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Master's Thesis Financial Economics

Commonalities in yield factors

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Abstract

This thesis investigates how much the yield factors of the Nelson-Siegel model have in common across countries. This is done by principal component analysis, which tells us how much of the variation in each yield factor can be summarized in the first few principal components. Using a broad dataset of ten countries, we find that the level and slope factors can both be reduced to two principal components while maintaining more than 90% of the variation. The curvature factors require three components. In first differences, the commonalities in the yield factors explain less of the variation and need more components to be fully captured. Many of the principal components contributing to the commonalities can be clearly interpreted as averages of either all or some countries. This can however not be said about the components of the curvature first differences. Finally, the principal components do not lead to significant improvements in forecasting, as the MCS procedure reveals.

Keywords: yield curve • Nelson-Siegel • principal component analysis • forecasting • model confidence set

Contents

1 Introduction

The return on a fixed income instrument is usually referred to as its yield. The yield depends on the price of the bond and its time to maturity. For a set of similar bonds, the yield curve summarizes the relation between yields and different maturities. The yield curve, also known as the term structure of interest rates, can be used for valuation and contains information about expected future interest rates. Therefore, modelling the yield curve of government bond yields, because of their low default risk, has been a popular research topic in finance and macroeconomics, especially in the areas of derivative pricing, portfolio management and monetary policy. Because yields are highly correlated cross-sectionally and over time, several approaches have been developed to estimate and predict the yield curve. These approaches can be divided into arbitrage-free models, equilibrium models or statistical methods, each with their own purpose.

In arbitrage-free models the key is to perfectly fit the yield curve such that no arbitrage opportunities exist. This is particularly important for derivative pricing. These methods are developed by Hull and White (1990) and Heath et al. (1992), which use option theory to value fixed income derivatives. The term structure of interest rates in equilibrium models is traditionally captured by affine term structure models where the price of a bond is an exponentially affine function of the short rate. Early contributions are made by Vasicek (1977) and Cox et al. (1985) specifying stochastic differential equations for the short rate. Later affine term structure models describe the short rate as a function of latent or macroeconomic factors, see for example Duffie and Kan (1996) and Ang and Piazzesi (2003). Although the in-sample fit of these models is generally quite well, they produce poor forecasts according to Duffee (2002). More recent advances in this area have focused on incorporating the current low interest rate environment in affine term structure models by introducing a shadow rate. This is because standard affine term structure models do not rule out negative nominal interest rates (Christensen & Rudebusch, 2015). While improving yield curve estimation for current interest rates, these shadow-rate models are difficult to solve (Bauer & Rudebusch, 2016).

The model of Nelson and Siegel (1987) belongs to the statistical models and offers a powerful yet relatively simple framework to yield curve modelling (Diebold et al., 2008). In this model the yields depend on three factors with predetermined factor loadings. The modified version of the Nelson-Siegel model, made by Diebold and Li (2006), provides a dynamic version in which the estimates of the time-varying factors are computed with high precision. Moreover, in their case, the factors can be interpreted as the level, slope and curvature factors of the yield curve and have proven to outperform random walk forecasts. However, like many models, the Nelson-Siegel model relates domestic yields with domestic factors ignoring linkages with foreign yield curves. It is apparent that financial markets are interconnected and highly complex, as shown by the Financial Crisis (BIS, 2009). Therefore it is important to investigate possible relations between country yield curves to obtain a better understanding of the interconnectedness of interest rate shocks. Because the Nelson-Siegel model summarizes the yield curve by three factors it seems natural to look at how similar or different the level, slope and curvature factors are of each country. Hence, comparing the yield factors instead of the yields themselves reduces the comparison to only three dimensions while maintaining an interpretable characterization of the yield curve. The research question of this thesis is therefore as follows

To answer the research question we make use of principal component analysis. Principal component analysis is usually used as a dimension reduction technique as it tries to store as much variation as possible of a certain dataset in as few as possible factors, called principal components. In our case, the extent to which the first few principal components capture the total amount of variation in each yield factor tells us whether the country yield factors are driven by a certain number of underlying components and whether there exist any commonalities in the yield factors. Because we are interested in the commonalities within each yield factor, principal component analysis will be performed on each factor separately, both in levels and first differences. Thereafter, a forecasting exercise using the first few principal components examines the usefulness of the common components for yield curve prediction.

Earlier work of, for example, Diebold et al. (2008) has focused on the links between yields across countries. Their method also relies on the Nelson-Siegel model but adjust the model by making the country yield factors depend on a global level and slope factor and additional country-specific factors[1]. In their results they show that the estimated global level factor explains most of the variation in the country level factors while the global slope factor primarily explains the slope factor of a single country. Moreover, they find that their global level and slope factor are linked to inflation and real activity. Coroneo et al. (2018) extends the model of Diebold et al. (2008) by adding the curvature factor in the Nelson-Siegel model and allowing for interactions between the different local factors. Their global factors explain on average 55% of the variation in yields. In an impulse response analysis, they find that shocks to global factors last longer than local shocks and have a larger impact. The impact is also more significant for yields with shorter maturities.

The paper of Modugno and Nikolau (2009) takes a different approach and uses the state space Nelson-Siegel model of Diebold et al. (2006), which adds a vector autoregressive process for the yield factors as transition equation to the version of Diebold and Li (2006). To capture international spillovers among yield curves, they allow the autoregressive process of each country's yield factor to also depend on the same yield factor of the other countries. This extension improves yield forecasting at longer horizons for Germany and the UK, but worsens US forecasts.

To study the presence of common factors, some papers have focused on bond returns. Driessen et al. (2003) has applied principal component analysis to a joint set of bond returns of the US, Germany and Japan and finds that five principal components explain 96.5% of the total variation. Similarly, Rodrigues (1997) found that three principal components explained at least 82% of variation at every maturity and more were necessary to explain most of the variation at every maturity. This study also covered a broader sample of seven countries. Pérignon et al. (2007) has criticized the use of principal component analysis to estimate the number of common factors. They argue that because principal component analysis maximizes explained variance it is not guaranteed that the extracted factors also represent factors that are common across countries. It might happen that local factors with high variance are selected which by definition are not a common factor. Therefore, Pérignon et al. (2007) studies US, German and Japanese

[1] The curvature factor is left out in the Nelson-Siegel model of Diebold et al. (2008) because the curvature factor lacks a link with macroeconomic fundamentals.

bond returns with inter-battery factor analysis, which first estimates common factors before extracting local factors. They find only a single common factor mostly related to the levels of the yield curves.

Principal component analysis on the yield factors has already been done by Diebold et al. (2008) and Coroneo et al. (2018), mainly to provide evidence of commonalities in these factors and to substantiate a role for global factors. Diebold et al. (2008) uses data on the US, UK, Germany and Japan from 1985 through 2005, while Coroneo et al. (2018) covers 1997 to 2010 for the US, UK and Germany. Both studies agree that for the level factor the first principal component is able to explain most of the variation in the level factors, around 85%. However, for the slope factors the result is quite different. In Diebold et al. (2008) the first component of the slope factors explains only 50% while this is 80% in Coroneo et al. (2018). Coroneo et al. (2018) reports another 80% for the amount of variation explained by the first component of the curvature factors, which Diebold et al. (2008) ignores. Hence, it seems the amount of variation explained by each component depends on the chosen countries in the dataset and the corresponding time period. Ideally, principal component analysis on the yield factors should be done on a dataset of many countries and a long time span to understand how much the yield factors have in common.

Several contributions have been made to the existing literature. First, we have investigated the existence of commonalities and carried out principal component analysis on a dataset of ten countries, whereas most studies have used three or four countries. Second, not only have we looked at commonalities in the yield factor levels but also in first differences. Third, a detailed overview has been given of each yield factor in terms of robustness and interpretation. Finally, a forecast study has shown that the principal components do not improve yield forecasting after applying the MCS procedure.

The results of this paper can be summarized as follows. The analysis in levels shows that for the level and slope factor, two principal components capture more than 90% of the total variation. The results for the curvature factor depend on whether the European Sovereign Debt Crisis is included. If included, the curvature factor has three principal components explaining a substantial amount of variation, while excluded, only two components seem to capture commonalities. In first differences, the results are somewhat different. For the level factor, the first two principal components are still the only significant components but now explain only 78%. The slope factor now has three components explaining more than 10% instead of two. These components also explain less, namely 83%. The first differences of the curvature factors now has four components with more than 10% explained variation for a total of 72%.

Overall, the Nelson-Siegel yield factors can be reduced to just a few principal components, more so in levels than in first differences. In terms of interpretation, most principal components contributing to the commonality seem to represent averages of either all countries or smaller subgroups. Only in case of the curvature first differences, is the interpretation ambiguous. Meanwhile, the total amount of variation explained by the components contributing to the commonality, in levels and first differences, is robust to normalization of the data and stable over time.

A forecasting study is conducted to see if the first few principal components, which can be regarded as common factors, are useful in a practical sense. Unfortunately, the principal component based forecasts rarely provide better predictions than using the estimated Nelson-Siegel factors themselves. The ARMSE is sometimes lower, especially at longer horizons, but the differences are not statistically significant because the standard Nelson-Siegel model also appears in the model confidence set. Therefore, commonalities in the yield factors cannot be exploited for out-of-sample forecasting.

The structure of this paper is outlined as follows. Section 2 contains the methodology and explains the Nelson-Siegel model and principal component analysis. Section 3 provides an overview of the yield dataset. It shows graphically the time series of country yield curves and gives initial evidence of commonalities. Thereafter, Section 4 presents the estimated Nelson-Siegel yield factors after which two different subsections give a detailed overview of the results of principal component analysis on the yield factors, in levels and first differences. Specifically, it shows how much the first five components explain of the total variation in each yield factor and interprets the components based on the factor loadings. A summary of the results, its shortcomings and suggestions for further research can be found in Section 6.

2 Methodology

2.1 Nelson-Siegel Model

The model of Nelson and Siegel (1987) is able to describe yields as a combination of three factors. The modified version of the Nelson-Siegel model, made by Diebold and Li (2006), is as follows

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right), \tag{1}$$

where $y_t(\tau)$ is the yield for maturity τ at time t and λ_t is the loading parameter. The three yield factors β_{1t}, β_{2t} and β_{3t} are latent and have to be estimated given the current set of yields. The expressions next to these factors are known as the factor loadings and depend on the loading parameter λ_t. To get more insight into the meaning of equation (1), consider first β_{1t}. The loading on this factor is equal to 1 for all maturities, hence it influences the yields at all maturities. Therefore, β_{1t} is usually referred to as the level factor of the yield curve. For β_{2t}, we have that

$$\lim_{\tau\to 0} y_t(\tau) = \beta_{1t} + \beta_{2t}, \qquad \lim_{\tau\to\infty} y_t(\tau) = \beta_{1t}, \tag{2}$$

thus its loading starts at 1 but goes to zero as maturity increases. A change in β_{2t} has a stronger effect on the short end of the yield curve than on the long end, affecting the slope of the curve. Consequently, this factor is interpreted as the slope factor. Finally, the third factor β_{3t} has no loading on the short rate or the long rate. It mostly influences the medium term yields and is therefore known as the curvature factor. Figure 1 illustrates the factor loadings at each maturity with a fixed loading parameter.

The last parameter of interest is the loading parameter. The loading parameter controls the exponential rate of decay and at which maturity the loading on β_{3t} reaches its maximum (Diebold & Li, 2006). In equation (1) the loading parameter is time-varying making the estimation procedure more difficult. Therefore, as suggested by Diebold and Li (2006), the loading parameter λ is fixed to 0.0609. This makes the estimation of the level, slope, and curvature factors much easier because the equation is now linear in the parameters. The value of the loading parameter is chosen such that the maximum of the loading on the curvature factor is at 30 months.

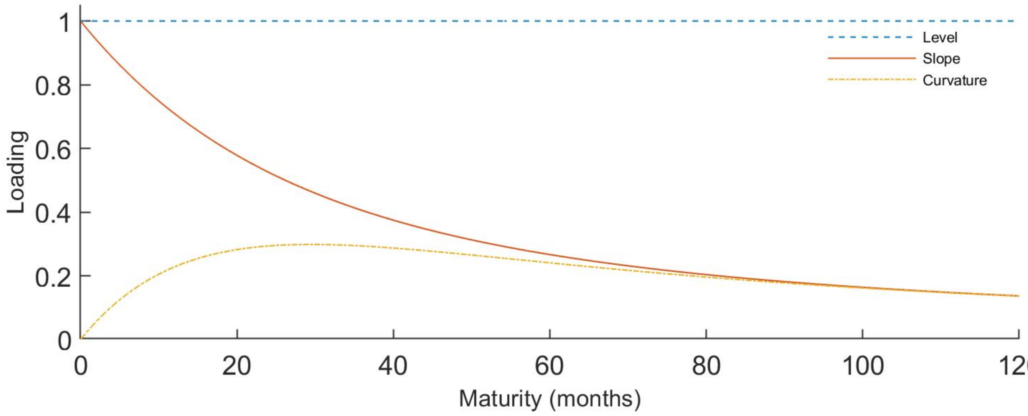


Figure 1: Nelson-Siegel factor loadings as a function of maturity, with $\lambda_t = 0.0609$ as in Diebold and Li (2006).

Although Koopman et al. (2010) provides evidence of a time-varying loading parameter, assuming a constant loading parameter is without too much loss of generality according to Diebold et al. (2008). Furthermore, having different values of λ for each country will complicate finding commonalities in the factors. This is because the relation between the factors and the yields will not be the same such that comparisons are troublesome. This holds especially for the curvature factor. It is questionable how much two curvature factors with different loading parameters would have in common. Therefore, λ remains the same for all countries in the dataset.

The Nelson-Siegel factors can conveniently be estimated using OLS. For each month t, the regression model

$$y_{it}(\tau) = \beta_{1it} + \beta_{2it}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3it}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \epsilon_{it}, \tag{3}$$

is used to obtain the country level, slope, and curvature factors. Here, $y_{it}, \beta_{1it}, \beta_{2it}, \beta_{3it}$, and ϵ_{it}, denote the yield at maturity τ, level, slope, curvature, and disturbance term for country i at time t respectively. This results in a panel of factor estimates and residuals. To investigate the commonalities in the yield factors, these factor estimates serve as inputs in the principal component analysis which will be explained next.

