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Forecasting macroeconomic variables for European countries using Markov-Switching Three-Pass Regression Filter

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Abstract

We are interested to learn whether the MS-3PRF methods perform well when we are forecasting series of macro-economic variables for various European countries. In particular, we wish to compare the results to American data forecasts and determine reasons if we observe any discrepancies. The MS-3PRF method splits up in three passes, which consist of a time-series regression in step 1, cross-section regression in step 2 and again a time-series regression and forecasting in step 3. We apply alterations to our weighing method of the estimated regime-specific factor loadings and thus create two versions of MS-3PRF. We also differentiate between a version that includes Markov switches only in the first pass, and a version that includes switching in both the first and third pass. We also compare the forecasting ability of the MS-3PRF procedure to earlier introduced approaches, such as linear 3PRF and PC-LARS and include a benchmark PCA and AR(1) forecasting approach. Overall, we found that the variants on MS-3PRF perform well. These promising results indicate that the new approach is relatively robust. For European data, this was generally the case as well. We are able to recommend MS-3PRF methods above PCA. Specifically, for the majority of variables and forecasting horizons, MSS-3PRF-1 works best.

Contents

1 Introduction

In this paper we attempt to estimate high-dimensional factor models with regime-switching factor loadings. We do so using the Markov-Switching Three-Pass Regression Filter (MS-3PRF) method as described by Guérin et al. (2020). Since they performed an application forecasting economic activity in the U.S., we want contribute by studying the effectiveness of the MS-3PRF approach on empirical forecasting outside the U.S. Therefore we applied the same methods to three new sets of data for the countries Germany, France and the Netherlands. We propose the following research question:

"How well does the MS-3PRF method perform as compared to other factor forecasting models for various U.S. and European macroeconomic data?"

Although the literature on estimating factor models is quite extensive (e.g. Fan et al. (2016) and Lam et al. (2012)), the MS-3PRF method central in this paper has multiple advantages over previous factor models. Firstly, in existing literature it is often assumed that co-movement among variables is constant over time (see Forni et al. (2000)). As we are interested in forecasting macroeconomic (and financial) variables, it is useful to challenge this notion by including a time-varying component to the factor loadings. Secondly, even when previous research included time-varying factor loadings, this time-variation was often modeled after a random-walk or with autoregressive behavior (see Eickmeier et al. (2015)). This makes the model more restricted or biased towards gradual changes, whilst this may not always be appropriate (e.g. in the case of a structural break caused by financial crises). For this reason, it is useful to consider a regime-switching process, where abrupt changes in factor loadings can be modelled more accurately. From an economic standpoint, these regimes could represent either a period of recession of expansion and from a more financial standpoint, we could differentiate between two regimes of bear and bull markets. Additionally, regime switches are not limited to merely two regimes, but can include any number of finite regimes that may help model changes in factor loadings. For example, a third regime may depict when the market is in its short term equilibrium: a situation where supply and demand are exactly equally matched. The chosen interpretation for these regimes follows naturally from the applied data.

An important work by Kelly and Pruitt (2015) forms the basis of the MS-3PRF approach. They introduce an estimator for factor models- the Three-Pass Regression Filter (3PRF), which relies on a series of ordinary least-square (OLS) regressions. This linear 3PRF filter has the ability to forecast using many predictors. It is useful to extend their method for non-linear cases. Thus, very recently Guérin et al. (2020) introduced a method which extended the 3PRF with switching regimes in the factor loadings, called Markov-Switching Three-Pass Regression Filter, the focal point of this paper. The main difference between 3PRF and MS-3PRF is by allowing for Markov regressions in two of the three passes. MS-3PRF is also computationally more efficient compared to other non linear factor models since it only considers univariate Markov-Switching regressions.

We will illustrate the effectiveness of the MS-3PRF approach by comparing it with some frequently used forecasting methods (e.g. PCA, TPCA, PCA-LARS and 3PRF, as well as a factor model where factor loadings are modelled after a random walk) in the forecasting application. For this empirical application we rely on the data-set from McCracken and Ng (2016) for U.S. macroeconomic variables and on various data-sets obtained from FRED and OECD for European macroeconomic variables. This study is relevant for forecasting experts who want a robust and time-efficient method for forecasting high-dimensional data with evidence of time-varying behavior. If this method can be proven to be consistently more accurate than its (occasionally) complex contemporaries this will result in more efficient forecasting and may prevent economic losses and improve economic policy making. For Guérin et al. (2020) the MS-3PRF approach appears to forecast more accurately than some of the alternative (linear) factor models, when considering Monte Carlo simulations, exchange rates and economic activity of the U.S. Our results could serve as additional evidence of the usefulness of the MS-3PRF approach. Furthermore, we are interested in the optimal number of factors we should include for the best forecasting results. If there are any discrepancies between the results for U.S. data and EU data, we would like to investigate the reason behind it as well.

Overall, our results do not seem to support the claim that the MS-3PRF approach result in more accurate forecasts, compared to the other methods. When forecasting for U.S. data, the variants on MS-3PRF outperformed all other methods more than 42% of the time. This is not in accordance with the results from Guérin et al. (2020), as they obtained that out of 40 cases, MS-3PRF approach performed the best in 22 of those cases (55% rate). For us, this number was 27 out of 64 cases. For German data, 26.5 cases out of 56 cases (47% rate) also indicates that the MS-methods are not always the best performing method. For French data, this is 37 out of 56 cases (66%). Although this is more than the threshold of 50%, this number is not conclusive as it is highly dependent on the other methods we consider. It is useful to note that within the different variations on the MS-3PRF model, the MSS-3PRF-1 model performs the best. We may recommend forecasters to use this model above others. The question remains whether this result is robust when we consider different macroeconomic variables or a different number of regimes.

The remainder of this proposal is structured as follows. Section 2 provides a short literature review and Section 3 shows an overview of the data. Section 4 discusses the Markov-Switching Three-Pass Regression Filter model and the empirical application for our research.

2 Literature Review

Firstly, we use the existing literature to briefly set the economic context for this paper. We will discuss the literature on the common factor model, as well as its derived models such as the Markov switching models. Furthermore, we will highlight important works which contributed to the creation of the MS-3PRF model. In particular, the paper by Kelly and Pruitt (2015) will be discussed in greater detail.

2.1 Literature on European macroeconomic activity

Stock and Watson (2016) contributed greatly to the literature on by providing an overview of dynamic factor models (DFMs), their estimation, and their uses in empirical macroeconomics.

They focus on how to extend methods for identifying shocks in structural vector auto-regression (SVAR) to structural DFMs. In this paper we are focused on Germany and France. These countries are closely related through business cycles and structural shocks (Karras, 1994). Karras (1994) investigate the sources of macroeconomic fluctuations in France and Germany since 1960. Aggregate demand and aggregate supply shocks are identified by assuming that in the long-run output and employment are only affected by aggregate supply innovations. The macro-economy in the U.S. and in Europe are correlated. Dolls et al. (2012) see a pattern of more often than not, activity in the U.S. causing an effect first in the U.S. and a latent response in European countries. The shocks will have a higher effect in the country of origin. Automatic stabilizers which are relevant after an economic shock, are stronger in Europe than in the U.S., this is especially the case for Central and Nothern European countries, thus Germany and France. This indicates that the regimes might be less prominent in the data and perhaps implementing Markov switches would be less relevant.

