Bachelor Thesis Econometrics & Management Science

The economic lot sizing problem with a remanufacturing option and separate setup cost

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Contents

1. Introduction

The subject of this thesis is a special variant of the economic lot sizing (ELS) problem, also known as dynamic lot sizing model. Production planning is one of the fields where the ELS problem is applied and it will be used to illustrate the problem in this thesis. In the ELS problem there is a discrete and finite time horizon with in each period a (possibly different) quantity of demand. This demand has to be satisfied by producing in such a way that total costs are minimized. Costs include a fixed setup cost associated with each production period , a unit cost for each item produced and a unit cost for each time period an item is held in stock.

The variant which will be discussed in this thesis is the ELS problem with a remanufacturing option. Remanufacturing is becoming more and more important in the recent years and not only for economical reasons. Also because of environmental legislation and societal pressure to use resources more efficiently for a better climate. In this thesis is assumed that the items which are taken back can be remanufactured (recovered) into items which are as good as new. So no distinction can be made between new manufactured items and items that are remanufactured. Examples of such items are pallets, disposable cameras, PET bottles and ink cartridges.

In Teunter et al. [2006] two models are considered with either a joint setup cost or separate setup cost for manufacturing and remanufacturing. Joint setup cost may be the case when remanufacturing and manufacturing are performed on the same production line. Separate setup cost is considered if in a period where both manufacturing and remanufacturing occurs there is both a manufacturing setup cost and a remanufacturing setup cost. The latter can be the case when manufacturing and remanufacturing are performed on different production lines. This case (economic lot sizing problem with a remanufacturing option and separate setup cost) is taken into account in this thesis and will be abbreviated as the ELSR problem.

There is very little literature on the ELS problem with a remanufacturing option and all models consider separate setup cost. In van den Heuvel [2006] some literature is discussed and a short overview of the results are summarized below:

- Richter and Sombrutzki [2000] and Richter and Weber [2001] show that two restricted models can be solved by a Wagner-Whitin-like recursion.
- Li et al. [2006] generalized these models, with the assumption of sufficient returns to satisfy demand, to the multiple product case and a heuristic procedure is developed to compute near-optimal solutions for the problem.
- Beltran and Krass [2002] show that, if returns can be immediately used to satisfy demand, the model can be solved in cubic time for general concave cost functions and in quadratic time for the case of non-speculative motives.

- Golany et al. [2001] and Yang et al. [2005] show that the problem with disposals is $\mathcal{NP}$-hard. Golany et al. [2001] show that the problem can be solved in polynomial time in the case of linear cost by representing the model as a network flow model. And Yang et al. [2005] develop a polynomial time dynamic programming heuristic for the case of time-invariant cost parameters.

In van den Heuvel [2006] is proved that the ELSR problem is $\mathcal{NP}$-hard in general, even in the case of time-invariant cost parameters. Therefor a genetic algorithm is proposed for the problem. The genetic algorithm is hard to implement and a simpler version (which uses a LP solver) is quite slow.

This thesis will focus on heuristics for solving the ELSR problem within a reasonable amount of time. In Teunter et al. [2006] three heuristics are developed and tested. These heuristics are straightforward extensions of the Silver-Meal (SM), Least Unit Cost (LUC) and Part Period Balancing (PPB) heuristics. Only the extensions of SM and LUC are considered in this thesis because they show an outperformance compared to the extension of PPB. The extensions of SM and LUC will simply be referred to as SM and LUC in the rest of this thesis. For details on SM and LUC see Teunter et al. [2006].

The goal of this thesis is to develop heuristics which perform better than the heuristics developed in Teunter et al. [2006]. This can be done with the use of construction or improvement heuristics. The heuristics will simultaneously determine the manufacturing and the remanufacturing order sizes. The reason for this choice is that Teunter et al. [2006] also tested heuristics that determine the order sizes sequentially, first for remanufacturing and then for manufacturing based on the remaining 'net demand'. And it turned out that their performances were very poor compared to those of the simultaneous heuristics.

The remaining of the thesis is organized as follows. In Section 2 the model is described. The different mathematical formulations of the model used and an explanation of all the heuristics are presented in Section 3. And in Section 4 sensitivity analyses are performed on the effect of system parameters on the heuristic performances. The thesis ends with conclusions and offers directions for future research in Section 5.

2. Model

2.1. Problem description

The problem discussed in this thesis is to satisfy the demand of items in each period at the lowest total cost possible. The demand is given for a finite planning horizon and is assumed to be not stationary. It can be satisfied by both manufactured new items and remanufactured returned items (referred to as serviceables in the rest of this thesis). Also the number of remanufacturable returned items (returns) is known for all periods and assumed to be not stationary.

A finite horizon is chosen because of the fact that is hard to predict the future demand and returns, otherwise it is better to choose stochastic demand and returns patterns. The setting in this thesis is not one where the lot sizing problem is over at the end of the finite planning horizon and any remaining stock of returns needs to be disposed of. Recent research on infinite horizon models by Fleischmann et al. [2003], van der Laan and Salomon [1997] and Teunter and Vlachos [2002] has shown that such an option will not lead to a considerable cost reduction, unless the remanufacturable return rate as a percentage of the demand rate is unrealistically high (above 90%) and the demand rate is very small (less than 10 per year) [Teunter et al., 2006]. In Figure 2.1, a simple sketch of the system can be found.

To calculate the lowest total cost, the optimal configuration of items (re)manufactured in each period should be determined. And this is dependent on the cost structure assumed. In this thesis only setup cost of manufacturing and remanufacturing and inventory holding costs for returns and serviceables will be considered. Both the setup cost and the inventory holding costs (shortened to holding costs in the rest of this thesis) are time-invariant. The variable production cost is left out due to the setting that the lot sizing problem is not over at the end. This can be done because in the long term all

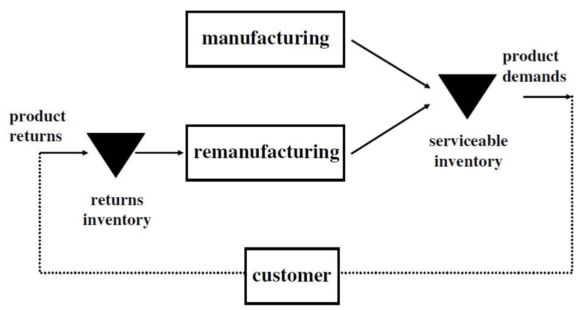

Figure 2.1.: Inventory system with remanufacturing

returns will be remanufactured and the total variable production costs won't be affected.

For economic reasons it is assumed that the inventory cost for serviceables is higher than for returns. This because the value added is higher for serviceables than for returns. If returns will be remanufactured in the period they arrive or serviceables are produced in the period they satisfy demand, no holding costs will be incurred. And if returns are left at the end of period T, then there are also holding costs at the end of period T.

Assume that in any period the sequence of events is as follows. First, returns come in and then manufacturing and remanufacturing take place. Next, demand occurs and finally holding costs are incurred for the serviceables and returns in stock at the end of the period [Teunter et al., 2006].

So zero lead times is assumed. Also assume that the initial stocks of serviceables and returns are both zero and that there is a positive demand in the first period. However, the model can easily be adjusted to any general lot sizing problem as Teunter et al. [2006] shows. So the non-zero lead times and/or non-zero initial stocks can be transformed to an equivalent problem with zero lead times and zero initial stocks and positive demand in the first period.

2.2. Mathematical model

To formulate the ELSR problem as a mathematical model, the notations of tables 2.1 and 2.2 are used.

Parameters	Description
T	Planning horizon
t	Index for periods in the planning horizon, t = 1, . . . , T
R_t	Number of returns received at the beginning of period t
D_t	Number of products demanded in period t
K^r	Setup cost for remanufacturing
K^m	Setup cost for manufacturing
h^r	Unit holding cost for returns per period
h^s	Unit holding cost of serviceables per period

Table 2.1.: Description of the parameters

Decision variables	Description
x_t^r	Number of products remanufactured in period t
x_t^m	Number of products manufactured in period t
I_t^r	Inventory level of returns at the end of period t
I_t^s	Inventory level of serviceables at the end of period t
y_t^r	0-1 indicator variable for remanufacturing setup in period t
y_t^m	0-1 indicator variable for manufacturing setup in period t

Table 2.2.: Description of the decision variables

The ELSR problem can be modeled as a mixed integer linear programming problem (MIP) as follows:

$$\min \quad \textstyle\sum_{t=1}^{T} \{K^r y_t^r + K^m y_t^m + h^r I_t^r + h^s I_t^s\}$$

subject to

$$I_{t-1}^r + R_t - x_t^r = I_t^r \text{ for } t = 1, \ldots, T \tag{2.1}$$

$$I_{t-1}^s + x_t^r + x_t^m - D_t = I_t^s \text{ for } t = 1, \ldots, T \tag{2.2}$$

$$x_t^r \leq M_t y_t^r \text{ for } t = 1, \ldots, T \tag{2.3}$$

$$x_t^m \leq M_t y_t^m \text{ for } t = 1, \ldots, T \tag{2.4}$$

$$y_t^r, y_t^m \in \{0,1\}, x_t^r, x_t^m, I_t^r, I_t^s \geq 0 \text{ and } M_t = \sum_{i=t}^{T} D_i \text{ for } t = 1, \ldots, T \tag{2.5}$$

In the objective function the total costs are minimized. The total costs consist of the total setup cost and holding costs. Constraints (2.1) and (2.2) take care that the inventories at the beginning and the end of every period are balanced for returns respectively serviceables. Constraints (2.3) and (2.4) force the indicator variables for (re)manufacturing setup to take the value 1 if (re)manufacturing takes place. And constraint (2.5) sets the indicator variables and prevents negative (re)manufacturing or inventory. The upper bound for (re)manufacturing in period t (M_t) is equal to the remaining demand at period t.

3. Methodology

As already mentioned in the introduction, the ELSR problem is $\mathcal{NP}$-hard in general and this thesis will focus on heuristics for solving the problem. The goal is to find a heuristics that generate solutions close to optimal and within a reasonable amount of time.

3.1. Different formulations

3.1.1. Mathematical model rewritten

First the mathematical model as described in section 2.2 will be rewritten as a model without inventory levels. This is done by using the substitutions $I_t^s = \sum_{i=1}^{t}(x_i^m + x_i^r - D_i)$ and $I_t^r = \sum_{i=1}^{t}(R_i - x_i^r)$, and redefining the unit cost for manufacturing and remanufacturing as $c_t^m = \sum_{i=t}^{T} h_i^s$ and $c_t^r = \sum_{i=t}^{T}(h_i^s - h_i^r)$, respectively. The new cost function now becomes $\sum_{t=1}^{T}\left\{K^r y_t^r + K^m y_t^m + c_t^r x_t^r + c_t^m x_t^m - h^s \sum_{i=1}^{t} D_i + h^r \sum_{i=1}^{t} R_i\right\}$, where the last two terms are constants. So the new mathematical model (referred to as normal formulation in the rest of this thesis) is modeled as follows:

$$\min \quad \sum_{t=1}^{T}\{K^r y_t^r + K^m y_t^m + c_t^r x_t^r + c_t^m x_t^m\}$$

subject to

$$\sum_{i=1}^{t}(x_i^r + x_i^m) \geq \sum_{i=1}^{t} D_i \text{ for } t = 1, \ldots, T \tag{3.1}$$

$$\sum_{i=1}^{t} x_i^r \leq \sum_{i=1}^{t} R_i \text{ for } t = 1, \ldots, T \tag{3.2}$$

$$x_t^r \leq M_t y_t^r \text{ for } t = 1, \ldots, T \tag{3.3}$$

$$x_t^m \leq M_t y_t^m \text{ for } t = 1, \ldots, T \tag{3.4}$$

$$y_t^r, y_t^m \in \{0,1\}, x_t^r, x_t^m \geq 0 \text{ and } M_t = \sum_{i=t}^{T} D_i \text{ for } t = 1, \ldots, T \tag{3.5}$$

The constant terms in the objective function are left out because they do not affect the optimal solution. Constraint (3.1) assures that the demand will be satisfied in every period. And constraint (3.2) ensures that the number of items remanufactured in a period can not exceed the number of returns available. The last three constraints do not change, only the variables I_t^r and I_t^s are now left out in constraint (3.5).

3.1.2. Facility location-based formulation

An optimal solution can of course be found with the help of a MIP solver. But if the number of periods becomes too large, it is not possible to solve the problem within a reasonable amount of time. However, the number of branch-and-bound nodes needed to solve a MIP depends heavily on the formulation used.

The tightest formulation of a set of possible solutions (X) is called the convex hull of X, denoted as conv(X). As all extreme points of conv(X) belong to X, all extreme optimal solutions to the linear relaxation belong to X, whatever the direction of minimization. Therefore it suffices to solve the linear relaxation to get an extreme optimal solution to LR, which is also an optimal solution to MIP [Pochet and Wolsey, 2006].

For the ELS problem Brahimi et al. [2006] suggest to use alternative problem formulations. One of the formulations is called the disaggregate formulation or facility location-based (FAL) formulation. The advantage of this formulation is that its LP relaxation has an optimal solution in which the variables y are integer. This was proven by Krarup and Bilde [1977] for an equivalent plant location formulation [Brahimi et al., 2006]. So this formulation is a convex hull for the ELS problem. Although it can not be a convex hull for the ELSR problem (because it is $\mathcal{NP}$-hard), it can create a tighter formulation. The disadvantage is that using this formulation increases the number of variables.

To use this formulation for the ELSR problem, the following variables have to be introduced. Define x^r_{qt} and x^m_{qt} as the number of items remanufactured respectively manufactured in period q to satisfy demand of period t ($q \leq t$). The ELSR problem modeled with the help of this formulation is shown below:

$$\min \quad \textstyle\sum_{q=1}^{T} \left\{ K^r y^r_q + K^m y^m_q + \sum_{t=q}^{T} \left\{ c^r_t x^r_{qt} + c^m_t x^m_{qt} \right\} \right\}$$

subject to

$$\sum_{i=1}^{t} \left\{ \sum_{q=1}^{i} (x^r_{qi} + x^m_{qi}) \right\} \geq \sum_{i=1}^{t} D_i \text{ for } t = 1, \dots, T \tag{3.6}$$

$$\sum_{i=1}^{q} \left\{ \sum_{t=i}^{T} x^r_{it} \right\} \leq \sum_{i=1}^{q} R_i \text{ for } q = 1, \dots, T \tag{3.7}$$

$$x^r_{qt} \leq D_t y^r_q \text{ for } q, t = 1, \dots, T \tag{3.8}$$

$$x^m_{qt} \leq D_t y^m_q \text{ for } q, t = 1, \dots, T \tag{3.9}$$

$$y^r_q, y^m_q \in \{0, 1\}, x^r_{qt}, x^m_{qt} \geq 0 \text{ for } q, t = 1, \dots, T \tag{3.10}$$

The objective function and constraints (3.6), (3.7) and (3.10) are the same as the objective function and constraints (3.1), (3.2) and (3.5) in the normal formulation. Only modified a little bit to implement the new variables x^r_{qt} and x^m_{qt} which replace x^r_t and x^m_t. Constraints (3.8) and (3.9) also assure in this formulation that the indicator variables for (re)manufacturing setup take the value 1 if (re)manufacturing takes place. But the difference is that using D_t as an upper bound instead of M_t creates a tighter formulation. Now (re)manufacturing in period q to satisfy demand of period t can not exceed the demand in period t.

3.2. Construction heuristics

The first type of heuristics that will be used are construction heuristics. That are heuristics that produce a feasible solution from scratch. The heuristics are based on the primal heuristics described in Pochet and Wolsey [2006]. Both the normal formulation as the FAL formulation of the ELSR problem will be used for all the construction heuristics. And these formulations form the basis for the construction heuristics. Note that only the notations y_q^r and y_q^m and $q \in \{1, \dots, q_1, \dots, q_2, \dots, T)$ are used. But for the normal formulation this should be replaced by y_t^r and y_t^m and $t \in \{1, \dots, t_1, \dots, t_2, \dots, T)$.

3.2.1. LP-and-Fix

The LP-and-Fix heuristic is a very simple heuristic. The heuristic will first solve the LP relaxation of the ELSR problem and the MIP afterwards, so it needs both a LP and a MIP solver. The heuristic is described in algorithm (3.1).

