Inventory Management
of Returnable Bottles of Brewery

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Preface

This thesis is done in order to finalise my study of master Econometrics and Management Science at the Erasmus University in Rotterdam. I have long lived under the impression that every assignment, especially this thesis, was a fight against time. The months that I have worked on this thesis have been a true journey. And as always with journeys, occasionally one might get lost or need some reliable guidance on the way in such a manner that the destination becomes clear and visible again. There are some people who I would like to thank.

The master's thesis that lies in front of you is the final product of my learning journey. This thesis would probably never have been possible to complete without help of certain persons. First of all I would like to express my gratitude to my supervisor, Prof. Dr. Ir. Rommert Dekker, with whom I had information, knowledge and sources. I would never have been able to write this thesis without the excellent support of my supervisor. I would like to thank to Drs. Eelco van Asperen who put the time to share about simulation program, Dr. Erwin van der Laan for sharing with me his knowledge and some papers, Cerag Pince MSc who put the time to review my thesis.

Without support and warmth of my husband, my children and my family, this thesis probably never have been possible to finish. Thank you for keeping me on the straight and narrow. Last but not least I would like to thank some peoples who have helped me to finish this thesis.

Sari Widi

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Chapter 1

Introduction

An important challenge arising in reverse logistics supply chains is the effective use of returns so as to maximize the value of this resource. Recondition is the process by which used products are recovered, processed, and used as new products. Product return has a major influence on inventory management. Since cleaned returned products enter the inventory of serviceable items, ordering strategies are affected both with respect to timing and quantity.

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In a brewing manufacturing company, they use kegs, bottles and crates that can be reused after inspection and cleaning. An adequate supply of empty kegs, bottles and crates must be on hand to satisfy the demand. Part of this supply is a result of the returns of previously issued ones. The time from issue to return of them however, is variable, without any prior information about it. This make the inventory control of those items a difficult task.

In order to allow a continuous production with respect to inventory control, the problems are as follows :
How to determine the number of kegs, bottles and crates needed to allow a continuous production with respect to inventory control.

Accordingly, the objective in this thesis is to develop a method to assist in inventory control of returnable items. Next to analyse data and apply the method in a case study.

To this end we model and determine the logistic process of returnable kegs, bottles and crates of a large brewery manufacturing company in order to attain required service levels with as low minimize average inventory levels as possible. We give advice on the number of them needed to allow a continuous production with a certain probability. Remanufacturing complicates inventory management by introducing return flow of used products. The random lead time in this stage (customer-use) equals the duration of the product's stay with the customer, and the yield is the proportion of products that are eventually returned. In production planning and inventory management decisions, the yield, the lead time, and the on-hand inventory associated with a given stage are key pieces of information.

In this thesis, we start with analyzing data on return cycles coming from a case study. We apply the distribution fitting to describe the return. Next we use the results in an inventory control model, and analyse it, both with analytical calculations and a simulation model that are suggested by the supply chain structure of remanufacturable products. We calculate and simulate with real data from the company.

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Chapter 1 presents introduction. In chapter 2, we discuss the background, problem formulation and literature review. Chapter 3 presents data description and analysis. We find a good distributional model for the return lead time. Distribution fitting has obtained by using SPSS vs. 15.00 program. Chapter 4 presents the models. Chapter 5 presents the analytical calculation. Chapter 6 deals with the simulation models. The purpose of this chapter is to give advice on the number of kegs / crates needed to allow a continuous production with a certain

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Chapter 2

Background, Problem Formulation and Literature Review

The problem originated from a beer brewery, we called the company with the fictitious name, Morbeef. Below we briefly introduce and describe the company as well as the circulation process of returnable beer items. Next we outline our approach and we provide a literature review.

2.1. Company and Process Description

Morbeef is one of brewer manufacturing company that sell products in containers (crates) that can be reused. An adequate supply chain of empty crates must be on hand to satisfy the demand. We model the returnable crates of a large brewer manufacturing company. Remanufacturing complicates inventory management by introducing return flow of used products. The random lead time in this stage (customer-use) equals the duration of the product's stay with the customer, and the yield is the proportion of products that are eventually returned.

Morbeef first entered Germany in 2001 with the acquisition of the Diebels Brewery, followed by Beck & Co in 2002. In 2003, Morbeef acquired the Gilde, Hasseroder and Spaten breweries. In 2006, Braunschweig, Zwickau and Stuttgart plants were sold. Volume of product sold in 2006 is 10.2 million hectoliters. The market position of Morbeef Germany is number 2 in the market, with market share 10.2%.

The beer supply chain in Morbeef is shown Figure 2.2 : (show the stocking points)

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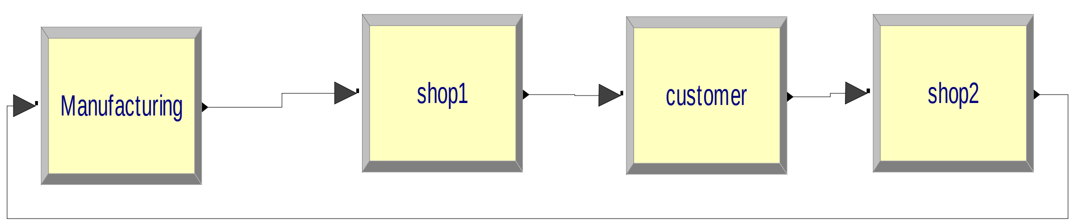
Figure 2.1. The beer Suply Chain in Morbeef



The final products (beer) are transported through the procurement Morbeef to various retails / shops (in figure 2.1 we called shop1). After a beer is purchased, the customer keeps the bottles / crates for a certain amount of time and then take the crates to various retails / shops (in figure 2.1 we called shop2) to be returned. The retails / shops are responsible for returning the bottles / crates to the procurement Morbeef. The return time in this process is the duration of the bottles / crates stay with the customer as well as the transportation time to the stocking point.

Using that distribution process , there is some interrelated sets of decision typically required on a daily basis. The decision is as follows :

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The quantity of bottles or crates should be ordered at which point in time.

2.2. Approach

The case study is related to the model the returnable bottles / crates, we employ data from the Supply Chain Planning Division of Morbeef in 2006 for Germany market. The details of our model are rooted in the supply chain for brewer manufacturing company, in this case is Morbeef. Our approach can be applied in a variety of manufacturing company.

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To model the returnable bottles, crates, we construct the queueing network in to model Morbeef's entire supply chain. A queueing network is a natural model to employ in the remanufacturing setting because it captures not only the flow of materials through the traditional stages of procurement, production, distribution, and sales, but also the dependence of the return flow of used products on past sales, the return delay, and the return probability. We find a good distributional model for return lead time. Distribution fitting has obtained by using SPSS vs. 15.00 program and the analytical calculation. Finally we simulate the probability distribution with the simulation models to give advice on the number of kegs / crates needed to allow a continuous production with a certain probability. The simulation will be obtained by using arena program. We compare the result between analytical calculation with simulation.

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2.3. Literature Review

In this thesis we conduct literature review to look at and study some theory and methodology on inventory control with return flows. We used some Journals publications as the source. The scope of this literature review are returnable items, inventory control, closed-loop queueing network and supply chain management. We seek the source using computerized methods, to identify a set of useful Journals, articles and books . Next we did reference and citation search. Below we will discuss the most important papers or Journals, which came out of the search.

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In a paper by Dekker and de Brito (2001) literature on inventory control models with returns can be distinguished into two streams : 1) typical repair models. 2) other models with imperfect correlation between demand and returns. The review follows in two sections : deterministic models and stochastic models.

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A. Deterministic models.

Schrady (1967) considers a deterministic inventory model in which a certain percentage of sold products come back, after a known period of time, to be repaired. Repaired items are put in inventory to be eventually re-used. Since the demand and return processes are assumed to be continuous deterministic flows, the dependency relationship between the demand and return process is not explicitly modelled. Later, Richter (1994) and Teunter (1998) extended this model with the option of product disposal. As regards the demand and return process, Schrady's assumptions remain the same in both extensions.

B. Stochastic models

The stochastic models can be divided into two groups that typically have different assumptions : periodic review models and continuous review models.

B1. Periodic review models.

This category of models typically focuses on proving the structure of the optimal policy rather than finding optimal parameter values. Simpson (1978) provides for instance the optimal policy structure for an inventory model with product return in which product demands and returns can be stochastically dependent within the same period only. Demand and returns are known through a joint probability function, which can differ from period to period. Inderfurth (1997) extends the previous model with non-zero (re)manufacturing and procurement lead times. All other assumptions equal the ones of Simpson. Inderfurth proves that there is a simple optimal control policy structure as long as the lead time for manufacturing and remanufacturing differ at most one period. Buchanan and Abad (1998) consider a system with partial returns. Each period, a fixed fraction of products is lost while a stochastic fraction is returned. The authors establish an optimal policy for the case that the time until return is exponentially distributed.

Toktay et al.(2000) study ordering policies for a business case of single-use Kodak's cameras. After using the camera, customers bring it to a photo laboratory where the film is developed. The laboratories next returns the used cameras to Kodak (but sometimes they go to the so-called jobbers). Kodak dismantles the used cameras and reuses the flash circuit broad of every camera in the manufacturing of new ones. A closed queueing network model is applied to decide on periodic ordering decisions. Custom demand is treated as a stationary Poisson process from which a known percentage is returned. Time until return is modelled by an infinite Customer and Lab server with general processing time. Another important feature of this paper is the identification of the information's value according to different scenario.

Kiesmuller and van der Laan (2001) develop a periodic review inventory model where product returns depend on the demand process. Both the demand and return streams follow a Poisson distribution. All returns depend on previous demands through a constant time until return, and two probabilities : the return probability (which underlying event is assumed to be known upon the demand), and the probability that a returned item is in a sufficiently good condition to be remanufactured. The authors compare this model with the situation of independent demands and returns. The outcome supports that it is worth to use information about the dependency structure between demands and returns.

B2. Continuous review models.

Heyman (1977) analysis different disposal policies for a single-item inventory system with returns. He uses a model where demands and returns are independent compound renewal processes and all lead times are zero. An explicit expression for the optimal disposal policy is given when the processes are Poisson. Muckstadt and Isaac (1981) investigate too the control of single-item inventory system with independent demands and returns following a Poisson Process and derive some approximations.

Fleischmann and Dekker (1997) derive an optimal policy and optimal control parameters for a basic inventory model with returns where demand and return are independent Poisson processes.

Van der Laan et al.(1999) deal with policies in the context of two inventory facilities, one of new products and the other of remanufactured items. The model considered is based on unit demand and unit returns with independent Poisson processes.

Apart from the papers mentioned by Dekker and De Brito (2001), there are some other papers relevant for this thesis.

Kelle and Silver (1989) describe forecasting methods the returns of reusable containers. The time from issue to return of an individual container is a random variable with a distribution that includes a finite probability of never being returned because of lost or damaged. In order to properly establish the reorder point for purchasing new containers, it is necessary to forecast the net demand (demand minus returns) during the purchasing lead time and the variability of this net demand. The problem studied in this paper was motivated by interactions (visits and consultations) with organizations selling products in returnable containers. These include : liquid gases (in cylinders), beer (in kegs) and non-alcoholic beverages (in returnable bottles and/or plastic cartons). Although detailed data on demands and returns were not readily available from these sources, approximate parameter values were obtained for use in simulation experiments. In this paper different forecasting methods, dependent upon the available data, are developed to estimate the returns and net demand during the lead time. A measure of forecast errors and the appropriate reorder point are also estimated for each of the forecasting methods.

