Volatility Timing: Dynamic Risk-Adjusted Scaling Strategy

Master Thesis: Financial Economics

Name student: Ioannis Niaros
Student ID number: 511636in

Supervisor: Rogier Quaedvlieg
Second assessor: Fabrizio Core
Date final version: 21/01/2021

Volatility Timing: Dynamic Risk-Adjusted Scaling Strategy

Ioannis Niaros

January 21, 2021

Abstract

This paper aims to explore further the additional value of volatility as a risk scaling factor. Literature has documented mixed results on volatility-managed portfolios' performance. This paper explores a volatility scaling method in various factor and multifactor portfolios and international indices and confirms the outperformance of the volatility managed portfolios. Specifically, a Dynamic Risk-Adjusted approach implementation indicates the robustness of the out-of-sample results and the potential real-life applications. Additionally, the GJR-GARCH model for forecasting conditional volatility improves the accuracy and the value of this approach.

Keywords: GJR-GARCH, Volatility Scaling, Dynamic Approach

Acknowledgements

I would like to thank my supervisor, Mr. R Quaedvlieg, for his guidance and support throughout this project, especially in our first meeting!

I would also like to thank the following people:

My parents Eleni and Spyros, who always help me reveal my own values and still support every step I decided to do.

My friends and cousins E. Liakopoulou, M. Dimopoulou and A. Christopoulos who encourage me to do a step and start this beautiful journey with them.

My master-soulmates V. Karikis and S. Rusi, who made this journey joyful and meaningful from the very beginning.

And M. Antonaki and M. del Pozo for being part of all of this!

Non-plagiarism

I condone that this report is fully based on my own personal study and research. I have referred properly to all sources used in my work, no matter the origin of the source. This assignment has not been previously assessed in any way or form. I have not copied or otherwise plagiarized work of other people and/or students.

Preface

Contents

1 Introduction

Asset allocation is difficult decision investors are calling to take. An optimal balance between risk and reward is an achievement that only a small percentage of participants accomplish. Fund managers and researchers make an effort to model the risk to forecast it and allocate their investments accordingly.

Dangerously, there are high-rewards opportunities in the market where their risk is also high. Additionally, some investments become dangerous in some particular moments while other moments, the same investments are considering safer, more predictable, and, hence, lower risk. Unfortunately, the aforementioned risk is not always linked with a relative change on the rewards. While some studies show that high risk(small) performs better (Banz, 1981), recent studies that the outperformance driver is possibly the mispricing either than the risk itself (Chaves, 2012).Asness(2012) and Baker(2012) documented that assets with low to moderate risk tend to perform better in all sampled financial environments. Hense, risk is a factor that has a non-linear relation to rewards and should be considared as seperate factor into investmenting models.

Moreira and Muir (2017), presented a way of control investment's exposure to a particular asset, with a relative to risk weight, variable over time. This portfolio formation strategy leads to a large investment reduction when the market is highly volatile, which happens during turmoil. Understandable, during volatile periods, high positive returns also happen; however, the risk-adjusted reward is low and in the long run, is generally preferable to be avoided. Given that deeps are more harmful for a portfolio than a jump pleased, a stable investment formation is expected to give a larger long-run reward.

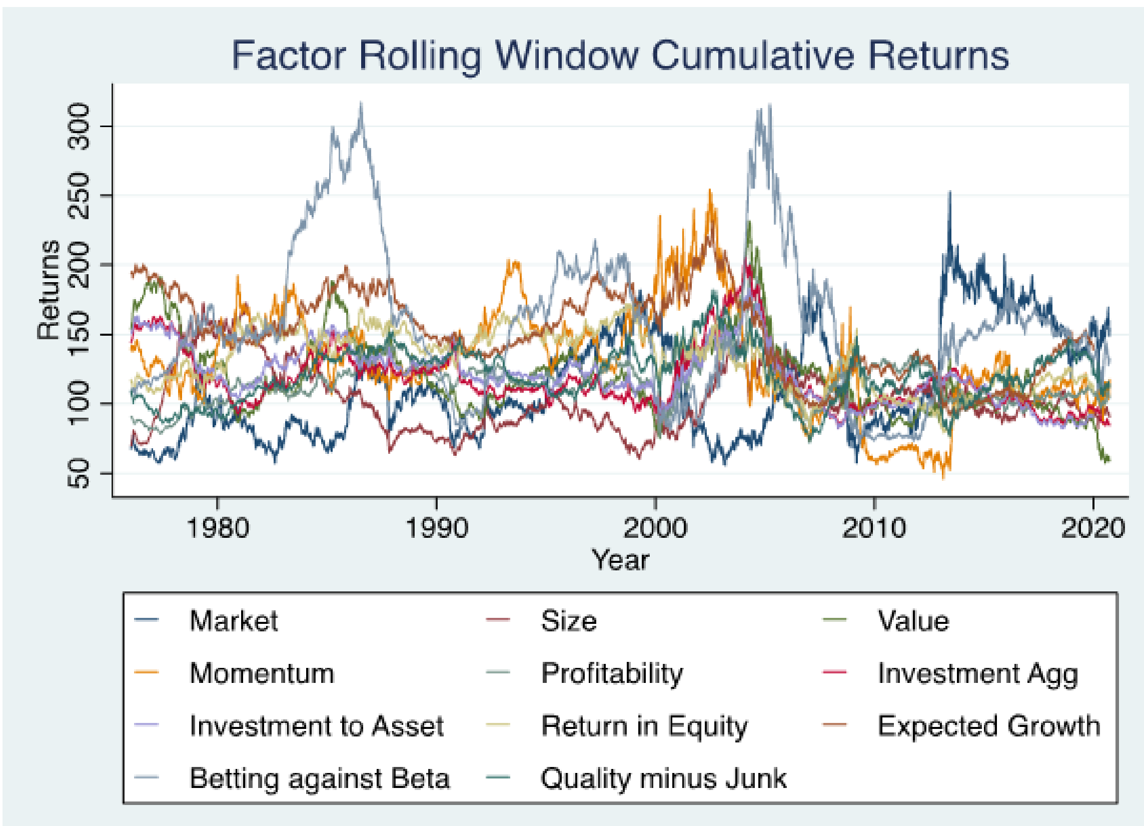
Importantly, an adjusted to risk capital allocation requires a smaller buffer, which leads to higher investment and hence, higher returns. This is crucial, especially for regulated institutions and pension funds, where a small percentage could have a large impact not only in their profits but also in society.

Given the importance of the topic, this paper researching the behaviour on volatility management of 19 portfolios, including several markets, indices and factors. It is useful for investors and academic to review real-time applicability, to extend the in-sample conclusions. A dynamic approach has been incorporating by Daniel and Moskowitz (2016), in their paper about volatility management of the momentum portfolio, with admirable results. Momentum is a factor portfolio with large profits over time; however, it suffers from extreme drawdowns, that moderate the overall performance. A financial behaviour like momentum's is common with various asset. On the Figure 1 someone can observe the volatility of 1000 trading days cumulative returns. All of them have significantly bad periods that question the value of the previously good ones while raising concerns about the future.

The latter question motivates this research to investigate the potential to reduce tails by improving the risk-reward relationship, with volatility managed dynamic approach. A sophisticated volatility model is incorporating, specifically, the GJR-GARCH model, which provide accurate volatility prediction. GARCH family models exploit a vast majority of documented characteristics of volatility, especially it's clustering. GJR-GARCH particularly can take into consideration the asymmetry effect of a bad or good event.

My research includes numerous turmoils including the Covid-19 pandemic that is still ongoing, providing insightful evidence that volatility managed portfolios outperform the buy-and-hold portfolio throughout periods with different characteristics and remain positively stable even when the underlying asset's rolling window cumulative returns sinks. This paper intends to illustrate the applicability of the theory additionally; thus, despite the dynamic approach, leverage restrictions are implemented. Access to high leverage trading is not available to all participants, while there are legal restrictions in some cases. Short selling is also possible to be prohibited. Therefore, I investigate the performance of different levels of leverage restrictions, from ±5 to 0-1, with the latter refers to the most common restriction among pension funds. The Sharpe Ratio is significantly improved in almost every application in the sample.

Figure 1: Factor Rolling Cumulative Returns represents the cumulative returns of the last 1000 trading days prior to the reported date, with the last day included. Initial investment is 100. Points below(above) the 100-line suggest 1000-days loss(gains). Factor portfolios seems to be profitable on average but with heavy tails.



In particular, results suggest an increase in Sharpe Ratio of the Market portfolio from 0.011 to 0.111 for the ±5 leverage restriction on the market portfolio, while a significant increase noticed in lower leverage portfolio. The ±1 leverage restricted portfolio enhance its Sharpe Ratio to 0.125 while the 0-1 portfolio achieves an economically interesting 0.101 in its risk adjustment rewards.

At the same time, cumulative returns of almost every managed portfolio in the sample are meaningfully improved, which suggests that volatility scaling approach should be considered by all type of risk aversion participants when comparing with the buy-and-hold portfolio. Repetitively, cumulative returns of the market portfolio during the 12460 sampled trading days are more than forty times up for the basic managed portfolio while it around four times up for the pensions' fund restricted leverage strategy, at 593%.

In order to make sure that the aforementioned figures are not time dependent, I constructed rolling window cumulative returns over 1000 trading days, for each unmanaged and managed portfolio. Mean numbers are almost always improved, however, the crucial point is the 4-year negative tails. While market unmanaged portfolio faced a hurting -45% during its worst one thousand trading days, the relaxed managed portfolio faced a symbolic -3.4% in its worst trading period. Results of the rest of the sampled assets are similarly improved (Appendix).

Although GJR-GARCH model is documented as one of the most suitable volatility forecasting models, I implemented an additional volatility model, the Random Walk, to find out the additional value of the former more advance model. The outperformance of the managed portfolio that formulated with the Random Walk were significantly improved in comparison with the unmanaged, but worst from those formulated with the GJR-GARCH, in terms of Sharpe Ratio. Specifically, Sharpe Ratio of the market portfolio by Random Walk is 0,077 where the one by GJR-GARCH is 0,111. Someone can also notice heavy tails with some largest negative daily returns exceed even those of the unmanaged.

The 114 portfolios that formulated are compared and presented with each other and with the 19 initial portfolios in details. The results suggest strong improvements in volatility managed portfolios' performance and applicability among a large percentage of the market participants.

2 Theoretical framework

This section has four parts. In the first part, I analyse why it is important for the research to adapt a dynamic profile. In the second part, I will discuss why modelling volatility is important for investors. In the third part, I describe some of the most common ways to model and forecast volatility. In the last part, I will analyse why volatility could help on scaling.

2.1 Dynamic Approach

A dynamic approach validates the applicability of the strategy in real-life and in out-of-sample results. Literature provides us with evidence that a static approach with a constant volatility target may produce better results (Fan, 2018). However, implied looking-forward bias risk occurs over the researcher's constant target choice; targeted volatility could significantly impact the overall performance and the comparison's conclusion. This research uses only data that are available the moment scaling decision is calculated. The dynamic approach is a step further into the real trading world where reasoning is combined with the applicability and real risk-adjusted returns prediction.

2.2 Modelling Volatility Importance

Prices in financial markets fluctuate strongly and with unpredicted directions. This fluctuation and uncertainty form what is considered a risk, and participants and researchers are constantly trying to reduce it.

Among others, pension funds and central banks rely their policies upon volatility models, as a bad unpredicted outcome is possible to crack their balance sheets hardly. Hence, institutions need to stay underinvested to have a safe amount of capital, in case of a negative unpredicted outcome. The latter makes the need for modelling the volatility, compulsory and robust, not only for the individual investors but also for the society.

To do so, an investor should predict the prices' fluctuation and direction as accurately as possible. However, prediction of the direction is way more difficult to be done than to predict the volatility. As a result, a strong effort is given to model volatility and subsequently to be forecasted, so that participants would act respectively.

Investors often prefer to form portfolios and diversify their asset to reduce their exposure to idiosyncratic risk. This approach can be easily achieved by investing in a predefined index that follows specific characteristics or combining a handful of indices. During the last decades, factor-investing has been proven a strong trend. It shares a lot of diversification characteristics with indices combined with other fundamental characteristics that each factor contains(Ang, 2014). Some factors have been proved more profitable than others, while some have shown robust profitability but with extreme heavy tails. Participants need to compare risk-adjusted results, considering the total risk, with the systemic risk included. Forecasted volatility is a major element of the total risk, while the use of the predicted value has still a huge potential.

2.3 Volatility Scaling

While classic asset pricing theory indicates that high risk is rewarded with corresponding high rewards, in practice, market participants suffer from asymmetrically large losses during volatile periods. The opposite is not a fact in the long run. Literature had documented that high volatility is linked more often with big losses than large gains (Easterling, 2020).

Hence, an investor should consider to reduce its exposure to a certain risk during highly volatile periods and increase it when volatility drops. In line with Markowitz(1952 1991) investors should value their asset allocations in accordance with their expectations. Therefore, future volatility should be forecasted, and the chosen model should be chosen carefully to implement all these characteristics weigh enough to affect the forecasting accuracy positively.

2.4 GJR-GARCH Volatility Forecasting Model

The variation of financial assets' returns is not constant; in contrast, it has been documented that there is a dependency between the following and the prior (Mandelbrot, 1963). Autocorrelation is easily observed in the stock markets as clustering in the volatility of returns. Additionally, Glosten(1993) clarified that volatility has different magnitude from a good and from a bad shock. Hence a model that captures autocorrelation leverage effect is required.

The ARCH model (Engle, 1982), modelling the variance of the residuals' error term, that econometric models leave without clarification, taking into consideration the aforementioned clustering volatility properties. The Generalized ARCH model, GARCH (Bollerslev, 1986), incorporates a moving average component to capture the reversal trend of volatility. However, they both fail to capture the leverage effect, and hence, they do not recognize the direction of volatility differently.

