Erasmus School of Economics
Erasmus University Rotterdam
March 2021

Supervisor: Dr. Kevin Spiritus
Second Assessor: Dr. Aart Gerritsen
Author: Henrika Meyer

Master thesis

To Work or not to Work

The optimal carbon tax
with an extensive margin of labor supply

Address:
Groen van Prinstererstraat 7BIII
3038RA Rotterdam

E-mail: henrika.meyer@posteo.de

Course of Studies: Msc Policy Economics
Student number: 538998

Contents

1 Abstract

This paper aims to build an optimal tax model with a carbon tax and an extensive margin of labor supply. According to optimal tax theory, labor supply responses strongly influence the optimal height of taxes and determine how to implement any tax efficiently. Usually, labor supply responses are modeled at the intensive margin. Due to norms and regulations, however, man people do not have the option to adjust the number of hours they work flexibly. The model developed in this paper allows to analyze the carbon tax whilst taking this inflexibility into account. It allows to carve out important mechanisms at the extensive margin, determine how the optimal carbon tax can differ according to circumstances, and develop policies to reduce the costs of implementing a carbon tax. The model shows that, next to the costs of the externality, the existing tax- and transfer system, different consumption needs of workers and non-workers, consumption elasticities and the number of people at the extensive margin, are crucial to determine the costs and benefits of a carbon tax.

2 Introduction

The aim of this paper is to build an optimal tax model with a carbon tax and an extensive margin of labor supply. Climate change is one of the biggest economic challenges of the next decades and “a result of the biggest market failure the world has ever seen” (Stern & Stern 2007). The carbon tax offers ”the most cost-effective lever to reduce carbon emissions at the scale and speed that is necessary” (Climate-Leadership-Council 2020). Therefore, it is a crucial economic question how to implement the carbon tax efficiently.

According to optimal tax theory, labor supply responses have a key influence on the optimal height of taxes and determine how to implement any tax efficiently. Usually, labor supply responses are modeled at the intensive margin. Due to norms and regulations, however, most people don´t have the option to adjust the number of hours they work flexibly, but can only choose between working, not working, and sometimes working half-time. Especially for European countries this is little controversial (Bargain et al. 2014) and shows in the distribution of working hours, which are extremely clustered around standard working times. Also, it has already been shown that the inclusion of an extensive margin can change the results of optimal tax models substantially, i.e. lowering the taxes rates derived (Diamond 1980) or making subsidies for low-income groups optimal (Saez 2002).

The aim of this paper is to develop a model that allows to analyze the carbon tax whilst taking into account the extensive margin of labor supply. It allows to carve out important mechanisms, determine how the optimal carbon taxes can differ from circumstance to circumstance, and think of policies to reduce the costs or increase the benefits of implementing it. It shows that, next to the costs of the externality, the existing tax system and transfer system, different consumption needs of workers and non-workers, consumption elasticities and the number of people at the extensive margin, hence the primary income distribution, are crucial to determine the costs and benefits of a carbon tax.

3 Literature Review

Most fundamentally speaking, the model built in this paper is based on the general framework of optimal taxation theory (Mirrlees 1971) and the concept of corrective externalities, which states that externalities should be taxed at their marginal costs in a first-best setting (Pigou 2020)

In the following, I will give a short overview of corrective taxes as well as the extensive margin in optimal tax theory. Sandmo was the first to analyze corrective taxes in an optimal tax framework, examining how corrective taxes interact with the rest of a (potentially distortionary) tax system (Sandmo 1975). Since then, much of the literature has been devoted to analyzing this question under different assumptions. Below, I will mention some exemplary papers in the field and give an idea of the broadness of baseline scenarios. Interestingly, the corrective tax has been modeled as a commodity tax on a dirty good in almost all papers and I will stick to this strand of the literature.

In the earliest papers which analyze the corrective tax, the revenue of the government is generated through commodity taxes which are collected in addition to the corrective commodity tax (Sandmo 1975). Later, optimal corrective commodity taxation with linear income taxes is examined (i.e. Pirttilä & Schöb (1999) and Jacobs & De Mooij (2015)). Recently, it has become more common to model tax systems with corrective taxation and optimal income redistribution with non-linear income taxes (see Micheletto (2008), Kaplow (2012), Jacobs & De Mooij (2015)). These papers focus on different aspects (i.e.the difference of atmospheric and non-atmospheric externalities, possible adaptations when starting from a non-optimal setting, and the question of what happens if either clean or polluting goods are more complementary to leisure). However, they all conclude that optimal corrective taxes are set at Pigouvian levels in a second-best setting if preferences are uniform and weakly separable between leisure and consumption.

Going into more detail, Jacobs has published several papers on corrective taxes in a setting with non-linear taxes. He supports the claim that a second-best corrective tax should not be corrected for the cost of marginal funds in such a setting because the marginal cost of the public funds equals marginal benefits from redistribution if the non-linear income tax is optimized. This is different if clean or dirty goods are more complementary to leisure (Jacobs 2011). In a second paper, Jacobs comes to the conclusion that the corrective tax can be above the Pigouvian tax if the externality hurts the poor disproportionally or below the Pigouvian tax if the poor spend a disproportionate amount of their income on the dirty good (Jacobs & van der Ploeg 2017). Furthermore, in an even more recent paper, Jacobs shows that if Engel curves are linear and pollution taxes are not at Pigouvian rates, a green tax reform, moving the tax closer to the Pigouvian rate, is Pareto improving (Jacobs & van der Ploeg 2019).

Since the early nineties, it has been discussed whether the government can use the revenue from corrective taxes to lower other, distortionary taxes, yielding a "double dividend" (Bovenberg & De Mooij 1994). For the case of an optimized tax system, the existence of a strong double dividend was rejected by Jacobs and de Mooji (Jacobs & De Mooij (2015)). However, Klenert et. al. show that the effect depends on the initial tax structure. If it is not optimized, there can be a double dividend in certain settings (Klenert et. al. 2018). Apart from this, the idea of a weak double dividend emerged, suggesting that it might be better to use revenues raised from correctives taxes to correct for other distortionary taxes than to pay them back in a lump-sum fashion. However, just as the strong dividend in an optimized setting, this idea was was rejected (i.e. Goulder (1995)).

A more recent strand of the literature analyses environmental tax reforms, the recycling of revenue, and the effects on income distribution (e.g. Klenert et al. (2018) and Aubert & Chiroleu-Assouline (2019)). These are interesting because they might provide answers to the distributional problems, which have so far mostly been discussed under the heading of the double dividend. All these papers conclude that the distributional consequences of corrective taxes are critically important to evaluate the consequences of green tax reforms.

In the following, I will give a quick overview of the most important contributions including an extensive margin, which are much fewer in number, even though the extensive margin turns out to be of quite some importance when looking more closely at labor supply (Heckman 1993). Diamond was the first to examine the influence of an extensive margin in an optimal tax framework and shows that in such a setting it is optimal to subsidize the working poor and thereby increase incentives to take the decision to work. (Diamond 1980). Later, Saez examines optimal income transfer programs including an intensive as well as extensive margin (Saez 2002). He comes to a similar conclusion stating that if labor supply responses are concentrated at the extensive margin, marginal tax rates need to be negative for small incomes. Later, when analyzing redistributional taxes, (Jacquet et al. 2013) conclude that even with an extensive margin, marginal tax rates are positive everywhere, but agree that they are strongly reduced by the extensive margin. Choné and Laroque further support these findings (Choné & Laroque 2011).

