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Master's Thesis

Operations Research and Quantitative Logistics

Assessing the wastefulness of online food retailing

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Abstract

In the coming years, online food retailing is expected to significantly impact the market, which raises new challenges for inventory control of perishable products. In online retailing, consumers do not observe expiration dates. Therefore, to maintain a high level of customer satisfaction, online retailers might only send products that are consumable for at least a week. As a result, food that is consumable for six more days is discarded. With sustainability issues high on the political agenda, more research is needed on the wastefulness of these online throw-out policies. This thesis compares the food waste in an offline and online food retailing environment. Based on a broad set of experiments, insights are gained on the wastefulness of online sales. It will be shown that online retailers can reduce food waste by adjusting their product range, lowering the fill rate, and optimizing policies suited for online retailing.

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Contents

1 Introduction

The Food and Agriculture Organization (FAO) of the United Nations states that one-third of all food produced globally is wasted[2]. This puts an unnecessary burden on the environment, e.g., the FAO attributes 8% of human greenhouse gas emission to food waste[3]. In response, as part of the 2030 sustainable development agenda, a target is set to halve the per capita food waste. Altogether, the food supply chain faces major challenges in improving sustainability and reducing food waste.

A considerable amount of food waste can be assigned to the decisions and actions of retailers. Roberti and Scheer (2005) argue that roughly 10% of perishable goods are thrown away before consumers purchase them. Therefore, efficient inventory management of food retailers is crucial to reduce global food waste. At the same time, we observe the advent of online food retailers that might manage inventory differently. Online food retailing is expected to significantly impact the market, with online sales expected to increase in the coming years[4].

There are major differences between traditional offline and modern online food retailers. In offline retailing, expiration dates are printed on the packaging of perishable food, or a consumer can inspect the product itself and determine whether it can still be consumed before perishing. Therefore, a consumer might decide to buy food that expires the next day, when intending to eat it the same day. In response, offline retailers usually discount food close to the expiration date and only discard expired food. On the other hand, in online retailing, consumers do not observe expiration dates, nor can they inspect the products. Therefore, to maintain a high level of customer satisfaction, online retailers might only send products that are consumable for at least a week. As a result, food that is still valid for six more days is discarded.

We provide theoretical results for the food waste as a function of the product shelf life and prove that the expected food waste under the optimal order policy is monotonically decreasing in the shelf life. Online retailers inherently decrease the shelf life if they throw out food before the expiration date. Therefore, online sales are more wasteful if online and offline retailers follow the same inventory depletion rule.

In practice, however, the inventory depletion of online and offline retailers differs. Consumers often buy the freshest products in an offline setting as this gives the highest level of utility. By keeping only older products on the shelf and discounting products close to the expiration date, offline retailers try to maintain First-In-First-Out (FIFO) inventory depletion. In practice, however, this is not always feasible due to high reviewing and labor cost (Broekmeulen and van Donselaar, 2009). Consequently, offline retailers often follow Last-In-First-Out (LIFO) inventory depletion (or a combination of FIFO and LIFO). On the other hand, online retailers can usually send the oldest products in stock, making it possible to maintain FIFO inventory depletion. FIFO inventory depletion is a considerable advantage that could offset the wastefulness of an

[2]http://www.fao.org/food-loss-and-food-waste/en/

[3]http://www.fao.org/fileadmin/templates/nr/sustainability_pathways/docs/FWF_and_climate_change.pdf

[4]https://www.statista.com/statistics/257532/us-food-and-beverage-e-commerce-revenue/

online throw-out policy.

Moreover, due to offline and online food retailers' different characteristics, they might diverge in their inventory control, for instance, by managing a different order-up-to level, or by effectively implementing a different policy. All in all, there is no common understanding of the impact on food waste by the move from offline to online retail.

In this thesis, we examine whether modern online retailing increases food waste with respect to traditional offline retailing. A simplified inventory control system is investigated to achieve this, focusing on practical applications for real-world food retailers. Modeling inventory control for a food retailer falls in the domain of inventory control of perishable goods with a fixed lifetime because expiration dates are fixed and known beforehand. Here, we exclude perishables without a printed expiration date (mainly fruit and vegetables).

In a retailing environment, orders are generally placed within a specific time interval. This gives the central warehouse time to process and group the orders to effectuate an efficient replenishment strategy. Therefore, we consider a periodic review, as it is sufficient to review inventory levels just before ordering. The review period is equal to one day because fresh products are usually ordered daily (van Donselaar et al., 2006). Furthermore, we incorporate a positive deterministic lead time and assume daily demand is a discrete stochastic variable, demand that cannot be satisfied is lost. Customers might come back later to check whether the product is available. However, a product being out of stock generally results in lost sales.

Food retailers usually try to minimize their costs, while at the same time maintaining a high level of product availability. Traditional cost components for perishable inventory control include fixed ordering cost, unit holding cost, unit outdate cost, and unit cost for lost sales. However, for food retailers, it is difficult to estimate the penalty cost of lost sales. Particularly for perishables, as a stock-out of fresh food not only impacts the short-term sales of other products but could even lead to changes in mid/long-term retailer preferences (Minner and Transchel, 2010). Therefore, a service level constraint is considered instead.

Furthermore, following Haijema and Minner (2016), the fixed ordering cost is excluded as perishables are usually ordered daily, and ordering costs are shared over a large number of goods. Moreover, due to the relatively short lifetime of perishable goods, the outdate cost outweighs the holding cost. This assumption is verified by Broekmeulen and van Donselaar (2009). As a result, as in van Donselaar and Broekmeulen (2012), the objective is to minimize the food waste under a service level constraint. The food waste is defined as the total outdated quantity divided by the total demand. As a service level, the product fill rate is used. The fill rate is defined as the fraction of the total demand that is satisfied.

Optimizing a perishable inventory control system is known for being difficult. Traditional stock-level dependent policies do not consider the inventory's age distribution when ordering, resulting in policies that are generally suboptimal for inventory control of perishables. Therefore, stock-age dependent policies are introduced, in which optimal order quantities are derived using dynamic programming. However, the state-space of the multi-dimensional dynamic programs increases in the shelf life because products need to be grouped by their remaining useful lifetime. Therefore, optimizing order quantities in these stock-age dependent policies suffers from the curse of dimensionality, resulting in derivations that are computationally impractical for real-world retailers.

Over the past decades, various research has been conducted to deal with this complexity. Either by modifying traditional stock-level dependent policies to better suit inventory control for perishables or obtaining computationally feasible stock-age dependent policies that take into account (partial) information on the inventory's age distribution.

This thesis focuses more on gaining insights into the wastefulness of online sales instead of analyzing new order policies or approximation techniques. To assess online sales' wastefulness, the food waste in an offline and online environment is estimated and compared by simulation, considering two order policies. van Donselaar et al. (2006) find that, in practice, the underlying policy of the automated ordering systems is usually based on some variant of the base-stock policy (BSP) in which an order is placed to bring the inventory position back to an order-up-to level. We implement the BSP as it is a relatively simple but effective policy with practical implications for most food retailers. Furthermore, we examine the impact of a more sophisticated policy by considering the EWA policy suggested by Broekmeulen and van Donselaar (2009). They argue that outdating affects the ability to satisfy demand. Therefore, when ordering, the inventory position should first be adjusted by the estimated outdated quantity during lead time and review period. The EWA policy uses only small calculations, making it possible to incorporate the policy in (daily) ordering decisions.

Our simulation addresses two major differences between offline and online retailing, namely, differences in inventory depletion and throw-out policy. For offline retailers, based on Haijema et al. (2007) and Haijema and Minner (2016), the total demand is split into FIFO and LIFO demand. By varying the fraction of the total demand that is FIFO demand, we investigate different offline management styles. If retailers display mainly older inventory on the shelf and discount products close to the expiration date (i.e., active management), the total demand is mainly FIFO demand. Contrarily, if no measures are taken to prevent customers from buying the freshest products (i.e., passive management), the total demand is mainly LIFO demand.

On the other hand, online retailers can easily maintain efficient inventory control as expiration dates are not observable to consumers. Therefore, in an online setting, the total demand is FIFO demand. Offline and online retailers also differ in their throw-out policy. Offline retailers only discard food that has expired, while online retailers might throw out food that is still valid for some days, to maintain a high level of customer satisfaction.

For various offline inventory depletion rules and product shelf lives, we derive the lowest online throw-out policy that significantly increases food waste, giving online retailers insights into their sales' wastefulness. Online sales' wastefulness is visible for all perishables with a short shelf life. Relatively small throw-out policies of 1-3 days increase the food waste for daily fresh (shelf life ≤ 7). However, perishables with a relatively long shelf life can be sold online quite well, especially under offline FIFO demand.

Furthermore, we analyze the shelf life range that is more wasteful when sold online. One main insight here is the impact of the offline management style on online sales' wastefulness. Active offline management makes the online sale of all perishables more wasteful. Contrarily, if offline retailers are passive, online retailers can maintain a high customer satisfaction level and reduce food waste. Therefore, a correct assumption on offline retailers' inventory management is crucial to assess online sales' wastefulness correctly.

Our research makes three suggestions to reduce online retailers' wastefulness. Firstly, we find that perishables in high demand and under low demand variation are ideally sold online. Contrarily, the online sale of perishables in low demand and under high demand variation is quite wasteful. By correctly adjusting the online product range, the sustainability of online retailing improves significantly. Secondly, by lowering the fill rate from 98% to 90%, a considerably wider range of perishables can be sold online without increasing the food waste, making fill rate adjustments an effective method to reduce the food waste of online sales. Thirdly, more work should be devoted to optimizing policies suited for online retailing. Hence, policies that perform well under FIFO inventory depletion and the sale of perishables with a relatively long shelf life, as online sales of daily fresh are generally not feasible.

2 Literature

Modeling inventory control of a food retailer falls in the domain of inventory control of perishable goods (Goyal and Giri, 2001). Numerous research is done on this topic. For extensive literature reviews we refer to Nahmias (1982) covering the years 1964-1982, Goyal and Giri (2001) covering the years 1990-2000, Bakker et al. (2012) covering the years 2001-2011 and Janssen et al. (2016) covering the years 2012-2015. In this thesis, we investigate a model where perishables have a fixed lifetime. Contrarily, various papers incorporate a stochastic lifetime distribution (e.g., Exponential or Erlang). The main contributions on this topic are by Liu and Shi (1999) and Lian et al. (2009), who investigate a model with exponential lifetime and Markovian renewal demand. By incorporating the Markov renewal theory, optimal policies can be derived fairly straightforward. For an extensive review of inventory control with stochastic lifetime distributions, we refer to Rafaat (1991).

Inventory control with a fixed lifetime can be divided into models with deterministic and stochastic demand. In a deterministic setting, under fairly general assumptions, optimal order quantities can be derived without food waste (Nahmias, 1982). However, deterministic demand is usually not a valid assumption with real-world applications. On the other hand, optimizing order policies in a stochastic setting is more complex. Particularly under periodic review and in case products have a lifetime longer than two periods (Nahmias, 1982). Optimal order quantities are derived by multi-dimensional dynamic programs in which the state-space increases in the shelf life because products need to be tracked by their remaining useful lifetime. Therefore, optimizing order quantities in these stock-age dependent policies suffers from the curse of dimensionality, resulting in derivations that are computationally impractical for real-world retailers. For more details concerning the complex structure of an optimal policy, we refer to Nahmias (1982), Haijema et al. (2007), and Karaesmen et al. (2011).

The academic literature on stochastic inventory control of perishable goods with a fixed lifetime can be subdivided into models in which inventory is reviewed continuously or periodically. Early research on continuous review models is conducted by Weiss (1980), who investigates the (s, S) policy under zero lead time and Poisson demand, incorporating all cost components traditionally inherent to perishable inventory control (i.e., fixed ordering, unit holding, unit outdate, unit shortage, with lost sales or backordering). Several related papers have appeared over the years. We refer to Karaesmen et al. (2011) for an extensive review of this topic. This review separates the literature based on ordering cost (yes/no) and lead time (zero/deterministic). Karaesmen et al. (2011) emphasize that zero lead time in a continuous review model simplifies the problem, as an order should only be placed when the inventory is depleted.

The foundation for periodic schemes is laid by Nahmias and Pierskalla (1973), who consider a model under zero lead time, stochastic demand, and a lifetime of two periods. Their research is extended to a general lifetime of m periods by Nahmias (1975b) and Fries (1975). Both papers investigate order-up-to policies under zero lead time, continuous demand, and FIFO inventory depletion. The difference is that Nahmias (1975b) assumes unsatisfied demand is backlogged, and Fries (1975) incorporates unmet demand as lost sales. However, computations for their proposed stock-age dependent policies exhibit high-dimensional state spaces and are, consequently, computationally intractable. Over the past decades, various research has been conducted to deal with this complexity. In general, research can be divided into papers that (i) simplify the lifetime assumption, (ii) propose stock-level dependent policies modified to better suit inventory control for perishables, and (iii) develop heuristics to obtain computationally feasible stock-age dependent policies that take into account (partial) information on the age distribution of the inventory.

2.1 Simplifying lifetime assumption

Cohen (1976) analyzes the class of policies with a single critical number and provides results for a product shelf life of two and three periods. Williams and Patuwo (1999) and Williams and Patuwo (2004) investigate a single perishable product with a lifetime of two periods. Their model operates under deterministic lead time and the lost sales assumption. Optimal order quantities are computed for various demand distributions and lead times up to four periods.

2.2 Stock-level dependent policies

Stock-level dependent policies can be 'modified' to better suit inventory control for perishables. Early work on these modified policies is by Nahmias (1975a), who decreases the dynamic order-up-to level by the current total stock on hand (independent of the age distribution), assuming zero lead time and full backordering. In recent years various papers propose models with a positive deterministic lead time, which enlarges the real-world applications. Haijema et al. (2007) investigate inventory management for blood platelets with practical applications for blood banks, assuming a deterministic lead time and weekday varying demand. Dynamic programming on a down-scaled version of the problem (platelets are distributed as units of 4 pools) shows the optimal policy's complex structure.

Furthermore, they develop an order-up-to policy that uses different critical values for the total and young inventory by considering two demand streams ('young' platelets vs. 'any' platelets). Using simulation, they find that their solutions are within 1% of the optimal solution. Haijema (2013) extends the research into inventory management for blood platelets and proposes a periodic stock-level dependent (s, S, q, Q) policy. A standard (s, S) policy in which the order quantity is bounded by a minimum q and maximum Q. By simulation, near-optimal parameter values are approximated, which vary over the weekdays. In general, they find an improved performance of 4-25% over the standard (s, S) policy and conclude that bounding order quantities may enhance the inventory control of perishables.

Haijema and Minner (2016) further explore the possibilities within the class of (s, S, q, Q) policies by investigating the impact of dropping one or multiple parameters. Furthermore, they introduce new ordering policies that reduce the risk of outdating by preventing many products close to the expiration date at the same time. Their model operates under a deterministic lead time, continuous demand, and the lost sales assumption. Moreover, they split demand into FIFO demand and LIFO demand. Following Nandakumar and Morton (1993) and Cooper (2001), they propose lower and upper bounds on the optimal parameter values based on the solution of the newsvendor problem. The optimal parameter values are approximated by simulation-based optimization. Based on a broad set of experiments, they find statistical evidence for the improved performance over the base-stock policy. Notably, the policies that reduce the risk of outdating perform well. Furthermore, Haijema and Minner (2016) derive a stock-age dependent policy using dynamic programming. They find that this policy still outperforms their 'best' modified stock-level dependent policy. Therefore, computationally feasible stock-age dependent heuristics could further enhance the inventory control of perishables.

2.3 Stock-age dependent policies

Various papers propose computationally feasible stock-age dependent heuristics. Nahmias (1977) approximates the state space of the dynamic programs under zero lead time, continuous demand, and full backordering. Only two states are considered, new inventory and all inventory older than one period. Chiu (1995) analyzes an order-up-to policy under a deterministic lead time, FIFO inventory depletion, and full backordering. He develops an approximation for the expected outdating by analyzing patterns in the perishability process. Minner and Transchel (2010) propose a dynamic replenishment model under a deterministic lead time, weekday varying Gamma demand, and the lost sales assumption. They investigate both FIFO and LIFO inventory depletion and multiple service level constraints. The order quantity in their dynamic model is conditional on the expected remaining inventory after lead time. Using simulation, they find statistical evidence for their policy's improved performance over the base-stock and constant order policy.

Broekmeulen and van Donselaar (2009) implement a periodic review (S, nQ) policy in which the replenishment quantity is limited to an integer multiple of the fixed case pack size Q. Their model operates under a deterministic lead time, Gamma demand, lost sales, and FIFO/LIFO inventory depletion. The order quantity in their model is first adjusted by the estimated outdated quantity. This quantity is derived via very simple recursive equations using the inventory's age distribution and assuming actual demand is equal to the expected demand. They find that incorporating the full age distribution of the inventory leads to significant cost reductions. Furthermore, the policy can easily be incorporated in (daily) ordering decisions as only small calculations are used.

van Donselaar and Broekmeulen (2012) investigate a similar (S, nQ) policy, under a deterministic lead time, continuous demand, and FIFO inventory depletion. Using a combination of dynamic programming and regression, they derive fast and well-performing approximations on the expected outdated quantity.