Glosten, Jagannathan, and Runkle (1993) modified the generalized model by relaxing the linear restriction on the conditional variance dynamics to capture the asymmetric effects of good and bad shocks.

3 Literature Review

Barroso and Santa-Clara (2015) found that while the unconditional Momentum anomaly strategy has a tremendous crash risk, its risk is highly predictable. Therefore, they eliminate that risk by scaling investor's exposure to the Momentum portfolio with volatility. To enhance the real-life findings' applicability, they apply a training sample of 20 years for their methodology. Afterwards, they scale investor's exposure based on the ratio between a constant target level of volatility and the forecasted, introducing the Constant Volatility Scaling Approach. However, they use a constant volatility target, which may cause forward-looking bias.

Daniel and Moskowitz (2016), in their effort to eliminate the momentum risk, using the literature insights about the forecastability of momentum payoffs and volatility (Henriksson and Merton, 1981; Grundy and Martin, 2001; Barroso and Santa-Clara, 2015), introduce the Dynamic Volatility Scaling Approach. Additionally, they employ a training period for their model; however, they scale investor's exposure only with forecasting variables. The use of maximizing the Sharpe Ratio strategy instead of a constant volatility target makes the dynamic scaling approach implementable in real-life strategies. Nevertheless, further research is needed to indicate the consistency across factors, financial markets, sectors, and other investment environment characteristics.

Fan et al.(2018) in their research compared the two previously mentioned volatility scaling strategies on the momentum factor portfolio. Their findings confirm the outperformance of volatility scaling throughout a well diversified portfolios formulated with 55 global liquid futures. The international presence of momentum implies that outperformance is not related with specific characteristics of an economy. Authors concluded in favor to the Constant Volatility Scaling Approach, however the looking forward bias risk has not eliminated.

Moreira and Muir (2017) extend documentation that Volatility-Managed portfolios yield a significant improvement in returns of market, value, momentum, profitability, return on equity, investment, and betting-against-beta factors, and the currency carries trade portfolios. They use a volatility scaling method like the previously mentioned authors to control the investor's exposure to each of the aforementioned portfolios to reduce their exposure during turmoil while increasing it when volatility is low. To achieve that, they use a constant volatility target, exploited from the full sample. Evidently, they reveal a strong potential for volatility-managed portfolios' outperformance that other market anomalies do not explain. Yet, there is a need for more comprehensive research for out-of-sample results.

Cederburg et al. (2020) reassessed the analysis of volatility-managed outperformance in the same nine equity strategies, and they further extend it to 94 more strategies. Furthermore, they assessed an out-of-sample analysis, and they concluded that the Moreira and Muir's (2017) results were looking forward biased. While there is evidence for further improvements, they claim that there is significant outperformance only in a few specific strategies (e.g., the market, momentum, and betting-against-beta factors).

4 Hypothesis Analysis

Investors and researchers wonder if the outperformance of volatility-managed portfolios could be an exploitable potential for their investment strategy and their research. Literature provides positive shreds of evidence; however, further research should be done, using advanced statistical models and methods in a dynamic approach.

H1: Volatility-Managed Portfolios outperform unmanaged

To expand the explorable potential, combinations of well-known investment strategies have been considered using multifactor portfolios and then scaling them with volatility.

H2: Multifactor Volatility-Managed Portfolios outperform unmanaged

While a vast investor majority is interested in the effectiveness of volatility scaling in portfolios, there is a lot more to explore in this paper. Volatility is already known as a crucial input for individual investor's (or departments') decisions, especially smaller investors that cannot hold a vast portfolio of assets. Instead, they choose to invest in indices and individual assets. This question derives the following testing hypothesis:

H3: Volatility-Managed scaling approach derive better decision for individual assets

Volatility forecasting is vital for investors and researchers in the finance world. Volatility is considering the main risk for investors, and hence, its forecastability has been analyzed excessively in literature. This paper explores the scaling properties of the volatility in different investing strategies by extending literature preexisting research, and therefore, the importance of a suitable forecasting model is paramount. GJR-GARCH and Random Walk forecasting volatility models are implementing in portfolio formation which their results are comparing, in order to conclude whether GJR-GARCH model derives better results.

H4: GJR-GARCH model improves the performance of the scaling method, in comparison with the Random Walk

For individual investors, to derive a definite conclusion implemented in real-life, it is necessary to measure and compare the scaling approach's performance in terms of return relative to risk. After all, the representative mean-variance investor decides based on the relationship between these two variables, and hence, Sharpe Ratio is employed as a performance metric in each hypothesis. Sharpe (1966) introduced a measure of risk-adjusted returns, which illustrates the price investors demand to bear an additional risk unit. Following the Sharpe Ratio tool's initial purpose, I apply it to measure and compare the managed and unmanaged portfolios' performance.

This paper explores Moreira and Muir's (2017) evidence of the volatility-managed portfolios' outperformance, implementing a dynamic volatility scaling approach, similar to that Daniel and Moskowitz (2016) applied on their momentum research. In this extension, I implement a different advanced statistical model, GJR-GARCH, to measure and forecast the volatility, which alone could reveal a different than an expected story about the volatility scaling performance. Additionally, research in multifactor portfolios and representative international indexes, gives an improved theoretical scope on the first hypothesis.

5 Data and Summary statistics

For this research, at first, the analysis uses daily data from the US market. Specifically, under Moreira and Muir (2017), data on factor excess returns have been retrieved from Kenneth French's, Andrea Frazzini's, and Lu Zhang's website. Data for the following equity factors were considered[1] :

Market (MKT), size (SMB), and value (HML), momentum factor (MOM), the profitability (RMW), and investment (CMA), the profitability (ROE), and investment (IA) factors and betting-against-beta factor (BAB). Furthermore, this paper investigates the volatility scaling performance in the additional quality factor (QMJ), representative equity indices from G7[2] countries around the world, and the case of the three and five Fama-French factor models.

Table 1: Summary Statistics:The sample contains 291341 observations, in three datasets: single factor, multi-factor and indexes.

Variables	Obs	Mean	Std. Dev.	Min	Max
Single Factors					
Market	12460	0,010	1,062	-17,467	11,346
Size	12460	0,002	0,553	-11,620	6,200
Value	12460	0,014	0,565	-4,730	4,770
Momentum	12460	0,028	0,767	-8,190	7,010
Profitability	12460	0,013	0,391	-3,020	4,490
Investment Aggresiveness	12460	0,015	0,380	-5,940	2,530
Investment To Assets	12275	0,019	0,379	-6,880	2,750
Returns on Equity	12275	0,026	0,414	-3,961	3,258
Expected Growth	12275	0,041	0,357	-2,910	2,973
Betting Against Beta	12460	0,041	0,627	-6,283	7,945
Quality minus Junk	12460	0,019	0,430	-3,743	5,035
BMI	12439	0,072	0,259	0	1
Multi Factors					
FF3	12.460	0,016	0,826	-13,389	9,334
FF5	12.460	0,028	0,624	-11,828	9,430
BMI	12.439	0,072	0,259	0	1
Indexes					
Global Market	8.902	0,021	0,898	-9,567	8,849
Canada	8.902	0,024	1,148	-12,619	12,662
Germany	8.902	0,021	1,266	-12,045	15,617
France	8.902	0,028	1,283	-13,088	11,207
Great Britain	8.902	0,020	1,176	-12,764	12,034
Italy	8.902	0,016	1,475	-17,412	11,672
Japan	8.902	0,011	1,319	-15,312	11,964
BMI	8.902	0,045	0,207	0	1

[1]Data on the US and Global MKT, SMB, HML, MOM, RMW, and CMA factors from Kenneth French's website: $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$

Data on BAB and QMJ from $http://people.stern.nyu.edu/afrazzin/data_library.htm$

Data on ROE and IA from $http://theinvestmentcapm.com/research.html$

$http://global-q.org/factors.html$

[2]United States, Canada, Germany, French, Great Britain, Italy and Japan

As an alternative, the implementation of the Smoothed Recession Probabilities (SRP) Index for the United States is considering (Chauvet, 1998). I expect this innovation to improve the accuracy of the bear market indication and the validity of the research significantly, as it incorporates other than returns data[3] , with great predictability properties.

All time-series are adjusted to start on the date that SPR is available, in 1967. The analysis is going on until the last available date, which is until the end of September of 2020. This extensive analysis is expected to illustrate the volatility of managed capabilities through different business cycles, several depression periods, and economic booms. On table 1 summary statistics of the unmanaged portfolios are available.

6 Methodology

In the Dynamic Volatility Scaling Approach, all calculations are based on data known before the forecasted period. Bear Market Indicator dummy variable is formulating in the first step. Expected returns are calculated in the second step, considering the market condition and the realized volatility. Afterwards, forecasted volatility is calculated by the GJR-GARCH model utilizing. Next, scalings are calculated by expected returns and predicted variance, and finally returns of the volatility managed portfolio are calculated based on the constructed scale and the actual returns of the underlying asset.

6.1 Bear-Market Indicator

Firstly, a bear-market indicator that captures the leverage effect considers having a value of either 1 indicating a bear market condition or 0 for a non-bear market. SRP has been chosen for its high out-of-sample predictability achievements, so that when SRP is higher than 80%, it is considered a recession, according to Chauvet (1998).

$$I_{SPR,t-1} = \begin{cases} 1 & \text{if} \quad SPR_{t-1} \geq T \\ 0 & \text{if} \quad SPR_{t-1} < T \end{cases} \tag{1}$$

where T is the aggressive 80% threshold

6.2 Betas indication

Secondly, the betas' determination that influences the realized returns is calculated on this step, based on ex-post realized returns and incorporating the bare-market indicator.

[3]"Smoothed recession probabilities for the United States are obtained from a dynamic-factor Markov-switching model applied to four monthly coincident variables: non-farm payroll employment, the index of industrial production, real personal income excluding transfer payments, and real manufacturing and trade sales." https://fred.stlouisfed.org/series/RECPROUSM156N

Following the Daniel and Moskowitz (2016) method, I run regression on the period before scaling, over past returns on expected volatility, and BMI. The fitted model used for next day returns prediction, which is a part of the volatility scaling model. This research does not focus on expected returns forecasting; hence this model could be considered basic, and further research could be done.

6.3 Forecasted volatility with GJR-GARCH

Afterwards, the forecasted volatility for the following period is calculated with the GJR-GARCH implementation over a rolling sample of 1000 observations (Christoffersen, 2012):

$$r_{u,t} = c + \beta_\sigma \sigma_{t-1}^2 + \beta_I I_{SPR,t-1} \quad (2)$$

where c is a constant, σ_{t-1} is the previous volatility prediction and $I_{SPR,t-1}$ is the last known SPR bear market indicator and $\beta_\sigma,\ \beta_I$ are their betas, and

$$\sigma_{u,t}^2 = \omega + (\alpha + \gamma \mathrm{I}_{B,t-1})\epsilon_{t-1}^2 + \beta \sigma_{u,t-1}^2 \quad (3)$$

where $\mathrm{I}_{B,t-1}$ is the nested in the model bear market indicator, $\omega,\ \alpha,\ \gamma$, and β are estimated simultaneously with the use of Quasi-Maximum Likelihood.

6.4 Scaling

Scaling weights w_t^* are calculating by the price of the risk for given expected returns. Using only the ex-post sample, the following formula derives:

$$w_t^* = \frac{\mu_{u,t}}{\sigma_{u,t}^2} \quad (4)$$

where $\mu_{u,t}$ and $\sigma_{u,t}^2$ is the conditional expected return and the conditional variance of the unmanaged portfolio, respectively, calculated on the previous two steps. Weights are restricted to be between -5 and 5 to prevent extremely rare cases of unrealistic weights.

6.5 Volatility managed returns calculation

After clarification of the relationship between market expected return and forecasted volatility and scaling method, managed portfolios' returns are calculated:

$$r_{u,t}^{managed} = w_t^* * \ r_{u,\ t} \quad (5)$$

where $r_{u,\ t}$ represents the unmanaged portfolio return, w_t^* is proposed methodology scaling over the next trading day.

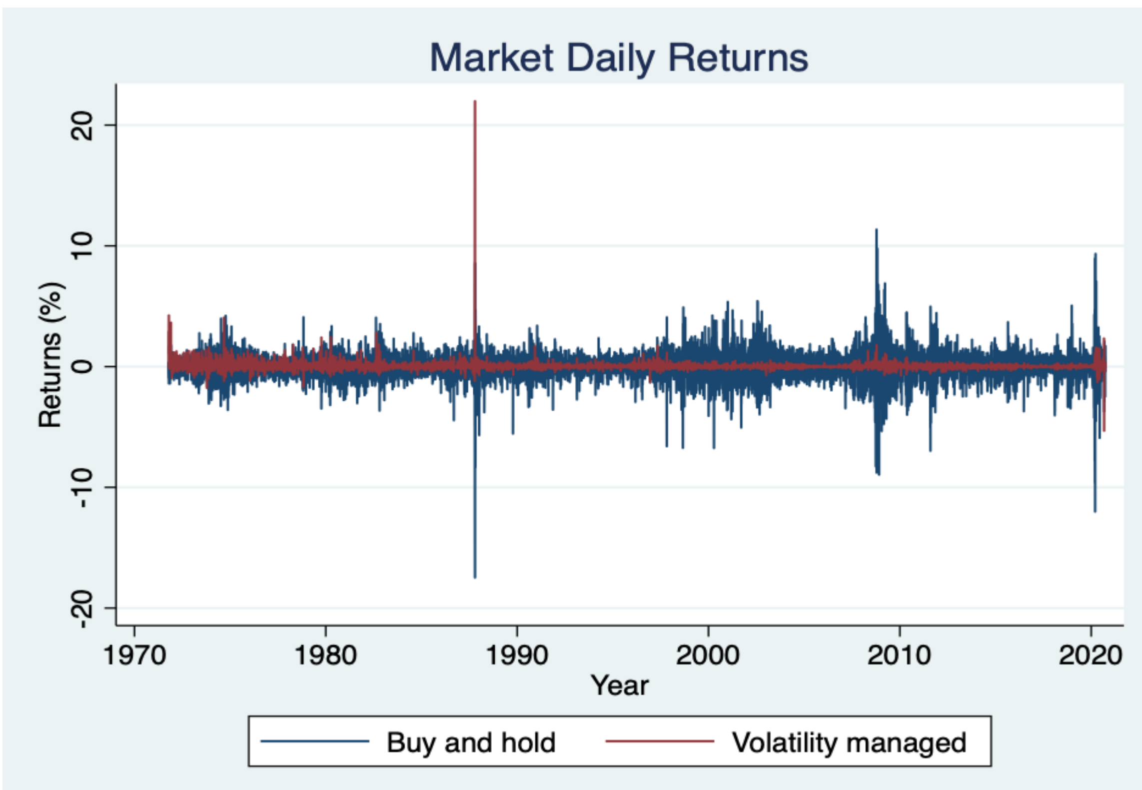
7 Results

7.1 Single-Factor Portfolios

Next, daily and cumulative returns, volatility, and Sharpe ratio of the volatility managed portfolios are calculated and compared with each of the unmanaged counterparts.