In more recent work it has been increasingly acknowledged that not all groups are equally likely to find themselves at the extensive margin. Recently, some life-cycle models have been introduced, which take into account that the choice to stop working is often a choice to retire (Michau 2014). Interestingly, they highlight the finding that retirement age, and hence the moment at which people take a decision at the extensive margin, should be strongly increasing with productivity. Similarly, more attention has been put to involuntary unemployment. The literature on unemployment also emphasizes that low wages should be subsidized rather than taxed (Lehmann et al. 2011). This literature is particularly important when evaluating possible policy implications which might arise from the conclusions drawn in this paper.

4 Why include the extensive margin?

Labor supply responses are one of the key parameters in optimal tax models and strongly influence their outcome. Labor supply responses can take place at the intensive and the extensive margin. A decision at the intensive margin encompasses how many hours to work and elasticities are calculated based on the variation in the number of hours worked per employed person. A decision at the extensive margin implies whether to work or not and elasticities at the extensive margin are determined by analyzing variation in the percentage of people who work.

In optimal tax models, it is usually assumed that there is only an intensive margin, and decisions at the extensive margin are left out. Mathematically this is convenient because it allows to analyze marginal changes in the tax system very well. However, since the conceptual distinction between intensive and extensive labor supply responses has been brought up, the empirics show that "the extensive margin is of fundamental importance" (Heckmann 1993). Later studies even suggest that the "extensive margin systematically dominates the intensive margin." (Bargain et al. 2014).

As a caveat, it has to be said that labor supply responses are extremely hard to quantify. Even though labor supply elasticities are a central parameter in public economics there is "little agreement among economists on the elasticity size that should be used in economic policy analyses" (Bargain et al. 2014). Variations across different studies come from the fact that they analyze different subgroups, but data years and decisions on estimation also account for part of the variation (Chetty et al. 2011). Still, there is good evidence that the extensive margin is at least as important as the intensive margin and should be taken into account. In the following, I want to give a quick overview of the most important insights from research.

Part of the literature tries to quantify labor supply responses at the extensive margin by exploiting quasi-experiments (among others Ljungqvist et al. (2006)). Most of these studies analyze changes in the incentives for different subgroups. Chetty et. al. summarize the findings of these studies and report an elasticity of 0,26 at extensive and 0,33 at the intensive margin (Chetty et al. 2011) These numbers give a rough idea of possible elasticities and indicate that responses at both margins play a role. However, they have to be taken with care, because the numbers reported are an unweighted mean and summarize findings from very different subgroups and in different periods. Having a closer look at the studies which are summarized, the elasticities found in the studies for the extensive margin range from 0.13 to 0.48.

Bargain et. Al. try to find consistent estimates across different subgroups and countries by employing a structural discrete choice model (Bargain et al. 2014). As a basis, they use data sets from all countries and simulations of the direct tax and transfer instruments. What they find is that the "extensive margin systematically dominates the intensive margin." (Bargain et al. 2014) They decompose labor supply responses and see that "most of the response is driven by the extensive margin." This holds systematically across almost all Western countries. "Even in the situation where the extensive margin is non-zero, the extensive margin is larger." (Bargain et al. 2014) Also, they find that small responses at the intensive margin are mainly due to the possibilities of working part-time, so even decisions counted as part of the intensive margin are mostly driven by discrete choices.

Apart from this, they find that the largest elasticities can be found at the bottom of the income distribution which is again due to participation elasticities. The only intensive margin responses that are found according to them are in the upper-income classes, more likely due to tax evasion than to actual adaptations in the number of working hours.

The studies on labor supply responses vary in their findings. However, quasi-experiments and estimations of a structural discrete-choice model point to the fact that the extensive margin has a big influence when it comes to labor supply responses. The estimation of the structural discrete-choice model even suggests that people might hardly be able to adjust their labor supply marginally and that all the responses are driven by the extensive margin. If this is true it has important implications for optimal tax design and more attention should be paid to models including such an extensive margin.

5 Crucial assumptions

Optimal tax models aim to simplify and abstract from reality to carve out the mechanisms of interest. The model developed in this paper aims to investigate the role of the extensive margin of labor supply for the optimal carbon tax. The model at hand mainly relies on the assumptions made in optimal tax theory, which have been explained and justified elsewhere. However, two assumptions aren´t found frequently in other models and are crucial for the model developed here. In the following, I will have a closer look at those assumptions.

The first assumption important assumption that needs to be highlighted is that workers consume differently from non-workers. Even though there have been a number of studies that analyze what determines the relative carbon emissions of a household and most of them mention that "factors such as the volume of employment are likely to be (...) important explanatory factors" (Preuß et al. 2019) there have been few attempts to pin down these differences and quantify them. In this regard, more research is needed. However, many workers have to commute to work and commuting is one of the most important factors in determining the relative carbon emission of an individual (Gechert et al. 2019). Also, the demand for fuel is very inelastic if somebody has to commute and public transport is not available. In such a case, not working can be the only way to avoid those costs and the extensive margin becomes particularly important.

A second assumption that deserves attention is that people react to price changes by adjusting their labor supply. In optimal tax theory, it is assumed that a labor income tax and a uniform commodity tax are equivalent in terms of the expected responses from individuals. This implies that people adapt their labor supply in the same way, regardless of whether their effective consumption is changed because of a change in the labor tax or because of a change in the price of products they want to buy. The assumptions made in this paper are in line with these considerations. However, further empirical research is needed to verify whether this holds empirically. Even though it would be rational to adopt labor supply in the same way, there might be behavioral biases, i.e. because price changes are less evident than wage changes. At least in the short run, this could cause a substantial difference in terms of adaptation.

6 Model

The model consists of heterogeneous individuals and a government. Individuals maximize their utility by choosing between working and not working and between consuming clean goods c and dirty goods x. If they work they earn an income z_n that depends on their abilities n, if they don´t work they receive a fixed government transfer g. Any disposable income can be divided between clean and dirty goods. The government maximizes social welfare by setting a linear income tax τ, a lump-sum tax s and a linear commodity tax t_x on the dirty good. The informational assumption is that the government is able to observe individual incomes, but only the aggregate consumption of dirty goods. It is assumed that the government has already optimized its income tax schedule.

6.1 Households

There is a total mass of individuals equal to M. Individuals differ by their ability $n \in N = [\underline{n}, \hat{n}]$. Their income z_n depends on their ability n and is defined by $z_n = n$. All individuals who work, have the same number of working hours, such that the number of working hours does not play a role in the model. All labor types are assumed to be perfect substitutes in aggregate production. The density of individual types with index n is denoted by $f(n)$ and the cumulative distribution function by $F(n)$. All individual-specific variables are indexed with subscript n if they work and subscript g if they don´t work. Each individual n derives utility from clean goods c and dirty goods x and a fixed disutility from working D. It is assumed that all individuals derive the same disutility from working.