The paper describes four different forecasting methods. Method 1 is the simplest case. It utilizes only the expected value and the variance of the demand during the lead time and the the probability of each container eventually being returned. Method 2 uses more detailed information, namely the actual issues during each previous period and the probability of return in 1, 2,….n periods for any given container. Method 3 uses the same issue and return probability data as Method 2, as well as the amount returned up to the present from each previous issue. This additional information permits us to obtain the appropriate conditional probabilities of return quantities during the lead time for the remaining outstanding portion of each previous issue. Method 4 requires, besides the issue and return probability data of method 2, only the total amounts returned in each of the recent periods without indentification of when the associated containers were issued (i.e.. only aggregate return data).

The reorder point is calculated as calculated in the form $s = ED + k^* \sqrt{VD}$ ensures an optimal policy if the lead time net demand ED and its variance VD are estimated correctly. If these estimates are incorrect, the reorder point will be set too high or too low. If the reorder point is set too low, the increase in the expected shortage cost is higher than the decrease in the expected holding cost. These properties are consequences of the choice of the appropriate safety factor k^* which minimizes the expected total of the two costs.

In a paper by Kroon and Vrijens (1994) Reverse logistics is an important issue. Reverse logistics refers to the logistic management skills and activities involved in reducing, managing and disposing of hazardous or non-hazardous waste from packaging and products. Reverse logistics may be applied to several stages of the logistic chain. Both the materials management part and the physical distribution part of the logistic chain are potential areas of application. The application of reverse logistics in the area of physical distribution : the reuse of secondary packaging material

Secondary packaging is packaging material used for packaging products during transport from a sender to a recipient, either in retail or in industry, Stock J.R. (1992). Traditionally, cardboard boxes are used as secondary packaging material. Since cardboxes can be used only once, they are defined as one way packaging material. In contrast, returnable packaging is a type of secondary packaging that can be used more than once in the same form.

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Although returnable packaging maybe of different types, such as returnable containers, irrespective of the actual type of the returnable packaging.

In 1993 the Frauenhofer Institut published the result of an ecological comparison of one way packaging and returnable containers. On the basis of four criteria, it concluded that returnable containers are less of a burden to the environment than one way packaging material, provided each container is used a certain minimum number of times during its lifetime. This minimum number is dependent on the type of container. The criteria taken into consideration in this study were energy consumption, emission to the atmosphere, water consumption and pollution, and solid waste.

The use of systems of returnable containers is being prompted by a growing concern for the environment and by regulations from the government. For example, in 1991 the Dutch government and industry signed the Packaging Covenant (1991) forcing industry to think of new ways to deal with packaging material. In broad terms, the Packaging Covenant requires that in the year 2000 the total amount of new packaging material in The Netherlands should be reduced by 10 percent (relative to 1986), and that the total amount of packaging waste to be dumped in the ecosystem should be reduced to zero.

Similarly, the German Packaging Order requires manufacturers to take responsibility for their packaging waste. In order to comply with this, German manufacturers and retailers created the non-profit organization Duals System Deutschland (DSD) to collect packaging material for recycling. Participating companies pay a per-item free based on the amount of packaging used and receive in return a green dot (grune Punkt) symbol that appears on their one-way packaging material. The system is still suffering from a number of growing pains, which, of course, works to the relative advantage of systems employing returnable containers.

A consequence of the use of returnable containers is that, after a container has been used for carrying products from a sender to a recipient, the container has to be transported from the recipient to the next sender, who need not be the same as the first one. In addition to transporting the containers, the return logistic system also involves the cleaning and maintenance of containers, as well as their storage and administration.

System with return logistics. In such a system the containers are owned by a central agency. This agency is also responsible for the return of the containers after they have been emptied by the recipient. Lutzebauer (1993) differentiates the following systems :

1. Transfer system : The sender always uses the same containers. The transfer system is only concerned with the return of containers from the recipient to the sender.
2. Depot system : In this system the containers that are not in use are stored at containers depots. From a container depot the sender is provided with the number of containers he needs. After having been transported to the recipient, the empty containers are collected and returned to a container depot.

System without return logistics. In this system the containers are also owned by a central agency. The user of this system, the sender, rents the containers from the agency. As soon as the sender no longer needs the containers, they are returned to the agency. The sender is responsible for all activities involving the containers, such as return logistics, cleaning, control, maintenance, and storage. By using this system, the sender can decrease his fixed costs by renting warying numbers of containers as required.

Whitt (1984) investigated the relationship between open and closed models for networks of queues. In open models, jobs enter the network from outside, receive service at one or more nodes, and eventually leave the network.. Thus, with a open model the total external arrival rate or throughput is an independent variable (specified as part of the model data), and the number of jobs in the system is a dependent variable (whose equilibrium distribution is described in the model solution). On the other hand, in a closed models there is a fixed population of jobs in the network. Hence, with a closed model the number of jobs in the system is an independent variable (specified as part of the model data), and the throughput (which may be defined, for example, as the departure rate from some designated node) is a dependent variable (to be calculated and described in the model solution). Since the individual service rate is part of the model data, knowing the throughput is equivalent to knowing the utilization, which is the expected proportion of the servers at the designated node that are busy in equilibrium.

It might seem that open models would be more appropriate for most applications because jobs do usually come from outside, flow through the system, and eventually depart. However, closed models are often used instead. The representation of flow through the system, i.e., the throughput, is easily handled in a closed model by assuming that a new job enters the system to replace an old one whenever the old one has received all of its required service. This can be represented in the closed model by a transition to a designated exit-entry node. At this node arriving jobs complete. All of their required service, and departures are new jobs. The rate of transitions through this node (which is both the arrival rate and the departure rate) can be regarded as the throughput. If no such exit-entry node exists originally, it is easy to add such a node. The modified if all jobs at this new exit-entry node have zero service time.

Closed model are often applied because it seems natural to regard the number of jobs in the system as the independent variable and the throughput as the dependent variable. The number of jobs in the system is often subject to control; the queueing analysis is desired to determine the associated throughputs and response times. For example, in production systems, new jobs usually do not arrive at random; they are scheduled.

Similarly, in computing systems the total number of jobs in device queues tends to be limited by resource constraints, so that it is natural to specify the number of jobs (the multiprogramming level) as a decision variable and then calculate the associated throughput. Also, in the time-sharing systems, so that the total number of jobs is not unbounded. Hence, even though closed model are significantly more difficult to analyze because of the normalization constant or partition function, there are good reasons for applying them.

A queueing network is a natural model to employ in the remanufacturing setting because it captures not only the flow of materials through the traditional stages of procurement, production, distribution, and sales, but also the dependence of the return flow of used products on past sales, the return delay, and the return probability, Toktay et al (2000).

Concluding, the review literature, in this thesis mainly we use literature by Dekker and De Brito (2001) and Kelle and Silver (1989), especially when uncertainty is modelled, the time from issue to return of kegs, bottles and crates is a random variable. In fact, to derive an optimal policy and optimal control parameters for a basic inventory model with returns where demand and return are independent Poisson processes.

Chapter 3
Data Description and Analysis

The data originated from a beer brewery, Morbeef. Below we briefly introduce and describe the data. Next we analyze the data. The goal of this chapter is to find a good distributional model for the return lead time. Once a good distributional model has been determined, various percent points for the return lead time will be computed. Moreover, we will be able to set-up in later chapters a model to calculate the number of bottles needed in the factory.

3.1. Data Description

A data set (refer to file Data Germany Cleaned 2006) was provided by Morbeef of the Supply Chain Planning Division in 2006 for Germany market. The data corresponds to how many days customer bring the bottles back to the shops (return lead time). Total data is 23.797 points, in excel format. The data includes 17 beer brands. The data came from 12 February 2006 until 13 Augustus 2006. We calculated data based on delivery time in 4 section.

- Data from 12 February 2006 until 17 February 2006, we assumed winter period.

- Data from 5 May 2006 until 20 may 2006, we assumed spring period.

- Data from 9 July 2006 until 21 July 2006, we assumed summer1 period

- Data from 13 Augustus 2006 until 25 Augustus, we assumed summer2 period.

Also, notice that data set has some shortcoming, for example, there is no information on what happened at the retailer, no information of kegs lost, only information on the items returned. In this case we can control the number of kegs lost from the serial number of the kegs or delivery date number. Beside that the data set has some incorrect number (maybe because human error from the input), example Fill Code always have relation with day code and year code and delivery date, and minimal have 3 digit number, 1 digit for day code, 1 digit for year code and the other digit for the code if they produce many products, so they need more fill code. In table below we saw incorrect number in fill code, 0016.

Delivery Date	Sort Number	Product	Fill Code	Customer Code	Day Code	Year Code	Fill Date
15-2-2006	23	Brand A GL 0,33	00166	308	1	6	1-1-2006
16-2-2006	23	Brand A GL 0,33	00166	410	1	6	1-1-2006
10-5-2006	23	Brand A GL 0,33	00166	413	1	6	1-1-2006
14-8-2006	8	Brand A Keg 50 l	0016	180	1	6	1-1-2006
15-8-2006	3	Brand A-AFP 0,33	00166	420	1	6	1-1-2006

15-8-2006	23	Brand A GL 0,33	00168	102	1	6	1-1-2006
15-8-2006	1	Brand A-Pils 0,33	00161	102	1	6	1-1-2006
15-8-2006	20	Brand A Gold 0,33	00161	351	1	6	1-1-2006

The data includes 14 columns. They are :

1. Column 1 : Delivery date, example : 12-JUL-2006

2. Column 2 : Sort number, example : 30, the sort number of Mini keg Home Draught

3. Column 3 : Product, example : Brand Mini keg Home Draught

Beer with type Draught and is served from mini keg. Draught beer is almost always un-pasteurized and therefore is more fragile. It should be consumed after being "tapped", and is generally truer to the flavours of the ingredients as pasteurization exposes the beer to heat and changes the flavour profile. A keg has a concentrically located down tube and a valve that allows beer in and gas out when filling and vice versa when beer is dispensed. Also kegs have a simple concave bottom. This aspect of keg design meant that all the beer in the keg was dispensed which therefore required that the beer be processed by filtration, fining or centrifuging, or some combination of these, to prevent sediment formation. Lastly, kegs have straight sides unlike the traditional barrel or cask shape. In order to get the beer out of a keg and into a customer's glass, it can be forced out with gas pressure, although if air or gas at low pressure is admitted to the top of the keg it can also be dispensed using a traditional hand pump at the bar. The mini keg is a 5-liter keg produced for retail sales. The example of keg is showed below :

4. Column 4 : Fill code, example : 0016.

The codes have relation with column 6 (days code), column 7 (years code) and column 8 (Fill data). Fill code 0016 means the product is filled at the first day in 2006. The code in column 6 should be 1 and the code in column 7 should be 6.

