ERASMUS UNIVERSITY ROTTERDAM
Erasmus School of Economics

Hawkes process approach to estimating *CoVaR*[1]

Master Thesis Quantitative Finance

D.W. van Klink (454140)

Supervisor:
Dr. A. Tetereva

Coreader:
Prof. Dr. C. Zhou

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Abstract

This paper investigates if the estimation of the systemic risk measures $CoVaR$ and $\Delta CoVaR$ proposed by Adrian and Brunnermeier (2011) can be improved using a multivariate Hawkes process instead of quantile regressions. The Hawkes process captures the contagion of negative shocks between the institutions with the ETAS model by Ogata (1988). The models are compared in a simulation study and by analysing historical data from 15 large financial institutions from the US in the period 2000 to 2010. To see which of the two methods is best at describing systemic risk, the two models are used to create an investment strategy where the stock with the lowest $\Delta CoVaR$ is selected. This paper finds that the Hawkes process cannot improve the quantile regressions as the quantile regressions give more reliable and accurate results. Therefore the quantile regressions result in a strategy that leads to returns with less risk than the Hawkes process-based investment strategy.

[1] The content of this thesis is the sole responsibility of the author and does not reflect the view of the supervisor, second assessor, Erasmus School of Economics or Erasmus University.

Contents

1 Introduction

During the financial crisis in 2008, it was observed that negative shocks in one financial institution could cause negative shocks in other financial institutions. These secondary shocks also harm the financial system. Which means a large negative shock can lead to a clustering of adverse events in different financial institutions. The consequences of these clusters of negative events resulted in the default of Lehman-Brothers and eventually, the financial crisis in 2008. Since then, the analysis of the financial system as a whole has become an important subject for researchers. Especially the topic regarding the contagion of negative events from one financial institution to another, and the additional risk these institutions undergo because of these spillovers of risk. Correctly modelling these spillovers of risk can prevent future collapses of financial institutions like Lehman-Brothers and can even prevent future economic crises.

Therefore it is critical we can model the relations between institutions in the financial system as accurately as possible, where the goal is to prevent the financial system from cascading into a recession during financial turmoil. Value at Risk (VaR) is currently an important measure in determining the companies and institutions' risks. The VaR is a risk measure which estimates how much a set of assets might lose with a given probability in a set period. An overview of the VaR risk measure is given in Jorion (2000) or Kupiec (2002). To model the financial system as accurately as possible, we need to look past the VaR of individual institutions. Individual risk measured by VaR may give a wrong impression of the state of the economy and the financial system's health. Because VaR as a risk measure does not explicitly account for the clustering of negative events. It can be implicitly accounted for through the model with which it is estimated. However, from VaR, it is not directly clear which events are caused by other negative shocks and which events are not. Therefore using it as the primary risk measure could result in the failing of important institutions, even though the risk the assets of the institutions undergo measured by their individual VaR was fine.

The VaR, which is typically the traditional workhorse of risk managers, needs to be reconsidered for measuring systemic risk. Instead, we turn to the Conditional Value at Risk ($CoVaR$) as defined by Adrian and Brunnermeier (2011). $CoVaR$ is defined as the VaR of an institution i given some event in another institution j. This risk measure can be used to measure how much the VaR of a financial institution or system changes given that another institution is in distress. In this paper, the focus lies on the influence of a bad event happening in one institution, for example, the losses reaching the 90% VaR, on the whole financial system. So, in this case, the event conditioned on will be the 90% VaR of an institution j. The other institution i will be the collection of all institutions considered. This collection of institutions will represent the financial system. We then can use this measure to compare the $CoVaR$ in two situations. The first is the $CoVaR$ conditional on an institution reaching the VaR_q for a set quantile q. In the second situation, the $CoVaR$ is conditional on the same institution in its median state. This difference in $CoVaR$ is measured by $\Delta CoVaR$. Adrian and Brunnermeier (2011) introduce the $\Delta CoVaR$ as a systemic risk measure, to capture externalities and fundamental comovement. It is a proposed quantification for the relation of contagion and spillovers. It is therefore related to other works on contagion and spillovers between financial institutes. Adrian and Brunnermeier (2011) focus mainly on the $\Delta CoVaR$ of the financial system in its entirety, given that one institution is in distress. This $\Delta CoVaR$ is interesting because it can function as an indicator of what financial institutions have the largest impact on the financial system as a whole when they fail. Thus identifying the institutions which have priority to get financial aid from a central bank or government. Adrian and Brunnermeier (2011) make use of quantile regressions to estimate the $CoVaR$. This method is relatively simple to implement but requires additional information in the form of state variables. This paper tries to model the contagion of negative shocks more accurately without including additional data in the model.

Constructing a more accurate model is where Hawkes processes can be of use. A Hawkes process is a counting process for events which depend on their past events. In this case, the event would be an exceedance of the VaR by an institution, which can lead to another institution reaching its VaR. Hawkes processes have a wide variety of applications in many different fields. They are mainly used to model contagion and earthquakes, but can also be used to model relations in a financial setting. Bacry, Mastromatteo, and Muzy (2015) give an overview of papers that apply Hawkes processes in the field of finance. In this thesis, the goal is to model systemic risk. Other papers can be found that implement Hawkes processes in the same field. For example, Errais, Giesecke, and Goldberg (2010) model credit default events as a correlated point process. They consider the dynamics of these default events to be described by a marked Hawkes process with exponential kernels. They are able to capture major defaults during the financial crisis in 2008 using this model. Another paper on a similar topic by Dassios and Zhao (2011) use a Dynamic Contagion Process to model default risk. This process is an exponential marked Hawkes process where the Poisson exogenous events known as immigrants are replaced by shot-noise, which is also modelled with a Hawkes process using a different intensity function. This model also proves to be suitable for modelling arriving default events. Aït-Sahalia, Cacho-Diaz, and Laeven (2015) model jumps that occur over a diffusion process using a Hawkes process. They model contagion between world markets using this process called a Mutually exciting Jump-Diffusion by the authors.

This paper aims to improve the modelling of $\Delta CoVaR$ by introducing Hawkes processes into the model to capture the contagion of negative shocks between institutions. In the literature, Hawkes processes have already been used to model VaR. For example, in Chavez-Demoulin, Davison, and McNeil (2005) and Chavez-Demoulin and McGill (2012) both use a Hawkes process to model the VaR. This paper will extend the methodology of Chavez-Demoulin et al. (2005) to estimate $CoVaR$ in a multivariate setting. The goal is to create a model that can accurately predict $\Delta CoVaR$ like the quantile regressions Adrian and Brunnermeier (2011) use but without the additional information of state variables. The models are evaluated by looking at the returns of an investment strategy where the investor is investing in the stock with the lowest $\Delta CoVaR$. This is an idea based on Rockafellar, Uryasev, et al. (2000) who evaluate estimated $\Delta CoVaR$ by constructing portfolios that minimise this measure. Constructing such a portfolio for the non-linear Hawkes process is a complicated optimisation problem. The strategy as described earlier is a simpler alternative which does not affect the comparison of the models. To my knowledge, $CoVaR$ and $\Delta CoVaR$ have not been modelled with a Hawkes process in the literature before. In fact, non-linear modelling of $CoVaR$ seems hardly explored at all.

Chavez-Demoulin et al. (2005) use an Epidemic Type Aftershock Sequence (ETAS) model to model the contagion of negative shocks. This functional form for Hawkes processes is introduced by Ogata (1988) and is mainly used to model earthquakes in papers such as Helmstetter and Sornette (n.d., 2002, 2003). The rate of occurrence of events caused by a single extreme event decays in time in this model. Chavez-Demoulin et al. (2005) use Maximum Likelihood Estimation (MLE) to estimate this model's parameters. Similarly, Chavez-Demoulin and McGill (2012) calculate intraday-VaR, but they use it on high-frequency log-returns data. They conclude that the model is capable of capturing volatility clustering in the high-frequency data. Computing the MLE of Hawkes processes can have extremely long computation times when the dimension increases. Veen and Schoenberg (2008) develop an alternative to MLE in the form of an Expectation-Maximisation (EM) algorithm to estimate ETAS models. The algorithm is based on a similar algorithm used in Ogata and Akaike (1982), where the Akaike information criterion is used as a measure of convergence. Other works get around problems regarding MLE in other ways. Examples are the simplex algorithm as well as genetic algorithms. Ogata, Matsu'ura, and Katsura (1993) present a 'fast' algorithm that can be applied to Markovian conditional intensity functions. In this study, we work around the problem of large dimensions by constructing a model with a maximum of two time series evaluated at a time. In this paper, daily return data are looked at to investigate the contagion of negative shocks. First, a simulated setting is studied and then historical data from large American financial institutions are analysed. The simulation study's goal is to find out if the methods in this paper improve upon the quantile regressions. The study on real data gives an example of how these measures can be used to analyse a data set.

This paper finds that the multivariate Hawkes process's estimation using MLE proves difficult due to this method getting stuck in local optima resulting in lengthy optimisations. To avoid long computation time calculations have to be cut short with the resulting parameters being sub-optimal. Compared to the quantile regressions, the method is unreliable, resulting in values of $\Delta CoVaR$ that differ from the quantile regression estimates. The results for the $CoVaR$ of the institutions conditional on the system are especially inaccurate. Minimum $\Delta CoVaR$ strategies based on both the Hawkes process and the quantile regressions perform both decently indicating there is some truth to the $CoVaR$ estimated by the Hawkes process. In the simulation study, the quantile regressions come out superior in this application against the Hawkes process's simplified version. From the historical data analysis, we also find that the two models' results contradict each other. The quantile regressions find that in general institutions that are not banks experience more risk during periods where the system is at risk than institutions that are banks. The quantile regressions are able to identify certain stocks that have a larger impact on the state of the system than others.

The rest of this paper is structured as follows: in section 2, the methodology is explained, including details on the likelihood function, ETAS model, estimation procedure, calculation of $CoVaR$ and how the model is evaluated. In section 3, a description of the data used and the transformations performed are given. In section 4 the parameters estimated with simulated data are given, and afterwards, in section 5, historical data is used to estimate the parameters. In section 6 our findings in this study will be summarised, and finally, in section 7, the limitations to this research and suggestions for further research will be discussed.

2 Methodology

This section describes how the $CoVaR$ and $\Delta CoVaR$ measures are calculated from the data using both a Hawkes process and quantile regressions. Furthermore, it explains what measures are used for the comparison of the two models.

2.1 Peaks over threshold

Hawkes processes are mostly used to describe the chance of some extreme event taking place. In order to adapt the model to the return data, it needs to be transformed first. To do this, we make use of Peaks Over Threshold (POT). The extreme events that need to be generated are the returns that are lower than a certain low threshold. For example, the 90% VaR of the losses is such a threshold. We consider a data set of losses with $T \times N$ random variables, one for every time point considered in N different time series: $X_{1,i} \dots X_{T,i}$ from a distribution $f_{X_i}(x_i)$ $\forall i = 1 \dots N$. Here T is the number of observations or time points in the data, and N is the number of different time series considered. In order to determine which losses are extreme, a certain high threshold u_i is chosen. In this paper, the thresholds u_i are the percentiles representing the VaR of the different time series i, which is denoted as VaR_t^i. Let $M_i(t)$ denote the number of times a time series i has exceeded this threshold u_i at time t. If the time series exceeds the threshold, the data point is called a mark. These marks occur at discrete times $t \in T$ such that we can define marks as: $w_{t,i} = x_{t,i} - u_i$ if $x_{t,i} > u_i$ and $w_{t,i} = 0$ if $x_{t,i} \leq u_i$. This results in the data $w_{1,i} \dots w_{T,i} \in M_i(T)$ $\forall i = 1 \dots N$. If we collect all marks in all time series up to time T; $\tilde{M} = \sum_{i=1}^{N} M_i(T)$ and order them by time of occurrence such that we have marks $1, \dots, m, \dots, \tilde{M}$ then the time t at which mark number m takes place is denoted as t_m. H_t denotes the entire history of the process up to time t.

2.2 Multivariate Hawkes process & likelihood function

A Hawkes process models the chance of an event happening with an intensity function. The chance of an event happening for time series i at time t is denoted as the intensity $\lambda_i(t)$. The most general formula for the intensity of a multivariate Hawkes process as described in Embrechts, Liniger, and Lin (2011) is the following:

$$\lambda_i(t) = \eta_i + \sum_{j=1}^{N} \nu_{i,j} \int_{\mathbb{R}} \int_{-\infty}^{t} \omega_i(t-s) g_j(w) M_j(s) \mathrm{d}s \mathrm{d}w, \quad t \in \mathbb{R} \tag{2.1}$$

In this formula, η_i denotes the background intensity or immigration intensity of time series i. It is the chance of the arrival of a new event. Furthermore, ω_i denotes the decay function. It determines the decrease of the intensity over time. The intensity is also dependent on g_j, which is the impact function. It determines the impact of the size of the marks w in time series j on the intensity of time series i. Often in the notation of ETAS models, the decay function and impact function are written as one function. The branching component $\nu_{i,j}$ determines how much the interaction between two time series i and j influences the intensity of time series i. In this general setting, the likelihood over an interval $[T_*, T^*]$ becomes as seen in the following function:

$$\log L = \sum_{i=1}^{N} \int_{\mathbb{R}} \int_{T_*}^{T^*} \log\lambda_i(t) M_i(t) \mathrm{d}t\mathrm{d}w + \sum_{i=1}^{N} \int_{\mathbb{R}} \int_{T_*}^{T^*} \log f_i(w) M_i(t) \mathrm{d}t\mathrm{d}w - \sum_{i=1}^{N} \Lambda_i(T^*) \tag{2.2}$$

where $f_i(w)$ represents the probability density function of the marks w_i and $\Lambda_i(T^*)$ is the compensator function, which is defined as:

$$\Lambda_i(T^*) = \int_{T_*}^{T^*} \lambda_i(s)ds = \eta_i(T^* - T_*) + \sum_{j=1}^{N} \nu_{i,j} \int_{\mathbb{R}} \int_{T_*}^{T^*} [\bar{\omega}_i(T^* - u) - \bar{\omega}_i(T_* - u)] g_j(w) M_j(u) \mathrm{d}u\mathrm{d}w \tag{2.3}$$

in which $\bar{\omega}$ is defined as:

$$\bar{\omega}_i(t) = \int_0^t \omega_i(s)\mathrm{d}s \; for \; i \in 1, \ldots, N \tag{2.4}$$

where $\omega_i(t)$ is the decay function that describes the decay of the intensity over time. In the framework to calculate conditional *VaR* provided by Chavez-Demoulin et al. (2005), they use the ETAS model introduced in Ogata (1988). In these papers, g and ω are denoted as one single function. The following function describes it:

$$g(t - t_m; w) = \frac{\psi e^{\beta w}}{(t - t_m + \gamma)^{\rho - 1}} \tag{2.5}$$

With parameters $\beta, \gamma, \mu, \rho, \psi > 0$ to ensure that the function is monotonic decreasing and the intensity depends not only on the time but also on the size of past events. This model by Ogata (1988) is primarily used for the modelling of earthquakes. It is constructed such that the rate of events triggered by one main event (the main shock) decays in time according to Omori law: $l(t) = \psi/(t + \gamma)^{\rho+1}$ and triggers after-events (aftershocks).

In this paper, the ω and g functions are separated to not lose their interpretation as decay and impact functions.

$$\omega(t) = \frac{\psi\gamma}{(1 + \gamma t)^{1+\rho}} \tag{2.6}$$

The impact functions $g_j(w)$ describe to which extent the size of a previous mark impacts the next mark. In the ETAS model it is of the following form:

$$g_j(w) = \exp(\beta_j w) \tag{2.7}$$

The likelihood also needs as input a density for the marks. The values generated by the POT model are values of a tail distribution and therefore are often well described by some form of the General Pareto Distribution (GPD). The size of the marks follow the following distribution:

$$F_i(w_{1,i} \ldots w_{T,i}, \xi, \sigma) = \begin{cases} 1 - (1 - \xi w/\sigma)^{1/\xi}, & \xi \neq 0, \quad \sigma > 0 \\ 1 - \exp(-w/\sigma), & \xi = 0, \quad \sigma > 0 \end{cases} \tag{2.8}$$

If $\xi = 0$ the distribution is a special case of the GPD: the exponential distribution.

Direct optimisation of the multivariate Hawkes process using the Maximum Likelihood Estimator as described before can be done. However, it is a very complicated and computationally heavy task because of its many parameters. Therefore it is often more manageable to rewrite the intensity function and make the computations differently. This alternative notation is described by Bacry et al. (2015), who introduce the kernels $\Phi(t)$:

$$\lambda_t^i = \eta_i + \sum_{j=1}^{N} \int_{\mathbb{R}} \mathrm{d}M_j(s)\phi_{i,j}(t-s) \tag{2.9}$$

Where η^i is still the background intensity of i, $M_j(s)$ is the number of marks up to time s and $\Phi(t)$ is a matrix-valued kernel. The following holds for the kernels:

$$\begin{aligned} \phi_{i,j}(t) \geq 0 \qquad & \forall\ 1 \leq i, j \leq N \\ t < 0 \implies & \Phi(t) = 0 \end{aligned} \tag{2.10}$$

Each component $\phi_{i,j}(t)$ has to belong to the space of L^1-integrable functions. If we introduce event times notation, that is the couples t_m, j_m are introduced, where t_m represents the time that mark number m takes place, and j_m represents the time series of origin of the mark. Such that $j_m \in [1, \ldots, N]$ can be rewritten to represent the kernels in equation (2.9) as follows:

$$\Phi(t) = \{\phi_{i,j}(t, m)\}_{i,j=1}^{N} \tag{2.11}$$

Rewriting leads to a different notation of the intensity function of the Hawkes process λ_t^i known as the event time notation:

$$\lambda_t^i = \eta^i + \sum_{m=1}^{\tilde{M}} \phi_{i,j_m}(t - t_m, m) \tag{2.12}$$

where the kernels $\phi_{i,j_m}(t - t_m, m)$ are described by the ω and g functions of the ETAS model:

$$\phi_{i,j_m}(t - t_m, m) = \phi_{i,j}(t - t_m) = \frac{\psi\gamma}{(1 + \gamma(t - t_m))^{(1+\rho)}} \exp\left(\beta_{i,j_m} m\right) \tag{2.13}$$

with ψ, γ, ρ and $\beta_{i,j} > 0$. These parameters need to satisfy the following conditions in order to ensure the stability of the process:

1. The spectral norm of the kernels need to be smaller than one: $\parallel \Phi \parallel < 1$.
2. The ψ parameter needs to be smaller than the gamma parameter: $\psi < \gamma$

The optimisation is again done by using the MLE. Where we need to find $\theta = (\psi, \gamma, \rho, \beta)$ the collection of all parameters in the model such that the likelihood function achieves the highest possible value within the constraints set above. This new likelihood function is as follows:

$$\log L(\eta, \Phi_\theta) = -\sum_{i=1}^{N} \int_0^T \lambda_t^i \mathrm{d}t + \sum_{m=1}^{\tilde{M}} \lambda_{t_m}^{j_m} \tag{2.14}$$

The problem of interest is defined as follows:

$$(\eta^*, \theta^*) = \mathrm{argmax}_{(\eta,\theta)} \log L(\eta, \Phi_\theta) \tag{2.15}$$

Estimating the MLE in this manner is still a computationally heavy task, but less so than before. This can be averted by using an algorithm to estimate the parameters. A possible algorithm is the EM algorithm, which is described in detail in section 8. The following steps summarise the EM Algorithm:

Step 0 (Initialization). n=1; set $\theta_{EM}^{(n)}$ to be strictly positive starting values.

Step 1 (E step). Based on $\theta_{EM}^{(n)}$ estimate probabilities $P^{(n+1)}(UQ_s = r)$ for all s, r as is shown in equation (8.3).

Step 2 (M step). Maximise equation (8.4) and find $\theta_{EM}^{(n+1)} = \text{argmax}_\theta E_{\hat{\theta}_{EM}^{(n)}}[l_c(\theta)]$

a.) Find $\hat{\eta}_{EM}^{(n+1)}$ using $\hat{\nu}_{k,EM}^{(n+1)} = \hat{N}_j^{(n+1)}$.

b.)Find $\hat{\gamma}_{EM}^{(n+1)}$ and $\hat{\rho}_{EM}^{(n+1)}$ by optimising equation (8.12) and equation (8.13) as described previously.

c.) Fix all parameters in θ to their current estimates in step $(n+1)$ except for ψ and β. Solve for these parameters using equation (8.8) and equation (8.9).