The utility of individuals is described by two different utility functions, which are strictly quasi-concave and additively separable between the potential disutility to work and consumption. This means that the proportion of clean and dirty goods does not depend on whether somebody works or not. u_1 is the utility function of individuals working, whereby utility depends on ability n. D is the disutility from working, which is the same for all individuals. u_0 is the utility function of individuals who don´t work and is independent of ability. Therefore, the utility of all individuals not working is the same.

$$u_1 = u_1(c_n, x_n) - D \tag{1}$$

$$u_0 = u_0(c_g, x_g) \tag{2}$$

The budget constraints of the two different groups can be described by the next equations, where $(1 - \tau)z_n$ - s is the disposable income of workers. Thereby, τ is the tax rate dependent on income, s is an additional lump-sum tax.

$$c_n + (1 + t_x)x_n = (1 - \tau)z_n - s \tag{3}$$

$$c_g + (1 + t_x)x_g = g \tag{4}$$

The indirect utility functions describe the utility of an individual who has optimized her consumption and labor supply. The indirect utilities depend on the carbon tax t_x, the income tax τ and the government transfer g. v_1 is the indirect utility of people who work, v_0 is the indirect utility of people who don´t work.

$$v_1 = (t_x, \tau) \tag{5}$$

$$v_0 = (t_x, g) \tag{6}$$

The optimization problem of individuals has to be solved by backward induction. In a first step, individuals maximize utility within their respective group of workers or non-workers. In a second step, the utility derived from optimal consumption when working and not working is compared, and individuals sort into the two groups. More precisely, from utility maximization follows that individuals work if:

$$v_1(t_x, \tau) > v_0(t_x, g) \tag{7}$$

This inequality determines their decision. The individual that is indifferent between working and not working, or rather the income that makes somebody indifferent between working and not working is denoted by n_e and defines the location of the extensive margin.

The Lagrangian for workers reads as follows, where λ Lagrange multiplier of the household budget constraint.:

$$L_1 = u_1 + \lambda\Big[(1-\tau)z_n - s - c - (1+t_x)x\Big] \tag{8}$$

And for non-workers:

$$L_0 = u_0 + \lambda\Big[g - c - (1+t_x)x\Big] \tag{9}$$

From these two equations, the marginal utilities of c and x can be derived. u_{1c} is the marginal utility of c for workers, u_{1x} is the marginal utility of x for workers. Similarly, u_{0c} is the marginal utility of c for non-workers, u_{0x} is the marginal utility of x for non-workers. In this benchmark case, the marginal utilities are the same for workers and non-workers.

For workers:

$$\frac{\partial L_1}{\partial c} = u_{1c} - \lambda = 0 \iff u_{1c} = \lambda \tag{10}$$

$$\frac{\partial L_1}{\partial x} = u_{1x} - \lambda(1+t_x) = 0 \iff u_{1x} = \lambda(1+t_x) \tag{11}$$

For non-workers:

$$\frac{\partial L_0}{\partial c} = u_{0c} - \lambda = 0 \iff u_{0c} = \lambda \tag{12}$$

$$\frac{\partial L_0}{\partial x} = u_{0x} - \lambda(1+t_x) = 0 \iff u_{0x} = \lambda(1+t_x) \tag{13}$$

In the optimized state, the individual benefit of the clean good c equals the cost of giving up one euro or the cost of buying that good. However, for the dirty good x, the environmental tax drives a wedge between individual benefit and individual cost. If the tax rate is at Pigouvian levels, this allows to internalize the social cost.

How does the optimized utility of somebody change when the government parameters change? One way to find these derivatives is to derive the Lagrangian with respect to the parameters that change. In the following, I will give expressions for these derivatives. All of them are found by deriving the Lagrangian with respect to the variable.

$$\frac{\partial v_0}{\partial t_x} = \frac{\partial L_0}{\partial t_x} = -\lambda x_g \tag{14}$$

$$\frac{\partial v_1}{\partial t_x} = \frac{\partial L_1}{\partial t_x} = -\lambda x_n \tag{15}$$

$$\frac{\partial v_0}{\partial g} = \frac{\partial L_0}{\partial g} = \lambda \tag{16}$$

$$\frac{\partial v_1}{\partial g} = \frac{\partial L_1}{\partial g} = 0 \tag{17}$$

These partial derivatives play a crucial role in determining the extensive margin.

6.2 Extensive margin

As already said, the utilities of individuals and their subsequent decisions determine the location of the extensive margin. However, if the government parameters change utilities change, individuals take different decisions, and the location of the extensive margin changes. Formally, n_e is a function of the government parameters, write $n_e(t_x, g.)$

The ability of the individual that is indifferent between working and not working, that is $n_e(t_x, g)$, is defined by $v_1(t_x, g, n_e) - v_0(t_x, g) = 0$. If a policy parameter t_x or g changes, n_e will change accordingly and the equation remains satisfied. $\frac{\partial n_e}{\partial t_x}$ can be derived from this reasoning:

$$\frac{\partial \left[v_1 \left(t_x, g, n_e(t_x, g)\right) - v_0 \left(t_x, g\right)\right]}{\partial t_x} = 0 \tag{18}$$

$$\Leftrightarrow \frac{\partial v_1(t_x, g, n_e)}{\partial t_x} + \frac{\partial v_1(t_x, g, n_e)}{\partial n}\frac{\partial n_e}{\partial t_x} - \frac{\partial v_0(t_x, g)}{\partial t_x} = 0 \tag{19}$$

$$\Leftrightarrow \frac{\partial n_e}{\partial t_x} = -\left[\frac{\partial v_1(t_x, g, n_e)}{\partial t_x} - \frac{\partial v_0(t_x, g)}{\partial t_x}\right]\left[\frac{\partial v_1(t_x, g, n_e)}{\partial n}\right]^{-1}. \tag{20}$$

The partial derivative $\frac{\partial n_e}{\partial t_x}$ of this equation describes how the extensive margin changes when the carbon tax is changed. The equation above is still very hard to interpret. To make it more clear, the derivatives of the indirect utility for workers and non-workers with respect to t_x, which have already been derived above (line 14 and 15), have to be inserted.

The last term from equation 20 can be written as follows:

$$\left[\frac{\partial v_1(t_x, g, n_e)}{\partial n}\right]^{-1} = \left[\frac{\partial L_1}{\partial n}\right]^{-1} \tag{21}$$

$$\frac{\partial L_1}{\partial n} = \lambda(1-\tau) \tag{22}$$

Putting these different equations together yields how the extensive margin changes when the carbon tax is changed:

$$\frac{\partial n_e}{\partial t_x} = -\left[\left(-\lambda x_n\right) - \left(-\lambda x_g\right)\right]\left[\frac{1}{\lambda(1-\tau)}\right] \tag{23}$$

$$\Leftrightarrow \frac{\partial n_e}{\partial t_x} = \frac{x_n - x_g}{(1-\tau)} \tag{24}$$

Since x_n is expected to be higher than x_g, the term is positive, which shows that the extensive margin increases when the carbon tax increases. Once there is a higher carbon tax, people with higher wages stop working. How much the extensive margin changes, depends on the difference in consumption of the dirty good x between the two groups. If workers consume much more of good x (i.e. because they have to commute), they lose relatively more through an increase of the tax t_x, compared to people who don´t work. Then, the option of not working becomes more attractive.

Also, the relative importance of this difference is determined by the net income of people who work. If the income tax is lower, and hence they have a higher net income, the difference between x_n and x_g is relatively less important and the extensive margin changes less. However, if their net income is lower, it is relatively more important and the extensive margin changes more. An alternative interpretation of the denominator is that if the tax is higher and working is more expensive compared to not-working, the extensive margin reacts more strongly.

How does this relate to the participation elasticity, which measures the percentage change in the fraction of people working caused by a percentage change in the tax? Here, $\frac{\partial n_e}{\partial t_x}$ is derived from assumed preferences and defined in terms of the variables used in this model. Hence, the term is endogenous to the model. The model could be either calibrated by using these variables and information on the number of people at the extensive margin $f(n_e)$ or by using participation elasticities from the empirical literature. If the assumptions on preferences are reasonable and the empirical estimates are reliable, the results should be similar. The method used here is more transparent on mechanisms, whereas the participation elasticity can be used to test the assumptions used.