2.2 Principal Component Analysis

Principal component analysis is a statistical technique to reduce the dimension of the data while keeping as much of the variation as possible (James et al., 2013). It does so by constructing linear combinations of the original variables, which are called the principal components. Unlike regression, there is no dependent variable, only a set of variables. The idea is that most of the information stored in the data can be summarized by a few uncorrelated components ignoring the dimensions with little variation. The principal components will be key in this research, as the amount of variation captured by the principal components determines the commonalities in each yield factor.

To obtain the principal components of, for example, the country level factors, a matrix X of size n x p is needed containing the demeaned level factor series of each country as its columns (Mardia et al., 1979). Using the eigenvalue decomposition, we obtain eigenvalues $\{\lambda_j\}_{j=1}^{p}$ and standardized eigenvectors $\{g_j\}_{j=1}^{p}$ of the covariance matrix of X denoted by Σ. The matrix of principal components is then equal to

$$Y = XG, \tag{4}$$

where G stores the eigenvectors in descending order of the corresponding eigenvalues. Hence, the principal components are a linear combination of the demeaned original variables where the weights of each variable in the linear combination is controlled by the values in the eigenvectors. Because the eigenvectors are ordered inside G by the eigenvalues, the amount of variation explained by each principal component follows the same ordering. Since the first principal component is related to the largest eigenvalue it also explains more variation than any other principal component. Specifically, the proportion of total variation explained by the kth principal component is

$$\frac{\lambda_k}{\lambda_1 + ... + \lambda_p}. \tag{5}$$

The total variation $\lambda_1 + ... + \lambda_p$ is also equal to the sum of the variances in Σ. The proportion of variance explained by the first few principal components indicates whether there are commonalities in the yield factors because it shows how much information can be retained in a lower dimension. Because we are interested in the commonalities within each yield factor, principal component analysis has to be performed on each factor separately.

The computation of principal components is relatively straightforward but also comes at a price. This is because principal component analysis is a purely statistical exercise. One of its issues is the sensitivity to the scaling of the variables (Mardia et al., 1979). Variables that appear on a larger scale have a higher variance and will therefore be the main target when constructing the principal components. Hence, these variables will be given a larger weight in the first few principal components. The solution to this problem is to normalize the data to have mean zero and unit standard deviation such that all the variables are on the same scale. Here, we will not let the existence of commonality depend on the scale of the yield factors and normalize the yield factors before performing principal component analysis. Thus, yield factors with a similar path will be regarded as having much in common ignoring their dissimilarity in terms of scale.

Another issue with principal components is that they can be difficult to interpret. The sole objective of principal component analysis is to put as much variation as possible in a principal component so there is no economic theory involved. Still, an interpretation can be given to the principal components by its relation with the original variables. Because the principal components are a linear combination of the original variables, the weights, also called loadings, of the variables in the combination determine their importance in the construction of the principal component (Mardia et al., 1979). The eigenvectors thus play a crucial role in the interpretation. Since the eigenvectors are standardized in such a way that $\sum_{j=1}^{p} g_{ij}^2 = 1$, with g_{ij} being the loading on variable j of principal component i, an interpretation can be given by using the size of each loading. This does not mean that the interpretations are straightforward but are subjective views made by the researcher. A common method is, however, to focus on the variables with the largest loadings (Mardia et al., 1979).

Besides investigating the commonalities in each yield factor, we will also concentrate on the commonalities in the (normalized) first differences of the factors. This is because of possible common trends in the yield factors. Since the yield factors are normalized, the covariances and correlations are equal. If there is a common trend then the product of two observations is almost always positive because both are either above or below zero at more or less the same time. This tilts the covariance upwards and overestimates the first few eigenvalues.

3 Data

The yield data is taken from Bloomberg and consists of end-of-month government bond yields at fixed maturities for Germany, the Netherlands, Belgium, France, Italy, Spain, the UK, Canada, the US, and Japan. With a time span ranging from August 2011 through October 2020, the number of observations is 111 per country and per maturity. Due to data availability the time span is relatively small, i.e. one could create a longer time series and leave out certain countries or maturities. However, the analysis focuses on commonalities in the cross-section of yields thus a broad coverage is preferred.

For all countries, except the US, we have yields for maturities of 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120 months. In case of the US, yields are missing at 48-, 72-, 96-, and 108-month maturity. Furthermore, the Dutch 12-month yield is not available from January 2015 until July 2018. These observations are filled before estimating the Nelson-Siegel factors. Similar to Brennan and Xia (2006), they are obtained by interpolating the existing yields with a cubic spline. A cubic spline provides a smooth function by fitting polynomials between the existing yields. For a comparison of cubic splines with other methods in fitting yield curves see Bliss (1996).

Figure 2 shows time series of yield curves for the US, the UK, Japan, and Germany. Time series of the other countries can be found in Appendix A.1. The yield curves show clear comovement in their levels and shapes. The yield curves were relatively high and steep around 2012, at the time of the European Sovereign Debt Crisis, while in 2020, amid the COVID-19 pandemic,

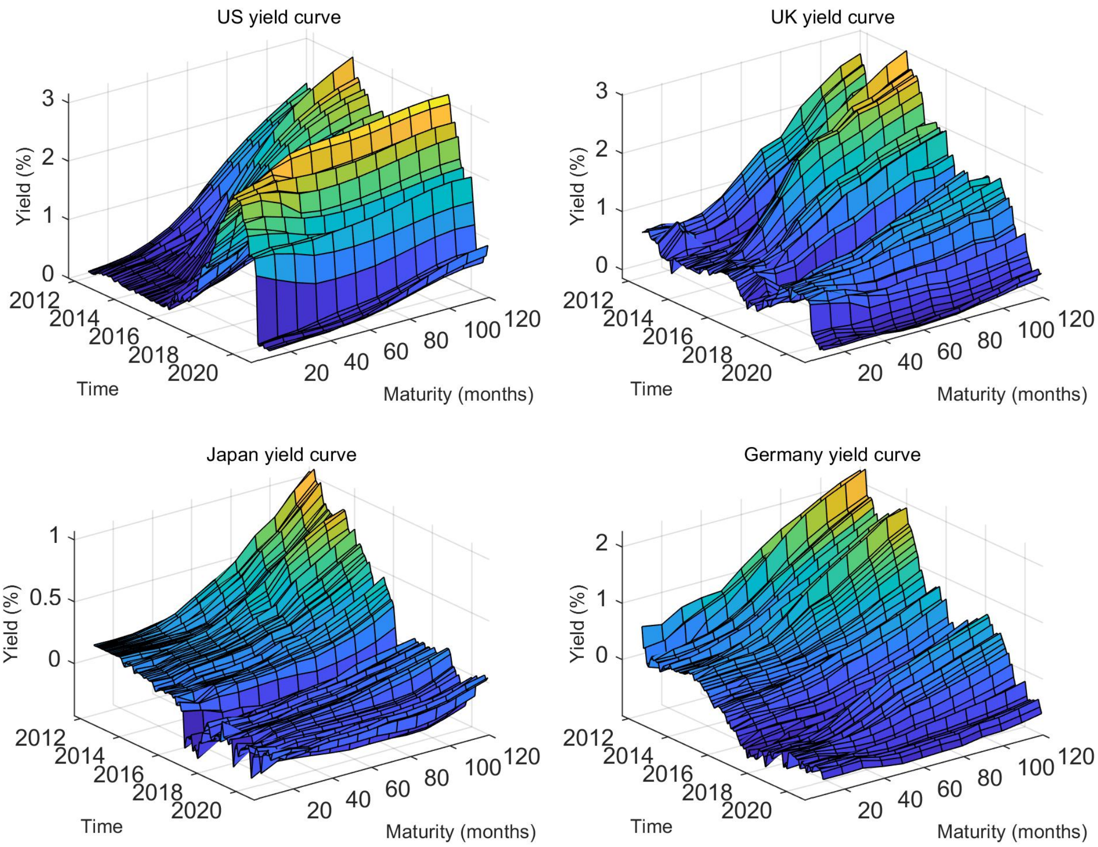


Figure 2: Yield curves across time and maturity. The bond yield data is monthly from 2011-08 through 2020-10.

yield curves had flattened off at lower levels. Moreover, the rise and fall in yields in the US and Canada from approximately 2017 through 2020 is very indicative of the commonalities displayed in the data.

Table I reports summary statistics of the bond yield data. It contains the mean, standard deviation, minimum, maximum, and first-, twelfth-, and thirtieth-order sample autocorrelation for several maturities. The average yield curves are upward sloping hence yields are on average higher at longer maturities. This is because the principal payment is due at a later date, which exposes bondholders to more risks. Although in general upward sloping, it is possible that yield curves become flat or downward sloping (inverted). A flat yield curve relates to high economic uncertainty, while an inverted curve usually precedes a recession. The table also shows that the long end of the yield curves is more volatile than the short end, which is in contrast to the stylized facts of the yield curve as listed in Diebold and Li (2006). This is most likely because the time span of the dataset is relatively short and is a few years after the Financial Crisis, when central banks started lowering the short rate significantly. In addition, the autocorrelations demonstrate that the yields are highly persistent for all countries with first-order autocorrelations being above 0.9.

For a preliminary analysis of commonalities in yield factors, Figure 3 plots a boxplot of the linear correlation in yields of different countries for each maturity separately. Thus, it shows the distribution of the off-diagonal elements of the correlation matrix for a given maturity. The correlations increase with maturity. At 3-month maturity the median correlation is about 0.6 and reaches 0.85 at 120-month maturity. Moreover, the variability is negatively related to maturity[2]. Correlations are mostly positive at the long end of the yield curve while at the short end yields of certain countries are negatively correlated. This happens between continental Europe and Japan on the one hand and the US and Canada on the other hand. To summarize, given that the level factor of the yield curve is the only factor that affects the long end and the with maturity increasing correlations, the country level factors have most likely the strongest commonalities.

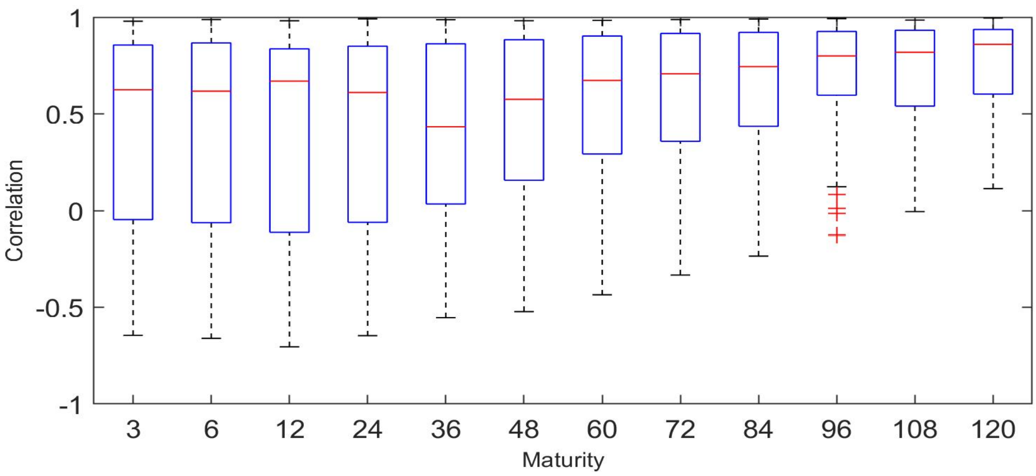


Figure 3: Boxplot of between-country yield correlation coefficients with a box for each maturity using monthly yield data from 2011-08 through 2020-10.

[2]The outliers displayed at 96 month maturity are correlations between Belgium, France, Italy, Spain and Japan on the one hand and the US on the other hand. These correlations are also below average for the other maturities but only at 96 month maturity are these correlations outside the interquartile range.

Table I

Descriptive Statistics of Bond Yields

This table gives an overview of the dataset. The bond yield data is monthly from 2011-08 through 2020-10. The last three columns report $\rho(\tau)$, the sample autocorrelations at displacement τ.

Maturity (Months)	Mean	St.dev.	Min	Max	$\rho(1)$	$\rho(12)$	$\rho(30)$
Germany							
3	−0.412	0.343	−0.995	0.490	0.929	0.688	0.122
12	−0.355	0.357	−0.915	0.639	0.935	0.684	0.195
60	−0.018	0.531	−0.927	1.309	0.926	0.505	0.145
120	0.638	0.774	−0.702	2.276	0.945	0.526	0.095
Netherlands							
3	−0.374	0.337	−1.107	0.504	0.909	0.692	0.149
12	−0.363	0.359	−0.927	0.671	0.929	0.694	0.188
60	0.112	0.660	−0.825	1.712	0.937	0.532	0.139
120	0.849	0.863	−0.551	2.647	0.946	0.530	0.099
Belgium							
3	−0.289	0.410	−0.918	1.373	0.890	0.533	0.203
12	−0.205	0.517	−0.757	2.580	0.860	0.413	0.184
60	0.393	0.986	−0.701	4.093	0.932	0.387	0.094
120	1.235	1.184	−0.389	4.971	0.948	0.474	0.092
France							
3	−0.307	0.341	−0.904	0.654	0.927	0.667	0.200
12	−0.258	0.374	−0.744	0.826	0.934	0.604	0.219
60	0.295	0.704	−0.771	2.198	0.939	0.480	0.110
120	1.111	0.965	−0.406	3.380	0.953	0.518	0.081
Italy							
3	0.108	0.731	−0.654	4.737	0.842	0.287	0.110
12	0.498	1.110	−0.461	5.863	0.913	0.346	0.064
60	1.775	1.604	0.190	7.499	0.951	0.430	−0.058
120	2.768	1.564	0.758	7.029	0.960	0.465	−0.069
Spain							
3	0.028	0.747	−0.763	4.092	0.844	0.486	0.123
12	0.415	1.200	−0.565	4.767	0.935	0.509	0.076
60	1.384	1.750	−0.373	6.203	0.965	0.595	0.044
120	2.386	1.841	0.100	6.831	0.972	0.607	0.036
United Kingdom							
3	0.402	0.202	−0.062	0.770	0.895	−0.137	−0.174
12	0.381	0.211	−0.036	0.809	0.880	−0.014	−0.269
60	0.913	0.496	−0.123	2.031	0.917	0.219	−0.071
120	1.542	0.707	0.103	3.021	0.935	0.427	0.122
United States							
3	0.639	0.822	0.000	2.450	0.982	0.543	−0.059
12	0.794	0.842	0.090	2.700	0.983	0.550	−0.048
60	1.480	0.670	0.210	2.980	0.946	0.249	−0.220
120	2.097	0.581	0.550	3.150	0.921	−0.002	−0.274
Canada							
3	0.896	0.433	0.087	1.732	0.931	0.158	−0.373
12	1.024	0.485	0.193	2.152	0.946	0.257	−0.449
60	1.314	0.502	0.314	2.427	0.922	0.151	−0.543
120	1.740	0.522	0.464	2.755	0.911	0.114	−0.307
Japan							
3	−0.076	0.142	−0.420	0.109	0.889	0.505	0.155
12	−0.069	0.137	−0.327	0.116	0.966	0.581	0.152
60	0.008	0.191	−0.361	0.380	0.955	0.560	0.175
120	0.299	0.373	−0.276	1.067	0.966	0.619	0.181

4 Results

4.1 Nelson-Siegel Model

This section presents an overview of the estimated yield factors using the Nelson-Siegel model of Section 2.1. These factors and its first differences will later be used to find possible commonalities. Table II reports summary statistics for the level, slope, and curvature factors. During the sample period 2011-08 until 2020-10, the level factor is lowest for Japan and highest for Italy. The yield curve of Italy is also on average the steepest and most curved of the ten countries in the dataset. The factor dynamics are very persistent given the high autocorrelations. This is a natural consequence of the persistence found in the yields in the previous section. Furthermore, Figure 4 shows the average estimation error of each country made by the Nelson-Siegel model at a maturity level. The model has negative residuals at the very short end and from mid to long term, hence at these maturities the Nelson-Siegel model overestimates the yields. At the other maturities the yields are underestimated. The residuals of all countries show this pattern and is very much like the residuals obtained by Diebold and Li (2006).