2.2 Literature on factor models

In order to forecast variables, a common way of approach is by using factor models, where a large number of (observed) variables are described in terms of a smaller number of (unobserved) factors. Note that in the context of this paper, the relevant factor models are macroeconomic factor models, where we use observable economic time series as measures of the factors. Generally, these factor models are the most intuitive. The difficulty lies in determining how many and which factors to include. Finance theory or context can provide some insight. As opposed to either statistical or fundamental factor models, these models do not require estimation of the unknown factors (Connor, 1995).

As mentioned in the introduction, a substantial amount of research has been done on high-dimensional factor modelling. Cheng et al. (2016) and Ait-Sahalia and Xiu (2017) are examples of such research. Cheng et al. (2016) also considers time-varying factor loadings by modelling for structural breaks and Ait-Sahalia and Xiu (2017) uses PCA for high-frequency data. The differentiating factor of our research compared to Cheng et al. (2016) lies in the fact that their research only accounts for one structural break, whilst MS-3PRF can include many factor loading regimes. But more importantly, MS-3PRF takes into account the probabilities of all possible regimes and in that sense gives a more smoothed structural break. MS-3PRF only provides the likelihood that a certain regime is occurring, that way the loading will be proportionally (with respect to the probability) move towards its value for that specific regime. In this sense, MS is less extreme and more flexible compared to structural break models.

2.3 Linear 3PRF

Kelly and Pruitt (2015) proposed a new method to estimate factor models using a linear 3PRF estimator, in order to forecast a single time-series using many predictor variables. One key advantage of their approach lies in the fact that it is only required to specify the number of relevant factors driving the forecast target, regardless of the total number of common factors driving the cross section of predictors. The 3PRF is a constrained least squares estimator and reduces to partial least squares as a special case. Simulation studies show that the 3PRF often outperforms alternatives across a variety of factor model specifications. In various empirical applications, they find that the 3PRF is a successful predictor of macroeconomic aggregates and equity market returns, and typically outperforms alternative methods as well.

2.4 Literature on Markov switching models

Camacho et al. (2012) provides a method that incorporates similar Markov regime-switching factor models, but not set in a filtering framework. Their research provides an expansive overview of previous researches on Markov-switching auto-regressive models. They developed distribution theory that is relevant for our research, as they draw inferences from smoothed probabilities. Such probabilities will be discussed later on in the paper. More recently, Barnett et al. (2016) also include a regime-switching dynamic factor model in their research. Due to computational difficulties, their model is only implemented on a rather small-scale, namely using fewer than 10 variables. In contrast, our approach is applicable to high-dimensional data as well.

3 Data

In our forecasting application, we will make use of three geographic sets of data, where we differentiate between American data and data for European countries. Each set contains various time-series corresponding with the different macroeconomic variables.

3.1 American data

For the economic activity in the U.S., we consider eight major quarterly variables. For some series, quarterly data was the highest available frequency, which conceivably will yield the forecasting results with the highest accuracy. The eight series of observations are obtained from the McCracken and Ng (2016) data-set. If monthly data was available for a particular series, quarterly factors were extracted such that quarterly data was taken as quarterly averages of monthly data before performing factor analysis. A few descriptive statistics for the relevant data-sets are given in Table 1. Note that the descriptive statistics apply to the data after the transformation.

3.1.1 Data statistics

Series	Data	Sample period	Length	Freq.	Min	Date (min)	Max	Date (max)
1	GDP	1960Q1 - 2016Q1	228	Q	-2.135	2010Q2	3.816	1979Q4
2	Consumption	1960Q1 - 2016Q1	228	Q	-2.276	1981Q4	2.772	1967Q2
3	Investment	1960Q1 - 2016Q1	228	Q	-17.562	1976Q3	11.026	1962Q3
4	Exports	1960Q1 - 2016Q1	228	Q	-12.760	1999Q4	20.229	1970Q4
5	Imports	1960Q1 - 2016Q1	228	Q	-10.572	1976Q3	18.006	1970Q4
6	Total Hours[a]	1960Q1 - 2016Q1	228	Q	-3.226	1976Q3	2.808	1979Q4
7	GDP inflation	1960Q1 - 2016Q1	228	Q	-0.156	2010Q4	3.004	1976Q1
8	PCE inflation[b]	1960Q1 - 2016Q1	228	Q	-1.444	2010Q2	2.947	1981Q3

[a]'Total Hours' here refers to the business sector: total hours worked to achieve GDP.
[b]Personal Consumption Expenditures (PCE) inflation here includes

Table 1: Descriptive statistics for macroeconomic series for the U.S.

All series are seasonally adjusted and the year 2009 is set to having index 100. Series 1 up to and including series 6 contain quantity indices, whereas series 7 and 8 contain chain price indices. Using this data-set, Guérin et al. (2020) calculated the natural logarithmic growth relative to the previous quarter and this data is eventually used in the forecasting exercise. In Figure 1 we plotted these growth series. Similarly to the paper by Guérin et al. (2020), we use a first sample period of 1960Q3 to 1984Q4 and recursively expand this as we progress in the forecasting exercise. The full evaluation sample runs from 1985Q1 to 2015Q3.

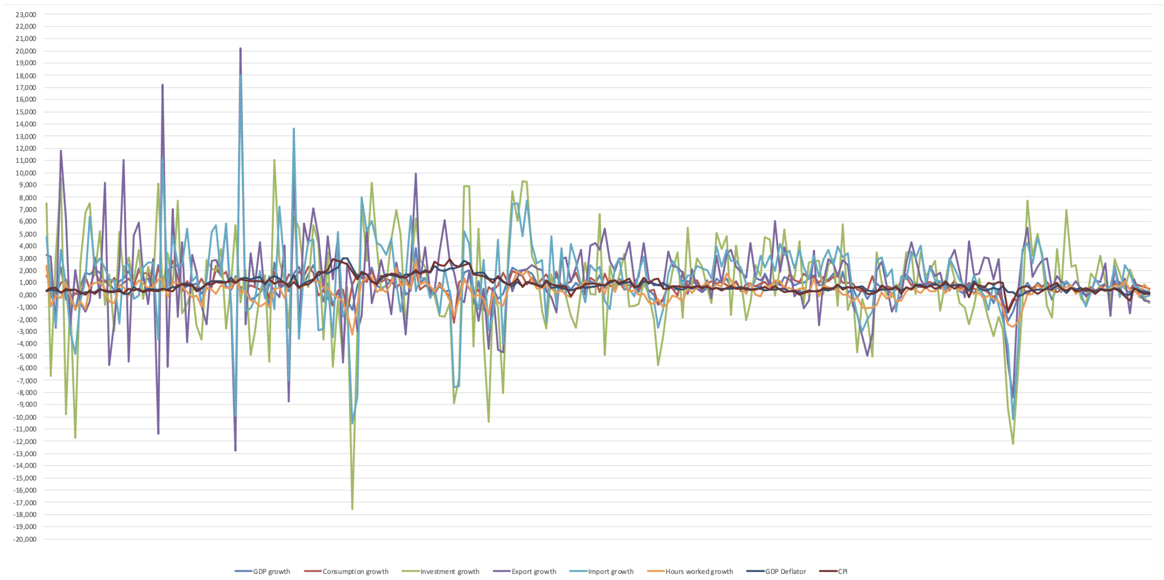

Figure 1: Graph with growth indices for different macroeconomic variables: U.S.