2.4 Food retailing environment

Our research differs from most researches as we compare retailing environments instead of a policy comparison. We address the main differences between offline and online retailing and compare their food waste. Therefore, this thesis provides insights into online sales' wastefulness rather than analyzing new order policies or approximation techniques. Assessing the wastefulness of online sales has hardly been analyzed in the literature. However, with the advent of online retail and the current worldwide sustainability issues in the food supply chain, more research on this topic is needed.

To compare traditional offline with modern online retailing, we concentrate on practical applications for real-world food retailers. For food retailers, it is difficult to estimate the penalty cost of lost sales. Particularly for perishables, as a stock-out of fresh food not only impacts the short-term sales of other products but could even lead to changes in mid/long-term retailer preferences (Minner and Transchel, 2010). Therefore, in practice, a service level constraint is usually considered. Little research is done into perishable inventory control under a service level constraint. Main contributions on this topic are by Minner and Transchel (2010) and van Donselaar and Broekmeulen (2012). Both papers incorporate a deterministic lead time, stochastic demand, the lost sales assumption, and full FIFO or LIFO inventory depletion. Our research differs from theirs as we analyze various combinations of FIFO and LIFO inventory depletion.

Research into inventory control of perishables for food retailers is conducted by van Donselaar et al. (2006). They find statistically significant differences in characteristics between perishables (shelf life ≤ 30) and non-perishables and argue that these groups demand a different kind of inventory control. However, in practice, the automated ordering systems' underlying policy is usually based on some variant of the base-stock policy (van Donselaar et al., 2006). In the base-stock policy, an order is placed to bring the inventory position back to an order-up-to level without considering the inventories' age distribution. Therefore, no distinction is made between perishables and non-perishables.

With the availability of new technologies like RFID, efficiently storing products' data (like age) becomes more feasible (Broekmeulen and van Donselaar, 2009). Therefore, implementing more sophisticated policies becomes interesting, for instance, the EWA policy proposed by Broekmeulen and van Donselaar (2009). They argue that outdating affects the ability to satisfy demand. Therefore, when ordering, the inventory position should first be adjusted by the estimated outdated quantity during lead time and review period. The EWA policy uses only small calculations, making it possible to incorporate the policy in (daily) ordering decisions.

We contribute to the existing literature in two primary ways. Firstly, we provide theoretical results for the food waste as a function of the shelf life. Secondly, to assess online sales' wastefulness, the food waste in an offline and online setting is compared by simulation under an extensive set of experiments. Here, we implement the standard base-stock policy and examine the impact of the more sophisticated EWA policy of Broekmeulen and van Donselaar (2009).

3 Problem description

3.1 Assumptions

We consider an infinite horizon, single-product, single-stage, periodic review model. Inventory is reviewed daily. Orders are also placed, and demand also occurs on Saturday and Sunday. There is a fixed order of events, as shown in Figure 1. After a positive deterministic lead time of L days, new products arrive at the start of the day with a fixed shelf life of m days. Without loss of generality, we assume that there is no aging during lead time (i.e., fresh products arrive in stock). Upon arrival, all new products are placed on the shelf, assuming ample shelf storage capacity. After stock refreshments, inventory is reviewed and a new order is placed.

During the day, inventory drops due to demand. If the inventory is insufficient to satisfy the demand, the excess demand is lost. Let D_t denote the demand during day t, which is an integer-valued random variable with mean μ and standard deviation σ. For offline retailing, based on Haijema et al. (2007) and Haijema and Minner (2016), the total demand is split into two demand distributions. Following the notation of Haijema and Minner (2016), we use f to denote the fraction of the total demand that is FIFO demand. Hence, daily FIFO demand has mean $f \cdot \mu$ and standard deviation $\sqrt{f} \cdot \sigma$ and daily LIFO demand has mean $(1-f) \cdot \mu$ and standard deviation $\sqrt{1-f} \cdot \sigma$. In the online setting, however, the total demand is FIFO demand ($f = 1$).

Let r denote the number of days before the expiration date that food is thrown out. Hence, the shelf life is first adjusted by the throw-out date

$$I = m - r, \tag{1}$$

where I is the adjusted shelf life. After demand has occurred, the remaining inventory of age $I-1$ is discarded at the end of the day. In the offline setting, only expired food is thrown out ($r = 0$). However, in the online setting, we implement different throw-out policies by considering $0 \leq r \leq m-1$.

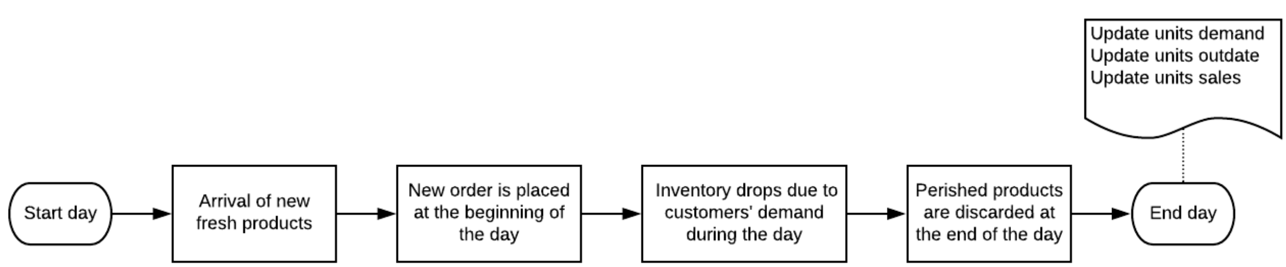


Figure 1: Flowchart daily order of events.

3.2 MDP formulation

Following Powell (2014), we present our stochastic optimization model by the five fundamental elements of any sequential decision problem. We refer to time t as the state on day t, after stock refreshments, but before inventory depletion due to demand.

The sequence of events is as follows. At time t, state $\mathbf{S}_t$ is observed and an action a_t is taken (bold notations represent vectors). During day t, exogenous information $\mathbf{W}_t$ is observed. At the end of day t, costs, rewards, and other measures of interest are evaluated.

State Information

State information $\mathbf{S}_t = \Big((\mathbf{X}_t, \widehat{\mathbf{X}}_t), \mathbf{OO}_t\Big)$ consists of inventory levels $\mathbf{X}_t$, $\widehat{\mathbf{X}}_t$ and outstanding orders $\mathbf{OO}_t$ at time t.

Inventory levels are classified by their age, i.e., $\mathbf{X}_t = (X_t^j \mid j = 0, 1, \ldots, I-1)$ where j indicates the age of an item and I is the adjusted shelf life. Furthermore, the 'fictive' inventory levels $\widehat{\mathbf{X}}_t = (\widehat{X}_t^j \mid j = 0, 1, \ldots, I-2)$ are stored. Inventory levels $\widehat{\mathbf{X}}_t$ disregard the oldest inventory and are, therefore, equivalent to the inventory levels of a system under $I-1$. All inventory levels are non-negative as we assume excess demand is lost (i.e., backordering is not allowed).

Outstanding orders are classified by their ordering moment, i.e., $\mathbf{OO}_t = (q_{t,i} \mid i = 1, \ldots, L-1)$ where $t-i$ is the time at which order quantity $q_{t,i}$ is placed, and thus delivered at the start of day $t-i+L$. Note that there are no outstanding orders for $L = 1$.

Actions

The action a_t is the order quantity at time t.

Exogenous information

The observed exogenous information $\mathbf{W}_t = (D_t^{FIFO}, D_t^{LIFO})$ is the FIFO and LIFO demand during day t, which is independent of the states and actions. Hence, the states, actions, and exogenous information evolve as follows

$$(\mathbf{S}_1, a_1, \mathbf{W}_1, \ldots, \mathbf{S}_t, a_t, \mathbf{W}_t, \ldots, \mathbf{S}_T, a_T, \mathbf{W}_T). \tag{2}$$

The total demand $D_t = D_t^{FIFO} + D_t^{LIFO}$ is the aggregate of the FIFO and LIFO demand. Note that under full FIFO demand $\mathbf{W}_t = (D_t, 0)$ and under full LIFO demand $\mathbf{W}_t = (0, D_t)$.

The transition function

The transition function can be written as $\mathbf{S}_{t+1} = S^M(\mathbf{S}_t, a_t, \mathbf{W}_t)$ where $S^M(\cdot)$ is the transition function that describes the evolution from state $\mathbf{S}_t$ to $\mathbf{S}_{t+1}$ after taking action a_t and facing exogenous information $\mathbf{W}_t$. We split the transition function into $T(\cdot)$, $U(\cdot)$, and $V(\cdot)$ where $\mathbf{X}_{t+1} = T(\mathbf{X}_t, a_t, \mathbf{W}_t)$, $\widehat{\mathbf{X}}_{t+1} = U(\widehat{\mathbf{X}}_t, a_t, \mathbf{W}_t)$, and $\mathbf{OO}_{t+1} = V(\mathbf{OO}_t, a_t)$.

The inventory withdrawal B_t^j of age j during day t can recursively be derived as the minimum of the remaining corresponding inventory and the sum of the FIFO demand that could not be satisfied from older aged inventory (i.e., age $j+1, \ldots, I-1$) and the LIFO demand that could not be satisfied from younger aged inventory (i.e., age $0, \ldots, j-1$)

$$B_t^j = \min\left\{X_t^j, \Big((D_t^{\text{FIFO}} - \sum_{i=j+1}^{I-1} X_t^i)^+ + (D_t^{\text{LIFO}} - \sum_{i=0}^{j-1} X_t^i)^+\Big)\right\}, \quad j = 0, \ldots, I-1, \tag{3}$$

in which $(x)^+ = \max\{x, 0\}$. The transition function $T(\cdot)$ is defined as

$$\Big(T(\mathbf{X}_t, a_t, \mathbf{W}_t)\Big)_j = X_{t+1}^j = \begin{cases} q_{t+1,L} & j = 0, \\ X_t^{j-1} - B_t^{j-1} & j = 1, \ldots, I-1, \end{cases} \tag{4}$$

where $(\cdot)_j$ is the j^{th} element of the vector inside the brackets. Note that if $L = 1$, we have $X_{t+1}^0 = q_{t+1,1} = a_t$ and there are no outstanding orders $\mathbf{OO}_t$. Furthermore, if $L > 1$, the transition function $T(\cdot)$ does not depend on a_t.

Similarly, the 'fictive' inventory withdrawal $\widehat{B}_t^j$ of age j during day t can recursively be derived as

$$\widehat{B}_t^j = \min\left\{\widehat{X}_t^j, \Big((D_t^{\text{FIFO}} - \sum_{i=j+1}^{I-2} \widehat{X}_t^i)^+ + (D_t^{\text{LIFO}} - \sum_{i=0}^{j-1} \widehat{X}_t^i)^+\Big)\right\}, \quad j = 0, \ldots, I-2. \tag{5}$$

The transition function $U(\cdot)$ is defined as

$$\Big(U(\widehat{\mathbf{X}}_t, a_t, \mathbf{W}_t)\Big)_j = \widehat{X}_{t+1}^j = \begin{cases} q_{t+1,L} & j = 0, \\ \widehat{X}_t^{j-1} - \widehat{B}_t^{j-1} & j = 1, \ldots, I-2. \end{cases} \tag{6}$$

Moreover, transition function $V(\cdot)$ is defined as

$$\Big(V(\mathbf{OO}_t, a_t)\Big)_i = q_{t+1,i} = \begin{cases} a_t & i = 1, \\ q_{t,i-1} & i = 2, \ldots, L-1. \end{cases} \tag{7}$$

The objective function

As in van Donselaar and Broekmeulen (2012), the objective is to minimize the food waste under a service level constraint. The food waste is defined as the total outdated quantity divided by the total demand. As a service level, the product fill rate β is used. It is defined as the fraction of the total demand that is satisfied. Note that the total demand is exogenous information independent of our states and actions. Hence, our measures of interest are the outdated quantity and the satisfied demand.

The outdated quantity O_t at the end of day t is the remainder of the batch of age $I-1$ after inventory withdrawal

$$O_t = X_t^{I-1} - B_t^{I-1}. \tag{8}$$

Note that the outdated quantity depends exclusively on the adjusted shelf life I. For example, a system under $m = 5$ and $r = 0$ is equivalent to a system under $m = 10$ and $r = 5$ as $I = 5$ in both cases.

The satisfied demand D_t^s during day t is the minimum of the inventory on-hand and the total demand

$$D_t^s = \min\{OH_t, D_t\}, \tag{9}$$

in which $OH_t = \sum_{j=0}^{I-1} X_t^j$. Note that inventory levels $\mathbf{X}_t$ (and not $\widehat{\mathbf{X}}_t$) are used to compute the performance measures, i.e., the outdated quantity and satisfied demand.

3.3 Policies

We consider two policies. The base-stock policy is implemented as a relatively simple but effective policy with practical implications for most food retailers. Furthermore, we examine the impact of a more sophisticated policy by considering the EWA policy of Broekmeulen and van Donselaar (2009). This stock-age dependent policy uses only small calculations, making it possible to incorporate the policy in (daily) ordering decisions.

3.3.1 Base-stock policy

In the BSP, an order is placed to bring the inventory position back to order-up-to level S

$$a_t = S - IP_t, \tag{10}$$

in which $IP_t = \sum_{j=0}^{I-1} X_t^j + \sum_{i=1}^{L-1} q_{t,i}$.

3.3.2 EWA policy

Broekmeulen and van Donselaar (2009) argue that outdating impacts the ability to satisfy demand. Therefore, when ordering, the inventory position should first be adjusted by the estimated outdated quantity during lead time and review period

$$a_t = S - IP_t + \sum_{i=t}^{t+L-1} \widehat{O}_i, \tag{11}$$

where $\widehat{O}_t$ is the expected outdated quantity at the end of day t. To compute the expected outdated quantity at the end of day t to $t + L - 1$, the following five-step procedure is applied, starting at time $i = t$:

1. Determine the expected inventory withdrawal during day i, using Eq. (3) and by assuming D_i^{FIFO} and D_i^{LIFO} are equal to their expected value.
2. Determine the outdated quantity at the end of day i, using Eq. (8) and by assuming the inventory withdrawal during day i is equal to the estimate as derived in Step 1.
3. If $i < t + L - 1$, go to Step 4, otherwise stop.
4. Determine the inventory levels at time $i + 1$, using Eq. (4) and by assuming the inventory withdrawal during day i is equal to the estimate as derived in Step 1.
5. Do $i = i + 1$ and go to Step 1.

4 Theoretical results

This section provides theoretical results for the food waste as a function of the shelf life. Section 4.1 gives some general theorems and proves that the food waste and fill rate are monotonic in the shelf life under a fixed sequence of actions. Section 4.2 transitions from a sequence of actions to a policy and derives expressions for the expected food waste and the expected fill rate.

Furthermore, we design two policy classes. The order quantity in a shelf-independent (state-dependent) policy is independent (dependent) of the shelf life (state-space). In Section 4.3, we prove that the expected food waste under the optimal policy in an arbitrary set of shelf-independent policies is monotonically decreasing in the shelf life. For the analysis of state-dependent policies, we construct the so-called 'mimic' policy that mimics the same policy's sequence of actions under shelf life $I-1$. In Section 4.4, we prove that the expected food waste under the optimal policy in an arbitrary set of policies is monotonically decreasing in the shelf life if the mimic policies are included. The set of all policies also includes all the mimic policies, and therefore, we conclude that the expected food waste under the optimal policy is monotonically decreasing in the shelf life.

Moreover, to support simulation search ranges, in Section 4.5 we analyze the food waste as a function of the policies' parameters.

4.1 General theorems

Theorem 4.1. *An additional unit of inventory X_t^j at time t increases the total outdated quantity, the total satisfied demand, or some inventory level X_T^k by one for all $t = 1, \ldots, T$, $T \in \mathbb{N}$, $T \geq t$, $j = 1, \ldots, I-1$, $k = 1, \ldots, I-1$, and $I \in \mathbb{N}$.*

Proof. Let us assume we have one additional inventory unit X_t^j at time t, for arbitrary $t \in \mathbb{N}$, $j = 1, \ldots, I-1$, and $I \in \mathbb{N}$, keeping all other things equal. Other aged inventories and exogenous information are independent of the additional unit of inventory. Let $\widetilde{\mathbf{X}}_i$, $\widetilde{OH}_i$, $\widetilde{D}_i^s$, $\widetilde{B}_i^j$ and $\widetilde{O}_i$ denote the inventory levels, on-hand inventory, satisfied demand, inventory withdrawal, and outdated quantity at time $i = t, t+1, \ldots, T$, respectively, when inventory level X_t^j is incremented by one at time t. We examine the effect of the additional unit and distinguish three cases:

1. The additional unit of inventory is outdated at the end of day $t + I - 1 - j$.

   The inventory levels are modified as follows

$$\widetilde{X}_i^{i-t+j} = X_i^{i-t+j} + 1, \text{ for } i = t, \ldots t + I - 1 - j. \tag{12}$$

   Therefore, at time $t + I - 1 - j$, we have $\widetilde{X}_{t+I-1-j}^{I-1} = X_{t+I-1-j}^{I-1} + 1$. Hence, by Eq. (8), the outdated quantity at the end of day $t + I - 1 - j$ increases by one

$$\widetilde{O}_{t+I-1-j} = \widetilde{X}^{I-1}_{t+I-1-j} - \widetilde{B}^{I-1}_{t+I-1-j} = X^{I-1}_{t+I-1-j} + 1 - B^{I-1}_{t+I-1-j} = O_{t+I-1-j} + 1. \tag{13}$$

There is no further effect on day $t+I-j$ as $\widetilde{\mathbf{X}}_t = \mathbf{X}_t$ for $t \geq t+I-j$.

