Profit based bonus reserve
Preferable compensation structure during the current economic crisis

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Abstract
This paper examines if a profit dependent reserve bonus structure is a more favourable option than straight salary payment to reduce the major negative contributions of a piece-rate bonus structure to the current economic crisis. The analysis is based on a model with two different compensation structures, which are both influenced by the state of the world. The overall conclusion is that the compensation structure with a profit dependent reserve is the most favourable option to reduce some major negative contributions of a piece-rate bonus structure to the current economic crisis. The findings in this paper show that this kind of compensation structure and straight salary payment can reduce the same amount of problems that are related with the current crisis. However, straight salary payment is incapable to maintain some advantages of bonuses, whereas a profit dependent reserve bonus structure is able to hold them.
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Introduction

In the general public opinion bonuses are seen as one of the major causes of the current economic crisis. This negative attitude towards bonuses is the underlying motive to conduct this paper. In this paper the usefulness of a proper bonus structure is highlighted.

Although the current crisis is a relative recent phenomenon, the reasons why bonuses can contribute to an economic crisis are already described in literature by, for example Baker, Gibbons and Murphy (1994) and Pearce and Robinson (2002). A negative contribution that they describe is that a bonus provides an employee an incentive to maximize the short term results of a firm. Such behaviour can reduce the firm’s survival chances in times of economic downturn, because the firm has fewer resources available that help the firm to continue its business. Another contribution that they describe is that these survival chances are further reduced, because bonuses make firms financially less flexible. This is caused by the fact that the bonus costs are often relatively fixed, which reduces the possibility that a firm survives a crisis. Their final argument is that it is socially seen as unfair when firms pay bonuses to their employees in times of economic downturn. Especially when at the same moment the same firm takes socially undesirable actions in order to survive an economic crisis.

However, there are positive effects of bonuses as well. Lazear (1986) describes the incentive that bonuses provide to the employee to produce more effort. The idea behind this theory is that an employee produces as little as possible effort, when he receives no direct incentive to produce more. Straight salary payment is not able to give this incentive, because the indicator on which the employee’s compensation is based is only indirectly linked to the total produced effort. In his analysis he shows that a bonus compensation provides the desirable incentive to produce more effort, because the indicator where the compensation is based on is directly linked with the total produced effort. There is empirical evidence that this bonus provides employees an incentive to produce more effort. An example of empirical research is Shearer (2004). He did an experiment to test the effects of a piece-rate bonus on the average productivity in the firm. His main finding was that employees showed an average productivity increase of approximately 21% when they worked under a piece-rate payment condition, instead of straight salary payment.

It is clear that an important purpose of bonuses is to provide the employee an incentive to produce more effort, but unfortunately bonuses do not only have positive effects. The purpose of this paper is to create a bonus structure that is able to increase the survival chance of the firm and reduce some of the described negative contributions of bonuses to the current economic crisis at the same moment. The social and scientific relevance of this paper is therefore that the social dissatisfaction with bonuses during an economic crisis is reduced and scientifically it is important that a compensation structure
which has less negative consequences is created. In order to find this theoretical reduction a bonus structure is designed in this paper which is based on a so called profit dependent bonus reserve. This reserve is influenced by the cyclical changes in order to create an automatic adjustment mechanism, which makes bonuses partly dependent on the state of the world. In this paper we will refer to this bonus structure as a reserve bonus. This reserve bonus is compared with a basic piece-rate bonus structure with a piece-rate bonus with a fixed reserve, this last structure is called a piece-rate bonus in the rest of the paper. It is important that the created compensation structure reduces the current occurring negative economic and social consequences of the current bonus systems in a more preferable way than straight salary payment can possibly do. The central question in this paper is therefore the following: Is a reserve bonus a more favourable option than straight salary payment to reduce the major negative contributions of a piece-rate bonus to the current economic crisis?

To answer the central question this essay will be conducted in the following manner: In chapter I the theoretical framework is made. The main purpose of this framework is to examine the main features of both straight salary payment and piece-rate bonus compensation. Furthermore the adverse selection and moral hazard problems are examined and how bonuses can reduce these problems. In addition a self-selection mechanism is examined where the desirable type of employee chooses the right firm. This section is followed by some major empirical findings and finished with the major disadvantages of bonuses in the context of the current economic crisis.

Chapter II is the method section. In this chapter a model is used as basis for both the reserve and piece-rate bonus. This model is developed to determine the value of several variables in order to optimize the utility and profit levels for respectively the employee and the employer. The first described compensation structure in this chapter that is influenced by the state of the world is the piece-rate bonus. The second described structure that is influenced by the state of the world is the reserve bonus.

Chapter III and IV are respectively the analysis and result chapter. In the analysis chapter the differences between the two models are examined. In the result chapter the major implications of replacement of a piece-rate bonus structure by a straight salary payment and a reserve bonus are described.

In the last two chapters a discussion and a conclusion will follow. In the discussion chapter, some additional and related research options are described. In the conclusion chapter the found implications of both straight salary payment and reserve bonus are compared.

In the appendix the common restrictions of the three states of the world are determined. It is important to determine these restrictions, because with these restrictions it is possible to prove that there are values possible for all three states of the world.
(I) Theoretical framework

In the theoretical framework the basic features of straight salary payment are compared with those of piece-rate compensation. It is important to examine the main features of straight salary payment before we can develop a different type of compensation structure. Furthermore it is necessary to describe the main characteristics of bonus compensation structures in order to create a proper junction between the existing literature and the model section. Finally the major disadvantages of bonus structures in the context of the current economic crisis have to be described in order to be able to analyse the effects of a different bonus structure.

The theoretical framework is conducted in the following manner. The first section of this chapter examines what an optimal compensation structure is and describes the main features of a piece-rate compensation structure. In the second section the main disadvantages of a straight salary payment are explained and how these disadvantages are reduced if a bonus structure is introduced. In the third section the reason why firms with straight salary and piece-rate payment can coexist is examined. In the fourth section empirical findings that are related with the previous described theory are examined. In the last section the main disadvantages of bonuses in the context of the current economic crisis are discussed. This section consists of two subsections. In subsection Va and Vb the main disadvantages of bonus structures are examined respectively with and without direct relations with the current economic crisis.

(I) Compensation structures and piece-rate bonus features

The purpose of a compensation structure is to compensate employees for their effort and time. According to Brown (1989: 5-7) there are different types of compensation structures, which can be roughly divided in a straight salary payment and different types of bonus compensation structures. Every type of compensation structure has its own features, but according to Baker, Gibbons and Murphy (1994 : 1127) only a system where employees are rewarded precisely for the value they have added to the firm, including both short and long run value, is the most efficient system from the firm’s point of view.

The first compensation structure Brown (1989: 6) mentioned is straight salary payment. This type of compensation structure is based on so called standard rates and has little to do with personal performance. With straight salary payment an employer assumes that one unit of a certain input based indicator is related with a given value that is added by the employee to the total value of the firm. This value is based on the average value employees add to the value of the firm for every unit of this indicator. The indicator can only give an indication of the value that is added and often measures the amount of time spent on an activity. As a result the basis upon what an employee is compensated is not the real value he or she has added to the firm, but the average value that is added by all employees. By using the description of Baker, Gibbons and Murphy (1994 : 1127) it is clear that a basic straight salary payment is not an optimal compensation system. The reason why a straight salary payment does
not fit in the description is that an employee is not precisely compensated for the value that he or she has added to the firm.

The purpose of bonuses is to increase the effectiveness of the straight salary payment compensation, by using an indicator that is directly related to the value an employee adds. This is done by a measurement system that is able to measure the total in- or output of an employee and the precise value that this in- or output has generated. In this situation the compensation mechanism is based on the actual value that an employee has generated and not on the average value. With a proper use of bonuses, an employer is at least in theory able to increase the optimal compensation system described by Baker, Gibbons and Murphy (1994: 1127).

Brown (1989: 5-7) has divided different types of bonus structures in two categories: piece-rate pay and merit pay. The first category described by Brown is piece rate payment. This compensation structure is based on a physical indicator that is perfectly measurable and directly related to a certain value that is added to the firm. Nowadays the contribution of an employee to the total added value to a firm with this type of measurement is increasingly harder to determine, due to the increasing complexity of production processes.

The second category is merit pay. This is a compensation structure where an employee’s personal performance is monitored by his or her supervisor and where the employee’s compensation is based on the supervisor’s perspectives of the employee’s performance. This type of compensation structure is often used in service industries, but also in other sectors where compensation is based on a physical indicator which makes measurement difficult or costly.

We will concentrate on the characteristics of the first category in the rest of this paper, because the purpose of this paper is to compare straight salary payment and a reserve bonus with a piece-rate bonus.

(IIa) Main disadvantages of straight salary compensation

The main two disadvantages of straight salary compensation are based on the classical principle agent problem. This principle assumes that in most cases an agent has better information about his own ability than the principle has.

An important disadvantage of straight salary compensation is based on the belief that employees differ from each other in their abilities and differ as a result of this in their productivity levels. Gibbons (1987 : 413-414) argues that the asymmetrical distribution of information between employees and employers about this productivity level can result in the adverse selection problem. The adverse selection problem occurs when an employer attracts employees with undesirable productivity levels as a result of their unobservable ability. This problem is caused by the fact that an employer can only attract employees with a desirable productivity level, when he offers a compensation that compensates employees sufficiently for their productivity. When an employer offers a straight salary that is high enough to attract the employees with the desired productivity level, he also attracts all employees with
a lower productivity than desired. These employees with a lower ability are attracted by the straight salary wage, because the wage is higher than the compensation they really deserve with their productivity level.