The results of Appendix 1 illustrate that the volatility scaling improves the overall performance of every factor of this research. It is observed that higher mean returns can be achieved simultaneously that the volatility is reduced.

FIGURE 2: Market daily returns for the unmanaged and managed market portfolio. Volatility of the managed portfolio is observably reduced and stable during the whole period including turmoils



In the Table 2 someone can observe a significant difference in the cumulative returns, which can be explained by the absence of heavy negative tails, in addition to the aforementioned improved mean results. The results' robustness can be described as even better than expected, with extreme daily negative returns around -5.3% for the managed Market portfolio, while the unmanaged counterpart reaches -17.46%. For the rest of the analyzed factors, the improvement is way better with extreme daily negative returns not exceeding -2.7%.

The Figure 2 demonstrates the daily market returns of the managed and unmanaged portfolio as a representative example. Comparing both portfolio's returns, someone can

TABLE 2: Single Factor Model:Sharpe Ratio and Rolling Window Cumulative Returns. Rolling Window Cumulative Returns represents the cumulative returns over the last 1000 trading days on each day of the sample. Both figures are improved in all single factor managed portfolios. Special attention requires the reduced negative extreme numbers which represents an important part of the total investment risk

	Sharpe	Ratio		Rolling	Window	Cumulative	Returns	
Variables	Unmanaged	Managed		Unmanaged			Managed	
			mean	min	max	mean	min	max
Market	0,011	0,111	112,970	55,540	252,854	135,701	96,621	406,019
Size	0,000	0,098	104,091	60,085	173,223	136,738	97,320	715,220
Value	0,024	0,210	120,393	57,046	231,804	185,297	99,542	581,758
Momentum	0,036	0,248	132,012	45,813	254,623	266,972	103,560	687,978
Profitability	0,034	0,157	115,712	75,856	201,853	151,463	97,237	279,925
Investment Aggresiveness	0,038	0,177	117,845	82,420	205,878	154,806	104,133	263,600
Investment To Assets	0,046	0,140	120,681	82,215	183,268	136,374	100,969	215,320
Returns on Equity	0,062	0,188	131,848	75,733	172,561	187,709	102,083	405,627
Expected Growth	0,114	0,197	151,185	90,818	232,184	160,567	105,318	284,249
Betting Against Beta	0,062	0,165	157,773	71,253	317,425	147,500	93,737	249,300
Quality minus Junk	0,042	0,174	120,711	71,978	182,375	166,918	95,772	270,668

easily conclude on the favour of the proposed scaling method.

For a more comprehensive illustration, Figure 3 shows the market rolling window cumulative returns which refers to cumulative returns over the previous 1000 trading days, over the full sample, for each trading day.

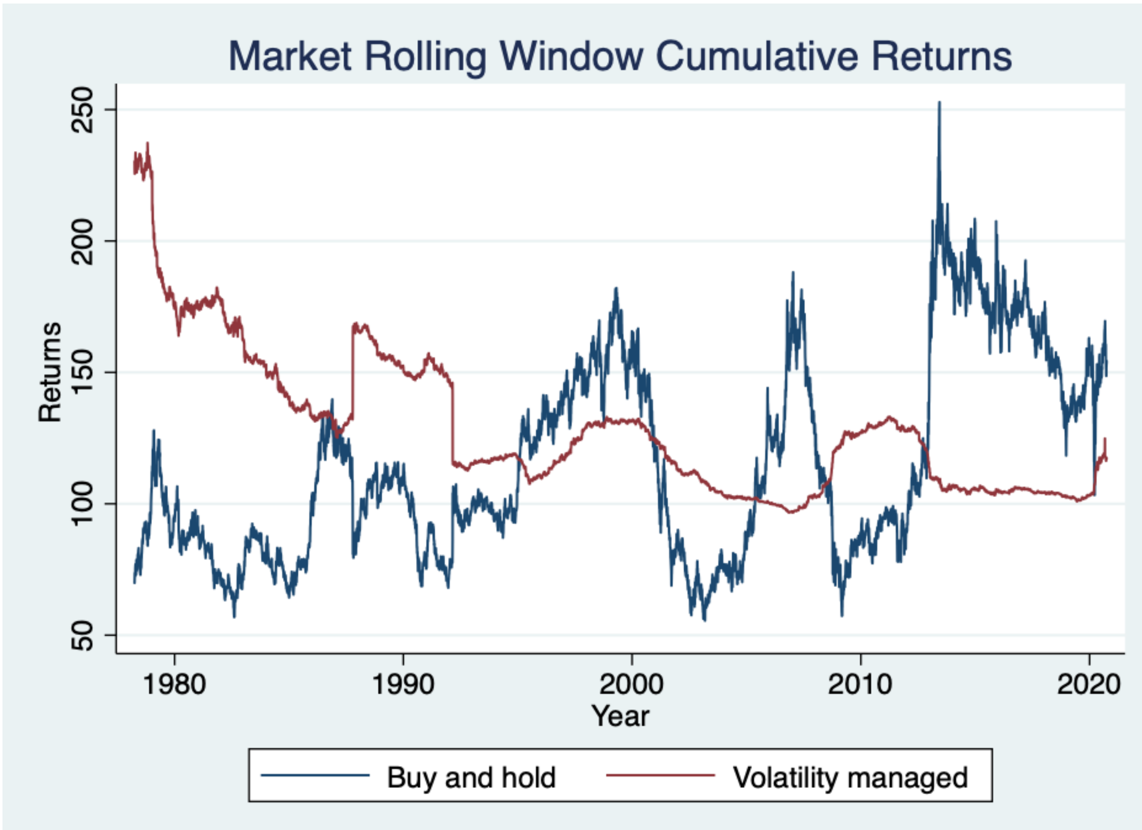
The volatility-managed portfolio of the market has a substantial reduction in the volatility of its daily returns. Additionally, the rolling window cumulative returns of the managed portfolio volatility is always between neutral and good performance. In other words, whenever the investment period starts, 1000 trading days later, the investment has at least no extreme negative returns, no matter the economic environment around the investment period.

Specifically, the worst 1000 cumulative return during the full sample is -3.4%. In contrast, the unmanaged buy and hold market portfolio, suffers from extreme negative rolling window cumulative returns, making investors face a painful -44.5% of their initial investment.

In particular, scaling procedure reducing dramatically the exposure on extreme daily returns during turmoil, as denominator increasing. As a result, aggressive volatility that endanger significant losses, does not meaningfully affect returns of the managed portfolios. The opposite happens when the market goes through a steady period, exposure increases when volatility risk is reduced. Market daily returns remains steady throughout the sampled period with only a few mainly positive exceptions. As a result, the first hypothesis has to be accepted.

It is representative that during the two most volatile period of the sample, the Tech bubble around 2000 and the Financial Crisis around 2009, the market portfolio suffered by huge losses demonstrating clearly on Figure 3. The long lasting of losses market portfolio lost almost 50% of it worth, while its managed counterpart earned more that 10% in both scenarios. These crises where fundamentally different one another, nonetheless, volatility

Figure 3: Rolling Window Cumulative returns for the unmanaged and managed market portfolio during the sampled period. The rolling window cumulative returns of the volatility managed portfolio confirms a stable performance without extreme negative values. In fact, the managed portfolio barely has any negative 4-year cumulative return. The buy-and-hold market portfolio has inevitably high volatility with both positive and negative extreme values



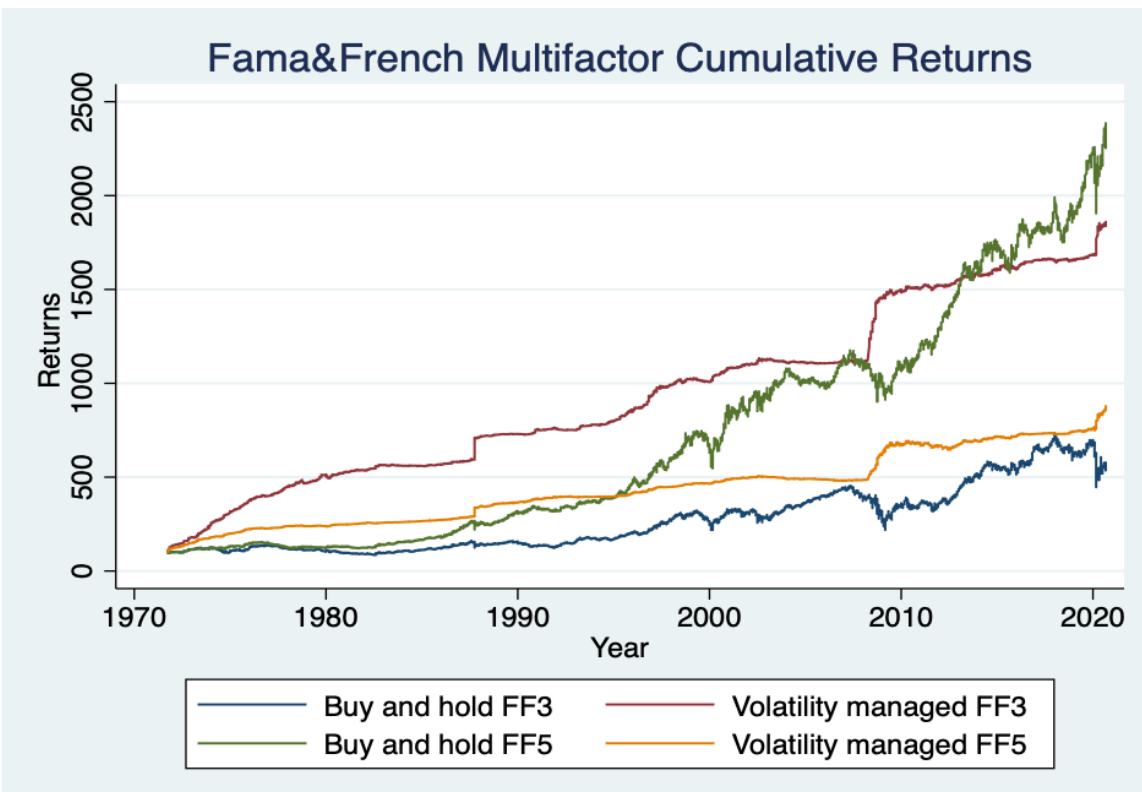
scaling would have similarly beneficial effect on investors' portfolio.

Despite the fact that Covid-19 crisis is ongoing, and that the fundamentals behind it were far from economics, there are already signs that volatility scaling will achieve to retain positive period cumulative returns and stable daily returns. Till the end of September, managed portfolios confirmed outperformance behavior, similar with all previous crises.

7.2 Multi-factor Portfolios

To investigate the proposed methodology's applicability range, data includes factor models and multiple international counties' indices. In this section, multifactor models' portfolios and individual assets are managed with the paper's dynamic volatility scaling approach. Volatility managed portfolios' results are compared with their unmanaged buy-and-hold versions, focusing on mean and cumulative return, volatility, and their relationship. Multifactor models have been constructed to represent the mean variance efficient (MVE) portfolio, using in-sample data. By construction, expectations of volatility timing performance improvement are moderate.

FIGURE 4: Cumulative returns for the unmanaged and managed Fama French 3- and 5-factor portfolios during the sampled period. The FF5 unmanaged portfolio achieves higher cumulative returns than the managed, however, its returns are not stable in comparison with the managed. FF3 unmanaged and managed portfolios follow the pattern of higher and more stable cumulative returns for the managed in comparison with the unmanaged.



Managed portfolio scaling follows a dynamic approach, using data only available before the trading moment. Hence, the scaling model uses only one-day predicted values.

Scaling weights are calculating using predicted values of expected returns and the volatility derived by GJR-GARCH model. The following formula calculates the returns of the multifactor managed portfolio:

$$r_{u,t}^{MF\ managed} = \frac{\mu_{u,t}^{MF}}{\sigma_{u,t}^{MF^2}} * r_{u,t}^{MF}$$

where $\mu_{u,t}^{MF}$ and $\sigma_{u,t}^{MF^2}$ is the conditional expected return and the conditional variance of the unmanaged portfolio, respectively, and is the actual return of the unmanaged multifactor portfolio. Weights are restricted to be between -5 and 5 to prevent sporadic cases of unrealistic weights.