Algorithm 3.1 LP-and-Fix

LP relaxation

- replace $y_q^r, y_q^m \in \{0, 1\}$ in restriction (3.5)/(3.10) by $0 \leq y_q^r, y_q^m \leq 1$
- solve the model with a LP solver, the resulting LP solution is noted as $(\hat{x}^r, \hat{x}^m, \hat{y}^r, \hat{y}^m)$

MIP$^{\text{LP-FIX}}$

- remove relaxation, so the original restriction (3.5)/(3.10) will be used
- add the following two restrictions to fix whatever is integral in y_q^r and y_q^m of the LP solution

$$y_q^r = \hat{y}_q^r \text{ for all } q \text{ with } \hat{y}_q^r \in \{0, 1\} \tag{3.11}$$
$$y_q^m = \hat{y}_q^m \text{ for all } q \text{ with } \hat{y}_q^m \in \{0, 1\} \tag{3.12}$$

- solve the remaining MIP$^{\text{LP-FIX}}$ model with a MIP solver to obtain the LP-and-Fix heuristic solution

In general, this heuristic produces better solutions when a tighter formulation is used, and when the corresponding LP solution has fewer fractional y_q^r and y_q^m variables [Pochet and Wolsey, 2006]. The more fractional variables y_q^r and y_q^m in the LP solution the more time is needed to solve the MIP$^{\text{LP-FIX}}$ model and the less useful this heuristic will be.

3.2.2. Diving LP-driven

The Diving LP-driven heuristic is the construction type of the Diving heuristics. At each node all the y_q^r and y_q^m variables that take value 0 or 1 in the linear programming solution are frozen at that value, and we need to create one branch by fixing one of the y_q^r and y_q^m variables that is fractional to an integer value [Pochet and Wolsey, 2006].

The advantage of this heuristic is that it only needs a LP solver and no MIP solver. A description of the heuristic is in algorithm 3.2.

Variants

Three variants of the Diving LP-driven heuristic are also considered here. The first two variants (the floor and ceil variants) are similar to the original heuristic, but have a different procedure of fixing fractional elements. The procedures are described below. The last variant (the best variant) simply selects the best solution of the normal and the floor and ceil variants of the Diving LP-driven heuristic.

Diving LP-driven floor is the first variant of the Diving LP-driven heuristic with a different procedure of fixing fractional elements. Instead of just rounding the elements that are closest to integer, now the element that is closest to zero will be rounded to zero. The procedure is the same as in algorithm 3.2, but the part of fixing fractional elements has to be replaced by the part shown in algorithm 3.3.

Diving LP-driven ceil is the second variant of the Diving LP-driven heuristic with a different procedure of fixing fractional elements. Instead of just rounding the elements that are closest to integer, now the element that is closest to one will be rounded to one. Just like the floor variant, only the part of fixing fractional elements in algorithm 3.2 has to be replaced by the part shown in algorithm 3.4.

Algorithm 3.2 Diving LP-driven

LP relaxation

1. replace $y_q^r, y_q^m \in \{0,1\}$ in restriction (3.5)/(3.10) by $0 \leq y_q^r, y_q^m \leq 1$
2. solve the model with a LP solver, the resulting LP solution is noted as $(\hat{x}^r, \hat{x}^m, \hat{y}^r, \hat{y}^m)$
   a) if all elements of $\hat{y}_q^r$ and $\hat{y}_q^m$ are integer, **the Diving LP-driven heuristic solution is found**
   b) else, fix whatever is integral in y_q^r and y_q^m of the LP solution by adding the following two restrictions to the model and go to **fixing fractional elements**

$$y_q^r = \hat{y}_q^r \text{ for all } q \text{ with } \hat{y}_q^r \in \{0,1\} \quad (3.13)$$
$$y_q^m = \hat{y}_q^m \text{ for all } q \text{ with } \hat{y}_q^m \in \{0,1\} \quad (3.14)$$

fixing fractional elements

create a set (F) with all fractional elements of $\hat{y}_q^r$ and $\hat{y}_q^m$, $F = \left\{\hat{y}_q^r, \hat{y}_q^m : \hat{y}_q^r, \hat{y}_q^m \notin \{0,1\}\right\}$

- While F is not empty:
  - choose the element of F that is closest to integer $(\hat{y}_k)$ and set $y_k = 0$ if $\hat{y}_k < 0.5$ and $y_k = 1$ otherwise
  - check whether a feasible solution is still possible:
    - if this is the case, go to step 2 of **LP relaxation**
    - else, convert y_k back to $\hat{y}_k$ and remove element from F

  End While
- Set the maximum of all fractional elements to 1, $\max\left\{y_q^r, y_q^m : \hat{y}_q^r, \hat{y}_q^m \notin \{0,1\}\right\} = 1$, and go to step 2 of **LP relaxation**

Algorithm 3.3 Diving LP-driven floor

fixing fractional elements

create a set (F) with all fractional elements of $\hat{y}_q^r$ and $\hat{y}_q^m$, $F = \left\{\hat{y}_q^r, \hat{y}_q^m : \hat{y}_q^r, \hat{y}_q^m \notin \{0,1\}\right\}$

- While F is not empty:
  - choose the element of F that is closest to zero $(\hat{y}_l)$ and set $y_l = 0$
  - check whether a feasible solution is still possible:
    - if this is the case, go to step 2 of **LP relaxation**
    - else, convert y_l back to $\hat{y}_l$ and remove element from F

  End While

- Set the maximum of all fractional elements to 1, $\max\left\{y_q^r, y_q^m : \hat{y}_q^r, \hat{y}_q^m \notin \{0,1\}\right\} = 1$, and go to step 2 of **LP relaxation**

Algorithm 3.4 Diving LP-driven ceil

fixing fractional elements

create a set (F) with all fractional elements of $\hat{y}_q^r$ and $\hat{y}_q^m$, $F = \left\{\hat{y}_q^r, \hat{y}_q^m : \hat{y}_q^r, \hat{y}_q^m \notin \{0,1\}\right\}$

- choose the element of F that is closest to one $(\hat{y}_u)$ and set $y_u = 1$
- go to step 2 of **LP relaxation**

3.2.3. Relax-and-Fix

The last construction heuristic is the Relax-and-Fix heuristic which decomposes large problems into several smaller subproblems. The variables y_q^r and y_q^m can be partitioned in R disjoint sets $Q^1, \ldots, Q^R$ of increasing time periods, with the same amount of elements of y_q^r and y_q^m in every set. Q^1 are all the y_q^r and y_q^m variables associated with time periods in $\{1, \ldots, q_1\}$, Q^2 those associated with periods in $\{q_1+1, \ldots, q_2\}$, and so on. So R should be chosen in such a way that $\frac{T}{R} \in \mathbb{Z}^+$. Then sequentially $R-1$ MIPs are solved, denoted MIP^r with $1 \le r \le R-1$, to find a heuristic solution to the original MIP.

In the first MIP^1, only impose the integrality of the variables in $Q^1 \cup Q^2$ and relax the integrality restrictions on all the other variables in Q. Now restriction (3.5)/(3.10) will be replaced by the following restrictions.

$$y_q^r, y_q^m \in \{0,1\} \text{ for all } q \in Q^1 \cup Q^2 \tag{3.15}$$

$$y_q^r, y_q^m, \ge 0 \text{ for } q \in Q \setminus Q^1 \cup Q^2 \tag{3.16}$$

$$x_{qt}^r, x_{qt}^m \ge 0 \text{ for } q, t = 1, \ldots, T \tag{3.17}$$

Let $(x^{r,1}, x^{m,1}, y^{r,1}, y^{m,1})$ be an optimal solution of MIP^1. Then the variables in Q^1 will be fixed at their values in $y^{r,1}$ and $y^{m,1}$, and move to MIP^2.

In the subsequent MIP^r, for $2 \leq r \leq R-1$, the values of the y_q^r and y_q^m variables will be fixed additionally with index in Q^{r-1} at their optimal values from MIP^{r-1}, and add the integrality restriction for the variables in $Q^r \cup Q^{r+1}$ [Pochet and Wolsey, 2006]. Restrictions (3.15) and (3.16) of MIP^1 will be replaced by the following restrictions.

$$y_q^r, y_q^m = y_q^{r,r-1}, y_q^{m,r-1} \text{ for all } q \in Q^1 \cup \ldots \cup Q^{r-1} \tag{3.18}$$

$$y_q^r, y_q^m \in \{0,1\} \text{ for all } q \in Q^r \cup Q^{r+1} \tag{3.19}$$

$$y_q^r, y_q^m, \geq 0 \text{ for } q \in Q \setminus Q^1 \cup \ldots \cup Q^r \cup Q^{r+1} \tag{3.20}$$

For solving MIP^r a MIP solver is needed, however if R is large enough it is possible to use a LP solver instead of a MIP solver. This is done by solving the subproblem r for every possible combination of y_q^r and y_q^m which are element of $Q^r \cup Q^{r+1}$ and select the best combination. The number of LP problems that has to be solved at every r is equal to $2^{2 \cdot 2T/R}$. The base indicates that every element of y_q^r and y_q^m has two possible outcomes (0 and 1) and in the exponent is indicated that in every MIP^r the integrality restriction is applied to two sets, with $2T/R$ elements of y_q^r and y_q^m in every set.

3.3. Improvement heuristics

The second type of heuristics that will be used are improvement heuristics. That are heuristics that try to improve a feasible solution.

3.3.1. ELSRG

Consider the ELSR problem for which only the manufacturing and remanufacturing periods (the periods with strictly positive production) are given and the manufacturing and remanufacturing quantities have to be determined such that the total costs are minimized (we denote this problem by ELSRG). From model (ELSR) it follows immediately that for given production periods, the remaining problem is a linear programming (LP) problem. Hence, the ELSRG problem can be simply solved by any LP solver [van den Heuvel, 2006].

So if the manufacturing and remanufacturing periods of the optimal solutions are known, the ELSRG will give the optimal solution. However, the problem is that it is unknown. But it could improve the solutions of some constructions heuristics. Namely the heuristics that do not have optimal manufacturing and remanufacturing quantities, given the manufacturing and remanufacturing periods. All the construction heuristic mentioned in section 3.2 do not have this problem, because at each step a MIP or LP solver is used, which will automatically result in optimal manufacturing and remanufacturing quantities. But for SM and LUC it could be useful.

3.3.2. Exchange

Just like the construction heuristics is the Exchange heuristic based on a primal heuristic described in Pochet and Wolsey [2006] and both the normal formulation as the FAL formulation of the ELSR problem can be used (results are the same because there are no relaxed variables). Here these formulations also form the basis for the heuristic. The information available is the best feasible solution found so far, which can be determined by one or more construction heuristics. And this solution is selected as starting solution.

The exchange heuristic is an improvement version of the Relax-and-Fix heuristic. The same decomposition of integer variables in sets Q^r is used, with $1 \leq r \leq R$. At each iteration r, all integer variables are fixed at their value in the best solution $(\bar{x}^r, \bar{x}^m, \bar{y}^r, \bar{y}^m)$ found so far (or in the last solution encountered), except the variables in the set Q^r which are restricted to take integer values. Restriction 3.5/3.10 will be replaced by the following restrictions.

$$y_q^r, y_q^m = \bar{y}_q^r, \bar{y}_q^m \text{ for all } q \in Q \setminus Q^r \tag{3.21}$$

$$y_q^r, y_q^m \in \{0, 1\} \text{ for all } q \in Q^r \tag{3.22}$$

This procedure will be repeated R times, starting at $r = 1$ and ending at $r = R$. Finally the variables y_q^r and y_q^m will be set to 0 if there is no (re)manufacturing at period $q \in \{1, \ldots, T\}$.

Just like the LP-and-Relax heuristic also for the exchange heuristic a MIP solver is needed for solving MIP^r. However, a LP solver could be used if R is large enough (see Subsection 3.2.3 for an explanation). The number of LP problems which have to be solved now at every r is equal to $2^{2T/R}$. A factor 2 disappeared in the exponent because in every MIP^r the integrality restriction is now applied to only one set.

4. Numerical results

The heuristics will be tested on the same data set as used in Teunter et al. [2006], so that a comparison can be made with the heuristics presented in that article. The description of this data set as it can be found in van den Heuvel [2006] is written below.

Four different types of demand and return patterns are considered: stationary, linearly increasing, linearly decreasing and seasonal. The return ratio, i.e., the mean return rate as a percentage of the mean demand rate, is set to either 30%, 50%, or 70%. The total number of demand and return patterns considered are 10 and 22, respectively. For each pattern, four series of realizations are generated, so that the total number of demand and return series are 40 and 88, respectively.

All costs are time-invariant and the serviceable holding cost per period is normalized at 1. The remanufacturing holding cost is relatively small (0.2), moderate (0.5), or large (0.8). For both the manufacturing and remanufacturing setup costs, 3 values are considered. We remark that based on some preliminary investigations, these cost values are chosen such that during the planning horizon, which is fixed at 12 periods, the number of periods with a setup for the optimal solution varies between 2 and 6. The details on the demand and return patterns, and on the cost parameter values are summarized in Table 4.1. A full factorial design is applied, so that the total number of problem instances is $40 \times 88 \times 3 \times 3 \times 3 = 95{,}040$.

For each problem instance the optimal solution is determined with Cplex. And the solutions of all the heuristics are also determined for each problem instance and compared with the optimal solutions. This is done by Matlab programming, with the TOMLAB Optimization Environment for solving the LPs/MIPs with the Cplex solver. The performance of a heuristic is measured by the percentage increase in the total cost compared to the optimal solution (referred to as error in the rest of this thesis).

4.1. Construction heuristics

The first type of heuristics tested are the construction heuristics. First the performance of the different formulations of all the heuristics in this section will be discussed. Second the performance of the new construction heuristics will be compared with SM and LUC. And finally a sensitivity analysis on the performance will be executed.

4.1.1. Formulations

All the construction heuristics use both the normal and the FAL formulation. It is clear from table 4.4 that the FAL formulation performs much better than the normal formulation. This is true for all heuristics and statistics that are evaluated here. The

Demand pattern					
μ	σ	τ	α	c	d
Stationary					
100	10	0	0	na	na
100	20	0	0	na	na
Positive trend					
100	10	10	0	na	na
100	10	20	0	na	na
Negative trend					
210	10	-10	0	na	na
320	10	-20	0	na	na
Seasonal (peak in the middle)					
100	10	0	20	12	1
100	10	0	40	12	1
Seasonal (valley in the middle)					
100	10	0	20	12	3
100	10	0	40	12	3

Return pattern					
μ	σ	τ	α	c	d
Stationary					
30	3	0	0	na	na
30	6	0	0	na	na
50	5	0	0	na	na
50	10	0	0	na	na
70	7	0	0	na	na
70	14	0	0	na	na
Positive trend					
30	3	3	0	na	na
30	3	6	0	na	na
70	7	7	0	na	na
70	7	14	0	na	na
Negative trend					
63	3	-3	0	na	na
96	3	-6	0	na	na
147	7	-7	0	na	na
224	7	-14	0	na	na
Seasonal (peak in the middle)					
30	3	0	6	12	1
30	3	0	12	12	1
70	7	0	14	12	1
70	7	0	28	12	1
Seasonal (valley in the middle)					
30	3	0	6	12	3
30	3	0	12	12	3
70	7	0	14	12	3
70	7	0	28	12	3

Cost parameters	
Parameter	Values
K^r, K^m	200, 500, 2000
h^r	0.2, 0.5, 0.8
h^s	1

Table 4.1.: Experimental Setting. The demand (and return) patterns are generated according to $d_t = \mu + \tau(t{-}1) + a\sin\left(\frac{2\pi t}{c} + d\frac{\pi}{2}\right) + \epsilon_t$ for $t = 1, \ldots, T$, where μ is the starting level of the pattern, τ is the trend level, a is the amplitude of the cycle, c is the cycle length, d is the location of the peak of the cycle and $\epsilon_t (t = 1, \ldots, T)$ are independently normally distributed random variables with standard deviation σ

tables of the sensitivity analysis on the performance (see Appendix A) show that the FAL formulation even has a better performance over all the instances.

As mentioned in Subsection 3.2.1, the LP-and-Fix heuristic produces better solutions when a tighter formulation is used, and the more fractional variables y_q^r and y_q^m in the LP solutions the less useful this heuristic will be. Table 4.2 also shows that the use of the FAL formulation for this heuristic is preferred because the average number of fixed elements after after solving the LP relaxation is higher than when the normal formulation is used. For more details, see Appendix B.