5. Column 5 : Customer code, example : 1

6. Column 6 : Day code, example : 1, the first day in that year.

7. Column 7 : Year code, example : 6, code for 2006.

8. Column 8 : Fill date, example : 27-JUN-2006

9. Column 9 : return lead time : 50 days

10. Column 10 : Correction number, example : 510 days. In this case we assumed 510 days is one year.

11. Column 11 : Delivery number, example : 132. The column has relation with column 1. Delivery number 132 should has delivery date at 13 February 2006, because At 13 February 2006 have delivery number between 1 until 136.

12. Column 12 : return lead time in week, example : 50 days = 7 weeks

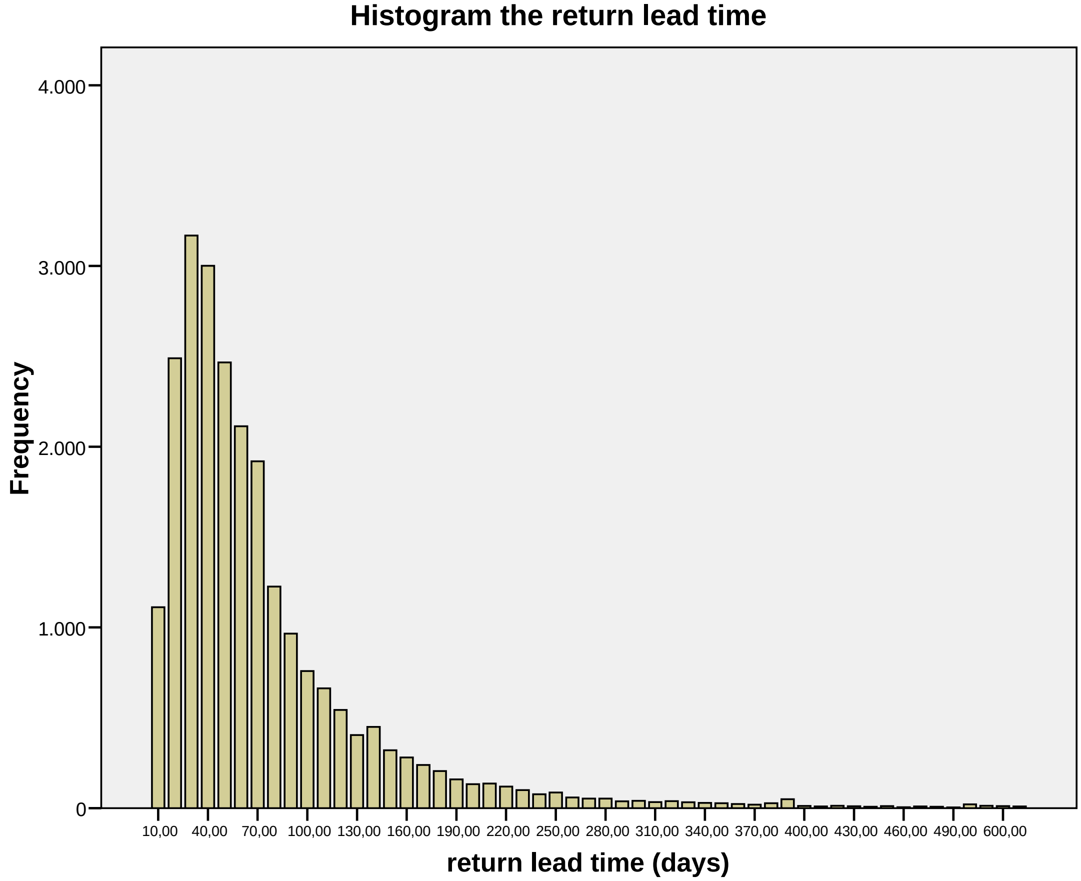
3.2 Data Analysis

3.2.1 Descriptive statistics

Graphical Output and Interpretation

The goal of this analysis is to determine a good distributional model for these data. The first step is to generate a histogram to get an overall feel for the data. The histogram is shown in figure 3.1. Figure 3.1 has been obtained by using the SPSS vs. 15.00 program.

Figure 3.1. Histogram The Return lead time



The histogram is shown in figure 3.1 is a skewed (non-symmetric) right distribution. A "skewed right" distribution is one in which the tail is on the right side. For a skewed distribution, however, there is no "centre" in the usual sense of the word. Be that as it may, several "typical value" metrics are often used for skewed distributions. The first metric is the mode of the distribution. Unfortunately, for severely-skewed distributions, the mode may be at or near the left or right tail of the data and so it seems not to be a good representative of the centre of the distribution. As a second choice, one could conceptually argue that the mean (the point on the horizontal axis where the distribution would balance) would serve well as the typical value. As a third choice, others may argue that the median (that value on the horizontal axis which has exactly 50% of the data to the left (and also to the right) would serve as a good typical value.

If the histogram indicates a right-skewed data set, the recommended next steps are to (Anscombe, Francis (1973) :

1. Quantitatively summarize the data by computing and reporting the sample mean, the sample median, and the sample mode.
2. Determine the best-fit distribution (skewed-right) from the
   - Weibull distribution

- Gamma distribution
- Lognormal distribution

Next we have calculated the sample mean, sample media and the sample mode and some number of statistics using SPSS vs 15.00. They are given in table 3.1

Tabel 3.1 **S**tatistic Return lead time (all data)

Mean	73.6799	days
Median	50.0000	days
Mode	30.00	days
Std. Deviation	68.13137	days
Variance	4641.883	points
Skewness	2.836	
Std. Error of Skewness	.016	
Kurtosis	11.627	
Std. Error of Kurtosis	.032	
Range	640.00	days
Minimum	10.00	days
Maximum	650.00	days

The definition of all characteristics calculated are in the appendix.

The normal distribution has a skewness of zero. But in reality, data points are not perfectly symmetric. So, an understanding of the skewness of the dataset indicates whether deviations from the mean are going to be positive or negative.

A distribution with a significant positive skewness has a long right tail. A distribution with a significant negative skewness has a long left tail. As a guideline, a skewness value more than twice its standard error is taken to indicate a departure from symmetry. The distribution has skewness value 2.836, the value indicates that the distribution has a long right tail and a departure from symmetry. The coefficient of variation is 0.92470 (standard deviation divided by the mean) equals almost one.

The distribution has positive kurtosis, 11.627 indicates that the observations cluster more and have longer tails than those in the normal distribution. The standard error of kurtosis is 0.32. The ratio of kurtosis to its standard error can be used as a test of normality (that is, you can reject normality if the ratio is less than -2 or greater than +2).

3.2.2 Fitting probability distributions

Distribution fitting is the procedure of selecting a statistical distribution that best fits to a data set generated by some random process. In other words, if we have some random data available, and would like to know what particular distribution can be used to describe our data.

Why should we use distribution in this thesis ? Random factors affect all areas of our life, and businesses striving to succeed in today's highly competitive environment need a tool to deal with risk and uncertainty involved. Using probability distributions is a scientific way of dealing with uncertainty and making informed business decisions.

Why is it important to select the best fitting distribution? Probability distributions can be viewed as a tool for dealing with uncertainty: you use distributions to perform specific calculations, and apply the results to make well-grounded business decisions.

The use of a distribution allow us to describe the data in a compact way, through its parameter. Moreover, it facilitates experiments by allowing drawing random numbers and it facilitates drawing general inferences. However, if we use a wrong tool, we will get wrong results. If we select and apply an inappropriate distribution (the one that doesn't fit to our data well), our subsequent calculations will be incorrect, and that will certainly result in wrong decisions.

In many industries, the use of incorrect models can have serious consequences such as inability to complete tasks or projects in time leading to substantial time and money loss, wrong engineering design resulting in damage of expensive equipment etc.

Distribution fitting allows us to develop valid models of random processes we deal with, protecting us from potential time and money loss which can arise due to invalid model selection, and enabling us to make better business decisions.

The next step is to try to fit various distributions to the data. To this end we will apply probability plots.

Probability Plot

The probability plot (Chambers 1983) is a graphical technique for assessing whether or not a data set follows a given distribution such as the normal or Weibull.

The data are plotted against a theoretical distribution in such a way that the points should form approximately a straight line. Departures from this straight line indicate departures from the specified distribution.

The correlation coefficient associated with the linear fit to the data in the probability plot is a measure of the goodness of the fit. Estimates of the location and scale parameters of the distribution are given by the intercept and slope. Probability plots can be generated for several competing distributions to see which provides the best fit, and the probability plot generating the highest correlation coefficient is the best choice since it generates the straightest probability plot.

For distributions with shape parameters (not counting location and scale parameters), the shape parameters must be known in order to generate the probability plot. For distributions with a single shape parameter, the probability plot correlation coefficient (PPCC) plot provides an excellent method for estimating the shape parameter.

There is a large number of distributions that would be distributional model candidates for the data. However, we will restrict ourselves to consideration of the following distributional models because these have proven to be useful in reliability studies.

1. Normal distribution
2. Gamma distribution
3. Weibull distribution
4. Lognormal distribution

There are two basic questions that need to be addressed.

1. Does a given distributional model provide an adequate fit to the data?
2. Of the candidate distributional models, is there one distribution that fits the data better than the other candidate distributional models?

The use of probability plots provide answers to both of these questions.

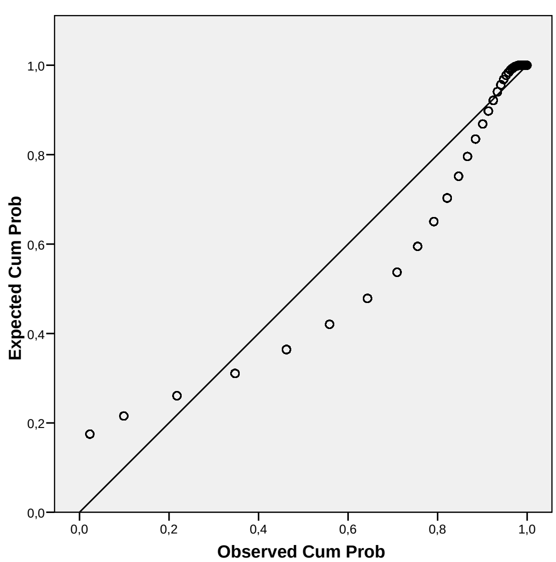
If the distribution does not have a shape parameter, we simply generate a probability plot.

1. If we fit a straight line to the points on the probability plot, the intercept and slope of that line provide estimates of the location and scale parameters, respectively.
2. The criteria for the "best fit" distribution is the one with the most linear probability plot.

We analyzed the data using the approach described above for the following distributional models. SPSS determined the parameters for a given distribution with maximum likelihood.

Using SPSS vs 15.00, we generated a normal probability plot. The result is shown in Figure 3.2.