Table 3 illustrates how volatility timing can largely enhance both Fama and French 3- and 5-factor models' performance in Sharpe Ratio. This is not clear for every aspect of comparison, as daily, cumulative or rolling window cumulative returns happen to be lower for the managed portfolio sometimes. However, the conclusion remains intact when comparing the risk-adjusted performance of each portfolio.

Table 3: Multi Factor Model. Sharpe Ratio and Rolling Window Cumulative Returns of the Multi-factor portfolios. Rolling Window Cumulative Returns represents the cumulative returns over the last 1000 trading days on each day of the sample. Sharpe Ratio has been improved in both portfolios. Rolling window cumulative returns for the FF5 portfolio has been reduced in its managed version however they have both reduced significantly the negative tails.

	Sharpe	Ratio		Rolling	Window	Cumulative	Returns	
Variables	Unmanaged	Managed		Unmanaged			Managed	
			mean	min	max	mean	min	max
Fama&French 3 factors	0,022	0,100	121,445	61,600	222,576	124,482	98,375	394,944
Fama&French 5 factors	0,046	0,094	133,058	81,929	193,322	116,406	95,265	224,017

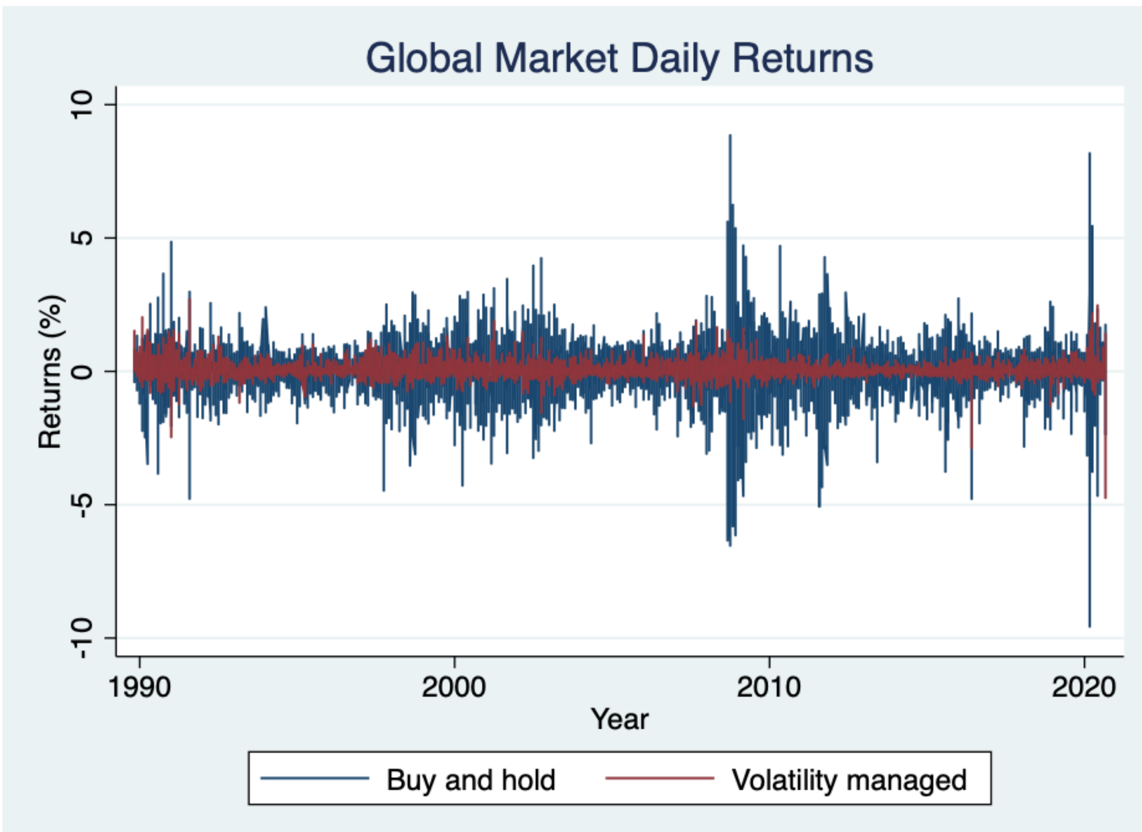
Sharpe Ratio for FF3 and FF5 is 0.022 and 0.046 for the buy and hold portfolios and 0.100 and 0.094 for the managed ones. This out-of-sample outperformance of the dynamic volatility scaling approach remains robust in multifactor models, which leads to the acceptance of the second hypothesis. As Appendix 5 analytically demonstrates, FF3 mean return has been increased for the managed portfolio from 0.019 to 0.026 while the standard deviation has been reduced from 0.85 to 0.26, signaling a jump in risk-adjusted returns. FF5's mean returns on the other hand have been reduced 0.029 to 0.019, however, standard deviation has been minimized even more from 0.64 to 0.20. Hence, latter Sharpe ratio indicates an outperformance of managed portfolio in the case of FF5 as well.

Alike with the market and the rest of the factor managed portfolios, the managed multifactor portfolios went though Tech bubble and the Financial Crisis unharmed, which is a motive that repeat itself during the current pandemic crisis. Unreported daily returns follow similar pattern with managed Market portfolio (Figure 2).

7.3 Individual Asset Portfolios

The volatility timing is an important factor for a vast part of the US market. It is, however, crucial for academia to investigate whether volatility is relevant for non-US markets. Hence, to include a representative sample of the rest of the developed world, the 7G countries have been chosen. Additional to the US market's main sample, I examine indices behaviour of Canada, Deutschland, France, Great Britain, Italy and Japan.

FIGURE 5: Global Market daily returns for the unmanaged and managed market portfolio. Volatility of the managed portfolio is observably reduced and stable during the whole period including turmoils.



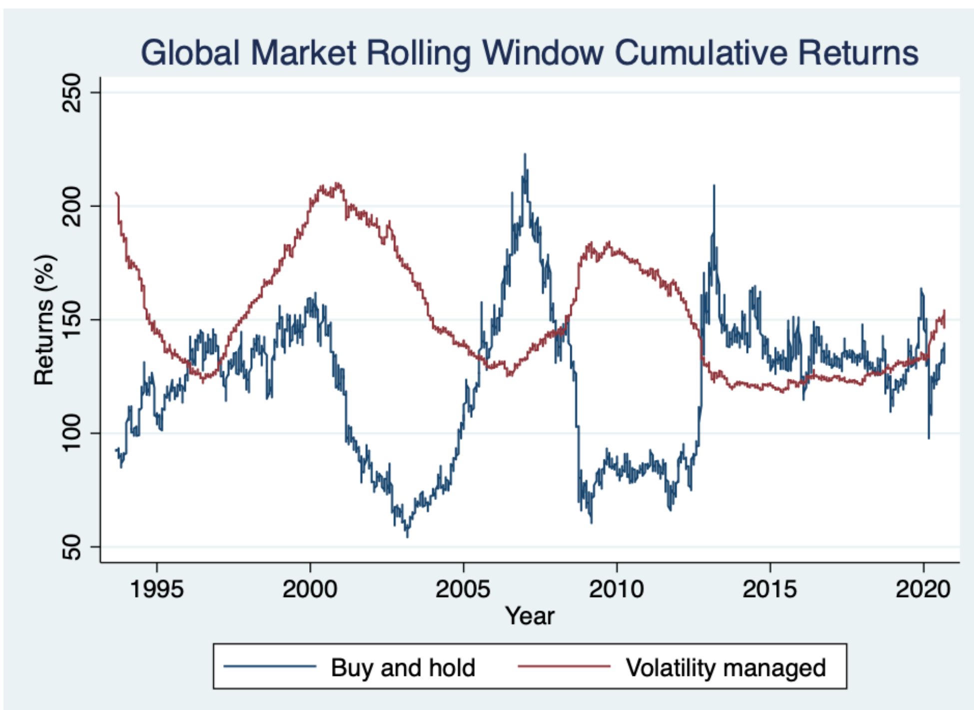
Volatility managed portfolio formation follows the same dynamic construction order with the rest of the paper, using only past data and predicted values. The following formula calculates the returns of the index's managed portfolio:

$$r_{u,t}^{C\ managed} = \frac{\mu_{u,t}^{C}}{\sigma_{u,t}^{C^2}} * r_{u,t}^{C}$$

where $\mu_{u,t}^{C}$ and $\sigma_{u,t}^{C^2}$ is the conditional expected return and the conditional variance of the unmanaged portfolio, respectively, and $r_{u,t}^{C}$is the actual return of the unmanaged index portfolio. Weights are restricted to be between -5 and 5 to prevent extremely rare cases of unrealistic weights.

Figures 5 and Figure 6 confirm the findings of the previous parts of the paper. Daily and rolling window cumulative returns imply stable returns throughout the sampled period. It is worth mentioning that the rolling window cumulative return remains positive with minimum 4-year cumulative return +18%, while the buy-and-hold portfolio sunk with almost -46% return, in its worst 4-year period. Table 4 is illustrative enough to confirm that the outperformance is robust through the sample. Looking directly into daily returns statistics in Appendix 3, someone can see a significant increase in mean returns and a sharp decrease in standard deviation. Nevertheless, as it is mentioned multiple times in this paper, it is worth mentioning that heavy negative tails have been eliminated from -9.6% in global market's worst day, to -4.74% in its managed counterpart.

FIGURE 6: Rolling Window Cumulative returns for the unmanaged and managed Global Market portfolio during the sampled period. The rolling window cumulative returns of the volatility managed portfolio confirms a stable performance without negative values. The buy-and-hold market portfolio has inevitably high volatility with both positive and negative extreme values.



Countries with common but also concrete differences behave financially similarly in volatility scaling investment approach. Sharpe Ratio has been improved in all sampled economies, which confirms that volatility timing is not a US financial market characteristic.

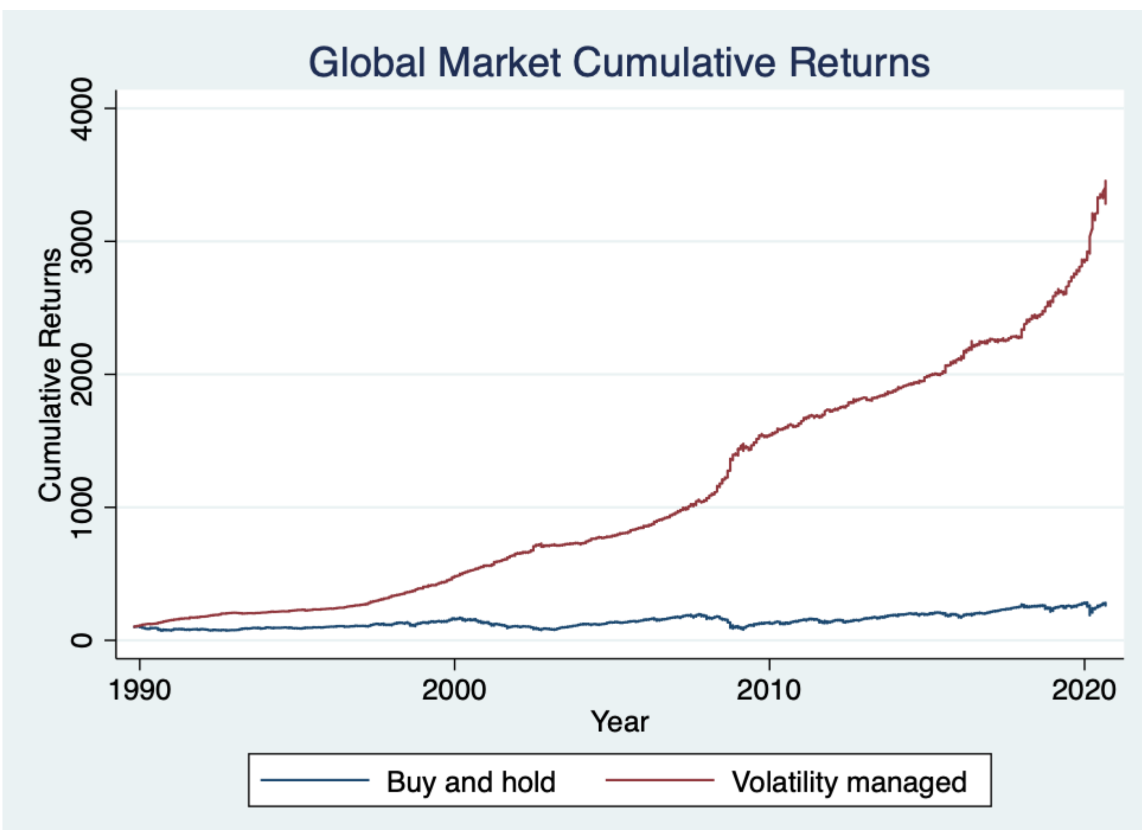
Without any difference, volatility scaling in international indexes portfolios behaves likewise factor and multifactor portfolios during crisis, however some countries achieve spectacular cumulative, rolling window and daily returns. This fact requires special attention, as crisis are not commonly treated by each country and their policies, namely the two most important, monetary and fiscal. Hypothesis three is accepted as volatility scaling portfolios

TABLE 4: Sharpe Ratio and Rolling Window Cumulative Returns of the Indexes portfolios. Rolling Window Cumulative Returns represents the cumulative returns over the last 1000 trading days on each day of the sample. Both figures are improved in all index managed portfolios. Special attention requires the reduced negative extreme numbers which represents an important part of the total investment risk

Variables	Sharpe Unmanaged	Ratio Managed	mean	Rolling Unmanaged min	Window max	Cumulative mean	Returns Managed min	max
Global Market	0,018	0,188	121,316	54,256	222,911	150,931	117,908	210,228
Canada	0,019	0,148	128,124	69,815	293,594	131,008	105,106	177,782
Germany	0,017	0,078	122,859	42,355	304,769	106,446	99,053	155,956
France	0,017	0,089	124,964	55,977	281,782	107,828	98,362	144,478
Great Britain	0,014	0,082	118,916	50,490	257,294	110,820	96,991	160,524
Italy	0,008	0,108	115,046	40,134	265,778	114,033	93,397	204,209
Japan	-0,001	0,109	100,210	37,741	220,064	115,995	101,373	183,391

outperforms the unmanaged in the indeces portfolio.

