A model for regime switching behavior in the joint distribution of stock and bond returns

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By: Christian Engelen

Erasmus University Student Number: 289779

Mentor: Thijs Markwat

Second review: Erik Kole

Erasmus University, Rotterdam

Abstract

The paper demonstrates the application of a Markov Switching Copula model on stock-bond relationships. This method is sufficiently flexible as it allows dependency to be modeled separately in two regimes, representing alternate bear and bull market climates. Each regime is described by a copula with asymmetric marginal density functions, allowing the so described *Flight to Quality*, a curious negative dependence in a bear market climate, to be described separately from the overall positive dependence in bull market climate. The optimization procedure combines Markov switching and copula theory to produce a well fitted description of the dependence structure between national stock and government bond indices. The model successfully identifies flight to quality movement and permanent shifts in market behavior.

Contents

1. Introduction

Investors diversify portfolios to reduce the risk of loss. For this reason the manner in which the asset classes in which they invest are dependent upon one another is of great interest. The measure mostly used to characterize co movements between assets, including the assets studied in this paper, stocks and government bonds, is the correlation measure, often assumed to be constant over time and by definition assuming linear dependence.

In most liquid national financial markets, equities tend to be (overall) positively correlated to government bonds. This is logically derived from economic theory: Government bonds tend to respond to the assumed probability of that government defaulting on its debt. The price of equities tend to drop when the expectation of future revenues fall. In this manner both are dependent on the national economic climate.

When studying the dependence between stocks and government bonds over time it is easy to demonstrate that the positive dependence described above is not constant. Figure 1 shows the change in correlation between long term government bonds and an equity index over the same period of time.

As equities experience extreme drops in value investors tend to shift away from equities and into safer assets, such as government bonds and real estate. This has the effect that extremely low returns in equities tend to be accompanied by increased returns in bonds at such times. This effect offers and exciting diversification opportunity in times of crises and bear markets. Note that this negative correlation is observed in the extreme case, when returns on equities are exceptionally low. This phenomenon labeled a *flight to quality* by Baur and Lucey (2008). A flight from quality is the opposite effect, when investors shift away from bonds and into equities, so that the price of equities increases as the price of bonds fall.

Contagion and flights to/from quality are effects that cast doubt on the normality of the distribution between financial assets and bivariate normality between stocks and bonds is disproven by Gulko (2002), Capiello et al. (2006) and Rodriguez (2007), among others.

A flight to quality tends to occur in times of economic turmoil. In this paper we shall describe such a situation as a bear market. Beside the bear market the model used in this research assumes that a market can only be in one other situation, or *regime.* This shall be called the bull regime.

Gulko (2002) finds that government bonds are positively correlated to the nation in question's stock markets in bull times. This correlation becomes a negative one when markets are in a bear regime, and suggests a flight to quality. This not only implies that the correlation that is considered in the diversification of portfolios is dangerously unstable, but also that the assumed dependence between stocks and government bonds will be the furthest from the actual short term dependence level and structure at exactly the time they are needed most (during a crisis). Note that the negative correlation found by Gulko does, however, suggest that when markets are in a bear situation, bonds become an interesting diversification mechanism if used correctly and can be considered a safeguard against extreme drops in asset values.

Having shown that national stock-bond relationships take on unique structures in different economic climates, the separate distributions should be modeled using separate copulas for each regime, rather than the same copulas with varying parameters as has been done so far. In this manner a more flexible model is achieved.

The goal of this research is to gain insight into the dependence structure between national stock and bond markets. Throughout this process we will consider the possibility of a change in economic climate so that the model may take on different parameter in a bear regime and in a bull regime. The dependence structure is modeled by a student t copula with skewed student's t marginal distributions and Markov switching parameters. A secondary aim is to detect changes in the economic environment such as a flight to/from quality, a permanent change at a certain point in time, or other phenomena that may arise.

A stock-bond portfolio that ignores regime switching behavior is not considered, is likely to utilize incorrect value weights for the different asset possibilities. Failure to consider a regime switching behavior can cause misspecification of the diversifying nature of government bonds in exceptionally bad economic climates and would underestimate the level of dependence between equities in these same periods. This will cause an overreliance on stocks in the portfolio. When the economic climate worsens drastically, the benefits of the diversifying nature of the bonds will be underused and the dependence will increase between equities causing this asset type to lose much of its diversifying effect due to contagion. Okimoto (2008) shows that ignoring asymmetry in bear markets caused VAR calculation at the 99% level to be undervalued by an average of 10%.

In both these situations a copula model with Markov switching parameters can fine tune these calculations so that an accurate risk assessment is possible, based on a risk model that does not rely on false the assumptions of normality and symmetry.

The remainder of this paper will be organized as follows: I will briefly explain research done on the relationship of similar datasets and research done with similar models. I will then explain the methodology and the theory that underlies it. This will include a description of the copula and its marginal density functions, Markov switching theory and this theory is combined to derive the overall model underlying this research. Following this I shall describe the data that is used and why I have chosen to structure it in a certain manner. I will then discuss the results and lastly I will analyze these results, draw conclusions on them and discuss the validity of these conclusions.

2. Literature

Ang and Bekaert (2002) have estimated dependence between US, UK and German equity markets using a Markov Switching Multivariate Normal (MSMVN) model. This has given weak evidence for the existence of a bear regime. Okimoto (2008) builds on Ang and Bekaert's method but suggests that the normal distribution of returns in bear markets is an assumption that leads to a poor match of this particular model and should be adapted with an asymmetric distribution. Okimoto proposes a flexible framework by introducing copulas in a Markov semi Switching Asymmetric Copula model and uses this to measure the effect that international equity markets have on one another. In this manner Okimoto describes the bear regime with a separate (asymmetric) copula.

The manner in which two financial markets affect one another is most often described by the (linear) correlation or the covariance measure. Correlation only measures the scale of the dependence but fails to describe the dependence structure. The contagion effect on international equity markets (Ang and Chen (2002)) and the flight to/from quality in stock – bond relationships (Baur and Lucey (2008)) show that the *structure* of the dependence should be described as well as the *scale*. In this manner one can achieve a much more accurate description of diversification possibilities, VAR calculations and other common financial applications.

Ang and Chen (2002) demonstrate how financial data is seldom (bivariate) normally distributed. The distribution between financial markets exhibits fat tails and conditional correlation tends to increase in times of exceptionally high returns and in times of extremely low returns.

A model that describes the stock-bond relationship thus needs to allow two (or more) separate regimes (market situations), each with a different error distribution and parameters. Furthermore the relationship in each regime should be able to take on a negative and/or skewed distribution.

Hamilton (1994) describes a Markov switching model that allows precisely this flexibility when creating univariate models. Zhou (2006) and Capiello et al. (2006) demonstrate the effectiveness of modeling with different regimes and allow the copulas that describe the distributions in these regimes to take on different parameters.

Rodriguez (2007) introduced a model that uses Markov switching to alternate between copulas. Okimoto (2008) allows a different copula type to be associated with each of two regimes.

Okimoto (2008) defines a model that describes the relationship between national equity markets. The author uses a Markov switching model that allows the error terms to take on separate asymmetrical distribution structures. As the error terms of each the models for each state could be negatively related and/or skewed in nature, the skewed student t copula is ideal to investigate the nature of this relationship.

A copula is a distribution function that links together two (or more) marginal distributions to form a joint distribution. First introduced by Sklar (1959), the approach has only historically recently been applied to finance, by Nelson (1999), Cherubini and Luciano (2002) and Patton (2004, 2006), among others. Copulas capture non linear dependence and contain all information about the dependence structure of a vector of random variables.

Hansen (1994) illustrates a skewed Student's t distribution. This distribution can take the form of a Student's t distribution if its parameter λ takes on the value zero. If in addition the parameter ν approaches infinity, the distribution approaches a normal distribution.

The relationship between stocks and government bonds tends to exhibit negatively conditional correlation in times of crisis. Baur and Lucey (2008) study the causality of these co movements and define the negative dependence as a flight to quality if stock markets are falling relative to bond markets and a flight from quality if stock markets are rising relative to bond markets. Cappiello et al. (2006) model this dependence by combining a Markov Switching Model with a GARCH model and extensions thereof. The aim of this paper is to use a methodology similar to that of Okimoto to allow a flexible model to be built that identifies bear and bull regimes and uses different copula structured to describe the distribution of the error term. As little research has been done into asymmetric negative dependence the challenge will be to find copulas that effectively describe the bivariate marginal distribution.

3. Method

The model will take the form of a copula with Markov switching parameters as described by Okimoto (2008). The main difference between this research and that of Okimoto is that the stock-government bond relationship is expected to be very different to the relationship between the international equity indices that Okimoto studied. Distributions such as the Gumbel copula are unable to present a negative relationship between data series. Typical copulas that are able to achieve this are the normal and Student's t copula, among others. As the normal copulas can be achieved at certain parameter values of Student's t copula, the latter shall be used to model the distribution of the data series.

A. The model

The overall model that will be used is given below. Its various elements and their underlying theories will be described in the remained of this section.

$$r_t = \mu(s_t) + \Sigma_1^{1/2}(s_t)e_t(s_t) \quad s_t \epsilon\ 1,2(\tag{1}$$

r_t is a 2x1 vector of returns of the stock and bond market at time t, $\mu_t(s_t)$ is a vector of each market's marginal mean in regime s_t and $\sum_1(s_t)$ is a diagonal matrix containing each variables' marginal variance in regime s_t. S_t takes on values equaling 1 and 2 representing regime 1 and regime 2 and $\sum_1(s_t)$ takes on the following form:

$$\Sigma_1 = \begin{pmatrix} \sigma_b^2(s_t) & 0 \\ 0 & \sigma_s^2(s_t) \end{pmatrix} \; s_t \epsilon\ 1,2 \qquad (2)$$

Each sigma above is dependent on s_t and is the standard deviation of the margins of the two markets in question and different for each regime.