Figure 3.2. Normal Probability Plot of Return lead time



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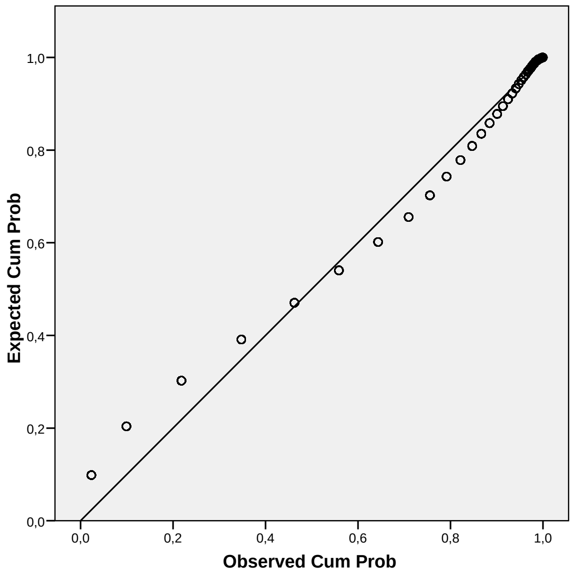
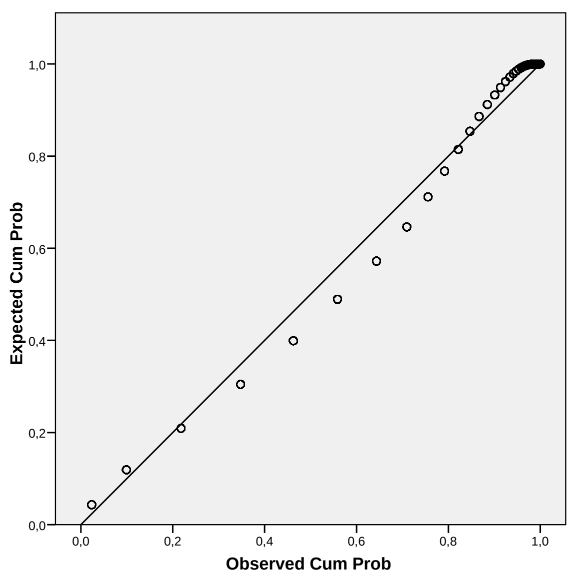
Using SPSS vs 15.00 we generated a gamma probability plot. The result is shown in Figure 3.3.

Figure 3.3. Gamma Probability Plot of Return lead time



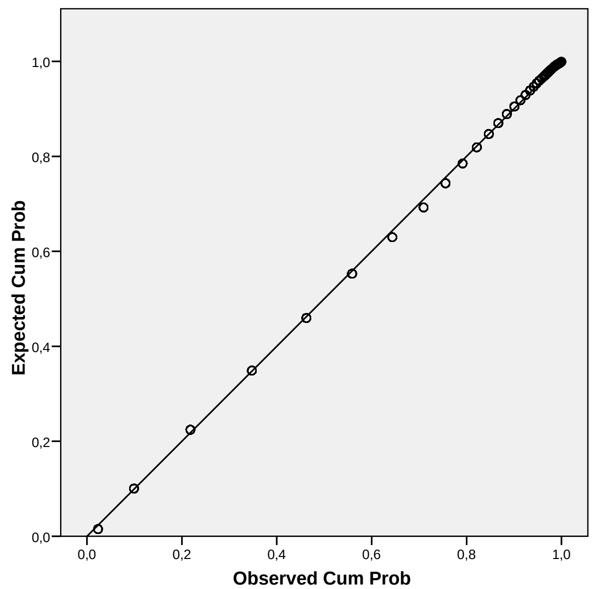
Using SPSS vs 15.00, we generated a Weibull probability plot. The result is shown in Figure 3.4.

Figure 3.4. Weibull Probability Plot of Return lead time



Using SPSS vs 15.00, we generated a Lognormal probability plot. The result is shown in Figure 3.5.

Figure 3.5. Lognormal Probability Plot of Return lead time



Using SPSS vs. 15.00, we determined the estimated distribution parameter. The result is given in Table 3.2.

Table 3.2. Estimated Distribution Parameters

Distribution	Scale	Location	Shape
Normal	73.6999 days	68.1314 days	
Gamma	.0160 days		1.1700 days
Weibull	77.9790 days		1.5180 days
Lognormal	54.1040 days		.7780 days

1. Normal distribution - from the probability-plot above, the normal probability plot has $\mu = 73.68$ days and $\sigma = 68.13$ days.
2. Gamma distribution - the optimal value, in the sense of having the most linear probability plot, of the shape parameter γ is 1.1700. At the optimal value of the shape parameter, the estimate of the scale parameter is 0.0160.
3. Weibull distribution - the optimal value, in the sense of having the most linear probability plot, of the shape parameter gamma is 1.5180. At the optimal value of the shape parameter, the estimate of the scale parameter is 77.9790.

4. Lognormal distribution - the optimal value, in the sense of having the most linear probability plot, of the shape parameter σ is 0.7780. At the optimal value of the shape parameter, the estimate of the scale parameter is 54.1040.

We choose the 2-parameter Lognormal distribution as the most appropriate model because it provides the best fit.

A variable X is lognormally distributed if Y = ln(x) is normally distributed with "ln" denoting the natural logarithm. The general formula for the probability density function of the lognormal distribution is

$$f(x) = \frac{e^{-((\ln(x-\phi)/m))^2/(2\sigma^2))}}{(x-\phi)\sigma\sqrt{2\pi}} \qquad x \geq \phi; m, \sigma > 0$$

Where σ is the shape parameter, ϕ is the location parameter and m is the scale parameter. The case where $\phi = 0$ and $m = 1$ is called the standard lognormal distribution. The case where ϕ equals zero is called the 2-parameter lognormal distribution.

The equation for the standard lognormal distribution is

$$f(x) = \frac{e^{-((\ln x)^2/2\pi^2)}}{x\sigma\sqrt{2\pi}} \qquad x \geq 0; \sigma > 0$$

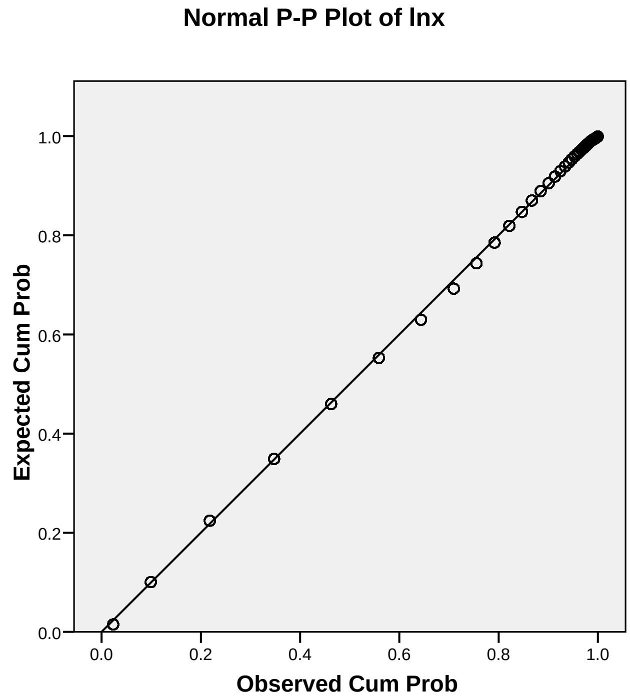
Since the general form of probability functions can be expressed in terms of the standard distribution, all subsequent formulas in this section are given for the standard form of the function.

The next step in this analysis is to make a normality test for Y = ln(x) with normal probability plot method (chamber 1983). Using SPSS vs. 15.00, we determined the estimated normal distribution parameter and generate probability plot. The result is given in table 3.3. and figure 3.6.

Table 3.3. Estimated Normal Distribution Parameter

		lnx
Normal Distribution	Location	3.9909
	Scale	.77815

Figure 3.6. Normal Probability Plot of ln x



The probability plot test indicate that Y=ln(x) is a straight line to the points on the probability plot with mean 3.9909 and standard deviation 0.77815.

The next step, we try to analyse with an other test. The One-Sample Kolmogorov-Smirnov Test (Chakravart, Laha, and Roy, 1967) procedure compares the observed cumulative distribution function for a variable with a specified theoretical distribution, which may be normal, uniform, or exponential. The Kolmogorov-Smirnov Z is computed from the largest difference (in absolute value) between the observed and theoretical cumulative distribution functions. This goodness-of-fit test tests whether the observations could reasonably have come from the specified distribution.

The Kolmogorov-Smirnov test assumes that the parameters of the test distribution are specified in advance. This procedure estimates the parameters from the sample. The sample mean and sample standard deviation are the parameters for a normal distribution, the sample minimum and maximum values define the range of the uniform distribution, the sample mean is the parameter for the Poisson distribution, and the sample mean is the parameter for the exponential distribution. The power of the test to detect departures from the hypothesized distribution may be seriously diminished.

The Kolmogorov-Smirnov test is performed using the SPSS 15.00 program. We have applied it to several distribution assumptions, like normal, uniform, and exponential. The results are presented in table 3.4., 3.5., and 3.6.

Table 3.4. One-Sample Kolmogorov-Smirnov Test - Normal

		lnx
N		23797
Normal Parameters(a,b)	Mean	3.9909
	Std. Deviation	.77815
Most Extreme Differences	Absolute	.073
	Positive	.062
	Negative	-.073
Kolmogorov-Smirnov Z		11.247

Table 3.5. One-Sample Kolmogorov-Smirnov Test - Uniform

		lnx
N		23797
Uniform Parameters(a,b)	Minimum	2.30
	Maximum	6.48
Most Extreme Differences	Absolute	.263
	Positive	.263
	Negative	-.119
Kolmogorov-Smirnov Z		40.588

Table 3.6. One-Sample Kolmogorov-Smirnov Test - Exponential

		Lnx
N		23797
Exponential parameter.(a,b)	Mean	3.9909
Most Extreme Differences	Absolute	.481
	Positive	.223
	Negative	-.481
Kolmogorov-Smirnov Z		74.232

The Kolmogorov-Smirov Z value is lowest for the normal distribution (11.247) compared to the value 40.588 for the uniform and 74.232 for the exponential distribution. The results of Kolmogorov-Smirov test thus indicates that Y=LN(X) is normally distributed. The test statistic for the normal distribution is noticeably higher than for exponential or uniform. This provides additional confirmation that a variable X is lognormally distributed. (Massey, F. J. Jr. (1951).

The Kolmogorov-Smirov test is a more powerful alternative to chi-square goodness-of-fit tests when its assumptions are met. Whereas the chi-square test of goodness-of-fit tests whether in general the observed distribution is not significantly different from the hypothesized one, the K-S test tests whether this is so even for the most deviant values of the criterion variable. Thus it is a more stringent test (Massey, F. J. Jr. (1951).

The Z value is the largest absolute difference between the cumulative observed proportion and the cumulative proportion expected on the basis of the hypothesized distribution. The computed Z is compared to a table of critical values of Z in the Kolmogorov-Smirnov One-Sample Test, for a given sample size (cf. Massey, 1951). For samples > 35, the critical value at the .05 level is approximately 1.36/SQRT(n), where n = sample size. If the computed Z is less than the critical value, the researcher fails to reject the null hypothesis that the distribution of the criterion variable is not different from the hypothesized (ex., normal) distribution. In practice, computer programs like SPSS compute the probability of Z directly without need to refer to such a table. SPSS prints the two-tailed significance level, testing the probability that the observed distribution is not significantly deviant from the expected distribution in either direction.

The next step, we calculated the mean return lead time and standard deviation of return lead time for every periods.

3.2.3 Analysis for every periods and every products.

We want to know the relation between the mean return lead time with periods and products. In this section we calculated the mean return lead time and standard deviation of return lead time for every periods and products using SPSS vs. 15.00.

We calculated data based on delivery time in 4 sections, because the close of the data revealed that a number of distinct periods can be distinguished. We want to know whether the return data is the same in all periods.

- Data from 12 February 2006 until 17 February 2006, we assumed winter period.

