Master of Science Thesis

Pension funds and economic crises

A scenario generating approach to incorporate economic crises in the asset liability management methodology

B. Masselink, M.Sc.

June 20, 2009

ERASMUS UNIVERSITEIT ROTTERDAM

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Department Of
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The undersigned hereby certify that they have read and recommend to the Erasmus School of Economics for acceptance a thesis entitled **"Pension funds and economic crises"** by **B. Masselink, M.Sc.** in partial fulfillment of the requirements for the degree of **Master of Science**.

Dated: June 20, 2009

Supervisor: ______________________

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Readers: ______________________

Preface

Den Haag, June 20, 2009

Dear Reader,

This report is the result of the graduation research at the Rotterdam School of Economics of the Erasmus University Rotterdam. For the last year the author was involved in the Finance Group of this faculty, by following courses and participating in seminars.

The purpose of this research is to investigate the possibility of including economic crisis into a scenario generating process used by the Asset Liability Management (ALM) approach of pension funds. In order to develop such a scenario generating tool, historical economic data are evaluated. The results of this analysis are used to generate economic scenarios using two different models, one including economic crisis and one without these.

I would like to thank the staff of the Finance Group of the Rotterdam School of Economics and the Erasmus Data Service Centre for all their support and the possibility to do most of the research at home. Especially, I would like to thank dr. Onno W. Steenbeek for his supervision.

I really enjoyed applying the control and simulation knowledge I learned at the Delft University of technology into the field of economics, especially into the field of pension funds and risk monitoring.

Sincerely yours,

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Abstract

The possibility of implementing economic crisis into the generation of economic scenarios using Vector AutoRegressive (VAR) models is studied. This allows pension funds using the Asset Liability Management (ALM) approach to get insight into risks associated with economic downturn. The economic scenarios are generated using two separate algorithms, one for the generation of *good* and one for *bad* times. These two are combined to create a scenario which includes economic crisis. This method is compared to a traditional method long term economic dynamics. The purpose of this thesis is to prove that the method introduced can be used, and not to gain a better insight into the risk profile of an individual pension fund The approach discussed in this thesis is a simplified ALM model, which does not incorporate demographic models and incorporates an investment space consisting of stocks and bonds only with different maturity. Despite these limitations, the results clearly demonstrate that it is possible to incorporate economic crisis into ALM models and therefore get a better insight in the pension fund risk profile.

Table of Contents

Preface v

Abstract vii

1 Introduction 1

2 Theory 5
2-1 Defined benefit vs. defined contribution . . . 5
2-2 Asset liability management . . . 6
2-3 Business cycles and frequency domain analysis . . . 8
2-4 Stochastic programming vs. scenario analysis . . . 11
2-5 Including economic dynamics . . . 12

3 Data and methodology 15
3-1 Data . . . 15
3-2 Methodology . . . 17
3-2-1 Step 1: Analyzing historical data . . . 17
3-2-2 Step 2: Generating economic scenarios . . . 18
3-2-3 Step 3: Calculating cash outflows . . . 20
3-2-4 Step 4: Determining asset returns . . . 22
3-2-5 Step 5: Calculating funding ratio . . . 22
3-2-6 Step 6: Evaluating results . . . 22
3-2-7 Initial conditions and assumptions . . . 23

4 Results and discussion 25
4-1 Liabilities . . . 25
4-2 Assets . . . 26
4-3 Funding ratio . . . 26

5 Conclusions and recommendations **31**
5-1 Conclusions 31
5-2 Recommendations 32
5-2-1 Depth 32
5-2-2 Width 33
5-3 Concluding remarks 33

References **35**
References 35

List of Figures

1-1 Major stock market performance, source: OECD 2
1-2 Importance of pension funds relative to the size of the economy 3

2-1 ALM approach: scenario analysis 7
2-2 Example observation of a filtered 100 seconds signal 9
2-3 Stochastic programming vs. scenario creation 11
2-4 Example of average return of different scenario generating processes 13

3-1 Historical data 15
3-2 Historical average yield curve based on Nelson Siegel formula 17
3-3 VAR approach to incorporate crisis 19
3-4 Nominal expected cash outflows as fraction of total cash outflow 21

4-1 Pension fund liabilities 25
4-2 Average pension fund assets 26
4-3 Average monthly stock returns and stock prices 27
4-4 Average funding ratio 27
4-5 Cross section of the average fund ratios 29
4-6 Probability of being under a certain funding ratio 30
4-7 Standard deviation of the funding ratio 30

List of Tables

2-1 Pension fund balance sheet 7

3-1 Data characteristics. Yearly returns and monthly standard deviation. 16
3-2 Data characteristics. Nelson Siegel parameters. 17
3-3 Autoregressive parameter matrices 20
3-4 Autoregressive noise matrices 21
3-5 Pension fund investment mix 23

4-1 Funding ratio characteristics 28

Chapter 1

Introduction

"The ongoing financial crisis has dealt a heavy blow to private pension systems. Between January and October this year [2008, ed.], private pensions in the OECD area have registered losses of nearly 20% of their assets (equivalent to USD 5 trillion)." While the same article, (Yermo & Salou, 2008), states that *"Although the short-term impact is evidently negative, pension funds, by their very nature, have to work with a long time horizon and their performance should also be evaluated on this basis. If one looks at returns over the last fifteen years up to October 2008 - a positive picture still emerges. For example, the average, annual real rate of return of pension funds was 8.5% in Sweden, and 6.1% in the United-States and the United Kingdom over this period."* So the question rises how to evaluate pension funds on the long term, while surfiving short term fluctuation in the funding ratio.

The trade-off between long term gains and short term losses should be made carefully, while anticipating future adjustments of the policy, (Kouwenberg, 2001).

Financial institutions own vast amounts of financial assets and liabilities and are therefore subject to changes in market values as stock prices and interest rates vary. But, not only financial institutions are subject to changes in valuations of assets and liabilities, other corporations as well. These institutions and corporations (should) use models to evaluate their investment strategies and risk profiles.

The institutions that use models are diverse and use them for different reasons. For example airlines use them to evaluate their risk profile with respect to fuel prices, banks mainly look at macroeconomic developments, credit risk and interest rate risks, hedge funds mainly look for investment opportunities. Pension funds on the other hand are interested in their policy decisions and investment mix and how they can mitigate interest rate, longevity, interest, and market risks.

A common used method to evaluate risks is Value at Risk (VaR). VaR estimates the probability and the amount of impact of certain risk factors and combines these to result in a currency amount of risk with a certain probability in a (part of) a portfolio. More details with respect to the VaR methodology can be found in (Duffie & Pan, 1997), (Jorion, 1997) and (Rockafellar & Uryasev, 2000). VaR is a straight forward method to evaluate risks which can be applied (to parts of) the portfolio. Asset Liability Management (ALM), on the other hand, is a holistic approach to evaluate implications on the complete portfolio of assets and liabilities. More details with respect to ALM will be given in the next chapter.

The different purposes of these economic models result in different requirements of the models in terms of variables and time horizon. Some models only look at market and/or individual stock price expectations. While others look at a macroeconomic level. This thesis focuses on macroeconomic scenario generating models, and how these models can incorporate sudden events like economic crises. These scenarios will be used in the ALM approach for pension funds. Currently, the main method of incorporating economic dynamics into the scenario generating process is by higher order Vector AutoRegression VAR(p)[1] models and by the use of spectral analysis, as will be discussed in the next chapter.

The ALM approach discussed in this thesis will provide pension fund stakeholders a better overview of the implications of economic crises. In contrast to traditional VAR models the proposed method includes a crisis in each of the generated macroeconomic scenario. By including this crisis in the ALM approach the effect of possible crisis can be evaluated.

To create economic scenarios, first the past is evaluated while the future is expected to have the same characteristics.[2] This might be a good starting point, while on the other hand investment commercials in the Netherlands should accompany the following warning: *Results obtained in the past are no guarantee for the future.* For example, Figure 1-1 shows the major stock market performance since 1993. It can be clearly seen that there exist significant long term up and downward movement in the market.

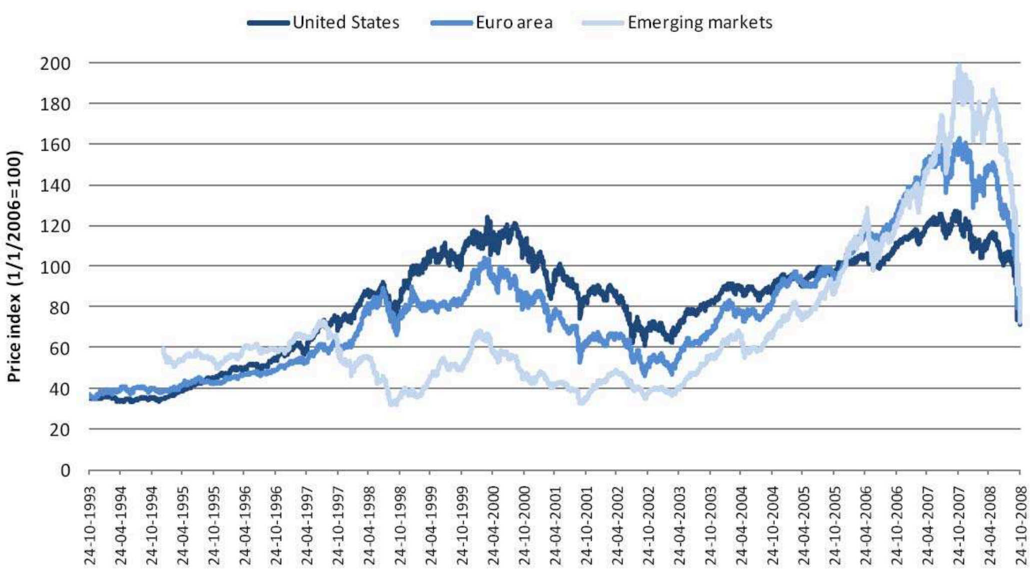


Figure 1-1: Major stock market performance, source: OECD

There is also a practical issue; what to do with exceptional periods in the past like economic crises? One way to summarize the past performance is the average and the standard deviation of the results of the past over a long period including these exceptional periods. Another

[1] Note the difference between the Value at Risk (VaR) and Vector AutoRegression (VAR)
[2] Details with respect to the creation of these economic scenarios will be given in Section 3-2

common used method is the ignore these *exceptional* periods and mark them as 'outliers'. Both methods have obvious drawbacks;(1) This assumes that these sub sequential periods of economic downturn will not occur in the future, while one would expect that the average result in the future is influenced by these exceptional periods. Method (2) assumes that the future will be without corrections and only consists of 'good' periods. A third method is to incorporate cycles, such as seasonal effects and business cycles by using spectral analysis as will be discussed in Section 2-3.