Ang and Berkaert (2002) created this model assuming that $\mu(s_t)$ and $\sum_1(s_t)$ are regime dependent only. Other factors that could affect these variables but are not taken up in the model are interest rates, their own past values. [1]

The states, or regimes, are an important element of the Markov switching model. In this case they take on the values 1 and 2 which are merely index numbers representing regimes. As the economic environment is often the most important factor in determining the next period's economic state (see Hamilton (1989)), the stochastic process can be described as a Markov chain. The EM algorithm, developed by Dempster, Laird and Rubin (1977), can then be used to get reliable Maximum Likelihood estimates.

Throughout this research it will be assumed that s_t will take on one of two states. The state affects the dependence structure between the two asset types, and it is assumed that the most significant difference in dependence structure is the change between bull and bear markets. If this is not the case then other changes in economic environment will maximize the likelihood of (1). If one supposed that the bear and bull regime are the most significant changes affecting the nature of the dependence structure, then a two state regime ignores the prospect of more specific states (recovery, depression or states that may represent a structural change). The returns of the time series are compared to the assumed states of the model to verify the validity of the bear/bull assumption and to determine if other changes, such as a permanent change in climate, take place.

B. Markov Switching Model

Markov switching models in finance are based on two or more regimes that represent different environments. This research assumes that two regimes exist, denoted $s_t = 1$ and $s_t = 2$. Several variables in a Markov Switching model are allowed to take on a separate values for each regime or environment at a certain time, t.

S_t will follow a two-state Markov Chain with transition probability:

[1] Okimoto (2008) attempted to model these variables using a General Auto Regressive Conditional Heteroskedastic model (GARCH) with multiple time lags, but could not find such a relationship at a significant level.

$$P = \begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix} \tag{3}$$

In (3), p_{11} denotes the probability that y_t was generated from regime 1 at time t assuming that y_{t-1} was also generated from regime 1. The only alternative to the dependent variable being generated from state 1 is for it to be generated from state 2. The value $1-p_{11}$ thus denotes being in state 2 at time t given that y_{t-1} was generated from regime 1 also.

$$p_{ij} = P[s_t = j \mid s_{t-1} = i] \qquad i, j \in 1{,}2 \tag{4}$$

As mentioned, we presume that the bivariate dataset is drawn from a separate distribution for each state. In this section we will refer to these distributions as $D_j(\theta_j)$ (for the distribution in regime j). θ_j represents the parameters that describe distribution j. In the following sections one will come to realize that these bivariate distributions are copula distributions. The difference in structure of these distributions can be expressed by the difference in θ_j. If $\theta_1 = \theta_2$ the distributions are identical. The density of y_t conditional on the regime is thus equal to the above described distribution.

$$f(y_t \mid s_t = j;\ \theta_j) = D_j(\theta_j) \qquad j \in 1{,}2 \tag{5}$$

The term π_j will refer to the probability that s_t is in regime j.

$$\pi_j = P\left[s_t = j;\ \theta_j\right] \qquad j \in 1{,}2 \tag{6}$$

The probability of being in regime j is generated by equation (6) assuming particular starting values. Following probability theory we can combine (5) and (6) to show that:

$$f(y_t; \theta) = \sum_{j=1}^{2} [\pi_j \cdot f(y_t \mid s_t = j; \theta)] \qquad \theta = (\theta_1\ \theta_2) \tag{7}$$

We now have an expression for the density of y_t. The maximum likelihood of the model, based on y_t, is thus described as follows:

$$\mathcal{L}(\theta) = \sum_{t=1}^{T} \log\,[f(y_t; \theta)] \qquad \theta = (\theta_1\ \theta_2) \tag{8}$$

C. The EM Algorithm

The probability that one is in regime j at time t is dependent on the distribution parameters of each state θ and of course on the observation itself in the following manner:

$$P[s_t = j \mid y_t;\ \theta] = \frac{P[y_t, s_j = j \mid \theta]}{f(y_t; \theta)} = \frac{\pi_j \cdot f(y_t \mid s_t = j; \theta)}{f(y_t; \theta)} \tag{9}$$

Hamilton (1994) describes the EM algorithm which helps to maximize the likelihood of a Markov Switching Model with parameter vector θ. Each variable in the optimization of the model described in (1) has a high dependence on matrix P, described in (3). Optimization with a large number of parameters becomes difficult in this case. The EM method separates the elements of P and estimates them separately from θ, the regime switching elements of the copula. this estimate is the taken as constant in the optimization of θ. This estimate of theta is then again used to determine a new estimate of P. This process is repeated until the parameter vector converges. The process is explained more specifically below.

By assuming the values of θ, one can derive a reasonable inference about the probability of each observation y_t having been generated by state j based on the density function of the distributions $D_j(\theta_j)$. Once we have an inference (denoted $\xi_{t|t-1}$) of the probability that y_t is generated in state j, we can form a forecast of a probability that each y_{t+1} is generated by state j. The transition matrix P can be used to calculate the probability of S_{t+1}=j for j = 1,2.

A starting value $\xi_{1|0}$ must be chosen for each state as these estimates cannot be based on historical data. Equations (10) and (11) are performed in succession for each t. Notice that these estimates are then used as the independent variables of the next equation. Logically the algorithm continues until t = T. The inference is calculated in (10);

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \odot \eta_t}{1'(\hat{\xi}_{t|t-1} \odot \eta_t)} \tag{10}$$

Note that the denominator of (10) is the sum of all $\xi_{t|t-1} \odot \eta_t$ values *up point t.* The forecast is calculated in (11);

$$\hat{\xi}_{t+1|t} = P \cdot \hat{\xi}_{t|t} \tag{11}$$

$\odot$ - denotes element by element multiplication

(12)

The likelihood is described in (8) and can now be estimated by using the forecast.

$$\mathcal{L}(\theta) = \sum_{t=1}^{T} \log\left[f(y_t | \mathcal{Y}_{t-1}; \theta)\right] = 1'(\hat{\xi}_{t|t-1} \odot \eta_t) \tag{13}$$

$\mathcal{Y}_{t-1}$ – denotes all observations before time t.

Smoothed inferences are denoted $\xi_{t|T}$. As the notation suggests, the estimate is based on information up to the end of the sample, time T. The last estimate $\xi_{t|t}$ can be denoted $\xi_{T|T}$ and is equal to the smoothed inference value as all information of the sample is included in the value. The smoothed inference of the probability that y_t was generated from regime j can be calculated as follows:

$$\hat{\xi}_{t|T} = \hat{\xi}_{t|t} \odot \{P' \cdot [\hat{\xi}_{t+1|T} (\div) \hat{\xi}_{t+1|t}]\} \tag{14}$$

Where (÷) is the sign for element by element division. This time one works backwards from time $\xi_{T|T}$ tot calculate $\xi_{T-1|T}$, then $\xi_{T-2|T}$, etc..

The smoothed inferences are used to guide the optimization function. The likelihood estimate is calculated for each regime and I multiplied by the chance of being in that regime. The total likelihood estimate is simple the sum of this result, as is described in sub section E. The smoothed inference are used to describe the probability of being in each regime, rather than the forecast (11) as the smoothed inference take on more extreme values, which helps guide each iteration of the optimization process, and as they are less dependent on the assumed initial inference value.

In the next section we will take a closer look at η_t and its components, $f(y_t|\mathcal{y}_t;\theta)$, for each state.

D. Copula theory

Copulas are standard distributions that describe the dependence structure between the marginal distributions of (in this case) two different markets of time length t. Once a copula type has been chosen the optimal parameters can be estimated by using maximum Likelihood (ML) theory. Copulas can be compared to one another by its likelihood measure (measure of model fit), Kendall's tau (τ) (measure of dependence), Spearman's rho (ρ) (measure of linear dependence) and Lower (Upper) lambda ($\lambda_{L\ (u)}$) (measure of tail dependence).

Theorem 1 (Sklar (1959)) Let *H* be a distribution function in R^k with margins F_s and F_b, then copula C exists such that for all x in R^2.

$$H(x_s, x_b) = P[F_s(X_s) \leq F_s(x_s), F_b(X_b) \leq F_b(x_b)] \tag{15}$$

If H is continuous, *C* is unique and can be expressed as

$$C(u_s, u_b) = H(F_s^{-1}(u_s), F_b^{-1}(u_b)) \tag{16}$$

Where u_s, $u_b \in R^2$ and $F^{-1}(u)$ is the inverse of the marginal distribution function of u.

Copulas thus consist of two parts. The marginal functions F_S, F_B , which describe the marginal behavior of each time series (error tem in (1)), and a copula C, which describes the dependence structure between F_S, and F_B, and thus the dependence structure between e_S and e_b. Due to this decomposition it becomes relatively easy to study the dependence structure between two markets.

The copula used is the student's t copula, which is based on student's t distribution. The probability density function and cumulative density function of this copula can be found in appendix A.

E. Marginal distributions

The marginal densities (F_s and F_B) of the student's t copula (in both regimes) are univariate skewed student t distributions, each with parameters lambda and nu. This function can take on parameter values so that it is identical to the (symmetric) student's t function (lambda is equal to zero) and to the normal function (if in addition to lambda equaling zero, the degrees of freedom approach infinity). The probability density function and cumulative density function of a skewed student's t distribution is shown in appendix A, as described by Hansen (1994).

The skew Student's t distribution gives flexibility in modeling the values, x and y (the state-dependent error terms of (1) and allows an asymmetrical distribution. Values in the upper left and lower right quadrants can be described by setting the parameter rho (of the student's t copula) to a negative value.