- Data from 5 May 2006 until 20 may 2006, we assumed spring period.

- Data from 9 July 2006 until 21 July 2006, we assumed summer1 period

- Data from 13 Augustus 2006 until 25 Augustus, we assumed summer 2 period.

Table 3.7 presents the mean return lead time and standard deviation of return lead time for every periods. Table 3.8 presents the mean return lead time and standard deviation of return lead time for products.

Table 3.7. The mean return lead time and standard deviation of return lead time for every period

Periods	mean	stdev
Winter	87.58	65.7732
Spring	85.03	75.0778
Summer1	70.58	64.6250
Summer2	75.92	66.0697

Table 3.8. The mean return lead time and standard deviation of return lead time for every product

Product	mean	Stdev
Brand A CO 0,33	71.72	61.3538
Brand A GL 0,33	76.32	64.5263
Brand A Gold 0,33	75.31	72.1766
Brand A Gold 0,5	71.80	60.6601
Brand A Keg 15 l ITA	95.94	61.8154
Brand A Keg 30 l	78.32	57.0696
Brand A Keg 30 l ITA	92.60	61.0480
Brand A Keg 50 l	73.94	46.2403
Brand A L7 0,33	79.93	64.7511
Brand A-AFP 0,33	97.35	76.9932
Brand A-Pils 0,33	78.70	73.5243
Brand A-Pils 0,5	82.39	75.2185
Brand B LN braun 0,33	81.99	76.1907
Brand B NRW 0,5	93.83	92.3764
Brand B Steinie 0,33	75.96	63.0066
Brand C keg 30L	67.80	45.6866
Minikeg Home Draught	77.15	62.7769

Discussion and conclusions statistical analysis.

We can draw the following conclusions from the results listed above.

The best distribution for return lead time data is log normal distribution.

The average return lead time in July – summer1 (70.58) and August – summer2 (75.92) are better than in February - winter (87.58) and May - spring (85.03). They give some implications :

1. The market is higher in the summer season (July and August) than in the winter season (February) and spring season (May).
2. The market in the winter season and spring season maybe not so different but in the winter (February) the customers keep longer the bottles in their house because they shop less frequent.

Table 3.8 showed the average return lead time for products is between 67.80 days (brand C keg 30L) until 97.35 days (brand A-AFP 0.33). If we compare with the result for all products, refer to table 3.1, we got the average return lead time for all products is 73,68 days. Those results give some implications :

1. Brand C keg 30L is the fastest moving product in this case, the average return lead time for this product is 67.80 days.
2. Brand A-AFP 0.33 is the slowest moving product in this case, the average return lead time for this product is 97.35 days.

Concluding, the data analysis indicates that the return lead time distribution is not same in every periods and for all products.

Chapter 4

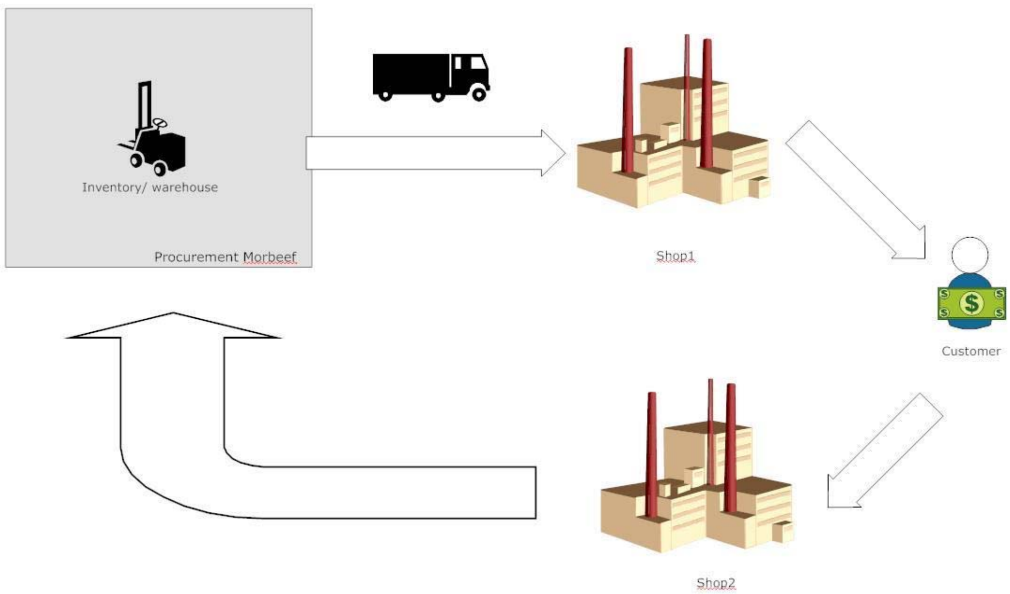
A Basic Formulation

In the previous chapter we analysed the return lead time data. In this chapter we introduce the model describing the circulation of the returnable items.

4.1. Descriptive Model

We can model the rotation of crates and kegs as follow.

Figure 4.1 The Model



Keep track of the inventory at Morbeef and of the number in the market (with the customer). We assume, as is sometimes done in practice, that each demand has a fixed probability P of an accompanying return of crates / kegs. The expected value $E(d_L)$ and variance $Var(d_L)$ of the random lead time demand d_L.

The expected return lead time, denoted by ER.

The variance of return lead time, denoted by VR.

The net demand is demand minus returns during the replenishment lead time. The expected lead time net demand denoted by ED (Kelle and Silver, 1989).

In the calculation of the variance of lead time net demand, VD, we have to account for the correlation between the random d_L and the random return lead time. At Morbeef, we check the inventory position and if it is below a level s, we order Q.

4.2. The Continuous-review (s,Q) inventory model

The assumption of this frequently used inventory model are as follows (Tijms, 1994) :

1. Continuous review of inventory, that is the stock status is continuously monitored and is updated each time a transaction occurs.
2. The individual demand transactions are small so that the inventory level can be treated as a continuous variable.
3. A replenishment order of size Q is placed each time the inventory position drops to the reorder point s.
4. The demands in disjoint time intervals can be treated as independent random variables.

In practice it is often reasonable to model the lead-time demand by a normal distribution. If the demand comes from a large number of independent sources, a justification for use of the normal distribution is provided by the central limit theorem. We assume that demand is normally distributed, with the following inputs :

D = average demand per period

σ_D = standard deviation of demand per period

L = average lead time for replenishment

The ROP represents the available inventory to meet demand during the lead time L. A stockout occurs if the demand during the lead time is larger than ROP. If demand across periods is independent, demand during the lead time is normally distributed with the following :

Mean demand during lead time, $D_L = DL$

Standard deviation of demand during lead time, $\sigma_L = \sqrt{L}\sigma_D$

Given the desire CSL, the required safety inventory (ss) are

$$ss = F_S^{-1}(CSL)x\sigma_L = NORMSINV(CSL)x\sigma_L, ROP = D_L + ss$$

Based on the continuous-review (s,Q) inventory model, Kelle and Silver in 1994 developed an (s,q) inventory system with return.

$$ROP = D_L + ss$$

$$s = ED + k^* \sqrt{VD}$$

s indicates the *ROP* on the continuous-review (s,Q) inventory model. ED is indicates mean demand during lead time on the continuous-review (s,Q) inventory model (D_L), and $k^* \sqrt{VD}$ indicates the safety stock, *ss* on the continuous-review (s,Q) inventory model. We can explain more detail about that in chapter 5.

Chapter 5

Analytical Reorder Point Calculations

In the chapter 3 we analysed the return lead time data. Refer to the result from chapter 3, we determine in this chapter the reorder level. Based on the continuous-review (s,Q) inventory model in chapter 4, Kelle and Silver in 1994 developed the (s,q) inventory system with return. We use a continuous review policy has to account only for the uncertainty of demand during the lead time. This is because the continuous monitoring of inventory allows us to adjust the timing of the replenishment order, depending on the demand experienced. If demand very high, inventory reaches the ROP quickly, leading to a quick replenishment order. If demand is very slow, inventory drops slowly to the ROP, leading to a delayed replenishment order. The available safety inventory thus must cover for the uncertainty demand over this period. The objective in this chapter is to give advice on the number of keg, crates needed to allow a continuous production with a certain probability.

5.1. Method

Which methods do we apply to determine the reorder level for purchasing new crates based on average behaviour the return delay ? We will use and adapt the methods developed by Kelle and Silver (1989). They called the method, Forecast based on average behaviour. The method is used based on available information. Forecast based on average behaviour utilizes only :

The expected value and the variance of the demand during the lead time, and
The probability of each kegs / crates being returned.

We assume, as is sometimes done in practice, that each demand has a fixed probability P of an accompanying return of crates.

Let $E(d_L)$ denote the expected value and $Var(d_L)$ denote the variance of the random lead time demand d_L.

The expected return lead time, denoted by ER, can be expressed as

$$ER = PE(d_L) \qquad (5.1.1)$$

The variance of return lead time, denoted by VR. The variance VR of return lead time has the form :

$$VR = P^2 Var(d_L) + P(1-P)E(d_L) \qquad (5.1.2)$$

The net demand is demand minus returns during the replenishment lead time. The expected lead time net demand denoted by ED, can be expressed as : (Kelle and Silver. 1989)

$$ED = E(D_L) - ER = (1-P)E(D_L) \qquad (5.1.3)$$

In the calculation of the variance of lead time net demand, VD, we have to account for the correlation between the random d_L and the random return lead time. Using the algebraic expression below : Kelle and Silver (1989)

For a mixed binomial random variable b with random n and known p

$$Var(n-b) = (1-p)^2 Var(n) + p(1-p)E(n)$$

$$g_i = P(n=i)$$

For

$$E(nb) = \sum_{k=1}^{i}\sum_{i=1}^{\infty} i g_i k \binom{i}{k} p^k (1-p)^{i-k}$$

$$= \sum_{i=1}^{\infty} i g_i \sum_{k=1}^{i} k \binom{1}{k} p^k (1-p)^{i-k} = \sum_{i=1}^{\infty} i^2 g_i p = pE(n^2)$$

$$= p[Var(n) + E^2(n)]$$

Thus

$$Cov(n.b) = E(nb) - E(n)E(b) = pVar(n) + pE^2(n) - pE^2(n) = pVar(n)$$

Further

$$Var(n-b) = Var(n) + Var(b) - 2Cov(n.b) = (1-p)^2 Var(n) + p(1-p)E(n)$$

Thus returning to VD, the variance of lead time net demand,

$$VD = (1-p)^2 Var(d_L) + P(1-P)E(d_L) \qquad (5.1.4)$$

The reorder point s is expressed in the common way used in the inventory control literature (e.g..Silver and Peterson (1985. Chapter 7) as :

$$s = ED + k^* \sqrt{VD} \qquad (5.1.5)$$

Where k^* is the appropriate safety factor (based on service considerations or on minimizing expected total relevant costs). In practice it is often reasonable to model the lead time demand by a normal distribution. Assume now that the lead time demand is normally distributed with mean μ_L and standard deviation σ_L. For detail, the safety factor k^* is calculated from the simple equation : Tijm (1994)

$$s = \mu_L + k^* \sigma_L$$

In this case let the target stock out probability is 1% or service level percentage is 99%. We use Excel function NORMSINV to convert service level percentage to service factor. We can find k^* is 2.32.