3.1.2 Data transformation

The code will do some data transformation before starting the series of regressions. Firstly, it will remove outliers. These are defined as observations of the transformed series with absolute median deviations larger than 6 times the inter quartile range. These are replaced with the median value of the preceding 5 observations.

3.2 European data

The European macroeconomic data we use originates from different sources: FRED and OECD. In particular, we will consider the following countries:

- Germany
- France

Our reasoning lies in the fact that Germany and France are respectively the largest and third-largest sovereign states within Europe with regard to highest GDP (nominal). The second-largest state would be the United Kingdom, but we would like to compare those countries which are part of the Euro Area 19, a collective of all countries with the Euro as their currency. We attempted to collect data closest to the American data described earlier. However, due to limitations for some of the eight macroeconomic variables, we omit some macroeconomic variables and/or included some other variables as a substitute. Similarly, we have listed summary statistics for the transformed data of the German and French macro-economic variables in Table 2 and 3 respectively.

3.2.1 Germany

Data	Sample period	Length	Freq.	Min	Date (min)	Max	Date (max)
GDP	1991Q1 - 2019Q4	116	Q	-4.091	2009Q1	3.440	1991Q3
PCE	1970Q1 - 2019Q4	200	Q	-2.207	1988Q1	5.749	1979Q2
Unemployment	1969M1 - 2020M4	616	M	-18.232	1970M1	28.768	1992M1
Exports	1991Q1 - 2019Q4	116	Q	-13.790	2009Q1	8.370	2010Q2
Imports	1970Q1 - 2019Q4	200	Q	-11.008	2009Q1	9.779	2010Q2
Interest Rate	1960M1 - 2020M4	724	M	-179.176	2019M3	154.044	2015M5
GDP inflation	1970Q1 - 2019Q4	200	Q	-0.540	2010Q2	4.716	1971Q1

Table 2: Descriptive statistics for macroeconomic series for Germany.

3.2.2 France

Data	Sample period	Length	Freq.	Min	Date (min)	Max	Date (max)
GDP	1975Q1 - 2020Q1	181	Q	-5.411	2020Q1	4.484	1976Q2
PCE	1980Q1 - 2020Q1	161	Q	-6.235	2020Q1	1.856	1999Q3
Unemployment	1983M1 - 2020M3	447	M	-3.681	1983M2	6.137	2020M3
Exports	1980Q1 - 2020Q1	161	Q	-9.518	2009Q1	7.019	1981M4
Imports	1960Q1 - 2020Q1	241	Q	-12.239	1963Q1	7.222	1973Q3
Interest Rate	1960M1 - 2020M3	723	M	-207.944	2019M11	158.045	2019M8
GDP inflation	1960Q1 - 2020Q1	241	Q	-0.674	1961Q2	4.513	1974Q3

Table 3: Descriptive statistics for macroeconomic series for France.

4 Methodology

This section consists of three main parts. First, we will introduce some specification and notation, as well as the steps of the Expectation-Maximization (EM) algorithm. Using this notation, we can introduce the Markov-Switching Three-Pass Regression Filter (MS-3PRF) algorithm as proposed by Guérin et al. (2020). Lastly, we will explain the specific methods used for our application: forecasting economic activity. Here, we will also give an overview of all the methods we are comparing.

4.1 Specification and the EM-algorithm

We assume that the asset return (or whichever data we are using) Y_t follows a distribution that depends on a latent process S_t. Here, we allow for $m = 2$ regimes, however it is possible to add regimes. At each point in time, the process S_t is in one out of two regimes, which we indicate by $S_t = 0$ and $S_t = 1$.

$$Y_t \sim \begin{cases} N(\beta_1' x_t, \sigma_1^2) & \text{if } S_t = 0. \\ N(\beta_2' x_t, \sigma_2^2) & \text{if } S_t = 1. \end{cases} \tag{1}$$

where β_i is an m-vector with regime-specific coefficients.

For both regimes, the return follows a normal distribution with varying means and variances. We use the function f to denote the normal pdf.

$$f(y;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} exp\bigg(-\frac{(y-\mu)^2}{2\sigma^2}\bigg). \tag{2}$$

In regime 0 and 1, it is possible to have different non-normal distributions. The latent process S_t follows a first order ergodic Markov chain.

For the ease of notation, we will introduce the separate parameter ζ, which denotes the probability that the first regime occurs,

$$\zeta = Pr[S_1 = 0] \tag{3}$$

and it follows naturally that $Pr[S_1 = 1] = 1 - \zeta$.

We also construct the following vector with the densities of observation y_t conditional on the regimes in a vector

$$\mathbf{f}_t \equiv \begin{pmatrix} f(y_t;\mu_0,\sigma_0^2) \\ f(y_t;\mu_1,\sigma_1^2). \end{pmatrix} \tag{4}$$

While implementing the Expectation Maximization-algorithm, in order to avoid long computational times, we update all above mentioned parameters updates directly instead of numerically. We do so in a similar fashion as described by Kole (2019). He notes that the steps of calculating the inference about and forecast for the states define a recursion. Following Kole (2019), we implement the following vector-matrix notation in order to write the recursions more compactly.

$$\xi_{t|t} \equiv \begin{pmatrix} Pr[S_t = 0|y_t, y_{t-1}, ..., y_1] \\ Pr[S_t = 1|y_t, y_{t-1}, ..., y_1]. \end{pmatrix} \tag{5}$$

From which the vector of inferences about the regimes at time t follow

$$\xi_{t+1|t} \equiv \begin{pmatrix} Pr[S_{t+1} = 0|y_t, y_{t-1}, ..., y_1] \\ Pr[S_{t+1} = 1|y_t, y_{t-1}, ..., y_1]. \end{pmatrix} \tag{6}$$

for the regime forecasts at time $t+1$, using all information up to time t. We introduce the so-called Hamilton filter (see Hamilton (1994)), which recursively constructs the series of inference and forecast probabilities in the following fashion:

$$\xi_{t|t} = \frac{1}{\xi'_{t|t-1} f_t} \xi_{t|t-1} \odot f_t \tag{7}$$

$$\xi_{t+1|t} = P\xi_{t|t} \tag{8}$$

with starting value $\xi_{1|0} = (\zeta, 1-\zeta)'$.

We also introduce the so-called Kim-smoother (see Kim (1994)), which determines the probability of a certain regime occurring at time t, using *all* available information. This includes information before *and* after time t, so-called smoothed inferences. Using the following recursion,

$$\xi_{t|T} = \xi_{t|t} \odot \big(P'(\xi_{t+1|T} \div \xi_{t+1|t})\big), \tag{9}$$

we can obtain

$$\xi_{t|T} \equiv \begin{pmatrix} Pr[S_t = 0|y_T, y_{T-1}, ..., y_1] \\ Pr[S_t = 1|y_T, y_{T-1}, ..., y_1]. \end{pmatrix} \tag{10}$$

The difference with the Hamilton filter lies in the fact that the Kim filter runs backwards starting with $\xi_{T|T}$. From the state and transition probabilities, we are able to forecast.