Another main disadvantage of straight salary payment is that with this type of compensation structure an important moral hazard problem occurs. The problem here is that straight salary payment provides no incentive to the employee to exert more effort than necessary to prevent being fired, which results in behaviour of the employee that is undesirable for the employer. The underlying theory of this disadvantage is the assumption that an employee bears a cost when he produces effort. This cost is often related with the trade-off between leisure and income. The cost is caused by the substitution mechanism where the employee must give up a certain positive utility generating leisure in order to earn some income. The major implication of this negative utility that is caused by producing effort is that an employee has an incentive to work as less as possible for a given wage level, which is the basis for the effort decision model of Shapiro and Stiglitz (1984 : 433-440). They assume that an employee has two options: the first option is to give the desired level of effort and receive a wage in return. The second option is to shirk or work as less as possible. The employee has a probability that he will get caught, in which case he will receive an unemployment benefit. Their idea can be summarized in a more simplified model, by assuming that the unemployment benefit is zero and shirking has no influence on the wage possibilities in the next period. In this model an employee will shirk when the utility of the fixed wages times the probability that he is caught is larger than the utility generated by the fixed wage minus the effort cost. The model shows that with a straight salary payment the employer can only prevent shirking when he offers a relatively high wage, otherwise an employee will produce as less as possible effort.

(IIb) Solutions offered by bonus structures

According to Gibbons (1987 : 413-414) offering a bonus to an employee can reduce the adverse selection problem. With a piece-rate bonus an employee receives compensation for every unit of output he produces. The higher the productivity of an employee, the lower the total effort he needs to produce for one unit of output. An employee bears a cost for every unit of effort he produces. The effort cost per unit of output becomes lower when the productivity of an employee increases. An employer can take advantage of this by offering a piece-rate that is high enough to attract employees with the desired level of productivity. For lower ability employees the piece rate is not high enough to compensate for their effort costs.¹

¹ Offering a bonus to an employee is not the only way to reduce the adverse selection problem. Other options are for example that an employer desires that an employee has a certain degree. If only capable employees can get this kind of degree, the policy is effective and only employees with the desired ability are attracted to the firm. However, demanding a degree is less efficient than installing a piece-rate bonus structure, because getting a degree creates a cost that is not made by installing a bonus structure.
The moral hazard problem that is described in section IIa of this chapter can also be reduced by installing a bonus structure. By installing a bonus structure the implication of the model of Shapiro and Stiglitz (1984: 433-440) changes. When an employee receives a piece-rate bonus he still has the two options: work or shirk. When he works he receives a certain compensation for every unit of effort. The employee is compensated only for the effort he really produces, so when he shirks he receives no compensation at all. With a piece-rate bonus compensation structure there are no more costs of shirking for the employer. With a bonus system an employer can motivate the employee to behave in a way that is favourable for both actors.

(III) Straight salary and piece-rate firms

Section IIa of this chapter shows that a straight salary compensation system has some serious disadvantages that can be reduced by installing a piece-rate bonus structure. A piece-rate bonus structure seems to be more favourable than straight salary compensation. However there is an important reason why both structures coexist. This reason is that in most cases productivity is not completely measurable and employers must invest in measurement tools to monitor the output an employee produces. These measurement costs are a disadvantage of bonuses and a reason to install a straight salary payment. The measuring costs are made by the employee when the information that is revealed is also useful elsewhere. These costs are paid by the employee, because when the employee is not prepared to pay these costs nobody is. Due to the universal usefulness of the information, every firm will wait until another firm has paid these costs and firms will only attract employees where this information is already revealed in order to save these measurement costs.

According to Lazear (1986: 412-416) employees will select themselves to work at either straight salary or piece-rate firms. This self-selection process is caused by the fact that different compensation structures are favourable for different types of employees, because the employees differ in their ability and productivity. It is possible to summarize these different types of employees in three categories. These different categories of employees are:

- The first category employee always works at a straight salary firm, because with his productivity level the fixed wage is higher than the piece-rate based compensation minus the monitoring cost.
- The second category employee always works at a piece-rate firm, because with his productivity level the piece-rate based compensation minus the monitoring is higher than the fixed wage.
- The third category employee is indifferent between the two compensation structures. For this employee with a certain productivity level, the level of the straight salary payment equals the piece-rate compensation.

In the case that the monitoring costs are higher than the maximum effort level of the most productive employee, it is optimal that only straight salary firms exist. In the case that the opposite is true, so if
monitoring costs are zero, it is optimal that most firms are based on bonus compensation. Only the employees with the lowest productivity are indifferent between a straight salary and bonus firm. However in most cases the monitoring costs are somewhere between these two extremes. In this case the optimal distribution is that the first category employees work at a bonus firm, the third category employees at a straight salary firm and the second category employees is indifferent between the two compensation structures.

(IV) Empirical findings

In sections II and III we have seen how the different compensation structures basically work and what the difference is between them. In this section the main implications of the previous sections are related with empirical findings.

Lazear (2000 : 1352-1360) used data of the Safelite Glass Cooperation to test if some theoretical effects of bonus structures also occur in reality. He found that after Safelite Glass Cooperation had switched from a straight salary payment to a piece-rate compensation structure, average productivity increased by 44 percent. This is a result of the previous described self-selection mechanism, where employees with a higher ability level became more likely to stay at Safelite Glass Cooperation and new hired employees were on average more productive than under the straight salary compensation. He also found evidence that besides the increased productivity the compensation costs rose as well. However the increase in compensation costs was much smaller and as a result both the employees and the employer were better off after installing the piece-rate system. Finally he found evidence that the variance of the height of the wage levels increased with piece-rate pay compared with straight salary payment, which is likely when the reward system is based on personal performance instead of average performance.

Shearer (2004 : 513-518) did empirical research about possible productivity gain when a straight salary payment is replaced by a piece-rate bonus. He did an experiment at a Canadian tree planting firm, where he observed the behaviour of nine randomly selected employees under the two compensation structures. Before the experiment the firm offered piece-rate compensation to the employee in most cases, but sometimes the employee received a straight salary. This is the result of the contractual structure between the firm and the government. As a result of this existing compensation structure it was possible to do the experiment without informing the employees about it. In the experiment the employees were observed for 120 planting days, with an equal distribution of straight salary and piece-rate payment. The main finding of his experiment is that employees showed a productivity increase of approximately 21% when they worked under a piece-rate payment condition, instead of under a straight salary payment. Another major finding is that employees earned more under piece-rate compensation, compared with straight salary payment, while at the same moment the production costs per unit decreased with an average of 13% under piece-rate compensation. The
implication of this finding is that both the employee and the employer are better off after installing a piece-rate compensation, which is in line with the findings of Lazear (2000: 1352-1360).

Groot and Oosterbeek (1995: 14-15) found more empirical evidence after a statistic analyses with data of the British Household Panel Survey. The most important finding in the context of the earlier described theory is that firms use incentive pay in order to reduce shirking behaviour. In line with these findings is that firms pay a higher fixed wage when monitoring costs increase and as a consequence bonus structures become less favourable. They pay a higher fixed wage in order to reduce shirking behaviour by increasing the costs of being caught.

Other empirical evidence is found by Brown (1989: 19-20). His research is based on data of the Industrial Wage Survey. He compared the theoretical effects of piece-rate payment with empirical findings. The most important finding of Brown in the context of this paper is that when monitoring costs decrease the chance that a bonus structure is used becomes more likely. This finding is in line with the described finding of Groot and Oosterbeek (1995: 14-15) about the effect of monitoring costs on the likeliness that piece-rate compensation is used.

(Va) Disadvantages with a direct link to the economic crisis

The purpose of this paper is not to find a solution for all the disadvantages of bonus compensation, but to find a compensation structure that reduce some major problems of a piece-rate bonus structure that have a direct contribution to the current economic crisis. Important arguments about the causes of the current economic crisis are in many cases related to effects that the current used bonus systems have. In this context there are three major arguments mentioned:

- The first argument is that bonuses give employees an incentive to maximize short-term results. This stimulation is often caused by the reward system of a firm and is seen in literature as one of the main disadvantages of bonus systems. Baker, Gibbons and Murphy (1994: 413-414) argue that this short-term vision is generally dangerous for the firm, because it can cause behaviour that harms the going concern of the firm in the long-run. Furthermore this type of vision is also dangerous for a firm in a period of economic downturn, because it can influence the firm’s survival chances negatively. An example of this negative influence is that when reserves that can help the firm in periods of economic downturn are used to stimulate short-term progress and are not longer available when they are really needed.

- Another argument is that the survival chances of a firm are reduced further by the structure most bonuses have. Bonuses are often contract based and even in a period of economic downturn paid to the employee. Bonuses can be seen as a component of the fixed wage costs, because an employer is forced to pay the bonuses when some goals are achieved by the

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2 See for example Rappaport (1978: 82-84)
employee. This reduces the flexibility of a firm, which is a major survival tool of a firm according to Pearce and Robinson (2002: 83-91).

- The final argument is more social than economic, but has important economic implications nevertheless. The argument is that high bonuses are paid to some employees of the firm even when costs must be reduced due to an economic downturn. This is socially seen as unfair and is often related to the relation between blue and white collar employees. Although this argument is more socially than economically relevant, it can become economically relevant when this compensation has a negative influence on the firm’s reputation and in the worst case sales decrease, because people start boycotting the firm’s products.