Formulation	***normal***			***FAL***		
Variable(s)	y^r	y^m	$y^r + y^m$	y^r	y^m	$y^r + y^m$
Average	2,6	3,2	5,8	2,9	6,9	9,8
Standard deviation	2,4	2,9	4,0	3,5	3,1	5,6
Minimum	0	0	0	0	1	1
Maximum	12	12	20	12	12	24

Table 4.2.: Number of fixed elements after after solving the LP relaxation of the LP-and-Fix heuristics, both formulations

4.1.2. Performance

The performance of SM and LUC are used as a benchmark in this thesis. The performance of these heuristics are shown in table 4.3. For more details on the demand and return patterns, and on the cost parameter values, see tables A.10 and A.11 in the Appendix. According to Teunter et al. [2006] it turned out that SM performs slightly better than LUC. A sensitivity study further revealed that this result is quite robust with respect to demand/return patterns and cost settings. The average performance of SM was fair for the separate setup cost model.

The performance of the LP-and-Fix, Diving LP-driven and Relax-and-Fix heuristics can be found in table 4.4. For more details on the demand and return patterns, and on the cost parameter values, see Appendix A. Only the FAL formulation will be discussed here because of its outperformance compared to the normal formulation.

All (best variants of) the new heuristics perform on average better, according to all the evaluated statistics, than SM and LUC. Especially LP-and-Fix and Relax-and-Fix show

Heuristic	SM	LUC
Average error (%)	8,3	9,0
Standard deviation (%)	8,7	8,7
Maximum error (%)	82,5	98,4
Percentage within 1%	23,2	15,7

Table 4.3.: Performance of SM & LUC

LP-and-Fix

Formulation	normal	FAL
Average error (%)	2,8	0,3
Standard deviation (%)	12,1	0,9
Maximum error (%)	230,9	65,2
Percentage within 1%	74,1	90,7

(a) Both formulations

Diving LP-driven

Variant	normal	ceil	floor	best
Average error (%)	73,6	110,3	112,5	52,1
Standard deviation (%)	70,9	44,7	91,3	37,3
Maximum error (%)	548,3	437,8	601,1	343,9
Percentage within 1%	2,5	0,1	2,2	2,9

(b) Normal formulation

Variant	normal	ceil	floor	best
Average error (%)	8.6	23.8	21.0	6.6
Standard deviation (%)	10.1	26.5	22.3	7.2
Maximum error (%)	128.3	311.2	262.2	77.3
Percentage within 1%	28.0	11.6	22.4	30.3

(c) FAL formulation

Relax-and-Fix, with different numbers of disjoint sets (R)

Disjoint sets (R)	R = 12	R = 6	R = 4	R = 3
Average error (%)	3.4	1.2	0.7	0.3
Standard deviation (%)	3.6	1.9	1.3	0.8
Maximum error (%)	36.7	34.1	24.0	14.1
Percentage within 1%	30.9	64.5	77.4	89.8

(d) Normal formulation

Disjoint sets (R)	R = 12	R = 6	R = 4	R = 3
Average error (%)	1.4	0.8	0.4	0.2
Standard deviation (%)	2.2	1.5	1.1	0.7
Maximum error (%)	36.7	34.1	19.2	14.1
Percentage within 1%	60.9	75.1	85.3	92.9

(e) FAL formulation

Table 4.4.: Performance of construction heuristics

much better performances on average and in all scenarios. For the Relax-and-Fix heuristic a clear relation is observed between the number of disjoint sets and the performance. The less disjoint sets used the better the performance. This can be explained by the fact that the problems are more similar to the original MIP problem (and become more complex) as the number of disjoint sets decreases.

The normal Diving LP-driven heuristic is, looking at the performance, comparable with SM and LUC. The average error is approximately the mean of these two heuristics, the standard deviation and maximum error are higher, but the percentage within 1% is also higher. The ceil and floor variant have a lot worser performance in almost all statistics. However, if these are combined with the normal LP-Diving heuristic and the best is chosen they are useful. The best variant is, as already mentioned before, better than SM and LUC on average. And in almost all instances this heuristic also outperforms SM and LUC.

4.1.3. Sensitivity analysis

Next, a sensitivity analysis is performed to determine the effects of the demand pattern, return pattern, return rate, and cost parameters on the performance of the heuristics. The results of all heuristics are shown in the tables in Appendix A. These tables report the average error, the standard deviation of the errors, the maximum error and the percentage of problem instances with error within 1% of optimality. The minimum error is always 0% and therefore omitted. Also here only the FAL formulation will be discussed.

The results observed by Teunter et al. [2006], for SM and LUC, are taken as a basis for the sensitivity analysis in this thesis. These results are summarized in table 4.5 together with the results of the other construction heuristics. The tables in the Appendix indicate that the demand fluctuation has little or no effect on the performance. And because it is hard to quantify these results (because of contrary observations between the different demand patterns) other signs are used for indicating this influence.

The sensitivity is determined in terms of average error. The signs $-, +\& =$ are used for indicating the influence of increasing demand fluctuation. $-, +\& =$ indicate a negative, a positive and no effect on the average error respectively. This effect won't be discussed in detail below because of the little influence of this effect.

The focus here will be on the return ratio, the setup cost for both remanufacturing and manufacturing and the unit holding cost of returns. The influence of these effects on the average error is indicated with $\downarrow, \Downarrow$ / $\uparrow, \Uparrow$ which stand for: affects the performances negatively/positively and influence is relative small ($\downarrow$ / $\uparrow$) or great ($\Downarrow$ / $\Uparrow$). No effect observed is again indicated with $=$.

No effect is observed if the difference in average error between the two 'extreme cases' (for example between a return ration of 0.3 and 0.7) is less than 5% or if there is a peak or valley in the performance in the middle case (for example a return ration of 0.5). If the difference in average error between the two 'extreme cases' is more than 5% and less than 35% it is defined as a small influence. All the other cases are assumed to have a great influence.

For the LP-and-Fix heuristic the results do not differ that much from SM and LUC.

Heuristic		demand fluctuation$\Uparrow$	return ratio $\Uparrow$	$K^m \Uparrow$	$K^r \Uparrow$	$h^r \Uparrow$
SM & LUC		$+$	$\downarrow$	$\Downarrow$	$\Uparrow$	$\Downarrow$
LP-and-Fix (*normal*)		$+$	$\Downarrow$	$\Uparrow$	$\Downarrow$	$\Uparrow$
LP-and-Fix (*FAL*)		$=$	$\downarrow$	$=$	$\Uparrow$	$\Downarrow$
Diving LP-driven	normal	$+$	$\downarrow$	$\Uparrow$	$\Uparrow$	$\uparrow$
(*normal*)	ceil	$+$	$\uparrow$	$\downarrow$	$\Downarrow$	$=$
	floor	$+$	$\downarrow$	$\Uparrow$	$\Uparrow$	$\uparrow$
	best	$=$	$\downarrow$	$\Uparrow$	$\uparrow$	$\uparrow$
Diving LP-driven	normal	$-$	$\Downarrow$	$=$	$\Uparrow$	$\Downarrow$
(*FAL*)	ceil	$=$	$\uparrow$	$\Uparrow$	$\Downarrow$	$\Downarrow$
	floor	$=$	$\Downarrow$	$\uparrow$	$\Uparrow$	$\Downarrow$
	best	$-$	$\Downarrow$	$\uparrow$	$\Uparrow$	$\Downarrow$
Relax-and-Fix	R = 12	$+$	$\Downarrow$	$=$	$=$	$\downarrow$
(*normal*)	R = 6	$+$	$\Downarrow$	$\Downarrow$	$\Uparrow$	$\Downarrow$
	R = 4	$=$	$\Downarrow$	$=$	$=$	$\downarrow$
	R = 3	$=$	$\Downarrow$	$=$	$\Uparrow$	$\downarrow$
Relax-and-Fix	R = 12	$+$	$\downarrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
(*FAL*)	R = 6	$=$	$\Downarrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
	R = 4	$=$	$\Downarrow$	$\Downarrow$	$\Uparrow$	$\Uparrow$
	R = 3	$=$	$\Downarrow$	$=$	$\Uparrow$	$\Downarrow$

Table 4.5.: Summary of the sensitivity, with respect to demand and cost settings of the construction heuristics, in terms of average error. $-/+$, $\downarrow$ / $\uparrow$ and $\Downarrow$ / $\Uparrow$ stand for: affects the performances negatively/positively and influence is relative small ($-/+$ & $\downarrow$ / $\uparrow$) or great ($\Downarrow$ / $\Uparrow$). No effect observed is indicated with $=$

The only difference is that some effects, increasing demand fluctuation and setup cost for manufacturing, are not observed for this heuristic.

The same is true for the normal Diving LP-driven heuristic. The only differences for this heuristic are the relative great influence of an increasing return ratio and that no effect is observed if the setup cost for manufacturing increases. The floor and best variants show both the same effects which only differ from the normal Diving LP-driven heuristic in the fact that a relative small effect is observed if the setup cost for manufacturing increases. This effect is contrary compared with all the other construction heuristics, here the performance improves if setup cost for manufacturing increases. Looking, however, at the ceil variant leads to almost totally opposite effects. Only the effect of increasing setup cost for manufacturing is similar but with a greater influence, and the influence of the return ratio becomes relative small. The influence of increasing demand fluctuation for the normal and best variant is the same as for SM and LUC and no influence is observed for the ceil and floor variant.

The effects observed for the Relax-and-Fix heuristic are almost similar for all choices of R except R = 3. And are again almost the same as those for SM and LUC. The return ratio has more influence for the cases R = 6 and R = 4 than SM and LUC. But the real difference compared to SM and LUC is the influence of increasing holding costs for returns. This affects the performance positively instead of negatively. For the case with R = 3 the results are the same as for LP-and-Fix with a greater influence of an increasing return ratio.

The following explanations which are given by Teunter et al. [2006] are also valid for almost all the new heuristics. An increase in the unit holding cost of returns affects the performances negatively, showing that the suboptimality is mainly due to the scheduling of remanufacturing orders. And in cases where remanufacturing setup is more costly than a manufacturing setup, the optimal solution often place no remanufacturing batches at all, and neither do the heuristic solutions. As is known from the literature, the heuristics perform well in pure manufacturing situations.

Furthermore, an increase in the return ratio will affect the performances negatively because of the same reason why an increase in the unit holding cost of returns does. This is due to the fact that the possibility of remanufacturing becomes bigger and as a result (more) remanufacturing has to be scheduled.

Two great contrary effects have been observed. The first is for the Diving LP-driven ceil heuristic. It is the great negative influence of increasing setup cost for remanufacturing. A possible explanation for this could be the fact that the heuristic forces to use too many remanufacturing periods by rounding the binary variables up. But the great positive influence of increasing setup cost for manufacturing contradicts this explanation. The second is that an increase in unit holding cost for returns affects the performance of the Relax-and-Fix heuristic positive for all R except for R = 3. An explanation for this is unknown.

4.2. Improvement heuristics

The second type of heuristics are the improvement heuristics. These heuristics need a feasible solution as start solution. As already mentioned, all the construction heuristics could be used as start solution. But in this section only SM and LUC will be used as start solutions. This is done because they can generate a feasible solution very fast and do not need any solver. Also these solutions are used as benchmark for the construction heuristic and will therefor be used here as benchmark again. First the performance and improvement of the heuristics will be discussed, and second the changes between the start solutions and improved solutions will be analyzed. In Appendix A more details and a sensitivity analysis can be found on the performance of the improvement heuristics.

4.2.1. Performance and improvement

The performance of both improvement heuristics are determined and compared with the performance of the start solutions. The improvement of the solutions is determined as the relative decrease (or the relative increase for the percentage within 1%) in error. All the observations described below are valid for both start solutions because they both show almost the same results.

The results of the ELSRG heuristic are shown in table 4.6. The improvements in standard deviation and maximum error are (almost) nil. However the improvements in average error and percentage within 1% are reasonable considered that the only improvement can be gained from a decrease in total holding costs.

	Performance		**Improvement (%)**	
Start solution	SM	LUC	SM	LUC
Average error (%)	7,9	8,5	5,3	5,3
Standard deviation (%)	8,7	8,6	0,2	0,6
Maximum error (%)	82,5	98,4	0,0	0,0
Percentage within 1%	24,8	17,1	7,0	9,2

Table 4.6.: Performance and relative improvement (compared to the start solution) of ELSRG heuristic, with start solutions SM & LUC

The Exchange heuristic can improve the solution by a whole new schedule of production periods. So unlike the ELSRG heuristic now both the total holding costs and total setup cost can change. The results can be found in table 4.7.

It is clear that the improvements are huge, almost all the statistics improve with at least 50%. The only remarkable difference is the fact that there is a difference in improvement of the percentage within 1% between the start solutions SM and LUC. This can be explained by the fact that this statistic is much lower for LUC than for SM so there is more space for improvement. However, the percentage within 1% after applying the Exchange heuristic is still higher for SM than for LUC but the gap has become smaller.

Start solution: SM

Disjoint sets (R)	R = 12	R = 6	R = 4	R = 3	R = 2
Average error (%)	3,8	3,1	2,6	2,0	1,3
Standard deviation (%)	4,4	3,7	3,3	2,8	2,4
Maximum error (%)	53,5	47,9	41,2	39,0	53,9
Percentage within 1%	35,1	39,5	44,4	50,4	65,7

(a) Performance

Disjoint sets (R)	R = 12	R = 6	R = 4	R = 3	R = 2
Average error (%)	53,6	62,9	68,9	75,4	84,7
Standard deviation (%)	48,9	57,3	62,4	67,3	72,7
Maximum error (%)	35,2	41,9	50,1	52,8	34,7
Percentage within 1%	51,4	70,6	91,4	117,5	183,4

(b) Improvement (%)

Start solution: LUC

Disjoint sets (R)	R = 12	R = 6	R = 4	R = 3	R = 2
Average error (%)	4,4	3,5	2,9	2,2	1,5
Standard deviation (%)	4,6	3,9	3,5	2,9	2,7
Maximum error (%)	58,3	50,7	45,2	40,0	53,9
Percentage within 1%	27,1	32,3	37,4	46,9	60,4

(c) Performance

Disjoint sets (R)	R = 12	R = 6	R = 4	R = 3	R = 2
Average error (%)	51,7	61,4	67,6	75,4	83,0
Standard deviation (%)	47,0	54,8	60,1	66,3	69,2
Maximum error (%)	40,7	48,5	54,0	59,3	45,2
Percentage within 1%	72,9	106,4	138,3	199,3	285,1

(d) Improvement (%)

Table 4.7.: Performance and relative improvement (compared to the start solution) of Exchange heuristic, with different numbers of disjoint sets (R) and with start solutions SM & LUC

The same relation between the number of disjoint sets and the performance is seen here as with the Relax-and-Fix heuristic. The less disjoint sets used the better the performance. This can again be explained by the fact that the problems are more similar to the original MIP problem (and become more complex) as the number of disjoint sets decreases.

4.2.2. Changes

The changes between the start solutions and improved solutions will be analyzed for both the ELSRG heuristic and the Exchange heuristic. The ELSRG heuristic can only change the number of items produced in production periods. The Exchange heuristic can also change the configuration of production periods, the periods with (re)manufacturing can be different than for the start solution.

The only costs that can change for the ELSRG heuristic are the holding costs. The changes in percentage remanufactured items of the total production and improvement of the average error after applying the ELSRG heuristic are shown in table 4.8. There is no significant difference between SM and LUC but there is a big difference between start solutions with only manufacturing and with remanufacturing.

Instances	all	pure manufacturing	with remanufacturing
Percentage of instances	100	30.43	69.57
Remanufactured items SM (%)	26.30	0	37.80
Remanufactured items ELSRG (%)	26.86	0	38.61
Increase of remanufactured items (%)	2.46	0	3.54
Improvement average error (%)	7.80	0	11.21

(a) For SM and ELSRG with SM as start solution

Instances	all	pure manufacturing	with remanufacturing
Percentage of instances	100	30.17	69.83
Remanufactured items LUC (%)	26.78	0	38.35
Remanufactured items ELSRG (%)	27.40	0	39.24
Increase of remanufactured items (%)	2.75	0	3.94
Improvement average error (%)	7.52	0	10.77

(b) For LUC and ELSRG with LUC as start solution

Table 4.8.: Changes in percentage remanufactured items of the total production and improvement of the average error after applying ELSRG heuristic

There can't be an increase of remanufactured items of course if there are no remanufacturing periods. But the table also shows that for pure manufacturing solutions no improvement is possible. So the heuristic can only be useful if the start solution has remanufacturing periods. This can be explained by the zero inventory property (ZIP) of SM and LUC. There can't be any postponement of production and so the holding costs of serviceables can't be lower. But the holding costs for returns can be lower by violating the ZIP and remanufacture more items. This will result in higher holding costs for serviceables but the total holding costs will be lower. Furthermore, table 4.9 shows that higher unit holding cost for returns will result on average in a better performance. This can be explained by the fact that the savings in holding costs will be higher.

h^r	0.2/0.5/0.8	0.2	0.5	0.8
SM	11.21	6.81	12.52	13.52
LUC	10.77	5.89	12.38	13.20

Table 4.9.: Improvement (%) in average error after applying the ELSRG heuristic for the start solutions SM and LUC, grouped by unit holding cost for returns (only for instances with remanufacturing)

The improvement of the solution after applying the Exchange heuristic is mainly due to the changes in configurations of the (re)manufacturing periods. Table 4.10 shows the percentage of instances where the configuration of (re)manufacturing periods has changed after applying the Exchange heuristic. And in table 4.11 the average number of changes are shown. A distribution of these changes over all problem instances can be found in figures C.1 and C.2 in the Appendix. Just like above, there is no important difference in the observations between the start solutions SM and LUC, and therefor the description of the observations below is valid for both of them.