5.2. Calculation

In this section we calculate the reorder level. Silver (1985) and Hadley (1963) use a normal distribution approximation for demand during lead time in their inventory models. In their case, where only demands are considered, demand during lead time is a positive random variable. Since a normally distributed random variable takes on negative values, the expected demand during lead time has got to be large so that the normal distribution can be reasonable approximation for a positive random variable. When also returns are considered, net demand (demands minus returns) during lead time takes on negative as well as positive values, so approximating net demand during lead time by a normal distribution seems reasonable for all values of expected net demand during lead time.

Refer to the data analysis in chapter 3, we found that the return lead time is normally distributed with mean 74 days and variance 68 days. We assume :

- The target stock out probability is 1% or service level percentage is 99%. We use Excel function NORMSINV to convert for a normal distribution the service level percentage to a service factor k^*. We find k^* 2.32.
- the probability P that a crate is ever returned, 90%.

Using the expression 5.1.1, we can calculate the expected value of the random lead time demand.

$$ER = PE(d_L)$$

$$E(d_L) = \frac{ER}{P} = \frac{74}{0.9} = 82$$

Using the expression 5.1.2, we can calculate the variance of the random lead time demand d_L.

$$VR = P^2 Var(d_L) + P(1-P)E(d_L) = 0.9^2 Var(d_L) + 0.9(1-0.9)82 = 68.1314$$
$$Var(d_L) = 75.0017$$

Using the expression 5.1.3, we can calculate the expected lead time net demand ED

$$ED = (1-P)E(D_L) = (1-0.9)82 = 8.2$$

Using the expression 5.1.4, we can calculate the variance of lead time net demand VD.

$$VD = (1-p)^2 Var(d_L) + P(1-P)E(d_L) = (1-0.9)^2 75.0017 + 0.9(1-0.9)82 = 8.1300$$

Using the expression 5.1.5, we can calculate the reorder point s.

Reorder point = $s = ED + k^* \sqrt{VD} = 8.2 + 2.32\sqrt{8.1300} = 14.8$

The results calculation for normal distribution are presented in table 5.1

ER : The expected return lead time

$E(d_L)$: The expected value of the random lead time demand d_L

VR : The variance of return lead time

$Var(d_L)$: variance of the random lead time demand d_L

ED : The expected lead time net demand

VD : the variance of lead time net demand

$k^* \sqrt{VD}$: safety stock

s : reorder Point

Table 5.1 The result calculation

ER	$E(d_L)$	VR	$Var(d_L)$	ED	VD	$k^* \sqrt{VD}$	s (crates)
74	82	68	75	8	8	6,6	14,8

We want to know the reorder point for every brand. In table 5.2 we calculated the reorder point for every brand. Refer to the data in table 3.8 (the mean return lead time and standard deviation of return lead time for every products) and using the expressions (5.1.1), (5.1.2), (5.1.3) and (5.1.4), we calculate for every brand. The results calculation for every brand are presented in table 5.2

Table 5.2 The results for every Brand

Brands	ER	$E(d_L)$	VR	$Var(d_L)$	ED	VD	$k^* \sqrt{VD}$	s (crates)
Brand A CO 0,33	72	80	61	67	8	8	6,50	14,47
Brand A GL 0,33	76	85	65	70	8	8	6,70	15,18
Brand A Gold 0,33	75	84	72	80	8	8	6,70	15,06
Brand A Gold 0,5	72	80	61	66	8	8	6,50	14,47
Brand A Keg 15 l ITA	96	107	62	64	11	10	7,42	18,08
Brand A Keg 30 l	78	87	57	61	9	8	6,74	15,44
Brand A Keg 30 l ITA	93	103	61	64	10	10	7,30	17,59
Brand A Keg 50 l	74	82	46	48	8	8	6,51	14,73

Brand A L7 0,33	80	89	65	70	9	9	6,84	15,72
Brand A-AFP 0,33	97	108	77	83	11	11	7,54	18,36
Brand A-Pils 0,33	79	87	74	81	9	9	6,84	15,58
Brand A-Pils 0,5	83	93	75	83	9	9	7,02	16,29
Brand B LN braun 0,33	82	91	76	84	9	9	6,97	16,08
Brand B NRW 0,5	94	104	92	102	10	10	7,48	17,91
Brand B Steinie 0,33	76	84	63	68	8	8	6,68	15,12
Brand C keg 30L	68	75	46	48	8	7	6,25	13,78
Minikeg Home Draught	77	86	63	68	9	8	6,72	15,29

We can get service level as table 5.3 below.

Table 5.3. Service Level for every Brand

	s	ED	VD	k^*	Service Level percentage
Brand A CO 0,33	14,47	8	8	2,29	0.9890
Brand A GL 0,33	15,18	8	8	2,54	0.9945
Brand A Gold 0,33	15,06	8	8	2,50	0.9938
Brand A Gold 0,5	14,47	8	8	2,29	0.9890
Brand A Keg 15 l ITA	18,08	11	10	2,24	0.9875
Brand A Keg 30 l	15,44	9	8	2,28	0.9887
Brand A Keg 30 l ITA	17,59	10	10	2,40	0.9918
Brand A Keg 50 l	14,73	8	8	2,38	0.9913
Brand A L7 0,33	15,72	9	9	2,24	0.9875
Brand A-AFP 0,33	18,36	11	11	2,22	0.9868
Brand A-Pils 0,33	15,58	9	9	2,19	0.9857
Brand A-Pils 0,5	16,29	9	9	2,43	0.9925
Brand B LN braun 0,33	16,08	9	9	2,36	0.9909
Brand B NRW 0,5	17,91	10	10	2,50	0.9938
Brand B Steinie 0,33	15,12	8	8	2,52	0.9941
Brand C keg 30L	13,78	8	7	2,18	0.9854
Minikeg Home Draught	15,29	9	8	2,22	0.9868

The calculations are repeated for every period for all brands, assuming a stationary demand within a period. The results are presented in table 5.4

Table 5.4. The results calculation for every period

Periods	ER	$E(d_L)$	VR	$Var(d_L)$	ED	VD	$k^*\sqrt{VD}$	s (crates)
Winter	88	97	66	70	10	9	7,14	16,87
Spring	85	94	75	82	9	9	7,08	16,53
Summer1	71	78	65	71	8	8	6,47	14,31
Summer2	76	84	66	72	8	8	6,69	15,13

We can get service level as table 5.5 below.

Table 5.5. Service Level for every period

Periods	s	*ED*	*VD*	k^*	Service Level percentage
Winter	16,87	10	9	2,29	0,9890
Spring	16,53	9	9	2,51	0,9940
Summer1	14,31	8	8	2,47	0,9932
Summer2	15,13	8	8	2,52	0,9941

In this chapter we have calculated the inventory reorder point for every brand and every period. Refer to the result in table 5.2, the lowest value inventory reorder point is 13,78 crates (Brand C keg 30L). The highest value inventory reorder point is 18,36 crates (Brand A-AFP 0,33). Refer to the result in table 5.3, the best service level is 99,45% and the lowest service level is 98,54%.

Refer to the result in table 5.4, we can draw the following conclusions :
The reorder point in the summer1 period and summer2 period, 14,31 crates and 15,13 crates) are better than in winter period, 16,87 crates and spring period, 16,53 crates). Given the continuous review policies, the purchasing department can order when the inventory drops to the ROP. The results above give some implication, in the winter season the customer keep longer the bottles in their house than in the summer season. Refer to the result in table 5.5, the best service level is 99,41% (summer2) and the lowest service level is 0.98,90% (winter).

Chapter 6

The Simulation Model

Simulated random historical data were used to estimate the statistical measures of relative performance, Kelton (2007). We simulated because we want to control the result from the analytical calculation and assumption.

6.1. Purpose

One of the main objective of this simulation is to understand how we may control the inventory into the front end and back end of the supply chain in order to minimize average inventory levels and maintain service levels.

We have calculated necessary safety stock that the return time had a normal distribution, although we found a lognormal distribution in chapter 3. In this simulation we simulate the return time with lognormal distribution. We want to give advice on the number of kegs, crates needed to allow a continuous production with a certain probability. Done at each type of container (crates, keg).

For the purpose of this thesis, the inputs are the demand distribution, return delay and loss percentage. The outputs are the required stock level and costs estimate.

6.2. Set-up of simulation model

In this thesis, the simulation is modelled using a very specific kind of simulation known as discrete event simulation. In this type of simulation, individual entities in the system are represented as unique work items, each with a appropriate set of attached identifying characteristics. In discrete event simulation, everything is event driven, and each event is treated individually. Because events are individualized, it is possible to have enormous control over the way in which each event and the associated items flow through the system. This control, in turn, makes it possible to create very accurate models.

Entities represent the objects moving through the system. In this simulation, the entities are crates. We determine when did each individual crate went into inventory, when was it made, when was it filled. Demand is according to a Poisson process with rate lambda. Figure 4.1 shows the processes, which consists of production, return process and new purchasing. Figure 6.1 contains the process flow diagram. Following the logic of the process flow diagram, demand orders are generated. When demand orders are generated, they are matched against the inventory. The level of inventory acts as a control on the rate at which crates orders are released. If the level inventory exceeds the reorder point, they can send the crates to the shops. But if the level inventory drops to the level reorder point, the process go to order crates. The return time is modeled using distributions, because the data is too big to simulate direct with the real data. Parameters for these distribution were based on historical data. (Morrice and Valdez, 2005).

The inputs are demand and return time.

Demand : demand generated crates entities into the system based on a Poisson process with a rate of the expected value of the random lead time demand, 82 crates. The Entities Per Arrival is based on a Poisson process. The first batch of documents is generated at the level of safety stock, 6.6 into the simulation run.

Return time : return time generate entities into the system based on an normal distribution with a mean of the return lead time, 73 days and variance 68 and based on lognormal distribution.

The Entities Per Arrival is based on a exponential distribution between 1 and 500 days refer to the historical data, file Germany cleaned 2006 by Morbeef, the range of return days between 1 day until 500 days. The first batch of documents is generated at 1 day into the simulation run.

Control process : in this section they check inventory position. If the inventory position drop below Reorder Point, then order one unit. Inventory level is number crates in stock. Inventory position is inventory level plus number of outstanding orders.

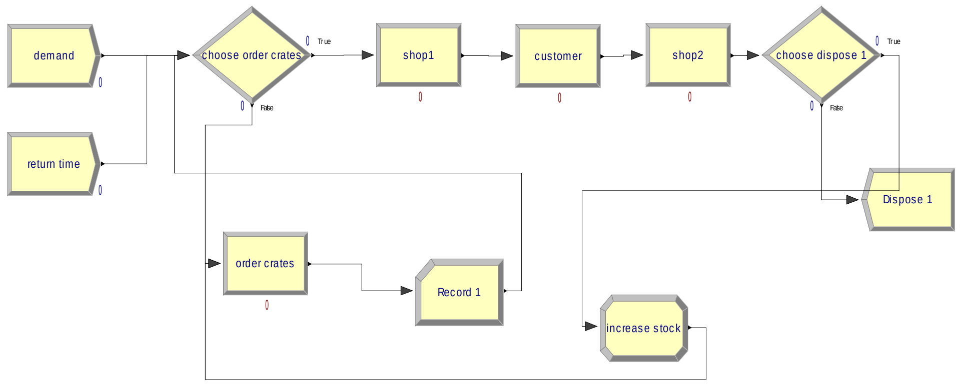
Order crates : the order crates processing area is an automatic process while the crates in storage is below the safety stock level. This is considered to be a value added process and the time incurred will be added to the entity's Entity. VA time (Value Added) attribute. Value added time Per Entity : The time each entity spent in any activity of a process, where the allocation is specified as value added. The delay time is determined by an expression, which distribution Refer to the historical data.