2. The additional unit of inventory is sold during day $v \in \{t, t+I-1-j\}$ and $OH_v < D_v$.

   The satisfied demand $D^s_v = \min\{OH_v, D_v\} = OH_v$. Furthermore, by Eq. (12), $\widetilde{X}^{v-t+j}_v = X^{v-t+j}_v + 1$, and therefore, $\widetilde{OH}_v = OH_v + 1$. Hence, by Eq. (9), the satisfied demand during day v increases by one

$$\widetilde{D}^s_v = \min\{\widetilde{OH}_v, D_v\} = \min\{OH_v + 1, D_v\} = OH_v + 1 = D^s_v + 1, \tag{14}$$

   in which $\min\{OH_v + 1, D_v\} = OH_v + 1$ as $OH_v < D_v$ and demand and inventory levels are integer-valued. There is no further effect on day $v+1$ as $\widetilde{\mathbf{X}}_t = \mathbf{X}_t$ for $t \geq v+1$.

3. The additional unit of inventory is sold during day $v \in \{t, t+I-1-j\}$ and $OH_v \geq D_v$.

   The satisfied demand $D^s_v = \min\{OH_v, D_v\} = D_v$. Hence, $\widetilde{D}^s_v = \min\{\widetilde{OH}_v, D_v\} = \min\{OH_v + 1, D_v\} = D_v$ (the satisfied demand does not increase). Instead, the inventory withdrawal B^w_v of some inventory of age $0 \leq w \leq I-1$ decreases by one, i.e., $\widetilde{B}^w_v = B^w_v - 1$. Hence, by Eq. (4)

$$\widetilde{X}^{w+1}_{v+1} = \widetilde{X}^w_v - \widetilde{B}^w_v = X^w_v - (B^w_v - 1) = X^{w+1}_{v+1} + 1. \tag{15}$$

Using the same reasoning, we can analyze the additional unit of inventory X^{w+1}_{v+1} by distinguishing the same three cases. This can be repeated for arbitrary $T \in \mathbb{N}$, $T \geq t$, at which the additional unit X^j_t either increases the total outdated quantity by one, the total satisfied demand by one, or some inventory level X^k_T of age $k = 0, \ldots, I-1$ by one. □

Definition 4.1. *Let $\boldsymbol{a} \in \mathbb{N}^T$ be a sequence of actions $(a_1, \ldots, a_T)$ for $T \in \mathbb{N}$.*

Definition 4.2. *Let $\boldsymbol{D} \in \mathbb{N}^T$ be a sample-path $(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_T)$ for $T \in \mathbb{N}$.*

Theorem 4.2. *Let $\overline{O}\Big(\boldsymbol{a}, \boldsymbol{D}, I\Big)$ and $\overline{D^s}\Big(\boldsymbol{a}, \boldsymbol{D}, I\Big)$ denote the total outdated quantity and total satisfied demand, respectively, under sequence of actions $\boldsymbol{a} \in \mathbb{N}^T$, sample-path $\boldsymbol{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$.*

The total outdated quantity and total satisfied demand are monotonically decreasing and monotonically increasing in the shelf life $I \in \mathbb{N}$ under arbitrary sequence of actions $\boldsymbol{a} \in \mathbb{N}^T$, sample-path $\boldsymbol{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$

$$\overline{O}\Big(\boldsymbol{a}, \boldsymbol{D}, I+1\Big) \leq \overline{O}\Big(\boldsymbol{a}, \boldsymbol{D}, I\Big), \tag{16}$$

$$\overline{D^s}\Big(\boldsymbol{a}, \boldsymbol{D}, I+1\Big) \geq \overline{D^s}\Big(\boldsymbol{a}, \boldsymbol{D}, I\Big). \tag{17}$$

Proof. Consider the quantity (batch) ordered at $t-L+1-I$ (i.e., $a_{t-L+1-I}$) for arbitrary $t \in \mathbb{N}$, $L \in \mathbb{N}$, and $I \in \mathbb{N}$. The remainder of this batch is outdated at the end of day t. We increase the shelf life of the batch from I to $\widetilde{I} = I + 1$, while keeping the sequence of actions and sample-path fixed. Furthermore, without loss of generality, we assume $T \geq t+1$ (note that increasing the shelf life does not affect the batch for $T < t+1$, hence there is no effect). Let $\widetilde{\mathbf{X}}_t$ denote the inventory levels at time t under shelf life $\widetilde{I}$. By incrementing the shelf life, outdated products at the end of day t (say $b \in \mathbb{N}$ units) are valid for one more day, i.e., $\widetilde{X}_{t+1}^{\widetilde{I}-1} = O_t$. Hence, at time $t+1$, the total outdated quantity has decreased by b and inventory $\widetilde{X}_{t+1}^{\widetilde{I}-1}$ has increased by b.

By Theorem 4.1, for arbitrary $T \in \mathbb{N}$, $T \geq t+1$, each of the b additional units of inventory $\widetilde{X}_{t+1}^{\widetilde{I}-1}$ either increases (i) the total outdated quantity by one (say in total $b^o \in \mathbb{N}$ units), (ii) the total satisfied demand by one (say in total $b^d \in \mathbb{N}$ units), (iii) some inventory level X_T^k of age $k = 0, \dots, I-1$ by one (say in total $b^i \in \mathbb{N}$ units). Hence, $b = b^o + b^d + b^i$, and thus, $b^o \leq b$. Under a fixed sequence of actions and sample-path, the total outdated quantity and total satisfied demand, respectively, are modified as follows

$$\overline{O}\Big(\mathbf{a}, \mathbf{D}, I+1\Big) = \overline{O}\Big(\mathbf{a}, \mathbf{D}, I\Big) + b^o - b, \tag{18}$$

$$\overline{D^s}\Big(\mathbf{a}, \mathbf{D}, I+1\Big) = \overline{D^s}\Big(\mathbf{a}, \mathbf{D}, I\Big) + b^d. \tag{19}$$

Therefore, by incrementing the shelf life of an arbitrary batch we have $\overline{O}\Big(\mathbf{a}, \mathbf{D}, I+1\Big) \leq \overline{O}\Big(\mathbf{a}, \mathbf{D}, I\Big)$, and $\overline{D^s}\Big(\mathbf{a}, \mathbf{D}, I+1\Big) \geq \overline{D^s}\Big(\mathbf{a}, \mathbf{D}, I\Big)$. We can repeat this analysis until the shelf life of all batches is incremented by one. $\square$

The food waste under sequence of actions $\mathbf{a} \in \mathbb{N}^T$, sample-path $\mathbf{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$ is defined as

$$\omega\Big(\mathbf{a}, \mathbf{D}, I\Big) = \frac{\overline{O}\Big(\mathbf{a}, \mathbf{D}, I\Big)}{\sum_{t=1}^{T} D_T}. \tag{20}$$

Corollary 4.2.1. *The food waste is monotonically decreasing in the shelf life $I \in \mathbb{N}$ under arbitrary sequence of actions $\boldsymbol{a} \in \mathbb{N}^T$, sample-path $\boldsymbol{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$*

$$\omega\Big(\boldsymbol{a}, \boldsymbol{D}, I+1\Big) \leq \omega\Big(\boldsymbol{a}, \boldsymbol{D}, I\Big). \tag{21}$$

The fill rate under sequence of actions $\mathbf{a} \in \mathbb{N}^T$, sample-path $\mathbf{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$ is defined as

$$\beta\Big(\mathbf{a}, \mathbf{D}, I\Big) = \frac{\overline{D^s}\Big(\mathbf{a}, \mathbf{D}, I\Big)}{\sum_{t=1}^{T} D_T}. \tag{22}$$

Corollary 4.2.2. *The fill rate is monotonically increasing in the shelf life $I \in \mathbb{N}$ under arbitrary sequence of actions $\boldsymbol{a} \in \mathbb{N}^T$, sample-path $\boldsymbol{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$*

$$\beta\Big(\boldsymbol{a}, \boldsymbol{D}, I+1\Big) \geq \beta\Big(\boldsymbol{a}, \boldsymbol{D}, I\Big). \tag{23}$$

4.2 Policies

Let π be a policy that decides on order quantity a_t using state $\mathbf{S}_t$ (or an aggregation of the state information). The inventory withdrawal is affected by the shelf life I, and therefore, state $\mathbf{S}_t$ is affected by I. Hence, policy π leads to a different sequence of actions for different I. Moreover, the transition from $\mathbf{S}_t$ to $\mathbf{S}_{t+1}$ depends on the exogenous information $\mathbf{W}_t$ (i.e., the FIFO and LIFO demand). Thus, the sequence of actions $\mathbf{a} \in \mathbb{N}^T$ under policy π generally depends on both the shelf life $I \in \mathbb{N}$ and the sample-path $\mathbf{D} \in \mathbb{N}^T$ for $T \in \mathbb{N}$.

Definition 4.3. *Let $\boldsymbol{a}^{\pi_{I,\boldsymbol{D}}} \in \mathbb{N}^T$ be the sequence of actions under policy π, shelf life $I \in \mathbb{N}$, sample-path $\boldsymbol{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$.*

Let $\mathbb{E}\Big[\omega(\pi, I)\Big]$ and $\mathbb{E}\Big[\beta(\pi, I)\Big]$ denote the expected food waste and expected fill rate, respectively, under policy π and shelf life $I \in \mathbb{N}$. The food waste $\omega\big(\mathbf{a}^{\pi_{I,\mathbf{D}}}, \mathbf{D}, I\big)$ is the waste under (the sequence of actions of) policy π, shelf life I, and sample-path $\mathbf{D}$. By conditioning on the probability that sample-path $\mathbf{D} \in \mathbb{N}^T$ is realized, the expected food waste under policy π and shelf life I is derived as

$$\mathbb{E}\Big[\omega(\pi, I)\Big] = \sum_{\mathbf{D} \in \mathbb{N}^T} \Big(\omega\big(\mathbf{a}^{\pi_{I,\mathbf{D}}}, \mathbf{D}, I\big) \cdot p(\mathbf{D})\Big), \tag{24}$$

in which $p(\mathbf{D})$ is the probability that sample-path $\mathbf{D} \in \mathbb{N}^T$ is realized. Similarly, the expected fill rate under policy π and shelf life I is derived as

$$\mathbb{E}\Big[\beta(\pi, I)\Big] = \sum_{\mathbf{D} \in \mathbb{N}^T} \Big(\beta\big(\mathbf{a}^{\pi_{I,\mathbf{D}}}, \mathbf{D}, I\big) \cdot p(\mathbf{D})\Big). \tag{25}$$

Theorem 4.3. *Let the food waste under policy $\widetilde{\pi}$ and shelf life i be less than or equal to the food waste under policy $\overline{\pi}$ and shelf life j, for all sample-paths $\boldsymbol{D} \in \mathbb{N}^T$ and horizon $T \in \mathbb{N}$*

$$\omega\big(\boldsymbol{a}^{\widetilde{\pi}_{i,\boldsymbol{D}}}, \boldsymbol{D}, i\big) \leq \omega\big(\boldsymbol{a}^{\overline{\pi}_{j,\boldsymbol{D}}}, \boldsymbol{D}, j\big). \tag{26}$$

The expected food waste under policy $\widetilde{\pi}$ and shelf life i is less than or equal to the expected food waste under policy $\overline{\pi}$ and shelf life j

$$\mathbb{E}\Big[\omega(\widetilde{\pi}, i)\Big] \leq \mathbb{E}\Big[\omega(\overline{\pi}, j)\Big]. \tag{27}$$

Proof. The exogenous information $\mathbf{W}_t$ is independent of our states and actions for all $t = 1, \ldots, T$, $T \in \mathbb{N}$. Hence, the probability $p(\mathbf{D})$ that sample-path $\mathbf{D} \in \mathbb{N}^T$ occurs is fixed and not affected by the policy, nor by the shelf life. Therefore, by Eq. (24) and Eq. (26), respectively

$$\mathbb{E}\Big[\omega(\widetilde{\pi}, i)\Big] = \sum_{\mathbf{D} \in \mathbb{N}^T} \Big(\omega\big(\mathbf{a}^{\widetilde{\pi}_{i,\mathbf{D}}}, \mathbf{D}, i\big) \cdot p(\mathbf{D})\Big) \leq \sum_{\mathbf{D} \in \mathbb{N}^T} \Big(\omega\big(\mathbf{a}^{\overline{\pi}_{j,\mathbf{D}}}, \mathbf{D}, j\big) \cdot p(\mathbf{D})\Big) = \mathbb{E}\Big[\omega(\overline{\pi}, j)\Big]. \tag{28}$$

□

Theorem 4.4. *Let the fill rate under policy* $\widetilde{\pi}$ *and shelf life* i *be larger than or equal to the fill rate under policy* $\overline{\pi}$ *and shelf life* j*, for all sample-paths* $\boldsymbol{D} \in \mathbb{N}^T$ *and horizon* $T \in \mathbb{N}$

$$\beta\big(\boldsymbol{a}^{\widetilde{\pi}_{i,\boldsymbol{D}}}, \boldsymbol{D}, i\big) \geq \beta\big(\boldsymbol{a}^{\overline{\pi}_{j,\boldsymbol{D}}}, \boldsymbol{D}, j\big). \tag{29}$$

The expected fill rate under policy $\widetilde{\pi}$ *and shelf life* i *is larger than or equal to the expected fill rate under policy* $\overline{\pi}$ *and shelf life* j

$$\mathbb{E}\Big[\beta(\widetilde{\pi}, i)\Big] \geq \mathbb{E}\Big[\beta(\overline{\pi}, j)\Big]. \tag{30}$$

Proof. By Eq. (25) and Eq. (29), respectively

$$\mathbb{E}\Big[\beta(\widetilde{\pi}, i)\Big] = \sum_{\mathbf{D} \in \mathbb{N}^T} \Big(\beta\big(\mathbf{a}^{\widetilde{\pi}_{i,\mathbf{D}}}, \mathbf{D}, i\big) \cdot p(\mathbf{D})\Big) \geq \sum_{\mathbf{D} \in \mathbb{N}^T} \Big(\beta\big(\mathbf{a}^{\overline{\pi}_{j,\mathbf{D}}}, \mathbf{D}, j\big) \cdot p(\mathbf{D})\Big) = \mathbb{E}\Big[\beta(\overline{\pi}, j)\Big]. \tag{31}$$

□

4.3 Shelf-independent policies

Definition 4.4. *Let* B^π *be the class of shelf-independent policies. Order quantity* a_t *under policy* $\pi \in B^\pi$ *is independent of the shelf life* $I \in \mathbb{N}$ *for all* $t = 1, \ldots, T$*,* $\boldsymbol{D} \in \mathbb{N}^T$*, and* $T \in \mathbb{N}$

$$\boldsymbol{a}^{\pi_{I+1,\boldsymbol{D}}} = \boldsymbol{a}^{\pi_{I,\boldsymbol{D}}}. \tag{32}$$

Example 4.1. *A constant order policy (COP) in which* $a_t = Q$ *is a shelf-independent policy.*

Proof. The order quantity a_t in the COP is equal to Q independent of the shelf life $I \in \mathbb{N}$ for all $t = 1, \ldots, T$, $\mathbf{D} \in \mathbb{N}^T$, and $T \in \mathbb{N}$. □

Definition 4.5. *Let* Π_I *be an arbitrary set of policies under shelf life* $I \in \mathbb{N}$*. The optimal policy* $\pi^{*(I)} \in \Pi_I$ *minimizes the expected food waste among all policies in* Π_I *that (in expectation) satisfy the fill rate constraint*

$$\mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big] \leq \mathbb{E}\Big[\omega(\pi, I)\Big], \text{ for all } \pi \in \Pi_I : \mathbb{E}\Big[\beta(\pi, I)\Big] \geq \beta. \tag{33}$$

Theorem 4.5. *Let* Π_I *be an arbitrary set of shelf-independent policies under shelf life* $I \in \mathbb{N}$*. The expected food waste under the optimal policy* $\pi^{*(I)} \in \Pi_I$ *is monotonically decreasing in the shelf life* $I \in \mathbb{N}$

$$\mathbb{E}\Big[\omega(\pi^{*(I+1)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big], \tag{34}$$

in which $\pi^{*(I+1)} \in \Pi_{I+1}$*.*

Proof. Consider the optimal policy $\pi^{*(I)}$ under arbitrary shelf life $I \in \mathbb{N}$, sample-path $\mathbf{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$. By Corollary 4.2.2 and Definition 4.4, respectively

$$\beta\Big(\mathbf{a}^{\pi^{*(I)}_{I+1,\mathbf{D}}}, \mathbf{D}, I+1\Big) \geq \beta\Big(\mathbf{a}^{\pi^{*(I)}_{I+1,\mathbf{D}}}, \mathbf{D}, I\Big) = \beta\Big(\mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}, \mathbf{D}, I\Big). \tag{35}$$

Hence, by Theorem 4.4 and Definition 4.5, respectively

$$\mathbb{E}\Big[\beta(\pi^{*(I)}, I+1)\Big] \geq \mathbb{E}\Big[\beta(\pi^{*(I)}, I)\Big] \geq \beta. \tag{36}$$

Thus, policy $\pi^{*(I)}$ satisfies the fill rate constraint (in expectation) under $I+1$. Hence, by Definition 4.5

$$\mathbb{E}\Big[\omega(\pi^{*(I+1)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I+1)\Big]. \tag{37}$$

Furthermore, by Corollary 4.2.1 and Definition 4.4, respectively

$$\omega\Big(\mathbf{a}^{\pi^{*(I)}_{I+1,\mathbf{D}}}, \mathbf{D}, I+1\Big) \leq \omega\Big(\mathbf{a}^{\pi^{*(I)}_{I+1,\mathbf{D}}}, \mathbf{D}, I\Big) = \omega\Big(\mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}, \mathbf{D}, I\Big). \tag{38}$$

Hence, by Theorem 4.3

$$\mathbb{E}\Big[\omega(\pi^{*(I)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big]. \tag{39}$$

□

Example 4.2. *Let Π_I^{COP} be the class of constant order policies (COP) under shelf life $I \in \mathbb{N}$. The expected food waste under the optimal constant order policy $\pi^{*(I)} \in \Pi_I^{COP}$ is monotonically decreasing in the shelf life $I \in \mathbb{N}$*

$$\mathbb{E}\Big[\omega(\pi^{*(I+1)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big], \tag{40}$$

in which $\pi^{*(I+1)} \in \Pi_{I+1}^{COP}$.