F. Likelihood function

The likelihood function is obtained by differentiating H (described in (18)) by x_s and x_b. This is equal to the density of $C(u_s,u_b)$ times the density functions of the marginal functions on which $C(u_s,u_b)$ is based:

$$h(x_s, x_b) = \frac{H(x_s,x_b)}{\partial x_s \partial x_b} \tag{17}$$

The likelihood function of a copula is simply the probability density function of the copula multiplied by the probability density functions of the marginal probabilities as shown below:

$$L(\theta) = c[F_s(x_s,\theta_s), F_b(x_b,\theta_b), \delta] \cdot f_s(x_s,\theta_s) \cdot f_b(x_b,\theta_b) \tag{18}$$

$$where\ \theta_s, \theta_b, \delta \epsilon\ \theta$$

As mentioned earlier, the likelihood function of each observation assumes that y_t is in a given regime j. The log likelihood value is therefore multiplied by the chance of being in regime j for each observation in each of the two regimes. The sum of these results is the likelihood estimate of the entire Markov switching model.

4. Data analysis

This research is based on the weekly returns of Morgan Stanley Country Indices (MCSI) and benchmark indices for 10 year government bonds of the United States, United Kingdom, Germany and Japan. Returns are based on the closing times every Thursday (to avoid end of week effects). These nations have been chosen as these have stable and liquid economies backed by separate, stable currencies. These markets have generally not suffered from major financial or economic blockades since the Second World War. German bonds may be affected by the Russian threat during the cold war, although they are now known as some of the most prudent bonds available due to German fiscal responsibility.

As this research does not consider international relationships, only the relationship between returns in the abovementioned national asset markets, we do not need to worry about time (hour lags) between closing times and all returns are based on local currency investments.

The data ranges from 1st January 1987 until 1st January 2009. This gives a total of 1174 original data points (index points).

Ideally short intervals are used as the purpose of this research is to separate extreme movements from calmer ones. If monthly returns were to be used the larger movements would be incorporated into the monthly average. The cause of the bear regime phenomena, flight to quality, is the aggregate selling of equities and buying of government bonds. Weekly returns are used rather than daily returns as this process could take one or two days as investors analyze different safe alternatives.

The basic descriptive statistics of the data are given below. Certain details and graphs will only be shown for the data pertaining to the United States. In such a case the similar graphs for the other nations can be found in Appendix B.

The weekly return of US Stock and US Bond indices

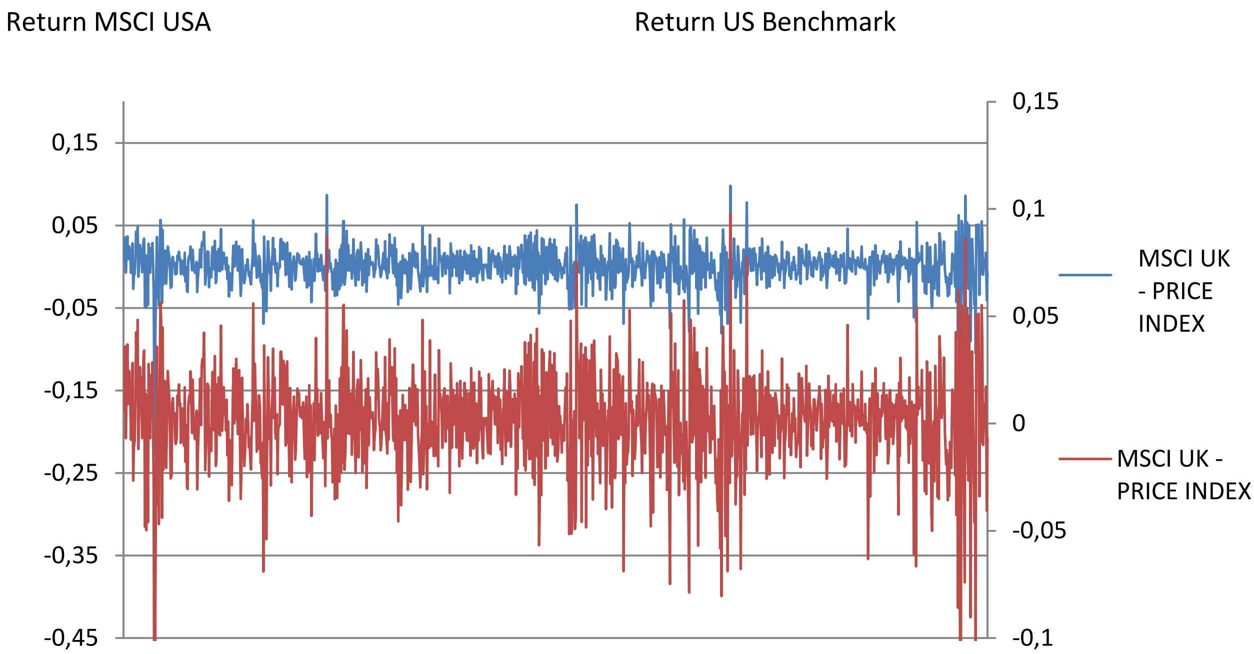


Figure 1. The weekly return of US stock and US Bond indices

Note that (1) models the *dependence* of two time series, so their joint movement is of interest. The moments of extreme returns coincide and closer inspection reveals that at certain time points the returns of equities fall as those of bonds rise. The question remains if the dependence structure differs to such an extent that these points in time constitute significantly different distribution parameters.

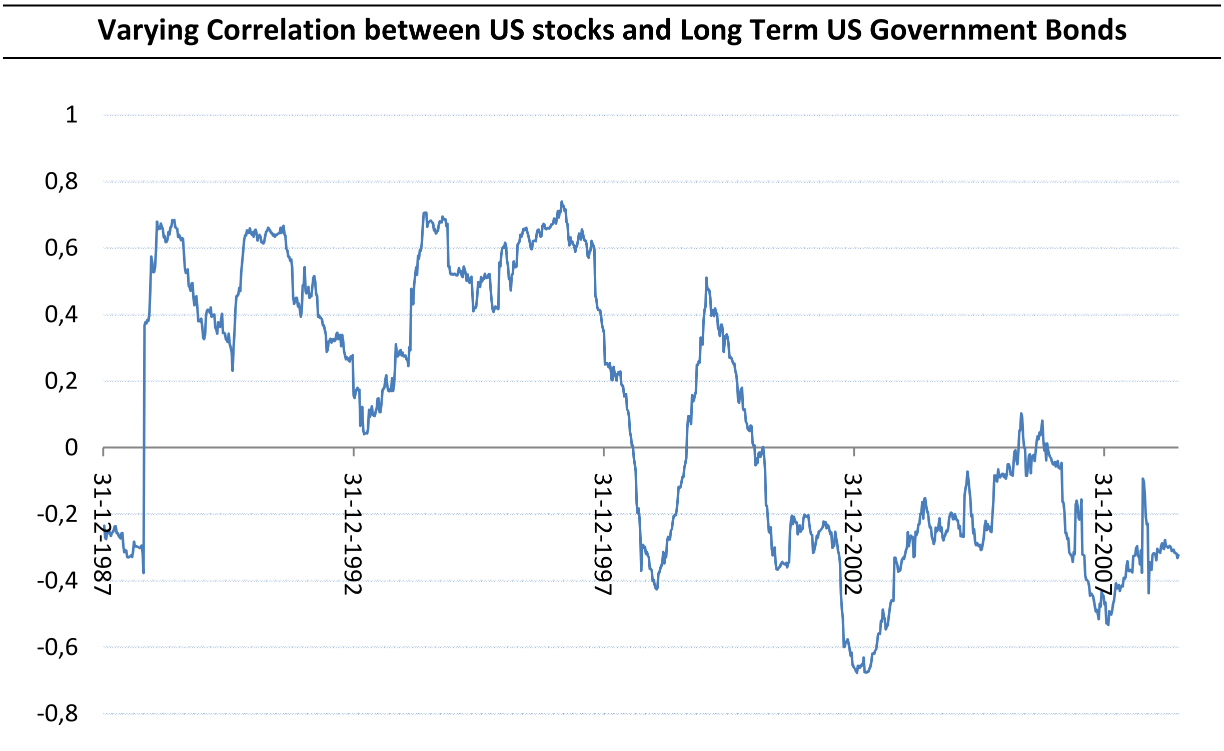


Figure 2. A (one year) moving window correlation of weekly returns of Morgan and Stanley Country Index USA and Government long term bond benchmark USA.

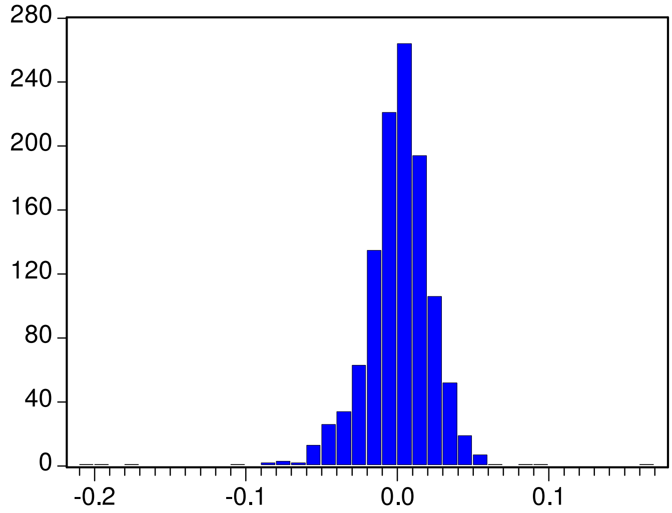
Not only does the correlation vary over time, it is also fails to describe the dependence structure in which two asset classes are related. Several assumptions underlying this measure are invalid for the equity-government bond relationship.

The descriptive statistics of the return of the mentioned asset classes are shown in figure 3 and figure 4. The results for the dataset of the USA are typical of stock and bond markets in all nations' studies. According to economic theory, stocks have a higher mean return than bonds and are more volatile.

Notice that US stocks exhibit a negative skew, and bonds a positive skew. This too is typical of these asset classes, and can also be seen in the distribution of the other national return indices in appendix B. Figure 3 and figure 4 give a sound indication of the marginal density distribution of the dataset that will be linked by the copula. Bare in mind that it is not the values shown below, but the error term, that will determine the marginal density F_s or F_b.