FIGURE 7: Cumulative returns for the unmanaged and managed Global Market portfolio during the sampled period. The volatility managed portfolio suggests a positive stable performance without extreme negative values. The buy-and-hold market portfolio suffers by strong negative tails that crucially affects the overall cumulative returns performance

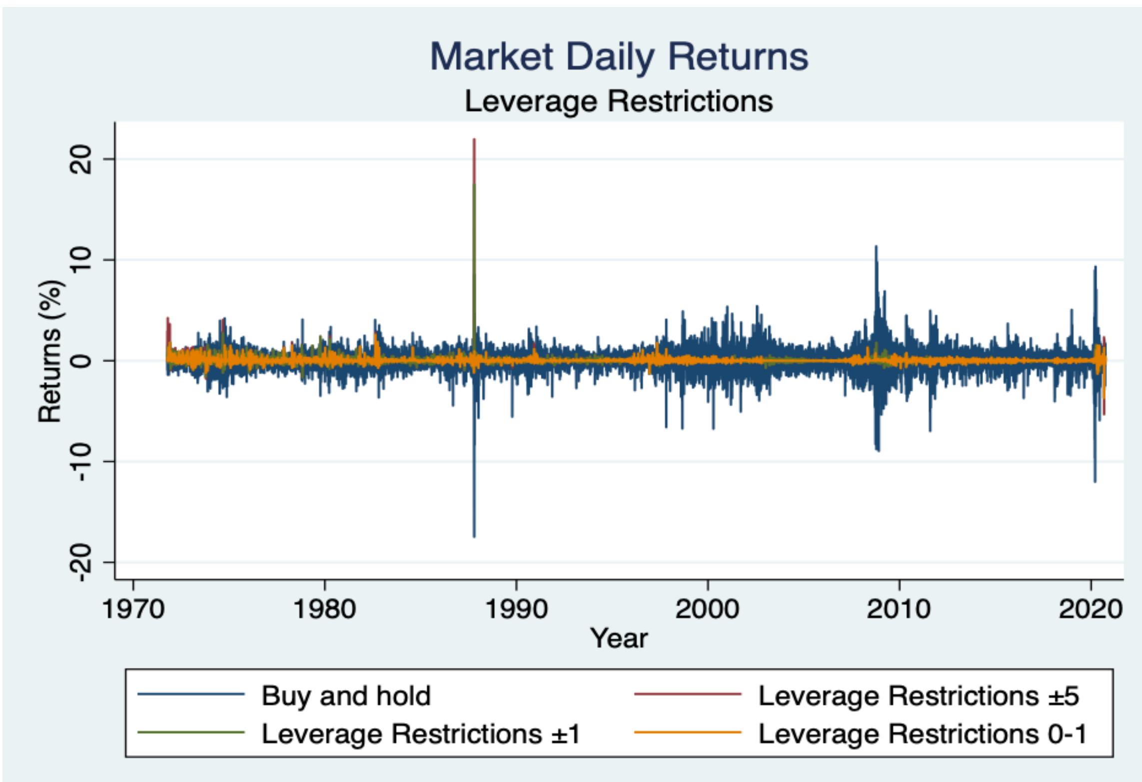


8 Robustness Check

8.1 Leverage Restrictions

Special attention takes the leverage restriction and the effect on the overall performance. To investigate the applicability of the scaling in real-time strategies from the vast majority of market participants, stricter leverage restriction is considered. Two separate cases are taken into consideration.

FIGURE 8: Market daily returns for the unmanaged and managed market portfolio with leverage restrictions. Leverage restriction of ±1 and 0-1 have been tested in addition to ±5 which serves as the basis of the analysis. Volatility of all three managed portfolio is significantly reduced and stable during the whole period including turmoils.



The first one relates to investors who cannot, or they do not want to hold large leveraged exposure related to the capital allocation for each strategy. For this case, a leverage restriction in between -1 and 1 is examined:

$$W_t^* = \begin{cases} 1 & \text{if} \quad W_t^* > 1 \\ -1 & \text{if} \quad W_t^* < -1 \end{cases}$$

The second situation I investigate is the short-selling restriction with leverage in between 0 and 1. This is largely common among pension funds and more risk-averse investors. Hence, scaling weight are normalized as follows:

$$W_t^* = \begin{cases} 1 & \text{if} \quad W_t^* > 1 \\ 0 & \text{if} \quad W_t^* < 0 \end{cases}$$

Table 5 illustrates the results of leverage restricted. Additional attention should be given to each case's extreme values, compared with the buy and hold and the basic -5 to 5 leverage restriction

Table 5: Robustness Check: Leverage RestrictionGlobal Market and US Market Sharpe Ratio and Rolling Window Cumulative Returns for the unmanaged and managed portfolio with leverage restrictions. Leverage restriction of ±1 and 0-1 have been tested in addition to ±5 which serves as the basis of the analysis. Sharpe Ratio of all managed portfolios is highly improved by almost 10 times. Regardless the leverage restriction unmanaged portfolios underperform their managed counterpart. All managed portfolios achieves a large reduction of heavy 4-year negative tails from around -45%to -3.4% or 0.3% for the 0-1 portfolio.)

Variables	Sharpe Ratio	Rolling Window	Cumulative	Returns
		mean	min	max
Unmanaged	0,011	112,970	55,539	252,854
Managed	0,111	135,701	96,621	406,019
Restrictions ±1	0,121	134,280	96,621	352,976
Restrictions 0-1	0,109	114,371	99,653	175,281

Sharpe Ratio remains significantly improved and close to the unrestricted version (Table 5). That is because unrestricted scaling weights are close to ±1. When comparing the buy-and-hold portfolio with the strictest 0-1 in cumulative returns, the latter has 200% outperformance (Appendix 2a). Given that the latter bears only 13% of the unmanaged portfolio's risk, the improvement is considering crucial. However, this is not the best throughout the sample (Appendix 1a). In Figure 9 rolling window, cumulative and cumulative returns are shown.
m
With all three restrictions tested, managed portfolios remain positive and stable across the sampled period, and cumulative returns are decisively larger. Figures and Tables illustrate a definitive superiority of the managed portfolios over the buy-and-hold one.

8.2 Forecasting Volatility with different model: Random Walk

This section investigates the research's innovation value of the chosen volatility forecasting model compared to the Random Walk model.

Managed portfolio's returns and volatility are recalculated based on the aforementioned model, and Table 6 illustrates the representative results. In Appendix' tables, a comprehensive, detailed analysis' results can be found.

FIGURE 9: Global Market and US Market cumulative and rolling window cumulative returns for the unmanaged and managed portfolio with leverage restrictions. Leverage restriction of ±1 and 0-1 have been tested in addition to ±5 which serves as the basis of the analysis. Volatility of all managed portfolios is significantly reduced and stable during the whole period including turmoils. Regardless the leverage restriction unmanaged portfolios underperform their managed counterpart. It is worth mentioning that while 0-1 portfolio is comparable underinvesting than the buy-and-hold invariant leverage of 1, still outperforms in every statistical aspect.

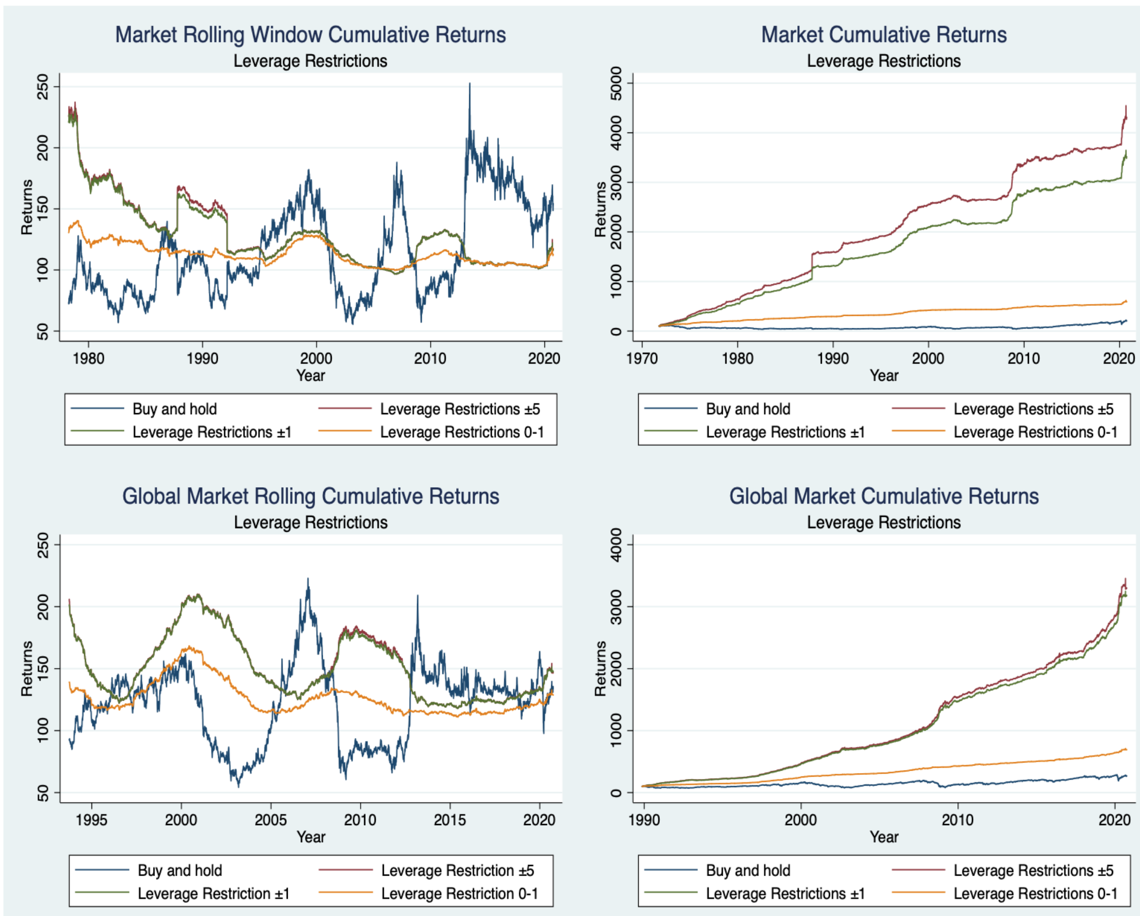
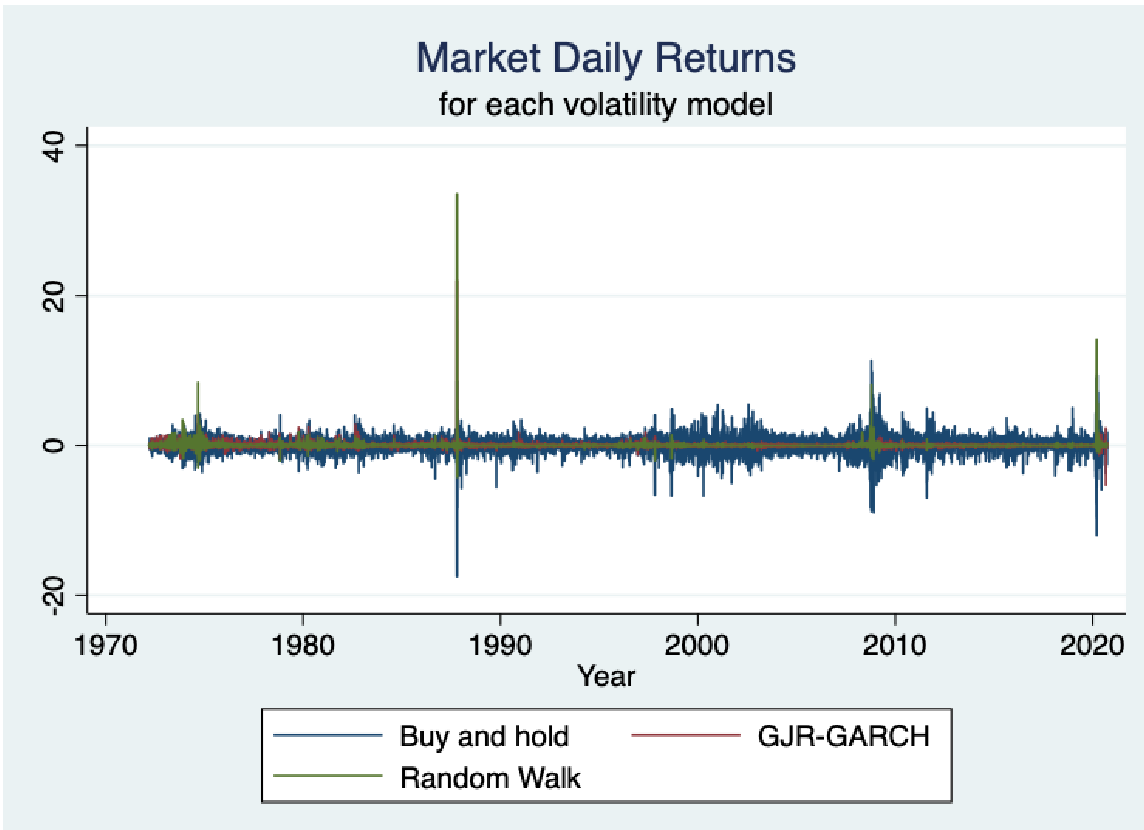


TABLE 6: Market Sharpe Ratio and Rolling Window Cumulative Returns for the unmanaged and managed portfolios formulated with GJR-GARCH and Random Walk forecasting volatility models. Random Walk volatility scaling improves market's portfolio Sharpe Ratio and rolling window cumulative returns. A loud reduction on negative tails to only -1.5% has been achieved. However, GJR-GARCH portfolios' Sharpe Ratio is significantly greater, while its cumulative returns are roughly similar.