According to (IMF, 2004) the role of pension funds increases in importance: *"The growth of funded pension and the growing emphasis on risk management should strengthen the role of pension funds as stable, long-term institutional investors."*

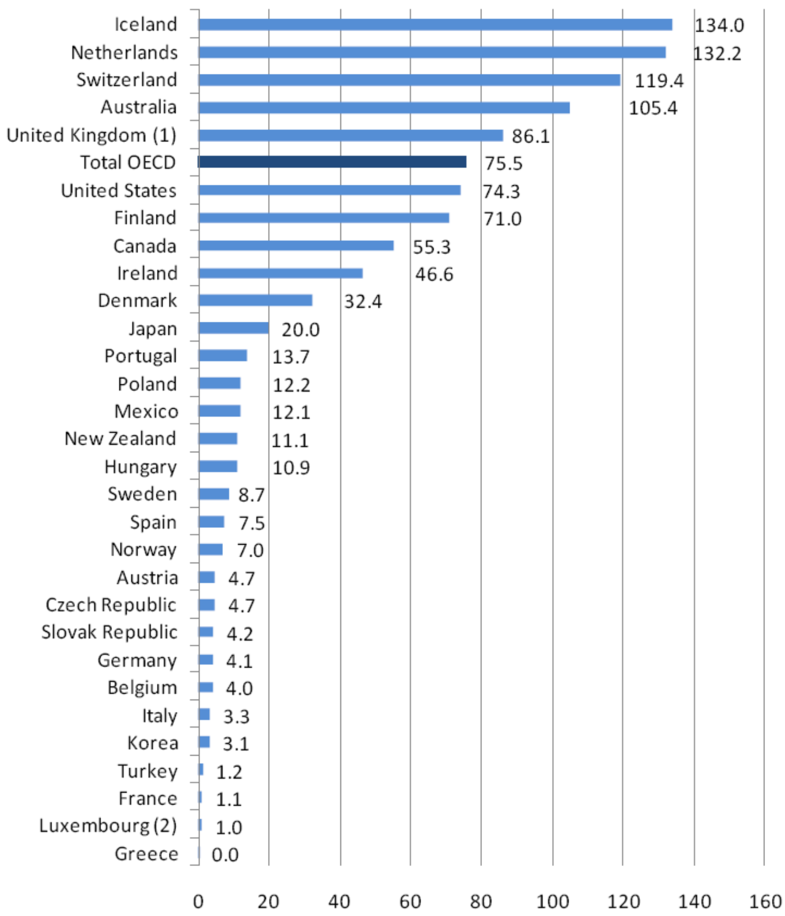


Figure 1-2: Importance of pension funds relative to the size of the economy in OECD countries (2007), source: OECD

In 2007 Dutch pension assets amount to about €770 billion, according to (CBS, 2009), equal to about 132% of the GDP, see Figure 1-2. Thus 3% more returns on assets results in more than 4% increase in GDP, which equals to about 8% of the national salaries, (Boender, Dert, Heemskerk, & Hoek, 2007). It can be concluded that the stakes are high, resulting in governance, justification, transparancy, efficiency, supervision and accountability of pension management are becoming more and more important.

A lot of research has been done in the field of portfolio optimization, for example determining the mean variance global solution portfolio, like (Huberman, Kandel, & Stambaugh, 1987) and (Fama, 1965). For example (Detemple, Garcia, & Rindisbacher, 2003) proposed a new simulation based approach for optimal portfolio allocation in realistic environments including complex dynamics and many state variables, using a Monte Carlo method.

Another interesting field of the ALM method, when applied to pension fund investment strategies, is the use of derivatives for hedging purposes. For example (Palin & Speed, 2003) discuss work in progress with respect to hedging the pension funds funding ratio. (Schotman & Schweitzer, 2000) show that stocks can be used as an inflation hedge even if the stock returns are negatively correlated with unexpected inflation shocks, depending on the investment horizon. Or (Engel, Kat, & Kocken, 2005) who studied how derivatives can hedge interest rates.

However it should be noted that the portfolio optimization and the use of derivatives is beyond the scope of this thesis, as the main purpose of this thesis is introducing a new methodology of generating economic scenarios and its implications of the ALM approach at pension funds. To prove the usability of this method a simplified ALM model is built.

The next chapter will discuss the theory of the ALM approach and how to incorporate business cycles and other dynamics. Different aspects with respect to stochastic programming using event trees and linear scenario generation will be elaborated on in the same chapter. Chapter 3 discusses the data used and the methodologies applied. Special interest is applied to ALM model and the scenario generating process. The results of this new method can be found in Chapter 4, and finally the conclusions and recommendations can be found in Chapter 5

Chapter 2

Theory

The theory discussed in this thesis can divided into several sub parts and is covered in this chapter. First, the theory on different pension schemes is discussed in Section 2-1. After that, the Asset Liability Management (ALM) method and the differences between business cycles and economic shocks are discussed in Section 2-2 and Section 2-3, respectively. Section 2-4 compares the the stochastic programming method to the scenario analysis method. Finally, the economic scenario generating method incorporated in this thesis is discussed in Section 2-5.

2-1 Defined benefit vs. defined contribution

National pension systems are typically represented by a *multi-pillar* structure, with different sources of retirement income like the government, employment and individual savings. The definitions of these pillars differs across academic literature, the following division can be found in (IMF, 2004); in *pillar 1* the state is the source of retirement income, often a combination of universal entitlement and a component related to earning. Occupational pension funds are the main source of income in *pillar 2*. Finally, *pillar 3* consists of private savings and individual financial products.

The relative importance of the contributions of pillars 1, 2, and 3 differ significantly from country to country. In the Netherlands pillar 1 contributes to about 50 percent of the retirement income and the other half consists of pillars 2 and 3, source (IMF, 2004). Currently, the Dutch pillar 1 is constructed as a pay as you go (PAYG) system, which is increasing the pressure on the working class due to the aging population.

Pillar 2 consists of the retirement saving built up during occupation and can be separated into defined benefit and defined contributions schemes, or a combination of these schemes, hybrid plans. Defined benefit (DB) schemes are those in which the employer commits to provide specific benefits related to individual wages and length of employment, while under defined contribution (DC) plans the commitment is to make specific contributions to a pension fund, where benefits depend on the level of contributions to the scheme and the investment return. In the Netherlands about 95% of pillar 2 consists of DB schemes.

One of the main differences between the DB and DC schemes from a employers perspective is the risk involved. In a DB plan the employer bears all the risks while in a DC plan the lower investment returns mean lower pension payments for the employees. For more information on pension schemes see (Ambachtsheer & Ezra, 1998), (Davis, 1994), (Modigliani & Muralidhar, 2004) and (Muralidhar, 2001).

2-2 Asset liability management

During the 2001-2005 period stock returns were falling and the interest rates were low resulting in a deterioration of the financial position of many pension funds, (Bauer, Hoevenaars, & Steenkamp, 2005). Regulations changed and more transparency was demanded by the participants of the pension plans. One of the consequences was that not only the assets, but also the liabilities were valued using fair valuation.

The purpose of the ALM approach consists of two parts; (1) to provide quantitative insight in the results of interaction of assets and liabilities over a certain evaluation period. And (2) to identify strategies to obtain an efficient policy mix. Important research with respect to ALM research can be found in (Boender, 1995), (Boender, Aalst, & Heemskerk, 1998), (Dert, 1995), (Mulvey, 1994), (Mulvey, 1996), (Mulvey, 2000), (Ziemba & Mulvey, 1998) and (Ziemba, 2003).

Most pension funds use the ALM methodology to study the effect of the investment, contribution and indexation decisions, the *pension deal*, for all stakeholders. These stakeholders are not only the retired, current and old employees, but also the employer and future generations. ALM is not only interesting for pension funds, but to all institutions with long term assets and liabilities like banks and insurance companies.

The ALM approach is an iterative process in which economic scenarios are generated using assumptions and data with respect to financial markets, participants and the company. These economic scenarios enter the pension funds company model, taking into account the ALM strategy and the pension deal, resulting in a score[1] of the pension deal with respect to the individual economic scenario. By evaluating a lot of (>1000) economic scenarios, the pension deal and ALM strategy can be evaluated and adjusted. This iterative process is shown in Figure 2-1, (Boender et al., 2007). The details of the scenario generating process introduced in this thesis can be found in Chapter 3.

In the ALM approach the policy makers try to influence the future balance sheet of the pension fund. The balance sheet consists on assets (A) on one side and liabilities (L) and the surplus[2] (S) on the other side, see Table 2-1. The surplus can be calculated by $L - A$, the funding ratio is defined as $1 + S/L$. As usually, the balance sheet is analysed on a liquidation basis, which means that only current assets and liabilities are taken into account[3].

The assets of the pension fund is the investment portfolio consisting of stocks, bonds, T-Bills, real estate, alternative investments, derivatives, etc.. The liabilities of a pension fund

[1] An economic scenario can be scored on basis of several variables like contributions, indexation, underfunding period, etc.. The scoring method depends on the relative importance to the different implications of the policy decisions defined by the board of the pension fund.

[2] or deficit as occurs more and more often nowadays

[3] Valuation on going concern is an alternative method which also takes future pension contributions (assets) and future pension payments (liabilities) into account

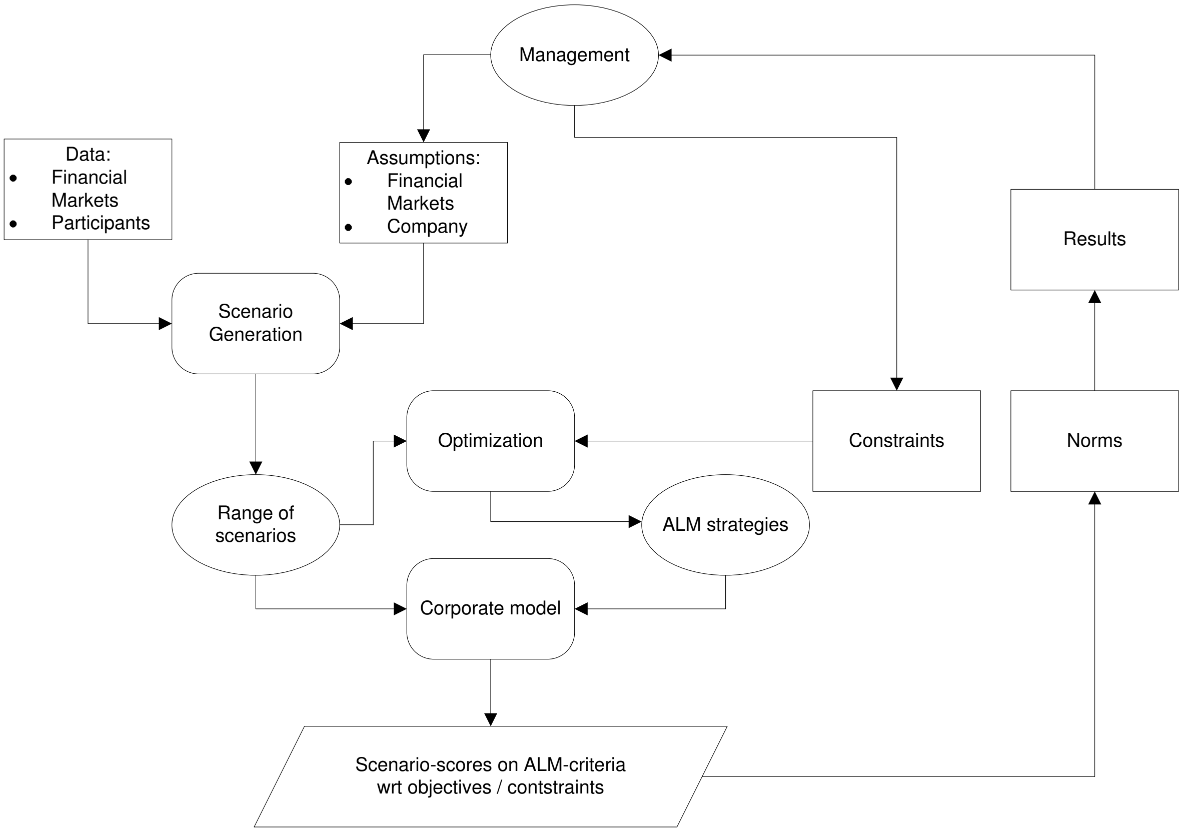


Figure 2-1: ALM approach: scenario analysis

are calculated by discounting the expected pension payments, and possibly other liabilities. These future assets and liabilities are calculated by using stochastic scenarios to construct probability distributions.