Step 3 (Convergence). If $\Delta\hat{\theta}_{EM}^{(n+1)} = \hat{\theta}_{EM}^{(n+1)} - \hat{\theta}_{EM}^{(n)}$ is smaller than some convergence criterion the algorithm is stopped. If the convergence criterion has not been reached, n is set to $n+1$ and steps 1-3 of the algorithm are repeated.

Although the steps of the algorithm are relatively straight forward, the probabilities that need to be calculated in step 1 are numerous. Although the estimation of individual probabilities does not necessarily lead to large errors, the accumulation of these small errors can mean that the estimated parameters do not result in an accurate or stable Hawkes process. The reason this alternative was considered was to reduce computation time. However, the amount of probabilities that need to be estimated is so large that it defeats this purpose.

Because of computation time the way the companies are analysed needs to be changed. Next to the data on stock returns, a new time series is created, which represents the whole financial system. The relation between the returns of each stock and the system is analysed individually with the multivariate Hawkes process. This way, only two time series need to be evaluated at a time, reducing the estimation procedure's dimensionality.

2.3 Conditional Value at Risk

The goal of using the Hawkes processes is to be able to more accurately model the Conditional Value at Risk ($CoVaR$) as described in Adrian and Brunnermeier (2011). Once we have estimated this model's parameters as accurately as possible, we calculate the VaR and $CoVaR$. The VaR for institution i at time t is defined as the quantile q for the losses $x_{t,i}$ and is denoted by $VaR_t^i(q)$. Its definition is as follows:

$$P\Big(x_{t,i} \leq VaR_t^i(q)\Big) = q \tag{2.16}$$

The $CoVaR$ for institution i given some event in institution j took place is defined as the quantile q in the following conditional probability distribution:

$$P\Big(x_{t,i} \leq CoVaR_t^{i|x_{t,j}}(q)|x_{t,j}\Big) = q \tag{2.17}$$

In this case, the $CoVaR$ is the probability that the VaR of company i is exceeded given that the return of company j is at its VaR level. We are looking for the predicted $CoVaR$ in $t+1$. It can be calculated by multiplying the following two probabilities:

$$\begin{aligned} P\Big(x_{t+1,i} \leq CoVaR_{t+1}^{i|x_{t,j}}|H_t; x_{t,j}\Big) =& P\Big(x_{t+1,i} - VaR_t^i \leq CoVaR_{t+1}^{i|x_{t,j}} - VaR_t^i|CoVaR_{t+1}^{i|x_{t,j}} \geq VaR_t^i; H_t; x_{t,j}\Big) \\ &\times P\Big(CoVaR_{t+1}^{i|x_{t,j}} \geq VaR_t^i|H_t\Big) \end{aligned} \tag{2.18}$$

The second term in this probability can be estimated by approximating the conditional probability that an event takes place in $(t, t+1]$. Once the parameters have been estimated, two intensities are calculated by replacing the institution of interest's marks by the VaR and the institution of interest's median value. These intensities lead to the second probability in equation (2.18):

$$1 - P\Big(N(t, t+1) = 0|H_t\Big) = 1 - \exp\Big(-\int_t^{t+1} \lambda_H(u)du\Big) \tag{2.19}$$

The first term in the probability of equation (2.18) is the probability that a mark exceeds the VaR, which can be estimated by drawing from a GPD distribution with the estimated parameters ξ and σ as defined in section 2.2.

$$CoVaR_{t+1}^{i|x_{t,j}} - VaR_t^i|H_t; CoVaR_{t+1}^{i|x_{t,j}} \geq VaR_t^i \sim GPD_{\xi,\sigma} \tag{2.20}$$

These parameters are estimated by taking the MLE over the marks of the previous year.

$\Delta CoVaR$ of institution i given institution j is defined by Adrian and Brunnermeier (2011) as the difference between the $CoVaR$ of institution i when institution j has reached the VaR for quantile q and the $CoVaR$ of institution i when institution j has a median value. This results in two time series of $CoVaR$ one conditional on the VaR: $CoVaR|VaR$ and one conditional on the median $CoVaR|Med$. Now we can calculate the $\Delta CoVaR$ for every institution and every time t by subtracting the $CoVaR$ conditional on the median values from the $CoVaR$ conditional on the VaR as in equation (2.21).

$$\Delta CoVaR_t^{i|j}(q) = CoVaR_t^{i|x_{t,j}=VaR_t^j(q)} - CoVaR_t^{i|x^j=\text{Median}_t^j} \tag{2.21}$$

2.4 Quantile Regression

The study by Adrian and Brunnermeier (2011) uses quantile regressions to estimate the $CoVaR$. The main advantage of quantile regressions to normal Ordinary Least Squares (OLS) regressions is that no distributional assumptions need to be made for the residuals ϵ_t. This is because estimation of the conditional mean and conditional volatility is incorporated in the quantile regressions. To estimate the $CoVaR$ first two regressions need to be done. The first regression in equation (2.22a) finds the relation between certain state variables SV_t and the 90% VaR of institution i. The state variables need to explain the time variation of the returns of the stocks well. The second regression in equation (2.22b) is to find the relation between the 90% *VaR* of the system and the 90% *VaR* of institution i and the state variables.

$$VaR_t^i(0.90) = \alpha^i + SV_{t-1}\gamma^i + \epsilon_t^i \tag{2.22a}$$

$$VaR_t^{system}(0.90) = \alpha^{system|i} + VaR_t^i(0.90)\beta^{system|i} + SV_{t-1}\gamma^{system|i} + \epsilon_t^{system|i} \tag{2.22b}$$

Finding the *CoVaR*(q) means constructing the conditional inverse cumulative distribution function of the returns $X_{t,System}$. This is the conditional quantile function shown in equation (2.23). The conditional quantile function for the *VaR* equals the *CoVaR* of the system.

$$CoVaR_t^{i|x_{t,j}=VaR_t^j(q)} = F_{X_{t,i}}^{-1}(q|SV_{t-1}, VaR_t^j(q)) = \alpha_q + SV_{t-1}\beta_q + VaR_t^j(q)\gamma_q \tag{2.23}$$

To find the optimal values for the parameters in equation (2.23) we need to solve the following minimisation problems: first, the optimisation problem in equation (2.24) is solved to estimate the parameters in equation (2.22a).

$$\min_{\alpha_q,\gamma_q} \sum_t^T \begin{cases} q|VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q| & \textit{if } (VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q) \geq 0 \\ (1-q)|VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q| & \textit{if } (VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q) < 0 \end{cases} \tag{2.24}$$

The optimisation problem in equation (2.25) is solved to estimate the parameters in equation (2.22b).

$$\min_{\alpha_q,\beta_q,\gamma_q} \sum_t^T \begin{cases} q|VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q - VaR_t^j(q)\beta_q| \\ \qquad \textit{if } (VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q - VaR_t^j(q)\beta_q) \geq 0 \\ (1-q)|VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q - VaR_t^j(q)\beta_q| \\ \qquad \textit{if } (VaR_t^i(q) - \alpha_q - SV_{t-1}\gamma_q - VaR_t^j(q)\beta_q) < 0 \end{cases} \tag{2.25}$$

Once the parameters have been estimated for every time point t, the 90% *VaR* of the institutions is calculated from the state variables as in equation (2.26a). Now, these values for the 90% *VaR* of the institutions are estimated, the *CoVaR* of the system can be calculated as in equation (2.26b).

$$\hat{VaR}_t^i(q) = \hat{\alpha}_q^i + SV_{t-1}\hat{\gamma}_q^i \tag{2.26a}$$

$$\hat{CoVaR}_t^{system}(q) = \hat{\alpha}_q^{system|i} + \hat{VaR}_t^i(q)\hat{\beta}_q^{system|i} + SV_{t-1}\hat{\gamma}_q^{system|i} \tag{2.26b}$$

For the *CoVaR* conditional on the median, the 90% *VaR* of the institutions is replaced by the institution's median. The same methods can be used to calculate the *CoVaR* of the institutions conditional on the systems by switching the dependent variable to be the *VaR* of the institutions and the explanatory variable the *VaR* of the system.

2.5 Model evaluation

As the *CoVaR* and Δ*CoVaR* measures are not observable, we cannot compare the model's results to true values of *CoVaR* and Δ*CoVaR*. The aim is to improve the methods of Adrian and Brunnermeier (2011) without using the information of state variables. To compare the two models, the summary statistics of the estimated time series of both $\Delta CoVaR$ and $CoVaR$ are reported for each model. The mean and standard deviations of the measures over the period considered are reported in tables in section 4 and section 5. Adrian and Brunnermeier (2011) compute covariances between financial institutions as a proxy of a realised risk measure. The $\Delta CoVaR$ is a measure that indicates the difference between the median state and a certain percentile. In a way, this is similar to the variance as the variance measures the average distance from the mean. As $\Delta CoVaR$ is intended as an indication of the systemic risk, it may be possible to create an investment strategy where the systemic risk of the stock invested in is minimal. It could then be compared to a similar strategy which selects the stock with the least variance, as the variance is also a measure which is supposed to capture risk. The stock with the lowest variance is considered as the "true" value for the stock with the least possible risk. The returns of the two minimum $\Delta CoVaR$ strategies can now be compared to this "true" value to evaluate the models. The weights will be adjusted every 50 days with a rolling window which includes the previous 50 days' data. The stock with the lowest $\Delta CoVaR$ is determined by minimising the sum of the $\Delta CoVaR$ over all stocks:

$$\min \sum_{t=1}^{t+50} \sum_{i=1}^{N} w_i \cdot \Delta CoVaR_t^i \; s.t. \; \mathbf{w}'\iota = 1 \tag{2.27}$$

This optimisation results in a corner solution of the feasible region. Meaning one of the weights is one while all the others are zero, selecting the desired stock. Similarly, the minimum variance strategy is determined by selecting the stock with the lowest variance over these 50 days.

$$\min \sum_{i=1}^{N} w_i \cdot \sigma_{ii}^2 \; s.t. \; \mathbf{w}'\iota = 1 \tag{2.28}$$

Where σ_{ii} are the diagonal elements of the empirical covariance matrix over the 50 days for which the strategy is implemented. The returns of this strategy are calculated by multiplying the weights assigned to asset i times asset's return for every time t. The returns that result from this strategy using the two models are compared by taking the difference between the different strategies' daily returns. This results in three measures that show the relative performance of the strategies:

$$\Delta R1(t) = R_{Hawkes}(t) - R_{Quantile}(t) \tag{2.29a}$$

$$\Delta R2(t) = R_{Hawkes}(t) - R_{Min.var.}(t) \tag{2.29b}$$

$$\Delta R3(t) = R_{Quantile}(t) - R_{Min.var.}(t) \tag{2.29c}$$

The strategies are also compared to two benchmark portfolios: the GMV portfolio and the 1/N portfolio. The 1/N portfolio acts as a benchmark for the strategy's return as the returns of this simple portfolio are often hard to beat. The GMV portfolio acts as a benchmark for the strategy's variance as the GMV portfolio minimises this measure. Thus we consider this the true achievable minimum risk a portfolio can achieve. The weights of the GMV portfolio are calculated in the following way:

$$\min \mathbf{w}'\mathbf{\Sigma}\mathbf{w} \;\; s.t. \;\; \mathbf{w}'\iota = 1 \implies \mathbf{w} = \frac{\mathbf{\Sigma}^{-1}\iota}{\iota'\mathbf{\Sigma}^{-1}\iota} \tag{2.30}$$

Where Σ^{-1} is estimated by the inverse of the empirical covariance matrix, and ι is a $N \times 1$ vector of ones. The mean, standard deviation and Sharpe ratio of all portfolios and strategies are reported. The Sharpe ratio is calculated with the risk-free rate represented by the historical three-month Treasury bill rate. As the US government issues these bonds, the risk of default is so low, it is negligible and can therefore be considered risk-free.

3 Data

Adrian and Brunnermeier (2011) base their analysis of the $CoVaR$ on data of an impressive list of more than 1200 companies. For each company they calculate the total market value of all assets. Considering the time this amount of information would take to gather, retrieving this much data was not an option. Instead, this paper looks at stock returns on a smaller number of companies. As the stock prices and the firm's value are closely related. The return on the stock is a good indicator of how the market value of the assets changes. Therefore the data used in this paper consists of daily stock prices and volumes. The closing prices and volumes of the stocks are taken into the model. The returns are calculated from the closing prices as follows:

$$R_{t,i} = \frac{P_{t,i} - P_{t-1,i}}{P_{t-1,i}} \qquad \forall \;\; t > 1, \;\; i = 1 \ldots N \tag{3.1}$$

The companies included are some of the largest banks, investment banks and insurance companies in North America. There is also one Government Sponsored Enterprise (GSE). This collection of large American financial institutions has been chosen because they are publically traded companies. So the data on stock prices can easily be retrieved. Secondly, because they belong to the largest financial institutions globally, they likely have a large impact on the financial system. Given below is a list of these companies:

Banks and thrifts: Bank of America (BAC), Citigroup (C), JPMorgan Chase (JPM), Wells Fargo (WFC), PNC financial services (PNC), Toronto-dominion bank (TD), Truist financial corporation (TFC), US Bancorp (USB), HSBC US (HSBC), Northern Trust Corporation (NTRS)

Investment banks: Goldman Sachs (GS), Morgan Stanley (MS)

GSEs: Freddie Mac (FMCC)

Insurance companies: American International Group (AIG), Metlife (MET)

The weights used to calculate the return of the System portfolio are proportional to the institution's market value of equity. The market value of equity is calculated by multiplying the number of outstanding shares a company has with the closing prices of the shares on that day:

$$ME_{t,i} = Price_{t,i} \times Volume_{t,i} \tag{3.2}$$

And then the weights are calculated as follows:

$$W_{t,i} = \frac{ME_{t,i}}{\sum_{j=1}^{N} ME_{t,j}} \tag{3.3}$$

These daily stock prices and volumes can be found on Yahoo finance. The summary statistics of the ME for all stocks considered are displayed in table 1. An interesting period to analyse losses

Table 1: This table displays the mean and standard deviation of the daily market value of equity for the 15 stocks in millions of dollars ($\times 10^6$) from 2000 to 2010.

Rank	Institution	Mean	Std.Dev.	Rank	Institution	Mean	Std.Dev.
1	BAC	1,139	1.359	9	FMCC	180	179
2	C	1,131	1.063	10	MET	160	170
3	GS	940	1.060	11	PNC	136	148
4	JPM	806	776	12	TFC	95	106
5	WFC	605	745	13	NTRS	84	66
6	AIG	509	464	14	HSBC	67	75
7	MS	415	319	15	TD	26	4
8	USB	219	201		System	6,510	5,338

is the financial crisis in 2008. This recession goes hand in hand with large negative returns. Therefore the data will range from 2000 to 2010. This period is chosen because it contains an interesting period, the financial crisis, and has enough observations before this period as a burn-in period for the model. Finally, there are not too many observations which would result in very long computation times. There are around 250 trading days a year which makes the time series approximately 2750 observations long. The time series of the institutions' returns can be found in the appendix in figure 12 and figure 13. From these graphs, it is immediately apparent that many marks occur during the financial crisis. The quantile regression uses information from state variables to estimate the parameters in the model. Adrian and Brunnermeier (2011) base their model on a list of state variables. From this list the following variables are used in this study: first to get an indication of what the volatility is like in the market the Volatility IndeX (VIX) is used as a state variable. Secondly, a short-term liquidity spread defined as the difference between the three-month repo and three-month bill rates. It is an indicator of short-term liquidity risk. Third, the change in the three-month bill rate is incorporated as a state variable as this does well in explaining the tails of financial sector returns. Lastly, the change in credit spread which is the difference between BAA rated bonds and the ten-year treasury constant maturity rate as it gives a long term outlook.

4 Simulation study

In this section, the results of a simulation study are presented. Instead of using historical data to estimate the parameters, the data is generated randomly. The estimated parameters and the resulting $CoVaR$ using the methods as in section 2 are discussed.

For this simulation study, the returns of four randomly generated institutions are analysed using the model presented in section 2. The time series contain 750 observations of which the first 250 observations are used as a burn-in period to start the model. The last 500 observations are what the parameters are based on. This is done 100 times in total, to calculate the mean and standard deviation of the parameters. After the parameters have been estimated 100 times, we use these estimates' mean values combined with the historical data described in section 3 to calculate the $CoVaR$ and $\Delta CoVaR$ of the four institutions and the system conditional on the different institutions. The simulated data are generated by drawing errors from a copula t-distribution. The copula is constructed using the correlation matrix from the historical data. The correlation matrix is based on data from the following four banks: Bank of America (BAC), Citigroup (C), Goldman Sachs (GS) and JP Morgan Chase (JPM). The conditions that a correlation matrix needs to satisfy are the following: the diagonals are all one and the matrix's spectral radius is smaller than one. Creating the correlation matrix this way is done to recreate volatility clusters present in the historical data. A GARCH model is used to generate the volatility of the errors.

The returns need to be weighed by their market value of equity and summed to create the system's returns. In this study, two cases are inspected: first, the case is inspected where the system returns are based on weights proportional to the market value of equity and secondly the case where the system returns are based on equal weights. This is done in order to see if creating the system returns based on ME is necessary. Next, the parameters η and θ are estimated as in section 2.2 using the MLE of a Hawkes process with two time series. The first time series contains the marks of the system, while the second time series represents the institution's marks being evaluated and thus changes for every institution. The burn-in period of the model is one year, which contains 250 daily observations. This is done to estimate the VaR and GDP parameters of the previous year such that we have meaningful parameters to calculate values with at $t = 1$. Before the resulting $CoVaR$ is discussed. First, the parameters of the model where the weights are proportional to the ME are discussed. In table 2, the mean values for the parameters are shown with their standard errors based on 100 simulations. δ describes what impact the marks have on the size of the next mark. There are four different δ parameters. $\delta_{1,1}$ and $\delta_{2,2}$ represent the self-exciting part of the process, while $\delta_{1,2}$ and $\delta_{2,1}$ represent the cross-exciting process. The first time series is always the time series of marks of the system while the second time series is a time series of marks over the different institutions. η_1 and η_2 are the background intensity of the two time series. It is the probability that an event happens when there is no impact from any marks.

The parameters concerning the decrease of the intensity over time are ψ, γ and ρ. They do not have an individual interpretation, but larger values for γ and ρ indicate the intensity drops faster while larger values for ψ mean the intensity stays high longer. After a certain period where no events have taken place, the intensity will return to the level of the background intensity η.

From table 2, it can be seen that in general, the parameters seem to be quite similar for the four banks. GS has the highest value for ψ and the lowest value for ρ and an average value for γ, indicating that the intensity stays high longer than the other banks. This would imply that after a big loss, GS is vulnerable for longer than the other banks. On the other side is C, which has the highest values for γ and ρ and the lowest value for ψ. This implies that large losses only have a short period in which it increases chances of additional large losses and is thus vulnerable for a shorter time than the other banks. BAC and JPM have similar parameters regarding the impact of time.

If the standard deviations of the parameters are considered we find that most of the time-related parameters ψ, γ and ρ are not significantly different from zero. The standard deviations of the parameters are much larger than the mean values. This would mean the intensities are not affected by a decrease of intensity over time.

The background intensities have one unexpected value. η_1 has a negative value for the GS stock. This is probably due to a few large negative values in the simulation, which is probably incorrect. These large negative values have led to a much higher standard deviation than for the other stocks.