(Vb) Disadvantages without a direct link to the economic crisis

In order to have a more complete overview of the disadvantages of bonus payment it is important that some major disadvantages of bonus structures are described that have no direct contribution, or at least no major contribution, with the current economic crisis. Besides the described monitoring costs, these disadvantages are:

- An important disadvantage is that bonuses are less preferable when the total bonus size is relatively variable and the size of the bonus is not completely under the control of the employee. Due to the increasing complexity of the production process the total bonus size often became more variable. This variability increases when an employee has no full control over all variables that influence the size of his bonus. According to Baker (1992: 600-606) the total bonus size is determined by the employee’s behaviour and so called uncontrollable random variables, that are beyond the control of the employee. These random variables can make bonuses less preferable in the case that an employee is risk averse. His findings can be well explained by the so called certainty effect described by Tversky and Kahneman (1981: 453-458). They describe the influence of risk on an actor’s behaviour. They found that risk-averse actors in most cases prefer a reward that they know they will definitely receive, than an uncertain reward that is on average higher. This effect is called the ‘certainty effect’ and occurs especially in cases when the reward is relatively large or important for the actor. This disadvantage can be reduced by replacing the risk from the employee to the on the on average more risk neutral firm, but by reducing the risk the incentive effect of bonuses are also reduced.

- Another disadvantage is that bonuses can give the wrong incentives when the bonus is based on several indicators. Baker, Gibbons and Murphy (1994: 413-414) argue that due to the increasing complexity of the production process most indicators became harder to measure. When the employee is aware of the fact that some of the indicators that are the basis for his bonus compensation are more difficult to measure than other indicators, he has according to Holmstrom and Milgrom (1991: 24-29) an incentive to concentrate his attention on the best
measurable indicators. He concentrates his attention on the best measurable indicators, because in that case the chance that he is actually rewarded for his performance is increased. Less measurable indicators are often measured with some error. This error increases the risk for the employee that he will be rewarded on a wrong base. Even when an employee is risk-neutral, he demands a compensation for this risk. As a result of this risk there is a fixed cost involved when an employer gives an employee a specific task: a so called risk-premium. Another effect of this fixed cost that is showed by Holmstrom and Milgrom (1991 : 33-49) is that it is suboptimal that employees share a relative small task, because in that case the employer has to pay two risk-premiums. An important underlying factor of this so called multi-task principle-agent problem is how the job is designed. When an employee has only one task, the multi-task principle-agent problem can be strongly reduced or even eliminated.

One often mentioned disadvantage of bonuses is the so called ratchet effect described by Clawson (1980 : 168-182). This disadvantage occurs when employees can not switch after taking a job easily to an outside option. The ratchet effect occurs when a firm uses the information on the employee’s ability found in the first period to set a higher target for the next period. As a consequence of this behaviour employees who work at a firm where the minimum desired quantity of output is based on the previous output, have an incentive to restrict their output. By restricting their output the target remains low and they can relatively easy earn a higher compensation. This effect is undesirable and reduces the effectiveness of bonus compensation compared to straight salary payment.

(II) Model

The purpose of the model chapter is to determine the behaviour of the employee and the employer under a different bonus compensation structures. The model deals with the optimization process for both the employer and the employee. It is designed to determine the optimal wage scheme, wherein the profit of the employer and the utility of the employee are optimized. The model is used to define...
these optimal values for two different compensation structures. The first structure is called ‘piece-rate scheme’ and is based on a fixed wage and a fixed bonus rate. In the model we refer to this fixed bonus rate as ‘piece-rate bonus’. The second structure is called ‘reserve scheme’ and is based on a fixed wage and a so called bonus reserve. This bonus reserve is based on a certain percentage of the total revenue of the firm and is will further be called: ‘reserve bonus’.

In order to determine the optimal utility and profit values this chapter is conducted in the following manner. In the first section the main features of the model, which will be used for both compensation structures, are explained. In subsection Ia of this first section the underlying features of the utility optimization process are described. In subsection Ib the concept and the features of the state of the world is explained. In subsection Ic the basic features of the profit optimization process are examined. In the second section the basic features of the model which were explained in the first section are used to explain the piece-rate bonus structure. Subsection IIa and subsection IIb concentrate respectively on the utility optimization and profit optimization processes. In the third section the basic features from the first section are used to explain the reserve bonus structure. Subsections IIIa and IIIb have the same layout as the subsections of the second section, but the purpose here is to determine the optimization under the reserve bonus.

(Ia) Basic features utility optimization process

In the utility optimization process it becomes clear how an employee optimizes his or her utility. To start the explanation from the employee’s point of view, the assumption has to be made that an employee always tries to optimize his profit. We also assume that an employee has an outside option with a given level of external utility. An employee can always receive this utility when he decides not to work for the firm. Nevertheless, after signing a contract with the firm, the outside option is no longer available. An employee is only attracted to the firm when the utility level that he can achieve there is at least as high as with the outside option. The relation is showed in the formula:

\[(1) \quad U \geq U_0\]

where \(U\) is the utility an employee can get at the firm and \(U_0\) the utility of the outside option

Producing effort generates a negative utility, this relation is described in a cost function. The most practical cost function is:

\[(2) \quad K(E) = \frac{1}{2}E^2\]

When the employee’s expected income function and the cost of effort formula are added together, the formula for expected utility is created. When the assumption is made that the employee is risk neutral the expected utility formula is:

\[(3) \quad U^e = W^e - \frac{1}{2}E^2\]

where \(U^e\) is expected utility and \(W^e\) is expected income
In this paper there are variables that are taken into account in the model that influence the bonus size which are beyond the control of the employee. These variables are combined into one major variable called: ‘the state of the world’ and it has three possible values in this paper. These three specific states of the world are:

- First state. In this state the variables beyond the control of the employee have a destructing influence on the firm’s revenue, so that the revenue becomes so low that the firm is not able to pay a part of the wage at-all. In this state the revenue is equal or smaller than zero. This state has a probability of \( \lambda^* \) to occur and is called \( \sigma^* \).

- Second state. In this state the influence of the variables on the firm’s income is so that the firm’s revenue is larger than zero, but too small to pay the total promised bonus to the employee in the case of a piece-rate bonus. This state has a probability of \( \lambda^* \) to occur and is called \( \sigma^* \).

- Third state. In this state the world is flourishing and the variables have a very positive effect on the firm’s profit. In order to achieve this the firm’s total income must be equal or larger than the total wage costs. This state has a probability of \( \lambda^{**} \) to occur and is called \( \sigma^{**} \).

We assume that all three state of the world have a chance to occur that is larger than zero that the following condition must hold:

\[
\lambda^* + \lambda^* + \lambda^{**} = 1
\]

In this paper the contract of the employee is only valid for one period and so only one of these states will occur in the period that the employee is employed. We assume that a firm goes bankrupt when it is not able to pay the promised wage to the employee.

An important assumption in this paper is that both the employer and the employee do not know which of the possible states of the world will occur until directly after signing the contract. However the influences of the three possible states are known at the moment of signing.

Another assumption that is made is that directly after signing the contract and, hence, before the employee exerts effort, it is clear which state of the world occurs. This assumption is preferred in this context above knowing the state of the world after the period has ended. This is based on the fact that there are a lot of indicators that can indicate changes in the economy and it is likely that an employee is capable of noticing at least some of these signs.

In the profit optimization process it becomes clear how an employer can optimize his profit. A good starting point to explain the model from this point of view is a basic micro-economic relation between profit and production. Profit is equal to the firm’s revenue minus the wage costs. These wage costs
compensate the employees for the effort they produce. When the firm only employs one employee the formula for the basic micro-economic relation is:

\[ n = PQ - W \]

where \( P \) is the price level, \( Q \) is total level of output, \( W \) are the total wage costs and \( n \) is total profit.

In order to simplify the model we assume that one unit of output is produced by precisely one unit of effort. This relation is showed by the formula:

\[ Q = E \]

It is possible to extend the profit function with the influence of the state of the world. It is reasonable that a negative influence of the state of the world, causes a decrease in the demand for output. This decreased demand is reflected by a decrease in price level. The opposite is true for an increase in demand. This influence of the state of the world on the profit condition is showed by the formula:

\[ n = (P + \sigma)E - W \]

where \( \sigma \) is the state of the world.

The total wage costs are the sum of two different parts. These two parts are a fixed wage and a bonus component. The bonus is a reward for every unit of effort an employee has produced. This reward per unit of effort times the total effort produced is the total wage cost. This relation is showed by the formula:

\[ W = \alpha + \beta E \]

where \( \alpha \) is the fixed cost component, \( \beta \) the reward per unit of effort and \( E \) is the total effort.

\( \text{IIa) Utility optimization under a piece-rate scheme} \)

The influences of the state of the world on the total compensation that is paid to the employee are known before signing the contract, so it is possible for the employee to create an expected income formula. The employee knows that in the first state there is no revenue to pay a bonus, so the expected income is equal to the fixed wage. Furthermore he knows that in the second state his bonus is larger than zero, but smaller than the total promised compensation. When the employee is forced to pay a negative fixed wage to the employee, this wage is used to pay the employee’s bonus. In fact he pays no negative wage to the employer and as a result the employee’s expected income formula is equal to the total sales value per product times his effort. Finally the employee knows that in the third state the revenue is large enough to pay the full promised wage. This knowledge results in the expected wage formula:

\[ W^e = \alpha(\lambda^* + \lambda^{**}) + \lambda^*E^*(P + \sigma^*) + \lambda^{**}\beta E^{**} \]
An employee takes the real state of the world into account when he aims to optimize his utility. Given that there are three possible states of the world, there are also three possible expected utilities. From the expected utility function of every state of the world it is possible to determine an optimal level of effort after taking the first derivative to $E$. In order to make the whole optimization process more clear we will start to derive the optimal effort for the first state and end with the optimal effort of the third state.