Table 2-1: Pension fund balance sheet

Balance sheet	
Assets (A)	Surplus (S)
	Liabilities (L)

The surplus depends on the assets and liabilities of the pension fund, which in turn are influenced by policy decisions and exogenous actuarial and economic factors. The policy decisions are for example the contributions, indexation and investment policy. Examples of exongenous factors are the inflation, interest rates, stock market returns, and the life expectancy of the participants.

The paper of (R. P. M. M. Hoevenaars, Molenaar, Schotman, & Steenkamp, 2007) discusses a long term investor with and without risky assets, subject to inflationary and interest rate risks. Hoevenaars et al. show that there are differences in the global minimum variance and liability hedge portfolio for the availability of alternative asset classes.

Besides the fact that liabilities alter the investment strategy, also the investment horizon alters the optimal portfolio. For example equity is less riskier in the long run than in the short run, according to (Campbell & Viceira, 2002). The time effect should also be incorporated into the ALM approach as this interacts with liabilities.

The influence of the pension deal to risk sharing and wealth transfers between young and old participants, generational accounting, is also studied using ALM analysis, like for example (Ponds, 2003). The subject of generational accounting is beyond the scope of this thesis.

2-3 Business cycles and frequency domain analysis

The objective of the scenario generating process is summarized by the definition given by (Bunn & Salo, 1993) who stated that a scenario is a possible evolution of the future that should be *consistent* with a *clear set of assumptions*. The clear set of assumptions is often translated to the empirical behavior of the economic variables which should resemble the past. This statement included some difficulties as to which behavior[4], and which interaction between economic variables should be taken into account, (R. P. M. M. Hoevenaars, 2008).

Common used methods to generate these scenarios is by the use of Vector AutoRegressive (VAR) models.The simplest form of a AutoRegressive (AR) model is the univariate first order AR(1) model;

$$x_t = v + \beta_1 x_{t-1} + \varepsilon_t \qquad \text{where} \qquad \varepsilon_t \; N(0, \sigma^2) \tag{2-1}$$

in which all variables are scalers ans v is a constant term and β the autoregressive parameter. The last term, ε is white noise with is assumed to be a Normal distribution;

$$\begin{aligned} E(\varepsilon_t) &= 0 \\ E(\varepsilon_t^2) &= \sigma^2 \\ E(\varepsilon_t \varepsilon_{t-k}) &= 0 \qquad \text{for} \qquad k \neq 0 \end{aligned} \tag{2-2}$$

If we extend the univariate AR(1) to a multidimensional equation, we obtain a VAR model. And if not only period $t-1$ is included we obtain a VAR(p) model of order p;

$$\underline{x}_t = \underline{v} + A_1 \underline{x}_{t-1} + A_2 \underline{x}_{t-2} + \cdots + A_p \underline{x}_{t-p} + \underline{\varepsilon}_t \tag{2-3}$$

in which the underlined variables represent $1 \times n$ vectors and the A is the autoregressive parameter matrix of $n \times n$ which combines the auto and cross covariance relationships between the economic variables.

In the article of (Campbell, Chan, & Viceira, 2003), the authors emphasize the importance of the cross covariance variables of the VAR model. Especially for the long term investors, like pension funds, the interaction of the economic variables cannot be ignored.

"Economic variables like GDP growth, employment, interest rates and consumptions show signs of cyclical behavior. Many variables display multiple cycles, with lengths ranging between five up to hundred years." (Groot & Franses, 2008) argue that *"multiple cycles can be associated with long-run stability of the economic system, provided that the cycle lengths are such that interference is rare or absent"*

[4]The question is "how is the behavior described?" is it just the mean and variance, or should cycles taken into account as well? And what about skewness and kurtosis?

By using higher order VAR models, dynamics in the economic system can be included as there is a clear interaction of the state of the economy in the past ($\underline{x}_{t-k}$) and the current state of the economy ($\underline{x}_t$) by the term A_k. The parameters in the model are $\underline{v}$ and $p \times A$, so the amount of variables in the model equals $n + n \times n \times p$. This means that a lot of data is needed to estimate the parameters accurately. Therefore this is an impractical way of including long term dynamics in the process of generating economic scenarios.

Another way to study stochastic signals is spectral analysis, a widely used technique in physical engineering. Instead of observing a signal as a value at each time step, the signal is studied by their characteristics at each frequency. In fact the signal is observed in the frequency domain instead of the time domain.

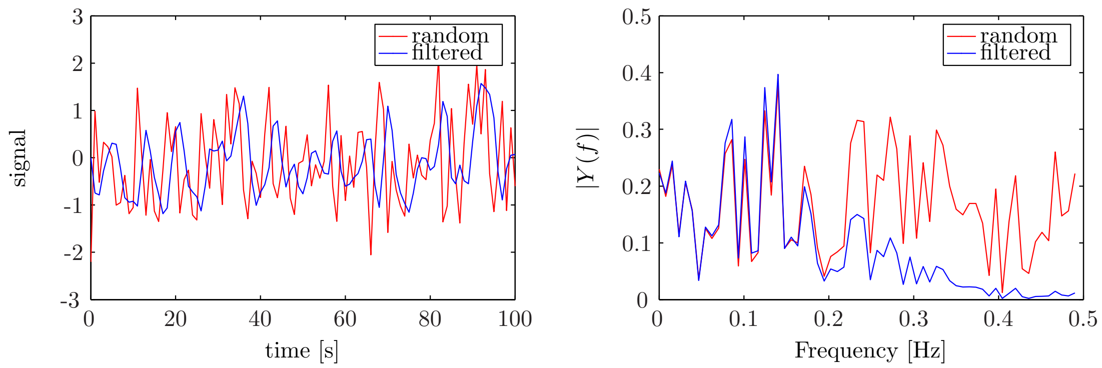
Fourier showed that any mathematical function can be written as an infinite sum of sines and cosines;

$$f(x) = \sum_{k=0}^{\infty} (A_k \cos(k\alpha x) + B_k \sin(k\alpha x)) \tag{2-4}$$

When transforming a stochastic signal using Fourier transformation a phase and amplitude is obtained for all frequencies ranging between 1/(observation time) and 1/(2 x time steps).

For example, suppose that we have observed a signal for 100 seconds with time steps of 1 second. This means that we have 100 data points. If a Fourier transformation is applied, 50 frequencies between 0.01 Hz ($\frac{1}{100s}$) and 0.50 Hz ($\frac{1}{2 \cdot 1s}$)can be observed, but at each frequency we have amplitude and phase information. This adds up to 100 data points as well. This means no detail is lost in this transformation as with averaging or filtering.

Figure 2-2(a) shows the values of two signals for 100 seconds sampled at 1 Hz. The red line is a random generated signal for which the signal is white noise for $t \to \infty$ with $\sigma^2 = 1$. The blue line is constructed by filtering the same signal with a second order low-pass filter[5].



(a) Random noise and filtered signal

(b) Fourier transform of the random noise and filtered signal

Figure 2-2: Example observation of a filtered 100 seconds signal

The Fourier transformation of these two signals can be found in Figure 2-2(b). As you can see, the power of the white noise is evenly distributed among all frequencies. While for the filtered signal, the power reduces for higher frequencies.

[5]The transfer function used for this filter is $H(s) = \frac{1}{s^2+s+1}$

The technique of spectral analysis for generating economic scenarios is thoroughly discussed by (Steehouwer, 2005). In his Ph.D. work also some drawbacks of this method are discussed, especially the uncertainty of the accuracy. If you want to include a 15 years business cycle by investigating 30 years of data, you only have two observations. Which means that the accuracy is limited.

This spectral analysis can be investigated using filtering techniques, in which certain frequency ranges can be closely studied. This allows researchers to closely look at seasonal effects or business cycles. The results of the spectral analysis can be directly used to recreate signals with the same properties. These recreated signals are the building blocks of the economic scenarios.

Unfortunately, everything that is measured included noise, the same is true for frequency domain analysis. This results in uncertainty in the amplitude and phase for especially the low frequencies as there are limited observations. These low frequency dynamics of the economy represent long term movement of the economy, and is therefor of major importance to pension funds with their long time horizons. Uncertainty in impact of these low frequency are therefore undesirable in the pension fund ALM analysis, and can be solved using different methods; (1) repetitive measurements, (2) frequency smoothing or (3) parameterizing the model.

All three methods have their advantages and limitations. A disadvantage of the first model is that in the economy it's impossible to repeat an independent measurement, as different economies are not completely independent. Another issue is that old historical data might be unavailable and/or irrelevant.
By averaging in the frequency domain different data points are combined to smoothen the function. This results in ignoring some dynamics and does not improve the accuracy at low frequencies in the second method.

Finally, the third method; by implementing a parameter model which is fitted onto the measured data might be the most interesting method, which is also used by (Steehouwer, 2005). In short, a parametric model is not a black box model, as spectral analysis, in which only the output is evaluated. A parametric model assumes certain relationships, but does not know the magnitude and direction of these relationships between variables. By fitting the parametric model onto the measured data, the value of the parameters can be determined, and the relationships revealed.

An advantage of this method is that the number of parameters and the value of these parameters can be limited. This results in less variables to determine compared to the spectral analysis, which results in higher accuracy and lower uncertainty. However, if (the number of) the parameters are determined incorrectly, the results change dramatically. This is discussed in the article of (R. Hoevenaars, Molenaar, Schotman, & Steenkamp, 2006), which investigated the influence of parameter uncertainty and prior information on the strategic asset allocation for long term investors.

However, the parametric model also has some disadvantages; (1) the decision the number and influence of the parameters needs to be made before fitting the model and (2) by using only one data set, the results cannot be checked. This mean that it is hard to prove if assumptions of the models are clear and correct.

