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A Bootstrap Strategy for Portfolio Rebalancing

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Abstract

The portfolio rebalancing decision is typically made based on a simple measure that neglects estimation uncertainty. To account for uncertainty and to provide a statistical rationale behind this decision, a bootstrap portfolio rebalancing strategy is proposed. This strategy applies the bootstrap to test whether an increase in utility is significant using a test statistic derived from the mean-variance framework. The performance of the bootstrap rebalancing strategy is evaluated in an empirical study with industry portfolio data from 2000 until 2019 in comparison to buy-and-hold, periodic, and threshold rebalancing strategies. Portfolios are rebalanced using global minimum variance weights and an optimisation method that accounts for transaction costs. The empirical study demonstrates there is a sub-linear relation between the average rebalancing time and the amount of incurred transaction costs. In addition, it shows that bootstrap rebalancing strategies can achieve higher Sharpe ratios and utility levels than other rebalancing strategies. Bootstrap rebalancing strategies are most effective when estimation uncertainty is high or global minimum variance weights are used. The optimisation method that accounts for transaction costs is already conservative with rebalancing, such that applying a bootstrap rebalancing strategy to further reduce transaction costs may be ineffective. However, this optimisation method often constructs significantly better portfolios than those obtained by using global minimum variance weights. The bootstrap rebalancing strategy has a hyperparameter that influences the amount of trading, which is optimally chosen to decrease with risk tolerance, transaction costs, and estimation uncertainty. The bootstrap rebalancing strategy is easily implementable in practice and can generate economic value for portfolio managers who apply mean-variance optimisation.

Contents

1 Introduction

A recent development in the field of portfolio optimisation is to take transaction costs into account in the optimisation, which can significantly reduce transaction costs and improve portfolio performance (Maurer, Pezzo, & Taylor, 2019). However, uncertainty in the optimisation inputs is often neglected. This paper proposes a novel rebalancing strategy that takes this uncertainty into account by testing for a gain in portfolio performance using the bootstrap. The portfolio is only rebalanced if the expected gain in performance is significant, which helps to further reduce transaction costs without sacrificing risk or return.

Applying an adequate portfolio rebalancing strategy is indispensable for any investor, because previously optimised portfolios can become sub-optimal for three reasons: (i) assets realise different returns, (ii) properties of assets change, or (iii) preferences of the investor alter (Dichtl, Drobetz, & Wambach, 2014). An investor would like to rebalance the portfolio as soon as such a change occurs. However, continuous trading is practically impeded by transaction costs. These costs are most relevant for portfolios that realise a high turnover, including mean-variance portfolios that may incur trading costs up to 30% per day (Garlappi, Uppal, & Wang, 2007).

To reduce the amount of transaction costs, these costs should be accounted for in the portfolio optimisation. Magill and Constantinides (1976) and Constantinides (1979) show that proportional transaction costs give rise to a no-trade region around the optimal asset allocation; if the current portfolio allocation is close to the optimum then rebalancing leads to a loss from transaction costs that exceeds the benefit of having a better allocation. Moreover, they show that the portfolio should be rebalanced to an allocation at the boundary of the no-trade region. The optimisation problem with proportional transaction costs and multiple risky assets is typically formulated as a non-linear programming problem, which can be solved to obtain an efficient portfolio allocation. However, the portfolio optimisation method is often combined with a naive rebalancing strategy that disregards estimation uncertainty.

This paper aims to fill this gap in the literature by proposing a rebalancing strategy that only rebalances the portfolio if there is a significant gain in the expected performance. The test uses a statistic that is derived from the mean-variance utility framework. The bootstrap is used to perform the test, because the distribution of the gain is generally unknown. The bootstrap rebalancing strategy proposed in this paper bears some resemblance to the rebalancing rule of Michaud and Michaud (2008), who make use of the bootstrap in the absence of transaction costs. Yet, the proposed strategy is more widely applicable as it works with mean-variance optimised portfolios rather than portfolios constructed using a bootstrap approach. The strategy is implemented both with global minimum variance (GMV) weights and by applying the optimisation procedure of Dybvig and Pezzo (2019) that accounts for proportional transaction costs.

The performance of the bootstrap rebalancing strategy is compared to several commonly applied strategies in an empirical study with industry portfolio data from 2000 until 2019. The study shows that bootstrap rebalancing strategies often simultaneously achieve better Sharpe ratios and higher utility levels than other rebalancing strategies, although performance differences are seldom large. Bootstrap rebalancing strategies enhance the portfolio performance of

GMV portfolios most, because transaction costs are neglected when applying GMV weights. The optimisation method that accounts for transaction costs is already conservative with portfolio rebalancing, such that bootstrap rebalancing strategies only become effective when estimation uncertainty is high. The performance of the bootstrap rebalancing strategies depends on a hyperparameter that impacts the amount of trading activity. This parameter is optimally set to decrease with risk tolerance, transaction costs, and estimation uncertainty. The performance of the bootstrap strategies is robust to the imposition of short sale constrains, changes in the bootstrap methodology, and the application of variable transaction costs.

This research further finds that the optimisation procedure of Dybvig and Pezzo (2019) greatly reduces transaction costs compared GMV optimisation, which often results in significantly higher post-transaction cost returns and Sharpe ratios. In addition, this paper observes a sub-linear relation between the amount of incurred transaction costs and the average rebalancing time. More concretely, the gains from rebalancing later decrease with the passing of time.

The relevance of this paper is twofold. From a scientific point of view, the rebalancing rule of Michaud and Michaud (2008) is extended to accommodate transaction costs. Moreover, the usefulness of a bootstrap procedure in portfolio rebalancing is empirically verified for the first time. Besides, new insights into the effectiveness of the optimisation method of Dybvig and Pezzo (2019) are provided by applying it to equity data. From a practical perspective, the proposed bootstrap rebalancing strategy is easily be implementable by portfolio managers who apply mean-variance optimisation. The empirical study demonstrates that the bootstrap strategy can generate genuine economic value.

The remainder of this paper is structured as follows. Section 2 describes the literature in which this research is embedded. The derivation and implementation of the bootstrap rebalancing strategy are outlined in Section 3. Section 4 first discusses the set-up of the empirical study with industry portfolio data and then provides its main results. The robustness of the findings is verified in Section 5. Section 6 applies the rebalancing strategy to a data set with stocks and relaxes the assumption of constant transaction costs. Finally, Section 7 concludes the paper.

2 Literature Review

This research is at the intersection of several strands of literature relating to transaction costs, portfolio optimisation, estimation uncertainty, and bootstrapping. The most relevant insights from portfolio optimisation with transaction costs and uncertainty are discussed in Sections 2.1 and 2.2, respectively. Literature related to the bootstrap is discussed in Section 2.3.

2.1 Portfolio Optimisation with Transaction Costs

Transaction costs consist of direct and indirect costs. Direct costs include brokerage commissions, the bid-ask spread, taxes, and costs related to submitting and optimising the trade. On the other hand, indirect costs are market impact costs that stem from large purchases (sales) adversely driving up (down) the asset price. Tóth et al. (2011) and Frazzini, Israel, and Moskowitz (2012) show that these costs grow approximately at a square root rate with the transaction size.

Historical estimates of transaction costs differ greatly due to the limited availability of data (Arnott & Wagner, 1990). For US stocks, Lesmond, Ogden, and Trzcinka (1999) estimate transaction costs to range from 1% to 10% for a round trip transaction (i.e., buy and sell), whereas Balduzzi and Lynch (1999) and Hasbrouck (2009) estimate them to be around 1%. These costs vary with a number of factors. Loeb (1983) finds that trading costs decrease with the market capitalisation and trading volume. Both Domowitz, Glen, and Madhavan (2001) and Frazzini et al. (2012) demonstrate that trading costs increase with volatility but decrease with the trading volume. Moreover, Hasbrouck (2009) observes that assets with higher gross returns generally incur more trading costs, such that asset net asset returns are more similar.

The presence of transaction costs and their magnitude make it relevant to take them into consideration, particularly for active investors. To this end, Magill and Constantinides (1976) are the first to coin the notion of a no-trade region in a continuous time framework with proportional transaction costs and a single risky asset. They show that it is only optimal to trade towards the boundary of the no-trade region. Davis and Norman (1990) further show that solving this problem involves a non-linear free boundary problem and propose an algorithm to solve it. Alternatively, Czichowsky and Schachermayer (2016) suggest solving the dual problem. The literature in continuous time usually works with a single risky asset due to the complexity of the problem, although some exceptions exist (e.g., Akian, Menaldi, and Sulem, 1995; Leland, 1999; Liu, 2004; Muthuraman and Kumar, 2006).

Despite the growing literature on modelling transaction costs in continuous time for multiple assets, this paper focuses on the discrete time setting where more analytical solutions are available. Constantinides (1979) finds similarly to Magill and Constantinides (1976) that proportional transaction costs in a multi-period model give rise to a no-trade region in discrete time. Lynch and Tan (2010) analyse the properties of the no-trade region for two risky assets. With unpredictable returns, there is a single no-trade region with the shape determined by the asset correlations. With predictable returns, the shape of the no-trade region is state-dependent, which results in more trading activity since there is more information to trade on.

Recent research relating to portfolio optimisation with transaction costs differs in the assumptions made regarding the type of costs and the predictability of returns. For example, Gârleanu and Pedersen (2013) assume there are quadratic transaction costs in a multi-period model with predictable returns. They find that rebalancing should occur towards a combination of the current and future portfolio that would be optimal in the absence of transaction costs. In this way, the investor saves costs now by only partially trading towards the current optimum, whereas costs are saved in the future by already trading towards the future optimum. Mei, DeMiguel, and Nogales (2016) derive a closed-form solution for multi-period investing with several unpredictable assets. They assume that trading costs are proportional for small trades but grow at a square root rate for large trades. In an application with commodity futures, they demonstrate that ignoring transaction costs is expensive and investing myopically is especially harmful for long-term investors. Mei and Nogales (2018) extend their work to include predictable returns, but they cannot find an analytical solution to the investment problem.

The most pertinent work for this research is that of Dybvig and Pezzo (2019), who find an analytical solution for a single-period model with fixed and proportional transaction costs. They show that it is optimal to trade to the interior of the no-trade region when costs are fixed, whereas it is only optimal to trade to the boundary when costs are proportional. In addition, they illustrate how the no-trade region is affected by changes in asset correlations, risk aversion, and transaction costs. Maurer et al. (2019) show that the method of Dybvig and Pezzo (2019) optimises currency portfolios better than a method that neglects trading costs. Their optimisation procedure is implemented in this paper for its tractability, which aids in verifying the usefulness of the bootstrap rebalancing strategy. Although the method induces myopic investing, it is still theoretically optimal in some cases. More specifically, Merton (1971) shows behaving myopically is optimal in the presence of constant investment opportunities. In addition, Hakansson (1971) finds that the same holds when investors have a log utility function.

2.2 Portfolio Optimisation under Uncertainty

The bootstrap rebalancing strategy proposed in this paper only rebalances the portfolio if the resulting gain in performance is significant, which may not be the case due to considerable estimation uncertainty. Jobson and Korkie (1980) demonstrate that accurate estimation of the mean-variance efficient frontier can be problematic due to errors in the sample mean and variance. Moreover, Michaud (1989) finds that mean-variance optimisation suffers from "error maximisation", because assets with the largest favourable estimation errors obtain the largest portfolio weights. Chopra and Ziemba (1993) further show that wrongful estimation of the expected return has the largest adverse impact on portfolio performance when applying mean-variance optimisation. Therefore, this paper restricts its attention to variance minimisation.

To enhance portfolio performance, several alternatives to the sample estimator have been proposed. For example, (stochastic) volatility models can be used to better capture the dynamics of the covariance matrix (e.g., Hansen and Lunde, 2005). In addition, Ledoit and Wolf (2003, 2004, 2012) propose a variety of shrinkage estimators for the covariance matrix. Furthermore, Fan, Fan, and Lv (2008) show that imposing a factor structure on the covariance matrix can improve the estimation. As an illustration, they show that the three-factor model of Fama and French (1993) can be used to better estimate the covariance matrix of stock returns. On the other hand, Fan, Liao, and Mincheva (2013) employ statistical factors obtained using principal component analysis (PCA) to estimate the covariance matrix. Both shrinkage and factor-based methods are employed in this research to improve covariance matrix estimation.

Estimation uncertainty also impacts the portfolio rebalancing decision. The distribution of portfolio weights is relevant to make a statistically sound rebalancing decision. GMV portfolio weights are of particular interest given that this research focuses on variance minimisation. Okhrin and Schmid (2006) derive the multivariate distribution of GMV portfolio weights under the assumption of normally distributed returns. This distribution is used by Golosnoy and Schmid (2007) to detect a significant change in the optimal portfolio weights. More generally, Bodnar and Schmid (2007) derive the distribution of the GMV portfolio's variance under the assumption of elliptically distributed returns. Subsequently, Bodnar and Schmid (2008) use this to construct a test for portfolio efficiency. Unfortunately, these results do not easily extent to a setting with transaction costs, because these costs affect the distribution of portfolio weights.

2.3 Bootstrap

To nevertheless make statistical inference on portfolio weights, a bootstrap approach is used. The bootstrap is introduced by Efron (1979) who repeatedly draws independent observations to generate new samples. Each newly generated sample is then used to estimate a statistic of interest. A large collection of these statistics forms a sampling distribution, which can be used to make inference about a population parameter of interest. This procedure can be performed by drawing observations directly from the data set, which is known as a non-parametric bootstrap. Alternatively, a parametric bootstrap is applied in which a distribution is fitted to the data and resampling occurs from the fitted model.

Applying the bootstrap of Efron (1979) can result in poor statistical inference when working with dependent data, because drawing independent observations removes the dependence structure (Singh, 1981). This issue is particularly relevant for asset returns that typically portray time-dependent volatility. To maintain the dependence structure, Carlstein (1986) suggests resampling non-overlapping blocks of data instead of individual observations. On the other hand, Kunsch (1989) and Liu and Singh (1992) advocate that bootstrapping overlapping blocks of data improves statistical inference. Their method is referred to as the moving block bootstrap. Politis and Romano (1992) note that observations at the start and the end of the sample are less likely to be drawn with the moving block bootstrap. Therefore, they propose the circular bootstrap that gives each observation the same probability to be drawn. Furthermore, Politis and Romano (1994) propose the stationary bootstrap that draws the block size from a geometric distribution in order to generate stationary time series.

The performance of the above-mentioned bootstrap methods is compared theoretically by Lahiri (1999). He finds that the overlapping block bootstrap is preferred to the non-overlapping block bootstrap, because it asymptotically has the same bias but a lower variance. This entails that the overlapping block bootstrap is more efficient for large samples. Nordman (2009) further shows that the stationary bootstrap and the non-overlapping block bootstrap have the same asymptotic variance. From an empirical perspective, Lahiri (1999) finds that overlapping block bootstrap methods perform best in estimating autoregressive and moving average models. Radovanov and Marcikić (2014) demonstrate that the stationary and overlapping block bootstrap are both well-suited to estimate volatility. Given that the circular and stationary bootstrap are equally applicable methods with different properties they are both implemented in this research.

A crucial element of the block bootstrap is the block size, because an improper choice of block size leads to erroneous estimates (Cogneau & Zakamouline, 2013). A larger block size often decreases bias at the cost of an increase in variance (Lahiri, 2003). Politis and White (2004) develop an algorithm to automatically select the block length for the circular and stationary bootstrap based on the work of Lahiri (1999). Patton, Politis, and White (2009) correct their algorithm after Nordman (2009) shed new light on the theoretical properties of the methods.

The bootstrap has several applications in portfolio optimisation, including the construction of robust portfolio weights. Michaud and Michaud (2008) pioneer this field by resampling asset returns to obtain bootstrap estimates of the mean return and covariance matrix. Subsequently, these estimates are used to bootstrap the efficient frontier. Finally, they obtain robust portfolio weights by taking the average of portfolio weights with similar mean-variance properties from the bootstrapped efficient frontiers. A similar method is applied by Shen and Wang (2017) who further restrict the number of assets held in the portfolio. They show their bootstrap method compares favourably to a variety of other commonly applied portfolio optimisation methods in an empirical study with equity and exchange traded funds data.

Besides for the construction of weights, bootstrapping is used to quantify portfolio uncertainty. Liang, Myer, and Webb (1996) use the independent bootstrap to obtain a confidence interval around the optimal portfolio weights for five asset classes. They find that the large amount of uncertainty in the mean-variance estimates practically renders them useless. Michaud and Michaud (2008) also use bootstrapped portfolio weights to obtain a confidence region around the optimal asset allocation. Srivatsa, Smith, and Lekander (2010) use a block bootstrap to obtain an acceptable region for an asset allocation problem with three assets.

The bootstrap is applied in this paper to perform a test that is used to make the portfolio rebalancing decision. To the best of my knowledge, Michaud and Michaud (2008) are the first and only to use such a test. They use the squared tracking error between the current portfolio and the target portfolio as a test statistic. The distribution of the test statistic is obtained by bootstrapping a series of optimal portfolios weight and computing the squared tracking error between the bootstrapped portfolio and the target portfolio. Then, the portfolio is rebalanced only if the test statistic exceeds a predetermined quantile of the distribution of bootstrapped test statistics.

The bootstrap rebalancing strategy applied in this paper is based on the method of Michaud and Michaud (2008), although there are some distinctions. The test statistic used in this research differs as it is derived from the mean-variance framework of Dybvig and Pezzo (2019) with transaction costs. In addition, the strategy is applied to mean-variance optimised portfolios rather than portfolios constructed by bootstrapping the efficient frontier. As a result, the proposed strategy is more widely applicable for portfolio managers. This paper is the first to empirically verify the usefulness of a bootstrap test in the context of portfolio rebalancing.

3 Bootstrap Rebalancing Strategy

This section introduces the bootstrap rebalancing strategy. The statistic that is used to test for a gain performance is derived from a mean-variance utility framework in Section 3.1. Subsequently, the implementation of the bootstrap rebalancing strategy is discussed in detail in Section 3.2.

3.1 Derivation Test Statistic

This paper builds on the portfolio optimisation method of Dybvig and Pezzo (2019) that takes transaction costs into account. It is assumed that the investor has linear mean-variance preferences and maximises the utility derived from terminal wealth in a single-period setting. In this case, the investor solves the utility maximisation problem

$$\begin{aligned} \max_{\Delta_+,\Delta_-} \quad & \theta'\mu - \frac{\lambda}{2}\theta'\Sigma\theta - c(\Delta_+,\Delta_-) \\ \text{s.t.} \quad & \theta = \theta_0 + \Delta_+ - \Delta_- \\ & \Delta_+ \geq 0 \\ & \Delta_- \geq 0, \end{aligned} \tag{3.1}$$

where Δ_+ and Δ_- are vectors denoting purchases and sales of assets, θ and θ_0 are vectors denoting the new and initial wealth allocation, μ is a vector of expected asset returns in excess of the risk-free rate, Σ is a positive definite covariance matrix, λ is a positive risk aversion parameter, and $c(\Delta_+,\Delta_-)$ is a transaction cost function. The cost function can be seen as a factor that negatively contributes to the return in the standard mean-variance optimisation problem. Dybvig and Pezzo (2019) use a cost function that allows for fixed and proportional costs that can be asset-specific and differ depending on whether an asset is purchased or sold.

In this research, the optimisation is restricted to variance minimisation with transaction costs, because the expected return is difficult to forecast for equity data and including it can result in poor portfolio performance (Chopra & Ziemba, 1993). Furthermore, a constraint is added to programming problem (3.1) to ensure that the investor remains fully invested at all times. Consequently, the optimisation problem becomes

$$\begin{aligned} \max_{\Delta_+,\Delta_-} \quad & -\frac{\lambda}{2}\theta'\Sigma\theta - c(\Delta_+,\Delta_-) \\ \text{s.t.} \quad & \theta = \theta_0 + \Delta_+ - \Delta_- \\ & \Delta_+ \geq 0 \\ & \Delta_- \geq 0 \\ & \Delta_+'\iota = \Delta_-'\iota, \end{aligned} \tag{3.2}$$

where ι is a vector of ones. The cost function in programming problem (3.2) is limited to include proportional transaction costs that can possibly vary for purchases and sales on an asset basis. Hence, the cost function can be expressed as

$$c(\Delta_+,\Delta_-) = \Delta_+'C_+ + \Delta_-'C_-, \tag{3.3}$$

where C_+ and C_- are non-negative vectors representing the transaction costs incurred when purchasing or selling assets, respectively. This cost function is employed in order to obtain a convex programming problem, which is considerably easier to solve than the non-convex optimisation problem obtained with fixed costs (Lobo, Fazel, & Boyd, 2007).

To derive the statistic to test for a significant improvement in portfolio performance, one should first note that maintaining the current portfolio allocation θ_0 in programming problem (3.2) yields an objective value of $-\frac{\lambda}{2}\theta_0'\Sigma\theta_0$, because $\theta = \theta_0$ implies $\Delta_+ = \Delta_- = 0$ and the cost function $c(\cdot)$ drops out of the equation. The gain from rebalancing is given by the difference between the objectives values realised when having the optimised allocation θ^* and the current portfolio allocation θ_0, which amounts to

$$D = -\frac{\lambda}{2}(\theta^{*\prime}\Sigma\theta^* - \theta_0'\Sigma\theta_0) - c(\Delta_+, \Delta_-). \tag{3.4}$$

This is a natural test statistic to test for an improvement in portfolio performance. This statistic can be augmented by $(\theta^* - \theta_0)'\mu$ in case the expected returns are estimated. In the absence of transaction costs, the gain in portfolio performance is given by

$$-\frac{\lambda}{2}(\theta^{*\prime}\Sigma\theta^* - \theta_0'\Sigma\theta_0), \tag{3.5}$$

which may be used as a test statistic if trading is assumed to be frictionless. After some scaling, test statistic (3.5) shows some similarity to the squared tracking error

$$TE^2 = (\theta^* - \theta_0)'\Sigma(\theta^* - \theta_0), \tag{3.6}$$

which is the test statistic of Michaud and Michaud (2008). The squared tracking error gives the *variance of the difference* between the optimal and the current portfolio allocation. This is generally unequal to test statistic (3.5), which gives the *difference of the variance* between the optimal and the current portfolio allocation. Test statistic (3.5) is a more complete measure than test statistic (3.6), because it computes the variance of the entire allocation rather than the change in the allocation.

3.2 Implementation

The test statistic derived in the previous section is used to test whether a gain in portfolio performance is statistically significant. The null and alternative hypothesis using test statistic D defined by equation (3.4) are formulated as follows:

H_0: rebalancing from the current portfolio allocation to the optimal allocation provides no gain in variance after deducting transaction costs ($D = 0$).

H_a: rebalancing from the current portfolio allocation to the optimal allocation provides a gain in variance after deducting transaction costs ($D > 0$).

Inspired by the rebalancing rule of Michaud and Michaud (2008), the outcome of this test is determines whether the portfolio is rebalanced. The bootstrap rebalancing strategy that applies this test proceeds as described below.

Bootstrap Rebalancing Strategy

1. Estimate the covariance matrix Σ^* of the asset returns.

2. Estimate the optimal portfolio allocation θ^* by solving programming problem (3.2) for Δ_+^* and Δ_-^* using $\hat{\Sigma}^*$ and current portfolio allocation θ_0.

3. Compute test statistic (3.4) denoted by D^* with θ^*, current portfolio allocation θ_0, $\Delta_+ = \Delta_+^*$, $\Delta_- = \Delta_-^*$, and $\Sigma = \hat{\Sigma}^*$.

4. For $b = 1, ..., B$:

   (a) Block bootstrap the asset returns.

   (b) Estimate the covariance matrix $\Sigma^{(b)}$ using the bootstrapped asset returns.

   (c) Estimate the optimal portfolio allocation $\theta^{(b)}$ by solving programming problem (3.2) for $\Delta_+^{(b)}$ and $\Delta_-^{(b)}$ using $\hat{\Sigma}^{(b)}$ and current portfolio allocation θ_0.