To investigate any commonalities in the yield factors before estimating the principal components, Figure 5 plots the level, slope, and curvature factor of each country, while Figure 6 plots the first differences of these yield factors. All yield factors exhibit a certain amount of commonality following a similar upward or downward trend. The level factor follows a downward trend while the slope and curvature factor follow an upward trend leading to flatter yield curves at lower levels. From Figure 5 it seems that the curvature factor is the one with the least amount of commonality given its more chaotic pattern caused by countries not following the general trend. Deviations form the general pattern happen especially early on in the sample period when the European Sovereign Debt Crisis caused a wedge between southern European countries like Italy and Spain and northern European countries like the Netherlands and Germany. At the end of the sample the yield factors have converged to very similar values suggesting the degree of commonality has increased over time.

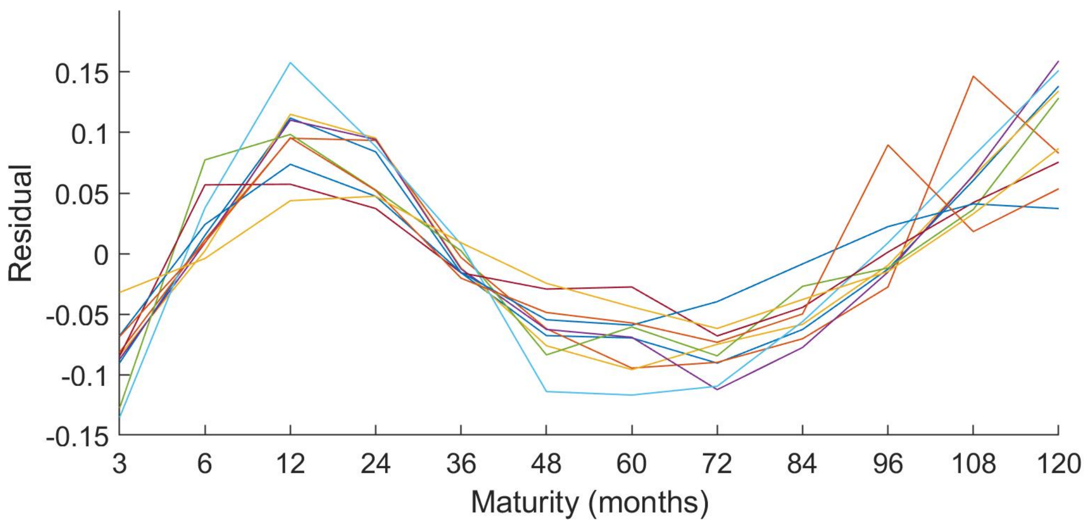


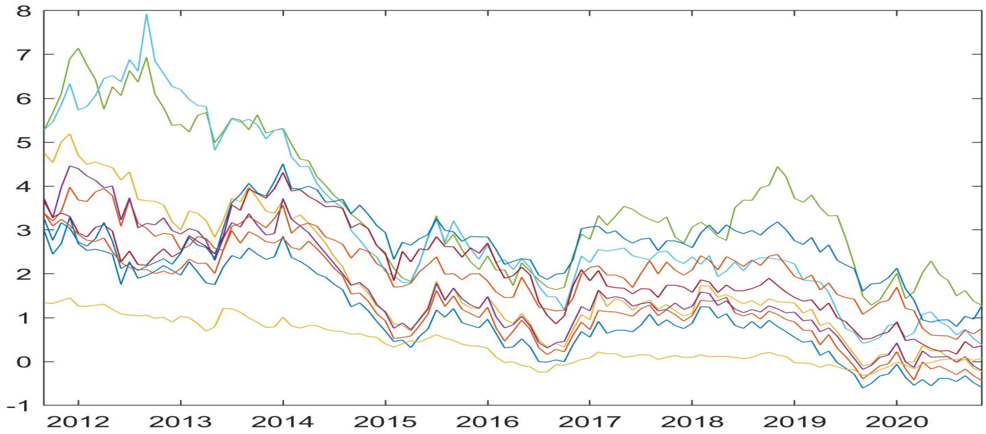
Figure 4: Yield residuals of the Nelson-Siegel model per maturity using monthly yield data from 2011-08 through 2020-10.

Table II

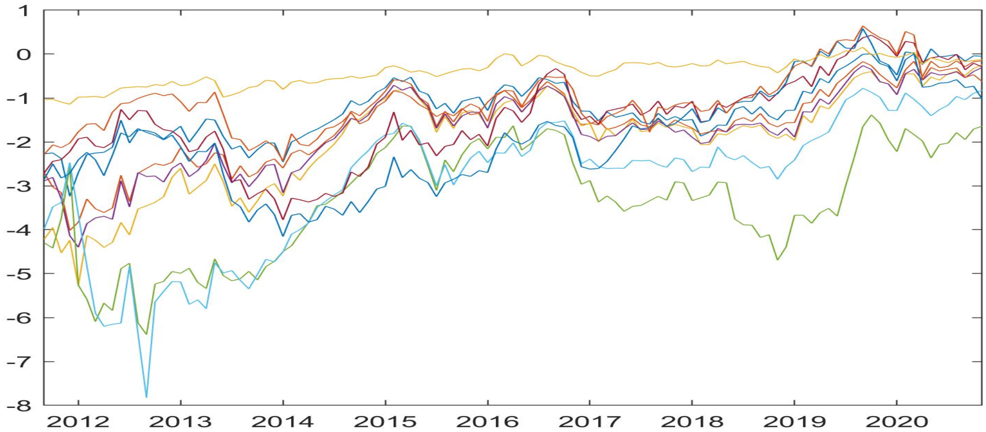
Descriptive Statistics of Nelson-Siegel Factors

This table gives an overview of the estimated country level, slope, and curvature factors. The bond yield data is monthly from 2011-08 through 2020-10. The last three columns report $\rho(\tau)$, the sample autocorrelations at displacement τ.

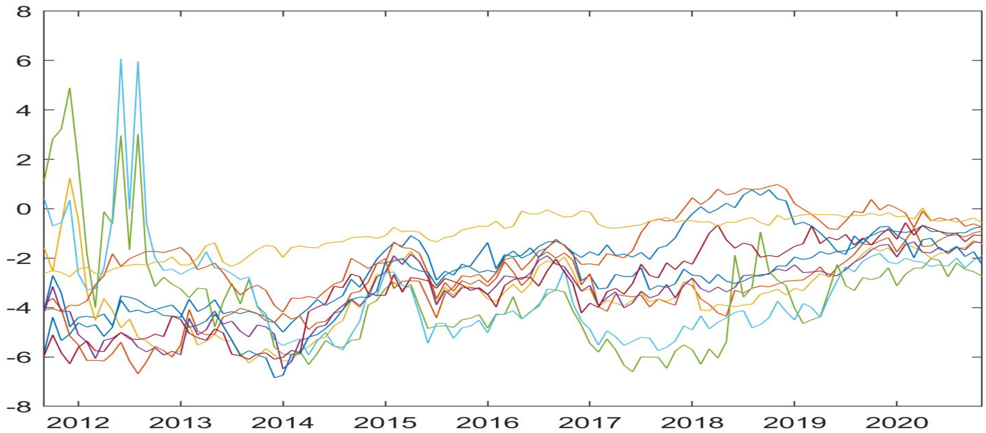
Factor	Mean	St.dev.	Min	Max	ρ_1	ρ_{12}	ρ_{30}
Germany							
Level	1.041	0.994	−0.607	3.287	0.942	0.484	0.038
Slope	−1.249	0.710	−3.239	0.121	0.921	0.313	−0.164
Curvature	−2.726	1.196	−5.000	−0.677	0.928	0.397	−0.127
Netherlands							
Level	1.441	1.203	−0.443	3.976	0.956	0.513	0.036
Slope	−1.599	0.919	−4.016	−0.155	0.944	0.396	−0.100
Curvature	−3.358	1.439	−6.684	−0.788	0.934	0.346	−0.191
Belgium							
Level	1.834	1.433	−0.230	5.194	0.956	0.502	0.041
Slope	−1.925	1.157	−5.274	−0.245	0.940	0.413	−0.077
Curvature	−3.458	1.469	−6.242	1.244	0.904	0.154	−0.255
France							
Level	1.660	1.234	−0.208	4.461	0.953	0.500	0.038
Slope	−1.751	0.947	−4.399	−0.262	0.939	0.380	−0.118
Curvature	−3.443	1.323	−6.484	−1.340	0.931	0.431	−0.100
Italy							
Level	3.578	1.589	1.278	7.145	0.961	0.443	−0.142
Slope	−3.344	1.289	−6.388	−1.385	0.925	0.354	−0.314
Curvature	−3.536	2.178	−6.602	4.895	0.802	0.116	−0.166
Spain							
Level	3.096	1.921	0.377	7.925	0.970	0.592	0.024
Slope	−2.907	1.553	−7.828	−0.778	0.947	0.513	−0.070
Curvature	−3.407	1.959	−5.925	6.063	0.744	0.172	−0.087
United Kingdom							
Level	2.102	1.028	0.167	4.311	0.945	0.490	0.156
Slope	−1.486	1.014	−3.776	0.430	0.950	0.492	0.098
Curvature	−3.179	1.701	−6.277	−0.563	0.947	0.574	0.109
United States							
Level	2.675	0.752	0.799	4.508	0.923	0.115	−0.107
Slope	−1.925	1.150	−4.156	0.581	0.965	0.591	0.077
Curvature	−2.586	1.880	−6.842	0.773	0.960	0.589	0.134
Canada							
Level	2.063	0.688	0.501	3.559	0.914	0.197	−0.045
Slope	−1.050	0.686	−2.436	0.642	0.921	0.237	0.008
Curvature	−1.709	1.469	−4.417	0.982	0.954	0.513	0.067
Japan							
Level	0.408	0.482	−0.322	1.451	0.964	0.605	0.164
Slope	−0.403	0.306	−1.143	0.156	0.929	0.507	0.118
Curvature	−1.029	0.790	−2.755	0.036	0.948	0.611	0.149



(a) Level factor

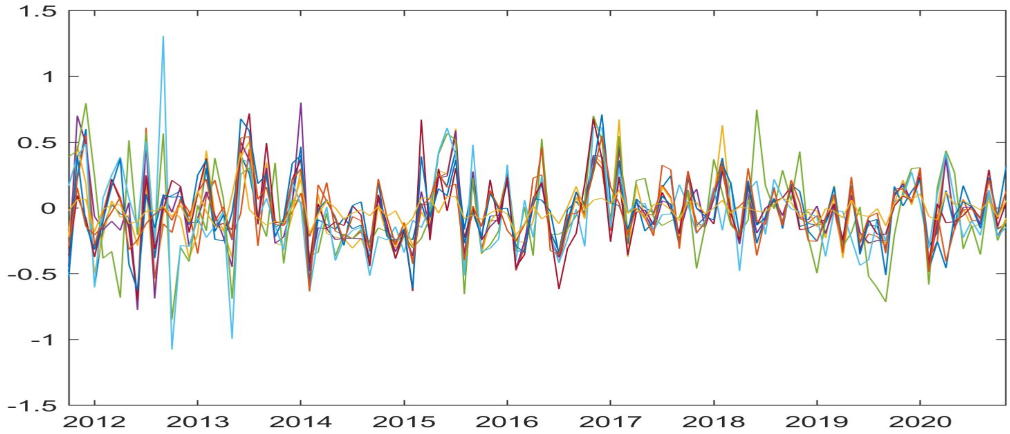


(b) Slope factor

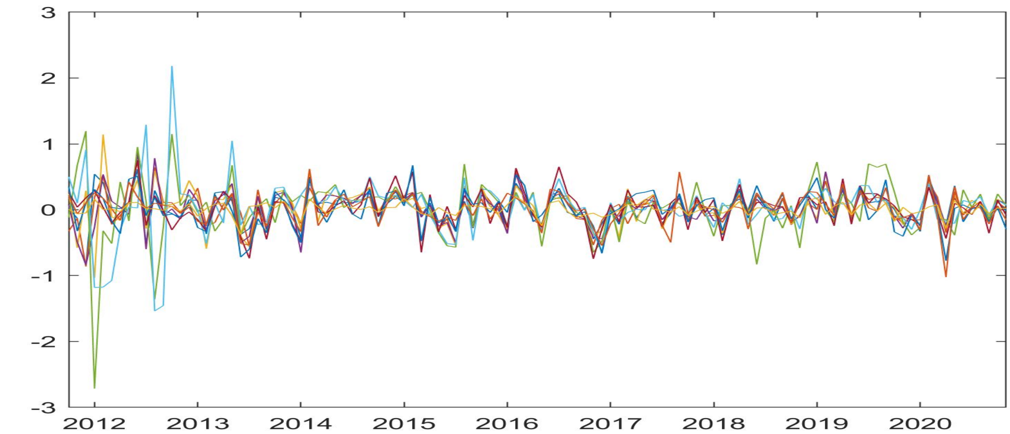


(c) Curvature factor

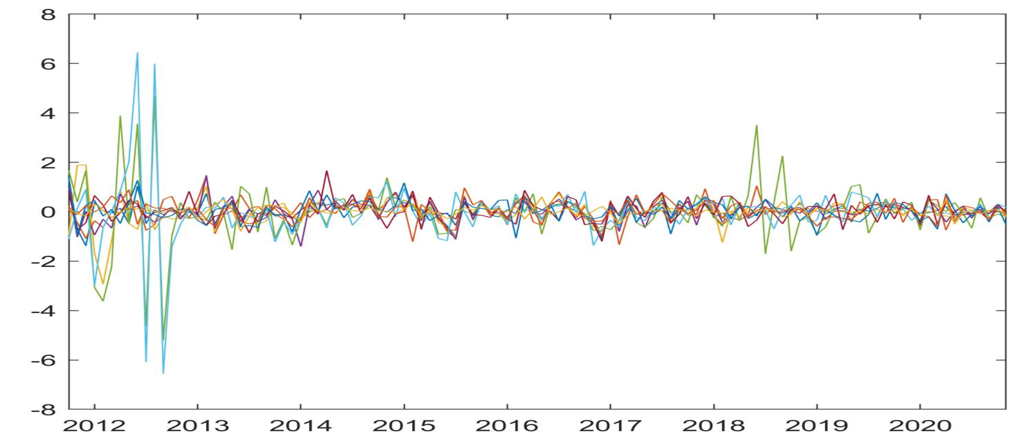
Figure 5: Estimated Nelson-Siegel level (a), slope (b), and curvature (c) factors using the first step of the Diebold and Li (2006) two-step procedure, for each of the ten countries reported in Table I, 2011-08 through 2020-10.