Values for δ are all quite similar, and it does not look like one time series has a much larger impact on the intensity than the other. In general, $\delta_{1,1}$ is higher than $\delta_{2,2}$, which would imply that the system is more likely to self-excite than the institutions are. The estimates of the

Table 2: Estimated values for the ETAS model parameters with the standard deviations in brackets beneath them. j indicates what institution is set as the second time series in the estimation. ψ, γ and ρ relate to the decrease of the intensity over time while δ determines the interaction of the marks between time series. η is the background intensity.

j:	ψ	γ	ρ	η_1	η_2	$\delta_{1,1}$	$\delta_{1,2}$	$\delta_{2,1}$	$\delta_{2,2}$
BAC	12.836	0.162	291.025	0.073	0.074	24.194	21.220	22.898	21.608
	(23.151)	(0.276)	(330.904)	(0.020)	(0.019)	(22.256)	(19.176)	(20.178)	(18.973)
C	10.886	0.203	317.061	0.063	0.085	25.796	19.524	24.022	19.228
	(23.134)	(0.305)	(342.698)	(0.062)	(0.018)	(23.683)	(19.3270)	(23.223)	(19.748)
GS	17.793	0.181	251.116	-0.007	0.071	20.797	22.092	20.525	21.828
	(31.785)	(0.284)	(314.697)	(0.362)	(0.019)	(21.162)	(19.592)	(18.704)	(21.598)
JPM	11.499	0.170	260.032	0.072	0.081	21.067	21.973	24.500	20.894
	(27.050)	(0.299)	(334.639)	(0.018)	(0.022)	(19.744)	(20.671)	(21.341)	(19.877)

scenario where the system returns are calculated using equal weights for all institutions are compared to the previous estimates. These estimates can be found in table 3.

In general values of δ have increased compared to the previous case. This means marks increase the intensity by a larger amount and are more likely to create an event in this scenario. This seems logical as the equal weights all company should have approximately the same impact on the system returns instead of the biggest institution having the largest impact.

The time-related parameters and background intensities are approximately the same as in the previous scenario, which is as expected as the change of the weights should not affect the timing of the marks that much, but are more likely to affect the size of the marks. Therefore the time-related parameters are still insignificant. Again there is an unexpected value for the background intensity involving GS, but this time it is η_2. From studying the parameters, it

Table 3: Estimated values for the ETAS model parameters with the standard deviations in brackets for the case where the weights are equal. j indicates what institution is set as the second time series in the estimation next to the systemic time series. ψ, γ and ρ relate to the decrease of the intensity over time while δ determines the interaction of the marks between time series. η is the background intensity.

j:	ψ	γ	ρ	η_1	η_2	$\delta_{1,1}$	$\delta_{1,2}$	$\delta_{2,1}$	$\delta_{2,2}$
BAC	11.236	0.199	296.701	0.070	0.075	24.743	22.077	24.182	22.365
	(23.643)	(0.312)	(355.528)	(0.018)	(0.016)	(22.570)	(18.773)	(21.968)	(20.651)
C	12.359	0.276	275.454	0.071	0.089	25.580	20.4550	24.101	21.521
	(22.439)	(0.335)	(337.962)	(0.019)	(0.018)	(19.081)	(18.044)	(18.0403)	(18.339)
GS	12.119	0.227	299.5270	0.070	0.006	25.843	28.742	30.587	28.508
	(24.436)	(0.307)	(355.170)	(0.019)	(0.317)	(21.233)	(20.284)	(23.366)	(20.672)
JPM	17.582	0.156	297.603	0.068	0.078	25.204	18.472	26.269	19.904
	(33.223)	(0.280)	(367.404)	(0.018)	(0.019)	(22.753)	(19.865)	(23.196)	(19.827)

was found that the differences between these four stocks are not that large. The time-related parameters do not seem to have a significant effect on the model. Between the two scenarios in calculating the systemic returns, the marks have more impact when the weights are equal. Based on the parameters presented in table 2 and table 3, two analyses can be done for both scenarios. The first is where the system's returns are conditioned on the situation in different institutions and secondly where the returns of the institutions are conditioned on the situation of the systemic returns.

4.1 System conditional on institutions

First, we look at the results of what happens to the systemic *CoVaR* if a certain institution reaches its *VaR* compared to when the institution is in its median state. On the left side of table 4, the summary statistics describing the time series of intensities, *CoVaR* and Δ*CoVaR* are displayed for the Hawkes process. The right part of table 4 presents summary statistics of the resulting *CoVaR* and Δ*CoVaR* from the quantile regressions.

An institution at its *VaR* level means it is at risk, which should be a situation which is more likely to produce a large loss for the system as a whole than when it is at its median level. Therefore the expectation is that the intensity λ, which is the chance of an event (a large loss) happening, is in general higher when conditioned on the institution's *VaR* than on the median.

As expected, the mean *CoVaR* conditional on the *VaR* is higher than the mean of the *CoVaR* conditional on the median. The same holds for the standard deviations. Because there is much uncertainty in the tail regions to which the *CoVaR* conditional on the *VaR* belongs. In comparison, *CoVaR* conditional on the median is more common and a less risky situation. Which is why the *CoVaR* conditioned on the median should be similar for all stocks as this is the 'normal' situation.

Table 4 shows that indeed for the quantile regressions, this is the case for all stocks. The Hawkes process does not give this result for the C stock. Conditioning on the C stock results in a *CoVaR* which is higher when the C stock is in the median state than when the C stock has large losses. And thus a negative Δ*CoVaR*. This is an unexpected result as this would mean that bad losses in this company would result in less risk for the system and benefit the financial system.

Both methods produce a mean $\Delta CoVaR$ of the system that is higher when conditioned on GS and JPM, which indicates that the difference between the $CoVaR$ conditional on the VaR and the median is larger. Recall that $\Delta CoVaR$ is an indication of the contribution to systemic risk. Therefore large negative returns in these two banks have a higher impact on the $CoVaR$ of the system than with BAC and C. Based on this one could conclude that these banks have a larger impact on the system and should be given priority in receiving financial aid in the case of a recession. Table 5 shows the summary statistics in the scenario with equal weights for

Table 4: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the system for the Hawkes process and the time series of $CoVaR$ and $\Delta CoVaR$ of the system estimated with the quantile regressions. The standard deviations are beneath the mean in brackets. j indicates the institution that is set as the variable conditioned on.

	Hawkes process					Quantile Regression		
j:	$\lambda\|VaR$	$\lambda\|Med$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$
BAC	0.0286	0.0212	0.0252	0.0198	0.0054	0.0030	0.0416	0.0385
	(0.0424)	(0.0412)	(0.0252)	(0.0258)	(0.0158)	(0.0004)	(0.0240)	(0.0240)
C	0.0225	0.0244	0.0199	0.0227	-0.0028	0.0057	0.0422	0.0366
	(0.0394)	(0.0423)	(0.0226)	(0.0277)	(0.0193)	(0.0110)	(0.0259)	(0.0149)
GS	0.0453	0.0281	0.0403	0.0263	0.0139	0.0075	0.0467	0.0392
	(0.0446)	(0.0432)	(0.0262)	(0.0264)	(0.0115)	(0.0009)	(0.0245)	(0.0249)
JPM	0.0445	0.0332	0.0396	0.0310	0.0086	0.0128	0.0509	0.0381
	(0.0434)	(0.0438)	(0.0251)	(0.0275)	(0.0139)	(0.0034)	(0.0248)	(0.0229)

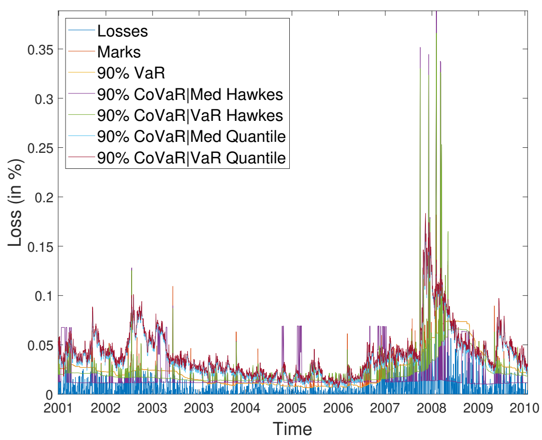
the intensities, $CoVaR$ and $\Delta CoVaR$ for both the Hawkes process and the quantile regressions. When comparing these results to the previous scenario's results, the mean intensity is in general higher for the Hawkes process, and the standard deviations are larger than before. Values for $\Delta CoVaR$ are higher, meaning the difference between levels of $CoVaR$ when the institutions are in distress and at a normal level is larger. These results are in line with what we found in the analysis of the estimated parameters. In the scenario with equal weights, the values for δ where slightly higher meaning marks have a larger impact on the intensities than in the previous scenario.

For the quantile regressions changes are not obvious from $\Delta CoVaR$. However, levels of $CoVaR$ are consistently lower in this scenario. Therefore from this comparison it seems the institution's market equity does play a role in the outcome of this model and should not be ignored. To get more intuition on how the models perform graphs of the different estimated $CoVaR$ are shown in figure 1, figure 2, figure 3 and figure 4. Figure 1 shows the $CoVaR$ over the returns of the system conditional on the BAC stock. The first thing that is immediately obvious is that the $CoVaR$ using the Hawkes process is less smooth than the $CoVaR$ estimated by the quantile regressions. This is due to the intensity function returning to the constant background intensity level after a certain period after an event. The quantile regressions have state variables as additional information and can adjust the mean $CoVaR$ level accordingly. The Hawkes process sometimes gives some unexpected spikes in the model. This could be due to two marks in the two time series shortly after each other. In general, the two different methods roughly get peaks in $CoVaR$ at the same time. It seems that for the system conditional on the BAC stock in volatile times, the $\Delta CoVaR$ increases in the Hawkes process. In contrast, the $\Delta CoVaR$ stays almost constant for the quantile regressions no matter the volatility. However,

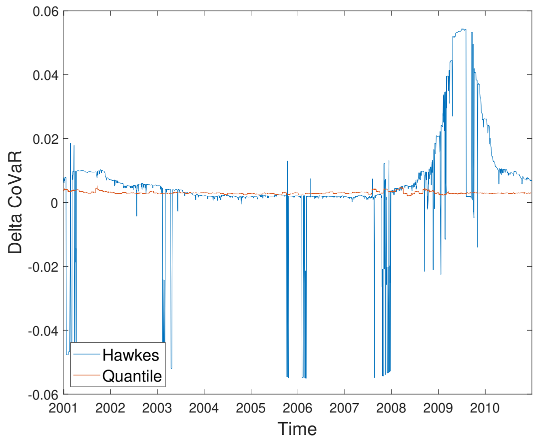
Table 5: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the system for the Hawkes process and the time series of $CoVaR$ and $\Delta CoVaR$ of the system estimated with the quantile regressions in the scenario where the weights to calculate systemic returns are equal. The standard deviations are beneath the means in brackets. j indicates the institution that is set as the variable conditioned on.

	Hawkes process					Quantile Regression		
j:	$\lambda\|VaR$	$\lambda\|Med$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$
BAC	0.0662	0.0423	0.0585	0.0398	0.0187	0.0037	0.0360	0.0324
	(0.0611)	(0.0453)	(0.0412)	(0.0286)	(0.0315)	(0.0011)	(0.0213)	(0.0204)
C	0.0607	0.0445	0.0541	0.0417	0.0123	0.0065	0.0372	0.0308
	(0.0542)	(0.0454)	(0.0359)	(0.0292)	(0.0274)	(0.0105)	(0.0233)	(0.0129)
GS	0.0619	0.0367	0.0555	0.0345	0.0210	0.0076	0.0402	0.0327
	(0.0505)	(0.0440)	(0.0329)	(0.0275)	(0.0206)	(0.0012)	(0.0218)	(0.0212)
JPM	0.0548	0.0416	0.0497	0.0391	0.0106	0.0119	0.0442	0.0322
	(0.0453)	(0.0450)	(0.0272)	(0.0284)	(0.0123)	(0.0034)	(0.0216)	(0.0192)

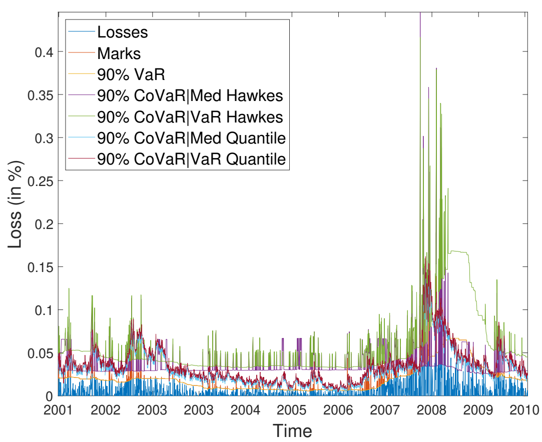
the Hawkes process might be overestimating this difference in one institution's impact on the whole system. It seems more logical that there is an impact than no impact especially with the system consisting of only four institutions. This difference becomes even more pronounced in the equal weight case as the $\Delta CoVaR$ is even more sensitive to volatility. Especially in the period around 2009, the Hawkes process shows a considerable gap between the two measures. Although the quantile regression method does react to more volatility in the system, it does so evenly for both scenarios resulting in a $\Delta CoVaR$ which is unaffected during the recession of 2008. If the results for the system conditional on the BAC stock in figure 1 are compared to the results for the system conditional on the C stock in figure 2. It can be seen that the $\Delta CoVaR$ does increase as the volatility in returns increases for both methods in figure 2. Where before this only happened for the Hawkes process. In times of volatility, the C stock has a lot more influence on the system's $CoVaR$ according to the quantile regressions than the BAC stock. Furthermore, the Hawkes process results in a lot of negative spikes in $\Delta CoVaR$ which indicate that somehow the intensity of the $CoVaR|Med$ gets triggered while the $CoVaR|VaR$ does not get triggered, which seems counter-intuitive as one would expect the 90% VaR of an institution to trigger the intensity function more than the median of the same institution. The $CoVaR$ of the system conditional on the GS stock estimated with the quantile regression shown in figure 3 is more similar to the $CoVaR$ conditional on the BAC stock than conditional on the C stock. An increase in volatility does not increase $\Delta CoVaR$. This also explains the low standard deviation for the $\Delta CoVaR$ estimated by the quantile regression for GS stock in table 4. If (b) and (d) of figure 3 are compared, the quantile regressions give quite a similar result with a slight increase in volatile times in (d) compared to the previous scenario in (b). Where the C stock had a lot of negative spikes in the $\Delta CoVaR$, the GS stock has a lot of positive spikes. This is because the $CoVaR|Med$ drops to zero sometimes, due to the low background intensity shown in table 2. From figure 4, it can be seen that the $\Delta CoVaR$ of JPM stock is different from the other simulated stocks. It is higher throughout the whole time series and looks like white noise of which the volatility increases in volatile times. Instead of having a peak in $\Delta CoVaR$ as the other stocks have. In the scenario where systemic returns are calculated with equal weights for the institutions, not much changes. Except for the fact that the peaks during volatile times are more extreme than in times with average volatility, especially for the Hawkes process.



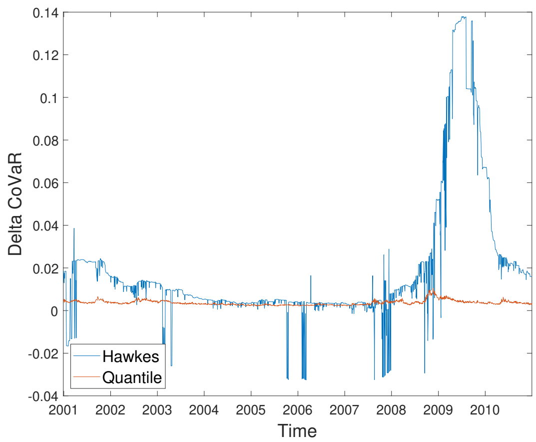
(a) Simulated $CoVaR$ System|BAC



(b) Simulated $\Delta CoVaR$ System|BAC

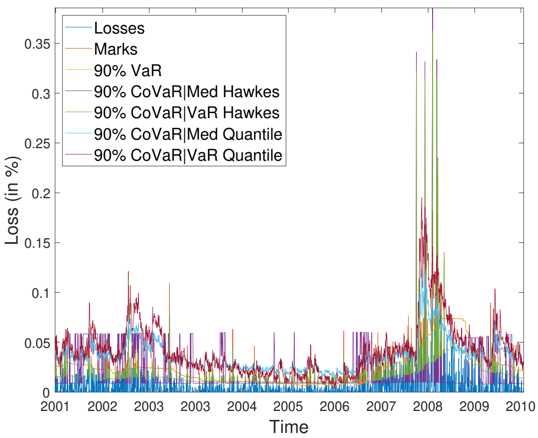


(c) Simulated $CoVaR$ System|BAC (equal)

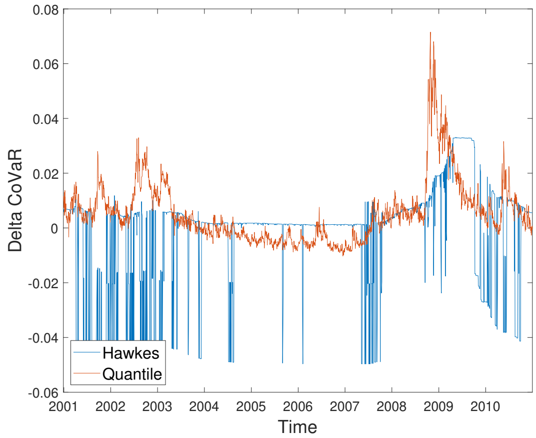


(d) Simulated $\Delta CoVaR$ System|BAC (equal)

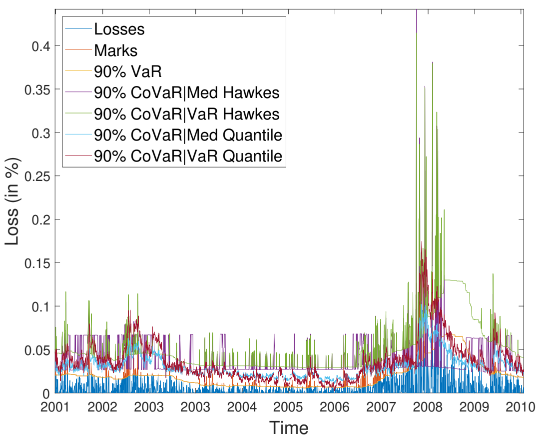
Figure 1: This figure displays results from the simulation of systemic returns conditional on data from the BAC stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the system. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.



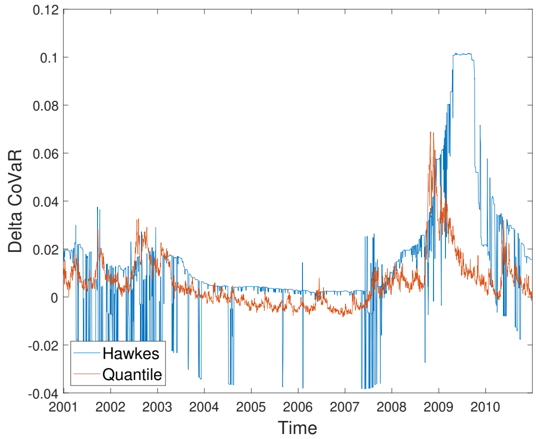
(a) Simulated *CoVaR* System|C



(b) Simulated Δ*CoVaR* System|C

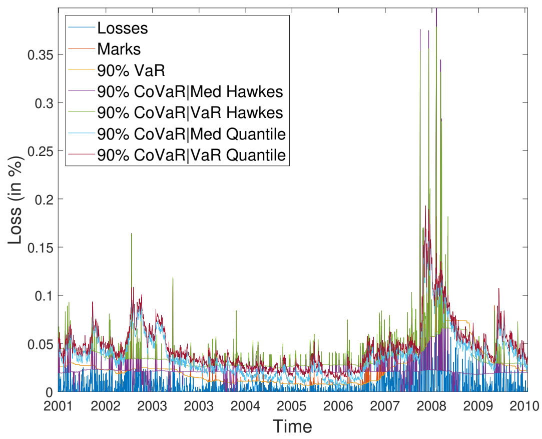


(c) Simulated *CoVaR* System|C (equal)

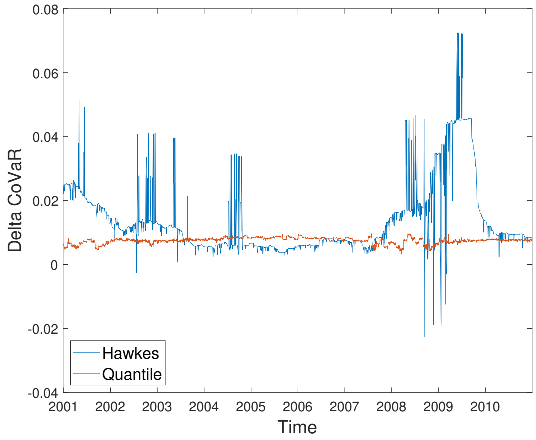


(d) Simulated Δ*CoVaR* System|C (equal)

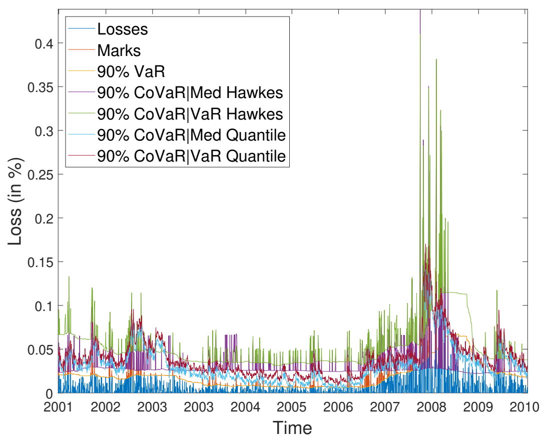
Figure 2: This figure displays results from the simulation of systemic returns conditional on data from the C stock. In (a) and (c) the estimated *CoVaR* using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the system. The orange line represents the marks. Red and light blue lines show the *CoVaR* estimated by the quantile regressions conditional on the *VaR* and median, respectively. Green and purple represent the *CoVaR* estimated by the Hawkes process conditional on the *VaR* and median, respectively. In (b) and (d) the Δ*CoVaR* is displayed for both methods. The orange line shows the Δ*CoVaR* calculated with quantile regressions while the blue line shows the Δ*CoVaR* from the Hawkes process.



