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Master Thesis Economics of Markets and Organizations

How to build decision-making teams?

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Abstract:

The power to decide is not always in the hands of those who have the best information to do so. In this paper I analyze which people to consult to maximize the probability of making the right decision. I assume that the optimal decision out of three possible options depends on two distinct pieces of evidence. For each piece of evidence, the decisionmaker can consult either a Biased Expert or a Neutral Agent. Biased Experts are more likely to find information, but also have an incentive to manipulate the information they find because they prefer one particular option. In contrast, Neutral Agents are less likely to find information but do not have an incentive to manipulate information. From the basic model with three possible decisions I conclude that it is never optimal to consult two different types of agents. Whether it is optimal to consult two Biased Experts or two Neutral Agents, depends on the ex-ante probability that there is information available and the extent of the expertise advantage of Biased Experts relative to Neutral Agents. If I alter the model to a situation in which there are two options, I find that there are situations in which it is optimal to consult two different types of agents.

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Introduction

The power to decide is not always in the hands of those with the necessary specialized knowledge about the consequences of the decision. Decisionmakers often need other experts to provide them with the relevant information to decide. In a lot of political decisions for example, different interest groups are involved in the decision-making process. One way in which interest groups can try to influence political decisions is by providing the decisionmakers with necessary information (Schneider & Naumann, 1982). When the Dutch government has to decide whether to replace gas by green energy as a main source of energy for example, the minister that has to decide does not have the necessary knowledge about the consequences of the possible decisions. Therefore, he will need different parties to gather the necessary information about oil and green energy as a main source of energy. The Dutch Oil Company (NAM) has a lot of knowledge about the benefits of using gas as a main source of energy while Green energy companies are able to provide the minister with information about the benefits of green energy. Another example of a situation in which the power to decide is not in hands of those with the necessary specialized knowledge is a CEO that has to decide whether or not to divest a division of the company that he works for. He needs advice from internal managers or external consultants to inform him about the profitability of a particular division. These types of decisions occur on every organizational level when decisions have to be made on how to use scarce production factors like time, money or manpower. The better-informed experts are seldom unbiased about the decision that has to be made (Krishna & Morgan, 2001). For example, the interest groups involved in political decisions obviously have preferences for a certain decision. If experts suffer from a bias in the direction of their own field of expertise, it is called a specialty bias (Correa & Yildirim, 2021). A specialty bias might be a consequence of intrinsic motivation to gather knowledge in a certain field. It is also possible that specialty bias is a consequence from monetary incentives to defend a specific cause. In the political context for example, interest groups are getting paid to influence the decision. They might manipulate the information they send to the decisionmaker to convince him to make their decision. Nevertheless, this information can still be valuable for the decisionmaker because of the specialized knowledge of the interest group. Therefore, the decisionmaker faces a tradeoff between the expertise and the preferences of the experts he is going to consult. If he consults

agents with a high level of expertise, the information he receives is biased. However, if he consults agents without a preference for a certain decision, the information he receives is of lower quality because of the lower expertise level of the agent.

In this paper, I will analyze the tradeoff the decisionmaker faces on which people to consult before he makes his final decision. Therefore, he has to choose between Biased Experts and Neutral Agents. Biased Experts have more expertise on a certain field but are also biased in the direction of their own expertise. Neutral Agents have less expertise but are neutral about the decision that has to be made. In the political environment for example, interest groups can be regarded as Biased Experts. The alternative for the decisionmaker is to consult neutral advisors with less specialized knowledge. In the example of a CEO deciding whether or not to split off a division, the internal manager can be seen as a Biased Expert while the external consultant is a Neutral Agent.

A lot of research has already been done on biased information collecting agents. Milgrom (1981) analyzes a situation in which a better-informed salesman is trying to convince a potential buyer to buy his product. The salesman possesses information about a random variable θ that determines the optimal decision of the buyer. The author argues that there is a sequential equilibrium in which the salesman reports the most favorite information about his product and the buyer takes a skeptical view of any information that is concealed. For example, when a salesman states that the quality of his product 'Meets or Exceeds a certain standard', the buyer might infer that the quality does not substantially exceed the standard. The assumption that the information cannot be misreported is essential to draw this conclusion.

In contrast, Crawford and Sobel (1982) assume that information can be misreported by the agent costless. They also conclude that the messages from the expert can still be valuable for the decisionmaker. However, perfect communication is not possible as long as the interests of the agent and the decisionmaker are not completely aligned. The informativeness of messages send by the agent depends on the difference between preferences of the agent and the decisionmaker. The closer their preferences are, the more informative the messages. A similar conclusion is drawn in the cheaptalk model presented in Farrell (1993; 1996). He argues that the language players use to communicate is rich and contains more possible messages than the ones often used in game theory. Cheap talk can help to avoid misunderstanding and coordination failures, but it does not necessarily lead to pareto efficiency.

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The insight that messages of better-informed agents with different preferences than the decisionmaker might still be informative, confirms the presumption that information from Biased Experts might be useful for the decisionmaker. However, Milgrom (1981) and Crawford and Sobel (1982) both analyze a situation in which one agent is providing the decisionmaker with information about the relevant state of the world. In the situations I described, the decisionmaker needs two pieces of information in order to make the best decision.

Milgrom and Roberts (1986) analyze a 'class of persuasion games' in which they analyze a situation where a decisionmaker has to decide based on information he receives from other agents. In one of these games there are two agents with opposing preferences that provide information. These agents both have private information about a random variable θ that determines the optimal decision of the decisionmaker. They conclude that competition between these agents can make sure that they will reveal their information.

In contrast, Gilligan and Krehbiel (1989) conclude that committee members cannot be induced to reveal all their information as long as the interests of the committee members and the decisionmaker are not completely aligned. They present a three-person game theoretic model of a legislature in which two heterogenous committee members are better-informed about a random variable ω than a third committee member that has the casting vote. The two better-informed committee members have opposing preferences. They find that extreme preference outlying committee members serve less of an informational role within the legislature than moderate committee members do. Krehbiel (1990) presents an empirical analysis about the composition of decision-making committees. He indeed finds no evidence that these committees are composed of extreme preference outliers.

In the model presented in Gilligan and Krehbiel (1990), the assumption that committee members have perfect information is relaxed. Before the game starts, committees can choose to gather information about ω at cost k. The committees can decide to acquire more relevant knowledge in order to give a better advice to the legislators. Once the committee has decided to gather information or not, the game is the same as in the model presented in Gilligan and Krehbiel (1989). They also find that extreme committee preferences have a negative impact on the informational efficiency gains. Therefore, they conclude that the legislature has to appoint perfectly representative committees to maximize the informational efficiency. This conclusion is comparable to

the conclusion of Crawford and Sobel (1982) and Gilligan and Krehbiel (1989) that full information revelation is not possible as long as the interests of the committee members and the decisionmaker are not completely aligned.

Krishna and Morgan (2001) present a model of expertise as an extension of the model of Crawford and Sobel (1982) to a setting with multiple experts. The decisionmaker is consulting two experts before he decides. These experts are perfectly informed about θ , the state of nature that determines the optimal decision. They conclude that two experts with 'like biases' are not informationally superior to consulting only one expert. When experts have opposing biases however, they conclude that there is always an equilibrium that is informationally superior to consulting only one expert. This conclusion is similar to the conclusion of Milgrom and Roberts (1986). At least one of the experts has to be a moderate expert rather than an extremist expert. If both experts are extremists, no information is transmitted in any equilibrium. This result is called the 'crossfire effect'. This conclusion is similar to the conclusion set in any equilibrium. This result is called the 'crossfire effect'. This conclusion is similar to the conclusion is similar to the conclusions of Crawford and Sobel (1982) and Gilligan and Krehbiel (1989; 1990). Experts are sending their messages sequentially and the biases of the experts are common knowledge in this model.

Bhattacharya and Mukherjee (2013) also did an attempt to find the optimal team composition in terms of quality (expertise) and agenda (preference) of the information collecting agents. They find that higher quality of agents is not necessarily better, extreme agendas are always preferred, and the optimal panel may consist of experts with identical agendas.

These papers analyzed situations in which two agents are providing the decisionmaker with relevant information. The two agents have information about the same variable that determines the optimal decision. The general conclusion is that competition between agents might lead to full information revelation but that the team of advisors should not be composed out of team members with extreme preference outliers. In the situations I described in the introduction however, there are two pieces of information that determine the optimal decision for the decisionmaker. In the example of the CEO, he needs information about the profitability of division A and about the profitability of division B. These are two different parameters. Another assumption in these models is that the better-informed agents have the same knowledge about the relevant state of the world. Therefore, it is not possible to analyze a difference in expertise levels of the agents.

Dewatripont and Tirole (1999) capture these issues in a model of decision-making under uncertainty where a decisionmaker has to decide either A, B or Status Quo. The optimal decision depends on a parameter $\theta \in \{-1, 0, 1\}$, where $\theta = \theta_A + \theta_B$. Decision A is optimal if $\theta = -1$, decision B is optimal if $\theta = 1$ and Status Quo is the optimal decision when $\theta = 0$. To decide, the decisionmaker needs agents to collect information regarding the values of θ_A and θ_B respectively. These agents are called advocates. Advocates have to incur a cost of effort K in order to investigate cause A or B. They assume that rewards are decision-based which results in advocates getting paid if their decision has been made. These advocates are therefore 'advocates by design of the game'. If an advocate incurs cost K there is a probability P that he finds information, given that it is available. If there is no information available, the advocate will find nothing. They conclude that it is better to hire two agents investigating the separate alternatives rather than having one agent investigating both alternatives. Two agents either find more information at the same cost of effort or the same amount of information at lower costs. Like Milgrom (1981), they find that competition between two agents has relative informational efficiency gains because one agent does not have an incentive to investigate both causes. This paper focuses on the agency problem where the decisionmaker wants to induce the agents to exert effort.

Dur and Swank (2005) analyzed the decision of the decisionmaker on which type of agents to select to collect information. They analyzed the uncertainty of the decisionmaker about the effort advisers put in the production of information and the risk that advisers manipulate information. They conclude that a biased decisionmaker faces a tradeoff between the quality of the recommendation and the quality of the information the recommendation is based on. To maximize the quality of the information, the policymaker should rely on unbiased advisers. To maximize the quality of the recommendation, the interests of the policy maker and the advisers should coincide. They do not distinguish between types of agents in terms of expertise.

To analyze the tradeoff between consulting Biased Experts and Neutral Agents, I will present a theoretical framework based on the model of Dewatripont and Tirole (1999). In this model, the decisionmaker has to choose between A, B or the Status Quo. His optimal decision depends on a variable $\theta = \theta_A + \theta_B$ where θ {-2, -1, 0, 1, 2}. The decisionmaker needs two different agents to inform him about the values of θ_A and θ_B . The model in this paper differs from the model of Dewatripont and Tirole (1999) in the sense that I analyze a tradeoff between two different types of agents while Dewatripont

and Tirole (1999) analyze a tradeoff between hiring one or two agents. To model the tradeoff between Biased Experts and Neutral Agents I allow the probability that agent *i* finds the information p_i to differ between different types of agents. This probability is higher for Biased Experts than for Neutral Agents such that $p_B > p_N$. I also added the possibility to manipulate information by adding an extra possible value for θ_A and θ_B . The agents are allowed to manipulate information by exaggerating information and present information that $|\theta_i| = 1$ as if it is information that $|\theta_i| = 2$. Biased Experts will manipulate the information they find because they have a biased preference in the direction of their own field of expertise. Neutral Agents do not have an incentive to manipulate information. Later, I alter the model by assuming that the decisionmaker only has to choose between option A and option B. I assume that option A is the optimal decision if $\theta \le 0$ and option B is the optimal decision if $\theta > 0$.

From the basic model with three possible decisions, I conclude that it is never optimal to consult two different types of agents because the probability that option A is the best decision is equal to the probability that option B is the best decision. If it is optimal to hire a Biased Expert as agent A, it is also optimal to hire a Biased Expert as agent B. Whether it is optimal to consult two Biased Experts or two Neutral Agents, depends on the ex-ante probabilities that there is information available and the extent of the expertise advantage of Biased Experts relative to Neutral Agents. If there is a relatively high probability that there is information, it is optimal to consult two Biased Experts because the absolute value of θ_i is important for the optimal decision. If there is a relatively low probability that there is information, it is optimal to consult two Biased Experts because it is more important to know if there is information available than to know the absolute value of θ_i .

If I alter the model to a situation in which there are two possible decisions, I find that there are situations in which it is optimal to consult two different types of Agents. If the decisionmaker believes that a Biased Expert tells the truth and the ex-ante probability that there is information is low, it might be optimal to consult a Neutral Agent about the Status Quo and a Biased Expert about the consequences of change. If the decisionmaker does not believe that a Biased Expert tells the truth and the ex-ante probability that there is information is relatively high, it might be optimal to consult a Biased Expert about the Status Quo and a matche the status Quo and a Neutral Agent about the consequences of change. If the decisionmaker does not believe that a Biased Expert tells the truth and the ex-ante probability that there is information is relatively high, it might be optimal to consult a Biased Expert about the Status Quo and a Neutral Agent about the consequences of change. It is possible that a mixed team is the optimal team composition in the model with two possible decisions because the probability that A is the best decision is not

equal to the probability that B is the best decision. A big difference between the two models is that there is no agent with a specialty bias in the direction of the Status Quo in the model with three possible decisions, while in the model with two possible decisions there are Biased Experts with a bias in the direction of the Status Quo. In the next section, I will present the theoretical model of decision-making under uncertainty based on the model of Dewatripont and Tirole (1999). Then I will analyze this model and interpret the results before I alter the model to a situation with two possible decisions. Finally, I will interpret the results and discuss some discussion points and topics for future research in the conclusion.

Model

The decision

To determine the best way to build a decision-making team, I will analyze a theoretical framework which is based on the model of Dewatripont and Tirole (1999). A decisionmaker has to make one of three decisions: A, B or Status Quo (SQ). In this framework, decisions A and B can be interpreted as two opposing decisions while SQ is an intermediate decision. For example, consider a company consisting of two different divisions that has to decide to divest one of the two divisions or to continue with both of them. Decision A would be equivalent to continue with division 1 only, decision B with division 2 only and the Status Quo would be equivalent to continuing with both divisions. The optimal decision depends on a parameter $\theta \in \{-2, -1, 0, 1, 2\}$, where $\theta = \theta_A + \theta_B$. θ_A takes on values -2 with probability α , -1 with probability β and 0 with probability $(1 - \alpha - \beta)$. α and β are both positive probabilities. This means that

$$\theta = \begin{cases} -2 \text{ with probability } \alpha(1 - \alpha - \beta) \\ -1 \text{ with probability } \alpha\beta + \beta(1 - \alpha - \beta) \\ 0 \text{ with probability } \alpha^2 + \beta^2 + (1 - \alpha - \beta)^2 \\ 1 \text{ with probability } \alpha\beta + \beta(1 - \alpha - \beta) \\ 2 \text{ with probability } \alpha(1 - \alpha - \beta) \end{cases}$$

The optimal decision is option A as long as $\theta < 0$ and option B if $\theta > 0$. The decisionmaker wants to decide Status Quo if $\theta = 0.^{1}$ This means that the probability that A is the optimal decision is equal to $\alpha\beta + (\alpha + \beta)(1 - \alpha - \beta)$. The same yields for the probability that B is the optimal decision. The probability that SQ is the optimal decision is equal to $\alpha^{2} + \beta^{2} + (1 - \alpha - \beta)^{2}$. All decisions are optimal with probability $\frac{1}{3}$ if $\alpha = \beta = (1 - \alpha - \beta) = \frac{1}{3}$.