(a) Level factor



(b) Slope factor



(c) Curvature factor

Figure 6: First differences of the estimated Nelson-Siegel level (a), slope (b), and curvature (c) factors using the first step of the Diebold and Li (2006) two-step procedure, for each of the ten countries reported in Table I, 2011-08 through 2020-10.

4.2 Commonality in Levels

The results of principal component analysis on the normalized yield factors is given in Table III below. It shows the eigenvalues of the first five principal components and their corresponding amount of variation explained. For the level factor, the first principal component already explains more than 85% of the total variation. The subsequent principal components explain much less variation per component, the second and third principal component explain only 11% and 2%. This means that there is one dominant factor driving the level factors. The level factors therefore have a lot in common. In case of the slope factors, more principal components are necessary to explain the same amount of variation in the level factors. The first principal component of the slope factors explains 79%, the second 14%, and the third only 3%. The first principal component of the slope factors explains much more than the 50% found in Diebold et al. (2008) but this is most likely because of the chosen countries in the dataset. Their principal component analysis is based on only the US, the UK, Japan, and Germany. The curvature factors have the least in common, the first principal component is responsible for just 61% of the total variation. The second and third 22% and 10% respectively. After the second or third principal component the variation explained is, in each case, only a few or less than one percent.

Table XIII in Appendix A.2 shows the outcomes of principal component analysis when using the raw yield factors. This is to show the effect of not normalizing the factors. The results are quite similar, especially for the slope factors. For the level and curvature factors, the first principal component explains a bit more but does so at the cost of the second component. The first five principal components explain just about as much as in Table III. Therefore, the scaling does not seem to have a large impact on the explained variation.

Table III

Principal Component Analysis - Levels

This table reports results of principal components analysis for the normalized Nelson-Siegel yield factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components. Panel A, B, and C give the results for the level, slope, and curvature factors, respectively.

Panel A: Level factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	8.559	1.047	0.208	0.075	0.047
Variance prop.	0.856	0.105	0.021	0.008	0.005
Cumulative prop.	0.856	0.961	0.982	0.990	0.995
Panel B: Slope factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	7.854	1.387	0.342	0.192	0.084
Variance prop.	0.785	0.139	0.034	0.019	0.008
Cumulative prop.	0.785	0.924	0.958	0.977	0.985
Panel C: Curvature factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	6.062	2.149	0.997	0.343	0.135
Variance prop.	0.606	0.215	0.100	0.034	0.013
Cumulative prop.	0.606	0.821	0.921	0.955	0.968

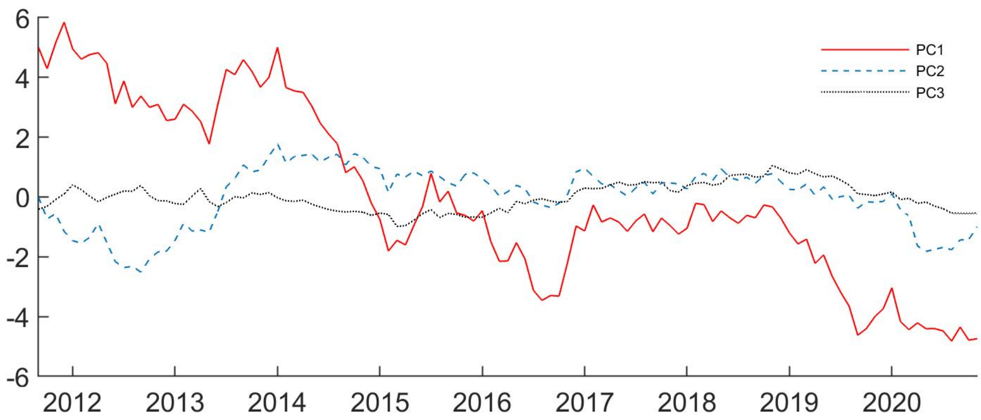


Figure 7: First three principal components of the Nelson-Siegel level factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10.

4.2.1 Level factors

Table IV provides the principal component loading vectors of the normalized country level factors. Meanwhile, Figure 7 plots the first three principal components. The loadings of the first principal component are all positive and with relatively equal weight. The first principal component therefore corresponds with an average of all level factors. It captures the global decreasing trend in interest rates since 2011. The second component weights heavily on the US, Canada, and the UK and can thus be considered a weighted average level factor of these countries. It captures the interest rate trend in these countries, which behave quite similar. The third and fourth components seem to capture contrasts between certain countries. Between Italy, the UK, and Japan, and between Spain, Canada, and Japan respectively. The fifth component is a measure of the level factors of Italy and Japan. Given the results in Table III and the factor loadings, the first and second principal components seem the only components capturing commonalities in the level factors. The other components focus on a small group of countries and explain only a small fraction of the total variation in the level factors, such that these can be deemed irrelevant.

Table IV

Level Factor - Loadings

This table reports the principal component loading vectors of the normalized Nelson-Siegel country level factors. The principal components are summarized in Table III, where the first three are shown graphically in Figure 7. The level factors are obtained using monthly yield data from 2011-08 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.340	0.008	−0.012	−0.029	−0.306
Netherlands	0.336	−0.146	0.037	0.040	−0.344
Belgium	0.334	−0.181	0.052	−0.184	−0.276
France	0.336	−0.135	0.016	−0.040	−0.315
Italy	0.313	−0.269	0.578	0.064	0.512
Spain	0.321	−0.276	−0.036	0.618	0.135
United Kingdom	0.307	0.341	−0.531	0.326	0.033
United States	0.239	0.687	0.203	0.187	0.219
Canada	0.306	0.390	0.260	−0.457	−0.056
Japan	0.319	−0.198	−0.519	−0.476	0.529

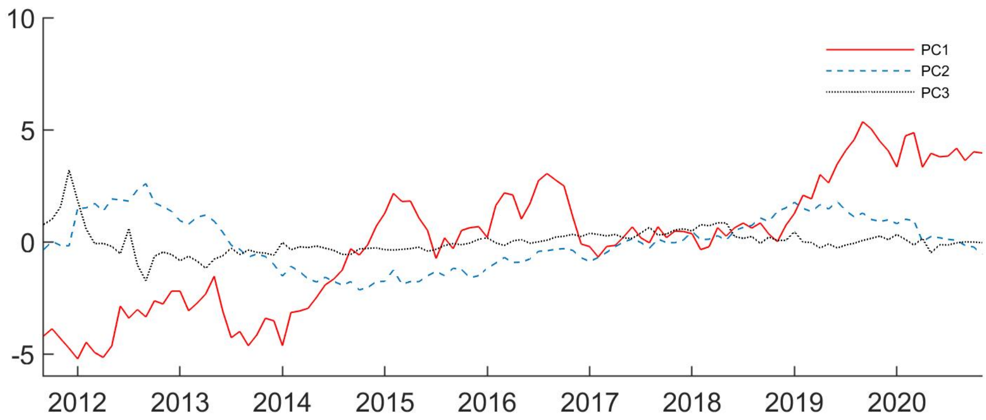


Figure 8: First three principal components of the Nelson-Siegel slope factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10.

4.2.2 Slope factors

The loading vectors of the principal components of the normalized slope factors are shown in Table V. The loadings of the first and second principal component are quite similar to the ones obtained for the level factors. Hence, the first component can be interpreted as an average of all slope factors and the second component as a weighted average of the UK, the US, and Canada. Figure 8 plots the first three principal components showing the first principal component has increased over time. This corresponds to the general pattern of flattening yield curves as observed in Figure 2. The second component follows the flattening of the US, UK, and Canada yield curves. The third component is a weighted average of Italy and Spain. Interestingly, the third component captures the diverging slopes of Italy and Spain from the other EU countries during the European Sovereign Debt Crisis. In the fourth principal component, Japan has a very large weight and can therefore be seen as a Japan factor. The fifth component contains relatively similar weights for Italy, Canada, and Japan such that this component corresponds to an average of these countries.

Table V

Slope Factor - Loadings

This table reports the principal component loading vectors of the normalized Nelson-Siegel country slope factors. The principal components are summarized in Table III, where the first three are shown graphically in Figure 7. The slope factors are obtained using monthly yield data from 2011-08 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.340	−0.034	−0.375	−0.375	−0.135
Netherlands	0.343	−0.170	−0.250	0.020	−0.266
Belgium	0.338	−0.222	−0.194	0.185	−0.094
France	0.342	−0.180	−0.258	0.018	−0.208
Italy	0.289	−0.412	0.401	−0.354	0.530
Spain	0.313	−0.229	0.614	0.037	−0.354
United Kingdom	0.293	0.439	0.235	−0.178	−0.112
United States	0.258	0.564	0.240	0.131	−0.202
Canada	0.301	0.394	−0.202	−0.293	0.466
Japan	0.333	0.019	−0.033	0.750	0.427

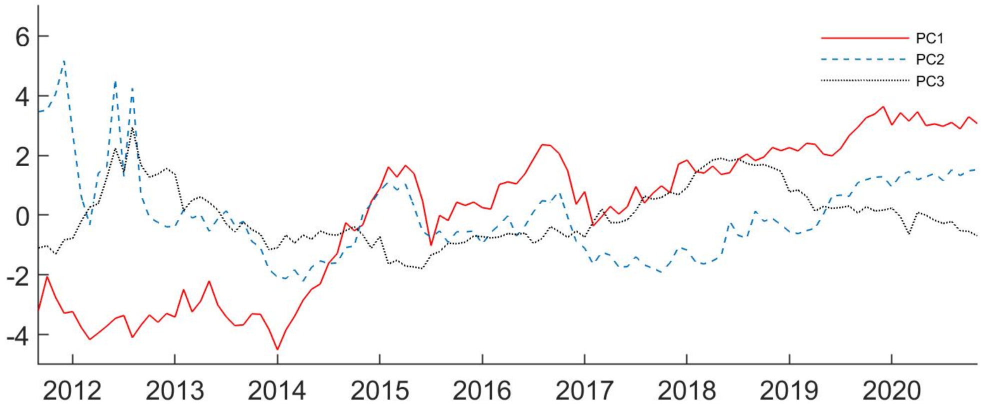


Figure 9: First three principal components of the Nelson-Siegel curvature factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10.

4.2.3 Curvature factors

Table VI reports the loadings of the first five principal components of the normalized curvature factors. Unlike the first principal component loadings of the level and slope factors, the first principal component of the curvature factors does not load on Italy and Spain. It therefore corresponds to an average excluding Italy and Spain. However, Italy and Spain dominate the second factor such that it can be regarded as an average of these. Apparently, they deviate too strongly from the rest and require a separate component. The third component seems to be a weighted average of the US and Canada. The fourth component measures the difference between Belgium and Spain, while the fifth component contrasts the US and Japan with the UK and Canada. Figure 9 depicts the first three principal components over time. The first principal component shows a clear increasing trend after 2014 capturing the rise in the curvature factor. This changed the shape of the yield curve from a more convex curve to a more concave curve. The second principal component captures the path of the curvature factors of Spain and Italy, which until mid 2013 deviated from the other countries.

Table VI

Curvature Factor - Loadings

This table reports the principal component loading vectors of the normalized Nelson-Siegel country curvature factors. The principal components are summarized in Table III, where the first three are shown graphically in Figure 7. The curvature factors are obtained using monthly yield data from 2011-08 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.378	0.124	−0.208	−0.292	0.060
Netherlands	0.357	0.072	−0.379	−0.334	−0.179
Belgium	0.270	0.382	−0.297	0.610	−0.131
France	0.388	0.121	−0.168	−0.050	−0.003
Italy	−0.072	0.645	0.121	0.223	0.049
Spain	−0.080	0.596	0.280	−0.527	0.195
United Kingdom	0.383	−0.055	0.175	−0.024	−0.397
United States	0.355	−0.062	0.365	0.299	0.570
Canada	0.290	−0.039	0.663	0.001	−0.446
Japan	0.371	−0.200	0.004	−0.105	0.475

4.2.4 Discussion

From the results in the beginning of this section we know that two or three principal components are enough to capture more than 90% of the variation in the yield factors. The dimension of the yield factors can be reduced significantly without losing much information. Hence, there is definitely commonality in the yield factors. However, the detailed overview of the last few subsections reveals there are subtle differences in the commonality structure of each yield factor. The first principal component of both the level and slope factors corresponds roughly to an average level and slope factor. In case of the curvature factor, the first principal component is also an average but excludes Italy and Spain. One could therefore say that only the first principal component of the level and slope factors represents a common factor because it relates to all countries. For the curvature factors, principal component analysis makes a clear distinction between Italy, Spain and the rest such that there is no single underlying factor driving all curvature factors. If one asks how much all curvature factors have in common, the answer is: (almost) nothing. Only if we also allow commonalities to exist in subgroups of the dataset, do all yield factors have a lot common.

