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Modeling Changing EU Carbon Price Dynamics and its Fundamental Drivers

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Abstract

To combat climate change the EU has developed the European Union Emissions Trading System (EU ETS) with the European Emission Allowances (EUAs) as the core carbon credits traded. The combination of the EU ETS being a policy driven market and the changing nature of its fundamentals builds a case for a more thorough investigation of carbon price dynamics. This paper investigates and identifies the dynamics between the EUAs and its fundamental drivers based on three different models: a Vector Autoregressive (VAR) model, a Vector Error Correction Model (VECM) and a time-varying VECM (TV-VECM). I investigate what the cointegration relationships are and whether they are constant. Contrary to existing literature, all models incorporate indicators for growth in clean technology and sustainability, in addition to the various traditional energy variables. The TV-VECM shows that there is a time-varying cointegration relation present between all the variables together. I find a significant cointegration relation between the EUA price, gas price, coal price and an indicator of renewable energy growth in the EU. Without this indicator, the relationship is not found, supporting the idea of adding various new renewable energy sources to the fundamental drivers. Based on a the time-varying cointegration likelihood ratio test, the constant cointegration relation is rejected.

Keywords | EU ETS, Carbon Price, EUA, Time-varying VECM, cointegration

Contents

1 Introduction & Literature Review

To combat climate change the European Union has developed an emissions trading system that launched in 2005: the European Union Emissions Trading System (EU ETS). (European Commission, 2015). At the base of the EU ETS lies the pricing of emissions. However, ever since its debut, there is an ongoing debate about the best way to establish a carbon price. It can be done either via carbon taxes or via cap-and-trade programs like the EU ETS (Goulder and Schein, 2013). In theory, the cap-and-trade programs ensure progress towards the goal of reducing emissions as it integrates a limit on both the emissions (using the cap) and the price (by trading) directly into the policy design, such that one has certainty on emissions rather than price through carbon taxes. The implementation of the system has been divided into distinct trading periods over time, known as phases: phase 1 (2005-2007), phase 2 (2008-2012), phase 3 (2013-2020), phase 4 (from 2021-2030), and beyond. At the time of writing, we are almost entering phase 4 of the system. Establishing a market determined price for carbon is crucial for a well functioning trading system. But since the policy for each phase is adjusted, it is especially difficult to capture long term carbon credit dynamics, stretching over multiple phases.

There is an extensive body of literature on modeling the price dynamics of carbon credits and in particular the European Emission Allowances(EUAs). Chevallier (2011) extensively analyses the influence of energy prices on the EUA price dynamics with simple linear regression models using energy variables and find that brent crude oil, gas and coal significantly influence EUA prices. Additionally, to capture the changes in policy between the phases, researchers have turned to more dynamic models. Lutz et al. (2013) estimates a Markov regime-switching GARCH model that classifies a low and a high volatility regime and show significant differences in the impact of the fundamentals across states. Chevallier (2011) identifies an economic recession and economic expansion based on data ranging from March 2007 to March 2009 to highlight the time-variation of uncertainty. Although regime-switching models allows one to use less underlying assumptions, it does not provide information on the fundamental drivers underlying the regimes.

Hence, this thesis investigates the long-run relationship between the EUAs and its possible price drivers using cointegration. To investigate the fundamental drivers between fuel, electricity, and EUA prices, Fell et al. (2012) investigates the relationship between electricity markets in multiple countries and continents and find different equilibria in all markets, due to the different price series in each market. Rickels et al. (2015) find similar results and argue that statistically significant cointegration equilibria might not explain long-term carbon price dynamics. This cointegration relationship only strengthens the need for more dynamic models, (Hintermann et al., 2016) as the previously mentioned methods do not allow for the relationship between prices to change over time.

Allowing the cointegration relation to change over time, could be crucial in the light of various new developments in the energy market (the Coronavirus/Covid-19 crisis, starting March 2020), as the dynamics of oil, energy and carbon price has drastically changed. First, more renewable energy players are entering the market for the past years and clean technology is rapidly evolving leading to lower costs. Most recently, (Lewis, 2020) argues that the need for green hydrogen points to higher carbon prices. Second, an article in the Financial Times (Riding, 2020) states that ESG funds saw a record inflow in Q2 of 2020, showing that trends in sustainable investing are changing. Third, the carbon market cap is still being continually adjusted and will keep being adjusted in the future (European Commission, 2015).

The combination of the EU-ETS being a policy driven market and the changing nature of its fundamentals builds a case for a more thorough investigation of carbon price dynamics and models that account for the changing relationships in the carbon market. This paper investigates these dynamics based on three different models. Contrary to existing literature, all models incorporate indicators for growth in clean technology and sustainability, in addition to the various energy indicators that influence the carbon credit prices. As a base case, I use a Vector Autoregressive (VAR) model to link the EUA returns to the other variables. To determine the long term equilibria between the various variables, I model the carbon prices using a Vector Error Correction Model (VECM) and investigate what, if any, the cointegration relationships are, based on the theoretical framework of Johansen (1988).

Lastly, in addition to existing literature, I extend this traditional model by investigating the hypothesis of time-variation in the cointegration relationship(s) based on a time-varying VECM (TV-VECM), proposed by Bierens and Martins (2010). When you think of these cointegration relationships as slowly shifting equilibria in different periods throughout time, rather than abruptly shifting equilibria with clear break points in time, the added value of time-varying cointegration over structural breaks becomes more clear. Considering the changing dynamics in the carbon market are not driven by specific events, but rather by gradually changing trends, policies and investor beliefs.

Bierens and Martins (2010) build this time-varying cointegration model via expansions in terms of Chebyshev time polynomials, allowing the cointegrating relationships to vary smoothly over time. Other notable research on the topic of time-varying cointegration is conducted by Koop et al. (2011), who develop a similar model comparable to the random walk variation used with time-varying VAR models. However, as Bierens and Martins (2010) propose a likelihood ratio test for time-varying cointegration, with time-invariant cointegration as the null hypothesis, the resulting extended VECM can be estimated similar to the maximum likelihood approach by Johansen (1988) and hence can be more easily compared to the regular, time-invariant VECM, explaining its use in this paper.

Based on the estimated VAR models and standard VECMs, in line with past literature, oil, coal and gas are significant EUA price drivers. However, I am the first to my knowledge, to find that sustainable investing indicators also significantly influence EUA prices. In addition, I find a statistically significant cointegration relation for the regular VECM between the EUA price, gas price, coal price and an indicator of renewable energy growth in the EU. This shows that gas and coal are amongst the fundamental drivers of EUA prices, however, without the indicator of renewable energy growth, the relationship is not found, indicating the increasing role of renewable energy sources in the fundamental drivers of the EUA prices. Lastly, the likelihood ratio test rejects constant cointegration relations over time in multiple models and finds that there is a time-varying cointegration relation present in the variables. Hence, past literature might wrongly conclude that there is a cointegration relation present between EUA prices and its fundamentals, which is the same at all points in time. Therefore these models could be misspecified.

These results are not only important to improve the policy design of the EU ETS, but also for traders and compliant companies under the EU ETS, as it is important to know what the market fundamentals are and how they interact for forecasting the allowance prices in the future.

The remainder of the paper is structured as follows. Section 2 describes the EU ETS and its regulations in more detail. Section 3 describes the choice of EUA prices drivers along with descriptive statistics. In Section 4 discussed the methods used to estimate the VAR models, VECMs and TV-VECMs. Section 5 summarizes the results for all considered models. Finally, Section 6 concludes and suggests areas of future research.

2 EU ETS market

The European Union Emissions Trading Scheme (EU-ETS) is the biggest compliance market in the world. In the cap-and-trade system, a company is paying someone else to reduce your green-house-gas emissions elsewhere: i.e. as the purchaser of a carbon credit, you compensate for your own companies' emissions.

The rights to emit are equivalent to the global warming potential of 1 tonne of CO_2 equivalent (tCO2e), these rights are called allowances. At the core of the scheme are the EUAs, which are either bought from auctions, received free of charge, or traded among participating companies. The price of the EUAs is determined by supply and demand, regulated by the cap-and-trade system. The level of the cap determines the number of allowances available in the whole system. The cap on the number of allowances is designed to decrease annually from 2013 by 1.74 % per year. This allows companies to gradually adjust their emissions. The system covers power and heat generation, energy-intensive industry and commercial aviation. The main purpose of the EU ETS is to achieve emission reduction targets at minimum costs and to promote global sustainable innovation.

Set up in 2005, the EU ETS is the world's first international emissions trading system. It remains the biggest one, accounting for over three-quarters of international carbon trading. The EU ETS has proved that putting a price on carbon and trading in carbon credits can work. Emissions from installations in the system are falling as intended by slightly over 8% compared to the beginning of phase 3. By the end of 2020, emissions from sectors covered by the system will be 21% lower than in 2005 (This estimation is based on pre-Covid forecasts). In 2030, under the regulations of phase 4 they will be 43% lower.

The EU is taking away rights volume by volume and the price is expected to increase because credits become more scarce. The buyers, suppliers and companies that are compliant with the actual scheme range from small to mid-size greenhouse type of companies (e.g. from tomato producers to companies like Tata Steel Europe). Not complying or miscalculation of the carbon emissions can get very expensive. In the first half of 2020 in the Netherlands alone, these fines ranged from 14 000 up to 376 490 EUR per company, with a total of ten fines handed out by the Dutch Emissions Authority [1]. A general remark to note about the system is that there used to be (or still is) an over-supply. Past literature argues that we have not reached the cap goal yet, but it is slowly being diminished. As a consequence, companies are increasingly starting to feel more pressured to transfer to green sources of energy. Together with the high fines, it adds to the system being taken more and more seriously and the price is expected to get higher and higher, mostly because of EU regulations and increasing participation. It seems that the goal of the European Commission is that it becomes more expensive and eventually too expensive to buy easily.

3 Data

The implementation of the EU ETS has been divided up into distinct trading periods over time, known as phases: phase 1 (2005-2007), phase 2 (2008-2012), phase 3 (2013-2020), phase 4 (from 2021-2030), and beyond.

An important time-period to note is between phases 1 and 2, where the EUA spot price hit zero since banking from phase I to phase II was not allowed and the allowances for phase I became worthless at some point. Figure 1 shows that the EUA spot price continued to move around zero for the first part of phase 2. Using a structural break in 2006, Creti et al. (2012) show that while a cointegration relationship exists for both phase 1 and 2, the relationship is different across the two periods with an increasing role of energy prices in phase 2.

After this time-period, the EUA spot price developed into a nonzero price toward the end of phase II, despite a nonbinding cap for the phase and despite the financial crisis of 2007-2008. This could reflect expectations of a cap on overall emissions that is binding

[1] www.emissieautoriteit.nl/over-de-nea/publicatie-van-boetes/gepubliceerde-boetebesluiten

in the long-term, given the opportunity to bank allowances (Hintermann et al., 2016).

Even though the EUA spot price rebounded more quickly from the first financial crisis as opposed to the market. The general view in the literature is that this period of the market is most probably still inefficient. Most recently, Ibikunle et al. (2016) suggest that trading quality in the EU-ETS matures from 2008 to 2011, in line with earlier research by Frino et al. (2010) who find that aggregate long-term liquidity improves throughout Phase I and during the early months of Phase II.

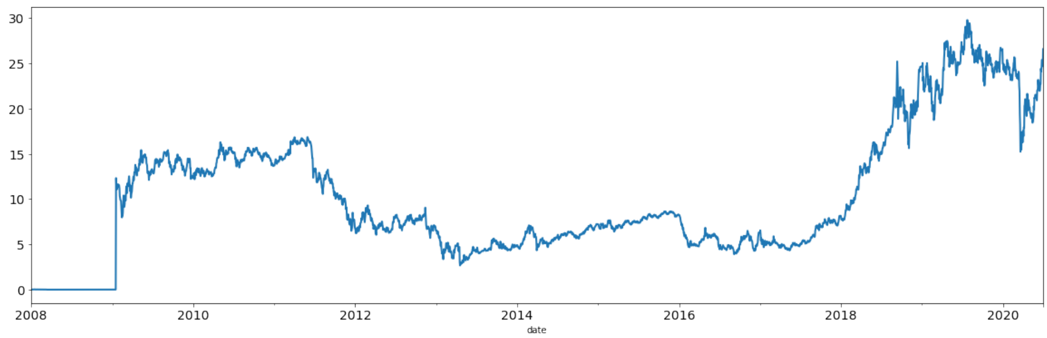


Figure 1: EUA prices from 01/01/2008 - 30/06/2020 - EU ETS phase 2 onward

Another important feature of the market is that every spring, participating companies must close the previous emissions year, i.e. they must report their greenhouse gas emissions and surrender allowances. At the time of writing, we are almost entering phase 4 of the system. It is important to note that the EUAs that will be allocated in phase 4 will not be valid for use in phase 3. However, any remaining emission allowances from phase 3 will continue to be valid in phase 4 and thereafter. The transitional year from phase 3 to phase 4 falls in 2021. In that year, companies will receive emission allowances from phase 4, while closing the trading year for phase 3. Phases 3 and 4 will overlap until the year-end closing in May 2021 (Dutch Emissions Authority). Previously it was possible to use the EUAs allocated to firms in February for the new 'emissions year' for the year-end closing in April of the same year. This will not be possible during the transition from phase 3 to phase 4, as phase 4 allowances are not valid anymore in phase 3. This makes the period leading up to it very interesting as companies might prepare for the transition more upfront.

The data set consists of 2217 daily time-series observations from 1 January 2012 to 30 June 2020, phase 2 onward. Given the fact that the market matures around the end of 2011 according to most literature, the carbon credit dynamics might be more established starting in 2012. Besides, the EUA-low of zero in 2008, mainly driven by poor initial construction of the system, will influence statistical models negatively and might overshadow the carbon credit dynamics.

Hintermann et al. (2016) analyse the influence of different factors on allowance price formation across different studies and find positive influences on allowance prices for economic activity indicators, growth indicators and oil prices. More specifically, Creti et al. (2012) find that in particular oil, gas and coal are cointegrated. Hintermann et al. (2016) find that it is important to also add electricity prices to the analysis, as otherwise some of the coal price results could be explained by omitted variables. This results in *Oil*, *Gas*, *Elec* and *Coal* as the most important energy prices to consider.[2] Rickels et al. (2015) show that regression results, in particular for coal and gas prices are extremely sensitive to choosing different price series of these potential influencing factors, hence *Gas*, *Elec* and *Coal* are all obtained from the same source, namely the European Energy Exchange (EEX), in an attempt to make the results as accurate as possible, as the data they provide comes from more closely related markets as opposed to using different data from different exchanges.

Overall, past literature suggests that macroeconomic variables and energy prices influence EUA prices. However, there is another important factor that intuitively could drive the EUA prices, namely factors involving sustainable growth and the development and implementation of renewable energy sources. In the light of recent events (the Covid crisis with its peak effect on the stock market in March 2020), these factors may be crucial as more renewable energy players are entering the market, clean technology is rapidly evolving leading to lower costs (Lewis, 2020), trends in sustainable investing are changing (Riding, 2020) and in less than a year, there has been a threefold increase in the number of companies committing to net zero (Tett et al., 2020). To account for these developments, in addition to the energy prices I will investigate two other indicators: *Sust* and *Ren*.

The remainder of the paper uses the natural logarithm of all series. This forms no issue for any time-series except *Elec*. Throughout the data set, electricity prices in Europe

[2] All data is obtained from Datastream. All variables together with their abbreviations used in this research and their Datastream tickers can be found in Table A1 in Section A in the Appendix.

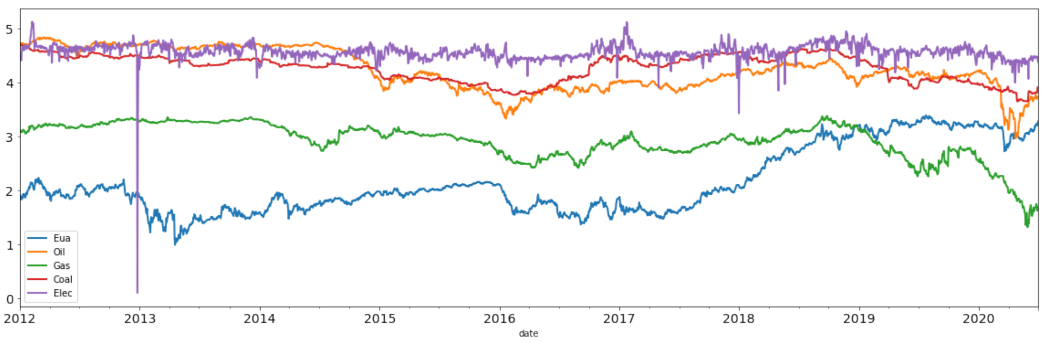
have turned negative, with the biggest dip into the negative for a total of exactly 97 hours between December 2012 and December 2013, with an average negative price of negative 57.46 euros per megawatt-hour (De Vos, 2014). One of the reasons of negative electricity prices are a result from a market distortion caused by renewable support mechanisms (De Vos, 2015). Treating the dips as outliers might therefore remove important information about the long term dynamics between the energy prices.

The new *Elec* variable is constructed by the following transformation on the entire series: adding 1.1 and subtracting the minimum value of the series and taking the natural logarithm after the transformation. Preliminary results show this does not affect model estimation at all or only slightly but does not influence the significance. A summary of the data is shown in Table 1 as well as a plot of all time-series in Figure 2. Even though the final data set only starts in 2012 and excludes the period of the EU-ETS where the EUA prices dropped to zero, it is important to note the high standard deviation of *Eua*, besides it is also interesting to note the high standard deviation of *Ren* as opposed to *Sust*.

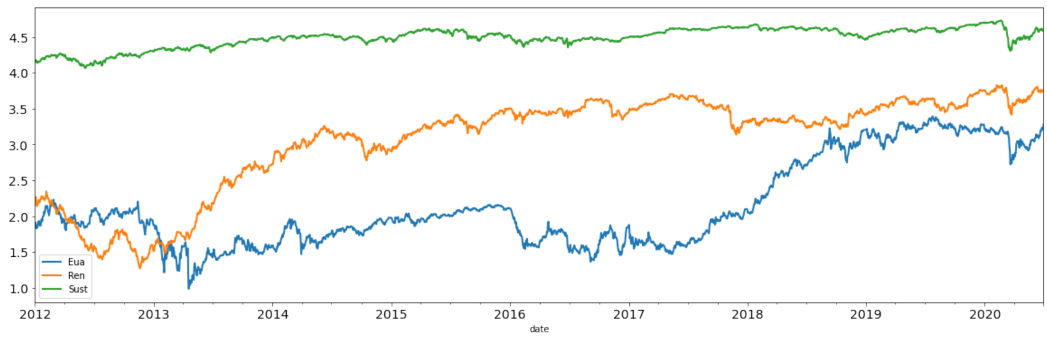
Table 1: Descriptive statistics of all variables

	Eua	*Oil*	*Gas*	*Coal*	*Elec*	*Ren*	*Sust*
Mean	2.1381	4.2383	2.9177	4.2719	4.5687	3.0586	4.4893
Std. dev.	0.6050	0.3760	0.3452	0.2457	0.1607	0.6668	0.1365
Min.	0.9858	2.9617	1.3137	3.6494	0.0953	1.2712	4.0647
Max.	3.3932	4.8380	3.3810	4.7023	5.1317	3.8285	4.7304

This Table shows the daily mean, standard deviation, minimum and maximum of all time series on the sample 1 January 2012 - 30 June 2020. The abbreviations of all variables are elaborated upon in Table A1 in the Appendix.



(a) *Eua* price in relation to the energy prices



(b) *Eua* price in relation to the sustainability indicators

Figure 2: Log time-series, 01/01/2012 - 30/06/2020

Table 2 shows that the correlations between the EUA spot price (*Eua*) and the two sustainability indicators (*Ren* and *Sust*) have increased drastically since the Covid-19 crisis (March 2020). Whereas in the past *Oil* price and *Eua* were negatively correlated, these dynamics have also drastically changed with the peak of the crisis on Black Monday (March 9th, 2020). The correlations between *Eua* and *Gas* remained rather constant, as well as between *Eu* and *Elec*. The correlation between *Eua* and *Coal*, turned from negative to positive after the peak of the crisis. The changing dynamics in the correlation between the variables for this single break further motivate the use of time-varying models in this research.