   (d) Compute test statistic (3.4) denoted by $D^{(b)}$ with θ^*, $\theta_0 = \theta^{(b)}$, $\Delta_+ = (\theta^* - \theta^{(b)})^+$, $\Delta_- = (\theta^* - \theta^{(b)})^-$, and $\Sigma = \hat{\Sigma}^*$.[1]

5. Rebalance the portfolio to θ^* if D^* is positive and larger than percentile $1 - \alpha$ of the empirical density function of $D^{(b)}$, else maintain portfolio allocation θ_0.

In this procedure, the distribution of the test statistic under H_0 is simulated by comparing the asset allocation optimised using the original data to a large number of allocations optimised using bootstrapped data. The significance level for rejecting H_0 is given by one minus the quantile of the largest bootstrapped test statistic $D^{(b)}$ that is below D^*. Within the framework of Dybvig and Pezzo (2019), this equals the probability that the portfolio is outside of the no-trade region. Increasing hyperparameter α increases the rebalancing frequency. Michaud and Michaud (2008) state that the optimal α varies considerably per investor. For example, an α of 90% might be appropriate for an active investor, whereas 10% suffices for a passive investor.

Besides the difference in the test statistic, this strategy also differs from the rebalancing rule of Michaud and Michaud (2008) in terms of the optimisation. They optimise the portfolio in step 2 and 4(c) by bootstrapping the efficient frontier rather than by solving programming problem (3.2). This non-linear programming problem can be modelled as a quadratic program that is solved efficiently using the `quadprog` function in `MATLAB` as suggested by Maurer et al. (2019) (see Appendix A for details). Similarly to Michaud and Michaud (2008), the covariance matrix that is an input for the optimisation problem can be estimated using a variety of methods, preferably chosen based on the structure of the data set.

For the percentile bootstrap in step 4 of the algorithm, both the circular block bootstrap of Politis and Romano (1992) and the stationary bootstrap of Politis and Romano (1994) are applicable. Using either method, the block size is selected based on the algorithm provided by Politis and White (2004) with the correction of Patton et al. (2009). Naturally, the sample size of the bootstrap is set equal to the sample size of the returns. The number of bootstrap iterations B is set to 1,000, which is sufficient for statistical inference at the 95% to 99% level (Davison & Hinkley, 1997).

[1] Here $(\cdot)^+$ and $(\cdot)^-$ denote vectors with zero for all non-positive and non-negative elements, respectively.

4 Empirical Study

This section describes the set-up of the empirical study and presents its main results. Section 4.1 discusses the applied industry portfolio data set. The methods that are used to estimate the covariance matrix are outlined in Section 4.2. Section 4.3 discusses the competing asset allocation strategies that are evaluated based on the performance measures listed in Section 4.4. The performance of the strategies is examined in Section 4.5. Finally, Section 4.6 closely investigates the functioning of the bootstrap rebalancing strategies.

4.1 Data

The data set contains 49 industry portfolios retrieved from Kenneth French's data library for the period from 2000 until 2019, which includes 5031 daily observations.[2] These industry portfolios are value-weighted and composed of stocks from the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotations (NASDAQ) based on the standard industrial classification code. The portfolios are non-tradeable, but similar portfolios could be obtained in practice using exchange traded funds. A full list of the industries with summary statistics can be found in Table C.1 in Appendix C.

Table 4.1 summarises the performance of the industry portfolios. The average return amounts to 0.04% per day with a modest standard deviation of 0.01%. The Shipbuilding and Railroad Equipment industry is a notable positive outlier that yields a return of 0.07% on a daily basis. The average volatility amounts to 1.59% per day with substantial differences across industries. The least volatile industries have an average daily volatility close to 1% and mainly sell fast-moving consumer goods. The most volatile industries realise a volatility of over 2% per day and include Steel Works, Precious Metals and Coal industries. The differences in the return and volatility across industries cause Sharpe ratios to vary widely. The best performing industries with respect to this measure are Defence and Tobacco Products industries with Sharpe ratios of 0.77, whereas the Printing and Publishing industry performs worst with a Sharpe ratio of just 0.13. The industry portfolios are highly correlated with each other as shown by the average cross-industry correlation of 0.55. The average correlation with the market amounts to 0.74, which indicates a large amount of covariance occurs through a market factor.

Table 4.1: Descriptive statistics of daily industry portfolio data from 2000-2019.
All statistics are daily in %, except for the annualised Sharpe ratio.

	Mean	Standard deviation	Minimum	Maximum
Mean return	0.043	0.013	0.015	0.073
Volatility	1.587	0.390	0.975	3.104
Sharpe ratio	0.413	0.151	0.128	0.769
Correlation across industries	0.549	0.151	0.096	0.868
Correlation with the market	0.737	0.137	0.195	0.925

[2] See `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

4.2 Covariance Estimation

The covariance matrix is an important input to portfolio optimisation problem (3.2). It is estimated using a rolling window of 250 trading days, which is in line with the methodology applied by Ledoit and Wolf (2017) to daily equity data. The covariance matrix is estimated using the sample estimator and two more robust estimation methods, because the sample estimator is prone to estimation error (Jobson & Korkie, 1980).

The first robust estimator is the shrinkage estimator of Ledoit and Wolf (2003). This estimator shrinks the sample covariance matrix to the covariance matrix implied by the capital asset pricing model. More specifically, the covariance matrix is estimated as

$$\hat{\Sigma}_{\text{LW}} = \frac{\kappa}{T}F + (1 - \frac{\kappa}{T})S, \tag{4.1}$$

where κ is a shrinkage constant, T is the sample size, F is the shrinkage target, and S is the sample estimator. The shrinkage constant κ is a term that increases with the error of the estimated sample covariance matrix, decreases with the misspecification of the shrinkage target, and decreases with covariance between the shrinkage target and the sample estimator.[3] The elements of shrinkage target F are estimated by performing the one-factor model regression

$$r_t - \bar{r} = \beta r^e_{m,t} + \varepsilon_t, \tag{4.2}$$

where r_t is the return vector at time t, $\bar{r}$ is the mean return vector, and $r^e_{m,t}$ is the market return in excess of the risk-free rate from Kenneth French's data library. After performing the regression, the structured estimator is computed as

$$F = s^2_{r^e_m}\hat{\beta}\hat{\beta}' + \text{diag}(\hat{\Sigma}_\varepsilon), \tag{4.3}$$

where $s^2_{r^e_m}$ is the sample variance of the excess market return, $\hat{\beta}$ is the estimated coefficient vector from regression (4.2), and $\text{diag}(\hat{\Sigma}_\varepsilon)$ is the diagonal of the regression residuals' estimated sample covariance matrix.

The second robust covariance estimator is the Principal Orthogonal complEment Thresholding (POET) method of Fan et al. (2013). This method uses statistical factors rather than an economic factor to estimate the covariance matrix. Assuming there are n assets, applying PCA yields n estimated eigenvalues $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq ... \geq \hat{\lambda}_n$ with corresponding estimated eigenvectors $\hat{e}_1, \hat{e}_2, ..., \hat{e}_n$. Then, the estimated covariance matrix can be decomposed as

$$\hat{\Sigma} = \sum_{i=1}^{k} \hat{\lambda}_i \hat{e}_i \hat{e}_i' + \sum_{i=k+1}^{n} \hat{\lambda}_i \hat{e}_i \hat{e}_i' = \sum_{i=1}^{k} \hat{\lambda}_i \hat{e}_i \hat{e}_i' + \hat{R}_k, \tag{4.4}$$

where k is a number of diverging eigenvalues and $\hat{R}_k$ is the estimated principal orthogonal complement. To induce sparsity in the orthogonal complement, Fan et al. (2013) suggest thresholding elements $\hat{r}_{ij}$ of $\hat{R}_k$ for $i, j = 1, ...n$. The elements of the thresholded orthogonal complement $\hat{R}^\tau_k$

[3] See equation (4) in Ledoit and Wolf (2003) for details.

are given by

$$\hat{r}^{\tau}_{ij} = \begin{cases} \hat{r}_{ij} & \text{if } i = j \\ s_{ij}(\hat{r}_{ij}) I(|\hat{r}_{ij}| \geq \tau_{ij}) & \text{if } i \neq j, \end{cases} \tag{4.5}$$

where $s_{ij}(\cdot)$ is a generalised shrinkage function and $\tau_{ij} \geq 0$ is a thresholding parameter. Following their recommendation for stock data, the soft thresholding function $s_{ij}(\hat{r}_{ij}) = \text{sign}(\hat{r}_{ij}) \cdot (|\hat{r}_{ij}| - \tau_{ij})^{+}$ is applied in combination with the adaptive thresholding parameter of Cai and Liu (2011). This thresholding parameter has the advantage that it differs per entry; the threshold becomes higher if there is more uncertainty in the covariance estimate. In addition, the threshold decreases with the sample size.[4] Using the thresholded principal orthogonal complement, the covariance matrix is estimated as

$$\hat{\Sigma}_{\text{POET}} = \sum_{i=1}^{\hat{k}} \hat{\lambda}_i \hat{e}_i \hat{e}_i' + \hat{R}^{\tau}_{\hat{k}}, \tag{4.6}$$

where $\hat{k}$ is an estimate of the number of diverging factors. As suggested by Fan et al. (2013), this number is selected based on the information criterion of Bai and Ng (2002) by solving

$$\min_{0 \leq k \leq M} \log\left(\frac{1}{nT}||Y - \hat{F}^k \hat{\beta}^k||_2\right) + k\left(\frac{n+T}{nT}\log\left(\frac{nT}{n+T}\right)\right), \tag{4.7}$$

where n is the number of assets, T is the sample size, Y is a matrix of asset returns, $\hat{F}^k = Y[\hat{e}_1, \hat{e}_2, ..., \hat{e}_k]$ is a matrix of estimated factor realisations, $\hat{\beta}^k$ is a vector of coefficients obtained by regressing Y on $\hat{F}^k$, and $|| \cdot ||_p$ denotes the L^p norm. A maximum of $M = 20$ factors is imposed to effectively shrink the non-diagonal elements of the covariance matrix. The optimal number of factors is estimated once using the entire data sample and then kept fixed over time for consistency and computational purposes.

4.3 Asset Allocation Strategies

The competing asset allocation strategies differ in two dimensions; the rebalancing strategy and optimisation method. The first rebalancing strategy is the buy-and-hold strategy, which only optimises the portfolio allocation at the beginning of the sample and then holds the allocation. Perold and Sharpe (1995) show this strategy performs well in case there is momentum. The buy-and-hold strategy serves as a benchmark to evaluate the benefit of rebalancing at all.

In addition, two naive rebalancing strategies are considered. The first is the periodic rebalancing strategy, which rebalances at a fixed frequency. This strategy is implemented with daily (1 trading day), weekly (5 trading days), monthly (20 trading days), quarterly (60 trading days), and annual (250 trading days) rebalancing. The second is the threshold rebalancing strategy, which only rebalances the portfolio if the current wealth allocation deviates from the optimal allocation by some threshold. Mathematically, this strategy rebalances the portfolio if

$$\frac{||\theta_0 - \theta^*||_2}{||\theta_0||_2} > \tau, \tag{4.8}$$

[4] See equation (3.2) in Fan et al. (2013) for details. In this equation, parameter C is set to one following their recommendation for stock data.

where θ_0 is the current portfolio allocation, θ^* is the optimal portfolio allocation, and τ is a pre-set threshold. This strategy is implemented with τ at the 5%, 10%, 20%, and 40% level.

Lastly, the bootstrap portfolio strategy proposed in Section 3 is considered. The strategy is implemented with α at the 1%, 25%, 50%, 75% and 99% level. By applying the bootstrap rebalancing strategy, it can be inferred to what extent taking estimation uncertainty into account enhances portfolio performance.

Portfolios are optimised using two methods that differ in their consideration of transaction costs. The first method ignores transaction costs and applies global minimum variance weights that are obtained by solving

$$\begin{aligned} \min_{w} \quad & w'\Sigma w \\ \text{s.t.} \quad & w'\iota = 1, \end{aligned} \tag{4.9}$$

which has the unique solution[5]

$$w^* = \frac{\Sigma^{-1}\iota}{\iota'\Sigma^{-1}\iota}. \tag{4.10}$$

When portfolio weights are GMV optimised, then transaction costs are also neglected in the bootstrap rebalancing strategy. More specifically, programming problem (4.9) is solved rather than (3.2) in step 2 and 4(c) of the algorithm presented in Section 3.2. In addition, the test for portfolio efficiency only considers the gain in variance while neglecting transaction costs, such that statistic (3.5) replaces (3.4) in step 3 and 4(d) of the bootstrap rebalancing strategy. This optimisation method serves as a benchmark to beat for the method that takes transaction costs into account.

The second optimisation method takes transaction costs into account by solving convex portfolio optimisation problem (3.2). To solve this problem, assumptions about the investor's risk aversion and the size of transaction costs are required. The risk aversion parameter λ determines the importance of minimising variance relative to realising high returns. Dybvig and Pezzo (2019) show that the amount of trading activity increases with the investor's risk aversion. Bodnar, Okhrin, Vitlinskyy, and Zabolotskyy (2018) indicate that λ regularly ranges from 1 to 10. A risk aversion parameter of three is chosen to represent an investor with moderate risk aversion. Another key component is the magnitude of the transaction costs since the amount of trading decreases with these costs (Dybvig & Pezzo, 2019). Provided that the industry portfolios are non-tradeable, there is no data on actual transaction costs. Yet, all industry portfolios consist of a large number of US stocks, such that the trading cost can be approximated by the cost of stock transactions. The transaction costs are set to 50 basis points (bps) for a single transaction (i.e., buy or sell) based on the estimates of Balduzzi and Lynch (1999) and Hasbrouck (2009), and the value used by Ledoit and Wolf (2017). This implies that the vectors in cost function (3.3) can be expressed as $C_+ = C_- = 0.0005 \cdot \iota$.

[5]Note that wealth allocation θ can be obtained using portfolio weights w as $\theta = w \cdot \theta_0'\iota$.

4.4 Performance Measures

The performance of the asset allocation strategies is assessed using various measures. The post-transaction cost return measures the return on investment after deducting trading costs. Despite that the optimisation methods focus on variance reduction, the realised return remains important to any investor. The volatility is used to quantify risk. The annualised Sharpe ratio measures the realised return relative to the risk exposure and is given by

$$\sqrt{250}\,\frac{\mu - r_f}{\sigma}, \tag{4.11}$$

where μ denotes the daily average post-transaction cost return, σ denotes the volatility, and r_f is the average risk-free rate.[6] The 3-month constant maturity treasury rate is used as a proxy for the risk-free rate provided that US equity data is used. Treasury rate data is retrieved from the Archival Federal Reserve Economic Data of the Federal Reserve Bank of St. Louis.[7] Moreover, the utility level that a mean-variance investor obtains is given by

$$\mu - \frac{\lambda}{2}\sigma^2, \tag{4.12}$$

where μ and σ^2 denote the net portfolio return and variance. The population parameters μ and σ^2 that are used to compute the Sharpe ratio and the utility level are estimated using the sample mean and variance estimator, respectively.

To quantify the effect rebalancing strategies have on trading, the transaction costs and the average rebalancing time (ART) are calculated. The transaction costs are expressed in basis points of the amount invested, such that the accumulation of wealth does not influence this measure. The ART is computed as the amount of times the portfolio is rebalanced divided by the number of trading days.[8] The ART indicates how transaction costs are incurred, as this both depends on the rebalancing frequency and the magnitude of trades.

To make statistical inference on the difference in performance of two asset allocation strategies, the robust hypothesis testing procedure of Ledoit and Wolf (2008) is used. They apply a studentised bootstrap to test for a significant difference in Sharpe ratios. With this method, the studentised test statistic is computed in each bootstrap iteration and the distribution of this statistic is subsequently used to construct a confidence interval. The studentised bootstrap is often more efficient than a standard bootstrap, especially when applying a non-parametric bootstrap (Davison & Hinkley, 1997). Ledoit and Wolf (2011) show a similar approach can be used to test for a significant difference in the variance of two portfolio strategies. This approach can also be adjusted to test for a difference in the realised post-transaction cost return and the utility level (see Appendix B for details). The bootstrap procedure is performed using 10,000 iterations of the circular bootstrap of Politis and Romano (1992) with the same block size as selected for the bootstrap rebalancing strategy.

[6] The Sharpe ratio is computed under the assumption that returns are independent and identically distributed.
[7] See `https://alfred.stlouisfed.org`.
[8] The portfolio is said to be rebalanced if the absolute change in portfolio weights exceeds 10^{-5}.

4.5 Performance of Asset Allocation Strategies

This section evaluates the performance of the asset allocation strategies. Bootstrap rebalancing strategies use the stationary bootstrap of with a block size of 20 trading days. This corresponds to bootstrapping blocks of monthly data, which is a robust choice based on the output of the automatic block length selection algorithm of Politis and White (2004) (see Table D.1 in Appendix D for details). However, Section 5.3 verifies that these choices are inessential for obtaining the results. For the POET covariance estimation method, five principal components are selected using the information criterion of Bai and Ng (2002).

Table 4.2 shows how the asset allocation strategies perform. First, differences across rebalancing strategies are examined. Clearly, there are large differences in the daily returns. These differences predominantly arise due to deviations in the incurred transaction costs. As expected, transaction costs decrease with the average rebalancing time. For this reason, bootstrap rebalancing strategies with a lower α achieve higher returns. Interestingly, the decline of transaction costs with the ART is non-linear; costs decrease more rapidly as the ART goes to zero. This phenomenon is best illustrated using the strategies optimised excluding transaction costs in Panel A. The daily rebalancing strategy incurs transaction costs of 12.2 bps per day, which is substantially more than the weekly rebalancing strategy with 6.1 bps per day. However, the transactions costs only decrease to 3.3 and 2.0 bps when further reducing the rebalancing frequency to monthly and quarterly, respectively. A similar pattern is also observed for other strategies that are optimised excluding transaction costs.

Table 4.2 evidently shows that volatility is positively correlated with transaction costs and negatively correlated with the ART. As a consequence, asset allocation strategies that rebalance daily achieve the lowest volatility for both optimisation methods in all panels. Nevertheless, bootstrap rebalancing strategies seem to be more efficient in reducing variance than periodic rebalancing strategies. For example, the volatility of Boot 99% strategies optimised excluding transaction costs is lower than that of monthly rebalancing strategies despite a similar ART.

The highest Sharpe ratios for strategies optimised excluding transaction costs are obtained by the annual rebalancing strategy and the Boot 1% strategy due to their high ART. Only these strategies outperform the buy-and-hold strategy in terms of the Sharpe ratio in Panels A and B. As shown in Table 4.3, strategies that rebalance frequently perform significantly worse than the buy-and-hold strategy. On the other hand, the differences in the Sharpe ratio are much smaller for strategies optimised including transaction costs, because they all incur low transaction costs (a maximum of 0.4 bps per day in Panel C). Table 4.3 shows that these strategies all have higher Sharpe ratios than the buy-and-hold strategy, although only the Boot 1% performs significantly better at the 10% level in Panels A and B.

In terms of utility, Table 4.3 shows that all rebalancing strategies significantly outperform the buy-and-hold strategy. Table 4.2 shows that the Boot 99% strategy performs best in Panel A for portfolios optimised excluding transaction costs. This strategy also performs well in Panels B and C, but the weekly rebalancing strategy performs marginally better. For portfolios optimised including transaction costs, Table 4.2 shows that the daily rebalancing strategy achieves the highest utility level in all panels. The reason for this is that portfolios optimised including

Table 4.2: Performance of asset allocation strategies applied on daily industry portfolio data from 2000-2019.

This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.038**	1.102	0.509	1.000	**0.000**	∞	0.038	1.102	0.509	1.000	**0.000**	∞
Daily	-0.082	**0.716**	-1.873	2.092	12.244	1.000	0.039	**0.742**	0.777	**2.266**	0.331	1.271
Weekly	-0.020	0.725	-0.503	2.204	6.135	5.000	0.040	0.747	0.787	2.237	0.300	5.494
Monthly	0.007	0.742	0.091	2.177	3.276	20.000	0.041	0.755	0.810	2.192	0.269	20.603
Quarterly	0.018	0.774	0.312	2.022	1.956	60.506	0.041	0.780	0.776	2.044	0.234	60.506
Annually	0.033	0.816	0.578	1.844	0.966	251.579	0.044	0.806	0.806	1.914	0.160	251.579
Thld 5%	-0.063	0.721	-1.454	2.111	10.387	1.708	0.040	0.747	0.789	2.235	0.293	14.984
Thld 10%	-0.036	0.725	-0.844	2.163	7.557	3.565	0.041	0.747	0.817	2.243	0.270	31.656
Thld 20%	-0.008	0.731	-0.242	2.198	4.702	9.835	**0.044**	0.766	**0.854**	2.134	0.255	70.294
Thld 40%	0.012	0.747	0.185	2.157	2.633	33.194	0.039	0.815	0.704	1.864	0.173	217.273
Boot 1%	0.033	0.790	**0.598**	1.975	1.124	177.037	0.043	0.759	0.842	2.171	0.309	2.178
Boot 25%	0.023	0.756	0.426	2.134	1.623	91.923	0.039	0.742	0.777	2.266	0.331	1.691
Boot 50%	0.020	0.751	0.370	2.160	1.935	64.595	0.039	0.742	0.777	2.266	0.331	1.691
Boot 75%	0.016	0.749	0.272	2.160	2.204	48.776	0.039	0.742	0.777	2.266	0.331	1.691
Boot 99%	0.005	0.732	0.052	**2.229**	3.217	21.339	0.039	0.742	0.777	2.266	0.331	1.691
Panel B: Shrinkage estimator												
B&H	**0.039**	1.084	0.524	1.000	**0.000**	∞	0.039	1.084	0.524	1.000	**0.000**	∞
Daily	-0.047	**0.689**	-1.154	2.270	8.771	1.000	0.039	**0.732**	0.783	**2.252**	0.290	1.306
Weekly	-0.003	0.695	-0.138	**2.365**	4.423	5.000	0.040	0.738	0.789	2.216	0.262	5.565
Monthly	0.016	0.711	0.292	2.322	2.397	20.000	0.041	0.746	0.807	2.172	0.236	20.603
Quarterly	0.024	0.744	0.455	2.140	1.465	60.506	0.040	0.770	0.768	2.029	0.202	60.506
Annually	0.035	0.780	0.652	1.961	0.749	251.579	0.043	0.794	0.808	1.911	0.138	251.579
Thld 5%	-0.030	0.693	-0.760	2.295	7.069	1.979	0.039	0.736	0.771	2.226	0.251	15.672
Thld 10%	-0.010	0.694	-0.283	2.355	4.978	4.426	0.040	0.743	0.784	2.187	0.232	33.194
Thld 20%	0.008	0.707	0.122	2.327	3.079	12.679	0.042	0.750	0.826	2.149	0.210	77.097
Thld 40%	0.021	0.717	0.401	2.300	1.729	43.063	0.040	0.802	0.731	1.863	0.161	217.273
Boot 1%	0.035	0.738	**0.681**	2.204	0.941	154.194	**0.044**	0.748	**0.862**	2.166	0.269	2.299
Boot 25%	0.026	0.728	0.502	2.241	1.298	81.017	0.039	0.732	0.783	2.251	0.290	1.790
Boot 50%	0.021	0.720	0.404	2.280	1.513	57.590	0.039	0.732	0.783	2.252	0.289	1.789
Boot 75%	0.022	0.708	0.421	2.358	1.717	44.259	0.039	0.732	0.783	2.252	0.289	1.789
Boot 99%	0.014	0.704	0.245	2.363	2.491	19.352	0.039	0.732	0.783	2.252	0.289	1.789
Panel C: POET estimator												
B&H	**0.039**	0.971	0.593	1.000	**0.000**	∞	0.039	0.971	0.593	1.000	**0.000**	∞
Daily	-0.039	**0.729**	-0.911	1.647	7.490	1.000	0.036	**0.756**	0.693	**1.678**	0.372	1.342
Weekly	-0.001	0.733	-0.085	**1.705**	3.817	5.000	0.037	0.760	0.704	1.658	0.325	5.677
Monthly	0.017	0.747	0.295	1.680	2.074	20.000	0.038	0.765	0.720	1.638	0.272	20.603
Quarterly	0.024	0.775	0.433	1.569	1.260	60.506	0.037	0.789	0.688	1.534	0.230	61.282
Annually	0.036	0.792	**0.663**	1.522	0.628	251.579	**0.044**	0.804	**0.804**	1.487	0.173	251.579
Thld 5%	-0.023	0.732	-0.550	1.666	5.798	2.286	0.036	0.758	0.689	1.664	0.319	13.855
Thld 10%	-0.005	0.735	-0.170	1.688	4.075	5.444	0.038	0.760	0.726	1.661	0.302	28.118
Thld 20%	0.008	0.740	0.119	1.692	2.617	15.127	0.039	0.768	0.740	1.629	0.245	69.275
Thld 40%	0.021	0.758	0.369	1.636	1.567	47.327	0.040	0.793	0.746	1.523	0.191	191.200
Boot 1%	0.032	0.785	0.583	1.543	0.857	140.588	0.040	0.777	0.747	1.588	0.296	3.359
Boot 25%	0.023	0.764	0.414	1.615	1.192	71.343	0.036	0.756	0.686	1.673	0.364	1.859
Boot 50%	0.020	0.749	0.362	1.674	1.452	48.283	0.036	0.756	0.693	1.678	0.371	1.766
Boot 75%	0.016	0.744	0.280	1.690	1.853	32.081	0.036	0.756	0.693	1.678	0.371	1.766
Boot 99%	0.007	0.739	0.095	1.696	2.633	14.184	0.036	0.756	0.693	1.678	0.371	1.766

Table 4.3: Differences in the Sharpe ratio and utility level between asset allocation strategies and the buy-and-hold strategy applied on daily industry portfolio data from 2000-2019.