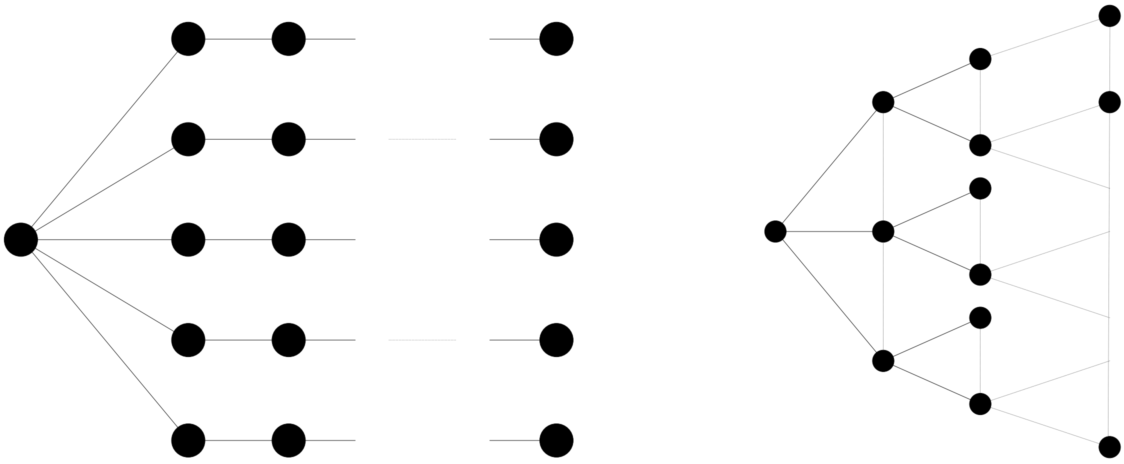
Both the VAR and the parametric models have the same disadvantage; the models assume highly deterministic behavior in which the state of the economy is only determined by the past and random noise, which remain constant over time. No external shocks like demographic changes and technological innovations are taken into account. Another disadvantage of using recreated signals observed by spectral analysis is the occurrence of peaks. As sines and cosines are combined these functions can interfere, resulting in high peaks, which might by impossible in real life.[6]

2-4 Stochastic programming vs. scenario analysis

The main purpose of the ALM approach is to evaluate investment and policy decisions made today on their possible outcomes in the future. As the exact outcome of the future cannot be predicted, otherwise I would not write this thesis, the future results have to be estimated with a certain probability.

There are two different methods to evaluate possible future outcomes; (1) is to generate a lot of independent economic scenarios for the future and (2) is to state that each time step consists of several possible outcomes which evolve from the previous one with a certain probability. The first method is called linear scenario structure, the second is stochastic programming, and summarized in Figures 2-3(a) and 2-3(b), respectively.

In the short run the difference is between a lot of computational effort versus elegant analytical investigation. However, during a long observation time, the number of possible outcomes in the stochastic programming method explodes[7]. This can be solved by limiting the number of time steps. However, this ignores high frequency behavior of the variables and therefore reduces accuracy because inter time step movements of the variable are ignored. To translate this to the pension fund case, this means that the short term underfunding risk is underestimated.



(a) linear scenario structure

(b) stochastic programming

Figure 2-3: Stochastic programming vs. scenario creation

[6]For example negative nominal interest rates.

[7]The number of end states is a multiplication of all the possible outcomes in the previous states; k^n, in which k is the number of possible outcomes per state and n is the number of time steps.

In the paper of (Kouwenberg, 2001), the author develops a stochastic model for an ALM analysis of a pension fund based on (Dert, 1995). Although he showed some interesting results, the increased complexity of these stochastic models and the reduced costs of computational power, made linear scenario generations more popular nowadays. Another example is the stochastic programming model of ALM of a Finnish pension company by (Hilli, Koivu, Pennanen, & Ranne, 2007)

As mentioned before, one of the main disadvantages of an event tree is the dimensions of the tree increases exponentially. To cope with this disadvantage (Bogentoft, Romeijn, & Uryasev, 2001) combines the linear and stochastic programming by grouping several scenarios, and evaluating investment decisions at each node.

Especially the method of (Bogentoft et al., 2001) is very interesting in optimizing the investment decisions made by the pension fund. However, implementing economic cycles and crises would make the model very complicated, reducing the advantages of combining scenarios. Just as the stochastic programming, this method will therefore not be used to study the dynamics of the economy in this thesis, but it might be recommended to investigate the possibilities.

2-5 Including economic dynamics

As discussed in Section 2-3 there are two commonly used methods to incorporate dynamics in the economic model. One is to use a higher order VAR(p) model, the other is by making use of (parametric) frequency domain models. Both with their specific advantages and limitations.

In this thesis, I will use a third model to incorporate economic crises in the ALM approach of pension funds. This approach is a modified VAR(p)[8] method and can be summarized by the following steps:

Step 1: Collecting historical data. Determine the variables needed for the economic scenarios and obtain historical data of these variables.

Step 2: Identifying economic crises. By studying the historical data, economic crises can be identified by for example falling stock prices or decline in GDP growth.

Step 3: Analyzing the periods. After determining the order of the VAR model[9] to be used, the parameters of the model can be determined using ordinary least square measures. This has to be done for two different types of periods, during crises and non crises. This enables the identification of the differences in average returns and (cross) covariances.

Step 4: Create economic scenarios. The economic scenarios are now created by varying the timing of the economic crisis. The VAR parameters will be different during the different periods. The complete scenario will be a combination of the two individually generated parts.

[8]The method used in this thesis will be denoted by *VAR 2*. The traditional VAR method will be denoted by *VAR 1*

[9]The order of the VAR model will be determined based on the Schwarz's Bayensian criterion.

Step 5: Evaluate ALM. After the generation of the economic scenarios, the influence of the policy decisions of the pension fund can be evaluated using the ALM approach.

Details with respect to the approach summarized above can be found in Chapter 3 in which the methodology will be discussed.

To clarify the differences between the different scenario generating processes, consider the following simplified example with only one variable, a stock price return, and no random factor (noise). First a AR(1) as in Eq. (2-1) is determined with a constant β of 0.005. Assuming no noise, would result in a constant stock price return of β.

Second is simplified model of (Steehouwer, 2005)is used with only cycles. One with a period of exactly the observation time and one with five cycles within the observation time. Including the same average return and adjusted amplitude the hypothetical model represent the following formula

$$r_t = 0.005 + 0.001 \sin\left(2\pi \cdot \frac{t}{120}\right) + 0.0002 \sin\left(10\pi \cdot \frac{t}{120}\right) + \varepsilon_t, \tag{2-5}$$

in which r_t is the stock price return at t. Assuming no noise would result in the same average stock price return, $\overline{r}_t$, of 0.005 per time step.

Finally consider a stock price return of 0.005545 during normal periods and -0.001 during a crises, in which the length of the crisis $\frac{1}{12}$ of the total observation time.

The three different approaches for economic scenarios generating are summarized by the example in Figure 2-4. Note that the average return of the three models is equal, and when applying the right amount of random noise, the volatility of the three scenarios will be equal as well.

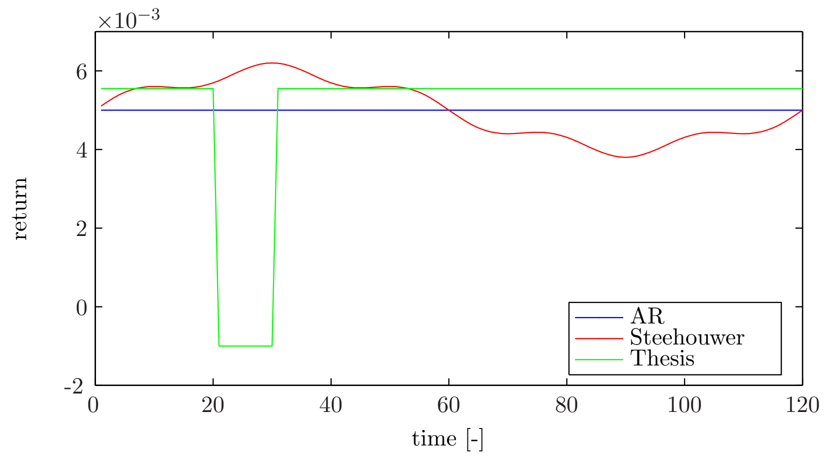


Figure 2-4: Example of average return of three different economic scenario generating processes

In the example three different scenarios are generated. When applying the right amount of noise, these scenarios will have the same *risk-return* characteristics defined by classic corporate finance, (Markowitz, 1952). However, note that their are significant differences during the observed time, which might result in short term underfunding of the pension fund. As

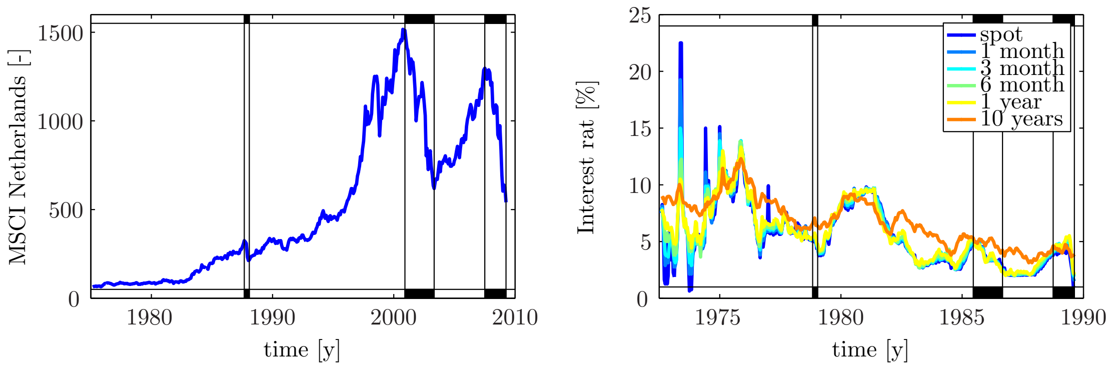
(R. P. M. M. Hoevenaars et al., 2007) clearly remarks there is a *"trade off between the long-term objective of maximizing the funding ratio while satisfying short-term risk constraints."*

By implementing a different method of generating economic scenarios with more emphasis on the the short term risk of underfunding, the short term risk constraints can be evaluated from a different perspective. An extensive version of the third model is introduced in this thesis.

Chapter 3

Data and methodology

This chapter discusses the data used and methodology applied. All the used data are obtained from DataStream. The software package used to analyse the data is Matlab[1], a computational software package mainly used by engineers and econometricians.



(a) MSCI Netherlands all share stock return

(b) Historical interest rates

Figure 3-1: Historical data (black indicating crisis)

3-1 Data

The historical period observed is between 1977M2 till 2009M3. For this period monthly data are collected for the Consumer Price Index (CPI), the Dutch market return, represented by the MSCI all share index, and interest rates ranging from the spot rate to the 10 year

[1]The MATrix LABoratory software package enables fast computations of (large) matrices. The scenario generating process will make use of large matrices, therefore this software package makes it very useful for this research.

government bond rate. The all market stock return is given in Figure 3-1(a). The interest rates are shown in Figure 3-1(b).