But principal component analysis is highly data dependent. Therefore the question arises whether the same results would be obtained if we would ignore the European Sovereign Debt Crisis. Appendix A.3 shows the results of principal component analysis on the curvature factors using only the data from 2014 onwards. This time the first principal component corresponds to an average of all curvature factors. The second component is now an average of the US and Canada, instead of the third. Moreover, the first principal component now also explains more of the total variation. Hence, the commonality structure depends on the events in the data leading to converging or diverging movements in the yield factors. The total amount of variation explained by the first three principal components has remained stable at 92%. It thus makes sense to not only consider the first principal component as the source of commonality.

Therefore, subgroups also contribute to the amount of commonality. Which of the principal components capturing subgroups of the data are important drivers of the commonality is determined by their amount of variation explained. For the level and slope factors the first two and in case of the curvature factor the first three components capture at least 10% of the data. This seems to be a natural cutoff point. The commonality in both the level and slope factors are therefore driven by an average and a factor related to the UK, the US, and Canada. The commonality in the curvature factor, over the whole sample, contains an average without Italy and Spain, an average of Italy and Spain, and an additional weighted average of the US and Canada. Overall the level factors have a bit more in common than the other yield factors.

Finally, Figure 14 in Appendix A.4 plots a 3-year moving window of the cumulative amount of variation explained by the first three principal components. This is to investigate how stable the results of Table III are. The figure shows that the variation explained by only the first component can vary quite a lot, but is more stable if we add the second and third component. The first three principal components always explain at least 85% of the yield factors in the moving window. These findings indicate the amount of commonality is quite stable over time.

4.3 Commonality in Changes

This section provides the results of principal component analysis of the first differences of the yield factors. By using the first differences, the effect of different base levels is removed. Table VII gives the eigenvalues, variation explained and cumulative variation explained of the first five principal components of the yield factor first differences. As before, the first differences are normalized to have zero mean and unit standard deviation. Compared to the previous section, the first principal component of the level factors explains much less variation (65%) than when using the level factors in levels (85%). The second and third component explain 12% and 7% respectively. There are also more principal components explaining only a few percent. The level factors therefore have less in common in first differences than in levels. For the slope factors the first principal component explains 54%, the second 18%, and the third component 11%. The other principal components explain only small fractions. In case of the curvature factors the difference is even larger. The first principal component of the first differences of the curvature factor explains just 30%, while the second and third component are responsible for 18% and 15% of the total variation. Here, much more components are necessary to explain a substantial amount of variation. Similar to the situation of the level factors, the first differences of the slope and curvature factors have less in common than their level counterparts.

Table XIV in Appendix A.2 presents the results of principal component analysis if the yield factors would not be normalized. The results are very similar for the level factors. The second principal component of the slope factors explains more variation in this case, at the cost of the first component. For the curvature factor the results are very different. The first principal component already explains 62% of the total variation, more than twice as much than in Table VII, because it captures the extreme curvature changes of Italy and Spain.

Table VII

Principal Component Analysis - Changes

This table reports results of principal components analysis for the normalized first differences of the Nelson-Siegel yield factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components. Panel A, B, and C give the results for the level, slope and curvature factors, respectively.

Panel A: Level factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	6.527	1.233	0.651	0.572	0.309
Variance prop.	0.653	0.123	0.065	0.057	0.031
Cumulative prop.	0.653	0.776	0.841	0.898	0.929
Panel B: Slope factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	5.438	1.753	1.069	0.509	0.359
Variance prop.	0.544	0.175	0.107	0.051	0.036
Cumulative prop.	0.544	0.719	0.826	0.877	0.913
Panel C: Curvature factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	2.947	1.792	1.469	1.007	0.912
Variance prop.	0.295	0.179	0.147	0.101	0.091
Cumulative prop.	0.295	0.474	0.621	0.722	0.813

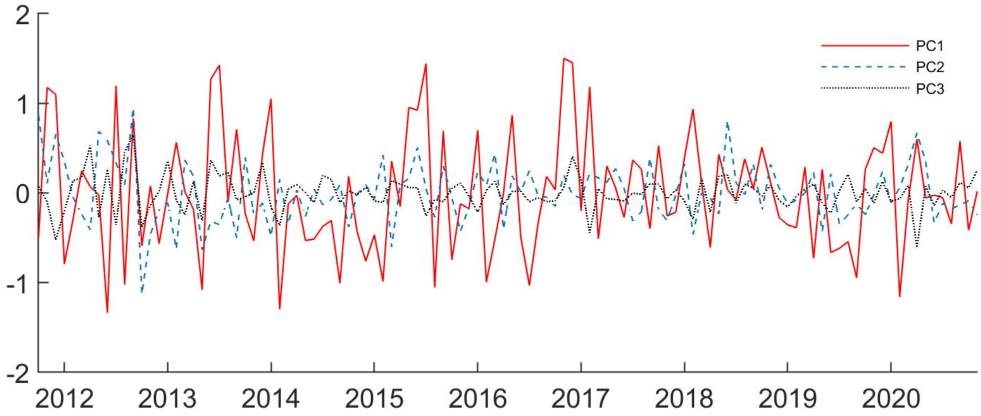


Figure 10: First three principal components of the first differences of the Nelson-Siegel level factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10.

4.3.1 Level factors

The loadings of each principal component on the level factor first differences are shown in Table VIII. The first three principal components are plotted in Figure 10. The first principal component corresponds to an average of all first differences and thereby represents the average change in the level of interest rates. The second principal component roughly measures an average of Italy and Spain. The third component loads heavily on Japan and is therefore a factor capturing mostly that country. The fourth principal component contrasts the US and Canada with Japan, while the last component captures the difference between Italy and Spain. Except for the first principal component, the loadings of the principal components are quite different from those obtained in Section 4.2.1, which were based on the level factors in levels instead of first differences. In levels, the second component was an average of the US, the UK, and Canada most likely because their level factors deviated from the others. Each country's own trend is removed by using first differences. Together with Table VII, the first two principal components seem to capture all commonalities. They explain 78% of the total variation, which is less than the 96% observed when using the level factors in levels.

Table VIII

Level Factor Changes - Loadings

This table reports the principal component loading vectors of the normalized first differences of the Nelson-Siegel country level factors. The principal components are summarized in Table VII, where the first three are shown graphically in Figure 10. The level factors are obtained using monthly yield data from 2011-08 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.360	−0.070	−0.182	−0.278	0.067
Netherlands	0.337	0.112	−0.359	−0.265	0.100
Belgium	0.358	0.088	−0.210	−0.109	−0.097
France	0.361	0.085	−0.155	−0.250	−0.099
Italy	0.232	0.617	0.172	0.220	−0.615
Spain	0.258	0.508	0.280	0.238	0.723
United Kingdom	0.334	−0.259	−0.225	0.196	0.062
United States	0.325	−0.371	0.139	0.332	0.102
Canada	0.314	−0.285	0.139	0.515	−0.214
Japan	0.253	−0.206	0.757	−0.509	−0.073

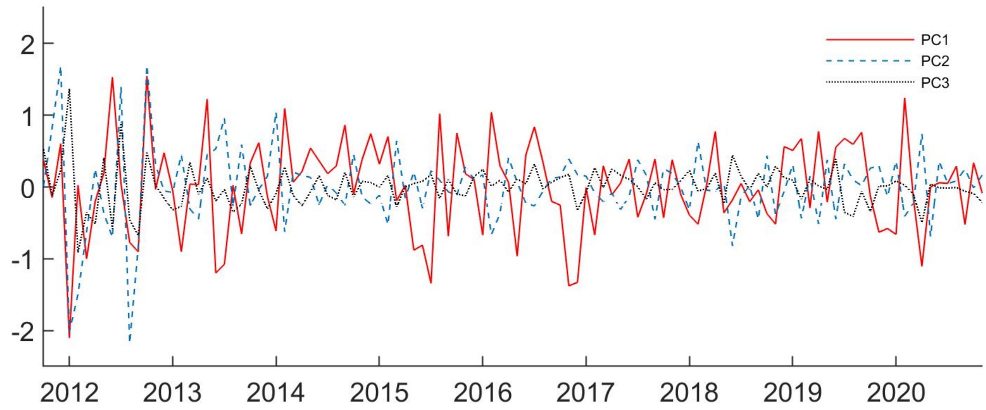


Figure 11: First three principal components of the first differences of the Nelson-Siegel slope factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10.

4.3.2 Slope factors

The loading vectors of each principal component of the normalized slope factors are given in Table IX. Figure 11 shows the first three principal components. The first component is an average ignoring Italy and Spain. The second principal component corresponds to an average of these two countries. The principal components thus make a clear distinction between Italy, Spain and the others. The third component roughly measures the difference between the UK, the US and Japan, and the Netherlands, Belgium, and France. It might be interpreted as the difference in slope changes between the EU and the rest. The fourth component corresponds to a Japan factor while the fifth component measures the difference between Belgium and Spain. These findings are very different from the results in levels, where the first principal component represented an average of all countries. It also seems that in first differences there are three principal components necessary to capture all commonalities, while explaining less than the first two principal components in case of levels (83% vs. 92%).

Table IX

Slope Factor Changes - Loadings

This table reports the principal component loading vectors of the normalized first differences of the Nelson-Siegel country slope factors. The principal components are summarized in Table VII, where the first three are shown graphically in Figure 11. The slope factors are obtained using monthly yield data from 2011-08 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.375	−0.225	−0.155	0.138	−0.317
Netherlands	0.354	−0.159	−0.337	0.222	−0.327
Belgium	0.334	0.147	−0.388	−0.153	0.576
France	0.378	−0.081	−0.344	0.190	0.034
Italy	0.096	0.669	−0.174	0.111	0.227
Spain	0.081	0.669	0.084	0.070	−0.536
United Kingdom	0.367	−0.006	0.108	−0.454	−0.191
United States	0.362	0.004	0.360	−0.334	−0.016
Canada	0.350	0.026	0.377	−0.214	0.184
Japan	0.278	−0.018	0.522	0.701	0.225

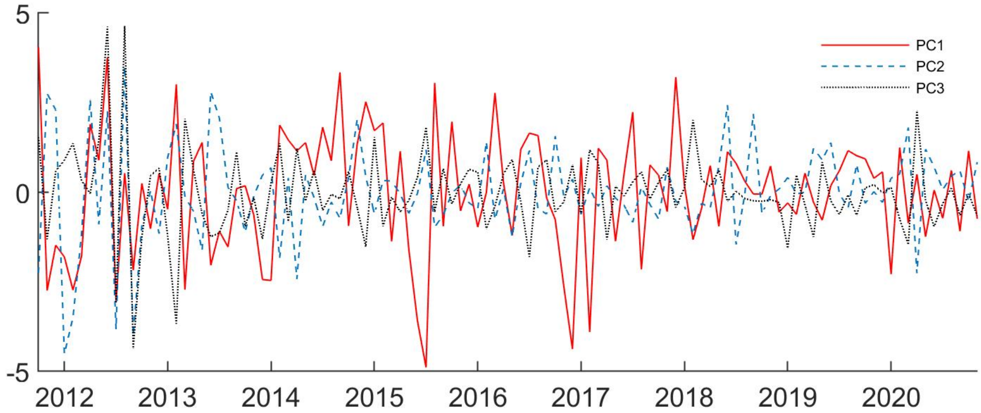


Figure 12: First three principal components of the first differences of the Nelson-Siegel curvature factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10.

4.3.3 Curvature factors

Table X gives the loadings of the first five principal components of the normalized curvature factors. Interpreting the principal components in this case is very ambiguous. The first component roughly corresponds to an average of Germany, the Netherlands, and France. However it could also be seen as a weighted average of all countries because the weights are all positive and relatively close to each other. The second component mostly captures an average of Italy and Spain but also loads negatively on all non-EU countries. The third component contrasts Italy, Spain, and the US with the Netherlands and Belgium. The fourth and fifth component are dominated by Belgium and Canada, and Japan, respectively. The first three principal components are shown in Figure 12. Given the loading vectors and the fraction of variation explained by each principal component, the curvature factors do not seem to have much in common or at least require four components. This is, for example, the only case where there is no dominant principal component explaining more than half the variation. Moreover, the interpretation of the components makes less sense because the components combine countries that do not seem to be related in a regional or monetary way.

Table X

Curvature Factor Changes - Loadings

This table reports the principal component loading vectors of the normalized first differences of the Nelson-Siegel country curvature factors. The principal components are summarized in Table VII, where the first three are shown graphically in Figure 12. The curvature factors are obtained using monthly yield data from 2011-08 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.471	−0.114	−0.035	−0.383	−0.020
Netherlands	0.378	0.157	−0.406	−0.284	−0.112
Belgium	0.260	0.270	−0.378	0.494	0.051
France	0.483	0.008	−0.256	−0.009	0.159
Italy	0.227	0.552	0.357	0.067	0.006
Spain	0.212	0.447	0.512	−0.042	0.037
United Kingdom	0.275	−0.373	0.183	−0.205	−0.256
United States	0.275	−0.331	0.419	0.113	−0.215
Canada	0.249	−0.245	0.044	0.678	−0.244
Japan	0.160	−0.277	0.159	0.085	0.887

4.3.4 Discussion

From Table VII we know how much information about the first differences of the yield factors can be stored in the first five principal components. The level factors need more principal components to explain most of the variation than in Section 4.2. Apparently, the level factors have less in common in their fist differences than in their levels. The first component is still an average of all countries. However, the second component is mostly an additional factor of Italy and Spain, while in levels the second component was an average of the US, the UK, and Canada. In levels, these three countries deviate from the others but were not so different in terms of first differences. Table XVII in Appendix A.5 reports the loading vectors of the level factor first differences after 2013, which shows that the second component still corresponds to an average of Italy and Spain. The results are thus robust to the crisis period. The first two components, which by their relevance and interpretation can be seen as common factors, explain 78% of the total variation.