It is clear that fewer number of disjoint sets leads to more instances changed. This is because every set contains more elements that can be changed. The larger number of instances changed leads of course to a better performance of the heuristic, as is already mentioned before, because changes are only made if they improve the solution.

The number of changes per instance also increases if the number of disjoint sets decreases. This can be observed by the higher average number of changes, and is also supported by the decreasing bars at no (0) changes and increasing bars for (>0) changes in figures C.1 and C.2. The same explanation as above (more element per set) can be used here. And this is also an explanation for the better performance because more changes can lead to more improvement.

Heuristic	R = 12	R = 6	R = 4	R = 3	R = 2
SM	59.39	66.72	70.90	73.73	77.29
LUC	65.73	73.57	76.96	81.12	83.73

Table 4.10.: Percentage of the instances where the configuration of (re)manufacturing periods has changed after applying the Exchange heuristic

The average number of changes, however, is not very high. All the average numbers are below one because in 16 to 40 percent of the instances there is no change at all. And the fact that there are changes does not mean that all the numbers used in the table change for this instance. But there are some obvious differences between the changes in remanufacturing and manufacturing periods. The average number of periods removed does not differ very much between remanufacturing and manufacturing for SM and only a little for LUC. But there is an obvious difference in the average number of (re)manufacturing periods added and the average number of periods where remanufacturing (manufacturing) is replaced by manufacturing (remanufacturing). The Exchange heuristic is in favour of remanufacturing. This means that more often remanufacturing periods are added instead of manufacturing periods, and that manufacturing periods are more often replaced by remanufacturing than the other way around.

Average number of	R = 12	R = 6	R = 4	R = 3	R = 2
Remanufacturing periods removed	0.24	0.36	0.45	0.55	0.64
Manufacturing periods removed	0.22	0.36	0.42	0.53	0.65
Remanufacturing periods added	0.50	0.60	0.64	0.75	0.79
Manufacturing periods added	0.04	0.13	0.19	0.27	0.38
Periods with remanufacturing instead of manufacturing	0.18	0.20	0.25	0.25	0.30
Periods with manufacturing instead of remanufacturing	0.00	0.03	0.07	0.09	0.12

(a) For SM

Average number of	R = 12	R = 6	R = 4	R = 3	R = 2
Remanufacturing periods removed	0.27	0.40	0.48	0.62	0.70
Manufacturing periods removed	0.30	0.45	0.55	0.68	0.81
Remanufacturing periods added	0.47	0.59	0.63	0.77	0.80
Manufacturing periods added	0.07	0.20	0.28	0.39	0.48
Periods with remanufacturing instead of manufacturing	0.17	0.21	0.25	0.26	0.31
Periods with manufacturing instead of remanufacturing	0.00	0.04	0.07	0.09	0.13

(b) for LUC

Table 4.11.: Changes in the configuration of (re)manufacturing periods after applying the Exchange heuristic

5. Conclusion

In this thesis the ELSR problem is discussed. It was proved by van den Heuvel [2006] that this problem is $\mathcal{NP}$-hard in general and therefor the focus of this thesis was on heuristics for solving the problem. The goal was to find heuristics that perform better than the SM and LUC heuristics presented by Teunter et al. [2006].

All the heuristics presented in this thesis are based on the MIP model presented in chapter 2. But before implementation the model was rewritten to get rid of the inventory variables. And an alternative tighter formulation (the FAL formulation) was introduced. Although this formulation needs more variables, the advantage of a tighter formulation outweighs the increase in the number variables. For all the tested heuristics the FAL formulation outperformed the rewritten model.

Two types of heuristics were developed for the ELSR problem. The first type are construction heuristics and the second improvement heuristics. Three construction heuristics were developed in this thesis: the LP-and-Fix, the Diving LP-driven and the Relax-and-Fix heuristics. And two improvement heuristics: the ELSRG and Exchange heuristics.

All the heuristics have been tested on the same data set as used in Teunter et al. [2006], so that a comparison could be made with SM and LUC. The results showed that all the (best variants of the) heuristics presented in this thesis outperformed the SM and LUC. And the sensitivity analyses showed that the different cost settings had (almost) the same effects on most of the new construction heuristics as on SM and LUC.

The LP-and-Fix heuristic produced very good solutions on average (average error of 0.3%) but the average number of fixed elements after solving the LP relaxation was only 9.8 out of 24. As a consequence the resulting MIP is still large and a powerful MIP solver is still needed.

The Diving LP-driven heuristic showed the worst performance of all the construction heuristics (but still only an average error of 6.6% for the best variant). The advantage is that the solution can be found fast and with a relative simple LP solver. It doesn't have to solve that much LP problems ($2T$ problems in the worst case).

The Relax-and-Fix heuristic showed the best results of all the heuristics. The best result is reached with three disjoint sets (average error of 0.2%) but it has the same disadvantage as LP-and-Fix heuristic because each MIP subproblem is large (it contains $\frac{4T}{3}$ variables). Fortunately the heuristic performs still very good with twelve disjoint sets (average error of 1.4%) and then only one period is fixed at every iteration. The resulting MIP subproblems are so small that they can easily be replaced by solving a small number of LP problems (16 problems at each MIP subproblem).

The improvement heuristics were tested with the SM and LUC solutions as start solutions. But the results did not differ significantly between these two start solutions. It was observed that the ELSRG heuristic is only useful if there are remanufacturing periods scheduled and the results are only a little bit better than the start solutions. However, the advantage is that it is a very simple and fast heuristic (solves only one LP problem). And it is therefor recommended to apply this heuristics if no other improvement heuristic is used. The improvements reached with the Exchange heuristic are much bigger. However it almost uses the same techniques as the Relax-and-Fix heuristic but the results are worser. Therefor the Relax-and-Fix heuristic is preferable to the Exchange heuristic.

Although these results look hopeful for solving the ELSR problem without a very powerful MIP solver more research has to be done. An issue for future research is to test the heuristics on larger data sets to test whether the results are still valid. And to test whether the speed of solving the heuristics with the FAL formulation is still comparable or better than with the normal formulation.

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A. Sensitivity analysis

Construction heuristics

- LP-and-Fix:
  - both formulations, see table A.1
- Diving LP-driven:
  - normal formulation, see tables A.2 & A.3
  - FAL formulation, see tables A.4 & A.5
- Relax-and-Fix:
  - normal formulation, see tables A.6 & A.7
  - FAL formulation, see tables A.8 & A.9

Improvement heuristics

- ELSRG:
  - both start solutions, see tables A.10 & A.11
- Exchange:
  - with SM as start solution, see tables A.12 & A.13
  - with LUC as start solution, see tables A.14 & A.15

LP-and-Fix	**Percentage Cost Error**							
	Average		**Standard deviation**		**Maximum**		**Percentage within 1%**	
formulation	***normal***	***FAL***	***normal***	***FAL***	***normal***	***FAL***	***normal***	***FAL***
All instances	2,8	0,3	12,1	0,9	230,9	65,2	74,1	90,7
Demand								
Stationary	2,4	0,2	10,3	0,9	170,3	16,8	71,8	91,5
-Small variance	2,5	0,2	9,9	0,9	130,3	11,6	70,7	91,6
-Large variance	2,3	0,2	10,7	0,8	170,3	16,8	73,0	91,3
Positive trend	2,7	0,3	13,1	0,8	230,9	10,7	81,8	87,9
-Small trend	2,9	0,3	14,4	0,8	230,9	8,5	84,1	87,8
-Large trend	2,5	0,3	11,7	0,8	183,9	10,7	79,5	88,1
Negative trend	1,0	0,2	6,8	0,7	147,9	12,4	81,2	90,6
-Small trend	1,7	0,3	9,6	0,8	147,9	12,4	79,2	89,4
-Large trend	0,4	0,2	0,9	0,6	10,0	5,0	83,2	91,8
Seasonal	3,9	0,2	14,2	1,0	181,7	65,2	67,8	91,7
-Small amplitude	3,1	0,2	12,0	0,8	170,4	23,9	69,7	92,5
-Large amplitude	4,8	0,3	16,2	1,2	181,7	65,2	65,9	90,9
Returns								
Stationary	0,8	0,2	4,0	0,6	118,4	10,2	78,9	92,2
-Small variance	0,9	0,2	4,5	0,6	118,4	10,2	78,7	92,5
-Large variance	0,8	0,2	3,5	0,6	114,1	6,5	79,1	91,9
Positive trend	1,5	0,2	8,5	0,6	153,8	9,9	76,7	93,2
-Small trend	1,3	0,2	6,9	0,6	153,8	6,5	76,9	93,0
-Large trend	1,8	0,2	9,8	0,6	138,4	9,9	76,6	93,5
Negative trend	10,0	0,5	23,8	1,6	230,9	65,2	57,8	83,8
-Small trend	10,8	0,4	26,2	1,6	230,9	65,2	61,8	87,1
-Large trend	9,2	0,6	21,2	1,5	173,9	17,6	53,9	80,4
Seasonal	1,3	0,2	6,4	0,6	181,7	13,4	77,3	91,8
-Small amplitude	1,1	0,2	5,5	0,6	175,3	10,4	78,2	91,8
-Large amplitude	1,5	0,2	7,2	0,7	181,7	13,4	76,3	91,8
Return ratio*								
0.3	0,5	0,2	0,9	0,5	6,1	5,5	80,2	93,2
0.5	0,6	0,2	1,7	0,6	67,1	6,4	79,5	92,9
0.7	1,5	0,3	6,6	0,7	118,4	10,2	77,1	90,4
Man. set-up cost K^m								
200.0	3,6	0,2	15,6	0,7	230,9	16,8	71,1	91,1
500.0	2,8	0,3	11,9	0,8	155,4	12,4	73,7	87,9
2000.0	1,9	0,2	7,5	1,1	119,8	65,2	77,5	93,0
Reman. set-up cost K^r								
200.0	0,6	0,4	1,4	1,1	16,8	65,2	82,5	87,0
500.0	1,2	0,3	3,6	0,9	66,2	34,2	76,7	88,1
2000.0	6,6	0,1	20,1	0,5	230,9	11,2	63,0	97,0
Returns hold. cost h^r								
0.2	3,9	0,2	16,2	0,8	230,9	65,2	67,4	93,4
0.5	2,6	0,3	10,3	0,9	138,6	30,7	72,4	90,4
0.8	1,9	0,3	8,6	0,9	131,8	17,6	82,5	88,3

* only examples with stationary returns are considered.

Table A.1.: Sensitivity analysis on the performance of the LP-and-Fix heuristic

Diving LP-driven ***(normal formulation)***	**Percentage Cost Error**							
	Average				**Standard deviation**			
	normal	**ceil**	**floor**	**best**	**normal**	**ceil**	**floor**	**best**
All instances	73,6	110,3	112,5	52,1	70,9	44,7	91,3	37,3
Demand								
Stationary	51,1	107,4	87,1	41,6	46,0	44,7	63,3	31,5
-Small variance	55,8	115,1	85,9	44,6	50,3	46,9	62,9	33,6
-Large variance	46,4	99,8	88,3	38,7	40,7	41,0	63,6	28,9
Positive trend	121,3	125,1	172,7	74,2	99,6	49,2	124,2	44,7
-Small trend	104,2	124,3	144,4	68,8	87,1	47,9	103,5	42,4
-Large trend	138,3	125,9	200,9	79,6	108,2	50,5	136,2	46,4
Negative trend	79,6	98,9	131,0	53,0	68,5	32,9	90,1	31,8
-Small trend	68,8	101,2	115,7	49,9	58,5	33,8	78,3	30,4
-Large trend	90,4	96,6	146,2	56,0	75,7	31,8	98,2	32,9
Seasonal	58,2	110,0	85,8	45,8	50,5	45,6	63,5	33,7
-Small amplitude	58,9	112,3	87,2	46,4	51,0	45,6	64,1	34,0
-Large amplitude	57,4	107,8	84,5	45,3	49,9	45,5	62,7	33,4
Returns								
Stationary	74,3	119,6	117,9	54,6	69,1	38,4	90,1	36,6
-Small variance	74,5	118,7	117,7	54,7	68,9	36,7	89,9	36,6
-Large variance	74,1	120,5	118,0	54,5	69,3	40,0	90,3	36,7
Positive trend	81,2	98,1	121,7	50,1	76,6	43,0	91,3	34,5
-Small trend	80,2	103,6	121,1	52,2	74,5	40,4	91,4	35,1
-Large trend	82,3	92,6	122,3	48,0	78,6	44,7	91,3	33,8
Negative trend	62,5	88,7	83,8	44,9	69,8	48,3	88,3	38,8
-Small trend	66,2	93,9	91,6	48,8	67,2	46,0	88,4	39,3
-Large trend	58,7	83,4	76,0	41,0	72,0	50,0	87,6	38,0
Seasonal	74,9	120,2	118,1	54,8	69,1	42,9	90,8	37,8
-Small amplitude	76,0	119,1	118,0	55,0	70,5	38,8	90,4	37,7
-Large amplitude	73,9	121,4	118,2	54,6	67,7	46,5	91,1	37,9
Return ratio*								
0.3	64,4	124,9	115,6	49,7	64,7	41,2	94,3	39,2
0.5	73,7	119,8	116,7	55,3	65,4	36,3	88,3	35,1
0.7	84,9	114,2	121,3	58,6	75,1	36,6	87,4	34,8
Man. set-up cost K^m								
200.0	120,2	102,0	186,8	73,3	84,6	50,6	97,3	37,5
500.0	68,3	105,3	106,0	54,2	55,3	37,4	64,0	32,6
2000.0	32,4	123,5	44,6	28,7	31,1	42,2	35,3	26,5
Reman. set-up cost K^r								
200.0	102,9	89,2	151,4	57,4	84,5	41,4	103,9	29,8
500.0	66,3	102,5	107,1	50,8	59,9	34,2	81,8	32,5
2000.0	51,8	139,2	78,9	48,0	54,4	42,1	69,4	46,7
Returns hold. cost h^r								
0.2	78,1	103,2	117,5	53,6	74,5	41,3	95,0	39,4
0.5	73,1	115,8	110,4	53,2	70,0	46,4	90,6	37,7
0.8	69,8	111,8	109,5	49,4	67,7	45,4	87,8	34,5

* only examples with stationary returns are considered.