The marginal densities will most likely be the standardized version of the distributions shown below (with a mean equal to 0 and a standard deviation equal to 1).

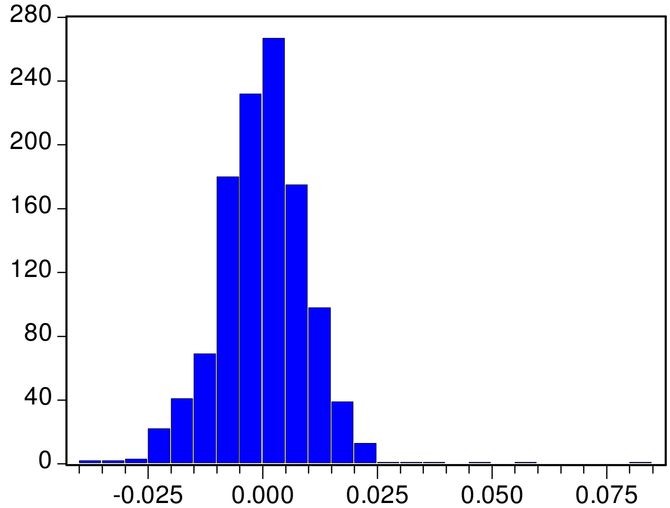
MSCI Return series USA - Histogram and basic statistics



Series: RETURNS_MSCI_INDEX_USA	
Sample 1 1150	
Observations 1149	
Mean	0.001132
Median	0.002560
Maximum	0.167014
Minimum	-0.202849
Std. Dev.	0.023711
Skewness	-1.391127
Kurtosis	16.61191
Jarque-Bera	9241.068
Probability	0.000000

Figure 3. A Histogram and basic distribution statistics of the full MSCI US return series.

US Benchmark Return Series - Histogram and basic statistics



Series: RETURN_GOVT_BONDS_USA	
Sample 1 1150	
Observations 1149	
Mean	0.000188
Median	0.000608
Maximum	0.082440
Minimum	-0.038506
Std. Dev.	0.009960
Skewness	0.502379
Kurtosis	8.369225
Jarque-Bera	1428.500
Probability	0.000000

Figure 4. A Histogram and basic distribution statistics of the full US Benchmark Long term Government Bond return series.

Following statistical theory, a Jarque-Bera recording follows a chi-squares distribution with two degrees of freedom and normality is rejected if the Jarque Bera statistic is higher than 10.60 (at a 95% significance level). Notice that for these distributions and for the distributions in appendix B, normality is rejected.

5. results

To demonstrate the presence of a dual state system, the US data has been split into extreme values for equity returns (in this case, below -0.02%) and regular returns (those above -0.03%). The scatter plots of the two supposed regimes and their lines of best fit are shown below. This separation helped to shape the initial value in the optimization of the maximum likelihood of the MSSAC model.

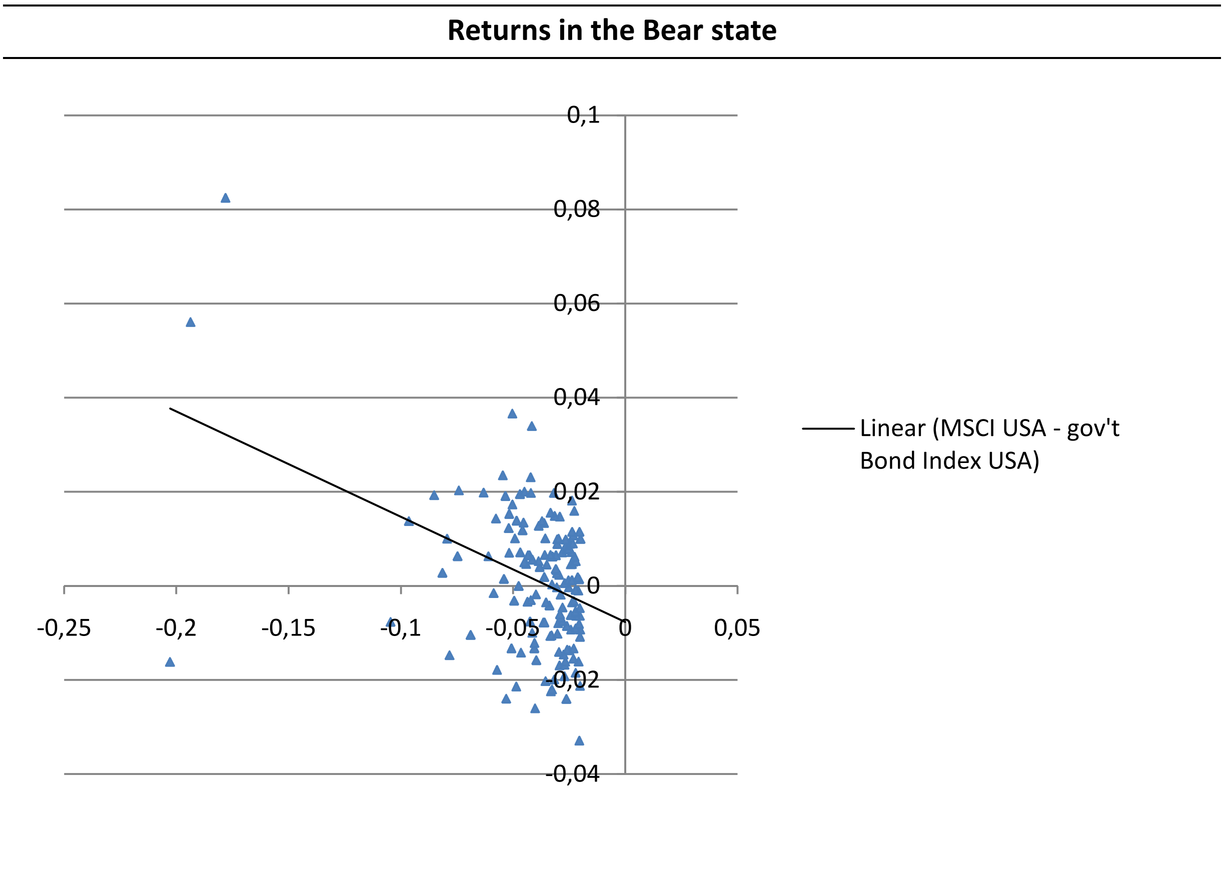


Figure 5. US equity returns in the supposed *"bear"* regime (<-2%) and the matching return of the 10Y government bond index.

Returns in the Bull state

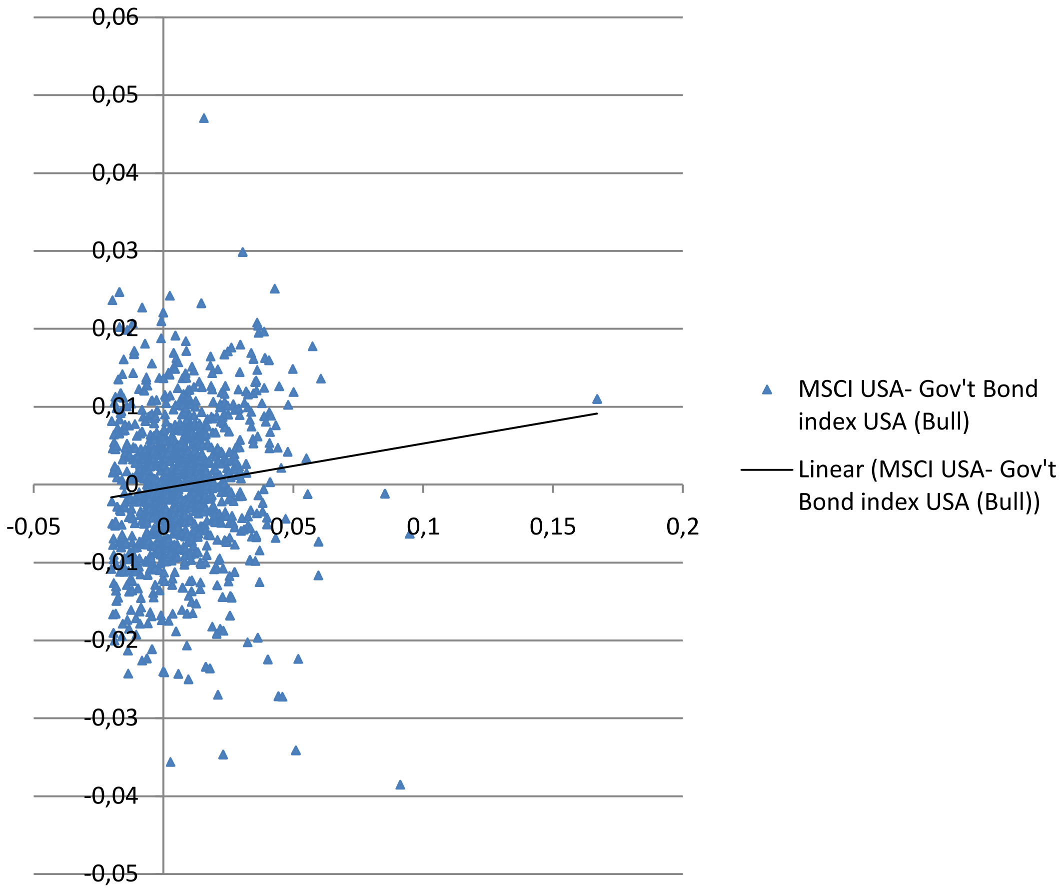


Figure 6. US equity returns in the supposed *"bull"* regime (>-2%) and the matching return of the 10Y government bond index.

Throughout this paper it has been assumed that states 1 and 2 of the Markov Switching Model represent a bear and a bull regime. As we know from econometric theory, a Markov Switching model can represent *any* change in model structure, and in this manner can also represent a split sample or other parameter changes. In this manner the two states represent different economic climates. The hypothesis that the bear and bull regimes are described by the two states can be tested by observing if extreme drops in market value coincide with the bear regime assumption. As mentioned earlier, the copula in each regime describes the dependence between the stock and bond time series. This implies that regimes will not necessarily change in extreme downturns or upturns, although this is suspected.