This table shows the difference in the annualised Sharpe ratio (ΔSR) and utility level multiplied by 100 (ΔU) of the periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies relative to the buy-and-hold strategy. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days (with the sample, shrinkage, and POET estimator) and assuming transaction costs of 50 basis points. The test for a difference in Sharpe ratio is performed using a studentised bootstrap based on Ledoit and Wolf (2008) and the test for a difference in utility is performed using an adaptation of that test. Significance for the test that $\Delta(\cdot) = 0$ is indicated at the 10%, 5% and 1% level by *, **, and ***, respectively. The best performing strategy within each column per covariance estimation method is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	Sample		**Shrinkage**		**POET**		**Sample**		**Shrinkage**		**POET**	
	ΔSR	ΔU	ΔSR	ΔU	ΔSR	ΔU	ΔSR	ΔU	ΔSR	ΔU	ΔSR	ΔU
Daily	-2.382***	93.009***	-1.678***	96.405***	-1.504***	54.078***	0.269	**99.571*** **	0.259	**95.776*** **	0.101	**55.611*** **
Weekly	-0.827***	97.076***	-0.531**	**99.070*** **	-0.567**	**57.014*** **	0.303	98.514***	0.285	94.647	0.116	53.943***
Monthly	-0.418*	96.351***	-0.232	98.076***	-0.298	55.725***	0.301	96.912***	0.283	92.975***	0.127	53.599***
Quarterly	-0.196	90.070***	-0.069	91.764***	-0.160	49.935***	0.268	91.034***	0.244	87.368***	0.095	47.940***
Annually	0.069	81.591***	0.128	84.433***	**0.071**	47.189***	0.297	85.108***	0.284	82.113***	**0.211**	45.083***
Thld 5%	-1.962***	93.798***	-1.284***	97.210***	-1.143***	55.015***	0.280	98.470***	0.246	94.876***	0.096	54.939***
Thld 10%	-1.353***	95.822***	-0.807***	99.141***	-0.762***	56.111***	0.308	98.756***	0.260	93.506***	0.133	54.794***
Thld 20%	-0.751***	97.123***	-0.403*	98.233***	-0.474**	56.276***	**0.345**	94.710***	0.301	92.133***	0.147	53.159***
Thld 40%	-0.323	95.578***	-0.123	97.371***	-0.223	53.491***	0.195	82.607***	0.207	79.818***	0.153	47.249***
Boot 1%	**0.090**	87.963***	**0.156**	94.123***	-0.010	48.430***	0.333*	96.110***	**0.338***	92.727***	0.154	50.969***
Boot 25%	-0.082	94.710***	-0.022	95.391***	-0.178	52.424***	0.269	99.569***	0.259	95.749***	0.093	55.372***
Boot 50%	-0.139	95.724***	-0.120	96.735***	-0.231	55.417***	0.269	99.569***	0.259	95.772***	0.101	55.607***
Boot 75%	-0.237	95.691***	-0.104	99.227***	-0.313	56.178***	0.269	99.569***	0.259	95.772***	0.101	55.607***
Boot 99%	-0.457*	**98.268*** **	-0.279	99.381***	-0.498**	56.505***	0.269	99.569***	0.259	95.772***	0.101	55.607***

transaction costs are already conservative with rebalancing, such that further reducing this in any way is not beneficial.

The performance of covariance estimation methods can be assessed by looking at the differences across panels in Table 4.2. Provided that all asset allocation strategies minimise variance, the accuracy of the estimation methods is best evaluated using the volatility. The shrinkage estimator evidently performs best with respect to this measure, since the achieved volatility in Panel B is lower than in Panel A and much lower than in Panel C. However, accuracy is not the only relevant property, because stability also helps to reduce the amount of rebalancing. In this respect, both the shrinkage and POET estimator perform well, because they are structured estimators. Overall, the shrinkage estimator is the preferred estimator, because the highest Sharpe ratio and utility level are both found in Panel B; the Boot 1% strategy optimised including transaction costs yields the highest Sharpe ratio of 0.86 and the weekly rebalancing strategy optimised excluding transaction costs realises the highest utility level.

The relative performance of the optimisation methods is evaluated by comparing the left- and right-hand side of Table 4.2. The large differences in the portfolio performance across the optimisation methods can largely be explained by the differences in the amount of rebalancing. For the same rebalancing strategy, the strategy optimised including transaction costs incurs much lower trading costs than the strategy optimised excluding transaction costs. Table 4.4 further shows how the choice of optimisation method affects the post-transaction cost returns, Sharpe ratio, and utility level. Portfolios optimised including transaction costs achieve a significantly higher return and Sharpe ratio at the 5% level for all strategies except for the annual and Boot 1% rebalancing strategy. This clearly illustrates that the method of Dybvig and Pezzo (2019) greatly improves upon GMV optimisation.[9] Strategies optimised including transaction costs usually achieve higher utility levels in Panel A but lower utility levels in Panels B and C. Table 4.2 shows that the reason behind this is that portfolios optimised excluding transaction costs are less volatile due to more frequent rebalancing. Finally, it should be noted that the bootstrap strategies behave very differently across optimisation methods; they are effective for strategies optimised excluding transaction costs but ineffective for strategies optimised including transaction costs. More specifically, bootstrap rebalancing strategies with a high α perform the same as the daily rebalancing strategy on the right-hand side of Table 4.2. This striking finding is further investigated in Section 4.6.

4.6 Functioning of Bootstrap Rebalancing Strategies

The previous section has shown that using a bootstrap test to make the portfolio rebalancing decision can enhance performance. However, the effectiveness of this approach rests on the selected hyperparameter and the portfolio optimisation method. To investigate the influence of both variables, Figure 4.1 plots the volatility, transaction costs, and utility level as a function of hyperparameter α for portfolios optimised excluding and including transaction costs.

[9] Experimenting with the optimisation method of Dybvig and Pezzo (2019) shows that even better portfolio performance can be realised shrinking $c(\cdot)$ in (3.2) towards zero, thereby increasing the amount of rebalancing.

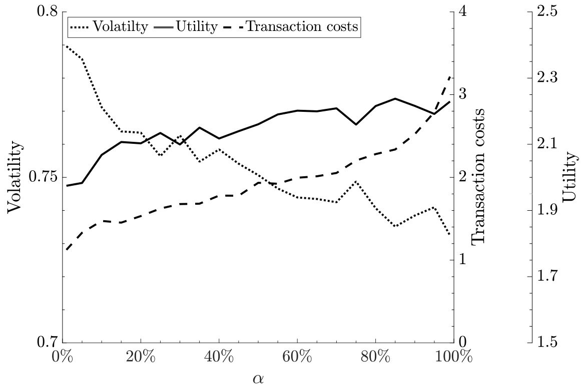
Table 4.4: Differences in the mean return, Sharpe ratio and utility level between asset allocation strategies optimised including and excluding transaction costs applied on daily industry portfolio data from 2000-2019.
This table shows the difference in the mean return in % ($\Delta\mu$), annualised Sharpe ratio (ΔSR) and utility level multiplied by 100 (ΔU) of the periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies optimised including transaction costs compared to those optimised excluding transaction costs. A stationary bootstrap is applied with an average block size of 20 trading days. The difference in performance is between portfolios rebalanced by solving a quadratic programming problem with transaction costs compared to global minimum variance weights. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days (with the sample, shrinkage, and POET estimator) and assuming transaction costs of 50 basis points. The test for a difference in Sharpe ratio is performed using the test of Ledoit and Wolf (2008) and the test for a differences in mean return and utility are performed using an adaptation of that test. Significance for the test that $\Delta(\cdot) = 0$ is indicated at the 10%, 5% and 1% level by *, **, and ***, respectively.

	Panel A: Sample			**Panel B: Shrinkage**			**Panel C: POET**		
	$\Delta\mu$	ΔSR	ΔU	$\Delta\mu$	ΔSR	ΔU	$\Delta\mu$	ΔSR	ΔU
Daily	0.121***	2.651***	6.563***	0.087***	1.937***	-0.629	0.075***	1.605***	1.532
Weekly	0.053***	1.131***	1.437	0.038***	0.816***	-4.423**	0.033***	0.683***	-3.072*
Monthly	0.034***	0.719***	0.561	0.025***	0.515***	-5.101**	0.021***	0.425***	-2.126
Quarterly	0.023***	0.464***	0.964	0.016***	0.313***	-4.396**	0.013**	0.256**	-1.995
Annually	0.011*	0.228*	3.517	0.008	0.156	-2.320	0.008	0.140	-2.106
Thld 5%	0.104***	2.242***	4.672**	0.069***	1.531***	-2.334	0.059***	1.239***	-0.076
Thld 10%	0.077***	1.661***	2.935	0.049***	1.067***	-5.635***	0.043***	0.896***	-1.317
Thld 20%	0.053***	1.096***	-2.412	0.034***	0.704***	-6.100***	0.030***	0.621***	-3.116*
Thld 40%	0.027***	0.518***	-12.971***	0.019***	0.330**	-17.553***	0.020***	0.377***	-6.242***
Boot 1%	0.011*	0.243**	8.147***	0.009*	0.182*	-1.396	0.008	0.163*	2.539
Boot 25%	0.016***	0.351***	4.859***	0.013***	0.281***	0.358	0.013***	0.272***	2.948*
Boot 50%	0.019***	0.408***	3.845**	0.018***	0.379***	-0.962	0.016***	0.332***	0.191
Boot 75%	0.024***	0.506***	3.878**	0.017***	0.362***	-3.455**	0.020***	0.414***	-0.571
Boot 99%	0.034***	0.726***	1.301	0.025***	0.538***	-3.609**	0.029***	0.598***	-0.898

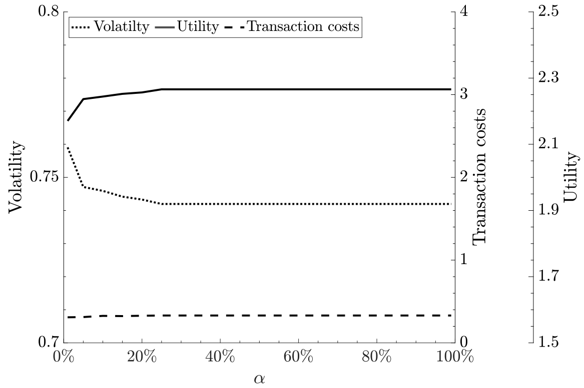
First examining strategies optimised excluding transaction costs, Figures 4.1a-c clearly demonstrate that volatility decreases with α. This decrease is the sharpest when α is close to zero, because it is difficult to reject the null hypothesis of no difference in portfolio performance at a very low significance level. This rejection is further complicated by the fat right tail in the distribution of the bootstrapped test statistics shown in Figures 4.2a-c. Figures 4.1a-c also display a fairly flat relation between the transaction costs and α. The reason for this is that rebalancing now helps to reduce transaction costs in the future, because the current optimal allocation is closer to the future optimal allocation than the allocation that is held now. As a result, rebalancing early is beneficial. Lastly, the utility function indicates that the highest utility level is obtained when α is close to 100%. This may be somewhat surprising given that transaction costs increase rapidly as α approaches 100%. However, a small gain in volatility is more important than a sizeable reduction in transaction costs for an investor with a risk aversion parameter of three. Yet, if the cost of trading is higher or if the investor is less risk-averse then a lower α is better, as shown in Tables D.2-5 in Appendix D.

Turning to portfolios optimised including transaction costs, Figures 4.1d-f show that performance is constant when α is high. The portfolio is always rebalanced if α is above 30% and the test statistic is positive. The explanation behind this is that most bootstrapped test statistics are at or below zero as shown in Figures 4.2d-f. The distribution of bootstrapped test statistics further shows a peak at zero, because there is zero utility gain if both the bootstrapped portfolio

Panel A: Sample estimator

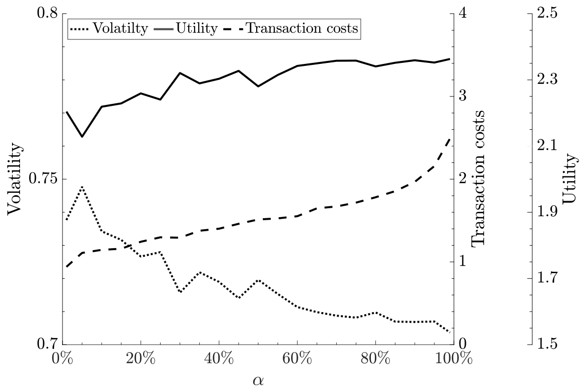


(a) Excluding transaction costs

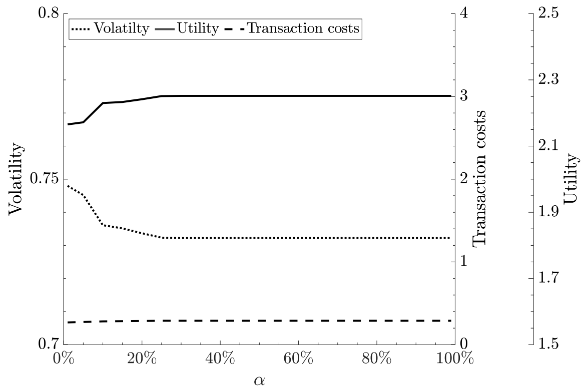


(d) Including transaction costs

Panel B: Shrinkage estimator

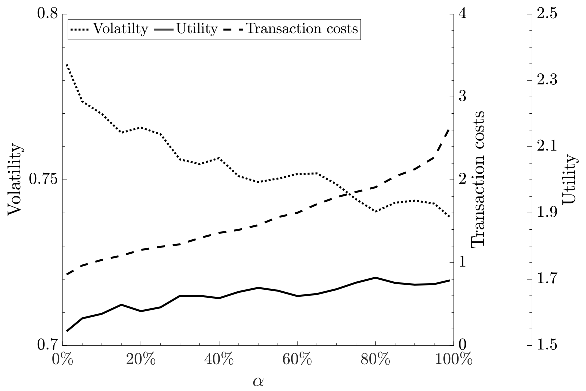


(b) Excluding transaction costs

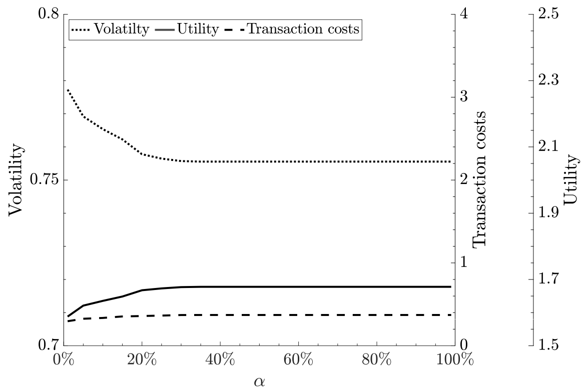


(e) Including transaction costs

Panel C: POET estimator



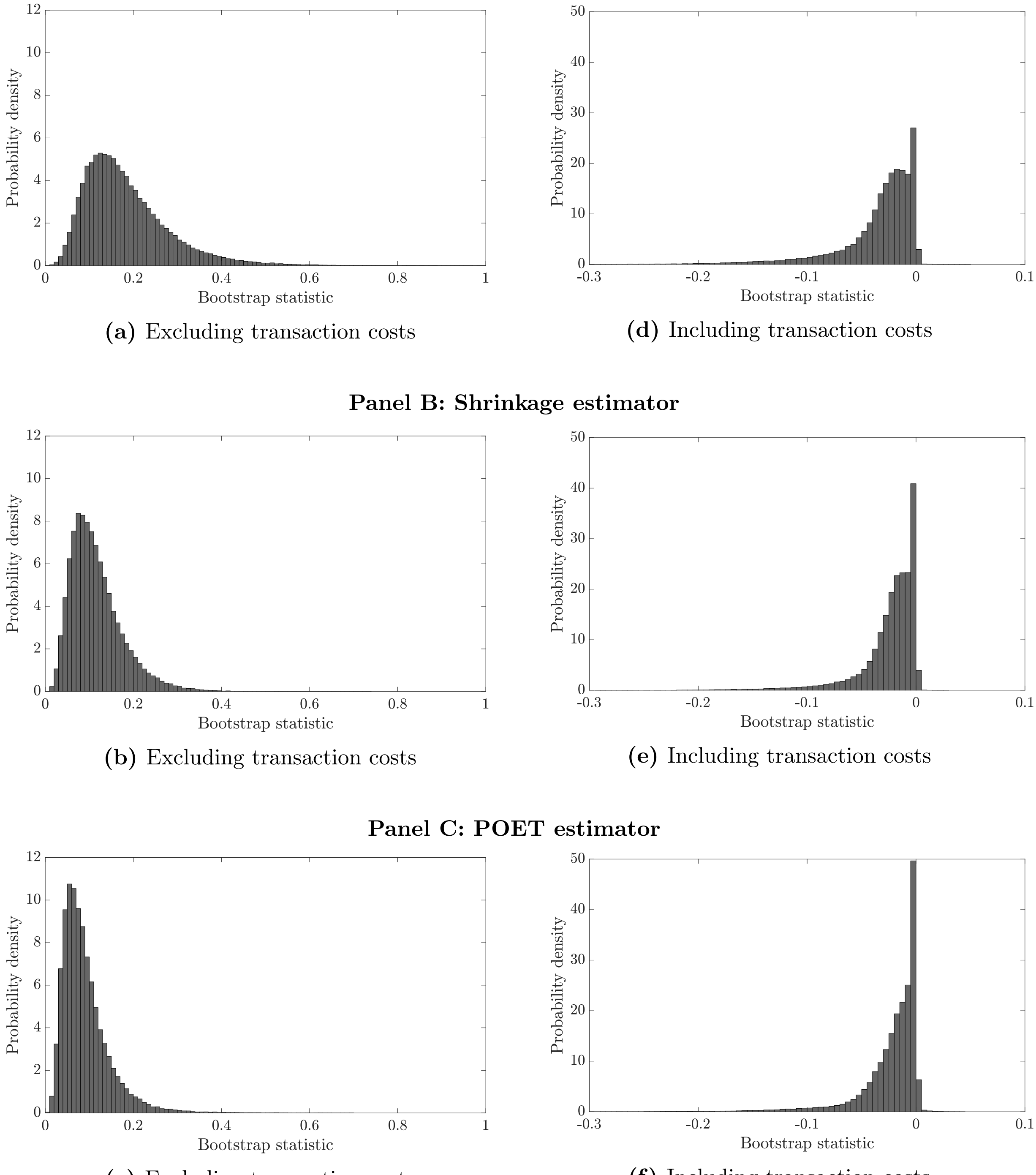
(c) Excluding transaction costs



(f) Including transaction costs

Figure 4.1: Performance of bootstrap rebalancing strategies as a function of hyperparameter α applied on daily industry portfolio data from 2000-2019.

This figure shows the performance of bootstrap rebalancing strategies as a function of hyperparameter α ranging from 1% to 99% with a step size of 5%. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the volatility in %, transaction costs in basis points, and utility of a mean variance investor with risk aversion parameter 3 relative to the buy-and-hold strategy.



Panel A: Sample estimator

(a) Excluding transaction costs

(d) Including transaction costs

Panel B: Shrinkage estimator

(b) Excluding transaction costs

(e) Including transaction costs

Panel C: POET estimator

(c) Excluding transaction costs

(f) Including transaction costs

Figure 4.2: Distribution of bootstrapped test statistics from the bootstrap rebalancing strategies applied on daily industry portfolio data from 2000-2019.

This figure shows the density of the bootstrapped test statistics from the first 100 trading days (100,000 observations) of bootstrap rebalancing strategies with α at 50%. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days and assuming transaction costs of 50 basis points. Note there is a difference in scaling of the figures' axes on the left- and the right-hand side.

allocation $\theta^{(b)}$ and the optimal portfolio allocation θ^* are equal to current allocation θ_0. These peaks are most notable for the structured estimators in Figures 4.2e-f, because there is often no gain from rebalancing after deducing transaction costs when the estimated covariance matrix changes little over time. The large amount of negative bootstrapped statistics indicates that if the portfolio is optimised using a bootstrapped input, then rebalancing causes a loss in utility. This loss results from a small variance gain that is more than offset by a loss from incurred transaction costs. As a result, an opportunity to rebalance nearly always provides a statistically significant gain in performance, such that the portfolio is rebalanced. Always rebalancing is also the optimal strategy as demonstrated in Section 4.5. The rebalancing decision may only be rejected for low levels of α, because there are a few positive bootstrapped test statistics.

Comparing the left- and right-hand side in Figure 4.1, there is a different pattern that arises due to the difference between test statistics (3.4) and (3.5) applied for strategies optimised including and excluding transaction costs, respectively. Test statistic (3.5) solely composes of the gain in variance realised by rebalancing from the current portfolio allocation θ_0 to the optimal portfolio allocation θ^*. Given the estimated covariance matrix Σ^*, the (bootstrapped) test statistic is always non-negative as shown in Figures 4.2a-c. On the other hand, Figures 4.2d-f show that the inclusion of transaction costs in test statistic (3.4) leads to a shift in the distribution of the bootstrapped test statistics to the left (due to a loss in utility from paying transaction costs) and more centring around zero (as transaction costs reduce the amount of rebalancing). The large dissimilarity in the distribution leads to diverging significance tests and hence different outcomes regarding the portfolio rebalancing decision.

To summarise, bootstrap rebalancing strategies often outperform other rebalancing strategies when the portfolio is optimised excluding transaction costs and the right hyperparameter is selected. On the other hand, bootstrap rebalancing strategies are ineffective when they are optimised including transaction costs, because they perform exactly the same as the daily rebalancing strategy when α is high. The reason for this is that the daily rebalancing strategy is already conservative in its rebalancing, such that there is no further gain from incorporating estimation uncertainty. Nevertheless, the bootstrap rebalancing strategy is useful as it provides a statistical rationale behind applying this rebalancing strategy.

5 Robustness Analysis

This section verifies the robustness of the results presented in Section 4. The performance of the asset allocation strategies is benchmarked against the 1/N portfolio in Section 5.1. The effect of imposing short sale constraints is demonstrated in Section 5.2. The impact of adjustments in the bootstrap methodology are evaluated in Section 5.3. The consequences of changes in the estimation window and data frequency are evaluated in Sections 5.4 and 5.5, respectively.

5.1 1/N Portfolio Strategy

The 1/N portfolio strategy is competitive with many asset allocation strategies in an out-of-sample setting, as it does not suffer from any estimation error (Garlappi et al., 2007). In addition, the 1/N strategy realises a low turnover, because the target allocation is fixed to an equal weighting scheme over time. Therefore, comparing the variance optimised strategies from Section 4 to the 1/N strategy provides a good indication of their effectiveness.