Table 2: Correlation EUA prices, energy and climate variables

Panel A: Correlation - Full Data Set							
	Eua	*Oil*	*Gas*	*Coal*	*Elec*	*Ren*	*Sust*
Eua	1.0000						
Oil	-0.2444	1.0000					
Gas	-0.4030	0.7378	1.0000				
Coal	-0.2437	0.6663	0.7328	1.0000			
Elec	0.0574	0.2633	0.3401	0.3594	1.0000		
Ren	0.4069	-0.7360	-0.6470	-0.4924	-0.1896	1.0000	
Sust	0.4406	-0.4686	-0.4037	-0.2973	-0.1223	0.8499	1.0000
Panel B: Correlation - Pre Covid							
	Eua	*Oil*	*Gas*	*Coal*	*Elec*	*Ren*	*Sust*
Eua	1.0000						
Oil	-0.1489	1.0000					
Gas	-0.2981	0.7021	1.0000				
Coal	-0.1466	0.6081	0.6846	1.0000			
Elec	0.1286	0.1943	0.2748	0.3078	1.0000		
Ren	0.3766	-0.7471	-0.7071	-0.4680	-0.1595	1.0000	
Sust	0.4541	-0.5214	-0.5015	-0.3166	-0.1256	0.8632	1.0000
Panel C: Correlation - Post Covid							
	Eua	*Oil*	*Gas*	*Coal*	*Elec*	*Ren*	*Sust*
Eua	1.0000						
Oil	0.6942	1.0000					
Gas	-0.2296	-0.1743	1.0000				
Coal	0.0406	0.1748	0.5462	1.0000			
Elec	0.2443	0.4472	0.1229	0.1650	1.0000		
Ren	0.7819	0.7579	-0.5421	-0.1144	0.2747	1.0000	
Sust	0.7995	0.6879	-0.4764	-0.0926	0.2725	0.9609	1.0000

This Table shows the cross-correlations between the EUA prices, energy and climate variables based on daily log of the time series. The abbreviations of all variables are elaborated upon in Table A1 in the Appendix. The correlations are calculated on three samples: Panel A shows the correlation for the full data set (31/12/2007 - 30/06/2020), panel B for Pre Covid (31/12/2007 - 08/03/2020) and panel C for Post Covid (09/03/2020 - 30/06/2020).

4 Methodology

I model the long-term carbon price behaviour of the EUAs by building further upon ideas found by Bredin and Muckley (2011), who examine the extent to which several theoretically founded factors including, economic growth, energy prices, and weather conditions determine the expected prices of the EUAs. They study both static and recursive versions of the Johansen multivariate cointegration likelihood ratio test as well as a variation on this test with a view to control for time-varying volatility effects. In addition I try to add ideas found by Chevallier (2011), who similarly to Bredin and Muckley (2011) proposes to estimate a VECM, but extends the Johansen Cointegration Test with a Structural Shift to explore the possibility of wrongly accepting a cointegration relationship. A thorough literature analysis on carbon price dynamics by Hintermann et al. (2016) finds that even though structural breaks are investigated, we need more dynamic models to investigate the carbon price dynamics because of its environments' changing nature. Even though time-varying volatility or structural breaks in the model might help forecast performance, it might be even more interesting to investigate a possible time-varying cointegration relationship. As a time-varying cointegration relationship could be more relevant from an economic and policy point of view. An example of a relationship like this that has already been determined and is used for forecasting of energy prices, is the fuel-switching constant (i.e. producing one unit of electricity based on coal-fired or gas-fired power- plants, and switching between inputs as their relative price vary). It would be increasingly important to establish fuel-switching constants between other energy sources, like switching from coal to a certain type of renewable energy. I use the theoretical framework proposed by Bierens and Martins (2010) to capture the time-varying dynamics of the carbon prices, who built a time-varying cointegration model via expansions in terms of Chebyshev time polynomials and propose a likelihood ratio test to test for time-varying cointegration, with time-invariant cointegration as the null hypothesis.

I start by investigating the relationship between the variables by a simple VAR model in section 4.1. I then explain how to extend this model to a VECM in section 4.2. Finally, section 4.3 gives a detailed overview of the methods adopted in Bierens and Martins (2010) to build the time-varying cointegration relation within the TV-VECM and describe how they compare the model to a regular VECM.

4.1 VAR Model

As a base model, I will start investigating the relationships between the variables using a Vector Autoregressive (VAR) model. VAR models are generally strong and flexible methods to linearly model multivariate time series and are appropriate for modeling stationary data, such as asset returns or growth rates of macroeconomic time series. A

$p - th$ order VAR, or VAR(p) is defined as follows:

$$Y_t = c + \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-2} + \ldots + \Gamma_p Y_{t-p} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

where Y_t is a $k \times 1$ vector with values of variables at time t, such that Y_{t-1} is the first lag of Y_t, c is the $k \times 1$ vector of constants, Γ_i is a $k \times k$ matrix of parameters for lag $i = 1, \ldots, p$ and ε_t is a k-dimensional white noise process.

The VAR model gives the ability to model the dynamics between the energy, climate variables and the EUA spot price. To choose the correct order p, I use the following model selection criteria in line with Bierens and Martins (2010): AIC (Akaike, 1969), BIC (Schwarz et al., 1978) and HQIC (Hannan and Quinn, 1979).

It is however important to note that the variables could be non-stationary. To test for the stationarity of the variables, I will conduct the Augmented-Dickey-Fuller test, which tests the presence of unit roots. A VAR(p) model is said to be stable, and the corresponding vector time series y_t and each of its components is said to be stationary if all solutions are outside the unit circle. If the variables are indeed non-stationary, a simple solution to make the series stationary would be taking first differences and estimate a new VAR, however, this would ignore a possible cointegration relation between the variables. To investigate the cointegration, the VAR model needs to be extended to a Vector Error Correction Model (VECM).

4.2 VECM & Cointegration

As the goal of this thesis is to investigate long-term carbon price behaviour it is interesting to find long-run equilibrium relationships between the energy, climate and carbon variables, i.e. cointegration.

It is firstly important to note that I use the definition of cointegration adopted by Engle and Granger (1987): The components of the vector $Y_t = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ are said to be cointegrated of order d, b, denoted by $Y_t \sim CI(d, b)$ if:

I All components of Y_t are integrated of order d.

II There exists a vector $\beta = (\beta_1, \beta_2, \ldots \beta_n)$ such that the linear combination $\beta Y_t = \beta_1 y_{1t} + \beta_2 y_{2t} + \ldots + \beta_n y_{nt}$ is integrated of order $(d - b)$ where $b > 0$.

The vector β is called the cointegrating vector and captures the long-term linear combinations of the variables. There may be multiple stationary linear combinations of them, meaning that there is more than one cointegrating vector. I first map the relation between variables more thoroughly by investigating their correlations, autocorrelation and perform the Granger Causality test. Engle and Granger (1987) discusses a cointegration test for a two-variable system, whereas the test proposed in Johansen (1988) is easily applicable for multiple variables. Hence I will test the cointegration relationships with the latter.

The cointegrated process Y_t can be written as a VECM (p) in Granger representation form (Engle and Granger, 1987) with deterministic terms as:

$$\Delta Y_t = \pi_0 + \pi_1 t + \Pi' Y_{t-p} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t, \tag{2}$$

Where $\Delta Y_t = Y_t - Y_{t-1}$ and $\pi_0 + \pi_1 t$ denote a constant and a trend. The Γ_j with $j > 0$, are $k \times k$ and $\Pi' = \alpha\beta'$, where α and β are fixed $k \times r$ matrices and have full column rank, with r the number of linearly independent cointegrating vectors (the columns of β), the ε_t are i.i.d. $N_k(0, \Sigma)$ errors.

Johansen (1995) considers five cases where he imposes different restrictions on the constant and trend term: $\pi_0 + \pi_1 t$ shown in parenthesis:

I The regression model contains no constants and no deterministic trends. ($\pi_0 = 0$ & $\pi_1 = 0$)

II The regression model contains constants but no deterministic trends while data do not display linear trending patterns. The parameters for the intercepts are restricted. ($\pi_0 = \alpha\gamma_0$ and $\pi_1 = 0$)

III The regression model contains constants but no deterministic trends while data display linear trending patterns. The parameters for the intercepts are unrestricted. ($\pi_1 = 0$)

IV The regression model contains constants and deterministic trends, while the data have linear trends but no quadratic trending patterns. The parameters for the trends are restricted. ($\pi_1 = \alpha\gamma_1$)

V The regression model contains constants and deterministic trends, while the data display quadratic trending patterns. The parameters for the trends are unrestricted. (none)

It is important to note that these restrictions mainly impact whether Y_t will have a drift or not, since this is a special focus of section 4.3, I elaborate more on the implications of the restrictions concerning the drift (or no drift) in Y_t below.

Since non of the time-series have zero mean (Table 1), case I will not be investigated at all. Case V, without any restrictions imposed, is used when data displays quadratic trending patterns. Visual inspection doesn't show this is evident for the time-series involved, and hence not investigated in this thesis.

Case II imposes a restriction on the intercept parameters and assumes no trend, i.e. $t = 0$ and $\exists \pi_0$, such that

$$\pi_0 = \alpha \gamma_0. \tag{3}$$

This reduces Equation 2 to:

$$\Delta Y_t = \alpha(\gamma_0 + \beta' Y_{t-p}) + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t. \tag{4}$$

Imposing a constant within the cointegration relations. Here the term $\gamma_0 + \beta' Y_{t-p}$ is zero-mean stationary, ensuring ΔY_t is zero-mean stationary and Y_t has no drift. It is not expected this model will perform best using our data as some variables such as *Eua* and *Gas* run approximately parallel without drift for some periods (from 2012 to 2018 in Figure 2), but this is not true for all variables. Also, most time series run upward sloping.

Case III, imposes no restrictions on the intercept parameter, leaving a constant outside the cointegration relation. However π_0 is also able to generate drift in Y_t because π_0 acts as a vector of drift parameters. Thus drift in the Y_t process is no reason to include a time trend in the VECM.

Hence I consider case IV, with a restriction on the trend parameter, resulting in a constant outside the cointegration relation and linear trend within the cointegration relation by assuming that $\exists \pi_1$, such that

$$\pi_1 = \alpha\gamma_1. \tag{5}$$

This reduces equation 2 to:

$$\Delta Y_t = \alpha(\gamma_1 t + \beta' Y_{t-p}) + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t. \tag{6}$$

Since ΔY_t is stationary, we must have that $\gamma_1 t + \beta' Y_{t-p}$ is stationary, hence $\beta' Y_{t-p}$ is trend stationary. In this case the time series involved have drift, and veer apart, like for *Eua* and *Ren* from mid 2013 to mid 2017.

Inference on standard cointegration for the rank of Π is done using two tests that both check how many eigenvalues equal the unit vector. I firstly use the Trace test statistic by Johansen (1988). The trace test, tests whether the the number of cointegration relations equals $r = r^* < k$, versus the alternative that $r = k$. The test statistic is given by:

$$Trace = -T \sum_{i=r+1}^{k} log(1 - \hat{\lambda}_i). \tag{7}$$

Where Testing proceeds sequentially for $r^* = 1, 2, \ldots, k$ and the first non-rejection of the null is taken as an estimate of r.

Secondly, I test the significance of the estimated eigenvalues themselves where the null hypothesis for the maximum eigenvalue test is that the number of cointegration relations equals $r = r^* < k$, versus $r = r^* + 1$. The test statistic is given by:

$$\lambda_{max} = -T\, log(1 - \hat{\lambda}_r). \tag{8}$$

Testing proceeds sequentially for $r^* = 1, 2, \ldots, k$ with the first non-rejection used as an estimator for r. For both tests, $\hat{\lambda}_1, \ldots \hat{\lambda}_k$ denote the ordered (generalized) eigenvalues resulting from solving the suitable generalized eigenvalue problem defined in Johansen (1988).

4.3 Time-Varying VECM & Cointegration

In addition to existing literature, I propose an alternative framework where the cointegrating vectors fluctuate over time by testing the time-invarying, regular cointegration hypothesis against a time-varying cointegration.

I first consider a time-varying VECM (p) with Gaussian errors, without intercepts and deterministic trends,

$$\Delta Y_t = \Pi_t' Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{9}$$

with $\varepsilon_t \sim i.i.d.\quad N_k[0, \Sigma]$, Y_t a $k \times 1$ vector containing all time-series, k the total number of variables and T is the number of observations. The objective is to test the null hypothesis of general cointegration, $\Pi_t' = \Pi' = \alpha\beta'$, where α and β are fixed $k \times r$ matrices with rank r, against TV cointegration, where only the cointegration relation fluctuates over time, with $\Pi_t' = \alpha\beta_t'$, α stays constant and the β_t 's are time-varying $k \times r$ matrices with constant rank r. Again, Σ and the Γ_j 's are fixed $k \times k$ matrices and $1 \leq r < k$. This form of time-varying cointegration is explicitly chosen as it could clarify more about the long-term dynamics between the time-series. An extension is briefly mentioned in the discussion in section 6.1.

For most cointegrated macroeconomic time series, ΔY_t, and $\beta_t Y_t$ are nonzero-mean stationary processes. If so, the tests will be conducted under the "drift case" assumptions. The time-varying cointegrating relation is expressed in $\beta_t' Y_t = \varepsilon_t$, where the process ε_t represents deviations from the cointegration relationship and
$\beta_t = (\beta_{Euat}, \beta_{Oilt}, \beta_{Gast}, \beta_{Coalt}, \beta_{Elect}, \beta_{Rent}, \beta_{Sustt})$ is an unknown function over time. We want this cointegration relationship to vary smoothly over time rather than abruptly change. To achieve this, Bierens and Martins (2010) uses lower-order Chebyshev polynomials and β_t will be approximated by $\beta_t(m) = \sum_{i=0}^{m} \xi_i P_{i,T}(t)$ where the ξ_i 's are the Fourier coefficients and m denotes the order of the Chebyshev polynomial. These polynomials are rather smooth functions of i, allowing for gradual change in the cointegration relationship.

The Chebyshev time polynomials $P_{i,T}(t)$ are orthonormal and hence can be represented by

$$P_{i,T}(t) = \sqrt{2}\cos(i\pi(t-0.5)/T), \quad P_{0,T}(t) = 1, \quad t = 1,2,\ldots,T, \quad i = 1,2,3,\ldots. \tag{10}$$

To model the time-varying β_t's, I substitute $\Pi_t' = \alpha\beta_t' = \alpha\left(\sum_{i=0}^{m}\xi_i P_{i,T}(t)\right)'$ in equation 9, which yields

$$\Delta Y_t = \alpha\left(\sum_{i=0}^{m}\xi_i P_{i,T}(t)\right)' Y_{t-1} + \sum_{j=1}^{p-1}\Gamma_j \Delta Y_{t-j} + \varepsilon_t, \tag{11}$$

for some $k \times r$ matrices ξ_i, which can be written more conveniently as

$$\Delta Y_t = \alpha\xi' Y_{t-1}^{(m)} + \Gamma X_t + \varepsilon_t, \tag{12}$$

where $\xi' = (\xi_0', \xi_1', \ldots, \xi_m')$ is an $r \times (m+1)k$ matrix of rank r, and $Y_{t-1}^{(m)}$ and X_t are defined by

$$Y_{t-1}^{(m)} = \left(Y_{t-1}', P_{1,T}(t)Y_{t-1}', P_{2,T}(t)Y_{t-1}', \ldots, P_{m,T}(t)Y_{t-1}'\right)', \tag{13}$$

$$X_t = \left(\Delta Y_{t-1}', \ldots, \Delta Y_{t-p+1}'\right)'. \tag{14}$$

The null hypothesis of general co-integration corresponds to $\xi' = (\beta', O_{r,k.m})$, where β is the $k \times r$ matrix of time-invarying co-integrating vectors from equation 9 and $\xi' Y_{t-1}^{(m)} = \beta' Y_{t-1}^{(0)}$, with $Y_{t-1}^{(0)} = Y_{t-1}$.

Similarly to the approach by Johansen (1995), the null hypothesis is tested via a likelihood ratio test.

$$LR^{tvc} = -2\left[\widehat{l}_T(r,0) - \widehat{l}_T(r,m)\right], \tag{15}$$

where $\hat{l}_T(r,0)$ is the log-likelihood of the regular VECM(p) (i.e. equation (12) in the case $m = 0$), so that $Y_{t-1}^{(m)} = Y_{t-1}$, and $\hat{l}_T(r,m)$ is the log-likelihood of the VECM(p) in equation (9) in the case where $Y_{t-1}^{(m)}$ is given by equation 13. I refer to this likelihood ratio test as the time-varying cointegration test (LR TVC) throughout the rest of the paper.

To estimate the time-varying model with Maximum Likelihood and perform the LR TVC, Bierens and Martins (2010) defines:

$$S_{00,T} = \frac{1}{T}\sum_{t=1}^{T} \Delta Y_t \Delta Y_t' - \widehat{\Sigma}_{X\Delta Y}' \widehat{\Sigma}_{XX}^{-1} \widehat{\Sigma}_{X\Delta Y}, \tag{16}$$

$$S_{11,T}^{(m)} = \frac{1}{T}\sum_{t=1}^{T} Y_{t-1}^{(m)} Y_{t-1}^{(m)\prime} - \widehat{\Sigma}_{XY^{(m)}}' \widehat{\Sigma}_{XX}^{-1} \widehat{\Sigma}_{XY^{(m)}}, \tag{17}$$

$$S_{01,T}^{(m)} = \frac{1}{T}\sum_{t=1}^{T} \Delta Y_t Y_{t-1}^{(m)\prime} - \widehat{\Sigma}_{X\Delta Y}' \widehat{\Sigma}_{XX}^{-1} \widehat{\Sigma}_{XY^{(m)}}, \tag{18}$$

$$S_{10,T}^{(m)} = \left(S_{01,T}^{(m)}\right)', \tag{19}$$

where $\widehat{\Sigma}_{XX} = \frac{1}{T}\Sigma_{t=1}^{T} X_t X_t'$, $\widehat{\Sigma}_{X\Delta Y} = \frac{1}{T}\Sigma_{t=1}^{T} X_t \Delta Y_t'$, $\widehat{\Sigma}_{XY^{(m)}} = \frac{1}{T}\Sigma_{t=1}^{T} X_t Y_{t-1}^{(m)\prime}$ and $\widehat{\lambda}_{m,1} \geq \widehat{\lambda}_{m,2} \geq \cdots \geq \widehat{\lambda}_{m,r} \geq \cdots \geq \widehat{\lambda}_{m,(m+1)k}$ are the ordered solutions of the generalized eigenvalue problem

$$\det\left[\lambda S_{11,T}^{(m)} - S_{10,T}^{(m)} S_{00,T}^{-1} S_{01,T}^{(m)}\right] = 0. \tag{20}$$

The rank of $S_{10,T}^{(m)} S_{00,T}^{-1} S_{01,T}^{(m)}$ is k, such that $\widehat{\lambda}_{m,k+1} = \cdots = \widehat{\lambda}_{m,(m+1)k} \equiv 0$.
The log-likelihood $\widehat{l}_T(r,m)$, given the cointegration rank r and Chebyshev polynomial of order m, takes the form:

$$\widehat{l}_T(r,m) = -0.5T \cdot \sum_{j=1}^{r} \ln\left(1 - \widehat{\lambda}_{m,j}\right) - 0.5T \cdot \ln\left(\det\left(S_{00,T}\right)\right) + C, \tag{21}$$

where C is a constant.

The LR TVC test now takes the form:

$$LR_T^{tvc} = -2\left[\widehat{l}_T(r,0) - \widehat{l}_T(r,m)\right] = T\sum_{j=1}^{r} \ln\left(\frac{1 - \widehat{\lambda}_{0,j}}{1 - \widehat{\lambda}_{m,j}}\right), \tag{22}$$

given the cointegration rank r and Chebyshev polynomial order m.

Using theorem 1 of Bierens and Martins (2010), we know that given $m \geq 1$ and $r \geq 1$, under the null hypothesis of standard cointegration the LR statistic LR_T^{tvc} defined in 22 follows an asymptotic χ^2 distribution with mkr degrees of freedom. The power of the LR TVC depends on the choice of the Chebyshev polynomial order m. The optimal choice for m can be compared to the optimal choice of the order of an AR model. Bierens and Martins (2010) recommend to use the minimum of the HQIC for ideal the choice of m, Martins (2018) recommends to estimate $m = 1, \ldots, M$ with the maximum Chebyshev order $M = T/10$ and choose the m corresponding to the minimum value of the HQIC.

For most cointegrated macroeconomic time series and energy variables, ΔY_t and $\beta' Y_t$ are nonzero-mean stationary processes, which correspond to a modification of the time-varying VECM. For the modified assumptions I refer to Bierens and Martins (2010) as they don't directly impact the previous results. These modified assumptions are be referred to as "the drift case." The updated equation 12 corresponding to time-varying VECM(p) with drift is now

$$\Delta Y_t = c + \alpha \xi' Y_{t-1}^{(m)} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \varepsilon t, \tag{23}$$

where c is a vector of intercept parameters.

5 Results

The results are presented in the following subsections. Section 5.1 shows the results of the Augmented Dickey-Fuller Test. Section 5.2 shows the VAR order selection criteria and regression results, section 5.3 discusses the Johansen cointegration rank tests and the VECM regression results. Finally, section 5.4 summarizes the estimated time-varying cointegration relationship.

5.1 Preliminary Results

The Augmented-Dickey-Fuller test tests all variables for the presence of unit roots, the results are shown in Table 3.

Table 3: Results Augmented Dickey-Fuller Test

	Eua	*Oil*	*Gas*	*Coal*	*Elec*	*Ren*	*Sust*
Constant, no trend							
ADF	-0.1358	-1.0640	-1.2010	-1.6044	-4.1262	-1.0686	-2.5467
P-value	0.9458	0.7292	0.6732	0.4814	0.0009***	0.7274	0.1045
Constant and trend							
ADF	-1.6807	-1.5762	-2.3810	-1.6217	-4.1566	-2.3883	-3.2210
P-value	0.7592	0.8016	0.3898	0.7838	0.0052***	0.3858	0.0803***

This table presents the results of the Augmented Dickey-Fuller Test. The Null Hypothesis corresponds to: the data has a unit root and is non-stationary. For a constant and no trend the critical values are -3.433 at 1% , -2.863 at 5% and -2.863 at 10%. For a constant and a trend the critical values are -3.963 at 1%, -3.412 at 5% and -3.412 at 10%. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. All calculations are made on the sample 1 January 2012 - 30 June 2020.