¹ I assume that the decisionmaker's payoff of making the right decision is equal to 1 and the payoff of making the wrong decision is equal to 0. There is no difference in the payoffs of the two wrong decisions.

Presentation of information

As in the model of Dewatripont and Tirole (1999), the decisionmaker has to rely on information regarding θ_A and θ_B presented by two agents. The message agent *i* sends about θ_i is denoted by $\hat{\theta}_i$. One agent has a certain amount of knowledge in the field of decision A while the other agent has some expertise in the field of decision B. I will refer to these agents as agent A and agent B. When θ_i (i = A, B) $\neq 0$, Agent *i* has a probability of finding the information p_i . This probability depends on the expertise level of the agent. The higher his expertise level, the higher the probability that he finds the information. There is a probability that an agent does not find the available information $(1 - p_i)$. If $\theta_i = 0$, there is nothing for the agent to find. After observing the messages, the decisionmaker updates his beliefs about θ following the Bayes rule. Agents do not have to present the same information as they find. Therefore, I introduce the possibility to manipulate information. If an agent learns nothing, there is no information to present but If he finds weakly positive information $|\theta_i| = 1$, an agent has the opportunity to exaggerate and send message $|\hat{\theta}_i| = 2$. In the next section I will explain why agents would want to manipulate information.

Team composition

Before the agents can present their information, the decisionmaker has to decide which agents he wants to consult. There are two types of agents, Biased Experts and Neutral Agents. The types of agents differ in their level of expertise and their preference for a certain decision. Biased Experts have a higher level of expertise which results in a higher probability to find information ($p_B > p_N$). They have a biased preference towards their own field of expertise because of a specialty bias. Therefore, they will manipulate the information if they find $|\theta_i| = 1$ and present it as $|\theta_i| = 2$. Neutral Agents do not have a biased preference and will not manipulate information². In the situation of a CEO that has to decide whether to divest a division for example, the CEO of a company has to choose which agents to consult in order to make the best decision on divesting one of the divisions or not. He can choose between the managers of the divisions or external consultants. Managers of the divisions are better informed about the profitability of their own division than external consultants are, but they also have

² Effectively, I assume that Biased Experts obtain a positive payoff if the option that corresponds to their field of expertise is chosen. In contrast, Neutral Agents are indifferent. I assume that they communicate truthfully.

an incentive to manipulate information to make the decisionmaker believe that their particular division must continue to exist. Under full information (agents have no possibility to exaggerate information), it is obviously optimal for the decisionmaker to hire two Biased Experts. One for each cause. These experts have a higher probability

that they find information and once they find it, there is no possibility to lie about it. Once there is a possibility to manipulate information, it might be optimal to hire a Neutral Agent with a lower probability of finding the right information. The goal of the decisionmaker is to build a team that

Team Composition	Notification
Two Biased Experts	BB
Two Neutral Agents	NN
Agent A Biased Expert Agent B Neutral Agent	BN
Agent A Neutral Agent Agent B Biased Expert	NB

Table 1. Different team compositions

maximizes the probability of making the right decision. He can choose between four different team compositions. To make the analysis understandable I will refer to the different compositions as stated in table 1.

Assumption minimum levels p_B and p_N

When agent *i* does not find any information and sends message $\hat{\theta}_i = 0$, this does not mean that $\theta_i = 0$. The decisionmaker updates his beliefs using the Bayes rule which results in the following Bayesian beliefs after observing $\hat{\theta}_i = 0$:

- $|\theta_i| = 0$ with probability $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_i) + (1-\alpha-\beta)}$
- $|\theta_i| = 1$ with probability $\tilde{\beta} = \frac{\beta(1-p_i)}{(\alpha+\beta)(1-p_i) + (1-\alpha-\beta)}, \ \tilde{\beta} < \beta$
- $|\theta_i| = 2$ with probability $\tilde{\alpha} = \frac{\alpha(1-p_i)}{(\alpha+\beta)(1-p_i) + (1-\alpha-\beta)}, \ \tilde{\alpha} < \alpha$

I assume that the Bayesian belief of the decisionmaker that $|\theta_i| = 0$ is high enough to affect the decision. Therefore, I assume that the decisionmaker believes that $\theta_i = 0$ after observing $|\hat{\theta}_i| = 0$. This results in the condition that the Bayesian belief that $|\theta_i| = 0$ exceeds the Bayesian beliefs that $|\theta_i| = 1$ and $|\theta_i| = 2$.

$$(1 - \alpha - \beta) > \alpha(1 - p_i)$$
$$(1 - \alpha - \beta) > \beta(1 - p_i)$$

If rewrite these conditions I find that $p_B > p_N > \frac{2\alpha+\beta-1}{\alpha}$ and $p_B > p_N > \frac{\alpha+2\beta-1}{\beta}$. If p_B and p_N are not high enough, the Bayesian belief that $|\theta_i| > 0$ after $\hat{\theta}_i = 0$ is too high and a message $\hat{\theta}_i = 0$ would be worthless. This might result in situations where the decisionmaker decides a case for which no information is found. For example, if $\hat{\theta}_A = 0$, and $\hat{\theta}_B = 1$, the decisionmaker might decide option A if his Bayesian belief that $\theta_A = -2$ exceeds his belief that $\theta_A = -1$ or $\theta_A = 0$. Especially the messages of Biased Experts would be worthless if these assumptions do not hold. The decisionmaker will decide SQ with every possible combination of messages he receives from team composition BB because he counts on his prior beliefs about α and β . Mathematical explanation of this assumptions can be found in appendix A.

Analysis

In order to find the optimal team composition, I will use backward induction. Figure 1 shows the different steps of the decision-making process described in the previous section. Before I can determine the probabilities of making the right decision for the different team compositions, I will take a look at the final decision of the decisionmaker and how he updates his beliefs following the Bayes rule.



Figure 1. Decision-making process

The decision

The final decision of the decisionmaker affects the welfare of the decisionmaker as well as the welfare of the agents representing one of the causes. In order to decide, the decisionmaker forms his Bayesian beliefs about θ based on the messages $\hat{\theta}_A$ and $\hat{\theta}_B$ he receives from agents *A* and *B*. The goal of the decisionmaker is to make the best decision possible for the organization.

- If the decisionmaker believes $\theta < 0$, he decides A
- If the decisionmaker believes $\theta > 0$, he decides B
- If the decisionmaker believes $\theta = 0$, he decides *Status Quo* (*SQ*)

I will consider the Bayesian beliefs of the decisionmaker about θ_i after messages from Neutral Agents and Biased Experts separately.

Neutral Agents

Since Neutral Agents do not have an incentive to exaggerate information, they will present the same information as they find. The Bayesian beliefs of the decisionmaker about θ_i after messages $\hat{\theta}_i$ are therefore as follows:

- If he receives message $|\hat{\theta}_i| = 2$ from a Neutral Agent, he believes that $|\theta_i| = 2$

- If he receives message $|\hat{\theta}_i| = 1$ from a Neutral Agent, he believes that $|\theta_i| = 1$
- If he receives message $|\hat{\theta}_i| = 0$ from a Neutral Agent, he believes that

$$\begin{array}{l} \circ \quad |\theta_i| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}, \ \tilde{\alpha} < \alpha \\ \\ \circ \quad |\theta_i| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}, \ \tilde{\beta} < \beta \\ \\ \circ \quad |\theta_i| = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \end{array}$$

Biased Experts

Biased Experts do have an incentive to exaggerate information if they find $|\theta_i| = 1$. Therefore, they will present $|\hat{\theta}_i| = 2$ if they find any positive information regarding the cause they are investigating. If they do not find anything they will present $|\hat{\theta}_i| = 0$. The Bayesian beliefs of the decisionmaker about θ_i after messages $\hat{\theta}_i$ are as follows:

- If he receives message $|\hat{\theta}_i| = 2$ from a Biased Expert, he believes that:
 - $|\theta_i| = 2$ with probability $\frac{\alpha}{\alpha + \beta}$

$$\circ |\theta_i| = 1$$
 with probability $\frac{\beta}{\alpha + \beta}$

 $\circ |\theta_i| \neq 0$

- If he receives message $|\hat{\theta}_i| = 1$ from a Biased Expert, he believes that $|\theta_i| = 1$

- This is an out-of-equilibrium situation. If the decisionmaker believes that a Biased Expert tells the truth, the Biased Expert has an incentive to send $|\hat{\theta}_i| = 2$ if he finds $|\theta_i| = 1$.
- If he receives message $|\hat{\theta}_i| = 0$ from a Biased Expert, he believes that

$$\circ \quad |\theta_i| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}, \ \tilde{\alpha} < \alpha$$

$$\circ \quad |\theta_i| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}, \, \tilde{\beta} < \beta$$

•
$$|\theta_i| = 0$$
 with probability $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}$

After observing both messages and updating his beliefs, the decisionmaker makes his final decision. For example, when a team composed of two Neutral agents present messages $\hat{\theta}_A = -2$ and $\hat{\theta}_B = 1$, the decisionmaker knows that both team members speak the truth and he will decide option A. When at least one of the agents is Biased

Expert, some situations are more complex because Biased Experts will always send message $|\theta_i| = 2$ if they find any positive information.

Table 2 shows the decisions the decisionmaker will make based on his updated beliefs after observing different combinations of messages with different team compositions. Mathematical proof of these decisions can be found in appendix B. A striking result is that decisions of the decisionmaker with teams that are composed of one Biased Expert and one Neutral Agent in some situations depend on the ex-ante probabilities that $|\theta_i| > 0 \alpha$ and β . This is a result of the uncertainty about the value of θ_i after a message $|\hat{\theta}_i| = 2$ from a Biased Expert. If $\alpha > \beta$, the decisionmaker believes that $|\theta_i| = 2$ while the decisionmaker believes that $|\theta_i| = 1$ if $\beta > \alpha$. Because of the symmetry in this model, this does not play a role with team composition BB. With a mixed team composition however, these different beliefs result in two different decisions in situations where both agents find information.

$\widehat{ heta}_A$	$\widehat{ heta}_B$	BB	NN	BN	NB
-2	2	SQ	SQ	B if $\beta > \alpha$	A if $\beta > \alpha$
				SQ if $\alpha > \beta$	SQ if $\alpha > \beta$
-2	1	A [*]	А	A if $\alpha > \beta$	A^*
				SQ if $\beta > \alpha$	
-2	0	А	А	А	А
-1	2	B*	В	B*	B if $\alpha > \beta$
					SQ if $\beta > \alpha$
-1	1	SQ [*]	SQ	SQ [*]	SQ [*]
-1	0	A*	А	A*	А
0	2	В	В	В	В
0	1	B*	В	В	B*
0	0	SQ	SQ	SQ	SQ

Table 2. Decisions after observing different combinations of messages. * = out-of-equilibrium situation

Presentation of information

Once the agents observed their information they have the opportunity to present it to the decisionmaker by sending messages $\hat{\theta}_A$ and $\hat{\theta}_B$. If they do not find information,

there is no information to present and they send message $\hat{\theta}_i = 0$. This does not necessarily mean that $\theta_i = 0$ because there is a probability that there is information available, but it is not found by the agent. If an agent finds that $|\theta_i| = 1$, he is sure that $|\theta_i| = 1$. However, there is a difference in the messages the different types of agents will send to the decisionmaker. A Neutral Agent will send $|\hat{\theta}_i| = 1$, while a Biased Expert will send $|\hat{\theta}_i| = 2$. If an agent finds that $|\theta_i| = 2$, he is sure that $|\theta_i| = 2$. In this case both types of agents will send a message $|\hat{\theta}_i| = 2$.

Team composition

The decisionmaker has to decide which type of agents he wants to consult before the agents are able to present their information. The goal of the decisionmaker is to maximize the probability of making the right decision *P*. I calculated the probability of making the right decision for different team compositions by calculating the probabilities for all nine possible combinations of θ_A and θ_B separately. The ex-ante probability that a team composition leads to the right decision is equal to the sum of these probabilities multiplied by the probability that the situation occurs. I found the following probabilities of making the right decision for the different team compositions.

Two Neutral Agents:

 $P(NN) = 2(\alpha^{2} + \beta^{2})p_{N}^{2} + 2(\alpha - 2\alpha^{2} + \beta - 2\beta^{2} - \alpha\beta)p_{N} + \alpha^{2} + \beta^{2} + (1 - \alpha - \beta)^{2}$

Two Biased Expert: $P(BB) = 2(\alpha^2 + \beta^2 - \alpha\beta)p_B^2 + 2(\alpha - 2\alpha^2 + \beta - 2\beta^2 - \alpha\beta)p_B + \alpha^2 + \beta^2 + (1 - \alpha - \beta)^2$

Mixed Team:

$$\begin{aligned} - & \text{If } \alpha \geq \beta : \\ P(Mix) &= (2\alpha^2 + \beta^2 - \alpha\beta)p_B p_N + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - \alpha\beta)(p_B + p_N) + \alpha^2 + \beta^2 + (1 - \alpha - \beta)^2 \\ - & \text{If } \beta > \alpha : \\ P(Mix) &= (\alpha^2 + 2\beta^2 - \alpha\beta)p_B p_N + (\alpha - 2\alpha^2 + \beta - 2\beta^2 + \alpha\beta)(p_B + p_N) + \alpha^2 + \beta^2 + (1 - \alpha - \beta)^2 \end{aligned}$$

Because of the symmetry in this model the results for one mixed team composition also yields for the other mixed team composition where the Biased Expert and the Neutral Agent are switched. Therefore, I only need to check one example of a mixed team. Table 2 showed that the final decision with a mixed team might depend on α and

 β . This results in two different equations for the probability of making the right decision with a mixed team, one if $\alpha > \beta$ and one if $\beta > \alpha$. When $\alpha = \beta$ these probabilities are the same. The mathematical explanations of these expressions can be found in appendix B.

From these equations I can conclude that the probability of making the right decision with a mixed team never exceeds the probability of making the right decision with two Biased Experts as long as $\alpha = \beta$. A mixed team only has a higher probability of making the right decision if $p_N > p_B$, which is not possible in this model. If $\alpha \neq \beta$, there are combinations of p_B and p_N for which a mixed team leads to a higher probability of making the right decision than a team with two Biased Experts. The expressions show that the optimal team composition depends on α , β , p_B and p_N but I cannot conclude much more about which team composition is the best. In the next section I will take a closer look at the results by looking at different situations in terms of values of α and β .

Numerical simulations

To draw conclusions about the optimal team composition, I plotted the combinations of p_B and p_N for which the decisionmaker is indifferent between two possible team compositions. At first, I will analyze the situation where $\alpha = \beta = (1 - \alpha - \beta) = \frac{1}{3}$. These results are to be found in figure 2. Later, I will analyze different situations in terms of α and β as well.