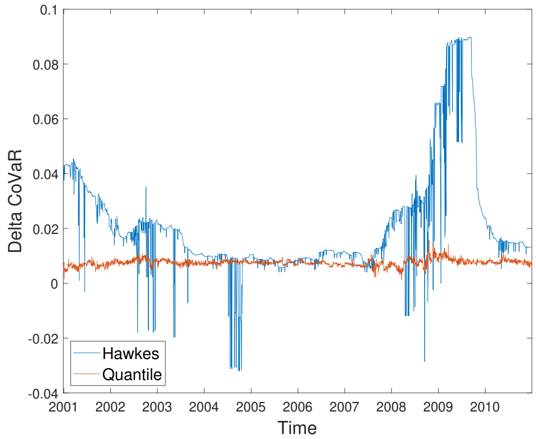
(a) Simulated $CoVaR$ System|GS



(b) Simulated $\Delta CoVaR$ System|GS

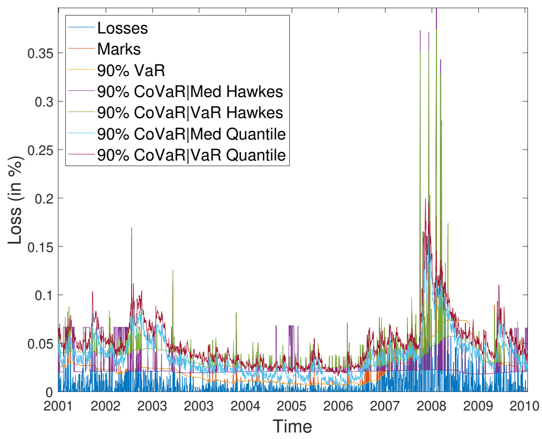


(c) Simulated $CoVaR$ System|GS (equal)

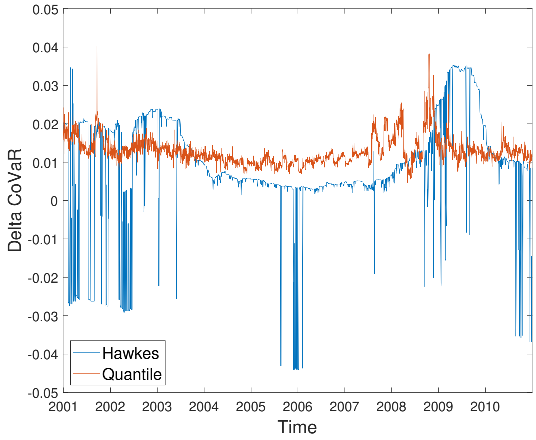


(d) Simulated $\Delta CoVaR$ System|GS (equal)

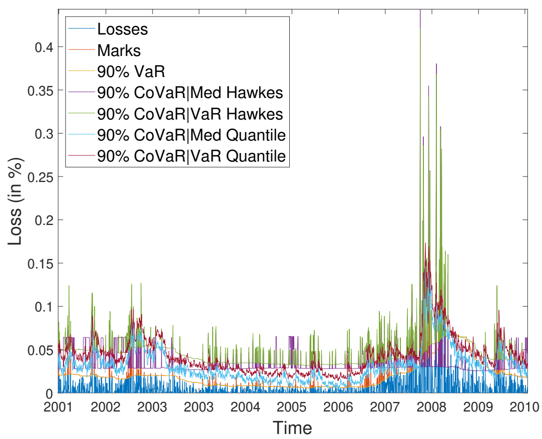
Figure 3: This figure displays results from the simulation of systemic returns conditional on data from the GS stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the system. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.



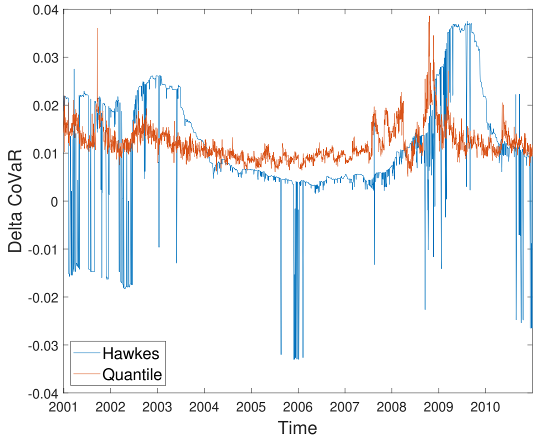
(a) Simulated $CoVaR$ System|JPM



(b) Simulated $\Delta CoVaR$ System|JPM



(c) Simulated $CoVaR$ System|JPM (equal)



(d) Simulated $\Delta CoVaR$ System|JPM (equal)

Figure 4: This figure displays results from the simulation of systemic returns conditional on data from the JPM stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the system. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.

In table 6, the performance of the investment strategies as described in section 2.5 are compared to the GMV portfolio and the 1/N portfolio. The different ΔR measures calculated as described in equation (2.29) are shown in the bottom rows. Positive values for $\Delta R1$ indicate that the Hawkes process-based strategy achieved higher returns while negative values show us that the quantile regression-based strategy performed best. Similarly, the $\Delta R2$ and $\Delta R3$ measures indicate how much the $\Delta CoVaR$ strategies differ from the minimum variance strategy. So values close to zero indicate they are similar. As can be seen in table 6 between the two $\Delta CoVaR$ strategies, the quantile regression method performs best. As it has the highest mean returns and lowest standard deviation, resulting in the highest Sharpe ratio of the two methods. However, the differences between the two strategies are not that large. Compared to the benchmark portfolios, they both perform worse when it comes to standard deviation, which is expected as diversification of a portfolio leads to less risk. Unexpectedly the Sharpe ratios are better for the minimum $\Delta CoVaR$ strategies. The $\Delta R1$ measure indicates that the quantile regressions performed better. Which is in accordance with $\Delta R2$ and $\Delta R3$ that indicate that both methods performed slightly worse than the minimum variance strategy, but the difference in returns is worse for the Hawkes process indicated by a larger negative value for $\Delta R2$ than $\Delta R3$. So again, these performance measures indicate that quantile regressions provide a slightly better strategy for minimising the $\Delta CoVaR$. The performance of the different portfolios and strategies

Table 6: The different performance measures for the constructed portfolios and strategies are displayed in this table, together with the ΔR measures. This table displays the results for the case where the weights to create the systemic returns are proportional to the market value of equity.

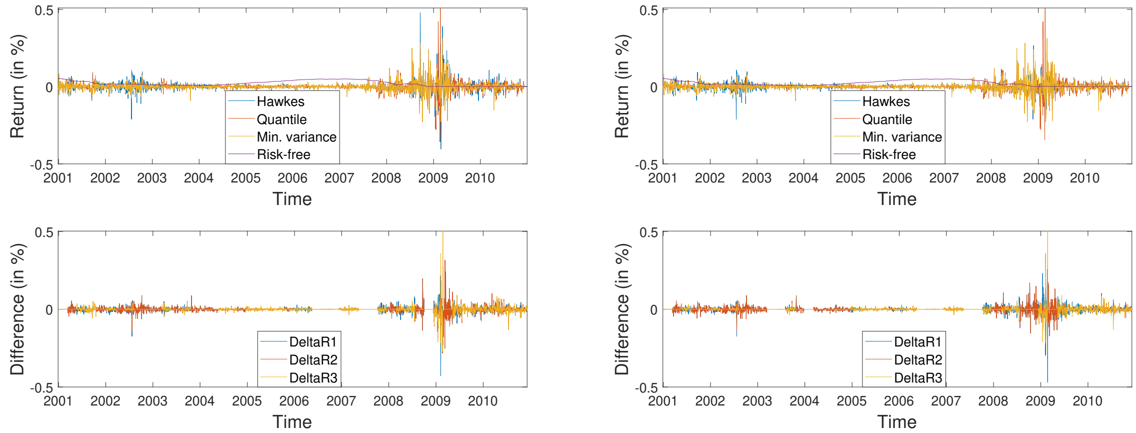
Strategy	Mean	Std. Dev.	Sharpe Ratio
GMV	0.0002	0.0237	-0.8957
1/N	0.0003	0.0276	-0.7612
Min. variance	0.0007	0.0291	-0.7116
Hawkes $CoVaR$	-0.0002	0.0354	-0.6085
Quantile $CoVaR$	0.0002	0.0353	-0.6000
ΔR1	-0.0004	0.0228	N.A.
ΔR2	-0.0009	0.0230	N.A.
ΔR3	-0.0005	0.0203	N.A.

are compared again in table 7. However, now for the scenario with equal weights to construct the system returns. The benchmark portfolios are the same as the weights are still the same regardless of how the system returns are constructed. The same applies to the minimum variance strategy. The minimum $\Delta CoVaR$ strategy returns are slightly different, especially the Hawkes process-based strategy. Where before it was the worst performing, in this scenario $\Delta CoVaR$ strategy is the best performing together with the 1/N portfolio for both mean returns and standard deviation of the returns. This means it performs better than the quantile regression-based strategy, resulting in a positive mean for $\Delta R1$. This change in performance compared to the previous scenario suggests that the market value of equity is an important factor in determining the returns of the system as a whole. The Sharpe ratios of the quantile regression-based minimum $\Delta CoVaR$ strategy is high compared to the others while the other summary statistics suggest its the worst performing portfolio. However, the differences in means are so small compared to the standard deviations that one can argue that they are not different. The different ΔR measures support this. $\Delta R1$ indicates that the Hawkes strategy's returns are slightly better, but the mean is only 0.0001. $\Delta R2$ and $\Delta R3$ are very similar, also indicating the two strategies are similar. In this scenario, the Hawkes process performs slightly better than the quantile regressions when minimising the risk. However, the increase in performance is minimal and therefore insignificant. Figure 5 shows the course of the two $\Delta CoVaR$ strategies' returns

Table 7: The different performance measures for the constructed portfolios and strategies are displayed in this table, together with the ΔR measures. This table displays the results for the scenario where the Systemic returns are calculated with equal weights for all institutions.

Strategy	Mean	Std. Dev.	Sharpe Ratio
GMV	0.0002	0.0237	-0.8957
1/N	0.0003	0.0276	-0.7612
Min. variance	0.0007	0.0291	-0.7116
Hawkes $CoVaR$	0.0003	0.0257	-0.8236
Quantile $CoVaR$	0.0002	0.0352	-0.6023
ΔR1	0.0001	0.0248	N.A.
ΔR2	-0.0005	0.0190	N.A.
ΔR3	-0.0005	0.0199	N.A.

in the top graphs together with the risk-free rate and the minimum variance strategy. The ΔR measures over time are displayed in the bottom graphs for the two different scenarios. From the top graphs of figure 5, the differences between the strategies in volatility come mainly from high volatility periods. Throughout the middle of the period, the returns of the different strategies are very similar. Between 2008 and 2010, the two $\Delta CoVaR$ strategies can be distinguished from the yellow graph of the minimum variance strategy. It can be seen from the bottom graphs of (a) and (b) of figure 5 that $\Delta R1$ is more often zero in (a). This implies that the two methods have approximately the same outcome for these periods where $\Delta R1$ is zero. This is in line with what could be seen in table 4 and table 5 earlier. Adjusting the weights mainly influences the Hawkes process, which gives it other results in the second scenario while the quantile regressions have a similar outcome. So far, the Hawkes process tends to overestimate the $\Delta CoVaR$ in volatile times compared to the quantile regressions. However, it leads to approximately the same conclusions regarding the importance of the institutions' influence on the system. How the system's returns are determined has a large impact on the outcome of the Hawkes process. Furthermore, the minimum $\Delta CoVaR$ strategies give decent returns compared to the benchmark portfolios, especially considering a large portion of the period considered is during a financial crisis. However, the quantile regression-based strategy produces slightly less risk in the first scenario, on which the Hawkes process could not improve.



(a) Simulated returns System|institutions **(b)** Simulated returns System|institutions (equal)

Figure 5: This figure displays the returns from the minimum $\Delta CoVaR$ strategies and the minimum variance strategy. In the top graphs, the returns created with these methods are shown. The orange line represents the quantile regression-based strategy and the blue line the Hawkes process-based strategy. In the bottom graphs, the ΔR measures over time are displayed.

4.2 Institutions conditional on system

In this section, the conditioned event is no longer an institution reaching its VaR, but the whole system is at the 90% VaR. This is done in order to find out what institutions are most vulnerable during recessions or economic downturns. In table 8, the summary statistics for the estimated intensity for the different stocks are presented. For the first three stocks in the table, the levels of $CoVaR$ estimated by the Hawkes process are too low and probably poorly estimated. Mean values lie between 0.1% and 0.7%, except for JPM which immediately stands out as the intensity values are way higher than for the other stocks and the mean of the $CoVaR$ values seem to make more sense. However, these values are also relatively low compared to the values estimated by the quantile regressions. Here the mean values of the $CoVaR$ lie between 3% and 5%. The Hawkes process does seem to have trouble to estimate reliable values in this reverse relation between the institutions and the system.

Suppose we assume that the quantile regressions are reliable. It can be said that values of $\Delta CoVaR$ are by far the highest for GS meaning this institution experiences the most change in risk during troubling times for the financial system. JPM and BAC are subject to much lower risk according to the quantile regressions. The Hawkes process contradicts the quantile regressions by estimating JPM to have the highest $\Delta CoVaR$ while the quantile regressions indicate it should be the lowest. The relative risk of the institutions measured by the two methods is not in accordance. The results for the second case where the weights are equal is presented in table 9. Here the same applies to the results of the Hawkes process as in the previous scenario. The first three stocks have very low $CoVaR$, and only the values for JPM seem to be somewhat right. As could be seen in the reverse relation of the institutions and the system, the levels of $CoVaR$ resulting from the quantile regressions increase in general in this scenario due to marks having more impact on the intensity. It seems $\Delta CoVaR$ slightly increases all around for this method too. For the Hawkes process the changes to the previous scenario seem the exact opposite of what we saw before. Levels of $CoVaR$ have decreased, and also the $\Delta CoVaR$ has decreased.

Table 8: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the institutions for the Hawkes process and the time series of $CoVaR$ and $\Delta CoVaR$ of the institutions estimated with the quantile regressions. The standard deviations are beneath the mean in brackets. i indicates the institution for which the measures are estimated.

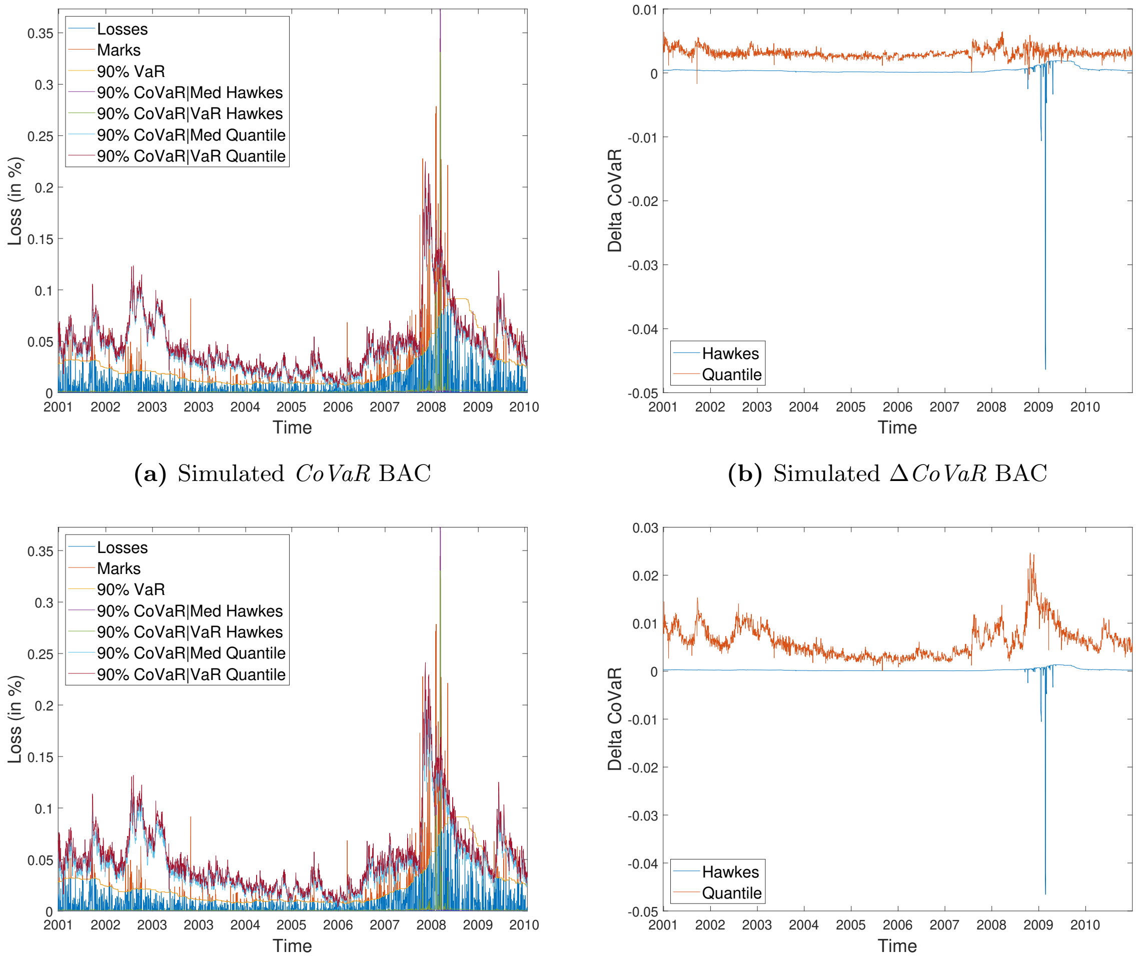
	Hawkes process					Quantile Regression		
i:	$\lambda\|VaR$	$\lambda\|Med$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$
BAC	0.0020	0.0015	0.0017	0.0013	0.0004	0.0031	0.0492	0.0462
	(0.0211)	(0.0211)	(0.0101)	(0.0114)	(0.0014)	(0.0007)	(0.0299)	(0.0297)
C	0.0017	0.0013	0.0014	0.0011	0.0002	0.0071	0.0593	0.0522
	(0.0218)	(0.0218)	(0.0100)	(0.0117)	(0.0017)	(0.0054)	(0.0383)	(0.0330)
GS	0.0075	0.0050	0.0067	0.0047	0.0020	0.0139	0.0529	0.0390
	(0.0211)	(0.0209)	(0.0108)	(0.0115)	(0.0024)	(0.0104)	(0.0292)	(0.0191)
JPM	0.0278	0.0165	0.0248	0.0156	0.0092	0.0030	0.0411	0.0381
	(0.0305)	(0.0270)	(0.0194)	(0.0166)	(0.0109)	(0.0039)	(0.0143)	(0.0180)

This means the $CoVaR|Med$ has been affected less than the $CoVaR|VaR$. In figure 6, the BAC

Table 9: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the institutions for the Hawkes process and the time series of $CoVaR$ and $\Delta CoVaR$ of the institutions estimated with the quantile regressions in the scenario where the weights to calculate systemic returns are equal. The mean values of the different measures are displayed with the standard deviations in brackets below. i indicates the institution for which the measures are estimated.