Table 4: Results Augmented Dickey-Fuller Test after taking first differences

	Eua	*Oil*	*Gas*	*Coal*	*Elec*	*Ren*	*Sust*
Constant, no trend							
ADF	-11.4444	-48.3843	-7.7134	-42.1811	-15.1485	-47.2285	-14.5218
P-value	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
Constant and trend							
ADF	-11.5171	-48.3733	-7.7483	-42.1727	-15.1417	-47.2179	-14.5405
P-value	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***

This table presents the results of the Augmented Dickey-Fuller Test on the first differences variables. The Null Hypothesis corresponds to: the data has a unit root and is non-stationary. For a constant and no trend the critical values are -3.433 at 1% , -2.863 at 5% and -2.863 at 10%. For a constant and a trend the critical values are -3.963 at 1%, -3.412 at 5% and -3.412 at 10%. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. All calculations are made on the sample 1 January 2012 - 30 June 2020.

All series are non-stationary, except *Elec*. However, in the presence of outliers, unit root tests lack robustness. In particular, Franses and Haldrup (1994) provide empirical evidence that additive outliers may produce spurious cointegration. Hence the Dickey-Fuller test will reject a unit root too frequently and as a consequence the Johansen test will indicate too many cointegration vectors. I choose to continue analysing the cointegration relationship by assuming that electricity is also non-stationary (i.e. the "Full model"), but will also estimate a second model by excluding electricity from the analysis (i.e. the "model without *Elec*"). Table 4 shows that after taking first differences all variables are stationary.

5.2 VAR Model

Table 5 shows the model selection criteria to determine the appropriate VAR order for the 4 models, as I consider analysing a VAR for both the Full model and the model without *Elec*, where for each one there are again two distinct models: one with solely a constant, and one including a constant and trend. When the AIC, BIC and HQIC indicate different VAR orders, the largest lag order is chosen, hence a VAR (4) model is chosen for all models.

Table 5: VAR Order Selection criteria

	Full model			w/o *Elec*		
	AIC	BIC	HQIC	AIC	BIC	HQIC
Constant, no trend						
0	-21.0562	-21.0382	-21.0496	-17.0572	-17.0417	-17.0515
1	-52.0003	-51.8560*	-51.9476*	-47.6945	-47.5863*	-47.6550*
2	-52.0245	-51.7539	-51.9257	-47.7258	-47.5247	-47.6523
3	-52.0387	-51.6418	-51.8937	-47.7332	-47.4393	-47.6259
4	-52.0456*	-51.5223	-51.8545	-47.7397*	-47.3530	-47.5984
5	-52.0283	-51.3787	-51.7910	-47.7307	-47.2513	-47.5556
Constant and trend						
0	-23.9709	-23.9348	-23.9577	-19.9719	-19.9410	-19.9606
1	-52.0093	-51.8469*	-51.9500*	-47.7041	-47.5804*	-47.6589*
2	-52.0332	-51.7445	-51.9278	-47.7351	-47.5186	-47.6560
3	-52.0461	-51.6311	-51.8945	-47.7414	-47.4321	-47.6284
4	-52.0530*	-51.5117	-51.8552	-47.7479*	-47.3458	-47.6010
5	-52.0351	-51.3675	-51.7912	-47.7383	-47.2434	-47.5575

This table shows different order selection criteria for the full model and model without *Elec*. Firstly calculated in a model with a constant shown in the first panel: *Constant, no trend*. Secondly calculated in a model with a constant and a trend shown in the second panel: *Constant and trend*. AIC: Akaike information criterion, BIC: Bayesian information criterion, HQIC: Hannan-Quinn information criterion. * indicates lag order selected by the respective criterion. All calculations are made on the sample 1 January 2012 - 30 June 2020.

Tables A2, A3, A4 and A5 show the VAR (4) regression results for the equation of *Eua*. The highest Log likelihood is found for the Full model with a trend included. Both adding a trend and adding a variable increases the Log likelihood, explaining the differences between the models.

For the full model including a constant, on a 5% significance level only the first, second and fourth lag of *Eua*, first and second lag of *Coal*, second lag of *Sust* and the fourth lag of *Oil* are significant. Adding a trend makes the second lag and fourth lag of *Oil* significant too.

Excluding *Elec* from the VAR results doesn't impact the significance of the variables. it is however notable that *Gas* become increasingly more significant in different lags by excluding *Elec* and adding a trend.

5.3 VECM & Cointegration

Table 6 shows the results of the Johansen LR tests for the full model and the model without *Elec*. For the full model, from the trace test results we can conclude that there is one cointegration relation present in the model. And that the null hypothesis that there are at most $r_0 = 0$ cointegrated vectors is rejected, whereas the null hypothesis that there are at most $r_0 = 1$ cointegrated vectors is not rejected. The Lambda-max test is not in line with the results of the trace test for the model with a trend, as the the null hypothesis that there are $r_0 = 0$ and $r_0 = 1$ cointegrated vectors is rejected, whereas the null hypothesis that there are $r_0 = 2$ cointegrated vectors is not rejected.

The results presented by Lüutkepohl et al. (2001) justify the common practice in empirical work of using either both types of tests simultaneously or applying the trace tests exclusively. Hence I proceed with estimating the VECM for the full model with only one cointegration relation, but it is important to note, that this cointegration might not exist.

After excluding *Elec* from the model, very different results appear. Based on the trace test, there is no cointegration relation present in the model. And that the null hypothesis that there are at most $r_0 = 0$ cointegrated vectors is not rejected. Again, the Lambda-max test is not in line with the results of the trace test for the model with a trend, as the the null hypothesis that there are $r_0 = 0$ cointegrated vectors is rejected, whereas the null hypothesis that there are $r_0 = 1$ cointegrated vectors is not rejected. Even though the tests indicate otherwise, I will proceed by estimating the VECM model excluding *Elec*, for better comparison with the TV-VECM.

Table 6: Johansen cointegration test results

	r_0	r_1	Test statistic	Critical value: 1% / 5%	Rank: 1% / 5%
Full model					
Constant, no trend					
Trace Test	0	7	314.3	136.0 / 125.6	
	1	7	82.02	105.0 / 95.75	1 / 1
λ_{max} Test	0	1	232.2	52.31 / 46.23	
	1	2	27.48	45.87 / 40.08	1 / 1
Constant and trend					
Trace Test	0	7	338.7	150.1 / 139.3	
	1	7	106.3	117.0 / 107.3	1 / 1
λ_{max} Test	0	1	232.4	55.82 / 49.59	
	1	2	44.16	49.41 / 43.42	
	2	3	23.52	- / 37.16	1 / 2
w/o *Elec*					
Constant, no trend					
Trace Test	0	6	82.28	105.0 / 95.75	0 / 0
λ_{max} Test	0	1	27.47	45.87 / 40.08	0 / 0
Constant and trend					
Trace Test	0	6	106.6	117.0 / 107.3	0 / 0
λ_{max} Test	0	1	44.29	49.41 / 43.42	
	1	2	23.81	- / 37.16	0 / 1

The table shows the results of two Johansen cointegration tests (Johansen (1988)): the trace test and the maximum eigenvalue test. The null hypothesis for both tests r_0 vs. the alternative r_1. The test statistic of the trace test and maximum eigenvalue test is given by equation 7 and 8 respectively with the results (column 4), with corresponding critical values on a 1% and 5% significance level (column 5) and the resulting suggested rank (column 6). Both tests are conducted for two different models, with a *Constant and no trend* or both a *Constant and trend* and with or without *Elec* included. All calculations are based on the sample 1 January 2012 - 30 June 2020

For a comparison of the three cases considered in Johansen (1995), (i.e. Case II, III and IV), the results of a regular VECM for the full model are presented in tables A6, A7 and A8 respectively. The lag order is chosen in accordance with the VAR lag order, namely one lag less. The tables show the parameter estimates for the regression of the lagged parameters on Eua and deterministic terms outside the cointegration relation in the upper panel. It also shows the loading coefficients for the equation of Eua, i.e. the error correction term. The third panel of the table shows the cointegration relations of the loading-coefficients and the last panel, the log likelihood.

The significance of the variables is similar to the the VAR Regression results. For case II, table A7 shows that the first and second lag of *Coal*, second lag *Eua* and third lag *Oil* are significant. Table 7 shows an overview of the estimated cointegration relations of all models including *Elec*. In the cointegration relation of case II, the deterministic constant and all variables are significant, except Oil, the relation is shown in figure A3. For case III, the significance of both the variables and the cointegration relation is the same, except, by definition, there is no deterministic term within the cointegration relation (figure A2). For case IV, again the variable significance is the same, but the cointegration relations include an insignificant deterministic trend (figure A4). The log likelihood is the smallest for Case II, case III and IV do not differ much.

Table 7: VECM(3) estimation results: Johansen Case II

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Cointegration relations Johansen Case II - Full model				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-0.2491	0.430	-0.580	0.562
β_{Gas}	2.2738	0.455	5.000	0.000***
β_{Coal}	1.9286	0.556	3.470	0.001***
β_{Elec}	-13.2832	0.802	-16.558	0.000***
β_{Ren}	0.7584	0.371	2.042	0.041**
β_{Sust}	-4.0979	1.356	-3.022	0.003***
Deterministic constant	60.9671	5.861	10.403	0.000***
Cointegration relations Johansen Case II - Full model				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-0.2495	0.430	-0.580	0.562
β_{Gas}	2.2717	0.455	4.991	0.000***
β_{Coal}	1.9341	0.556	3.477	0.001***
β_{Elec}	-13.2951	0.803	-16.558	0.000***
β_{Ren}	0.7581	0.372	2.040	0.041**
β_{Sust}	-4.0985	1.357	-3.020	0.003***
Cointegration relations Johansen Case IV - Full model				
β_{Eua}	1.000	0.000	0.000	0.000***
β_{Oil}	-0.1111	0.515	-0.216	0.829
β_{Gas}	2.8004	0.643	4.353	0.000***
β_{Coal}	1.9525	0.694	2.814	0.005***
β_{Elec}	-15.3357	0.97	-15.81	0.000***
β_{Ren}	0.8407	0.428	1.963	0.050**
β_{Sust}	-5.2569	1.847	-2.846	0.004***
Deterministic trend	0.0003	0.000	0.887	0.375

This table shows the cointegration relations of the loading-coefficients in the VECM(3) estimation results of Johansen (1995) for the full model, estimated based on the sample 1 January 2012 - 30 June 2020. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. β_x correspond to the estimates specified by equation 9 The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table 8: VECM(3) estimation results: Johansen Case II

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Cointegration relations Johansen Case II - Model excluding *Elec*				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-0.7692	1.553	-0.495	0.620
β_{Gas}	6.3835	1.619	3.942	0.000***
β_{Coal}	-8.5331	1.957	-4.359	0.000***
β_{Ren}	-0.889	1.339	-0.664	0.507
β_{Sust}	5.1093	4.894	1.044	0.297
Deterministic constant	-0.8048	17.835	-0.045	0.964
Cointegration relations Johansen Case III - Model excluding *Elec*				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-1.1977	2.827	-0.424	0.672
β_{Gas}	10.0592	2.947	3.414	0.001***
β_{Coal}	-15.4448	3.562	-4.336	0.000***
β_{Ren}	-3.1165	2.437	-1.279	0.201
β_{Sust}	17.7418	8.907	1.992	0.046***
Cointegration relations Johansen Case IV - Model excluding *Elec*				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-1.1242	0.325	-3.455	0.001***
β_{Gas}	-1.9382	0.388	-4.998	0.000***
β_{Coal}	3.284	0.438	7.49	0.000***
β_{Ren}	0.2851	0.271	1.052	0.293
β_{Sust}	3.0574	1.145	2.669	0.008***
Deterministic trend	-0.0022	0.000	-9.626	0.000***

This table shows the cointegration relations of the loading-coefficients in the VECM(3) estimation results of Johansen (1995) for the model excluding *Elec*, estimated based on the sample 1 January 2012 - 30 June 2020. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. β_x correspond to the estimates specified by equation 9 The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

The results of the regular VECM for the model excluding *Elec* are presented in tables A9, A10 and A11 respectively. The significance of all variables across all cases is almost the same compared to the full model, except two small differences, as *Eua* lag 3 is also significant for case IV in table A11. However, quite a lot changes in the cointegration relations, Table 8 shows an overview. For case II, the cointegration relations change completely, only *Gas* and *Coal* remain significant (figure A5). For case III, there is a significant deterministic constant outside the cointegration and within only *Gas*, *Coal* and *Sust* are significant. The cointegration relation is plotted in A6 For case IV, the cointegration relation includes an insignificant deterministic trend, but all variables are significant except *Ren* with the resulting cointegration relation plotted in figure A7. The log likelihood is again smallest for Case II, however, this time Case IV is higher relatively.

5.3.1 Reduced dimensional model

Tests for cointegration become oversized and their power becomes low in case of large systems, hence I also consider a lower dimensional case. I investigated all possible combinations of the variables in different dimentions and tested them for possible cointegration. All combinations that included *Elec* in the considered cointegration relation, found a cointegration rank equal to one. Since *Elec* is found stationary by the Augmented Dickey Fuller test, the Johansen test might indicate too many cointegration vectors, hence the results of the remaining combinations including *Elec* are not presented. When *Elec* is excluded, there are only a couple instances where the trace test and maximum eigenvalue test indicate cointegration. Table 9 shows the test results for the one instance that actually produces a significant cointegration relation. This cointegration relation includes *Eua*, *Gas*, *Coal* and *Ren* and follows Johansen case IV, it will be referred to as the reduced model.

Table 9: Johansen cointegration test results reduced model

	r_0	r_1	Test statistic	Critical value: 1% / 5%	Rank: 1% / 5%
Constant, no trend					
Trace Test	0	4	40.57	54.68 / 47.85	0 / 0
λ_{max} Test	0	1	22.00	32.72 / 27.59	0 / 0
Constant and trend					
Trace Test	0	4	59.98	62.52 / 55.25	
	1	4		- / 35.01	0 / 1
λ_{max} Test	0	1	36.71	36.19 / 30.82	
	1	2	16.70	29.26 / 24.25	1 / 1

The table shows the results of two Johansen cointegration tests (Johansen (1988)): the trace test and the maximum eigenvalue test for the reduced model containing: *Eua*, *Gas*, *Coal* and *Ren*. The null hypothesis for both tests r_0 vs. the alternative r_1. The test statistic of the trace test and maximum eigenvalue test is given by equation 7 and 8 respectively with the results (column 4), with corresponding critical values on a 1% and 5% significance level (column 5) and the resulting suggested rank (column 6). Both tests are conducted for two different models, with a *Constant and no trend* or both a *Constant and trend*. All calculations are based on the sample 1 January 2012 - 30 June 2020

Table 10 shows the results of the VECM(2) corresponding to the reduced model and Figure 3 shows the corresponding cointegration relation.

Table 10: VECM(2) estimation results: Johansen Case IV, reduced model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Deterministic constant	-0.0236	0.015	-1.538	0.124
ΔEua_{-1}	0.0304	0.021	1.417	0.156
ΔGas_{-1}	-0.0177	0.029	-0.619	0.536
$\Delta Coal_{-1}$	0.2823	0.056	5.03	0.000***
ΔRen_{-1}	-0.0245	0.032	-0.769	0.442
ΔEua_{-2}	-0.0877	0.021	-4.1	0.000***
ΔGas_{-2}	0.0152	0.028	0.536	0.592
$\Delta Coal_{-2}$	-0.1349	0.057	-2.347	0.019**
ΔRen_{-2}	0.0676	0.032	2.129	0.033**
Error Correction Term	0.0019	0.001	1.585	0.113
Cointegration relations				
β_{Eua}	1.000	0.000	0.000	0.000***
β_{Gas}	-2.2243	0.527	-4.219	0.000***
β_{Coal}	3.6578	0.617	5.93	0.000***
β_{Ren}	1.3109	0.275	4.762	0.000***
Deterministic trend	-0.0022	0.000	-7.354	0.000***
Log likelihood	21451.0443			

This table shows the VECM(2) estimation results of Johansen (1995) Case IV, estimated based on the sample 1 January 2012 - 30 June 2020 with *Oil*, *Elec* and *Sust* not included. Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on *Eua* and deterministic terms outside the cointegration relation in the upper panel. It also shows the loading coefficients for the equation of *Eua*, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9 The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

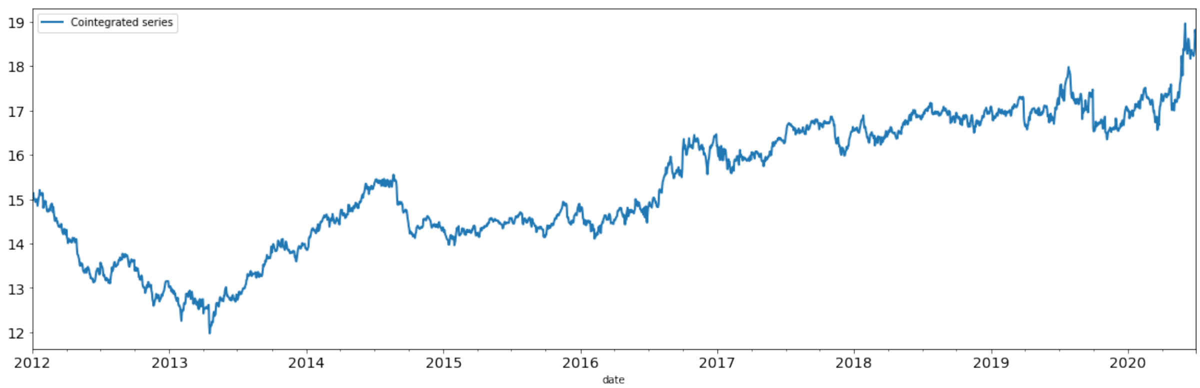


Figure 3: Cointegration relation ($\beta' Y_t$) for the VECM(2) Johansen Case IV specified in Table 10 - 1 January 2012 - 30 June 2020.

The model shows that in the equation of *Eua*, different lags of each variable are significant, except Gas. However, in the cointegration relations all variables and the deterministic trend are significant, including Gas.

5.4 Time-Varying VECM & Cointegration

Figure A8 presents the plots of the time-varying cointegration vector and figure A9 the corresponding cointegration relation ($\beta_t' Y_t$) for
$\beta_t = (\beta_{Eua,t}, \beta_{Oil,t}, \beta_{Gas,t}, \beta_{Coal,t}, \beta_{Elec}, \beta_{Ren,t}, \beta_{Sust,t})$ for different Chebyshev polynomial orders m. It is important to note, that β_t is normalized with respect to the first beta estimate corresponding to Eua, i.e. $\beta_{Eua1} = 1$, without loss of generality in all figure plotting the time-varying cointegration vector.

Figure A10 presents the plots of the time-varying cointegration vector and figure A11 the corresponding cointegration relation ($\beta_t' Y_t$) for $\beta_t = (\beta_{Eua,t}, \beta_{Oil,t}, \beta_{Gas,t}, \beta_{Coal,t}, \beta_{Ren,t}, \beta_{Sust,t})$ for different Chebyshev polynomial orders m.

The ideal Chebyshev polynomial order m is considered 2 in accordance with the Hannan-Quinn criterion. The VECM order p was chosen in line with the regular VECM and VAR: i.e. $p = 3$. For the standard time series, the Johansen approach indicated $r = 1$, hence the number of cointegrating vectors is also chosen 1.

The results of the LR TVC Statistics and corresponding p-values, shown in table 11, indicate to reject the null hypothesis of time-invariant cointegration strongly for the full model and across all m. It also shows the effect of the Chebyshev polynomial order m in more detail. As the order increases, the log likelihood does as well.

Table 11: Results Statistical Tests Time-Varying VECM(3)

	Full model			w/o *Elec*		
	$m = 2$	$m = 3$	$m = 4$	$m = 2$	$m = 3$	$m = 4$
LR TVC Statistics	39.257	50.256	51.897	44.719	59.123	87.050
P-Value	0.000	0.000	0.004	0.000	0.000	0.000
Log Likelihood	35593.928	35599.427	35600.248	34128.573	34135.775	34149.739
AIC	-32.134	-32.097	-32.114	-30.811	-30.778	-30.780
BIC	-32.036	-31.881	-31.944	-30.718	-30.572	-30.543
HQC	-32.098	-32.018	-32.052	-30.777	-30.703	-30.693

The table shows the TVC Statistic, with corresponding p-values using the $\chi^2(mrk)$, the time-varying log likelihood, the AIC, BIC and HQIC for different Chebishev orders m, $r = 1$.

For the full model, figure 4 shows that there is more volatility in the coefficients around 2013 Q3 and 2020 Q1. This could be explained by the methodology, but also as a consequence of structural changes in the carbon dynamics. Whereas the volatility around 2013 Q3, is less clear, but could possibly be explained by the increasing Eua and Ren prices, the 2020 Q1 volatility can most logically be linked to the start of the Covid-19 crisis.