Proof. Π^{COP} is a set of shelf-independent policies as a constant order policy is a shelf-independent policy (by Example 4.1). Hence, the result follows from Theorem 4.5. □

4.4 State-dependent policies

Definition 4.6. *Let S^π be the class of state-dependent policies. Policy $\pi \in S^\pi$ determines order quantity a_t using state information $\boldsymbol{S}_t$ for all $t = 1, \ldots, T$, $T \in \mathbb{N}$.*

Example 4.3. *A base-stock policy (BSP) in which $a_t = S - IP_t$ is a state-dependent policy.*

Proof. In the BSP, the order quantity a_t is determined using the inventory position IP_t for all $t = 1, \ldots, T$, $T \in \mathbb{N}$ which is an aggregate of the state information $\mathbf{S}_t$. □

State $\mathbf{S}_t$ is affected by the shelf life I, and therefore, the sequence of actions under policy $\pi \in S^\pi$ depends on $I \in \mathbb{N}$. Thus, in general, for arbitrary sample-path $\mathbf{D} \in \mathbb{N}^T$

$$\mathbf{a}^{\pi_{I+1,\mathbf{D}}} \neq \mathbf{a}^{\pi_{I,\mathbf{D}}}. \tag{41}$$

Therefore, for state-dependent policies, Eq. (35) and Eq. (38) generally do not hold. Hence, Theorem 4.5 might not hold if we include state-dependent policies in the policy set Π_I.

Consider an arbitrary set of policies Π_I under shelf life $I \in \mathbb{N}$. For policy $\pi \in \Pi_I$, we construct a 'mimic' policy $\widehat{\pi}$ under shelf life $I+1$ that mimics the sequence of actions of π. For each policy $\pi \in \Pi_I$, we construct a mimic policy $\widehat{\pi} \in \widehat{\Pi}_{I+1}$ where $\widehat{\Pi}_{I+1}$ is the set of mimic policies under $I+1$. Policy $\widehat{\pi}$ incorporates the same order rule as policy π. However, $\pi \in \Pi_I$ determines order quantity a_t based on state information $(\mathbf{X}_t, \mathbf{OO}_t)$, while $\widehat{\pi} \in \widehat{\Pi}_{I+1}$ uses state information $(\widehat{\mathbf{X}}_t, \mathbf{OO}_t)$. The 'fictive' inventory levels $\widehat{\mathbf{X}}_t$ disregard the oldest inventory and are, therefore, equivalent to the inventory levels of a system under $I-1$ (i.e., the system under policy π). Thus, policy $\pi \in \Pi_I$ and policy $\widehat{\pi} \in \widehat{\Pi}_{I+1}$ incorporate the same state information. Hence, under arbitrary shelf life $I \in \mathbb{N}$, sample-path $\mathbf{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$

$$\mathbf{a}^{\widehat{\pi}_{I+1,\mathbf{D}}} = \mathbf{a}^{\pi_{I,\mathbf{D}}}. \tag{42}$$

Theorem 4.6. *Let Π_I be an arbitrary set of policies under shelf life $I \in \mathbb{N}$. The expected food waste under the optimal policy $\pi^{*(I)} \in \Pi_I$ is monotonically decreasing in the shelf life $I \in \mathbb{N}$*

$$\mathbb{E}\Big[\omega(\pi^{*(I+1)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big], \tag{43}$$

in which $\pi^{*(I+1)} \in \Big(\Pi_{I+1}, \widehat{\Pi}_{I+1}\Big)$.

Proof. For each policy $\pi \in \Pi_I$, there exists a mimic policy $\widehat{\pi} \in \widehat{\Pi}_{I+1}$ that mimics the sequence of actions of π. Hence, there exists a policy $\overline{\pi} \in \widehat{\Pi}_{I+1}$ that mimics the sequence of actions of the optimal policy $\pi^{*(I)} \in \Pi_I$ under arbitrary shelf life $I \in \mathbb{N}$, sample-path $\mathbf{D} \in \mathbb{N}^T$, and horizon $T \in \mathbb{N}$

$$\mathbf{a}^{\overline{\pi}_{I+1,\mathbf{D}}} = \mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}. \tag{44}$$

By Eq. (44) and Corollary 4.2.2, respectively

$$\beta\Big(\mathbf{a}^{\overline{\pi}_{I+1,\mathbf{D}}}, \mathbf{D}, I+1\Big) = \beta\Big(\mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}, \mathbf{D}, I+1\Big) \geq \beta\Big(\mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}, \mathbf{D}, I\Big). \tag{45}$$

Hence, by Theorem 4.4 and Definition 4.5, respectively

$$\mathbb{E}\Big[\beta(\overline{\pi}, I+1)\Big] \geq \mathbb{E}\Big[\beta(\pi^{*(I)}, I)\Big] \geq \beta. \tag{46}$$

Thus, policy $\overline{\pi} \in \widehat{\Pi}_{I+1}$ satisfies the fill rate constraint (in expectation) under $I+1$. Hence, by Definition 4.5, for the optimal policy $\pi^{*(I+1)} \in \Big(\Pi_{I+1}, \widehat{\Pi}_{I+1}\Big)$ we have

$$\mathbb{E}\Big[\omega(\pi^{*(I+1)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\overline{\pi}, I+1)\Big]. \tag{47}$$

Furthermore, by Eq. 44 and Corollary 4.2.1, respectively

$$\omega\Big(\mathbf{a}^{\overline{\pi}_{I+1,\mathbf{D}}}, \mathbf{D}, I+1\Big) = \omega\Big(\mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}, \mathbf{D}, I+1\Big) \leq \omega\Big(\mathbf{a}^{\pi^{*(I)}_{I,\mathbf{D}}}, \mathbf{D}, I\Big). \tag{48}$$

Hence, by Theorem 4.3

$$\mathbb{E}\Big[\omega(\overline{\pi}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big]. \tag{49}$$

□

Example 4.4. *Let Π_I^A be the set of all policies under shelf life $I \in \mathbb{N}$. The expected food waste under the optimal policy $\pi^{*(I)} \in \Pi_I^A$ is monotonically decreasing in the shelf life $I \in \mathbb{N}$*

$$\mathbb{E}\Big[\omega(\pi^{*(I+1)}, I+1)\Big] \leq \mathbb{E}\Big[\omega(\pi^{*(I)}, I)\Big], \tag{50}$$

in which $\pi^{*(I+1)} \in \Pi_{I+1}^A$.

Proof. The set of all policies also includes all the mimic policies, i.e., $\Pi_{I+1}^A = \Big(\Pi_{I+1}^A, \widehat{\Pi}_{I+1}^A\Big)$. Hence, the result follows from Theorem 4.6. □

4.5 Parameter optimization

To support the search range for the optimal parameter value of the base-stock policy, we analyze the food waste as a function of the policies' parameters. Note that this analysis can be extended to support search ranges for a variety of policies, for instance, for the constant order policy (Example 4.6).

Definition 4.7. *Policy class A^π denotes the class of policies in which the order quantity a_t is monotonically increasing in the policies' parameters $P_i \in \mathbb{N}$, $i = 1, \ldots, n$ for all $t = 1, \ldots, T$ under arbitrary sample-path $\boldsymbol{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$.*

Theorem 4.7. *The total outdated quantity is monotonically increasing in order quantity a_t for all $t = 1, \ldots, T$ under arbitrary sample-path $\boldsymbol{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$*

$$\overline{O}\Big((a_1, \ldots, (a_t+1), \ldots, a_T), \boldsymbol{D}, I\Big) \geq \overline{O}\Big(a_1, \ldots, a_t, \ldots, a_T), \boldsymbol{D}, I\Big). \tag{51}$$

Proof. We increment order quantity a_t by one for arbitrary t, while keeping the sample-path and shelf life fixed. Hence, at time $t+L$ we have an additional inventory unit X^0_{t+L}. Without loss of generality, we assume $T \geq t+L$ (note that increasing order quantity a_t has no effect for $T < t+L$). By theorem 4.1, for arbitrary $T \in \mathbb{N}$, $T \geq t+L$, the additional unit X^0_{t+L} either increases (i) the total outdated quantity by one, (ii) the total satisfied demand by one, or (iii) some inventory level X^k_T of age $k = 0, \dots, I-1$ by one. Hence, by incrementing a_t by one for arbitrary t, the total outdated quantity never decreases. □

Corollary 4.7.1. *The food waste is monotonically increasing in order quantity a_t for all $t = 1, \dots, T$ under arbitrary sample-path $\boldsymbol{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$*

$$\omega\Big((a_1, \dots, (a_t+1), \dots, a_T), \boldsymbol{D}, I\Big) \geq \omega\Big(a_1, \dots, a_t, \dots, a_T), \boldsymbol{D}, I\Big). \tag{52}$$

Corollary 4.7.2. *The food waste is monotonically increasing in the policies' parameters $P_i \in \mathbb{N}$, $i = 1, \dots, n$ for policy $\pi \in A^\pi$ under arbitrary sample-path $\boldsymbol{D} \in \mathbb{N}^T$, horizon $T \in \mathbb{N}$, and shelf life $I \in \mathbb{N}$.*

Example 4.5. *The base-stock policy $\pi^{BSP} \in A^\pi$.*

Proof. The BSP incorporates one parameter $P_1 = S$ with $S \in \mathbb{N}$ and $a_t = S - IP_t$. At arbitrary time i we increase the order-up-level from S to $\widetilde{S} = S+1$, while keeping the sample-path and shelf life fixed

$$\widetilde{S} = \begin{cases} S, & \text{for } 1 \leq t < i, \\ S+1, & \text{for } t \geq i. \end{cases} \tag{53}$$

Let $\widetilde{a}_t$ and $\widetilde{IP}_t$, respectively, denote the order quantity and inventory position at time t under $\widetilde{S}$. At time i we have $\widetilde{IP}_i = IP_i$ as $\widetilde{S} = S$ for $t < i$, hence

$$\widetilde{a}_i = \widetilde{S} - \widetilde{IP}_i = S + 1 - IP_i = a_i + 1. \tag{54}$$

Therefore, at time $i+L$ we have an additional inventory unit X^0_{i+L}. Without loss of generality, we assume $T \geq i+L$ (note that increasing S at time i has no effect for $T < i+L$). By Theorem 4.1, for arbitrary $T \in \mathbb{N}$, $T \geq i+L$, the additional unit X^0_{i+L} either increases (i) the total outdated quantity by one, (ii) the total satisfied demand by one, (iii) some inventory level X^k_T of age $k = 0, \dots, I-1$ by one. We assume that option (i) or (ii) occurs during day j which decreases the inventory position by one. Hence, the inventory positions and order quantities, respectively, are modified as follows

$$\widetilde{IP}_i = \begin{cases} IP_i, & \text{for } 1 \leq t \leq i, \\ IP_i + 1, & \text{for } i < t \leq j, \end{cases} \tag{55}$$

$$\widetilde{a}_i = \begin{cases} a_i, & \text{for } 1 \leq t < i, \\ a_i + 1, & \text{for } t = i, \\ a_i, & \text{for } i < t \leq j. \end{cases} \tag{56}$$

If $T \leq j$ the proof is finished as $\widetilde{a}_t \geq a_t$ for all $t = 1, \ldots, T$. If $T > j$, we have $\widetilde{IP}_{j+1} = IP_{j+1}$, and therefore, $\widetilde{a}_{j+1} = a_{j+1} + 1$. We can repeat the analysis of the additional inventory unit X^0_{j+1+L} for arbitrary T and conclude that a_t is monotonically increasing in S for all $t = 1, \ldots, T$. □

Example 4.6. *The constant order policy* $\pi^{COP} \in A^\pi$.

Proof. The COP incorporates one parameter $P_1 = Q$ with $Q \in \mathbb{N}$. Furthermore, $a_t = Q$, hence, increasing Q by one increases a_t by one for all $t = 1, \ldots, T$. □

5 Methodology

We aim to find the optimal value of the policies' parameter by simulation-based optimization. This section provides search ranges for the optimal value, describes the optimization procedure, and explains the computation of our measure for the food waste. Furthermore, to assess online sales' wastefulness, we define two performance measures; the minimum throw-out date and the online wasteful shelf life range. We describe the derivation of these measures and provide some inherent properties.

5.1 Simulation

5.1.1 Search ranges

For the EWA policy, following van Donselaar and Broekmeulen (2012), we simulate each integer value of S, starting with zero and ending when the fill rate constraint is satisfied. For the BSP, we show that the food waste is never decreasing in S (by Corollary 4.7.2). Therefore, we apply the same search range as the lowest value for S that satisfies the fill rate constraint is optimal.

5.1.2 Optimization procedure

To ensure that the model is stationarity, a warm-up phase of 100 days is initialized, starting with zero inventories and the mean demand for the outstanding orders. The initialized outstanding orders ensure that the fill rate can be satisfied. The warm-up phase is followed by one or more consecutive 'batches' during which the performance is measured. A batch consists of 10 runs, and a run has a length of 1000 days. For each run, we compute a measure for the fill rate. Hence, each batch adds ten measures for the fill rate. We initialize a single batch and increment the number of batches by one until the length of the 95% confidence interval for the mean of the fill rate < 0.004. This approach is suggested by Minner and Transchel (2010) and van Donselaar and Broekmeulen (2012). If the mean of the confidence interval for the fill rate exceeds the required level of β, the optimal value for S is found. Otherwise, we increment S by one and rerun the simulation using the same seed and starting with a warm-up phase. The optimal value for S is simulated at a higher level of accuracy as obtained by 1000 runs (each with a length of 1000 days). For each run, we compute a measure for the food waste. Hence, an estimate for the food waste is the average of 1000 simulated measures for the food waste. To acquire a larger sample for the paired sample t-test, ten estimates for the food waste are computed. An additional estimate is obtained by simulating the optimal value for S for another 1000 subsequent runs (using the same seed) and taking the average of 1000 new simulated measures for the food waste.

The choice for the relatively small number of runs per batch allows for fast optimization of S—moreover, 1000 runs yields more accurate estimates for the expected food waste at an acceptable speed.

5.2 Performance measures

5.2.1 Minimum throw-out date

Experiment $e \in E$ is defined by the parameters in Table 1. We define setting $E(m, f) \subseteq E$ as the subset of experiments for a fixed shelf life m and fraction offline FIFO demand f.

Table 1: List of the parameters considered.

Parameter	Description
L	Lead time
μ	Mean demand
$cvr = \sqrt{\frac{\sigma^2}{\mu}}$	Coefficient of variation
β	Required fill rate
m	Shelf life
f	Fraction offline FIFO demand

For each setting $E(m, f)$, we derive the minimum online throw-out date $r(m, f)$ that leads to a statistically significant increase in food waste. Hence, under setting $E(m, f)$, an online retailer incorporating a throw-out policy $r \geq r(m, f)$ is statistically more wasteful than an offline retailer.

