 for the US and Sweden. Now, after the assumption of the REH is relaxed, we find that the coefficients for the US and Sweden are well above 1. The result are also in support for the tax adjusted restriction $\beta_1 = 1.35$, as hypothesized by Crowder and Hoffman (1996). Recall that the tests from section 3.2, where the REH assumption was implied by the use of future inflation realizations π^e_t, resulted in estimates of 0.69 and 0.88 for the US and Sweden respectively. Hence, it appears that the long run coefficient estimates are sensitive to the assumption of the REH. The results in *Table 7* are also in contrast to the evidence of some previous literature, including Mehra (1998) and Evans and Lewis (1995), who, relying on the assumption of the REH, find the inflation coefficients below 1.

In the previous two sections we have found that the results on the Fisher hypothesis may be sensitive to the assumption how inflation expectations are formed. To gain further insights into the problem, we study how inflation expectations are actually formed. We conduct an inflation forecast test and study whether rational expectations about future inflation are contained in the current bond rate.

4.4 Inflation Forecast Test *(Test 3)*

In this section we study whether bond rates contain useful information about expected inflation. For this purpose, we employed an adjusted version of the error correction model.

According to Mehra (1998), the short run movements in the bond rate do not always reflect the movements in the expected inflation, particularly if the real interest rate is variable. In this setting it is necessary to control for the influences of other variables that contribute to the movement of real interest rates. Mehra (1998) controls for the monetary policy and the state of the economy in the inflation forecast test and finds that results are sensitive to this conditioning.

We follow Mehra's approach and revise our model to control for the effects of monetary policy and the economic state, both of which may have a significant influence on the real interest rate component of the bond rate. The model (25-27) is adjusted as follows:

(35) $$i_t = \beta_0 + \beta_1 \pi_t + U_t$$

(36) $$cbr_t = \lambda_0 + \lambda_1 \pi_t + V_t$$

(37) $$\Delta i_t = \alpha_0 + \alpha_1 (U_{t-1}) + \alpha_2 (V_{t-1}) + \sum_{k=1}^{j-1} \alpha_{3k} \Delta i_{t-k} + \sum_{k=1}^{j-1} \alpha_{4k} \Delta \pi_{t-k} + \sum_{k=1}^{j-1} \alpha_{5k} \Delta gdp_{t-k} + \alpha_6 \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \alpha_7 \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t$$

(38) $$\Delta \pi_t = \gamma_0 + \gamma_1 (U_{t-1}) + \gamma_2 (V_{t-1}) + \sum_{k=1}^{j-1} \gamma_{3k} \Delta \pi_{t-k} + \sum_{k=1}^{j-1} \gamma_{4k} \Delta i_{t-k} + \sum_{k=1}^{j-1} \gamma_{5k} \Delta gdp_{t-k} + \gamma_6 \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \gamma_7 \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_t$$

where we define *cbr* as a central bank rate and *gdp* as the real output growth. The system is adopted in line with Mehra's model. It is defined by two cointegrating vectors which identify the interaction between inflation, the bond rate and the central bank rate. The change in the output growth is stationary so it only enters the VAR section of the model. Note that π_t is the current realized inflation computed as expressed in (39). All other specifications are as defined above.

(39) $$\pi_t = \ln\left(\frac{p_t}{p_{t-y}}\right)$$

Central Bank rates and Other Data

The system (35-38) relies on the realized values of the price indexes and the model is not restricted by the limited length of the dataset in the same way as the survey data of expectations. This allows for expanding the sample period, which in turn may eliminate the statistical problems associated with the limited number of observations. The objective is to maintain the focus of the study on the recent period. This is to avoid having very significant monetary regime changes in the selected sample period, which would further complicate the analysis. Hence we focus on the period which is mostly characterized by a monetary policy of price stability, at least for the selected countries. The selected sample runs from 01/1990 to 07/2007.

Concerning the measure of US monetary policy, we follow Mehra's approach and use the realized federal funds rate. Extending the analysis on three other European countries (markets), it is required that the most representative measures for monetary policy are selected for each one. We take the following approach. We plot the official central bank rates and the short term money market rates in charts for each country and visually examine the relationships. The idea is to select the rate that the bank uses as a main tool in steering the short term interest rates. Hence, we analyze which measure most closely corresponds to the short term money market rates. The charts are reported in Appendix B. The selected rates are ECB Repo Rate for Eurozone, Swedish Riksbank Repo rate and 3 month SNB LIBOR target rate for Switzerland. All of these rates are key official target interest rates as declared by each of the respective central banks.

Open market operations serve as one of the key monetary policy tools of the ECB and Riksbank, and the Repo rate is the general name of the signaling rate of both banks. Interbank rates are slightly higher than the official Repo rates. This is because repurchase agreements are issued against collateral, unlike interbank loans, which include the market risk premium. At certain points in time it appears that the rates deviate from the official target when the market appears to expect a rise in central bank rate. Yet in general, the data plots suggest that the banks are prepared to defend their targets quite firmly. At least in monthly frequency, the official target closely corresponds to the short term interbank rates. The Swiss National Bank also conducts open market operations, but their approach specifically relies on targeting 3-month LIBOR. Hence, the official LIBOR target is somewhat more representative of the policy stance, because the realized interbank rate barely deviates from the target, while the realized Swiss Repo rate may be more commonly influenced by liquidity shocks or other factors.

The selected policy measures from all three countries have been introduced only recently. In Switzerland, the 3 month LIBOR target was instituted only in 2000, after the Swiss National Bank reformed its operations. Prior to that, SNB had a policy of targeting money supply and the Discount rate was the key official interest rate. The Repo rate was introduced by Riksbank in May 1994, when it replaced the Marginal rate, which previously served as the key signaling rate. Another obstacle is that the aggregate data on the Euro Area is generally not available prior to the introduction of ERM II.

We substitute the missing part of the sample for the Eurozone by the German Repo rate. For Sweden and Switzerland, we employ the original policy rates to fill in for the missing data. The official bank rates for Sweden, Switzerland and the Euro Area are obtained from Swedish Riksbank, the Swiss National Bank and the European Central Bank respectively. The central bank rates for Germany and US are obtained from DataStream.

It is important to mention that the new specification of the model no longer allows for the use of one year bond rates, in the same way as carried out in the two previous tests. We observe that short bond term rates are highly correlated with central bank rates and the high level of multicollinearity may prevent a sensible statistical analysis. Therefore, we follow Mehra's approach and employ the yields on 10 year Government bonds instead. We estimate the system (35 – 38) using the yield on 10 year US Treasury bills, Swedish government bonds, Swiss Confederation bonds, German bunds and Eurobond composite, all of which are collected from DataStream. Last, as the measure of economic state *(gdp),* we implement annual output growth in quarterly frequency. The estimated results on system (35-38) are reported below.

Test 3 Results and Interpretation

Unit Root Test

As we extend our dataset to cover a considerably longer period and include two more variables, it is necessary to reconsider our conclusion regarding the evidence of a unit root. Table 9 reports the results of the unit root analysis for the extended dataset. Focusing first on the interest rate variables (i_t, cbr_t), the results suggest a strong evidence of unit root for all countries and both measures. Perhaps the only exception is the cbr_t in United States, where the results are slightly conflicting as the ADF test nearly rejects the null hypothesis at 5% critical level. The evidence of unit root in output growth gdp_t is also somewhat conflicting, particularly for the US and the Euro Area where the results of the KPSS test indicate no rejection of stationarity. Finally, we find a unit root in inflation for the Euro Area and Switzeralnd, as clearly indicated by both tests. In the US and Sweden the KPSS test lends support for unit root in inflation. However, the unit root for both the US and Sweden is clearly rejected by the ADF test at 1% level which yields conflicting results. Lacking the clear evidence of unit root in the US and Swedish inflation, we still proceed with the cointegration analysis for all countries. At any rate, the results for the US and Sweden should be interpreted with caution.

Table 9: Unit Root Analysis (Test 3)

	i_t - bond rates		π_t - inflation		cbr_t – central bank rate		gdp_t - gdp growth	
	ADF	KPSS	ADF	KPSS	ADF	KPSS	ADF	KPSS
Critical values								
1%	-3.461	0.739	-3.463	0.739	-3.462	0.739	-3.462	0.739
5%	-2.875	0.463	-2.876	0.463	-2.875	0.463	-2.875	0.463
10%	-2.574	0.347	-2.575	0.347	-2.574	0.347	-2.574	0.347
US	-2.174 (0.216)	1.549***	-3.730*** (0.004)	0.488**	-2.840* (-0.055)	0.512**	-2.135 (0.231)	0.180
Euro Area	-1.094 (0.718)	1.628***	-1.975 (0.298)	0.619**	-1.790 (0.385)	1.317***	-2.748* (0.068)	0.245
Switzerland	-1.434 (0.565)	1.601***	-1.894 (0.335)	1.060***	-1.766 (0.397)	1.185***	-1.238 (0.658)	0.385*
Sweden	-2.410 (0.140)	1.643***	-5.400*** (0.000)	0.879***	-1.841 (0.360)	1.591***	-1.822 (0.369)	0.669**

*,**,*** indicate the rejection of the null hypothesis at 10,5, and 1% level respectively. The null hypothesis of the ADF is unit root. The null hypothesis of the KPSS test is stationarity. The table displays test values for the respective test with p-values below in brackets.

Trace and Maximum Eighenvalue Tests

We proceed with the approach as described in the previous sections. Initially, we conduct a sensitivity[13] analysis to the lag length using the Trace test. Like in the first two tests, we observe that above a certain minimum lag level, the results of the tests are fairly consistent. Here again we consider only the lag levels which are supported by the Trace tests. It is necessary that at least 2 cointegrating vectors are identified as significant, because the model (35-38) is specified to contain two cointegrating vectors. Finally, the model with the lowest Schwarz information criteria is selected.