Variables	Sharpe Ratio	Rolling Window	Cumulative	Returns
		mean	min	max
Unmanaged	0,011	112,970	55,540	252,854
GJR-GARCH	0,111	135,701	96,621	406,019
Random Walk	0,077	134,855	98,550	444,725

Sharpe Ratio has been significantly improved with the Random Walk too; however, Sharpe Ration achieved with GJR-GARCH volatility prediction is 44% better. This can be easily observed in the next section's daily returns figure (Figure 11) where the Random Walk's portfolio returns are observably higher.

FIGURE 10: Market daily returns for the unmanaged and managed market portfolio formulated with GJR-GARCH and Random Walk forecasting volatility models. Volatility is reduced for both managed portfolio, still the GJR-GARCH is less volatile which is more noticeable during turmoil.

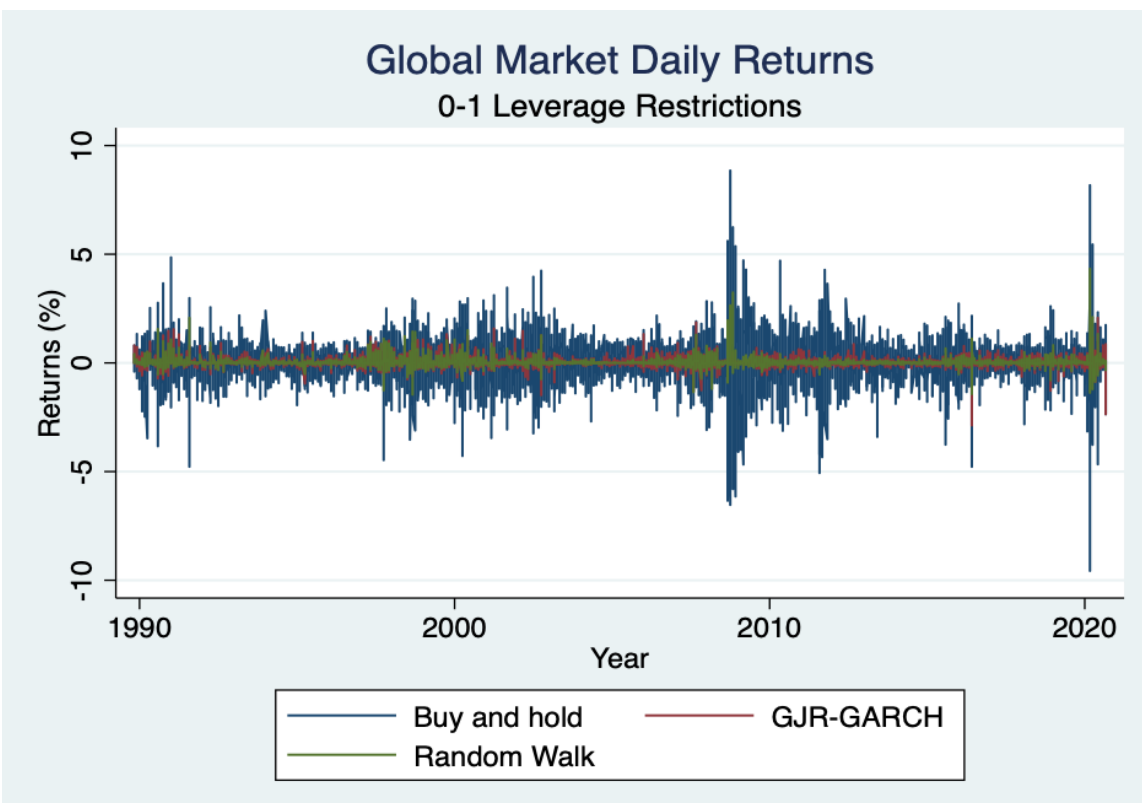


8.3 Vertical Line

In this final section, I compare all 114 formulated portfolios with the unmanaged ones. Using the US and the Global Market portfolio as a basis, I found that every volatility managed portfolio outperforms the unmanaged. The Sharpe Ratio is improved eight to ten times while the cumulative returns soar. Rolling Window Cumulative Returns barely reach the 4-year cumulative returns -7%, in the worst starting day over the last roughly 50 years. Cumulative returns are almost always better even for the most restricted version of the managed portfolios.

Heavy negative tails have been minimized more than the positive tails, while the total cumulative returns growing stably through the time. That is less obvious when it comes to Random Walk, where the returns are mixed compared to GJR-GARCH, but an increased risk. The section's figures demonstrate the increased volatility of the Global Market managed portfolio.

Figure 11: Global Market daily returns: unmanaged portfolio and the managed portfolio constructed with GJR-GARCH volatility forecasting model, are comparing with managed portfolio constructed with Random Walk model. Leverage restriction of 0-1 have been chosen in this figure as the most conservative leverage restriction. Both managed portfolios achieve robust reduction in volatility with stable results throughout the sample. However, portfolio formulated with Random Walk is slightly more volatile the GJR-GARCH portfolio, with larger extreme prices.



Same can be read in Table 7, where the Sharpe Ratio of all three portfolios formulated with Random Walk volatility model is below the ones formulated with GJR-GARCH by at least 20%.

Extreme prices of rolling window cumulative returns of the Global Market are slightly improved in most cases, but not always (Appendix 2, 4 and 6).

TABLE 7: Market Sharpe Ratio and Rolling Window Cumulative Returns for the unmanaged and managed portfolios formulated with GJR-GARCH and Random Walk forecasting volatility models and leverage restriction ±1 and 0-1.

Variables	Sharpe Ratio	Rolling Window	Cumulative	Returns
		mean	min	max
Unmanaged	0,011	112,970	55,540	252,854
GJR-GARCH	0,111	135,701	96,621	406,019
GJR-GARCH (Leverage Restriction ±1)	0,121	134,280	96,621	352,976
GJR-GARCH (Leverage Restriction 0-1)	0,109	114,371	99,653	175,281
Randow Walk	0,077	134,855	98,550	444,725
Randow Walk (Leverage Restriction ±1)	0,099	129,954	98,550	383,717
Randow Walk (Leverage Restriction 0-1)	0,085	111,330	100,121	164,416

9 Conclucion

This analysis explores the additional value an investor can achieve when volatility timing is incorporating in an investment model. The scaling approach that this paper follows is completely dynamic, inspired by Daniel and Moskowitz (2016) dynamic approach in momentum scaling, and confirms their findings. Models are chosen carefully to make out-of-sample predictions based only in past data, which implies a high level of applicability in real-time investments.

Comparing the buy-and-hold portfolio with the volatility managed portfolio, someone can observe significant differences favouring the latter one. The mean and cumulative returns are well improved, which is not against a possible high risk, which, in contrast, is greatly reduced. The results hold across most of the 9 factors, the two multifactor models, 3 and 5-factor models of Fama French, and across the 7G countries' main indices. This is also true over the whole sampled period, with only a few moments where the rolling window cumulative return of the managed portfolios was slightly under its buy-and-hold counterpart.

Despite the overall outperformance of the volatility scaling strategy, it is still worth mentioning, that the volatility managed momentum factor has an extreme outperformance with a Sharpe Ratio of 0.035 for the unmanaged and 0.248 for the managed and a Sharpe Ratio of 0.024 for the unmanaged Value portfolio and 0.209 for the managed. In the multifactor models, the outperformances were less extreme but still significant with a Sharpe Ratio for FF3 of 0.021 for the unmanaged and 0.100 for the managed and FF5 of 0.045 for the unmanaged and 0.094 for the managed. In country indices, the picture is stable, with almost all unmanaged portfolios' Sharper Ratio between 0.008 and 0.018, and managed ones between 0.078 and 0.187.

Robustness tests reveal that volatility timing improves performance in multiple situations of financial market participants regarding leverage restrictions. Moreover, volatility managed portfolios formulated with other than GJR-GARCH model are probably able to improve buy-and-hold portfolio's performance, but further research required for each model. This paper tests Random Walk performance and confirms that volatility timing outperformance, still GJR-GARCH superiority is undeniable throughout the sampled assets and period.

The outperformance would be less impressive if transaction costs were incorporated; however, it is not expected this approach would lose its appeal completely, given the level of improvement in risk-adjusted returns. It is also noticeable that factor investing is not always available to investors, and hence, not all of the revealed opportunities are exploitable from all investors. On the other hand, indices investing is widely available and low-cost investing.

Furthermore, the analysis shows that volatility scaling investment approach is suitable for short- and long-term investors, with different restrictions on leverage and short-selling, making it appealing for a wide range of investors from individuals and hedge funds to institutional investors and pension funds.

Further research that includes more economies in the main indices, factor, multifactor and sector level would be insightful; however, computational requirements would exponentially increase.

References

[1] Clifford S. Asness, Andrea Frazzini, and Lasse H. Pedersen. Leverage aversion and risk parity. *Financial Analysts Journal*, 2012.

[2] Clifford S. Asness, Andrea Frazzini, and Lasse Heje Pedersen. Quality minus junk. *Review of Accounting Studies*, 2019.

[3] Nardin L. Baker and Robert A. Haugen. Low Risk Stocks Outperform within All Observable Markets of the World. *SSRN Electronic Journal*, 2012.

[4] Rolf W. Banz. The relationship between return and market value of common stocks. *Journal of Financial Economics*, 1981.

[5] Pedro Barroso and Pedro Santa-Clara. Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120, 2015.

[6] Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307–327, 1986.

[7] Mark M. Carhart. On persistence in mutual fund performance. *Journal of Finance*, 1997.

[8] Scott Cederburg, Michael S. O'Doherty, Feifei Wang, and Xuemin (Sterling) Yan. On the performance of volatility-managed portfolios. *Journal of Financial Economics*, 2020.

[9] Marcelle Chauvet. An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching. *International Economic Review*, 1998.

[10] Denis B. Chaves, Jason C. Hsu, Vitali Kalesnik, and Yoseop Shim. What Drives the Value Effect? Risk versus Mispricing: Evidence from International Markets. *SSRN Electronic Journal*, 2012.

[11] Peter Christoffersen. *Elements of Financ. Risk Management*. 2012.

[12] Kent Daniel and Tobias J. Moskowitz. Momentum crashes. *Journal of Financial Economics*, 122(2):221–247, 2016.

[13] Robert F. Engle. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, Vol. 50(No. 4):pp. 987–1007, 1982.

[14] E.Easteling. Volatility in Perspective. *Crestmont Research (www.CrestmontResearch.com)*, 2020.

[15] Eugene F. Fama and Kenneth R. French. The Cross-Section of Expected Stock Returns. *The Journal of Finance*, 47(2), 1992.

[16] Eugene F. Fama and Kenneth R. French. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56, 1993.

[17] Eugene F. Fama and Kenneth R. French. Multifactor explanations of asset pricing anomalies. *Journal of Finance*, 1996.

[18] Eugene F. Fama and Kenneth R. French. Size, value, and momentum in international stock returns. *Journal of Financial Economics*, 2012.

[19] Eugene F. Fama and Kenneth R. French. A five-factor asset pricing model. *Journal of Financial Economics*, 116(1):1–22, apr 2015.

[20] Eugene F. Fama and Kenneth R. French. Dissecting Anomalies with a Five-Factor Model. *Review of Financial Studies*, 2016.

[21] Minyou Fan, Youwei Li, and Jiadong Liu. Risk adjusted momentum strategies: A comparison between constant and dynamic volatility scaling approaches. *Research in International Business and Finance*, 46(November 2017):131–140, 2018.

[22] Andrea Frazzini and Lasse Heje Pedersen. Betting against beta. *Journal of Financial Economics*, 111(1):1–25, 2014.

[23] Bruce D. Grundy and J. Spencer Martin. Understanding the nature of the risks and the source of the rewards to momentum investing. *Review of Financial Studies*, 2001.

[24] Roy D. Henriksson and Robert C. Merton. On Market Timing and Investment Performance. II. Statistical Procedures for Evaluating Forecasting Skills. *The Journal of Business*, 1981.

[25] Narasimhan Jegadeesh and Sheridan Titman. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *The Journal of Finance*, 48(1):65–91, 1993.

[26] Glosten Lawrence, Jagannathan Ravi, and Runkle David. On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*, 48(5):1779–1801, 1993.

[27] Harry Markowitz. Portfolio Selection. *The Journal of Finance*, 1952.

[28] Harry M. Markowitz. Foundations of Portfolio Theory. *The Journal of Finance*, 1991.

[29] Alan Moreira and Tyler Muir. Volatility-Managed Portfolios. *Journal of Finance*, 72(4):1611–1644, 2017.

[30] Andrew Ang. *Asset Management: A Systematic Approach to Factor Investing*. Oxford University Press, 2014.

[31] William F. Sharpe. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance*, 1964.

[32] William F. Sharpe. Mutual Fund Performance. *The Journal of Business*, Vol. 39(No. 1):pp. 119–138, 1966.