First state of the world
In this case the expected utility function is:

$$U^{e_1} = \alpha - \frac{1}{2}E^{e_1}^2$$

In order to determine the optimal level of effort given the expected income, we start by taking the first order derivative to $E^{e_1}$:

$$-E^{e_1} = 0$$

The optimal level of effort is found by extracting the variable $E^{e_1}$ from the rest of the formula. The formula of the optimal level of effort in the case of $\lambda^{e_1}$ is:

$$E^{e_1} = 0$$

Second state of the world
In this case the expected utility function is:

$$U^{e_2} = E^{e_2}(P + \sigma^*) - \frac{1}{2}E^{e_2}^2$$

In order to determine the optimal level of effort given the expected income, we start by taking the first order derivative to $E^{e_2}$:

$$(P + \sigma^*) - E^{e_2} = 0$$

The optimal level of effort is found by extracting the variable $E^{e_2}$ from the rest of the formula. The formula of the optimal level of effort in the case of $\lambda^{e_2}$ is:

$$E^{e_2} = (P + \sigma^*)$$

Third state of the world
In this case the expected utility function is:

$$U^{e_3} = (\alpha + \beta E^{e_3}) - \frac{1}{2}E^{e_3}^2$$

In order to determine the optimal level of effort given the expected income, we start by taking the first order derivative to $E^{e_3}$:

$$\beta - E^{e_3} = 0$$

The optimal level of effort is found by extracting the variable $E^{e_3}$ from the rest of the formula. The formula of the optimal level of effort in the case of $\lambda^{e_3}$ is:

$$E^{e_3} = \beta$$
With these three individual values of expected utility known, it is possible to determine the total expected utility function. This is done by multiplying the expected utilities with their likeliness to occur and then adding them together:

\[-(13a) \quad U^e = a(\lambda^* + \lambda^{**}) - \frac{1}{2}\lambda^*E^* + \lambda^*[E^*(P + \sigma^*) - \frac{1}{2}E^*] + \lambda^{**}E^{**} - \frac{1}{2}\lambda^{**}E^{**} \]

The undetermined levels of effort in the previous expected utility formula can be replaced by the three individual values of optimal level of effort. After some rearranging the formula of expected utility can be written as:

\[-(13b) \quad U^e = a(\lambda^* + \lambda^{**}) + \frac{1}{2}[\lambda^*(P + \sigma^*)^2 + \lambda^{**}E^{**}] \]

The implication of this formula is that the expected utility increases when either one or more of the following components increases:

- the fixed wage component
- the price of the product
- the bonus rate
- the probability that the first state of the world occurs
- the probability that the second state of the world occurs
- the probability that the third state of the world occurs
- the value of the second state of the world

\[(IIb) \quad \text{Profit optimization under a piece-rate scheme} \]

Although the employer does not know which state of the world will occur before he signs the contract, he is aware of the optimal levels of the employee’s effort in the different possible states. In order to optimize his own profit the employer must motivate the employee to produce his optimal level of effort in every state. This can be done by giving each variable the employer can influence a specific value that results in the desired motivation of the employee. The only two variables an employer can influence are the fixed wage and the fixed bonus rate. In order to optimize it’s own profits the firm should set these two values so that the outside option at the time of signing the contract is no more favourable for the employee than the firm’s option. The optimal value of this participation constraint is when the employee is indifferent between the two options. This relation is showed by the formula:

\[-(14) \quad U_o = a(\lambda^* + \lambda^{**}) + \frac{1}{2}[\lambda^*(P + \sigma^*)^2 + \lambda^{**}E^{**}] \]

By rearranging the participation constraint it is possible to extract the value of the fixed wage. Rearranging results in the formula:

\[-(15) \quad a = \frac{[U_o - \frac{1}{2}\lambda^*(P + \sigma^*)^2 - \frac{1}{2}\lambda^{**}E^{**}]}{[\lambda^* + \lambda^{**}]} \]
The implication of this formula is that the fixed wage increases when the following component increases:
- the outside option
Furthermore the fixed wage decreases when either one or more of the following components increases:
- the price of the product
- the bonus rate
- the probability that the first state of the world occurs
- the probability that the second state of the world occurs
- the probability that the third state of the world occurs
- the value of the second state of the world

The firm’s profit is influenced by the state of the world. There are three possible profit levels, because there are three possible states of the world. These three schemes are all based on the profit scheme described in subsection Ic.

In the case of $\lambda^*$ the profit level is:
-(16a) $\pi = (P + \sigma^*)E^* - \alpha - \beta E^*$
By replacing $E^*$ for its previous found value, the profit formula in the first state is:
-(16b) $\pi = -\alpha$

In the case of $\lambda^{**}$ the profit level is:
-(17a) $\pi = (P + \sigma^{**})E^{**} - \alpha - \beta E^{**}$
Given the conditions about the second state described in section I, the employer must pay all it’s income as wage to the employee. As a result of the assumption that the total income of the firm is not sufficient to pay the promised wage, the total profit must be zero. Therefore the profit formula in the second state is:
-(17b) $\pi = 0$

In the case of $\lambda^*$ the profit level is:
-(18a) $\pi = (P + \sigma^{**})\beta - \alpha - \beta^2$
By replacing $E^{**}$ for its previous found value, the profit formula in the third state is:
-(18b) $\pi = (P + \sigma^{**})\beta - \alpha - \beta^2$

When all these profit conditions are added together it is possible to determine the total expected profit formula. This expected profit formula is:
-(19a) $\pi_e = \alpha[\lambda^* + \lambda^{**}] + \lambda^{**}[(P + \sigma^{**})\beta - \beta^2]$
When the fixed wage variable is replaced by the previous found value for the fixed wage, the only variable under control of the firm is the fixed bonus rate. After some rearranging the profit formula is:

\[-(19b) \quad \pi^e = U_0 - \lambda^*\frac{1}{2}(P + \sigma^*)^2 + \lambda^{**}\left[\frac{1}{2}\beta^2 + (P + \sigma^{**})\beta\right]\]

In order to find the bonus level where profit is maximized, we have to take the first order derivative to \( \beta \). The first order condition is:

\[-(20a) \quad \lambda^{**}(P + \sigma^{**}) - \lambda^{**}\beta = 0\]

After some rearranging it is possible to find the optimal level of bonus where profit is maximized. This optimal bonus level is:

\[-(20b) \quad \beta = (P + \sigma^{**})\]

The implication of this formula is that a firm must pay the employee an one hundred percent bonus rate in order to motivate him to produce his optimal level of effort.

With the optimal bonus level determined, it is possible to determine what the profit level of the firm is under these conditions in the third state of the world. After some rearranging the formula for expected profit under these optimal conditions is:

\[-(21a) \quad \pi^e = \alpha[\lambda^* + \lambda^{**}]\]

When \( \alpha \) is replaced by the determined fixed wage value the formula for expected income is:

\[-(21b) \quad \pi^e = U_0 - \frac{1}{2}[\lambda^*(P + \sigma^*)^2 + \lambda^{**}(P + \sigma^{**})^2]\]

The implication of this formula is that a firm can only make a profit by installing a negative fixed wage component, because the employee’s effort is optimized when he receives an one hundred percent bonus rate. This fixed wage component can be seen as a fee that the employee must pay to the employer for the use of the equipment he needs to produce his output.

(IIIa) Utility optimization under a reserve scheme

Under a reserve scheme a certain percentage of the revenue is paid as bonus. The bonus reserve formula is:

\[-(22) \quad tE(P + \sigma)\]

where \( t \) is the percentage of the total revenue that is paid as bonus.

With the influence of the three possible states known it is possible for the employee to create an expected income formula. The employee knows that in the first state there is no revenue to pay a
bonus, so the expected income is equal to the fixed wage. Furthermore he knows that in the second and third state of the world the firm’s revenue is large enough to pay the compensation that is promised to him and a fixed bonus. This knowledge results in the expected wage formula:

\[-(23) \quad W^e = \alpha + t[\lambda^*E^*(P + \sigma^*) + \lambda^{**}E^{**}(P + \sigma^{**})] \]

After signing the contract the employee knows which state of the world occurs and takes this knowledge into account when he aims to optimize his utility. From the expected utility function of every state of the world it is possible to determine the optimal level of effort after taking the first derivative to E. In order to make the whole optimization process more clear we will start to derive the optimal effort for the first state and end with the optimal effort of the third state.

First state of the world
In this case the expected utility function is:

\[-(24a) \quad U^e = \alpha - \frac{1}{2}E^2 \]

In order to determine the optimal level of effort given the expected income, we start by taking the first order derivative to E:

\[-(24b) \quad -E = 0 \]

The optimal level of effort is found by extracting the variable E from the rest of the formula. The formula of the optimal level of effort in the case of \(\lambda^*\) is:

\[-(24c) \quad E^* = 0 \]

Second state of the world
In this case the expected utility function is:

\[-(25a) \quad U^e* = \alpha + tE^*(P + \sigma^*) - \frac{1}{2}E^2 \]

In order to determine the optimal level of effort given the expected income, we start by taking the first order derivative to E:

\[-(25b) \quad t(P + \sigma^*) - E^* = 0 \]

The optimal level of effort is found by extracting the variable E from the rest of the formula. The formula of the optimal level of effort in the case of \(\lambda^*\) is:

\[-(25c) \quad E^* = t(P + \sigma^*) \]

Third state of the world
In this case the expected utility function is:

\[-(26a) \quad U^{**} = \alpha + tE^{**}(P + \sigma^{**}) - \frac{1}{2}E^{**2} \]

In order to determine the optimal level of effort given the expected income, we start by taking the first order derivative to E:

\[-(26b) \quad t(P + \sigma^{**}) - E^{**} = 0 \]
The optimal level of effort is found by extracting the variable $E^{**}$ from the rest of the formula. The formula of the optimal level of effort in the case of $\lambda^{**}$ is:

\[-(26c)\quad E^{**} = t(P + \sigma^{**})\]