The results for the Trace and Maximum Eigenvalue tests for the selected lag lengths are displayed in *Table 10*, where the last row indicates the *r* number of cointegrating vectors at 5% level. The result of the Trace test suggests an evidence of three vectors for Switzerland and Sweden and only two for the US and the Euro Area. We also observe that the hypothesis of three cointegrating vectors fails to be rejected only marginally for the Euro Area. The Maximum Eigenvalue test indicates yet a lower number of cointegrating vectors for the US, Sweden and Switzerland.

[13]Following Mehra's (1998) specification, the system (35-38) is estimated without a trend in the cointegrating equations. The Trace test lag sensitivity analysis generally lends support for the model with no trend.

Table 10: Trace and Maximum Eigenvalue tests;

System (i, π_t, cbr_t, gdp_t)			United States		Euro Area		Switzerland		Sweden	
K			3		6		5		1	
	Trace	Max. Eigen.	Trace	Max. Eigen.	Trace	Max. Eigenv.	Trace	Max. Eigenv.	Trace	Max. Eigenv.
H0	5% critical values									
r=0	47.9	27.6	55.2** (0.01)	26.3* (0.07)	61.2*** (0.00)	28.3** (0.04)	73.8*** (0.00)	40.1*** (0.00)	56.4** (0.01)	25.7* (0.09)
r≤1	29.8	21.1	28.9* (0.06)	18.1 (0.12)	32.9** (0.02)	21.1** (0.05)	33.7** (0.02)	17.6 (0.14)	30.7** (0.04)	14.2 (0.35)
r≤2	15.5	14.3	10.9 (0.22)	7.2 (0.46)	12.6 (0.13)	7.7 (0.40)	16.1** (0.04)	12.4* (0.09)	16.5** (0.04)	12.0 (0.12)
r≤3	3.8	3.8	3.6** (0.05)	3.6** (0.05)	4.9** (0.02)	4.9** (0.03)	3.7** (0.05)	3.7** (0.05)	4.5** (0.03)	4.5** (0.03)
Number of vectors at 5% level			1-2	0	2	2	3	1	3	0

*,**,*** indicate the rejection of the null hypothesis at 10,5, and 1%. The trace test has the null hypothesis of r cointegrating vectors. The Maximum Eigenvalue test has the null hypothesis of r cointegrating vectors against the alternative of *r+1* vectors. The first row displays eigenvalues with the test statistics below and p-values in brackets. The last row reports the VAR lag length *k*, which was chosen using the Schwarz criterion.

As for the Euro Area and Switzerland, the above results lend support for the use of the model with 2 cointegrating equations. The evidence is somewhat less consistent for Sweden, where the Maximum Eigenvalue test indicates no cointegration at 5% level. Yet lower evidence of cointegration is found for the US.

Coefficients of the Cointegrating Vectors

Next, we estimate the VEC model using the lag levels displayed in the table 10. Mehra (1998) hypothesizes that the inflation coefficients in both equations are 1. Hence, we also adopt this approach and test for the restrictions $\beta_1=1$, $\lambda_1=1$. For the simplicity, we ignore the tax effect in this test. The coefficient estimates of the cointegrating vectors are reported in *Table 11*.

Consider first the results for the US and Sweden. The first glance suggests that the results for Sweden are somewhat irrational. The coefficient estimates from the inflation term are very high in both equations (5.75 and 5.27). Moreover the constant β_0 is negative. For the United States the inflation coefficient in the interest rate equation is 0.47 and the coefficient from the *cbr* equation is -1.08. The latter indicates a negative relationship between policy response and inflation, which suggests that low federal funds rate coincides with high inflation and vice versa. However, we also note that the coefficients for the US are estimated highly imprecisely as indicated by the standard errors and the

Table 11: Coefficients of the Cointegrating Vectors

Cointegrating regressions: $i_t = \beta_0 + \beta_1 \pi_t + U_t$; $cbr_t = \lambda_0 + \lambda_1 \pi_t + V_t$

	EuroArea		Switzeralnd		US		Sweden	
k	6		5		3		1	
Equation	*i*	*cbr*	*I*	*cbr*	*i*	*cbr*	*i*	*cbr*
$\beta_1(\pi_{t-1})$ *and* $\lambda_1(\pi_{t-1})$	0.68	1.55	0.85	1.14	0.47	-1.08	5.75	5.27
st. Error	(0.22)	(0.21)	(0.13)	(0.18)	(0.46)	(0.98)	(1.33)	(1.15)
t-test	[3.02]	[7.23]	[6.34]	[6.45]	[1.01]	[1.10]	[4.32]	[4.57]
β_0 *- const.*	3.84	0.58	2.46	0.63	4.31	7.24	-6.56	-6.57
Restrictions								
β_1*=1,*λ_1*=1, Chi-square st.*	5.59*		1.37		4.33		8.27**	
(P-value)	(0.06)		(0.50)		(0.11)		(0.016)	

The remarks *; **; *** indicate that the coefficient restrictions are rejected at 10, 5 and 1% levels, respectively. P-values from the Chi-square test are displayed in brackets. The results are estimated using the Johansen method.

t-test values of both β_1 and λ_1. The joint restriction on β_1 and λ_1 is strongly rejected for Sweden and almost rejected for the US. In brief, no strong conclusion can be drawn from the results of these countries. The results for Sweden appear nearly nonsensical and the coefficient estimates for the US are estimated highly imprecisely.

We assume that the results for this period might be suffering from the fact that the variables are possibly of different orders of cointegration. Recall that in *Table 9* we have found some evidence that inflation is I(0) and strong evidence that interest rates are I(1). The different orders of integration in the variables might be preventing us from a sensible cointegration analysis. Hence, for the remainder of the section we focus on the remaining two markets, the Euro Area and Switzerland.

As for the Euro Area and Switzerland, the cointegration coefficient β_1 estimates are 0.68 and 0.85 respectively. This is somewhat lower than the hypothesized tax adjusted level of 1.35, but not necessarily in conflict with the Fisher hypothesis. This is because β_1 informs about the relationship between the bond rate and the current rate of inflation, which is in contrast to the hypothesis which assumes the expected future values.

The estimated values of the coefficients λ_1 are 1.55 and 1.14 for the Euro Area and Switzerland, respectively. The result indicates that the relationship between inflation and central bank rate is positive, and central bank rate fluctuates somewhat more rapidly than inflation. The restrictions $\beta_1=1$, $\lambda_1=1$ are nearly rejected by the Chi-square test for the Euro Area but not for Switzerland.

Inflation Forecast Test

Finally, we conduct the inflation forecast test. The test is estimated with the restrictions $\beta_1=1$, $\lambda_1=1$, where the restrictions are supported by the data. The test for Sweden is conducted without the restrictions as the restrictions are rejected at the 5% level. *Table 12* presents the selected results from the inflation forecast test. For the inflation equation we also conduct the Granger Causality Wald Test on the joint significance of the lags.

The first row of the table displays the estimates of the coefficients α_1 which describe the correction mechanism for U_{t-1}, which stands for the deviations from equilibrium between inflation and bond rate. Recall that, if the deviations are corrected by the subsequent adjustment in the inflation at time t, the term U_{t-1} is significantly positive in the inflation equation. If the deviations are corrected via an adjustment in bond rate, then the term is significantly negative in the bond rate equation. Hence, the expected sign for U_{t-1} is negative in the bond rate equation and positive in the inflation equation.

The estimated coefficients on the U_{t-1} term are as expected, for all countries. However, not all coefficients are significant. For one, in Sweden both coefficients α_1 and γ_1 are insignificant, suggesting that the evidence for cointegration is rather low. In the US, only the coefficient α_1 from the bond rate equation is significant. This suggests evidence of the so-called weak causality, meaning that in the US, the inflation weakly causes the bond rate. A converse conclusion is found in the Euro Area, where the coefficient results indicate that the bond rate weakly causes inflation. In Switzerland, both coefficients α_1 and γ_1 are significant, suggesting that the deviation from the long run equilibrium value were corrected partially through the adjustment of the bond rate and partially through inflation, meaning the causality works both ways. Comparing the two coefficients α_1 and γ_1 , we note that γ_1 is almost twice larger, which suggests that inflation responds to the bond rate faster than the other way around.

The results for the Euro Area and Switzerland can also be described as an evidence of a predictive content of inflation found in the bond rate. In other words, lenders seem to rationally expect some shocks to inflation even over such a short horizon as one month. On the other hand, in Switzerland the bond rate also responds to past inflation, suggesting that not all inflationary shocks are expected. Unexpected shocks simply materialize in the bond rate over the next period. In the US, there is little evidence of forecastability of the inflation, and inflation shocks take at least one month to materialize in the bond rates.