6.3 Government

The government sets taxes such that they maximize social welfare. The welfare function is composed of the indirect utility of people who work and people who don´t work. Utilities of people are weighted according to their welfare weight Ψ, which is based on ability or earnings potential n. Apart from this, the number of people at a particular level of ability, which is denoted by f(n), has to be taken into account. The size of the group of workers and non-workers is determined by the location of the extensive margin.

The externality can be modeled in different ways. Here, it will be assumed that the government has to pay for the damages caused by the externality. They encompass damages to the public infrastructure as well as compensations that might have to be given to individuals. The externality is defined in terms of these costs for the government and enters the government budget constraint. It is denoted by E. The revenue requirement that is independent of these damages is denoted by G.

Apart from this, the maximization problem of the government is influenced by assumptions about the households. As already mentioned, here it is assumed that the proportion of individual consumption is not influenced by whether they work or not.

The social welfare function looks as follows:

$$W = \int_{\underline{n}}^{n_e} \Psi v_0(t_x, g)\, f(n)dn + \int_{n_e}^{\hat{n}} \Psi v_1(t_x, \tau, n)\, f(n)dn \tag{25}$$

The budget constraint is:

$$\int_{\underline{n}}^{n_e} \Big[t_x x_g(t_x) - g\Big] f(n)dn + \int_{n_e}^{\hat{n}} \Big[t_x x(t_x, n) + \tau z_n + s\Big] f(n)dn = G + E(X) \tag{26}$$

To solve the government maximization problem the Lagrangian is needed. It consists of several parts and has to be differentiated by using the Leibniz rule. x_g denotes the dirty goods consumption of somebody who does not work. x_n is the dirty good consumption of somebody with ability n who does work. η is the shadow price of the government budget constraint.

$$L = \int_{\underline{n}}^{n_e} \Psi v_0(t_x, g) f(n)dn + \int_{n_e}^{\hat{n}} \Psi v_1\big(t_x, \tau, n\big) f(n)\, dn \tag{27}$$

$$+\eta\Bigg\{\int_{\underline{n}}^{n_e} \Big[t_x x_g(t_x) - g\Big] f(n)dn + \int_{n_e}^{\hat{n}} \Big[t_x x(t_x, n) + \tau z_n + s\Big] f(n)dn - G - E(X)\Bigg\} \tag{28}$$

For simplicity, the Lagrangian will be split into several sub-equations and the partial derivatives with respect to t_x are taken separately. Due to the summation rule, the derivatives can be added afterward.

$$A = \int_{\underline{n}}^{n_e} \Psi v_0\left(t_x, g\right) f(n)dn + \int_{n_e}^{\hat{n}} \Psi v_1\left(t_x, \tau, n\right) f(n)dn \tag{29}$$

$$B = \eta \int_{\underline{n}}^{n_e} \Big[t_x x_g(t_x) - g\Big] f(n)dn \tag{30}$$

$$C = \eta \int_{n_e}^{\hat{n}} \Big[t_x x(t_x, n) + \tau z_n + s\Big] f(n)dn \tag{31}$$

$$D = \eta \Big[- G - E(X)\Big] \tag{32}$$

In the next section, the partial derivatives are taken one by one.

6.3.1 Individual utilities

This first part consists of the individual utilities, weighted by the welfare weights Ψ. When partially deriving A with respect to t_x the Leibniz rule is used.

$$A = \int_{\underline{n}}^{n_e} \Psi v_0\left(t_x, g\right) f(n)dn + \int_{n_e}^{\hat{n}} \Psi v_1\left(t_x, \tau, n\right) f(n)dn \tag{33}$$

The Lagrangian looks as follows:

$$\frac{\partial A}{\partial t_x} = \left[\frac{\partial n_e}{\partial t_x}\Psi v_0(t_x, g) f(n_e)\right] - \left[\frac{\partial \underline{n}}{\partial t_x}\Psi v_0\left(t_x, g\right) f(\underline{n})\right] + \int_{\underline{n}}^{n_e} \Psi \frac{\partial v_0}{\partial t_x} f(n)dn \tag{34}$$

$$+\left[\frac{\partial \hat{n}}{\partial t_x}\Psi v_1\left(t_x, \tau, \hat{n}\right) f(\hat{n})\right] - \left[\frac{\partial n_e}{\partial t_x}\Psi v_1\left(t_x, \tau, n_e\right) f(n_e)\right] + \int_{n_e}^{\hat{n}} \Psi \frac{\partial v_1}{\partial t_x} f(n)dn \tag{35}$$

The first two summands describe the change in welfare caused by a change in the size of the integral encompassing those people who don´t work. The third summand describes the change in utility of those people who work neither before nor after a change in t_x. Analogously, the second line describes the partial derivative of the second summand of A.

This partial derivative can be simplified further. The term $-\frac{d\underline{n}}{dt_x}\Psi v_0\left(t_x, g\right)$f($\underline{\text{n}}$) equals 0, because what is the lowest ability $\underline{n}$ does not change as the results of a change in the tax. The same is true for $\frac{d\hat{n}}{dt_x}\Psi v_1\left(t_x, \tau, \hat{n}\right) f(\hat{n})$, because the highest ability isn´t influenced by the height of t_x either.

$$\frac{\partial A}{\partial t_x} = \left[\frac{\partial n_e}{\partial t_x}\Psi v_0\left(t_x, g\right) f\left(n_e\right)\right] + \int_{\underline{n}}^{n_e} \Psi \frac{\partial v_0}{\partial t_x} f(n) dn \tag{36}$$

$$-\left[\frac{\partial n_e}{\partial t_x}\Psi v_1\left(t_x, \tau, n_e\right) f\left(n_e\right)\right] + \int_{n_e}^{\hat{n}} \Psi \frac{\partial v_1}{\partial t_x} f(n) dn \tag{37}$$

Apart from this, $\frac{\partial n_e}{\partial t_x}\Psi v_0\left(t_x, g\right) f\left(n_e\right)$ and $\frac{\partial n_e}{\partial t_x}\Psi v_1\left(t_x, \tau n_e\right) f\left(n_e\right)$ cancel out. This is because the extensive margin is defined by the individual that is indifferent between working. The difference between the utility of the two groups directly at the extensive margin equals 0.

Now, the derivative can be written as:

$$\frac{\partial A}{\partial t_x} = \int_{\underline{n}}^{n_c} \Psi \frac{\partial v_0}{\partial t_x} f(n) dn + \int_{n_e}^{\hat{n}} \Psi \frac{\partial v_1}{\partial t_x} f(n) dn \tag{38}$$

The partial derivatives of indirect utility $\frac{\partial v_0}{\partial t_x}$ and $\frac{\partial v_1}{\partial t_x}$, which have been derived before, are than put into this equation.

This yields:

$$\frac{\partial A}{\partial t_x} = \int_{\underline{n}}^{n_e} -\Psi \lambda x_g f(n) dn + \int_{n_e}^{\hat{n}} -\Psi \lambda x_n f(n) dn \tag{39}$$

The equation describes the welfare loss at the individual level which is caused by higher prices of the dirty good. It is divided into two integrals because the loss of income is likely to be different for people who work and people who don´t work.

6.3.2 Government revenue non-workers

In the following, the partial derivative of part B will be derived. It describes the government revenue collected from non-workers.

$$B = \eta \int_{\underline{n}}^{n_e} \Big[\big(t_x x_g(t_x) - g\big) f(n)\Big] dn \tag{40}$$

Using the Leibniz rule, the following equation arises:

$$\frac{\partial B\left(t_x\right)}{\partial t_x}=\eta\left\{\frac{\partial n_e}{\partial t_x}\left[t_x x_g\left(t_x\right)-g\right]f(n_e)+\int_{\underline{n}}^{n_e(t_x)}\left[x_g\left(t_x\right)+t_x\frac{\partial x_g}{\partial t_x}\right]f(n)dn\right\} \tag{41}$$

This equation describes the change in overall welfare that is caused by a change in government revenue that can be attributed to people who stop working and people who work neither before nor after, but pay a different amount of tax for what they consume and also change the amount of x they consume.