The graph below shows the smoothed inferences of Markov Switching state and the weekly returns of equities, separately for the US and the Japan. Recall that the smoothed inference is a probability assumption of being in a certain state, in this case state 1. The smoothed inference probability of being in state 2 is by definition equal to 1 minus the smoothed inference probability of being in state 1.

US MSSAC Model - Smoothed Inferences & Equity Returns

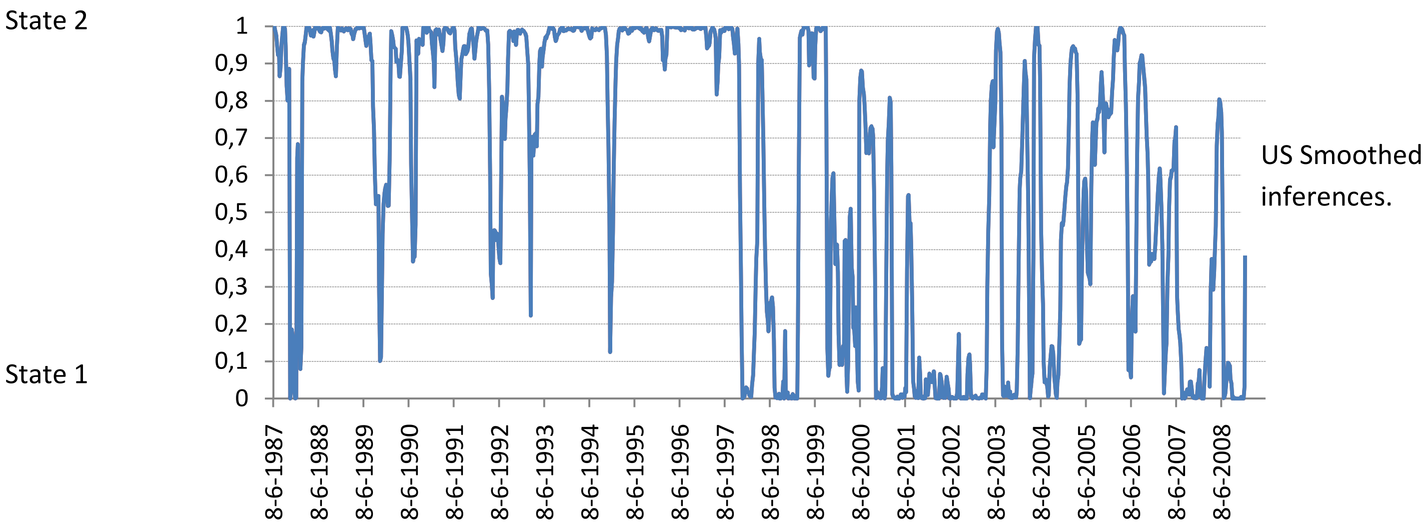


Figure 7. US MSCI index returns and smoothed inferences – a comparison.

$$P_{US} = \begin{bmatrix} 0.93 & 0.04 \\ 0.07 & 0.96 \end{bmatrix}$$

In the case of the US the change in economic climate that is detected seems to have happened around 1997. The suggested change in economic climate seems to be a change over time.

The economic climate of the United Kingdom also appears to change in 1997. This suggests that a large scale impact, such as the Asian crisis, has changed the way in which investors behave after this time.

UK MSSAC Model - Smoothed Inferences & Equity Returns

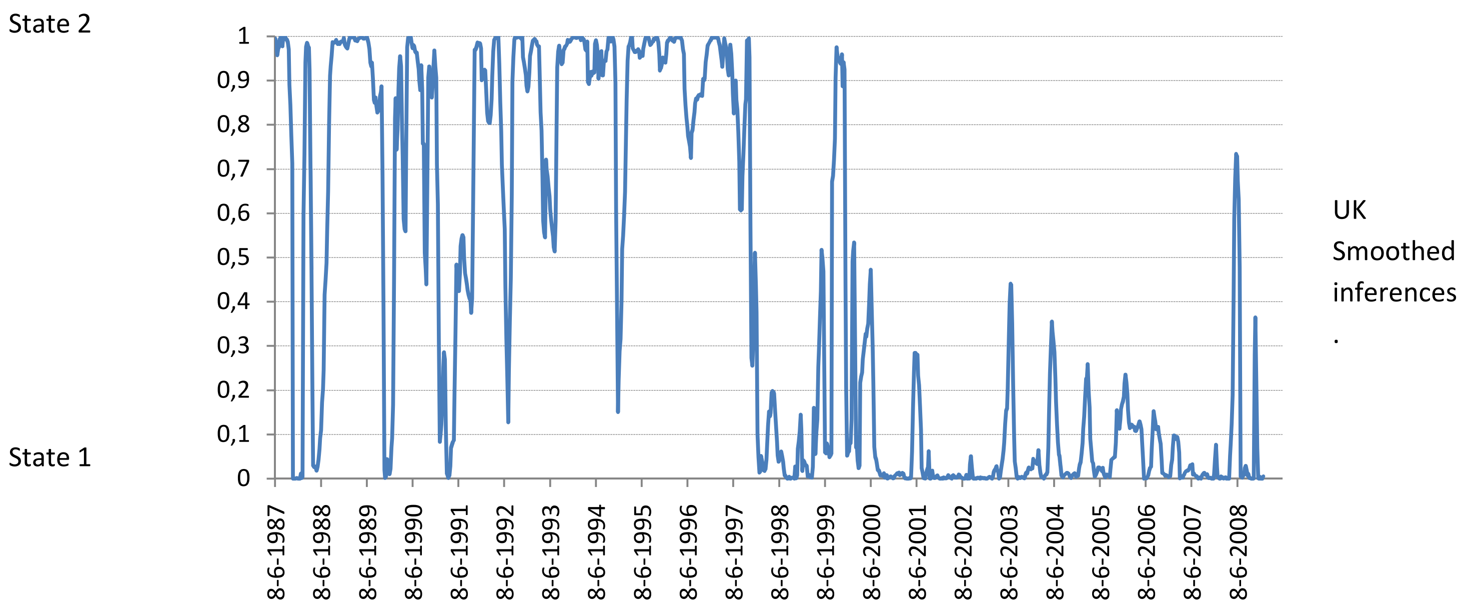


Figure 8. UK MSCI index returns and smoothed inferences – a comparison.

$$P_{UK} = \begin{bmatrix} 0.97 & 0.04 \\ 0.03 & 0.96 \end{bmatrix}$$

German MSSAC Model - Smoothed Inferences & Equity Returns

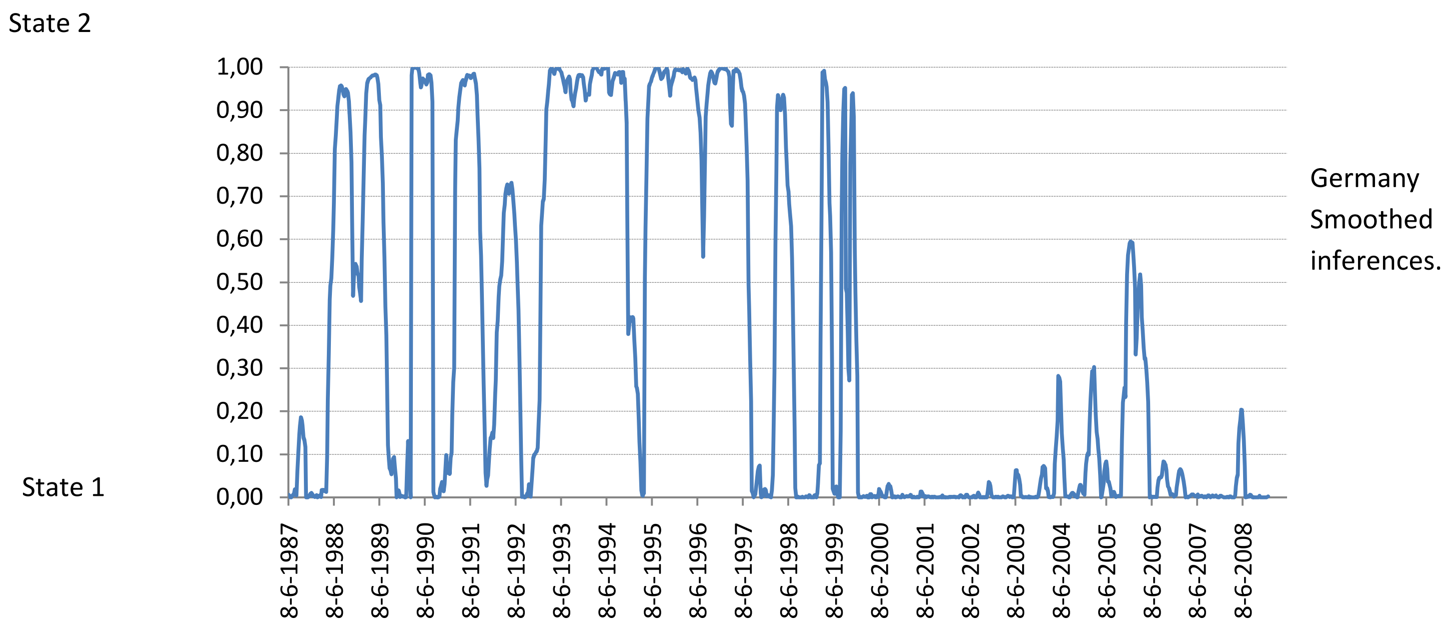


Figure 9. German MSCI index returns and smoothed inferences – a comparison.

$$P_{GER} = \begin{bmatrix} 0.98 & 0.04 \\ 0.02 & 0.96 \end{bmatrix}$$

Figure 9 indicates that Germany most clearly breaks with its past dependence structure as it fails to return to its past economic state after 1999.