The performance of 1/N strategies is shown in Table 5.1. Clearly, the performance is barely influenced by the choice of rebalancing strategy or optimisation method. 1/N strategies incur up to 12 bps less transaction costs per day than the variance optimised portfolios from Section 4. As a result, 1/N strategies achieve higher Sharpe ratios than most strategies that are variance optimised excluding transaction costs. On the other hand, 1/N strategies perform worse than strategies that are variance optimised including transaction costs. This once again highlights the benefit of taking these costs into account in the portfolio optimisation. Finally, 1/N portfolio strategies realise much lower utility levels than all variance optimised strategies, because their volatility is approximately 0.5% higher per day. This shows that it is better to estimate the covariance matrix to construct portfolio weights rather than to give all assets the same weight.

Table 5.1: Performance of 1/N portfolio strategies applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, and threshold (Thld) rebalancing strategies.[10] Portfolios are rebalanced to equal weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the identity matrix as the estimated covariance matrix and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
B&H	0.041	1.200	0.508	1.000	**0.000**	∞	0.041	1.200	0.508	1.000	**0.000**	∞
Daily	0.040	1.201	0.485	0.997	0.343	1.000	**0.041**	**1.199**	**0.508**	**1.001**	0.000	199.167
Weekly	0.041	1.200	0.508	1.000	0.157	5.000	0.041	1.200	0.508	1.000	0.000	478.000
Monthly	0.042	1.197	0.513	1.004	0.077	20.000	0.041	1.200	0.508	1.000	0.000	1593.333
Quarterly	0.042	1.193	0.520	1.013	0.046	60.506	0.041	1.200	0.508	1.000	0.000	4780.000
Annually	0.042	1.181	0.530	1.033	0.022	251.579	0.041	1.200	0.508	1.000	0.000	4780.000
Thld 5%	0.042	1.199	0.520	1.002	0.077	25.561	0.041	1.200	0.508	1.000	**0.000**	∞
Thld 10%	0.043	1.198	0.531	1.003	0.041	91.923	0.041	1.200	0.508	1.000	**0.000**	∞
Thld 20%	0.044	1.194	0.540	1.011	0.020	367.692	0.041	1.200	0.508	1.000	**0.000**	∞
Thld 40%	**0.044**	**1.153**	**0.570**	**1.085**	0.010	1593.333	0.041	1.200	0.508	1.000	**0.000**	∞

5.2 Short Sale Constraints

Portfolios may include short positions in some assets in Section 4. In practice, investors may be impeded from short selling, because assets are difficult to short or there are regulatory constraints at the institutional level (Jones & Lamont, 2002). Jagannathan and Ma (2003) find that imposing portfolio constraints can improve the performance, because it reduces the amount of risk. They further show that the imposition of short sale constraints can be regarded as a form of shrinkage of the covariance matrix. Moreover, they show that this type of shrinkage improves portfolio performance when the sample covariance estimator is applied, whereas it is ineffective

[10]Bootstrap rebalancing strategies are inapplicable, because the covariance matrix is fixed to the identity matrix.

with a shrinkage or factor-based covariance estimator. To investigate how short sale constraints affect the performance of the asset allocation strategies, the empirical study is performed again with constraint $\Delta_{-} \leq \theta_0$ added to optimisation problem (3.2) and constraint $w \geq 0$ added to optimisation problem (4.9).

Table 5.2 shows the performance of long-only asset allocation strategies. Compared to Section 4, all strategies achieve much lower transaction costs but higher volatility. The imposition of short sale constraints further leads to higher Sharpe ratios for most rebalancing strategies, both when optimised excluding and including transaction costs. In contrast to the findings of Jagannathan and Ma (2003), the imposition of short sale constraints also improves the performance of strategies for which the covariance matrix is estimated using shrinkage and POET estimators (Panels B and C). This suggests that "double shrinkage" of the covariance matrix is advantageous. Nevertheless, the largest gains in Sharpe ratios are realised by strategies optimised using the sample covariance estimator; the threshold 40% strategy optimised including transaction achieves the highest Sharpe ratio. The Boot 1% strategy remains performing well for strategies optimised excluding transaction costs. Imposing short sale constraints only improves the utility of the strategies in Panel C. Bootstrap rebalancing strategies do not achieve the highest utility levels, although the Boot 99% strategy is always competitive with the best strategy. The unconstrained portfolio strategies that apply the shrinkage estimator of Ledoit and Wolf (2003) from Section 4 remain the best performing overall in terms of utility.

5.3 Bootstrap Methodology

This section investigates the robustness of results regarding the bootstrap block size and method. Table 5.3 shows the performance of bootstrap rebalancing strategies applied with the stationary bootstrap and block sizes of 1 and 10 trading days. Table D.1 in Appendix D indicates that these smaller block sizes may be preferred for some industry portfolios. Applying a smaller block size leads to more variation in the bootstrap samples, such that bootstrap rebalancing strategies have a lower ART when α is low but a higher ART when α is high. The largest performance differences are realised by Boot 1% and Boot 99% strategies, because they depend most on observations in the tail of the bootstrapped test statistics' distribution. Compared to the findings in Section 4, Boot 1% strategies achieve lower Sharpe ratios but higher utility levels, whereas the opposite is observed for Boot 99% strategies. Yet, these differences are not large.

Table 5.4 shows how bootstrap rebalancing strategies perform when a circular rather than a stationary bootstrap is applied. In comparison to the results shown in Section 4, the performance of the Boot 1% strategy has slightly deteriorated, whereas the performance of the Boot 99% strategy has improved marginally. The performance differences of bootstrap rebalancing strategies with α ranging from 25% to 75% is small, which shows that the choice of bootstrap method is not of major influence. Overall, it can be concluded that changes in the bootstrap methodology have little impact on the results. However, it is expected that the bootstrap method is more important when applying dynamic rather than static covariance estimators, because they heavily rely on the dependence structure in the data.

Table 5.2: Performance of asset allocation strategies with short sale constraints applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	0.040	1.098	0.531	1.000	**0.000**	∞	0.040	1.098	0.531	1.000	**0.000**	∞
Daily	0.021	0.806	0.366	1.857	1.968	1.000	0.045	**0.825**	0.811	**1.814**	0.147	1.668
Weekly	0.030	**0.805**	0.539	**1.879**	1.045	5.000	0.045	0.826	0.805	1.809	0.140	6.566
Monthly	0.035	0.815	0.619	1.840	0.595	20.000	0.045	0.829	0.807	1.796	0.130	22.762
Quarterly	0.039	0.821	0.691	1.817	0.386	60.506	0.046	0.830	0.823	1.790	0.117	61.282
Annually	**0.044**	0.828	**0.784**	1.798	0.210	251.579	0.049	0.844	0.859	1.735	0.080	251.579
Thld 5%	0.027	0.806	0.470	1.866	1.418	2.987	0.045	0.826	0.811	1.807	0.137	16.370
Thld 10%	0.030	0.807	0.541	1.869	1.021	6.742	0.046	0.829	0.823	1.797	0.131	31.447
Thld 20%	0.034	0.809	0.606	1.865	0.658	18.819	0.046	0.831	0.826	1.786	0.115	68.286
Thld 40%	0.036	0.820	0.647	1.818	0.421	56.905	**0.049**	0.838	**0.871**	1.760	0.097	170.714
Boot 1%	0.040	0.825	0.720	1.802	0.238	170.714	0.048	0.826	0.859	1.812	0.121	5.968
Boot 25%	0.037	0.823	0.666	1.808	0.367	77.097	0.045	0.827	0.811	1.805	0.147	2.878
Boot 50%	0.037	0.818	0.660	1.827	0.423	54.318	0.045	0.825	0.811	1.814	0.147	2.698
Boot 75%	0.036	0.812	0.641	1.856	0.476	37.937	0.045	0.825	0.811	1.814	0.147	2.696
Boot 99%	0.033	0.808	0.596	1.872	0.755	12.781	0.045	0.825	0.811	1.814	0.147	2.696
Panel B: Shrinkage estimator												
B&H	0.040	1.090	0.539	1.000	**0.000**	∞	0.040	1.090	0.539	1.000	**0.000**	∞
Daily	0.023	0.804	0.402	1.842	1.801	1.000	0.045	0.824	0.813	1.793	0.136	1.687
Weekly	0.031	**0.803**	0.559	**1.863**	0.960	5.000	0.045	0.824	0.808	1.789	0.129	6.928
Monthly	0.035	0.812	0.629	1.827	0.551	20.000	0.045	0.827	0.811	1.777	0.122	22.762
Quarterly	0.039	0.819	0.696	1.802	0.364	60.506	0.046	0.829	0.821	1.770	0.110	61.282
Annually	**0.044**	0.825	**0.785**	1.786	0.201	251.579	**0.048**	0.842	0.856	1.717	0.076	251.579
Thld 5%	0.029	0.804	0.505	1.850	1.260	3.236	0.045	0.824	0.817	1.790	0.126	17.256
Thld 10%	0.032	0.805	0.570	1.857	0.895	7.516	0.046	0.827	0.823	1.777	0.121	32.966
Thld 20%	0.034	0.807	0.618	1.847	0.581	20.965	0.047	0.828	0.841	1.774	0.104	75.873
Thld 40%	0.038	0.818	0.681	1.805	0.372	62.895	0.048	0.842	0.842	1.716	0.083	199.167
Boot 1%	0.042	0.824	0.744	1.786	0.231	170.714	0.048	**0.822**	**0.865**	**1.803**	0.106	6.136
Boot 25%	0.037	0.820	0.651	1.793	0.340	77.097	0.045	0.826	0.811	1.781	0.135	2.924
Boot 50%	0.035	0.818	0.628	1.802	0.385	56.235	0.045	0.824	0.813	1.793	0.136	2.761
Boot 75%	0.035	0.811	0.633	1.833	0.447	38.240	0.045	0.824	0.813	1.793	0.136	2.758
Boot 99%	0.034	0.806	0.615	1.855	0.686	13.025	0.045	0.824	0.813	1.793	0.136	2.758
Panel C: POET estimator												
B&H	0.040	1.109	0.531	1.000	**0.000**	∞	0.040	1.109	0.531	1.000	**0.000**	∞
Daily	0.023	0.803	0.391	1.910	1.852	1.000	0.045	**0.822**	0.815	**1.866**	0.160	1.738
Weekly	0.031	**0.802**	0.557	**1.933**	0.972	5.000	0.045	0.823	0.814	1.859	0.150	6.968
Monthly	0.036	0.810	0.640	1.901	0.561	20.000	0.046	0.826	0.823	1.845	0.139	22.441
Quarterly	0.038	0.817	0.684	1.873	0.373	60.506	0.046	0.830	0.828	1.829	0.122	62.078
Annually	**0.043**	0.824	**0.777**	1.853	0.202	251.579	0.048	0.841	0.853	1.781	0.080	251.579
Thld 5%	0.028	0.803	0.494	1.920	1.310	3.128	0.045	0.824	0.816	1.856	0.148	15.672
Thld 10%	0.032	0.803	0.574	1.930	0.927	7.354	0.046	0.826	0.827	1.846	0.141	30.063
Thld 20%	0.035	0.806	0.634	1.922	0.614	19.671	0.047	0.828	0.853	1.841	0.119	67.324
Thld 40%	0.038	0.818	0.682	1.871	0.390	58.293	**0.049**	0.839	**0.862**	1.793	0.105	159.333
Boot 1%	0.042	0.818	0.748	1.878	0.220	183.846	0.047	0.824	0.846	1.857	0.123	7.673
Boot 25%	0.038	0.823	0.668	1.847	0.335	81.017	0.045	0.824	0.817	1.855	0.154	3.333
Boot 50%	0.036	0.816	0.647	1.875	0.389	54.943	0.045	0.822	0.815	1.866	0.160	2.704
Boot 75%	0.037	0.810	0.659	1.907	0.482	34.388	0.045	0.822	0.815	1.866	0.160	2.705
Boot 99%	0.033	0.804	0.599	1.927	0.733	11.832	0.045	0.822	0.815	1.866	0.160	2.705

Table 5.3: Performance of bootstrap rebalancing strategies with a block size 1 and 10 trading days applied on daily industry portfolio data from 2000-2019.
This table shows the performance of bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 1 and 10 trading day(s). Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance estimated on a rolling-window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold if it outperforms buy-and-hold, periodic, and threshold rebalancing strategies.

Table 5.3a: Block size of one trading day

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
Boot 1%	0.029	0.775	0.540	2.045	1.359	129.189	0.039	0.744	0.775	2.251	0.331	1.768
Boot 25%	0.023	0.757	0.422	2.132	1.627	90.189	0.039	0.742	0.777	2.266	0.331	1.691
Boot 50%	0.023	0.754	0.431	2.147	1.827	73.538	0.039	0.742	0.777	2.266	0.331	1.691
Boot 75%	0.023	0.746	0.421	2.193	1.960	61.282	0.039	0.742	0.777	2.266	0.331	1.691
Boot 99%	0.014	0.738	0.235	**2.217**	2.384	40.508	0.039	0.742	0.777	2.266	0.331	1.691
Panel B: Shrinkage estimator												
Boot 1%	0.030	0.740	0.587	2.178	1.143	108.636	0.039	0.733	0.783	2.250	0.286	1.853
Boot 25%	0.029	0.724	0.562	2.273	1.308	75.873	0.039	0.732	0.783	2.252	0.289	1.789
Boot 50%	0.024	0.721	0.460	2.278	1.491	61.282	0.039	0.732	0.783	2.252	0.289	1.789
Boot 75%	0.025	0.719	0.483	2.294	1.570	54.943	0.039	0.732	0.783	2.252	0.289	1.789
Boot 99%	0.020	0.704	0.389	**2.382**	1.849	35.940	0.039	0.732	0.783	2.252	0.289	1.789
Panel C: POET estimator												
Boot 1%	0.030	0.755	0.570	1.669	0.876	132.778	0.037	0.760	0.713	1.659	0.299	2.552
Boot 25%	0.022	0.757	0.398	1.645	1.203	70.294	0.036	0.756	0.693	1.677	0.369	1.817
Boot 50%	0.017	0.750	0.303	1.666	1.438	50.316	0.036	0.756	0.693	1.678	0.371	1.766
Boot 75%	0.017	0.746	0.291	1.681	1.700	37.638	0.036	0.756	0.693	1.678	0.371	1.766
Boot 99%	0.011	0.743	0.174	1.685	2.267	20.783	0.036	0.756	0.693	1.678	0.371	1.766

Table 5.3b: Block size of ten trading days

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
Boot 1%	0.029	0.790	0.522	1.964	1.215	159.333	0.040	0.757	0.782	2.177	0.303	2.083
Boot 25%	0.026	0.747	0.495	2.197	1.527	97.551	0.039	0.742	0.777	2.266	0.331	1.691
Boot 50%	0.018	0.748	0.321	2.171	1.778	72.424	0.039	0.742	0.777	2.266	0.331	1.691
Boot 75%	0.018	0.748	0.323	2.169	2.055	56.235	0.039	0.742	0.777	2.266	0.331	1.691
Boot 99%	0.009	0.730	0.131	**2.253**	2.787	29.325	0.039	0.742	0.777	2.266	0.331	1.691
Panel B: Shrinkage estimator												
Boot 1%	0.030	0.763	0.569	2.046	1.045	132.778	0.040	0.748	0.778	2.152	0.262	2.174
Boot 25%	0.029	0.725	0.577	2.270	1.275	83.860	0.039	0.732	0.783	2.252	0.289	1.789
Boot 50%	0.024	0.716	0.468	2.310	1.472	60.506	0.039	0.732	0.783	2.252	0.289	1.789
Boot 75%	0.023	0.711	0.440	2.343	1.618	49.792	0.039	0.732	0.783	2.252	0.289	1.789
Boot 99%	0.018	0.704	0.333	**2.370**	2.185	26.409	0.039	0.732	0.783	2.252	0.289	1.789
Panel C: POET estimator												
Boot 1%	0.029	0.782	0.520	1.551	0.833	144.849	0.038	0.783	0.705	1.559	0.296	2.874
Boot 25%	0.021	0.764	0.385	1.611	1.170	73.538	0.036	0.756	0.692	1.678	0.369	1.835
Boot 50%	0.018	0.753	0.316	1.653	1.413	51.398	0.036	0.756	0.693	1.678	0.371	1.766
Boot 75%	0.016	0.750	0.278	1.662	1.801	34.638	0.036	0.756	0.693	1.678	0.371	1.766
Boot 99%	0.012	0.739	0.192	**1.705**	2.402	17.574	0.036	0.756	0.693	1.678	0.371	1.766

Table 5.4: Performance of bootstrap rebalancing strategies with the circular bootstrap applied on daily industry portfolio data from 2000-2019.
This table shows the performance of bootstrap (Boot) rebalancing strategies. A circular bootstrap is applied with a block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance estimated on a rolling-window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), daily transaction costs in % (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold if it outperforms buy-and-hold, periodic, and threshold rebalancing strategies.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
Boot 1%	0.032	0.796	0.571	1.937	1.154	177.037	0.044	0.759	0.850	2.171	0.312	2.201
Boot 25%	0.026	0.762	0.489	2.108	1.558	97.551	0.039	0.742	0.777	2.266	0.331	1.691
Boot 50%	0.018	0.747	0.328	2.180	1.844	70.294	0.039	0.742	0.777	2.266	0.331	1.691
Boot 75%	0.018	0.747	0.328	2.176	2.060	54.318	0.039	0.742	0.777	2.266	0.331	1.691
Boot 99%	0.009	0.738	0.143	**2.208**	2.964	25.699	0.039	0.742	0.777	2.266	0.331	1.691
Panel B: Shrinkage estimator												
Boot 1%	0.035	0.741	**0.687**	2.184	0.961	149.375	0.043	0.748	**0.855**	2.162	0.273	2.309
Boot 25%	0.030	0.724	0.583	2.278	1.262	83.860	0.039	0.732	0.783	2.252	0.289	1.789
Boot 50%	0.024	0.717	0.462	2.304	1.489	60.506	0.039	0.732	0.783	2.252	0.289	1.789
Boot 75%	0.024	0.710	0.465	2.349	1.669	47.327	0.039	0.732	0.783	2.252	0.289	1.789
Boot 99%	0.016	0.704	0.302	**2.369**	2.272	23.900	0.039	0.732	0.783	2.252	0.289	1.789
Panel C: POET estimator												
Boot 1%	0.031	0.789	0.573	1.526	0.831	144.849	0.039	0.780	0.736	1.575	0.300	3.397
Boot 25%	0.024	0.761	0.445	1.632	1.162	73.538	0.036	0.755	0.693	**1.681**	0.368	1.848
Boot 50%	0.019	0.754	0.335	1.653	1.443	49.792	0.036	0.756	0.693	1.678	0.371	1.766
Boot 75%	0.016	0.748	0.282	1.673	1.788	34.388	0.036	0.756	0.693	1.678	0.371	1.766
Boot 99%	0.009	0.738	0.124	**1.705**	2.504	16.314	0.036	0.756	0.693	1.678	0.371	1.766

5.4 Estimation Window

To assess how including more short- or long-term information impacts portfolio performance, the empirical analysis is repeated with estimation windows of 100 and 500 trading days. The block size is decreased from 20 to 10 observations in order to obtain sufficient variation in the bootstrap samples when an estimation window of 100 trading days is used.

Table 5.5 shows the performance of the asset allocation strategies with the covariance matrix estimated using a rolling window of 100 trading days. A reduction in the estimation window from 250 to 100 trading days leads to three to four times higher transaction costs, because the estimated covariance matrix changes more over time. Together with an increase in volatility, this is detrimental for portfolio performance. The smaller estimation window also leads to more estimation uncertainty. This makes bootstrap rebalancing strategies more effective when applied with portfolios optimised including transaction costs; for two out of three covariance estimators the Boot 25% strategy achieves the highest utility level. Bootstrap rebalancing strategies of portfolios optimised excluding transaction costs also perform relatively well with a low α.

The performance of strategies optimised with an estimation window of 500 trading days is displayed in Table 5.6. Trading costs are approximately halved when using a two-year estimation

Table 5.5: Performance of asset allocation strategies with an estimation window of 100 days applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 100 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.041**	1.135	**0.530**	1.000	**0.000**	∞	0.041	1.135	0.530	1.000	**0.000**	∞
Daily	-0.421	**0.902**	-7.429	1.152	44.813	1.000	0.029	0.769	0.544	2.208	0.867	1.223
Weekly	-0.190	0.980	-3.109	1.161	21.896	5.000	0.032	0.773	0.594	2.192	0.813	5.335
Monthly	-0.079	0.990	-1.309	1.222	10.675	20.041	0.036	0.785	0.658	2.131	0.723	20.205
Quarterly	-0.025	0.990	-0.448	1.265	5.396	60.122	**0.041**	0.808	**0.752**	2.017	0.604	60.122
Annually	0.031	0.970	0.465	**1.371**	1.480	259.474	0.038	0.846	0.657	1.829	0.174	259.474
Thld 5%	-0.413	0.906	-7.263	1.152	44.015	1.101	0.031	0.773	0.565	2.187	0.807	7.888
Thld 10%	-0.379	0.923	-6.538	1.142	40.571	1.408	0.032	0.777	0.586	2.167	0.774	15.311
Thld 20%	-0.292	0.959	-4.859	1.133	31.738	2.496	0.034	0.795	0.620	2.072	0.669	35.725
Thld 40%	-0.168	0.992	-2.721	1.152	19.282	6.895	0.037	0.840	0.638	1.852	0.478	100.612
Boot 1%	0.041	1.135	0.530	1.000	**0.000**	∞	0.032	0.790	0.584	2.094	0.716	2.034
Boot 25%	-0.015	0.986	-0.288	1.285	4.808	71.449	0.029	**0.769**	0.545	**2.208**	0.866	1.487
Boot 50%	-0.041	1.005	-0.687	1.217	7.465	37.923	0.029	0.769	0.544	2.208	0.867	1.478
Boot 75%	-0.082	0.992	-1.360	1.214	10.497	21.068	0.029	0.769	0.544	2.208	0.867	1.478
Boot 99%	-0.318	0.947	-5.361	1.139	34.516	1.954	0.029	0.769	0.544	2.208	0.867	1.478
Panel B: Shrinkage estimator												
B&H	**0.038**	1.100	0.510	1.000	**0.000**	∞	0.038	1.100	0.510	1.000	**0.000**	∞
Daily	-0.137	**0.713**	-3.106	1.977	16.921	1.000	0.033	**0.724**	0.653	**2.362**	0.650	1.269
Weekly	-0.052	0.731	-1.191	2.085	8.533	5.000	0.035	0.731	0.689	2.318	0.588	5.546
Monthly	-0.010	0.748	-0.285	2.094	4.390	20.041	0.037	0.748	0.714	2.215	0.493	20.372
Quarterly	0.008	0.779	0.108	1.971	2.554	60.122	**0.042**	0.770	**0.810**	2.097	0.428	60.864
Annually	0.032	0.816	**0.561**	1.837	0.798	259.474	0.038	0.808	0.688	1.891	0.124	259.474
Thld 5%	-0.128	0.716	-2.889	1.983	15.978	1.272	0.033	0.728	0.663	2.338	0.594	8.726
Thld 10%	-0.103	0.724	-2.307	2.002	13.478	2.036	0.035	0.733	0.686	2.308	0.564	16.942
Thld 20%	-0.062	0.733	-1.401	2.046	9.373	4.629	0.038	0.741	0.737	2.259	0.510	37.348
Thld 40%	-0.021	0.735	-0.507	**2.142**	5.447	14.331	0.036	0.792	0.663	1.964	0.323	126.410
Boot 1%	0.016	0.780	0.254	1.980	2.181	79.516	0.034	0.752	0.646	2.182	0.516	2.225
Boot 25%	0.006	0.754	0.060	2.100	3.300	37.348	0.033	0.724	0.653	2.362	0.650	1.579
Boot 50%	-0.010	0.746	-0.273	2.104	4.163	24.049	0.033	0.724	0.653	2.362	0.650	1.578
Boot 75%	-0.018	0.742	-0.455	2.108	5.094	15.852	0.033	0.724	0.653	2.362	0.650	1.578
Boot 99%	-0.100	0.722	-2.253	2.015	13.152	2.003	0.033	0.724	0.653	2.362	0.650	1.578
Panel C: POET estimator												
B&H	**0.039**	1.064	0.530	1.000	**0.000**	∞	0.039	1.064	0.530	1.000	**0.000**	∞
Daily	-0.114	**0.744**	-2.485	1.759	14.550	1.000	0.029	0.750	0.557	2.039	0.846	1.290
Weekly	-0.039	0.759	-0.868	1.841	7.155	5.000	0.031	0.756	0.589	2.013	0.700	5.496
Monthly	-0.003	0.772	-0.122	1.851	3.594	20.041	0.035	0.774	0.657	1.922	0.602	20.714
Quarterly	0.016	0.809	0.250	1.722	2.086	60.122	0.041	0.798	0.756	1.819	0.481	60.122
Annually	0.034	0.800	**0.616**	1.795	0.637	259.474	0.039	0.809	0.707	1.762	0.165	259.474
Thld 5%	-0.105	0.747	-2.296	1.762	13.691	1.271	0.031	0.753	0.581	2.027	0.796	6.671
Thld 10%	-0.081	0.751	-1.777	1.791	11.231	2.219	0.030	0.758	0.560	1.995	0.658	14.457
Thld 20%	-0.046	0.761	-1.013	1.815	7.800	5.217	0.035	0.766	0.669	1.964	0.668	29.000
Thld 40%	-0.009	0.773	-0.254	1.836	4.265	17.059	**0.042**	0.789	**0.789**	1.865	0.511	83.559
Boot 1%	0.020	0.770	0.344	1.910	1.827	70.429	0.036	0.783	0.667	1.880	0.589	3.002
Boot 25%	0.007	0.763	0.088	**1.917**	2.894	31.806	0.030	**0.747**	0.569	**2.060**	0.774	1.702
Boot 50%	-0.006	0.771	-0.185	1.850	3.800	19.720	0.030	0.749	0.566	2.044	0.839	1.569
Boot 75%	-0.017	0.759	-0.424	1.885	4.861	12.545	0.029	0.750	0.558	2.039	0.847	1.540
Boot 99%	-0.081	0.754	-1.754	1.780	11.117	2.082	0.029	0.750	0.557	2.039	0.846	1.540