4.2 The Markov-Switching Three-Pass Regression Filter Model

As described in the previous section, using two regimes which represent different Markovian states, we are able to include time variation in the model parameters. We introduce the model as denoted by Guérin et al. (2020) and follow the same notation for the sake of convenience.

$$y_t = \beta_0(S_{yt}) + \boldsymbol{\beta}(S_{yt})\boldsymbol{f}_{t-1} + \eta_t, \quad t = 1, ..., T, \tag{11}$$

$$z_{jt} = \lambda_{0,j}(S_{z_jt}) + \boldsymbol{\lambda}_j(S_{z_jt})\boldsymbol{f}_t + \omega_{jt}, \quad j = 1, ..., k_{\boldsymbol{f}}, \tag{12}$$

$$x_{it} = \phi_{0,i}(S_{x_it}) + \boldsymbol{\phi}_{f,i}(S_{x_it})\boldsymbol{f}_t + \boldsymbol{\phi}_{g,i}(S_{x_it})\boldsymbol{g}_t + \epsilon_{it}, \quad i = 1, ..., N, \tag{13}$$

where y is the scalar target variable of interest for forecasting; $\boldsymbol{f}_t = (f_{1t}, ..., f_{k_{\boldsymbol{f}}t})'$ is a $k_{\boldsymbol{f}} \times 1$ vector of unobservable factors, with associated slope coefficient $\boldsymbol{\beta}(S_{yt}); z_{jt}, j = 1, ..., k_{\boldsymbol{f}}$, are so-called proxy variables driven by the same factors as $y, \boldsymbol{f}_t$, with variable specific loadings $\boldsymbol{\lambda}_j(S_{z_jt}); x_{it}, i = 1, ..., N$, are variables driven by the $\boldsymbol{f}_t$ factors but also by the $k_{\boldsymbol{g}}$ unobservable factors in the vector $\boldsymbol{g}_t$, with associated variable specific loadings $\boldsymbol{\phi}_{f,i}(S_{x_it})$ and $\boldsymbol{\phi}_{g,i}(S_{x_it})$, respectively; $\beta_0(S_{yt}), \lambda_{0,j}(S_{z_jt}), \phi_{0,i}(S_{x_it})$ are intercepts. As anticipated, the coefficients in (11) to (13) are time-varying and driven by variable specific and independent across variables M-state Markov chains (latent processes): S_{yt}, S_{z_jt} and $S_{x_it}, j = 1, ..., k_{\boldsymbol{f}}$ and $i = 1, ..., N$. Each Markov chain is governed by its own $M \times M$ transition probability ergodic matrix,

$$P_q = \begin{pmatrix} p_{q,11} & p_{q,21} & \cdots & p_{q,M1} \\ p_{q,12} & p_{q,22} & \cdots & p_{q,M2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{q,1M} & p_{q,2M} & \cdots & p_{q,MM} \end{pmatrix} \tag{14}$$

for $q = y, z_1, ..., z_{k_{\boldsymbol{f}}}, x_1, ..., x_N$.

We will also define the variables $\boldsymbol{\phi}_{A,it}$ and $\boldsymbol{\phi}_{B,it}$ beforehand, to keep the three-step algorithm uncluttered. We have

$$\boldsymbol{\phi}_{A,it} = \sum_{j=1}^{M} \boldsymbol{\phi}_i(S_{x_it} = j)P(S_{x_it} = j|\Omega_T), \tag{15}$$

such that $\boldsymbol{\phi}_{A,it}$ are a weighted-average of the estimated regime-specific factor loadings and $P(S_{x_it} = j|\Omega_T)$ is the smoothed probability of being in regime j given the full sample information Ω_T. We will denote this weighing method A. On the other hand, weighing method B

$$\boldsymbol{\phi}_{B,it} = \sum_{j=1}^{M} \boldsymbol{\phi}_i(S_{x_it} = j)I(P(S_{x_it} = j|\Omega_T)), \tag{16}$$

gives **selected** factors loadings, where $I(\cdot)$ is an indicator function that selects the regime with

the highest smoothed probability at time t. This parameter in essence contains the factor loadings which correspond to the most likely regime at time t. The **smoothed** probability used in this model is defined as the probability of the occurrence of a specific regime at time t, using all available information, i.e., information before and after time t.
The algorithm for the MS-3PRF model consists of the following three steps:

- **Step 1**: Univariate time-series regressions of each x_{it} on the proxy variables z_{jt}, $j = 1, ..., k_f$. Now we have defined the vector $\boldsymbol{z}_t = (z_{1t}, ..., z_{k_{\boldsymbol{f}}t})'$, we run N Markov-switching regressions
$$x_{it} = \phi_{0,i}(S_{x_it}) + \boldsymbol{\phi}_i(S_{x_it})\boldsymbol{z}_t + \epsilon_{it}, \quad t = 1, ..., T, \tag{17}$$
where $\epsilon_{it} \sim \text{NID}(0, \sigma^2_{x_i}(S_{x_it}))$, and we keep the (variable-specific) estimates of $\boldsymbol{\phi}_i(S_{x_it})$, denoted by $\hat{\boldsymbol{\phi}}_i(S_{x_it})$.

- **Step 2**: Cross-section regressions of the x_it on either $\hat{\boldsymbol{\phi}}_{A,it}$ or $\hat{\boldsymbol{\phi}}_{B,it}$. Hence, we run T linear regressions
$$x_{it} = \alpha_{0,t} + \hat{\boldsymbol{\phi}}_{q,it}\boldsymbol{f}_t + \upsilon_{it}, \quad i = 1, ..., N, \tag{18}$$
$\theta_{it} \sim \text{IID}(0, \sigma^2_{\theta_i})$, with $t = 1, ..., T, q = A, or q = B$, and we keep $(\forall t)$ the OLS estimates $\hat{\boldsymbol{f}}_t$, where $\hat{\boldsymbol{f}}_t$ is a $k_{\boldsymbol{f}} \times 1$ vector.

- **Step 3**: Time-series regression of y_t on $\hat{\boldsymbol{f}}_{t-1}$. Hence, we run one MS regression:
$$y_t = \beta_0(S_{yt}) + \boldsymbol{\beta}(S_{yt})\hat{\boldsymbol{f}}_{t-1} + \eta_t, \quad t = 1, ..., T, \tag{19}$$
$\eta_t \sim \text{NID}(0,\sigma^2_\eta(S_{yt}))$, and we keep the maximum likelihood estimates $\hat{\beta}_0(S_{yt})$ and $\hat{\boldsymbol{\beta}}(S_{yt})$. The forecast equation is as follows:
$$\hat{y}_{T+1|T} = \sum_{j=1}^{M} \big(P(S_{yT+1} = j|\Omega_T)\hat{\beta}_0(S_{yT+1} = j) + P(S_{yT+1} = j|\Omega_T)\hat{\boldsymbol{\beta}}(S_{yT+1} = j)\hat{\boldsymbol{f}}_T\big), \tag{20}$$
where $P(S_{yT+1} = j|\Omega_T)$ is the predicted probability of being in regime j in period $T + 1$ given the information available up to time T, Ω_T.