Figure 2 illustrates the derivation of $r(m, f)$, results in the figure are based on the base-stock policy. Let $\omega(m, f)$ be the average food waste in setting $E(m, f)$. The blue horizontal line presents $\omega(15, 0.5)$, i.e., the average food waste in $E(15, 0.5)$. The black curved line presents the food waste as a function of the corresponding online throw-out policies, i.e., $\omega(15 - r, 1)$. Note that we incorporate full FIFO inventory depletion ($f = 1$) for online sales. The expected food waste is monotonically decreasing in the adjusted shelf life $I = m - r$ (by Theorem 4.6). Therefore, the wastefulness of online sales increases in r. The red dot in the graph displays the lowest online throw-out date that leads to a statistically significant increase in food

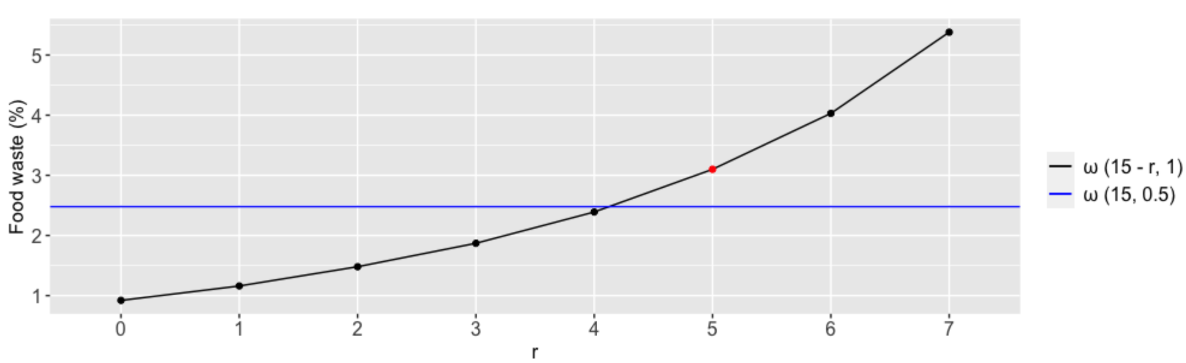


Figure 2: Illustration of the derivation of $r(m, f)$.

waste, i.e., $r(15, 0.5) = 5$. Hence, under setting $E(15, 0.5)$, an online retailer that incorporates a throw-out policy $r \geq 5$ is significantly more wasteful than an offline retailer.

To test statistical significance, the paired sample t-test is performed (using the same seed for the offline and online setting). As a significance level, $\alpha = 0.05$ is implemented. The full description of the paired sample t-test can be found in Appendix A. For a given setting $E(m, f)$, the following five-step procedure is applied to compute $r(m, f)$:

1. Determine $\omega(m, f)$.
2. Set $r = 0$.
3. Determine $\omega(m - r, 1)$.
4. If $\omega(m - r, 1) > \omega(m, f)$, determine the p-value of the paired sample t-test for equal means $\Big(H_0 : \omega(m - r, 1) = \omega(m, f)\Big)$ and go to Step 5.
   Else if $r = m - 1$, stop, $r(m, f)$ does not exist.
   Else, do $r = r + 1$ and go to Step 3.
5. If the p-value ≤ 0.05, set $r(m, f) = r$ and stop.
   Else if $r = r(m + 1, f)$, replicate the experiments in the paired sample t-test until the p-value ≤ 0.05, set $r(m, f) = r$ and stop.
   Else if $r = m - 1$, stop, $r(m, f)$ does not exist.
   Else, do $r = r + 1$ and go to Step 3.

Note that by definition $0 \leq r \leq m - 1$. Hence, if $r = m - 1$ does not lead to a statistically significant increase in food waste, we conclude that $r(m, f)$ does not exist. Also note that the expected food waste is monotonically decreasing in m (by Theorem 4.6), i.e., $\omega(m + 1, f) \leq \omega(m, f)$. Therefore, $r(m + 1, f)$ exists if $r(m, f)$ exists for all $m \in \mathbb{N}$. For instance, the throw-out policy $r(m + 1, f) = r(m, f) + 1$ is always a (feasible) wasteful policy.

Furthermore, in case $r = r(m + 1, f)$, we extend the sample in the paired sample t-test until throw-out policy r is statistically more wasteful (see Step 5). The sample is extended by additional estimates for the food waste as described in the simulation optimization procedure. This step is added to ensure that $r(m + 1, f) \geq r(m, f)$ for all $m \in \mathbb{N}$.

5.2.2 Online wasteful shelf life range

If the fraction offline FIFO demand f and online throw-out policy r are known, we can examine the online wasteful shelf life range. That is, the range in terms of m that is (statistically significant) more wasteful when sold online.

Let $\Phi(f, r)$ denote the online wasteful shelf life range under f and r, which is derived as follows

$$\Phi(f, r) = \Big\{ m \in \mathbb{N} \;\Big|\; r \geq r(m, f) \Big\}. \tag{57}$$

This equation can be interpreted as follows. $r(m, f)$ is the minimum online throw-out date that leads to a statistically significant increase in food waste. Therefore, under f, the online sale of a perishable with a shelf life m is more wasteful if an online throw-out policy $r \geq r(m, f)$ is implemented.

The derivation of $r(m, f)$ ensures that $r(m+1, f) \geq r(m, f)$ for all $m \in \mathbb{N}$. Thus, if $m + 1 \in \Phi(f, r)$ $\big($i.e., $r \geq r(m+1, f)\big)$ and $r(m, f)$ exists, then $r \geq r(m, f)$ and $m \in \Phi(f, r)$. Therefore, the online wasteful shelf life range is an interval, i.e., $\Phi(f, r) = \big[l, u\big]$ in which l and u, respectively, are the smallest and largest (integer) value of the shelf life that is more wasteful when sold online.

Furthermore, the wastefulness of online sales is monotonically increasing in r (by Theorem 4.6), and therefore, the online wasteful shelf life range is a nested interval

$$\Phi(f, r) \subseteq \Phi(f, r+1). \tag{58}$$

6 Results

This section presents the main findings. Firstly, we describe the experiment design and define several scenarios to perform our analysis. Secondly, we analyze the wastefulness of online sales under the base-stock policy and the EWA policy.

6.1 Setup

6.1.1 Experiment design

To compare food waste in an offline and online setting, we simulate a wide range of perishable inventory systems. By combining all parameter values in Table 2, in total 30,000 experiments can be designed (or 150 settings for $E(m, f)$ each containing 200 experiments). Parameter values are based on Haijema and Minner (2016); however, adjusted for this research. We exclude the cost components as a fill rate constraint is considered instead. A fill rate between 90% and 98% is usually acceptable for perishables (Broekmeulen and van Donselaar, 2009). Therefore, we add $\beta = 0.9, 0.92, 0.95, 0.98$ to the experiment design. Furthermore, van Donselaar et al. (2006) characterize perishables as products with a shelf life smaller than or equal to 30 days. Hence, we consider $1 \leq m \leq 30$.

Table 2: List of the simulation experiments considered.

Parameter	Values
L	1, 2
μ	1, 2, 4, 6, 10
$cvr = \sqrt{\frac{\sigma^2}{\mu}}$	0.5, 0.7, 1, 1.5, 2
β	0.90, 0.92, 0.95, 0.98
m	1, . . . , 30
f	0, 0.2, 0.5, 0.8, 1

For each experiment, we fit two discrete demand distributions (one for FIFO demand and one for LIFO demand). For this, the fitting procedure of Adan et al. (1995) is used in which a discrete distribution is fitted on the first two moments. Based on the variable $y = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu}$, one out of four discrete distributions is selected. The selected discrete distribution has the same mean and standard deviation as the continuous distribution in the experiment design. For instance, if $y = 0$, the discrete distribution is a Poisson distribution with rate parameter $\lambda = \mu$. The distributions included in the fitting procedure are the Poisson, binomial, negative binomial, and geometric distribution.

Following the notation of Haijema and Minner (2016), FIFO demand is fitted on mean $\mu(\text{FIFO}) = f \cdot \mu$, and standard deviation $\sigma(\text{FIFO}) = cvr\sqrt{f \cdot \mu}$. LIFO demand is fitted on mean $\mu(\text{LIFO}) = (1 - f) \cdot \mu$ and standard deviation $\sigma(\text{LIFO}) = cvr\sqrt{(1-f)\cdot\mu}$. Adan et al. (1995) show that there does not exist a discrete demand distribution if $y < -1$. For 2,400 experiments, the combination of f, μ, and cvr results in $y < -1$. Therefore, we remove these experiments.

6.1.2 Scenarios

We define several scenarios, which are listed in Table 3. Each scenario incorporates a (sub)set of the considered simulation experiments (see Table 2). Scenario A incorporates the full experiment design, i.e., the aggregate of all products. We introduce Scenario B (high demand, low variance-perishables) and Scenario C (low demand, high variance-perishables) to analyze the wastefulness of different product categories.

Scenario B and C are the extremes cases in which demand and variance are noticeably high/low. In addition, Scenario D is used to examine the individual impact of the mean and variance of the demand on online sales' wastefulness. In this scenario, we incorporate one-to-one FIFO/LIFO demand for offline sales ($f = 0.5$) as a setting between the worst-case and best-case scenarios. For $f = 0.5$, $\mu = 1$, and cvr 0.5/0.7 there does not exist a discrete demand distribution (i.e., the combination of f, μ, and cvr results in $y = -1$). Hence, to make a fair comparison, $\mu = 1$ is removed in Scenario D.

Moreover, we examine the impact of a less restrictive fill rate on online sales' food waste by analyzing Scenario $E - H$. For the offline setting, we maintain a constant fill rate constraint of 98%. For the online setting, however, we vary the fill rate between 98-90%.

Table 3: List of the considered scenarios.
The table displays the scenarios considered. For each parameter in the experiment design, the (sub)set of the considered parameter values is listed.

Scenario	L	μ	cvr	β (offline)	β (online)	m	f
A	all	all	all	all	all	all	all
B	all	10	0.5	all	all	all	all
C	all	1	2	all	all	all	all
D	all	2,4,6,10	all	all	all	all	0.5
E	all	all	all	0.98	0.98	all	all
F	all	all	all	0.98	0.95	all	all
G	all	all	all	0.98	0.92	all	all
H	all	all	all	0.98	0.90	all	all

6.2 Base-stock policy

6.2.1 Minimum throw-out date

Figure 3 shows the results for the BSP under Scenario A. Setting $E(m, f)$ is defined by a product shelf life m, and a fraction offline FIFO demand f. For each setting, the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste is plotted. By assuming f, online retailers gain insights into the wastefulness of their throw-out policies for all perishables with a shelf life of $2 \leq m \leq 30$. For instance, $r(20, 0.5) = 7$. Hence, under one-to-one offline FIFO/LIFO demand, online retailers can discard a product with a shelf life of 20 days up to 6 days before it expires without being more wasteful than an offline retailer.

Note that $0 \leq r \leq m - 1$, and therefore, $r = 0$ is the only feasible throw-out policy under $m = 1$. Furthermore, under $m = 1$, there is no difference between FIFO and LIFO inventory depletion as all inventory is of the same age, i.e., the food waste is equal for all values of f. Hence, offline retailing and online retailing (under $r = 0$) is equally wasteful for products with a shelf life of one day. Moreover, note that $r(m, f)$ possibly does not exist. For instance, $r(2, 0)$ (the corresponding red dot is missing in Figure 3). Hence, under full offline LIFO inventory depletion and a product with $m = 2$, there is no online throw-out policy significantly more wasteful.

The impact of throwing out food is visible for small values of m. Relatively small throw-out policies of 1-3 days increase the food waste for daily fresh (m $\leq$ 7). Furthermore, online retailing seems more wasteful for larger values of f. Online retailers can easily maintain efficient inventory control as expiration dates are

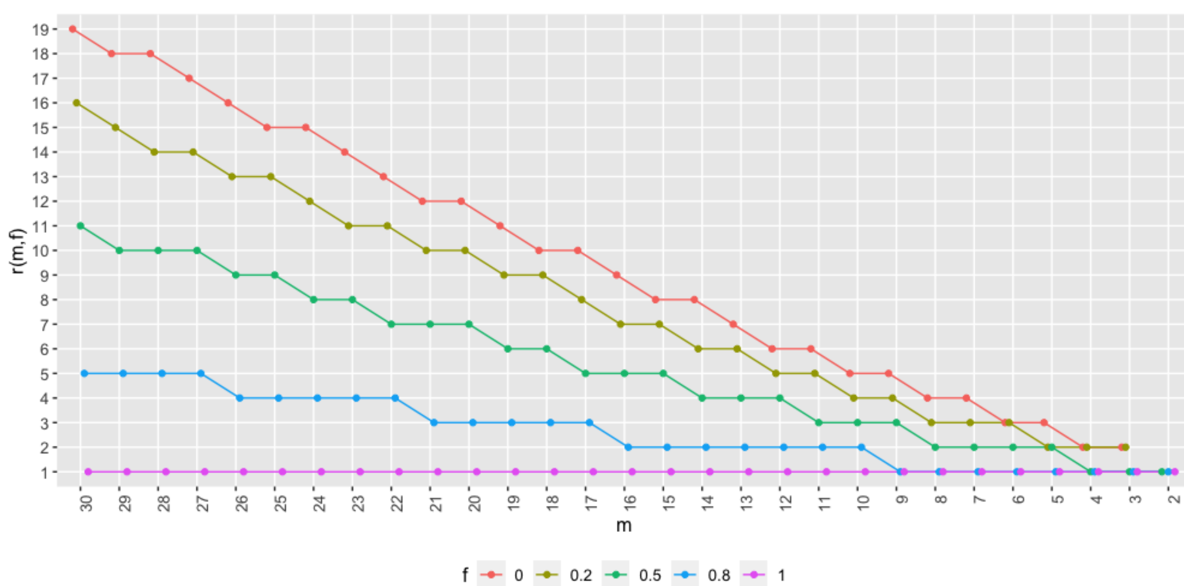


Figure 3: Results for $r(m, f)$ for the BSP under Scenario A.

The figure presents results for the BSP under Scenario A. For each setting $E(m, f)$, the figure plots the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste.

not observable to customers. Therefore, the total demand is FIFO demand, which can be a considerable advantage. However, if the fraction offline FIFO demand increases (for instance, by better shelf management and more discounting close to the expiration date), the advantage of online efficiency decreases. As a result, online throw-out policies are relatively more wasteful.

Throw-out policies are far less wasteful for large m and small f. Consider, for instance, the minimum throw-out date $r(30, 0) = 19$. Hence, under offline LIFO inventory depletion, an online retailer can discard a product with a shelf life of 30 days up to 18 days before it expires without increasing the food waste.

6.2.2 Online wasteful shelf life range

By assuming an offline fraction FIFO demand f and online throw-out policy r, we examine the shelf life range $\Phi\big(f, r\big)$ that is more wasteful when sold online. The online wasteful shelf life range for the BSP under Scenario A is displayed in Figure 4. We vary the online throw-out policy r between one and seven as it is assumed that a throw-out policy of seven days is sufficient to ensure a high level of customer satisfaction. In the worst-case scenario for online retailers, we find $\Phi\big(1, 1\big) = \big[2, 30\big]$. Hence, under offline FIFO demand, an online throw-out policy of just one day makes the online sale of all perishables more wasteful. For small f, the wasteful shelf life range is considerably tighter. In the best-case scenario (i.e., $f = 0$), we find $\Phi\big(0, 2\big) = \big[3, 4\big]$, which increases to $\Phi\big(0, 7\big) = \big[3, 13\big]$ under a relatively large throw-out policy of seven days.

Therefore, a correct assumption on offline retailers' inventory management is crucial to assess online sales' wastefulness correctly. If offline retailers actively try to maintain FIFO inventory depletion (i.e., f is large), the online wasteful shelf life range is wide. Thus, online sales are more wasteful, even under a relatively moderate throw-out policy. On the other hand, if offline retailers are passive (i.e., f is small), online retailers can maintain a high level of customer satisfaction and reduce food waste. However, even in the best-case scenario, the online sale of perishables with $m \leq 13$ is more wasteful if customer satisfaction is highly valued (i.e., $r = 7$).

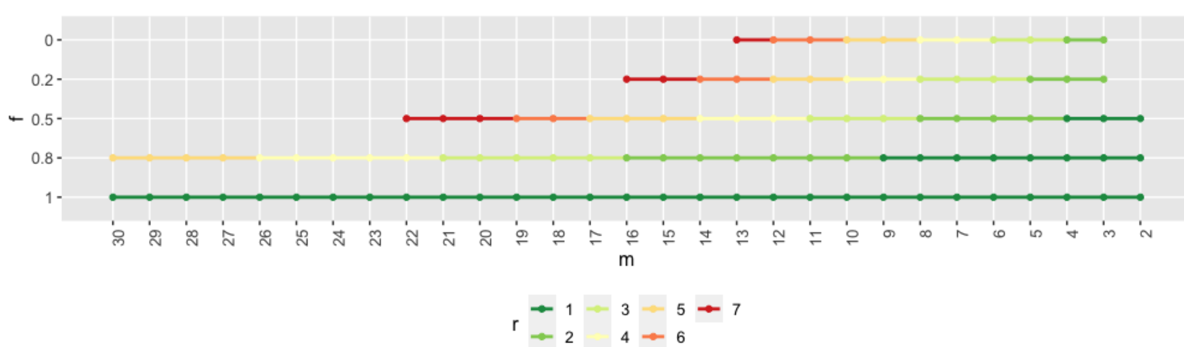


Figure 4: Results for $\Phi\big(f, r\big)$ for the BSP under Scenario A.