With these three individual values of expected utility known, it is possible to determine the total expected utility function. This is done in the same way as in IIa, so by multiplying the expected utilities with their likeliness to occur and than adding them together:

\[-(27a)\quad U^e = \alpha - \lambda^{*}tE^{*2} + \lambda^{*}[tE^{*}(P + \sigma^{*}) - \frac{1}{2}E^{*2}] + \lambda^{**}[tE^{**}(P + \sigma^{**}) - \frac{1}{2}E^{**2}]\]

The undetermined levels of effort in the previous expected utility formula can be replaced by the three individual values of optimal level of effort. After some rearranging the formula of expected utility can be written as:

\[-(27b)\quad U^e = \alpha + \frac{1}{2}t^2[\lambda^{*}(P + \sigma^{*})^2 + \lambda^{**}(P + \sigma^{**})^2]\]

The implication of this formula is that the expected utility increases when either one or more of the following components increases:

- the fixed wage component
- the price of the product
- the probability that the second state of the world occurs
- the probability that the third state of the world occurs
- the value of the second state of the world
- the value of the third state of the world
- the percentage of the total revenue that is paid as bonus

(IIIb) **Profit optimization under a reserve scheme**

In order to optimize his own profit the employer must motivate the employee to produce his optimal level of effort in every state of the world. This is done by determining the optimal values of the only variables an employer can influence. These two variables are the fixed wage and the percentage of the total revenue that is paid as bonus. The optimal value of this participation constraint is when the employee is indifferent between the two options. This relation is showed by the formula:

\[-(28)\quad U_0 = \alpha + \frac{1}{2}t^2[\lambda^{*}(P + \sigma^{*})^2 + \lambda^{**}(P + \sigma^{**})^2]\]

By rearranging the participation constraint it is possible to extract the value of the fixed wage. Rearranging results in the formula:

\[-(29)\quad \alpha = U_0 - \frac{1}{2}t^2[\lambda^{*}(P + \sigma^{*})^2 + \lambda^{**}(P + \sigma^{**})^2]\]
The implication of this formula is that the fixed wage increases when the following component increases:

- the outside option

Furthermore the fixed wage decreases when either one or more of the following components increases:

- the price of the product
- the probability that the second state of the world occurs
- the probability that the third state of the world occurs
- the value of the second state of the world
- the value of the third state of the world
- the percentage of the total revenue that is paid as bonus

The firm’s profit is influenced by the state of the world and because there are three states possible there are also three possible profit-wage schemes. These three schemes are all based on the profit scheme described in subsection Ia.

In the case of $\lambda^*$ the profit level is:

-(30a) $\pi = (P + \sigma^*)E^* - \alpha - t(P + \sigma^*)E^*$

By replacing $E^*$ its previous found value, the profit formula in the first state is:

-(30b) $\pi = -\alpha$

In the case of $\lambda^*$ the profit level is:

-(31a) $(P + \sigma^*)E^* - \alpha - t(P + \sigma^*)E^*$

By replacing $E^*$ for its previous found value, the profit formula in the second state is:

-(31b) $\pi = t(P + \sigma^*)^2 - \alpha - t^2(P + \sigma^*)^2$

In the case of $\lambda^{**}$ the profit level is:

-(32a) $(P + \sigma^{**})E^{**} - \alpha - t(P + \sigma^{**})E^{**}$

By replacing $E^{**}$ for its previous found value, the profit formula in the third state is:

-(32b) $\pi = t(P + \sigma^{**})^2 - \alpha - t^2(P + \sigma^{**})^2$

When the profit level formulas that are added together it is possible to create a expected profit formula. This can be done by taking the probability that a state occurs into account. The expected profit level formula is:

-(33a) $\pi^e = -\alpha + \lambda^*(P + \sigma^*)^2[t - t^2] + \lambda^{**}(P + \sigma^{**})^2[t - t^2]$
When the fixed wage variable is replaced by the previous found value for the fixed wage the only variable under control of the firm is the percentage of the total revenue that is paid as bonus. After some rearranging the expected profit formula is:

\[-(33b) \quad E = - U_o + \lambda^* (P + \sigma^*)^2 [t - \frac{1}{2} \sigma^2] + \lambda^{**} (P + \sigma^{**})^2 [t - \frac{1}{2} \sigma^{**}] \]

In order to find the optimal level of this percentage where profit is maximized, we have to take the first order derivative to t. The first order condition is:

\[-(34a) \quad \lambda^* (P + \sigma^*)^2 [1 - t] + \lambda^{**} (P + \sigma^{**})^2 [1 - t] = 0 \]

After some rearranging it is possible to find the optimal level of t where profit is maximized. This optimal level is

\[-(34b) \quad t = 1 \]

The implication of this formula is that a firm must pay the employee one hundred percent of the total revenue as bonus in order to motivate him to produce his optimal level of effort.

With the optimal bonus level determined it is possible to determine what the profit level of the firm is under these conditions in the second and the third state of the world. After some rearranging the formula for expected profit under these optimal is:

\[-(35a) \quad E = - \alpha \]

When \(\alpha\) is replaced by the determined fixed wage value the formula for expected income is:

\[-(35b) \quad E = - U_o + \frac{1}{2} [\lambda^* (P + \sigma^*)^2 + \lambda^{**} (P + \sigma^{**})^2] \]

The implication of this formula is that a firm can only make a profit by installing a negative fixed wage component, because the employee’s effort is optimized when the percentage of the total revenue he receives is one hundred percent. This fixed wage component can be seen as a fee that the employee must pay to the employer for the use of the equipment he needs to produce his output.

\(V) \quad \text{Chapter summary} \]

In this chapter two compensation schemes are described, these schemes are based on a piece-rate bonus and a reserve bonus. For both compensation schemes the optimal utility, effort, straight salary and bonus level of the employee and the expected profit for the firm is determined. These findings are analysed in the next chapter in order to make the interpretation of the findings easier.
(III) Analysis

The purpose of this chapter is to analyse the effects of the different states of the world on several aspects that are relevant for the employee and the employer. The bases for this analysis are the two bonus schemes described in the model section. In this chapter the main similarities and differences between the two compensation schemes are described.

This chapter is conducted in the following manner. Every section describes the similarities and differences of several aspects between the piece-rate and reserve scheme. This comparison is done for respectively utility and effort, fixed wage, bonus level and profit and bankruptcy chances.

(I) Utility and effort

In the model the employee tries to optimize his utility level in every state. The model shows that this optimization process results in different effort levels that are optimal for the employee under the different states of the world. For both compensations structures the total expected effort and utility is equal. Nevertheless the optimal utility levels differ between the two structures under the different states of the world.

The model shows that under the piece-rate scheme the employee maximizes his utility in the first state of the world when he produces no effort at all. In this state the firm is unable to pay the promised bonus to the employee when he produces effort, so there is no chance that the employee will earn any bonus when he produces effort. In this case the employee has no negative utility that is caused by producing effort, only the negative utility that is caused by the negative fixed wage obligation. In the second and third state of the world the employee maximizes his utility by equalizing his produced effort with the total sales value per product. In the case of the second state of the world his utility is equal to the total sales revenue minus the effort costs, because the he receives this fixed wage back as bonus. In the case of the third state of the world his utility is equal to the total sales revenue minus the negative fixed wage component and the effort costs.

The model shows that under the reserve scheme it is optimal for the employee to produce no effort at all when the first state of the world occurs. He optimizes his utility by producing no effort, because there is no chance that he is rewarded for his effort. In this case his utility is equal to the negative fixed wage component. In the second and third state of the world the firm’s revenue is large enough to pay the promised bonus. As a consequence the employee maximizes his utility by equalizing his produced effort with the total sales value per product. As a result his utility level in both the second and third state of the world is equal to the total sales revenue of a firm minus the fixed wage and effort costs.

The model shows that the employee’s expected utility is equal under both compensation schemes. The different effort levels that the employee produces in every state are equal under both schemes as well. However the expected utility in every state differs among the two compensation schemes. This is caused by a different size of the fixed wage component and as a consequence the employee’s utility
under the piece-rate scheme is smaller in the first and third state and larger in the second state than it is under the reserve scheme.

**(II) Fixed wage**

Both the piece-rate and reserve scheme show that the employee’s wage is conducted from two components. The first component is a negative fixed wage that the employee has to pay. This wage component can be seen as a fee that the employee must pay to the employer for the use of the equipment he needs to produce his output.

The model shows that under both schemes the chance that an employee has to pay a fixed wage to the employer differs. Under the piece-rate bonus the employee has to pay a negative fixed wage to the employer only in the first and third state. Officially there is the same negative wage obligation in the second period as in the other periods. However the employee receives every penny that he pays as negative fixed wage back form the firm as a part of his bonus. In the reserve bonus case the model shows that the employee has a negative wage obligation in every period.

Although the chance that the employee has to pay a fixed wage differs in both schemes, the expected height of the negative fixed wage is equal in both schemes. The only way to achieve this result is if the fixed wage component of a piece-rate bonus is higher than with a reserve bonus. This relation is showed in the model section by the formulas (15) and (29). These formulas show that the first part of the fixed wage equation is equal under both bonus schemes. The only difference is that the first part of the piece-rate bonus equation is divided by the sum of the chance that the first and third state of the world occur, where the reserve bonus equation is divided by one. The total value of this first and third state is smaller than one and as a result the fixed wage component under a piece-rate bonus must be larger than it is under a reserve bonus.

**(III) Bonus**

The second wage component that is showed in the model is in the case of a piece-rate bonus, a certain compensation for every unit of effort that is produced by the employee and in the case of a reserve bonus a certain percentage of the firm’s revenue. In the case of a reserve bonus the compensation is also linked with the total effort the employee produces, because without effort the firm can never make any revenue.