The above mentioned period is a compromise between long dated historical data, and the availability of this data.

Figure 3-1 also indicates the periods marked as crisis. These are the periods from 1987M8 to 1988M1 in which the stock markets declined during the oil crisis. From 2000M11 to 2003M4 when the Dutch stock market declined by more than 59% during the dotcom crisis. And, finally during the credit crunch from 2007M6 to 2009M3, which is at the time of writing the most up to date data, the stocks again declined with 58%.

In the analysis in this thesis two different scenario analysis methods are used. One, in which to complete period from 1977M2 to 2009M3 is evaluated without any alterations, this method is referred to as "VAR 1". The second method ("VAR 2") splits the observed period in *good* and *bad* periods in which the *bad* periods are defined as the above mentioned periods. The *good* period is the remaining time. This means that both methods use the same dataset.

The characteristics of the data displayed in Figure 3-1 can be found in Table 3-1. In this table, all data are given as yearly change/return [%].

Table 3-1: Data characteristics. Yearly returns and monthly standard deviation.

	complete period (N = 410)		non crisis (N = 352)		crisis (N = 58)	
	mean	(st.dev.)	mean	(st.dev.)	mean	(st.dev.)
CPI	3.0147	(0.4313)	3.1096	(0.4254)	2.4403	(0.4966)
MSCI return	8.9935	(5.3829)	17.7290	(4.6507)	-32.4777	(7.5100)
spot rate	5.4753	(2.9463)	5.7433	(3.0799)	3.8491	(0.8616)
1 month	5.5432	(2.7825)	5.8031	(2.9025)	3.9659	(0.8555)
3 month	5.6181	(2.6933)	5.8705	(2.8053)	4.0863	(0.9005)
6 month	5.6876	(2.6230)	5.9485	(2.7198)	4.1043	(0.9194)
1 year	5.7915	(2.5272)	6.0589	(2.6059)	4.1682	(0.9423)
10 years	6.7620	(2.1114)	7.0880	(2.0841)	4.7831	(0.7748)

The interest rates are analysed using the Nelson Siegel formula, see (Nelson & Siegel, 1987). This formula enables to determine the yield curve with a limited number of variables, see Eq. (3-1);

$$r(t) = \beta_0 + (\beta_1 + \beta_2)\frac{1 - e^{-t/\tau}}{t/\tau} - \beta_2 e^{-t/\tau} \tag{3-1}$$

In the Nelson Siegel formula, β_0, represents the long term interest rate. The short term interest rate is defined by $\beta_0 + \beta_1$. The intermediate interest rate and the time constant are defined by β_2 and τ, respectively. The observed Nelson Siegel variables for the historical period can be found in Table 3-2.

The values displayed in Table 3-2 are obtained using a nonlinear optimization method explained in Section 3-2-1.

The average historical interest rate yield curves can be found in Figure 3-2. The lines indicate the average yield curve according to the Nelson Siegel methodology for the different periods. The crosses mark the average observed interest rates for the different periods as defined in Table 3-1.

Table 3-2: Data characteristics. Nelson Siegel parameters.

	complete period (N = 410)		non crisis (N = 352)		crisis (N = 58)	
	mean	(st.dev.)	mean	(st.dev.)	mean	(st.dev.)
β_0	5.7887	(1.6985)	6.0703	(1.6435)	4.0798	(0.7792)
β_1	-0.3004	(2.2194)	-0.3226	(2.3578)	-0.1658	(1.0435)
β_2	3.5015	(1.2847)	3.6302	(1.3199)	2.7199	(0.6247)
τ	4.3070	(1.0188)	4.3808	(1.0436)	3.8593	(0.7093)

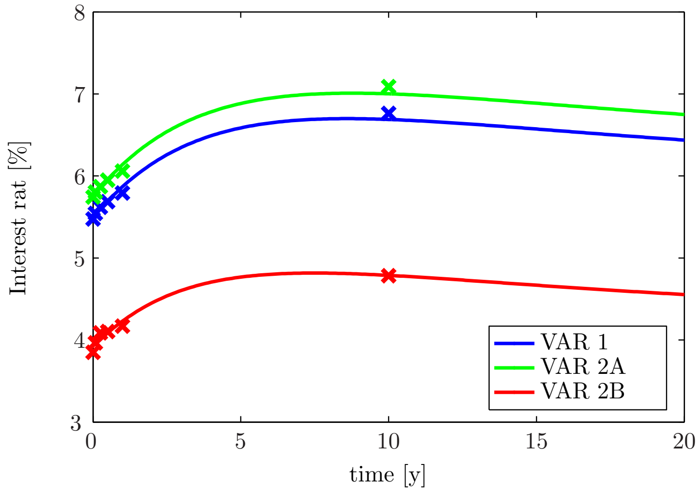


Figure 3-2: Historical average yield curve based on Nelson Siegel formula

One of the things that can be noted instantly is that the average interest rates are lower during periods of declining stock market. These lower interest rates result in higher discounted values for the pension fund liabilities. As stocks are declining during crisis and interest rates fall, this means significant funding risks for pension funds.

3-2 Methodology

The evaluation of the pension fund funding ratio consists of six sub steps. First, the data discussed in Section 3-1 are analysed. Second, future economic scenarios are generated. Third, the pension fund nominal future cash outflows are determined. Fourth, using the economic scenarios, the pension fund asset returns are calculated. Fifth, the pension fund funding ratios are determined by calculating the fair values of the asset and liabilities. Finally, the results are evaluated. These sub steps will be discussed in the next subsections.

3-2-1 Step 1: Analyzing historical data

To enable economic scenarios which are *consistent* with *a clear set of assumptions*, as stated by (Bunn & Salo, 1993), the first step is to analyse historical interest rates, stock returns and inflation. This is needed to generate signals that have the same characteristics, defined by their mean and standard deviation as the historical data. As mentioned before the data are obtained from DataStream and loaded into the Matlab software package from the excel sheet.

At each time step, the Nelson Siegel yield curve has to be determined. This is done by fitting the non linear Nelson Siegel formula given in Eq. (3-1) by minimizing the following cost function[2]:

$$\begin{aligned} C &= \sum(r(t) - r_{NS}(t))^2 \\ C &= \sum(r(t) - \beta_0 + (\beta_1 + \beta_2)\tfrac{1-e^{-t/\tau}}{t/\tau} - \beta_2 e^{-t/\tau})^2 \end{aligned} \tag{3-2}$$

in which C is the cost function, t is the time vector of the maturities of the different bonds, ranging form the spot rate to 10 years. The last two terms, r and r_{NS} are the measured interest rates and the yield according to the Nelson Siegel formula, respectively.

The raw data obtained consist of consumer price index and MSCI All Share market index. These two data ranges were converted into monthly returns and monthly price changes. This will be used in the next step to generate future economic scenarios as can be read in the next subsection.

3-2-2 Step 2: Generating economic scenarios

The economic generating process uses the vector autoregressive model. As can be seen in Eq. (2-3) on page 8 their are several decisions to be made with respect to VAR model. First, the number of state variables have to be determined. These are the variables that determine the state of the economy. As mentioned in the previous subsection, the state of the economy in this model is represented by the yield curve, the CPI and the all share market index. The yield curve is represented by the four Nelson Siegel variables, therefore the economic state is represented by vector $\underline{x}$;

$$\underline{x} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \tau \\ r_{stocks} \\ \Delta CPI \end{bmatrix} \tag{3-3}$$

in which r_{stocks} and ΔCPI is the monthly returns on the stocks and the change in the CPI.

The second decisions is the order of the VAR model. In fact, the question is how many historical steps should be incorporated to determine the current state. Both the VAR 1 and the VAR 2 model use the Schwarz's Bayensian criterion to determine the order of the VAR model, see (Schwarz, 1978). By calculating the final prediction error, the influence of one extra order can be evaluated. In this case, the optimal order appeared to be one.

In this research two different models are examined, *VAR 1* and *VAR 2*. *Var 1* is a model in which the future data are generated using the full range of historical data. *VAR 2* generates the future by modeling economic steady periods (*VAR 2A*) and crisis (*VAR 2B*). This second model will generate these two separate scenarios and combine into one.

[2] In mathematics, the term *cost function* refers to the function to minimize in order to obtain the optimal solution.

The VAR 2 model includes *one crisis* in *each scenario*. The length of each crisis is 85 months, the length of the total scenario is 600 month (50 years). The assumption underlying the VAR 2 model is that it is *certain* that a crisis will occur, but that it is *uncertain* when the crisis will occur. Figure 3-3 displays the procedure to combine the two parts of the VAR model.

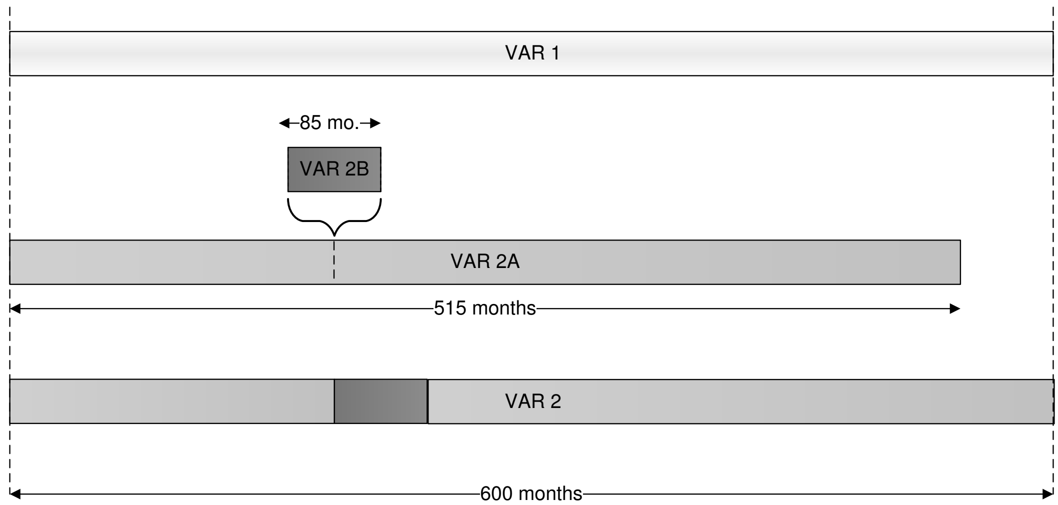


Figure 3-3: VAR approach to incorporate crisis

The start of the crisis is uniformly distributed over the total time span. This means that probability that the crisis will start in month each is 0.167% (1/600).
If the crisis would start later than 85 months before the end of the observation time, the crisis would be shorter, and scenario would have different characteristics than desired. To prevent this, in these cases the 'remaining months' will be placed at the beginning of the economic scenario. This results in two separate crises, one at the beginning and one at the end.

As can be seen in Figure 3-1 on page 15, in the observed data, the crisis accounts to a total of 14.15% (58 out of 410 months) of the total time. In the generation of the economic scenarios this will remain the same. Looking at a 50 years time horizon, consisting of 600 months (data points) of which 85 (14.17%) are generated using the *VAR 2B* model and the remaining part using the *VAR 2A* model.