	Hawkes process					Quantile Regression		
i:	$\lambda\|VaR$	$\lambda\|Med$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR\ \|VaR$	$CoVaR\ \|Med$
BAC	0.0016	0.0013	0.0013	0.0011	0.0002	0.0063	0.0519	0.0456
	(0.0210)	(0.0210)	(0.0100)	(0.0113)	(0.0013)	(0.0033)	(0.0322)	(0.0293)
C	0.0010	0.0009	0.0008	0.0007	0.0000	0.0080	0.0601	0.0521
	(0.0214)	(0.0214)	(0.0097)	(0.0114)	(0.0017)	(0.0061)	(0.0394)	(0.0333)
GS	0.0020	0.0013	0.0017	0.0012	0.0005	0.0166	0.0553	0.0386
	(0.0202)	(0.0202)	(0.0100)	(0.0109)	(0.0012)	(0.0131)	(0.0318)	(0.0189)
JPM	0.0312	0.0196	0.0279	0.0186	0.0093	0.0058	0.0441	0.0383
	(0.0317)	(0.0279)	(0.0203)	(0.0173)	(0.0115)	(0.0015)	(0.0177)	(0.0171)

stock losses are presented with the estimated $CoVaR$ using the different methods in (a) and (c). The $\Delta CoVaR$ is shown in (b) and (d) of figure 6. It is very clear that the two methods give very different results for this stock. It seems that the Hawkes process is most of the time at the value of its background intensity, only to shoot up in the most volatile time in 2008 and 2009. Especially the $CoVaR|Med$ has a huge spike here, resulting in the large negative spike that can be seen in the $\Delta CoVaR$ in graph (b) of figure 6 during this time. This, of course, is not what the $CoVaR$ should look like and we can confidently say the Hawkes process fails to estimate the $CoVaR$ in this case correctly. Results for the second scenario are very similar to the previous scenario. Only the $\Delta CoVaR$ of the quantile regression seems to be more volatile during the financial crisis, which implies that in the second scenario, the BAC stock experiences more risk during a recession than in the first scenario. Table 8 and table 9 find that the C stock has the lowest average $\Delta CoVaR$ for the Hawkes process. In figure 7, it is clear why. The values of $CoVaR|VaR$ and $CoVaR|Med$ hardly differ. Only during the recession, there are some large negative spikes in $\Delta CoVaR$. The quantile regressions show a $CoVaR$ that is way more volatile than for the BAC stock. Especially during the 2008 - 2010 period. Between the two scenarios the results for the C stock do not change much. The adjusted weights seem to have



(a) Simulated $CoVaR$ BAC

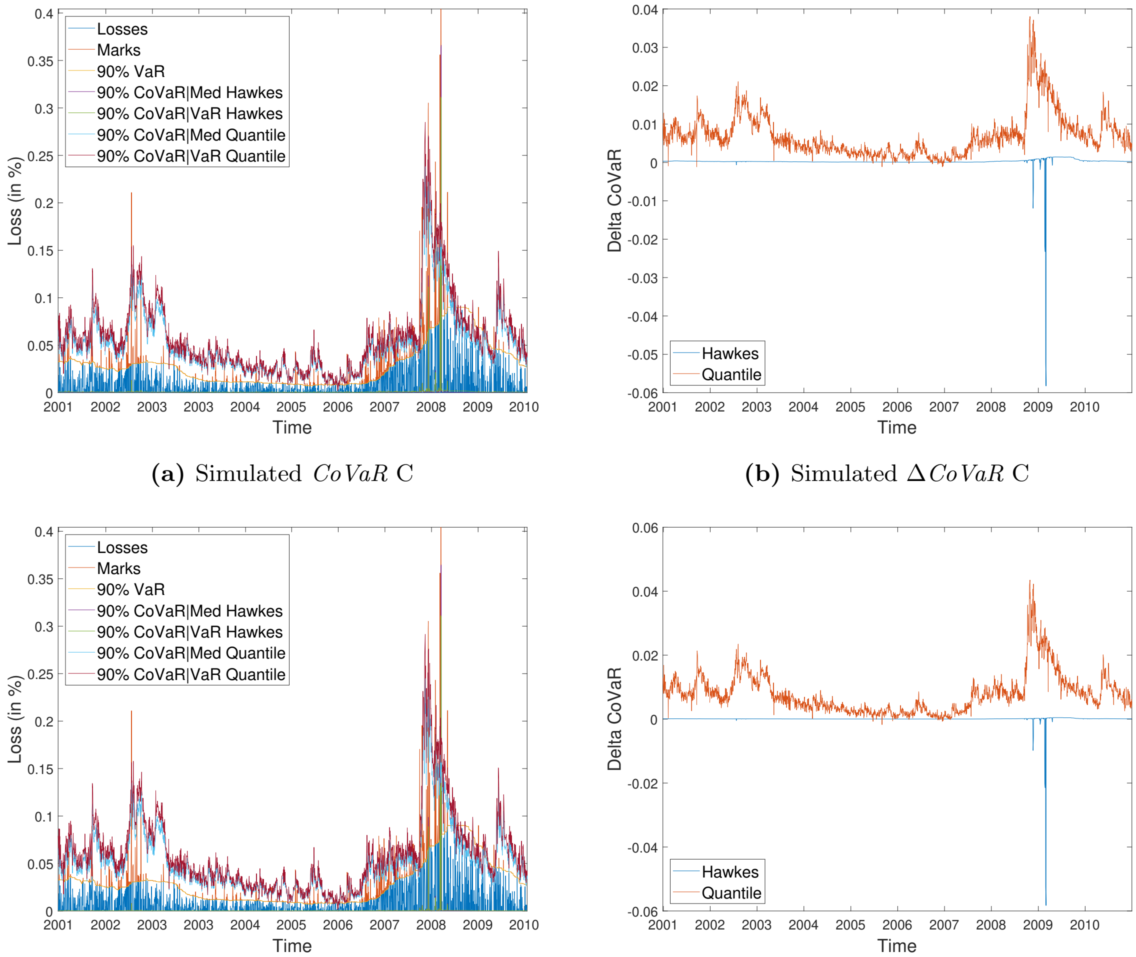
(b) Simulated $\Delta CoVaR$ BAC

(c) Simulated $CoVaR$ BAC (equal weights)

(d) Simulated $\Delta CoVaR$ BAC (equal weights)

Figure 6: This figure displays results from the simulation based on data from BAC stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the BAC stock. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.

almost no influence at all. This is because the ME-based weight of the C stock for calculating the systemic returns is on average approximately the same as the equal weight. In figure 8, the resulting $CoVaR$ for the simulated GS stock is displayed. For the GS stock there is a clearer difference between the two $CoVaR$ values estimated by the quantile regressions. This means that this stock is affected more by the system's state of the system compared to the other stocks. However, the $\Delta CoVaR$ estimated by the Hawkes process makes slightly more sense than the previous stocks. The $CoVaR$ is still very badly estimated. Based on this observation, one could state that the GS stock is more vulnerable to large losses during a recession than the stocks investigated previously in this section. In the second scenario, the differences in $CoVaR$ are again more pronounced, resulting in a higher peak in $\Delta CoVaR$ in (d) than (b) of figure 8. From table 8 and table 9 it could already be seen that the estimation by the Hawkes process of the



(a) Simulated $CoVaR$ C

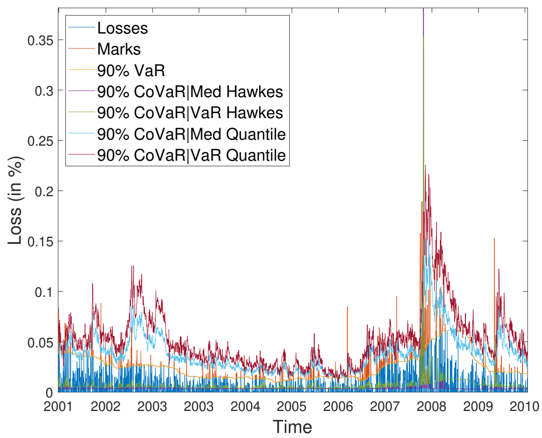
(b) Simulated $\Delta CoVaR$ C

(c) Simulated $CoVaR$ C (equal weights)

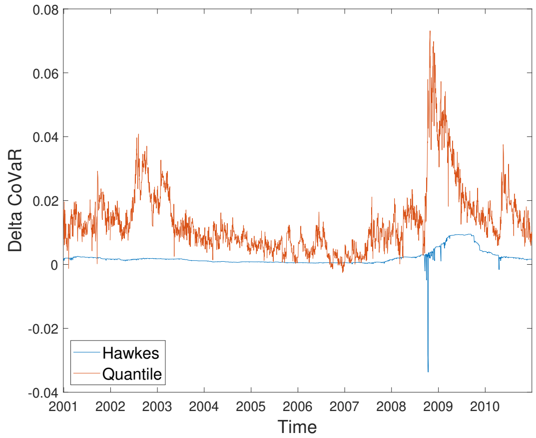
(d) Simulated $\Delta CoVaR$ C (equal weights)

Figure 7: This figure displays results from the simulation based on data from C stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the C stock. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.

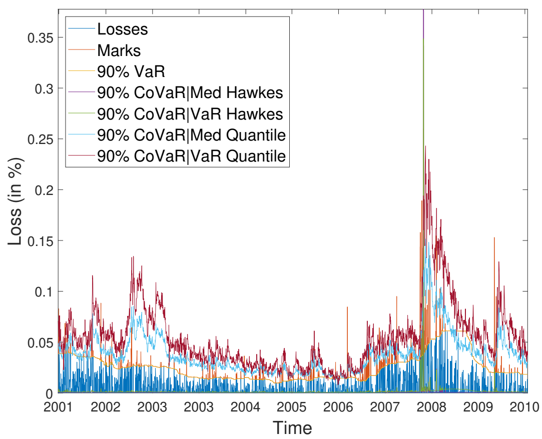
$CoVaR$ for the JPM stock was more successful than the other three stocks. Figure 9 confirms this. The background intensity is at a more realistic value, and the $CoVaR$ not only changes during the financial crisis in 2008 but reacts to all periods with more volatility. The graph of $\Delta CoVaR$ in (b) also looks more familiar. The institution comes under more risk during volatile times. Where the Hawkes process estimates are more like the results we expected, the quantile regressions give results that are not familiar. It gives negative values for $\Delta CoVaR$ for this stock during the financial crisis. This would suggest that the JPM stock experiences less risk from the system being in a worse state than when it is in its median state during recessions. The second scenario results in (c) and (d) are similar, except that the $\Delta CoVaR$ of the quantile regression does not reach the same large negative values in this case. $\Delta CoVaR$ during the recession is higher for both methods than in the first scenario, which is also seen for the other stocks.



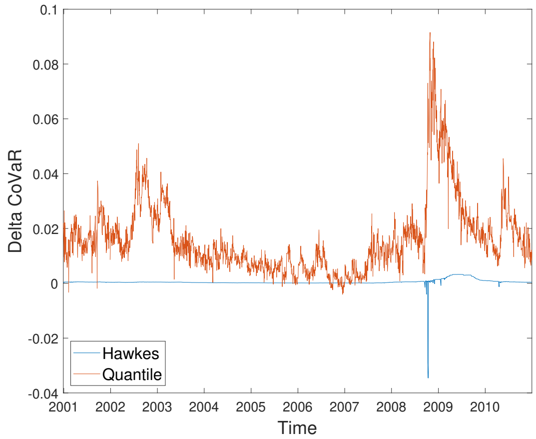
(a) Simulated $CoVaR$ GS



(b) Simulated $\Delta CoVaR$ GS

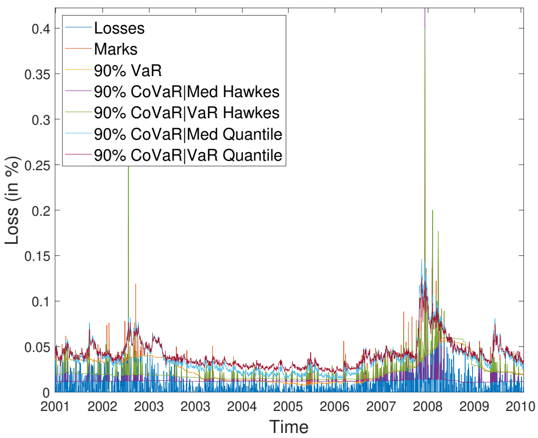


(c) Simulated $CoVaR$ GS (equal weights)

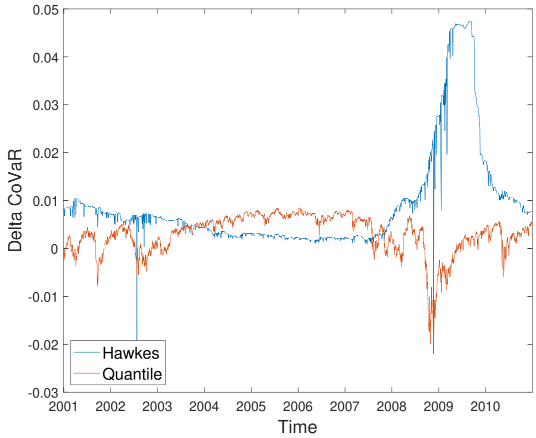


(d) Simulated $\Delta CoVaR$ GS (equal weights)

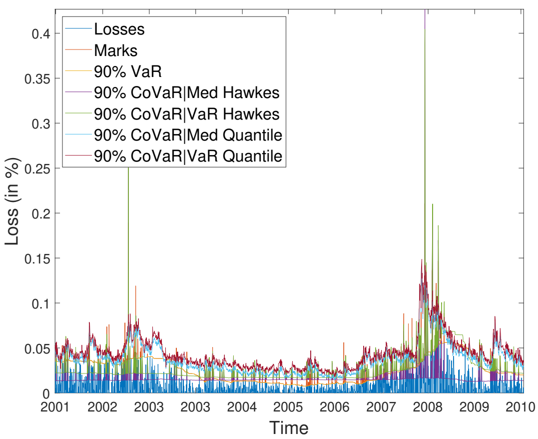
Figure 8: This figure displays results from the simulation based on data from GS stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the GS stock. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.



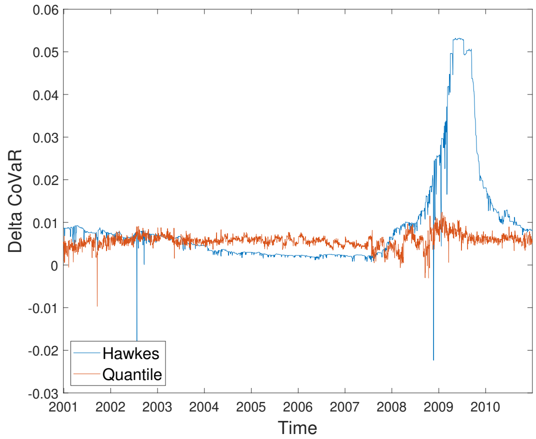
(a) Simulated $CoVaR$ JPM



(b) Simulated $\Delta CoVaR$ JPM



(c) Simulated $CoVaR$ JPM (equal weights)



(d) Simulated $\Delta CoVaR$ JPM (equal weights)

Figure 9: This figure displays results from the simulation based on data from JPM stock. In (a) and (c) the estimated $CoVaR$ using both the Hawkes process and the quantile regressions are displayed. The dark blue line represents the loss of the JPM stock. The orange line represents the marks. Red and light blue lines show the $CoVaR$ estimated by the quantile regressions conditional on the VaR and median, respectively. Green and purple represent the $CoVaR$ estimated by the Hawkes process conditional on the VaR and median, respectively. In (b) and (d) the $\Delta CoVaR$ is displayed for both methods. The orange line shows the $\Delta CoVaR$ calculated with quantile regressions while the blue line shows the $\Delta CoVaR$ from the Hawkes process.

Table 10 shows the summary statistics of the different portfolios and strategies calculated with the institutions' $\Delta CoVaR$ conditional on the system. The statistics for the benchmark portfolios in table 10 are the same as in table 6 and table 7. The weights of the benchmark portfolios are also the same as in the previous section. This means they perform the same as before because they are the same portfolios. According to the mean returns, the minimum $\Delta CoVaR$ strategies perform worse than both benchmark portfolios. Even though the Hawkes process-based strategy has the highest standard deviation and the lowest mean of all portfolios, the Sharpe ratio is the best. The quantile regression-based strategy performs very well, considering the standard deviation. It even beats the 1/N portfolio in this regard.

As expected from the statistics in the top half of table 10, the different ΔR measures slightly favour the quantile regressions. $\Delta R1$ has a negative mean which indicating that the quantile regression-based strategy performs best between the two minimum $\Delta CoVaR$ strategies. When comparing the values of $\Delta R2$ and $\Delta R3$, it can also be seen that the quantile regressions are closer to the minimum variance strategy. Again, the differences are not that big, but the Hawkes process fails to improve on the quantile regressions. In the other scenario, not much has changed

Table 10: The different performance measures for the constructed portfolios and strategies are displayed in this table, together with the ΔR measures. These are the results for the case where the weights to create the systemic returns are proportional to the market value of equity.

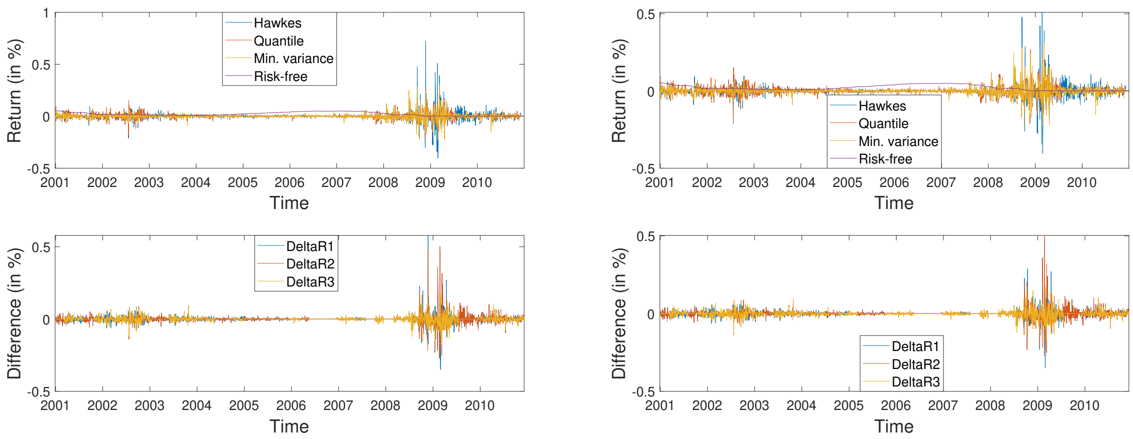
Strategy	Mean	Std. Dev.	Sharpe Ratio
GMV	0.0002	0.0237	-0.8957
1/N	0.0003	0.0276	-0.7612
Min. variance	0.0007	0.0291	-0.7116
Hawkes $CoVaR$	-0.0001	0.0419	-0.5128
Quantile $CoVaR$	0.0000	0.0271	-0.7885
ΔR1	-0.0002	0.0303	N.A.
ΔR2	-0.0008	0.0299	N.A.
ΔR3	-0.0007	0.0200	N.A

for the performance of the portfolios and strategies. The different ΔR measures indicate that the results of the two methods lie slightly closer together than before. The mean of $\Delta R1$ is closer to zero, which would suggest that the returns of the two $\Delta CoVaR$ strategies are similar. The most significant change is in the Hawkes process-based strategy, where the main change is that the standard deviation is greatly reduced compared to the previous scenario. It still has the highest standard deviation out of all the portfolios and strategies. The Sharpe ratio has become slightly lower as well. So it is still not a favourable strategy in minimising risk. From the graphs in figure 10, it is clear that there is a difference in volatility between the quantile regression-based strategy returns and the Hawkes process-based strategy returns, which was also clear from the summary statistics presented before in table 10 and table 11. Especially during volatile times like the period around 2009, the difference is noticeable. The blue graph representing the Hawkes process-based strategy's returns has much larger peaks than the other strategies. The same applies to the $\Delta R2$ measure that captures the difference between this method and the minimum variance strategy. The standard deviation of the quantile regression-based strategy is lower than the Hawkes process-based strategy. Because the strategy tries to

Table 11: The different performance measures for the constructed portfolios and strategies are displayed in this table, together with the ΔR measures. This table displays the results for the scenario where the Systemic returns are calculated with equal weights for all institutions.