The loadings of the principal components of the slope factors are also quite different in first differences. Again, there is no component specifically related to the US, the UK, and Canada. The first component is an average of all except Italy and Spain. These two countries are captured by the second component. The third component seems to be the difference in slope changes between EU and non-EU countries. Hence, three the first three component seem to capture all commonalities explaining 83% of the variation, which is less than in levels. If we ignore the European Sovereign Debt Crisis, the first component changes to an average of all countries and the second component mostly remains an average of Italy and Spain. This is shown in Appendix A.6. The loadings and interpretations of the principal components thus change depending on the data at hand.

The same can be said for the curvature factors. The first component is a weighted average of all countries, the second focuses on Italy and Spain. The third and fourth component combine countries that do not really make a lot of sense. Appendix A.7 shows that without the crisis period the first component becomes an average of Germany, the Netherlands, Belgium, and France. The second component is now the difference between Spain and Italy, and the UK and the US. The third component is an average of these four. However, while the loadings and interpretation change, the fraction of variation explained by each component remains more or less the same. In terms of first differences, the curvature factors do not have much in common and the loadings are unstable. There are four components, which can be seen as common factors, explaining more than 10% for a total of 72%, compared to the three common components in levels explaining 92%.

Moreover, Figure 15 in Appendix A.4 plots a 3-year moving window of the cumulative amount of variation explained by the first four principal components when using the yield factor first differences. Similar to the results in levels, the total explained variation becomes less volatile if we add more components such that the total amount of commonality is very stable.

5 Forecasting

5.1 Design

Now that the commonalities in the yield factors have been analyzed, this section investigates the usefulness of the principal components. To obtain the principal components in the yield factors we first need the yield factors themselves so the question is whether the principal components can replace the yield factors and improve out-of-sample yield curve forecasting. For that reason, forecasts are made at each available maturity using a 5-year moving window. To forecast with the principal components, remember from Section 2.2, that the principal components are a linear combination of the demeaned yield factors ($Y = XG$). This relation is also true the other way around, i.e. the demeaned yield factors are a linear combination of the principal components ($X = YG'$) (Mardia et al., 1979). Therefore, forecasts will be constructed by first approximating the original variable by its mean and the first few, say q, principal components. Specifically,

$$z_i = \mu_i + g_{i1}y_1 + ... + g_{iq}y_q, \tag{6}$$

where z_i is the approximation of the yield factor of interest of country i and μ_i is the yield factor mean. Then, as in Diebold and Li (2006), an autoregressive model of order one is used to predict the future value of z_i. Hence, the h-step ahead forecast is equal to

$$\hat{z}_{i,t+h} = c_i + \gamma_i z_{it}, \tag{7}$$

Here, c_i and γ_i are the estimated constant and slope parameter of country i. To obtain forecasts of the yield curve two methods will be used. The first method forecasts the yield curve with the level, slope, and curvature factor as z_i. These forecasts can then be used in the Nelson-Siegel model of equation (1) to derive forecasts $\hat{y}_{i,t+h}(\tau)$ at all available maturities. The second method uses the first differences of each yield factor as z_i with the additional step of adding the last known value of the yield factor to this forecast before calculating the yield forecasts with the Nelson-Siegel model.

In both methods the value of q will be set at 1, 3 and 5, thereby varying the amount of commonalities relative to insignificant components stored in z_i. Furthermore, $h = \{1, 6, 12\}$ to assess the performance of the predictions at several horizons. To evaluate the accuracy of the predicted yields, the average root mean squared prediction error (ARMSE) of each country will be reported. The country ARMSE is

$$ARMSE = \frac{1}{T}\sum_t \left(\sqrt{\frac{1}{12}\sum_\tau (\hat{y}_{i,t+h}(\tau) - y_{i,t+h}(\tau))^2}\right), \tag{8}$$

where T is the number of predicted yield curves over time and $y_{i,t+h}(\tau)$ is the actual yield. The 12 comes from the fact that we are averaging over twelve different maturities. A lower ARMSE means that the forecasts are more accurate. To see whether the principal component based predictions improve yield curve forecasts over using the original yield factors, predictions will be included using an autoregressive model for the yield factors in each method, which is the same approach as in (6) and (7) but with $q = 10$.

Finally, to test for any significant differences in performance between the three principal component based models and the one using the original yield factors, we apply the model confidence set (MCS) procedure of Hansen et al. (2011). The MCS is a set of models that contains the best models with a certain confidence. Given a collection of models, the MCS is obtained by comparing the relative performance of each model and sequentially dropping models from the set with a relatively bad performance. It does so until the null hypothesis of equal performance among the models in the set cannot be rejected. In our case, the performance of each model is measured by the time series of RMSEs. Denote $L_{i,t}$ as the RMSE of model i at time t, then

$$d_{ij,t} = L_{i,t} - L_{j,t} \quad \text{for all} \quad i, j \in M^0 \tag{9}$$

defines the relative performance variables of the starting set of models M^0. The superior set of models is defined by $M^* \equiv \{i \in M^0 : E(d_{ij}) \leq 0 \text{ for all } j \in M^0\}$. The MCS procedure tries to find this superior set M^*. To test the hypothesis whether $E(d_{ij}) = 0$, we construct the test statistic $T_R = max|t_{ij}|$, where

$$t_{ij} = \frac{\bar{d}_{ij}}{\sqrt{var(\bar{d}_{ij})}}, \tag{10}$$

with $\bar{d}_{ij} = T^{-1} \sum_t^T d_{ij,t}$. Hence, T_R is the largest standardized loss differential and the model responsible for this will be removed from the set if the test statistic is too large. The distribution of the test statistic is estimated with the block bootstrap method as suggested by Hansen et al. (2011). In the results, the MCS p-values of the models, denoted by p_{MCS}, will be reported. These p-values are equal to the maximum p-value of the deleted models and the current worst model in the sequential testing procedure. This makes it easier to see whether a certain model is in the superior model set. For example, if model 1,2 and 3 would be removed in ascending order and the p-value of model 3 is 0.03, below the 0.07 p-value of model 2, then at a 5% significance level both model 2 and 3 stay in the superior set because of the sequential nature of testing. Conveniently, the MCS p-value is set at 0.07 for model 3, such that at a 10% significance level both would be deleted from the set.

Here, we will set the significance level at 10% and work with a block size of 3 and 5000 bootstrap replications in the block bootstrap method. The MCS is determined for each country and each horizon separately because the MCS procedure can only compare time series of losses, not panels. It would therefore be nonsensical to mix up forecasts of different countries and different horizons. The MCS procedure is implemented using Sheppard (2018).

5.2 Results

This section reports the forecasting performance of the models described in Section 5.1. All models are estimated with a 5-year moving window, where the first window is from 2011-08 until 2016-08. Because the last possible window depends on the forecast horizon, the number of forecasts decreases with longer horizons. Specifically, the results of the 1-month, 6-month, and 12-month forecasts are based on 51, 45, and 39 observations respectively.

Table XI reports the ARMSEs and MCS p-values of the forecasts made using the yield factor levels, whereas Table XII reports the results of forecasting the first differences of the yield factors. For simplicity, we refer to the forecasts made with the yield factors and the first 1, 3 and 5 principal components as the NS, PC1, PC1-3 and PC1-5 forecasts respectively. In Table XI, for the 1-month-ahead forecasts, the NS forecasts have the lowest ARMSE. However, when the value of q, the number of principal components used to approximate the yield factors, increases, the ARMSE approaches the ARMSE of the yield factor predictions. The MCS p-values confirm that the NS model performs best, as it is always in the model confidence set. PC1 is never in the MCS, while PC1-3 and PC1-5 appear once and four times in the MCS. Hence, for some countries the NS forecasts are not significantly better than these other forecasts. Given that PC1 is never in the MCS and PC1-3 only once, using the common factors does not lead to better forecasts at the 1-month horizon. One reason for this could be that the approximations are biased because information is lost by using only the first q principal components. This information is probably still important in the short-run.

At longer horizons, the 6-month-ahead and 12-month-ahead forecasts, the ARMSE of the principal component based predictions is in some cases lower than their NS counterpart. The reduction is nevertheless very small. The presence of PC1-3 and PC1-5 in the MCS is also more prominent, whereas PC1 is mostly absent from the MCS. At longer horizons the forecasts of the principal component models improves relative to the NS model, such that significant differences in forecasts cannot be determined by the MCS procedure and consequently more models are in the MCS. Remember from moving window principal component analysis of Figure 14 that the explained variance of using only one component can vary substantially, which indicates not too many information can be lost to produce accurate forecasts. Because PC1-3 and PC1-5 now appear just as frequently in the MCS as the NS model, it is ambiguous which of these models is best. The role of the one common factor model is limited.

The second method which forecasts the first differences of the yield factors to make yield predictions makes a less clear distinction. At all horizons the improvements over the NS forecasts, if any, are negligible. This is recognized by the MCS procedure since in most settings the MCS contains all models, no matter the forecasting horizon or the country under consideration. On the other hand, forecasting with first differences results in better predictions in general than with levels. Thus, using only common factor(s) provides no improvement over the NS model in first differences.

The aforementioned results show that approximating the yield factors by their first few principal components does not improve yield forecasts, except for a few instances where the NS forecasts are not in the MCS. Given the relatively small number of observations on which the results in Table XI and XII are based, these instances should have no significant impact on the general finding. This means that the commonalities in the yield factors cannot be exploited for out-of-sample forecasting and relying on the estimated Nelson-Siegel factors directly is still the preferred procedure.

Table XI

Forecasting Results - Levels

This table reports the average root mean squared error of all maturities and MCS p-values using 1-month-ahead, 6-month-ahead, and 12-month-ahead, out-of-sample forecasting. The forecasts are based on the levels of the estimated Nelson-Siegel factors and the models are estimated with a 5-year moving window, where the initial estimation sample is from 2011-08 to 2016-08. All models assume AR(1) factor dynamics, where NS uses the estimated Nelson-Siegel factors, PC1 approximates each Nelson-Siegel factor with its first principal component, PC1-3 approximates each Nelson-Siegel factor with its first three principal components, and PC1-5 approximates each Nelson-Siegel factor with its first five principal components. The bold MCS p-values indicate the model is part of the model confidence set.

Model	NS	PC1	PC1-3	PC1-5
1 - Month	ARMSE (p_{MCS})	ARMSE (p_{MCS})	ARMSE (p_{MCS})	ARMSE (p_{MCS})
Germany	0.125 (**1.000**)	0.250 (0.000)	0.160 (0.000)	0.135 (0.025)
Netherlands	0.125 (**1.000**)	0.267 (0.000)	0.163 (0.000)	0.125 (**0.689**)
Belgium	0.141 (**1.000**)	0.288 (0.000)	0.151 (**0.105**)	0.159 (0.009)
France	0.130 (**1.000**)	0.252 (0.000)	0.153 (0.002)	0.144 (0.002)
Italy	0.236 (**1.000**)	0.497 (0.000)	0.291 (0.002)	0.240 (**0.239**)
Spain	0.165 (**1.000**)	0.326 (0.000)	0.191 (0.021)	0.171 (**0.267**)
United Kingdom	0.130 (**1.000**)	0.286 (0.000)	0.192 (0.003)	0.154 (0.010)
United States	0.161 (**1.000**)	0.620 (0.000)	0.245 (0.003)	0.198 (0.018)
Canada	0.153 (**1.000**)	0.435 (0.000)	0.215 (0.001)	0.160 (**0.532**)
Japan	0.053 (**1.000**)	0.119 (0.000)	0.078 (0.000)	0.085 (0.000)
6 - Months				
Germany	0.225 (**1.000**)	0.326 (0.000)	0.264 (0.002)	0.246 (0.002)
Netherlands	0.258 (**1.000**)	0.341 (0.000)	0.281 (**0.161**)	0.263 (**0.218**)
Belgium	0.301 (**1.000**)	0.354 (0.013)	0.305 (**0.844**)	0.304 (**0.844**)
France	0.289 (**0.833**)	0.376 (0.000)	0.291 (**0.833**)	0.287 (**1.000**)
Italy	0.552 (**1.000**)	0.588 (**0.595**)	0.595 (0.052)	0.552 (**0.938**)
Spain	0.385 (**0.542**)	0.522 (0.000)	0.376 (**1.000**)	0.383 (**0.542**)
United Kingdom	0.301 (**1.000**)	0.513 (0.000)	0.319 (**0.563**)	0.320 (**0.273**)
United States	0.773 (**1.000**)	1.108 (0.000)	0.813 (0.023)	0.787 (**0.356**)
Canada	0.576 (**0.494**)	0.721 (0.003)	0.586 (**0.429**)	0.567 (**1.000**)
Japan	0.096 (**0.497**)	0.146 (0.005)	0.088 (**1.000**)	0.119 (**0.192**)
12 - Months				
Germany	0.353 (**0.223**)	0.378 (**0.223**)	0.342 (**1.000**)	0.342 (**0.974**)
Netherlands	0.373 (**0.744**)	0.386 (**0.744**)	0.364 (**1.000**)	0.372 (**0.744**)
Belgium	0.377 (**1.000**)	0.401 (**0.333**)	0.401 (0.091)	0.378 (**0.895**)
France	0.341 (**1.000**)	0.419 (0.038)	0.371 (0.052)	0.353 (0.052)
Italy	0.575 (0.007)	0.751 (0.000)	0.576 (**0.264**)	0.561 (**1.000**)
Spain	0.467 (**0.628**)	0.583 (0.003)	0.474 (**0.366**)	0.464 (**1.000**)
United Kingdom	0.479 (**1.000**)	0.597 (**0.630**)	0.495 (**0.771**)	0.482 (**0.771**)
United States	1.162 (**1.000**)	1.356 (0.000)	1.181 (**0.124**)	1.182 (0.000)
Canada	0.773 (**1.000**)	0.965 (0.005)	0.812 (**0.114**)	0.799 (**0.114**)
Japan	0.142 (0.083)	0.155 (0.041)	0.097 (**1.000**)	0.185 (0.041)

Table XII

Forecasting Results - Changes

This table reports the average root mean squared error of all maturities and MCS p-values using 1-month-ahead, 6-month-ahead, and 12-month-ahead, out-of-sample forecasting. The forecasts are based on the first differences of the estimated Nelson-Siegel factors and the models are estimated with a 5-year moving window, where the initial estimation sample is from 2011-08 to 2016-08. All models assume AR(1) factor dynamics for the first differences, where NS uses the first differences of the estimated Nelson-Siegel factors, PC1 approximates the first differences of each Nelson-Siegel factor with the first principal component of the factors' first differences, PC1-3 approximates the first differences of each Nelson-Siegel factor with the first three principal components of the factors' first differences, and PC1-5 approximates the first differences of each Nelson-Siegel factor with the first five principal components of the factors' first differences. The bold MCS p-values indicate the model is part of the model confidence set using