Japan MSSAC Model - Smoothed Inferences & Equity Returns

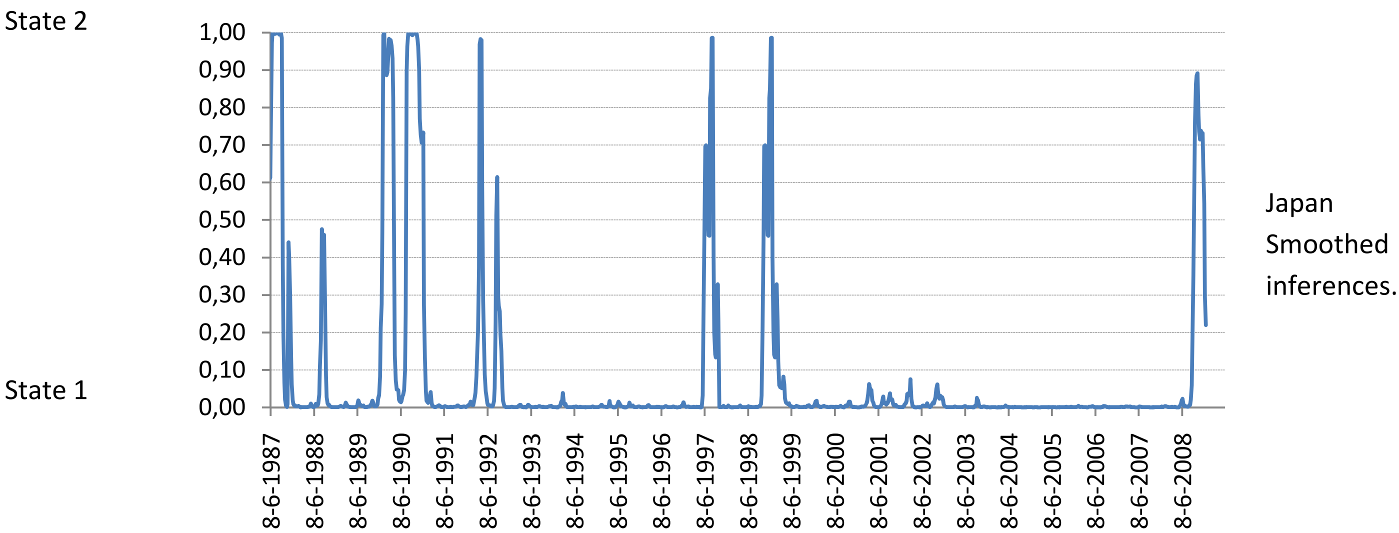


Figure 10. Japanese MSCI index returns and smoothed inferences – a comparison.

$$P_{JAP} = \begin{bmatrix} 0.99 & 0.1 \\ 0.01 & 0.89 \end{bmatrix}$$

The Japanese economy most clearly shows momentary switches in regime before returning to its regular state. Despite having a lower maximum likelihood value than its counterparts, Japan appears to show a clear change in dependence structure in times of turmoil in the financial markets. Notice that the spike in 1997 corresponds to the Asian crisis. The spike in 1998 potentially points to default of the Russian government on its bonds at that time.

Results of MSSAC Model

Nation	Regime		μ	σ	ν	λ	ρ	Log-Likelihood
US	1	Stock	-0.088	0.503	5.32	-0.307		2276
		Bond	0.050	0.210	7.38	0.708		
		Copula			20.00		-0.921	
	2	Stock	0.082	0.085	3.27	0.496		
		Bond	0.091	0.354	2.20	0.802		
		Copula			20.00		0.982	
UK	1	Stock	-0.080	0.603	10.54	-0.257		1897
		Bond	0.070	0.300	11.20	0.700		
		Copula			20.43		-0.930	
	2	Stock	0.076	0.401	6.20	0.417		
		Bond	0.072	0.301	9.71	0.718	0.830	
		Copula			27.83			
Germany	1	Stock	-0.055	0.703	5.48	-0.123		1977
		Bond	0.054	0.270	7.78	0.851		
		Copula			9.25		-0.999	
	2	Stock	0.091	0.401	6.90	0.417		
		Bond	0.060	0.201	6.91	0.850		
		Copula			20.27		0.999	
Japan	1	Stock	-0.039	0.553	5.37	-0.188		1622
		Bond	0.060	0.270	7.44	0.851		
		Copula			5.30		-0.998	
	2	Stock	0.071	0.401	7.00	0.317		
		Bond	0.052	0.171	14.35	0.810		
		Copula			10.02		0.900	

Table 1. The optimal parameter values for the MSSAC model for each nation.

The models based on US, UK and Germany data find (relatively) low values for the first regime stocks while bonds in the same regime derive an average value of around 0.05. For the second regime stock markets attain an average stock return value around 0.08. One notices that the standard deviation of the second regime is typically lower than that of the first regime. This in combination with a higher average and a strong positive rho of the copula in the second regime is in sharp contrast to the low average and strong negative rho of the copula in the second regime. This is consistent with the flight to quality theory. The second regime, containing a higher average and lower standard deviation consistently also displays a strong positive dependence between the stock and bond returns.
The lambda values indicate the level of skewness. Notice that these tend to be positive and high in the second regime and are typically, though not always, slightly negative in the second regime.

The Japanese indices most clearly demonstrate a flight to quality pattern. Also here regime one (relative to regime 2) has a lower average combined with a higher volatility and a negative dependency between the asset types in question.

6. Conclusion.

The Copula based Markov Switching model provides interesting insight into the dependence between stocks and bonds. A flight to quality situation was discovered in the Japanese markets and a definite permanent change in economic climate is evident for the United Kingdom, United States and Germany at the start of the 1990's. Major changes taking place at this time were the Asian financial crisis, the impact of which was strongly felt in the western world due to an increase in globalization, the ruble crash and default on Russian Government bonds. The permanent changes in economic climate are most likely more significant than a flight to quality, which would involve relatively few data points. Incorporating another state in the model would make optimization exceptionally difficult (the two state model already has 22 parameters). In this case it would be most sensible to slit the data into two groups, the boundary being the point where the permanent change appears to take place. This would allow us to inspect each smaller set without significant change in economic climate.

It is only natural that the economic environment and the manner in which financial markets behave change over time (consider the last few decades). For this reason further research should go into determining an effective time span so that enough periods of negative equity returns are present in the dataset but that the time span is not so great that the economic climate undergoes permanent structural changes.

Another method would be to incorporate explanatory variables into the model (currently only an average is used.) The disadvantage of this procedure is once again that optimization would be more difficult yet due to the number of parameters involved.

Furthermore one could consider the effect of interest rates on the model. Interest rates tend to move lower in bear markets and yet have a positive effect on bonds. This provides a further basis for the flight to quality phenomenon, with the additional advantage that the interest rate is measured and can be used as an independent variable in the model.

An interesting pursuit for further research is most certainly the economic significance of ignoring the asymmetric nature of the distribution and the changes in dependency. One might then wish to know what assumptions can be based on phenomena such as the flight to quality. This paper provides a method for quantifying such observations. The next step is logically to use the parameter value and their standard deviations to demonstrate what assumptions one can make and possibly explore more effective risk management and newfound diversification opportunities.

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Appendix A

Student's t copula

$$C_t(u,v;\rho,\nu) = \int_{-\infty}^{x_t}\int_{-\infty}^{y_t} p_t(s,t;\rho,\nu)\,ds\,dt$$

$$c_t(u,v;\rho,\nu) = \frac{1}{\sqrt{1-\rho^2}}\frac{\Gamma\left(\frac{\nu+2}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)^2}\frac{\left[\left(1+\frac{x_t^2}{\nu}\right)\left(1+\frac{y_t^2}{\nu}\right)\right]^{-\frac{\nu+1}{2}}}{\left[1+\frac{x_t^2-2\rho x_t y_t+y_t^2}{\nu}\right]^{-\frac{\nu+2}{2}}},$$

where $x_t = t_\nu^{-1}(u)$, $y_t = t_\nu^{-1}(v)$, $\rho \in (0,1)$, and $\nu > 0$. Special cases are $C_t(u,v;-1,\nu) = W(u,v)$, $C_t(u,v;0,\nu) = \Pi(u,v)$, and $C_t(u,v;1,\nu) = M(u,v)$. Furthermore, we have $C_t(u,v;\rho,\infty) = C_N(u,v;\rho)$.

Skewed student's t distribution.

The probability density function of Hansen's (1994) skewed t distribution is given by

$$g(z;\nu,\lambda) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{bz+a}{1-\lambda}\right)^2\right)^{-\frac{\nu+1}{2}}, & \text{if } z < -a/b \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{bz+a}{1+\lambda}\right)^2\right)^{-\frac{\nu+1}{2}}, & \text{if } z \geq -a/b, \end{cases}$$

where the degrees of freedom parameter $\nu \in (2,\infty)$ and the skewness parameter $\lambda \in (-1,1)$. The constants a, b, and c are given by

$$a = 4\lambda c\frac{\nu-2}{\nu-1},$$
$$b^2 = 1+3\lambda^2-a^2,$$
$$c = \frac{1}{\sqrt{\pi(\nu-2)}}\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}.$$

This is the standard skewed student's t distribution, which implies that the mean is zero and the variance equal to one. In this paper the skewed student's t copula is multiplied by the state dependent standard deviation and shifted by mu (equation (1)).