Similarly, we calculate the h-step ahead forecasts are directly, instead of recursively updating the forecast origin T. This is more efficient, as the parameters in the three steps do not require re-estimation in each update. We replace the third step with the following Markov-switching regression:
$$y_t = \gamma_0(S_{yt}) + \boldsymbol{\gamma}(S_{yt})\hat{\boldsymbol{f}}_{t-h} + \eta_t, \quad t = 1, ..., T, \tag{21}$$
where we keep the maximum likelihood estimates $\hat{\gamma}_0(S_{yt})$ and $\hat{\boldsymbol{\gamma}}(S_{yt})$. The h-step ahead forecasts are then:
$$\hat{y}_{T+h|T} = \sum_{j=1}^{M} \big(P(S_{yT+h} = j|\Omega_T)\hat{\gamma}_0(S_{yT+h} = j) + P(S_{yT+h} = j|\Omega_T)\hat{\boldsymbol{\gamma}}(S_{yT+h} = j)\hat{\boldsymbol{f}}_T\big), \tag{22}$$

Partial least squares is obtained as a special case of the 3PRF by Kelly and Pruitt (2015), therefore this method can also be adopted to add Markov-switching to PLS regression. In step one, the reason that we assume normality for the error term is because in order to define the maximum likelihood function, we need the underlying distribution. By assuming normality we can specify the likelihood as a regular normal distribution. For step two, normality is not required since we do not need to derive or assume anything about the distribution of v (and consequently x). Since step two uses OLS estimation, homoskedastic error terms and zero mean for the errors are required. In the third step, we model time variation in the intercept of the forecast regression. Changes in the slope parameters $\boldsymbol{\beta}$ are relevant in that they allow us to model time variation in the predictive power of the estimated factors $\hat{\boldsymbol{f}}_t$ for the target variable y_{t+1}. However, this is a common source of failure. Therefore, we will distinguish between MS-3PRF (first pass) and MS-3PRF (first and third pass), referring to whether we include regime changes in the third pass.

4.3 Forecasting Economic Activity

There are numerous different approaches which are interesting for comparison, such as variations on PCA, linear 3PRF and variations on MS-3PRF. We will provide a brief overview of the methods considered by Guérin et al. (2020).

1. PCA: five factors are extracted from the underlying dataset (but only the first factor is used in the forecasting equation).

2. Linear 3PRF: as described by Kelly and Pruitt (2015).

3. MS-3PRF-1: implementing weighing method A and with regime-switching parameters in the first pass only.

4. MSS-3PRF-1: implementing weighing method B and with regime-switching parameters in the first pass only.

5. MS-3PRF-13: implementing weighing method A and with regime-switching parameters in the first and third passes.

6. MSS-3PRF-13: implementing weighing method B and with regime-switching parameters in the first and third passes.

From Guérin et al. (2020) we gather that Monte Carlo simulations and their applications generate similar results when comparing forecasting ability. In general, the MS-3PRF approaches perform better than their linear counterparts. Under certain circumstances PCA, in particular PC-LARS, seems to perform well. The MSS-3PRF method is a variation on the MS-3PRF where the loadings are selected instead of averaged; described earlier as weighing method B. It can be interesting to assess which weighing method provides the best forecasting results. One can also investigate whether including regime switching only in the first (the factor loadings) as opposed to including regime switching in both the first and third steps (the factor loadings and in the parameters of the forecasting equation respectively) gives significantly different results. Due to time limitations, it is difficult to compare all eight different methods. For the reasons mentioned above, we focus on comparing the following methods: Linear 3PRF, MS-3PRF-1, MS-3PRF-13, MSS-3PRF-1 and MSS-3PRF-13. We will also include normal PCA as well as

AR(1). Further research can be conducted in order to assess the effectiveness of TPCA and PC-LARS in comparison to the 3PRF methods as well. We will use the following h-step ahead forecasting equations:

$$y_{T+h|T} = \hat{\alpha} + \hat{\beta}(L)\hat{f}_T + \hat{\gamma}(L)y_T \tag{23}$$

where $\beta(L)$ and $\gamma(L)$ are finite-order lag polynomials. The lag lengths are obtained with the SIC, using a maximum lag length of 6 for $\gamma(L)$ and 3 for $\beta(L)$. We will compute the out-of-sample mean square forecasting errors (MSFE) and use this as the forecasting performance criterion. We will test significant outperformance by using the Diebold and Mariano (2002) test of equal out-of-sample predictive accuracy. One frequently used method models the factor loadings after a random walk, which is an AR(1) process with the parameter θ set to 1. We use the direct approach as described in Section 4.3 to calculate forecasts from the AR model. For this reason we use this relatively straight-forward model as a benchmark model for time varying parameters.

The choice of the number of factors $k_{\boldsymbol{f}}$ to include is important. Using too many factors reduces forecast efficiency in finite samples, while using too little risks generating an omitted-variable bias. Guérin et al. (2020) apply PCA from which they extract five factors from the underlying dataset, but only use only the first factor in their application. Kelly and Pruitt (2015) introduce information criteria with asymptotic optimality properties. However, empirically it can be more informative to assess the performance of different numbers of factors. We will compare the methods while using either only the first factor or both the first and second factor. We could also vary the number of regimes m. Guérin et al. (2020) use only two different regimes in their applications. Although we would like to investigate whether using more regimes improves the forecasting results, due to our interpretation of the data, we will follow in their footsteps and only consider two regimes for the American data set. For European data however, it is interesting to investigate this matter further.

We use the program *GAUSS 20* to run the code written by Guérin et al. (2020). We apply some modifications to the original code in terms of data transformation, as well as scale the function to correct for the computationally complex optimizer. The function overall is relatively flat, meaning that large changes (by the standard of the optimization algorithm) create small changes or no change in the objective function. We also added a starting random number seed in order to run the code successfully. Moreover, we modify the rather computationally complex MIDAS-procedure used by the original authors in such a way that the algorithm takes less time to optimize. The modifications rely on the more direct procedure is described by Kole (2019) in Section 4.3. The algorithm has the advantage that it is relatively simple to adjust for the number of forecast horizons and regimes we wish to include. It includes clear division of the three passes. In the third pass, we distinguish between different forecasting approaches. We can also assess the co-variance between macroeconomic data of these countries. Eventually, the Root Squared Mean Error (RSME) for the different methods mentioned above is returned. In the table, we provide the MSFE of each approach relative to that of PCA for forecast horizons ranging from one quarter to eight quarters ahead.