The first thing that stands out in figure 2 is that a mixed team is indeed never preferred over BB. The points where the decisionmaker is indifferent between BB and Mix are on the grey line that depicts $p_B = p_N$. A mixed team is preferred over BB in the area to the right of this line, where $p_N > p_B$. These situations are not possible in this model. The other indifference curves ensure that there are three areas left in which the decisionmaker has a different



order of preferences on the team compositions. I numbered these areas 1-3 in figure 2. To the left of the black solid line, in area 1, P(Mix) exceeds P(NN). However, to the left of the interrupted line, P(BB) exceeds P(NN). This means that in area 1 P(BB) >P(Mix) > P(NN). The order of preferences of team compositions in the different areas in this situation can be found in table 3. A mixed team is never the optimal team composition because there is always at least one team composition that leads to a

higher probability of making the right decision. Team composition BB leads to the probability highest of making the right decision in

Area	Order of preferences
1	P(BB) > P(Mix) > P(NN)
2	P(BB) > P(NN) > P(Mix)
3	P(NN) > P(BB) > P(Mix)

composition NN leads to

area 1 and 2 while team Table 3. Probabilities making the right decision for team compositions when $\alpha = \beta = \frac{1}{3}$

the highest probability of making the right decision in the shaded area 3. I plotted these indifference curves for all possible combinations of α and β with steps of 0.1. I present eight of them in figure 3-10 to say something about different situations and draw some general conclusions. The first four figures show situations in which $\alpha = \beta$. In figure 7-10, I analyze situations for which $\alpha \neq \beta$. The situations to the right of the grey line that depicts $p_B = p_N$ are not relevant because there $p_N > p_B$, which is not possible in this model.







Figure 5. Three possible decisions, $\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$









Figure 3-6 show that a mixed team is indeed never preferred over BB as long as $\alpha = \beta$. The points where the decisionmaker is indifferent are on the grey line that depicts $p_B = p_N$. A mixed team is preferred over BB in the area to the right of this line, when $p_N > p_B$, but these situations are not possible in this model.

There are some more conclusions that I can draw by analyzing these figures. It is possible that a mixed team is preferred over a team that consists of two members of the same type, but a mixed team is never preferred over both team compositions with two agents of the same type at the same time. To illustrate this, I will analyze figure 7 where $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{4}$ in the same way as I analyzed figure 2. I numbered the relevant areas in figure 11 with numbers 1-4. There is one more relevant area to analyze because there is a probability that P(Mix) exceeds P(BB) now. If I analyze the situation

within area 1, I find that a mixed team is preferred over NN to the left of the black solid line that shows the points for which the decisionmaker is indifferent between a mixed team and NN. However, I also find that BB is preferred over NN in the areas to the left of the dash dotted line that depicts the points for which the decisionmaker is indifferent between BB and NN. Therefore, I can conclude that a mixed team is preferred over NN but not over BB within area 1. This means that



Figure 11. Figure 7. with numbered area's

P(BB) > P(MIX) > P(NN). The results for the other areas can be found in table 4. From these results I can conclude that it is never optimal to hire two different types of team members and compose a mixed team, even if $\alpha \neq \beta$. These results also yield for the other figures where $\alpha \neq \beta$, which results in proposition 1.

Proposition 1: In the model with three possible decisions, it is never optimal hire a team that consists of two different types of team members because at least one of the two possible team compositions with two members of the same type leads to a higher probability of making the right decision.

If a Biased Expert as agent A does not find any information and sends a message $\hat{\theta}_A = 0$, the decisionmaker believes that $\theta_A = 0$. It does not matter whether $\theta_B = 1$ or $\theta_B = 2$. As long as the decisionmaker believes that $\theta_B > 0$, he will decide option B.

Area	Order
1	P(BB) > P(MIX) > P(NN)
2	P(BB) > P(NN) > P(MIX)
3	P(NN) > P(BB) > P(MIX)
4	P(NN) > P(MIX) > P(BB)

Table 4. Preferences for different team compositions

Therefore, it is optimal to hire a Biased Expert as agent B as well. However, if a Biased Expert does find information and sends a message $\hat{\theta}_A = -2$, the absolute value of θ_B does matter and the decisionmaker might prefer a Neutral Agent as Agent B if p_N is not too low. If a Neutral Agent as agent A does not find any information and sends a

message $\hat{\theta}_A = 0$, the decisionmaker will prefer a Biased Expert as agent B for the same reason as before. He does not care whether $\theta_B = 1$ or $\theta_B = 2$. As long as the decisionmaker believes that $\theta_B > 0$, he will decide option B. If a Neutral Agent as agent A does find information, it does matter whether $\theta_B = 1$ or $\theta_B = 2$ and the decisionmaker will prefer a Neutral Agent as agent B as well if p_N is not too low. In situations where there is no information regarding at least one of the two alternatives, a mixed team composition might be preferred over team composition NN, but not over team composition BB. In situations where there is positive information available regarding both alternatives, a mixed team might be preferred over team composition BB but not over team composition NN.

This means that either team composition BB or team composition NN leads to the highest probability of making the right decision. In figure 7, BB is the optimal team composition in area 1 and 2 while NN is the optimal team composition in area 3 and 4. As in figure 2, the shaded area's in figure 3-10 show the combinations for p_B and p_N for which NN is the optimal team composition. BB is the optimal team composition in the non-shaded areas.

If I take a look at the different figures, there are a few things that pop out. The first thing that stands out is that NN is preferred over BB if p_B and p_N are close to each other. For BB to be the optimal team composition, the difference between p_B and p_N must be big enough to compensate for the fact that Biased experts will not always tell the truth about the information they find. The second thing that pops out is that the minimal difference between p_B and p_N that makes BB the optimal team composition increases with p_N . In other words, a higher value of p_N means that the difference between p_B and p_N must be bigger to make BB the optimal team composition. The last thing that stands out is that higher values of α and β results in an increase of the surface of the shaded area. This means that a higher probability of information being available makes it more attractive to hire two Neutral Agents. When $\alpha + \beta = 1$, NN will be preferred over BB for every combination of p_B and p_N . Since the probability that there is information available is 1 in that situation, messages of Biased Experts are worthless because they do not tell anything about the specific value of θ_i . However, these extreme cases do not exist because of the assumption that $(1 - \alpha - \beta) > \alpha(1 - p_N)$ and $(1 - \alpha - \beta) > \beta(1 - p_N)$. When the probability that there is no information available $(1 - \alpha - \beta)$ approximates 1, team composition BB will be preferred over NN for almost all the possible combinations of p_B and p_N . Biased Experts have a higher probability of finding information if it is available. Once it is found, the probability that there is information available for the opposite case as well is relatively low. Therefore, it does not matter if the Biased Expert exaggerates his information.

Proposition 2: Whether an optimal team consists of two Biased Experts or two Neutral Agents depends on the difference between p_B and p_N and the ex-ante probability that there is information available $\alpha + \beta$. The minimal difference between p_B and p_N to make two Biased Experts optimal increases with the absolute value of p_N . A higher probability of information being available results in a higher probability that an optimal team consists of two Neutral Agents.

In the example of a CEO of a company that has to decide whether to divest one of two divisions or to continue with both of them, the CEO can choose to consult external consultants or the managers of the two divisions respectively. The external consultants are Neutral Agents that have the same goal as the CEO, maximizing the profit of the company. The managers of the divisions are Biased Experts that have a lot of knowledge about their own division but are biased in the direction of their own division. The CEO is better off by hiring two external consultants both investigating the profitability of one of the two divisions than consulting the managers of the divisions if the information about the profitability of the divisions is publicly available and no specific knowledge about the division is needed to interpret it. If the information is hard to find and understand for outsiders, it might be a better idea to consult the managers of the divisions. The ex-ante probabilities that divesting one of the divisions is the optimal decision also plays a role in the decision which agents to consult. If the ex-ante probability that divesting one of the two divisions is the optimal decision is high, it is better to hire external consultants but if the ex-ante probability that continuing with both divisions is high it might be better to consult the managers of the divisions themselves. Hiring an external consultant for one division while consulting a manager for the other will never lead to the highest probability of making the right decision. If the ex-ante probability that the optimal decision is to divest one of the two divisions is relatively high, consulting two different agents might be preferred over consulting two managers. However, it will not be preferred over consulting two external consultants. If the exante probability that the optimal decision is to divest one of the two divisions is relatively low, there are situations in which consulting two different agents is preferred over consulting two external consultants. However, in these situations it is optimal to consult two internal managers.

Mixed teams are never preferred over teams consisting of two agents of the same type because of the symmetry in this model. The ex-ante probability that A is the optimal decision is always equal to the ex-ante probability that B is the optimal decision. In the next section I will alter the model to analyze a situation in which these probabilities are different to see if this creates situations in which a mixed team leads to the highest probability of making the right decision.

Two possible decisions

In this section, I am going to alter the model in a way that the decisionmaker faces a different type of decision. Rather than choosing between A, B and Status Quo, the decisionmaker now has to choose between options A and B only. Therefore, I assume that A is the optimal decision if $\theta \leq 0$ and B is the optimal decision when $\theta > 0$. The ex-ante probability that A is the optimal decision is then $\alpha^2 + \beta^2 + (1 - \alpha - \beta) + \alpha\beta$. The ex-ante probability that B is the optimal decision is still $(\alpha + \beta)(1 - \alpha - \beta) + \alpha\beta$. When a manager faces a certain decision in which he has to implement a new program or not, he has to change something or stay at the status quo for example. The status quo is option A in this framework while change is option B. When $\theta = 0$ the status quo is preferred because there are some small costs of implementing change. Think about switching costs or risk of failure. I do not consider these costs in the model because I am not working with utility outcomes of the decision, but these costs make sure that the status quo (option A) is preferred when $\theta = 0^3$. Another example is the Dutch government deciding whether to use gas or green energy as the main source of energy. There is no 'in-between' option, the government has to choose one of the two options.

Assumption

I assumed that in situations where one agent finds information and the other agent does not, the decisionmaker will make the decision for which information is found. In the previous model with three possible decisions this resulted in the assumption that the Bayesian belief that $|\theta_i| = 0$ given that $|\hat{\theta}_i| = 0$ must exceed the Bayesian beliefs that $|\theta_i| = 2$ or $|\theta_i| = 1$. In this model with two possible decisions, I need a stronger assumption. The Bayesian belief that $|\theta_i| = 0$ given that $|\hat{\theta}_i| = 0$ must exceed the belief that $|\theta_i| \neq 0$ now. Therefore, I need to make the following assumption in the model with two possible decisions: $(1 - \alpha - \beta) > (\alpha + \beta)(1 - p_N) > (\alpha + \beta)(1 - p_B)$. If I rewrite this I find that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$. Mathematical explanation of this stronger assumption can be found in the second part of appendix A.

³ The utilities of the decisionmaker and the agents are still equal to 1 if the right decision is made and 0 if not.

Analysis

The beliefs of the decisionmaker and the messages that agents will send are the same as in the situation with three possible decisions before. A Neutral Agent will present the same information as he finds. A Biased Expert will present $|\hat{\theta}_i| = 0$ if he finds nothing and $|\hat{\theta}_i| = 2$ otherwise. This results in the decisions after observing the possible combinations of messages received from different team compositions presented in table 5.

$\widehat{ heta}_A$	$\widehat{oldsymbol{ heta}}_B$	BB	NN	BN	NB
-2	2	A	А	B if $\beta > \alpha$	А
				A if $\alpha > \beta$	
-2	1	A*	А	А	A [*]
-2	0	А	А	А	А
-1	2	B if $\alpha > \beta^*$	В	Β*	B if $\alpha > \beta$
		A if $\beta > \alpha$			A if $\beta > \alpha$
-1	1	A [*]	А	A [*]	A [*]
-1	0	A*	А	A*	А
0	2	В	В	В	В
0	1	B*	В	В	B*
0	0	A	А	А	А

Table 5. Decisions after observing different combinations of messages. * = Out-of-equilibrium situation

I calculated the probabilities of making the right decision for the different team compositions in the same way as I did in the previous model with three possible decisions. Because the model is not symmetric anymore, I have to consider both types of mixed team compositions. In the first mixed team agent A is a Biased Expert while in the second mixed team agent A is a Neutral Agent. Table 5 shows that decisions after observing messages from mixed teams might depend on α and β again. As in the model with three possible decisions, this is a result of the uncertainty about the value of θ_i after a message $|\hat{\theta}_i| = 2$ from a Biased Expert. The probabilities of making the right decision for the different team composition are as follows:

Two Neutral Agents:

 $P(NN) = (\alpha^2 + \beta^2 + \alpha\beta)p_N^2 + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_N + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$

Two Biased Experts: $P(BB) = (\alpha^2 + \beta^2)p_B^2 + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_B + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$

Agent A Biased Expert, Agent B Neutral Agent:

- If
$$\alpha \ge \beta$$
:

$$P(BN) = (\alpha^{2} + \beta^{2})p_{B}p_{N} + (\alpha - 2\alpha^{2} + \beta - 2\beta^{2} - 2\alpha\beta)p_{N} + (1 - \alpha - \beta) + \alpha^{2} + \beta^{2} + \alpha\beta$$
- If $\beta > \alpha$:

$$P(BN) = (\alpha\beta + \beta^{2})p_{B}p_{N} + (\alpha - 2\alpha^{2} + \beta - 2\beta^{2} - 2\alpha\beta)p_{N} + (1 - \alpha - \beta) + \alpha^{2} + \beta^{2} + \alpha\beta$$

Agent A Neutral Agent, Agent B Biased Expert:

- If
$$\alpha \ge \beta$$
:

$$P(NB) = (\alpha\beta + \alpha^2)p_Bp_N + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_B + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$$
- If $\beta > \alpha$:

$$P(NB) = (\alpha^2 + \beta^2)p_Bp_N + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_B + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$$

Mathematical explanation of these expressions can be found in appendix C. As in the model with three possible decisions, the expressions show that the optimal team composition depends on α , β , p_B and p_N . It is not possible to draw a conclusion about the optimal team composition by looking at these expressions. However, in every situation I can eliminate at least one possible mixed team composition because team composition BB leads to a higher probability of making the right decision. To show this, I will compare the expressions for P(BB) with the expressions for P(BN) and P(NB) separately in situations where $\alpha > \beta$, $\beta > \alpha$ and $\alpha = \beta$.

 $\alpha > \beta$

If I look at the tradeoff between BB and BN, I see that the optimal team composition depends of α , β , p_B and p_N . To take a closer look at these relationships, I calculate the difference between P(BB) and P(BN). If P(BB) - P(BN) > 0, team composition BB leads to a higher probability of making the right decision than team composition BN. Team composition BN leads to a higher probability of making the right decision than team team team composition BB if P(BB) - P(BN) < 0. If I calculate this difference I find that:

$$P(BB) - P(BN) = (\alpha^{2} + \beta^{2})(p_{B}^{2} - p_{B}p_{N}) + (\alpha - 2\alpha^{2} + \beta - 2\beta^{2} - 2\alpha\beta)(p_{B} - p_{N})$$

I can rewrite this to:

$$(p_B - p_N)(\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta + \alpha^2 p_B + \beta^2 p_B)$$

When I fill in the minimal value of $p_B = \frac{2\alpha+2\beta-1}{\alpha+\beta}$, I find that P(BB) - P(BN) > 0 because $p_B - p_N > 0$ and $(\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta + \left(\frac{2\alpha+2\beta-1}{\alpha+\beta}\right)(\alpha^2 + \beta^2)) = \frac{2\alpha\beta(1-\alpha-\beta)}{\alpha+\beta} > 0$. The difference between P(BB) and P(BN) increases with p_B , which results in the conclusion that P(NB) will never exceed P(BB). Therefore, I can conclude that team composition BN will never lead to a higher probability of making the right decision than team composition BB as long as $\alpha > \beta$.