The figure presents results for the BSP under scenario A. For each offline fraction FIFO demand f, the figure plots the shelf life range $\Phi\big(f, r\big)$ that is more wasteful when sold online under an online throw-out policy r.

6.2.3 Wastefulness of different product categories

We examine the impact of throw-out policies on different product categories by analyzing Scenario B and Scenario C; the results are displayed in Figure 6. Under Scenario B, there is a considerable advantage for online retailers. Inventory control efficiency (i.e., FIFO inventory depletion) is highly valuable for the high-demand category (i.e., $\mu = 10, cvr = 0.5$) and results in a negligible food waste for $3 \leq m \leq 30$ (see Figure 5). The food waste under LIFO inventory depletion, on the other hand, is strictly decreasing in the shelf life. Consequently, perishables in the high-demand category can always be sold online under a throw-out policy $r = m - 3$ without increasing the food waste. With other words, the online throw-out date that increases food waste $r(m, f) \geq m - 2$, independent of f (see Panel A of Figure 6). Hence, the results under Scenario B are similar for all considered f.

Under Scenario C, the advantage of inventory control efficiency is less significant. We find that the minimum throw-out date under Scenario C is (always) lower than the minimum throw-out date under Scenario A (i.e., the aggregate of all products). Hence, perishables in the low-demand category (i.e., $\mu = 1, cvr = 2$) are more wasteful when sold online.

Furthermore, we find considerable differences between the online wasteful shelf life range under Scenario B and Scenario C. For instance, $\Phi(0, 7) = [3, 8]$ under Scenario B, and $\Phi(0, 7) = [3, 17]$ under Scenario C. Hence, under full offline LIFO demand, the online sale of the low-demand category is considerably more wasteful than the online sale of the high-demand category. The high-demand category is only wasteful for daily fresh, while the low-demand category is wasteful for shelf lives up to 17 days. Note also the online wasteful shelf life range $\Phi(1, 7) = [2, 9]$ under Scenario B. Even in the worst-case scenario, perishables in the high-demand category can be sold online extremely well.

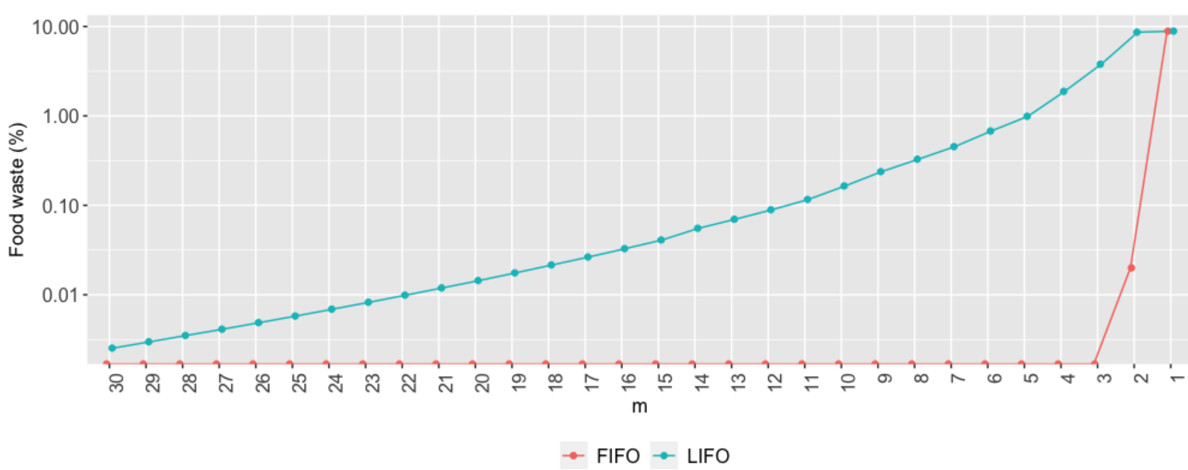
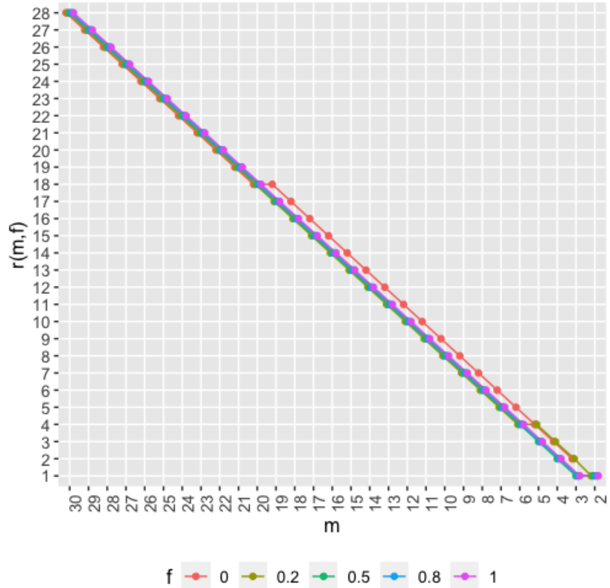
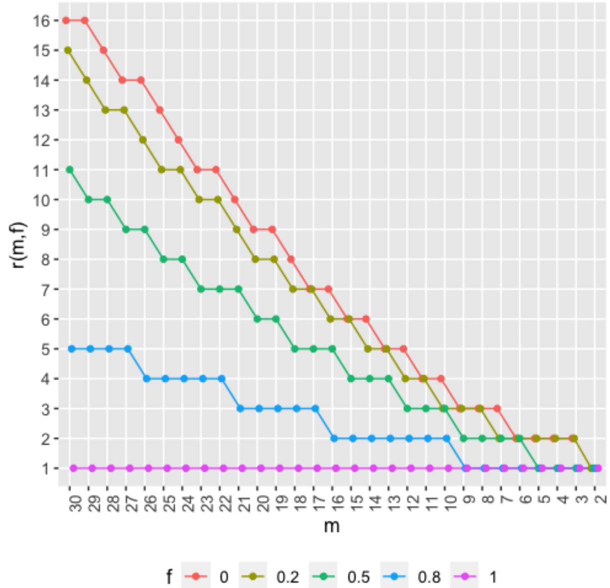


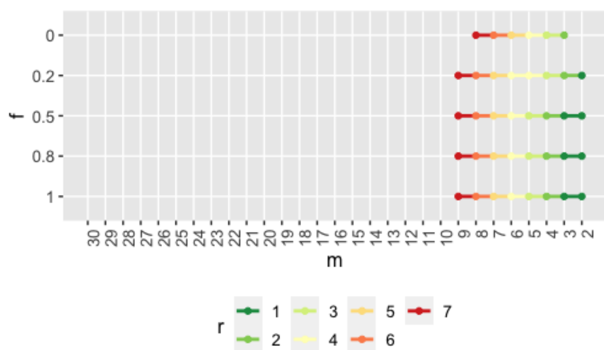
Figure 5: Food waste under FIFO/LIFO inventory depletion, $\mu = 10$, and cvr = 0.5 on a logarithmic scale.



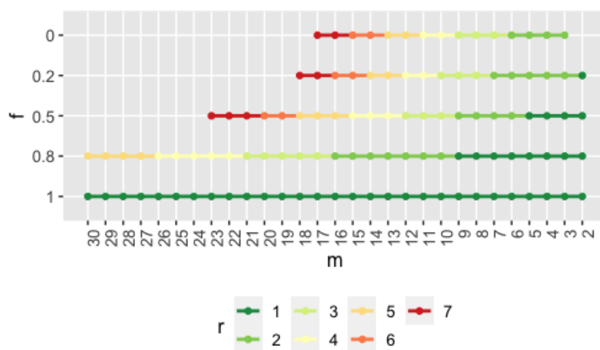
(a) Results for $r(m, f)$ under Scenario B.



(b) Results for $r(m, f)$ under Scenario C.



(c) Results for $\Phi(f, r)$ under Scenario B.



(d) Results for $\Phi(f, r)$ under Scenario C.

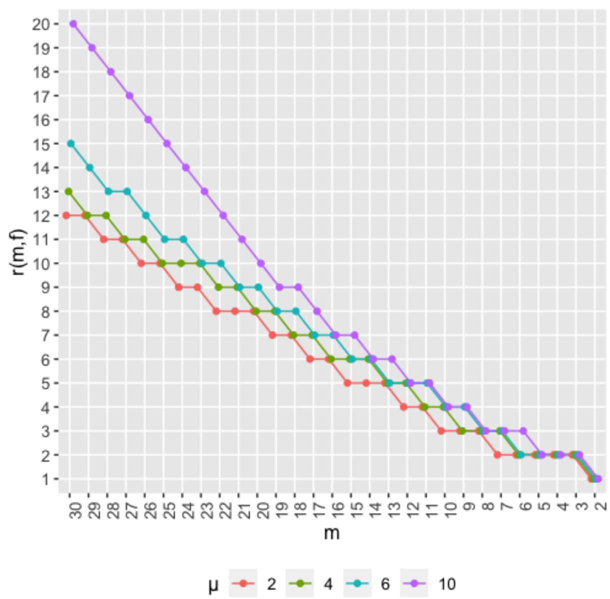
Figure 6: Results for the BSP under Scenario $B - C$.

The figure presents results for the BSP under Scenario B (panel A and C) and Scenario C (panel B and D). Panel A and B plot the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste. Panel C and D plot the shelf life range $\Phi(f, r)$ that is more wasteful when sold online.

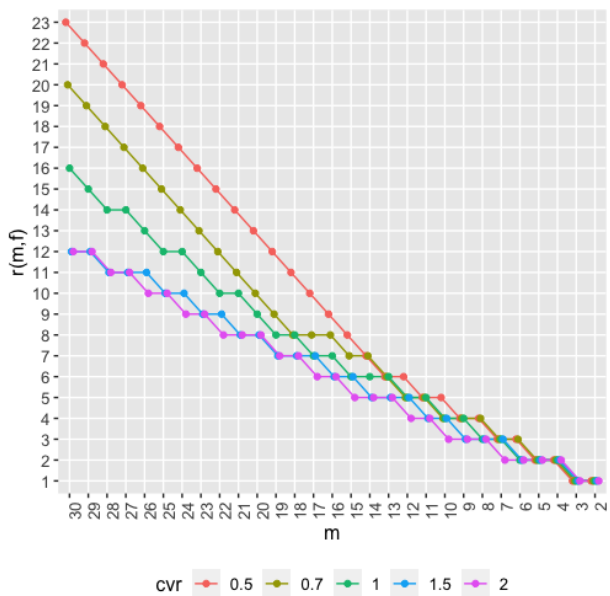
6.2.4 Impact of demand mean and variation

We examine the individual impact of the mean and variation of demand on online sales' wastefulness; the results are displayed in Figure 7. In Scenario D, we incorporate one-to-one FIFO/LIFO demand for offline sales ($f = 0.5$) as a setting between the best-case and the worst-case scenarios. Panel A and C group the experiments by the mean demand (Scenario D^1). Panel B and D group the experiments by the coefficient of variation (Scenario D^2).

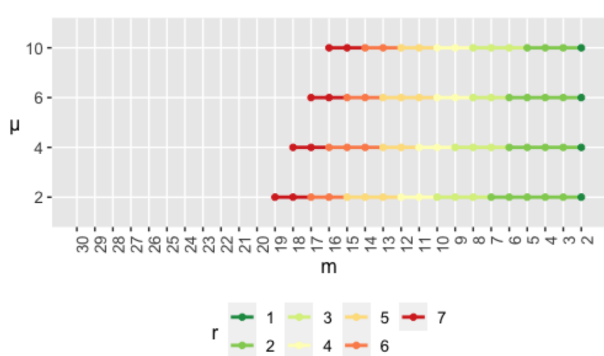
Under Scenario B, we find a substantial advantage for the online sale of perishables in the high-demand category (i.e., $\mu = 10$, cvr $= 0.5$). However, Scenario D^1 does not exhibit the same results. For all perishables with $m \leq 20$, we find that the minimum throw-out date is hardly affected by the mean demand. For instance, under Scenario D^1 and $\mu = 2$ we find $r(20, 0.5) = 7$, and under Scenario D^1 and $\mu = 10$ we find $r(20, 0.5) = 9$. Consequently, under Scenario D^1, the online wasteful shelf life range is hardly affected by μ. Note also the



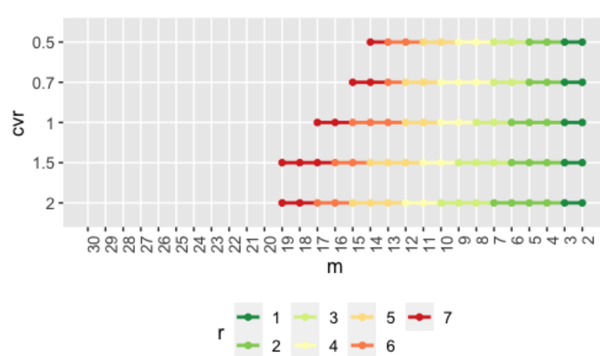
(a) Results for $r(m, f)$ under Scenario D^1.



(b) Results for $r(m, f)$ under Scenario D^2.



(c) Results for $\Phi(f, r)$ under Scenario D^1.



(d) Results for $\Phi(f, r)$ under Scenario D^2.

Figure 7: Results for the BSP under Scenario D.

The figure presents results for the BSP under Scenario D. Panel A and C group the experiments by the mean demand (Scenario D^1). Panel B and D group the experiments by the coefficient of variation (Scenario D^2). Furthermore, Panel A and B plot the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste. Panel C and D plot the shelf life range $\Phi(f, r)$ that is more wasteful when sold online.

difference in the wasteful shelf life range under Scenario B and Scenario D^1. For instance, $\Phi\big(7, 0.5\big) = \big[2, 9\big]$ under Scenario B, and $\Phi\big(7, 0.5\big) = \big[2, 16\big]$ under Scenario D^1 and $\mu = 10$. It seems that high demand alone does not decrease online sales' wastefulness, but rather the combination of high demand and low demand variation. However, perishables with $m \geq 20$ in high demand (i.e., $\mu = 10$) can be sold online fairly well. Compare, for instance, $r(30, 0.5) = 12$ under Scenario D^1 and $\mu = 2$, with $r(30, 0.5) = 20$ under Scenario D^1 and $\mu = 10$.

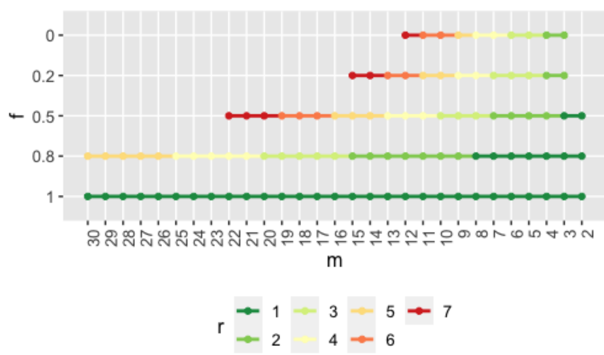
Similar to our findings under Scenario D^1, under Scenario D^2 we find minor differences in the minimum throw-out date for all perishables with $m \leq 14$. Therefore, the online wasteful shelf life range is only moderately affected by the cvr. Low demand variation (i.e., cvr = 0.5/0.7) does seem beneficial for the online sale of perishables with a relatively long shelf life of $m \geq 15$. For instance, under Scenario D^2 and $cvr = 2$ we find $r(30, 0.5) = 12$, and under Scenario D^2 and $cvr = 0.5$ we find $r(30, 0.5) = 23$.

We compare the wasteful shelf life range under Scenario B and Scenario D^2. Under Scenario B we find $\Phi\big(7, 0.5\big) = \big[2, 9\big]$, and under Scenario D^2 and cvr $= 0.5$ we find $\Phi\big(7, 0.5\big) = \big[2, 14\big]$. This difference in wasteful shelf life range again shows the superiority of the high-demand category of Scenario B. Perishables in high demand and under low demand variation are ideally sold in an online environment.

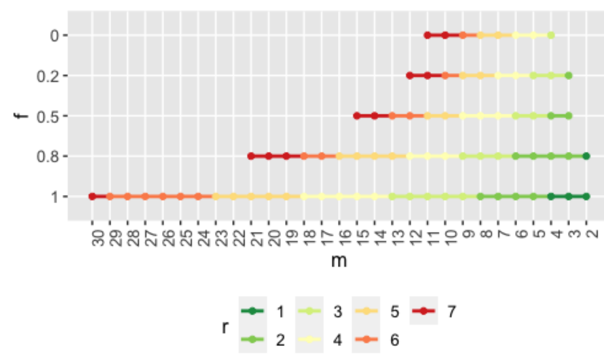
6.2.5 Lowering the online fill rate

We analyze the effect of lowering the fill rate constraint on online sales' wastefulness. For the offline setting, we maintain a constant fill rate of 98%. For the online setting, however, we vary the fill rate between 98-90% in Scenario $E - H$ (see Table 3). The results for the online wasteful shelf life range are displayed in Figure 8; see Appendix B for the results for the minimum throw-out date. Under Scenario E, we find similar results as under Scenario A; this is expected as Scenario E incorporates the same fill rate for online and offline retailing. However, by lowering the fill rate for online sales, we find substantial improvements in the online wasteful shelf life range, especially for larger f. For instance, under Scenario E we find $\Phi\big(1, 1\big) = \big[2, 30\big]$, and under Scenario H we find $\Phi\big(1, 1\big) = \big[2\big]$, which increases to $\Phi\big(1, 7\big) = \big[2, 18\big]$ under a relatively large throw-out policy of seven days. Therefore, by decreasing the fill rate constraint from 98% to 90%, online retailers can sell a considerably wider range of perishables (in terms of m) without increasing the food waste.