To verify the above findings on the forecastability of inflation, we also conduct the Granger causality test. This test examines the joint significance of the lagged bond rate in the inflation equation. The

Table 12: VEC Granger Causality/Block Exogeneity Wald Tests

Cointegrating regressions: $i_t = \beta_0 + \beta_1 \pi_t + U_t$; $cbr_t = \lambda_0 + \lambda_1 \pi_t + V_t$

$$\Delta i_t = \alpha_0 + \alpha_1 (U_{t-1}) + \alpha_2 (V_{t-1}) + \sum_{k=1}^{j-1} \alpha_{3k} \Delta i_{t-k} + \sum_{k=1}^{j-1} \alpha_{4k} \Delta \pi_{t-k} + \sum_{k=1}^{j-1} \alpha_{5k} \Delta gdp_{t-k} + \alpha_6 \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \alpha_7 \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_1$$

$$\Delta \pi_t = \gamma_0 + \gamma_1 (U_{t-1}) + \gamma_2 (V_{t-1}) + \sum_{k=1}^{j-1} \gamma_{3k} \Delta \pi_{t-k} + \sum_{k=1}^{j-1} \gamma_{4k} \Delta i_{t-k} + \sum_{k=1}^{j-1} \gamma_{5k} \Delta gdp_{t-k} + \gamma_6 \Delta \operatorname{cov}[\Delta c_{t+m}, \pi_{t+m}] + \gamma_7 \Delta \operatorname{var}[\pi_{t+m}] + \varepsilon_2$$

	Euro Area		Switzerland		US		Sweden	
k	6		5		3		1	
	Δi	$\Delta \pi$	Δi	$\Delta \pi$	Δi	$\Delta \pi$	Δi	$\Delta \pi$
U_{t-1}	-0.004	0.068**	-0.030**	0.065**	-0.055**	0.032	-0.004	0.009
	[0.27]	[2.68]	[2.11]	[2.54]	[3.20]	[1.28]	[0.36]	[0.341]
V_{t-1}		0.030		-0.035		-0.01		0.001
		[1.46]		[-1.26]		[0.24]		[0.037]
$\Sigma\Delta i_{t-k}$		-0.456		-0.145		-0.138		-0.058
Chi^2 (p-value)		(0.18)		(0.54)		(0.55)		(0.68)

The dependant variables are shown in heading row. The first two rows of the table displays coefficient estimates on the error terms U_{t-1} and V_{t-1}. The third row displays sum of the lagged coefficients $\Sigma\Delta i_{t-k}$ followed by the Chi-square test on the joint significance of the bond rate lags. The 1%, 5% and 10% critical value for the t-tests of given equations are 2.76, 2.05, 1.70 respectively.

results are found in the third row of *Table 12*. The first line displays the sum of all the lagged bond rate coefficients $\Sigma\Delta i_{t-k}$, and the Chi-square statistics from the Granger causality test is reported below. We expect the coefficient sum to be positive, if the bond rate includes the predictive content about inflation. On the contrary, we observe that the lagged coefficients are negative for all countries. On the other hand, the coefficients are jointly insignificant. Hence, no strong inferences can be made on the basis of this test, as there is not enough evidence to reject the hypothesis of inflation forecastability.

We also briefly consider the effect of monetary policy, by analyzing the coefficient estimates on the error term V_{t-1}. The interpretation of this coefficient should be as follows. If V_{t-1} is positive, the deviation from the long run equilibrium between *cbr* and inflation is corrected by an adjustment in the inflation. This means that inflation rises at time *t* after an increase in *cbr* occurs at time *t-1*. Alternatively, if the coefficient has negative sign, then inflation decreases after a rise in *cbr*. The results show that the coefficient on V_{t-1} is negative for Switzerland and the US and positive for the Euro Area and Sweden. However, the coefficients are very close to zero for the US and Sweden. In the Euro Area and Switzerland, the coefficients are slightly higher, but still insignificantly different from zero.

In addition, we find that the inflation forecast coefficients U_{t-1} are very low in absolute values. Notice in the first row of *Table 12* that γ_1 is 0.068 and 0.065 for the Euro Area and Switzerland, respectively.

However, the magnitude of the coefficients is expected to be low, at least for two reasons. First, we measure an immediate response to the rise in the long term bond rate over the period of one month, which is a very short period of time, particularly because the expectations contained in the bond rate have a ten year horizon. Second, inflation is measured as an annual change in the price level and hence it contains a considerable moving average component, which makes the monthly inflation changes appear even smaller. Still, these estimates are comparable to the results of Mehra (1998), who reported an inflation forecast coefficient 0.32 for United States over the period 1961 to 1979. The obvious reason why Mehra's coefficient estimate is more than 4 times larger is that he uses a quarterly frequency of observations and hence the response period of inflation is 3 times longer.

In brief, we find some evidence that rational expectations about future inflation are contained in the bond rates of the Euro Area and Switzerland. The evidence suggests that inflation responds to the rise in the bond rate even with a period of only one month. However, it may be of interest to further examine how inflation responds over an longer horizon. From a theoretical prospective, if rational expectations on inflation are contained in the 10 year bond rate, they may require up to 10 years to fully materialize. To analyze inflation forecastability over an longer horizon, we examine the impulse response function between the inflation and the bond rate.

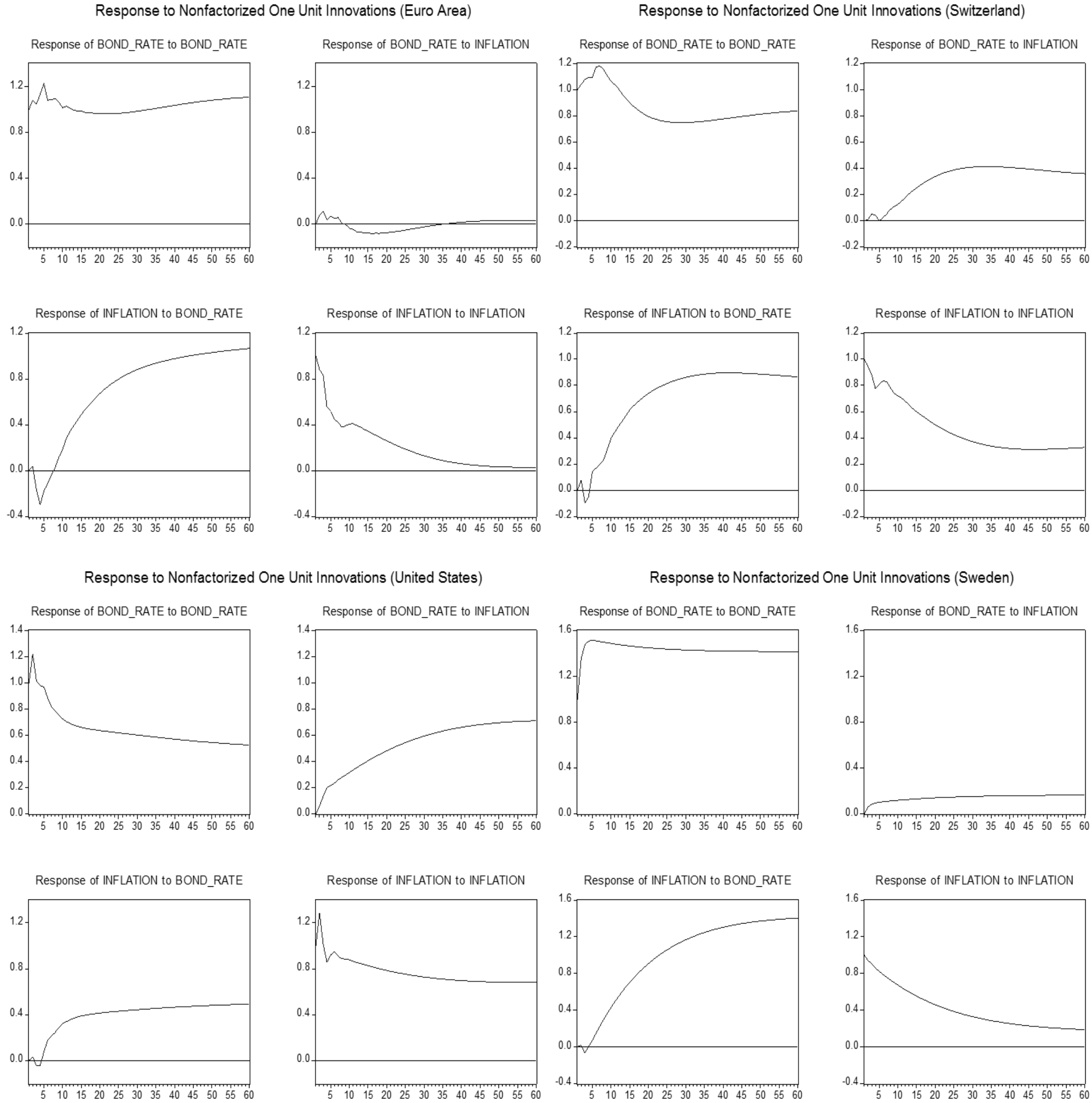
Impulse Response Function

We test the response of inflation and nominal rates to the 'non-factorized one unit' innovation in the other variable. The impulse response graphs are found in Figure 2. Each impulse response function is estimated on the model as specified in the inflation forecast test.

Figure 2 reports a 2x2 diagram of charts for each country. Each chart measures the response function over 60 periods, which is equivalent to 5 years. The first row of the charts in each diagram displays the response function of bond rate. The second row displays the response function of inflation.

Let us first focus on the results for the Euro Area. The first chart indicates that the shocks to the bond rates are permanent. Conversely, the fourth chart indicates that shocks to inflation die out completely over a period of around 4 years. The third chart indicates that a one unit shock to the inflation generates a one unit response of inflation. The response of inflation to the bond rate is gradual and it takes approximately three years to fully reach the level of one unit. On the other hand, bond rate does not respond to inflation nearly at all. These results indicate that some inflation shocks die out, but the shocks that do not die out are fully anticipated. Hence, the bond rate includes complete informational content about future inflation.

Figure 2: Impulse Response Function without Factorization



Similar results are also found for Switzerland. We observe that the shocks to the bond rate are fully persistent, while the shocks to inflation gradually die out, although not completely. Around 30% of one inflation shock persists. Again, the response of inflation to a shock in the bond rate is very close to unity, within a horizon of 3 years. A shock to inflation also generates a response from the bond rate and the response is stabilized to the level of around 30%. This result indicates that shocks to inflation are partially anticipated, and partially unexpected. The unexpected part of a shock to inflation is smaller than the expected part. The unexpected component of a shock is reflected in the bond rate with some time lag.

In the US, more than half of the shocks to inflation and bond rates persist. One unit of a shock to interest rates generates response of 0.4 units in inflation, which suggest that inflation shocks are anticipated only partially. The response of the bond rate to inflation is more rapid and stabilizes around the level of 0.6. This result suggests that around 0.4 of one shock to inflation is expected, as an information content of the bond rate. The remaining part of the shock is unexpected, and the bond rate adjusts for this information, shortly after the shocks occur.

In Sweden, shocks to inflation die out, while the shocks to bond rates persist. Inflation shocks also appear to be fully expected via the bond rate.