10 Appendix

TABLE 8: Appendix 1: Summary statistics of the single-factor portfolios and each of the constructed counterparts. Number of observations mean returns, standard deviation, minimum and maximum daily return as well as Sharpe Ratio displayed as follows: a) variables start with single r represents the unmanaged portfolios, while variables start with double rr represents the volatility managed portfolios. b) variables contain GJR have been formulated with GJR-GARCH volatility model, while those that contains $Random_{W}alk$ have been formulated with Random Walk volatility model. c) the last part of naming represents the leverage restriction. 5 represents the ±5 restriction, 1 represents the ±1 restriction while pf represents the 0-1 restriction. d) Finally: Market (MKT), size (SMB), value (HML), momentum factor (MOM), profitability (RMW), investment (CMA), profitability (ROE), investment (IA), betting-against-beta factor (BAB) and quality factor (QMJ).

Variables	Obs	Mean	sd	Min	Max	Sharpe Ratio
rMkt	11.460	0,012	1,087	-17,467	11,346	0,011
rr_GJR_rMkt_5	11.460	0,033	0,300	-5,312	21,975	0,111
rr_GJR_rMkt_1	11.460	0,031	0,259	-3,700	17,467	0,121
rr_GJR_rMkt_pf	11.460	0,016	0,144	-3,700	2,555	0,109
rr_Random_Walk_rMkt_5	11.460	0,035	0,457	-4,200	33,516	0,077
rr_Random_Walk_rMkt_1	11.460	0,031	0,319	-3,997	17,467	0,099
rr_Random_Walk_rMkt_pf	11.460	0,014	0,168	-3,997	8,543	0,085
rSMB	11.460	0,000	0,566	-11,620	6,200	0,000
rr_GJR_rSMB_5	11.460	0,034	0,348	-1,994	26,613	0,098
rr_GJR_rSMB_1	11.460	0,024	0,202	-1,491	11,620	0,117
rr_GJR_rSMB_pf	11.460	0,012	0,121	-1,491	2,939	0,101
rr_Random_Walk_rSMB_5	11.460	0,033	0,575	-4,195	55,054	0,058
rr_Random_Walk_rSMB_1	11.460	0,021	0,188	-1,980	11,620	0,113
rr_Random_Walk_rSMB_pf	11.460	0,011	0,101	-1,516	2,444	0,105
rHML	11.460	0,014	0,577	-4,730	4,770	0,024
rr_GJR_rHML_5	11.460	0,057	0,273	-2,702	6,150	0,210
rr_GJR_rHML_1	11.460	0,046	0,216	-1,700	2,743	0,215
rr_GJR_rHML_pf	11.460	0,028	0,171	-1,700	2,743	0,165
rr_Random_Walk_rHML_5	11.460	0,056	0,385	-9,750	16,683	0,144
rr_Random_Walk_rHML_1	11.460	0,044	0,240	-2,370	4,040	0,182
rr_Random_Walk_rHML_pf	11.460	0,027	0,190	-2,370	4,040	0,141
rMom	11.460	0,028	0,784	-8,190	7,010	0,036
rr_GJR_rMom_5	11.460	0,090	0,361	-2,447	13,700	0,248
rr_GJR_rMom_1	11.460	0,072	0,266	-1,600	4,650	0,270
rr_GJR_rMom_pf	11.460	0,048	0,202	-1,324	1,936	0,237
rr_Random_Walk_rMom_5	11.460	0,094	0,585	-9,743	20,209	0,161
rr_Random_Walk_rMom_1	11.460	0,072	0,347	-3,350	5,880	0,209
rr_Random_Walk_rMom_pf	11.460	0,042	0,225	-3,350	4,072	0,186
rRMW	11.460	0,014	0,401	-3,020	4,490	0,034

rr_GJR_rRMW_5	11.460	0,040	0,259	-2,492	11,598	0,157
rr_GJR_rRMW_1	11.460	0,028	0,160	-0,940	2,400	0,175
rr_GJR_rRMW_pf	11.460	0,018	0,127	-0,843	2,400	0,138
rr_Random_Walk_rRMW_5	11.460	0,034	0,238	-4,720	7,846	0,142
rr_Random_Walk_rRMW_1	11.460	0,027	0,179	-4,490	2,740	0,148
rr_Random_Walk_rRMW_pf	11.460	0,017	0,131	-2,055	2,400	0,126
rCMA	11.460	0,014	0,377	-5,940	2,530	0,038
rr_GJR_rCMA_5	11.460	0,042	0,235	-1,485	6,200	0,177
rr_GJR_rCMA_1	11.460	0,032	0,172	-0,990	1,779	0,183
rr_GJR_rCMA_pf	11.460	0,020	0,137	-0,990	1,779	0,149
rr_Random_Walk_rCMA_5	11.460	0,034	0,223	-6,117	5,050	0,153
rr_Random_Walk_rCMA_1	11.460	0,028	0,158	-1,320	2,250	0,174
rr_Random_Walk_rCMA_pf	11.460	0,018	0,130	-1,292	2,250	0,140
rIA	11.275	0,017	0,380	-6,880	2,750	0,046
rr_GJR_rIA_5	11.460	0,030	0,213	-2,239	4,999	0,140
rr_GJR_rIA_1	11.460	0,023	0,159	-1,172	1,495	0,148
rr_GJR_rIA_pf	11.460	0,017	0,133	-1,172	1,278	0,131
rr_Random_Walk_rIA_5	11.460	0,024	0,173	-1,981	3,476	0,139
rr_Random_Walk_rIA_1	11.460	0,021	0,137	-1,245	2,212	0,151
rr_Random_Walk_rIA_pf	11.460	0,015	0,114	-1,245	2,212	0,130
rROE	11.275	0,026	0,422	-3,961	3,258	0,062
rr_GJR_rROE_5	11.460	0,057	0,304	-2,308	7,889	0,188
rr_GJR_rROE_1	11.460	0,039	0,197	-1,113	2,040	0,201
rr_GJR_rROE_pf	11.460	0,029	0,165	-1,101	1,921	0,179
rr_Random_Walk_rROE_5	11.460	0,043	0,254	-2,221	6,615	0,169
rr_Random_Walk_rROE_1	11.460	0,034	0,186	-1,708	2,827	0,185
rr_Random_Walk_rROE_pf	11.460	0,025	0,155	-1,708	2,827	0,159
rEG	11.275	0,041	0,359	-2,910	2,973	0,114
rr_GJR_rEG_5	11.460	0,045	0,231	-2,024	2,299	0,197
rr_GJR_rEG_1	11.460	0,035	0,180	-1,190	1,499	0,196
rr_GJR_rEG_pf	11.460	0,032	0,173	-1,070	1,499	0,187
rr_Random_Walk_rEG_5	11.460	0,027	0,173	-2,401	3,508	0,158
rr_Random_Walk_rEG_1	11.460	0,025	0,146	-1,678	1,878	0,171
rr_Random_Walk_rEG_pf	11.460	0,023	0,139	-1,678	1,878	0,162
rBAB_US	11.460	0,040	0,643	-6,283	7,945	0,062
rr_GJR_rBAB_US_5	11.460	0,039	0,235	-2,748	2,441	0,165
rr_GJR_rBAB_US_1	11.460	0,034	0,204	-2,748	2,353	0,167
rr_GJR_rBAB_US_pf	11.460	0,027	0,174	-2,748	1,381	0,155
rr_Random_Walk_rBAB_US_5	11.460	0,029	0,260	-4,378	8,868	0,111
rr_Random_Walk_rBAB_US_1	11.460	0,026	0,204	-4,378	6,181	0,128
rr_Random_Walk_rBAB_US_pf	11.460	0,016	0,133	-4,378	2,656	0,119
rQMJ_US	11.460	0,018	0,436	-3,743	5,035	0,042
rr_GJR_rQMJ_US_5	11.460	0,050	0,286	-1,736	14,353	0,174
rr_GJR_rQMJ_US_1	11.460	0,037	0,189	-1,429	2,871	0,194
rr_GJR_rQMJ_US_pf	11.460	0,025	0,158	-1,429	2,871	0,160

rr_Random_Walk_rQMJ_US_5	11.460	0,041	0,300	-4,578	14,353	0,138
rr_Random_Walk_rQMJ_US_1	11.460	0,032	0,193	-2,051	2,871	0,167
rr_Random_Walk_rQMJ_US_pf	11.460	0,021	0,157	-2,051	2,871	0,131

Table 9: Appendix 2: Cumulative and Rolling Window Cumulative Returns of the single-factor portfolios and each of the constructed counterparts displayed as follows: a) variables that following the cum(cumulative) start with single r* represents the unmanaged portfolios, while variables start with double rr* represents the volatility managed portfolios. b) variables contain GJR have been formulated with GJR-GARCH volatility model, while those that contains $Random_{W}alk$ have been formulated with Random Walk volatility model. c) the last part of naming represents the leverage restriction. 5 represents the ±5 restriction, 1 represents the ±1 restriction while pf represents the 0-1 restriction. d) Finally: Market (MKT), size (SMB), value (HML), momentum factor (MOM), profitability (RMW), investment (CMA), profitability (ROE), investment (IA), betting-against-beta factor (BAB) and quality factor (QMJ)

Variables	Cumulative Returns	Rolling	Window	Cumulative	Returns
		mean	sd	min	max
cum_rMkt	209	113	38	56	253
cum_rr_GJR_rMkt_5	4311	136	42	97	406
cum_rr_GJR_rMkt_1	3516	134	39	97	353
cum_rr_GJR_rMkt_pf	594	114	11	100	175
cum_rr_Random_Walk_rMkt_5	5041	135	51	99	445
cum_rr_Random_Walk_rMkt_1	3483	130	42	99	384
cum_rr_Random_Walk_rMkt_pf	505	111	10	100	164
cum_rSMB	85	104	23	60	173
cum_rr_GJR_rSMB_5	4620	137	78	97	715
cum_rr_GJR_rSMB_1	1475	124	34	97	314
cum_rr_GJR_rSMB_pf	400	112	16	96	160
cum_rr_Random_Walk_rSMB_5	3850	136	83	100	695
cum_rr_Random_Walk_rSMB_1	1110	120	32	101	307
cum_rr_Random_Walk_rSMB_pf	338	110	15	99	169
cum_rHML	402	120	28	57	232
cum_rr_GJR_rHML_5	68128	185	91	100	582
cum_rr_GJR_rHML_1	19849	163	56	100	331
cum_rr_GJR_rHML_pf	2510	134	30	101	231
cum_rr_Random_Walk_rHML_5	53447	182	88	101	496
cum_rr_Random_Walk_rHML_1	14378	157	51	101	319
cum_rr_Random_Walk_rHML_pf	2122	132	30	99	223
cum_rMom	1720	132	37	46	255
cum_rr_GJR_rMom_5	2671070	267	124	104	688
cum_rr_GJR_rMom_1	357242	216	80	104	410
cum_rr_GJR_rMom_pf	23194	165	42	102	259
cum_rr_Random_Walk_rMom_5	4051351	298	185	100	837

cum_rr_Random_Walk_rMom_1	374849	222	97	100	486
cum_rr_Random_Walk_rMom_pf	11854	156	44	95	269
cum_rRMW	433	116	17	76	202
cum_rr_GJR_rRMW_5	9955	151	42	97	280
cum_rr_GJR_rRMW_1	2455	133	22	97	174
cum_rr_GJR_rRMW_pf	742	120	13	98	154
cum_rr_Random_Walk_rRMW_5	4622	143	41	99	266
cum_rr_Random_Walk_rRMW_1	2054	133	33	99	243
cum_rr_Random_Walk_rRMW_pf	656	119	15	99	163
cum_rCMA	473	118	22	82	206
cum_rr_GJR_rCMA_5	11453	155	44	104	264
cum_rr_GJR_rCMA_1	3644	139	30	102	212
cum_rr_GJR_rCMA_pf	1030	124	19	96	171
cum_rr_Random_Walk_rCMA_5	4888	145	38	100	251
cum_rr_Random_Walk_rCMA_1	2312	134	26	100	208
cum_rr_Random_Walk_rCMA_pf	803	121	17	99	169
cum_rIA	661	121	20	82	183
cum_rr_GJR_rIA_5	3034	136	30	101	215
cum_rr_GJR_rIA_1	1445	127	21	101	178
cum_rr_GJR_rIA_pf	728	120	16	97	156
cum_rr_Random_Walk_rIA_5	1568	129	21	101	174
cum_rr_Random_Walk_rIA_1	1057	124	17	101	161
cum_rr_Random_Walk_rIA_pf	539	117	13	99	146
cum_rROE	1731	132	21	76	173
cum_rr_GJR_rROE_5	66346	188	68	102	406
cum_rr_GJR_rROE_1	8995	153	31	102	207
cum_rr_GJR_rROE_pf	2889	138	23	101	180
cum_rr_Random_Walk_rROE_5	13011	158	41	99	280
cum_rr_Random_Walk_rROE_1	5026	145	28	99	192
cum_rr_Random_Walk_rROE_pf	1658	131	19	97	176
cum_rEG	9176	151	30	91	232
cum_rr_GJR_rEG_5	17741	161	45	105	284
cum_rr_GJR_rEG_1	5686	144	27	104	214
cum_rr_GJR_rEG_pf	4028	140	25	103	201
cum_rr_Random_Walk_rEG_5	2251	132	23	102	203
cum_rr_Random_Walk_rEG_1	1722	129	20	102	177
cum_rr_Random_Walk_rEG_pf	1309	126	17	101	169
cum_rBAB_US	7601	158	54	71	317
cum_rr_GJR_rBAB_US_5	8404	148	32	94	249
cum_rr_GJR_rBAB_US_1	4920	141	25	93	206
cum_rr_GJR_rBAB_US_pf	2186	133	22	102	206
cum_rr_Random_Walk_rBAB_US_5	2657	133	25	100	206
cum_rr_Random_Walk_rBAB_US_1	1956	130	24	100	206
cum_rr_Random_Walk_rBAB_US_pf	608	117	11	96	142
cum_rQMJ_US	715	121	19	72	182

cum_rr_GJR_rQMJ_US_5	29329	167	38	96	271
cum_rr_GJR_rQMJ_US_1	6459	146	26	96	214
cum_rr_GJR_rQMJ_US_pf	1788	129	15	99	171
cum_rr_Random_Walk_rQMJ_US_5	11052	155	38	100	275
cum_rr_Random_Walk_rQMJ_US_1	3956	141	29	100	235
cum_rr_Random_Walk_rQMJ_US_pf	1037	124	16	100	174

TABLE 10: Appendix 3: Summary statistics of the indexes portfolios and each of the constructed counterparts. Number of observations mean returns, standard deviation, minimum and maximum daily return as well as Sharpe Ratio displayed as follows: a) variables start with single r* represents the unmanaged portfolios, while variables start with double rr* represents the volatility managed portfolios. b) variables contain GJR have been formulated with GJR-GARCH volatility model, while those that contains $Random_{W}alk$ have been formulated with Random Walk volatility model. c) the last part of naming represents the leverage restriction. 5 represents the ±5 restriction, 1 represents the ±1 restriction while pf represents the 0-1 restriction. d) Finally: Canada(CAN), Germany(DEU), French(FRA), Great Britain(GBR), Italy(ITA) and Japan(JPN).