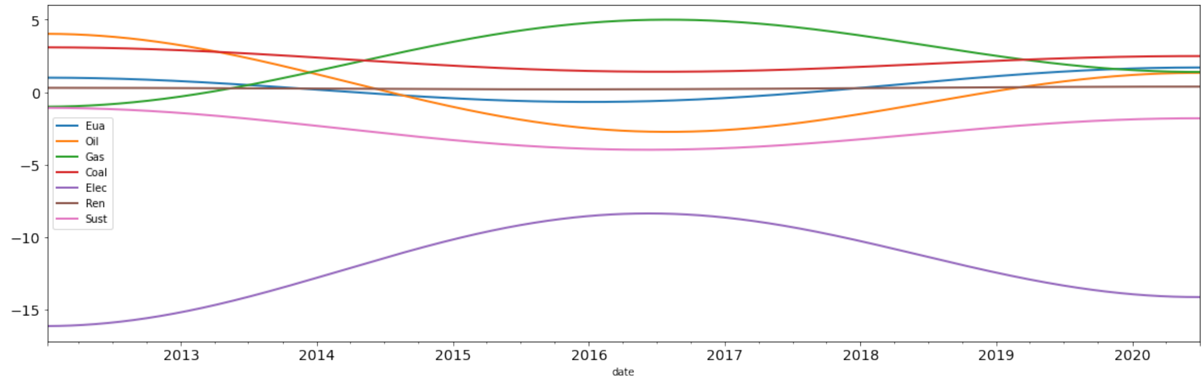


Figure 4: Estimates of $(\beta_t' Y_t)$ for the TV-VECM(3) including $Elec$ - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order $m = 2$

However, as the Chebyshev order m is increased, the volatility in figures A8 (b) and (c) cannot be traced back easily to signature events in the market, supporting the ideal Chebyshev order of $m = 2$.

The cointegration relation for $m = 2$ is shown in figure 5 and for comparison the same graph is also plotted along with $m = 3$ and $m = 4$ in figure A9 in the Appendix. Whereas a visual inspection would suggest very similar results, with the large negative electricity price drop in 2013, there are very large differences in the range of the cointegrated series, about 70, 120 and 40 respectively for $m = 2, m = 3$ and $m = 4$.

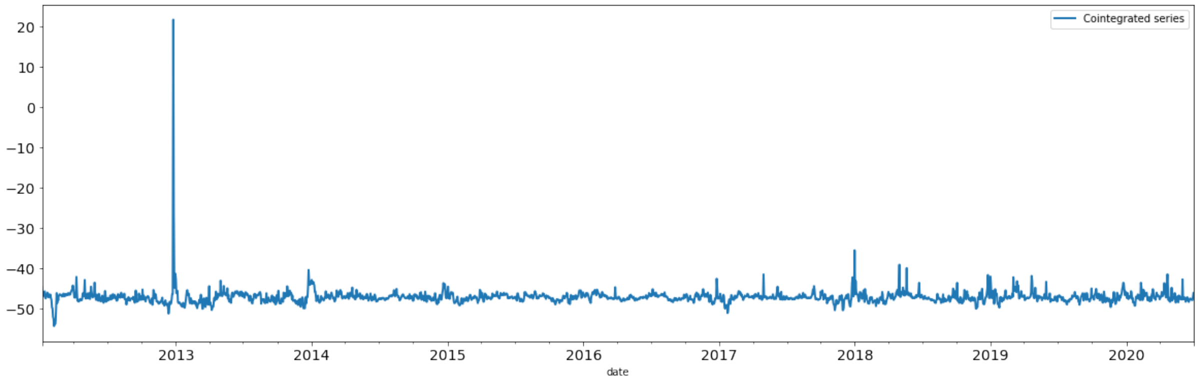


Figure 5: Cointegration relation ($\beta_t Y_t$) for the TV-VECM(3) including *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order $m = 2$

For the model excluding *Elec*, Figure 6 shows that there is again more volatility in the coefficients around 2013 Q3. However the 2020 Q1 volatility is not visible at all.

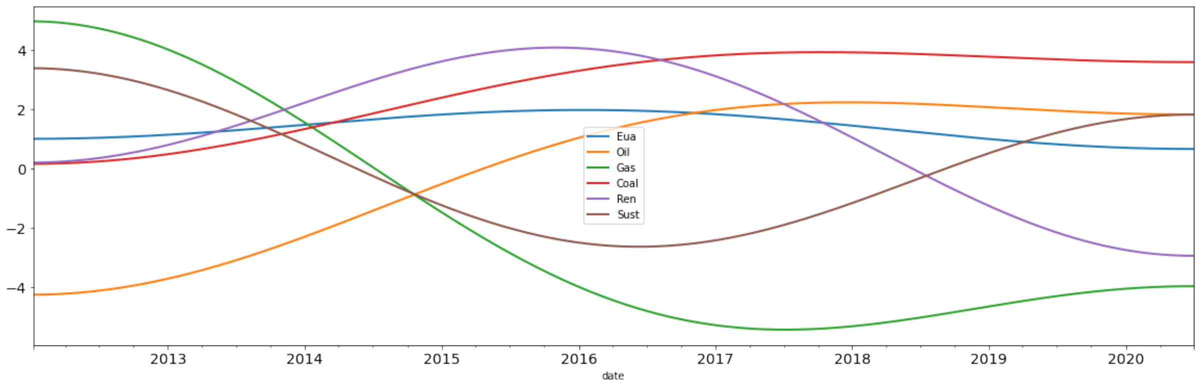


Figure 6: Estimates of ($\beta_t' Y_t$) for the TV-VECM(3) excluding *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order $m = 2$

As the Chebyshev order m is increased, the volatility in figures A10 (b) and (c) is much higher (especially evident, taking into account their range), supporting the ideal Chebyshev order of $m = 2$.

The cointegration relation for $m = 2$ is shown in figure 7 and for comparison the same graph is also plotted for $m = 3$ and $m = 4$ in figure A11 in the Appendix. The resulting time-series differ much more compared to the models including electricity, with very different patterns.

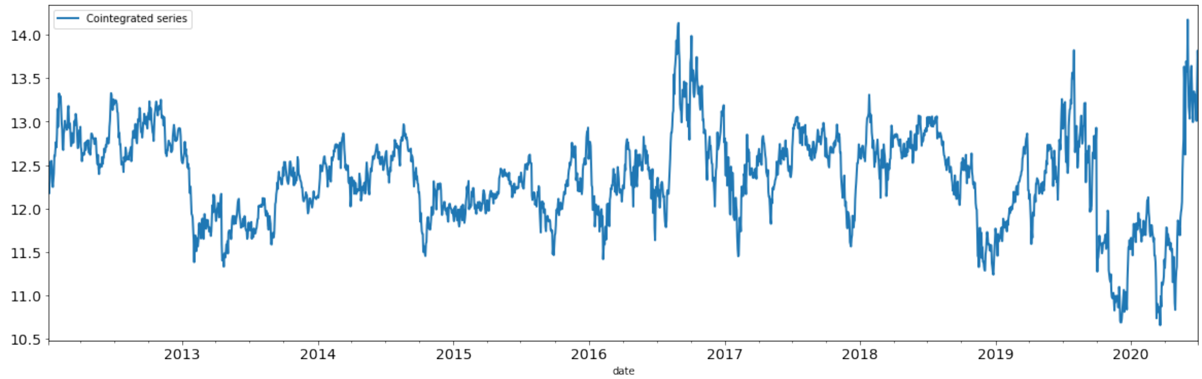


Figure 7: Cointegration relation ($\beta_t' Y_t$) for the TV-VECM(3) excluding *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order $m = 2$

5.4.1 Reduced dimensional model

Figure 8 presents the plots of the time-varying cointegration vector and figure 9 the corresponding cointegration relation ($\beta_t' Y_t$) for
$\beta_t = (\beta_{Eua,t}, \beta_{Gas,t}, \beta_{Coal,t}, \beta_{Ren,t})$ for Chebyshev polynomial order $m = 1$.

The ideal Chebyshev polynomial order m is considered 1 in accordance with the Hannan-Quinn criterion shown in 12. The VECM order p was chosen in line with the regular VECM in 10: i.e. $p = 2$. For the standard time series, the Johansen approach indicated $r = 1$, hence the number of cointegrating vectors is also chosen 1.

The results of the LR TVC Statistics and corresponding p-values, shown in table 12, indicate to reject the null hypothesis of time-invariant cointegration strongly for the full model and across all m on a 5% siginifance level, however, contrary to the full model, the time-varying cointegration for $m = 1$ is not significant.

Table 12: Results Statistical Tests Time-Varying VECM(2) - Reduced model

	$m = 1$	$m = 2$	$m = 3$
LR TVC Statistic	12.9893	37.5980	41.3088
P-Value	0.0113	0.0000	0.0000
Log likelihood	21450.3177	21462.6221	21464.4775
AIC	-19.3625	-19.3520	-19.3609
BIC	-19.3213	-19.2489	-19.2784
HQIC	-19.3475	-19.3143	-19.3308

The table shows the likelihood ratio time varying-cointegration statistic (LR TVC Statistic), with corresponding p-value using the $\chi^2(mrk)$, the time-varying log likelihood, the AIC, BIC and HQIC for Chebishev order $m = 1$ and cointegration rank $r = 1$.

Figure 8 shows that throughout the whole sample, *Gas* seems to be constant withing the cointegration. *Eua* and *Coal* run parallel to each other and drift apart slightly. One thing is very clear from the graph, the dynamics between *Eua* and *Ren* take completely opposite directions. In line with previous results for the full model the cointegration relations seems to be changing starting in 2013 Q3. This could be explained by the methodology, but also as a consequence of structural changes in the carbon dynamics. However contrary to the previous results, the 2020 Q1 volatility is not present.

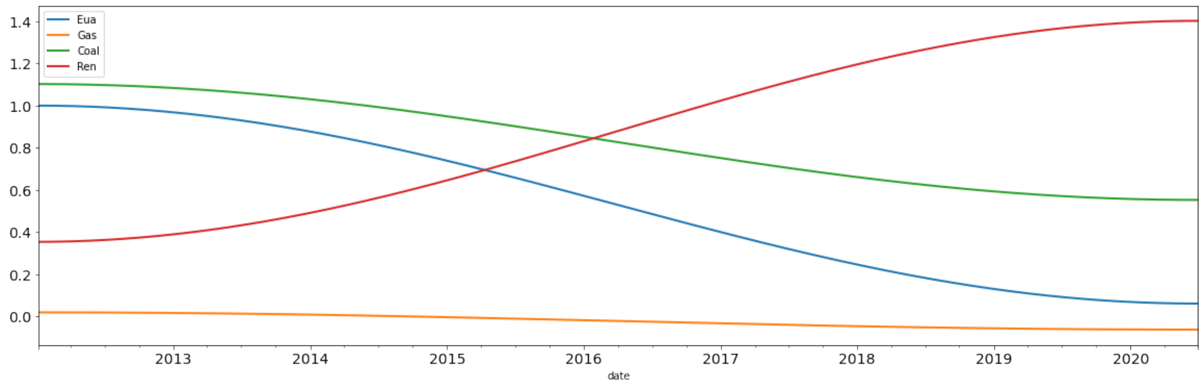


Figure 8: Estimates of $(\beta_t' Y_t)$ for the TV-VECM(2) reduced model - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order $m = 1$

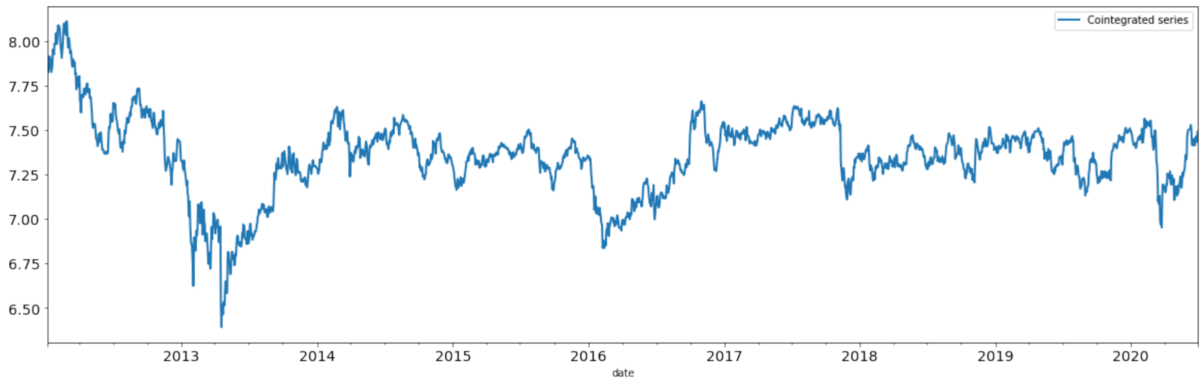


Figure 9: Cointegration relation $(\beta_t' Y_t)$ for the TV-VECM(2) reduced model - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order $m = 1$

Overall, for all three models, I find support for the TV hypothesis where the long-run coefficients smoothly transition over the sample period.

6 Conclusion

This paper analyses the long-term price dynamics of the European Emission Allowances in the European Union Emissions Trading System. The EUAs are influenced by several energy variables and the influence of sustainability and renewable energy market indicators is tested in addition. I propose to model the long term carbon dynamics by a VECM and find a cointegration relation. In addition to existing literature, I test whether this cointegration relation could be time-varying.

There are three main findings. In line with past literature, the VAR models and VECM show that coal and oil prices significantly influence the EUA price drivers, based on a VAR and VECM. But in addition I also find that sustainability indicators are significant, and play an increasingly important role in establishing the EUA price.

I do not find significant cointegration relations for the regular VECM including all variables. However, my extension, the TV-VECM showed that there is a time-varying cointegration relation present between all the variables together.

Lastly, I find a significant cointegration relation between the EUA price, gas price, coal price and variables to indicate the growth in renewable energy based on a regular VECM. Without the indicator of renewable energy growth in the EU, this relationship is not found, supporting the idea of adding various renewable energy sources to the fundamental drivers of EUA prices. Based on a TV-VECM this cointegration relation is rejected, and hence time-varying.

The energy prices changed drastically in the first quarter of 2020 and it is evident from the peaks in the TV cointegration relations that this event did distort the dynamics temporarily. However, the changing dynamics between the EUA prices, energy prices and sustainability indicators evident from the estimated time-varying cointegration relations have not changed abruptly, but are rather part of a slowly changing cointegration relation.

Although it could be that these time-varying cointegration relations are found because the Time-Varying Cointegration likelihood ratio test by Bierens and Martins (2010), rejects the null hypothesis of regular cointegration too often.

6.1 Discussion

For future research, there could be improved in the following three fields: identifying carbon price drivers, estimating/testing TVC and assessing/fine-tuning the discussed models more.

A recent article of the Financial Times argues how important the role of green hydrogen might be in the drivers of carbon price (Lewis). For future research it would be increasingly interesting to differentiate between the different types of renewable energy technologies and measure their effect separately, as has been done in the past with the more traditional energy sources.

To estimate the time varying VECM, a more general form of TV cointegration could be considered. $Y_t = C_t Z_t$, where C_t is a sequence of nonsingular $k \times k$ matrices and $Z_t \in \mathbb{R}^k$ is a cointegrated $I(1)$ process with a VECM (p) representation. This way, all the parameters are functions of t, which could especially be relevant for the error correction term.

To test the TVC better, Martins (2018) considers two alternative bootstrap algorithms to the time-varying cointegration test with the same first-order asymptotic distribution under the null hypothesis of standard cointegration. The bootstrap procedures did not show severe size distortions and Monte Carlo results suggest that the bootstrap approximation to the finite-sample distribution is very accurate, in particular for the wild bootstrap case.

As mentioned by Martins (2018) proposing a test for restrictions on the cointegrating space under the TV model could provide valuable insights. Especially to test whether there are smaller equilibria within the cointegration relation overall. For example, between energy sources that are closely connected due to fuel switching, such as substituting coal for natural gas.

Lastly, I suggest a very elaborate out-of-sample analysis. It would be interesting to develop forecast methods for the TV-VECM approach and test the forecast performance in comparison to regular VECM. Another extension would be to investigate the additive outliers within the model more accurately, in particular Franses and Haldrup (1994) propose to use outlier robust estimation techniques to reduce the effect of aberrant data points.

References

Akaike, H. (1969). Fitting autoregressive models for prediction. *Annals of the institute of Statistical Mathematics 21*(1), 243–247.

Bierens, H. J. and L. F. Martins (2010). Time-varying cointegration. *Econometric Theory*, 1453–1490.

Bredin, D. and C. Muckley (2011). An emerging equilibrium in the eu emissions trading scheme. *Energy Economics 33*(2), 353–362.

Chevallier, J. (2011). *Econometric analysis of carbon markets: the European Union emissions trading scheme and the clean development mechanism.* Springer Science & Business Media.

Creti, A., P.-A. Jouvet, and V. Mignon (2012). Carbon price drivers: Phase i versus phase ii equilibrium? *Energy Economics 34*(1), 327–334.

De Vos, K. (2014). Factsheet on negative electricity market prices.

De Vos, K. (2015). Negative wholesale electricity prices in the german, french and belgian day-ahead, intra-day and real-time markets. *The Electricity Journal 28*(4), 36–50.

Engle, R. F. and C. W. Granger (1987). Co-integration and error correction: representation, estimation, and testing. *Econometrica: journal of the Econometric Society*, 251–276.

European Commission (2015). Eu ets handbook. *Brussels, Belgium. URL: https://ec. europa. eu/clima/sites/clima/files/docs/ets_ handbook_ en. pdf*.

Fell, H., H. Vollebergh, and B. Hintermann (2012). Estimation of carbon cost pass-through in electricity markets. In *Conference paper, CESifo Area Conference on Energy and Climate Economics.*

Franses, P. H. and N. Haldrup (1994). The effects of additive outliers on tests for unit roots and cointegration. *Journal of Business & Economic Statistics 12*(4), 471–478.

Frino, A., J. Kruk, and A. Lepone (2010). Liquidity and transaction costs in the european carbon futures market. *Journal of Derivatives & Hedge Funds 16*(2), 100–115.

Goulder, L. H. and A. R. Schein (2013). Carbon taxes versus cap and trade: a critical review. *Climate Change Economics 4*(03), 1350010.

Hannan, E. J. and B. G. Quinn (1979). The determination of the order of an autoregression. *Journal of the Royal Statistical Society: Series B (Methodological) 41*(2), 190–195.

Hintermann, B., S. Peterson, and W. Rickels (2016). Price and market behavior in phase ii of the eu ets: A review of the literature. *Review of Environmental Economics and Policy 10*(1), 108–128.

Ibikunle, G., A. Gregoriou, A. G. Hoepner, and M. Rhodes (2016). Liquidity and market efficiency in the world's largest carbon market. *The British Accounting Review 48*(4), 431–447.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of economic dynamics and control 12*(2-3), 231–254.

Johansen, S. (1995). Likelihood-based inference in cointegrated vector autoregressive models oxford university press. *New York*.

Koop, G., R. Leon-Gonzalez, and R. W. Strachan (2011). Bayesian inference in a time varying cointegration model. *Journal of Econometrics 165*(2), 210–220.

Lewis, M. (2020). Why the need for green hydrogen points to higher carbon prices.

Lutz, B. J., U. Pigorsch, and W. Rotfuß(2013). Nonlinearity in cap-and-trade systems: The eua price and its fundamentals. *Energy Economics 40*, 222 – 232.

Lüutkepohl, H., P. Saikkonen, and C. Trenkler (2001). Maximum eigenvalue versus trace tests for the cointegrating rank of a var process. *The Econometrics Journal 4*(2), 287–310.

Martins, L. F. (2018). Bootstrap tests for time varying cointegration. *Econometric Reviews 37*(5), 466–483.

Rickels, W., D. Görlich, and S. Peterson (2015). Explaining european emission allowance price dynamics: evidence from phase ii. *German Economic Review 16*(2), 181–202.

Riding, S. (2020). Esg funds attract record inflows during crisis.

Schwarz, G. et al. (1978). Estimating the dimension of a model. *The annals of statistics 6*(2), 461–464.

Tett, G., P. Temple-West, and K. Talman (2020). Carbon removal demand soars, eu warms to natural gas, japan joins net-zero club.

A Appendix

A.1 Tables

Table A1: Detailed information EUA prices, energy and climate variables

Symbol	Name	Source	Mnemonic
Eua	EU CO2 Emissions E/EUA	EEX	EEXEUAS
Oil	Brent Crude Oil Continuous	ICE	LLCCS00
Coal	Coal ARA Month - Continuous	EEX	LFUTECMC
Gas	Egix Gaspool Index European Union/Mw Hour	EEX	EEXGSPD
Elec	Indicator, weighted average of		
	1. Elix Base Index	EEX	EEXIXBS
	2. Phelix Base Index	EEX	EEXBASE
Ren	Europe Total Market Renewable Energy Equipment Index	STOXX	S4TMRQE
Sust	Indicator, weighted average of		
	1. EURO Sustainability Index	STOXX	DJEZSUE
	2. EUROPE ESG LEADERS 50 Index	MSCI	MSEUSG$

This table presents the symbol, name, source and Datastream Mnemonics of all variables used in this research. All symbols represent the natural logarithm of the time-series.