To take a closer look at the relationship between team compositions BB and NB, I calculate the difference between P(BB) and P(NB). If P(BB) - P(NB) > 0, team composition BB leads to a higher probability of making the right decision than team composition NB. Team composition NB leads to a higher probability of making the right decision than team composition BB if P(BB) - P(NB) < 0. If I calculate this difference I find that:

$$P(BB) - P(NB) = (\alpha^{2} + \beta^{2})p_{B}^{2} - (\alpha\beta + \alpha^{2})p_{B}p_{N} = \alpha^{2}p_{B}(p_{B} - p_{N}) + \beta p_{B}(\beta p_{B} - \alpha p_{N})$$

From this equation I can conclude that the larger the difference between p_B and p_N , the larger the difference between α and β has to be for NB to lead to a higher probability of making the right decision than BB. In other words, the smaller the difference between p_B and p_N , the higher the probability that team composition NB is preferred over team composition BB. The assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ does not rule out the possibility that there are combinations of α, β, p_B and p_N for which team composition NB leads to a higher probability of making the right decision than team composition BB.

$\beta > \alpha$

If I look at the tradeoff between BB and BN when $\beta > \alpha$, I see that the optimal team composition depends on α , β , p_B and p_N again. I will calculate the difference between P(BB) and P(BN) to take a closer look at these relationships. If P(BB) - P(BN) > 0, team composition BB leads to a higher probability of making the right decision than team composition BN. Team composition BN leads to a higher probability of making the right decision than the right decision than team composition BB if P(BB) - P(BN) < 0. If I calculate this difference I find that:

$$P(BB) - P(BN) = (\alpha^{2} + \beta^{2})p_{B}^{2} - (\alpha\beta + \beta^{2})p_{B}p_{N} + (\alpha - 2\alpha^{2} + \beta - 2\beta^{2} - 2\alpha\beta)(p_{B} - p_{N})$$

I can rewrite this to:

$$(\beta^2 p_B + \alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)(p_B - p_N) + \alpha p_B(\alpha p_B - \beta p_N)$$

It is not possible to draw a similar conclusion as in the previous situations where $\alpha > \beta$. The assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ does not rule out the possibility that there are combinations of α , β , p_B and p_N for which team composition BN leads to a higher probability of making the right decision than team composition BB.

To take a look at the relationship between team composition BB and team composition NB, I will calculate the difference between P(BB) and P(NB) again. If P(BB) - P(NB) > 0, team composition BB leads to a higher probability of making the right decision than team composition NB. Team composition NB leads to a higher probability of making the right decision than team composition BB if P(BB) - P(NB) < 0. If I calculate this difference I find that:

$$P(BB) - P(NB) = (\alpha^{2} + \beta^{2})p_{B}^{2} - (\alpha^{2} + \beta^{2})p_{B}p_{N}$$

From this equation I can draw the conclusion that P(BB) > P(NB) as long as $\beta > \alpha$ because $p_B > p_N$. In other words, team composition NB will never lead to a higher probability than team composition BB as long as $\beta > \alpha$. $\alpha = \beta$

If $\alpha = \beta$, both mixed team compositions will never lead to a higher probability of making the right decision than team composition BB. Both equations for P(BN) are equal to each other if $\alpha = \beta$. To calculate P(BB) - P(BN), I can use the same equations as in the situations where $\alpha > \beta$. From these equations I can draw the conclusion that P(BB) - P(BN) > 0 as long as the assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ holds.

For the same reason, I can use the same expressions to calculate P(BB) - P(NB) as in the situations where $\beta > \alpha$. From these equations I can draw the conclusion that P(BB) - P(NB) > 0 because $p_B > p_N$. Therefore, I can conclude that a mixed team composition will never be the optimal team composition as long as $\alpha = \beta$.

Proposition 3: In the model with two possible decisions, team composition BN never leads to a higher probability of making the right decision than team composition BB as long as $\alpha \ge \beta$. Team composition NB never leads to higher probability of making the right decision than team composition BB as long as $\beta \ge \alpha$.

To explain this result, I will look at the situations where $\alpha \ge \beta$ and $\beta \ge \alpha$ separately.

$\alpha \geq \beta$

As long as $\alpha \ge \beta$, team composition BN will never lead to a higher probability of making the right decision than team composition BB. If a Biased Expert as agent A finds information and sends a message $\hat{\theta}_A = -2$, the decisionmaker will believe that $\theta_A =$ -2. It does not matter what information agent B presents, the decisionmaker will decide option A anyway. If a Biased Expert as agent A does not find any information and sends a message $\hat{\theta}_A = 0$, the decisionmaker believes that $\theta_A = 0$. The decisionmaker does not care whether $\theta_B = 1$ or $\theta_B = 2$. As long as he believes that there is positive information regarding θ_B , he will decide option B. Since Biased Experts have a higher probability of finding this information if it is available, it is better to hire a Biased Expert for cause B. Therefore, team composition BB will always lead to a higher probability of making the right decision than team composition BN as long as $\alpha \ge \beta$.

$\beta \geq \alpha$

As long as $\beta \ge \alpha$, team composition NB will never lead to a higher probability of making the right decision than team composition BB. If a Biased Expert as Agent B finds information and sends a message $\hat{\theta}_B = 2$, the decisionmaker believes that $\theta_B = 1$. It does not matter whether agent A sends message $\hat{\theta}_A = -1$ or $\hat{\theta}_A = -2$, the decisionmaker will decide option A if he believes that there is positive information regarding θ_A . Since Biased Experts have a higher probability of finding that information if it is available, it is better to consult a Biased Expert for cause A as well. If a Biased Expert as agent B sends a message $\hat{\theta}_B = 0$, the decisionmaker believes that $\theta_B = 0$. It does not matter what information agent A presents, the decisionmaker will decide option A. Therefore, team composition BB will always lead to a higher probability of making the right decision than team composition NB as long as $\beta \ge \alpha$.

It is not possible to compare the equations for P(NN) with the equations of the other team compositions analytically in the same way. As in the model with three possible decisions, I will take a closer look at the results for these team compositions by looking at different situations in terms of α and β graphically in the next section.

Numerical simulations

As in the model with three possible decisions, I plotted the combinations of p_B and p_N for which the decisionmaker is indifferent between two possible team compositions. I did it for all possible combinations of α and β with steps of 0.1. As in the previous model, I present eight of them here to say something about different situations and draw some general conclusions. I will analyze situations in which $\alpha = \beta$, $\alpha > \beta$ and $\beta > \alpha$ separately.





Figure 12-14 show that team composition BB indeed always leads to a higher probability of making the right decision than both mixed team compositions BN and NB. The points where the decisionmaker is indifferent between a mixed team composition and team composition BB are on the grey line that depicts $p_B = p_N$. Both mixed team compositions are preferred over team composition BB in the area to the right of this line, when $p_N > p_B$. These situations are not possible in this model.

Figure 15 shows a more complex situation because it looks like it is possible that team compositions BN is preferred over team composition BB. In order to analyze this situation, I numbered the relevant area's and analyzed which team composition is

preferred in the different areas. The numbered areas can be found in figure 19, the order in which the team compositions are preferred in table 3. The figure shows that BN is the optimal team composition in area 2 and area 3 where $p_B < \frac{1}{2}$ and p_N is not too high. However, these areas cannot be reached if I look at the assumption that $p_{B} > p_{N} >$ $\frac{2\alpha+2\beta-1}{\alpha+\beta}$. If I solve this for $\alpha = \beta = \frac{2}{5}$, I find that $p_B > p_N > \frac{3}{4}$. This means that only area 1, 5



Figure 20. Figure 15 with numbered areas

and 6 have to be considered and the decisionmaker will hire two agents of the same type. In area 1 and area 6 he will decide to hire team composition BB and in area 5 he will decide to hire team composition NN. I looked at the other situations in which $\alpha = \beta$ as well and I found that BN is never optimal because of the assumption that $p_B > p_N >$ $\frac{2\alpha+2\beta-1}{\alpha+\beta}$. Just as in the previous model, it is possible that BN is preferred over NN, but

Area	Order
1	P(BB) > P(BN) > P(NN)
2	P(BN) > P(BB) > P(NN)
3	P(BN) > P(NN) > P(BB)
4	P(NN) > P(BN) > P(BB)
5	P(NN) > P(BB) > P(BN)
6	P(BB) > P(NN) > P(BN)

it will never be the optimal team composition. Therefore, the decisionmaker will hire a team with two team members of the same type in these situations. In the situations in the shaded areas in figure 12-14, he will decide to hire NN. In the non-shaded

areas, he will decide to hire BB. These results are similar to the results in the previous model with three possible decisions. NN is preferred over BB if p_B and p_N are close, the minimal difference between p_B and p_N that makes BB the optimal team composition increases with p_N and a higher probability of information being available $(\alpha + \beta)$ makes it more attractive to hire two Neutral Agents. I can conclude that both mixed team compositions are never optimal as long as $\alpha = \beta$ and NN is preferred over BB if p_B and p_N are close and the probability of information being available $(\alpha + \beta)$ is high enough. The minimal difference between p_B and p_N that makes BB the optimal team composition increases with p_N .

Table 6. Preferences for different team compositions in figure 20



- $\alpha > \beta$

If $\alpha > \beta$, the decisionmaker believes that a Biased Expert tells the truth if he sends a message that $|\hat{\theta}_i| = 2$. He will make his decisions based on this belief. For example, when a Biased Expert as agent A sends message $\hat{\theta}_A = -2$ the decisionmaker will decide A no matter what agent B finds. In proposition 3, I concluded that the probability of making the right decision with team composition BN will never exceed the probability of making the right decision with team composition BB as long as $\alpha > \beta$. I will analyze figure 16 and 17 separately to see if it is possible that mixed team composition NB might lead to the highest probability of making the right decision if $\alpha > \beta$.

In figure 16 it looks like it is possible that BN is the optimal team composition but in the previous section I found that the assumption ensures that these situations are not possible. If I consider the assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$, I find that $p_B > p_N > \frac{2}{3}$. If I take this into account I see that P(BB) > P(BN) for every possible combination of p_B and p_N . If I look at the preferences for team composition NB I find that it is possible that NB is preferred over NN as well as over BB, but NB is never preferred over both team compositions with two members of the same type at the same time. In figure 16, the decisionmaker will therefore choose team composition BB or NN again. NN is preferred over BB if p_B and p_N are relatively close to each other. The minimal difference between p_B and p_N that makes BB the optimal team composition increases with p_N . In figure 17, there are situations in which NB is the optimal team composition. In the shaded area within this figure team composition NB is preferred over BB as well as
over NN. This result also holds if I consider the assumption that $p_B > p_N > \frac{2\alpha+2\beta-1}{\alpha+\beta}$. BB is optimal in the non-shaded area left to the shaded area, NN is optimal in the non-shaded area right to the shaded area. It is not possible that BN is the optimal team composition since team composition BB is preferred over team composition BN because $p_B > p_N$.

Proposition 4: In the model with two possible decisions, there are situations in which it is optimal to consult a Neutral Agent as agent A and a Biased Expert as agent B. This is possible if and only if $\alpha > \beta$ and the ex-ante probability that there is information available, $\alpha + \beta$, is relatively low.

If $\alpha > \beta$, the decisionmaker believes that $\theta_B = 2$ if a Biased Expert as agent B finds information and sends a message $\hat{\theta}_B = 2$. The decisionmaker will decide option B if he believes that $\theta_A = 1$. If the decisionmaker believes that $\theta_A = 2$, he will decide option A. In other words, it does matter for the decisionmaker whether $\theta_A = 1$ or $\theta_A = 2$. Therefore, it is optimal to hire a Neutral Agent as agent A if p_N is not too low. If the difference between p_B and p_N is too big, it is optimal to consult a Biased Expert as agent A as well. If a Biased Expert as agent B does not find information and sends a message $\hat{\theta}_B = 0$, the type of agent A and the message he sends does not matter. The decisionmaker will decide option A anyway.

If a Neutral Agent as agent A finds information and sends a message $\hat{\theta}_A = -2$, the decisionmaker is sure that $\theta_A = -2$. The type of agent B and the message he sends does not matter because the decisionmaker will decide option A anyway. When a Neutral Agent as agent A finds information and sends a message $\hat{\theta}_A = -1$, the decisionmaker is sure that $\theta_A = -1$. If the decisionmaker believes that $\theta_B \leq 1$, he will decide option A. The decisionmaker will decide option B if he believes that $\theta_B = 2$. Therefore, it does matter whether $\theta_B = 1$ or $\theta_B = 2$. If p_N is not too low, the decisionmaker might prefer a Neutral Agent as Agent B in these cases. When a Neutral Agent does not find any information however, the decisionmaker believes that $\theta_A = 0$. He does not care whether $\theta_B = 1$ or $\theta_B = 2$. As long as he believes that there is positive information regarding θ_B , he believes that $\theta_B \neq 0$ and he will decide option B. In these situations, a Biased Expert as agent B will lead to a higher probability of making the right decision than a Neutral Agent.

When the probability that there is information available $\alpha + \beta$ is relatively low, there is a higher probability that there is no information available. This means that the probability that a Neutral Agent as agent A does not find any information is higher and the probability that a Biased Expert as agent B is preferred over a Neutral Agent increases. If $\alpha + \beta$ is too high the decisionmaker will consult two agents of the same type.



- $\beta > \alpha$

If $\beta > \alpha$, the decisionmaker believes that $|\theta_i| = 1$ if a Biased Expert sends a message $|\hat{\theta}_i| = 2$. He will make his decisions based on this belief. For example, when a Biased Expert as agent A sends a message $\hat{\theta}_A = -2$, the decisionmaker's decision will depend on his beliefs about θ_B . If the decisionmaker believes that $\theta_B \leq 1$, he will decide option A but the decisionmaker will decide option B if he believes that $\theta_B = 2$. As I found in the analysis, figure 18 and 19 show that NB is indeed never preferred over BB as long as $\beta > \alpha$. The points where the decisionmaker is indifferent between NB and BB are on the grey line that depicts $p_B = p_N$. NB is preferred over BB in the area to the right of this line, when $p_N > p_B$. These situations are not possible in this model. I will analyze figure 18 and 19 separately to see if there are situations in which BN is the optimal team composition if $\beta > \alpha$.

In figure 18, there are situations in which mixed team composition BN is the optimal team composition. In the shaded area within this figure, team composition BN is preferred over BB as well as over NN. This result also holds if I consider the

assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$. Team composition BB is optimal in the non-shaded area left to the shaded area, team composition NN is optimal in the non-shaded area right to the shaded area.

In figure 19, there is no possibility that BN is the optimal team composition. It is possible that BN is preferred over NN as well as over BB, but BN is never preferred over both team compositions with two team members of the same type at the same time. NN is the optimal team composition in the shaded area, BB is the optimal team composition in the non-shaded area.

Proposition 5: In the model with two possible decisions, there are situations in which it is optimal to consult a Biased Expert as agent A and a Neutral Agent as agent B. This is possible if and only if $\beta > \alpha$ and the ex-ante probability that there is information available, $\alpha + \beta$, is relatively high.