Table 5.6: Performance of asset allocation strategies with an estimation window of 500 days applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 500 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.035**	1.011	0.504	1.000	**0.000**	∞	0.035	1.011	0.504	1.000	**0.000**	∞
Daily	-0.025	**0.698**	-0.628	1.986	6.164	1.000	0.039	0.767	0.745	1.778	0.139	1.450
Weekly	0.006	0.703	0.066	2.038	3.100	5.000	0.039	0.770	0.741	1.762	0.128	5.906
Monthly	0.020	0.719	0.378	1.986	1.667	20.044	0.040	0.779	0.758	1.721	0.125	22.098
Quarterly	0.028	0.758	0.525	1.796	1.044	60.400	0.041	0.793	0.774	1.664	0.111	60.400
Annually	0.034	0.792	**0.631**	1.655	0.576	251.667	**0.042**	0.808	0.777	1.602	0.085	251.667
Thld 5%	-0.005	0.700	-0.171	2.024	4.108	3.105	0.039	0.772	0.735	1.752	0.121	28.312
Thld 10%	0.009	0.702	0.141	**2.056**	2.711	7.947	0.039	0.777	0.738	1.731	0.113	59.605
Thld 20%	0.019	0.714	0.358	2.010	1.618	24.486	0.038	0.794	0.714	1.654	0.099	141.562
Thld 40%	0.023	0.753	0.438	1.815	0.970	78.103	0.040	0.809	0.738	1.592	0.076	411.818
Boot 1%	0.032	0.771	0.611	1.746	0.635	205.909	0.040	**0.764**	**0.777**	**1.795**	0.132	2.660
Boot 25%	0.030	0.738	0.590	1.906	0.775	125.833	0.039	0.767	0.745	1.778	0.139	2.347
Boot 50%	0.028	0.733	0.557	1.931	0.893	96.383	0.039	0.767	0.745	1.778	0.139	2.347
Boot 75%	0.025	0.722	0.493	1.982	0.978	74.262	0.039	0.767	0.745	1.778	0.139	2.347
Boot 99%	0.021	0.715	0.414	2.013	1.283	40.811	0.039	0.767	0.745	1.778	0.139	2.347
Panel B: Shrinkage estimator												
B&H	**0.036**	1.018	0.516	1.000	**0.000**	∞	0.036	1.018	0.516	1.000	**0.000**	∞
Daily	-0.016	**0.691**	-0.432	2.074	5.275	1.000	0.039	**0.762**	0.750	**1.822**	0.133	1.456
Weekly	0.010	0.696	0.167	2.119	2.660	5.000	0.039	0.766	0.748	1.806	0.123	6.032
Monthly	0.022	0.711	0.438	2.062	1.441	20.044	0.040	0.775	0.767	1.764	0.121	21.990
Quarterly	0.029	0.749	0.562	1.868	0.910	60.400	0.042	0.787	**0.788**	1.711	0.108	61.216
Annually	0.035	0.781	**0.658**	1.723	0.512	251.667	**0.043**	0.804	0.787	1.637	0.083	251.667
Thld 5%	0.002	0.694	-0.009	2.110	3.393	3.416	0.039	0.767	0.751	1.798	0.118	27.622
Thld 10%	0.014	0.694	0.251	**2.141**	2.239	8.813	0.039	0.771	0.735	1.778	0.107	58.077
Thld 20%	0.021	0.706	0.414	2.087	1.347	27.622	0.039	0.779	0.731	1.740	0.093	141.562
Thld 40%	0.029	0.729	0.565	1.976	0.802	88.824	0.040	0.796	0.749	1.666	0.070	411.818
Boot 1%	0.031	0.753	0.598	1.854	0.560	205.909	0.040	0.764	0.779	1.818	0.127	2.755
Boot 25%	0.030	0.730	0.590	1.971	0.711	116.154	0.039	0.762	0.750	1.822	0.133	2.404
Boot 50%	0.027	0.724	0.537	2.001	0.821	85.472	0.039	0.762	0.750	1.822	0.133	2.404
Boot 75%	0.028	0.713	0.565	2.067	0.874	70.781	0.039	0.762	0.750	1.822	0.133	2.404
Boot 99%	0.024	0.706	0.475	2.095	1.151	39.052	0.039	0.762	0.750	1.822	0.133	2.404
Panel C: POET estimator												
B&H	**0.035**	0.944	0.546	1.000	**0.000**	∞	0.035	0.944	0.546	1.000	**0.000**	∞
Daily	-0.014	**0.737**	-0.363	1.570	4.645	1.000	0.035	**0.785**	0.657	**1.463**	0.168	1.420
Weekly	0.008	0.744	0.123	1.582	2.349	5.000	0.036	0.787	0.664	1.457	0.155	5.929
Monthly	0.019	0.760	0.349	1.536	1.308	20.044	0.037	0.798	0.678	1.419	0.146	22.098
Quarterly	0.027	0.793	0.489	1.423	0.849	60.400	**0.039**	0.808	**0.709**	1.383	0.131	62.917
Annually	0.033	0.809	**0.594**	1.371	0.499	251.667	0.039	0.822	0.690	1.337	0.095	251.667
Thld 5%	0.003	0.739	0.000	**1.595**	2.864	4.114	0.036	0.787	0.665	1.458	0.150	24.754
Thld 10%	0.011	0.745	0.185	1.584	1.924	10.811	0.035	0.795	0.640	1.428	0.134	55.244
Thld 20%	0.019	0.757	0.339	1.548	1.252	30.816	0.036	0.809	0.648	1.377	0.113	133.235
Thld 40%	0.026	0.758	0.488	1.559	0.753	96.383	0.036	0.817	0.636	1.349	0.064	453.000
Boot 1%	0.029	0.787	0.527	1.447	0.513	226.500	0.036	0.793	0.661	1.436	0.150	3.707
Boot 25%	0.024	0.766	0.437	1.521	0.735	100.667	0.035	0.785	0.657	1.463	0.168	2.276
Boot 50%	0.022	0.763	0.401	1.529	0.879	69.692	0.035	0.785	0.657	1.463	0.168	2.276
Boot 75%	0.021	0.760	0.380	1.541	0.969	52.674	0.035	0.785	0.657	1.463	0.168	2.276
Boot 99%	0.016	0.749	0.283	1.579	1.309	26.337	0.035	0.785	0.657	1.463	0.168	2.276

window rather than a one-year window that is used in Section 4. On the other hand, the volatility is often higher due to infrequent rebalancing and possibly the incorporation of less relevant older observations in the covariance estimation. As a result, the Sharpe ratios and utility levels of the strategies have not improved relative to Section 4. However, the performance is generally better than with the estimation window of 100 days due to both lower transaction costs and volatility. The large estimation window leads to low estimation uncertainty, which causes bootstrap rebalancing strategies to perform weakly compared to other strategies. The performance of strategies in Panels B and C has declined relative to Panel A for the same reason.

5.5 Data Frequency

The results in the empirical study are obtained using data at a daily frequency, which may not be available for all financial instruments. To evaluate the performance of the asset allocation strategies with low-frequency data, the analysis is repeated using monthly industry portfolio data. A covariance estimation window of 12 months is used. The bootstrap block size is set to one month as suggested by the algorithm of Politis and White (2004) (see Table D.6 in Appendix D for details). The number of assets exceeds the number of observations, hence the sample covariance estimator is no longer applicable. For the POET estimator, the number of principal components selected by the information criterion of Bai and Ng (2002) remains five.

The performance of the rebalancing strategies with monthly data is summarised in Table 5.7. The ART of the strategies has increased considerably due to the lower data frequency, which results in lower transaction costs but higher volatility. As found in Section 4, transaction costs decrease non-linearly with the ART. Bootstrap rebalancing strategies with α from 1% to 75% perform best in terms of the volatility, Sharpe ratio, and utility level. Bootstrap rebalancing strategies achieve much higher Sharpe ratios than other rebalancing strategies, because there is large estimation uncertainty paired with estimating a covariance matrix of 49 assets with 12 observations. Bootstrap rebalancing strategies effectively enhance the performance of portfolios optimised including transaction costs, such that these strategies perform best overall. The out-performance of bootstrap rebalancing strategies is particularly large in Panel B, which indicates that these strategies can provide great economic value to an investor.

5.6 Summary

The robustness analysis has provided several interesting insights. A comparison between the variance optimised strategies and 1/N strategies shows that portfolio performance is much better when the (co)variance of asset returns is taken into account in the portfolio optimisation. In addition, the imposition of short sale constraints improves the Sharpe ratios of portfolio strategies for any covariance estimation method, although it usually yields lower utility levels. Furthermore, the performance of bootstrap rebalancing strategies is robust to changes in both the bootstrap block size and method. Lastly, bootstrap portfolio rebalancing strategies with a low α fare much better relative to other strategies when there is more uncertainty due to either a smaller estimation window or a lower data frequency.

Table 5.7: Performance of asset allocation strategies applied on monthly industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 1 month. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 12 months and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), monthly volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The statistics have been transformed from monthly to daily measures based on the number of trading days in the daily sample. The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Shrinkage estimator												
B&H	**0.037**	0.903	0.601	1.000	**0.000**	∞	**0.037**	0.903	0.601	1.000	**0.000**	∞
Monthly	0.014	0.765	0.231	1.372	2.306	20.969	0.024	0.761	0.447	1.404	1.354	20.969
Quarterly	0.023	0.774	0.418	1.354	1.369	63.467	0.027	0.771	0.499	1.372	0.935	63.467
Annually	0.032	0.795	0.587	1.295	0.589	264.446	0.033	0.770	0.621	1.385	0.466	264.446
Thld 5%	0.014	0.765	0.231	1.372	2.306	20.969	0.024	0.761	0.447	1.403	1.352	21.442
Thld 10%	0.014	0.765	0.230	1.371	2.304	21.156	0.024	0.760	0.446	1.407	1.339	23.107
Thld 20%	0.014	0.763	0.229	1.378	2.241	23.448	0.026	0.756	0.473	1.426	1.281	29.565
Thld 40%	0.019	0.747	0.351	1.451	1.853	37.481	0.029	0.758	0.539	1.424	1.091	52.308
Boot 1%	0.035	0.788	0.637	1.321	0.166	952.006	0.036	0.739	**0.711**	1.515	0.190	595.004
Boot 25%	0.032	0.728	**0.641**	1.553	0.873	136.001	0.031	**0.707**	0.620	**1.647**	0.753	79.334
Boot 50%	0.031	**0.723**	0.621	**1.573**	1.051	99.167	0.028	0.751	0.529	1.450	0.975	47.129
Boot 75%	0.023	0.783	0.403	1.323	1.391	62.632	0.027	0.750	0.515	1.453	1.099	36.336
Boot 99%	0.014	0.766	0.236	1.368	2.256	22.559	0.024	0.761	0.446	1.405	1.353	21.062
Panel B: POET estimator												
B&H	0.037	0.891	0.613	1.000	**0.000**	∞	0.037	0.891	0.613	1.000	**0.000**	∞
Monthly	0.006	0.708	0.076	1.547	3.064	20.969	0.022	0.708	0.427	1.580	1.288	20.969
Quarterly	0.022	0.731	0.424	1.482	1.612	63.467	0.024	0.717	0.465	1.542	0.843	63.467
Annually	**0.040**	0.766	**0.771**	1.371	0.578	264.446	0.040	0.752	0.787	1.428	0.414	264.446
Thld 5%	0.006	0.708	0.076	1.547	3.064	20.969	0.022	0.708	0.426	1.578	1.287	21.156
Thld 10%	0.006	0.708	0.076	1.547	3.064	20.969	0.022	0.708	0.427	1.579	1.282	22.453
Thld 20%	0.006	0.708	0.077	1.547	3.061	21.062	0.022	0.703	0.425	1.601	1.208	28.334
Thld 40%	0.008	**0.705**	0.121	1.562	2.818	25.319	0.023	0.704	0.459	1.603	0.957	52.308
Boot 1%	0.037	0.891	0.613	1.000	**0.000**	∞	**0.041**	0.737	**0.813**	1.491	0.014	4760.031
Boot 25%	0.036	0.752	0.694	1.421	0.557	264.446	0.026	**0.700**	0.514	**1.625**	0.742	79.334
Boot 50%	0.029	0.743	0.566	1.445	0.985	128.649	0.030	0.704	0.602	1.616	0.911	52.889
Boot 75%	0.029	0.712	0.573	**1.576**	1.337	82.069	0.024	0.713	0.471	1.559	1.063	33.056
Boot 99%	0.007	0.714	0.090	1.522	2.932	23.107	0.022	0.708	0.427	1.580	1.288	20.969

6 Analysis with S&P 500 Data

This section performs an empirical study using a data set containing S&P 500 stocks to verify that the results are not data-driven and to evaluate how strategies perform with variable rather than fixed transaction costs. The S&P 500 data set is described in Section 6.1. Subsequently, the performance of the asset allocation strategies is analysed with fixed and variable transaction costs in Sections 6.2 and 6.3, respectively.

6.1 Data

The S&P 500 data set is retrieved from the Center for Research in Security Prices (CRSP) for the years 2000 until 2019 by selecting companies that are part of the index at the end of the sample period. The sample consists of 5031 daily observations. In the data set, there are 605 companies as identified by the permanent company number (PERMCO) of which 372 have daily return data available throughout the entire sample.[11] Then, dropping companies with stocks that have missing closing bid or ask prices for more than 1% of the observations leaves a sample of 142 companies. Some of these companies have multiple share classes outstanding. For these companies, the data relating to the most traded share class is used. A list of the companies and their descriptive statistics is provided in Table E.1 in Appendix E.

Table 6.1 summarises the performance of the S&P 500 stocks. The average return of 0.06% per day is rather high due to survivorship bias; the sample only consists of companies that have either remained large or grown large over the sample of twenty years. Nevertheless, the data can still be used to assess the relative performance of strategies with similar average rebalancing times. There is a large disparity in stock returns and volatility, because the sample includes stocks of large well-established companies (e.g., Coca-Cola Company) as well as some quickly growing companies (e.g., Monster Beverages Corporation). Stocks are highly correlated with each other as indicated by the average correlation across stocks of 0.31. In addition, stocks have a high average correlation with the market of 0.55.

Table 6.1: Descriptive statistics of daily S&P 500 data from 2000-2019.
All statistics are daily, except for the annualised Sharpe ratio. The return and volatility are in %. Trading volume is denoted in million US dollars. The bid-ask spread is denoted in basis points.

	Mean	Standard deviation	Minimum	Maximum
Average return	0.064	0.029	0.001	0.175
Volatility	2.357	0.731	1.178	4.304
Sharpe ratio	0.422	0.168	-0.013	0.907
Correlation across stocks	0.307	0.099	0.030	0.830
Correlation with market	0.549	0.094	0.294	0.779
Average trading volume	333.637	462.343	7.962	4014.409
Average bid-ask spread	15.047	9.468	3.042	57.979

6.2 Performance of Asset Allocation Strategies with Fixed Transaction Costs

Asset allocation strategies are compared using the same set-up as in Section 4. A block size of 20 trading days is applied for the bootstrap, because the recommended block size for stock data is similar to that for industry portfolio data (see Table F.1 in Appendix F for details). The number of factors selected for the POET estimator using the information criterion of Bai and Ng (2002) amounts to four, which is one less than with the industry portfolio data.

The performance of the rebalancing strategies is displayed in Table 6.2. Noteworthily, the buy-and-hold strategy achieves an outstanding return and Sharpe ratio. This strategy fixes its investments at the start of the sample, such that the proportion invested in quickly growing companies becomes very large over time. Therefore, this strategy benefits most from the fact that the sample includes companies that are small at the beginning of the sample but grow

[11] The number of companies with a PERMCO exceeds the 500 listed on the S&P 500 due to mergers and acquisitions.

rapidly to become listed on the S&P 500 at the end of the sample. Yet, the large proportion of wealth invested in growing companies also leads to extremely volatile returns and hence a low utility level. Portfolios optimised excluding transaction costs incur higher transaction costs than with the industry portfolio data in Panel A in Section 4, because the estimated sample covariance matrix changes more over time. These changes occur due to an increase in the number of assets and more variation in the volatility of individual stocks than portfolios of stocks. As a result, strategies in Panels B and C yield much higher utility levels than strategies in Panel A. Bootstrap rebalancing strategies achieve relatively high utility levels independent of whether transaction costs are taken into account in the optimisation. There is some dispersion in the best α across the panels for strategies optimised including transaction costs. Yet, the choice of α is not crucial, because the performance of these bootstrap rebalancing strategies is comparable.

6.3 Performance of Asset Allocation Strategies with Variable Transaction Costs

Transaction costs have hitherto been fixed at the same rate for all assets, whereas these costs likely differ per stock and change over time in practice. To better mimic reality, this section estimates transaction costs using a simple model that includes a fixed trading commission and a variable spread. Market impact costs are ignored, because the method of Dybvig and Pezzo (2019) does not allow for costs growing at a non-linear rate. However, these costs are negligible when less than 1% of the daily volume is traded (Frazzini et al., 2012). Table 6.1 shows that the average daily trading volume is \$334 million, which implies the investor should trade less than \$3 million of a single stock per day to avoid market impact costs. Consequently, the results in this section are most realistic for smaller investors.

The first component of the transaction costs is a fixed trading commission that amounts to 24 bps based on the estimate of Jones (2002) for stocks listed on the NYSE.[12] Half of the bid-ask spread is added to this commission in order to obtain the total transaction costs. Following the methodology of Chung and Zhang (2014), the bid-ask spread is estimated as

$$\text{bid-ask spread}_t = \frac{\text{ask}_t - \text{bid}_t}{\frac{1}{2}(\text{bid}_t + \text{ask}_t)}, \tag{6.1}$$

where bid_t and ask_t are daily closing bid and ask prices at day t obtained from CRSP. As daily bid and ask prices fluctuate considerably, a more robust estimate is obtained by taking the average spread over the last 20 trading days. Observations with a negative spread or a spread higher than 50% of the quote midpoint are excluded as they likely result from data errors.

Table 6.1 shows that average bid-ask spread estimated using this procedure amounts to 15 bps. The average spread ranges widely from 3 to 58 bps. Stocks with a higher trading volume generally have a lower bid-ask spread. There is a large difference in the trading volume across shares, which is mainly attributable to large differences in the companies' market capitalisation. The sample includes several companies that are not listed on a major index throughout the

[12]The sample includes 67 companies that are listed on the NYSE. The remaining 75 are listed on the NASDAQ where the commission is comparable.

Table 6.2: Performance of asset allocation strategies with fixed transaction costs applied on daily S&P 500 stock data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.099**	1.478	**1.024**	1.000	**0.000**	∞	**0.099**	1.478	**1.024**	1.000	**0.000**	∞
Daily	-0.282	**0.954**	-4.718	1.929	33.290	1.000	0.037	0.787	0.693	3.562	0.784	1.005
Weekly	-0.111	1.007	-1.786	1.948	16.335	5.000	0.038	0.791	0.711	3.533	0.681	5.000
Monthly	-0.035	1.002	-0.599	2.061	8.118	20.000	0.041	0.791	0.762	3.537	0.607	20.000
Quarterly	-0.003	1.003	-0.092	**2.102**	4.342	60.506	0.043	0.802	0.784	3.444	0.488	60.506
Annually	0.034	1.041	0.480	2.000	1.619	251.579	0.047	0.857	0.821	3.016	0.342	251.579
Thld 5%	-0.270	0.959	-4.493	1.929	32.038	1.177	0.038	0.791	0.705	3.529	0.716	7.636
Thld 10%	-0.226	0.971	-3.728	1.936	27.610	1.739	0.036	0.796	0.654	3.478	0.620	17.638
Thld 20%	-0.145	0.990	-2.355	1.970	19.167	3.855	0.037	0.801	0.674	3.439	0.515	45.094
Thld 40%	-0.062	1.011	-1.013	1.994	10.508	12.679	0.045	0.822	0.809	3.281	0.501	111.163
Boot 1%	**0.099**	1.478	1.024	1.000	**0.000**	∞	0.039	**0.781**	0.727	**3.628**	0.630	1.280
Boot 25%	0.039	1.064	0.539	1.918	0.826	478.000	0.037	0.786	0.693	3.575	0.747	1.056
Boot 50%	0.023	1.044	0.303	1.970	2.034	183.846	0.037	0.787	0.693	3.562	0.784	1.032
Boot 75%	0.023	1.021	0.310	2.061	3.027	103.913	0.037	0.787	0.693	3.562	0.784	1.032
Boot 99%	-0.027	1.015	-0.460	2.023	6.487	30.641	0.037	0.787	0.693	3.562	0.784	1.032
Panel B: Shrinkage estimator												
B&H	**0.095**	1.479	**0.987**	1.000	**0.000**	∞	**0.095**	1.479	**0.987**	1.000	**0.000**	∞
Daily	-0.036	**0.696**	-0.885	4.174	8.528	1.000	0.041	0.725	0.825	4.267	0.450	1.013
Weekly	0.006	0.710	0.075	4.247	4.353	5.000	0.041	0.729	0.825	4.212	0.395	5.000
Monthly	0.024	0.724	0.454	4.180	2.324	20.000	0.042	0.738	0.840	4.118	0.358	20.000
Quarterly	0.031	0.747	0.589	3.954	1.392	60.506	0.042	0.756	0.826	3.913	0.287	60.506
Annually	0.041	0.799	0.753	3.480	0.674	251.579	0.049	0.804	0.900	3.458	0.227	251.579
Thld 5%	-0.023	0.698	-0.593	4.225	7.221	1.723	0.041	0.728	0.835	4.231	0.415	8.968
Thld 10%	-0.004	0.702	-0.162	4.288	5.303	3.627	0.041	0.733	0.820	4.165	0.370	20.873
Thld 20%	0.014	0.708	0.249	**4.320**	3.266	10.529	0.041	0.749	0.803	3.984	0.317	54.318
Thld 40%	0.022	0.737	0.409	4.018	1.844	36.212	0.041	0.782	0.779	3.642	0.243	159.333
Boot 1%	0.035	0.769	0.664	3.743	0.926	140.588	0.043	0.742	0.867	4.078	0.398	1.340
Boot 25%	0.031	0.739	0.612	4.043	1.192	85.357	0.040	0.726	0.816	4.249	0.428	1.131
Boot 50%	0.032	0.726	0.629	4.199	1.364	63.733	0.041	**0.725**	0.826	**4.267**	0.450	1.103
Boot 75%	0.025	0.728	0.490	4.142	1.606	46.863	0.041	**0.725**	0.826	**4.267**	0.450	1.103
Boot 99%	0.021	0.712	0.408	4.308	2.321	21.532	0.041	**0.725**	0.826	**4.267**	0.450	1.103
Panel C: POET estimator												
B&H	**0.091**	1.475	**0.942**	1.000	**0.000**	∞	**0.091**	1.475	**0.942**	1.000	**0.000**	∞
Daily	-0.017	**0.735**	-0.431	3.834	5.595	1.000	0.035	0.777	0.653	3.639	0.381	1.100
Weekly	0.009	0.749	0.140	3.812	2.891	5.000	0.035	0.785	0.652	3.565	0.328	5.156
Monthly	0.022	0.760	0.405	3.759	1.559	20.000	0.037	0.786	0.694	3.570	0.274	20.000
Quarterly	0.028	0.785	0.502	3.541	0.948	60.506	0.039	0.805	0.720	3.406	0.229	60.506
Annually	0.035	0.841	0.614	3.092	0.507	251.579	0.044	0.868	0.742	2.921	0.185	251.579
Thld 5%	-0.005	0.736	-0.176	3.878	4.371	2.175	0.034	0.784	0.639	3.575	0.332	10.170
Thld 10%	0.007	0.742	0.079	3.869	3.093	5.213	0.035	0.791	0.644	3.507	0.299	23.663
Thld 20%	0.018	0.756	0.319	3.776	1.913	15.621	0.035	0.805	0.629	3.381	0.241	60.506
Thld 40%	0.025	0.805	0.437	3.348	1.087	52.527	0.039	0.857	0.664	2.981	0.207	170.714
Boot 1%	0.032	0.802	0.569	3.402	0.679	132.778	0.036	0.792	0.660	3.508	0.256	4.574
Boot 25%	0.029	0.780	0.528	3.592	0.906	68.286	0.035	0.782	0.654	3.599	0.348	1.462
Boot 50%	0.023	0.768	0.423	3.682	1.127	45.962	0.035	0.778	0.651	3.629	0.363	1.348
Boot 75%	0.022	0.758	0.410	3.780	1.300	33.427	0.035	**0.777**	0.653	**3.640**	0.381	1.285
Boot 99%	0.017	0.745	0.298	**3.887**	1.817	15.419	0.035	0.777	0.653	3.639	0.381	1.283

Table 6.3: Performance of asset allocation strategies with variable transaction costs applied on daily S&P 500 stock data from 2000-2019.