As mentioned before, for all three VAR models the order appeared to be one, according to the Schwarz's Bayensian criterion. The autoregressive parameter matrices are displayed in Table 3-3.

The VAR formula used in this thesis is slightly different from the one used in some other research. Some literature uses a vector to scale the white noise to the correct standard deviation, this thesis uses a matrix. To incorporate not only the past links between the economic variables using the autoregressive parameter matrix A, the noise is generated using covariance matrices:

$$\underline{x}_t = \underline{v} + A\underline{x}_{t-1} + C\underline{\varepsilon}_t \tag{3-4}$$

in which $\underline{\varepsilon}_t$ is a 6 x 1 white noise vector with $\sigma^2 = 1$. The noise covariance matrices can be found in Table 3-4.

Table 3-3: Autoregressive parameter matrices

	A						$\underline{v}$
VAR 1	0.8529	0.0625	0.1857	0.1145	0.7187	12.3575	−0.3158
	−0.2327	1.0579	0.4797	0.1486	−2.2947	−10.8126	−0.9213
	0.2551	−0.1597	0.4680	−0.0152	0.8015	5.8219	0.3839
	0.1936	−0.0090	−0.1803	0.5231	0.7262	−16.1119	1.5907
	0.0094	−0.0077	−0.0144	0.0008	0.0752	−0.5552	−0.0031
	0.0003	0.0002	0.0002	−0.0003	0.0016	0.2514	0.0004
VAR 2A	0.8372	0.0742	0.2130	0.1082	0.6611	11.4070	−0.2818
	−0.2878	1.0827	0.5437	0.1568	−4.6417	−14.0521	−0.7936
	0.3254	−0.2073	0.3664	−0.0080	1.6319	7.3132	0.2506
	0.1651	0.0209	−0.1224	0.5159	1.1003	−22.8384	1.6076
	0.0008	−0.0031	−0.0040	0.0011	−0.0515	−1.1224	0.0199
	0.0003	0.0003	0.0005	−0.0003	0.0005	0.2304	−0.0002
VAR 2B	0.6393	0.0688	0.2166	0.3098	0.0163	17.2812	−0.3809
	0.6418	0.8198	−0.3967	−0.6248	1.9480	−5.5060	0.9024
	−0.3580	0.0601	1.1579	0.4112	0.1229	9.0300	−0.5722
	0.7092	−0.2837	−0.8677	−0.0144	−0.5583	11.2036	3.2647
	0.0745	−0.0507	−0.1482	−0.0913	0.0134	0.8867	0.4076
	0.0024	−0.0022	−0.0055	−0.0041	0.0030	0.2330	0.0224

As mentioned before a scenario including the crisis is a combination of a steady or growth period and a declining period. As the VAR model only generates the yield curve and monthly returns, the two parts can be combined as displayed in Figure 3-3.

The last step in the scenario generating process is to convert the monthly returns into a stock price index and a CPI, using the following formula;

$$P(t) = \prod_{\tau=1}^{t} (1 + r(\tau))) \tag{3-5}$$

The CPI is only used to include the cross covariances between the other variables. The CPI is not used to determine the expected future pension payments. By including this variable, it can be proved that the methodology can incorporate more variables than stock and bond returns only.

3-2-3 Step 3: Calculating cash outflows

Before the pension fund funding ratios can be determined two more steps need to be executed. First, the expected cash outflows need to be determined. This will be discussed here, subsequently, the asset returns are calculated, this will be discussed in the next subsection.

The main focus of this thesis is the economic scenario generating process, therefore the calculations of the expected cash outflows are using some simplifying assumptions instead of generating demographic scenarios as well. The assumptions made in this research are discussed below.

The first assumption is that the expected cash outflows in the first year of the evaluation are distributed by a normal distribution with its maximum at 20 years and a standard deviation

Table 3-4: Autoregressive noise matrices

	C					
VAR 1	0.1213	−0.0183	0.0308	−0.0392	−0.0011	0.0002
	−0.0183	1.0867	−0.5508	−0.1218	−0.0033	0.0001
	0.0308	−0.5508	0.3929	−0.0270	0.0009	0.0001
	−0.0392	−0.1218	−0.0270	0.5879	0.0031	0.0000
	−0.0011	−0.0033	0.0009	0.0031	0.0026	0.0000
	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
VAR 2A	0.1331	−0.0037	0.0287	−0.0470	−0.0026	0.0002
	−0.0037	1.2232	−0.6139	−0.1461	−0.0047	0.0001
	0.0287	−0.6139	0.4279	−0.0107	0.0015	0.0001
	−0.0470	−0.1461	−0.0107	0.6471	0.0025	0.0000
	−0.0026	−0.0047	0.0015	0.0025	0.0018	0.0000
	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000
VAR 2B	0.0792	−0.1165	0.0760	0.0110	0.0053	−0.0001
	−0.1165	0.2857	−0.1354	−0.0234	−0.0061	−0.0001
	0.0760	−0.1354	0.1585	−0.1086	0.0057	−0.0002
	0.0110	−0.0234	−0.1086	0.2217	0.0015	0.0000
	0.0053	−0.0061	0.0057	0.0015	0.0053	−0.0001
	−0.0001	−0.0001	−0.0002	0.0000	−0.0001	0.0000

of 20 years, which is in accordance to (James, 2004). Off course, the fractions of the cash outflows which are placed below zero are ignored, the result can be seen in Figure 3-4. This figure shows that 1.5024% of the total nominal liabilities will be paid next year and 1.5814% in the second year, and so on.

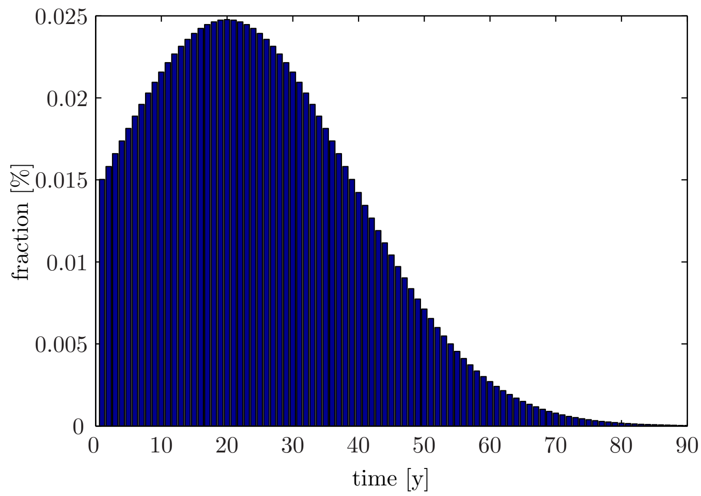


Figure 3-4: Nominal expected cash outflows as fraction of total cash outflow

Further more, it is assumed that this distribution remains constant during the 50 years evaluation period and that during the current year no contributions are paid that have to be paid back the next year. This means that if, for example, in the first year there are liabilities worth 1,000 EUR nominal, of which 15.814 EUR (1.5814%) will be paid in the second year from now. After the first year this 15.814 EUR represents 1.5024% of the nominal liabilities, representing an annual growth of 5.26% of the nominal liabilities. Therefore, it is assumed that the nominal cash outflows will grow with 5.26% annualy.

3-2-4 Step 4: Determining asset returns

The next step is to determine the asset returns. As mentioned before, the investment mix is limited to bonds with different maturities and the market index. The bonds have maturities of 1, 2, 3, 5, 10, 20 and 30 years.

First, the monthly return on stocks and 1/12 of the interest is added to the nominal values of the bonds. After that, the new market values of the individual assets are determined. To determine the market value of the bonds, the interest and principal payments are discounted using the yield curve determined by the Nelson Siegel parameters.

The next step is the reallocation of the assets to assure a constant investment mix. The asset distribution and other initial conditions can be found in Section 3-2-7. Note that during the asset reallocation the nominal values are taken into account and that this reallocation is done at each time step, and that transaction are costs ignored.

3-2-5 Step 5: Calculating funding ratio

In order to calculate the funding ratio, the fair value of the liabilities and the market value of the assets are needed at each time step. To obtain these two values, some calculation steps need to be executed.

First, the yearly expected cash outflows need to be transformed to monthly data, as the economic scenarios use monthly data points. To transform this yearly data into monthly data a linear two dimensional interpolation method was used.[3] Second, these cash flows are discounted using the yield curve resulting in fair value for the liabilities at each time step.

To calculate the market value of assets the current level of the market index is used. The market value of the bonds is calculated by discounting the future interest and principal payments of the bonds using the same yield curve as the liabilities.

Finally, the funding ratio is calculated using the following formula:

$$\begin{aligned} FR &= 1 + \frac{A-L}{L} \\ &= 1 + \frac{S}{L} \end{aligned} \tag{3-6}$$

In which FR, S, A and L, represent the funding ratio, the surplus, the market value of the assets, and the market value of the liabilities, respectively.

3-2-6 Step 6: Evaluating results

The results obtained during the previous five steps are evaluated on a number of aspects. Besides the average funding ratio, also the distribution of the funding ratio is of high importance. For example what is the probability of becoming underfunded below certain threshold?

[3] A two dimensional interpolation method is used, to transform the 50 years of data to 600 monthly data points. To incorporate more detail, the evaluated 90 years of expected cash outflows are transformed to monthly data as well.

Another aspect is the average growth of total assets and the average recovery time. The results of these analysis will be discussed in the next chapter.

3-2-7 Initial conditions and assumptions

The previous subsections clearly discussed the complexity and the number of variables that is encountered in this analysis. This subsection discusses the initial conditions with respect to this analysis as well as the assumptions made.

The ALM is evaluated for 50 years consisting of 12 months. This results into 600 data points per scenario. For each data point the expected cash outflows are evaluated on a monthly basis for the next 90 years.

As mentioned before, the investment mix consists of the market index and bonds with several maturities. In this model the investment mix can be found in Table 3-5. As mentioned before, this is rebalanced during each time step. The initial investment mix is based on information of (Kakes & Broeders, 2006).

Table 3-5: Pension fund investment mix

	stocks	bonds						
maturity [y]	-	1	2	3	5	10	20	30
fraction [%]	50	5	5	5	10	10	5	10

The initial market value of the assets and liabilities are 1,250 and 1,000, respectively. These are all scenarios for a general case and not a case study of an individual pension plan, therefore these number represent initial index number, and necessary currencies.

The above mentioned assets and liabilities result in an initial funding ratio of 125%, which is a desirable level according to (Kakes & Broeders, 2006) and (DNB, 2009). To calculate the nominal value of the bonds and liabilities using the market value, the average yield curve, as displayed by the blue line in Figure 3-2, was used.