Strategy	Mean	Std. Dev.	Sharpe Ratio
GMV	0.0002	0.0237	-0.8957
1/N	0.0003	0.0276	-0.7612
Min. variance	0.0007	0.0291	-0.7116
Hawkes $CoVaR$	-0.0001	0.0385	-0.5585
Quantile $CoVaR$	0.0000	0.0272	-0.7852
ΔR1	-0.0001	0.0278	N.A.
ΔR2	-0.0008	0.0285	N.A.
ΔR3	-0.0007	0.0202	N.A.

minimise risk, this is a desirable trait. Therefore the conclusion can be drawn that in minimising the risk, the quantile regression-based strategy does a better job than the Hawkes process-based strategy in this scenario. In this part of the study the Hawkes process does not reliably estimate



(a) Simulated returns institutions|System **(b)** Simulated returns institutions|System (equal)

Figure 10: This figure displays the returns from the minimum $\Delta CoVaR$ strategies and the minimum variance strategy. In the top graphs, the returns of these methods are shown. The orange line represents the quantile regression-based strategy, the blue line the Hawkes process-based strategy and the yellow line the minimum variance strategy. In the bottom graphs, the ΔR measures are displayed.

the $CoVaR$ for all stocks. The returns of the Hawkes process-based strategy could not improve upon the quantile regression-based strategy. So far, the quantile regressions seem superior to the Hawkes process in this analysis. Overall the simulation study shows that estimating the $CoVaR$ of the system conditional on the institutions is better suited to the Hawkes process than the reverse relation. Ultimately the quantile regressions give more accurate estimates of $CoVaR$ and $\Delta CoVaR$. The minimum $\Delta CoVaR$ strategies, in general, do a good job of selecting stocks with low risk. Depending on the accuracy of the estimation of $\Delta CoVaR$ the Hawkes process beats all other portfolios and strategies when it comes to Sharpe ratios, but has the highest standard deviations and the lowest mean returns most of the time. The quantile regression-based minimum $\Delta CoVaR$ strategy is, therefore, superior when it comes to minimising risk.

5 Historical data analysis

In the previous section, we looked at the simulation study's results where the data used to estimate the parameters were randomly generated. In this section, the parameters are directly estimated from historical data for 15 different institutions. In this data set some of the largest financial institutions in the United States from 2000 to 2010 are included. The Hawkes process results are presented and compared to the results of the quantile regressions using the state variables in section 3.

5.1 System conditional on institutions

First, the $CoVaR$ of the system conditioned on the different institutions is analysed. Previously in the simulation study, only four banks were considered. Now a total of 15 institutions are considered. The VaR of the system is conditioned on two different events, similar to before. The institutions being at their 90% VaR level and the institutions being at the median level. The results of this study are displayed in table 12. The intensity λ should be higher conditional on the 90% VaR than when conditioned on the median. From table 12, it can be seen that this is indeed the case for most stocks. The BAC and USB stocks are exceptions to this rule. Therefore the $\Delta CoVaR$ of these stocks is negative and thus have the lowest mean values of $\Delta CoVaR$ of all stocks. These results do not seem correct and must be the result of the estimation of the parameters being stuck in a local optimum. If these results are compared to the outcome of the quantile regressions, it seems that the estimated values there do not deviate much from the other stocks. There is no reason to suggest that the Hawkes process produces a correct estimation in this case. The $CoVaR$ for the system estimated by the Hawkes process is in general much higher than the $CoVaR$ estimated by the quantile regressions. This goes for all stocks. Since the differences between values of $CoVaR|VaR$ and $CoVaR|Med$ are also larger, values of $\Delta CoVaR$ are higher for the Hawkes process. Section 4 found that the increased $CoVaR$ estimated by the Hawkes process was due to the model estimating a large increase in volatile periods. This could explain why the mean $CoVaR$ is higher in this case as well.

The simulation study in section 4 found that the quantile regressions give more reliable results than the Hawkes process. Therefore the quantile regressions are used to give an analysis of the systemic risk. Then the Hawkes process' results can be compared to see if it can come to the same conclusions. Among the resulting values of $\Delta CoVaR$ from the quantile regressions, the ones that stand out with the highest mean values are FMCC followed by JPM, NTRS and USB. Interestingly the institution that has the largest impact on the system seems to be the only GSE in the data set, while this stock has a mean ME that is not that large. The same goes for NTRS. It is one of the smaller banks in the list of companies evaluated. There does not seem to be a common factor between these stocks. According to the quantile regressions the stocks with the least impact on the system are BAC and PNC followed closely by MET and WFC. This is also an unexpected result as BAC is the stock with the largest mean ME, and therefore one would expect it to have a massive impact on the system. The opposite seems to be true. Using the Hawkes process, the same results cannot be found. Here PNC, MS, BAC and USB have smaller values of mean $\Delta CoVaR$. However, as was found earlier, the values for BAC and

USB do seem strange. The standard deviations of these time series are much larger than for the other stocks. This indicates that some values are being estimated way too low. Therefore the only stock of which can be said that both methods assign low $\Delta CoVaR$ to is the PNC stock. Other values of $\Delta CoVaR$ estimated by the Hawkes process that stand out are C and AIG. They have the highest mean of all stocks. However, they also have very high standard deviations. This could mean that a small number of values deviate greatly from the mean and increase its value. Table 13 shows the results of the different portfolios and strategies in this scenario.

Table 12: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the system for the Hawkes process and the time series of $CoVaR$ and $\Delta CoVaR$ of the system estimated with the quantile regressions. The standard deviations are beneath the mean values in brackets. j indicates the institution that is set as the variable conditioned on.

	Hawkes process					Quantile Regression		
j:	$\lambda \| VaR$	$\lambda \| Med$	$CoVaR \| VaR$	$CoVaR \| Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR \| VaR$	$CoVaR \| Med$
BAC	0.2193	0.2428	0.1762	0.1956	-0.0194	0.0031	0.0417	0.0386
	(0.1316)	(0.1948)	(0.0937)	(0.1253)	(0.1545)	(0.0003)	(0.0242)	(0.0241)
C	0.2158	0.1531	0.1742	0.1319	0.0423	0.0058	0.0424	0.0366
	(0.1242)	(0.1170)	(0.0886)	(0.0912)	(0.0537)	(0.0112)	(0.0261)	(0.0150)
GS	0.2234	0.1918	0.1777	0.1592	0.0185	0.0073	0.0465	0.0392
	(0.1482)	(0.1554)	(0.1059)	(0.1093)	(0.0651)	(0.0007)	(0.0245)	(0.0250)
JPM	0.2374	0.1992	0.1887	0.1665	0.0222	0.0127	0.0509	0.0382
	(0.1461)	(0.1425)	(0.1025)	(0.1060)	(0.0411)	(0.0033)	(0.0249)	(0.0231)
AIG	0.2497	0.1632	0.1970	0.1414	0.0556	0.0075	0.0455	0.0381
	(0.1687)	(0.1145)	(0.1132)	(0.0912)	(0.0745)	(0.0019)	(0.0219)	(0.0227)
FMCC	0.1603	0.1343	0.1353	0.1183	0.0170	0.0146	0.0498	0.0353
	(0.1075)	(0.1003)	(0.0853)	(0.0810)	(0.0289)	(0.0091)	(0.0287)	(0.0197)
MET	0.2584	0.2143	0.1994	0.1770	0.0224	0.0038	0.0413	0.0375
	(0.1744)	(0.1495)	(0.1156)	(0.1088)	(0.0296)	(0.0040)	(0.0254)	(0.0216)
MS	0.2197	0.2016	0.1648	0.1584	0.0064	0.0082	0.0467	0.0385
	(0.2259)	(0.2226)	(0.1236)	(0.1334)	(0.0335)	(0.0055)	(0.0296)	(0.0242)
PNC	0.2066	0.1835	0.1674	0.1552	0.0122	0.0040	0.0435	0.0395
	(0.1259)	(0.1296)	(0.0898)	(0.0984)	(0.0220)	(0.0016)	(0.0262)	(0.0248)
TD	0.2198	0.1785	0.1765	0.1536	0.0229	0.0089	0.0470	0.0381
	(0.1293)	(0.1045)	(0.0880)	(0.0805)	(0.0393)	(0.0018)	(0.0254)	(0.0237)
TFC	0.2160	0.1792	0.1759	0.1542	0.0217	0.0070	0.0454	0.0384
	(0.1081)	(0.1053)	(0.0733)	(0.0781)	(0.0318)	(0.0034)	(0.0263)	(0.0230)
USB	0.1676	0.1972	0.1408	0.1593	-0.0185	0.0112	0.0480	0.0368
	(0.0887)	(0.2126)	(0.0657)	(0.1331)	(0.1250)	(0.0063)	(0.0284)	(0.0224)
WFC	0.2625	0.2078	0.2002	0.1739	0.0263	0.0044	0.0424	0.0380
	(0.2208)	(0.1501)	(0.1430)	(0.1116)	(0.0471)	(0.0018)	(0.0250)	(0.0233)
HSBC	0.1868	0.1552	0.1558	0.1362	0.0196	0.0067	0.0410	0.0343
	(0.0968)	(0.0924)	(0.0736)	(0.0736)	(0.0333)	(0.0077)	(0.0246)	(0.0173)
NTRS	0.1872	0.1466	0.1554	0.1284	0.0269	0.0113	0.0500	0.0387
	(0.1231)	(0.1014)	(0.0953)	(0.0818)	(0.0453)	(0.0074)	(0.0299)	(0.0243)

These are constructed using the $\Delta CoVaR$ of the system conditional on the institutions. They are compared to each other with the ΔR measures and the benchmark portfolios with the presented performance measures. The Hawkes process-based minimum $\Delta CoVaR$ strategy has the highest mean return and Sharpe ratio. This indicates that it is the best performing portfolio. However, it also has the highest standard deviation. As the goal is to minimise risk with these strategies, this is not a result favouring the Hawkes process. The $\Delta R1$ statistic is in favour of the Hawkes process-based strategy as it has a positive mean. $\Delta R2$ and $\Delta R3$ also indicate the Hawkes process-based strategy's mean returns are closer to the minimum variance strategy. However, when we look at the standard deviations, the Hawkes process gives a much larger standard deviation than the quantile regression and the minimum variance-based strategy. Therefore the quantile regressions still come out on top as the mean values are not significantly different, but the standard deviations favour the quantile regressions. The $CoVaR$ was used to analyse the

Table 13: The different performance measures for the constructed portfolios and strategies are displayed in this table, together with the ΔR measures. Based on the case where the $\Delta CoVaR$ is calculated for the system conditional on the institutions.

Strategy	Mean	Std. Dev.	Sharpe Ratio
GMV	-0.0001	0.0155	-1.3634
1/N	0.0002	0.0224	-0.9291
Min. variance	0.0003	0.0216	-0.9619
Hawkes $CoVaR$	0.0004	0.0318	-0.6491
Quantile $CoVaR$	-0.0001	0.0250	-0.8457
$\Delta R1$	0.0005	0.0288	N.A.
$\Delta R2$	0.0002	0.0252	N.A.
$\Delta R3$	-0.0004	0.0194	N.A.

impact of the 15 different institutions on the system. Although the methods are both able to assign different levels of systemic risk for different companies. There seems to be no trend among these results. It is confirmed once more in table 13 that the quantile regressions deliver more accurate systemic risk evaluations.

5.2 Institutions conditional on system

In this section, the reverse relation from the previous section is studied. The $CoVaR$ is again estimated using the two methods for two situations: the returns of the 15 institutions conditioned on the system reaching its 90% VaR level and conditioned on the system at its median level. The results are displayed in table 14. Again the expectation is that levels of $CoVaR$ are higher when the system is at risk than when the system is in a normal state. Therefore in the Hawkes process $\lambda|VaR$ should have a higher mean than $\lambda|Med$. From table 14, it can be seen that this is indeed the case for most stocks, but there are some exceptions. These exceptions are the mean intensities for BAC, GS, MS and USB. From the quantile regressions, it seems that institutions that are not banks have in general higher values of $CoVaR$. AIG, FMCC, MET and MS all have mean $CoVaR$ that are consistently higher than 0.04. In contrast, most banks are below this value. This would indicate that institutions that are non-banks are subject to more risk when the whole financial system has reached its 90% VaR than banks. The stock with the largest mean values for $\Delta CoVaR$ estimated by the Hawkes process is by far TD, followed by MET, TFC and HSBC. The quantile regressions result in a different order. According to this method, the stocks with the highest $\Delta CoVaR$ are FMCC and AIG, followed by MS and GS. Four institutions that again are not banks. The lowest values of $\Delta CoVaR$ for Hawkes process-based results $\Delta CoVaR$ are the stocks with negative results: BAC, GS, USB and MS. The quantile regressions' outcome suggests that the lowest values of $\Delta CoVaR$ are for BAC, JPM, HSBC and NTRS. So of these stocks, only BAC is suggested by both methods as a stock with low systemic risk. In the simulation study in section 4.2, it was found that for the BAC, C and GS stocks, the $\Delta CoVaR$ is estimated too low. Here it is found that BAC and GS, two of these three stocks even have negative values of $\Delta CoVaR$. In table 15, the summary statistics of the returns of the

Table 14: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the institutions for the Hawkes process and the $CoVaR$ and $\Delta CoVaR$ of the institutions estimated with the quantile regressions. The standard deviations are beneath the mean values in brackets. i indicates the institution for which the measures are estimated.

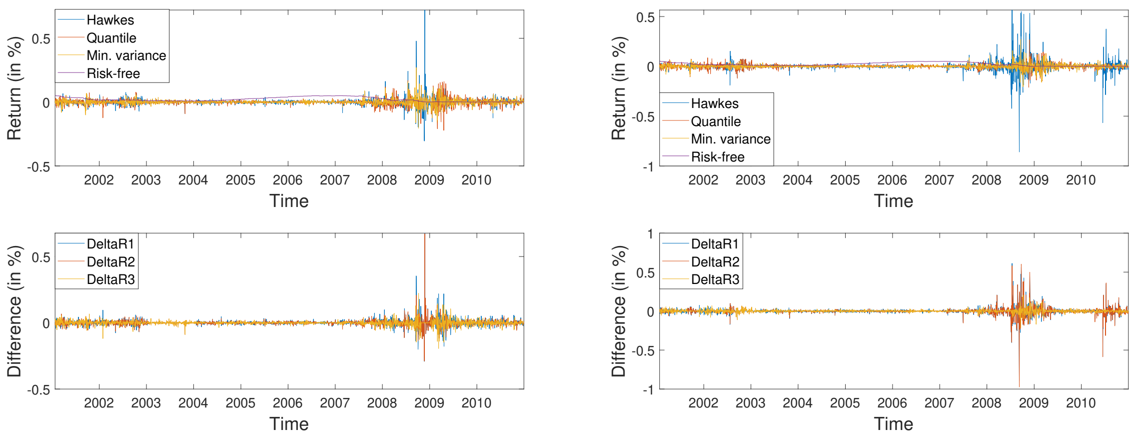
	Hawkes process					Quantile Regression		
i:	$\lambda\|VaR$	$\lambda\|Med$	$CoVaR\|VaR$	$CoVaR\|Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR\|VaR$	$CoVaR\|Med$
BAC	0.1996	0.2632	0.1593	0.2065	-0.0472	0.0029	0.0495	0.0466
	(0.1255)	(0.2252)	(0.0859)	(0.1394)	(0.1628)	(0.0009)	(0.0302)	(0.0301)
C	0.2321	0.1957	0.1830	0.1645	0.0185	0.0074	0.0596	0.0522
	(0.1087)	(0.1258)	(0.0699)	(0.0922)	(0.0346)	(0.0057)	(0.0388)	(0.0331)
GS	0.2141	0.2240	0.1680	0.1754	-0.0074	0.0080	0.0469	0.0389
	(0.1552)	(0.2267)	(0.0986)	(0.1354)	(0.0585)	(0.0061)	(0.0250)	(0.0191)
JPM	0.2934	0.2424	0.2299	0.2018	0.0281	0.0030	0.0411	0.0381
	(0.1329)	(0.1345)	(0.0856)	(0.0940)	(0.0415)	(0.0039)	(0.0145)	(0.0181)
AIG	0.2216	0.1967	0.1731	0.1649	0.0082	0.0088	0.0775	0.0687
	(0.1438)	(0.1403)	(0.1062)	(0.1078)	(0.0226)	(0.0079)	(0.0504)	(0.0428)
FMCC	0.1641	0.1433	0.1330	0.1241	0.0089	0.0109	0.0761	0.0652
	(0.1197)	(0.1066)	(0.0924)	(0.0853)	(0.0252)	(0.0084)	(0.0495)	(0.0412)
MET	0.296	0.1995	0.2277	0.1696	0.0581	0.0047	0.0462	0.0415
	(0.1466)	(0.1478)	(0.0857)	(0.1151)	(0.1230)	(0.0023)	(0.0267)	(0.0245)
MS	0.1914	0.1947	0.1460	0.1515	-0.0055	0.0077	0.0545	0.0468
	(0.1471)	(0.2156)	(0.0862)	(0.1313)	(0.0622)	(0.0031)	(0.0233)	(0.0262)
PNC	0.2575	0.1960	0.2023	0.1681	0.0342	0.0068	0.0436	0.0368
	(0.1344)	(0.1102)	(0.0728)	(0.0789)	(0.0328)	(0.0050)	(0.0259)	(0.0209)
TD	0.1283	0.0374	0.0351	-0.1748	0.2099	0.0031	0.0287	0.0255
	(0.3705)	(0.5225)	(0.5472)	(1.0726)	(0.5890)	(0.0023)	(0.0140)	(0.0118)
TFC	0.2553	0.2013	0.2073	0.1725	0.0348	0.0058	0.0393	0.0335
	(0.1220)	(0.1123)	(0.0827)	(0.0809)	(0.0593)	(0.0034)	(0.0221)	(0.0188)
USB	0.1970	0.2134	0.1602	0.1669	-0.0066	0.0072	0.0424	0.0352
	(0.1220)	(0.2521)	(0.0807)	(0.1555)	(0.1432)	(0.0056)	(0.0254)	(0.0200)
WFC	0.2663	0.2212	0.2097	0.1837	0.0260	0.0041	0.0428	0.0387
	(0.1828)	(0.1614)	(0.1163)	(0.1185)	(0.0711)	(0.0024)	(0.0261)	(0.0237)
HSBC	0.2142	0.1611	0.1774	0.1428	0.0346	0.0025	0.0312	0.0287
	(0.1120)	(0.0796)	(0.0825)	(0.0626)	(0.0340)	(0.0007)	(0.0157)	(0.0163)
NTRS	0.2252	0.1743	0.1838	0.1509	0.0329	0.0120	0.0432	0.0313
	(0.1199)	(0.1161)	(0.0831)	(0.0845)	(0.0397)	(0.0093)	(0.0238)	(0.0148)

minimum $\Delta CoVaR$ strategies are shown. They are compared to the benchmark portfolios and the minimum variance strategy using the different ΔR measures. The result that stands out the most is that the quantile regression-based minimum $\Delta CoVaR$ strategy has the highest mean return. The standard deviation is not that bad either compared to the two benchmark portfolios and is less than half of the standard deviation of the Hawkes process-based strategy. The Sharpe ratios suggest that the Hawkes process-based strategy performs best, which is unexpected as it has the highest standard deviation and the lowest mean returns. This usually should result in a worse Sharpe ratio. From the Sharpe ratios, it can be seen that it was better to invest in a risk-free product throughout this period. Which is understandable as the financial crisis in 2008 hit the stock market hard. If this is taken into consideration, these portfolios and strategies do not perform badly at all, especially the quantile $CoVaR$-based strategy as it outperforms even the 1/N portfolio in mean returns and Sharpe ratio. However, the increase in performance is very modest. The difference in the performance of the two methods is even more clear from the ΔR measures. $\Delta R1$ is in favour of the quantile regressions, and the big difference between $\Delta R2$ and $\Delta R3$ also confirms this. Again the quantile regressions prove superior in describing the systemic risk and thus result in less risky investment strategy returns. In figure 11, the

Table 15: The different performance measures for the constructed portfolios and strategies are displayed in this table, together with the ΔR measures. Based on the case where the $\Delta CoVaR$ is calculated for the institutions conditional on the system.