This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance matrix estimated with a rolling window of 250 trading days and assuming transaction costs consisting of a fixed commission of 24 basis points plus half of the estimated bid-ask spread. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.099**	1.478	**1.024**	1.000	**0.000**	∞	**0.099**	1.478	**1.024**	1.000	**0.000**	∞
Daily	-0.140	**0.940**	-2.410	2.171	19.136	1.000	0.038	0.801	0.688	3.440	0.842	1.000
Weekly	-0.041	0.965	-0.725	2.210	9.380	5.000	0.039	0.798	0.725	3.473	0.708	5.000
Monthly	-0.001	0.958	-0.056	2.308	4.663	20.000	0.038	0.799	0.703	3.457	0.538	20.000
Quarterly	0.016	0.963	0.215	**2.311**	2.449	60.506	0.039	0.807	0.707	3.388	0.403	60.506
Annually	0.042	1.017	0.603	2.104	0.897	251.579	0.050	0.847	0.871	3.095	0.287	251.579
Thld 5%	-0.133	0.941	-2.290	2.174	18.429	1.177	0.038	0.798	0.690	3.462	0.733	5.438
Thld 10%	-0.109	0.945	-1.870	2.194	15.885	1.739	0.037	**0.793**	0.678	**3.508**	0.596	12.781
Thld 20%	-0.063	0.952	-1.099	2.236	11.042	3.855	0.036	0.802	0.654	3.422	0.482	32.740
Thld 40%	-0.017	0.963	-0.328	2.257	6.027	12.679	0.039	0.818	0.709	3.297	0.397	85.357
Boot 1%	**0.099**	1.478	1.024	1.000	**0.000**	∞	0.039	0.801	0.723	3.443	0.502	1.949
Boot 25%	0.045	1.090	0.618	1.831	0.445	531.111	0.037	0.794	0.680	3.496	0.660	1.210
Boot 50%	0.030	0.988	0.432	2.215	1.042	199.167	0.038	0.797	0.692	3.472	0.758	1.123
Boot 75%	0.033	0.969	0.495	2.310	1.706	106.222	0.038	0.799	0.691	3.453	0.811	1.064
Boot 99%	-0.002	0.972	-0.071	2.243	3.758	30.641	0.038	0.801	0.688	3.440	0.843	1.006
Panel B: Shrinkage estimator												
B&H	**0.095**	1.479	**0.987**	1.000	**0.000**	∞	**0.095**	1.479	**0.987**	1.000	**0.000**	∞
Daily	-0.001	**0.695**	-0.089	4.399	5.016	1.000	0.041	0.714	0.835	4.406	0.406	1.001
Weekly	0.024	0.706	0.479	4.409	2.552	5.000	0.041	0.719	0.834	4.340	0.350	5.000
Monthly	0.033	0.719	0.668	4.290	1.360	20.000	0.040	0.729	0.815	4.210	0.297	20.000
Quarterly	0.037	0.742	0.718	4.037	0.802	60.506	0.042	0.747	0.827	4.008	0.243	60.506
Annually	0.044	0.794	0.816	3.536	0.379	251.579	0.048	0.788	0.917	3.615	0.176	251.579
Thld 5%	0.006	0.696	0.077	4.431	4.261	1.723	0.040	0.718	0.811	4.348	0.362	6.751
Thld 10%	0.017	0.698	0.328	4.472	3.134	3.627	0.040	0.725	0.801	4.260	0.329	15.127
Thld 20%	0.027	0.702	0.553	**4.478**	1.927	10.529	0.038	0.736	0.754	4.109	0.280	38.548
Thld 40%	0.030	0.731	0.578	4.132	1.082	36.212	0.042	0.759	0.813	3.879	0.245	101.702
Boot 1%	0.042	0.758	0.809	3.891	0.532	140.588	0.044	0.739	0.873	4.115	0.296	1.731
Boot 25%	0.037	0.737	0.723	4.094	0.682	85.357	0.042	0.715	0.864	4.393	0.360	1.229
Boot 50%	0.037	0.723	0.758	4.270	0.802	62.895	0.040	0.715	0.827	4.393	0.384	1.137
Boot 75%	0.034	0.725	0.672	4.218	0.957	46.863	0.041	0.714	0.835	4.406	0.404	1.054
Boot 99%	0.030	0.704	0.605	4.469	1.390	21.244	0.041	**0.714**	0.835	**4.406**	0.406	1.037
Panel C: POET estimator												
B&H	**0.091**	1.475	**0.942**	1.000	**0.000**	∞	**0.091**	1.475	**0.942**	1.000	**0.000**	∞
Daily	0.006	**0.734**	0.073	3.960	3.255	1.000	0.035	0.762	0.659	3.789	0.322	1.026
Weekly	0.022	0.747	0.396	3.889	1.679	5.000	0.035	0.771	0.650	3.697	0.269	5.032
Monthly	0.029	0.759	0.542	3.800	0.907	20.000	0.035	0.775	0.649	3.660	0.218	20.000
Quarterly	0.032	0.783	0.584	3.570	0.545	60.506	0.037	0.792	0.675	3.508	0.173	60.506
Annually	0.038	0.839	0.657	3.116	0.285	251.579	0.043	0.849	0.742	3.057	0.135	251.579
Thld 5%	0.013	0.734	0.218	**3.986**	2.539	2.175	0.035	0.766	0.661	3.749	0.285	7.636
Thld 10%	0.020	0.739	0.358	3.964	1.793	5.213	0.033	0.774	0.608	3.665	0.243	17.704
Thld 20%	0.026	0.753	0.489	3.848	1.110	15.621	0.035	0.783	0.640	3.580	0.195	48.776
Thld 40%	0.030	0.801	0.529	3.397	0.633	52.527	0.035	0.814	0.629	3.308	0.164	132.778
Boot 1%	0.035	0.798	0.633	3.446	0.399	132.778	0.035	0.791	0.644	3.512	0.188	10.529
Boot 25%	0.032	0.777	0.604	3.636	0.534	68.286	0.035	0.770	0.662	3.711	0.268	1.621
Boot 50%	0.027	0.763	0.498	3.746	0.667	45.962	0.034	0.765	0.648	3.759	0.293	1.298
Boot 75%	0.028	0.755	0.529	3.834	0.761	33.901	0.035	0.763	0.658	3.785	0.318	1.172
Boot 99%	0.026	0.742	0.487	3.968	1.062	15.672	0.035	**0.762**	0.659	**3.789**	0.321	1.129

entire 20-year period, because they have a low market capitalisation at the beginning of the sample period. On the other hand, it also includes large companies that have been around for more than a century.

The performance of the asset allocation strategies with variable transaction costs is shown in Table 6.3. The relative performance of the rebalancing strategies generally aligns with the results shown for fixed transaction costs in Table 6.2. However, the incurred transaction costs are slightly lower due to a decrease in the average trading cost from 50 to 40 bps per transaction. Hence, most strategies achieve higher utility levels. The performance of strategies optimised including transaction costs is disappointing compared to those optimised excluding transaction costs when considering that variable transaction costs should make this method more attractive; this method can pick the most cost-efficient companies to trade. An explanation behind this is that the incurred transaction costs are already low when these costs are excluded in the optimisation; transaction costs are below 5 bps per day in Panels B and C. For this reason, only marginal gains can be realised by further reducing transaction costs. Nevertheless, this section demonstrates that the bootstrap rebalancing strategy is flexible to work with variable transaction costs.

7 Conclusion

The bootstrap rebalancing strategy proposed in this research can improve portfolio performance relative to traditional rebalancing techniques, because it helps to account for uncertainty in the portfolio rebalancing decision. The economic value derived from this strategy is highest when it is applied with a portfolio optimisation method that does not account for transaction costs. Taking transaction costs into account in the optimisation already severely reduces the rebalancing activity, such that there is only a benefit to applying the bootstrap rebalancing strategy when estimation uncertainty is high. The performance of the bootstrap rebalancing strategy depends on a hyperparameter that increases the willingness to trade. This hyperparameter optimally decreases with the investor's risk appetite, the size of transaction costs, and estimation certainty. The performance of the bootstrap rebalancing strategy is robust to the imposition of short sale constraints and changes in the bootstrap methodology. In addition, the strategy also performs well when proportional transaction costs change over time and differ per asset.

This paper further shows that the optimisation method of Dybvig and Pezzo (2019) yields significantly higher post-transaction cost returns and Sharpe ratios than a method that ignores these costs. Therefore, it can be concluded that this optimisation method is highly effective in improving portfolio performance. Moreover, the empirical study demonstrates that there exists a non-linear relation between the average rebalancing time and the amount of transaction costs incurred for any rebalancing strategy. More specifically, reducing the rebalancing frequency from often to sometimes leads to a large reduction in transaction costs, whereas reducing this from seldom to never only yields marginal gains in terms of transaction costs.

This research contributes to the existing literature by extending the bootstrap procedure of Michaud and Michaud (2008) to handle transaction costs and by empirically verifying that this method can enhance portfolio performance. Furthermore, it is shown that the optimisation method of Dybvig and Pezzo (2019) also works well with equity data. From a practical perspective, the bootstrap rebalancing strategy can easily be implemented by a portfolio manager who applies mean-variance optimisation. Implementation of this strategy requires selecting a hyperparameter, which can be done by performing a historical simulation or based on the findings of this paper. In case uncertainty is high, applying this strategy yields considerable economic gains due to a reduction in transaction costs while maintaining the same level of risk and return.

The methodology used in this paper has some limitations. The gains from incorporating transaction costs into the optimisation are limited by the single-period horizon assumed by the method of Dybvig and Pezzo (2019), since this leads to myopic investing. The performance of this optimisation technique can be improved by shrinking the costs function to zero. However, the optimal amount of shrinkage and the performance gains that come with this have not been investigated. Therefore, future research may be dedicated to improving the method of Dybvig and Pezzo (2019). Alternatively, a similar analysis can be conducted using a multi-period utility optimisation method, such as that of Mei et al. (2016).

Another limitation is that the transaction cost function has been restricted to include proportional costs. In practice, some costs are fixed rather than proportional and thus decrease with the amount traded. On the other hand, large investors suffer from market impact costs that increase with the transaction size. Ideally, these more complex cost structures are also taken into account. This research further assumed trading costs to be known, which is usually not the case in practice. An interesting direction for future research would be to estimate transaction costs using a model and to take the uncertainty in this estimation into account. This can conveniently be done using the bootstrap methodology applied in this research.

Finally, this paper restricted itself to variance minimisation, although mean-variance optimisation may be preferred when asset returns are more predictable. Incorporating expected returns into the optimisation leads to additional estimation uncertainty, which likely makes the bootstrap rebalancing strategy even more effective. This hypothesis should be tested in follow-up research. In addition, it would be of interest to investigate how bootstrap rebalancing strategies perform in combination with dynamic covariance estimation methods, because then the bootstrap methodology becomes more important.

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Appendix A: Quadratic Programming Problem

This appendix formulates programming problem (3.2) with n assets and the cost function defined by equation (3.3) as a constrained quadratic program. Following the method of Maurer et al. (2019), this problem can be expressed as

$$\begin{aligned} \min_{x} \quad & q'x + \frac{1}{2}x'Hx \\ \text{s.t.} \quad & Ax = b \\ & l \leq x \leq u, \end{aligned} \tag{A.1}$$

where $x = \begin{bmatrix} \Delta_+ \\ \Delta_- \end{bmatrix}$, $q = b + \lambda \bar{\bar{I}}' \Sigma \theta_0$ with $b = \begin{bmatrix} C_+ \\ C_- \end{bmatrix}$ and $\bar{\bar{I}} = [I, -I]$. Here, I denotes an $n \times n$ identity matrix. In addition, $H = \lambda \bar{\bar{I}}' \Sigma \bar{\bar{I}}$, $A = [\iota, -\iota]$ with ι being an $n \times 1$ vector, and $b = 0$. The lower bound $l = [0, ..., 0]'$ and upper bound $u = [\infty, ..., \infty]'$ are both $2n \times 1$ vectors. In case short sale constraints are imposed, then the upper bound is given by $u = [\infty, ..., \infty, \theta_0']'$ with ∞ for the first n elements. The optimal portfolio allocation is given by

$$\theta^* = \theta_0 + \bar{\bar{I}}x. \tag{A.2}$$

Appendix B: Bootstrap Test for Mean Return and Utility Level

This appendix shows how the robust hypothesis test of Ledoit and Wolf (2008) can be used to test for a significant difference in the mean return and utility level. To apply this method, it is necessary to (i) express the difference in performance measures as a function of the moments of the returns and to (ii) derive the gradient of this function.

To derive the necessary expressions, the excess returns of portfolio investment strategies i and j at time t are denoted by r_{it} and r_{jt}, respectively. It is assumed the time series are strictly stationary, which means that the bivariate distribution does not change over time. The mean return vector μ and covariance matrix Σ are given by

$$\mu = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_i^2 & \sigma_{ij} \\ \sigma_{ij} & \sigma_j^2 \end{bmatrix}. \tag{B.1}$$

The differences in mean return and utility level can both be expressed in terms of elements of the mean vector and covariance matrix as

$$\Delta\mu = \mu_i - \mu_j \quad \text{and} \quad \Delta\text{U} = \mu_i - \mu_j - \frac{\lambda}{2}(\sigma_i^2 - \sigma_j^2), \tag{B.2}$$

where λ is a given risk aversion parameter. The method uses the first two uncentred moments, which are given by $\mu_i = \mathbb{E}(r_{1i})$ and $\gamma_i = \mathbb{E}(r_{1i}^2)$ for strategy i and are defined analogously for strategy j. Let $[a, b, c, d] = [\mu_i, \mu_j, \gamma_i, \gamma_j]$, then the difference in mean return and utility level can be expressed as

$$\Delta\mu = f(a, b, c, d) \quad \text{and} \quad \Delta\text{U} = g(a, b, c, d), \tag{B.3}$$

with

$$f(a, b, c, d) = a - b \quad \text{and} \quad g(a, b, c, d) = a - b - \frac{\lambda}{2}\left((c - a^2) - (d - b^2)\right). \tag{B.4}$$

The gradient of these functions are given by

$$\nabla' f(a, b, c, d) = [1,\ -1,\ 0,\ 0]' \quad \text{and} \quad \nabla' g(a, b, c, d) = [1 + a\lambda,\ -1 - b\lambda,\ -\frac{\lambda}{2},\ \frac{\lambda}{2}]', \tag{B.5}$$

which can be used to apply the delta method in the computation of the standard error (see Ledoit and Wolf (2008) for details). All other elements of the robust performance hypothesis testing procedure remain unaltered.

Appendix C: Description Industry Portfolio Data

This appendix provides additional descriptive statistics of the industry portfolio data set described in Section 4.1.

Table C.1: Descriptive statistics of daily industry portfolio data and the market from 2000-2019.
All statistics are daily in %, except for the annualised Sharpe ratio. The correlation with the market is denoted by Mkt.

Industry	Mean	Volatility	Sharpe ratio	Minimum	Maximum	Mkt
Agriculture	0.045	1.639	0.405	-15.270	18.240	0.582
Food Products	0.039	0.975	0.577	-7.250	8.800	0.658
Candy & Soda	0.054	1.381	0.588	-19.220	11.680	0.525
Beer & Liquor	0.040	1.072	0.540	-7.700	10.120	0.526
Tobacco Products	0.073	1.445	0.765	-13.380	14.990	0.400
Recreation	0.038	1.555	0.352	-9.960	9.690	0.688
Entertainment	0.059	1.957	0.453	-13.390	16.580	0.744
Printing and Publishing	0.015	1.490	0.128	-11.240	19.450	0.787
Consumer Goods	0.032	1.071	0.427	-17.980	9.440	0.665
Apparel	0.059	1.540	0.578	-11.520	12.690	0.785
Healthcare	0.045	1.349	0.494	-13.970	8.300	0.638
Medical Equipment	0.049	1.184	0.618	-7.210	11.680	0.760
Pharmaceutical Products	0.036	1.172	0.447	-6.530	11.140	0.743
Chemicals	0.044	1.517	0.427	-11.380	13.070	0.820
Rubber and Plastic Products	0.041	1.357	0.443	-10.140	8.090	0.802
Textiles	0.043	1.850	0.343	-18.310	19.500	0.695
Construction Materials	0.045	1.504	0.437	-10.670	9.620	0.833
Construction	0.057	1.945	0.436	-12.390	15.580	0.790
Steel Works	0.026	2.205	0.161	-15.900	20.260	0.813
Fabricated Products	0.032	1.868	0.241	-15.450	11.440	0.714
Machinery	0.052	1.675	0.460	-12.260	13.890	0.897
Electrical Equipment	0.035	1.598	0.317	-13.120	14.080	0.871
Automobiles and Trucks	0.026	1.771	0.203	-12.890	11.700	0.808
Aircraft	0.054	1.504	0.534	-18.370	13.570	0.787
Shipbuilding and Railroad Equipment	0.072	1.686	0.649	-11.280	10.620	0.657
Defence	0.073	1.448	0.769	-10.290	14.920	0.491
Precious Metals	0.048	2.519	0.283	-14.160	25.560	0.195
Non-Metallic and Industrial Metal Mining	0.056	2.131	0.389	-16.990	19.850	0.717
Coal	0.041	3.104	0.193	-19.340	21.360	0.566
Petroleum and Natural Gas	0.039	1.649	0.347	-15.380	19.270	0.697
Utilities	0.043	1.136	0.561	-8.920	14.430	0.653
Communication	0.019	1.328	0.187	-9.670	14.470	0.854
Personal Services	0.035	1.402	0.355	-9.400	8.950	0.748
Business Services	0.033	1.281	0.375	-8.800	8.230	0.925
Computer Hardware	0.029	1.924	0.212	-11.680	21.650	0.789
Computer Software	0.030	1.601	0.262	-8.950	14.820	0.866
Chips	0.034	1.949	0.251	-10.930	15.880	0.815
Measuring and Control Equipment	0.047	1.621	0.428	-9.310	12.700	0.860
Business Supplies	0.034	1.253	0.392	-9.610	8.630	0.825
Shipping Containers	0.047	1.438	0.488	-9.040	10.920	0.774
Transportation	0.047	1.416	0.485	-14.040	9.330	0.826
Wholesale	0.038	1.185	0.471	-8.500	9.740	0.861
Retail	0.041	1.300	0.457	-8.310	11.750	0.805
Restaurants, Hotels and Motels	0.054	1.211	0.662	-8.260	8.900	0.740
Banking	0.039	1.859	0.307	-16.970	16.940	0.817
Insurance	0.044	1.435	0.450	-11.530	17.840	0.840
Real Estate	0.045	1.853	0.354	-16.190	19.190	0.755
Trading	0.042	1.950	0.313	-16.280	17.960	0.888
Other	0.022	1.450	0.204	-10.220	15.240	0.796
Market	0.032	1.203	0.376	-8.950	11.354	1.000

Appendix D: Additional Results Empirical Study

This appendix provides additional results relating to the empirical study shown in Section 4.5.

Table D.1 shows the suggested bootstrap block size acquired by minimising the mean squared error of the variance estimate for each industry portfolio. The suggested block size has a wide range from 1 to more than 30 trading days. A smaller block size generally leads to more bias but more efficiency (Lahiri, 2003). Therefore, a block size of one month (20 trading days) is chosen to obtain an appropriate balance between bias and variance.

Table D.1: Optimal bootstrap block size selected by the algorithm of Politis and White (2004) for daily industry portfolio data from 2000-2019.
The algorithm selects the block size by minimising the long-run variance estimate based on the mean squared error. They suggest using a block size of one if the optimal estimated block size is below one.

Bootstrap	Mean	Standard deviation	Minimum	Maximum
Stationary	6.062	6.995	0.420	33.728
Circular	6.939	8.007	0.481	38.608

Tables D.2 and D.3 show how the asset allocation strategies perform when the risk aversion parameter is varied to 1 and 10, respectively. This parameter was fixed at 3 in Section 4. Importantly, Table D.2 shows that bootstrap rebalancing strategies with a lower α optimised excluding transaction costs perform relatively better when the investor is more risk seeking; the Boot 75% strategy achieves the highest utility level in all three panels. For portfolios optimised including transaction costs the Boot 1% strategy consistently performs best. Furthermore, Table D.3 shows that increased risk aversion leads to a sharp decline in the rebalancing activity for portfolios optimised including transaction costs. As a result, transaction costs decrease at the cost of an increase in volatility. This is in line with the findings of Dybvig and Pezzo (2019).

Tables D.4 and D.5 show the performance of the asset allocation strategies when trading costs have been changed to 25 and 100 bps per transaction, respectively. Table D.4 shows that strategies optimised excluding transaction costs perform relatively better when transaction costs are low, because there is a smaller benefit to taking trading costs into account. In line with the findings of Dybvig and Pezzo (2019), strategies that take trading costs into account rebalance more when these costs decrease, which leads to a volatility reduction. Table D.5 shows that if transaction costs are 100 bps then strategies optimised including transaction costs achieve much higher Sharpe ratios. On the other hand, strategies optimised excluding transaction costs achieve higher utility levels in Panels B and C. This may be unexpected, as the importance of taking trading costs into account would logically increase with its magnitude. Yet, the myopic investment behaviour induced by the method of Dybvig and Pezzo (2019) causes a large decrease in rebalancing activity, which leads to inferior portfolio performance. For portfolios optimised excluding transaction costs, bootstrap rebalancing strategies with a low α perform relatively well when transaction costs are high. These bootstrap rebalancing strategies outperform other strategies. The bootstrap rebalancing strategy is most valuable in Panel A, because the sample covariance estimator suffers most from estimation uncertainty.