Table A2: VAR(4) model estimation results. Full model with a constant

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.0823	0.0514	-1.6023	0.1091
ΔEua_{-1}	1.0333	0.0219	47.0934	0.0000***
ΔOil_{-1}	-0.0410	0.0315	-1.3011	0.1932
ΔGas_{-1}	-0.0256	0.0290	-0.8824	0.3775
$\Delta Coal_{-1}$	0.2797	0.0563	4.9710	0.0000***
$\Delta Elec_{-1}$	0.0019	0.0061	0.3147	0.7530
ΔRen_{-1}	0.0097	0.0377	0.2583	0.7961
$\Delta Sust_{-1}$	-0.1245	0.0808	-1.5405	0.1234
ΔEua_{-2}	-0.1272	0.0315	-4.0398	0.0001***
ΔOil_{-2}	0.0791	0.0446	1.7732	0.0762*
ΔGas_{-2}	0.0355	0.0409	0.8675	0.3857
$\Delta Coal_{-2}$	-0.3977	0.0852	-4.6706	0.0000***
$\Delta Elec_{-2}$	0.0003	0.0067	0.0380	0.9697
ΔRen_{-2}	0.0248	0.0530	0.4677	0.6400
$\Delta Sust_{-2}$	0.2433	0.1121	2.1703	0.0300**
ΔEua_{-3}	0.0487	0.0313	1.5540	0.1202
ΔOil_{-3}	0.0542	0.0445	1.2163	0.2239
ΔGas_{-3}	-0.0695	0.0409	-1.6985	0.0894*
$\Delta Coal_{-3}$	0.0822	0.0862	0.9537	0.3402
$\Delta Elec_{-3}$	-0.0003	0.0067	-0.0512	0.9592
ΔRen_{-3}	-0.0203	0.0529	-0.3838	0.7011
$\Delta Sust_{-3}$	-0.0716	0.1123	-0.6375	0.5238
ΔEua_{-4}	0.0430	0.0218	1.9681	0.0491**
ΔOil_{-4}	-0.0892	0.0314	-2.8364	0.0046***
ΔGas_{-4}	0.0517	0.0288	1.7989	0.0720*
$\Delta Coal_{-4}$	0.0453	0.0582	0.7780	0.4366
$\Delta Elec_{-4}$	0.0028	0.0061	0.4677	0.6400
ΔRen_{-4}	-0.0128	0.0376	-0.3413	0.7329
$\Delta Sust_{-4}$	-0.0402	0.0808	-0.4976	0.6187
Log likelihood:	35811.5	BIC:	-51.5223	
AIC:	-52.0456	HQIC:	-51.8545	

This table shows the VAR(4) estimation results, estimated with a constant added to the model based on the sample 1 January 2012 - 30 June 2020. Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The bottom panel shows the Log Likelihood, AIC: Akaike information criterion, BIC: Bayesian information criterion and HQIC: Hannan-Quinn information criterion. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A3: VAR(4) model estimation results.. Full model with a constant and trend

	Coefficient	Std. Error	t-Statistic	Prob.
Contant	-0.0711	0.0615	-1.1560	0.2477
Trend	0.0000	0.0000	0.3312	0.7405
ΔEua_{-1}	1.0332	0.0219	47.0736	0.0000***
ΔOil_{-1}	-0.0406	0.0315	-1.2892	0.1973
ΔGas_{-1}	-0.0250	0.0291	-0.8592	0.3903
$\Delta Coal_{-1}$	0.2790	0.0563	4.9536	0.0000***
$\Delta Elec{-1}$	0.0019	0.0061	0.3112	0.7556
ΔRen_{-1}	0.0088	0.0378	0.2337	0.8153
$\Delta Sust_{-1}$	-0.1248	0.0809	-1.5438	0.1226
ΔEua_{-2}	-0.1272	0.0315	-4.0411	0.0001***
ΔOil_{-2}	0.0792	0.0446	1.7744	0.0760**
ΔGas_{-2}	0.0355	0.0409	0.8687	0.3850
$\Delta Coal_{-2}$	-0.3979	0.0852	-4.6713	0.0000***
$\Delta Elec_{-2}$	0.0002	0.0067	0.0356	0.9716
ΔRen_{-2}	0.0248	0.0530	0.4679	0.6399
$\Delta Sust_{-2}$	0.2434	0.1121	2.1705	0.0300**
ΔEua_{-3}	0.0486	0.0313	1.5524	0.1206
ΔOil_{-3}	0.0542	0.0446	1.2156	0.2241
ΔGas_{-3}	-0.0693	0.0409	-1.6947	0.0901*
$\Delta Coal_{-3}$	0.0821	0.0862	0.9519	0.3411
$\Delta Elec_{-3}$	-0.0004	0.0067	-0.0552	0.9560
ΔRen_{-3}	-0.0202	0.0529	-0.3812	0.7030
$\Delta Sust_{-3}$	-0.0718	0.1124	-0.6390	0.5228
ΔEua_{-4}	0.0425	0.0219	1.9404	0.0523*
ΔOil_{-4}	-0.0887	0.0315	-2.8194	0.0048**
ΔGas_{-4}	0.0517	0.0288	1.7988	0.0720*
$\Delta Coal_{-4}$	0.0451	0.0582	0.7747	0.4385
$\Delta Elec_{-4}$	0.0028	0.0061	0.4606	0.6451
ΔRen_{-4}	-0.0122	0.0376	-0.3248	0.7453
$\Delta Sust_{-4}$	-0.0423	0.0810	-0.5221	0.6016
Log likelihood:	35826.7	BIC:	-51.5117	
AIC:	-52.0530	HQIC:	-51.8552	

This table shows the VAR(4) estimation results, estimated with a constant and trend added to the model based on the sample 1 January 2012 - 30 June 2020. Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The bottom panel shows the Log Likelihood, AIC: Akaike information criterion, BIC: Bayesian information criterion and HQIC: Hannan-Quinn information criterion. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A4: VAR(4) model estimation results. Model with a constant, without *Elec*

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.0619	0.0413	-1.4986	0.1340
ΔEua_{-1}	1.0334	0.0219	47.1404	0.0000***
ΔOil_{-1}	-0.0410	0.0315	-1.3041	0.1922
ΔGas_{-1}	-0.0247	0.0289	-0.8524	0.3940
$\Delta Coal_{-1}$	0.2805	0.0562	4.9956	0.0000***
ΔRen_{-1}	0.0108	0.0376	0.2883	0.7731
$\Delta Sust_{-1}$	-0.1258	0.0807	-1.5577	0.1193
ΔEua_{-2}	-0.1270	0.0314	-4.0386	0.0001***
ΔOil_{-2}	0.0792	0.0446	1.7753	0.0758*
ΔGas_{-2}	0.0352	0.0408	0.8631	0.3881
$\Delta Coal_{-2}$	-0.3985	0.0849	-4.6942	0.0000***
ΔRen_{-2}	0.0236	0.0529	0.4472	0.6547
$\Delta Sust_{-2}$	0.2438	0.1120	2.1776	0.0294**
ΔEua_{-3}	0.0485	0.0313	1.5487	0.1215
ΔOil_{-3}	0.0545	0.0445	1.2245	0.2208
ΔGas_{-3}	-0.0692	0.0409	-1.6937	0.0903*
$\Delta Coal_{-3}$	0.0826	0.0859	0.9610	0.3365
ΔRen_{-3}	-0.0206	0.0528	-0.3909	0.6959
$\Delta Sust_{-3}$	-0.0708	0.1122	-0.6310	0.5280
ΔEua_{-4}	0.0434	0.0218	1.9874	0.0469**
ΔOil_{-4}	-0.0895	0.0314	-2.8514	0.0044***
ΔGas_{-4}	0.0515	0.0287	1.7939	0.0728*
$\Delta Coal_{-4}$	0.0456	0.0581	0.7842	0.4329
ΔRen_{-4}	-0.0122	0.0375	-0.3260	0.7444
$\Delta Sust_{-4}$	-0.0415	0.0806	-0.5145	0.6069
Log likelihood:	34133.6	BIC:	-47.3530	
AIC:	-47.7397	HQIC:	-47.5984	

This table shows the VAR(4) estimation results, estimated with a constant added to the model based on the sample 1 January 2012 - 30 June 2020 and *Elec* not included. Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The bottom panel shows the Log Likelihood, AIC: Akaike information criterion, BIC: Bayesian information criterion and HQIC: Hannan-Quinn information criterion. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A5: VAR(4) model estimation results. Model with a constant and trend, without *Elec*

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.0502	0.0525	-0.9560	0.3391
Trend	0.0000	0.0000	0.3617	0.7176
ΔEua_{-1}	1.0332	0.0219	47.1201	0.0000***
ΔOil_{-1}	-0.0407	0.0315	-1.2911	0.1967
ΔGas_{-1}	-0.0240	0.0290	-0.8280	0.4077
$\Delta Coal_{-1}$	0.2797	0.0562	4.9766	0.0000***
ΔRen_{-1}	0.0098	0.0377	0.2609	0.7941
$\Delta Sust_{-1}$	-0.1261	0.0808	-1.5612	0.1185
ΔEua_{-2}	-0.1271	0.0315	-4.0402	0.0001***
ΔOil_{-2}	0.0792	0.0446	1.7765	0.0756*
ΔGas_{-2}	0.0353	0.0408	0.8645	0.3873
$\Delta Coal_{-2}$	-0.3987	0.0849	-4.6949	0.0000***
ΔRen_{-2}	0.0237	0.0529	0.4477	0.6544
$\Delta Sust_{-2}$	0.2439	0.1120	2.1779	0.0294**
ΔEua_{-3}	0.0484	0.0313	1.5472	0.1218
ΔOil_{-3}	0.0545	0.0445	1.2236	0.2211
ΔGas_{-3}	-0.0691	0.0409	-1.6898	0.0911**
$\Delta Coal_{-3}$	0.0824	0.0859	0.9591	0.3375
ΔRen_{-3}	-0.0205	0.0528	-0.3881	0.6979
$\Delta Sust_{-3}$	-0.0710	0.1122	-0.6328	0.5269
ΔEua_{-4}	0.0428	0.0219	1.9571	0.0503*
ΔOil_{-4}	-0.0891	0.0314	-2.8327	0.0046***
ΔGas_{-4}	0.0515	0.0287	1.7939	0.0728*
$\Delta Coal_{-4}$	0.0454	0.0581	0.7803	0.4352
ΔRen_{-4}	-0.0116	0.0376	-0.3082	0.7579
$\Delta Sust_{-4}$	-0.0438	0.0809	-0.5410	0.5885
Log likelihood:	34148.8	BIC:	-47.3458	
AIC:	-47.7479	HQIC:	-47.6010	

This table shows the VAR(4) estimation results, estimated with a constant and trend added to the model based on the sample 1 January 2012 - 30 June 2020 and *Elec* not included. Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The bottom panel shows the Log Likelihood, AIC: Akaike information criterion, BIC: Bayesian information criterion and HQIC: Hannan-Quinn information criterion. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A6: VECM(3) estimation results: Johansen Case II

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ΔEua_{-1}	0.0388	0.022	1.781	0.075*
ΔOil_{-1}	-0.0410	0.031	-1.31	0.190
ΔGas_{-1}	-0.0208	0.029	-0.724	0.469
$\Delta Coal_{-1}$	0.2851	0.056	5.108	0.000***
$\Delta Elec_{-1}$	-0.0027	0.007	-0.373	0.709
ΔRen_{-1}	0.0062	0.037	0.165	0.869
$\Delta Sust_{-1}$	-0.1250	0.080	-1.56	0.119
ΔEua_{-2}	-0.0891	0.022	-4.112	0.000***
ΔOil_{-2}	0.0387	0.031	1.234	0.217
ΔGas_{-2}	0.0157	0.029	0.55	0.583
$\Delta Coal_{-2}$	-0.1162	0.058	-2.011	0.044**
$\Delta Elec_{-2}$	-0.0025	0.007	-0.366	0.714
ΔRen_{-2}	0.0309	0.037	0.827	0.408
$\Delta Sust_{-2}$	0.1188	0.080	1.48	0.139
ΔEua_{-3}	-0.0400	0.022	-1.842	0.065*
ΔOil_{-3}	0.0930	0.031	2.979	0.003***
ΔGas_{-3}	-0.0524	0.029	-1.835	0.066*
$\Delta Coal_{-3}$	-0.0333	0.058	-0.576	0.565
$\Delta Elec_{-3}$	-0.0028	0.006	-0.457	0.648
ΔRen_{-3}	0.0110	0.037	0.296	0.767
$\Delta Sust_{-3}$	0.0466	0.080	0.581	0.562
Error Correction Term	-0.0004	0.001	-0.699	0.484
Cointegration relations				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-0.2491	0.430	-0.580	0.562
β_{Gas}	2.2738	0.455	5.000	0.000***
β_{Coal}	1.9286	0.556	3.470	0.001***
β_{Elec}	-13.2832	0.802	-16.558	0.000***
β_{Ren}	0.7584	0.371	2.042	0.041**
β_{Sust}	-4.0979	1.356	-3.022	0.003***
Deterministic constant	60.9671	5.861	10.403	0.000***
Log likelihood	35766.6945			

This table shows the VECM(3) estimation results of Johansen (1995) Case II, estimated based on the sample 1 January 2012 - 30 June 2020.Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on Eua. It also shows the loading coefficients for the equation of Eua, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9 The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A7: VECM(3) estimation results: Johansen Case III

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Deterministic constant	-0.0221	0.032	-0.696	0.486
ΔEua_{-1}	0.0383	0.022	1.758	0.079*
ΔOil_{-1}	-0.0402	0.031	-1.285	0.199
ΔGas_{-1}	-0.0201	0.029	-0.699	0.484
$\Delta Coal_{-1}$	0.2865	0.056	5.133	0.000***
$\Delta Elec_{-1}$	-0.0029	0.007	-0.387	0.699
ΔRen_{-1}	0.0053	0.037	0.142	0.887
$\Delta Sust_{-1}$	-0.1257	0.080	-1.569	0.117
ΔEua_{-2}	-0.0895	0.022	-4.132	0.000***
ΔOil_{-2}	0.0394	0.031	1.256	0.209
ΔGas_{-2}	0.0163	0.029	0.572	0.567
$\Delta Coal_{-2}$	-0.1148	0.058	-1.986	0.047**
$\Delta Elec_{-2}$	-0.0025	0.007	-0.378	0.705
ΔRen_{-2}	0.0300	0.037	0.804	0.421
$\Delta Sust_{-2}$	0.1179	0.080	1.469	0.142
ΔEua_{-3}	-0.0405	0.022	-1.866	0.062*
ΔOil_{-3}	0.0939	0.031	3.006	0.003***
ΔGas_{-3}	-0.0516	0.029	-1.808	0.071*
$\Delta Coal_{-3}$	-0.0320	0.058	-0.554	0.579
$\Delta Elec_{-3}$	-0.0028	0.006	-0.464	0.642
ΔRen_{-3}	0.0102	0.037	0.273	0.785
$\Delta Sust_{-3}$	0.0457	0.080	0.570	0.568
Error Correction Term	-0.0004	0.001	-0.719	0.472
Cointegration relations				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-0.2495	0.430	-0.580	0.562
β_{Gas}	2.2717	0.455	4.991	0.000***
β_{Coal}	1.9341	0.556	3.477	0.001***
β_{Elec}	-13.2951	0.803	-16.558	0.000***
β_{Ren}	0.7581	0.372	2.040	0.041**
β_{Sust}	-4.0985	1.357	-3.020	0.003***
Log likelihood	35770.4461			

This table shows the VECM(3) estimation results of Johansen (1995) Case III, estimated based on the sample 1 January 2012 - 30 June 2020
Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on Eua in the upper panel. It also shows the loading coefficients for the equation of Eua, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9 The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A8: VECM(3) estimation results: Johansen Case IV

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Deterministic constant	-0.0227	0.033	-0.692	0.489
ΔEua_{-1}	0.0383	0.022	1.756	0.079*
ΔOil_{-1}	-0.0402	0.031	-1.284	0.199
ΔGas_{-1}	-0.0201	0.029	-0.699	0.485
$\Delta Coal_{-1}$	0.2865	0.056	5.132	0.000***
$\Delta Elec_{-1}$	-0.0028	0.007	-0.384	0.701
ΔRen_{-1}	0.0053	0.037	0.143	0.886
$\Delta Sust_{-1}$	-0.1259	0.080	-1.571	0.116
ΔEua_{-2}	-0.0896	0.022	-4.134	0.000***
ΔOil_{-2}	0.0394	0.031	1.257	0.209
ΔGas_{-2}	0.0163	0.029	0.573	0.567
$\Delta Coal_{-2}$	-0.1148	0.058	-1.987	0.047**
$\Delta Elec_{-2}$	-0.0025	0.007	-0.376	0.707
ΔRen_{-2}	0.0301	0.037	0.806	0.420
$\Delta Sust_{-2}$	0.1177	0.080	1.467	0.142
ΔEua_{-3}	-0.0406	0.022	-1.868	0.062*
ΔOil_{-3}	0.0939	0.031	3.007	0.003***
ΔGas_{-3}	-0.0516	0.029	-1.808	0.071*
$\Delta Coal_{-3}$	-0.0320	0.058	-0.555	0.579
$\Delta Elec_{-3}$	-0.0028	0.006	-0.463	0.643
ΔRen_{-3}	0.0102	0.037	0.274	0.784
$\Delta Sust_{-3}$	0.0456	0.080	0.569	0.570
Error Correction Term	-0.0003	0.000	-0.714	0.475
Cointegration relations				
β_{Eua}	1.000	0.000	0.000	0.000***
β_{Oil}	-0.1111	0.515	-0.216	0.829
β_{Gas}	2.8004	0.643	4.353	0.000***
β_{Coal}	1.9525	0.694	2.814	0.005***
β_{Elec}	-15.3357	0.97	-15.81	0.000***
β_{Ren}	0.8407	0.428	1.963	0.050**
β_{Sust}	-5.2569	1.847	-2.846	0.004***
Deterministic trend	0.0003	0.000	0.887	0.375
Log likelihood	35770.552			

This table shows the VECM(3) estimation results of Johansen (1995) Case IV, estimated based on the sample 1 January 2012 - 30 June 2020
Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on Eua in the upper panel. It also shows the loading coefficients for the equation of Eua, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9 The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A9: VECM(3) estimation results: Johansen Case II, model excluding *Elec*

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ΔEua_{-1}	0.0379	0.022	1.742	0.082*
ΔOil_{-1}	-0.0434	0.031	-1.386	0.166
ΔGas_{-1}	-0.0185	0.029	-0.643	0.520
$\Delta Coal_{-1}$	0.2734	0.056	4.881	0.000***
ΔRen_{-1}	0.0121	0.037	0.323	0.747
$\Delta Sust_{-1}$	-0.1289	0.080	-1.61	0.107
ΔEua_{-2}	-0.0895	0.022	-4.133	0.000***
ΔOil_{-2}	0.0363	0.031	1.16	0.246
ΔGas_{-2}	0.0172	0.028	0.605	0.545
$\Delta Coal_{-2}$	-0.1261	0.058	-2.181	0.029**
ΔRen_{-2}	0.0357	0.037	0.956	0.339
$\Delta Sust_{-2}$	0.1149	0.080	1.432	0.152
ΔEua_{-3}	-0.0407	0.022	-1.878	0.060*
ΔOil_{-3}	0.0907	0.031	2.906	0.004***
ΔGas_{-3}	-0.0517	0.028	-1.814	0.070*
$\Delta Coal_{-3}$	-0.0429	0.058	-0.742	0.458
ΔRen_{-3}	0.0152	0.037	0.408	0.683
$\Delta Sust_{-3}$	0.0439	0.080	0.549	0.583
Error Correction Term	-0.0009	0.000	-2.041	0.041**
Cointegration relations				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-0.7692	1.553	-0.495	0.620
β_{Gas}	6.3835	1.619	3.942	0.000***
β_{Coal}	-8.5331	1.957	-4.359	0.000***
β_{Ren}	-0.889	1.339	-0.664	0.507
β_{Sust}	5.1093	4.894	1.044	0.297
Deterministic constant	-0.8048	17.835	-0.045	0.964
Log likelihood	34103.19034			

This table shows the VECM(3) estimation results of Johansen (1995) Case II, estimated based on the sample 1 January 2012 - 30 June 2020
Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on *Eua* in the upper panel. It also shows the loading coefficients for the equation of *Eua*, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9 The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A10: VECM(3) estimation results: Johansen Case III, model excluding *Elec*

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Deterministic constant	0.0159	0.007	2.200	0.028**
ΔEua_{-1}	0.0370	0.022	1.698	0.089*
ΔOil_{-1}	-0.0419	0.031	-1.340	0.180
ΔGas_{-1}	-0.0175	0.029	-0.609	0.542
$\Delta Coal_{-1}$	0.2760	0.056	4.935	0.000***
ΔRen_{-1}	0.0113	0.037	0.302	0.763
$\Delta Sust_{-1}$	-0.1286	0.080	-1.607	0.108
ΔEua_{-2}	-0.0903	0.022	-4.170	0.000***
ΔOil_{-2}	0.0378	0.031	1.207	0.227
ΔGas_{-2}	0.0181	0.028	0.635	0.525
$\Delta Coal_{-2}$	-0.1238	0.058	-2.142	0.032**
ΔRen_{-2}	0.0350	0.037	0.938	0.348
$\Delta Sust_{-2}$	0.1147	0.080	1.430	0.153
ΔEua_{-3}	-0.0417	0.022	-1.923	0.054*
ΔOil_{-3}	0.0924	0.031	2.962	0.003***
ΔGas_{-3}	-0.0506	0.029	-1.776	0.076*
$\Delta Coal_{-3}$	-0.0407	0.058	-0.705	0.481
ΔRen_{-3}	0.0144	0.037	0.387	0.698
$\Delta Sust_{-3}$	0.0440	0.080	0.550	0.582
Error Correction Term	-0.0005	0	-2.111	0.035**
Cointegration relations				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-1.1977	2.827	-0.424	0.672
β_{Gas}	10.0592	2.947	3.414	0.001***
β_{Coal}	-15.4448	3.562	-4.336	0.000***
β_{Ren}	-3.1165	2.437	-1.279	0.201
β_{Sust}	17.7418	8.907	1.992	0.046***
Log likelihood	34106.2137			