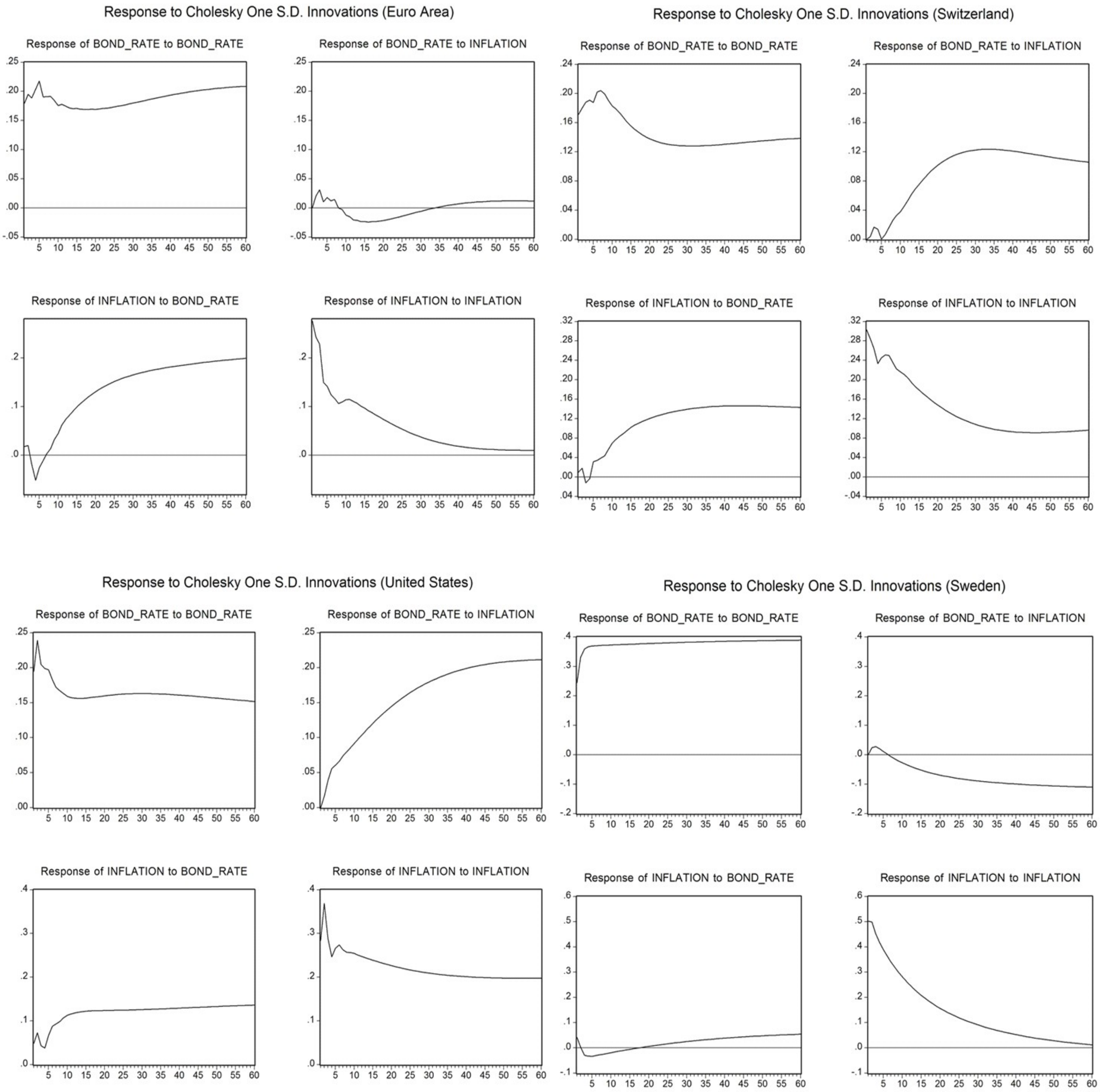
It has been suggested by Brooks (2002) that the above test is highly sensitive to the ordering of variables. Hence, we also test for different ordering and the results do not change visibly. Brooks (2002) also suggest that the results may be misleading, if the responses are not factorized. We reconsider the above results by means of Cholesky d.f. factorization, which uses a Cholesky factor of the residual covariance matrix to factorize the shocks. The factorized impulse response analysis reveals the following. The scaling of responses is different for all countries. However, the interpretation for the Euro Area and Switzerland is nearly the same.

We take the Euro Area as an example. The initial shock to inflation and interest rate is around 0.20. The inflation shock dies out at nearly the same pace as before, the shock to the interest rate persists. The persistent shocks to inflation are anticipated in the one to one fashion, as the inflation response to the bond rate stabilizes at the level of 0.20 within three years. Hence, we conclude that the bond rate contains predictive information about future inflation and the permanent shocks to the inflation are expected.

Also for Switzerland the same conclusion applies as above. It still holds that one inflation shock is partly expected and partly unexpected. It seems that around half of the shock is unexpected and the unexpected component translates into the bond rate with some time lag. The factorization has also no apparent effect on the interpretation of the US results.

The factorization changes the interpretation only for Sweden, where it becomes clear that inflation and bond rate share nearly no relationship. Expectations about the future inflation are not contained in the bond rate. Moreover, when inflation shocks occur, the bond rate responds negatively. Hence, the results provide no evidence of any positive relationship. Again, we suspect this may be a consequence of the fact that Swedish variables are possibly of different orders of integration.

Figure:3 Impulse Response Function with the Factorization



5. Concluding Remarks

In this paper we have examined the Fisher hypothesis on the markets: the US, the Euro Area, Switzerland and Sweden. First, we tested the tax adjusted level of the long run coefficients, using two different assumptions about the formation of expectations. Second, we tested whether the long term bond rate includes a predictive content regarding future inflation. Based on the results, we summarize in three main points.

First, we find that results on the tax adjusted Fisher hypothesis are sensitive to the assumption of the rational expectation hypothesis. When we relax the REH assumption by employing survey expectations, we observe that the interest rates adjust for inflation at, or above, the hypothesized tax adjusted range.

Second, we find that the evidence on the Fisher hypothesis is sensitive to the period of choice as well as the choice of the bond rate variable. We observe that evidence of the Fisher effect for the US and Sweden is strong for the short term bond rates, but weak using the long term bond rates. Conversely, the evidence on the hypothesis in the Euro Area and Switzerland is found only using the long term bond rates. Partly, these differences can be explained by the fact that the order of integration in the inflation and bond rates is sensitive to the choice of a particular bond rate variable and the time period selected for inflation. We record that when the unit root analysis is inconclusive, the cointegration results provide a little support for the Fisher effect.

Third, the bond rate is found to include a clear predictive content for the Euro Area and Switzerland. For both markets, the expectations contained in the 10 year bond rates are rationally formed, and may take up to three years to materialize. Permanent shocks to inflation in the Euro Area are almost fully anticipated. Shocks to the Swiss inflation are anticipated at least partially. In the US and Sweden, inferences about the predictive content of the bond rate are affected by the weak evidence of unit root in the inflation.

Further research may be directed at extending the analysis on a longer time horizon for a larger panel of countries, to expand the analysis of inflation forecastability under different monetary regimes.