If $\beta > \alpha$, the decisionmaker believes that $\theta_A = -1$ if a Biased Expert as agent A finds information and sends a message $\hat{\theta}_A = -2$. The decisionmaker will decide option A if he believes that $\theta_B \le 1$. If the decisionmaker believes that $\theta_B = 2$, he will decide option B. In other words, it does matter for the decisionmaker whether $\theta_B = 1$ or $\theta_B = 2$. Therefore, it is optimal to hire a Neutral Agent as agent B if p_N is not too small. If a Biased Expert as agent A does not find any information and sends a message $\hat{\theta}_A = 0$, the decisionmaker believes that $\theta_A = 0$. The decisionmaker will decide option A if he believes that $\theta_B = 0$. The decisionmaker does not care whether $\theta_B = 1$ or $\theta_B = 2$, he will decide option B as long as he believes that $\theta_B > 0$. Therefore, a Biased Expert as agent B would be optimal. This explains why it is possible that a mixed team composition BN is optimal if and only if $\beta > \alpha$ and the probability that there is information available, $\alpha + \beta$, is relatively high. A high probability of information being available means that the probability that a Biased Expert as agent A finds information increases. I showed that a Neutral Agent is optimal as agent B if agent A finds information and p_N is not too low.

If a Neutral Agent as agent B finds information and sends a message $\hat{\theta}_B = 2$, the decisionmaker is sure that $\theta_B = 2$. If the decisionmaker believes that $\theta_A \ge -1$, he will decide option B. The decisionmaker will decide option A if he believes that $\theta_A = -2$. It matters whether $\theta_A = -2$ or $\theta_A = -1$. Therefore, it is optimal to consult a Neutral Agent

as agent A in this situation if p_N is not too low. If a Neutral Agent as agent B finds information and sends a message $\hat{\theta}_B = -1$ however, the decisionmaker believes that $\theta_B = 1$. He does not care whether $\theta_A = -2$ or $\theta_A = -1$. If the decisionmaker believes that $\theta_A < 0$, he will decide option A. When a Neutral Agent as agent B does not find any information and sends a message that $\hat{\theta}_B = 0$, the decisionmaker believes that $\theta_B = 0$. It does not matter what message agent A sends, the decisionmaker will decide option A.

When the probability that there is information available, $\alpha + \beta$, is relatively high and $\beta > \alpha$ the decisionmaker expect that a Neutral Agent as agent B will find that $\theta_B = 1$. I showed that it is optimal to hire a Biased Expert as agent A in that situation. Therefore, it is possible that a mixed team composition BN leads to the highest probability of making the right decision if $\alpha + \beta$ is relatively high and $\beta > \alpha$.

If I look at the example of the Dutch government deciding whether to use gas or green energy as the main source of energy, I can conclude that there are situations in which it is optimal for the decisionmaker to hire two different agents in some situations. If the probability that there is information available is relatively low and the ex-ante probability that the information is strong exceeds the probability that the information is weak, there are situations in which it is optimal to hire an external consultant to investigate the benefits of the green energy as the main source of energy and consulting the NAM on the benefits of using gas. If the probability that there is information available is relatively high and the ex-ante probability that the information is weak exceeds the probability that the information is strong, there are situations in which it is optimal to consult green energy companies on the benefits of green energy and hire an external consultant to investigate the benefits of using gas.

Conclusion

I analyzed a model of decision-making under uncertainty in which a decisionmaker needs information collecting agents to decide. The decisionmaker faces a tradeoff between hiring different types of agents, Biased Experts and Neutral Agents. Biased Experts have a higher level of expertise and therefore their information is of a higher quality. However, they suffer from specialty bias which means they are biased in the direction of their own expertise. Neutral Agents have a lower level of expertise, but they are unbiased about the final decision. At first, I analyzed a situation in which a decisionmaker had to choose between three options. Later, I altered the model to a situation in which the decisionmaker had to choose between two options.

In the model with three possible decisions, a mixed team never leads to the highest probability of making the right decision. Whether an optimal team consists of two Biased Experts or two Neutral Agents depends on the difference between p_B and p_N and the ex-ante probability that there is information available $\alpha + \beta$. The minimal difference between p_B and p_N increases with the absolute value of p_N . A higher probability of information being available $\alpha + \beta$ results in a higher probability that an optimal team consists of two Neutral Agents.

If I take a look at the political context in the introduction, I can conclude that interest groups can help decisionmakers to make better decisions if their expertise advantage compared to Neutral Agents is sufficiently large and there is a relatively low probability there is some information to find. In other words, the probability that Status Quo is the optimal decision is relatively high. If the expertise level of Neutral Agents increases, the interest group needs a larger expertise advantage to help the decisionmaker make better decisions.

In the altered model with two possible decisions, there are situations in which it is optimal to hire two different types of team members. If $\alpha > \beta$ and the ex-ante probability that there is information available $\alpha + \beta$ is relatively low, there are combinations of p_B and p_N for which it is optimal to hire a Neutral Agent as agent A and a Biased Expert as agent B. if $\beta > \alpha$ and the ex-ante probability that there is information available $\alpha + \beta$ is relatively that there is information available $\alpha + \beta$ is relatively high, there are combinations of p_B and p_N for which it is optimal to hire a Biased Expert as agent B. If $\beta > \alpha$ and the ex-ante probability that there is information available $\alpha + \beta$ is relatively high, there are combinations of p_B and p_N for which it is optimal to hire a Biased Expert as agent A and a Neutral Agent as agent B. If $\alpha > \beta$, the decisionmaker believes that a Biased Expert tells the truth if he sends a message $|\hat{\theta}_i| = 2$. This means that the value of θ_B does not matter if agent A is a

Biased Expert and sends a message $\hat{\theta}_A = -2$. If agent B is a Biased Expert, the value of θ_A does matter after receiving a message $\hat{\theta}_B = 2$. If the decisionmaker believes that $\theta_A = -2$, the decisionmaker will decide option A while he will decide option B if he believes that $\theta_A = -2$. Therefore, team composition NB might lead to the highest probability of making the right decision, but team composition BN will not.

If $\beta > \alpha$, the decisionmaker believes that a Biased Expert does not tell truth if he sends a message $|\hat{\theta}_i| = 2$. This means that the value of θ_B does matter if agent A is a Biased Expert and sends a message $\hat{\theta}_A = -2$. If the decisionmaker believes that $\theta_B = 2$, the decisionmaker will decide option B while he will decide option A if he believes that $\theta_B =$ 1. If agent B is a Biased Expert, the value of θ_A does matter after receiving a message

 $\hat{\theta}_B = 2$. As long as the decisionmaker believes that $\theta_A < 0$, he will decide option A.

In the example I used in the introduction where the Dutch government needs to decide about using gas or green energy as a main source of energy, using gas can be seen as the status quo (option A) and using green energy is option B. $|\theta_i| = 2$ can be seen as hard evidence regarding option *i* while $|\theta_i| = 1$ can be seen as weak information. For example, $\theta_A = -2$ would mean that using gas is a good idea while $\theta_A = -1$ means that there is uncertainty and using gas might be a good idea. If the decisionmaker consults the Dutch Oil Company (NAM) about using gas, they will always say that it is a good idea if they have some information to present. Still, the information from the NAM might be useful for the decisionmaker in order to decide. If the prior probability that there is hard evidence exceeds the prior probability that there is weak evidence, it might be a good idea to consult an external consult on the benefits of using gas and a green energy company on the benefits of using green energy. Hiring the NAM to gather information on the benefits of using gas as a main source of energy and an external consultant on the benefits of green energy will not lead to the highest probability of making the right decision. If the prior probability that there is weak information exceeds the prior probability that there is hard evidence however, it might be useful to consult the NAM and an external consultant. In these cases, it will not be optimal to consult an external consultant on the benefits of gas while consulting a green energy company on the benefits of using green energy as a main source of energy.

The assumptions that $p_B > p_N > \frac{2\alpha+\beta-1}{\alpha}$ and $p_B > p_N > \frac{\alpha+2\beta-1}{\beta}$ in the model with three possible decisions and $p_B > p_N > \frac{2\alpha+2\beta-1}{\alpha+\beta}$ in the model with two possible decisions are strong assumptions, especially in the model with two possible decisions. If they do not hold, agent *i* finding no information does not tell anything about the value of θ_i . In situations where an agent sends a message $\hat{\theta}_i = 0$, the decisionmakers' beliefs about α and β are not changed and he counts on his ex-ante beliefs to make the decision. The informativeness of the messages of the agents if the assumption does not hold is an interesting topic for further investigation. In some situations, it might be better to ignore agents because their messages are not informative enough. If hiring agents to collect information would be costly it might even be better to not hire them at all if these assumptions do not hold.

In future research it might be interesting to take a look at the absolute values of the utilities of the decisionmaker and the agents. In this model I assumed that the payoff for the decisionmaker is 1 if the right decision is made and 0 if not. In reality however, there might be a difference in utility between the two 'wrong' decisions. When decision A is the optimal decision for example, it does not matter whether the decisionmaker makes a 'small mistake' and decides the status quo, or a 'big mistake' and decides B in this set-up. Dewatripont and Tirole (1999) accounted for this difference by analyzing the 'loss of inertia' and 'loss of extremism'. It might be interesting to approach the model in this paper in the same way.

Dewatripont and Tirole (1999) also analyzed the utility function of the agents. They assume that agents have to incur a cost *K* in order to investigate a certain cause. If agents do not exert effort, they find nothing and have to send a message $\hat{\theta}_i = 0$. Because I wanted to analyze the tradeoff between hiring different types of agents, I did not model the tradeoff agents face to exert effort or not. If I would take the effort tradeoff into account this might result in an extra reason to hire Biased Experts because they are intrinsically motivated to exert effort. Therefore, it would be cheaper to discipline Biased Experts to incur cost *K*.

In this framework, the game stops after one period. If it would be a repeated game, there might be reputational issues that play a role as well. Krishna and Morgan (2001) investigated this by adding a second stage to the game. Reputational incentives might discipline Biased Experts to tell the truth. Reputational concerns might mitigate the problem of Biased Experts manipulating information.

Appendix A – Mathematical Explanation Assumption

In this appendix I will give a mathematical explanation for the assumption that $p_B > p_N > \frac{2\alpha + \beta - 1}{\alpha}$ and $p_B > p_N > \frac{\alpha + 2\beta - 1}{\beta}$. This assumption only matters in situations where one agent presents information and the other agent does not. I assume that the decisionmaker will make the decision for which information is found in these situations. To explain the assumption, I will take a look at these situations separately. Because of the symmetry in this model I will replace decisions A and B with *i* and *j*. It does not matter whether i = A of i = B. $\hat{\theta}_i$ denotes the message agent *i* sends to the decisionmaker.

With two Neutral Agents:

$$\left|\hat{\theta}_{i}\right| = 2, \left|\hat{\theta}_{j}\right| = 0$$

Bayesian beliefs of the decisionmaker:

$$\begin{array}{l} \circ \quad |\theta_i| = 2 \\ \circ \quad |\theta_j| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad |\theta_j| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad |\theta_j| = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \end{array}$$

The decisionmaker decides *i* if and only if he believes that $|\theta_j| \neq 2$. Therefore, the probability that $|\theta_j| \neq 2$ must exceed the probability that $|\theta_j| = 2$.

The probability that $|\theta_j| \neq 2$ exceeds the probability that $|\theta_j| = 2$ if:

$$\frac{\beta(1-p_N)+(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)} > \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)}$$

$$p_N > \frac{2\alpha - 1}{\alpha - \beta}$$

$$- |\hat{\theta}_i| = 1, |\hat{\theta}_j| = 0$$

Bayesian beliefs of the decisionmaker:

$$|\theta_i| = 1$$

$$|\theta_j| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$$

$$|\theta_j| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$$

$$|\theta_j| = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$$

The decisionmaker chooses option *i* if and only if he believes that $|\theta_j| = 0$. Therefore, the probability that $|\theta_j| = 0$ must exceed the probability that $|\theta_j| = 1$ and the probability that $|\theta_j| = 2$.

The probability that $|\theta_j| = 0$ exceeds the probability that $|\theta_j| = 1$ if: $(1 - \alpha - \beta)$ $\beta(1 - p_N)$

 $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} > \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$

$$p_N > \frac{\alpha + 2\beta - 1}{\beta}$$

The probability that $|\theta_j| = 0$ exceeds the probability that $|\theta_j| = 2$ if:

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} > \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$$
$$p_N > \frac{2\alpha+\beta-1}{\alpha}$$

With two Biased experts:

$$- |\hat{\theta}_i| = 2, |\hat{\theta}_j| = 0$$

Bayesian beliefs of the decisionmaker:

$$\begin{array}{l} \circ \quad |\theta_i| = 2 \text{ with probability } \frac{\alpha}{\alpha + \beta} \\ \circ \quad |\theta_i| = 1 \text{ with probability } \frac{\beta}{\alpha + \beta} \\ \circ \quad |\theta_j| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1 - p_B)}{(\alpha + \beta)(1 - p_B) + (1 - \alpha - \beta)} \\ \circ \quad |\theta_j| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1 - p_B)}{(\alpha + \beta)(1 - p_B) + (1 - \alpha - \beta)} \\ \circ \quad |\theta_j| = 0 \text{ with probability } \frac{(1 - \alpha - \beta)}{(\alpha + \beta)(1 - p_B) + (1 - \alpha - \beta)} \end{array}$$

The decisionmaker chooses option *i* if and only if he believes that $|\theta_i| > |\theta_j|$. Therefore, the probability that $|\theta_i| > |\theta_j|$ must exceed the probability that $|\theta_i| = |\theta_j|$ and the probability that $|\theta_i| < |\theta_j|$.

The probability that $|\theta_i| > |\theta_j|$ exceeds the probability that $|\theta_i| = |\theta_j|$ if:

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} + \frac{\alpha}{\alpha+\beta} * \tilde{\beta} > \frac{\alpha}{\alpha+\beta} * \tilde{\alpha} + \frac{\beta}{\alpha+\beta} * \tilde{\beta}$$
$$p_B > \frac{2\alpha^2 - \alpha + 2\beta^2 - \beta + \alpha\beta}{\alpha^2 + \beta^2 - \alpha\beta}$$

The probability that $|\theta_i| > |\theta_j|$ exceeds the probability that $|\theta_i| < |\theta_j|$ if:

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)\,+\,(1-\alpha-\beta)}\,+\frac{\alpha}{\alpha+\beta}*\tilde{\beta}>\frac{\beta}{\alpha+\beta}*\tilde{\alpha}$$

This inequality holds for every possible combination of the values p_B , α and β .

With agent i Biased Expert and agent j Neutral Agent:

$$- |\hat{\theta}_i| = 2, |\hat{\theta}_i| = 0$$

Bayesian beliefs of the decisionmaker:

$$|\theta_i| = 2 \text{ with probability } \frac{\alpha}{\alpha + \beta}$$

$$|\theta_i| = 1 \text{ with probability } \frac{\beta}{\alpha + \beta}$$

$$|\theta_j| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1 - p_N)}{(\alpha + \beta)(1 - p_N) + (1 - \alpha - \beta)}$$

$$|\theta_j| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1 - p_N)}{(\alpha + \beta)(1 - p_N) + (1 - \alpha - \beta)}$$

$$|\theta_j| = 0 \text{ with probability } \frac{(1 - \alpha - \beta)}{(\alpha + \beta)(1 - p_N) + (1 - \alpha - \beta)}$$

The decisionmaker decides option *i* if and only if he believes that $|\theta_i| > |\theta_j|$. Therefore, the probability that $|\theta_i| > |\theta_j|$ must exceed the probability that $|\theta_i| = |\theta_j|$ and the probability that $|\theta_i| < |\theta_j|$.