This table shows the VECM(3) estimation results of Johansen (1995) Case III, estimated based on the sample 1 January 2012 - 30 June 2020
Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on *Eua* in the upper panel. It also shows the loading coefficients for the equation of *Eua*, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9 The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

Table A11: VECM(3) estimation results: Johansen Case IV, model excluding *Elec*

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Deterministic constant	-0.0367	0.0300	-1.239	0.215
ΔEua_{-1}	0.0355	0.0220	1.6240	0.104
ΔOil_{-1}	-0.0392	0.0310	-1.254	0.210
ΔGas_{-1}	-0.0190	0.0290	-0.659	0.510
$\Delta Coal_{-1}$	0.2819	0.0560	5.045	0.000***
ΔRen_{-1}	0.0105	0.0370	0.279	0.780
$\Delta Sust_{-1}$	-0.1342	0.0800	-1.670	0.095*
ΔEua_{-2}	-0.0918	0.0220	-4.225	0.000***
ΔOil_{-2}	0.0402	0.0310	1.283	0.200
ΔGas_{-2}	0.0169	0.0280	0.594	0.553
$\Delta Coal_{-2}$	-0.1190	0.0580	-2.060	0.039**
ΔRen_{-2}	0.0340	0.0370	0.908	0.364
$\Delta Sust_{-2}$	0.1093	0.0810	1.357	0.175
ΔEua_{-3}	-0.0429	0.0220	-1.974	0.048**
ΔOil_{-3}	0.0950	0.0310	3.045	0.002***
ΔGas_{-3}	-0.0515	0.0290	-1.806	0.071*
$\Delta Coal_{-3}$	-0.0355	0.0580	-0.614	0.539
ΔRen_{-3}	0.0132	0.0370	0.354	0.723
$\Delta Sust_{-3}$	0.0390	0.0800	0.485	0.628
Error Correction Term	0.0021	0.0020	1.263	0.206
Cointegration relations				
β_{Eua}	1.0000	0.000	0.000	0.000***
β_{Oil}	-1.1242	0.325	-3.455	0.001***
β_{Gas}	-1.9382	0.388	-4.998	0.000***
β_{Coal}	3.284	0.438	7.49	0.000***
β_{Ren}	0.2851	0.271	1.052	0.293
β_{Sust}	3.0574	1.145	2.669	0.008***
Deterministic trend	-0.0022	0.000	-9.626	0.000***
Log likelihood	34114.93722			

This table shows the VECM(3) estimation results of Johansen (1995) Case IV, estimated based on the sample 1 January 2012 - 30 June 2020
Δx_p indicates the first-differenced variable x at lag p. The columns show the estimated coefficient, corresponding standard error, t-statistic and p-value. The table shows the parameter estimates for the regression of the lagged parameters on *Eua* in the upper panel. It also shows the loading coefficients for the equation of *Eua*, i.e. the error correction term. The bottom panel of the table shows the cointegration relations of the loading-coefficients. β_x correspond to the estimates specified by equation 9. The last row shows the log likelihood of the model. The abbreviations of the variables are elaborated upon in Table A1 in the Appendix. ***, ** or *, asterisk indicates significance of a variable at the 1%, 5% and 10% level respectively.

A.2 Figures

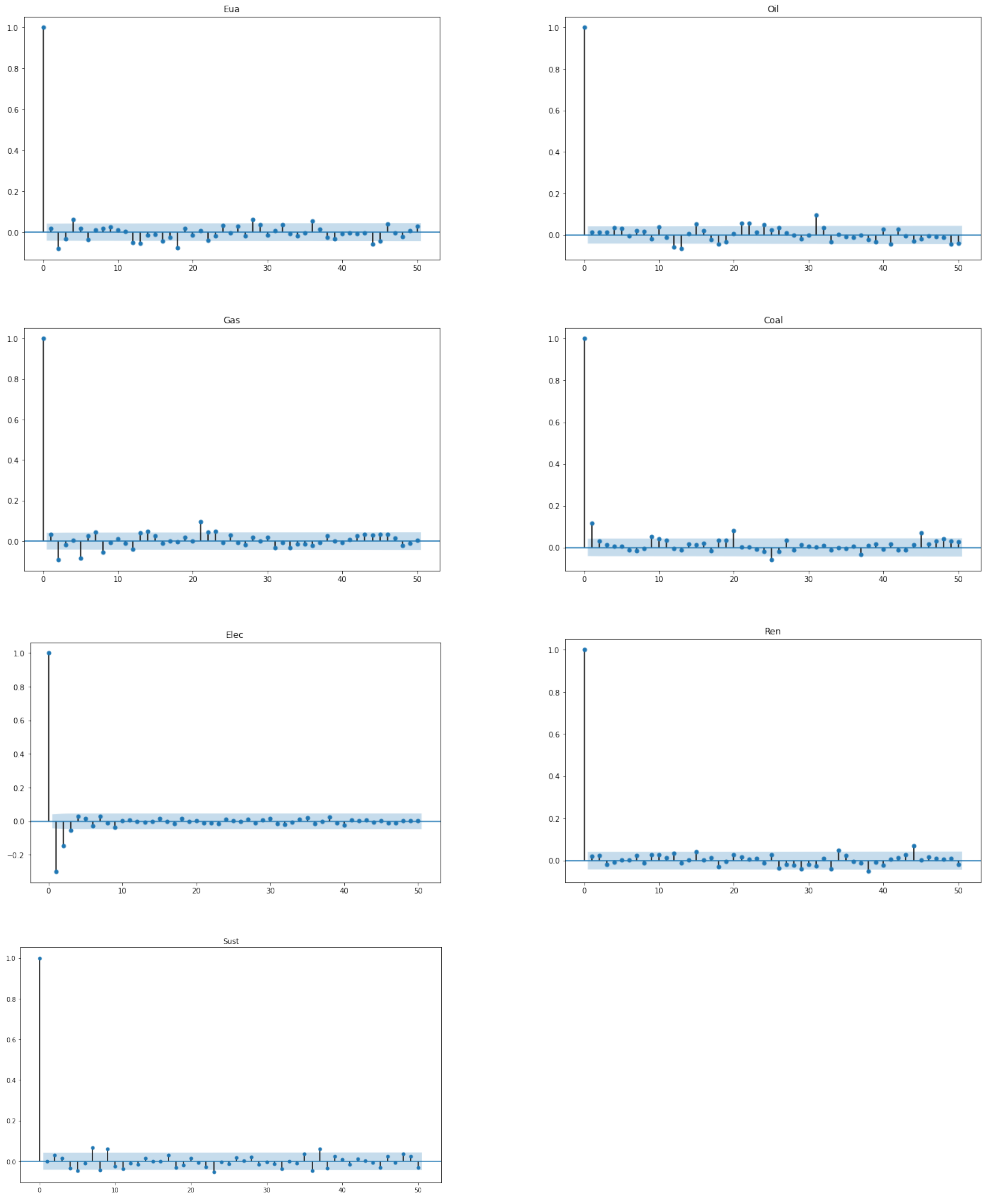


Figure A1: Empirical auto-correlation function of the daily first-differenced variables. The abbreviations of all variables are elaborated upon in Table A1 in the Appendix. (31/12/2007 - 30/06/2020).

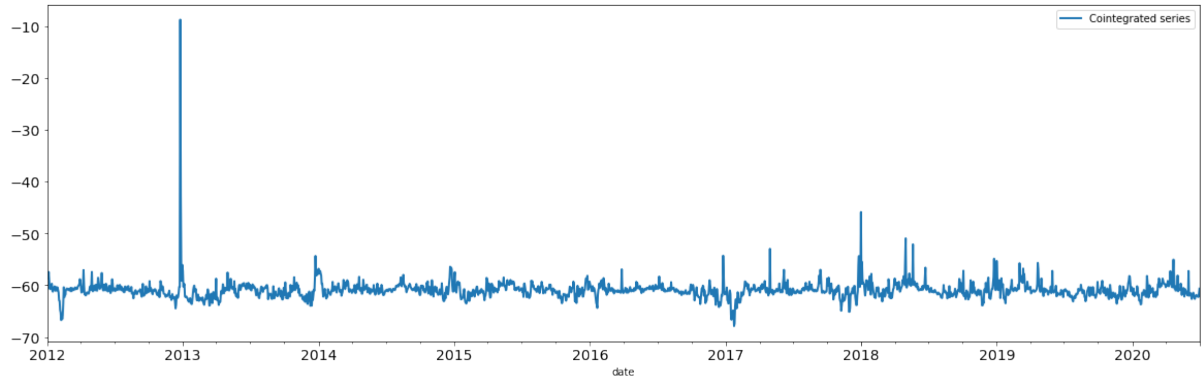


Figure A2: Cointegration relation ($\beta_t' Y_t$) for the VECM(3) Johansen Case II specified in Table A6 - 1 January 2012 - 30 June 2020.

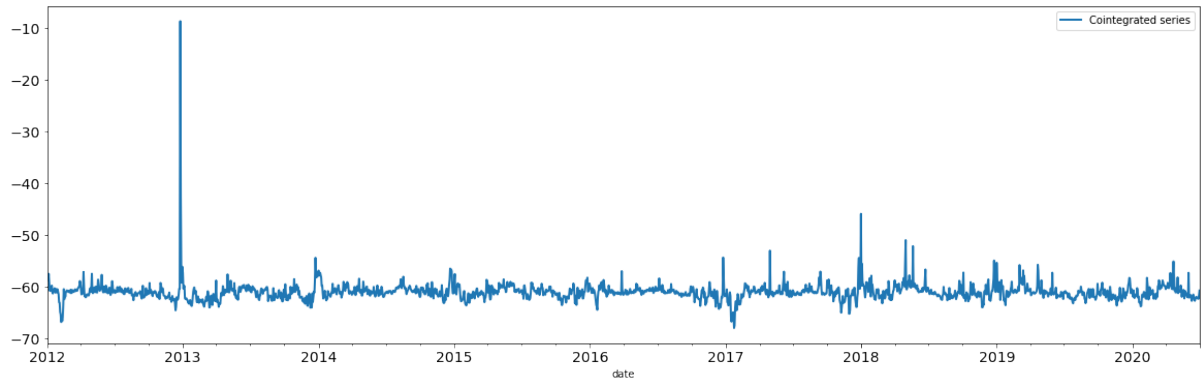


Figure A3: Cointegration relation ($\beta' Y_t$) for the VECM(3) Johansen Case III specified in Table A7 - 1 January 2012 - 30 June 2020.

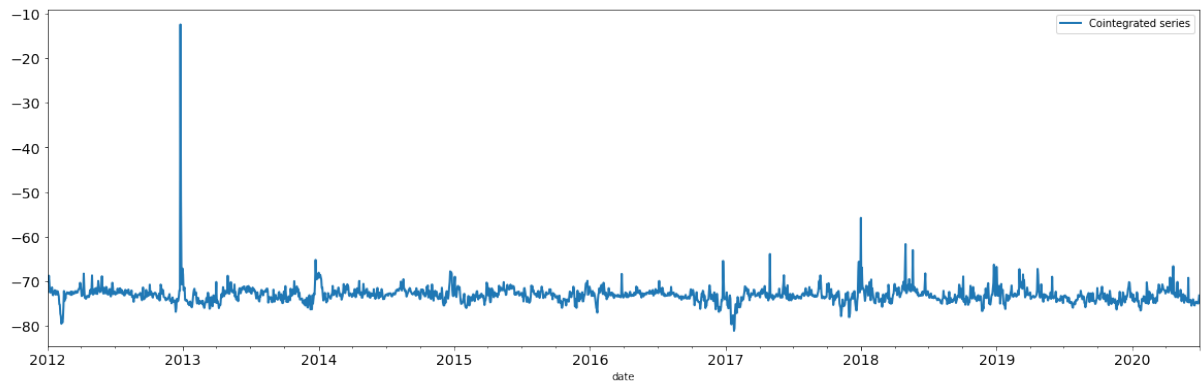


Figure A4: Cointegration relation ($\beta_t' Y_t$) for the VECM(3) Johansen Case IV specified in Table A8 - 1 January 2012 - 30 June 2020.

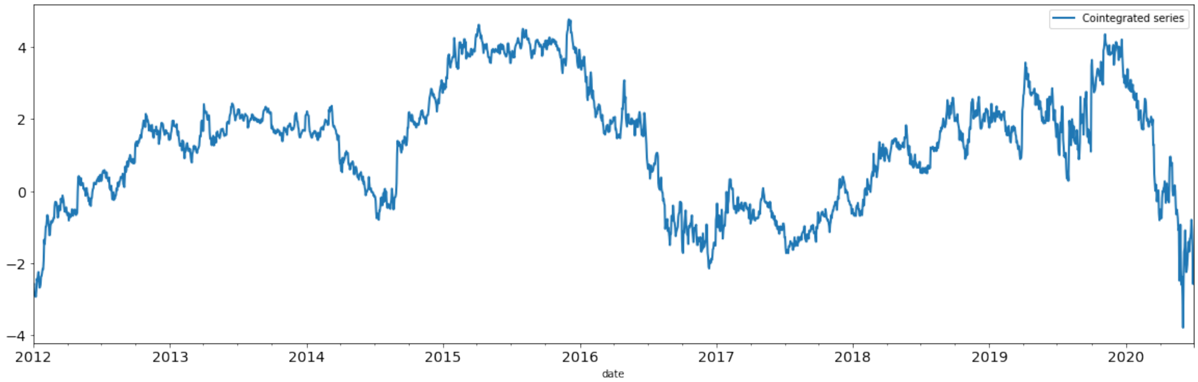


Figure A5: Cointegration relation ($\beta_t' Y_t$) for the VECM(3) Johansen Case II specified in Table A9 - 1 January 2012 - 30 June 2020.

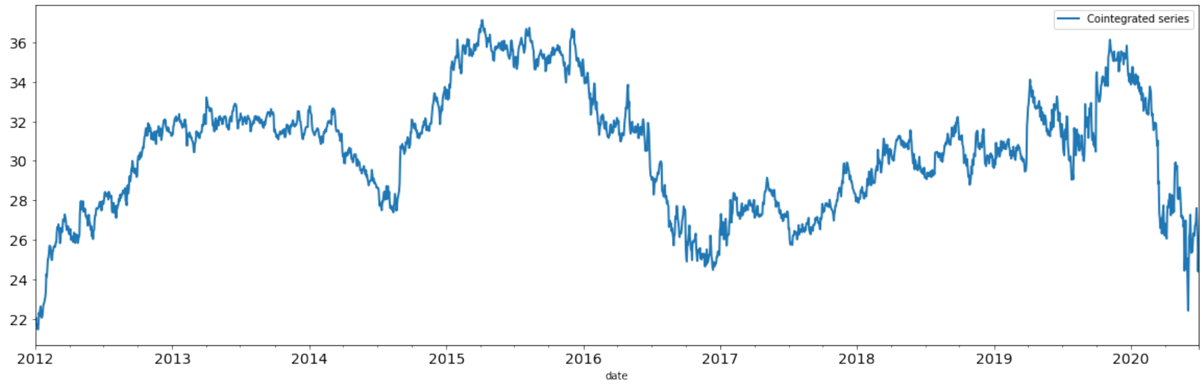


Figure A6: Cointegration relation ($\beta_t' Y_t$) for the VECM(3) Johansen Case III specified in Table A9 - 1 January 2012 - 30 June 2020.

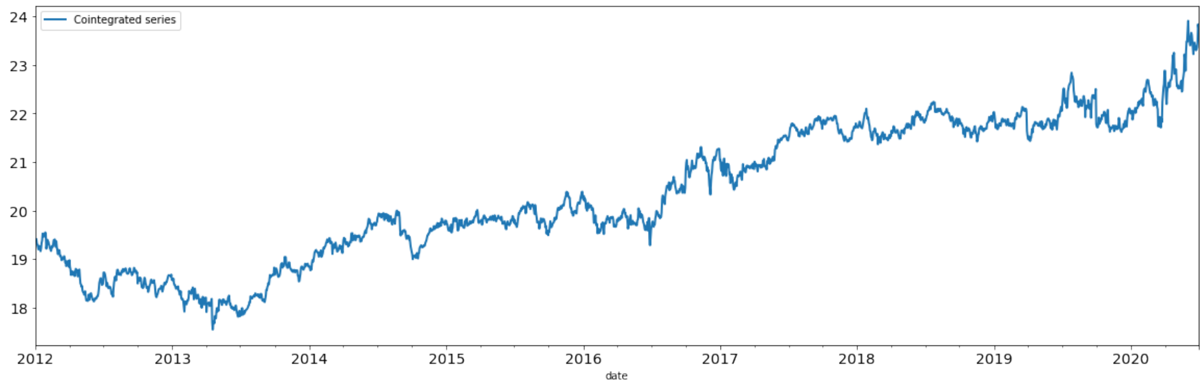
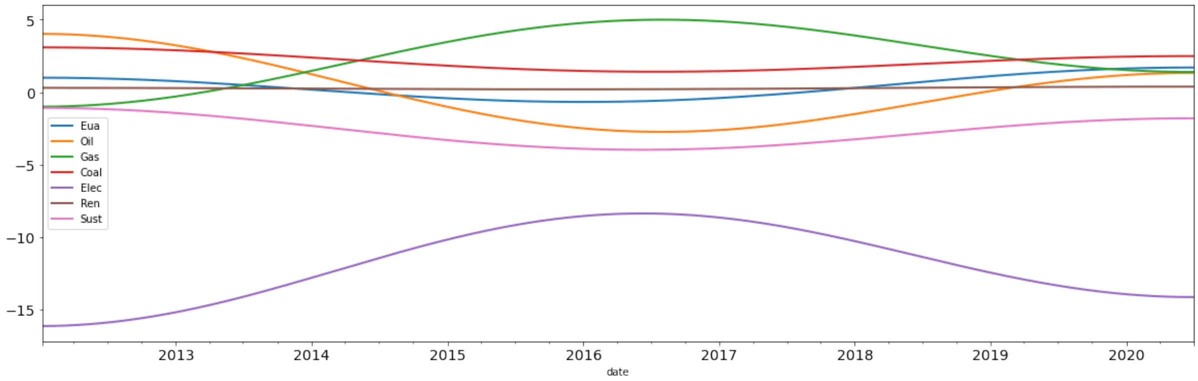
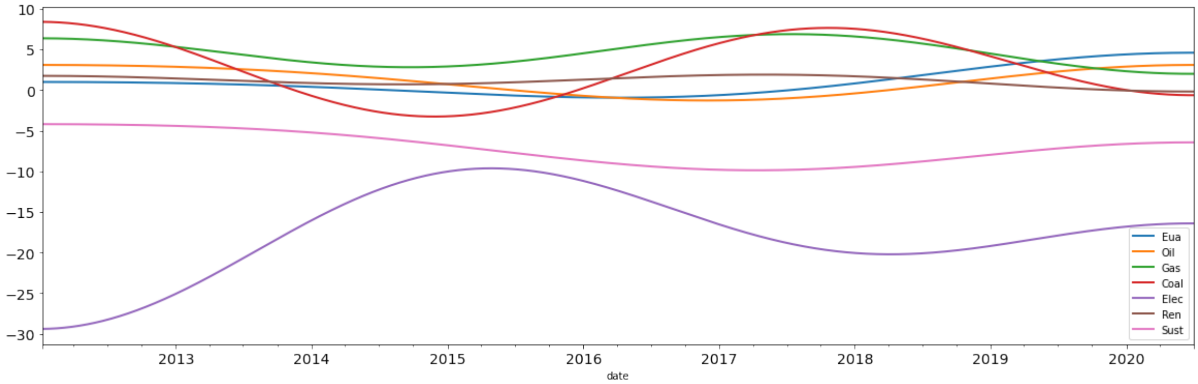


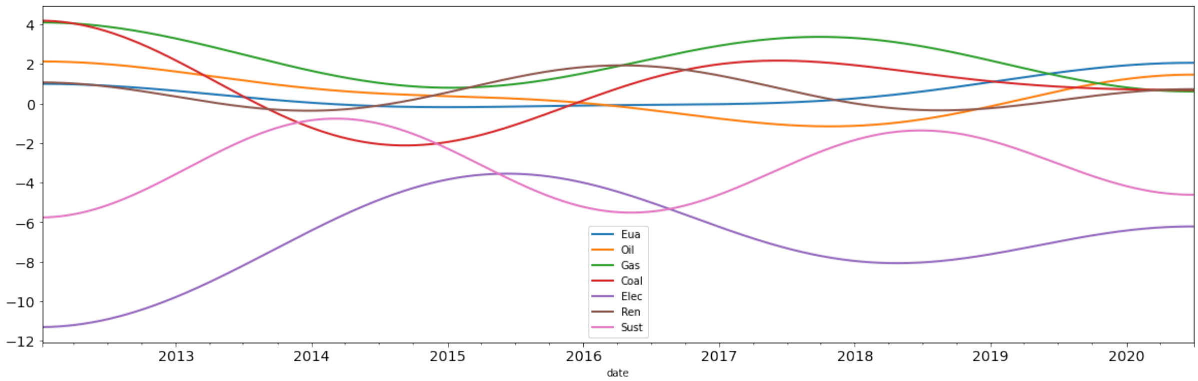
Figure A7: Cointegration relation ($\beta' Y_t$) for the VECM(3) Johansen Case IV specified in Table A9 - 1 January 2012 - 30 June 2020.