Table A.2.: Sensitivity analysis on the performance of the Diving LP-driven heuristics (normal formulation)

Diving LP-driven *(normal formulation)*	Percentage Cost Error							
	Maximum				Percentage within 1%			
	normal	**ceil**	**floor**	**best**	**normal**	**ceil**	**floor**	**best**
All instances	548,3	437,8	601,1	343,9	2,5	0,1	2,2	2,9
Demand								
Stationary	266,3	422,5	306,7	191,6	3,5	0,1	3,1	4,2
-Small variance	266,3	416,4	297,4	191,6	3,1	0,0	3,0	3,7
-Large variance	264,5	422,5	306,7	184,6	4,0	0,2	3,2	4,6
Positive trend	548,3	437,8	601,1	343,9	0,5	0,0	0,3	0,6
-Small trend	541,3	386,4	541,3	297,7	0,7	0,0	0,5	0,8
-Large trend	548,3	437,8	601,1	343,9	0,3	0,0	0,1	0,3
Negative trend	460,4	322,3	463,6	223,0	0,9	0,0	0,3	1,0
-Small trend	347,9	322,3	370,7	207,9	0,7	0,0	0,6	1,0
-Large trend	460,4	224,2	463,6	223,0	1,0	0,0	0,1	1,1
Seasonal	279,4	411,7	327,4	232,6	3,8	0,1	3,7	4,5
-Small amplitude	279,4	409,1	327,4	232,6	3,7	0,0	3,9	4,6
-Large amplitude	255,1	411,7	307,0	197,1	3,9	0,2	3,5	4,3
Returns								
Stationary	490,8	336,7	601,1	273,7	1,3	0,0	0,8	1,5
-Small variance	479,0	296,8	479,0	263,6	1,3	0,0	0,8	1,5
-Large variance	490,8	336,7	601,1	273,7	1,3	0,0	0,9	1,6
Positive trend	548,3	320,7	548,3	288,5	1,4	0,3	1,0	1,6
-Small trend	524,1	320,7	542,4	288,5	0,9	0,0	0,7	0,9
-Large trend	548,3	259,7	548,3	259,7	1,9	0,6	1,4	2,3
Negative trend	479,2	326,8	489,7	325,2	7,4	0,1	8,0	8,8
-Small trend	473,4	274,1	475,0	257,5	7,4	0,1	8,7	9,3
-Large trend	479,2	326,8	489,7	325,2	7,3	0,1	7,4	8,2
Seasonal	506,4	437,8	582,0	343,9	1,5	0,0	0,9	1,8
-Small amplitude	482,1	304,8	582,0	267,0	1,5	0,0	0,8	1,6
-Large amplitude	506,4	437,8	512,9	343,9	1,6	0,0	1,1	1,9
Return ratio*								
0.3	451,6	336,7	469,4	273,7	0,6	0,0	0,5	0,8
0.5	470,2	265,5	560,4	256,1	0,7	0,0	0,3	0,8
0.7	490,8	262,3	601,1	230,2	2,7	0,0	1,8	3,0
Man. set-up cost K^m								
200.0	548,3	437,8	601,1	343,9	0,8	0,0	1,0	1,0
500.0	487,8	321,6	487,8	188,7	1,3	0,1	1,4	1,5
2000.0	326,4	300,8	326,4	259,7	5,4	0,1	4,3	6,4
Reman. set-up cost K^r								
200.0	548,3	300,8	601,1	259,7	2,1	0,1	2,3	2,4
500.0	409,5	268,0	426,2	158,1	1,9	0,1	1,9	2,3
2000.0	409,5	437,8	426,2	343,9	3,5	0,0	2,4	4,1
Returns hold. cost h^r								
0.2	511,0	437,8	511,1	343,9	2,0	0,0	1,8	2,4
0.5	548,3	422,5	548,3	273,7	2,6	0,1	2,2	3,0
0.8	541,3	364,9	601,1	259,7	2,9	0,1	2,6	3,4

* only examples with stationary returns are considered.

Table A.3.: Sensitivity analysis on the performance of the Diving LP-driven heuristics (normal formulation)

Diving LP-driven ***(FAL formulation)***	**Percentage Cost Error**							
	Average				**Standard deviation**			
	round	**ceil**	**floor**	**best**	**round**	**ceil**	**floor**	**best**
All instances	8.6	23.8	21.0	6.6	10.1	26.5	22.3	7.2
Demand								
Stationary	8.4	24.0	21.1	6.5	10.2	26.1	22.6	7.2
-Small variance	8.5	24.7	21.2	6.6	10.3	26.6	22.7	7.4
-Large variance	8.4	23.3	21.0	6.4	10.1	25.5	22.5	7.1
Positive trend	8.7	26.2	21.5	6.9	9.3	31.2	21.8	7.1
-Small trend	8.9	23.1	21.9	6.9	9.3	25.7	21.9	7.0
-Large trend	8.4	29.3	21.1	6.9	9.2	35.6	21.8	7.2
Negative trend	8.6	22.5	21.1	6.3	10.0	25.0	22.6	6.7
-Small trend	9.1	23.3	21.8	6.5	10.5	26.9	23.6	6.9
-Large trend	8.1	21.6	20.4	6.0	9.5	23.0	21.6	6.6
Seasonal	8.6	23.2	20.7	6.7	10.4	24.7	22.3	7.3
-Small amplitude	8.6	23.7	20.5	6.7	10.3	25.1	21.9	7.3
-Large amplitude	8.7	22.6	20.9	6.6	10.5	24.3	22.7	7.4
Returns								
Stationary	8.1	25.3	20.6	6.4	9.7	26.7	21.3	7.1
-Small variance	8.2	25.2	20.8	6.5	9.6	26.6	21.6	7.1
-Large variance	8.1	25.5	20.4	6.4	9.8	26.8	21.1	7.0
Positive trend	9.5	18.3	23.5	6.6	10.9	16.2	24.3	6.9
-Small trend	9.6	20.1	23.1	6.8	11.0	17.5	24.8	7.0
-Large trend	9.5	16.5	23.9	6.4	10.9	14.5	23.7	6.8
Negative trend	9.2	21.4	21.2	7.3	10.5	21.1	24.2	7.7
-Small trend	9.1	22.4	21.2	7.1	10.3	22.1	23.0	7.5
-Large trend	9.4	20.4	21.1	7.5	10.8	20.0	25.3	7.9
Seasonal	8.2	26.6	20.0	6.4	9.6	31.9	21.0	7.0
-Small amplitude	8.1	25.8	20.2	6.4	9.5	28.9	21.2	7.0
-Large amplitude	8.2	27.4	19.8	6.5	9.6	34.6	20.7	7.1
Return ratio*								
0.3	5.6	27.5	13.6	4.3	7.0	34.9	15.2	5.4
0.5	8.1	24.8	21.8	6.6	9.4	23.6	21.6	7.1
0.7	10.6	23.8	26.4	8.4	11.4	18.9	24.2	7.9
Man. set-up cost K^m								
200.0	8.6	30.4	22.1	7.0	10.5	35.1	25.1	7.7
500.0	9.0	23.3	21.2	6.9	10.6	23.7	22.9	7.1
2000.0	8.2	17.7	19.7	6.0	8.9	15.1	18.4	6.6
Reman. set-up cost K^r								
200.0	11.6	14.3	29.3	7.8	11.3	10.7	24.8	6.9
500.0	10.6	20.9	24.5	8.7	9.9	13.6	21.2	7.4
2000.0	3.7	36.1	9.2	3.3	6.4	39.4	14.5	5.8
Returns hold. cost h^r								
0.2	4.4	9.8	10.5	3.4	5.5	10.3	12.2	4.4
0.5	8.3	29.9	20.1	6.7	8.5	27.0	18.8	6.7
0.8	13.1	31.8	32.5	9.7	12.8	31.2	27.5	8.3

* only examples with stationary returns are considered.

Table A.4.: Sensitivity analysis on the performance of the Diving LP-driven heuristics (FAL formulation)

Diving LP-driven	Percentage Cost Error							
(FAL formulation)	Maximum				Percentage within 1%			
	round	ceil	floor	best	round	ceil	floor	best
All instances	128.3	311.2	262.2	77.3	28.0	11.6	22.4	30.3
Demand								
Stationary	112.4	311.2	227.4	51.4	30.5	12.6	23.6	32.9
-Small variance	112.4	309.9	184.0	51.4	30.2	12.5	21.6	32.7
-Large variance	85.1	311.2	227.4	48.1	30.7	12.8	25.5	33.1
Positive trend	99.4	311.2	184.4	56.1	25.3	10.9	21.0	27.3
-Small trend	89.6	268.3	184.4	46.3	24.3	10.9	20.9	26.7
-Large trend	99.4	311.2	173.5	56.1	26.4	10.8	21.1	27.8
Negative trend	111.6	284.0	262.2	50.9	26.3	11.5	19.4	29.3
-Small trend	111.6	275.1	262.2	47.4	26.4	11.5	21.6	29.4
-Large trend	85.9	284.0	234.6	50.9	26.3	11.5	17.3	29.1
Seasonal	128.3	306.4	197.6	77.3	29.0	11.5	24.1	31.0
-Small amplitude	128.3	304.5	175.2	48.1	29.2	11.2	24.1	30.9
-Large amplitude	125.8	306.4	197.6	77.3	28.9	11.9	24.1	31.2
Returns								
Stationary	125.8	255.1	227.4	52.8	30.1	10.9	23.4	31.9
-Small variance	83.7	255.1	227.4	46.5	30.0	10.9	23.4	31.8
-Large variance	125.8	213.4	175.6	52.8	30.1	11.0	23.4	31.9
Positive trend	101.4	166.0	184.4	51.4	26.3	11.6	19.7	29.7
-Small trend	93.0	166.0	175.1	46.3	27.2	10.6	22.0	29.9
-Large trend	101.4	104.2	184.4	51.4	25.5	12.5	17.4	29.6
Negative trend	118.0	203.8	262.2	77.3	23.2	13.9	20.5	26.1
-Small trend	118.0	203.8	175.2	77.3	24.1	11.9	20.9	27.1
-Large trend	111.6	176.5	262.2	51.4	22.2	15.8	20.1	25.1
Seasonal	128.3	311.2	197.6	56.1	29.8	11.0	24.1	31.5
-Small amplitude	112.4	295.6	197.6	56.1	29.9	11.0	24.1	31.4
-Large amplitude	128.3	311.2	197.5	51.0	29.8	11.1	24.0	31.6
Return ratio*								
0.3	60.5	255.1	135.7	32.3	40.8	13.8	33.6	42.9
0.5	76.2	205.3	162.6	43.2	28.4	10.5	20.8	30.6
0.7	125.8	172.2	227.4	52.8	21.0	8.5	15.8	22.1
Man. set-up cost K^m								
200.0	128.3	311.2	262.2	56.1	32.9	13.0	28.2	34.4
500.0	112.3	221.5	200.8	50.9	28.5	12.6	23.5	29.8
2000.0	90.1	107.2	202.5	77.3	22.8	9.2	15.6	26.8
Reman. set-up cost K^r								
200.0	128.3	82.1	262.2	77.3	8.7	5.5	3.7	12.6
500.0	118.0	116.3	234.6	56.1	11.9	2.1	6.2	13.3
2000.0	67.1	311.2	167.2	51.4	63.6	27.2	57.4	65.0
Returns hold. cost h^r								
0.2	77.3	82.1	131.1	77.3	42.0	30.7	33.3	45.0
0.5	81.2	284.0	165.7	53.9	26.5	2.7	22.0	28.9
0.8	128.3	311.2	262.2	56.1	15.6	1.5	12.0	17.0

* only examples with stationary returns are considered.

Table A.5.: Sensitivity analysis on the performance of the Diving LP-driven heuristics (FAL formulation)

Relax-and-Fix *(normal formulation)*	**Percentage Cost Error**							
	Average				**Standard deviation**			
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 12**	**R = 6**	**R = 4**	**R = 3**
All instances	3.4	1.2	0.7	0.3	3.6	1.9	1.3	0.8
Demand								
Stationary	3.7	1.3	0.7	0.3	3.8	1.9	1.4	0.9
-Small variance	3.7	1.3	0.7	0.3	3.7	1.9	1.3	0.9
-Large variance	3.6	1.2	0.7	0.3	3.9	2.0	1.4	0.9
Positive trend	2.7	1.2	0.8	0.2	3.0	1.9	1.5	0.7
-Small trend	3.1	1.4	0.8	0.2	3.4	2.2	1.8	0.8
-Large trend	2.3	1.0	0.8	0.2	2.6	1.5	1.2	0.6
Negative trend	3.4	1.2	0.5	0.2	3.5	2.0	1.0	0.7
-Small trend	3.4	1.2	0.6	0.2	3.6	1.9	1.2	0.7
-Large trend	3.5	1.3	0.3	0.2	3.5	2.1	0.8	0.6
Seasonal	3.7	1.2	0.6	0.3	3.7	1.9	1.3	0.9
-Small amplitude	3.8	1.3	0.7	0.3	3.6	2.0	1.3	0.8
-Large amplitude	3.6	1.1	0.6	0.3	3.7	1.9	1.3	0.9
Returns								
Stationary	3.4	1.1	0.7	0.3	3.3	1.8	1.3	0.8
-Small variance	3.4	1.1	0.7	0.3	3.3	1.8	1.2	0.8
-Large variance	3.4	1.2	0.7	0.3	3.3	1.8	1.3	0.8
Positive trend	3.8	1.3	0.7	0.3	4.1	2.0	1.4	0.8
-Small trend	3.7	1.3	0.7	0.3	3.9	2.0	1.3	0.8
-Large trend	3.9	1.2	0.7	0.3	4.3	2.1	1.4	0.9
Negative trend	3.5	1.4	0.7	0.3	3.8	2.2	1.4	1.0
-Small trend	3.5	1.5	0.7	0.3	3.7	2.3	1.4	1.0
-Large trend	3.5	1.3	0.7	0.3	4.0	2.2	1.4	1.0
Seasonal	3.3	1.2	0.6	0.3	3.3	1.8	1.2	0.7
-Small amplitude	3.3	1.1	0.7	0.3	3.4	1.8	1.2	0.7
-Large amplitude	3.3	1.2	0.6	0.2	3.3	1.9	1.2	0.7
Return ratio*								
0.3	2.4	0.9	0.5	0.1	2.4	1.4	0.9	0.4
0.5	3.2	1.1	0.6	0.3	2.9	1.6	1.1	0.7
0.7	4.4	1.5	0.9	0.4	4.1	2.2	1.6	1.0
Man. set-up cost K^m								
200.0	2.7	0.8	0.5	0.2	3.2	1.4	1.0	0.6
500.0	3.9	1.4	0.8	0.5	3.8	2.0	1.4	1.0
2000.0	3.8	1.5	0.6	0.2	3.6	2.2	1.4	0.8
Reman. set-up cost K^r								
200.0	3.8	1.5	0.8	0.4	3.4	2.1	1.5	0.9
500.0	4.8	1.5	0.9	0.4	3.9	2.0	1.4	0.9
2000.0	1.8	0.6	0.3	0.1	2.6	1.5	0.8	0.5
Returns hold. cost h^r								
0.2	2.9	0.9	0.6	0.2	3.1	1.7	1.2	0.7
0.5	3.5	1.2	0.7	0.3	3.6	1.9	1.3	0.8
0.8	3.9	1.5	0.7	0.3	3.9	2.1	1.4	0.9

* only examples with stationary returns are considered.

Table A.6.: Sensitivity analysis on the performance of the Relax-and-Fix heuristic (normal formulation), with different numbers of disjoint sets (R)

Relax-and-Fix *(normal formulation)*	**Percentage Cost Error**							
	Maximum				**Percentage within 1%**			
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 12**	**R = 6**	**R = 4**	**R = 3**
All instances	36.7	34.1	24.0	14.1	30.9	64.5	77.4	89.8
Demand								
Stationary	36.7	34.1	24.0	13.8	31.7	62.6	76.5	89.1
-Small variance	33.8	15.3	14.0	8.7	30.4	61.5	76.4	89.2
-Large variance	36.7	34.1	24.0	13.8	33.1	63.7	76.5	89.1
Positive trend	25.8	15.4	14.3	8.9	37.7	64.2	72.7	91.3
-Small trend	25.8	15.4	14.3	8.9	32.4	61.6	78.1	92.2
-Large trend	20.0	14.0	11.1	5.2	43.1	66.8	67.3	90.3
Negative trend	25.9	16.5	9.1	7.6	29.1	64.3	82.7	91.8
-Small trend	25.9	16.5	9.1	7.6	27.4	64.3	77.4	92.4
-Large trend	25.5	14.1	8.8	7.0	30.8	64.4	88.0	91.2
Seasonal	34.7	19.6	14.6	14.1	27.9	65.6	77.5	88.4
-Small amplitude	34.7	19.5	14.6	14.1	27.0	63.5	74.5	88.5
-Large amplitude	30.2	19.6	12.0	10.8	28.8	67.8	80.5	88.3
Returns								
Stationary	28.3	15.5	14.0	9.9	30.0	65.0	76.9	89.7
-Small variance	28.3	14.9	12.8	9.1	30.0	65.1	76.8	89.7
-Large variance	22.9	15.5	14.0	9.9	29.9	65.0	77.0	89.7
Positive trend	34.7	19.6	11.1	8.7	31.5	64.6	77.2	89.4
-Small trend	34.7	12.9	11.1	8.0	29.9	62.9	77.5	89.5
-Large trend	29.1	19.6	10.1	8.7	33.1	66.2	76.9	89.2
Negative trend	36.7	34.1	24.0	14.1	34.1	62.9	77.7	88.8
-Small trend	36.7	34.1	24.0	14.1	32.5	60.0	76.6	88.5
-Large trend	25.9	14.4	16.0	10.2	35.7	65.8	78.9	89.1
Seasonal	29.8	16.3	14.3	10.8	29.6	64.8	77.7	90.6
-Small amplitude	25.7	15.1	12.4	10.8	30.0	64.7	76.9	90.3
-Large amplitude	29.8	16.3	14.3	8.0	29.3	64.9	78.4	90.9
Return ratio*								
0.3	16.9	9.1	7.0	4.8	35.5	70.0	80.9	94.8
0.5	16.2	12.6	9.8	6.3	29.8	65.6	77.2	88.8
0.7	28.3	15.5	14.0	9.9	24.5	59.5	72.6	85.4
Man. set-up cost K^m								
200.0	27.0	12.5	9.8	6.2	37.5	74.0	80.2	93.4
500.0	34.7	19.6	14.0	10.8	28.4	58.4	71.5	82.6
2000.0	36.7	34.1	24.0	14.1	26.8	61.0	80.5	93.4
Reman. set-up cost K^r								
200.0	28.3	16.3	14.3	10.8	24.0	56.3	73.8	86.8
500.0	36.7	34.1	24.0	14.1	16.4	55.1	70.0	86.6
2000.0	30.9	18.3	16.0	10.2	52.3	82.0	88.3	96.0
Returns hold. cost h^r								
0.2	36.7	34.1	24.0	14.1	35.4	72.4	80.7	91.8
0.5	34.7	26.6	18.6	12.4	29.6	63.9	76.7	89.0
0.8	33.8	20.0	16.0	10.2	27.7	57.2	74.8	88.6

* only examples with stationary returns are considered.