6.3.3 Government revenue workers

The term C is also derived by applying the Leibniz rule.

$$C=\eta\int_{n_e}^{\hat{n}}\left[t_x x(t_x,n)+\tau z_n+s\right]f(n)dn \tag{42}$$

The derivative looks as follows, when taking into account that $\hat{n}$ does not change as a result of a change in the tax:

$$\frac{dC}{dt_x}=\eta\left\{\left[-\frac{\partial n_e}{\partial t_x}\left(t_x x(t_x,n_e)+\tau z_{ne}+s\right)f(n_e)\right]+\int_{n_e(t_x)}^{\hat{n}}\left(x\left(t_x,n\right)+t_x\frac{\partial x\left(\hat{n}\right)}{\partial t_x}\right)f(n)dn\right\} \tag{43}$$

Adding up the derivatives of B and C, the following equation arises:

$$\frac{\partial(B+C)}{\partial t_x}=\eta\left\{\frac{\partial n_e}{\partial t_x}\cdot\left[\left(t_x x_g\left(t_x\right)-g\right)-\left(t_x x\left(t_x,n_e\right)+\tau z_{ne}+s\right)\right]f(n_e)\right. \tag{44}$$

$$\left.+\int_{\underline{n}}^{n_e(t_x)}\left(x_g\left(t_x\right)+t_x\frac{\partial x}{\partial t_x}\right)f(n)dn+\int_{n_e(t_x)}^{\hat{n}}\left(x\left(n,t_x\right)+t_x\frac{\partial x\left(n\right)}{\partial t_x}\right)f(n)dn\right\} \tag{45}$$

Line 44 describes the marginal costs of a carbon tax in terms of a loss in welfare, because of people who stop working. This cost occurs in terms of a loss in tax revenue and in terms of the transfer, which the government has to pay additionally. In short, it is the revenue that the government loses at the extensive margin. Line 45 describes costs/benefits in terms of the loss/gain of revenue within each of the groups, because the revenue generated through every good that is consumed, changes. In addition to that, the change in consumption of each of the groups changes revenues. This effect could be called an effect at the intensive margin of consumption. Somebody who works before and after the change in the tax might still commute, but try to commute less regularly or avoid rush hour.

6.3.4 Expenditures

The last part of the Lagrangian that has to be derived are the summands which describe the costs faced by the government; the public resource requirement G and costs which arise through the externality $E(X)$.

The public resource requirement does not change as a result of the change in the tax because any potential reform is assumed to be revenue neutral. Therefore, the partial derivative with respect t_x is 0 and can be dropped.

$$\frac{\partial G}{\partial t_x} = 0 \tag{46}$$

The costs caused by the externality change, depending on the change in consumption induced by the change in the tax.

$$\frac{\partial E}{\partial t_x} = \frac{\partial E\left(X\right)}{\partial X} \cdot \frac{\partial X}{\partial t_x} \tag{47}$$

Thereby, X is the overall consumption of the dirty good defined and derived as follows, again assuming that lowest ability $\underline{n}$ and highest ability $\hat{n}$ are not changed by the tax t_x.:

$$X = \int_{\underline{n}}^{n_e} x_g f(n)\ dn + \int_{n_e}^{\hat{n}} x\left(n\right) f(n) dn \tag{48}$$

$$\frac{\partial X}{\partial t_x} = \left(\frac{\partial n_e}{\partial t_x} x_g f(n_e)\right) + \int_{\underline{n}}^{n_e} \frac{\partial x_g}{\partial t_x} f(n)dn - \left(\frac{\partial n_e}{\partial t_x} x\left(n_e\right) f\left(n_e\right)\right) + \int_{n_e}^{\hat{n}} \frac{\partial x\left(n\right)}{\partial t_x} f(n)dn \tag{49}$$

This can be summarized as follows:

$$\frac{\partial X}{\partial t_x} = \frac{\partial n_e}{\partial t_x}\Big[x_g - x(n_e)\Big] f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n)dn \tag{50}$$

Again, when it comes to consumption there is a change at the intensive and the extensive margin. People change their consumption when switching groups, but also adapt their consumption within the group of workers and non-workers.

6.4 Optimal tax

Shortly summarized, the following derivatives, all with respect to t_x, are important:

$$\frac{\partial A}{\partial t_x} = \int_{\underline{n}}^{n_e} -\Psi\lambda x_g f(n)dn + \int_{n_e}^{\hat{n}} -\Psi\lambda x_n f(n)dn \tag{51}$$

$$\frac{\partial(B+C)}{\partial t_x} = \eta\Bigg\{\frac{\partial n_e}{\partial t_x}\Big[\Big(t_x x_g\left(t_x\right) - g\Big) - \Big(t_x x(t_x, n_e) + \tau z_{ne} + s\Big)\Big] f(n_e) \tag{52}$$

$$+ \int_{\underline{n}}^{n_e(t_x)} \left(x\left(t_x\right) + t_x \frac{\partial x}{\partial t_x}\right) f(n)dn + \int_{n_e(t_x)}^{\hat{n}} \left(x\left(n\right) + t_x \frac{\partial x\left(n\right)}{\partial t_x}\right) f(n)dn\Bigg\} \tag{53}$$

$$\frac{\partial D}{\partial t_x} = \eta\left\{\frac{\partial E}{\partial X}\frac{\partial X}{\partial t_x}\right\} = \eta\left\{\frac{\partial E}{\partial X}\left[\frac{\partial n_e}{\partial t_x}\Big(x_g - x(n_e)\Big) f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n)dn\right]\right\} \tag{54}$$

Putting all these partial derivatives together into one equation, the partial derivative of the Lagrangian looks as follows:

$$\frac{\partial L}{\partial t_x} = -\left\{\int_{\underline{n}}^{n_e} \Psi\lambda x_g f(n)dn + \int_{n_e}^{\hat{n}} \Psi\lambda x_n f(n)dn\right\} \tag{55}$$

$$+\eta\Bigg\{\frac{\partial n_e}{\partial t_x}\Bigg[\Big(t_x x_g\,(t_x)-g\Big)-\Big(t_x x(t_x,n_e)+\tau z_{ne}+s\Big)\Bigg]f(n_e) \tag{56}$$

$$+\int_{\underline{n}}^{n_e(t_x)}\left(x\,(t_x)+t_x\frac{\partial x}{\partial t_x}\right)f(n)dn+\int_{n_e(t_x)}^{\hat{n}}\left(x\,(n)+t_x\frac{\partial x\,(n)}{\partial t_x}\right)f(n)dn \tag{57}$$

$$-\frac{\partial E}{\partial X}\left[\frac{\partial n_e}{\partial t_x}\Big(x_g-x(n_e)\Big)f(n_e)+\int_{\underline{n}}^{\hat{n}}\frac{\partial x}{\partial t_x}f(n)dn\right]\Bigg\}=0 \tag{58}$$