(a) Chebyshev polynomial order $m = 2$

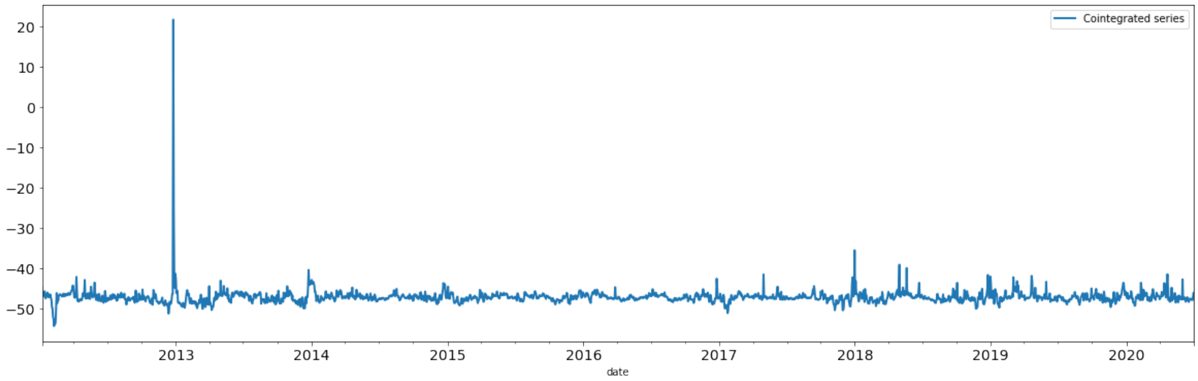


(b) Chebyshev polynomial order $m = 3$

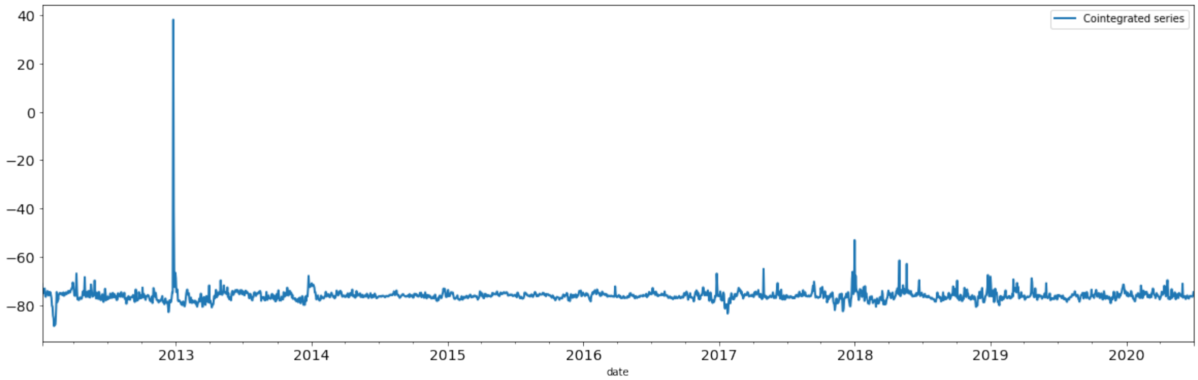


(c) Chebyshev polynomial order $m = 4$

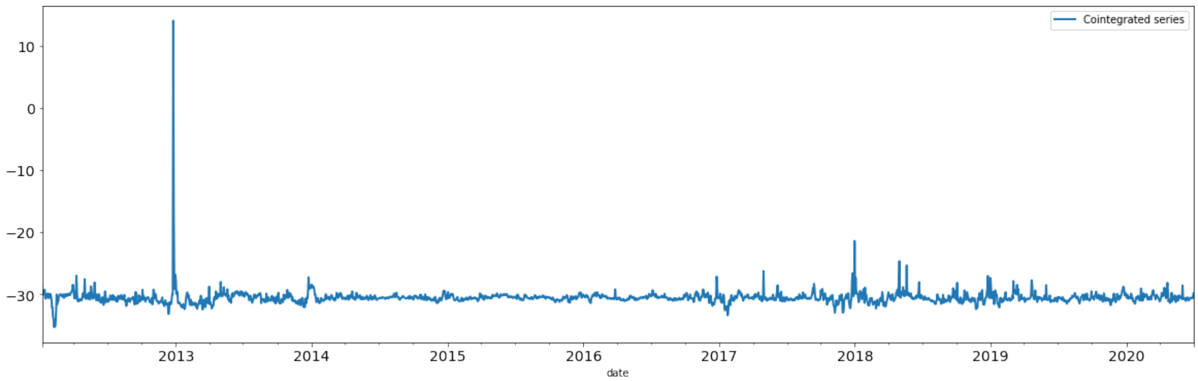
Figure A8: Estimates of β_t' for the TV-VECM(3) including *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order m.



(a) Chebyshev polynomial order $m = 2$

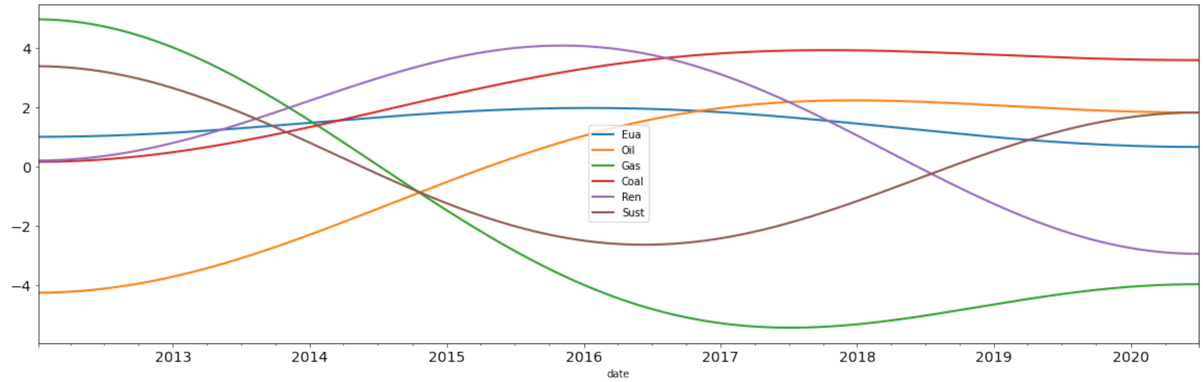


(b) Chebyshev polynomial order $m = 3$

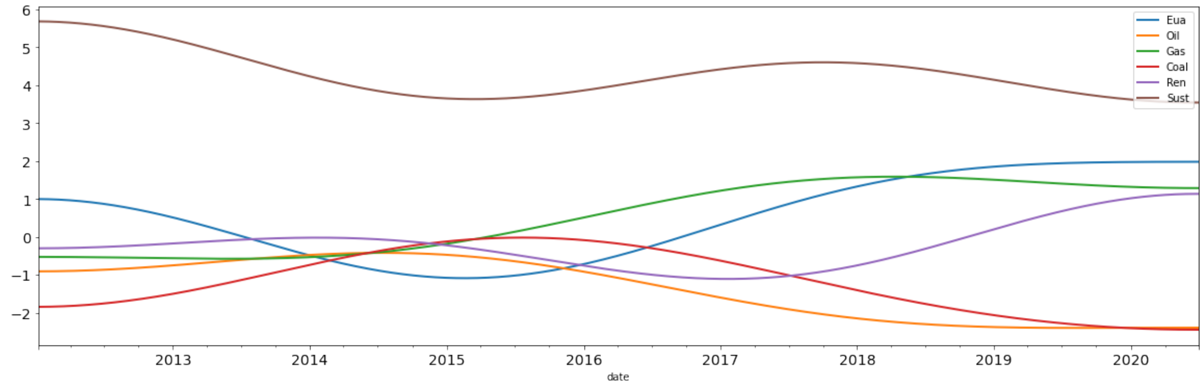


(c) Chebyshev polynomial order $m = 4$

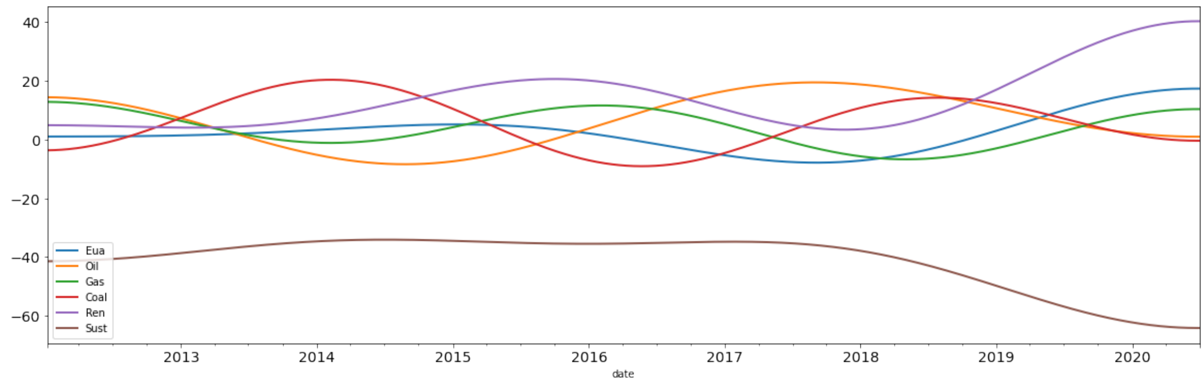
Figure A9: Estimates of $(\beta_t' Y_t)$ for the TV-VECM(3) including *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order m.



(a) Chebyshev polynomial order $m = 2$

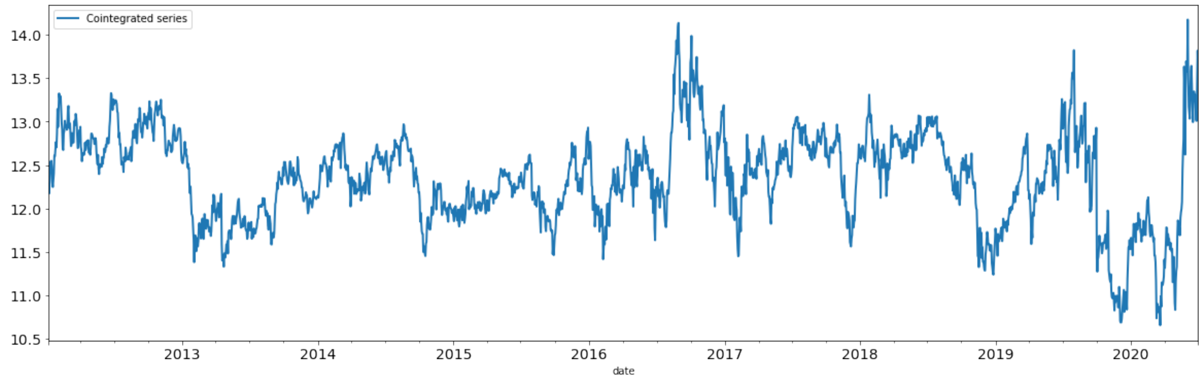


(b) Chebyshev polynomial order $m = 3$

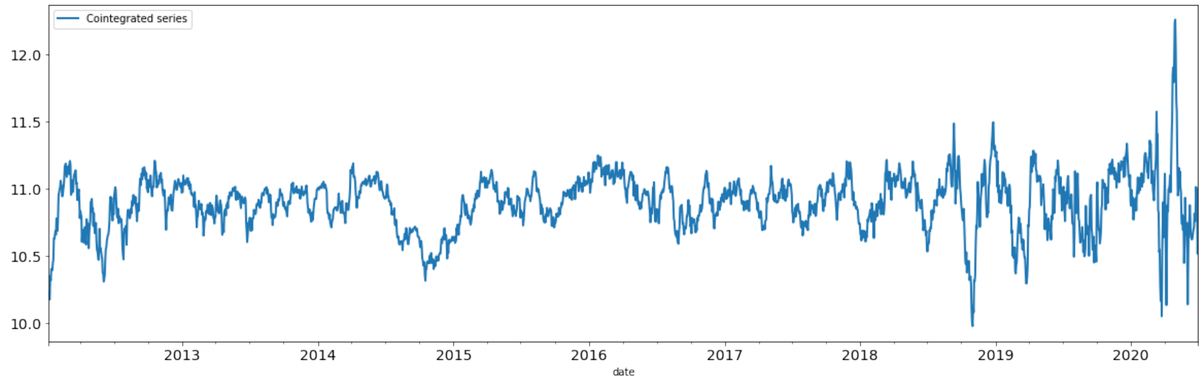


(c) Chebyshev polynomial order $m = 4$

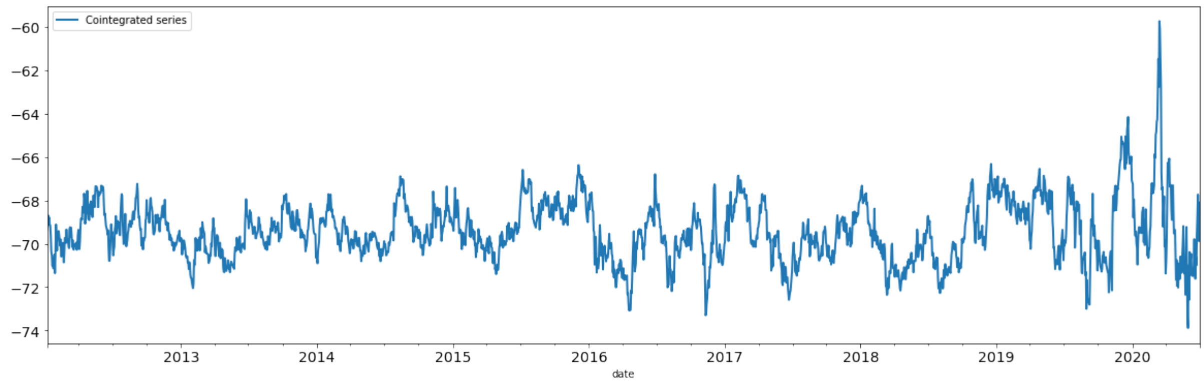
Figure A10: Estimates of β_t' for the TV-VECM(3) excluding *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order m.



(a) Chebyshev polynomial order $m = 2$



(b) Chebyshev polynomial order $m = 3$



(c) Chebyshev polynomial order $m = 4$

Figure A11: Estimates of $(\beta_t' Y_t)$ for the TV-VECM(3) excluding *Elec* - 1 January 2012 - 30 June 2020 for Chebyshev polynomial order m.

A.3 Python Code

Descriptive Statistics & Preliminary Analysis

```
[ ]: # IMPORT TOOLKITS

import time as time
import pandas as pd
import numpy as np
import datetime as dt

# Import Statsmodels
import statsmodels.api as sm
import statsmodels.formula.api as smf
from statsmodels.tsa.api import VAR
from statsmodels.tsa.stattools import adfuller
from statsmodels.tools.eval_measures import rmse, aic
from statsmodels.tsa.vector_ar import vecm
from statsmodels.tsa.api import VECM

# Import plotting tools
import seaborn as sns
import matplotlib.pyplot as plt
from matplotlib.pylab import rcParams
rcParams['figure.figsize']=10,6
import seaborn as sns
from scipy.stats import boxcox
```

```
[ ]: #IMPORT DATA
path = '/Users/sophiahummelman/Documents/Master Thesis/Data/
  ↪Data.xlsx'
df = pd.read_excel(path, parse_dates=['date'],␣
  ↪index_col='date')
df = df['01/01/2008' : '30/06/2020']
```

```
# Plot full EUA series
df_EUA = df['Eua']
df_EUA.plot(figsize=(20,6), linewidth=2, fontsize=14)
plt.show()

df = df['01/01/2012' : '30/06/2020']

# Make a new indicator for sustainability, by taking an␣
↪equally weighted average of Esg and Sust
Sust= (df.Esg+df.Sust)/2
df = df.drop('Esg',axis=1)
df = df.drop('Sust',axis=1)
df['Sust'] = Sust

# Make a new indicator for Elec and transform on Elec to␣
↪account for negative electricity prices
df['Elec'] = (df.Elec+df.Elec2)/2
df = df.drop('Elec2',axis=1)
df.Elec = (df.Elec + 1.1 - df['Elec'].min())


# Transform data with natural log
df = np.log(df)

# Transform dataset by taking first differences
df_diff =df.diff().dropna()


variablenames = df.columns
pd.set_option('display.float_format', lambda x: '%.4f' % x)
```

[ ]:
```
# Checking the correlations between X(t) and X(t-1)
fig, (ax1, ax2) = plt.subplots(nrows=2, ncols=1,␣
↪figsize=(10,6))
df.plot(ax=ax1) # series plot
```

```
pd.plotting.lag_plot(df) # lag plot
plt.show()
```

```
[ ]: descr=df.describe()
print(descr.to_latex())

# Plot Energy series
df_energy = df.drop('Ren',axis=1)
df_energy = df_energy.drop('Sust',axis=1)
df_energy.plot(figsize=(20,6), linewidth=2, fontsize=14)
plt.show()



# Plot Sustainability indicators
# Plot Energy series
df_sust = df.drop('Coal',axis=1)
df_sust = df_sust.drop('Elec',axis=1)
df_sust = df_sust.drop('Oil',axis=1)
df_sust = df_sust.drop('Gas',axis=1)
df_sust.plot(figsize=(20,6), linewidth=2, fontsize=14)
plt.show()

# Plot all variables together
df.plot(figsize=(20,6), linewidth=2, fontsize=14)
plt.show()
```

```
[ ]: #Full data set correlation
dfcorr=df.corr()

#Correlation Pre-Covid
dfprec = df['01/01/2012' : '03/03/2020']
dfprec_corr = dfprec.corr()

#Correlation Post-Covid
dfpostc = df['03/03/2020' : '30/06/2020']
```

```
dfpostc_corr = dfpostc.corr()

print(dfcorr.to_latex(),"\n",dfprec_corr.
 ↪to_latex(),"\n",dfpostc_corr.to_latex())
```

[ ]:
```
# Plot auto-correlations
for i, ax in enumerate(axes.flatten()):
    data = df_diff[df_diff.columns[i]]
    title = df_diff.columns[i]
    sm.graphics.tsa.plot_acf(data.values.squeeze(), lags=50,␣
 ↪title=title)
plt.tight_layout();
```

[ ]:
```
#GRANGER CAUSALITY TESTS

# Constant is added by default

from statsmodels.tsa.stattools import grangercausalitytests
maxlag=12
test = 'ssr_chi2test'
def grangers_causation_matrix(data, variables,␣
 ↪test='ssr_chi2test', verbose=False):
    """Check Granger Causality of all possible combinations of␣
 ↪the Time series.
    The rows are the response variable, columns are predictors.
 ↪ The values in the table
    are the P-Values. P-Values lesser than the significance␣
 ↪level (0.05), implies
    the Null Hypothesis that the coefficients of the␣
 ↪corresponding past values is
    zero, that is, the X does not cause Y can be rejected.
    A constant is added by default

    data      : pandas datafrja ame containing the time series␣
 ↪variables
```

```
    variables : list containing names of the time series␣
↪variables.
    """
    df = pd.DataFrame(np.zeros((len(variables),␣
↪len(variables))), columns=variables, index=variables)
    for c in df.columns:
        for r in df.index:
            test_result = grangercausalitytests(data[[r, c]],␣
↪maxlag=maxlag, verbose=False)
            p_values = [round(test_result[i+1][0][test][1],4)␣
↪for i in range(maxlag)]
            if verbose: print(f'Y = {r}, X = {c}, P Values =␣
↪{p_values}')
            min_p_value = np.min(p_values)
            df.loc[r, c] = min_p_value
    df.columns = [var + '_x' for var in variables]
    df.index = [var + '_y' for var in variables]
    return df

print('Full Data set','\n', grangers_causation_matrix(df,␣
↪variables = df.columns).to_latex())
print('Pre Covid Data set','\n',␣
↪grangers_causation_matrix(dfprec, variables = df.columns).
↪to_latex())
print('Post Covid Data set','\n',␣
↪grangers_causation_matrix(dfpostc, variables = df.columns).
↪to_latex())
```