Table A.7.: Sensitivity analysis on the performance of the Relax-and-Fix heuristic (normal formulation), with different numbers of disjoint sets (R)

Relax-and-Fix *(FAL formulation)*	**Percentage Cost Error**							
	Average				**Standard deviation**			
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 12**	**R = 6**	**R = 4**	**R = 3**
All instances	1.4	0.8	0.4	0.2	2.2	1.5	1.1	0.7
Demand								
Stationary	1.4	0.8	0.5	0.2	2.2	1.5	1.2	0.8
-Small variance	1.3	0.8	0.5	0.2	2.0	1.5	1.1	0.8
-Large variance	1.4	0.8	0.5	0.2	2.3	1.6	1.2	0.8
Positive trend	1.2	0.8	0.5	0.2	1.8	1.5	1.2	0.6
-Small trend	1.3	0.9	0.5	0.2	2.0	1.7	1.4	0.7
-Large trend	1.1	0.7	0.5	0.2	1.6	1.3	1.0	0.5
Negative trend	1.6	0.7	0.3	0.2	2.5	1.5	0.7	0.5
-Small trend	1.8	0.7	0.3	0.2	2.6	1.4	0.9	0.6
-Large trend	1.4	0.7	0.2	0.1	2.4	1.5	0.6	0.5
Seasonal	1.4	0.7	0.4	0.2	2.1	1.4	1.0	0.7
-Small amplitude	1.4	0.8	0.5	0.2	2.2	1.5	1.1	0.7
-Large amplitude	1.4	0.7	0.4	0.2	2.1	1.4	1.0	0.7
Returns								
Stationary	1.4	0.8	0.4	0.2	2.1	1.4	1.0	0.6
-Small variance	1.4	0.8	0.4	0.2	2.1	1.4	1.0	0.6
-Large variance	1.4	0.8	0.4	0.2	2.1	1.4	1.0	0.6
Positive trend	1.4	0.8	0.4	0.2	2.2	1.5	1.1	0.6
-Small trend	1.4	0.8	0.4	0.2	2.0	1.5	1.1	0.7
-Large trend	1.5	0.7	0.5	0.2	2.5	1.5	1.1	0.6
Negative trend	1.4	0.8	0.4	0.3	2.4	1.6	1.1	0.9
-Small trend	1.5	0.9	0.4	0.3	2.4	1.8	1.1	0.9
-Large trend	1.2	0.7	0.4	0.3	2.3	1.5	1.1	0.9
Seasonal	1.4	0.8	0.4	0.2	2.1	1.4	1.0	0.6
-Small amplitude	1.4	0.8	0.5	0.2	2.1	1.4	1.1	0.6
-Large amplitude	1.4	0.7	0.4	0.2	2.1	1.5	1.0	0.6
Return ratio*								
0.3	1.2	0.6	0.3	0.1	1.9	1.1	0.7	0.3
0.5	1.3	0.7	0.4	0.2	1.9	1.3	0.8	0.6
0.7	1.6	1.0	0.6	0.3	2.4	1.7	1.4	0.8
Man. set-up cost K^m								
200.0	0.8	0.5	0.3	0.1	1.4	1.0	0.7	0.4
500.0	1.4	0.8	0.5	0.3	2.2	1.5	1.2	0.9
2000.0	1.9	1.0	0.5	0.1	2.6	1.8	1.2	0.6
Reman. set-up cost K^r								
200.0	1.9	1.0	0.6	0.3	2.3	1.6	1.3	0.8
500.0	1.6	1.0	0.5	0.3	2.3	1.6	1.1	0.7
2000.0	0.7	0.3	0.1	0.0	1.7	1.0	0.6	0.4
Returns hold. cost h^r								
0.2	1.0	0.5	0.3	0.1	2.0	1.2	0.9	0.6
0.5	1.4	0.8	0.4	0.2	2.2	1.5	1.1	0.7
0.8	1.8	1.0	0.5	0.2	2.3	1.6	1.1	0.7

* only examples with stationary returns are considered.

Table A.8.: Sensitivity analysis on the performance of the Relax-and-Fix heuristic (FAL formulation), with different numbers of disjoint sets (R)

Relax-and-Fix *(FAL formulation)*	Percentage Cost Error							
	Maximum				Percentage within 1%			
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 12**	**R = 6**	**R = 4**	**R = 3**
All instances	36.7	34.1	19.2	14.1	60.9	75.1	85.3	92.9
Demand								
Stationary	36.7	34.1	19.2	13.8	61.9	74.2	83.1	92.0
-Small variance	19.1	13.6	13.3	9.2	62.2	73.8	82.5	91.9
-Large variance	36.7	34.1	19.2	13.8	61.6	74.6	83.8	92.2
Positive trend	20.3	15.4	12.7	12.9	60.6	72.8	83.4	93.4
-Small trend	20.3	15.4	12.7	12.9	59.8	73.2	85.9	94.4
-Large trend	14.2	14.0	9.8	5.2	61.3	72.3	80.9	92.4
Negative trend	17.9	13.6	9.1	7.1	59.1	76.6	89.7	94.4
-Small trend	17.3	13.6	8.2	7.1	52.8	76.2	87.9	94.5
-Large trend	17.9	12.1	9.1	6.5	65.5	77.0	91.5	94.3
Seasonal	25.5	19.5	14.6	14.1	61.4	75.9	85.2	92.3
-Small amplitude	22.9	19.5	14.6	14.1	60.8	74.4	84.2	92.1
-Large amplitude	25.5	11.3	10.6	9.9	61.9	77.5	86.2	92.4
Returns								
Stationary	17.9	13.6	12.3	9.9	60.3	74.4	85.2	93.1
-Small variance	17.6	13.6	11.6	9.2	60.5	74.7	85.0	93.3
-Large variance	17.9	13.5	12.3	9.9	60.2	74.2	85.4	92.8
Positive trend	18.4	15.4	11.0	8.0	62.2	76.1	84.9	92.6
-Small trend	14.2	9.6	11.0	8.0	60.5	74.2	85.3	91.8
-Large trend	18.4	15.4	10.1	6.3	64.0	78.0	84.6	93.4
Negative trend	36.7	34.1	19.2	14.1	63.2	75.7	86.4	91.3
-Small trend	36.7	34.1	19.2	14.1	59.6	73.3	85.7	91.1
-Large trend	22.2	14.4	16.0	9.9	66.7	78.2	87.0	91.5
Seasonal	20.3	14.2	13.3	9.7	59.4	74.8	85.1	93.6
-Small amplitude	19.1	13.7	13.3	9.7	59.2	73.7	84.1	93.3
-Large amplitude	20.3	14.2	12.7	6.8	59.6	75.9	86.1	94.0
Return ratio*								
0.3	17.3	8.7	5.5	3.7	62.5	78.3	87.8	96.8
0.5	17.9	10.9	6.3	6.0	61.5	73.9	86.3	92.2
0.7	17.6	13.6	12.3	9.9	57.0	71.1	81.4	90.2
Man. set-up cost K^m								
200.0	15.8	10.8	9.2	7.2	71.4	82.1	89.6	95.2
500.0	30.5	13.6	13.3	9.9	60.7	72.0	81.2	88.1
2000.0	36.7	34.1	19.2	14.1	50.5	71.1	85.2	95.3
Reman. set-up cost K^r								
200.0	19.3	15.4	13.3	9.9	47.6	66.2	80.2	89.3
500.0	36.7	34.1	19.2	14.1	54.4	68.2	80.5	90.6
2000.0	25.5	14.4	16.0	9.9	80.6	90.9	95.3	98.6
Returns hold. cost h^r								
0.2	36.7	34.1	19.2	14.1	71.1	82.9	89.8	94.8
0.5	30.5	26.6	18.6	10.9	60.3	74.0	84.5	92.1
0.8	25.5	20.0	16.0	12.9	51.2	68.4	81.6	91.7

* only examples with stationary returns are considered.

Table A.9.: Sensitivity analysis on the performance of the Relax-and-Fix heuristic (FAL formulation), with different numbers of disjoint sets (R)

SM, LUC & ELSRG	**Percentage Cost Error**							
	Average				**Standard deviation**			
	normal		**ELSRG**		**normal**		**ELSRG**	
	SM	**LUC**	**SM**	**LUC**	**SM**	**LUC**	**SM**	**LUC**
All instances	8,3	9,0	7,9	8,5	8,7	8,7	8,7	8,6
Demand								
Stationary	8,1	8,7	7,6	8,2	9,1	9,0	9,0	8,9
-Small variance	7,6	7,8	7,1	7,3	9,0	8,9	9,0	8,8
-Large variance	8,6	9,5	8,2	9,1	9,1	9,0	9,1	8,9
Positive trend	7,4	7,7	6,9	7,3	7,3	7,1	7,2	7,1
-Small trend	7,4	7,8	6,9	7,3	7,6	7,1	7,6	7,1
-Large trend	7,4	7,7	7,0	7,3	6,9	7,1	6,8	7,1
Negative trend	9,5	9,3	9,1	8,7	8,1	8,0	8,1	8,0
-Small trend	9,4	9,0	8,9	8,4	7,9	7,5	7,9	7,4
-Large trend	9,6	9,6	9,2	9,0	8,3	8,5	8,3	8,5
Seasonal	8,3	9,6	7,8	9,2	9,3	9,4	9,3	9,4
-Small amplitude	7,8	8,5	7,3	8,1	9,0	9,0	9,0	8,9
-Large amplitude	8,8	10,8	8,3	10,3	9,6	9,7	9,6	9,6
Returns								
Stationary	7,5	8,2	7,0	7,7	7,3	7,3	7,3	7,3
-Small variance	7,5	8,2	7,0	7,7	7,3	7,2	7,2	7,2
-Large variance	7,4	8,2	7,0	7,7	7,4	7,4	7,4	7,4
Positive trend	10,9	11,6	10,5	11,2	11,5	11,7	11,5	11,7
-Small trend	10,4	11,1	10,0	10,7	11,4	11,3	11,4	11,3
-Large trend	11,4	12,1	11,1	11,7	11,6	12,1	11,6	12,0
Negative trend	8,9	9,9	8,5	9,4	9,8	9,7	9,8	9,6
-Small trend	8,1	9,2	7,6	8,8	8,5	9,1	8,5	9,1
-Large trend	9,8	10,5	9,4	10,0	10,9	10,2	10,9	10,1
Seasonal	7,3	7,9	6,9	7,4	6,9	6,7	6,8	6,7
-Small amplitude	7,3	7,8	6,8	7,3	6,8	6,7	6,7	6,6
-Large amplitude	7,3	7,9	6,9	7,4	6,9	6,8	6,9	6,7
Return ratio*								
0.3	5,6	6,5	5,2	6,0	5,5	6,0	5,4	5,9
0.5	8,0	8,9	7,4	8,3	8,1	8,3	8,1	8,2
0.7	8,9	9,3	8,4	8,7	7,8	7,2	7,8	7,2
Man. set-up cost K^m								
200.0	6,2	7,6	5,6	6,9	7,3	7,8	7,2	7,6
500.0	6,9	7,2	6,4	6,7	6,8	6,7	6,7	6,6
2000.0	11,8	12,2	11,6	11,9	10,3	10,2	10,3	10,2
Reman. set-up cost K^r								
200.0	11,3	11,3	10,5	10,5	9,9	9,7	10,1	9,8
500.0	9,0	9,6	8,4	9,0	7,7	8,1	7,7	7,9
2000.0	4,7	6,1	4,6	6,0	6,8	7,3	6,7	7,2
Returns hold. cost h^r								
0.2	5,5	6,0	5,4	5,9	7,2	7,1	7,2	7,1
0.5	8,3	9,1	7,7	8,5	8,7	8,5	8,7	8,5
0.8	11,2	11,9	10,4	11,1	9,1	9,2	9,2	9,3

* only examples with stationary returns are considered.

Table A.10.: Sensitivity analysis on the performance of the normal SM & LUC heuristics and the ELSRG heuristic (with start solutions SM & LUC)

SM, LUC & ELSRG	Percentage Cost Error							
	Maximum				Percentage within 1%			
	normal		ELSRG		normal		ELSRG	
	SM	LUC	SM	LUC	SM	LUC	SM	LUC
All instances	82,5	98,4	82,5	98,4	23,2	15,7	24,8	17,1
Demand								
Stationary	76,3	66,0	76,3	66,0	27,2	21,1	29,3	22,9
-Small variance	61,4	62,4	61,4	62,4	30,1	27,4	32,6	29,9
-Large variance	76,3	66,0	76,3	66,0	24,4	14,7	25,9	15,9
Positive trend	82,5	68,4	82,5	68,4	23,3	21,9	25,2	23,4
-Small trend	82,5	68,4	82,5	68,4	22,0	21,4	23,8	22,9
-Large trend	39,3	39,1	39,3	39,1	24,7	22,3	26,5	23,9
Negative trend	60,3	64,4	60,3	64,4	15,7	8,2	16,6	9,5
-Small trend	60,3	64,4	60,3	64,4	16,5	8,2	17,3	9,5
-Large trend	54,7	61,6	54,7	61,6	14,8	8,2	15,9	9,5
Seasonal	79,9	98,4	79,9	98,4	24,8	13,6	26,5	14,9
-Small amplitude	79,9	85,0	79,9	85,0	28,3	19,5	30,4	21,3
-Large amplitude	69,1	98,4	68,7	98,4	21,4	7,7	22,6	8,5
Returns								
Stationary	53,5	50,6	53,5	50,6	23,5	15,9	25,6	17,8
-Small variance	48,9	48,3	48,9	48,3	23,7	15,6	25,8	17,7
-Large variance	53,5	50,6	53,5	50,6	23,4	16,2	25,3	17,9
Positive trend	76,3	98,4	76,3	98,4	19,5	13,5	20,5	14,2
-Small trend	76,3	90,4	76,3	90,4	22,5	16,0	23,5	16,6
-Large trend	70,9	98,4	70,9	98,4	16,6	11,1	17,6	11,9
Negative trend	82,5	68,6	82,5	68,6	25,9	16,5	27,3	17,7
-Small trend	66,1	63,1	66,1	63,1	27,5	18,9	29,2	20,4
-Large trend	82,5	68,6	82,5	68,6	24,3	14,2	25,3	15,0
Seasonal	57,1	53,1	56,9	53,1	23,4	16,1	25,1	17,7
-Small amplitude	56,5	47,3	56,5	47,3	23,5	16,0	25,3	17,7
-Large amplitude	57,1	53,1	56,9	53,1	23,2	16,3	24,9	17,8
Return ratio*								
0.3	26,8	29,9	26,8	29,9	30,8	22,5	33,1	24,3
0.5	44,0	42,9	44,0	42,9	22,9	15,0	25,1	16,8
0.7	53,5	50,6	53,5	50,6	16,9	10,3	18,5	12,3
Man. set-up cost K^m								
200.0	82,5	68,6	82,5	68,6	31,2	20,3	33,7	22,0
500.0	53,6	56,3	53,6	56,3	23,0	16,0	25,0	18,2
2000.0	76,3	98,4	76,3	98,4	15,4	10,7	15,7	11,1
Reman. set-up cost K^r								
200.0	76,3	98,4	76,3	98,4	9,3	7,6	12,4	10,3
500.0	82,5	68,6	82,5	68,6	11,8	9,9	13,4	11,4
2000.0	48,4	67,9	48,4	67,9	48,4	29,6	48,5	29,7
Returns hold. cost h^r								
0.2	69,1	57,8	68,7	57,8	35,2	27,1	35,7	27,5
0.5	82,5	75,1	82,5	75,1	21,6	12,6	23,8	14,7
0.8	79,9	98,4	79,9	98,4	12,7	7,3	14,9	9,1

* only examples with stationary returns are considered.