Variables	Obs	mean	sd	min	max	Sharpe Ratio
rMkt	7902	0,017	0,908	-9,567	8,849	0,018
rr_GJR_rMkt_5	7902	0,045	0,238	-4,739	2,707	0,188
rr_GJR_rMkt_1	7902	0,044	0,229	-2,877	2,707	0,192
rr_GJR_rMkt_pf	7902	0,025	0,165	-2,877	2,070	0,149
rr_Random_Walk_rMkt_5	7902	0,044	0,376	-8,399	12,386	0,117
rr_Random_Walk_rMkt_1	7902	0,042	0,330	-8,399	8,861	0,126
rr_Random_Walk_rMkt_pf	7902	0,020	0,156	-1,458	4,331	0,126
rCAN	7902	0,023	1,173	-12,619	12,662	0,019
rr_GJR_rCAN_5	7902	0,029	0,195	-2,214	4,231	0,148
rr_GJR_rCAN_1	7902	0,028	0,186	-1,994	4,231	0,150
rr_GJR_rCAN_pf	7902	0,015	0,131	-1,994	1,807	0,118
rr_Random_Walk_rCAN_5	7902	0,029	0,318	-3,366	12,647	0,093
rr_Random_Walk_rCAN_1	7902	0,029	0,300	-3,366	9,512	0,096
rr_Random_Walk_rCAN_pf	7902	0,013	0,130	-2,473	4,695	0,099
rDEU	7902	0,022	1,251	-12,045	15,617	0,017
rr_GJR_rDEU_5	7902	0,011	0,147	-1,274	6,749	0,078
rr_GJR_rDEU_1	7902	0,011	0,130	-1,274	3,389	0,084
rr_GJR_rDEU_pf	7902	0,007	0,110	-1,274	3,389	0,066
rr_Random_Walk_rDEU_5	7902	0,010	0,147	-2,913	6,805	0,067
rr_Random_Walk_rDEU_1	7902	0,009	0,132	-2,913	3,712	0,072
rr_Random_Walk_rDEU_pf	7902	0,004	0,069	-0,980	1,766	0,063
rFRA	7902	0,022	1,282	-13,088	11,207	0,017
rr_GJR_rFRA_5	7902	0,012	0,133	-0,812	3,779	0,089
rr_GJR_rFRA_1	7902	0,011	0,121	-0,812	2,747	0,092

rr_GJR_rFRA_pf	7902	0,007	0,094	-0,812	2,747	0,075
rr_Random_Walk_rFRA_5	7902	0,012	0,163	-1,932	7,559	0,075
rr_Random_Walk_rFRA_1	7902	0,012	0,145	-1,932	4,280	0,080
rr_Random_Walk_rFRA_pf	7902	0,005	0,074	-1,332	2,732	0,071
rGBR	7902	0,016	1,179	-12,540	12,034	0,014
rr_GJR_rGBR_5	7902	0,014	0,169	-2,284	7,454	0,082
rr_GJR_rGBR_1	7902	0,013	0,143	-1,562	4,907	0,093
rr_GJR_rGBR_pf	7902	0,008	0,112	-1,562	4,907	0,072
rr_Random_Walk_rGBR_5	7902	0,014	0,177	-2,284	6,688	0,080
rr_Random_Walk_rGBR_1	7902	0,014	0,165	-1,934	4,267	0,085
rr_Random_Walk_rGBR_pf	7902	0,007	0,100	-1,272	4,139	0,068
rITA	7902	0,012	1,468	-17,412	11,672	0,008
rr_GJR_rITA_5	7902	0,016	0,153	-1,482	3,801	0,108
rr_GJR_rITA_1	7902	0,016	0,146	-0,897	3,773	0,111
rr_GJR_rITA_pf	7902	0,009	0,104	-0,897	2,623	0,087
rr_Random_Walk_rITA_5	7902	0,014	0,145	-2,331	3,758	0,094
rr_Random_Walk_rITA_1	7902	0,014	0,144	-2,331	3,758	0,095
rr_Random_Walk_rITA_pf	7902	0,006	0,076	-0,646	2,983	0,081
rJPN	7902	-0,002	1,331	-8,656	11,964	-0,001
rr_GJR_rJPN_5	7902	0,019	0,176	-2,233	2,937	0,109
rr_GJR_rJPN_1	7902	0,019	0,176	-2,233	2,937	0,109
rr_GJR_rJPN_pf	7902	0,008	0,102	-1,501	1,947	0,082
rr_Random_Walk_rJPN_5	7902	0,015	0,159	-1,480	5,863	0,096
rr_Random_Walk_rJPN_1	7902	0,015	0,147	-1,480	4,303	0,100
rr_Random_Walk_rJPN_pf	7902	0,008	0,094	-1,420	3,178	0,088

TABLE 11: Appendix 4 Cumulative and Rolling Window Cumulative Returns of the indexes portfolios and each of the constructed counterparts displayed as follows: a) variables that following the cum(cumulative) start with single r* represents the unmanaged portfolios, while variables start with double rr* represents the volatility managed portfolios. b) variables contain GJR have been formulated with GJR-GARCH volatility model, while those that contains $Random_{W}alk$ have been formulated with Random Walk volatility model. c) the last part of naming represents the leverage restriction. 5 represents the ±5 restriction, 1 represents the ±1 restriction while pf represents the 0-1 restriction. d) Finally: Canada(CAN), Germany(DEU), French(FRA), Great Britain(GBR), Italy(ITA) and Japan(JPN))

Variables	Cumulative Returns	Rolling mean	Window sd	Cumulative min	Returns max
cum_rMkt	270	121	32	54	223
cum_rr_GJR_rMkt_5	3302	151	26	118	210
cum_rr_GJR_rMkt_1	3182	150	25	118	210
cum_rr_GJR_rMkt_pf	692	126	14	111	168
cum_rr_Random_Walk_rMkt_5	2989	146	37	106	222

cum_rr_Random_Walk_rMkt_1	2571	145	35	106	218
cum_rr_Random_Walk_rMkt_pf	468	120	13	105	157
cum_rCAN	346	128	42	70	294
cum_rr_GJR_rCAN_5	937	131	21	105	178
cum_rr_GJR_rCAN_1	882	131	21	105	177
cum_rr_GJR_rCAN_pf	333	117	13	103	151
cum_rr_Random_Walk_rCAN_5	961	128	22	103	181
cum_rr_Random_Walk_rCAN_1	932	128	22	103	174
cum_rr_Random_Walk_rCAN_pf	274	113	9	101	139
cum_rDEU	299	123	42	42	305
cum_rr_GJR_rDEU_5	244	106	7	99	156
cum_rr_GJR_rDEU_1	234	106	7	99	150
cum_rr_GJR_rDEU_pf	176	103	3	99	139
cum_rr_Random_Walk_rDEU_5	216	107	8	99	128
cum_rr_Random_Walk_rDEU_1	209	107	8	99	126
cum_rr_Random_Walk_rDEU_pf	140	102	2	100	113
cum_rFRA	301	125	40	56	282
cum_rr_GJR_rFRA_5	247	108	7	98	144
cum_rr_GJR_rFRA_1	238	108	7	98	139
cum_rr_GJR_rFRA_pf	172	105	3	100	124
cum_rr_Random_Walk_rFRA_5	256	109	9	99	132
cum_rr_Random_Walk_rFRA_1	248	109	9	99	130
cum_rr_Random_Walk_rFRA_pf	151	104	2	100	112
cum_rGBR	212	119	37	50	257
cum_rr_GJR_rGBR_5	303	111	9	97	161
cum_rr_GJR_rGBR_1	285	111	9	97	151
cum_rr_GJR_rGBR_pf	189	106	4	100	129
cum_rr_Random_Walk_rGBR_5	308	111	9	100	137
cum_rr_Random_Walk_rGBR_1	300	111	9	100	134
cum_rr_Random_Walk_rGBR_pf	172	106	3	101	113
cum_rITA	108	115	46	40	266
cum_rr_GJR_rITA_5	370	114	21	93	204
cum_rr_GJR_rITA_1	359	114	20	93	198
cum_rr_GJR_rITA_pf	205	107	8	100	144
cum_rr_Random_Walk_rITA_5	294	112	14	101	157
cum_rr_Random_Walk_rITA_1	294	112	14	101	157
cum_rr_Random_Walk_rITA_pf	162	105	6	100	128
cum_rJPN	43	100	34	38	220
cum_rr_GJR_rJPN_5	448	116	13	101	183
cum_rr_GJR_rJPN_1	448	116	13	101	183
cum_rr_GJR_rJPN_pf	192	107	5	99	127
cum_rr_Random_Walk_rJPN_5	327	112	9	101	164
cum_rr_Random_Walk_rJPN_1	316	112	9	101	164
cum_rr_Random_Walk_rJPN_pf	192	107	5	100	128

TABLE 12: Appendix 5: Summary statistics of the multi-factor portfolios and each of the constructed counterparts. Number of observations mean returns, standard deviation, minimum and maximum daily return as well as Sharpe Ratio displayed as follows: a) variables start with single r* represents the unmanaged portfolios, while variables start with double rr* represents the volatility managed portfolios. b) variables contain GJR have been formulated with GJR-GARCH volatility model, while those that contains $Random_{W}alk$ have been formulated with Random Walk volatility model. c) the last part of naming represents the leverage restriction. 5 represents the ±5 restriction, 1 represents the ±1 restriction while pf represents the 0-1 restriction. d) Finally: FamaFrench 3-factor portfolio(FF3), FamaFrench 5-factor portfolio(FF5)

Variable	Obs	Mean	sd	Min	Max	Sharpe Ratio
FF3	11.460	0,018	0,854	-13,389	9,334	0,022
rr_GJR_FF3_5	11.460	0,026	0,258	-3,247	17,212	0,100
rr_GJR_FF3_1	11.460	0,022	0,200	-1,673	13,389	0,109
rr_GJR_FF3_pf	11.460	0,013	0,121	-1,289	1,963	0,107
rr_Random_Walk_FF3_5	11.460	0,032	0,358	-2,728	15,088	0,090
rr_Random_Walk_FF3_1	11.460	0,027	0,281	-2,640	13,389	0,096
rr_Random_Walk_FF3_pf	11.460	0,014	0,172	-2,640	7,186	0,082
FF5	11.460	0,029	0,642	-11,828	9,430	0,046
rr_GJR_FF5_5	11.460	0,019	0,204	-1,472	12,114	0,094
rr_GJR_FF5_1	11.460	0,017	0,186	-1,472	11,828	0,093
rr_GJR_FF5_pf	11.460	0,013	0,123	-1,472	2,605	0,104
rr_Random_Walk_FF5_5	11.460	0,023	0,373	-1,875	30,578	0,061
rr_Random_Walk_FF5_1	11.460	0,018	0,197	-1,875	9,430	0,093
rr_Random_Walk_FF5_pf	11.460	0,012	0,166	-1,014	9,430	0,073

TABLE 13: Appendix 6: Cumulative and Rolling Window Cumulative Returns of the multi-factor portfolios and each of the constructed counterparts displayed as follows: a) variables that following the cum(cumulative) start with single r* represents the unmanaged portfolios, while variables start with double rr* represents the volatility managed portfolios. b) variables contain GJR have been formulated with GJR-GARCH volatility model, while those that contains $Random_{W}alk$ have been formulated with Random Walk volatility model. c) the last part of naming represents the leverage restriction. 5 represents the ±5 restriction, 1 represents the ±1 restriction while pf represents the 0-1 restriction. d) Finally: FamaFrench 3-factor portfolio(FF3), FamaFrench 5-factor portfolio(FF5)

Variables	Cumulative Returns	Rolling mean	Window sd	Cumulative min	Returns max
cum_FF3	544	121	28	62	223
cum_rr_GJR_FF3_5	1860	124	33	98	378
cum_rr_GJR_FF3_1	1199	121	27	98	278
cum_rr_GJR_FF3_pf	432	112	12	100	178
cum_rr_Random_Walk_FF3_5	3669	130	42	100	396
cum_rr_Random_Walk_FF3_1	2099	125	33	100	288
cum_rr_Random_Walk_FF3_pf	492	112	12	101	172
cum_FF5	2275	133	25	82	193
cum_rr_GJR_FF5_5	879	116	16	95	224
cum_rr_GJR_FF5_1	704	116	14	95	185
cum_rr_GJR_FF5_pf	427	111	8	99	153
cum_rr_Random_Walk_FF5_5	1285	121	27	99	199
cum_rr_Random_Walk_FF5_1	798	116	19	99	180
cum_rr_Random_Walk_FF5_pf	397	110	10	99	146