Shop 1 : the process is an automatic process where no resources are necessary. Considered to be a value added process, the time incurred will be added to the entity's Entity.VATime (Value Added) attribute. The delay type is constant.

Customer : the entity enters the Process module to undergo a shop 1. Considered to be a value added process, the time incurred will be added to the entity's Entity.VATime (Value Added) attribute. The delay type is normal.

Shop 2 : the entity enters the Process module to undergo a customer. Considered to be a value added process, the time incurred will be added to the entity's Entity.VATime (Value Added) attribute. The delay type is normal

Figure 6.1. The Process Flow Diagram of Simulation



6.3. Implementation

The next stage of this thesis involves conducting the simulation. The simulation for this problem is developing by using the Arena vs. 10.00 program. Arena is an advanced simulation system that provides an interactive environment for building, graphically animating, verifying and analyzing simulation models. Arena combines the ease of use found in high-level simulators with the flexibility of simulation language and even all the way down to general-purpose procedural language (Kelton, W(2007)). The simulation is developing using basic process template.

A scenario is defined by a specific set of values for the parameters. Ten simulation replications were made for each scenario in order to generate confidence intervals. Each replicate is simulated for 2000 days after 10 days warm-up period.The warm-up period was chosen by visual inspection using an approach similar to Welch's procedure (Law and Kelton 2000). By experimentation, we determined that a 2000 days simulation replication was sufficient because statistics had stabilized indicating that we were approximating longrun steady state results. Customers arrive with inter arrival times distributed as exponential, with the first arrival occurring not at time zero but after one of these inter arrival times past zero.

The simulation results can be used as test for the analytical calculations. If the results are almost the same as those from the analytical calculations, it means that the latter are good.

6.4. Output & Analysis

We first report the main simulation results, and then perform sensitivity analysis with respect to several key parameters. The running time of the simulation is 2000 days. The data cover a period of 194 days.

The main simulation results for distributions are presented in table 6.1

Table 6.1 The simulated average order crates (95% confidence intervals) for distributions

Distribution	ED	VD	Half Width of value added time	s	Time spent in the inventory (days)	k^*	Service Level percentage
Normal distribution	8,1889	8,1201	0,021	15	3.24	2.39	0.9916
Lognormal distribution	8,1889	8,1201	0,020	16	2.25	2.74	0.9969

k^* is the appropriate safety factor (based on service considerations or on minimizing expected total relevant costs). *. In this case let the target stock out probability is 1% or service level percentage is 99%. We use Excel function NORMSINV to convert service level percentage to service factor. We can find k^* is 2.32. We determined s from the input data. We applied an algorithm with start s = 1 and increase until we surpassed the service level. After we got s we calculated the really k^* based on the number of reorder level. Next we converted the service factor to service level.

ED is the expected lead time net demand.

VD is the variance of lead time net demand.

s is the reorder level. The number of reorder level is output from the simulation.

The target stock out probability is 1% or which corresponds to a k^* = 2.3. We got result k^* = 2.39 and 2.74.

k^* is calculated from reorder level minus the expected lead time demand and then divided with square root of the variance of lead time net demand.

95% Confidence intervals :
Value is returned in the Half Width category, this value may be interpreted by saying "in 95% of repeated trials, the sample mean would be reported as within the interval sample mean ± half width". The half width can be reduced by increasing the number of replications.

Refer to the result in chapter 3, using spss v15 program, we have calculated the estimated distribution parameters. The results are presented in table 6.2.

Table 6.2 The Estimated distribution parameter normal distribution

		Winter (days)	Spring (days)	Summer1 (days)	Summer2 (days)
Normal Distribution	Location	87.5800	85.0300	70.5800	75.9200
	Scale	65.7733	75.0778	64.6251	66.0697

Refer to the result in chapter 3, using spss v15 program, we have been calculated the estimated distribution parameters. The result is presented in table 6.3.

Table 6.3 The Estimated distribution parameter lognormal distribution

		Winter (days)	Spring (days)	Summer1 (days)	Summer2 (days)
Lognormal Distribution	Scale	71.206	64.628	53.414	58.844
	Shape	.634	.720	.716	.687

Refer to data in the table 6.2 (the estimated distribution parameter normal distribution) we simulated using arena. The main simulation results for periods are presented in table below.

Table 6.4 The simulated average order crates (95% confidence intervals) for periods with lognormal distribution

Periods	ED	VD	s	Half Width of value added time	Time spent in the inventory (days)	k^*	Service Level percentage
Winter	9,7311	9,4619	19	0,034	3.24	3,01	0.9987
Spring	9,4478	9,3249	17	0,064	2.25	2.57	0.9949
Summer1	7,8422	7,7687	15	0,055	3.09	2.53	0.9943
Summer2	8,4356	8,3139	17	0,058	3.31	2.97	0.9985

Refer to data in the table 6.3 (the estimated distribution parameter lognormal distribution) we simulated using arena. The main simulation results for periods are presented in table below.

From the results in table 6.4 and table 6.5 we can see clearly why should we simulate for periods (seasons). We got result k^* = 2,74 and 2,39 from the simulation of all year. But with simulation for periods we can see detail in which periods we need more number of reorder level.

Table 6.5 The simulated average order crates (95% confidence intervals) for periods with normal distribution

Periods	ED	VD	s	Half Width of value added time	Time spent in the inventory (days)	k^*	Service Level percentage
Winter	9,7311	9,4619	18	0,0005	2.96	2.69	0.9964
Spring	9,4478	9,3249	17	0,0005	2.67	2.57	0.9949
Summer1	7,8422	7,7687	14	0,0009	2.21	2.21	0.9864
Summer	8,4356	8,3139	17	0,0005	2.42	2.97	0.9985

In table 6.6 we can compare the safety factor using simulation and analytical calculation.

Table 6.6 The Safety factor

Periods	Analytical calculation	Simulation with lognormal distribution parameter	Simulation with normal distribution parameter
February	2.36	3,01	2.69
May	2.47	2.57	2.57
July	2.21	2.53	2.21
August	2.28	2.97	2.97

We want to know the effect if the return time constant, instead of a distribution take a constant value . The result of them are showed in table 6.7 and table 6.8 below. In table 6.7 the result simulation with the return time is constant, assume lead time is 1 month = 30 days.

Table 6.7 Simulation with the return time constant for all periods

Lead time/ return time	ED	VD	s	Half Width of value added time	Time spent in the inventory	k^*	Service Level Percentage
30 days	8,1889	8,1201	20	0,047	3.29	4.14	0.9999

In table 6.8 the result simulation with the return time is constant, assume lead time is 1 month = 30 days for months.

Table 6.8 Simulation with the return time constant (30 days) for every period

Periods	ED	VD	s	Half Width of value added time	Time spent in the inventory (days)	k^*	Service Level percentage
Winter	9,7311	9,4619	23	0,042	3.29	4.31	0.9999
Spring	9,4478	9,3249	21	0,048	3.29	3.78	0.9999
Summer1	7,8422	7,7687	19	0,047	3.29	3.00	0.9987
Summer2	7,8422	8,3139	21	0,048	3.29	3.36	0.9909

Refer to the conclusion above, with the variation demand distribution, we can adapt the inventory policy.

$$ROP = D_L + ss$$

$$s = ED + k^* \sqrt{VD}$$

s indicates the *ROP* on the continuous-review (s,Q) inventory model. *ED* is indicates mean demand during lead time on the continuous-review (s,Q) inventory model (D_L), and $k^* \sqrt{VD}$ indicates the safety stock, *ss* on the continuous-review (s,Q) inventory model.

Safety stock = ss = $k^* \sqrt{VD}$.

Table 6.9 is showed the safety stock for every periods with the variation demand distribution.

Table 6.9 The safety stock for every periods with the variation demand distribution

Periods	ED	k^*	VD	Ss (crates)
Winter	9,7311	4.31	9,4619	13
Spring	9,4478	3.78	9,3249	12
Summer1	7,8422	3.00	7,7687	8
Summer2	7,8422	3.36	8,3139	10

From the result in table 6.9 above we can see how many crate we need for the safety stock. If demand high we need safety stock high too. The problem is if safety stock high, the cost will be high too. But with a continuous review policy we can adjust the timing of the replenishment order, so we can reduce the safety stock. If demand very high, inventory reaches the ROP quickly, leading to a quick replenishment order. If demand is very slow, inventory drops slowly to the ROP, leading to a delayed replenishment order. The available safety inventory thus must cover for the uncertainty demand over this period.

Refer to the result in table 6.6, we can draw the following conclusions :
The safety factor from the simulation is better than the safety factor from the analytical calculation. The result from the simulation is more accurate. Analytical results can be highly precise and in most cases do not take very long to compute. Simulated results often take longer to calculate and their accuracy depends on the number of simulation iterations performed. Additionally, simulated results can vary slightly from run to run due to the randomness of the analysis

Chapter 7

Summary and Concluding Remarks

The objective in this thesis is to develop a method to assist in inventory control of returnable items. We analysed data and applied the method in a case study. To this end we model and determine the logistic process of returnable kegs, bottles and crates of a large brewery manufacturing company in order to attain required service levels with as low minimize average inventory levels as possible.

In this thesis we used mainly literature from Dekker and De Brito (2001) and Kelle and Silver (1989), especially when uncertainty is modelled. In fact, to derive an optimal policy and optimal control parameters for a basic inventory model with returns where demand and return are independent Poisson processes.

Fitting a probability distribution have been done. The best distribution for the return lead time for the data given appears to be the log normal distribution.

The analytical calculation model have been done. the lowest value inventory reorder point is 13,78 crates (Brand C keg 30L). The highest value inventory reorder point is 18,36 crates (Brand A-AFP 0,33. Refer to the result in table 5.3, the best service level is 99,45% and the lowest service level is 98,54%.

The reorder point in the summer1 period and summer2 period, 14,31 crates and 15,13 crates are better than in winter period, 16,87 crates and spring period, 16,53 crates). Given the continuous review policies, the purchasing department can order when the inventory drops to the ROP. The results above give some implication, in the winter season the customer keep longer the bottles in their house than in the summer season. Refer to the result in table 5.5, the best service level is 99,41% (summer2) and the lowest service level is 98,90% (winter).

To assess the result of analytical calculation, a simulation has been done. Some simulations have been done, both with the normal distribution and lognormal distribution. The results of simulation with lognormal distribution parameter yield lower costs than the results of simulation with normal distribution, which we can see from the appropriate safety factor k^* (based on service considerations or on minimizing expected total relevant costs). We have 0.35 point less in case of the more with lognormal than normal distribution. Simulation with the best fitting distribution is important, so we can get better result. Of course simulation with the real data directly is better, but sometimes data is too much, and so we have problems to simulate.