Table A.11.: Sensitivity analysis on the performance of the normal SM & LUC heuristics and the ELSRG heuristic (with start solutions SM & LUC)

Exchange *(start solution: SM)*	Percentage Cost Error									
	Average					Standard deviation				
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 2**	**R = 12**	**R = 6**	**R = 4**	**R = 3**	**R = 2**
All instances	3,8	3,1	2,6	2,0	1,3	4,4	3,7	3,3	2,8	2,4
Demand										
Stationary	3,7	3,1	2,7	2,1	1,4	4,7	4,0	3,6	3,0	2,8
-Small variance	3,4	2,9	2,5	2,0	1,3	4,3	3,7	3,4	2,8	2,6
-Large variance	4,1	3,3	2,9	2,2	1,5	5,0	4,2	3,8	3,2	3,0
Positive trend	3,3	2,5	2,1	1,6	0,9	3,7	3,0	2,6	2,1	1,7
-Small trend	3,3	2,6	2,1	1,6	1,0	3,9	3,2	2,7	2,2	2,0
-Large trend	3,3	2,5	2,1	1,5	0,9	3,6	2,9	2,5	2,0	1,3
Negative trend	4,4	3,3	2,7	2,2	1,2	4,4	3,6	3,2	2,8	2,0
-Small trend	4,5	3,6	2,9	2,3	1,4	4,5	3,6	3,3	2,9	2,1
-Large trend	4,2	3,1	2,6	2,0	1,1	4,4	3,5	3,1	2,7	1,8
Seasonal	3,9	3,2	2,7	2,2	1,4	4,6	3,9	3,4	3,0	2,6
-Small amplitude	3,7	3,1	2,7	2,1	1,4	4,5	3,8	3,4	3,0	2,6
-Large amplitude	4,1	3,4	2,7	2,2	1,5	4,7	4,0	3,4	3,1	2,6
Returns										
Stationary	3,6	2,9	2,4	1,9	1,2	4,1	3,4	3,0	2,5	2,1
-Small variance	3,6	2,9	2,4	1,9	1,2	4,0	3,3	3,0	2,4	2,1
-Large variance	3,6	2,9	2,4	1,9	1,2	4,2	3,4	3,0	2,6	2,2
Positive trend	4,8	3,8	3,4	2,6	1,6	5,5	4,7	4,2	3,8	3,1
-Small trend	4,7	3,7	3,2	2,7	1,6	5,6	4,8	4,2	3,8	3,4
-Large trend	4,9	3,8	3,5	2,6	1,6	5,4	4,5	4,1	3,8	2,8
Negative trend	3,9	3,0	2,5	2,0	1,3	4,6	3,7	3,2	2,9	2,3
-Small trend	3,6	3,0	2,3	1,9	1,2	4,3	3,8	3,1	2,9	2,3
-Large trend	4,2	3,0	2,6	2,0	1,3	4,8	3,7	3,3	2,9	2,3
Seasonal	3,6	2,9	2,4	1,9	1,2	3,9	3,3	2,9	2,4	2,1
-Small amplitude	3,6	2,9	2,4	1,9	1,2	3,9	3,3	2,9	2,4	2,1
-Large amplitude	3,5	2,9	2,4	1,9	1,2	3,9	3,4	2,9	2,5	2,1
Return ratio*										
0.3	2,7	2,1	1,7	1,5	0,7	3,1	2,5	2,2	1,8	1,1
0.5	3,6	2,8	2,3	1,9	1,1	4,3	3,3	2,9	2,5	1,8
0.7	4,5	3,8	3,2	2,4	1,7	4,6	4,0	3,5	2,9	2,9
Man. set-up cost K^m										
200.0	3,3	2,5	2,0	1,6	0,9	4,0	3,3	2,8	2,3	1,8
500.0	3,6	2,8	2,2	1,9	1,1	4,1	3,3	2,8	2,6	2,0
2000.0	4,7	3,9	3,5	2,6	1,8	5,0	4,3	3,8	3,4	3,1
Reman. set-up cost K^r										
200.0	4,6	3,6	3,1	2,4	1,5	4,6	3,9	3,5	3,0	2,6
500.0	4,9	4,0	3,1	2,6	1,7	4,5	3,9	3,3	2,9	2,7
2000.0	2,0	1,6	1,5	1,2	0,7	3,5	2,8	2,7	2,4	1,6
Returns hold. cost h^r										
0.2	2,3	1,9	1,7	1,3	0,8	3,1	2,7	2,5	2,2	1,5
0.5	3,9	3,3	2,8	2,2	1,4	4,3	3,8	3,3	2,9	2,5
0.8	5,3	4,0	3,3	2,6	1,6	5,1	4,2	3,7	3,2	2,9

* only examples with stationary returns are considered.

Table A.12.: Sensitivity analysis on the performance of the Exchange heuristic (SM is used as start solution), with different numbers of disjoint sets (R)

Exchange *(start solution: SM)*	Percentage Cost Error									
	Maximum					Percentage within 1%				
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 2**	**R = 12**	**R = 6**	**R = 4**	**R = 3**	**R = 2**
All instances	53,5	47,9	41,2	39,0	53,9	35,1	39,5	44,4	50,4	65,7
Demand										
Stationary	53,5	41,4	36,7	34,1	45,5	40,9	43,8	48,0	52,6	67,0
-Small variance	36,6	34,3	31,9	24,2	37,4	43,2	45,6	49,9	53,7	69,2
-Large variance	53,5	41,4	36,7	34,1	45,5	38,6	42,1	46,2	51,5	64,8
Positive trend	48,0	47,9	41,2	27,4	53,9	35,9	41,3	47,1	53,8	69,3
-Small trend	48,0	47,9	41,2	27,4	53,9	35,5	39,8	47,6	53,8	71,0
-Large trend	26,9	26,3	19,0	26,3	12,6	36,3	42,9	46,7	53,8	67,6
Negative trend	38,8	28,4	30,5	28,4	21,4	28,3	34,2	39,6	46,8	63,5
-Small trend	38,8	28,4	24,0	28,4	19,4	28,0	31,8	39,5	45,2	60,3
-Large trend	33,6	27,5	30,5	25,6	21,4	28,7	36,7	39,7	48,4	66,7
Seasonal	50,6	38,7	39,0	39,0	41,4	35,1	39,2	43,5	49,4	64,4
-Small amplitude	50,6	38,7	37,5	36,2	41,4	39,0	42,2	45,4	51,2	65,3
-Large amplitude	38,8	37,8	39,0	39,0	39,0	31,3	36,1	41,7	47,7	63,5
Returns										
Stationary	53,5	40,2	33,2	29,3	33,1	36,2	40,2	45,0	50,4	65,6
-Small variance	33,2	33,2	33,2	23,4	33,1	36,1	40,2	44,8	50,3	65,6
-Large variance	53,5	40,2	28,6	29,3	28,6	36,4	40,1	45,2	50,4	65,6
Positive trend	50,6	47,9	41,2	36,2	53,9	32,3	36,6	40,7	48,5	63,3
-Small trend	50,6	37,5	32,5	26,7	41,4	34,0	38,0	43,0	48,1	64,3
-Large trend	48,0	47,9	41,2	36,2	53,9	30,7	35,3	38,3	49,0	62,2
Negative trend	41,4	41,4	39,0	39,0	45,5	36,5	42,3	47,0	53,5	67,3
-Small trend	41,4	41,4	39,0	39,0	45,5	38,0	42,1	47,8	54,1	67,4
-Large trend	39,8	39,8	32,6	31,5	23,3	35,0	42,5	46,2	52,9	67,1
Seasonal	45,9	37,9	37,5	23,1	37,5	34,9	39,1	44,5	49,8	66,2
-Small amplitude	45,9	37,9	37,5	23,1	37,5	34,9	39,0	44,3	49,9	66,0
-Large amplitude	37,9	35,9	31,7	22,4	31,7	34,8	39,2	44,6	49,7	66,4
Return ratio*										
0.3	17,8	16,0	14,9	9,2	6,7	43,0	47,1	53,9	55,5	73,9
0.5	28,2	23,4	21,7	19,9	12,3	38,0	41,9	45,9	51,6	65,1
0.7	53,5	40,2	33,2	29,3	33,1	27,7	31,5	35,2	44,1	57,9
Man. set-up cost K^m										
200.0	39,8	39,8	32,6	31,5	23,3	42,2	47,0	52,9	56,9	70,8
500.0	38,7	38,7	36,2	36,2	28,3	35,5	39,4	46,0	50,3	67,9
2000.0	53,5	47,9	41,2	39,0	53,9	27,6	32,2	34,2	44,0	58,4
Reman. set-up cost K^r										
200.0	53,5	47,9	41,2	39,0	53,9	21,4	27,2	32,5	40,9	58,7
500.0	41,4	41,4	36,7	34,1	32,1	22,1	26,1	33,2	38,7	56,2
2000.0	33,6	21,6	20,3	19,9	14,1	61,8	65,3	67,4	71,6	82,2
Returns hold. cost h^r										
0.2	41,4	41,4	36,7	34,1	24,0	47,4	51,9	56,0	61,7	74,5
0.5	39,8	39,8	33,2	32,6	33,1	33,7	37,5	41,8	47,9	62,5
0.8	53,5	47,9	41,2	39,0	53,9	24,2	29,2	35,4	41,7	60,0

* only examples with stationary returns are considered.

Table A.13.: Sensitivity analysis on the performance of the Exchange heuristic (SM is used as start solution), with different numbers of disjoint sets (R)

Exchange (start solution: LUC)	Percentage Cost Error									
	Average					Standard deviation				
	R = 12	R = 6	R = 4	R = 3	R = 2	R = 12	R = 6	R = 4	R = 3	R = 2
All instances	4,4	3,5	2,9	2,2	1,5	4,6	3,9	3,5	2,9	2,7
Demand										
Stationary	4,4	3,5	3,1	2,3	1,5	4,8	4,1	3,7	3,1	2,7
-Small variance	3,8	3,1	2,6	2,1	1,4	4,6	3,9	3,5	2,9	2,8
-Large variance	4,9	3,9	3,5	2,5	1,6	5,1	4,2	3,9	3,3	2,7
Positive trend	3,7	2,9	2,3	1,9	1,2	4,0	3,4	3,0	2,5	2,2
-Small trend	3,9	3,0	2,5	2,0	1,4	4,3	3,8	3,4	2,8	2,8
-Large trend	3,6	2,8	2,1	1,7	1,0	3,6	2,9	2,5	2,1	1,4
Negative trend	4,5	3,4	2,8	2,2	1,2	4,4	3,6	3,2	2,8	2,2
-Small trend	4,5	3,6	2,8	2,3	1,3	4,1	3,5	3,0	2,8	2,0
-Large trend	4,4	3,2	2,8	2,1	1,2	4,8	3,6	3,3	2,7	2,3
Seasonal	4,6	3,8	3,2	2,3	1,9	4,8	4,2	3,7	3,1	3,0
-Small amplitude	4,1	3,4	2,9	2,2	1,6	4,6	4,1	3,6	3,0	3,1
-Large amplitude	5,1	4,2	3,4	2,5	2,1	4,9	4,3	3,7	3,2	3,0
Returns										
Stationary	4,0	3,2	2,7	2,0	1,4	4,0	3,4	3,1	2,4	2,3
-Small variance	4,0	3,2	2,7	2,0	1,4	4,0	3,3	3,0	2,4	2,3
-Large variance	4,0	3,3	2,7	2,0	1,5	4,1	3,5	3,1	2,5	2,4
Positive trend	5,3	4,1	3,6	2,8	2,0	5,8	4,9	4,4	3,9	3,7
-Small trend	5,1	4,0	3,3	2,7	1,9	5,8	4,9	4,1	3,8	3,7
-Large trend	5,4	4,2	3,9	2,9	2,1	5,8	5,0	4,6	4,0	3,8
Negative trend	4,7	3,6	3,0	2,1	1,5	5,1	4,2	3,6	3,1	2,6
-Small trend	4,4	3,6	2,8	2,1	1,5	5,0	4,3	3,6	3,1	2,6
-Large trend	4,9	3,5	3,1	2,2	1,6	5,1	4,1	3,6	3,1	2,6
Seasonal	4,0	3,3	2,7	2,1	1,4	3,9	3,5	3,1	2,5	2,3
-Small amplitude	4,0	3,3	2,7	2,1	1,3	3,9	3,4	3,1	2,5	2,2
-Large amplitude	4,0	3,3	2,7	2,1	1,4	4,0	3,6	3,0	2,6	2,3
Return ratio*										
0.3	3,2	2,5	2,2	1,6	1,0	3,3	2,8	2,5	1,9	1,4
0.5	3,9	3,1	2,5	1,9	1,4	4,0	3,2	2,8	2,3	2,0
0.7	4,8	4,1	3,4	2,5	2,0	4,5	4,0	3,6	2,9	3,1
Man. set-up cost K^m										
200.0	4,1	3,1	2,5	1,9	1,2	4,5	3,7	3,3	2,5	2,2
500.0	3,9	3,1	2,6	2,0	1,3	4,1	3,4	3,0	2,8	2,1
2000.0	5,1	4,3	3,7	2,7	2,1	5,0	4,4	3,9	3,3	3,4
Reman. set-up cost K^r										
200.0	5,0	3,8	3,3	2,4	1,7	4,8	4,0	3,7	3,1	3,0
500.0	5,3	4,4	3,5	2,7	1,8	4,9	4,2	3,7	3,0	2,8
2000.0	2,8	2,2	2,0	1,5	1,1	3,6	3,1	2,8	2,4	2,1
Returns hold. cost h^r										
0.2	2,8	2,3	2,0	1,5	1,0	3,3	2,8	2,6	2,2	1,9
0.5	4,5	3,7	3,1	2,4	1,7	4,5	4,0	3,5	2,9	2,8
0.8	5,8	4,4	3,6	2,8	1,9	5,3	4,5	3,9	3,3	3,1

* only examples with stationary returns are considered.

Table A.14.: Sensitivity analysis on the performance of the Exchange heuristic (LUC is used as start solution), with different numbers of disjoint sets (R)

Exchange *(start solution: LUC)*	Percentage Cost Error									
	Maximum					Percentage within 1%				
	R = 12	**R = 6**	**R = 4**	**R = 3**	**R = 2**	**R = 12**	**R = 6**	**R = 4**	**R = 3**	**R = 2**
All instances	58,3	50,7	45,2	40,0	53,9	27,1	32,3	37,4	46,9	60,4
Demand										
Stationary	42,2	41,8	40,3	39,8	35,9	33,2	37,5	41,0	48,8	63,7
-Small variance	36,8	34,6	34,0	39,3	35,9	39,0	41,4	47,3	51,4	66,4
-Large variance	42,2	41,8	40,3	39,8	28,1	27,4	33,5	34,6	46,1	61,0
Positive trend	50,7	50,7	45,2	40,0	53,9	31,4	35,5	41,7	48,1	63,3
-Small trend	50,7	50,7	45,2	40,0	53,9	32,4	36,5	42,0	47,5	63,5
-Large trend	25,5	21,8	20,5	19,1	16,1	30,5	34,5	41,5	48,6	63,1
Negative trend	38,8	34,5	29,7	23,6	36,8	22,7	28,8	34,5	45,3	63,1
-Small trend	38,8	23,2	23,0	20,9	18,2	18,4	24,1	32,6	43,3	60,5
-Large trend	32,6	34,5	29,7	23,6	36,8	27,0	33,4	36,3	47,3	65,6
Seasonal	58,3	42,6	39,7	37,2	51,7	24,1	30,0	34,8	46,2	55,9
-Small amplitude	58,3	42,5	39,7	37,2	51,7	31,0	36,3	40,0	49,2	61,7
-Large amplitude	44,8	42,6	38,4	33,5	33,6	17,2	23,7	29,6	43,3	50,0
Returns										
Stationary	33,9	28,0	29,3	28,0	28,0	28,4	33,0	37,8	47,2	59,9
-Small variance	29,3	28,0	29,3	28,0	28,0	28,1	32,6	37,3	47,2	60,2
-Large variance	33,9	26,6	25,2	19,8	27,8	28,8	33,4	38,3	47,3	59,6
Positive trend	58,3	50,7	45,2	40,0	53,9	24,0	29,6	34,2	44,0	57,8
-Small trend	58,3	42,5	37,6	39,3	51,7	25,2	30,8	37,0	45,3	59,3
-Large trend	50,7	50,7	45,2	40,0	53,9	22,7	28,4	31,5	42,7	56,2
Negative trend	44,8	42,6	39,7	34,1	26,6	27,3	34,1	39,5	50,3	61,4
-Small trend	44,8	42,6	39,7	34,1	26,6	28,9	34,2	40,0	50,6	61,9
-Large trend	38,8	33,1	31,5	29,1	22,5	25,6	33,9	39,1	49,9	60,9
Seasonal	43,8	39,9	39,5	22,1	39,5	27,6	32,4	37,5	46,5	61,5
-Small amplitude	30,7	30,7	25,5	22,1	25,3	27,5	32,3	37,0	46,1	61,0
-Large amplitude	43,8	39,9	39,5	21,3	39,5	27,7	32,5	38,0	46,8	62,0
Return ratio*										
0.3	20,5	17,1	17,8	12,1	8,7	33,9	38,9	43,2	52,0	64,9
0.5	27,8	19,9	22,7	19,0	19,1	29,6	33,6	38,2	47,9	59,9
0.7	33,9	28,0	29,3	28,0	28,0	21,8	26,5	32,1	41,8	54,8
Man. set-up cost K^m										
200.0	44,8	42,6	39,7	29,8	28,9	30,4	37,4	42,9	50,1	66,1
500.0	40,5	40,5	40,3	39,8	29,7	28,5	33,6	38,6	48,7	62,7
2000.0	58,3	50,7	45,2	40,0	53,9	22,4	26,0	30,6	42,0	52,4
Reman. set-up cost K^r										
200.0	58,3	50,7	45,2	40,0	53,9	18,1	24,3	30,4	39,2	56,5
500.0	44,8	42,6	39,7	34,1	34,0	20,6	23,4	30,0	37,8	53,9
2000.0	31,4	23,8	23,8	23,8	18,0	42,6	49,4	51,7	63,7	70,7
Returns hold. cost h^r										
0.2	41,4	41,4	36,7	34,1	29,7	38,9	44,2	47,2	57,8	69,2
0.5	40,6	34,5	31,2	31,4	36,8	24,3	29,3	34,6	43,6	57,4
0.8	58,3	50,7	45,2	40,0	53,9	18,1	23,5	30,3	39,3	54,6

* only examples with stationary returns are considered.