Strategy	Mean	Std. Dev.	Sharpe Ratio
GMV	-0.0001	0.0155	-1.3634
1/N	0.0002	0.0224	-0.9291
Min. variance	0.0003	0.0216	-0.9619
Hawkes $CoVaR$	-0.0005	0.0542	-0.3979
Quantile $CoVaR$	0.0004	0.0236	-0.8713
$\Delta R1$	-0.0010	0.0510	N.A.
$\Delta R2$	-0.0008	0.0508	N.A.
$\Delta R3$	0.0002	0.0199	N.A.

returns of the minimum $\Delta CoVaR$ and the minimum variance strategies are displayed to support the findings of table 13 and table 15. From these graphs, it is quite clear that the Hawkes process-based strategy creates more volatile returns than the quantile regression-based strategy. Especially from 2008 to 2010 large positive and negative returns are made by the strategy. The $\Delta R2$ measure is in accordance with this result. The quantile regression-based strategy even seems more similar to the minimum variance strategy than the Hawkes process is to the quantile regression-based strategy. For the institutions conditional on the system in (b) of figure 11, the Hawkes process-based strategy performs even worse when it comes to risk. This shows from the large positive and negative returns around the financial crisis for the Hawkes process-based strategy. This is expected since both the simulation study and table 14 find that the Hawkes process describes the $\Delta CoVaR$ badly in this scenario. Looking at the different ΔR measures confirms these observations. Therefore it can be concluded that in both scenarios the quantile regressions perform better. From the analysis of the historical data using the two methods, it was found that the Hawkes process does not reliably estimate levels of $\Delta CoVaR$ as the values for $CoVaR|VaR$ relative to $CoVaR|Med$ are unexpected for some stocks. The findings using the Hawkes process often contradict the findings from the quantile regressions. As the quantile regressions are the established method and the Hawkes process is heavily compromised in this study, the quantile regressions are used to draw conclusions. From a short analysis using the quantile regressions, it was found that FMCC, JPM, NTRS and USB have the highest impact on the system as a whole and should be the first to receive financial aid in trying times. The non-banks in the data set have higher levels of $CoVaR$ in general and are more susceptible to risk than banks. Surprisingly the BAC stock that has on average the largest share in the systemic returns does not stand out among the other stocks when it comes to impacting the system or being impacted by the system. The different strategies' returns all point towards the quantile regression-based strategy describing the systemic risk of the stocks better than the Hawkes process.



(a) system|institutions

(b) institutions|system

Figure 11: The top figures show the course of the returns of the constructed minimum $\Delta CoVaR$ and minimum variance strategies. The bottom figures display the ΔR measures over time. On the left in (a), the system's returns conditional on the institutions are displayed. On the right in (b), the institutions' returns conditional on the system is shown. The purple graph is the yield of the bond set as the risk-free rate. The orange graphs show the quantile regression-based strategy returns over time. The blue graphs represent the Hawkes process-based strategy returns over time, and the yellow graph is the minimum variance strategy return over time.

6 Conclusion

This paper investigates whether multivariate Hawkes processes accurately model the risk measures $CoVaR$ and $\Delta CoVaR$. The type of intensity function used in the Hawkes process is the ETAS model by Ogata (1988). After unsuccessful estimation of the multivariate Hawkes process using the EM algorithm shown in section 8, the MLE is used to estimate the ETAS model parameters. To evaluate the models; first, a simulation study was performed, in which, four sets of returns are simulated 100 times. Next, an analysis of historical data of 15 different stocks of large financial institutions from the US was performed. For each study, two cases were inspected. The system conditional on the institutions and the institutions conditional on the system. These studies' results are evaluated by strategies that select the stock with the lowest $\Delta CoVaR$ resulting from the two models.

From the simulation study, it was found that the Hawkes process is able to estimate the $CoVaR$ of the system conditional on events in an institution quite well. However, not as well as the quantile regressions. The Hawkes process gives less accurate results than the quantile regressions because the Hawkes process tends to overestimate the $\Delta CoVaR$ in volatile times compared to the quantile regressions. Furthermore, the minimum $\Delta CoVaR$ strategies give decent returns compared to the benchmark portfolios, especially considering a large portion of the period analysed is during a financial crisis. Of the two minimum $\Delta CoVaR$ strategies, the Hawkes process-based strategy results in more volatile returns than the quantile regression-based strategy. Therefore the quantile regression-based $\Delta CoVaR$ strategy is superior when it comes to selecting the stock with the least systemic risk. In the simulation study, the $CoVaR$ of the institutions conditional on the system is also analysed. In this reverse relation, the Hawkes process cannot accurately describe the $CoVaR$ resulting in values that are way smaller compared to the more reliable quantile regressions. So the estimation of the systems' $CoVaR$ conditional on the institutions is better suited to the Hawkes process than the reverse relation.

From the analysis of the historical data, it was found that the Hawkes process estimates levels of $\Delta CoVaR$ less accurately than the quantile regressions. The findings using the Hawkes process often contradict the findings from the quantile regressions. As the quantile regressions are the established method, and the Hawkes process is heavily compromised in this study, conclusions of the historical data analysis are drawn from the quantile regressions. It was found that FMCC, JPM and USB have the highest impact on the system as a whole and should therefore have priority in receiving financial aid in trying times. The non-banks in the data set have higher levels of $CoVaR$ in general and are more susceptible to risk than banks. The returns of the minimum $\Delta CoVaR$ strategy based on the quantile regressions prove to have higher means and lower standard deviations, making it the best strategy between the two to select the stock with the least systemic risk and is, therefore, the method that describes the systemic risk best.

7 Discussion

This study of the multivariate Hawkes process was severely limited by a lack of computation power. To avoid extremely lengthy computations, the multivariate Hawkes process had to be compromised to two variables at a time. This can of course not capture all intricate relations between the multiple institutions that a fully-fledged multivariate Hawkes process potentially could describe. The estimation of the parameters is computationally heavy due to the MLE used. The MLE also tends to get stuck in a local maximum, resulting in estimated intensities of size 10^{30} even though it is supposed to lie between 0 and 1 as it is a probability. Alternatives to the MLE for estimating the univariate Hawkes process exist, such as the EM algorithm by Veen and Schoenberg (2008). Adapting this to a multivariate setting could fix the computational burden of the Hawkes processes. However, this paper was not successful in adapting this algorithm to a multivariate setting. Another limitation due to the computation time is the number of institutions evaluated. As a result, only 4 out of 15 institutions could be evaluated in the simulation study with a limited amount of 100 simulations. Future research could be done on larger sets of data with more observations, institutions and simulations. The Hawkes process based on observations of the POT model has a specific range of observations for which it works well. The 90% VaR was chosen because 10% of the observations being marks suits the model well. For example, choosing the 5% VaR and 1% VaR means the number of observations in a data set is heavily reduced resulting in a less accurate estimation of the model. On the other hand, too many observations often result in the intensity function going to infinity. The quantile regressions in this study were based on four state variables. To further increase quantile regressions' accuracy, one could improve the model by changing or adding state variables. Although the $CoVaR$ and $\Delta CoVaR$ contain much useful information, they are sometimes hard to interpret and can be confusing. This study evaluated the models by constructing strategies where the stock with the lowest $\Delta CoVaR$ is selected. As this paper was not interested in these strategies' absolute performance, but the relative performance between the two methods, the $\Delta CoVaR$ strategies sufficed as a tool for comparison. In theory, portfolios have less risk due to diversification. Constructing a minimum $\Delta CoVaR$ portfolio for both methods would find returns similar to those of the GMV portfolio and could have more practical implications. However, finding such a portfolio for a non-linear model like the Hawkes process is not straightforward as the minimisation is a non-linear optimisation problem. A grid search could be a solution to this problem. However, for every grid point in the search, the parameters need to be re-estimated, which would result in a tremendous amount of computation time needed to get the results for every different scenario. Therefore it is out of the scope of this paper.

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List of Tables

List of Figures

8 Appendix

8.1 Estimation using Expectation Maximisation algorithm

Maximum Likelihood Estimation (MLE) can lead to a variety of problems. Firstly MLE may have trouble converging because the likelihood function can be very flat. Secondly the outcome may depend on the starting values. The likelihood function can be multimodal and end up in a local maximum instead of the global maximum depending on which initial values one puts into an optimisation procedure such as the Newton-Raphson routine. Veen and Schoenberg (2008) suggest using an algorithm based on the Expectation Maximisation algorithm (EM). The ETAS model described in section 2.2 can be seen as an incomplete data problem where there is an unobservable quantity UQ_i which acts as a indicator if shock s is a main event ($UQ_s = 0$) or whether shock s is an aftershock of another shock r ($UQ_s = r$) where $s, r \in S$ the set of all shocks . As Veen and Schoenberg (2008) consider actual earthquakes, they split up the spatial observation window into cells. Here the data is already split up since there are m time series considered in this paper. Therefore background intensities $\mu = (\mu_1, \dots, \mu_m)$ can be defined for each time series. We also define $\nu_j = \mu_j \cdot T$ where T is the time length window. The actual number of main events is denoted as N_j and is modelled as a Poisson random variable with expectation ν_j. Under the assumption that the complete branching structure of the observed ETAS process is known, that is UQ_s is known for all s. We can write the complete data log likelihood as follows:

$$\ell_c(\theta) = \sum_{j=1}^{m} \Big(-\log(N_j!) - \nu_j + N_j \log(\nu_j) \Big) + \sum_{s} \Big(-\log(L_s!) - G_s(\theta) + L_s \log(G_s(\theta)) \Big) + \sum_{s:\, UQ_s \neq 0} \Big(\log(\rho) + \rho \log(\gamma) - (1+\rho)\log(t_s - t_{UQ_s} + c) - \log(\pi) \Big) \tag{8.1}$$

In this equation the first sum relates to the actual number of main events in each of the m time series. The second sum relates to the number of aftershocks L_s triggered by main event s. This process follows a Poisson distribution with expectation $G_s(\theta)$ using the triggering function g as in equation (2.5).

$$G_s(\theta) = \int_0^\infty g(t - t_j; w_j)dt = \frac{\psi e^{\beta w_j} \gamma^{-\rho}}{\rho} \tag{8.2}$$

The steps in the EM Algorithm are as follows:

We first calculate the probabilities $P^{(n+1)}(UQ_s = r) \forall s, r$ based on the current estimate $\theta_{EM}^{(n)}$

$$P^{(n+1)}(UQ_s = r) = g(t_s - t_r; w_r | \theta_{EM}^{(n)}) / \Big(\hat{\mu}^n_{j:s \in j} + \sum_{p=1}^{s-1} g(t_s - t_p; w_p | \theta_{EM}^{(n)}) \Big) \tag{8.3}$$

These are the probabilities that shock s is an aftershock of shock r. With these probabilities the expected number of main events and aftershocks can be expressed.

$$\begin{aligned}\hat{N}_j^{(n+1)} &= \sum_{s\in j, s\geq 2}\Big(1-\sum_{r=1}^{s-1}P^{(n+1)}(UQ_s=r)\Big),\\ \hat{L}_s^{(n+1)} &= \sum_{p\geq s+1}P^{(n+1)}(UQ_p=s)\end{aligned} \tag{8.4}$$

Using these expressions the expected complete data log likelihood can be written in the following way:

$$\begin{aligned}E_{\theta^{(n)}_{EM}}[\ell_c(\theta)] =& \sum_j\Big(-\log(\Gamma(\hat{N}_j^{(n)}+1))-\nu_j+\hat{N}_j^{(n)}\log(\nu_j)\Big)\\ &+\sum_s\Big(-\log(\Gamma(\hat{L}_s^{(n)}+1))-G_s(\theta)+\hat{L}_s^{(n)}\log(G_s(\theta))\Big)+\\ &\sum_{s\geq 2}\sum_{r=1}^{s-1}P^{(n)}(UQ_s=r)\\ &\times\Big(\log(\rho)+\rho\log(\gamma)-(1+\rho)\log(t_s-t_{UQ_s}+c)-\log(\pi)\Big)\end{aligned} \tag{8.5}$$

The factorials of equation (8.1) are replaced by the gamma function $x! = \Gamma(x+1)$ in this equation. Equation (8.5) is maximised in the M step of the EM algorithm. This can be done in a similar way to Maximum Likelihood estimation, namely by setting the partial derivatives of parameters γ and ρ in θ representing the aftershock distribution equal to 0.

$$0=\frac{\partial\ell_c(\theta)}{\partial\gamma}=\sum_{s:UQ_s\neq 0}\Big(\frac{\rho}{\gamma}-\frac{1+\rho}{t_s-t_{UQ_s}+\gamma}\Big)+\frac{\rho}{\gamma}\sum_s\Big(G_s(\theta)-L_s\Big) \tag{8.6}$$

$$0=\frac{\partial\ell_c(\theta)}{\partial\rho}=\sum_{s:UQ_s\neq 0}\Big(\frac{1}{\rho}+\log(\gamma)-\log(t_s-t_{UQ_s}+\gamma)\Big)+\Big(\frac{1}{\rho}+\log(\gamma)\Big)\sum_s\Big(G_s(\theta)-L_s\Big) \tag{8.7}$$

$$0=\frac{\partial\ell_c(\theta)}{\partial\psi}=-\frac{1}{\psi}\sum_s\Big(G_s(\theta)-L_s\Big) \tag{8.8}$$

$$0=\frac{\partial\ell_c(\theta)}{\partial\beta}=-\sum_s\Big((G_s(\theta)-L_s)w_s\Big) \tag{8.9}$$

Because the partial derivative to ψ implies that $\sum_s(G_s(\theta)-L_s)$ is equal to zero. This means these terms can be ignored in equation (8.6) and equation (8.7). This gives us the following solutions for the maximisation which only depend on ρ and γ:

$$\frac{\rho}{(1+\rho)\gamma}=\frac{1}{L}\sum_{s:UQ_s\neq 0}\frac{1}{t_s-t_{UQ_s}+\gamma} \tag{8.10}$$

$$\frac{1}{\rho}+\log(\gamma)=\frac{1}{L}\sum_{s:UQ_s\neq 0}\log(t_s-t_{UQ_s}+\gamma) \tag{8.11}$$

This equation system is solved by choosing a strictly positive starting value for γ and iterate between computing the right sides of equation (8.10) and equation (8.11) and updating the values of γ and ρ by solving the equation system using the values gotten in the first step.

When maximising the expected likelihood in equation (8.5) the parameters are replaced by their estimates in the current step and include the estimated probabilities into the solution such that equation (8.10) and equation (8.11) become:

$$\frac{\hat{\rho}^{(n)}}{(1+\hat{\rho}^{(n)})\hat{\gamma}^{(n)}} = \frac{1}{\hat{L}^{(n)}} \sum_{s\geq 2} \sum_{r=1}^{s-1} P^{(n)}(UQ_s = r) \times \frac{1}{t_s - t_r + \hat{\gamma}^{(n)}} \tag{8.12}$$

$$\frac{1}{\hat{\rho}^{(n)}} + \log(\hat{\gamma}^{(n)}) = \frac{1}{\hat{L}^{(n)}} \sum_{s\geq 2} \sum_{r=1}^{s-1} P^{(n)}(UQ_s = r) \times \log(t_s - t_r + \hat{\gamma}^{(n)}) \tag{8.13}$$

Where $\hat{L}^{(n)} = \sum_s \hat{L}_s^{(n)}$ is equal to the expected amount of triggered earthquakes. We repeat the algorithm till some convergence criterion is reached. Veen and Schoenberg (2008) use the criterion that all parameters have converged to 4 significant digits.

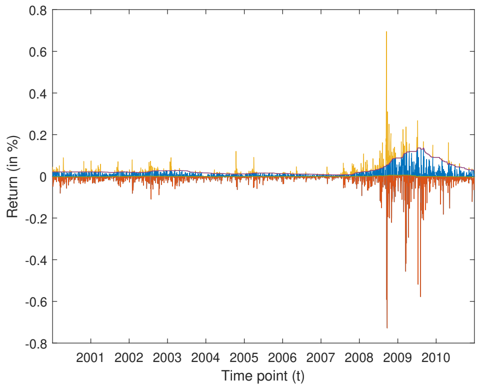
8.2 Tables & Graphs

Table 16: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the system for the Hawkes process and the time series of $CoVaR$ and $\Delta CoVaR$ of the system estimated with the quantile regressions. This is the scenario where the weights to calculate systemic returns are equal. The standard deviations are beneath the mean values in brackets. j indicates the institution that is set as the variable conditioned on.

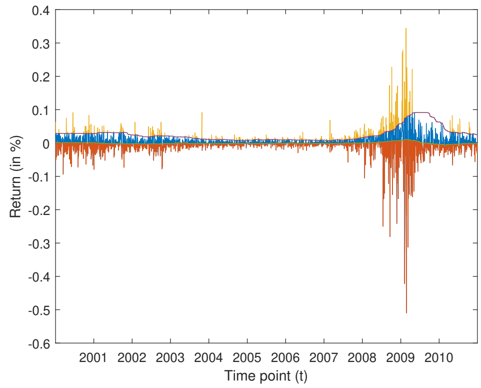
	Hawkes process					Quantile Regression		
j:	$\lambda \vert VaR$	$\lambda \vert Med$	$CoVaR \vert VaR$	$CoVaR \vert Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR \vert VaR$	$CoVaR \vert Med$
BAC	0.2656	0.2230	0.2154	0.1890	0.0264	0.0062	0.0381	0.0319
	(0.1228)	(0.1204)	(0.0854)	(0.0888)	(0.0460)	(0.0050)	(0.0189)	(0.0144)
C	0.2587	0.2096	0.2102	0.1770	0.0332	0.0081	0.0396	0.0315
	(0.1237)	(0.1352)	(0.0852)	(0.0984)	(0.1105)	(0.0067)	(0.0194)	(0.0132)
GS	0.2997	0.2302	0.2386	0.1929	0.0457	0.0032	0.0349	0.0317
	(0.1518)	(0.1447)	(0.1063)	(0.0985)	(0.0636)	(0.0007)	(0.0196)	(0.0194)
JPM	0.3539	0.2480	0.2676	0.2054	0.0622	0.0084	0.0419	0.0336
	(0.2145)	(0.1531)	(0.1219)	(0.1024)	(0.0809)	(0.0082)	(0.0218)	(0.0147)
AIG	0.2153	0.1613	0.1789	0.1414	0.0375	0.0074	0.0390	0.0315
	(0.1315)	(0.1047)	(0.1028)	(0.0874)	(0.0605)	(0.0047)	(0.0180)	(0.0163)
FMCC	0.2293	0.1652	0.1894	0.1430	0.0464	0.0082	0.0382	0.0300
	(0.1481)	(0.1315)	(0.1094)	(0.0973)	(0.0626)	(0.0099)	(0.0218)	(0.0130)
MET	0.2813	0.2197	0.2235	0.1850	0.0386	0.0063	0.0348	0.0285
	(0.1629)	(0.1378)	(0.1058)	(0.0990)	(0.0500)	(0.0060)	(0.0200)	(0.0156)
MS	0.1868	0.1779	0.1541	0.1502	0.0040	0.0019	0.0342	0.0322
	(0.1333)	(0.1468)	(0.0953)	(0.1051)	(0.0287)	(0.0008)	(0.0198)	(0.0198)
PNC	0.2769	0.2173	0.2247	0.1855	0.0392	0.0038	0.0394	0.0356
	(0.1297)	(0.1161)	(0.0896)	(0.0828)	(0.0516)	(0.0013)	(0.0199)	(0.0195)
TD	0.2248	0.1904	0.1862	0.1632	0.0230	0.0054	0.0398	0.0344
	(0.1116)	(0.1176)	(0.0812)	(0.0916)	(0.0417)	(0.0035)	(0.0201)	(0.0187)
TFC	0.2735	0.1982	0.2194	0.1717	0.0477	0.0078	0.0413	0.0334
	(0.1544)	(0.1105)	(0.1018)	(0.0816)	(0.0734)	(0.0042)	(0.0199)	(0.0177)
USB	0.2278	0.1820	0.1888	0.1568	0.0320	0.0078	0.0409	0.0331
	(0.1222)	(0.1213)	(0.0875)	(0.0917)	(0.0471)	(0.0081)	(0.0218)	(0.0158)
WFC	0.2978	0.2110	0.2300	0.1804	0.0496	0.0035	0.0368	0.0332
	(0.2056)	(0.1167)	(0.1146)	(0.0831)	(0.0731)	(0.0025)	(0.0191)	(0.0180)
HSBC	0.2687	0.2106	0.2199	0.1796	0.0402	0.0062	0.0360	0.0298
	(0.1215)	(0.1245)	(0.0872)	(0.0915)	(0.0494)	(0.0086)	(0.0199)	(0.0129)
NTRS	0.2681	0.1942	0.2191	0.1681	0.0510	0.0065	0.0383	0.0319
	(0.1382)	(0.1155)	(0.0992)	(0.0849)	(0.0621)	(0.0054)	(0.0219)	(0.0176)

Table 17: This table displays the summary statistics of the estimated time series from 2001 to 2010 of intensities, $CoVaR$ and $\Delta CoVaR$ of the institutions for the Hawkes process and the $CoVaR$ and $\Delta CoVaR$ of the institutions estimated with the quantile regressions. This is the scenario where the weights to calculate the systemic returns are equal. The standard deviations are beneath the mean values in brackets. i indicates the institution for which the measures are estimated.