5 Results

5.1 American data

Guérin et al. (2020) noticed that across all series, there is substantial time variation in the smoothed probability and they suggest that this can be interpreted as evidence in favor of regime shifts in the factor loadings. Table 2 shows the out-of-sample forecasting results. We report the Root Mean Squared Errors for the different variables and for horizons spanning from $h = 1$ quarter to $h = 8$ quarters. To easily assess the forecasting ability of the various approaches, we take the RSME of each approach relative to that of the PCA approach. Thus, the lower the ratio, the better the approach performed. If the ratio is smaller than 1, it means that the approach performed better than the benchmark PCA approach. It is also of our interest to compare the linear 3PRF and MS-3PRF methods to a simple AR(1) regression model. Note that the linear 3PRF, MS-3PRF and MSS-3PRF variants use GDP growth as a target variable. From running the programs in GAUSS 20 as described in methodology, we obtained RSME values which we then transformed into Mean Square Prediction Error (MSFE) values. These estimator measures are closely related since RMSE is the square root of the mean square error (MSE). Comparing the values to those obtained in the paper from Guérin et al. (2020), we see that the values are very similar. We consider the RMSE of each approach relative to that of PCA and draw comparisons and conclusions of the performance of each method between their work with ours. The boldfaced entries indicate the best-performing procedure for each specific horizon and variable. Overall, across all forecast horizons and predicted variables (64 cases), it seems that the various MS-3PRF approaches performed best in 27 of the cases. The AR(1) and linear 3PRF methods perform best in respectively 16 and 19 of the cases. In the remaining 3 cases, PCA appeared to be best-performing.

A few interesting points to note

- We expected the MSS-3PRF-13 to be the best performing approach amongst the variants on MS-3PRF. However, this is not the case.
- For GDP inflation and PCE inflation, the MSS-3PRF-13 method seems to outperform every other method, regardless of the chosen forecast horizon. This seems to be the case for both Guérin et al. (2020) as well as in our results. The only exception to this is when forecasting with h = 1 for GDP inflation.

Forecast horizon	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
	1: GDP								2: Consumption							
AR(1)	0.947	0.902	0.887	0.791	0.911	0.903	**0.781**	**0.897**	1.029	0.943	0.801	**0.849**	**0.837**	**0.828**	0.805	0.917
Linear 3PRF	**0.849**	**0.854**	**0.838**	**0.766**	**0.888**	0.922	0.865	0.924	**0.949**	**0.573**	**0.785**	0.819	0.838	0.877	0.885	0.942
MS-3PRF-1	1.37	0.976	1.003	**0.795**	**0.913**	**0.877**	**0.781**	**0.912**	0.966	1.034	0.916	0.824	0.875	**0.872**	**0.804**	0.923
MS-3PRF-13	1.162	1.01	0.992	0.965	1.004	0.987	1.062	1.273	1.101	1.581	1.517	2.049	2.207	1.807	1.94	1.852
MSS-3PRF-1	1.341	0.974	1.002	0.796	0.917	**0.877**	0.783	0.911	0.964	1.036	**0.919**	**0.816**	0.875	0.86	**0.806**	**0.915**
MSS-3PRF-13	1.166	0.981	1.315	0.974	1.29	1.203	1.038	1.085	1.052	1.392	1.194	1.521	1.567	1.604	1.363	1.588
	3: Investment								4: Exports							
AR(1)	0.935	0.975	0.974	0.83	1.011	1.049	**0.986**	**1.024**	0.842	**0.938**	**0.889**	**0.966**	**0.97**	0.998	0.996	0.994
Linear 3PRF	**0.891**	**0.945**	**0.961**	0.822	1.001	1.024	1.026	1.087	0.93	1	1.024	0.996	0.986	**0.983**	**0.992**	1
MS-3PRF-1	0.92	0.951	0.962	0.8	1.004	1.053	1.065	1.05	0.727	**0.938**	0.937	1.002	0.998	0.996	0.995	0.99
MS-3PRF-13	0.959	1.028	1.041	1.118	1.781	1.412	1.539	1.271	**0.467**	0.997	0.993	1.044	1.014	1.03	1.061	1.063
MSS-3PRF-1	0.917	0.95	0.971	**0.797**	1.006	1.048	1.066	1.057	0.738	0.942	0.961	1.015	0.976	0.993	0.996	**0.974**
MSS-3PRF-13	0.977	1.015	1.051	0.875	1.146	1.161	1.102	1.107	0.47	1.049	1.058	1.093	0.997	1.06	1.096	1.09
	5: Imports								6: Total Hours							
AR(1)	**0.996**	**0.891**	**0.920**	**0.790**	**0.823**	0.897	0.806	**0.861**	0.953	**0.927**	0.996	0.982	0.996	**0.945**	**0.974**	**0.970**
Linear 3PRF	0.912	0.895	0.918	0.805	0.834	**0.893**	**0.891**	0.990	**0.880**	0.958	**0.658**	**0.955**	**0.986**	0.980	1.028	1.006
MS-3PRF-1	0.937	0.893	0.884	0.852	0.878	0.933	**0.780**	0.872	0.924	1.012	0.980	1.034	1.067	1.030	1.117	1.000
MS-3PRF-13	**0.704**	**0.803**	0.964	1.082	1.067	**1.004**	0.882	0.925	0.933	1.038	1.481	2.295	4.924	3.729	3.519	2.484
MSS-3PRF-1	0.949	0.897	**0.882**	0.848	0.880	0.935	0.787	0.870	0.933	1.016	0.980	0.970	1.063	1.020	1.115	1.000
MSS-3PRF-13	0.706	0.852	1.090	1.049	1.016	1.028	**0.889**	**0.986**	0.943	1.026	1.245	1.355	1.659	2.011	2.589	2.554
	7: GDP Inflation								8: PCE Inflation							
AR(1)	**1.014**	**0.976**	**0.884**	**0.821**	**0.792**	0.741	0.731	0.724	1.000	1.000	1.012	0.988	0.972	0.893	0.882	0.857
Linear 3PRF	**0.986**	1.051	1.020	0.874	0.865	**0.870**	**0.929**	0.992	1.000	0.982	1.008	1.000	1.105	1.067	1.164	1.169
MS-3PRF-1	1.075	1.000	0.951	0.834	0.887	0.970	1.051	1.162	1.016	0.970	0.982	0.955	0.980	0.970	0.998	1.113
MS-3PRF-13	1.297	0.817	0.980	0.832	1.232	**0.757**	0.671	0.721	1.334	0.884	0.964	0.986	0.964	1.049	1.053	1.237
MSS-3PRF-1	1.075	1.004	0.960	0.854	0.899	0.978	1.063	1.177	1.022	0.970	0.974	0.953	0.980	0.968	1.000	1.117
MSS-3PRF-13	1.100	**0.590**	**0.527**	**0.433**	**0.333**	**0.255**	**0.261**	**0.263**	**0.939**	**0.733**	**0.759**	**0.637**	**0.607**	**0.573**	**0.506**	**0.503**

Table 4: MSFE of each forecasting approach for the macro-economic variables applied to the American market, relative to MSFE of the PCA forecasting approach. The best method for each forecasting horizon is marked in bold. A column without a bold value indicates PCA performing better than any of the displayed methods.