The probability that $|\theta_i| > |\theta_j|$ exceeds the probability that $|\theta_i| = |\theta_j|$ if:

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)}+\frac{\alpha}{\alpha+\beta}*\tilde{\beta}>\frac{\alpha*\tilde{\alpha}+\beta*\tilde{\beta}}{\alpha+\beta}$$

 $p_N > \frac{2\alpha^2 - \alpha + 2\beta^2 - \beta + \alpha\beta}{\alpha^2 + \beta^2 - \alpha\beta}$

The probability that $|\theta_i| > |\theta_j|$ exceeds the probability that $|\theta_i| < |\theta_j|$ if:

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)}+\frac{\alpha}{\alpha+\beta}*\tilde{\beta}>\frac{\beta}{\alpha+\beta}\tilde{\alpha}$$

This inequality holds for every possible combination of the values p_N , α and β .

$$- |\hat{\theta}_i| = 0, |\hat{\theta}_j| = 2$$

Bayesian beliefs of the decisionmaker are:

$$|\theta_j| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$$

$$|\theta_j| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$$

$$|\theta_j| = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$$

$$|\theta_j| = 2$$

The decisionmaker chooses option *j* if and only if he believes that $|\theta_j| > |\theta_i|$. Therefore, the probability that $|\theta_j| > |\theta_i|$ must exceed the probability that $|\theta_j| = |\theta_i|$ and the probability that $|\theta_j| < |\theta_i|$.

The probability that $|\theta_j| > |\theta_i|$ exceeds the probability that $|\theta_j| = |\theta_i|$ if:

$$\frac{(1-\alpha-\beta)+\beta(1-p_N)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)} > \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}$$

If $\beta > \alpha$, this equation always holds. If $\alpha > \beta$: $p_B > \frac{2\alpha - 1}{\alpha - \beta}$

The probability that $|\theta_j| < |\theta_i|$ is 0.

- $\left| \hat{\theta}_i \right| = 0$, $\left| \hat{\theta}_j \right| = 1$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \left|\theta_{j}\right| = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)} \\ \circ \quad \left|\theta_{j}\right| = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)} \\ \circ \quad \left|\theta_{j}\right| = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)} \\ \circ \quad \left|\theta_{j}\right| = 1 \end{array}$

The decisionmaker chooses option *j* if and only if he believes that $|\theta_j| > |\theta_i|$. Therefore, the probability that $|\theta_j| > |\theta_i|$ must exceed the probability that $|\theta_j| = |\theta_i|$ and the probability that $|\theta_j| < |\theta_i|$.

The probability that $|\theta_j| > |\theta_i|$ exceeds the probability that $|\theta_j| = |\theta_i|$ if: $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} > \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$

 $p_B > \frac{\alpha + 2\beta - 1}{\beta}$

The probability that $|\theta_j| > |\theta_i|$ exceeds the probability that $|\theta_j| < |\theta_i|$ if:

 $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} > \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$

$$p_B > \frac{2\alpha + \beta - 1}{\alpha}$$

With the assumptions that $p_B > p_N > \frac{2\alpha+\beta-1}{\alpha}$ and $p_B > p_N > \frac{\alpha+2\beta-1}{\beta}$ all these conditions hold. This can also be written as $(1 - \alpha - \beta) > \alpha(1 - p_N) > \alpha(1 - p_B)$ and $(1 - \alpha - \beta) > \beta(1 - p_N) > \beta(1 - p_B)$. In other words, the Bayesian belief that $\theta_i = 0$ given that $\hat{\theta}_i = 0$ must exceed the Bayesian belief that $|\theta_i| = 1$ and the Bayesian belief that $|\theta_i| = 2$.

$$P(\theta_i = 0 | \hat{\theta}_i = 0) > P(|\theta_i| = 2 | \hat{\theta}_i = 0)$$

 $P(\theta_i = 0 | \hat{\theta}_i = 0) > P(|\theta_i| = 1 | \hat{\theta}_i = 0)$

Assumption in the model with Two possible decisions

In the model with two assumptions, I need a stronger assumption. Here the conditional probability $\theta_i = 0$ given that $\hat{\theta}_i = 0$ must exceed the conditional probability that $|\theta_i| \neq 0$. I will prove this by analyzing the situations in which one agent finds information and the other does not again. In this model, the decisionmaker will decide option A if he believes that $\theta \leq 0$ and option B if he believes that $\theta > 0$.

With two Neutral Agents:

- $\hat{\theta}_A = -2, \hat{\theta}_B = 0$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -2 \\ \circ \quad \theta_B = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \end{array}$

There is no probability that $\theta > 0$ since $\theta_A = -2$. The decisionmaker will decide option A.

 $- \quad \hat{\theta}_A = -1, \hat{\theta}_B = 0$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -1 \\ \circ \quad \theta_B = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \end{array}$

The decisionmaker will decide A if the probability that $\theta_B \leq 1$ exceeds the probability that $\theta_B = 2$.

$$\frac{(1-\alpha-\beta)+\beta(1-p_N)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)} > \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)}$$

 $p_N > \frac{2\alpha - 1}{\alpha - \beta}$

$$\hat{\theta}_A = 0, \hat{\theta}_B = 2$$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 2 \end{array}$

The decisionmaker will decide B if the probability that $\theta_A \leq 1$ exceeds the probability that $\theta_A = 2$.

$$\frac{(1-\alpha-\beta)+\beta(1-p_N)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)} > \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)}$$
$$p_N > \frac{2\alpha-1}{\alpha-\beta}$$

- $\hat{\theta}_A = 0$, $\hat{\theta}_B = 1$

Bayesian beliefs of the decisionmaker:

$$\theta_{A} = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{A} = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 1$$

The decisionmaker will decide B only if the probability that $\theta_A = 0$ exceeds the probability that $\theta_A \neq 0$.

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} > \frac{(\alpha+\beta)(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$$

$$p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$$

With two Biased Experts:

 $- \quad \hat{\theta}_A = -2, \hat{\theta}_B = 0$

Bayesian beliefs of the decisionmaker:

 $\theta_{A} = -2 \text{ with probability } \frac{\alpha}{\alpha + \beta}$ $\theta_{A} = -1 \text{ with probability } \frac{\beta}{\alpha + \beta}$ $\theta_{B} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1 - p_{B})}{(\alpha + \beta)(1 - p_{B}) + (1 - \alpha - \beta)}$ $\theta_{B} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1 - p_{B})}{(\alpha + \beta)(1 - p_{B}) + (1 - \alpha - \beta)}$ $\theta_{B} = 0 \text{ with probability } \frac{(1 - \alpha - \beta)}{(\alpha + \beta)(1 - p_{B}) + (1 - \alpha - \beta)}$

The decisionmaker will decide A if the probability that $\theta \leq 0$ exceeds the probability that $\theta > 0$.

$$\frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta} * \frac{(1-\alpha-\beta)+\beta(1-p_B)}{(\alpha+\beta)(1-p_N)} \ge \frac{\beta}{\alpha+\beta} * \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}$$

- $\hat{\theta}_A = 0, \hat{\theta}_B = 2$

Bayesian beliefs of the decisionmaker:

$$\begin{array}{l} \circ \quad \theta_A = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)} \\ \circ \quad \theta_A = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)} \\ \circ \quad \theta_A = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)} \\ \circ \quad \theta_B = 2 \text{ with probability } \frac{\alpha}{\alpha+\beta} \\ \circ \quad \theta_B = 1 \text{ with probability } \frac{\beta}{\alpha+\beta} \end{array}$$

The decisionmaker will decide B if the probability that $\theta > 0$ exceeds the probability that $\theta \le 0$.

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}+\frac{\alpha}{\alpha+\beta}*\tilde{\beta}\geq\tilde{\alpha}+\frac{\beta}{\alpha+\beta}*\tilde{\beta}$$

With agent A Biased Expert and agent B Neutral Agent:

$$- \hat{\theta}_A = -2, \hat{\theta}_B = 0$$

Bayesian beliefs of the decisionmaker:

$$\theta_{A} = -2 \text{ with probability } \frac{\alpha}{\alpha+\beta}$$

$$\theta_{A} = -1 \text{ with probability } \frac{\beta}{\alpha+\beta}$$

$$\theta_{B} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{N})}{(\alpha+\beta)(1-p_{N})+(1-\alpha-\beta)}$$

$$\theta_{B} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{N})}{(\alpha+\beta)(1-p_{N})+(1-\alpha-\beta)}$$

$$\theta_{B} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{N})+(1-\alpha-\beta)}$$

The decisionmaker will decide A if the probability that $\theta \leq 0$ exceeds the probability that $\theta > 0$.

$$\frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta} * \frac{\beta(1-p_N) + (1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \ge \frac{\beta}{\alpha+\beta} * \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$$

 $- \quad \hat{\theta}_A = 0, \hat{\theta}_B = 2$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 2 \text{ with probability } \frac{\alpha}{\alpha+\beta} \\ \circ \quad \theta_B = 1 \text{ with probability } \frac{\beta}{\alpha+\beta} \end{array}$

The decisionmaker will decide B if the probability that $\theta > 0$ exceeds the probability that $\theta \le 0$.

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N)+(1-\alpha-\beta)}+\frac{\alpha}{\alpha+\beta}*\tilde{\beta}\geq \tilde{\alpha}+\frac{\beta}{\alpha+\beta}*\tilde{\beta}$$

- $\hat{\theta}_A = 0$, $\hat{\theta}_B = 1$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \end{array}$

The decisionmaker will decide B if the probability that $\theta_A = 0$ exceeds the probability that $\theta_A \neq 0$.

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)} \ge \frac{(\alpha+\beta)(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}$$

With agent A Neutral Agent and agent B Biased Expert:

-
$$\hat{\theta}_A = -2$$
, $\hat{\theta}_B = 0$

Bayesian beliefs of the decisionmaker:

•
$$\theta_A = -2$$

• $\theta_B = 2$ with probability $\tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$

 $\begin{array}{l} \circ \quad \theta_B = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \end{array}$

The decisionmaker will decide A because there is no probability that $\theta > 0$ since $\theta_A = -2$.

 $- \quad \hat{\theta}_A = -1, \hat{\theta}_B = 0$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -1 \\ \circ \quad \theta_B = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \end{array}$

The decisionmaker will decide A if the probability that $\theta \leq 0$ exceeds the probability that $\theta > 0$.

$$\frac{(1-\alpha-\beta)+\beta(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)} > \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}$$

- $\hat{\theta}_A = 0, \hat{\theta}_B = 2$

Bayesian beliefs of the decisionmaker:

 $\begin{array}{l} \circ \quad \theta_A = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 2 \text{ with probability } \frac{\alpha}{\alpha+\beta} \\ \circ \quad \theta_B = 1 \text{ with probability } \frac{\beta}{\alpha+\beta} \end{array}$

The decisionmaker will decide B if the probability that $\theta > 0$ exceeds the probability that $\theta \le 0$.

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)\,+\,(1-\alpha-\beta)}+\frac{\alpha}{\alpha+\beta}*\tilde{\beta}\geq\tilde{\alpha}+\frac{\beta}{\alpha+\beta}*\tilde{\beta}$$

All these conditions hold if I assume that the Bayesian belief that there is no information regarding option *i* exceeds the Bayesian belief that there is information, given that the decisionmaker received a message $|\hat{\theta}_i| = 0$:

$$\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_i)+(1-\alpha-\beta)} > \frac{(\alpha+\beta)(1-p_i)}{(\alpha+\beta)(1-p_i)+(1-\alpha-\beta)}$$

If I fill in p_B and p_N and rewrite this I find that:

$$p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$$

$\widehat{oldsymbol{ heta}}_A$	$\widehat{oldsymbol{ heta}}_B$	BB	NN	BN	NB
-2	2	SQ	SQ	B if $\beta > \alpha$	A if $\beta > \alpha$
				SQ if $\alpha > \beta$	SQ if $\alpha > \beta$
-2	1	A [*]	А	A if $\alpha > \beta$	A [*]
				SQ if $\beta > \alpha$	
-2	0	А	А	А	А
-1	2	B*	В	B*	B if $\alpha > \beta$
					SQ if $\beta > \alpha$
-1	1	SQ [*]	SQ	SQ [*]	SQ [*]
-1	0	A*	А	A [*]	А
0	2	В	В	В	В
0	1	B*	В	В	B*
0	0	SQ	SQ	SQ	SQ

Appendix B – Calculations model with three possible decisions

Table 2. Decisions after observing different combinations of messages in the model with three possible decisions. * = out-of-equilibrium situation

Team Composition

The decisionmaker has to decide on the team composition before the agents are able to present their information to the decisionmaker. At first, I will analyze teams with two members of the same type. Later I will take a look at what happens when the team consists of one Neutral Agent and one Biased Expert. Since the decisions do not have monetary values, the decisionmaker chooses the team composition in order to maximize the probability of making the right decision.

Two Neutral Agents

At first, I take a look at the probability of making the right decision with two Neutral Agents. First, I analyze the situations in which either A or B is the optimal decision, then I will analyze the situations in which the Status Quo is the optimal decision. Finally, I can determine the total probability that a team with two Neutral Agents will lead to the optimal decision. Because of the symmetry in this model I will replace decisions A and B with *i* and *j*. It does not matter whether i = A of i = B. The decisions after observing different combinations of messages can be found in table 2.

In situations where decision *i* is the best decision, it does not matter what agent *j* finds. As long as agent *i* finds the information, the right decision is made. The probability of making the right decision is therefore p_N . The same yields for situations in which decision *j* is the best decision. The probability that the *i* or *j* is the optimal decision is equal to $2(\alpha(1 - \alpha - \beta) + \alpha\beta + \beta(1 - \alpha - \beta))$.

It is more complex to determine the probability of making the right decision when the right decision is SQ because there is a chance that the right decision is made accidentally when both agents do not find information when it is available. There are three situations in which the optimal situation is SQ. I will analyze these situations separately.

$$|\theta_i| = 2, \, |\theta_j| = 2$$

When this situation occurs, there are two possible ways in which the right decision is made. If both agents find positive information or if they both do not find anything. The probability that both agents find the information is equal to p_N^2 , the probability that both agents do not find the information is equal to $(1 - p_N)^2$. The total probability of making the right decision is therefore $p_N^2 + (1 - p_N)^2 = 2p_N^2 - 2p_N + 1$. This result also yields for the situations in which $|\theta_i| = 1$ and $|\theta_j| = 1$. The probability that $|\theta_i| = |\theta_j| \neq 0$ is $(\alpha^2 + \beta^2)$.

- $\theta_i = 0, \ \theta_j = 0$

If there is no information available, agents will not find any information. In this situation both agents will find nothing, and the right decision will be made with probability 1. The probability that this situation occurs is $(1 - \alpha - \beta)^2$.