As all models represent a simplified version of reality, assumptions have to be made. Most of the assumptions have been discussed before and are summarized below:

- The first assumption is that the future state of the economy is represented by a six dimensional state, which is generated using a first order VAR model.
- The future consists of good and bad periods, just as the past, in which the state of the economy is generated using the VAR methodology to replicate similar behavior as the past good and bad periods.
- The length of these good and bad periods is in the same proportion as the historical observation period.
- The *VAR 2* model includes one crisis of 85 months in each scenario. The start of the crisis is uniformly distributed over the total length of the scenario.
- The calculation of future liabilities and assets is done without incorporating a demographic and actuary model. It is assumed that the initial cash outflows can be represented by Figure 3-4 and the distribution remains constant.

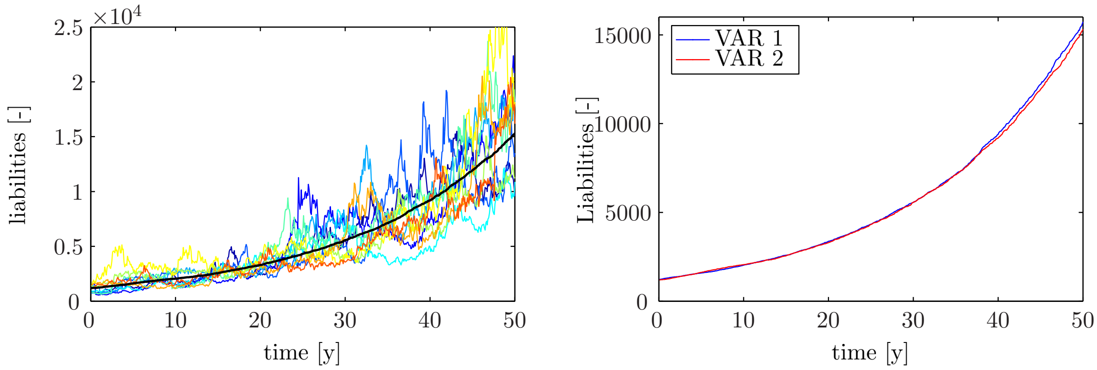
- The investment mix only consists of the market index and bonds with several maturities. Alternative investments, like private equity, hedge funds, commodities, real estate, etc. are not excluded. Derivatives like options, swaps, futures, etc. to hedge certain risks like interest rate, currency, longevity risks are not included in this analysis.
- Recovery plans, including dynamic indexation and contribution schemes are beyond the scope of this thesis. Just as the investment mix remains constant.
- Transaction and other costs are not included in this analysis.

Chapter 4

Results and discussion

This chapter discusses the results of a thousand random generated scenarios for the VAR 1 model as well as thousand scenarios for the VAR 2 model. The VAR 1 model generates scenarios using a direct first order Vector AutoRegressive methodology. The VAR 2 model includes crises, as explained in Section 3-2-2.

This took a 1.66 GHz Intel T2300 Dell Inspiron 640m laptop 3h 32m 52s. Unfortunately, more independent scenarios were not possible using this hardware due to large matrices involved.



(a) First ten scenarios of the liabilities of VAR 1

(b) Pension fund average liabilities

Figure 4-1: Pension fund liabilities

4-1 Liabilities

As discussed in Section 3-2-3 the liabilities are calculated by discounting the expected future cash outflows. As the nominal value of these cash outflows are similar for the two models, only the interest rate can imply differences. Figure 4-1(a) displays the evolution of the first ten economic scenarios of the VAR 1 model.

The liabilities are the discounted cash outflows. The cash outflows are similar for each scenario and only the yield curve can influence differences in the market value of the liabilities. As Figure 4-1(a) displays large fluctuations in the market value of the liabilities, it can be concluded that the influence of the interest rates are huge.

The black line in Figure 4-1(a) represents the average liability level for the VAR 1 model. The average liabilities of the two models can be found in Figure 4-1(b). As expected, these two are nearly similar. The slight difference at the end of the observation time can be explained by the limited number of crisis near the end. This results in higher average interest rates, and thus lower liabilities.

4-2 Assets

The evolution of the average asset value for the two different models can be found in Figure 4-2. It can be concluded form this picture that the average return on the assets are equal for the two models. This is expected, as the scenarios of the two models reproduce historical signals obtained from the same data range.

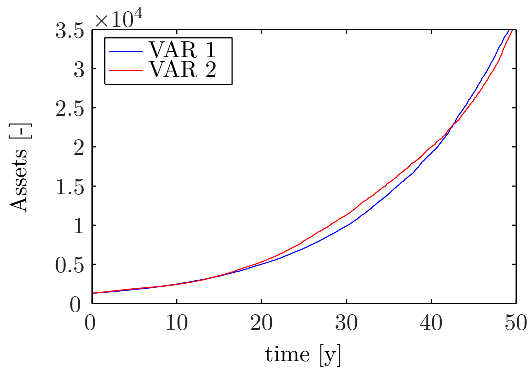


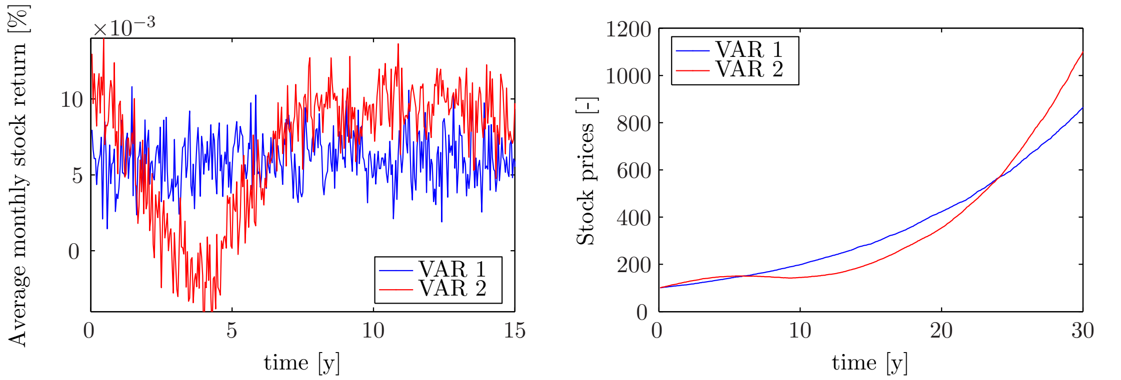
Figure 4-2: Average pension fund assets

However, Figure 4-3(a) displays the average monthly stock returns. This figure clearly displays the differences between the VAR 1 and VAR 2 model. The average returns of the VAR 1 model fluctuate around a steady mean, while the VAR 2 model clearly shows some long term dynamics.

The same economic dynamics can be seen in Figure 4-3(b) where the average market index is displayed. During the period between five and ten years, the stock prices remain nearly level. After this period, it catches up with the VAR 1 model.

4-3 Funding ratio

After analyzing the assets and liabilities, the funding ratio can be examined more closely. The average funding ratio is displayed in Figure 4-4. The first thing to notice is that the average funding ratios at the start and end are (nearly) similar. As explained in Section 4-2 this is



(a) Average monthly stock returns (b) Fourier transform of the random noise and filtered signal

Figure 4-3: Average monthly stock returns and stock prices

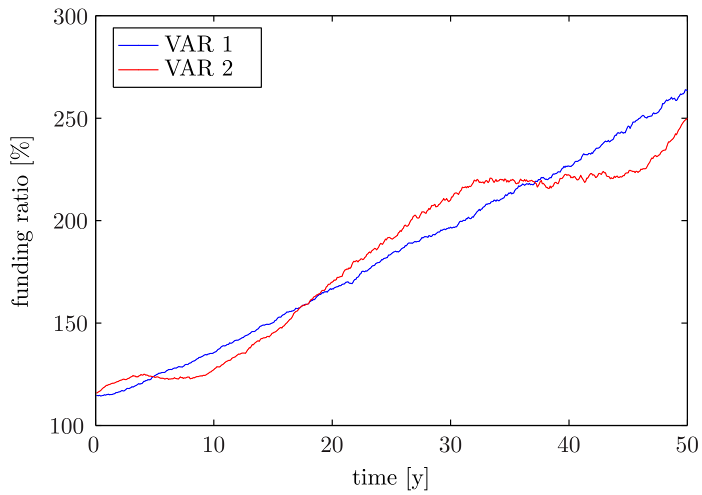


Figure 4-4: Average funding ratio

due to the fact that the scenarios are generated using the same historical data, which they reproduce.

It should be noted that the funding ratios obtained in this thesis are not representative for a particular pension fund. Therefore the absolute values of the average funding ratio are not as interesting as the differences in average funding ratios of the two models, *VAR 1* and *VAR 2*.

The second thing to notice is that between 20 and 35 years form now, it is expected that the average funding ratio will be larger in the VAR 2 model, the model including the crisis. At first sight this might by counter intuitive. However if we look at a cross section of the distribution of the funding ratios in Figure 4-5, some different conclusions can be made. It should be noted that this figure only displays funding ratios up to 300%, but higher funding ratios are present.

Figure 4-5 displays the histograms of the probability that the funding ratio is a certain value at times ranging from 5 to 30 years from now. Blue represents the fractions of the VAR 1 model, while red displays the VAR 2 model. Especially after 10, 15 and 20 years the red bars are higher than the blue for low funding ratios, meaning that the probability of the pension fund to have a certain low funding ratio is higher in the VAR 2 model than in the VAR 1 model.

The same conclusion can be drawn from Table 4-1. This table summarizes the signal characteristics of the funding ratios after 5 to 30 years. Although the average funding ratio of the VAR 2 model is similar or higher, the median of the VAR 2 model is clearly lower. The same can be noted for the first quartile. These two aspects mean that the VAR 2 model incorporates more risk.

A second thing to notice is that the standard deviation of the VAR 2 model is higher, ranging up to almost 50% after 30 years. Meaning that by incorporating the specific crisis into an ALM approach, more risk is incorporated, according to the classical risk return trade off.

Table 4-1: Funding ratio characteristics

		5 years	10 years	15 years	20 years	25 years	30 years
VAR 1	mean	124.2290	135.4793	150.1143	166.7477	183.8248	196.7414
	st. dev.	53.3526	72.3798	98.2894	133.9895	160.9110	183.9411
	skewness	1.1135	1.3463	2.1074	4.0939	3.1087	3.1766
	kurtosis	4.3348	5.5839	11.0582	43.1909	21.0534	19.9451
	min	23.8797	21.6470	13.1708	17.2813	10.3482	15.4582
	25 %	86.2632	82.1160	82.6257	82.0399	80.9803	85.0934
	median	112.1364	121.4445	124.8187	128.6432	132.8405	141.0304
	75 %	151.1828	171.5219	193.7312	212.3142	232.3858	250.5166
	max	345.6455	527.2016	883.8048	2029.7775	1811.1198	2002.4993
VAR 2	mean	123.4668	127.2568	145.1021	169.9077	191.1724	210.0961
	st. dev.	52.8432	92.2839	128.8811	176.4540	217.2371	258.3521
	skewness	0.0279	0.5104	1.0338	1.6692	2.1878	2.6141
	kurtosis	2.6803	2.2841	3.5549	6.4869	9.8902	12.5938
	min	10.1999	7.4574	7.7512	9.2206	11.7763	13.9052
	25 %	40.4794	57.9283	77.1271	86.4588	140.9542	193.2463
	median	128.8526	118.5814	87.5583	85.0893	90.1845	102.7292
	75 %	36.6963	42.3294	44.5755	50.9986	74.1584	144.1159
	max	310.0014	441.8953	672.8036	1246.6778	1905.7068	2360.5050

Both the VAR 1 and VAR 2 model show positive skewness. This is expected because the funding ratio cannot be symmetrically distributed as the funding ratio cannot be negative, but theoretically infinitively positive. Just as skewness also kurtosis increases with time. The distributions in Figure 4-5 clearly display non normal distributions as the tails represent a lot of observations. This is also represented by the kurtosis numbers in Table 4-1.