A few important questions can be answered when analyzing which of the methods performs best for each case. Guérin et al. (2020) did not provide a conclusive answer to the question whether or not we should include weighing method A or weighing method B. This is also the case for this paper, as the number of times that MS-1 and MSS-1 were the best methods is tied. We see that it is variable-specific. The question whether we should include regime-switching parameters in only the first pass or in both the first and third pass can be answered. From the results from Guérin et al. (2020) we see that more often than not, only including Markov switches in the first pass outperforms the approach where Markov switches are included in both the first and the third pass. We suspect that the reason for this lies in the fact that changes in the slope parameter β are sensitive to failure. This is also the case for our results, where MS-1 was the best performing method in 12.5 cases, MS-13 in 1 case, MSS-1 in 12.5 cases and MSS-13 in 1 case out of 27. When a tie occurred, this is denoted by 0.5. We can conclude that only including regime-switching in the first pass generally gives better forecasting results. Lastly, we discuss the question whether or not we recommend implementing the Markov-switching element in factor models for forecasting economic activity. In order to answer this question, we sum number of times the four variants of the MS-3PRF model perform the best and compare this number to the total number of cases. From the results from Guérin et al. (2020), we see that the MS-methods performed best in 24 out of 40 cases. If we work with a recommendation threshold of 50%, from their results we can conclude that it is recommendable to include MS. However, it is important to note the extra computational costs may not always outweigh this slight outperformance. More importantly, which of the methods is the best is dependent on the variable in question and on the forecasting horizon. We are also not able to give a clear conclusion which of the four methods works best in which situations. After doing the same forecasting exercise for the same data, the MS-methods were only best performing in 42% of the cases. These numbers are all dependent on which alternative methods we are comparing with, therefore should not be taken as any more than an indication. It is safer to conclude that the MS-methods at least generally outperform PCA.

5.2 European data

For Germany, we follow the same structure as for the American forecasts, with the exception that we use the macroeconomic variables as described in Section 3.2.1.

5.2.1 Germany

Forecast horizon	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
	1: GDP								2: PCE							
AR(1)	0.891	0.805	0.701	0.635	**0.709**	0.848	0.746	0.852	0.902	0.970	0.799	0.940	0.784	0.949	0.762	0.858
Linear 3PRF	**0.748**	**0.759**	**0.664**	**0.579**	0.887	**0.764**	**0.608**	**0.830**	**0.768**	**0.696**	0.965	0.767	0.798	0.940	0.943	0.960
MS-3PRF-1	1.605	1.210	0.982	0.632	**0.797**	**0.972**	**1.126**	**1.218**	0.938	0.969	**0.789**	**0.710**	0.764	0.943	**0.673**	**0.669**
MS-3PRF-13	1.100	1.355	1.602	0.937	1.432	1.067	0.812	1.030	1.276	1.149	1.197	1.323	1.393	1.333	0.926	1.173
MSS-3PRF-1	1.588	1.208	0.971	0.628	0.789	0.965	1.127	1.215	0.947	0.974	0.793	0.729	**0.762**	**0.938**	0.696	0.681
MSS-3PRF-13	1.174	1.309	1.654	0.917	1.421	1.014	0.808	1.003	1.238	1.114	1.187	1.297	1.280	1.287	0.897	1.045
	3: Unemployment								4: Exports							
AR(1)	**0.949**	**0.573**	**0.785**	0.819	**0.838**	0.877	0.885	0.942	0.842	0.890	**0.784**	**0.953**	0.978	**0.823**	0.976	1.044
Linear 3PRF	0.966	1.034	0.916	**0.816**	0.875	**0.828**	**0.804**	**0.915**	0.895	0.990	1.016	0.986	0.976	0.976	0.965	1.061
MS-3PRF-1	1.164	1.236	1.119	1.024	1.075	1.06	1.006	1.023	**0.618**	**0.822**	0.903	1.003	**0.800**	0.986	0.986	0.965
MS-3PRF-13	1.201	1.581	1.517	2.049	2.207	1.807	1.940	1.852	1.280	1.232	1.184	1.134	1.044	1.231	1.475	1.350
MSS-3PRF-1	1.152	1.222	1.114	1.021	1.067	1.058	0.983	1.028	0.620	0.823	0.913	0.996	0.818	0.984	**0.921**	**0.961**
MSS-3PRF-13	1.229	1.343	1.301	1.949	2.137	1.672	1.705	1.617	1.160	1.123	1.048	1.083	1.100	1.030	1.106	1.102
	5: Imports								6: Interest rate							
AR(1)	**0.988**	0.936	**0.888**	**0.909**	**0.862**	0.926	0.830	0.896	1.112	1.061	0.913	0.894	0.883	**0.863**	**0.783**	**0.726**
Linear 3PRF	0.912	**0.896**	**0.867**	**0.835**	**0.815**	**0.917**	**0.926**	0.978	1.051	1.121	1.087	0.931	0.921	0.882	0.793	1.057
MS-3PRF-1	1.018	1.386	1.062	1.138	1.098	1.012	1.019	1.007	1.149	0.973	**0.858**	**0.710**	**0.882**	0.973	1.121	1.107
MS-3PRF-13	**0.790**	0.857	0.984	0.885	0.870	0.874	0.873	0.899	1.326	1.218	1.045	1.077	1.009	1.007	0.991	0.869
MSS-3PRF-1	1.021	1.392	**1.071**	1.148	1.103	1.015	1.024	1.012	1.147	**0.949**	0.865	0.713	0.902	0.981	1.114	1.104
MSS-3PRF-13	0.825	0.892	0.963	0.884	0.857	**0.852**	**0.832**	**0.856**	1.345	1.198	1.022	1.047	0.974	0.984	0.892	0.885
	7: GDP Inflation															
AR(1)	**0.846**	**0.851**	**0.834**	**0.761**	**0.885**	0.920	0.862	0.922								
Linear 3PRF	1.377	0.975	1.002	0.791	0.911	**0.874**	**0.776**	0.910								
MS-3PRF-1	1.165	1.010	1.329	0.964	1.303	1.286	1.063	1.178								
MS-3PRF-13	0.948	0.973	0.901	0.792	0.815	**0.678**	0.728	0.699								
MSS-3PRF-1	1.169	0.980	1.321	0.973	1.296	1.287	1.058	1.176								
MSS-3PRF-13	0.945	0.900	0.884	**0.723**	**0.809**	**0.675**	**0.721**	**0.694**								

Table 5: MSFE of each forecasting approach for the macro-economic variables applied to the German market, relative to MSFE of the PCA forecasting approach. The best method for each forecasting horizon is marked in bold. A column without a bold value indicates PCA performing better than any of the displayed methods.

The results for these macroeconomic variables are in some way similar to those when considering the U.S. variables. The various MS-methods performed best in 26.5 cases out of 56 and AR(1) and linear 3PRF performed best in respectively 13 and 14.5 cases. In the last two cases, PCA was the preferred method. Since 26.5 out of 56 corresponds to less than 50%, we cannot give an initial conclusion that the MS-methods perform overwhelmingly well. When we analyse the number of times that the methods outperform PCA respectively, we cannot conclude that this new method is recommendable to implement above standard PCA. Respectively, MS-1, MS-13, MSS-1 and MSS-13 outperform PCA in 32, 19, 33 and 29 out of 56 cases. On average, this is 50% of the time.

We also analyzed how the different MS-methods compare relative to each other. Out of 56 cases, MS-1 was the best performing method among the four in 16 cases. For MS-13 this number is 3, for MSS-1 and MSS-13 it is 21 and 17 respectively. Therefore, we can conclude that only including Markov switches in the first pass is again generally the recommendable method. We can also include that weighing method B generally gives better results. Thus the best performing MS-method among the four variants is the MSS-1 method.