The total probability that a team with two Neutral Agents will lead to making the decision is equal to:

$$2p_N * (\alpha(1 - \alpha - \beta) + \alpha\beta + \beta(1 - \alpha - \beta)) + (\alpha^2 + \beta^2)(2p_N^2 - 2p_N + 1) + (1 - \alpha - \beta)^2$$

I can rewrite this to: $2(\alpha^2 + \beta^2)p_N^2 + 2(\alpha - 2\alpha^2 + \beta - 2\beta^2 - \alpha\beta)p_N + \alpha^2 + \beta^2 + (1 - \alpha - \beta)^2$

Two Biased Experts

The analysis of the team composition with two Biased Experts is more complex because the Biased Experts have an incentive to exaggerate the information they have. Therefore, a message from a Biased Expert that says $|\theta_i| = 2$ says nothing about the value of θ_i . The decisionmaker only knows that there is positive information $(|\theta_i| > 0)$, but not if this information has value 1 or 2. I will analyze the Biased Experts case in the same way as the Neutral Agents case. First, I will look at situations in which either A or B is the optimal decision. Then I will take look at situations in which the Status Quo is the optimal decision. After that I can calculate the total probability that a team with two Biased Experts will result in the optimal decision. The decisions after observing different combinations of messages can be found in table 2.

There are three situations in which decision A is the best decision:

- $\theta_A = -2, \ \theta_B = 0 \rightarrow$ With probability $\alpha(1 \alpha \beta)$
- $\theta_A = -1, \ \theta_B = 0 \rightarrow$ With probability $\beta(1 \alpha \beta)$
- $\theta_A = -2, \ \theta_B = 1 \rightarrow$ With probability $\alpha\beta$

In the first two situations, the probability that the decisionmaker makes the right decision is equal to the probability that agent A finds the information. Agent B will not find any information since it is not available. The probability of making the right decision is therefore equal to the probability that agent A finds the available information, p_B .

In the third situation, there is a risk that both agents find the information and that agent B presents it as if $\theta_B = 2$. The decisionmaker is not able to distinguish the two types of information and will choose to stay at the Status Quo. To make the right decision in this situation, Agent A must find his information and agent B must not. The probability of making the right decision in a team with two Biased Experts is therefore $p_B * (1 - p_B) = p_B - p_B^2$.

The same results yield for situations in which decision B is the optimal decision. In the two situations where $\theta_A = 0$, the probability of making the right decision is equal to the probability that agent B finds the available information, p_B . In the situation where there is positive information available for both causes but stronger information for cause B, the probability of making the right decision is $p_B * (1 - p_B) = p_B - p_B^2$.

To analyze the situations in which the Status Quo is the optimal decision, I will analyze the three situations in which this is the case separately again.

$$\theta_A = -2, \ \theta_B = -2 \rightarrow$$
 With probability α^2

In this situation, there are two ways in which the optimal decision can be made. If both agents find their positive information or if they both find nothing. The probability that both agents find the information is equal to p_B^2 , the probability that both agents do not find the information is equal to $(1 - p_B)^2$. The total probability of making the right decision (given this situation) is therefore $p_B^2 + (1 - p_B)^2 = 2p_B^2 - 2p_B + 1$.

- $\theta_A = -1, \ \theta_B = 1 \rightarrow$ With probability β^2

If both Biased Experts find their information, they will both present it as $|\theta_i| = 2$ and the optimal decision will still be made. Therefore, there are two ways to reach the optimal decision here as well. The right decision is made when both agents find their information or if they both find nothing. The probability of making the right decision is therefore the same as before: $p_B^2 + (1 - p_B)^2 = 2p_B^2 - 2p_B + 1$.

- $\theta_A = 0, \ \theta_B = 0 \rightarrow$ With probability $(1 - \alpha - \beta)^2$

If there is no information available for both causes, both agents will find nothing, and the right decision will be made with probability 1.

The total probability that a team of two Biased Experts results in the optimal decision is therefore:

$$2p_B(\alpha + \beta)(1 - \alpha - \beta) + 2\alpha\beta(p_B - p_B^2) + (\alpha^2 + \beta^2)(2p_B^2 - 2p_B + 1) + (1 - \alpha - \beta)^2$$

I can rewrite this to:

 $2(\alpha^2+\beta^2-\alpha\beta)p_B{}^2+2(\alpha-2\alpha^2+\beta-2\beta^2-\alpha\beta)p_B+\alpha^2+\beta^2+(1-\alpha-\beta)^2$

Mixed team

Beliefs

To calculate the probability of making the right decisions with a mixed team, I need to start reconsidering the beliefs of the decisionmaker after he received messages of the agents. There are six possible combinations of messages, for each combination I will take a look at the Bayesian beliefs of the decisionmaker and determine his decision.

Due to the symmetry of this model, I only need to consider one type of mixed team. In this analysis, I assume that agent A is Biased Expert and agent B is a Neutral Agent. The probability of making the right decision I calculate is equal to the probability of making the right decision with a Neutral Agent as agent A and a Biased Expert as agent B.

$$\hat{\theta}_A = -2, \hat{\theta}_B = 2$$

The beliefs of the decisionmaker are:

• $\theta_A = -2$ with probability $\frac{\alpha}{\alpha + \beta}$ • $\theta_A = -1$ with probability $\frac{\beta}{\alpha + \beta}$ • $\theta_B = 2$

The decisionmaker will decide B if $\frac{\beta}{\alpha+\beta} > \frac{\alpha}{\alpha+\beta}$. If $\frac{\alpha}{\alpha+\beta} > \frac{\beta}{\alpha+\beta}$, the decisionmaker will decide Status Quo.

 $- \quad \widehat{\theta}_A = -2, \, \widehat{\theta}_B = 1$

The beliefs of the decisionmaker are:

• $\theta_A = -2$ with probability $\frac{\alpha}{\alpha + \beta}$ • $\theta_A = -1$ with probability $\frac{\beta}{\alpha + \beta}$ • $\theta_B = 1$

The decisionmaker will decide option A if $\frac{\alpha}{\alpha+\beta} > \frac{\beta}{\alpha+\beta}$. If $\frac{\beta}{\alpha+\beta} > \frac{\alpha}{\alpha+\beta}$, the decisionmaker will decide Status Quo.

- $\hat{\theta}_A = -2$, $\hat{\theta}_B = 0$

The beliefs of the decisionmaker are:

 $\theta_{A} = -2 \text{ with probability } \frac{\alpha}{\alpha+\beta}$ $\theta_{A} = -1 \text{ with probability } \frac{\beta}{\alpha+\beta}$ $\theta_{B} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$ $\theta_{B} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$ $\theta_{B} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$

The assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ makes sure that the decisionmaker will decide option A in this case.

$$- \hat{\theta}_A = 0, \hat{\theta}_B = 2$$

The beliefs of the decisionmaker are:

$$\theta_{A} = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{A} = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 2$$

The assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ makes sure that the decisionmaker will decide option B in this case.

-
$$\hat{\theta}_A = 0, \hat{\theta}_B = 1$$

The beliefs of the decisionmaker are:

$$\theta_{A} = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{A} = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 1$$

The assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ makes sure that the decisionmaker will decide option B in this case.

$$\widehat{ heta}_A=0$$
 , $\widehat{ heta}_B=0$

-

The beliefs of the decisionmaker are:

$$\theta_{A} = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)}$$

$$\theta_{A} = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)}$$

$$\theta_{B} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{N})}{(\alpha+\beta)(1-p_{N})+(1-\alpha-\beta)}$$

$$\theta_{B} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{N})}{(\alpha+\beta)(1-p_{N})+(1-\alpha-\beta)}$$

•
$$\theta_B = 0$$
 with probability $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$

The assumption that $p_B > p_N > \frac{2\alpha + 2\beta - 1}{\alpha + \beta}$ makes sure that the decisionmaker will decide the Status Quo.

These decisions after different possible combinations of messages can be found in table 7 and table 2.

$\widehat{oldsymbol{ heta}}_A$	$\widehat{\boldsymbol{ heta}}_B$	Decision
-2	2	B if $\beta \ge \alpha$, SQ if $\alpha > \beta$
-2	1	A if $\alpha \ge \beta$, SQ if $\beta > \alpha$
-2	0	А
0	2	В
0	1	В
0	0	Status Quo

Table 7. Decisions with agent A Biased Expert and agent B Neutral Agent in the model with three possible decisions

Probability of making the right decision

To find the probability of making the right decision with this team composition I need to calculate the probability of making the right decision in each situation and multiply it by the probability this situation occurs. There are nine different possible situations.

$$\theta_A = -2, \theta_B = 2$$

The possible combinations of messages that agents can receive are:

- \circ $\hat{\theta}_A = -2, \hat{\theta}_B = 2$ → The decisionmaker decides Status Quo if $\alpha > \beta$
- $\hat{\theta}_A = -2, \hat{\theta}_B = 0 \rightarrow$ The decisionmaker decides A
- $\hat{\theta}_A = 0, \hat{\theta}_B = 2$ → The decisionmaker decides B
- $\hat{\theta}_A = 0, \hat{\theta}_B = 0 \Rightarrow$ The decisionmaker decides Status Quo

The probability of making the right decision is $p_B p_N + (1 - p_B)(1 - p_N)$ if $\alpha > \beta$. If $\beta > \alpha$, the right decision is made with probability $(1 - p_B)(1 - p_N)$.

- $\theta_A = -2, \theta_B = 1$

The possible combinations of messages that agents can receive are:

 \circ $\hat{\theta}_A = -2, \hat{\theta}_B = 1$ → The decisionmaker decides A if $\alpha > \beta$

- $\hat{\theta}_A = -2, \hat{\theta}_B = 0 \rightarrow$ The decisionmaker decides A
- $\hat{\theta}_A = 0, \hat{\theta}_B = 1 \rightarrow$ The decisionmaker decides B
- $\hat{\theta}_A = 0, \hat{\theta}_B = 0 \rightarrow$ The decisionmaker decides Status Quo

The probability that the decisionmaker makes the right decision is p_B if $\alpha > \beta$. If $\beta > \alpha$, the probability of making the right decision is $p_B(1 - p_N)$.

- $\theta_A = -2, \theta_B = 0$

The decisionmaker will only make the right decision when agent A finds the information. When both agents find nothing, the decisionmaker will choose the status quo. The probability of making the right decision is therefore equal to the probability that agent A finds the available information, p_B .

- $\theta_A = -1, \theta_B = 2$

The possible combinations of messages that agents can receive are:

◦ $\hat{\theta}_A = -2, \hat{\theta}_B = 2 \rightarrow$ The decisionmaker decides B if $\beta > \alpha$

○ $\hat{\theta}_A = -2, \hat{\theta}_B = 0 \rightarrow$ The decisionmaker decides A

○ $\hat{\theta}_A = 0, \hat{\theta}_B = 2 \rightarrow$ The decisionmaker decides B

○ $\hat{\theta}_A = 0, \hat{\theta}_B = 0 \rightarrow$ The decisionmaker decides Status Quo

The right decision is made with probability p_N if $\beta > \alpha$. If $\alpha > \beta$, the probability of making the right decision is $p_N(1 - p_B)$.

- $\theta_A = -1, \theta_B = 1$

The possible combinations of messages that agents can receive are:

 $\circ \quad \widehat{\theta}_A = -2, \widehat{\theta}_B = 1 \twoheadrightarrow \text{The decisionmaker decides SQ if } \beta > \alpha$

 $\circ \quad \widehat{\theta}_A = -2, \widehat{\theta}_B = 0 \twoheadrightarrow \text{The decisionmaker decides A}$

○ $\hat{\theta}_A = 0, \hat{\theta}_B = 1 \rightarrow$ The decisionmaker decides B

○ $\hat{\theta}_A = 0, \hat{\theta}_B = 0 \rightarrow$ The decisionmaker decides Status Quo

If $\beta > \alpha$, the probability of making the right decision is $p_B p_N + (1 - p_B)(1 - p_N)$. If $\alpha > \beta$, the probability of making the right decision is $(1 - p_N)(1 - p_B)$.

- $\theta_A = -1$, $\theta_B = 0$

The probability that the decisionmaker makes the right decision is equal to the probability that agent A finds the information again. The probability that the right decision is made is p_B .

- $\theta_A = 0, \theta_B = 2$

The probability that the decisionmaker makes the right decision is equal to the probability that agent B finds the available information. The probability that the right decision is made is p_N . The same yields for the situation in which $\theta_A = 0$ and $\theta_B = 1$.

$$- \theta_A = 0, \theta_B = 0$$

Both agents will not find any information, the right decision is made with probability 1.

The total probability of making the right decision with a team that consists of a Biased Expert as agent A and a Neutral Agent as agent B is:

If
$$\alpha \ge \beta$$
:
 $\alpha^{2}(p_{B}p_{N} + (1 - p_{B})(1 - p_{N})) + \alpha\beta p_{B} + (\alpha + \beta)(1 - \alpha - \beta)p_{B} + \alpha\beta p_{N}(1 - p_{B})$
 $+ \beta^{2}(1 - p_{B})(1 - p_{N}) + (\alpha + \beta)(1 - \alpha - \beta)p_{N} + (1 - \alpha - \beta)^{2}$
 $=$
 $(2\alpha^{2} + \beta^{2} - \alpha\beta)p_{B}p_{N} + ((\alpha + \beta)(1 - \alpha - \beta) - \alpha^{2} - \beta^{2} + \alpha\beta)(p_{B} + p_{N}) + \alpha^{2} + \beta^{2} + (1 - \alpha - \beta)^{2}$

If
$$\beta > \alpha$$
:

$$\begin{aligned} \alpha^{2}(1-p_{B})(1-p_{N}) + \alpha\beta p_{B}(1-p_{N}) + (\alpha+\beta)(1-\alpha-\beta)p_{B} + \alpha\beta p_{N} \\ + \beta^{2}(p_{B}p_{N} + (1-p_{B})(1-p_{N})) + (\alpha+\beta)(1-\alpha-\beta)p_{N} + (1-\alpha-\beta)^{2} \\ = \\ (\alpha^{2} + 2\beta^{2} - \alpha\beta)p_{B}p_{N} + ((\alpha+\beta)(1-\alpha-\beta) - \alpha^{2} - \beta^{2} + \alpha\beta)(p_{B} + p_{N}) + \alpha^{2} + \beta^{2} + (1-\alpha-\beta)^{2} \end{aligned}$$

$\widehat{ heta}_A$	$\widehat{oldsymbol{ heta}}_B$	BB	NN	BN	NB
-2	2	A	А	B if $\beta > \alpha$	А
				A if $\alpha > \beta$	
-2	1	A [*]	А	А	A [*]
-2	0	А	А	А	А
-1	2	B if $\alpha > \beta^*$	В	Β*	B if $\alpha > \beta$
		A if $\beta > \alpha$			A if $\beta > \alpha$
-1	1	A*	А	A [*]	A*
-1	0	A*	А	A*	А
0	2	В	В	В	В
0	1	B*	В	В	B *
0	0	A	А	А	А

Appendix C – Calculations model with two possible decisions

Table 7. Decisions after observing different combinations of messages in the model with two possible decisions. * = Out-of-equilibrium situation

At first, I will analyze teams with two members of the same type again. Then I will take a look at what happens when the team consists of one Neutral Agent and one Biased Expert. In this model I have to analyze both types of mixed teams.

Two Neutral Agents

The decisions after observing different combinations of messages can be found in table 5. The ex-ante belief of the decisionmaker is that $\theta = 0$. This means that the agent investigating option B will have to present at least some positive information for the decisionmaker to decide option B. If there is no positive information available regarding option B, the decisionmaker will decide option A. The right decision is therefore made with probability 1 in the following situations:

- $\theta_A = -2, \ \theta_B = 0$
- $\quad \theta_A = -1, \ \theta_B = 0$
- $\theta_A = 0, \, \theta_B = 0$

When there is positive information regarding option B, it is a bit more complex. I will analyze these situations separately.