The derivative above can be used to find an expression that illustrates which mechanisms determine the height of the optimal tax. The right-hand side can be reordered, such that it can be interpreted more easily. Line 56 is rearranged into line 60, separating transfer and income tax from the carbon tax. Line 57 is rearranged into line 61, respectively merging the two integrals into one:

$$0=\frac{\partial L}{\partial t_x}=-\left\{\int_{\underline{n}}^{n_e}\Psi\lambda x_g f(n)dn+\int_{n_e}^{\hat{n}}\Psi\lambda x_n f(n)dn\right\} \tag{59}$$

$$+\,\eta\frac{\partial n_e}{\partial t_x}\Big(-g-\tau z_n-s\Big)f(n_e)+\eta\frac{\partial n_e}{\partial t_x}\Big(t_x x_g(t_x)-t_x x(t_x,n_e)\Big)f(n_e) \tag{60}$$

$$+\,\eta\int_{\underline{n}}^{\hat{n}}x(g,n)f(n)dn+\eta\int_{\underline{n}}^{\hat{n}}t_x\frac{\partial x}{\partial t_x}f(n)dn \tag{61}$$

$$-\,\eta\frac{\partial E}{\partial X}\left[\frac{\partial n_e}{\partial t_x}\Big(x_g x(n_e)\Big)f(n_e)+\int_{\underline{n}}^{\hat{n}}\frac{\partial x}{\partial t_x}f(n)dn\right] \tag{62}$$

The Slutsky decomposition $\frac{\partial x}{\partial t_x}=\frac{\partial x^*}{\partial t_x}-x\frac{\partial x}{\partial I}$ is used to include the substitution and the income effect separately into the equation. Substituting the Slutsky decomposition into the equation, results in the following:

$$0 = \frac{\partial L}{\partial t_x} = -\left\{ \int_{\underline{n}}^{n_e} \Psi \lambda x_g f(n) dn + \int_{n_e}^{\hat{n}} \Psi \lambda x_n f(n) dn \right\} \tag{63}$$

$$+ \eta \frac{\partial n_e}{\partial t_x} \Big(-g - \tau z_n - s \Big) f(n_e) + \eta \frac{\partial n_e}{\partial t_x} \Big(t_x x_g(t_x) - t_x x(t_x, n_e) \Big) f(n_e) \tag{64}$$

$$+ \eta \int_{\underline{n}}^{\hat{n}} x(g,n) f(n) dn + \eta \int_{\underline{n}}^{\hat{n}} t_x \left(\frac{\partial x^*}{\partial t_x} - x \frac{\partial x}{\partial I} \right) f(n) dn \tag{65}$$

$$- \eta \frac{\partial E}{\partial X} \left[\Big(x_g - x(n_e) \Big) \frac{\partial n_e}{\partial t_x} f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n) dn \right] \tag{66}$$

This can be rearranged by putting together the integrals of the individual indirect utilites and deviding by η:

$$0 = \frac{\partial L}{\partial t_x} = \left\{ \int_{\underline{n}}^{\hat{n}} -\frac{1}{\eta} \Psi \lambda x f(n) dn \right\} \tag{67}$$

$$+ \int_{\underline{n}}^{\hat{n}} x(g,n) f(n) dn + \int_{\underline{n}}^{\hat{n}} t_x \left(\frac{\partial x^*}{\partial t_x} - x \frac{\partial x}{\partial I} \right) f(n) dn \tag{68}$$

$$+ \frac{\partial n_e}{\partial t_x} \Big(-g - \tau z_n - s \Big) f(n_e) + \frac{\partial n_e}{\partial t_x} \Big(t_x x_g(t_x) - t_x x(t_x, n_e) \Big) f(n_e) \tag{69}$$

$$- \frac{\partial E}{\partial X} \left[\Big(x_g - x(n_e) \Big) \frac{\partial n_e}{\partial t_x} f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n) dn \right] \tag{70}$$

In a next step, the first and the second summand can be put together.

$$0 = \frac{\partial L}{\partial t_x} = \Bigg\{ \int_{\underline{n}}^{\hat{n}} x - \frac{1}{\eta}\Psi\lambda x f(n)dn \Bigg\} + \int_{\underline{n}}^{\hat{n}} t_x \Big(\frac{\partial x^*}{\partial t_x} - x\frac{\partial x}{\partial I} \Big) f(n)dn \tag{71}$$

$$+ \frac{\partial n_e}{\partial t_x}\Big(-g - \tau z_n - s \Big) f(n_e) + \eta \frac{\partial n_e}{\partial t_x}\Big(t_x x_g(t_x) - t_x x(t_x, n_e) \Big) f(n_e) \tag{72}$$

$$- \frac{\partial E}{\partial X}\Bigg[\Big(x_g - x(n_e) \Big) \frac{\partial n_e}{\partial t_x} f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n)dn \Bigg] \tag{73}$$

The expression is adapted towards a more readable notation by factorizing the first summand, including the income effect, multiplying everything by $\frac{1+t_x}{1+t_x}$ and $\frac{x}{x}$ and rearranging the second summand most conveniently.

$$0 = \frac{\partial L}{\partial t_x} = \Bigg\{ \int_{\underline{n}}^{\hat{n}} (1 - \frac{\Psi\lambda}{\eta} - \frac{\partial x}{\partial I}) x f(n)dn \Bigg\} + \int_{\underline{n}}^{\hat{n}} \frac{t_x}{1+t_x} \frac{1+t_x}{x} \frac{\partial x*}{\partial 1 + t_x} x f(n)dn \tag{74}$$

$$+ \frac{\partial n_e}{\partial t_x}\Big(-g - \tau z_n - s \Big) f(n_e) + \eta \frac{\partial n_e}{\partial t_x}\Big(t_x x_g(t_x) - t_x x(t_x, n_e) \Big) f(n_e) \tag{75}$$

$$- \frac{\partial E}{\partial X}\Bigg[\Big(x_g - x(n_e) \Big) \frac{\partial n_e}{\partial t_x} f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n)dn \Bigg] \tag{76}$$

Now, this can easily be turned into a more readable expression, by using two additional concepts: The compensated elasticity and the social welfare weight. The compensated elasticity describes the percentage change in the overall consumption of x as a reaction to a percentage change in the tax t_x.

$$\varepsilon^*_{xt_x} \equiv \frac{1+t_x}{x} \frac{\partial x^*}{\partial(1+t_x)} \tag{77}$$

The social welfare describes the effect of giving an additional euro to an individual in terms of social welfare. It is composed of the relation between private and social marginal utility and the change in the consumption, caused by the income effect.

$$\beta \equiv \frac{\Psi\lambda}{\eta} + t_x \frac{\partial x}{\partial I}. \tag{78}$$

When substituting these two concepts into the equation above, the following equation arises:

$$0 = \frac{\partial L}{\partial t_x} = \int_{\underline{n}}^{\hat{n}} (1-\beta)\, x f(n) dn + \int_{\underline{n}}^{\hat{n}} \frac{t_x}{1+t_x} \varepsilon_{xt_x} x f(n) dn \tag{79}$$

$$+ \frac{\partial n_e}{\partial t_x}\Big(- g - \tau z_n - s\Big) f(n_e) + \frac{\partial n_e}{\partial t_x}\Big(t_x x_g(t_x) - t_x x(t_x, n_e)\Big) f(n_e) \tag{80}$$

$$- \frac{\partial E}{\partial X}\left[\Big(x_g - x(n_e)\Big)\frac{\partial n_e}{\partial t_x} f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n) dn\right], \tag{81}$$

Assuming that the labor tax, the intercept of the labor tax s, and the transfer g are optimal, the following property holds. The reason for this is that in an optimized tax system - per definition - the marginal benefit of transferring one additional euro to an individual equals the marginal benefit of giving one euro to the state. Otherwise, it would not be optimal.

$$\int_{\underline{n}}^{\hat{n}} (1-\beta)\, f(n) dn = 0 \tag{82}$$

Under this assumption, the following equation can be derived:

$$0 = \frac{\partial L}{\partial t_x} = - \, cov\,(\beta, x) + \frac{t_x}{1+t_x} \int_{\underline{n}}^{\hat{n}} \varepsilon^*_{xt_x} x f(n) dn \tag{83}$$

$$+ \frac{\partial n_e}{\partial t_x}\Big(- g - \tau z_n - s\Big) f(n_e) + \frac{\partial n_e}{\partial t_x}\Big(t_x x_g(t_x) - t_x x(t_x, n_e)\Big) f(n_e) \tag{84}$$

$$- \frac{\partial E}{\partial X}\left[\frac{\partial n_e}{\partial t_x}\Big(x_g - x(n_e)\Big) f(n_e) + \int_{\underline{n}}^{\hat{n}} \frac{\partial x}{\partial t_x} f(n) dn\right] \tag{85}$$

In the following section, this expression will be interpreted.