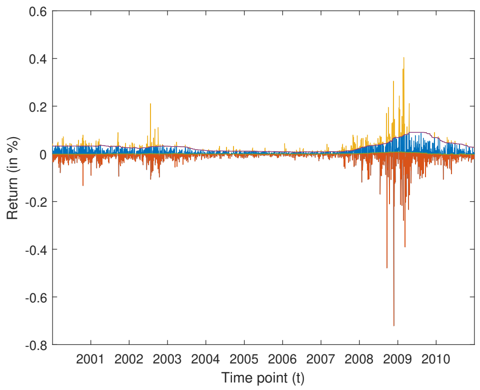
	Hawkes process					Quantile Regression		
j:	$\lambda \mid VaR$	$\lambda \mid Med$	$CoVaR \mid VaR$	$CoVaR \mid Med$	$\Delta CoVaR$	$\Delta CoVaR$	$CoVaR \mid VaR$	$CoVaR \mid Med$
BAC	0.2369	0.2167	0.1873	0.1802	0.0072	0.0024	0.0527	0.0502
	(0.1289)	(0.1366)	(0.0874)	(0.1010)	(0.0234)	(0.0091)	(0.0309)	(0.0258)
C	0.2325	0.2519	0.1797	0.2042	-0.0245	0.0057	0.0583	0.0526
	(0.1468)	(0.1485)	(0.0976)	(0.1037)	(0.1026)	(0.0124)	(0.0358)	(0.0265)
GS	0.2682	0.2521	0.2075	0.2002	0.0073	0.0052	0.0421	0.0369
	(0.1679)	(0.2104)	(0.0981)	(0.1227)	(0.0593)	(0.0024)	(0.0188)	(0.0177)
JPM	0.3136	0.2645	0.2420	0.2150	0.0270	0.0039	0.0417	0.0379
	(0.1507)	(0.1662)	(0.0864)	(0.1064)	(0.0377)	(0.0036)	(0.0210)	(0.0176)
AIG	0.2176	0.1893	0.1714	0.1621	0.0093	0.0025	0.0670	0.0645
	(0.1180)	(0.1049)	(0.0890)	(0.0859)	(0.0240)	(0.0098)	(0.0390)	(0.0346)
FMCC	0.1593	0.1469	0.1277	0.1241	0.0035	0.0049	0.0665	0.0616
	(0.1400)	(0.1415)	(0.1043)	(0.1085)	(0.0186)	(0.0153)	(0.0427)	(0.0322)
MET	0.2798	0.2317	0.2140	0.1883	0.0258	0.0019	0.0466	0.0447
	(0.1642)	(0.1847)	(0.0901)	(0.1194)	(0.0527)	(0.0036)	(0.0247)	(0.0227)
MS	0.2123	0.2088	0.1589	0.1628	-0.0039	0.0044	0.0517	0.0472
	(0.1712)	(0.2157)	(0.0938)	(0.1285)	(0.0541)	(0.0013)	(0.0248)	(0.0240)
PNC	0.2489	0.1915	0.1947	0.1653	0.0294	0.0050	0.0435	0.0386
	(0.1458)	(0.1004)	(0.0789)	(0.0727)	(0.0265)	(0.0077)	(0.0233)	(0.0175)
TD	0.2260	0.1743	0.1872	0.1524	0.0348	0.0031	0.0284	0.0253
	(0.1071)	(0.0926)	(0.0792)	(0.0745)	(0.0448)	(0.0032)	(0.0123)	(0.0100)
TFC	0.2607	0.2129	0.2130	0.1817	0.0313	0.0030	0.0388	0.0358
	(0.1059)	(0.1159)	(0.0705)	(0.0840)	(0.0388)	(0.0073)	(0.0207)	(0.0163)
USB	0.2356	0.1685	0.1871	0.1469	0.0402	0.0044	0.0397	0.0353
	(0.1445)	(0.0995)	(0.0905)	(0.0783)	(0.0493)	(0.0039)	(0.0173)	(0.0183)
WFC	0.2636	0.2182	0.2097	0.1827	0.0270	0.0040	0.0439	0.0399
	(0.1334)	(0.1405)	(0.0764)	(0.0954)	(0.0412)	(0.0078)	(0.0254)	(0.0196)
HSBC	0.2759	0.2035	0.2198	0.1764	0.0434	0.0018	0.0315	0.0296
	(0.1494)	(0.0963)	(0.0939)	(0.0690)	(0.0404)	(0.0014)	(0.0157)	(0.0144)
NTRS	0.2359	0.1745	0.1934	0.1521	0.0412	0.0022	0.0328	0.0307
	(0.0893)	(0.1001)	(0.0576)	(0.0719)	(0.0403)	(0.0009)	(0.0138)	(0.0139)



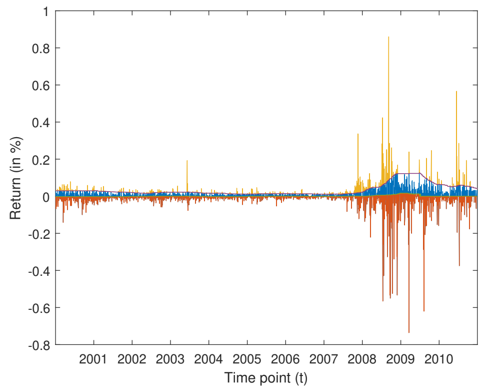
(a) AIG



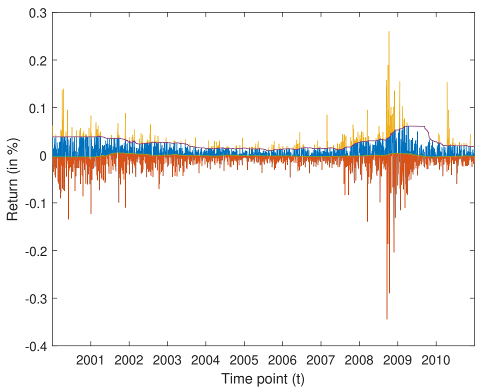
(b) BAC



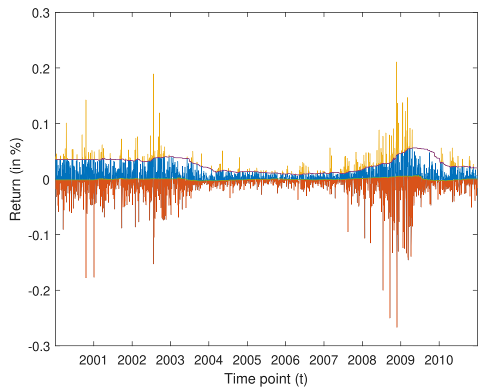
(c) C



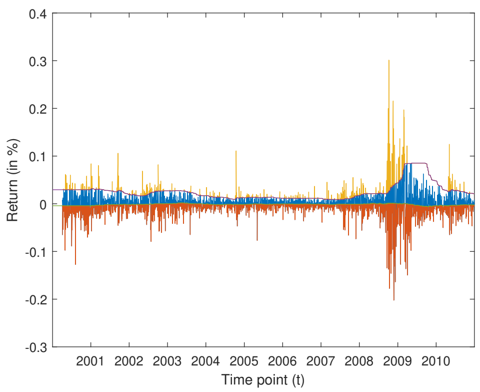
(d) FMCC



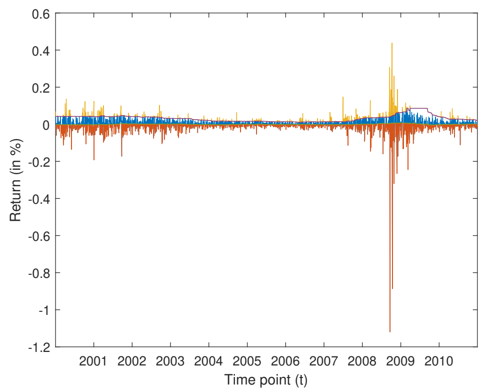
(e) GS



(f) JPM

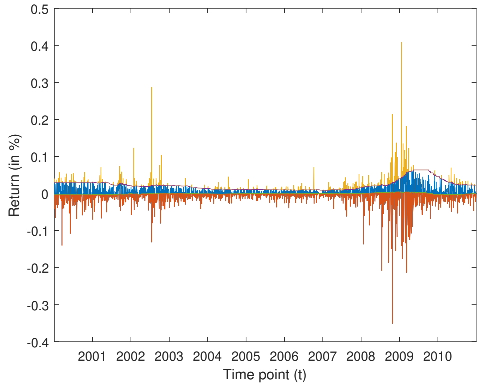


(g) MET

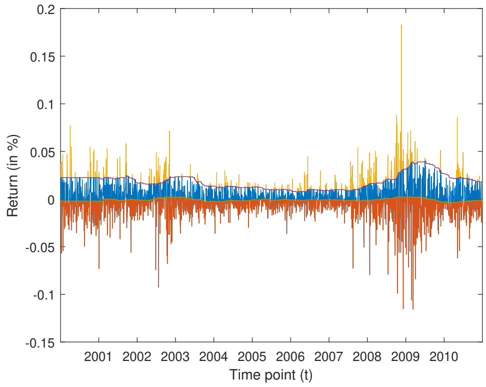


(h) MS

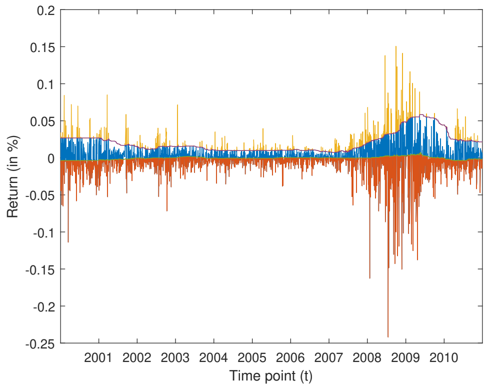
Figure 12



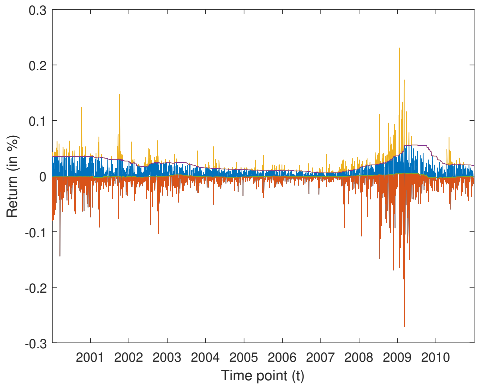
(a) PNC



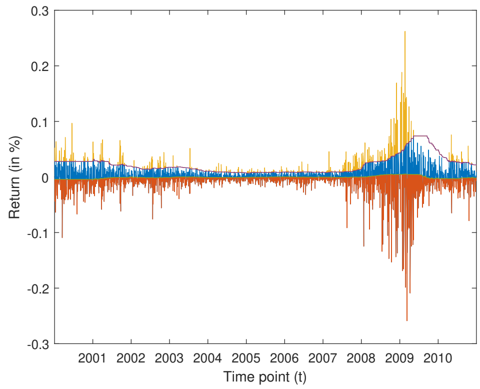
(b) TD



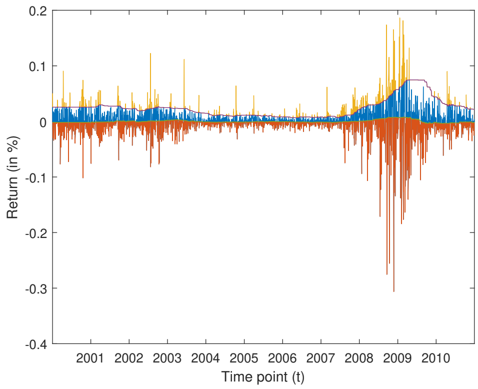
(c) TFC



(d) USB



(e) WFC



(f) System

Figure 13: In figure 12 and figure 13 the losses of the different institutions are displayed. The different colors indicate the following: The purple line is the 90% *VaR* of the institution. Yellow indicates that the loss is greater than the *VaR*. The green line represents the median of the losses. Blue indicates values larger than the median and red indicates values smaller than the median. In figure 13 (f) The losses of the system are shown.

Matlab Codes

Main estimation script

This Matlab script runs the estimation of the parameters of the Hawkes process for the simulation study and the historical data analysis.

```
%% Main script for estimating Hawkes process
clear
close all
simulationstudy=0;
global timeseries
timeseries = 2;
global burnin
burnin =250;
institutions = 15;
statevar = 4;
%% Historical data
% This part of the script runs the estimation of the parameters for the
% historical data.
    [Yreal90, Qreal90, Medreal90, Xreal90, Statevarreal90 ,timeWindowreal90]= datamutator(institutions, statevar, 0.90, burnin, false);
    [Yreal90eq, Qreal90eq, Medreal90eq, Xreal90eq, Statevarreal90eq ,timeWindowreal90eq]= datamutator(institutions, statevar, 0.90, burnin, true);
    % Split the estimation of the historical data into 11 parts to reduce computation time (per 250 days).
    interval = [1 (1:11)*250];
    RealResults90(10).Results = [];
    RealResults90eq(10).Results = [];
    for i=1:10
    t1 = interval(i);
    t2 = interval(i+2);
    [Realoutput90] = Scriptfunction(Yreal90(t1:t2,:), Qreal90(t1:t2,:), Medreal90(t1:t2,:), Xreal90(t1:t2,:), 0.90, [], Statevarreal90(t1:t2,:));
    RealResults90(i).Results = Realoutput90;
    [Realoutput90eq] = Scriptfunction(Yreal90eq(t1:t2,:), Qreal90eq(t1:t2,:), Medreal90eq(t1:t2,:), Xreal90eq(t1:t2,:), 0.90, [], Statevarreal90eq(t1:t2,:));
    RealResults90eq(i).Results = Realoutput90eq;
    end
    save('realresults1')
    [muThetaReal90, sigmaThetaReal90, comptimeReal90] = distributioncalc(institutions, 10, RealResults90);
    [muThetaReal90eq, sigmaThetaReal90eq, comptimeReal90eq] = distributioncalc(institutions, 10, RealResults90);
    Totalcomptime = comptimeReal90 + comptimeReal90eq;

%% simulate data
%This part of the script runs the simulation for the parameters of the
%Hawkes process.
if simulationstudy
global observations
burnin = 250;
observations = 250;
simulations = 100;
institutions =4;
SimResults90(simulations).Results = [];
SimResults90eq(simulations).Results = [];
    for i=1:simulations
    [Ysim90, Qsim90, Medsim90, Xsim90]= simulatedata(observations, institutions, burnin, 0.90, Xreal90(100:end,:) ,false);
```

```
    [Ysim90eq, Qsim90eq, Medsim90eq, Xsim90eq]= simulatedata(observations, institutions, burnin, 0.90, Xreal90eq(100:end,:) ,true);

    [output90] = Scriptfunction(Ysim90, Qsim90, Medsim90, Xsim90, 0.90, [], Statevarreal90);
    SimResults90(i).Results = output90;

    [output90eq] = Scriptfunction(Ysim90eq, Qsim90eq, Medsim90eq, Xsim90eq, 0.90, [], Statevarreal90eq);
    SimResults90eq(i).Results = output90eq;
    end
    [muTheta90, sigmaTheta90, comptime90] = distributioncalc(institutions, simulations, SimResults90);
    [muTheta90eq, sigmaTheta90eq, comptime90eq] = distributioncalc(institutions, simulations, SimResults90eq);
    Totalcomptime = comptime90 + comptime90eq;
    save('simulationp11.mat');

%Main function used to operate other functions
function [output] = Scriptfunction(Y, Q, Med, X, quant, theta, statevariables)

%% Calculate intensity
function [lambda, phisumSelf, phisumOther] = intensityCalc(theta, marks)

%Calculates the distributions of the parameters over the simulations.
function [muTheta, sigmaTheta, Totalcomptime] = distributioncalc(institutions, simulations, Results)

%Simulates data used in the simulation study
function [Yq, Q, Med, simreturns] = simulatedata(observations, institutions, burnin, quant, returns, equal)

%Converts the cleaned data in the variables needed for the Hawkes process estimation.
function[Yq, Q, Med, X, StateVar, timeWindow]= datamutator(noinstitutions, nostatevar, q, burnin, equal)

%Converts datatables to arrays
function[Marks, Returns, timeWindow, dailyData, dailyVolume] =StockdataCleaner(dailyTable)

%Converts the data of losses into a vector of marks.
function [Y, xQuantile] = MarksGenerator(x, q)

%Executes the Maximum likelihood estimation using fmincon.
function [theta_hat] = Maximum_likelihood(marks)

%Likelihood function to be optimised.
function [output] = likelihood(marks ,theta)

%Describes the intensity function used in the likelihood function.
function [output] = intensity(i, marks, theta, T)

%Kernels used in the intensity
function [output] = exp_phi(i,k, t, mark, theta)

%Constraints for the optimisation problem.
function [c, ceq] = constraints(theta)
```

Results calculator

This Matlab function calculates the *CoVaR* and performance measures from the results of the different scenarios from the Main estimation script.

```
%This function takes the parameters estimated in Masterthesisscript and
%calculates both the Hawkes process results and the quantile regression
%results by operating the other functions. It also calculates the different portfolios, plots the graphs and makes the tables.
function [Outcome, tab1, tab2, tab3, tab4] = resultscalculator(Results, X, Y, Q, Med, statevariables, timeWindow)

    %This function integrates the intensity and calculates the CoVaR from it.
    function [VaR, p_mark, p_zu] = integrateintensity(lambda, marks, q)

    %This function rescales lambda to a probability if necessary.
    function [newlambda] = changelambda(lambda)

    %This function creates the returns and weights of the investment strategies and
    %portfolios. It also calculates the summary statistics.
    function [performanceMatrix, GMVreturn, GMVweights, CoVaRreturn, CoVaRweights, varreturn, varweights] = CoVaRPortfolio(returns, deltaCoVaR ,burnin, riskfree)

    %Function to be optimised for the minimum Delta CoVaR strategy.
    function [output] = CoVaRopt(deltaCoVaR, weights)

    %Function to be optimised for the minimum variance strategy.
    function [output] = varopt(Returns, weights)

    %Estimates the CoVaR with the quantile regressions.
    function [DeltaCoVaRsi, CoVaRsiq, CoVaRsimed, DeltaCoVaRis, CoVaRisq, CoVaRismed] = quantileregression(returns, Q, Med, statevariables, quant)

    %First quantile regression optimisation problem.
    function [value] = quantilefunction1(theta, Xj, Xi, M, q)

    %Second optimization problem quantile regressions
    function [value] = quantilefunction2(theta, Xj, M, q)

    Collects the results to create tables.
    function [tab1, tab2, tab3, tab4] = Covartabmaker(Outcome)
    %This function
end
```