Table D.2: Performance of asset allocation strategies with a risk aversion parameter of 1 applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance estimated on a rolling-window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 1 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.038**	1.102	0.509	1.000	**0.000**	∞	0.038	1.102	0.509	1.000	**0.000**	∞
Daily	-0.082	**0.716**	-1.873	1.678	12.244	1.000	0.042	0.817	0.751	1.948	0.098	2.265
Weekly	-0.020	0.725	-0.503	2.009	6.135	5.000	0.042	**0.813**	0.762	1.969	0.090	8.739
Monthly	0.007	0.742	0.091	2.120	3.276	20.000	0.043	0.815	0.782	1.966	0.084	29.325
Quarterly	0.018	0.774	0.312	2.018	1.956	60.506	0.043	0.831	0.758	1.879	0.067	78.361
Annually	0.033	0.816	0.578	1.893	0.966	251.579	0.044	0.846	0.772	1.813	0.045	298.750
Thld 5%	-0.063	0.721	-1.454	1.756	10.387	1.708	0.042	0.820	0.760	1.935	0.086	37.937
Thld 10%	-0.036	0.725	-0.844	1.904	7.557	3.565	0.043	0.831	0.769	1.884	0.082	78.361
Thld 20%	-0.008	0.731	-0.242	2.061	4.702	9.835	0.044	0.842	0.775	1.831	0.077	164.828
Thld 40%	0.012	0.747	0.185	2.123	2.633	33.194	0.044	0.847	0.776	1.807	0.039	531.111
Boot 1%	0.032	0.785	**0.590**	2.061	1.071	191.200	**0.045**	0.814	**0.814**	**1.986**	0.096	4.609
Boot 25%	0.023	0.760	0.416	2.139	1.609	91.923	0.042	0.817	0.751	1.947	0.097	4.110
Boot 50%	0.021	0.749	0.384	2.188	1.905	66.389	0.042	0.817	0.751	1.947	0.097	4.110
Boot 75%	0.017	0.743	0.308	**2.200**	2.139	49.792	0.042	0.817	0.751	1.947	0.097	4.110
Boot 99%	0.003	0.736	0.005	2.125	3.281	20.873	0.042	0.817	0.751	1.947	0.097	4.110
Panel B: Shrinkage estimator												
B&H	**0.039**	1.084	0.524	1.000	**0.000**	∞	0.039	1.084	0.524	1.000	**0.000**	∞
Daily	-0.047	**0.689**	-1.154	1.927	8.771	1.000	0.042	0.807	0.769	1.937	0.089	2.445
Weekly	-0.003	0.695	-0.138	2.238	4.423	5.000	0.042	0.805	0.777	1.950	0.083	9.175
Monthly	0.016	0.711	0.292	2.317	2.397	20.000	0.043	0.809	0.788	1.933	0.077	30.446
Quarterly	0.024	0.744	0.455	2.174	1.465	60.506	0.043	0.822	0.778	1.862	0.062	77.097
Annually	0.035	0.780	**0.652**	2.035	0.749	251.579	**0.044**	0.840	0.774	1.774	0.042	298.750
Thld 5%	-0.030	0.693	-0.760	2.027	7.069	1.979	0.042	0.811	0.772	1.916	0.081	37.937
Thld 10%	-0.010	0.694	-0.283	2.192	4.978	4.426	0.044	0.816	0.796	1.897	0.077	74.688
Thld 20%	0.008	0.707	0.122	2.273	3.079	12.679	0.042	0.832	0.753	1.807	0.066	177.037
Thld 40%	0.021	0.717	0.401	2.327	1.729	43.063	0.043	0.851	0.754	1.721	0.032	597.500
Boot 1%	0.032	0.737	0.621	2.285	0.951	154.194	0.044	**0.799**	**0.808**	**1.989**	0.081	5.173
Boot 25%	0.028	0.724	0.552	2.345	1.294	81.017	0.042	0.807	0.769	1.936	0.089	4.434
Boot 50%	0.022	0.720	0.428	2.318	1.513	57.590	0.042	0.807	0.769	1.936	0.089	4.434
Boot 75%	0.022	0.708	0.426	**2.396**	1.728	43.455	0.042	0.807	0.769	1.936	0.089	4.434
Boot 99%	0.013	0.699	0.219	2.366	2.498	19.274	0.042	0.807	0.769	1.936	0.089	4.434
Panel C: POET estimator												
B&H	**0.039**	0.971	0.593	1.000	**0.000**	∞	0.039	0.971	0.593	1.000	**0.000**	∞
Daily	-0.039	**0.729**	-0.911	1.420	7.490	1.000	0.039	0.822	0.694	1.447	0.104	2.563
Weekly	-0.001	0.733	-0.085	1.603	3.817	5.000	0.040	0.823	0.705	1.445	0.098	9.560
Monthly	0.017	0.747	0.295	1.652	2.074	20.000	0.040	0.820	0.715	1.460	0.082	32.081
Quarterly	0.024	0.775	0.433	1.566	1.260	60.506	**0.040**	0.841	0.701	1.379	0.064	91.923
Annually	0.036	0.792	**0.663**	1.559	0.628	251.579	0.039	0.865	0.670	1.293	0.045	318.667
Thld 5%	-0.023	0.732	-0.550	1.489	5.798	2.286	0.038	0.825	0.681	1.432	0.087	38.548
Thld 10%	-0.005	0.735	-0.170	1.572	4.075	5.444	0.040	0.828	0.709	1.430	0.083	73.538
Thld 20%	0.008	0.740	0.119	1.629	2.617	15.127	0.038	0.842	0.657	1.365	0.055	207.826
Thld 40%	0.021	0.758	0.369	1.622	1.567	47.327	0.037	0.865	0.628	1.283	0.046	531.111
Boot 1%	0.033	0.791	0.594	1.543	0.861	140.588	0.040	**0.813**	**0.720**	**1.487**	0.096	5.323
Boot 25%	0.025	0.758	0.470	1.653	1.202	70.294	0.039	0.822	0.693	1.446	0.104	4.509
Boot 50%	0.020	0.748	0.372	1.668	1.424	50.316	0.039	0.822	0.693	1.446	0.104	4.509
Boot 75%	0.017	0.743	0.305	**1.673**	1.821	32.297	0.039	0.822	0.693	1.446	0.104	4.509
Boot 99%	0.008	0.738	0.120	1.641	2.639	14.226	0.039	0.822	0.693	1.446	0.104	4.509

Table D.3: Performance of asset allocation strategies with a risk aversion parameter of 10 applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance estimated on a rolling-window of 250 trading days and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 10 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	0.038	1.102	0.509	1.000	**0.000**	∞	0.038	1.102	0.509	1.000	**0.000**	∞
Daily	-0.082	**0.716**	-1.873	2.276	12.244	1.000	0.029	0.711	0.580	2.416	0.892	1.025
Weekly	-0.020	0.725	-0.503	**2.277**	6.135	5.000	0.032	0.713	0.636	2.400	0.807	5.021
Monthly	0.007	0.742	0.091	2.196	3.276	20.000	0.035	0.722	0.698	2.343	0.713	20.000
Quarterly	0.018	0.774	0.312	2.023	1.956	60.506	0.035	0.751	0.674	2.168	0.561	60.506
Annually	0.033	0.816	0.578	1.829	0.966	251.579	**0.040**	0.794	**0.735**	1.937	0.399	251.579
Thld 5%	-0.063	0.721	-1.454	2.262	10.387	1.708	0.030	0.713	0.606	2.401	0.805	7.735
Thld 10%	-0.036	0.725	-0.844	2.264	7.557	3.565	0.032	0.715	0.643	2.386	0.745	16.950
Thld 20%	-0.008	0.731	-0.242	2.247	4.702	9.835	0.032	0.728	0.639	2.304	0.632	39.833
Thld 40%	0.012	0.747	0.185	2.168	2.633	33.194	0.034	0.751	0.659	2.166	0.512	106.222
Boot 1%	**0.039**	0.793	**0.717**	1.940	1.051	191.200	0.034	0.746	0.657	2.192	0.649	2.459
Boot 25%	0.025	0.758	0.471	2.115	1.624	91.923	0.031	0.714	0.616	2.391	0.823	1.306
Boot 50%	0.021	0.751	0.387	2.156	1.907	65.479	0.029	0.711	0.579	2.412	0.876	1.150
Boot 75%	0.016	0.743	0.287	2.196	2.188	48.776	0.029	0.711	0.580	2.416	0.892	1.096
Boot 99%	0.004	0.738	0.025	2.216	3.260	21.057	0.029	**0.711**	0.580	**2.416**	0.892	1.096
Panel B: Shrinkage estimator												
B&H	**0.039**	1.084	0.524	1.000	**0.000**	∞	0.039	1.084	0.524	1.000	**0.000**	∞
Daily	-0.047	**0.689**	-1.154	2.412	8.771	1.000	0.031	0.694	0.648	2.452	0.765	1.034
Weekly	-0.003	0.695	-0.138	2.410	4.423	5.000	0.033	0.698	0.688	2.426	0.681	5.016
Monthly	0.016	0.711	0.292	2.323	2.397	20.000	0.035	0.709	0.723	2.352	0.588	20.000
Quarterly	0.024	0.744	0.455	2.129	1.465	60.506	0.035	0.739	0.693	2.162	0.476	60.506
Annually	0.035	0.780	0.652	1.938	0.749	251.579	**0.041**	0.775	**0.775**	1.968	0.344	251.579
Thld 5%	-0.030	0.693	-0.760	2.399	7.069	1.979	0.033	0.697	0.674	2.432	0.689	8.102
Thld 10%	-0.010	0.694	-0.283	**2.415**	4.978	4.426	0.033	0.701	0.688	2.408	0.619	17.836
Thld 20%	0.008	0.707	0.122	2.345	3.079	12.679	0.033	0.718	0.668	2.292	0.528	43.455
Thld 40%	0.021	0.717	0.401	2.291	1.729	43.063	0.033	0.735	0.647	2.185	0.385	122.564
Boot 1%	0.033	0.734	**0.653**	2.190	0.971	149.375	0.036	0.727	0.732	2.237	0.548	2.566
Boot 25%	0.026	0.733	0.501	2.192	1.301	81.017	0.033	0.696	0.683	2.441	0.683	1.365
Boot 50%	0.023	0.720	0.443	2.270	1.514	57.590	0.031	0.695	0.648	2.446	0.745	1.207
Boot 75%	0.023	0.712	0.455	2.320	1.725	43.853	0.031	**0.694**	0.648	**2.452**	0.764	1.130
Boot 99%	0.014	0.701	0.243	2.385	2.493	19.352	0.031	0.694	0.648	2.452	0.765	1.119
Panel C: POET estimator												
B&H	**0.039**	0.971	0.593	1.000	**0.000**	∞	0.039	0.971	0.593	1.000	**0.000**	∞
Daily	-0.039	**0.729**	-0.911	1.737	7.490	1.000	0.027	**0.734**	0.514	**1.754**	0.934	1.054
Weekly	-0.001	0.733	-0.085	**1.741**	3.817	5.000	0.029	0.739	0.551	1.730	0.777	5.021
Monthly	0.017	0.747	0.295	1.690	2.074	20.000	0.030	0.745	0.586	1.706	0.615	20.000
Quarterly	0.024	0.775	0.433	1.571	1.260	60.506	0.030	0.768	0.561	1.605	0.483	60.506
Annually	0.036	0.792	**0.663**	1.510	0.628	251.579	**0.040**	0.788	**0.745**	1.526	0.362	251.579
Thld 5%	-0.023	0.732	-0.550	1.732	5.798	2.286	0.027	0.737	0.524	1.741	0.807	7.480
Thld 10%	-0.005	0.735	-0.170	1.729	4.075	5.444	0.028	0.739	0.540	1.729	0.709	16.040
Thld 20%	0.008	0.740	0.119	1.713	2.617	15.127	0.029	0.744	0.557	1.710	0.589	39.180
Thld 40%	0.021	0.758	0.369	1.640	1.567	47.327	0.035	0.758	0.676	1.648	0.461	111.163
Boot 1%	0.030	0.797	0.538	1.487	0.839	144.849	0.035	0.775	0.663	1.579	0.476	10.552
Boot 25%	0.024	0.760	0.433	1.632	1.198	70.294	0.028	0.738	0.536	1.735	0.717	1.794
Boot 50%	0.020	0.748	0.363	1.685	1.482	47.327	0.027	0.736	0.526	1.747	0.835	1.399
Boot 75%	0.017	0.742	0.298	1.709	1.837	31.867	0.027	0.735	0.518	1.750	0.900	1.278
Boot 99%	0.009	0.738	0.132	1.725	2.646	14.311	0.027	0.734	0.514	1.754	0.934	1.138

Table D.4: Performance of asset allocation strategies with transaction costs of 25 bps applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance estimated on a rolling-window of 250 trading days and assuming transaction costs of 25 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.038**	1.102	0.509	1.000	**0.000**	∞	0.038	1.102	0.509	1.000	**0.000**	∞
Daily	-0.021	**0.711**	-0.527	2.290	6.122	1.000	0.035	0.721	0.711	2.394	0.287	1.079
Weekly	0.010	0.715	0.168	**2.359**	3.068	5.000	0.036	0.725	0.732	2.369	0.261	5.085
Monthly	0.023	0.730	0.447	2.296	1.638	20.000	0.038	0.735	0.751	2.308	0.225	20.000
Quarterly	0.028	0.763	0.520	2.111	0.978	60.506	0.038	0.762	0.740	2.140	0.193	60.506
Annually	0.037	0.804	0.681	1.911	0.483	251.579	**0.044**	0.795	**0.811**	1.972	0.147	251.579
Thld 5%	-0.012	0.713	-0.319	2.305	5.193	1.708	0.035	0.726	0.710	2.362	0.252	10.814
Thld 10%	0.002	0.714	-0.020	2.338	3.779	3.565	0.036	0.728	0.715	2.346	0.225	23.547
Thld 20%	0.015	0.719	0.270	2.343	2.351	9.835	0.038	0.743	0.753	2.255	0.210	53.708
Thld 40%	0.025	0.735	0.471	2.268	1.317	33.194	0.040	0.761	0.777	2.151	0.172	140.588
Boot 1%	0.038	0.779	**0.707**	2.042	0.554	183.846	0.039	0.748	0.758	2.229	0.239	2.144
Boot 25%	0.031	0.744	0.595	2.227	0.807	91.923	0.035	0.721	0.709	2.392	0.284	1.334
Boot 50%	0.030	0.738	0.577	2.262	0.953	65.479	0.035	**0.721**	0.711	**2.394**	0.287	1.323
Boot 75%	0.028	0.729	0.556	2.317	1.074	49.792	0.035	**0.721**	0.711	**2.394**	0.287	1.323
Boot 99%	0.021	0.724	0.394	2.327	1.639	20.873	0.035	**0.721**	0.711	**2.394**	0.287	1.323
Panel B: Shrinkage estimator												
B&H	0.039	1.084	0.524	1.000	**0.000**	∞	0.039	1.084	0.524	1.000	**0.000**	∞
Daily	-0.004	**0.685**	-0.148	2.436	4.385	1.000	0.036	**0.707**	0.751	**2.412**	0.253	1.090
Weekly	0.019	0.689	0.368	2.483	2.212	5.000	0.037	0.712	0.769	2.382	0.230	5.140
Monthly	0.028	0.704	0.564	2.406	1.199	20.000	0.038	0.723	0.777	2.313	0.198	20.084
Quarterly	0.032	0.736	0.616	2.203	0.732	60.506	0.039	0.748	0.773	2.153	0.170	60.506
Annually	**0.039**	0.773	0.735	2.010	0.374	251.579	**0.043**	0.780	**0.814**	1.984	0.127	251.579
Thld 5%	0.005	0.687	0.047	2.450	3.535	1.979	0.036	0.711	0.738	2.383	0.219	11.039
Thld 10%	0.015	0.687	0.287	**2.489**	2.489	4.426	0.037	0.714	0.760	2.368	0.200	23.900
Thld 20%	0.024	0.697	0.473	2.445	1.540	12.679	0.038	0.729	0.769	2.272	0.181	54.943
Thld 40%	0.030	0.708	0.599	2.382	0.864	43.063	0.039	0.746	0.777	2.167	0.140	149.375
Boot 1%	0.038	0.728	**0.767**	2.274	0.484	149.375	0.039	0.737	0.779	2.223	0.205	2.261
Boot 25%	0.033	0.719	0.655	2.318	0.656	79.667	0.036	0.708	0.748	2.406	0.247	1.418
Boot 50%	0.031	0.713	0.625	2.355	0.754	58.293	0.036	0.707	0.751	2.412	0.253	1.361
Boot 75%	0.031	0.700	0.638	2.445	0.855	44.259	0.036	0.707	0.751	2.412	0.253	1.361
Boot 99%	0.027	0.695	0.541	2.471	1.252	19.274	0.036	0.707	0.751	2.412	0.253	1.361
Panel C: POET estimator												
B&H	**0.039**	0.971	0.593	1.000	**0.000**	∞	0.039	0.971	0.593	1.000	**0.000**	∞
Daily	-0.002	**0.724**	-0.099	1.747	3.745	1.000	0.034	**0.741**	0.663	**1.744**	0.321	1.089
Weekly	0.018	0.729	0.329	1.766	1.908	5.000	0.034	0.745	0.669	1.726	0.273	5.096
Monthly	0.027	0.742	0.517	1.721	1.037	20.000	0.035	0.749	0.690	1.707	0.222	20.000
Quarterly	0.030	0.771	0.564	1.600	0.630	60.506	0.034	0.771	0.646	1.604	0.178	60.506
Annually	0.039	0.787	**0.731**	1.549	0.314	251.579	**0.043**	0.791	**0.809**	1.536	0.144	251.579
Thld 5%	0.006	0.726	0.077	1.756	2.899	2.286	0.034	0.743	0.656	1.733	0.280	9.264
Thld 10%	0.015	0.728	0.271	1.767	2.038	5.444	0.035	0.746	0.673	1.720	0.256	19.197
Thld 20%	0.021	0.734	0.402	1.752	1.309	15.127	0.035	0.746	0.690	1.722	0.211	47.327
Thld 40%	0.028	0.751	0.538	1.684	0.783	47.327	0.038	0.764	0.732	1.643	0.167	132.778
Boot 1%	0.033	0.781	0.605	1.559	0.416	144.849	0.035	0.773	0.663	1.598	0.199	4.238
Boot 25%	0.030	0.753	0.578	1.679	0.604	70.294	0.033	0.746	0.647	1.717	0.281	1.647
Boot 50%	0.028	0.744	0.530	1.715	0.716	49.278	0.034	0.741	0.660	1.742	0.314	1.446
Boot 75%	0.026	0.733	0.504	1.763	0.907	32.517	0.034	0.741	0.663	1.744	0.321	1.335
Boot 99%	0.022	0.730	0.415	**1.772**	1.301	14.573	0.034	0.741	0.663	1.744	0.321	1.334

Table D.5: Performance of asset allocation strategies with transaction costs of 100 bps applied on daily industry portfolio data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, threshold (Thld), and bootstrap (Boot) rebalancing strategies. A stationary bootstrap is applied with an average block size of 20 trading days. Portfolios are rebalanced to global minimum variance weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the covariance estimated on a rolling-window of 250 trading days and assuming transaction costs of 100 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column per panel is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
Panel A: Sample estimator												
B&H	**0.038**	1.102	**0.509**	1.000	**0.000**	∞	0.038	1.102	0.509	1.000	**0.000**	∞
Daily	-0.204	**0.736**	-4.456	1.754	24.489	1.000	0.039	**0.785**	0.735	2.014	0.325	1.907
Weekly	-0.082	0.764	-1.747	1.861	12.271	5.000	0.040	0.785	0.752	**2.015**	0.300	7.029
Monthly	-0.026	0.787	-0.572	1.866	6.552	20.000	0.042	0.787	0.779	2.006	0.274	22.762
Quarterly	-0.001	0.821	-0.082	1.762	3.913	60.506	0.042	0.804	0.779	1.921	0.234	66.389
Annually	0.023	0.861	0.370	1.636	1.933	251.579	0.044	0.825	0.788	1.822	0.156	265.556
Thld 5%	-0.167	0.751	-3.585	1.760	20.773	1.708	0.041	0.788	0.761	2.002	0.297	23.547
Thld 10%	-0.111	0.763	-2.367	1.809	15.115	3.565	0.041	0.795	0.751	1.964	0.265	51.957
Thld 20%	-0.055	0.776	-1.186	1.858	9.405	9.835	0.043	0.800	0.788	1.944	0.238	113.809
Thld 40%	-0.015	0.795	-0.350	1.853	5.266	33.194	**0.046**	0.828	**0.826**	1.814	0.209	298.750
Boot 1%	0.015	0.847	0.225	1.681	2.249	183.846	0.043	0.790	0.799	1.995	0.321	2.940
Boot 25%	0.007	0.807	0.089	1.839	3.268	91.923	0.039	0.785	0.735	2.013	0.325	2.691
Boot 50%	0.003	0.790	0.013	**1.912**	3.761	66.389	0.039	0.785	0.735	2.013	0.325	2.691
Boot 75%	-0.004	0.794	-0.134	1.879	4.296	49.792	0.039	0.785	0.735	2.013	0.325	2.691
Boot 99%	-0.028	0.790	-0.620	1.847	6.582	20.873	0.039	0.785	0.735	2.013	0.325	2.691
Panel B: Shrinkage estimator												
B&H	**0.039**	1.084	**0.524**	1.000	**0.000**	∞	0.039	1.084	0.524	1.000	**0.000**	∞
Daily	-0.135	**0.701**	-3.114	1.977	17.542	1.000	0.040	0.777	0.750	1.992	0.292	1.977
Weekly	-0.047	0.718	-1.108	2.098	8.846	5.000	0.041	0.777	0.767	1.991	0.271	7.365
Monthly	-0.008	0.737	-0.233	2.094	4.795	20.000	0.042	0.780	0.794	1.981	0.249	24.264
Quarterly	0.010	0.772	0.138	1.950	2.929	60.506	0.043	0.797	0.792	1.892	0.214	66.389
Annually	0.028	0.809	0.483	1.804	1.497	251.579	**0.044**	0.818	0.803	1.798	0.146	281.176
Thld 5%	-0.101	0.712	-2.309	1.999	14.138	1.979	0.040	0.781	0.759	1.970	0.268	24.896
Thld 10%	-0.059	0.718	-1.370	2.071	9.956	4.426	0.042	0.784	0.783	1.955	0.249	50.851
Thld 20%	-0.023	0.738	-0.543	2.053	6.159	12.679	0.044	0.792	**0.815**	1.918	0.224	116.585
Thld 40%	0.004	0.746	0.019	2.073	3.457	43.063	0.044	0.857	0.754	1.627	0.175	341.429
Boot 1%	0.024	0.765	0.437	2.017	1.945	149.375	0.041	**0.776**	0.787	**2.000**	0.282	3.180
Boot 25%	0.014	0.765	0.231	1.992	2.642	79.667	0.040	0.777	0.750	1.992	0.291	2.825
Boot 50%	0.008	0.749	0.115	2.070	3.026	57.590	0.040	0.777	0.750	1.992	0.291	2.825
Boot 75%	0.004	0.737	0.015	**2.125**	3.471	43.455	0.040	0.777	0.750	1.992	0.291	2.825
Boot 99%	-0.013	0.733	-0.337	2.106	4.984	19.352	0.040	0.777	0.750	1.992	0.291	2.825
Panel C: POET estimator												
B&H	**0.039**	0.971	**0.593**	1.000	**0.000**	∞	0.039	0.971	0.593	1.000	**0.000**	∞
Daily	-0.114	**0.743**	-2.489	1.462	14.980	1.000	0.036	0.796	0.654	1.506	0.357	2.086
Weekly	-0.039	0.750	-0.887	1.559	7.634	5.000	0.037	0.797	0.685	1.503	0.343	7.563
Monthly	-0.004	0.765	-0.141	1.563	4.147	20.000	0.039	0.797	0.714	**1.507**	0.307	24.264
Quarterly	0.011	0.794	0.172	1.474	2.519	60.506	0.038	0.824	0.669	1.403	0.249	68.286
Annually	0.030	0.812	0.525	1.434	1.256	251.579	**0.042**	0.831	**0.751**	1.386	0.185	251.579
Thld 5%	-0.081	0.751	-1.756	1.485	11.596	2.286	0.037	0.798	0.684	1.500	0.334	22.762
Thld 10%	-0.046	0.758	-1.014	1.516	8.150	5.444	0.037	0.807	0.667	1.464	0.302	45.094
Thld 20%	-0.018	0.764	-0.426	1.541	5.234	15.127	0.037	0.820	0.666	1.416	0.251	111.163
Thld 40%	0.005	0.784	0.041	1.501	3.133	47.327	0.041	0.829	0.725	1.391	0.196	298.750
Boot 1%	0.021	0.819	0.347	1.396	1.733	140.588	0.040	0.801	0.731	1.493	0.368	3.806
Boot 25%	0.011	0.783	0.166	1.515	2.432	70.294	0.036	**0.796**	0.655	1.506	0.357	2.925
Boot 50%	0.006	0.772	0.073	1.551	2.862	49.792	0.036	**0.796**	0.655	1.506	0.357	2.925
Boot 75%	-0.001	0.763	-0.089	**1.572**	3.648	32.081	0.036	**0.796**	0.655	1.506	0.357	2.925
Boot 99%	-0.019	0.765	-0.441	1.535	5.295	14.184	0.036	**0.796**	0.655	1.506	0.357	2.925

Table D.6 shows the optimal block size for industry portfolios with monthly data. The average recommended block size is between one and two months. For most industry portfolios a block size of one month is preferred. Therefore, a block size of one month is used to perform the bootstrap. This choice is also in line with the 20 trading days selected for daily industry portfolio data based on Table D.1.