6.5 Interpretation of results

The term $cov(\beta, x)$ is a distributional characteristic: The covariance of the tax base with the social welfare weight will affect how this good should be taxed. The negative sign shows that if the relation between the two is stronger and people with a higher welfare weight consume more of the dirty good, the costs in terms of redistribution are higher and the tax should be lower. If people with a higher welfare weight consume less of the dirty good, the costs in terms of redistribution are lower and the tax should be higher.

The compensated elasticity also plays an important role. As in standard indirect taxation, a higher elasticity is an argument to have a lower tax, because if people react to it more strongly, the costs of the tax in terms of a dead-weight loss of the tax are higher. However, in the case of Pigouvian taxation, a higher elasticity also leads to higher tax benefits through a stronger decrease in the externality. How the compensated elasticity influences the optimal height of the tax, is therefore determined by the relation between the overall dirty good consumption X and the externality E because this relation determines the difference in the weight of the two channels. The same is true for the difference in dirty good consumption between workers and non-workers. A big difference is an argument to have a lower tax because the loss in revenue and therefore the costs are higher if people stop working, but it can also be an argument to have a higher tax because the externality decreases more and the gains are higher. As in the case of the compensated elasticity it depends on $\frac{\partial E}{\partial X}$.

Line 71 reflects the change in the government budget due to the change in the extensive margin. It is determined by the difference in revenue between workers and non-workers, regarding income tax and transfer as well as dirty good consumption and thereby carbon tax t_x. Apart from this, it is determined by the relation between the location of the extensive margin and t_x and by the number of people who find themselves at the extensive margin. If the extensive margin moves more strongly as a reaction to a change in t_x or if there are more people at the extensive margin, t_x has to be lower, because the costs of implementing the tax are higher if more people stop working.

How does all of the above relate to the standard Pigouvian tax? The standard Pigouvian tax is set such that the tax equals exactly the costs of the externality. However, depending on the compensated elasticity, the location of the extensive margin, the number of people at the extensive margin, and the differences between workers and non-workers in terms of revenue and dirty good consumption, it can be higher or lower. The short intuition for this difference is that the Pigouvian tax interacts with other parts of the tax system and distributional concerns. These interactions influence the costs and benefits of the tax and have to be taken into account when determining its optimal height.

7 Limitations

There are a number of limitations to the model developed here. Some of them come from the general framework of optimal tax theory, others limitations are more specific. Before coming to the conclusion and applying results to policy-making, I want to pay some attention to the limitations of the framework as well as the limitations of this specific model.

7.1 General Framework

Optimal tax theory can be useful to carve out mechanisms that are hidden because of non-trivial interactions and to weigh them according to their importance. However, the framework has a number of limitations that need to be considered carefully before drawing policy advice from the results. In the following, I want to bring up a number of limitations of optimal tax theory, which are particularly relevant for this paper.

In optimal tax models, labor supply responses are driven by a substitution and an income effect, and outcomes are determined by these channels. However, the studies which are used to quantify the relation between wages and labor supply are only evidence for the quantitative relation, not for the channels. The main drivers could be different and heterogeneous across subgroups. This is important because the composition of the channels matters for the outcomes of optimal tax models. If a large fraction of certain income groups does not optimize, but try to keep their income level constant or stick to their full-time job because of security, status, norms, or social commitment, the conclusions drawn from optimal tax models with respect to these groups are misleading. To give an example, if some groups stick to their job no matter what, there is no deadweight-loss with respect to these groups and - following the reasoning of the optimal tax framework - taxes could be much higher.

Optimal tax theory rests on the assumption that there is a trade-off between labor supply and government revenue. The aim is to set taxes such, that they optimize welfare in light of this trade-off. However, there are good arguments to support the claim that this trade-off does not exist. First, a big part of the government revenue is spent on wages. If people adjust their labor supply little (as the empirical literature suggests), and have a lower proportion to spend money and thereby create jobs than the government, a higher government revenue can even increase labor supply. Second, optimal tax models assume that taxes have to balance labor supply and government revenue optimally under additional constraints and aims. However, the government can influence labor supply and revenue through other channels, i.e. by taking up debt. If these mechanisms were included and labor supply and government revenue would not entirely rest on the height of taxes, optimal taxes might look different and depend on other policies. Therefore, it is unclear to what extent the results of optimal tax models can be used to formulate optimal policies.

Last, optimal tax models are very sensitive to alternative specifications. Depending on whether the income tax is linear or not, the relation between leisure and consumption, the question of whether savings are incorporated or whether there are several generations, results can differ substantially. If the aim is to get an intuition for different potential mechanisms, this is not a problem. However, in many cases, optimal tax theory is used to derive specific policy recommendations. This can be misleading if recommendations depend on individual model specifications and are not robust to alternative specifications because the policies recommended are not generally optimal, but only optimal in a very specific setting, that might differ from reality in key aspects, determining the policy.

7.2 Model with externality and extensive margin

Of course, the limitations mentioned above affect the model developed in this paper. In the following, I want to name a number of limitations, which do not necessarily affect all optimal tax models, but rather are more specific to the model at hand.

There is little empirical research on how people react to price changes, as has been mentioned above. In theory, people react to changes in wages by substituting income and leisure, whilst there is no difference between a change in income and a change in prices. In practice, the relation between wages and changes in labor supply might be different (as explained above) and people could react to changes in prices entirely differently. To name one potential difference, changes in wages might be more obvious to people than changes in prices, such that the reaction could be much stronger. Until further research is done, this remains an issue of speculation. Once there is more empirical evidence on these relations, behavioral findings might need to be incorporated into the model.

The model at hand captures differences in the absolute amount of consumption of different income groups and differences in the patterns of consumption between workers and non-workers. However, other interactions in terms of utility that influence decisions and consumption patterns are left out, i.e. the relation between income and the amount of dirty good consumption. This limits the extent to which distributional concerns can be taken into account. To be able to investigate these more properly, it might be valuable to include these into further research on the extensive margin. The same is true for the question how the externality affects different groups and how the externality might cause changes in working behavior and consumption.

8 Conclusion

This paper shows that the extensive margin is decisive in determining the costs and benefits of a carbon tax. The existing tax system, the primary income distribution, consumption elasticities, and consumption patterns all affect its optimal height. In the following, costs and benefits will be summarized. As an outlook, a number of policies that might allow to move to a better optimum by reducing the costs or increasing the benefits of a carbon tax, are suggested.

The existing tax system is an important factor in determining the costs of a carbon tax. First, it determines the incentives to work. If the income tax is higher, the number of people who potentially stop working is higher and the costs of the carbon tax increase. Second, it determines the loss in revenue if an individual at the extensive margin indeed stops working. Economically speaking, laissez-fair governments should therefore implement higher carbon taxes.