5.2.2 France

In this case, we see that the AR(1) performs relatively well. The MS-3PRF methods perform better for France than for the other countries, they were the best performing in 37 out of 56 cases. When we consider the four different variants, we see that it is again the case that generally MSS-methods were preferred above MS-methods, indicating that weighing method B gave a more accurate depiction. Additionally, MS-1 outperformed MS-13 and MSS-1 outperformed MSS-13 as well. These results are in accordance to those obtained by Guérin et al. (2020) using U.S. data. It seems that the MS-3PRF method is quite robust, as it performs the best compared to the other methods we reviewed. However, we cannot give a clear recommendation to always use MS-3PRF above for example PCA. Under more economic context and information on the relevant forecast horizon, we can make a more safe recommendation.

Forecast horizon	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
	1: GDP								2: PCE							
AR(1)	0.989	1.058	1.062	1.034	0.971	**0.967**	**0.942**	1.027	0.850	0.770	1.068	0.849	0.883	1.041	1.044	1.063
Linear 3PRF	**0.965**	1.044	1.047	1.026	0.965	0.972	0.967	0.976	**0.745**	**0.675**	0.936	**0.744**	0.774	**0.912**	0.915	0.931
MS-3PRF-1	1.097	1.08	1.111	1.036	0.926	1.09	0.943	0.973	0.927	0.954	**0.776**	0.746	**0.714**	0.918	**0.681**	**0.667**
MS-3PRF-13	1.088	1.039	1.094	1.089	1.023	0.989	0.993	1.063	1.857	1.671	1.781	1.946	1.931	1.920	1.346	1.562
MSS-3PRF-1	1.077	1.078	1.111	1.036	**0.916**	0.973	0.946	**0.961**	1.004	1.037	0.817	0.760	0.844	1.009	0.720	0.716
MSS-3PRF-13	1.08	1.04	1.168	1.064	0.98	0.969	1.007	1.018	1.876	1.689	1.760	1.945	1.960	1.028	1.361	1.701
	3: Unemployment								4: Exports							
AR(1)	**0.923**	0.974	**0.886**	**0.811**	**0.855**	**0.820**	**0.803**	**0.885**	1.019	1.057	1.056	1.064	0.912	0.926	0.924	0.924
Linear 3PRF	1.175	**0.817**	1.011	1.065	1.083	1.102	1.120	1.176	1.000	1.029	1.047	1.027	0.884	0.883	0.884	0.885
MS-3PRF-1	1.293	1.369	1.257	1.174	1.213	0.904	0.933	0.976	0.897	**0.965**	**0.977**	**0.980**	0.845	0.864	0.863	0.862
MS-3PRF-13	1.371	1.479	1.439	1.366	2.235	1.792	1.824	2.026	**0.803**	0.980	1.011	1.010	0.849	**0.852**	**0.857**	0.857
MSS-3PRF-1	1.261	1.334	1.226	1.145	1.183	0.947	1.000	1.015	0.900	0.966	0.983	0.983	**0.841**	0.864	0.864	**0.849**
MSS-3PRF-13	1.344	1.706	1.645	1.451	2.302	1.921	1.981	2.059	0.804	0.992	1.007	1.002	0.845	0.858	0.864	0.863
	5: Imports								6: Interest rate							
AR(1)	1.260	1.123	1.161	0.992	1.033	1.130	1.012	1.083	0.867	0.960	1.012	1.087	0.808	0.762	0.800	**0.756**
Linear 3PRF	0.909	0.999	0.994	1.013	1.009	0.991	1.106	1.151	1.075	1.090	1.151	1.185	1.110	0.990	1.041	1.082
MS-3PRF-1	0.839	**0.904**	**0.898**	**0.850**	**0.881**	0.932	**0.782**	**0.888**	1.195	1.099	1.124	1.216	1.263	1.126	1.207	1.122
MS-3PRF-13	0.955	0.943	0.937	0.910	0.930	0.956	0.878	1.013	**0.822**	0.807	**0.783**	0.837	0.796	**0.746**	0.801	0.888
MSS-3PRF-1	**0.833**	0.920	0.923	0.915	0.892	**0.912**	0.841	0.998	1.195	1.099	1.122	1.192	1.262	1.129	1.210	1.144
MSS-3PRF-13	0.939	0.923	1.038	1.015	1.002	1.012	0.943	0.998	**0.822**	**0.801**	**0.783**	**0.835**	**0.794**	**0.746**	**0.768**	0.919
	7: GDP Inflation															
AR(1)	0.987	0.982	0.891	0.945	1.102	1.081	0.972	0.982								
Linear 3PRF	1.312	1.298	1.275	1.238	1.329	1.274	1.185	1.190								
MS-3PRF-1	1.498	1.323	1.327	1.231	1.189	1.172	0.834	0.836								
MS-3PRF-13	0.705	0.702	**0.686**	0.720	0.855	0.821	0.792	**0.712**								
MSS-3PRF-1	1.495	1.321	1.398	1.291	1.293	1.182	0.823	0.838								
MSS-3PRF-13	**0.701**	**0.699**	0.692	**0.714**	**0.791**	**0.803**	**0.745**	0.723								

Table 6: MSFE of each forecasting approach for the macro-economic variables applied to the French market, relative to MSFE of the PCA forecasting approach. The best method for each forecasting horizon is marked in bold. A column without a bold value indicates PCA performing better than any of the displayed methods.

6 Conclusion

We investigated the forecasting ability of the Markov-switching three-pass regression filter where parameters can vary according to Markov processes. In our paper, we show an empirical example where we forecast economic activity for the U.S. and for Germany and France. We find that the MS-3PRF method compares favorably with existing alternatives in terms of forecasting performance. Since our results are similar to those obtained by Guérin et al. (2020), this demonstrates the robustness of the MS-3PRF approach.

There are some limitations to our research. First of all, we only consider cases with two regimes. Adding more regimes could expand our research to a wider field of possible applications. Secondly, although we concluded that MSS-3PRF-1 is generally the best among the four, this results is highly situational and the question remains which method is the most suitable for which variable. Thirdly, the estimation method does not work perfectly, in the sense that running it multiple times can sometimes give different results (depending on the random seed used). It must be noted that our computation capacity and running time is limited. Fourthly, due to data limitations, we were unable to directly compare the forecasting results between the U.S. and European data-sets. Therefore we could not draw conclusions regarding the forecasting performance for the missing time-series. One solution can be to find time series that have high correlation with other economic variables of which data is available. Finally, we rely on certain assumptions, such as data being normally distributed. However, assuming an alternative distribution may result in better predictions.

The MSS-3PRF approach seems to fail more times than the approach with weighing method A. Further research could be conducted in order to optimize MSS-3PRF. We can also adjust the number of included (relevant) factors as an attempt to increase forecasting accuracy. Having more accurate out-of-sample forecasts can contribute greatly to the interest of investors and policy makers alike, as they can better anticipate economic activity in various countries. Another avenue for more in-depth research could be attempting to make the model more computationally efficient. One example is applying the Hamilton and Kim filter directly, instead of doing numerical optimization.

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7 Appendix