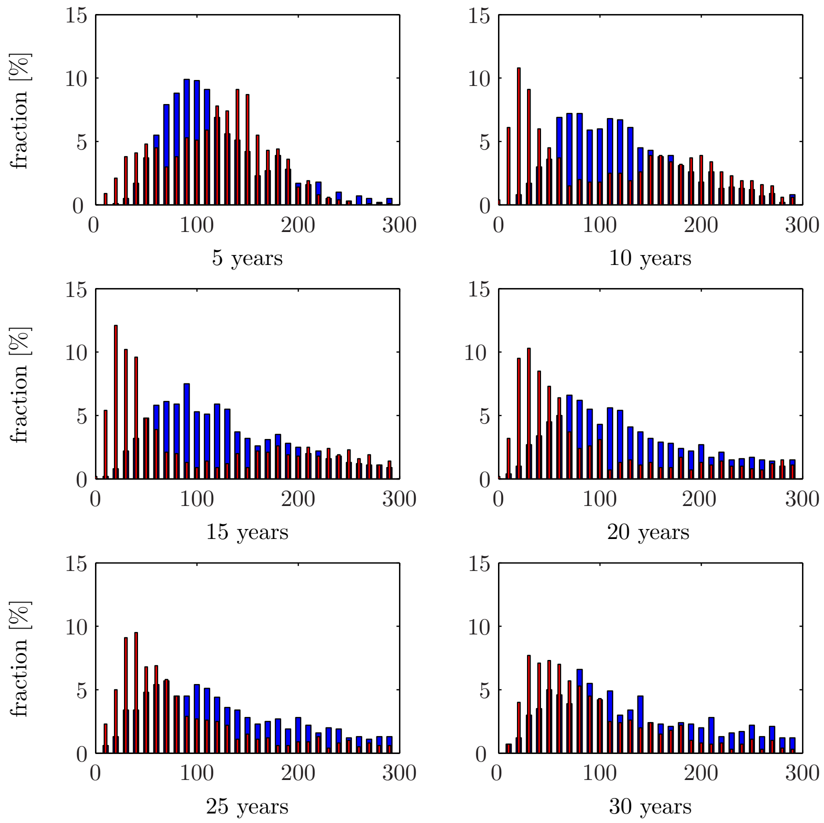


Figure 4-5: Cross section of the average fund ratios

Figure 4-6 represent the probability that the funding ratio is below four different thresholds, ranging from 80% to 110%, and how this evolves over time. Two conclusions can be drawn from this figure. First, in the first phase, up to five years, the probability underfundeding, below 100% is higher according to VAR 1 model. However, if the pension fund is underfunded, the level of being underfunded is lower for the VAR 2 model, as can be seen in the upper two subfigures.

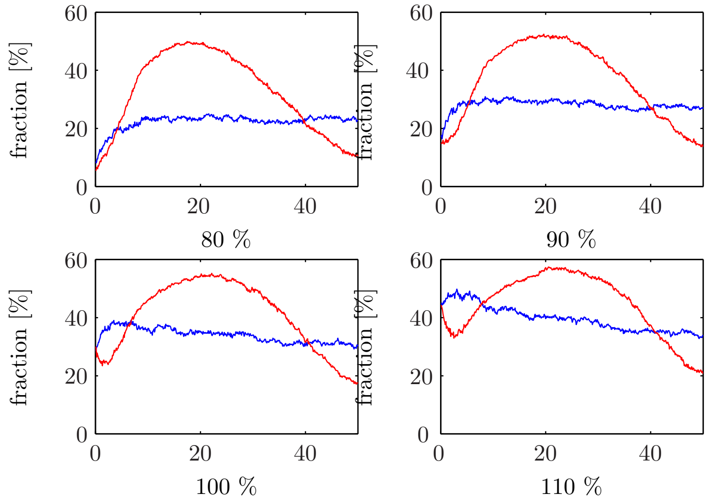


Figure 4-6: Probability of being under a certain funding ratio (lower is better).

The second thing to notice is that during the middle range, from five to around thirty five years, the pension funds perform better according to the VAR 1 model. This can be explained by the fact that more risk is present in the VAR 2 model, and therefore the chance of unlikely performance increases.

It can therefore be concluded that using the VAR 2 model in the long run, the same returns are expected. But the risk, denoted by the chance of being underfunded, and the standard deviation of the funding ratio, changes dynamically over time. Figure 4-7 clearly demonstrates the differences in risk incorporated in the two models.

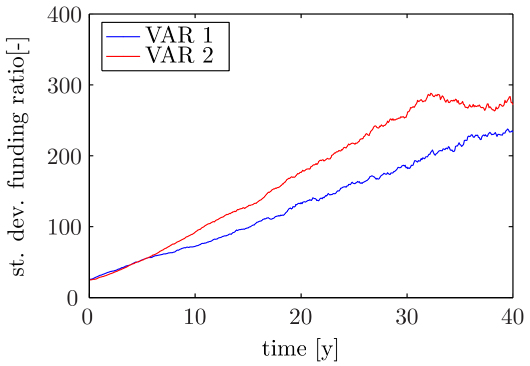


Figure 4-7: Standard deviation of the funding ratio

Chapter 5

Conclusions and recommendations

This chapter discusses the conclusions and recommendations with respect to the introduction of economic crisis in the scenario generating process in the Asset Liability Management (ALM) approach of pension funds. The conclusions can be found in Section 5-1. The recommendations with respect to the approach discussed in this thesis, as well as some limitations are discussed in Section 5-2.

5-1 Conclusions

Most economic scenarios are generated using Vector AutoRegressive (VAR) methodology in which the generation of economic scenarios should be *consistent* with a *clear set of assumptions*, according to (Bunn & Salo, 1993). Historically, this meant that the generated stock returns and interest rates should have the same characteristics, defined by their mean and standard deviation.

The above mentioned approach ignores long term economic dynamics. Therefore (Steehouwer, 2005) uses spectral analysis and parametric models to incorporate these dynamics. Unfortunate, these techniques provide limited certainty on the accuracy of the long term dynamics.

To cope with this, a new approach is proposed. To prove that this new method can be applied, this thesis uses a simple six dimensional first order VAR approach which differentiates between good and bad periods. The basic assumption in this method that in each generated scenario one crisis will occur, but the timing of this crisis is random. The length of the crisis is proportional to the sum of the crisis observed in historical data.

The first conclusion is that it is possible to include crisis in the ALM approach by using VAR methodology of generating economic scenarios. This allows for including long term dynamics into the analysis in an accessible way.If one expect crisis to occur in the future as it did occur in the past, this is a valuable result.

Applying the traditional VAR methodology to generate economic scenarios, long term economic dynamics are ignored. Therefore, the second conclusion is that the risk profile of the pension fund is underestimated. Figure 4-6 on page 30 clearly displays this underestimated risk, especially for the period between five to thirty years from now.

The results of this thesis clearly show that it is possible to include long term economic dynamics into a simplified model to observe the risk profile of a pension fund. The purpose of this thesis is not to obtain a perfect insight into this risk profile, but to introduce a new way of analysing.

5-2 Recommendations

As this is only the first step, there are a lot of recommendations to be made. The recommendations are split up into two parts which are called depth and width. Recommendations with respect to extending the model to give a complete overview of the risk profile of an individual pension fund is meant by depth. The width recommendations are recommendations that apply to different directions into which the results of this research can be used as well.

5-2-1 Depth

The programming done for the research in this paper applies to a simple generalized pension fund. To really map the risk profile of an individual pension fund, the model should be extended, and at least the following aspects should be taken into account.

- As mentioned before, this model does not include any demographic or actuary models. To investigate the risk profile, the migration of individual members should be simulated, including their life expectancy and individual contributions and pension rights. For example, indexation is not taken into account.
- The second step to complete the model is to extend the investment mix. Not only stocks and bonds should be evaluated. Also private equity, hedge funds, real estate etc. should be available and therefore monitored and regenerated.
- If the above mentioned points are implemented hedging strategies should be evaluated. The effect of derivatives, futures, options, etc. can be analysed during crisis.
- All the economic scenarios generated in the proposed model include *one crisis of constant length*, corresponding to the proportion of crisis observed in historical data. It might by interesting to extend the model to incorporate multiple crises and distribute these over the evaluated period, and varying the length of the crisis.
- The next thing to look at is the number of different kinds of periods. This research identified two types of periods *good* and *bad* periods. As this is a clear simplification, the amount of periods might be extended to four or more different kind of periods, each with their own particular behavior with respect to (cross)covariances between the economic signals.
- As mentioned before, transaction and other costs are not taken into account. It is recommended to incorporate these as well.

5-2-2 Width

To extend the obtained results to other fields of interest or other methodologies, some recommendations can be made. First, the methodology of implementing different kinds of economic periods in the analysis of a pension fund risk profile can be used at other institutes as well. Institutions that are using, or should use, the ALM methodology can use this approach. This means that insurance companies, (central) banks, but also airlines and energy corporations can gain more insight by using this approach.

Not only companies that use the ALM methodology can incorporate this approach, every institute that monitors macroeconomic developments can implement it. For example governments as well as stock analysts.

As mentioned in Chapter 4 the calculation of a thousand repetitions of this simplified model took more than three and a half hour. It should therefore be recommended to look at the possibilities to implement the more elegant method of stochastic programming. This might not be possible for all parts of the software, but a combination of stochastic programming and the linear scenario structure might be interesting. Doing this, the advantages of both technologies can be combined, like (Bogentoft et al., 2001).

By incorporating macroeconomic dynamics, the influence of generational accounting can be evaluated. This allows pension fund managers to evaluate the pension deal by taking the expected generational transfers during economic crisis into account.

5-3 Concluding remarks

The main purpose of this thesis is to introduce a new VAR methodology, which could be incorporated into an ALM analysis for a pension fund. As it was not the purpose to map the risk profile of an individual pension fund, no sensitivity analysis is executed. It is recommended to investigate the influences of changing the investment mix, the initial funding ratio, the length of the crisis and the probability distribution of the timing of the crisis.

This thesis clearly proves that there exist a possibility of including long term economic dynamics into an ALM analysis using the VAR methodology. As stated by (Kouwenberg, 2001), pension funds should always make the trade off between long term performance and the short term risk of underfunding. This new method allows better insight into this risk.

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