Situations where A (Status Quo) is the optimal decision:

$$\theta_A = -2, \, \theta_B = 1$$

The right decision in this situation is option A. The decisionmaker will choose option A when agent A finds the information available and when both agents find nothing. Therefore, the right decision is made with probability $p_N + (1 - p_N)^2 = p_N^2 - p_N + 1$. The same yields for the situation where $\theta_A = -2$, $\theta_B = 2$ and the situation where $\theta_A = -1$, $\theta_B = 1$.

Situations where B is the optimal decision:

In these situations, $|\theta_B| > |\theta_A|$. It does not matter whether agent A finds available information or not. When agent B finds his information, the right decision is made. The probability of making the right decision is therefore equal to the probability that agent B finds the available information, p_N .

The total probability is of making the right decision with two Neutral Agents is therefore: $(1 - \alpha - \beta) + (\alpha^2 + \beta^2 + \alpha\beta)(p_N^2 - p_N + 1) + ((\alpha + \beta)(1 - \alpha - \beta) + \alpha\beta)p_N$

I can rewrite this to:

 $(\alpha^2 + \beta^2 + \alpha\beta)p_N^2 + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_N + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$

Two Biased Experts

In situations where $\theta_B = 0$, the probability of making the right decision is 1 because of the same reason as in the case with two Neutral Agents. The ex-ante belief of the decisionmaker is that $\theta = 0$, which means that if agent B does not have any information to present the decisionmaker will decide option A. The only difference with the previous team composition is in the situation where there is positive information available for both causes, but the information of agent B is stronger than the information of agent A; $\theta_A = -1$, $\theta_B = 2$. Because the agents are Biased Experts, agent A will exaggerate and send message $\hat{\theta}_A = -2$ if he finds the information. The right decision is only made when agent B finds the information, and agent A does not. The probability of making the right decision is therefore $p_B(1 - p_B) = p_B - p_B^2$. The total probability of making the right decision with two Biased Experts is:

 $(1-\alpha-\beta)+(\alpha^2+\beta^2+\alpha\beta)(p_B{}^2-p_B+1)+(\alpha+\beta)(1-\alpha-\beta)p_B+\alpha\beta(p_B-p_B{}^2)$

I can rewrite this to: $(\alpha^2 + \beta^2)p_B^2 + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_B + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$

Mixed teams

Because the ex-ante belief about the best decision is different than in the previous model with three possible decisions, I have to take a look at both mixed team compositions.

Agent A Biased Expert, Agent B Neutral Agent

Beliefs

To analyze these cases, I need to start reconsidering the beliefs of the decisionmaker after he received messages of the agents. There are six possible combinations of messages, for each combination I will take a look at the beliefs of the decisionmaker and determine his decision.

 $- \quad \hat{\theta}_A = -2, \hat{\theta}_B = 2$

The beliefs of the decisionmaker are:

•
$$\theta_A = -1$$
 with probability $\frac{\beta}{\alpha + \beta}$
• $\theta_A = -2$ with probability $\frac{\alpha}{\alpha + \beta}$

$$\circ \quad \theta_B = 2$$

The decisionmaker decides option A if $\alpha > \beta$. If $\beta > \alpha$ the decisionmaker decides B.

$$\hat{\theta}_A = -2, \hat{\theta}_B = 1$$

The beliefs of the decisionmaker are:

•
$$\theta_A = -1$$
 with probability $\frac{\beta}{\alpha + \beta}$
• $\theta_A = -2$ with probability $\frac{\alpha}{\alpha + \beta}$
• $\theta_B = 1$

The decisionmaker will decide option A if he receives these messages.

$$- \quad \hat{\theta}_A = -2, \, \hat{\theta}_B = 0$$

The beliefs of the decisionmaker are:

$$\begin{array}{l} \circ \quad \theta_A = -1 \text{ with probability } \frac{\beta}{\alpha+\beta} \\ \circ \quad \theta_A = -2 \text{ with probability } \frac{\alpha}{\alpha+\beta} \\ \circ \quad \theta_B = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \end{array}$$

The assumption that $(1 - \alpha - \beta) > (\alpha + \beta)(1 - p_N)$ makes sure that the decisionmaker decides A if he receives these messages.

 $- \quad \hat{\theta}_A = 0, \hat{\theta}_B = 2$

The beliefs of the decisionmaker are:

$$\theta_{A} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)}$$

$$\theta_{A} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{B})+(1-\alpha-\beta)}$$

$$\theta_{B} = 2$$

The assumption that $(1 - \alpha - \beta) > (\alpha + \beta)(1 - p_B)$ makes sure that the decisionmaker decides B if he receives these messages.

- $\hat{\theta}_A = 0, \hat{\theta}_B = 1$

The beliefs of the decisionmaker are:

$$\begin{array}{l} \circ \quad \theta_A = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_A = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \end{array}$$

The assumption that $(1 - \alpha - \beta) \ge (\alpha + \beta)(1 - p_B)$ makes sure that the decisionmaker decides B.

$$- \quad \hat{\theta}_A = 0, \hat{\theta}_B = 0$$

The beliefs of the decisionmaker are:

0	$\theta_A = 2$ with probability $\tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$
0	$\theta_A = 1$ with probability $\tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$
0	$\theta_A = 0$ with probability $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)}$
0	$\theta_B = 2$ with probability $\tilde{\alpha} = \frac{\alpha(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$
0	$\theta_B = 1$ with probability $\tilde{\beta} = \frac{\beta(1-p_N)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$
0	$\theta_B = 0$ with probability $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)}$

The assumption that $(1 - \alpha - \beta) > (\alpha + \beta)(1 - p_N)$ makes sure that the decisionmaker chooses option A.

These decisions after observing different combinations of messages can be found in table 8 and table 5.

$\widehat{oldsymbol{ heta}}_A$	$\widehat{\boldsymbol{ heta}}_B$	Decision
-2	2	A if $\alpha > \beta$
		B if $\beta > \alpha$
-2	1	A
-2	0	A
0	2	В
0	1	В
0	0	A

Table 8. Decisions with agent A Biased Expert and agent B Neutral Agent in the model with two possible decisions

Probability of making the right decision

As in appendix B for the model with three possible decisions, I need to calculate the probability of making the right decision in each situation and multiply it by the probability this situation occurs to find the total probability of making the right decision. There are nine different possible situations.

- $\theta_A = -2$, $\theta_B = 2$

The right decision is made with probability $p_B + (1 - p_B)(1 - p_N)$ if $\alpha > \beta$. If $\beta > \alpha$, the right decision is made with probability $(1 - p_N)$.

$$\theta_A = -2, \theta_B = 1$$

_

The right decision is made with probability $p_B + (1 - p_B)(1 - p_N) = 1 - p_N + p_B p_N$.

-
$$\theta_A = -2$$
, $\theta_B = 0$

The probability of making the right decision is 1 in this situation.

- $\theta_A = -1$, $\theta_B = 2$

The right decision is made with probability $(1 - p_B)p_N$ if $\alpha > \beta$. If $\beta > \alpha$, the right decision is made with probability p_N .

- $\theta_A = -1, \theta_B = 1$

The right decision is made with probability $p_B + (1 - p_B)(1 - p_N) = 1 - p_N + p_B p_N$.

$$\theta_A = -1, \theta_B = 0$$

The right decision is made with probability 1.

- $\theta_A = 0, \theta_B = 2$

The right decision is made with probability p_N .

- $\theta_A = 0, \theta_B = 1$

The right decision is made with probability p_N .

$$-\theta_A=0, \theta_B=0$$

The right decision is made with probability 1.

The total probability of making the right decision with a Biased Expert as agent A and a Neutral Agent as agent B is:

If $\alpha \ge \beta$: $(\alpha^2 + \beta^2 + \alpha\beta)(1 - p_N + p_B p_N) + (\alpha + \beta)(1 - \alpha - \beta) + \alpha\beta(p_N - p_B p_N) + (\alpha + \beta)(1 - \alpha - \beta)p_N + (1 - \alpha - \beta)^2$

I can rewrite this to: $(\alpha^2 + \beta^2)p_B p_N + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_N + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$

If $\beta > \alpha$: $\alpha^2(1-p_N) + (\alpha\beta + \beta^2)(1-p_N + p_B p_N) + (\alpha + \beta)(1-\alpha - \beta) + \alpha\beta p_N + (\alpha + \beta)(1-\alpha - \beta)p_N + (1-\alpha - \beta)^2$ I can rewrite this to:

 $(\alpha\beta+\beta^2)p_Bp_N+(\alpha-2\alpha^2+\beta-2\beta^2-2\beta)p_N+(1-\alpha-\beta)+\alpha^2+\beta^2+\alpha\beta$

Agent A Neutral Agent, Agent B Biased Expert

Beliefs

In this section I will take a look at the results when agent A is Neutral Agent and agent B is Biased Expert. Therefore, I need to start by reconsidering the beliefs of the agent for the six possible combinations of messages again.

- $\hat{\theta}_A = -2, \hat{\theta}_B = 2$

The beliefs of the decisionmaker are:

$$\circ \quad \theta_A = -2$$

$$\circ \quad \theta_B = 1$$
 with probability $\frac{\beta}{\alpha + \beta}$

$$\circ \quad \theta_B = 2$$
 with probability $\frac{\alpha}{\alpha + \beta}$

The decisionmaker decides option A if he receives these messages because there is no probability that $\theta > 0$.

$$\hat{\theta}_A = -2$$
, $\hat{\theta}_B = 0$

-

The beliefs of the decisionmaker are:

$$\begin{array}{l} \circ \quad \theta_A = -2 \\ \circ \quad \theta_B = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_B)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \\ \circ \quad \theta_B = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B) + (1-\alpha-\beta)} \end{array}$$

The decisionmaker will decide A if he receives these messages.

$$- \hat{\theta}_A = -1, \hat{\theta}_B = 2$$

The beliefs of the decisionmaker are:

 $\circ \quad \theta_A = -1$

•
$$\theta_B = 1$$
 with probability $\frac{\beta}{\alpha + \beta}$

$$\circ \quad \theta_B = 2$$
 with probability $\frac{\alpha}{\alpha + \beta}$

The decisionmaker decides option A if $\beta > \alpha$. If $\alpha > \beta$ the decisionmaker decides B.

$$- \quad \hat{\theta}_A = -1, \hat{\theta}_B = 0$$

The beliefs of the decisionmaker are:

$$\theta_{A} = -1$$

$$\theta_{B} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

The decisionmaker decides option A if $(1 - \alpha - \beta) + \beta(1 - p_B) \ge \alpha(1 - p_B)$ which is always the case because of the assumption that $(1 - \alpha - \beta) > (\alpha + \beta)(1 - p_B)$.

-
$$\hat{\theta}_A = 0, \hat{\theta}_B = 2$$

The beliefs of the decisionmaker are:

$$\theta_{A} = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{A} = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 1 \text{ with probability } \frac{\beta}{\alpha+\beta}$$

$$\theta_{B} = 2 \text{ with probability } \frac{\alpha}{\alpha+\beta}$$

The decisionmaker decides option B if $\tilde{\beta} * \frac{\alpha}{\alpha+\beta} + \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_N) + (1-\alpha-\beta)} \ge \tilde{\alpha} + \tilde{\beta} * \frac{\beta}{\alpha+\beta}$. The assumption that $(1-\alpha-\beta) \ge (\alpha+\beta)(1-p_N)$ makes sure that the decisionmaker decides B.

$$- \hat{\theta}_A = 0, \hat{\theta}_B = 0$$

The beliefs of the decisionmaker are:

$$\theta_{A} = -2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{A} = -1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{N})}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{A} = 0 \text{ with probability } \frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_{N}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 2 \text{ with probability } \tilde{\alpha} = \frac{\alpha(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

$$\theta_{B} = 1 \text{ with probability } \tilde{\beta} = \frac{\beta(1-p_{B})}{(\alpha+\beta)(1-p_{B}) + (1-\alpha-\beta)}$$

•
$$\theta_B = 0$$
 with probability $\frac{(1-\alpha-\beta)}{(\alpha+\beta)(1-p_B)+(1-\alpha-\beta)}$

The assumption makes sure that the decisionmaker decides option A.

These decisions after observing different combinations of messages can be found in table 9 and table 5.

$\widehat{oldsymbol{ heta}}_A$	$\widehat{\boldsymbol{ heta}}_B$	Decision
-2	2	A
-2	0	A
-1	2	A if $\beta > \alpha$
		B if $\alpha > \beta$
-1	0	A
0	2	В
0	0	A

Table 9. Decisions with agent A Neutral Agent and agent B Biased Expert in the model with two possible decisions.

Probability of making the right decision

To find the probability of making the right decision with this team composition I need to calculate the probability of making the right decision in each situation and multiply it by the probability this situation occurs. There are nine different possible situations.

$$\theta_A = -2, \theta_B = 2$$

The right decision is made with probability $p_N + (1 - p_N)(1 - p_B) = 1 - p_B + p_N p_B$.

$$\theta_A = -2, \theta_B = 1$$

The right decision is made with probability $p_N + (1 - p_N)(1 - p_B) = 1 - p_B + p_N p_B$.

- $\theta_A = -2, \theta_B = 0$

The right decision is made with probability 1.

$$\theta_A = -1, \theta_B = 2$$

The right decision is made with probability p_B when $\alpha > \beta$. If $\beta > \alpha$, the right decision is made with probability $p_B(1 - p_N)$.

-
$$\theta_A = -1, \theta_B = 1$$

The right decision is made with probability $(1 - p_B)$ when $\alpha > \beta$. When $\beta > \alpha$, the right decision is made with probability $p_N + (1 - p_B)(1 - p_N) = 1 - p_B + p_B p_N$.
- $\theta_A = -1$, $\theta_B = 0$

The right decision is made with probability 1.

-
$$\theta_A = 0, \theta_B = 2$$

The right decision is made with probability p_B .

$$\theta_A = 0, \theta_B = 1$$

The right decision is made with probability p_B .

-
$$\theta_A = 0, \theta_B = 0$$

Both agents will not find information and the right decision is made with probability 1.

The total probability of making the right decision with a team that consists of a Neutral Agent as agent A and a Biased Expert as agent B is:

If
$$\alpha \ge \beta$$
:
 $(\alpha^2 + \alpha\beta)(1 - p_B + p_N p_B) + (\alpha + \beta)(1 - \alpha - \beta) + \alpha\beta p_B + \beta^2(1 - p_B) + (\alpha + \beta)(1 - \alpha - \beta)p_B$
 $+ (1 - \alpha - \beta)^2$

I can rewrite this to:

 $(\alpha\beta+\alpha^2)p_Bp_N+(\alpha-2\alpha^2+\beta-2\beta^2-2\alpha\beta)p_B+(1-\alpha-\beta)+\alpha^2+\beta^2+\alpha\beta$

If
$$\beta > \alpha$$
:
 $(\alpha^2 + \beta^2 + \alpha\beta)(1 - p_B + p_N p_B) + (\alpha + \beta)(1 - \alpha - \beta) + \alpha\beta(p_B - p_B p_N) + (\alpha + \beta)(1 - \alpha - \beta)p_B$
 $+ (1 - \alpha - \beta)^2$

I can rewrite this to: $(\alpha^2 + \beta^2)p_B p_N + (\alpha - 2\alpha^2 + \beta - 2\beta^2 - 2\alpha\beta)p_B + (1 - \alpha - \beta) + \alpha^2 + \beta^2 + \alpha\beta$

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