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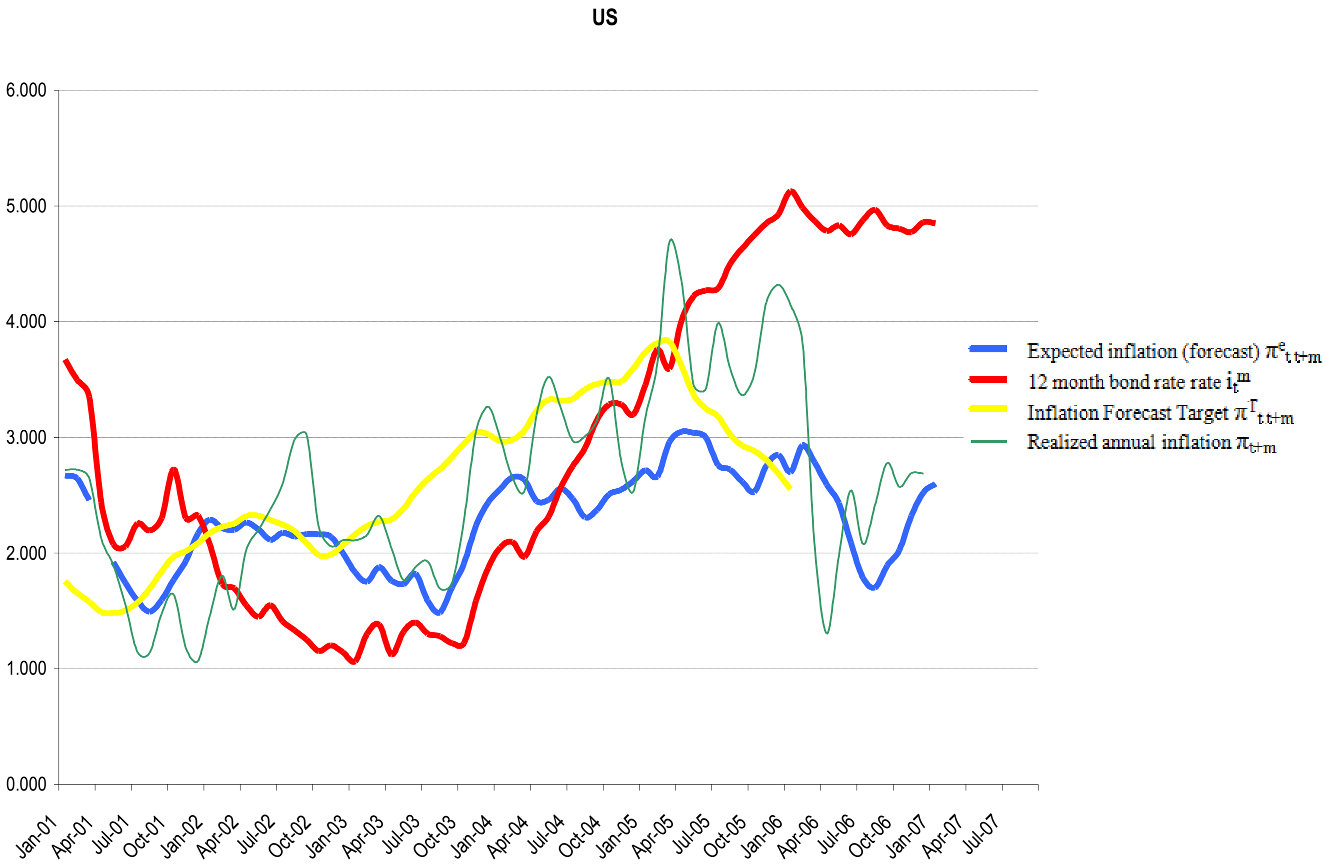
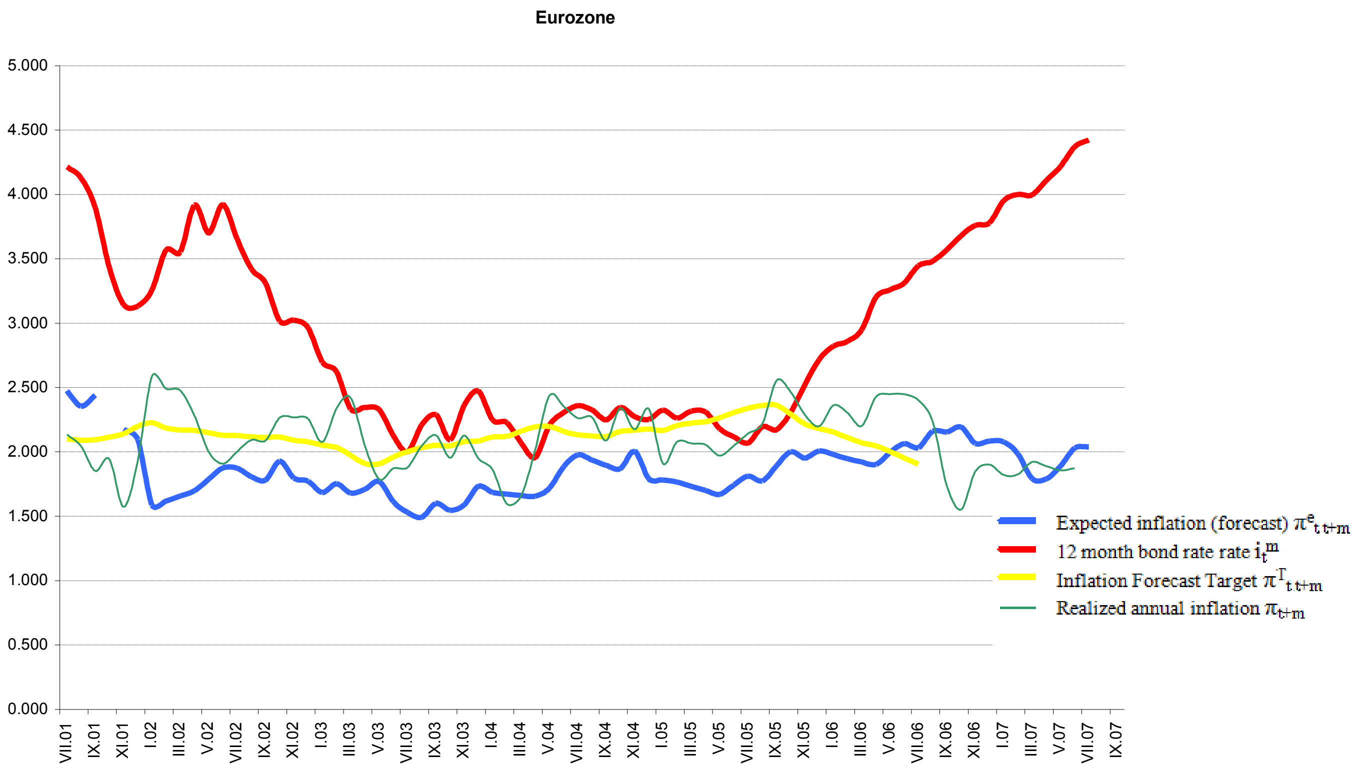
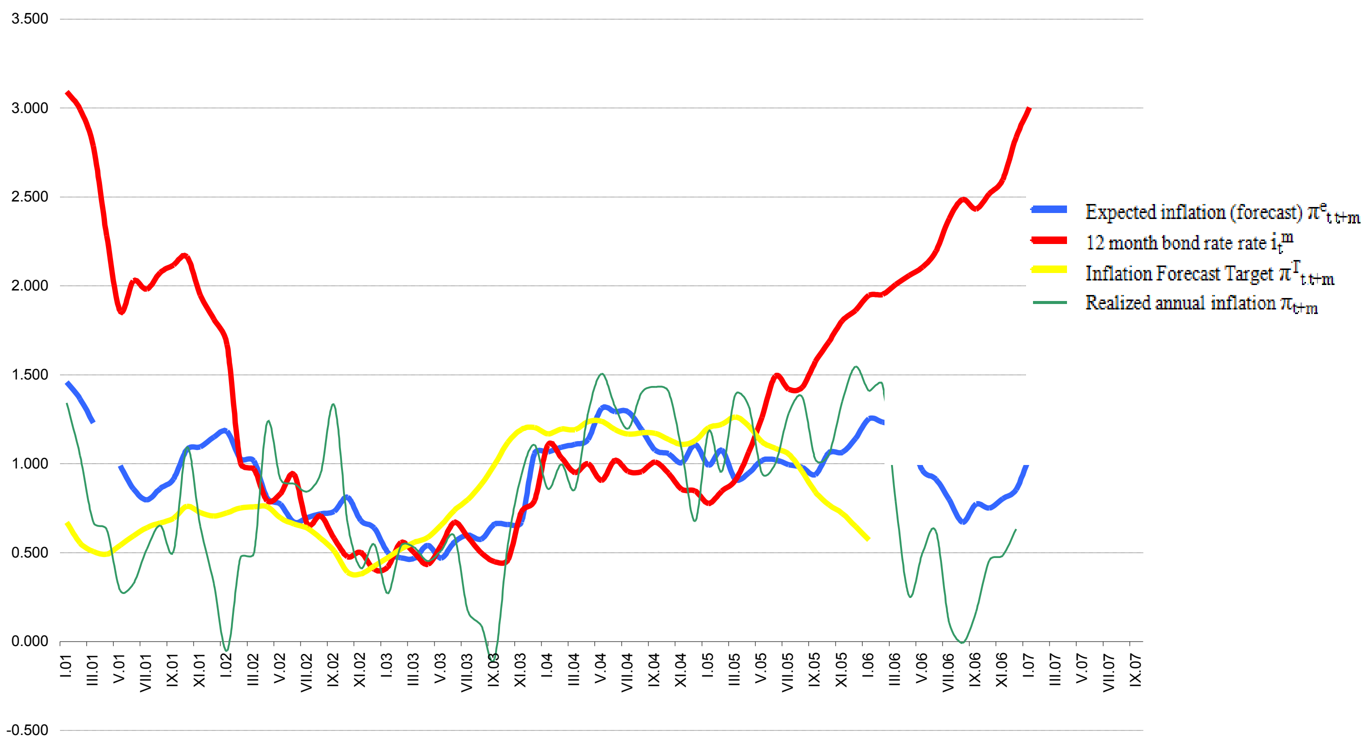
APPENDIX A: INFLATION AND NOMINAL RATE CHARTS (TEST 1 & 2)