[ ]:
```
# AUGMENTED DICKEY FULLER TEST

def adfuller_test(series ,regression,signif=0.05, name='',␣
↪verbose=False):
    """Perform ADFuller to test for Stationarity of given␣
↪series and print report"""
```

```
    r = adfuller(series, autolag='AIC',regression=regression)
    output = {'test_statistic':round(r[0], 4), 'pvalue':
 ↪round(r[1], 4), 'n_lags':round(r[2], 4), 'n_obs':r[3]}
    p_value = output['pvalue']
    def adjust(val, length= 6): return str(val).ljust(length)

    teststat = output["test_statistic"]

    critvals = []
    for key,val in r[4].items():
        new_critval= f' Critical value {adjust(key)} =␣
 ↪{round(val, 3)}'
        critvals = [critvals, new_critval]

    ADF = pd.
 ↪DataFrame(data=[teststat,p_value],index=["ADF","P-value"],columns=[name

    return ADF,critvals


# RESULTS ADF

# ADF Test with "c" : constant only
ADF = pd.
 ↪DataFrame(data=[0,0],index=["ADF","P-value"],columns=['test'])
for name, column in df.iteritems():
    new_ADF,critvals=adfuller_test(column,␣
 ↪regression='c',name=column.name)
    ADF = pd.concat([ADF, new_ADF],axis=1)
ADF=ADF.drop('test',axis=1)

print("ADF test ADF Test with "c" : constant only",'\n',ADF.
 ↪to_latex())
print('\n',"Critial Values",'\n',critvals,'\n')
```

```
# ADF Test with “ct” : constant and trend
ADF = pd.
  ↪DataFrame(data=[0,0],index=["ADF","P-value"],columns=['test'])
for name, column in df.iteritems():
    new_ADF,critvals=adfuller_test(column,␣
  ↪regression='ct',name=column.name)
    ADF = pd.concat([ADF, new_ADF],axis=1)
ADF=ADF.drop('test',axis=1)

print("ADF test ADF Test with #‘‘ct’’ : constant and␣
  ↪trend",'\n',ADF.to_latex())
print('\n',"Critial Values",'\n',critvals,'\n')
```

VAR Model & VECM

[ ]:
```
# IMPORT TOOLKITS

import time as time
import pandas as pd
import numpy as np
import datetime as dt

# Import Statsmodels
import statsmodels.api as sm
import statsmodels.formula.api as smf
from statsmodels.tsa.api import VAR
from statsmodels.tsa.stattools import adfuller
from statsmodels.tools.eval_measures import rmse, aic
from statsmodels.tsa.vector_ar import vecm
from statsmodels.tsa.api import VECM

# Import plotting tools
import matplotlib.pyplot as plt
import seaborn as sns
from matplotlib.pylab import rcParams
rcParams['figure.figsize']=10,6
import seaborn as sns
from scipy.stats import boxcox
```

[ ]:
```
#IMPORT DATA
path = '/Users/sophiahummelman/Documents/Master Thesis/Data/
    ↪Data.xlsx'
df = pd.read_excel(path, parse_dates=['date'],␣
    ↪index_col='date')
df = df['01/01/2012' : '30/06/2020']

# Make a new indicator for sustainability, by taking an␣
    ↪equally weighted average of Esg and Sust
Sust= (df.Esg+df.Sust)/2
```

```
df = df.drop('Esg',axis=1)
df = df.drop('Sust',axis=1)
df['Sust'] = Sust

# Make a new indicator for Elec and transform on Elec to␣
↪account for negative electricity prices
df['Elec'] = (df.Elec+df.Elec2)/2
df = df.drop('Elec2',axis=1)
df.Elec = (df.Elec + 1.1 - df['Elec'].min())

#Exclude Electricity from the analysis
df = df.drop('Elec',axis=1)

# Transform data with natural log
df = np.log(df)

# Transform dataset by taking first differences
df_diff =df.diff()

# Check input variables
variablenames = df.columns
print(variablenames)

# Set output to four decimal places significance
pd.set_option('display.float_format', lambda x: '%.4f' % x)
```

[ ]:

```
# LAG ORDER SELECTION VAR MODEL
# VAR model must be indicated in trend , by "c" for constant␣
↪only and "ct" for a constant and trend

#Fit VAR model
model = VAR(endog=df,freq=None)
res = model.select_order(maxlags=5, trend='ct')
varorder=res.summary()
```

```
#Print VAR order selection criteria summary
print(varorder)

# Determine VAR order by taking the maximum of the order␣
↪selection criteria
ressum = res.ics
varorder =(res.aic,res.bic,res.hqic)

#Print recommended varorder
print("recommended varorder:",varorder,"\n")
varorder =max(res.aic,res.bic,res.hqic)
ressum = pd.DataFrame(ressum)
ressum = ressum.drop('fpe', axis=1)

#Print latex table order selection criteria
print(ressum.to_latex())
```

[ ]:
```
# FIT VAR model
# VAR model must be indicated by “c” for constant only and␣
↪“ct” for a constant and trend

model_fit = model.fit(maxlags=varorder, trend='ct')
varorder=model_fit.k_ar

#Print a summary of the VAR
print(model_fit.summary())
varsumm = model_fit.summary()
```

[ ]:
```
# DETERMINE IDEAL VECM RANK

# if det_order =
#-1 - no deterministic terms
#0 - constant term
#1 - linear trend

#Order of VECM = VAR order -1
```

```
varorder = varorder-1

input_data = df

vecm_order = vecm.select_order(input_data, maxlags=8, deterministic='c', seasons=0, exog=None, exog_coint=None)
print((vecm_order))

# Initialize cointegration rank
cointrank = 0

from statsmodels.tsa.vector_ar import vecm
i=0
while i <=1:

    # VEC Rank Test
    vec_trace = vecm.select_coint_rank(input_data, det_order = i, k_ar_diff = varorder, method = 'trace', signif=0.01)
    print(vec_trace.summary())

    # Output and results VEC Rank test
    print("Deterministic order:",i,"\n","Vec rank suggested by trace test:",vec_trace.rank)
    if vec_trace.rank>cointrank:
        cointrank = vec_trace.rank

    if i == -1:
        tracesumm_no_det = vec_trace.summary()
        with open('/Users/sophiahummelman/Documents/Master Thesis/Data/tracesumm_no_det.tex','w') as fh:
            fh.write(tracesumm_no_det.as_latex_tabular() )
    if i == 0:
        tracesumm_constant = vec_trace.summary()
        with open('/Users/sophiahummelman/Documents/Master Thesis/Data/tracesumm_constant.tex','w') as fh:
```

```
            fh.write(tracesumm_constant.as_latex_tabular() )
    if i == 1:
        tracesumm_trend = vec_trace.summary()
        with open('/Users/sophiahummelman/Documents/Master␣
↪Thesis/Data/tracesumm__trend.tex','w') as fh:
            fh.write(tracesumm_trend.as_latex_tabular() )

    #Maximum-eigenvalue Statistic
    vec_maxeig = vecm.select_coint_rank(input_data, det_order␣
↪= i, k_ar_diff = varorder, method = 'maxeig', signif=0.05)
    print(vec_maxeig.summary())

    # Output and results Maximum-eigenvalue test
    print("Deterministic order:",i,"\n","Vec rank suggested by␣
↪maxeig test:",vec_maxeig.rank)
    if vec_maxeig.rank>cointrank:
        cointrank = vec_maxeig.rank
    if i == -1:
        maxeigsumm_no_det = vec_maxeig.summary()
        with open('/Users/sophiahummelman/Documents/Master␣
↪Thesis/Data/maxeigsumm_no_det.tex','w') as fh:
            fh.write(maxeigsumm_no_det.as_latex_tabular() )
    if i == -0:
        maxeigsumm_constant = vec_maxeig.summary()
        with open('/Users/sophiahummelman/Documents/Master␣
↪Thesis/Data/maxeigsumm_constant.tex','w') as fh:
            fh.write(maxeigsumm_constant.as_latex_tabular() )
    if i == 1:
        maxeigsumm_trend = vec_maxeig.summary()
        with open('/Users/sophiahummelman/Documents/Master␣
↪Thesis/Data/maxeigsumm__trend.tex','w') as fh:
            fh.write(maxeigsumm_trend.as_latex_tabular() )
    i = i+1
```

```
[ ]: input_data = df

     # Uses cointegration rank suggest by maximum eigenvalue test␣
      ↪by default. To change add line: cointrank = X
     print("Cointegration Rank equals:",cointrank)
     print("VECM Order:",varorder)

     # Change deterministic terms for the VECM in determ

     #"nc" - no deterministic terms
     #"co" - constant outside the cointegration relation
     #"ci" - constant within the cointegration relation
     #"lo" - linear trend outside the cointegration relation
     #"li" - linear trend within the cointegration relation

     # The four combinations of command used are
     #ci
     #co
     #coli

     # Current deterministic term settings
     determ = 'ci'

     #VECM on the prices with "varorder" lags, "cointrank"␣
      ↪co-integrating relationship
     vecm = VECM(endog = input_data, k_ar_diff = varorder,␣
      ↪coint_rank = cointrank, deterministic = determ)
     vecm_fit = vecm.fit()
     vecmsumm=vecm_fit.summary()
     #Print a summary of the model results
     print(vecmsumm)

     #Print the log-likelihood of the model
     print("Log-likelihood",vecm_fit.llf)
```

[ ]:
```
# PLOT RESULTING COINTEGRATING RELATION
dfm=df
portfolio_insample = np.dot(dfm,beta)
portfolio_insample = pd.
 ↪DataFrame(portfolio_insample,columns=["Cointegrated␣
 ↪series"])
portfolio_insample.index=dfm.index

portfolio_insample.plot(figsize=(20,6), linewidth=2,␣
 ↪fontsize=14)
plt.show()
```

Time-varying VECM[1]

```
[ ]: # IMPORT TOOLKITS

import time as time
import pandas as pd
import numpy as np
import datetime as dt
import matplotlib.pyplot as plt
import math

# Import Statsmodels
import statsmodels.api as sm
import statsmodels.formula.api as smf
from statsmodels.tsa.api import VAR
from statsmodels.tsa.stattools import adfuller
from statsmodels.tools.eval_measures import rmse, aic
from statsmodels.tsa.vector_ar import vecm
```

```
[ ]: #IMPORT DATA

path = '/Users/sophiahummelman/Documents/Master Thesis/Data/
  ↪Data.xlsx'
df = pd.read_excel(path, parse_dates=['date'],␣
  ↪index_col='date')
df = df['01/01/2012' : '30/06/2020']
variablenames = df.columns
pd.set_option('display.float_format', lambda x: '%.4f' % x)

# Make a new indicator for sustainability, by taking an␣
  ↪equally weighted average of Esg and Sust
Sust = (df.Esg+df.Sust)/2
df = df.drop('Esg',axis=1)
df = df.drop('Sust',axis=1)
df['Sust'] = Sust
```

[1] Code Based on pseudocode by Luis Filipe Martins: http://home.iscte-iul.pt/Ĩfsm/

```
# Make a new indicator for Elec and transform on Elec to␣
↪account for negative electricity prices
df['Elec'] = (df.Elec+df.Elec2)/2
df = df.drop('Elec2',axis=1)
df.Elec = (df.Elec + 1.1 - df['Elec'].min())

#Exclude Electricity from the analysis
df = df.drop('Elec',axis=1)

# Transform data with natural log
df = np.log(df)

# Variable and data check
variablenames = df.columns
print("Final model contains variables:",variablenames)
```

[ ]:
```
# INITIALIZE VARIABLES

data = df.rename_axis('ID').values

y = data

n = len(y)
k = len(y[0])

m=1

mmax= 4                         # Set maximum dimension of␣
↪Chebishev Polynomials

p=3                             # Set VECM order

lrtvc = np.zeros((mmax,k))        # TVC Stat (row) m=1,...,mmax␣
↪and (col) r=1,..,k
```

```
lrtvcpv=np.zeros((mmax,k))      # P-values using the Asymp␣
↪Distr (chisquare(mrk))
betat=np.zeros((n-p-1,k*mmax)) # beta_t for cointegration rank␣
↪1, i.e. r=1. The rows display observation i and the columns␣
↪1 up to k= (b1,...,bk) for m=1, columns k+1 up to 2k=(b1,...
↪,bk) for m=2
lnlikm=np.zeros((mmax,k))       # Log likelihood for different␣
↪m and r
aic=np.zeros((mmax,k))          # AIC Akaike model selection␣
↪criterion
bic=np.zeros((mmax,k))          # BIC Schwarz model selection␣
↪criterion
hann=np.zeros((mmax,k))         # Hannan-Quinn model selection␣
↪criterion
```

[ ]:
```
# Create vector normalization function
def normalize(v):
    norm = np.linalg.norm(v)
    if norm == 0:
       return v
    return v / norm
```

[ ]:
```
# Create function for Chebychev time polynomials

def ycheb(data, varord, chebdim):
    nn = len(data)
    k  = len(data[0])
    yst=np.zeros((nn-varord-1,(chebdim+1)*k))
    yst[0:len(yst),0:k]=data[varord:nn-1,0:len(yst[0])]
    if chebdim==0:
        yst = yst
    else:
        n=len(yst[0:len(yst),1])
        ind=np.arange(varord+2, n+varord+2, 1).tolist()
        ind=np.array(ind)
    i=1
```

```
    while i<=chebdim:
        matrix1 = math.sqrt(2)*np.cos(i*math.pi*(ind-0.5)/n)
        matrix2 = yst[0:len(yst),0:k]
        yst[0:len(yst),i*k:(i+1)*k] = matrix2 * matrix1[:, np.
 ↪newaxis]
        i=i+1
    return yst
```

[ ]:
```
# Function to take different lags of the data

def varlags(var,lags):
    i=1
    dy = var[lags:len(var),:]
    b=var[0:len(var)-lags]
    dylags = b
    while i<lags:
        af=var[i:len(var)-lags+i]
        newaf = np.concatenate((af, b), axis=1)
        b = newaf
        i = i+1
        dylags = newaf
    return dy, dylags
```

[ ]:
```
# Function to calculate eigenvalues, eigenvectors and␣
 ↪determinants to calculate the time varying cointegration␣
 ↪test statistic

def tvcoint(y,p,m):
    ystar_1 = ycheb(y,p,m)
    ysub,y_1=varlags(y,1)
    dy=ysub-y_1
    dy,dylags=varlags(dy,p)
    T=len(dylags)
    dylags = np.concatenate((np.ones((T,1)), dylags), axis=1)
    betau,a,b,c = np.linalg.lstsq(dylags,dy)
    resu=dy-dylags@betau
```

```
    betav,a,b,c=np.linalg.lstsq(dylags,ystar_1)
    resv=ystar_1-dylags@betav
    S00=(resu.T@resu)/T
    S01=(resu.T@resv)/T
    S10=S01.T
    S11=(resv.T@resv)/T
    S00inv=np.linalg.inv(S00)
    S11inv=np.linalg.inv(S11)
    A=S11inv@S10@S00inv@S01
    evals,evec=np.linalg.eig(A)
    idx = evals.argsort()[::-1]
    evals = evals[idx]
    evec = evec[:,idx]
    detS00 = np.linalg.det(S00)

    return evals, evec, detS00
```

[ ]:
```
# MAIN FUNCTION

import scipy as scipy
from scipy.stats import chi2

ev0,evec0,det0=tvcoint(y,p,0)     #perform standard␣
 ↪cointegration with the Chebychev order equal to 0
ll0=np.log(1-ev0)
m=1                                # Start with Chebychev order =␣
 ↪1

while m<=mmax:
    evals,evect,detm=tvcoint(y,p,m) # Output gives the␣
 ↪eigenvalues, eigenvectors q1...qr...q(m+1)k and␣
 ↪determinant(S00)
    llm=np.log(1-evals)
    #for r=1
    ind = np.arange(p+2, n+1, 1).tolist()
```

```
    ind = np.array(ind)
    c=1
    while c <= k:
        beta1sum = np.zeros((m,len(ind)))
        beta2sum = np.zeros((m,len(ind)))
        beta3sum = np.zeros((m,len(ind)))
        beta4sum = np.zeros((m,len(ind)))
        beta5sum = np.zeros((m,len(ind)))
        beta6sum = np.zeros((m,len(ind)))
        beta7sum = np.zeros((m,len(ind)))
        c = c+1
    mm = 1
    while mm <=m:
        cosin = np.sqrt(2)*np.cos(mm* math.pi*(ind-0.5)/
↪(n-p-1))

        beta1sum[mm-1,:]=evect[k*mm,0]*cosin
        beta2sum[mm-1,:]=evect[k*mm+1,0]*cosin
        beta3sum[mm-1,:]=evect[k*mm+2,0]*cosin
        beta4sum[mm-1,:]=evect[k*mm+3,0]*cosin
        beta5sum[mm-1,:]=evect[k*mm+4,0]*cosin
        beta6sum[mm-1,:]=evect[k*mm+5,0]*cosin
        beta7sum[mm-1,:]=evect[k*mm+6,0]*cosin

        mm = mm +1
        # Add beta estimates together in a matrix
        matrix2=np.column_stack((np.sum(beta1sum,axis=0),np.
↪sum(beta2sum,axis=0),np.sum(beta3sum,axis=0),np.
↪sum(beta4sum,axis=0),np.sum(beta5sum,axis=0),np.
↪sum(beta6sum,axis=0),np.sum(beta7sum,axis=0)))
        eigenvectors = evect[0:k,0]
        betat[:,k*(m-1):k*m]=eigenvectors.T+matrix2

    r=2
    while r<=k+1:
```

```
        lrtvc[m-1,r-2]=(n-p-1)*np.sum(ll0[0:
↪r-1],axis=0)-(n-p-1)*np.sum(llm[0:r-1],axis=0)
        lrtvcpv[m-1,r-2]=1-scipy.stats.chi2.
↪cdf(lrtvc[m-1,r-2],(m*(r-1)*k))
        lnlikm[m-1,r-2]=(np.log(r-1)-k-k*np.log(2*math.
↪pi))*(n-p-1)/2 - np.sum(llm[0:r-1],axis=0)*(n-p-1)/2 - (np.
↪log(detm))*(n-p-1)/2
        npar=(m+1)*k*(r)+(r)*k+k^2+(k+(p-1)*k^2)
        aic[m-1,r-2]= -2*lnlikm[m-1,r-2]/(n-p-1)+npar*2/(n-p-1)
        bic[m-1,r-2]= -2*lnlikm[m-1,r-2]/(n-p-1)+npar*(np.
↪log(n-p-1))/(n-p-1)
        hann[m-1,r-2]= -2*lnlikm[m-1,r-2]/(n-p-1)+npar*(np.
↪log(np.log(n-p-1)))*2/(n-p-1)

        r=r+1
    m = m + 1
```

[ ]:
```
# Analyze output

hannmax = pd.DataFrame(hann)
# Convert beta estimates to a DataFrame
df2=pd.DataFrame((betat))

# Determine ideal chebichev order with Hannan-Quinn Criteria␣
↪and re-run if necessary
hannmax=hannmax.idxmin(axis=0, skipna=True)+1
print("Ideal Chebichev order: ", hannmax)

#PRINT OUTPUT
print("Time Varying Cointegration Empirical Analysis - DRIFT␣
↪CASE - Cointegration rank = 1")
print("Number of time series =",k, '\n',"Number of␣
↪observations =",n, '\n'," Order of VECM =",p,'\n',"Current␣
↪Dimensiion of Chebishev time-polynomial =",mmax,'\n')
```

```
print("Likelihood Ratio Time-Varying Cointegration Test␣
  ↪Statistics",'\n',lrtvc,'\n', "P-Value",'\n',lrtvcpv,'\n',␣
  ↪"Using the Asymp Distr (chisquare(mrk)) with row = m =1,...
  ↪,mmax and col = r = 1,...,k")
print("Time-varying Cointegration log likelihood =  (row=m,␣
  ↪col=r)",'\n',lnlikm,'\n',)
print("AIC Akaike =  (row=m, col=r)",'\n',aic,'\n',)
print("BIC Schwarz =  (row=m, col=r)",'\n',bic,'\n',)
print("Hannan-Quinn =  (row=m, col=r)",'\n',hann,'\n',)

#Create a DataFrame for a Latex summary table
A=(lrtvc[mmax-1],lrtvcpv[mmax-1],lnlikm[mmax-1],aic[mmax-1],bic[mmax-1],ha
A=np.stack(A, axis = 0)
TVsummary = pd.DataFrame(A, index=["LR TVC␣
  ↪Statistics","P-Value","TV Coint. log␣
  ↪likelihood","AIC","BIC","HQIC"])

cointrank = 1
print("For chosen cointegration rank ,'", cointrank,"',")
print("Summary of time varying cointegration",TVsummary[0].
  ↪to_latex())
```

[ ]:
```
# Displays non-normalized beta estimates

df2=pd.DataFrame((betat))

df4=df2[df2.columns[-k:]]



# Insert some empty values to match the dates of the original␣
  ↪dataframe to create a new Dataframe containing time-stamps␣
  ↪for the beta estimates
data = []

data.insert(0, {'Eua': 0})
```

```
data.insert(0, {'Eua': 0})
data.insert(0, {'Eua': 0})
data.insert(0, {'Eua': 0})

df4.columns=variablenames
df4=pd.concat([pd.DataFrame(data), df4], ignore_index=True)
df4.index=df.index

df4_tail = df4.iloc[4:]

# Plot the beta estimates in one graph for m = mmax
df4_tail.plot(figsize=(20,6), linewidth=2, fontsize=14)

plt.show()
```

[ ]:
```
# Displays normalized beta's in one graph for m = mmax

df4_norm=normalize(df4_tail)
df4_norm=df4_norm*(1/df4_norm['Eua'][0])

df4_norm.plot(figsize=(20,6), linewidth=2, fontsize=14)

plt.show()
```

[ ]:
```
# Displays the normalized cointegration relation for m = mmax

dfm=df
dfm = dfm.iloc[4:]
portfolio_insample = np.dot((dfm),(df4_norm).T)

portfolio_insample=np.diagonal(portfolio_insample)
portfolio_insample=pd.
  ↪DataFrame(portfolio_insample,columns=["Cointegrated␣
  ↪series"])
portfolio_insample.index=dfm.index
portfolio_insample.column='Cointegration Relation'
```

```
portfolio_insample.plot(figsize=(20,6), linewidth=2, fontsize=14)
```