Appendix B

The weekly return of UK Stock and UK Bond indices

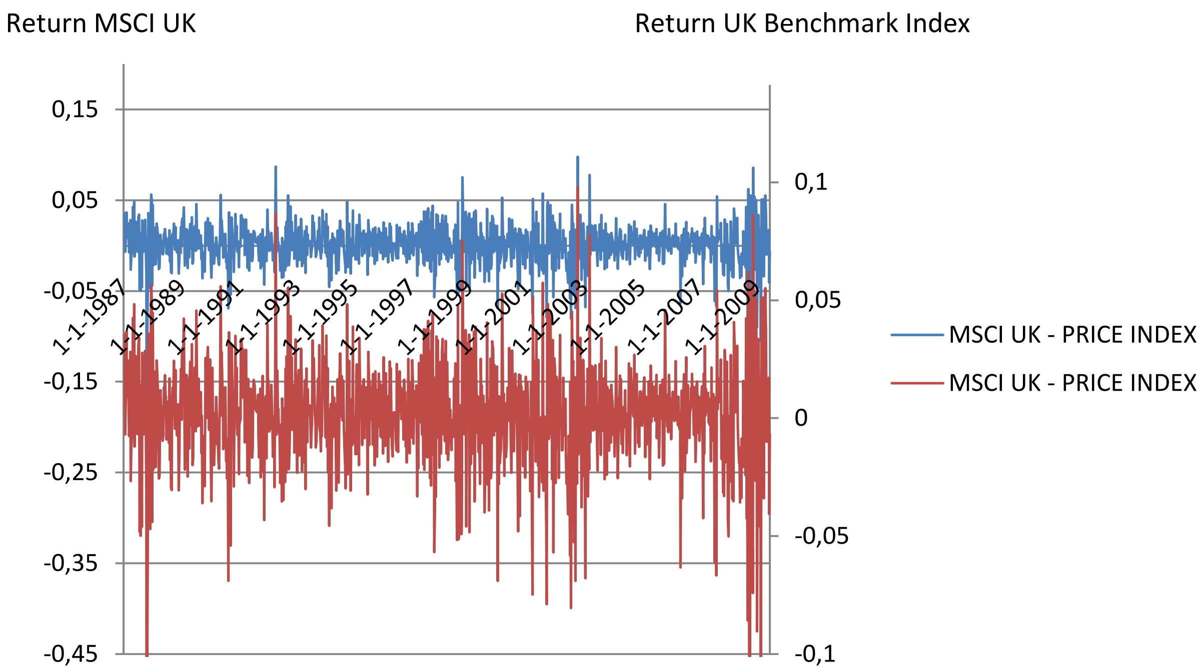


Figure B1. The weekly return of US stock and US Bond indices

The weekly return of German Stock and German Bond indices

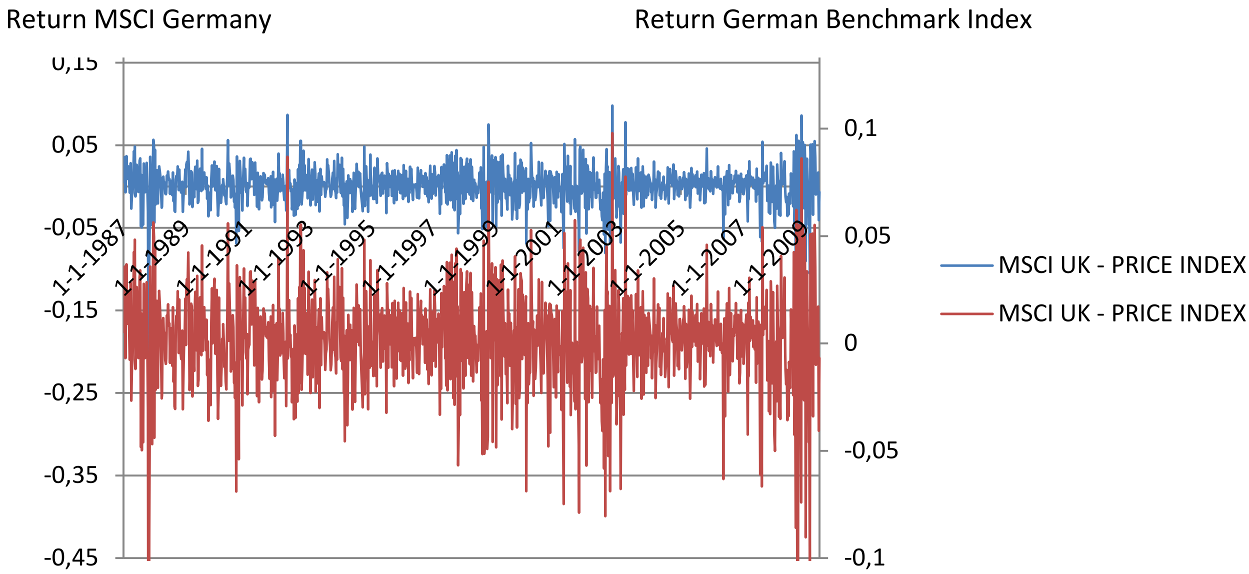


Figure B2. The weekly return of US stock and US Bond indices

The weekly return of Japanese Stock and Japanese Bond indices

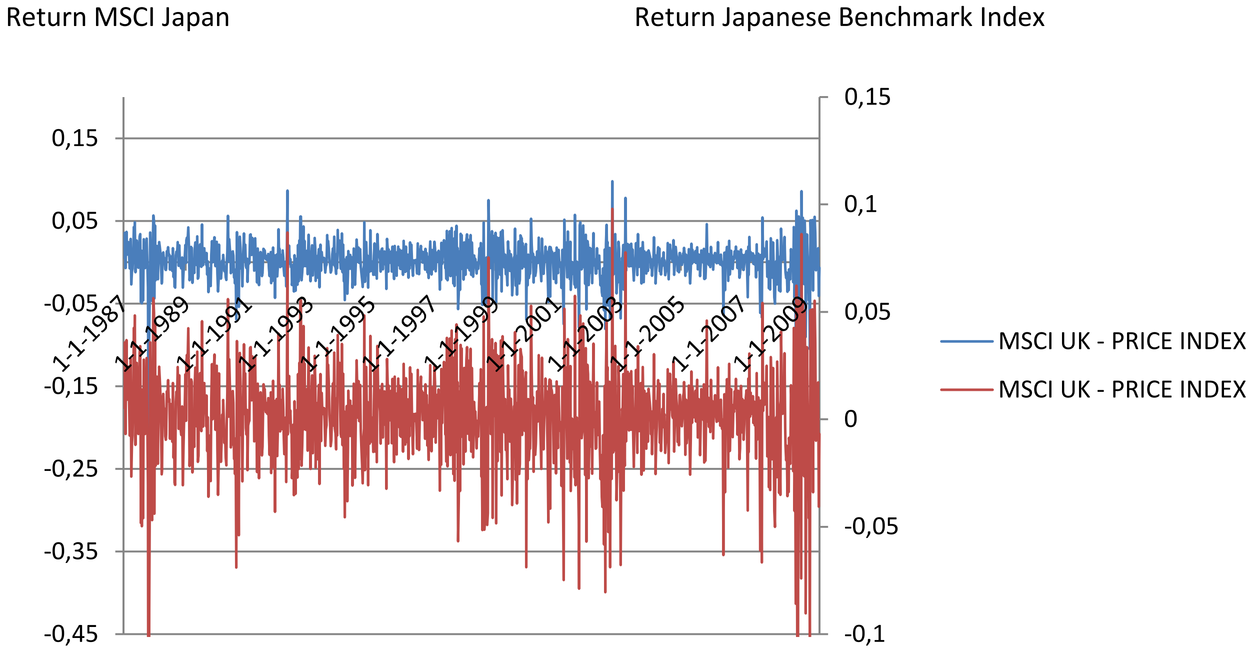


Figure B3. The weekly return of US stock and US Bond indices

MSCI Return series UK - Histogram and basic statistics

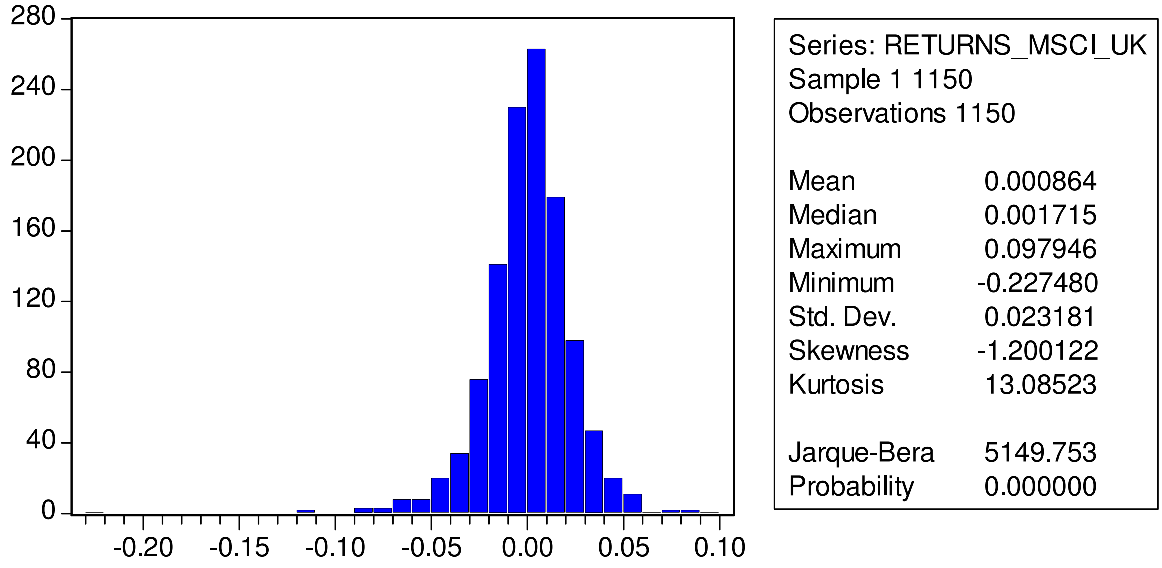


Figure B4. A Histogram and basic distribution statistics of the full MSCI UK return series.