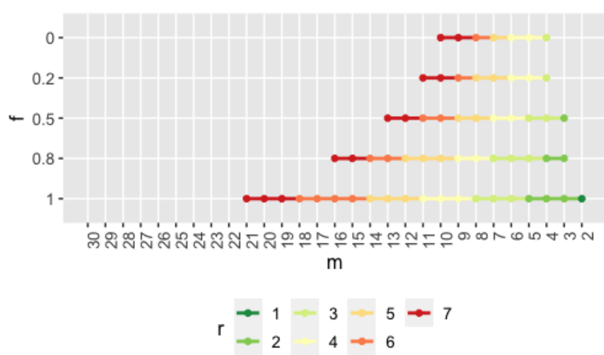
Lowering the fill rate is less effective for smaller f. Compare, for instance, $\Phi\big(0, 7\big) = \big[3, 12\big]$ under Scenario E with $\Phi\big(0, 7\big) = \big[4, 9\big]$ under Scenario H. Hence, lowering the fill rate constraint is mainly effective if offline retailers actively try to maintain FIFO inventory depletion (for instance, by discounting products close to the expiration date).



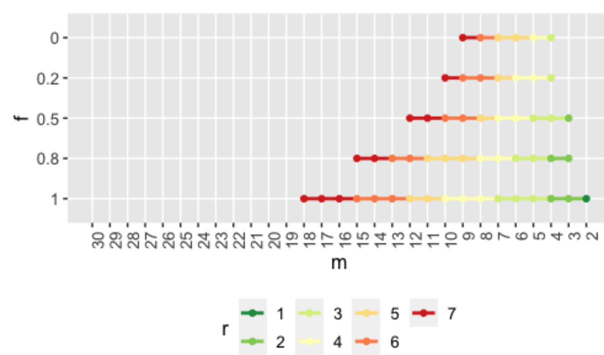
(a) Scenario E.



(b) Scenario F.



(c) Scenario G.



(d) Scenario H.

Figure 8: Results for $\Phi(f, r)$ for the BSP under Scenario $E - H$.

The figure presents results for the BSP under Scenario $E - H$. For each offline fraction FIFO demand f, the figure plots the shelf life range $\Phi(f, r)$ that is more wasteful when sold online under an online throw-out policy r.

6.3 EWA policy

6.3.1 Performance comparison with the BSP

We compare the food waste under the EWA policy with the food waste under the BSP. Let $\omega^i(m, f)$ denote the average food waste in setting $E(m, f)$ under policy i. The food waste reduction by the EWA policy, compared to the BSP, is measured by

$$\omega_\Delta(m, f) = 100 \cdot \frac{\omega^{\text{BSP}}(m, f) - \omega^{\text{EWA}}(m, f)}{\omega^{\text{BSP}}(m, f)}. \tag{59}$$

The results for the food waste reduction $\omega_\Delta(m, f)$ under Scenario A are displayed in Figure 9. We find that the EWA policy outperforms the BSP under nearly all settings. For $m \geq 10$, there is a relatively small decrease in food waste of 0-10%. However, for small m, the food waste reduction by the EWA policy is considerably larger. Here, the impact of FIFO/LIFO demand is also clearly visible. The EWA policy is mainly effective for small f (i.e., LIFO inventory depletion); this is disadvantageous for online retailers following FIFO inventory depletion. Offline retailers implementing the EWA policy can reduce their food waste by up to 50% (under LIFO demand). Contrarily, the maximum food waste reduction in an online setting (i.e., under FIFO demand) is around 10%.

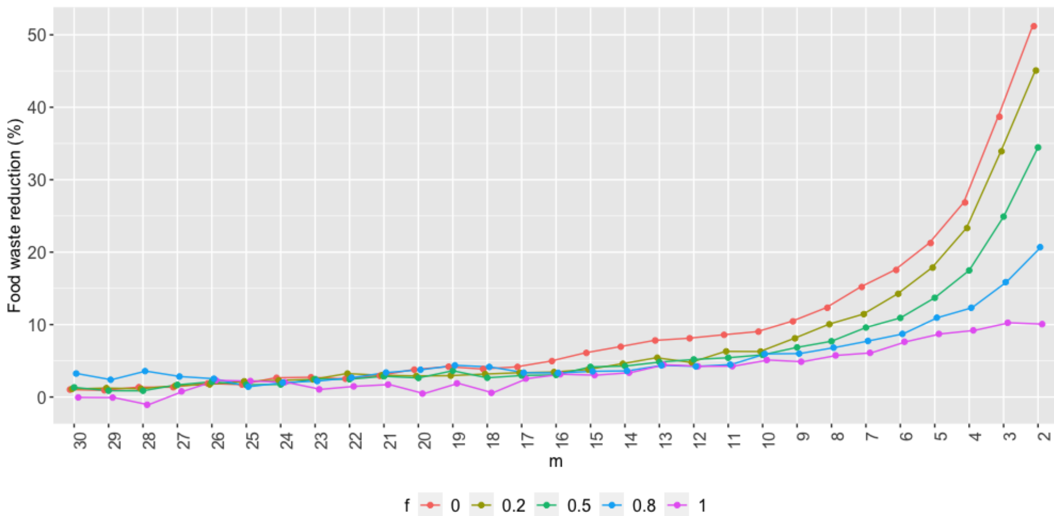


Figure 9: Food waste reduction by the EWA policy.

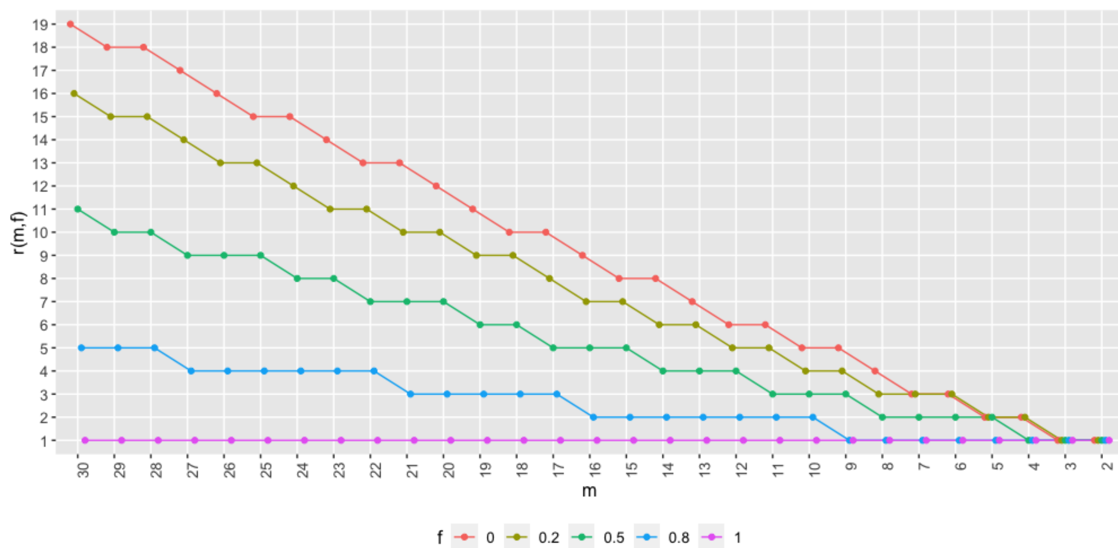
The figure displays results under Scenario A. For each setting $E(m, f)$, the figure plots the food waste reduction by the EWA policy $\omega_\Delta(m, f)$, compared to the BSP.

6.3.2 Wastefulness comparison with the BSP

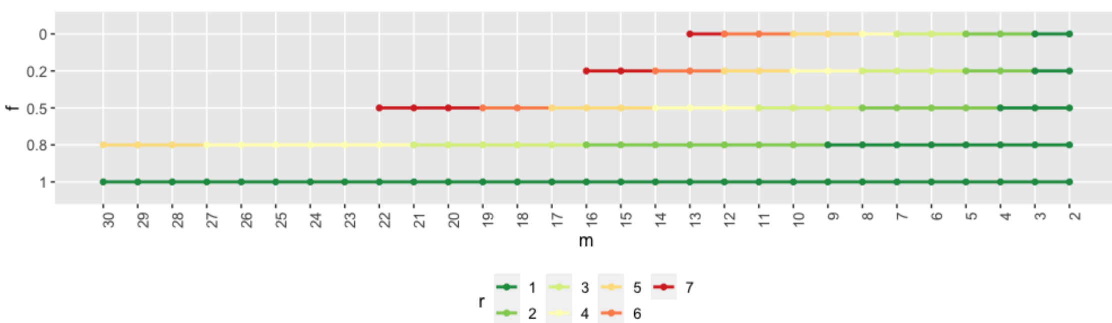
The results for the EWA policy under Scenario A are displayed in Figure 10. In general, we find that the results for the minimum throw-out date under the EWA policy and the BSP are similar. Hence, the online wasteful shelf life range is also similar. A notable exception is the wider wasteful range for the EWA policy under $f = 0.2$ and $f = 0$ as the online sale of daily fresh ($m \leq 7$) is more wasteful. The EWA policy's food waste reduction is largest for the offline sale of daily fresh, especially under full LIFO inventory depletion (see Figure 9). As a result, under small f, online throw-out policies for daily fresh are relatively more wasteful under the EWA policy.

The results for the EWA policy under Scenario B and Scenario C are displayed in Appendix C. Our findings are similar to the results under the BSP. However, for small f and small m, we find that online throw-out policies are more wasteful under the EWA policy. This is in line with our findings under Scenario A. The increase in online sales' wastefulness is most considerable under Scenario C. The low-demand category, a wasteful product category under the BSP, is even more wasteful when the more sophisticated EWA policy is implemented.

Appendix D displays the results for the EWA policy under Scenario $E - H$. The results are similar to our findings for the BSP. However, for the EWA policy, we find that the online wasteful shelf life range under



(a) Results for $r(m, f)$.



(b) Results for $\Phi(f, r)$.

Figure 10: Results for the EWA policy under Scenario A.

The figure presents results for the EWA under Scenario A. Panel A plots the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste. Panel B plots the shelf life range $\Phi(f, r)$ that is more wasteful when sold online.

Scenario $F-H$ is wider for $f \leq 0.8$. Lowering the fill rate is a less effective method to reduce the wastefulness for daily fresh. Notably, lowering the fill rate is also less effective if offline demand is mainly FIFO demand (i.e., $f = 0.8$).

To summarize, we find that the online sale of perishables with a relatively long shelf life is equally wasteful under the BSP and the EWA policy. However, for daily fresh, online throw-out policies under the EWA policy are relatively more wasteful. This is especially true for small f (i.e., offline LIFO demand) as the EWA policy's food waste reduction is largest under these settings.

7 Conclusions and managerial insights

In the coming years, the food supply chain faces major challenges to improve sustainability and reduce food waste. A considerable amount of the total food waste can be assigned to the decisions and actions of retailers. Therefore, efficient inventory management of food retailers is crucial to reduce global food waste.

At the same time, we observe the advent of online food retailers that might manage inventory differently. In online retailing, consumers do not observe expiration dates. Therefore, to maintain a high level of customer satisfaction, online retailers might only send products that are consumable for at least a week and discard food that is still valid for six more days.

We prove that the expected food waste under the optimal order policy is monotonically decreasing in the product shelf life. Online retailers inherently decrease the shelf life if they throw out food before the expiration date. Therefore, online sales are more wasteful if online and offline retailers follow the same inventory depletion rule (e.g., FIFO or LIFO). In practice, however, the inventory depletion of online and offline retailers differs. In an offline setting, consumers often buy the freshest products, as this gives the highest level of utility. Consequently, offline retailers often follow LIFO inventory depletion (or a combination of FIFO and LIFO). On the other hand, online retailers can usually send the oldest products in stock and thus maintain FIFO inventory depletion. FIFO inventory depletion is a considerable advantage that could offset the wastefulness of an online throw-out policy.

We examine the impact on food waste by moving from offline to online retail. A simplified inventory control system is investigated to achieve this, focusing on practical applications for real-world food retailers. The food waste in an offline and online environment is estimated and compared by simulation under an extensive set of experiments. By varying the offline inventory depletion rule, we analyze different management styles. If retailers display mainly older inventory on the shelf and discount products close to the expiration date (i.e., active management), they follow mainly FIFO inventory depletion. On the other hand, if no measures are taken to prevent customers from buying the freshest products (i.e., passive management), offline retailers follow mainly LIFO inventory depletion.

Online sales' wastefulness is visible for perishables with a short shelf life. Relatively small throw-out policies of 1-3 days increase the food waste for daily fresh (shelf life ≤ 7). Moreover, the online sale of these perishables is wasteful under both passive and active offline management. Therefore, to reduce food waste, online retailers might avoid the sale of daily fresh.

On the other hand, the online sale of perishables with a relatively long shelf is far less wasteful. One main insight here is the impact of the offline management style on online sales' wastefulness. Active offline management decreases the advantage of online inventory efficiency, and thus, makes online throw-out policies relatively more wasteful. Therefore, if offline retailers actively manage their inventory, the online sale of all perishables with a shelf life ≤ 30 is wasteful, even under a relatively moderate throw-out policy. Contrarily, if offline retailers are passive, online retailers can maintain a high customer satisfaction level and reduce food waste. Under full offline LIFO inventory depletion, an online retailer can discard a perishable with a shelf life of 30 days up to 18 days before it expires without increasing the food waste. Therefore, a correct assumption on offline retailers' inventory management is crucial to assess online sales' wastefulness correctly; this is an area for future research.

Furthermore, our research makes three suggestions for online retailers to reduce their food waste. Firstly, the online sale of perishables in high demand and under low demand variation is preferred. Online inventory control efficiency is highly valuable for this product category. Therefore, online retailers can sell perishables with a relatively short shelf life (even some daily fresh) without increasing food waste. The wastefulness of these online sales is also hardly affected by the offline management style. Contrarily, we find that perishables in low demand and under high demand variation are quite wasteful when sold online. Therefore, by correctly adjusting the online product range, the sustainability of online retailing improves significantly.

Secondly, lowering the online fill rate constraint is an effective method to reduce online sales' wastefulness. By lowering the fill rate from 98% to 90%, a considerably wider range of perishables can be sold online without increasing the food waste. A lower fill rate for online sales might not be an issue. In an online setting, it is more convenient to provide substitute goods, i.e., goods that a consumer perceives as similar or comparable. Furthermore, it often takes a couple of days to deliver a customer's order. Hence, an online retailer might satisfy some of these orders with product arrivals of the next days.

By considering the EWA policy, we examine a more sophisticated policy that considers the inventory's full age distribution. Although the EWA policy outperforms the BSP for online sales, it is mainly effective for offline sales of daily fresh (shelf life ≤ 7). As a result, online throw-out policies for daily fresh are relatively more wasteful under the EWA policy. Hence, our final suggestion; more work should be devoted to optimizing policies suited for online retailing. Online retailers follow FIFO inventory depletion, and it is shown that the online sale of daily fresh is usually not feasible. Therefore, policies that perform well under FIFO inventory depletion and the sale of perishables with a relatively long shelf life could enhance inventory control in an online environment.