The overall loss of revenue from people who stop working is determined by the number of people who find themselves at the extensive margin. If more people find themselves at the extensive margin, the costs of the tax are higher. In the short-term, this means that countries with more people at the extensive margin should implement lower carbon taxes. In the long run, the costs of a carbon tax could be lowered by pushing people away from the extensive margin by changing the primary income distribution i.e. by educating people well or introducing a minimum wage.

Apart from this, the costs and benefits depend on the consumption patterns of the different groups. First, depending on the distributional characteristic of the dirty good consumption, the tax can have costs or benefit in terms of redistribution. If poor households spend more on dirty goods, the carbon tax is associated with costs in terms of redistribution. If rich households spend more on dirty goods, the carbon tax is associated with benefits in terms of redistribution. Second, just as the income tax, the consumption patterns influence the extensive margin. If the tax has a bigger effect on people who work, i.e. because they have to commute, it becomes more attractive not to work. Third, if people who don´t work, consume less of the dirty good and people stop working, this is an additional cost in terms of revenue, but a benefit in terms of the externality. In the following, this trade-off will be explained in more detail.

If a Pigouvian tax is implemented in an existing tax system and internalizing externalities is not the only aim, high elasticities imply a higher benefit in terms of reducing the externality, but also a higher cost, in terms of reducing the revenue. Similarly, it has costs, but also benefits, if people stop working and potentially decrease their dirty good consumption. In optimal taxation there is little research on this problem, whilst politicians seem to have a strong focus on the challenges that come along with a volatile tax base. To analyze this problem more closely, it might be helpful to use dynamic models which can represent how revenue and the externality evolve over time, and how the benefit of this revenue, the cost of the externality, but also the role of other taxes, changes.

This paper has been all about the role of the extensive margin in optimal carbon taxation. To pursue this question further, it might be interesting to get a better understanding of the behavior of different subgroups at the extensive margin. Here, it has been assumed that the responses are homogeneous, to get a first impression of the most important mechanisms. However, the motivation to potentially stop working might differ from group to group. Understanding these motivations might be one of the most important ways of decreasing the costs of a carbon tax and making use of the awareness that the extensive margin crucially drives labor supply and the costs that come along with taxes.

References

Aubert, D. & Chiroleu-Assouline, M. (2019), 'Environmental tax reform and income distribution with imperfect heterogeneous labour markets', *European Economic Review* **116**, 60–82.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0014292119300467*

Bargain, O., Orsini, K. & Peichl, A. (2014), 'Comparing Labor Supply Elasticities in Europe and the United States: New Results', *Journal of Human Resources* **49**(3), 723–838.
URL: *http://jhr.uwpress.org/lookup/doi/10.3368/jhr.49.3.723*

Bovenberg, A. L. & De Mooij, R. A. (1994), 'Environmental taxes and labor-market distortions', *European Journal of Political Economy* **10**(4), 655–683.

Chetty, R., Guren, A., Manoli, D. & Weber, A. (2011), 'Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins', *American Economic Review* **101**(3), 471–75.

Choné, P. & Laroque, G. (2011), 'Optimal taxation in the extensive model', *Journal of Economic Theory* **146**(2), 425–453.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0022053110001651*

Climate-Leadership-Council (2020), 'Economists´statement'.

Diamond, P. (1980), 'Income taxation with fixed hours of work', *Journal of Public Economics* **13**(1), 101–110.
URL: *https://linkinghub.elsevier.com/retrieve/pii/0047272780900249*

Gechert, S., Rietzler, K., Schreiber, S. & Stein, U. (2019), Wirtschaftliche instrumente für eine klima-und sozialverträgliche co2-bepreisung: Gutachten im auftrag des bundesministeriums für umwelt, naturschutz und nukleare sicherheit, Technical report, IMK Study.

Goulder, L. H. (1995), 'Environmental taxation and the double dividend: a reader's guide', *International tax and public finance* **2**(2), 157–183.

Heckman, J. J. (1993), 'What has been learned about labor supply in the past twenty years?', *The American Economic Review* **83**(2), 116–121.

Jacobs, B. & De Mooij, R. A. (2015), 'Pigou meets mirrlees: On the irrelevance of tax distortions for the second-best pigouvian tax', *Journal of Environmental Economics and Management* **71**, 90–108.

Jacobs, B. & van der Ploeg, F. (2019), 'Redistribution and pollution taxes with non-linear engel curves', *Journal of Environmental Economics and Management* **95**, 198–226.

Jacobs, B. & van der Ploeg, R. (2017), 'Should pollution taxes be targeted at income redistribution?'.

Jacquet, L., Lehmann, E. & Van der Linden, B. (2013), 'Optimal redistributive taxation with both extensive and intensive responses', *Journal of Economic Theory* **148**(5), 1770–1805.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0022053113001373*

Kaplow, B. L. (2012), 'OPTIMAL CONTROL OF EXTERNALITIES IN THE PRESENCE OF INCOME TAXATION*: optimal control of externalities', *International Economic Review* **53**(2), 487–509.
URL: *http://doi.wiley.com/10.1111/j.1468-2354.2012.00689.x*

Klenert, D., Mattauch, L., Combet, E., Edenhofer, O., Hepburn, C., Rafaty, R. & Stern, N. (2018), 'Making carbon pricing work for citizens', *Nature Climate Change* **8**(8), 669–677.
URL: *http://www.nature.com/articles/s41558-018-0201-2*

Lehmann, E., Parmentier, A. & Van Der Linden, B. (2011), 'Optimal income taxation with endogenous participation and search unemployment', *Journal of Public Economics* **95**(11-12), 1523–1537.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0047272711000843*

Ljungqvist, L., Sargent, T. J., Blanchard, O. & Prescott, E. C. (2006), 'Do taxes explain european employment? indivisible labor, human capital, lotteries, and savings [with comments and discussion]', *NBER macroeconomics annual* **21**, 181–246.

Michau, J.-B. (2014), 'Optimal redistribution: A life-cycle perspective', *Journal of Public Economics* **111**, 1–16.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0047272713002521*

Micheletto, L. (2008), 'Redistribution and optimal mixed taxation in the presence of consumption externalities', *Journal of Public Economics* **92**(10-11), 2262–2274.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0047272708001114*

Mirrlees, J. A. (1971), 'An Exploration in the Theory of Optimum Income Taxation', *The Review of Economic Studies* **38**(2), 175.
URL: *https://academic.oup.com/restud/article-lookup/doi/10.2307/2296779*

Pigou, A. C. (2020), *The economics of welfare*, Palgrave Macmillan.

Pirttilä, J. & Schöb, R. (1999), 'Redistribution and Internalization: The Many-Person Ramsey Tax Rule Revisited', *Public Finance Review* **27**(5), 541–560.
URL: *http://journals.sagepub.com/doi/10.1177/109114219902700505*

Preuß, M., Reuter, W. H. & Schmidt, C. M. (2019), Verteilungswirkung einer co2-bepreisung in deutschland, Arbeitspapier 08/2019, Wiesbaden.
URL: *http://hdl.handle.net/10419/204442*

Saez, E. (2002), 'Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses', *The Quarterly Journal of Economics* **117**(3), 1039–1073.
URL: *https://academic.oup.com/qje/article-lookup/doi/10.1162/003355302760193959*

Sandmo, A. (1975), 'Optimal Taxation in the Presence of Externalities', *The Swedish Journal of Economics* **77**(1), 86.
URL: *https://www.jstor.org/stable/3439329?origin=crossref*

Stern, N. & Stern, N. H. (2007), *The economics of climate change: the Stern review*, cambridge University press.