The model shows that it is optimal for the firm with a piece-rate bonus to equal the bonus rate with the total sales revenue that a firm will earn if the third state of the world occurs. The firm must pay the employee this one hundred percent bonus rate in order to motivate the employee to produce his optimal level of effort. When the employer pays an one hundred percent bonus rate his own profits don’t longer depend on the behaviour of the employee. The found value for the bonus rate is optimal, because in the case that the bonus rate is larger than the found value, the employee is compensated for more than he produces and in the case that the bonus rate is smaller than the found value, the firm can
still motivate the employee to work harder by increasing his compensation. The bonus rate motivates the employee to produce his optimal level of effort in the second and third state, however the bonus rate is ineffective in the first state. This is due to the feature of the first state that the revenue is so low that the employer can never pay any bonus and as a consequence the height of the bonus has no influence on the employee, because it is always zero.

The model shows that it is optimal for the firm with a reserve bonus to pay one hundred percent of its revenue as bonus to the employee. With this percentage the employer motivates the employee to produce his optimal level of effort in the second and third state of the world. As a result of this percentage the employer’s profit doesn’t longer depend on the behaviour of the employee. The found value for the percentage is optimal for the same two reasons as described for a piece-rate bonus. The bonus is only effective in the second and third state. In the first state of the world the firm’s bonus has no influence on the employee for the same reason as described under a piece-rate bonus.

Under the piece-rate scheme the bonus size is as big as under the reserve scheme in the first and third period, but has a different size in the case that the second state occurs. Nevertheless it is reasonable that the total expected bonus size is equal under both schemes. In the second state the piece-rate bonus is larger than the bonus under the reserve scheme by precisely as much as the fixed wage. This negative fixed wage that is paid by the employee to the employer is used as a source to pay the promised bonus. Due to the fact that the employee pays the fixed wage and receives it back as bonus, it is reasonable that his real bonus in the second state is equal to the bonus under the reserve scheme.

(IV) Profit and bankruptcy

In the model section the condition for bankruptcy is described. A firm goes bankrupt when it is not able to pay the total promised wage to the employee. This assumption is reasonable, because in the case that a firm cannot fulfil its obligations it is forced out of business.

The model shows that under the piece-rate scheme the firm makes a profit in the first and third state of the world. In the second state the firm’s total revenue is not sufficient to pay the promised wage to the employee and so the profit is zero. The firm has to use its negative fixed wage income to fulfil its obligations to the employee as well as possible. Nevertheless he is unable to pay the whole wage. As a result of the bankruptcy condition the firm goes bankrupt in the second state of the world.

The model shows that under the reserve scheme a firm is able to pay the promised wage to the employee in all the three states. As a result the firm always makes a profit under a reserve bonus, because the firm is always able to pay the promised wage to the employee from its sales revenue. The firm makes a profit in every state and survives all different states of the world.

Although the two compensation schemes have the same efficiency, expected utility and income, the bankruptcy chances are unequal. Furthermore the piece-rate scheme shows that the firm only makes a profit in the first and third state, where the reserve scheme shows that the firm always makes a profit.
The profit with a reserve bonus is smaller in the first and third state than with a piece-rate bonus, however the total expected profit is equal with both schemes.

(V) Chapter summary

In this chapter the implications of the two compensations schemes that are described in the model section are examined. The main implication in respect to the optimal utility and effort level of the employee is that the expected values of these variables are equal in both schemes. However the exact optimal utility per state of the world differs under both schemes. Furthermore the implication of the fixed wages determined in the model section is that the expected fixed wage is equal under both schemes, but that the size of it is different among the states under both schemes. The implications of the determined bonus rate and bonus reserve are examined as well. For both schemes the expected bonus is equal and they have the same distribution. Finally the bankruptcy chances of the firm under both schemes have been analysed. In the piece-rate scheme the firm goes bankrupt in the second state and in the reserve scheme the firm survives all periods. The expected profit levels of the firm are equal under both schemes, only their distribution differs. The implications that are described in this chapter are used in the next chapter, together with the information that is described in the theoretical framework, to examine what the effects of a piece-rate bonus replacement by straight salary payment and a reserve bonus are.

(IV) Results

The purpose of this chapter is to examine what the effects of a piece-rate bonus replacement by straight salary payment and a reserve bonus are. Important in this context is that the purpose of the replaced compensation structure is to reduce the negative influences of a piece-rate bonus, while maintaining its positive effects.

This chapter is therefore conducted in the following manner. In the first section a straight salary payment and a piece-rate bonus are compared with each other, to examine the desirable and undesirable effects that can occur if a piece-rate bonus structure is replaced by a straight salary payment. In section II a piece-rate bonus is compared with a reserve bonus for the same purpose as the comparison in the previous section.

(I) Straight salary as solution

An often used argument in the discussion about bonuses is that with straight salary payment the current economic crisis would not have occurred. If this argument is true, straight salary payment could be more preferable than changing the bonus structure altogether. In order to be able to make such a comparison the possibility of straight salary payment to reduce the negative contributions of a piece-rate bonus to the current economic crisis must be examined. Together with this it must also be examined if straight salary payment doesn’t cause the loss of the positive influences of a piece-rate
bonus. This is done for every one of the three major negative influences and then for the positive aspects.

The introduction of straight salary payment can reduce the drive of employees to maximize short-term results. This is simply achieved by the structure of a straight salary payment, where an employee is rewarded for the amount of time spent on a certain activity in most cases. The employee has no direct incentive to maximize short-term results, due to the merely indirect link between produced output and the size of the compensation as described in the theoretical framework.

If a piece-rate bonus is replaced by a straight salary payment it is unlikely that the flexibility of a firm will increase. The major problem in this case of a piece-rate bonus is that the total wage costs are relatively fixed, which becomes even worse if it is replaced by a straight salary payment where the complete compensation structure is fixed.

Straight salary payment can reduce the socially unfair situation, caused by the fact that the firm will pay bonuses even when at the same time the firm takes socially undesirable actions in economic downturn. Due to the fact that bonuses create a larger variance between the heights of the compensations that are paid to the employees. There are empirical results that this variance is lower with straight salary payment, which is socially seen as more fair, especially in times of economic downturn.

In section II of the theoretical framework chapter, two major advantages of bonuses are examined compared with a straight salary payment. By offering the employees a bonus the firm can reduce the adverse selection and moral hazard problems. By replacing this compensation structure for a straight salary payment, the firm loses this major advantage. Furthermore it is important to mention that a straight salary gives no incentive at all to the employee to produce effort in the described model. It is undesirable to replace a piece-rate bonus by a straight salary payment when after the replacement the employee will not produce effort any more. This is true, even when straight salary payment can reduce the negative contributions of a piece-rate bonus to the current economic crisis. However it is likely that in reality the features where the compensation schemes in the model are based in are less narrow and that there is space for straight salary firms to exist, especially when monitoring costs are taken into account.

(II) Reserve bonus as solution

The purpose of this paper is to create a bonus structure that reduces, at least theoretically, the major negative influences that are related with the current economic crisis that a piece-rate bonus has. In this section the two bonus schemes that are described in the model are compared in order to identify in what extent the reserve bonus can reduce the negative contributions of a piece-rate bonus to the current economic crisis. At the same time it must be examined if the positive effects of a piece-rate bonus structure are maintained.
There is no indication that the introduction of a reserve bonus structure can reduce the drive of employees to maximize short-term results. This drive occurs when a firm has a compensation structure based on a piece-rate bonus. The analyse chapter shows that the employee optimizes his utility for both compensation schemes by equalizing his effort level in the first state of the world to zero and in the other states to the total sales value per product. In the first state the utility optimization behaviour of the employee for both compensation schemes provides little indication of a possible short- or long-term results maximization drive, because he produces no effort. In the other two states of the world it is optimal for the employee to equalize his effort level to the total sales value per product with both compensation schemes, irrespectively of the long-run implications for the firm. The model shows that a reserve bonus compared with a piece-rate bonus provides no solution for the problem that bonuses give an incentive to the employee to maximize the short-term results, because both compensation schemes are based on the same indicator: the total produced effort.

If a piece-rate bonus is replaced by a reserve bonus it is clear that the firm’s flexibility increases and as a result also the survival chances of the firm. A firm with a piece-rate bonus goes bankrupt in the second state of the world, where a firm with a reserve bonus survives all states of the world. The firm under a reserve bonus can survive the second state of the world as a result of its relative financial flexibility. This flexibility is a result of the structure of the bonus compensation, where the state of the world influences the size of the bonus directly.

A reserve bonus can reduce the socially unfair situation, caused by the fact that the firm will pay bonuses even when at the same time the firm takes socially undesirable actions in economic downturn, when we see bankruptcy as the social undesirable action. For the first and third state of the world both compensation schemes have the same result and show no socially undesirable actions. With a piece-rate bonus the firm pays a bonus to the employee and goes bankrupt, which is an example of the examined social unfair situation. This undesirable situation does not occur in the second state with a reserve bonus, because the firm survives this state and does not pay bonuses that it can not afford. As a result a reserve bonus can reduce the socially unfair situation, caused by the fact that bonuses are paid at the same time as socially undesirable actions are taken by the firm, because there are no states under this bonus structure where such situations can occur.

The purpose of the paper is to reduce some major negative aspects of a piece-rate bonus, while maintaining the positive effects. There is no indication that a reserve bonus will reduce one of the positive effects of bonuses that are described in the theoretical framework.

(III) Chapter summary

In this chapter the effects of a piece-rate bonus replacement by straight salary payment and a reserve bonus have been examined. An important finding is that straight salary payment can reduce two out of the three major problems, but is incapable to maintain the two major positive effects of a bonus structure. Another important finding is that a reserve bonus can also reduce two out of the three major
problems and is capable to maintain the two major positive effects of a bonus structure. In the next chapter possible additional research options are described that can investigate if the results of this chapter hold under other conditions and in reality.