Table A.15.: Sensitivity analysis on the performance of the Exchange heuristic (LUC is used as start solution), with different numbers of disjoint sets (R)

B. Number of fixed elements

- normal formulation, see table B.1
- FAL formulation, see table B.2

(normal formulation)	Number of fixed elements					
	Average			Standard deviation		
	y^r	y^m	$y^r + y^m$	y^r	y^m	$y^r + y^m$
All instances	2,6	3,2	5,8	2,4	2,9	4,0
Demand						
Stationary	2,8	3,7	6,4	2,5	3,2	4,2
-Small variance	2,7	3,5	6,3	2,5	3,2	4,2
-Large variance	2,8	3,8	6,6	2,5	3,2	4,1
Positive trend	2,1	1,8	3,9	2,4	2,2	3,4
-Small trend	2,2	2,2	4,4	2,5	2,4	3,5
-Large trend	2,0	1,4	3,4	2,4	1,9	3,2
Negative trend	2,6	2,4	5,0	2,3	1,7	3,0
-Small trend	2,5	2,6	5,1	2,3	2,0	3,2
-Large trend	2,6	2,2	4,8	2,2	1,3	2,9
Seasonal	2,9	4,1	6,9	2,5	3,2	4,1
-Small amplitude	2,7	3,7	6,4	2,5	3,2	4,1
-Large amplitude	3,0	4,4	7,4	2,5	3,2	4,0
Returns						
Stationary	2,6	2,2	4,9	2,5	1,6	3,3
-Small variance	2,6	2,2	4,8	2,5	1,6	3,3
-Large variance	2,6	2,3	4,9	2,5	1,6	3,3
Positive trend	2,5	3,9	6,4	2,2	3,1	3,9
-Small trend	2,5	3,4	5,9	2,3	2,7	3,5
-Large trend	2,5	4,4	6,9	2,1	3,4	4,1
Negative trend	2,7	5,7	8,4	2,3	4,3	4,9
-Small trend	2,5	5,1	7,6	2,4	4,2	4,7
-Large trend	2,8	6,4	9,2	2,3	4,2	5,1
Seasonal	2,6	2,3	5,0	2,5	1,7	3,3
-Small amplitude	2,6	2,3	4,9	2,5	1,7	3,3
-Large amplitude	2,7	2,4	5,0	2,5	1,7	3,3
Return ratio*						
0.3	3,0	1,9	4,8	2,8	1,2	3,4
0.5	2,7	2,0	4,7	2,5	1,4	3,2
0.7	2,3	2,7	5,0	2,2	2,0	3,1
Man. set-up cost K^m						
200.0	2,7	2,3	5,0	2,6	2,8	4,0
500.0	2,4	2,8	5,1	2,4	2,9	4,0
2000.0	2,8	4,6	7,4	2,4	2,7	3,4
Reman. set-up cost K^r						
200.0	1,7	3,2	4,8	2,3	3,3	4,4
500.0	1,9	3,3	5,2	1,9	3,0	3,8
2000.0	4,3	3,1	7,4	2,3	2,4	3,1
Returns hold. cost h^r						
0.2	4,0	3,3	7,3	3,0	2,9	4,1
0.5	2,3	3,2	5,5	1,8	2,9	3,7
0.8	1,6	3,2	4,7	1,5	3,0	3,8

* only examples with stationary returns are considered.

Table B.1.: Number of fixed elements after solving the LP relaxation of the LP-and-Fix heuristic (normal formulation)

(FAL formulation)	Number of fixed elements					
	Average			Standard deviation		
	y^r	y^m	$y^r + y^m$	y^r	y^m	$y^r + y^m$
All instances	2,9	6,9	9,8	3,5	3,1	5,6
Demand						
Stationary	3,3	7,8	11,1	3,7	2,8	5,4
-Small variance	3,2	7,8	11,0	3,7	2,8	5,4
-Large variance	3,4	7,8	11,2	3,7	2,8	5,4
Positive trend	2,2	5,4	7,6	3,3	3,0	5,5
-Small trend	2,3	5,9	8,2	3,3	3,0	5,4
-Large trend	2,0	5,0	7,0	3,2	2,9	5,5
Negative trend	2,4	5,7	8,2	3,1	2,7	4,7
-Small trend	2,5	6,5	9,0	3,2	2,6	4,7
-Large trend	2,4	5,0	7,4	2,9	2,7	4,6
Seasonal	3,4	7,7	11,1	3,6	3,0	5,5
-Small amplitude	3,2	7,6	10,8	3,6	3,0	5,6
-Large amplitude	3,6	7,8	11,4	3,6	3,0	5,4
Returns						
Stationary	2,7	6,4	9,0	3,4	2,9	5,4
-Small variance	2,7	6,3	9,0	3,5	2,9	5,4
-Large variance	2,7	6,4	9,1	3,4	2,9	5,4
Positive trend	3,0	7,6	10,7	3,3	3,0	5,2
-Small trend	2,9	7,3	10,2	3,3	3,0	5,2
-Large trend	3,2	7,9	11,1	3,3	3,0	5,1
Negative trend	3,7	7,8	11,5	3,7	3,4	6,2
-Small trend	3,0	7,5	10,5	3,4	3,3	5,6
-Large trend	4,3	8,2	12,4	3,9	3,4	6,5
Seasonal	2,7	6,4	9,1	3,4	2,9	5,4
-Small amplitude	2,7	6,4	9,1	3,4	2,9	5,4
-Large amplitude	2,7	6,4	9,1	3,4	2,9	5,3
Return ratio*						
0.3	2,7	6,2	9,0	3,4	2,9	5,4
0.5	2,7	6,3	8,9	3,4	2,9	5,4
0.7	2,6	6,6	9,2	3,5	2,9	5,3
Man. set-up cost K^m						
200.0	3,8	5,1	9,0	3,9	3,4	6,7
500.0	3,0	6,6	9,6	3,7	2,7	5,8
2000.0	1,9	8,9	10,9	2,5	1,7	3,5
Reman. set-up cost K^r						
200.0	0,7	6,2	6,9	1,8	3,1	4,2
500.0	1,8	6,5	8,3	1,9	2,9	3,5
2000.0	6,3	8,0	14,3	3,5	3,0	5,7
Returns hold. cost h^r						
0.2	4,8	8,0	12,7	4,6	3,2	7,0
0.5	2,4	6,4	8,8	2,7	2,9	4,2
0.8	1,6	6,3	7,9	1,8	3,0	3,6

* only examples with stationary returns are considered.

Table B.2.: Number of fixed elements after solving the LP relaxation of the LP-and-Fix heuristic (FAL formulation)

C. Changes

Changes in the configuration of (re)manufacturing periods after applying the Exchange heuristic

- start solution SM, see figure C.1
- start solution LUC, see figure C.2

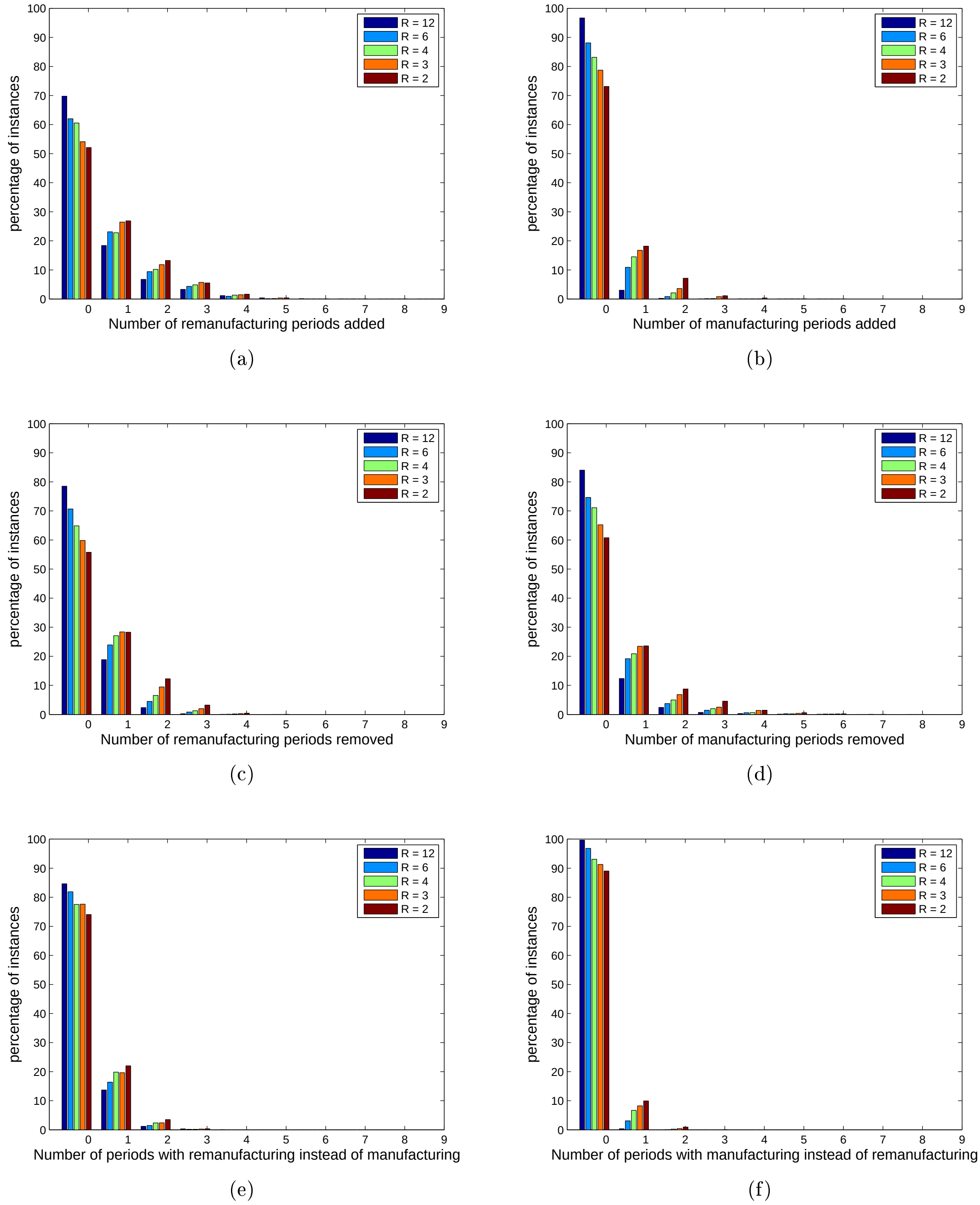

Figure C.1.: Changes in the configuration of (re)manufacturing periods after applying the Exchange heuristic to SM

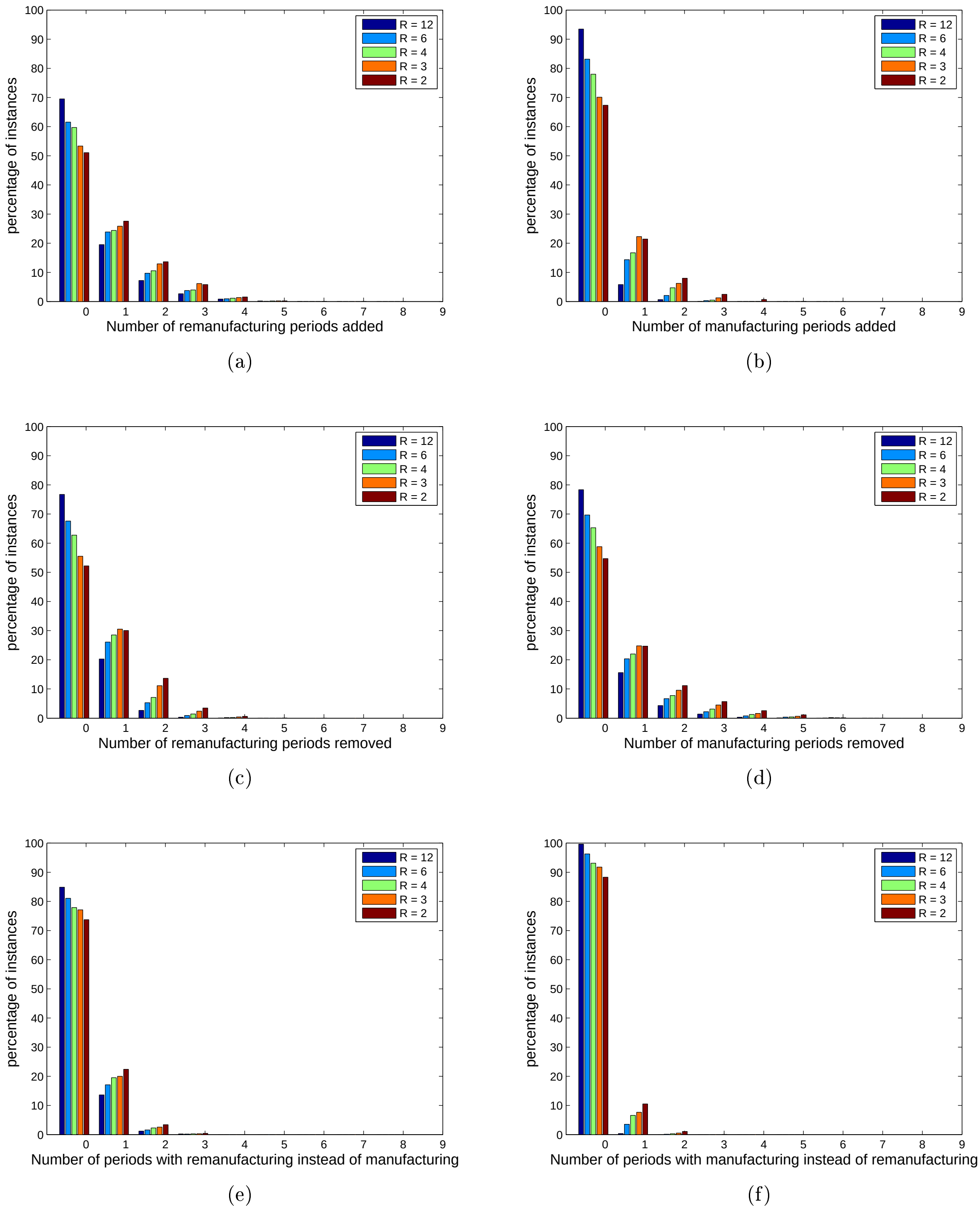

Figure C.2.: Changes in the configuration of (re)manufacturing periods after applying the Exchange heuristic to LUC