Model	NS	PC1	PC1-3	PC1-5
1 - Month	ARMSE (p_{MCS})	ARMSE (p_{MCS})	ARMSE (p_{MCS})	ARMSE (p_{MCS})
Germany	0.123 (**0.313**)	0.124 (0.079)	0.122 (**0.930**)	0.122 (**1.000**)
Netherlands	0.121 (**0.846**)	0.122 (**0.652**)	0.121 (**0.846**)	0.121 (**1.000**)
Belgium	0.136 (**0.890**)	0.138 (**0.256**)	0.136 (**1.000**)	0.137 (**0.478**)
France	0.127 (**0.398**)	0.127 (**0.793**)	0.127 (**0.793**)	0.127 (**1.000**)
Italy	0.244 (**0.943**)	0.244 (**1.000**)	0.244 (**0.969**)	0.244 (**0.969**)
Spain	0.158 (**0.822**)	0.158 (**0.822**)	0.158 (**0.822**)	0.158 (**1.000**)
United Kingdom	0.126 (**0.944**)	0.126 (**0.944**)	0.126 (**1.000**)	0.126 (**0.944**)
United States	0.157 (0.046)	0.156 (**0.932**)	0.155 (**1.000**)	0.155 (**0.932**)
Canada	0.148 (**1.000**)	0.149 (**0.774**)	0.149 (**0.774**)	0.149 (**0.774**)
Japan	0.054 (**0.593**)	0.054 (**0.622**)	0.054 (**0.593**)	0.053 (**1.000**)
6 - Months				
Germany	0.194 (0.018)	0.191 (**0.327**)	0.191 (**1.000**)	0.191 (**0.327**)
Netherlands	0.190 (**0.269**)	0.188 (**0.370**)	0.187 (**1.000**)	0.191 (**0.211**)
Belgium	0.221 (**0.677**)	0.219 (**1.000**)	0.220 (**0.677**)	0.221 (**0.252**)
France	0.215 (**1.000**)	0.216 (**0.649**)	0.216 (0.097)	0.215 (**0.649**)
Italy	0.577 (**0.532**)	0.572 (**1.000**)	0.574 (**0.532**)	0.576 (**0.532**)
Spain	0.262 (**0.814**)	0.262 (**1.000**)	0.265 (0.018)	0.263 (0.018)
United Kingdom	0.269 (**0.413**)	0.271 (**0.413**)	0.268 (**1.000**)	0.268 (**0.978**)
United States	0.579 (0.031)	0.572 (**0.549**)	0.571 (**1.000**)	0.573 (**0.472**)
Canada	0.463 (**0.906**)	0.463 (**0.906**)	0.463 (**1.000**)	0.466 (**0.422**)
Japan	0.088 (**0.626**)	0.087 (**1.000**)	0.087 (**0.626**)	0.087 (**0.880**)
12 - Months				
Germany	0.289 (**0.737**)	0.288 (**0.824**)	0.287 (**1.000**)	0.288 (**0.824**)
Netherlands	0.257 (**1.000**)	0.260 (**0.405**)	0.259 (**0.339**)	0.257 (**0.486**)
Belgium	0.271 (**1.000**)	0.273 (**0.672**)	0.271 (**0.876**)	0.272 (**0.560**)
France	0.266 (**0.870**)	0.267 (**0.663**)	0.265 (**1.000**)	0.266 (**0.760**)
Italy	0.802 (**0.993**)	0.805 (**0.962**)	0.802 (**0.993**)	0.802 (**1.000**)
Spain	0.310 (**0.220**)	0.309 (**0.532**)	0.307 (**1.000**)	0.311 (**0.116**)
United Kingdom	0.421 (**0.820**)	0.431 (0.000)	0.426 (0.028)	0.421 (**1.000**)
United States	0.988 (**1.000**)	0.993 (0.025)	0.988 (**0.866**)	0.988 (**0.988**)
Canada	0.766 (0.050)	0.767 (0.050)	0.762 (**1.000**)	0.765 (0.050)
Japan	0.108 (**1.000**)	0.108 (**0.983**)	0.108 (**0.983**)	0.109 (**0.957**)

6 Conclusion

This thesis has investigated to what extend there are commonalities in the Nelson-Siegel level, slope and curvature factors. By applying principal component analysis on the different yield factors, the amount of variation explained by the principal components describes how many components are required to explain most of the variation in the yield factors and thereby how much the yield factors have in common. The analysis in levels shows that for the level and slope factor, two principal components capture more than 90% of the total variation. The results for the curvature factor depend on whether the European Sovereign Debt Crisis is included. If included, the curvature factor has three principal components explaining a substantial amount of variation, while excluded, only two components seem to capture commonalities. In first differences, the results are somewhat different. For the level factor, the first two principal components are still the only significant components but now explain only 78%. The slope factor now has three components explaining more than 10% instead of two. These components also explain less, namely 83%. The first differences of the curvature factors now has four components with more than 10% explained variation for a total of 72%.

Overall, the Nelson-Siegel yield factors can be reduced to just a few principal components, more so in levels than in first differences. In terms of interpretation, most principal components contributing to the commonality seem to represent averages of either all countries or smaller subgroups. Only in case of the curvature first differences is the interpretation ambiguous. Meanwhile, the total amount of variation explained by the components contributing to the commonality, in levels and first differences, is robust to normalization of the data and stable over time.

A forecasting study is implemented to see if the first few principal components, which can be regarded as common factors, are useful in a practical sense. Unfortunately, the principal component based forecasts rarely provide better predictions than using the estimated Nelson-Siegel factors themselves. The ARMSE is sometimes lower, especially at longer horizons, but the differences are not statistically significant because the standard Nelson-Siegel model also appears in the model confidence set. Therefore, commonalities in the yield factors cannot be exploited for out-of-sample forecasting.

Several contributions have been made to the existing literature. First, we have investigated the existence of commonalities and carried out principal component analysis on a dataset of ten countries, whereas most studies have used three or four countries. Second, not only have we looked at commonalities in the yield factor levels but also in first differences. Third, a detailed overview has been given of each yield factor in terms of robustness and interpretation. Finally, a forecast study has shown that the principal components do not improve yield forecasting after applying the MCS procedure.

The applied methodology also has a few shortcomings. The relatively small time period covered in the dataset, 2011-8 through 2020-10, is quite limited. As a result, the outcomes of principal component analysis and the forecasting study might not hold in other samples. Specifically, the exclusion of the sovereign debt crisis has shown that the number of important principal components and their interpretation can vary depending on the data at hand. Moreover, the forecasting accuracy is evaluated over just a few years time. From a methodological perspective, the use of principal components to find commonalities is not without debate. The components do not necessarily have to be common factors, yet based on their importance and loadings an attempt has been made. Therefore, other studies have focused on extracting actual common/global factors and local factors.

Further research in this area can investigate whether the principal components are linked to macroeconomic variables or if there exist factors belonging to different subgroups in the data using methodologies similar to Ando and Bai (2017). Since principal component analysis is a linear method, nonlinear methods like kernel principal component analysis can be used to analyze potential nonlinear commonalities in the yield factors.

References

Ando, T., & Bai, J. (2017). Clustering huge number of financial time series: A panel data approach with high-dimensional predictors and factor structures. *Journal of the American Statistical Association*, *112*(519), 1182–1198.

Ang, A., & Piazzesi, M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary economics*, *50*(4), 745–787.

Bauer, M. D., & Rudebusch, G. D. (2016). Monetary policy expectations at the zero lower bound. *Journal of Money, Credit and Banking*, *48*(7), 1439–1465.

BIS. (2009). *79th annual report* (tech. rep.). Bank for International Settlements.

Bliss, R. R. (1996). *Testing term structure estimation methods* (tech. rep.). Working Paper.

Brennan, M. J., & Xia, Y. (2006). International capital markets and foreign exchange risk. *The Review of Financial Studies*, *19*(3), 753–795.

Christensen, J. H., & Rudebusch, G. D. (2015). Estimating shadow-rate term structure models with near-zero yields. *Journal of Financial Econometrics*, *13*(2), 226–259.

Coroneo, L., Garrett, I., & Sanhueza, J. (2018). Dynamic linkages across country yield curves: The effects of global and local yield curve factors on us, uk and german yields. *New methods in fixed income modeling* (pp. 205–222). Springer.

Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica*, *53*(2), 385–407.

Diebold, F. X., & Li, C. (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics*, *130*, 337–364.

Diebold, F. X., Li, C., & Yue, V. Z. (2008). Global yield curve dynamics and interactions: A dynamic nelson–siegel approach. *Journal of Econometrics*, *146*(2), 351–363.

Diebold, F. X., Rudebusch, G. D., & Aruoba, S. B. (2006). The macroeconomy and the yield curve: A dynamic latent factor approach. *Journal of econometrics*, *131*(1-2), 309–338.

Driessen, J., Melenberg, B., & Nijman, T. (2003). Common factors in international bond returns. *Journal of International Money and Finance*, *22*(5), 629–656.

Duffee, G. R. (2002). Term premia and interest rate forecasts in affine models. *The Journal of Finance*, *57*(1), 405–443.

Duffie, D., & Kan, R. (1996). A yield-factor model of interest rates. *Mathematical finance*, *6*(4), 379–406.

Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The model confidence set. *Econometrica*, *79*(2), 453–497.

Heath, D., Jarrow, R., & Morton, A. (1992). Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica: Journal of the Econometric Society*, 77–105.

Hull, J., & White, A. (1990). Pricing interest-rate-derivative securities. *The review of financial studies*, *3*(4), 573–592.

James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). *An introduction to statistical learning* (Vol. 112). Springer.

Koopman, S. J., Mallee, M. I., & Van der Wel, M. (2010). Analyzing the term structure of interest rates using the dynamic nelson–siegel model with time-varying parameters. *Journal of Business & Economic Statistics*, *28*(3), 329–343.

Maltritz, D. (2012). Determinants of sovereign yield spreads in the eurozone: A bayesian approach. *Journal of International Money and Finance*, *31*(3), 657–672.

Mardia, K., Kent, J., & Bibby, J. (1979). *Multivariate analysis.* Academic Press.

Modugno, M., & Nikolau, K. (2009). The forecasting power of international yield curve linkages. *ECB Working Paper Series.*

Nelson, C. R., & Siegel, A. F. (1987). Parsimonious modeling of yield curves. *Journal of business*, 473–489.

Pérignon, C., Smith, D. R., & Villa, C. (2007). Why common factors in international bond returns are not so common. *Journal of International Money and Finance*, *26*(2), 284–304.

Rodrigues, A. P. (1997). Term structure and volatility shocks. *Manuscript, Federal Reserve Board*, *500.*

Sheppard, K. (2018). *Matlab function reference financial econometrics.*

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, *5*(2), 177–188.

A Appendix

A.1 Yield Curves

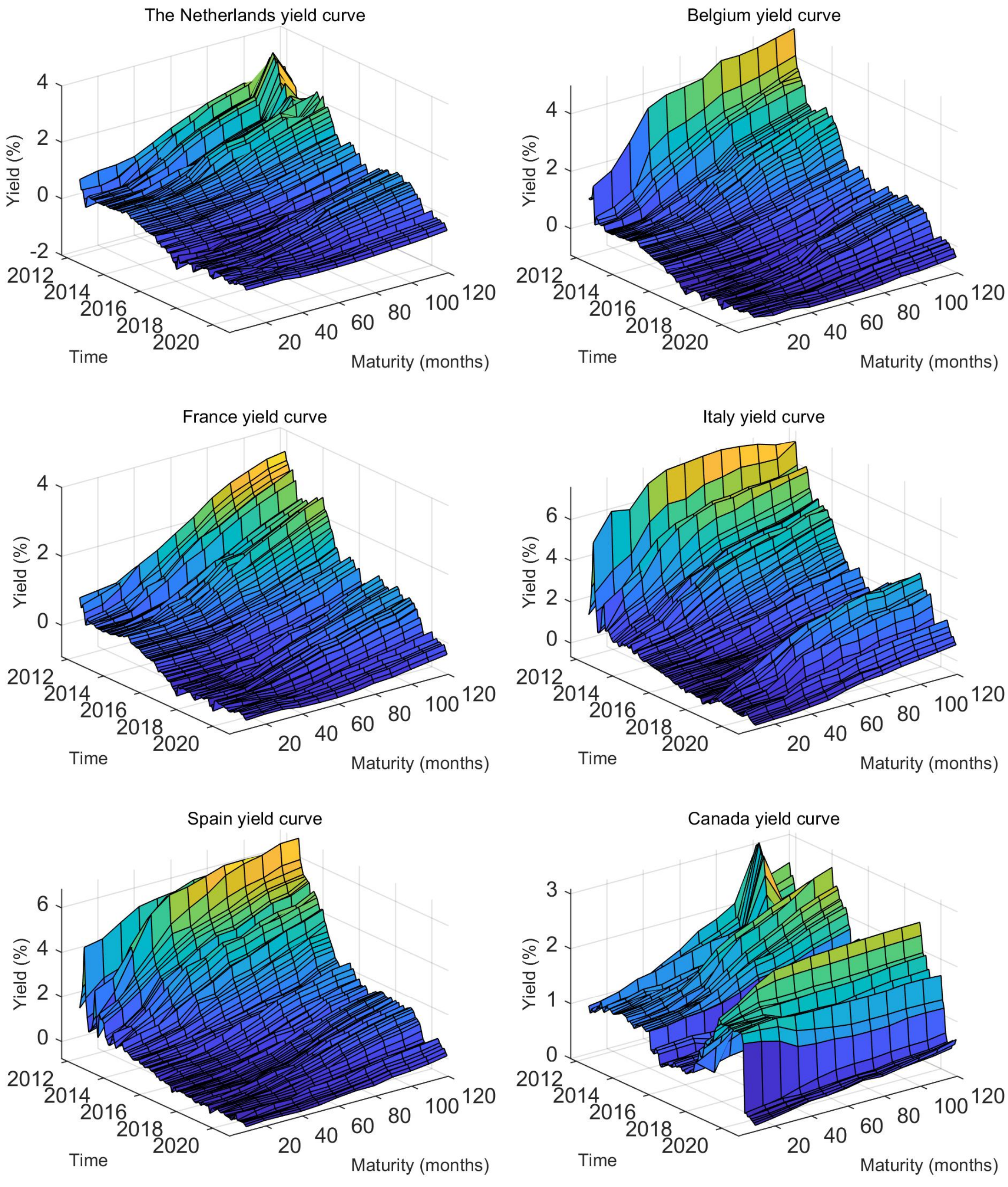


Figure 13: Yield curves of the remaining countries across time and maturity. The bond yield data is monthly from 2011-08 through 2020-10.