Table D.6: Optimal bootstrap block size selected by the algorithm of Politis and White (2004) for monthly industry portfolio data from 2000-2019 .
The algorithm selects the block size by minimising the long-run variance estimate based on the mean squared error. They suggest using a block size of one if the optimal estimated block size is below one.

Bootstrap	Mean	Standard deviation	Minimum	Maximum
Stationary	1.435	1.208	0.269	7.016
Circular	1.643	1.383	0.308	8.031

Appendix E: Description S&P 500 Data

This appendix provides additional descriptive statistics of the S&P 500 data set described in Section 6.1.

Table E.1: Descriptive statistics of daily S&P 500 stock data and the market from 2000-2019. Mean return (Mean), volatility (Vol.), minimum return (Min.), maximum return (Max.) are daily in %. The Sharpe ratio (SR) is annualised. Volume is the average trading volume in million US dollars. The bid-ask spread (BidAsk) is the average spread in basis points. The correlation with the market is denoted by Mkt.

Stock	Mean	Vol.	SR	Min.	Max.	Volume	BidAsk	Mkt
Apple Inc	0.124	2.542	0.751	-51.869	13.905	4014.409	4.173	0.540
American Express Co	0.048	2.198	0.324	-17.595	20.649	338.482	17.135	0.736
American International Group Inc	0.009	3.647	0.024	-60.791	66.000	472.302	16.411	0.479
Applied Materials Inc	0.055	2.713	0.302	-14.018	25.633	458.672	5.593	0.631
Advanced Micro Devices Inc	0.103	4.013	0.394	-32.402	52.290	368.532	42.474	0.495
Amgen Inc	0.051	1.997	0.383	-13.412	15.102	543.374	3.373	0.517
Berkley W R Corp	0.083	1.622	0.779	-13.451	16.240	25.081	13.667	0.516
Cincinnati Financial Corp	0.054	1.690	0.479	-20.068	18.252	30.913	6.362	0.693
Regions Financial Corp New	0.056	3.073	0.271	-41.071	48.410	116.991	15.178	0.567
Fedex Corp	0.046	1.888	0.360	-14.483	11.832	221.277	19.240	0.635
Fifth Third Bancorp	0.051	3.208	0.237	-43.626	60.366	146.758	6.980	0.550
Huntington Bancshares Inc	0.059	3.290	0.267	-30.586	50.074	74.218	13.966	0.516
Westamerica Bancorporation	0.045	1.838	0.363	-14.071	15.235	7.962	11.550	0.661
Intel Corp	0.043	2.316	0.273	-22.033	20.123	1328.938	3.994	0.662
K L A Tencor Corp	0.075	2.853	0.398	-16.987	25.092	218.861	4.217	0.610
Limited Inc	0.047	2.433	0.284	-18.690	21.906	104.727	32.379	0.556
Medtronic Plc	0.041	1.562	0.384	-13.235	11.184	300.648	26.248	0.522
Bank Of America Corp	0.058	2.870	0.302	-28.969	35.269	1401.296	18.412	0.649
Northern Trust Corp	0.044	2.147	0.303	-18.811	30.913	89.332	5.419	0.719
Paccar Inc	0.081	2.169	0.565	-12.310	18.843	104.318	5.652	0.728
Zions Bancorporation N A	0.042	2.802	0.220	-24.544	27.557	70.926	6.764	0.574
Home Depot Inc	0.049	1.912	0.382	-28.736	14.067	513.315	18.018	0.636
Cintas Corp	0.063	1.874	0.504	-15.753	21.339	52.856	5.492	0.643
Paychex Inc	0.051	1.821	0.414	-13.182	14.286	104.690	4.771	0.608
Hunt J B Transport Services Inc	0.099	2.253	0.672	-24.615	16.604	53.821	12.515	0.551
Lam Resh Corp	0.091	3.121	0.447	-17.384	20.000	164.010	7.486	0.611
Expeditors International Wa Inc	0.065	2.118	0.463	-12.149	18.277	63.277	7.222	0.603
Autodesk Inc	0.096	2.615	0.563	-20.949	16.895	120.798	7.093	0.593
Ross Stores Inc	0.104	2.106	0.759	-11.896	23.383	101.564	7.627	0.487
Henry Jack & Assoc Inc	0.072	1.994	0.547	-26.792	14.570	21.339	10.750	0.594
Costco Wholesale Corp New	0.059	1.753	0.502	-21.539	15.007	276.378	4.014	0.550
Monster Beverage Corp New	0.175	2.993	0.907	-25.839	30.482	89.167	53.934	0.294
Oracle Corp	0.045	2.439	0.272	-21.053	21.327	749.009	4.697	0.625
Microsoft Corp	0.046	1.897	0.362	-15.598	19.565	1963.216	3.042	0.677
T Rowe Price Group Inc	0.073	2.290	0.481	-17.939	18.167	89.314	6.099	0.779
Adobe Systems Inc	0.097	2.729	0.542	-29.758	23.972	267.332	4.231	0.615
Fiserv Inc	0.076	1.919	0.601	-19.460	16.515	93.327	4.920	0.634
Cerner Corp	0.101	2.572	0.602	-45.061	28.000	73.396	8.624	0.450
Dentsply Sirona Inc	0.055	1.616	0.504	-18.652	17.649	46.201	7.592	0.534
Fastenal Company	0.081	2.128	0.576	-14.574	17.150	75.395	6.820	0.609
Silicon Valley Bancshares	0.087	2.837	0.466	-33.256	26.771	41.761	8.911	0.622
Peoples United Financial Inc	0.057	1.655	0.518	-15.632	17.881	40.797	12.756	0.595
Symantec Corp	0.069	2.690	0.386	-36.552	27.778	191.653	6.684	0.497
Electronic Arts Inc	0.068	2.693	0.384	-17.851	21.034	212.663	4.992	0.509
Cisco Systems Inc	0.032	2.424	0.188	-16.211	24.388	1187.332	4.306	0.660

Table E.1 (cont.) Descriptive statistics of daily S&P 500 stock data and the market from 2000-2019.

Stock	Mean	Vol.	SR	Min.	Max.	Volume	BidAsk	Mkt
Hologic Inc	0.111	2.823	0.605	-20.192	24.590	57.340	30.436	0.417
Xilinx Inc	0.061	2.807	0.327	-21.090	18.437	213.191	4.493	0.603
I D E X X Laboratories Inc	0.104	2.039	0.780	-17.074	17.233	35.906	11.198	0.482
Zebra Technologies Corp	0.073	2.357	0.470	-23.846	22.845	27.728	9.836	0.515
Old Dominion Freight Line Inc	0.133	2.663	0.770	-15.397	23.781	25.065	57.979	0.444
Qualcomm Inc	0.042	2.663	0.233	-16.848	23.207	794.700	3.256	0.566
Perrigo Co Plc	0.065	2.229	0.437	-29.278	18.390	72.760	13.126	0.402
Synopsys Inc	0.051	2.130	0.359	-31.156	15.953	49.876	6.529	0.543
Starbucks Corp	0.092	2.104	0.669	-18.423	18.380	305.026	5.102	0.577
Intuit Inc	0.077	2.527	0.461	-29.438	29.561	153.849	4.611	0.519
Microchip Technology Inc	0.077	2.651	0.443	-15.631	23.782	107.320	5.810	0.597
O Reilly Automotive Inc New	0.096	2.119	0.694	-18.892	20.202	91.834	11.297	0.464
Flir Systems Inc	0.108	2.845	0.581	-41.606	33.019	30.975	24.601	0.388
Activision Inc New	0.115	2.693	0.655	-27.119	23.270	157.814	12.350	0.464
Tractor Supply Co New	0.120	2.374	0.781	-17.241	22.183	53.730	23.253	0.441
Copart Inc	0.088	2.162	0.618	-27.450	19.481	26.981	12.910	0.443
Dollar Tree Stores Inc	0.074	2.440	0.458	-43.814	17.745	110.842	6.503	0.396
Echostar Communications Corp New	0.040	2.761	0.211	-19.126	24.642	98.827	6.293	0.542
Schein Henry Inc	0.081	1.829	0.675	-13.746	19.111	46.962	9.170	0.487
Network Appliance Inc	0.077	3.670	0.319	-20.909	40.929	202.875	5.615	0.554
Citrix Systems Inc	0.065	3.062	0.318	-45.979	24.561	132.175	6.507	0.528
Alexion Pharmaceuticals Inc	0.111	3.424	0.498	-31.217	45.124	105.775	13.849	0.431
Ansys Inc	0.118	2.363	0.769	-15.544	17.357	29.086	21.423	0.507
E Trade Group Inc	0.043	3.905	0.160	-58.673	50.000	83.431	40.116	0.592
Amazon Com Inc	0.117	3.295	0.545	-24.766	34.471	1684.095	5.519	0.492
R F Micro Devices Inc	0.087	4.263	0.312	-38.044	43.149	85.935	15.197	0.533
Ch Robinson Worldwide Inc	0.067	1.971	0.514	-15.643	21.572	75.737	6.876	0.529
Verisign Inc	0.060	3.344	0.269	-45.779	30.410	145.219	5.552	0.549
Cognizant Technology Sols Corp	0.105	2.780	0.580	-23.965	20.235	151.198	12.395	0.550
Crown Castle Intl Corp New	0.077	2.935	0.397	-38.830	31.220	108.853	32.656	0.448
Ebay Inc	0.074	2.818	0.397	-20.576	30.383	472.692	4.131	0.559
Nvidia Corp	0.156	3.823	0.631	-35.234	42.415	595.150	6.813	0.527
Priceline Group Inc	0.118	3.945	0.461	-42.330	35.876	395.974	16.234	0.418
F 5 Networks Inc	0.089	3.777	0.358	-24.834	39.706	100.474	11.350	0.457
S B A Communications Corp New	0.125	3.861	0.498	-48.750	45.536	66.862	24.775	0.385
Juniper Networks Inc	0.047	3.520	0.197	-21.053	31.071	288.533	5.502	0.538
Akamai Technologies Inc	0.064	4.304	0.222	-26.278	45.599	111.758	15.600	0.485
Abbott Laboratories	0.054	1.473	0.544	-16.138	12.466	302.687	20.090	0.461
Skyworks Solutions Inc	0.110	4.041	0.420	-22.239	35.749	104.432	11.590	0.505
American Electric Power Co Inc	0.051	1.529	0.498	-22.785	19.842	115.849	21.444	0.440
Baker Hughes Inc New	0.048	2.491	0.285	-22.080	26.982	199.993	23.867	0.526
Bank Of New York Mellon Corp	0.040	2.334	0.250	-27.158	24.807	193.669	18.782	0.700
Baxter International Inc	0.052	1.602	0.486	-26.284	9.144	181.927	14.376	0.427
Verizon Communications Inc	0.032	1.533	0.295	-11.846	14.632	545.334	16.119	0.536
Boeing Co	0.067	1.881	0.540	-17.625	15.463	510.024	16.895	0.607
Bristol Myers Squibb Co	0.032	1.771	0.255	-22.414	14.644	354.175	18.083	0.450
Cigna Corp New	0.069	2.287	0.455	-38.066	23.540	159.724	12.628	0.487
Campbell Soup Co	0.026	1.441	0.256	-12.366	14.286	66.861	21.099	0.352
Caterpillar Inc	0.067	2.001	0.506	-14.518	14.723	425.619	20.905	0.685

Table E.1 (cont.) Descriptive statistics of daily S&P 500 stock data and the market from 2000-2019.

Stock	Mean	Vol.	SR	Min.	Max.	Volume	BidAsk	Mkt
JPMorgan Chase & Co	0.060	2.425	0.373	-20.727	25.097	1011.750	19.721	0.735
ChevronTexaco Corp	0.047	1.584	0.437	-12.489	20.854	640.554	12.347	0.635
Coca Cola Co	0.032	1.286	0.352	-10.061	13.880	467.346	14.562	0.450
Colgate Palmolive Co	0.032	1.362	0.342	-15.462	20.319	185.647	14.476	0.434
Citigroup Inc	0.021	3.037	0.095	-39.024	57.825	1162.873	17.570	0.660
D X C Technology Co	0.034	2.405	0.202	-39.556	42.076	79.080	18.515	0.521
Target Corp	0.053	2.035	0.387	-12.500	20.426	322.898	22.815	0.556
Disney Walt Co	0.054	1.863	0.434	-18.363	15.972	475.402	25.464	0.669
Exxon Mobil Corp	0.033	1.509	0.314	-13.953	17.191	1183.566	13.311	0.639
Ford Motor Co Del	0.023	2.609	0.122	-25.000	29.518	407.785	30.121	0.532
General Dynamics Corp	0.058	1.561	0.555	-12.377	11.728	156.536	13.256	0.555
General Electric Co	0.001	1.978	-0.013	-12.789	19.703	1039.892	13.422	0.683
Halliburton Company	0.047	2.690	0.257	-42.446	24.352	384.086	21.622	0.535
L3Harris Technologies Inc	0.082	2.026	0.614	-17.037	17.407	57.021	32.912	0.555
Hewlett Packard Co	0.033	2.341	0.202	-20.027	17.288	418.451	24.567	0.592
International Business Machs Cor	0.025	1.621	0.218	-15.542	13.044	740.056	11.502	0.641
International Flavors & Frag Inc	0.046	1.602	0.425	-15.964	16.116	38.793	22.307	0.568
International Paper Co	0.034	2.233	0.217	-18.520	21.884	129.557	22.724	0.634
Johnson & Johnson	0.040	1.200	0.487	-15.846	12.229	689.929	11.486	0.499
Lilly Eli & Co	0.039	1.619	0.354	-29.303	17.647	313.505	13.961	0.489
McDonalds Corp	0.052	1.434	0.537	-12.817	9.390	421.225	24.600	0.450
Merck & Co Inc New	0.035	1.688	0.302	-26.781	13.033	538.877	12.212	0.484
Entergy Corp New	0.057	1.476	0.577	-18.098	14.198	97.102	19.910	0.396
Minnesota Mining & Mfg Co	0.046	1.454	0.465	-12.945	11.071	321.664	9.383	0.668
Norfolk Southern Corp	0.074	2.042	0.551	-12.915	15.413	151.682	29.037	0.602
Wells Fargo & Co New	0.058	2.373	0.366	-23.822	32.765	775.722	17.269	0.636
Occidental Petroleum Corp	0.060	2.034	0.446	-18.493	18.108	302.531	28.506	0.596
PepsiCo Inc	0.044	1.218	0.530	-11.931	14.869	373.893	19.046	0.443
Pfizer Inc	0.029	1.544	0.266	-11.146	10.172	809.717	19.198	0.546
P E C O Energy Co	0.046	1.589	0.430	-11.794	17.201	188.836	14.641	0.443
Philip Morris Cos Inc	0.077	1.525	0.767	-13.844	16.375	420.509	20.105	0.355
Procter & Gamble Co	0.036	1.320	0.395	-31.380	10.214	621.785	12.750	0.422
Schlumberger Ltd	0.039	2.218	0.256	-18.404	14.914	511.127	14.763	0.584
Southern Co	0.055	1.178	0.699	-8.473	11.066	157.060	22.751	0.366
S B C Communications Inc	0.028	1.606	0.246	-12.661	16.280	701.929	17.648	0.548
Texas Instruments Inc	0.056	2.472	0.339	-18.224	24.068	392.926	21.346	0.625
Tyco International Plc Ireland	0.042	2.390	0.258	-30.822	45.818	238.530	19.703	0.513
United Technologies Corp	0.052	1.655	0.470	-28.248	13.647	318.582	14.825	0.691
Viacomcbs Inc	0.035	2.474	0.201	-20.757	26.581	221.014	18.418	0.674
Walmart Inc	0.029	1.485	0.276	-10.183	11.073	659.399	15.149	0.498
Weyerhaeuser Co	0.035	1.992	0.252	-17.161	14.039	108.077	17.825	0.652
Williams Cos	0.078	3.577	0.332	-61.047	87.736	171.503	25.096	0.470
Honeywell International Inc	0.051	1.918	0.393	-17.784	28.223	247.749	19.178	0.681
Allstate Corp	0.059	1.920	0.459	-21.179	21.687	146.845	26.101	0.619
Incyte Pharmaceuticals Inc	0.110	4.220	0.402	-41.022	49.427	56.920	18.378	0.471
Hartford Financial Svcs Grp Inc	0.072	3.532	0.311	-51.561	102.358	131.964	15.336	0.521
Rockwell International Corp New	0.080	2.146	0.568	-20.131	16.492	82.533	24.630	0.673
Goldman Sachs Group Inc	0.048	2.305	0.311	-18.960	26.468	814.538	15.636	0.725
Market	0.032	1.203	0.376	-8.950	11.354			1.000

Appendix F: Additional Results S&P 500 Data

This appendix provides additional results relating to the analysis with S&P 500 data presented in Sections 6.2 and 6.3.

Table F.1 displays the suggested block size for the selected S&P 500 stocks based on the algorithm of Politis and White (2004). The results are comparable to those found in Table D.1 in Appendix D, although there is more dispersion in the suggested block size as the data set contains more assets. A block size of 20 trading days is chosen to be consistent with the empirical study that uses industry portfolio data.

Table F.1: Optimal bootstrap block size selected by the algorithm of Politis and White (2004) for daily S&P 500 stock data from 2000-2019 .
The algorithm selects the block size by minimising the long-run variance estimate based on the mean squared error. They suggest using a block size of one if the optimal estimated block size is below one.

Bootstrap	Mean	Standard deviation	Minimum	Maximum
Stationary	6.600	7.798	0.183	47.980
Circular	7.555	8.926	0.210	54.924

Tables F.2 and F.3 display the performance of 1/N strategies applied to S&P 500 data with fixed and variable transaction costs, respectively. Compared to the findings in Sections 6.2 and 6.3, there is a sizeable reduction in transaction costs in both cases. Yet, the volatility of the 1/N portfolios is much higher, which leads to inferior Sharpe ratios and utility levels. Hence, it can be concluded that 1/N strategies are greatly outperformed by variance optimised strategies.

Table F.2: Performance of 1/N portfolio strategies with fixed transaction costs applied on daily S&P 500 stock data from 2000-2019.
This table shows the performance of buy-and-hold (B&H), periodic, and threshold (Thld) rebalancing strategies. Portfolios are rebalanced to equal weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the identity matrix as the estimated covariance matrix and assuming transaction costs of 50 basis points. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
B&H	**0.074**	1.334	**0.843**	1.000	**0.000**	∞	**0.074**	1.334	0.843	1.000	**0.000**	∞
Daily	0.058	1.295	0.671	1.057	0.576	1.000	0.072	1.282	0.853	1.085	0.012	2.407
Weekly	0.060	1.293	0.702	1.061	0.266	5.000	0.072	1.284	0.853	1.081	0.012	10.670
Monthly	0.060	1.288	0.703	1.070	0.136	20.000	0.072	1.286	0.854	1.078	0.012	35.940
Quarterly	0.061	1.281	0.714	1.082	0.079	60.506	0.072	1.287	0.856	1.077	0.011	99.583
Annually	0.062	**1.258**	0.741	**1.123**	0.040	251.579	0.073	1.298	0.850	1.058	0.011	341.429
Thld 5%	0.061	1.293	0.712	1.060	0.228	8.490	0.072	1.284	0.852	1.082	0.011	93.725
Thld 10%	0.062	1.290	0.724	1.067	0.122	30.446	0.072	1.282	0.854	1.084	0.011	191.200
Thld 20%	0.062	1.282	0.726	1.081	0.060	122.564	0.072	1.284	0.856	1.082	0.011	434.545
Thld 40%	0.063	1.260	0.757	1.120	0.031	434.545	0.073	**1.281**	**0.862**	**1.087**	0.010	1195.000

Table F.3: Performance of 1/N portfolio strategies with variable transaction costs applied on daily S&P 500 stock data from 2000-2019.

This table shows the performance of buy-and-hold (B&H), periodic, and threshold (Thld) rebalancing strategies. Portfolios are rebalanced to equal weights on the left-hand side and by solving a quadratic programming problem with transaction costs on the right-hand side. The optimisation is performed using the identity matrix as the estimated covariance matrix and assuming transaction costs consisting of a fixed commission of 24 basis points plus half of the estimated bid-ask spread. The performance is measured by the daily post-transaction cost return in % (μ), daily volatility in % (σ), annualised post-transaction cost Sharpe ratio (SR), utility of a mean-variance investor with risk aversion parameter 3 relative to B&H (Utility), daily transaction costs in basis points (TC), and average rebalancing time in days (ART). The best performing strategy within each column is indicated in bold.

	Excluding transaction costs						Including transaction costs					
	μ	σ	SR	Utility	TC	ART	μ	σ	SR	Utility	TC	ART
B&H	**0.074**	1.334	**0.843**	1.000	**0.000**	∞	**0.074**	1.334	**0.843**	1.000	**0.000**	∞
Daily	0.060	1.295	0.694	1.057	0.394	1.000	0.068	**1.259**	0.823	**1.123**	0.007	1.629
Weekly	0.061	1.293	0.713	1.061	0.179	5.000	0.069	1.261	0.824	1.121	0.007	7.399
Monthly	0.061	1.288	0.709	1.070	0.090	20.000	0.069	1.261	0.826	1.120	0.007	27.471
Quarterly	0.061	1.281	0.717	1.082	0.051	60.506	0.069	1.262	0.829	1.118	0.007	78.361
Annually	0.062	**1.258**	0.743	**1.123**	0.025	251.579	0.069	1.271	0.829	1.104	0.006	298.750
Thld 5%	0.062	1.293	0.719	1.061	0.172	8.490	0.069	1.261	0.826	1.121	0.007	73.538
Thld 10%	0.062	1.290	0.728	1.067	0.092	30.446	0.069	1.260	0.830	1.122	0.006	159.333
Thld 20%	0.062	1.282	0.728	1.081	0.044	122.564	0.069	1.260	0.827	1.123	0.006	398.333
Thld 40%	0.063	1.260	0.758	1.120	0.022	434.546	0.070	1.264	0.837	1.116	0.006	956.000