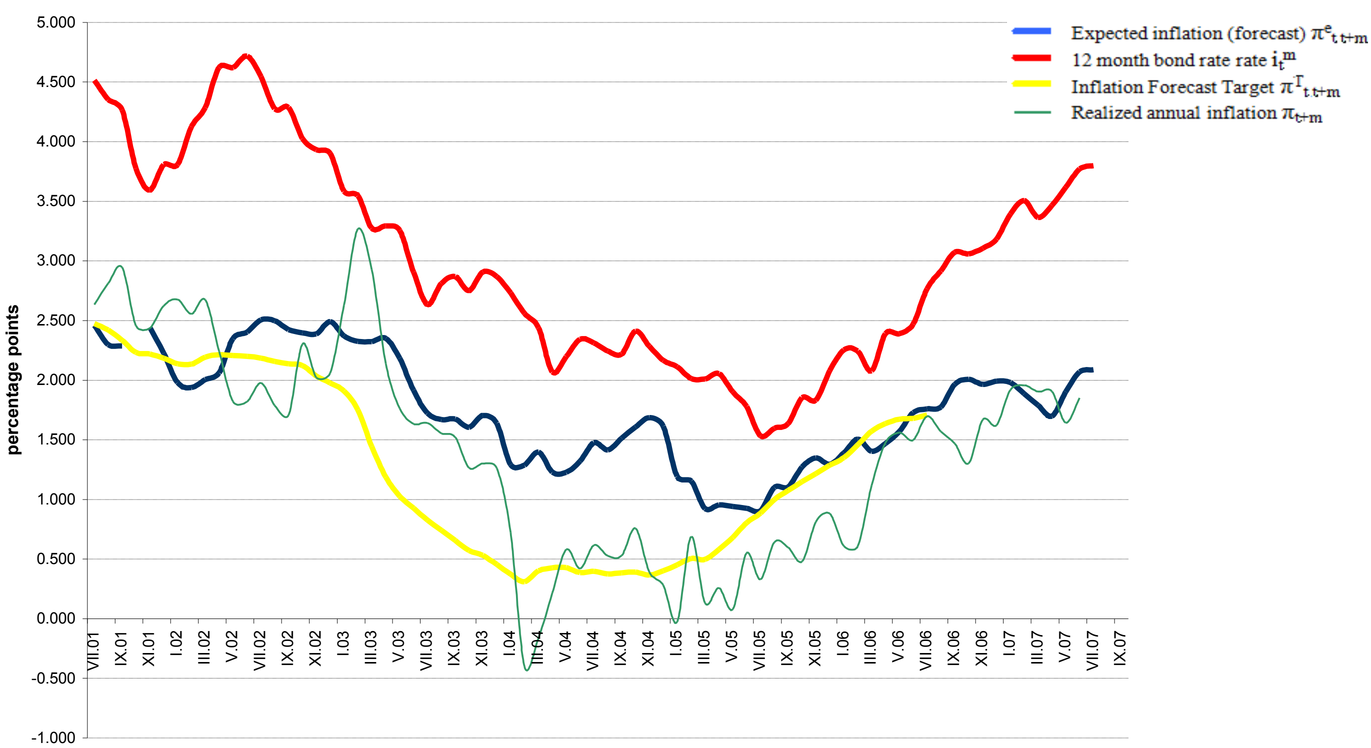




Switzerland

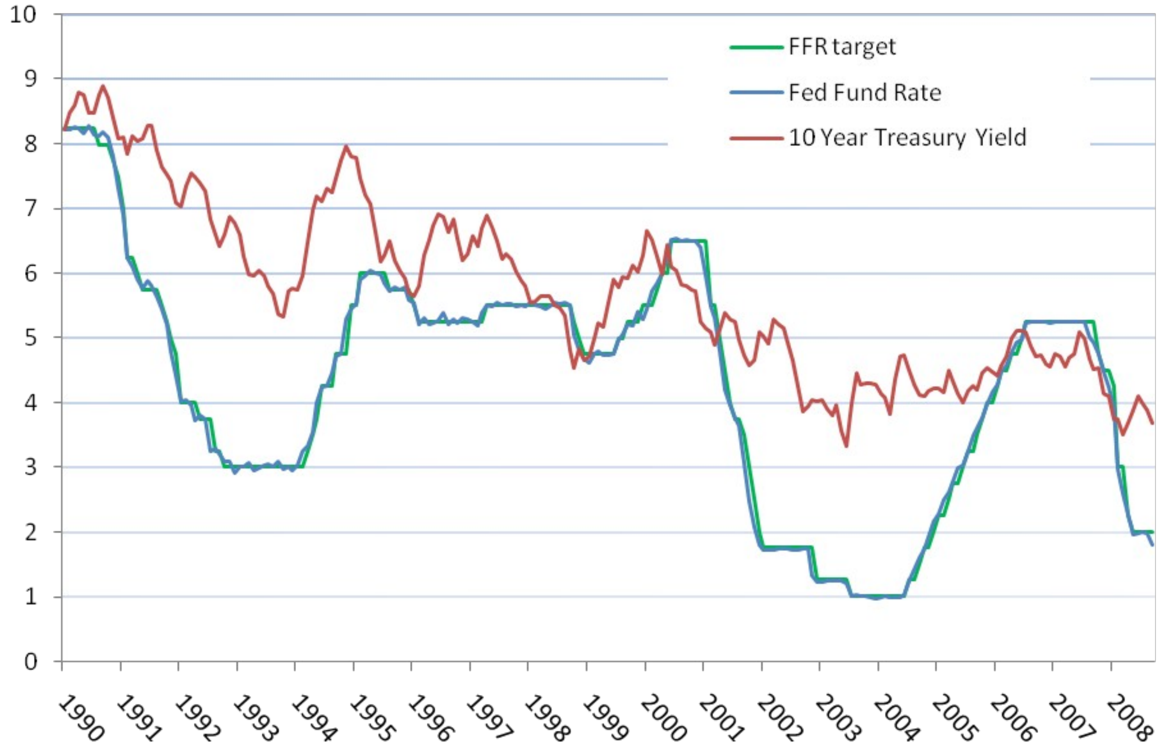


Sweden

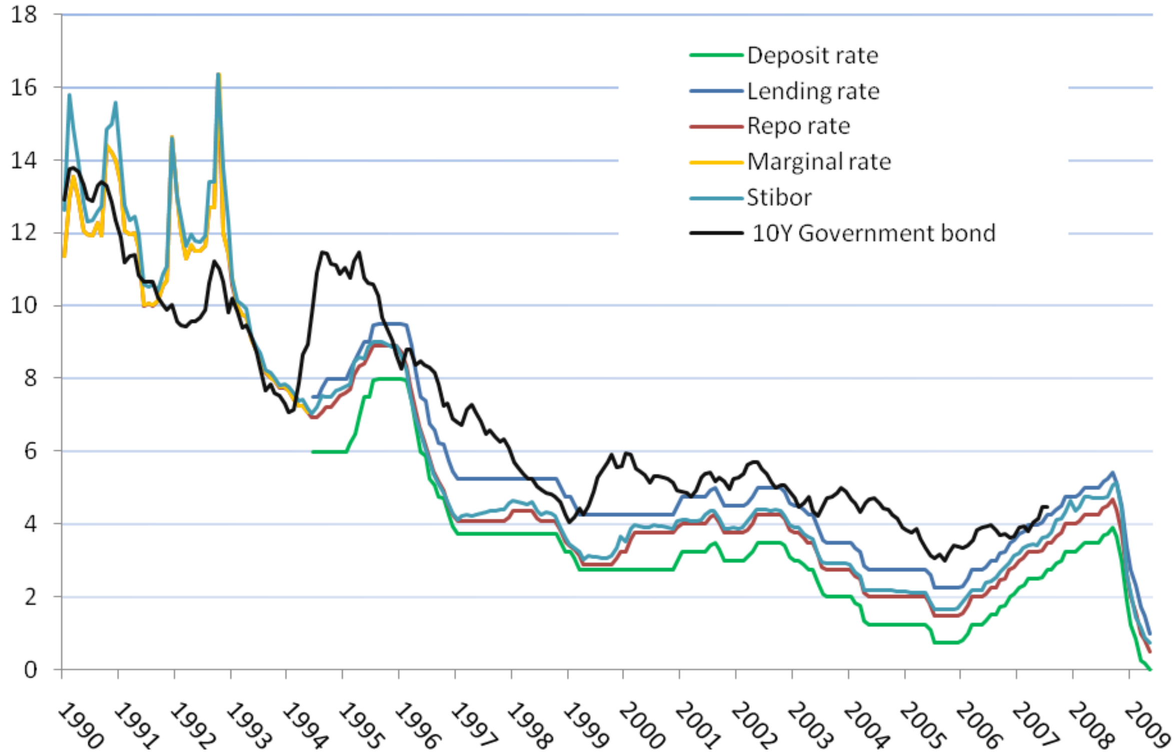


APPENDIX B: ANALYSIS OF THE CENTRAL BANK RATES (TEST 3)