The safety stock for every period using the variation of the demand distribution has been determined. . If demand high (in winter 9.7 ≈ 10) we need safety stock high too, 13 crates. The problem is if safety stock high, the cost will be high too. But with a continuous review policy we can adjust the timing of the replenishment order, so we can reduce the safety stock. If demand very high, inventory reaches the ROP quickly, leading to a quick replenishment order. If demand is very slow, inventory drops slowly to the ROP, leading to a delayed replenishment order. The available safety inventory thus must cover for the uncertainty demand over this period.

The safety factor from the simulation is better than the safety factor from the analytical calculation. The result from the simulation is more accurate. Analytical results can be highly precise and in most cases do not take very long to compute. Simulated results often take longer to calculate and their accuracy depends on the number of simulation iterations performed. Additionally, simulated results can vary slightly from run to run due to the randomness of the analysis

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Appendix 3
The Definition of all characteristics calculated.

Skewness

A measure of the asymmetry of distribution. Skewness is written as γ_1 and defined as :

$$\gamma_1 = \frac{\mu_3}{\sigma^3}$$

Where μ_3 is the third moment about the mean and σ is the standard deviation. Equivalently, skewness can be defined as the ratio of the third cumulant κ_3 and the third power of the square root of the second cumulant κ_2 :

$$\gamma_1 = \frac{\kappa_3}{\kappa_2^{3/2}}$$

This is analogous to the definition of kurtosis, which is expressed as the fourth cumulant divided by the fourth power of the square root of the second cumulant.

For a sample of n values the *sample skewness* is

$$g_1 = \frac{m_3}{m_2^{3/2}} = \frac{\sqrt{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{(\sum_{i=1}^{n}(x_i - \overline{x})^{3/2}}$$

Where x_i is the i^{th} value, $\overline{x}$ is the sample mean, m_3 is the sample third central moment, and m_2 is the sample variance.

Given samples from a population, the equation for the sample skewness g_1 above is a biased estimator of the population skewness. The usual estimator of skewness is

$$G_1 = \frac{k_3}{k_2^{3/2}} = \frac{\sqrt{n(n-1)}}{n2} g_1$$

Where k_3 is the unique symmetric unbiased estimator of the third cumulant and k_2 is the symmetric unbiased estimator of the second cumulant. Unfortunately G_1 is, nevertheless, generally biased. Its expected value can even have the opposite sign from the true skewness.

Skewness has benefits in many areas. Many simplistic models assume normal distribution i.e. data is symmetric about the mean. The normal distribution has a skewness of zero. But in reality, data points are not perfectly symmetric. So, an understanding of the skewness of the dataset indicates whether deviations from the mean are going to be positive or negative.

A distribution with a significant positive skewness has a long right tail. A distribution with a significant negative skewness has a long left tail. As a guideline, a skewness value more than twice its standard error is taken to indicate a departure from symmetry. The distribution has skewness value 2.836, the value indicates that the distribution has a long right tail and a departure from symmetry.
The coefficient of variation is 0.92470 (standard deviation divided by the mean) equals almost one. There is quite a variability in residence time.

Std. Error of skewness
The ratio of skewness to its standard error can be used as a test of normality (that is, you can reject normality if the ratio is less than -2 or greater than +2). A large positive value for skewness indicates a long right tail; an extreme negative value indicates a long left tail.

Kurtosis
A measure of the extent to which observations cluster around a central point. For a normal distribution, the value of the kurtosis statistic is zero. Positive kurtosis indicates that the observations cluster more and have longer tails than those in the normal distribution, and negative kurtosis indicates that the observations cluster less and have shorter tails.

Std. Error of Kurtosis
The ratio of kurtosis to its standard error can be used as a test of normality (that is, you can reject normality if the ratio is less than -2 or greater than +2). A large positive value for kurtosis indicates that the tails of the distribution are longer than those of a normal distribution; a negative value for kurtosis indicates shorter tails (becoming like those of a box-shaped uniform distribution).

Range
The difference between the largest and smallest values of a numeric variable, the maximum minus the minimum.

Appendix 4

List of Outputs of Simulation

Process Detail Summary normal distribution- all data
Process Detail Summary lognormal distribution- all data
Process Detail Summary season February-normal distribution
Process Detail Summary season May-normal distribution
Process Detail Summary season July-normal distribution
Process Detail Summary season August-normal distribution
Process Detail Summary season February-lognormal distribution
Process Detail Summary season May-lognormal distribution
Process Detail Summary season July-lognormal distribution
Process Detail Summary season August-lognormal distribution
Process Detail Summary with return time constant for 1 year
Process Detail Summary with return time constant for February
Process Detail Summary with return time constant for May
Process Detail Summary with return time constant for July
Process Detail Summary with return time constant for August

Normal Distribution for all periods

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3,24	3,24

Crates

ED	8,1889

	Number In	Number Out
crates	15.427,00	15.378,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelnormaldistribution   Page 1 of 2

crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3,7498	0,021180488	0	14,0527
VA Time Per Entity	3,7498	0,021180488	0	14,0527

Accumulated Time	Value
Accum VA Time	125.408,23
Total Accum Time	125.408,23

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelnormaldistribution   Page 2 of 2

LogNormal Distribution for all periods

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	2,25	2,25

Crates

ED 8,1889

	Number In	Number Out
crates	15.952,00	15.915,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modellognormaldistribution Page 1 of 2

crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	2,6239	0,020018721	0	15,0520
VA Time Per Entity	2,6239	0,020018721	0	15,0520

Accumulated Time	Value
Accum VA Time	121.312,17
Total Accum Time	121.312,17

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modellognormaldistribution Page 2 of 2

February normal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.75	3.75

Crates

ED	9,7311

	Number In	Number Out
crates	33.514,00	33.444,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonfeb

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crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.7474	0,046489019	0	14.0527
VA Time Per Entity	3.7517	0,034770794	0	13.9059

Accumulated Time	Value
Accum VA Time	125.408,23
Total Accum Time	125.408,23

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonfeb

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May normal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.76	3.76

Crates

ED	9,4478

	Number In	Number Out
crates	19.829,00	19.764,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonmay

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crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.7596	0,060073662	0	14.9718
VA Time Per Entity	3.7517	0,064790232	0	14.6754

Accumulated Time	Value
Accum VA Time	74.232,33
Total Accum Time	74.232,33

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonmay

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July normal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.09	3.09

Crates

ED	9,4478

	Number In	Number Out
crates	15.269,00	15.223,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonjuly Page 1 of 2

crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.0828	0,056869839	0	12.6704
VA Time Per Entity	3.1025	0,055585836	0	12.3854

Accumulated Time	Value
Accum VA Time	47.090,54
Total Accum Time	47.090,54

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonjuly Page 2 of 2

August normal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.31	3.31

Crates

ED	9,4478

	Number In	Number Out
crates	16.625,00	16.568,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonaugust

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crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.3048	0,072335489	0	12.9227
VA Time Per Entity	3.2845	0,058017551	0	13.1835

Accumulated Time	Value
Accum VA Time	40.806,40
Total Accum Time	40.806,40

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonaugust

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February lognormal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	2.96	2.96

Crates

ED 9,7311

	Number In	Number Out
crates	18.996,00	18.943,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonfeblognormal Page 1 of 2

crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	2.9584	0,000513810	0	3.0511
VA Time Per Entity	2.9587	0,000517906	0	3.0486

Accumulated Time	Value
Accum VA Time	56.044,25
Total Accum Time	56.044,25

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonfeblognormal Page 2 of 2

May lognormal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	2.67	2.67

Crates

ED		9,4478

	Number In	Number Out
crates	18.996,00	18.943,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonmaylognormal   Page 1 of 2

crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	2.6668	0,000813501	2.5649	2.7670
VA Time Per Entity	2.6671	0,000537651	2.5477	2.7753

Accumulated Time	Value
Accum VA Time	47.935,72
Total Accum Time	47.935,72

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonmaylognormal   Page 2 of 2

July lognormal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	2.21	2.21

Crates

ED	7,8422

	Number In	Number Out
crates	18.996,00	18.943,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonjulylognormal

Page 1 of 2

crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	2.2082	0,001020531	2.1165	2.3090
VA Time Per Entity	2.2081	0,000924607	2.1069	2.3030

Accumulated Time	Value
Accum VA Time	18.499,77
Total Accum Time	18.499,77

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonjulylognormal

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August lognormal Distribution

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	2.42	2.42

Crates

ED	8,4356

	Number In	Number Out
crates	12.878,00	12.873,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonaugustlognormal   Page 1 of 2

Crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	2.4164	0,000813765	2.3146	2.5097
VA Time Per Entity	2.4167	0,000518773	2.3346	2.5073

Accumulated Time	Value
Accum VA Time	31.022,35
Total Accum Time	31.022,35

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelseasonaugustlognormal   Page 2 of 2

1 year with return time constant

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.29	3.29

Crates

ED	8,1889

	Number In	Number Out
crates	12.465,00	12.399,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Model1yearreturntimeconstant   Page 1 of 2

Crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.2146	0,000813765	0	3.6171
VA Time Per Entity	3.2146	0,000813765	0	3.6171

Accumulated Time	Value
Accum VA Time	42.133,21
Total Accum Time	42.133,21

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Model1yearreturntimeconstant   Page 2 of 2

February with return time constant

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.29	3.29

Crates

ED	9,7311

	Number In	Number Out
crates	12.477,00	12.349,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelfebreturntimeconstant   Page 1 of 2

Crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.0013	0,0429823	0	3.5201
VA Time Per Entity	3.0013	0,0429823	0	3.5201

Accumulated Time	Value
Accum VA Time	42.101,13
Total Accum Time	42.101,13

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelfebreturntimeconstant   Page 2 of 2

May with return time constant

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.29	3.29

Crates

ED	9,4478

	Number In	Number Out
crates	12.451,00	12.390,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelmayreturntimeconstant Page 1 of 2

Crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.3251	0,0487210	0	3.7641
VA Time Per Entity	3.3251	0,0487210	0	3.7641

Accumulated Time	Value
Accum VA Time	42.357,10
Total Accum Time	42.357,10

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelmayreturntimeconstant Page 2 of 2

July with return time constant

Process Detail Summary

Time per Entity

	Total Time	VA Time
crates	3.29	3.29

Crates

ED	7,8422

	Number In	Number Out
crates	12.330,00	12.281,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modeljulyreturntimeconstant

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Crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.6741	0,0478612	0	3.7780
VA Time Per Entity	3.6741	0,0478612	0	3.7780

Accumulated Time	Value
Accum VA Time	42.167,31
Total Accum Time	42.167,31

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modeljulyreturntimeconstant

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August with return time constant

Process Detail Summary

Time per Entity

	Total Time	VA Time
Crates	3.29	3.29

Crates

ED	7,8422

	Number In	Number Out
crates	12.430,00	12.371,00

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelaugustreturntimeconstant   Page 1 of 2

Crates

Time per Entity	Average	Half Width	Minimum	Maximum
Total Time Per Entity	3.8971	0,0488321	0	3.9981
VA Time Per Entity	3.8971	0,0488321	0	3.9981

Accumulated Time	Value
Accum VA Time	42.399,31
Total Accum Time	42.399,31

Model Filename: C:\Documents and Settings\Administrator\Mijn documenten\inbev\Modelaugustreturntimeconstant   Page 2 of 2

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