(V) Discussion
The purpose of this chapter is to discuss possible additional research that can extend and contribute to this research. For every additional research option, possible implications and difficulties are described. This is done by using related literature about theoretical and/or empirical findings that are related with the possible additional research.

The chapter is conducted in the following manner. Section I describes additional research that can be done to determine what the influence of a risk-averse employee and/or employer can be on the findings of this paper. In the second section a possible extension of this paper with a multiple period condition is described. In this section the possible theoretical and empirical implication of such an extension is examined. In the third section possible additional research is described that investigates what the influence of a multiple employee assumption can be. This section gives some theoretical and empirical findings that can be relevant for such a research. In the fourth section, possible empirical research in order to examine if the theoretical implications of the model hold in reality is described.

(I) Risk-aversion
Additional research can investigate if the current findings hold when the assumption of risk-neutrality of the employee is replaced by a risk-averse employee. The model shows that the total expected utility is identical under both compensation schemes. However the utility (and income) is under a reserve bonus more equally distributed than under a piece-rate bonus. As a result of the certainty effect of Tversky and Kahneman (1981 : 453-458), that is described in the theoretical framework section, it is likely that a risk averse employee desires a reserve bonus instead of a piece-rate bonus.

The same kind of research can be done by replacing the risk-neutrality of the employer by risk-aversion. The model shows that the total expected profit is identical under both compensation schemes. The firm in the reserve scheme makes always a profit, regardless of the state of the world. Nevertheless the firm in the piece-rate scheme makes only a profit when the first or third state occurs. According to the theory of Tversky and Kahneman (1981 : 453-458) it is likely that the employee will prefer a reserve bonus instead of a piece-rate bonus structure. This remains the case when is taken into account that the employer has in generally a better position than the employee to reduce his risk, by diversifying risky activities. Even with this possibility to diversify risks the reverse bonus remains more favourable than a piece-rate bonus. Due to the fact that it is more preferable for the firm that it’s survival chances are hundred percent, than that there is a chance on bankruptcy.
A possible extension can be research that investigates if the findings of the described model hold if the one period assumption is replaced for a multiple period model. An important advantage of a multiple period model can be that it provides the possibility to analyse the effect of long-term contracts in the model that is described in this paper.

According to Bose (1993: 248-256 and 272) long-term preferable behaviour of the tenancy can be stimulated by providing the tenancy an incentive that depends on the implications of the tenancy’s behaviour in the long run. His analysis is concentrated on incentives that improve the long-run investments in the case of an agricultural enterprise. Despite this narrow view an important finding of his model can be used in a more general purpose. This important finding is that by offering an tenancy a long-term contract the tenancy provides the employee an incentive to take the long run implications of his behaviour into account. Important is that with this type of contract the positive long run implications of the employee’s behaviour must result in a higher compensation for the employee in the long-run. In the case of an agricultural enterprise this can be achieved by offering the tenancy a part of the produced crops. A more general option to achieve this is providing the employee stocks of the firm.

There is empirical evidence that long-term contracts are more likely to be used when it is important that the employee takes the long run implications of his behaviour into account. Bandiera (2005: 2-8 and 21-22) used information of more than seven hundred tenancy contracts that have been made up between 1870 and 1880 in Italy as a source for his empirical research. Her main finding in the context of this paper is that both the duration and the structure of the contract influences the behaviour of the employee. She found that employers provide their tenancies a long run contract and a stake in the futures production in order to give them an incentive to take the long run implications of their behaviour into account. She found another important advantage of long-term contracts and that it reduces the contract and monitoring costs. An important negative aspect of long-term contracts is that the firm becomes less flexible and that the long-run implications of the tenancy’s behaviour are also influenced by factors beyond the tenancy’s control. This last aspect is especially important when the tenancy is risk averse.

Other research can investigate if the found positive effects hold if the one employee structure of the model is replaced by a multiple employee structure. The analysis and result section show that the effort indicator can stimulate undesirable behaviour. With team incentives, there is a chance that this undesirable behaviour is reduced. Besides this, important advantages of team production are the advantages of specialization and knowledge transfers among employees. An important disadvantage is that when output is not verifiable to a specific employee, some employees will shirk. Although they

(II) Multiple periods

(III) Multiple employees
produce as less as possible effort, they are rewarded according to the total production of the team. This undesirable behaviour is called the free-rider effect.

Lazear and Rosen (1981: 841-850) describe rank-order tournaments as a reward structure that can reduce the free-rider effect by rewarding the employees on their relative performance. In their rank-order tournament they assume that there are two risk-neutral employees and that the winner of the tournament is rewarded with a certain desirable compensation, such as a promotion. Their basic assumptions can be summarized as following: The win chances of an employee are influenced by his own performance, the other employee’s performance and luck. In their model they make the assumption that employees made their decisions that is described in a Nash-Cournot decision structure. The employees optimize their investment by taking only the optimal investment of their opponent into account, because they have no influence on the luck factor. In their analysis they show that the rank-order tournament can result under certain conditions in the same incentive for employees to optimize their effort as under piece-rate payment. Their rank-order tournament structure is based on relative performance instead of produced effort. It is possible that installing relative performance pay makes the model described in this paper more preferable.

Empirical evidence for the advantage of team production is described by Hamilton, Nickerson and Owan (2003: 11-27). They used data about the weekly productivity over the years 1995-1997 for 288 employees at Koret Corporation. Traditionally the manufacturing process at the firm was broken down in a number of specific individual tasks. An employee was compensated for his individual performance according to piece-rate compensation. In 1995 the firm asked volunteers for team production, with a team result based compensation. The result of team production was a productivity increase of 18%. Roughly 14% was directly linked with the installed team production and the other 4% was a result of the self-selection mechanism of higher ability employees, who voluntarily joined the team production. As a result of these positive effects the firm introduced more team production. The productivity increase diminished for these later created teams. There was even a decline in productivity for the last team that was created, although this decline was not significant. An explanation for this diminished increase can be that the bargaining power of members of the first teams was higher than of the members in the later teams, because the threat that they could leave the team and move back to an individual compensation structure was more credible. Another important finding of their research was that heterogeneous teams were more productive than homogeneous teams. This finding is an indication that the theoretical advantages such as specialization and knowledge transfer occur in reality.

(IV) Empirical research

Additional related research that can be done is research that investigates if the theoretical implications of the created bonus structure are in line with the empirical findings when a firm replaces a piece-rate
bonus structure for a reserve bonus structure. The major advantage of such related research is that it maybe can prove that the implications of the bonus structure occur in reality too. Even when not all theoretical implications occur, it can give more insight in unforeseeable features, advantages and disadvantages of the reserve bonus structure.

In this paper several empirical analyses are described. It is reasonable that the empirical analysis by Shearer (2004) has the most desirable structure of these described empirical researches for an empirical research with the results of this paper. His structure is preferable, because employees were not aware of the fact that they were a part of an experiment, which eliminates undesirable tactical behaviour of the employees and due to the experimental structure other influences besides the influence of the compensation structure were reduced. There are several difficulties that occur when such empirical analysis is done. One of these difficulties with an empirical analysis on the findings of this paper is that it is unlikely that there is a firm where a reserve and piece-rate bonus already coexist. Another more important difficulty is how to measure the influence of the state of the world. It is possible to use macro-economic indicators to measure this influence, but it is likely that there will remain some errors in this measurement. It is likely that the three described states of the world need a more universal character before empirical analyse is possible, because it is unlikely that only three categories of the state of the world can occur.

(V) Chapter summary
In this chapter important possible additional research is described that can investigate if the findings of this paper hold under other conditions and in reality. A possible other condition is when the employer and/or the employee is/are risk-avers. It is likely that they will prefer a reserve bonus above a piece-rate bonus when they are risk-averse. Another possible change in the conditions is that the firm operates in a multiple period structure. It is likely that under specific conditions such structure can reduce the drive of employees to maximize the short-term results by installing long-term contracts. Furthermore it is possible to install a multiple employee structure. With such a structure it is possible to use other indicators than effort to determine the compensation of the employee. Finally additional research can be done to the possibilities of empirical research on the findings of this paper. It is likely that the main problem for such research is the determination of the influence of the state of the world.

(VI) Conclusion
The purpose of this paper is to compare straight salary payment and a reserve bonus with a piece-rate bonus, in order to determine which structure is most preferable to reduce some negative contributions of a piece-rate bonus to the current economic crisis. Major negative influences of a piece-rate bonus are that it provides the employee an incentive to maximize short-term results, it decreases the financial flexibility of a firm and that it causes a socially unfair situation, because the firm will pay bonuses even when at the same time the firm takes socially undesirable actions in economic downturn.
A major finding of the comparison that is done is that the introduction of a straight salary payment can reduce the drive of many employees to maximize short term results, whereas a reserve bonus is unlikely to reduce this effect. This is caused by the structure of a straight salary compensation that gives the employee no direct incentive to maximize short-term results, because there is only an indirect link between output and the size of the employee’s compensation. The inability of the reserve bonus to reduce the result maximization behaviour of the employee is caused by the used indicator that determines the employee’s compensation. This indicator triggers the employee to maximize his utility for a given period, regardless of the long-term consequences for the firm.

Another important finding is that the introduction of a straight salary payment is unlikely to increase the firm’s financial flexibility, whereas a reserve bonus can increase this flexibility. Straight salary payment will only deteriorate the financial flexibility of a firm further, because the whole compensation structure is completely fixed. The opposite is true for a reserve bonus structure which increases the firm’s flexibility and as a result it’s survival chances. This flexibility is a result of the structure of the reserve bonus, where the state of the world influences the size of the bonus directly.