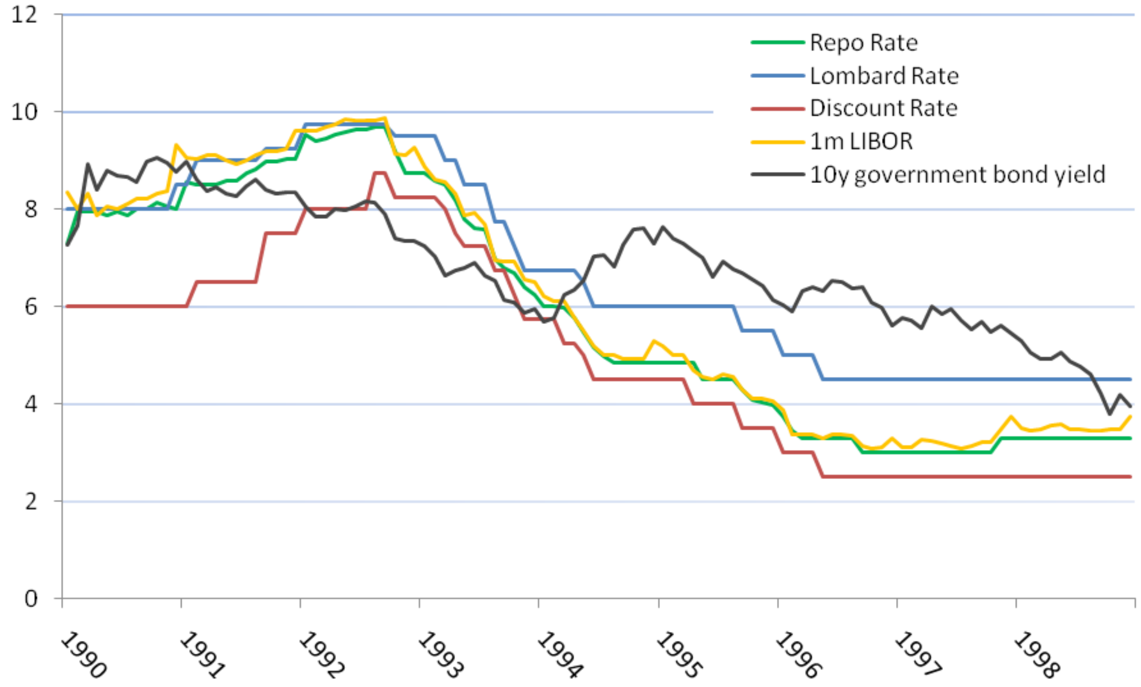
United States



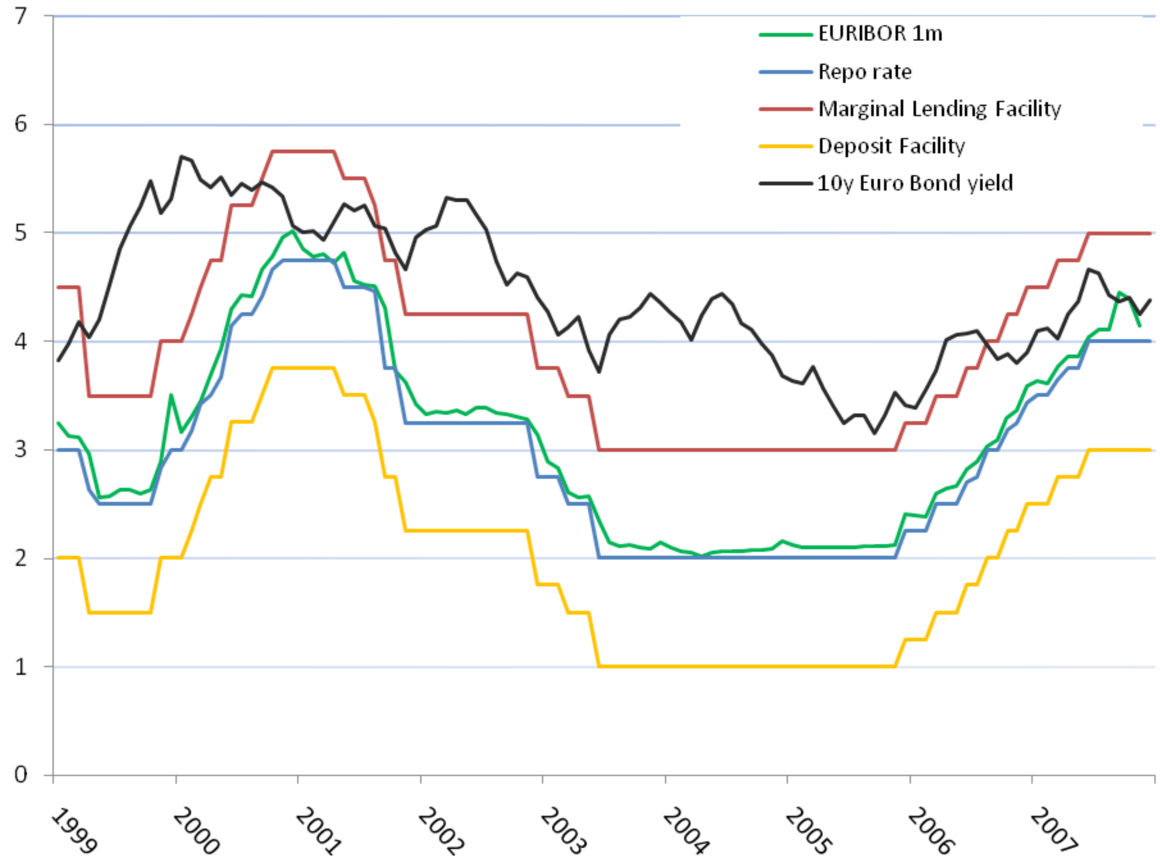
Sweden



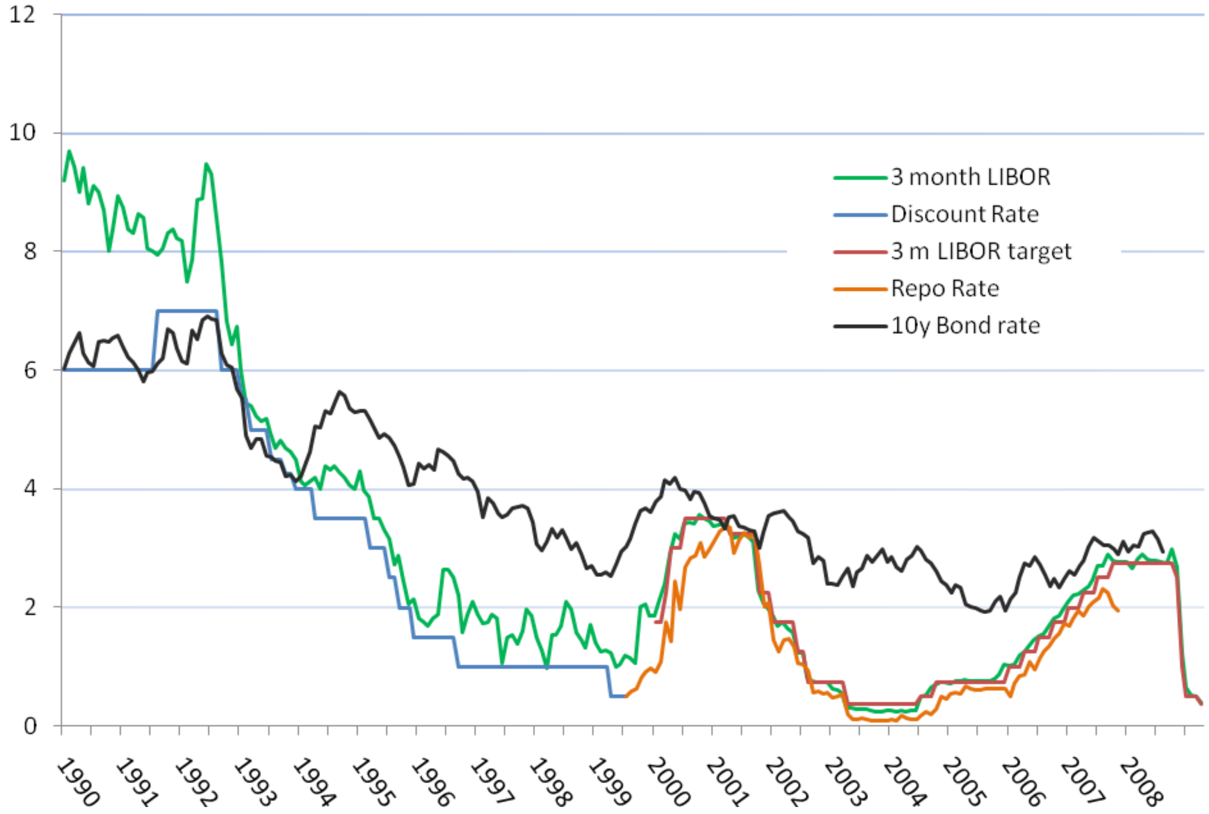
Germany



Euro Area



Switzerland



APPENDIX C: EVIEWS PROGRAM CODE

Note: The appendix C displays the program code for the United States. The program code for the remaining three countries is practically identical.

@ TEST 1

```
create Test1 m 2001:1 2008:7

read(B8,s=US) dataset.xls current_year next_year GDP Actual_CPI_Year_on_Year_inflation Var_CPI
Cov_CPI_GDP Interest_Rates
read(K8,s=US) dataset.xls Known_Inflation
read(M8,s=US) dataset.xls Weights
read(P8,s=US) dataset.xls Crate

pagerename untitled US
series cp=Actual_CPI_Year_on_Year_(+11)


series log_current_year=log(1+current_year/100)*100
series log_next_year=log(1+next_year/100)*100
series log_known_inflation=log(1+known_inflation/100)*100
series log_inflation=log(1+cp/100)*100

series log_irate=log(1+interest_rates/100)*100

series us_real_rate_spread=log_irate-log_inflation
group spread us_real_rate_spread log_inflation

freeze(uroot1_irate) log_irate.uroot
freeze(uroot1_inflation) log_inflation.uroot
freeze(uroot2_irate) log_irate.uroot(pp)
freeze(uroot2_inflation) log_inflation.uroot(pp)
freeze(uroot3_irate) log_irate.uroot(kpss)
freeze(uroot3_inflation) log_inflation.uroot(kpss)

group group_1 log_irate log_inflation

freeze(c1_1) group_1.coint(d,1)  log_irate log_inflation
freeze(c1_2) group_1.coint(d,2)  log_irate log_inflation
freeze(c1_3) group_1.coint(d,3)  log_irate log_inflation
freeze(c1_4) group_1.coint(d,4)  log_irate log_inflation
freeze(c1_5) group_1.coint(d,5)  log_irate log_inflation
freeze(c1_6) group_1.coint(d,6)  log_irate log_inflation
freeze(c1_7) group_1.coint(d,7)  log_irate log_inflation
freeze(c1_8) group_1.coint(d,8)  log_irate log_inflation
freeze(c1_9) group_1.coint(d,9)  log_irate log_inflation
freeze(c1_10) group_1.coint(d,10)  log_irate log_inflation
freeze(c1_11) group_1.coint(d,11)  log_irate log_inflation
freeze(c1_12) group_1.coint(d,12)  log_irate log_inflation
freeze(c1_13) group_1.coint(d,13)  log_irate log_inflation
freeze(c1_14) group_1.coint(d,14)  log_irate log_inflation
freeze(c1_15) group_1.coint(d,15)  log_irate log_inflation

freeze(c2_1) group_1.coint(f,1)  log_irate log_inflation
freeze(c2_2) group_1.coint(f,2)  log_irate log_inflation
freeze(c2_3) group_1.coint(f,3)  log_irate log_inflation
freeze(c2_4) group_1.coint(f,4)  log_irate log_inflation
freeze(c2_5) group_1.coint(f,5)  log_irate log_inflation
freeze(c2_6) group_1.coint(f,6)  log_irate log_inflation
```

```
freeze(c2_7) group_1.coint(f,7)  log_irate log_inflation
freeze(c2_8) group_1.coint(f,8)  log_irate log_inflation
freeze(c2_9) group_1.coint(f,9)  log_irate log_inflation
freeze(c2_10) group_1.coint(f,10)  log_irate log_inflation
freeze(c2_11) group_1.coint(f,11)  log_irate log_inflation
freeze(c2_12) group_1.coint(f,12)  log_irate log_inflation
freeze(c2_13) group_1.coint(f,13)  log_irate log_inflation
freeze(c2_14) group_1.coint(f,14)  log_irate log_inflation
freeze(c2_15) group_1.coint(f,15)  log_irate log_inflation

freeze(c6_12) group_1.coint(d,12)  log_irate log_inflation


var v1_12.ec(d) 1 12 log_irate log_inflation @ d(var_cpi) d(cov_cpi_gdp)
freeze(vec1_12) v1_12
v1_12.append(coint) b(1,1)=1,  b(1,2)=-1.35,
var v1_12.ec(d,restrict) 1 12 log_irate log_inflation@ d(var_cpi) d(cov_cpi_gdp)
freeze(vec1_12_restricted) v1_12


var v2_12.ec(d) 1 12 log_irate log_inflation @ d(var_cpi) d(cov_cpi_gdp)
freeze(vec2_12) v2_12
v2_12.append(coint) b(1,1)=1,  b(1,2)=-1,
var v2_12.ec(d,restrict) 1 12 log_irate log_inflation @ d(var_cpi) d(cov_cpi_gdp)
freeze(vec2_12_restricted) v2_12
```