A.2 PCA Raw Data

Table XIII
Principal Component Analysis - Levels
Raw data

This table reports results of principal components analysis for the original Nelson-Siegel yield factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components. Panel A, B, and C give the results for the level, slope and curvature factors, respectively.

Panel A: Level factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	13.156	0.915	0.258	0.123	0.039
Variance prop.	0.904	0.063	0.018	0.008	0.003
Cumulative prop.	0.904	0.967	0.985	0.993	0.996
Panel B: Slope factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	8.170	1.636	0.459	0.162	0.055
Variance prop.	0.773	0.155	0.043	0.015	0.005
Cumulative prop.	0.773	0.928	0.971	0.986	0.991
Panel C: Curvature factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	12.751	8.105	2.347	0.972	0.357
Variance prop.	0.506	0.322	0.093	0.039	0.014
Cumulative prop.	0.506	0.828	0.921	0.960	0.974

Table XIV
Principal Component Analysis - Changes
Raw data

This table reports results of principal components analysis for the first differences of the original Nelson-Siegel yield factors of the ten countries mentioned in Table I using monthly yield data from 2011-08 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components. Panel A, B, and C give the results for the level, slope and curvature factors, respectively.

Panel A: Level factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	0.438	0.114	0.039	0.033	0.014
Variance prop.	0.648	0.169	0.058	0.049	0.020
Cumulative prop.	0.648	0.817	0.875	0.924	0.944
Panel B: Slope factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	0.424	0.303	0.082	0.049	0.027
Variance prop.	0.453	0.325	0.088	0.053	0.029
Cumulative prop.	0.453	0.778	0.866	0.919	0.948
Panel C: Curvature factors					
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	3.102	0.600	0.472	0.272	0.200
Variance prop.	0.620	0.112	0.094	0.054	0.040
Cumulative prop.	0.620	0.732	0.826	0.880	0.920

A.3 PCA Curvature Factors - Levels 2014-2020

Table XV
Principal Component Analysis - Levels
2014-2020

This table reports results of principal components analysis for the normalized Nelson-Siegel curvature factors of the ten countries mentioned in Table I using monthly yield data from 2014-01 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components.

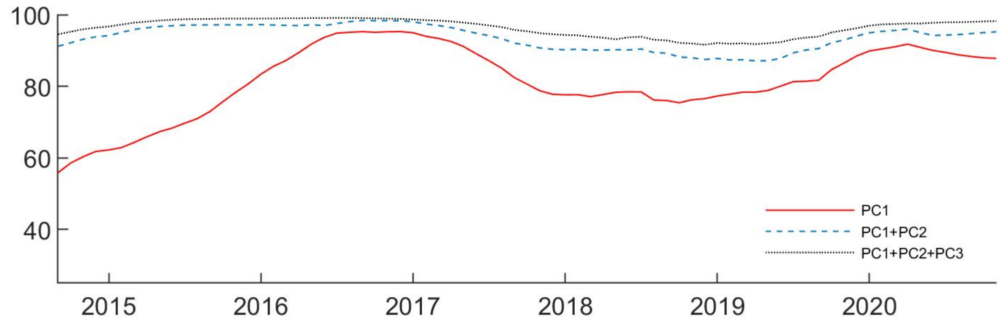
	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	6.821	1.849	0.498	0.326	0.235
Variance prop.	0.682	0.185	0.050	0.033	0.024
Cumulative prop.	0.682	0.867	0.917	0.950	0.974

Table XVI
Curvature Factor - Loadings
2014-2020

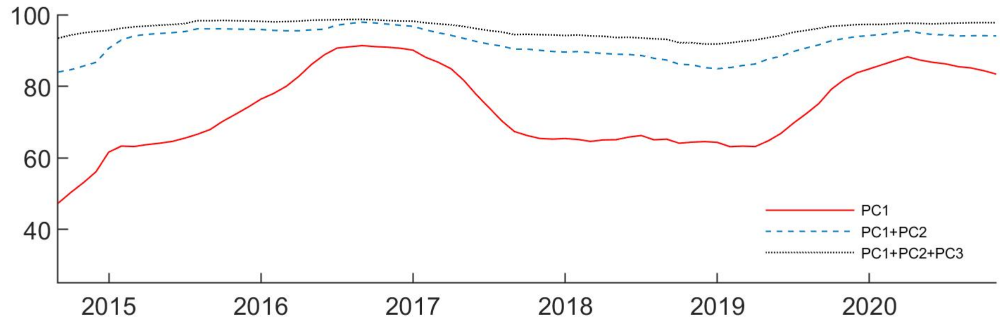
This table reports the principal component loading vectors of the normalized Nelson-Siegel country curvature factors. The level factors are obtained using monthly yield data from 2014-01 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.354	−0.224	0.182	−0.179	−0.134
Netherlands	0.338	−0.317	0.121	0.010	−0.002
Belgium	0.353	−0.205	0.251	−0.169	−0.165
France	0.372	−0.066	0.123	−0.060	−0.267
Italy	0.309	−0.179	−0.519	0.649	−0.107
Spain	0.332	−0.249	−0.275	−0.029	0.483
United Kingdom	0.310	0.276	−0.331	−0.593	−0.024
United States	0.241	0.511	0.117	0.271	−0.530
Canada	0.237	0.526	−0.308	−0.060	0.243
Japan	0.286	0.308	0.558	0.292	0.547

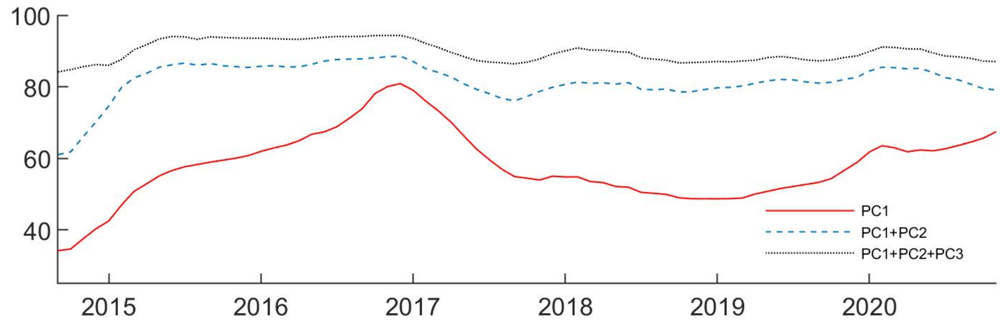
A.4 Moving Window PCA



(a) Level factor

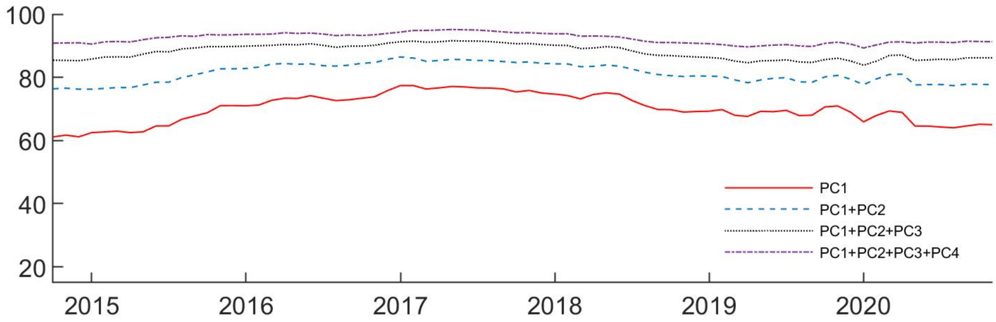


(b) Slope factor

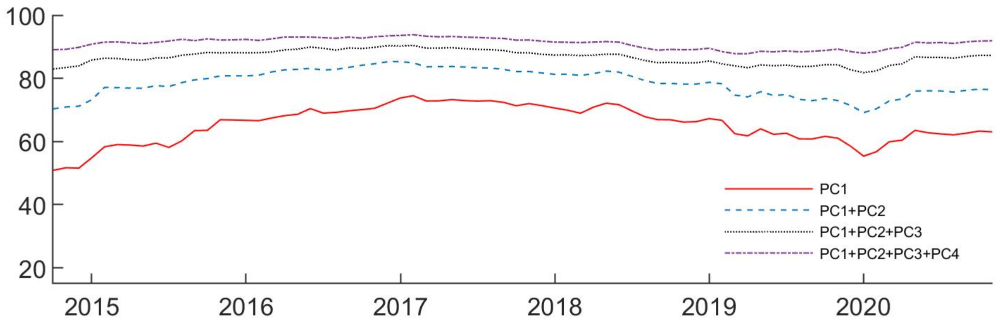


(c) Curvature factor

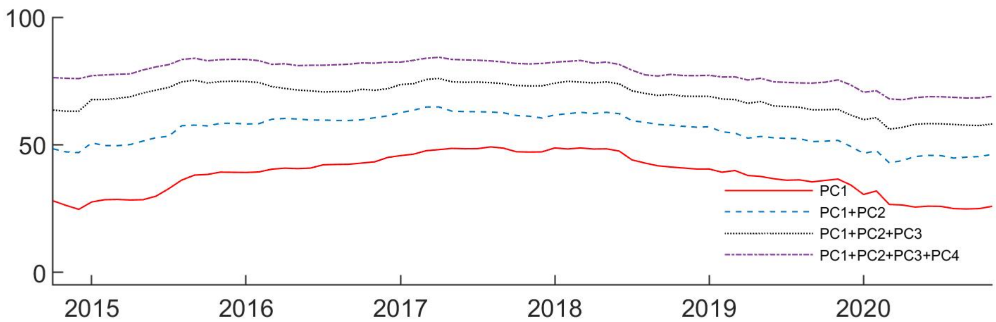
Figure 14: 3-year moving window of total variance explained by the first, first and second, and first second and third principal component for the normalized level factor (a), slope factor (b), and curvature factor (c).



(a) Level factor



(b) Slope factor



(c) Curvature factor

Figure 15: 3-year moving window of total variance explained by the first, first and second, and first second and third principal component for the normalized first differences of the level factor (a), the slope factor (b), and the curvature factor (c).

A.5 PCA Level Factors - Changes 2014-2020

Table XVII
Level Factor - Loadings
2014-2020

This table reports the principal component loading vectors of the normalized first differences of the Nelson-Siegel country level factors. The level factors are obtained using monthly yield data from 2014-01 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.355	−0.050	0.061	−0.322	−0.156
Netherlands	0.350	0.021	0.229	−0.267	−0.164
Belgium	0.349	0.156	0.026	−0.339	−0.192
France	0.358	0.109	0.130	−0.266	0.080
Italy	0.270	0.507	−0.220	0.469	−0.067
Spain	0.277	0.534	−0.138	0.179	0.205
United Kingdom	0.316	−0.263	−0.253	−0.126	0.736
United States	0.304	−0.448	−0.207	0.324	0.102
Canada	0.302	−0.346	−0.396	0.202	−0.551
Japan	0.263	−0.170	0.772	0.476	0.058

A.6 PCA Slope Factors - Changes 2014-2020

Table XVIII
Principal Component Analysis - Changes
2014-2020

This table reports results of principal components analysis for the normalized first differences of the Nelson-Siegel slope factors of the ten countries mentioned in Table I using monthly yield data from 2014-01 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	6.592	1.206	0.776	0.433	0.308
Variance prop.	0.659	0.121	0.078	0.043	0.031
Cumulative prop.	0.659	0.780	0.858	0.901	0.932

Table XIX
Slope Factor - Loadings
2014-2020

This table reports the principal component loading vectors of the normalized first differences of the Nelson-Siegel country slope factors. The slope factors are obtained using monthly yield data from 2014-01 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.354	−0.036	−0.354	0.102	0.009
Netherlands	0.348	0.014	−0.236	0.317	−0.173
Belgium	0.348	0.189	−0.300	0.057	0.163
France	0.361	0.127	−0.237	0.170	−0.047
Italy	0.237	0.545	0.461	−0.291	−0.098
Spain	0.286	0.483	0.265	0.081	0.181
United Kingdom	0.330	−0.176	−0.133	−0.528	−0.430
United States	0.309	−0.377	0.182	−0.436	−0.031
Canada	0.306	−0.353	0.192	−0.043	0.762
Japan	0.258	−0.343	0.550	0.543	−0.364

A.7 PCA Curvature Factors - Changes 2014-2020

Table XX
Principal Component Analysis - Changes
2014-2020

This table reports results of principal components analysis for the normalized first differences of the Nelson-Siegel curvature factors of the ten countries mentioned in Table I using monthly yield data from 2014-01 through 2020-10. It shows the eigenvalues, variance proportions and cumulative variance proportions of the first five principal components.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Eigenvalue	3.850	1.525	1.053	0.999	0.789
Variance prop.	0.385	0.153	0.105	0.100	0.079
Cumulative prop.	0.385	0.538	0.643	0.743	0.822

Table XXI
Curvature Factor - Loadings
2014-2020

This table reports the principal component loading vectors of the normalized first differences of the Nelson-Siegel country curvature factors. The curvature factors are obtained using monthly yield data from 2014-01 through 2020-10.

	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$
Germany	0.354	−0.036	−0.354	0.102	0.009
Netherlands	0.348	0.014	−0.236	0.317	−0.173
Belgium	0.348	0.189	−0.300	0.057	0.163
France	0.361	0.127	−0.237	0.170	−0.047
Italy	0.237	0.545	0.461	−0.291	−0.098
Spain	0.286	0.483	0.265	0.081	0.181
United Kingdom	0.330	−0.176	−0.133	−0.528	−0.430
United States	0.309	−0.377	0.182	−0.436	−0.031
Canada	0.306	−0.353	0.192	−0.043	0.762
Japan	0.258	−0.343	0.550	0.543	−0.364