UK Benchmark Return Series - Histogram and basic statistics

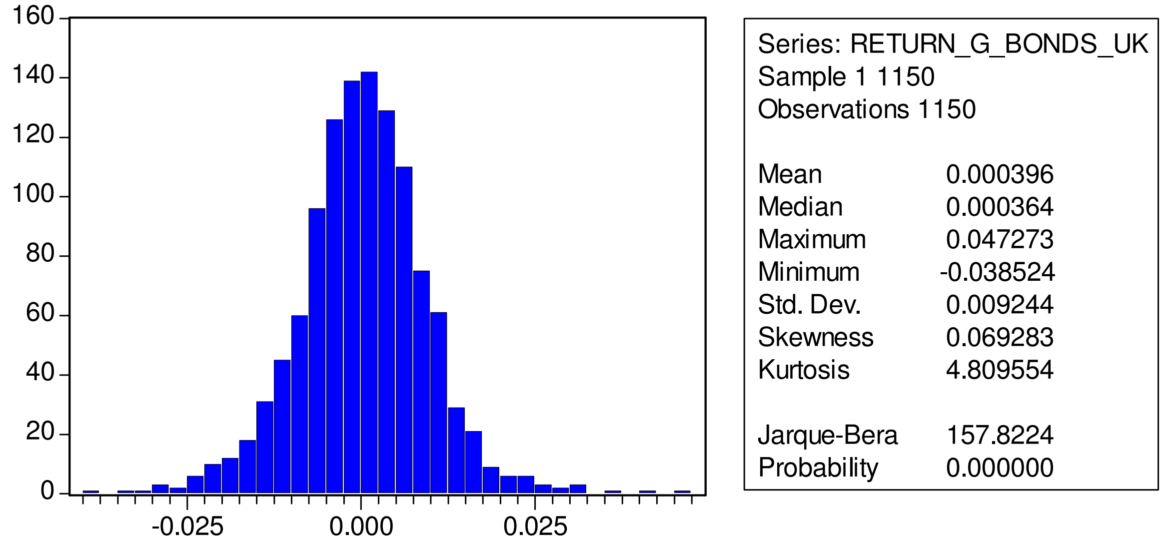
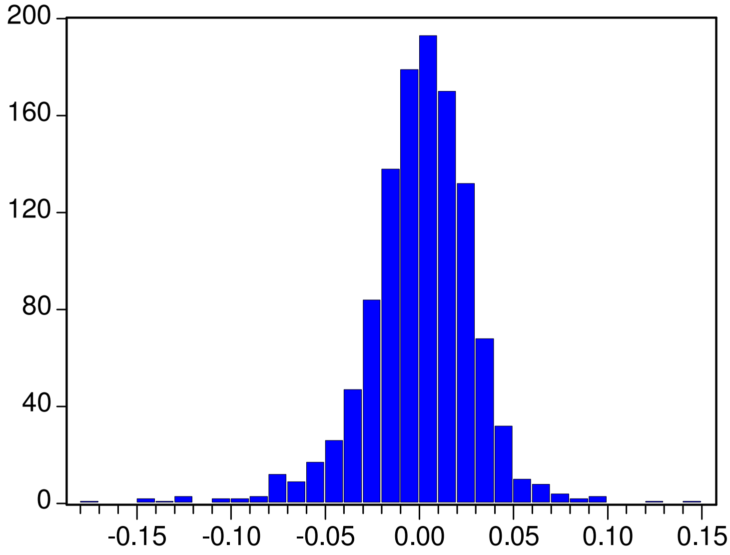


Figure B5. A Histogram and basic distribution statistics of the full UK Benchmark Long term Government Bond return series.

MSCI Return series Germany - Histogram and basic statistics

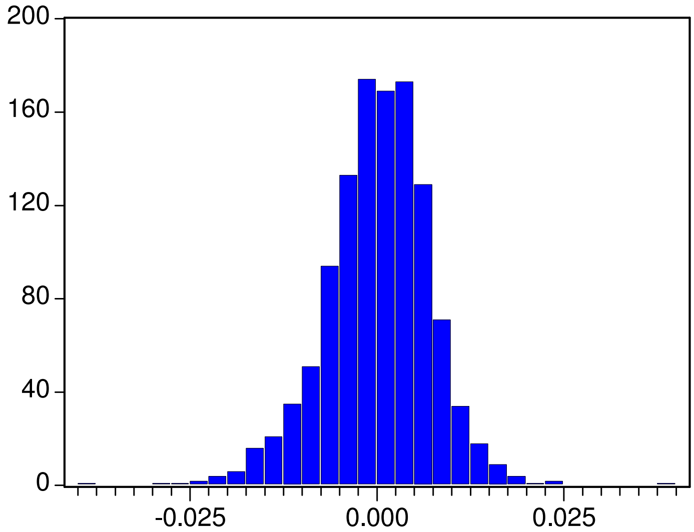


Series: RETURNS_MSCI_GER
Sample 1 1150
Observations 1150

Mean	0.000603
Median	0.002446
Maximum	0.147616
Minimum	-0.173245
Std. Dev.	0.029398
Skewness	-0.752010
Kurtosis	7.256457
Jarque-Bera	976.5180
Probability	0.000000

Figure B6. A Histogram and basic distribution statistics of the full MSCI German return series.

German Benchmark Return Series - Histogram and basic statistics



Series: RETURN_G_BONDS_GER
Sample 1 1150
Observations 1150

Mean	0.000157
Median	0.000510
Maximum	0.039896
Minimum	-0.038544
Std. Dev.	0.007099
Skewness	-0.288989
Kurtosis	5.055613
Jarque-Bera	218.4811
Probability	0.000000

Figure B7. A Histogram and basic distribution statistics of the full German Benchmark Long term Government Bond return series.

MSCI Return series Japan - Histogram and basic statistics

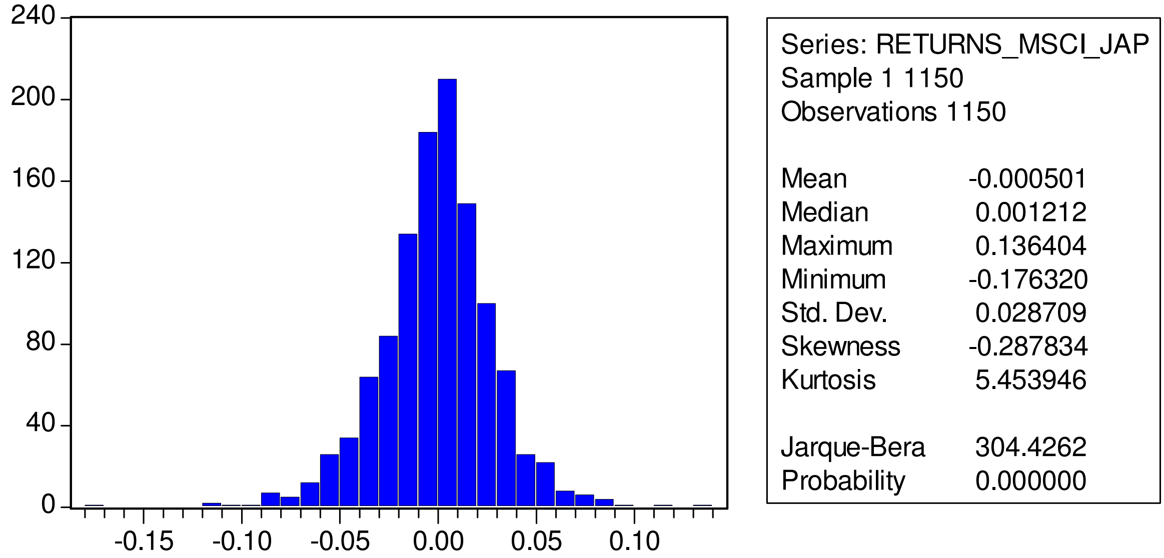


Figure B8. A Histogram and basic distribution statistics of the full MSCI Japan return series.

Japan Benchmark Return Series - Histogram and basic statistics

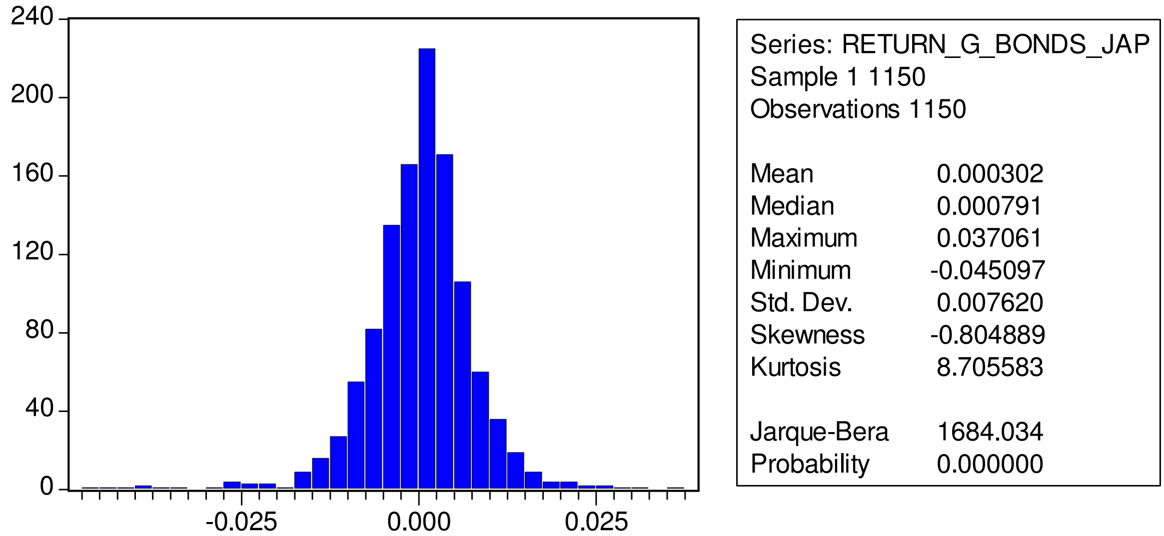


Figure B9. A Histogram and basic distribution statistics of the full Japan Benchmark Long term Government Bond return series.