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A Paired sample t-test

To test the null hypothesis of equal means for sample j and k, we use the paired sample t-test. Let x_{ij} (x_{ik}) denote the value for observation i of sample j (sample k). The difference d between each pair of values is derived as follows

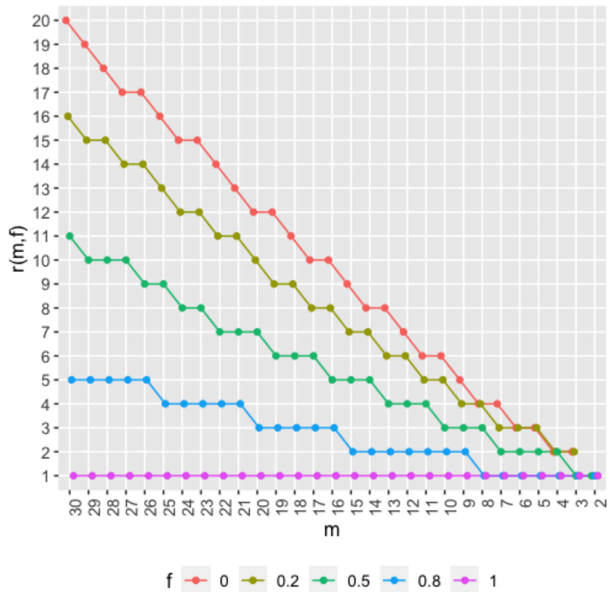
$$d = \sum_{i=1}^{n} \Big(x_{ij} - x_{ik} \Big), \tag{60}$$

in which n is the sample size. The test statistic is computed as

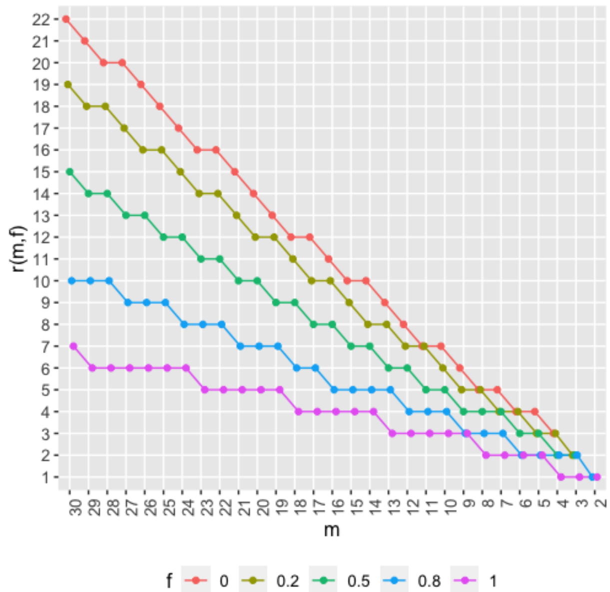
$$t = \frac{m}{\frac{s}{\sqrt{n}}}, \tag{61}$$

in which m and s are the mean and standard deviation of the difference d, respectively. We reject the null hypothesis that the two samples have equal means if $|t| > t_{(1-\frac{\alpha}{2},v)}$. Here, $t_{(1-\frac{\alpha}{2},v)}$ is the critical value of the t-distribution with significance level α and degrees of freedom $v = n - 1$.

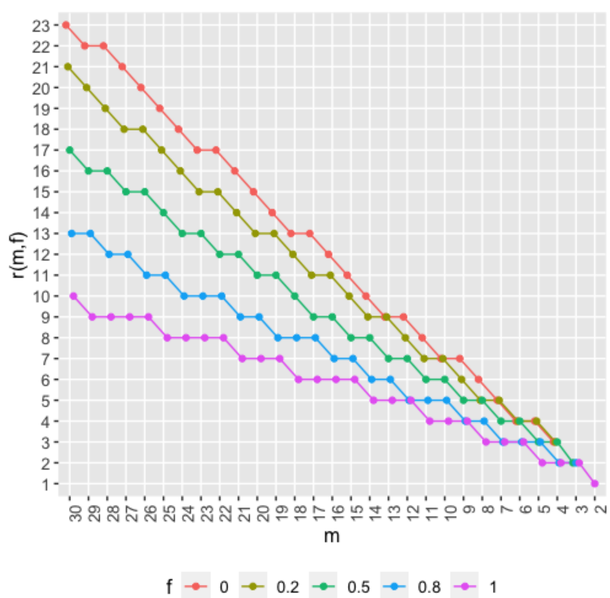
B Results for the BSP under Scenario $E - H$



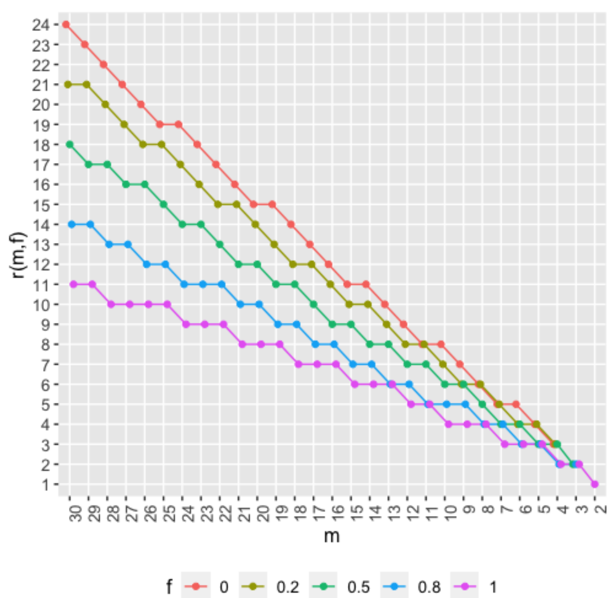
(a) Scenario E.



(b) Scenario F.



(c) Scenario G.

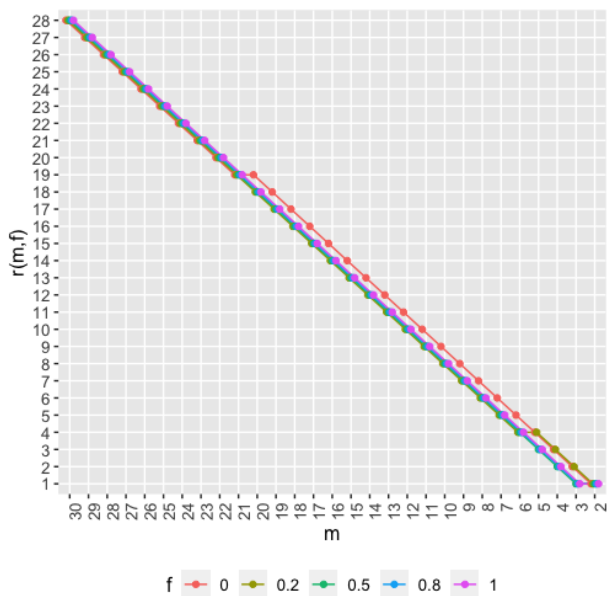


(d) Scenario H.

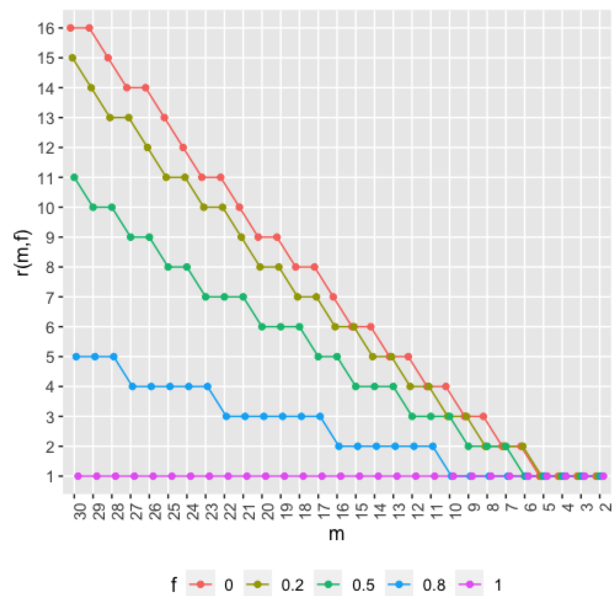
Figure 11: Results for $r(m, f)$ for the BSP under Scenario $E - H$

The figure presents results for the BSP under Scenario $E - H$. For each setting $E(m, f)$, the figure plots the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste.

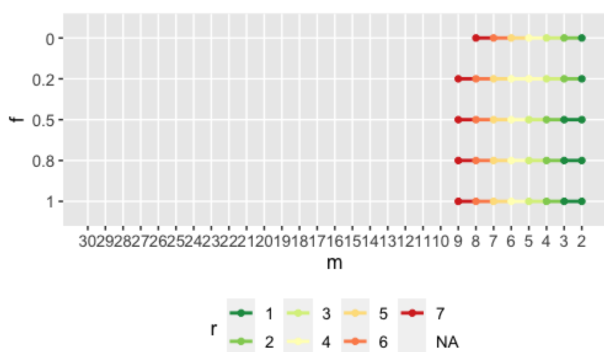
C Results for the EWA policy under Scenario $B - C$



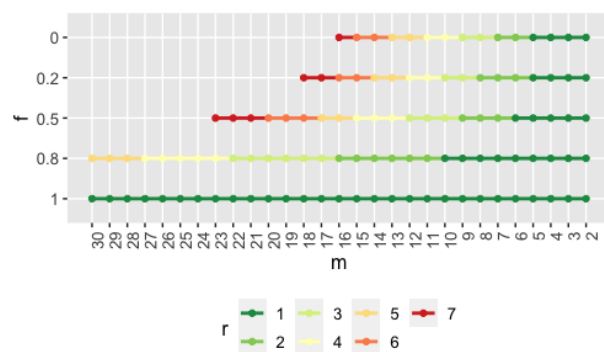
(a) Results for $r(m, f)$ under Scenario B.



(b) Results for $r(m, f)$ under Scenario C.



(c) Results for $\Phi(f, r)$ under Scenario B.

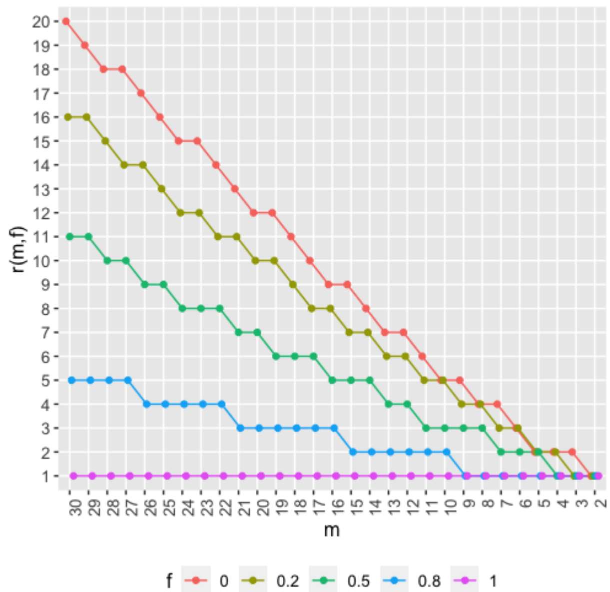


(d) Results for $\Phi(f, r)$ under Scenario C.

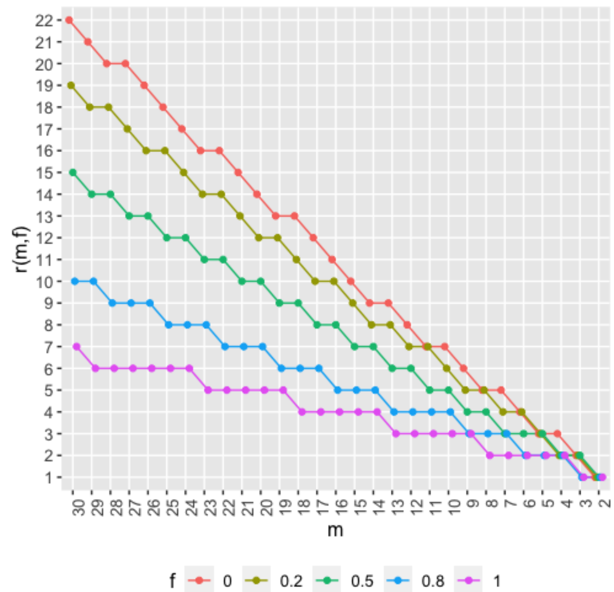
Figure 12: Results for the EWA policy under Scenario $B - C$.

The figure presents results for the EWA policy under Scenario B (panel A and C) and Scenario C (panel B and D). Panel A and B plot the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste. Panel C and D plot the shelf life range $\Phi(f, r)$ that is more wasteful when sold online.

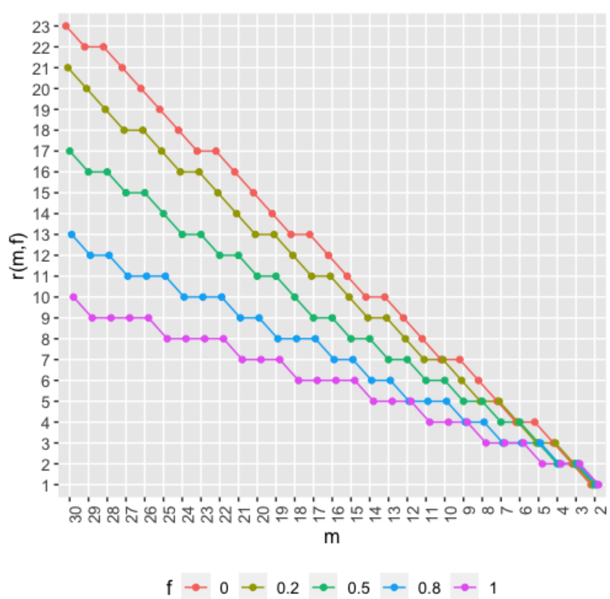
D Results for the EWA policy under Scenario $E - H$



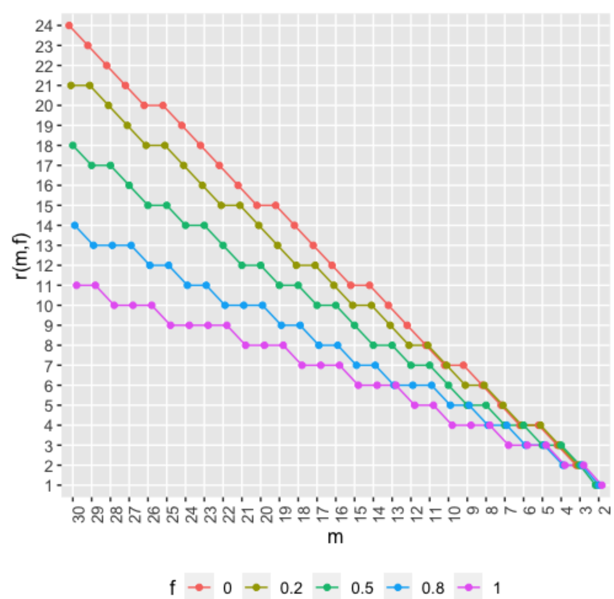
(a) Scenario E.



(b) Scenario F.



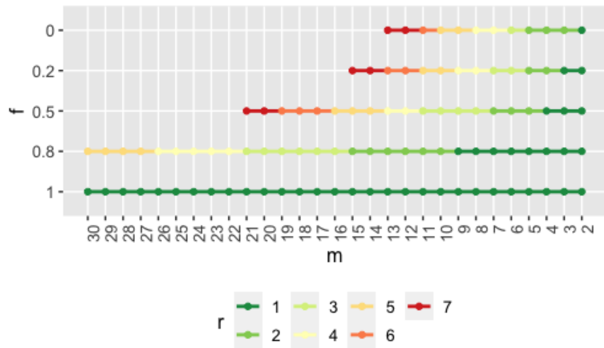
(c) Scenario G.



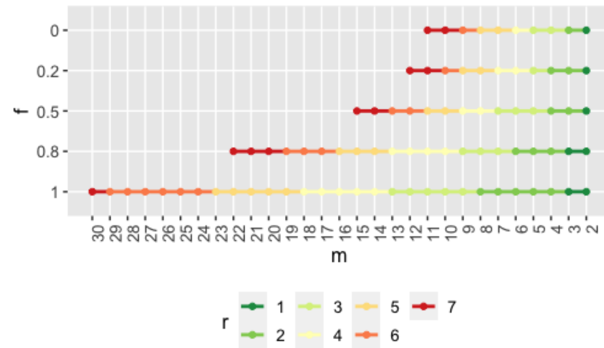
(d) Scenario H.

Figure 13: Results for $r(m, f)$ for the EWA policy under Scenario $E - H$.

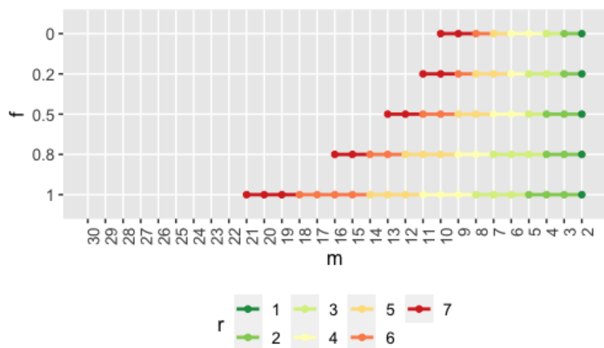
The figure presents results for the EWA policy under Scenario $E-$. For each setting $E(m, f)$, the figure plots the minimum online throw-out date $r(m, f)$ that leads to a (statistically significant) increase in food waste.



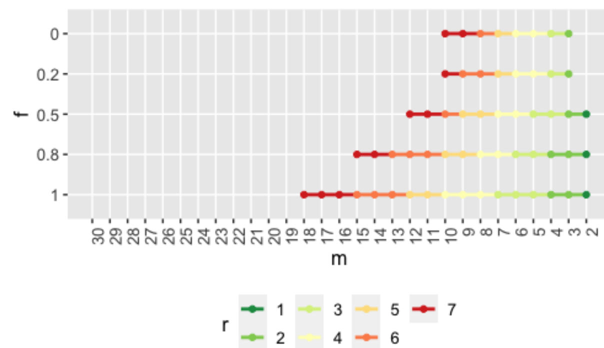
(a) Scenario E.



(b) Scenario F.



(c) Scenario G.



(d) Scenario H.

Figure 14: Results for $\Phi(f, r)$ for the EWA policy under Scenario $E - H$.

The figure presents results for the EWA policy under Scenario $E - H$. For each offline fraction FIFO demand f, the figure plots the shelf life range $\Phi(f, r)$ that is more wasteful when sold online under an online throw-out policy r.