The result section shows that the introduction of both a straight salary payment and a reserve bonus can reduce the socially unfair situation, caused by the fact that the firm will pay bonuses even when at the same time the firm takes socially undesirable actions in economic downturn. For straight salary payment this is caused by the relative low variance between the size of the compensation under this structure, which is socially seen as more fair, especially in times of economic downturn. In the case of a reserve bonus the undesirable situation can not occur, because the firm survives every state of the world in the reserve scheme.

There are two major positive effects of a bonus structure that are described in the theoretical framework. However, with the introduction of a straight salary payment the positive effects disappear and so the adverse selection and the moral hazard problems can occur. The model also shows that an employee has no incentive to produce effort with straight salary compensation, which is of course undesirable. When a firm has the same feature as described in the model, straight salary payment is not desirable. However it is likely that in reality these features are less narrow and that there is space for straight salary firms to exist. These negative side effects do not occur when a piece-rate bonus is replaced by a reserve bonus.

We conclude that a reserve bonus is a more favourable option than straight salary payment to reduce the major negative influences of a piece-rate bonus, which are related with the current economic crisis. Due to the fact that a reserve bonus can reduce two out of the three major problems and maintain the two major positive effects of a bonus structure. A straight salary payment can also reduce two out of the three major problems, but is incapable to maintain the two major positive effects of a bonus structure.
References


-(5) Brown, C, (1989), Firms’ choice of method of pay, p. 5-7 and 19-20

-(6) Clawson, D, (1980), Bureaucracy and the labor process, p. 168-182


Appendix

In the appendix the common restrictions for all three states of the world are determined. It is important to determine these restrictions, because with these restrictions it is possible to prove that there are values possible for all three states and that there is no overlap between them. In section I the restrictions for the piece-rate scheme are examined and in section II this is done for the reserve scheme. In section III the restrictions that are found in sections I and II are compared to determine which restrictions of the different states of the world are held for both schemes.

(I) Restrictions states of the world under the piece-rate scheme

In the first section of the model chapter the necessary characteristics of each of the three states of the world were explained. Every state has certain restrictions, otherwise the states of the world won’t suffice to their own descriptions.

In the first state the firm’s revenue is equal or smaller than zero. This is needed to have a condition where the firm can not afford to pay a part of the fixed wage or the bonus. This relation is described in the formula:

\[-(36a) \quad (P + \sigma^*)E^* \leq 0\]

The variable $E^*$ in the formula is replaced by the optimal level of effort in the first state and after some rearranging we can extract the value of $\sigma^*$, this formula is:

\[-(36b) \quad \sigma^* \leq -P\]

In the second state the firm’s sales revenue is larger than zero, but not sufficient to pay the promised bonus completely. This state of the world needs two restrictions. The first part describes the restriction that the firm’s revenue is larger than zero. This relation is described in the formula:

\[-(37a) \quad (P + \sigma^*)E^* > 0\]

When the variable $E$ in the formula is replaced by the optimal level of effort in the second state the following formula is found:

\[-(37b) \quad (P + \sigma^*)^2 > 0\]

The second part of the restrictions for the second state of the world shows the assumption that the firm doesn’t have enough sales revenue to pay the promised bonus. This relation is showed in the formula:

\[-(38a) \quad (P + \sigma^*)E^* < \beta E^* + \alpha\]

The variables $E$, $\alpha$ and $\beta$ in the formula are replaced by the optimal level of effort in the second state. After some rearranging we find the relation:
In the third state of the world the firm must make a profit, so the total wage cost must be equal or smaller than the total income of the firm. This relation is described in the formula:

\[-(39a) \quad (P + \sigma^*)(E^* - \alpha - \beta E^*) \geq 0\]

The values of the optimal level of the bonus rate and effort that were found in the second section of the model chapter can be used in the restriction. When we use these values we find after some rearranging the following relation:

\[-(39b) \quad \alpha \leq 0\]

This value of the fixed wage is consistent with the earlier found relation between the fixed wage and profit. When the third state of the world occurs the firm’s profit is at least zero, which is also showed in this relation. The only profit a firm can make is to the negative fixed wage that appears when contracting an employee. When we use the found value for the fixed wage to determine what the restriction of the third state is we find:

\[-(39c) \quad \sigma^{**} \geq \sqrt{2U_o/\lambda^{**} - \lambda^*(P + \sigma^*)^2/\lambda^{**}} - P\]

(II) Restrictions states of the world under the reserve scheme

The restrictions of the three states of the world are also based on the description of their characteristics in the first section of the model chapter.

In the first state the firm’s revenue is at least zero. This relation is described in the formula:

\[-(40a) \quad (P + \sigma^*)E^* \leq 0\]

It is possible to rearrange the formula so that we can find the value for \(\sigma^*\) under a profit based bonus reserve:

\[-(40b) \quad \sigma^* \leq -P\]

In the second state of the world the firm’s revenue must be large than zero. The bonus reserve depends on the revenue of a firm, so in every situation that the revenue is larger than zero there is a bonus paid to the employee. This relation is described in the formula:

\[-(41a) \quad (P + \sigma^*)E^* > 0\]

When the value of the optimal effort in the second state of the world is used to replace \(E\), the following relation is found:

\[-(42b) \quad (P + \sigma^*)^2 > 0\]
In the third state of the world the firm’s profit must be equal or larger than zero, so the firm’s total cost must be smaller than the total income. This relation is described in the formula:

\[-(43a) \quad (P + \sigma^{**})E - \alpha - t(P + \sigma^{**}) \geq 0\]

When we use the founded values of the optimal effort and the value of the percentage of the profit that is paid to the employee to replace the variable E and t in order to find the following relation:

\[-(43b) \quad \alpha \leq 0\]

The implication of this relation is that with a compensation reserve bonus the firm can only make a profit when the fixed wage component is negative.

It is possible to determine the value of $\sigma^{**}$ where profit is larger than zero:

\[-(43c) \quad \sigma^{**} \geq \sqrt{[2U_{\alpha}/\lambda^{**} - \lambda^*(P + \sigma^*)^2/\lambda^{**}]} - P\]

(III) Restrictions states of the world that are common for both schemes

The two compensation schemes can only be compared with each other when there are specific values of the first, second and third state of the world that fit in the restrictions of both schemes. In this section we use the previous found restrictions to determine the values of the different states of the world that hold for both schemes.

The common restriction for the first state of the world is clear because the restrictions in both compensation structures for this state are equal in both structures. The common value of $\sigma^*$ must be:

\[-(44) \quad \sigma^* \leq - P\]

The common restriction for the second state is also clear. For both compensation structures the following minimum condition must hold:

\[-(45) \quad (P + \sigma^*)^2 > 0\]

The maximum value of the second state of the world is showed by the restriction that is found in section I of the appendix. The reason why there is only a maximum value under the piece-rate scheme is that there is no real maximum for the value of the second state in the case of the reserve scheme as long as the second state is smaller than the third state. The formula for the maximum value is:

\[-(46) \quad (P + \sigma^*)^2 < \frac{[(\lambda^* + \lambda^{**})(P + \sigma^*)(P + \sigma^{**}) + U_{\alpha} - \lambda^{**1/2}(P + \sigma^{**})^2]}{\lambda^* + \lambda^{**} + 1/2\lambda^*}\]
The common restriction for the third state of the world is clear as well. This restriction is equal to the value of the third state of the compensation structure with a fixed bonus reserve and is every value of the state of the world that is larger than the maximum of the second state. The common third state of the world restriction is:

\[-(47) \quad \sigma^{**} \geq \sqrt{2U_o/\lambda^{**} - \lambda^*(P + \sigma^*)^2/\lambda^{**}} - P\]

An important aspect of these restrictions is that all restrictions can occur and that there is no overlap of the different states. It is clear that the restriction of the first state of the world is smaller and has no overlap with the minimum value of the second state of the world. This is showed by rearranging formula (45):

\[-(48) \quad \sigma^* > - P\]

However it is not directly clear if this is true for the lower and upper bound of the second state of the world. In order to determine if there are possible values between the lower and upper bounds of the restriction for the second state of the world we must prove that the restriction for the minimum value is smaller than the restriction of the maximum value of the second state of the world. This relation is showed by:

\[-(49) \quad 0 < (\lambda^* + \lambda^{**})(P + \sigma^*)(P + \sigma^{**}) + U_o - \frac{1}{2}\lambda^{**}(P + \sigma^{**})^2\]

This relation must hold when the right part of the equation is larger than zero. When we look at the formula for the third state of the world it is possible to prove that the condition must hold. The third state of the world shows a square root in the equation. As a consequence the first part of the equation must be positive and larger than zero. It is clear that the first part of the equation is positive, because it is impossible to take a square root of a negative number. In order to be sure that the first part of the equation is larger than zero, the following condition must hold:

\[-(50) \quad 2U_o > \lambda^*(P + \sigma^{**})^2\]

Due to this condition the formula for the lower and upper bound of the second state of the world hold when the following condition hold:

\[-(51) \quad (\lambda^* + \lambda^{**})(P + \sigma^*)(P + \sigma^{**}) > 0\]

Formula (48) shows that the second state of the world is larger than a negative price. As a consequence the second and third state of the world are always smaller than the price. As a result the condition for the upper and lower bound holds.

Due to the fact that there are values of the second state of the world that are positioned somewhere within the range of the common second state restriction it is possible to define a case where the desired
characteristic of the states of the world occur. A possible value for the second state of the world is when:

\[-(52)\]

\[(P + \sigma^*)^2 = \frac{\frac{1}{2}(\lambda^* + \lambda^{**})(P + \sigma^*)(P + \sigma^{**}) + U_0 - \lambda^{**}\frac{1}{2}(P + \sigma^{**})^2}{(\lambda^* + \lambda^{**} + \frac{1}{2}\lambda^*)}\]

This value is larger than the lowest bound and smaller than the upper bound that the value of \(\sigma^*\) can possibly have.