@ TEST 2

```
create Test2 m 2001:1 2008:7

read(B8,s=US) dataset.xls current_year next_year GDP Actual_CPI_Year_on_Year_inflation Var_CPI
Cov_CPI_GDP Interest_Rates
read(K8,s=US) dataset.xls Known_Inflation
read(M8,s=US) dataset.xls Weights

pagerename untitled US

series log_current_year=log(1+current_year/100)*100
series log_next_year=log(1+next_year/100)*100
series log_known_inflation=log(1+known_inflation/100)*100



series log_inflation=log_current_year-(weights*log_known_inflation)+log_next_year*weights

series log_irate=log(1+interest_rates/100)*100

series us_real_rate_spread=log_irate-log_inflation
group spread us_real_rate_spread log_inflation

series cp=log(1+Actual_CPI_Year_on_Year_/100)*100
series actual_inflation=(cp(-2)+cp(-1)+cp+cp(1)+cp(2)+cp(3)+cp(4)+cp(5)+cp(6)+cp(7)+cp(8)+cp(9))/12

group accuracy actual_inflation log_inflation

freeze(uroot1_irate) log_irate.uroot
freeze(uroot1_inflation) log_inflation.uroot
```

```
freeze(uroot2_irate) log_irate.uroot(pp)
freeze(uroot2_inflation) log_inflation.uroot(pp)
freeze(uroot3_irate) log_irate.uroot(kpss)
freeze(uroot3_inflation) log_inflation.uroot(kpss)


group group_1 log_irate log_inflation


freeze(c1_1) group_1.coint(d,1)  log_irate log_inflation
freeze(c1_2) group_1.coint(d,2)  log_irate log_inflation
freeze(c1_3) group_1.coint(d,3)  log_irate log_inflation
freeze(c1_4) group_1.coint(d,4)  log_irate log_inflation
freeze(c1_5) group_1.coint(d,5)  log_irate log_inflation
freeze(c1_6) group_1.coint(d,6)  log_irate log_inflation
freeze(c1_7) group_1.coint(d,7)  log_irate log_inflation
freeze(c1_8) group_1.coint(d,8)  log_irate log_inflation
freeze(c1_9) group_1.coint(d,9)  log_irate log_inflation
freeze(c1_10) group_1.coint(d,10)  log_irate log_inflation
freeze(c1_11) group_1.coint(d,11)  log_irate log_inflation
freeze(c1_12) group_1.coint(d,12)  log_irate log_inflation
freeze(c1_13) group_1.coint(d,13)  log_irate log_inflation
freeze(c1_14) group_1.coint(d,14)  log_irate log_inflation
freeze(c1_15) group_1.coint(d,15)  log_irate log_inflation

freeze(c2_1) group_1.coint(f,1)  log_irate log_inflation
freeze(c2_2) group_1.coint(f,2)  log_irate log_inflation
freeze(c2_3) group_1.coint(f,3)  log_irate log_inflation
freeze(c2_4) group_1.coint(f,4)  log_irate log_inflation
freeze(c2_5) group_1.coint(f,5)  log_irate log_inflation
freeze(c2_6) group_1.coint(f,6)  log_irate log_inflation
freeze(c2_7) group_1.coint(f,7)  log_irate log_inflation
freeze(c2_8) group_1.coint(f,8)  log_irate log_inflation
freeze(c2_9) group_1.coint(f,9)  log_irate log_inflation
freeze(c2_10) group_1.coint(f,10)  log_irate log_inflation
freeze(c2_11) group_1.coint(f,11)  log_irate log_inflation
freeze(c2_12) group_1.coint(f,12)  log_irate log_inflation
freeze(c2_13) group_1.coint(f,13)  log_irate log_inflation
freeze(c2_14) group_1.coint(f,14)  log_irate log_inflation
freeze(c2_15) group_1.coint(f,15)  log_irate log_inflation

freeze(c6_7) group_1.coint(d,7)  log_irate log_inflation



var v1_7.ec(d) 1 7 log_irate log_inflation @ d(var_cpi) d(cov_cpi_gdp)
freeze(vec1_7) v1_7
v1_7.append(coint) b(1,1)=1,  b(1,2)=-1.35,
var v1_7.ec(d,restrict) 1 7 log_irate log_inflation@ d(var_cpi) d(cov_cpi_gdp)
freeze(vec1_7_restricted) v1_7


var v2_7.ec(d) 1 7 log_irate log_inflation @ d(var_cpi) d(cov_cpi_gdp)
freeze(vec2_7) v2_7
v2_7.append(coint) b(1,1)=1,  b(1,2)=-1,
var v2_7.ec(d,restrict) 1 7 log_irate log_inflation @ d(var_cpi) d(cov_cpi_gdp)
freeze(vec2_7_restricted) v2_7
```

@ TEST 3

```
create Test3 m 1988:12 2007:8

read(B8,s=US) dataset_test_3.xls gdp Inflation_yoy Var_CPI Cov_CPI_gdp Interest_Rates
read(H8,s=US) dataset_test_3.xls Crate
read(J8,s=US) dataset_test_3.xls Cpiindex


pagerename untitled US

series inflationlow=log(Cpiindex(+1)/Cpiindex(-2))*100


series log1_inflation=log(Cpiindex(+1)/Cpiindex(-11))*100
series inflation=log(1+Inflation_yoy(+1)/100)*100
series log_crate=log(1+crate/100)*100
series bond_rate=log(1+interest_rates/100)*100

series log_gdp=log(1+gdp/100)*100
series lgdp=log_gdp-log_gdp(-3)

series new= lgdp


freeze(uroot1_irate) bond_rate.uroot
freeze(uroot1_inflation) inflation.uroot
freeze(uroot1_crate) log_crate.uroot
freeze(uroot1_gdp) log_gdp.uroot


freeze(uroot3_irate) bond_rate.uroot(kpss)
freeze(uroot3_inflation) inflation.uroot(kpss)
freeze(uroot3_crate) log_crate.uroot(kpss)
freeze(uroot3_gdp) log_gdp.uroot(kpss)

group group_1 bond_rate inflation
group group_2 bond_rate inflation log_crate
group group_3 bond_rate inflation log_crate log_gdp


freeze(c5_3) group_3.coint(c,3)  bond_rate inflation log_crate log_gdp
freeze(c5_6) group_3.coint(c,6)  bond_rate inflation log_crate log_gdp
freeze(c5_9) group_3.coint(c,9)  bond_rate inflation log_crate log_gdp
freeze(c5_12) group_3.coint(c,12)  bond_rate inflation log_crate log_gdp
freeze(c5_15) group_3.coint(c,15)  bond_rate inflation log_crate log_gdp


freeze(c1_1) group_3.coint(c,1)  bond_rate inflation log_crate log_gdp
freeze(c1_2) group_3.coint(c,2)  bond_rate inflation log_crate log_gdp
freeze(c1_3) group_3.coint(c,3)  bond_rate inflation log_crate log_gdp
freeze(c1_4) group_3.coint(c,4)  bond_rate inflation log_crate log_gdp
freeze(c1_5) group_3.coint(c,5)  bond_rate inflation log_crate log_gdp
freeze(c1_6) group_3.coint(c,6)  bond_rate inflation log_crate log_gdp
freeze(c1_7) group_3.coint(c,7)  bond_rate inflation log_crate log_gdp
freeze(c1_8) group_3.coint(c,8)  bond_rate inflation log_crate log_gdp
freeze(c1_9) group_3.coint(c,9)  bond_rate inflation log_crate log_gdp
freeze(c1_10) group_3.coint(c,10)  bond_rate inflation log_crate log_gdp
freeze(c1_11) group_3.coint(c,11)  bond_rate inflation log_crate log_gdp
```

```
freeze(c1_12) group_3.coint(c,12)  bond_rate inflation log_crate log_gdp
freeze(c1_13) group_3.coint(c,13)  bond_rate inflation log_crate log_gdp
freeze(c1_14) group_3.coint(c,14)  bond_rate inflation log_crate log_gdp
freeze(c1_15) group_3.coint(c,15)  bond_rate inflation log_crate log_gdp

freeze(c6_3) group_3.coint(c,3)  bond_rate inflation log_crate log_gdp

var v2_3.ec(c,2) 1 3 bond_rate inflation log_crate @ d(var_cpi) d(Cov_CPI_gdp) lgdp lgdp(-1)
v2_3.append(coint) b(1,1)=1, b(1,3)=0, b(2,1)=0, b(2,3)=1,

var v2_3.ec(c,2,restrict) 1 3 bond_rate inflation log_crate @ d(var_cpi) d(Cov_CPI_gdp) lgdp lgdp(-1)
freeze(vec2_3) v2_3

var v2_3r.ec(c,2) 1 3 bond_rate inflation log_crate @ d(var_cpi) d(Cov_CPI_gdp) lgdp lgdp(-1)
v2_3r.append(coint) b(1,1)=1, b(1,2)=-1, b(1,3)=0, b(2,1)=0, b(2,2)=-1, b(2,3)=1,

var v2_3r.ec(c,2,restrict) 1 3 bond_rate inflation log_crate @ d(var_cpi) d(Cov_CPI_gdp) lgdp lgdp(-1)
freeze(vec2_3r) v2_3r

freeze(imp2) v2_3r.impulse(60,imp=unit)
freeze(imp1) v2_3r.impulse(60,imp=unit) bond_rate inflation @ bond_rate inflation
freeze(imp1a) v2_3r.impulse(60) bond_rate inflation @ bond_rate inflation

freeze(granger2) v2_3r.testexog(name=g2)

series US_real_rate=bond_rate-inflation
freeze(US_real_rateG) US_real_rate.line
```