Master Thesis

ESTIMATING BETA IN BETTING AGAINST BETA STRATEGY

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ABSTRACT

This study examines the relationship between the low-beta and expected stock returns based on two different beta estimation techniques: I) novel procedure by Frazzini and Pedersen (2014) and II) standard method by Novy-Marx and Velikov (2018). Both estimation techniques are compared to an actual beta. Results indicate that the novel procedure is a better predictor than the standard method since it has a lower MSE. Moreover, when using the novel procedure betas and actual betas, the BAB factor becomes statistically significant, indicated by a statistically significant CAPM alpha, three-factor alpha, four-factor alpha, and five-factor alpha. Contrary, no evidence is found for a statistically significant alpha when using the standard beta estimation technique. Results of robustness checks indicate that using quintile portfolios or control for market volatility increases the magnitude of the results Consequently, there is evidence to believe that the low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, when controlling for market volatility.

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SECTION 1: INTRODUCTION

One of the most prominent models considered in finance is the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1969). The CAPM describes the relationship between risk and return, where investors holding riskier assets are compensated with higher returns. The main idea of CAPM is that not all risks should affect asset prices. Specifically, diversification leads to a reduction in risk without sacrificing any expected return. The CAPM makes a distinction between idiosyncratic and systematic risk. Idiosyncratic risk can be diversified away, where systematic risk cannot be. Consequently, systematic risk is resembling market risk and is denoted with beta. Although the CAPM is the foundation of asset pricing models, it has its implications. Empirical studies have found that the relation between beta and return is flatter than CAPM suggests. Haugen and Heins (1975) found that portfolios with lower variance produce higher returns, while the CAPM state that higher volatility should realize a higher return, constituting the low volatility anomaly. Other studies also found a flatter relationship when adjusting for size and value effects (Fama and French (1992). Expanding the CAPM with size risk and value risk in addition to the market risk of the CAPM, resulting in the three-factor model. Later, they expanded their model by constructing a five-factor model (Fama & French, 2015). Frazzini and Pedersen (2014) constituted the betting against beta factor. Their main finding is that low beta assets outperform high beta assets. They construct a portfolio that goes long in low beta assets and short high beta assets, achieving an excess return of 0.73% per month. They argue that constraints on leverage are a cause of overinvestment in high-beta stocks, which leads to an increase in price, consequently leading to a lower return.

Empirical studies have shown that low beta assets outperform high beta stocks. However, some studies have criticized the paper of Frazzini and Pedersen (2014). Novy-Marx and Velikov (2018) criticize the non-standard method for estimating the beta, therefore not yielding market betas. According to Novy-Marx and Velikov, the Frazzini and Pedersen (FP betas) are highly predictable by market volatility and are therefore biased. Besides, Novy-Marx and Velicov believe that any variable correlated with market volatility will predict beta compression when betas are estimated using a non-standard method. However, Novy-Marx and Velikov do not examine whether using the novel procedure controlled for market volatility provides different results than using a standard method to calculate the ex-ante beta. Furthermore, the research of Novy-Marx and Velikov does not examine whether the novel procedure to calculate beta is just a better predictor of beta than the standard method. This research examines whether the novel method to calculate beta is a better predictor compared to the standard method, whether it matters in performance when calculating beta differently and what happens when controlling for market volatility when using the novel procedure. Consequently, consider whether Novy-Marx and Velikov (2018) criticism is justified. This results in the following research question:

"Which beta estimation technique is the best beta predictor concerning the betting against beta factor?"

First, this research compares beta estimations by comparing the predicted betas with the actual betas for both estimation techniques: novel procedure and standard method. Then, this study compares the performances of the BAB factor to compare whether the differences in beta estimation matter. Novy-Marx and Velikov (2018) state that beta is always compressed to one since beta correlates to market volatility. Therefore, when using the novel procedure, this study controls for market volatility as a robustness check.

The importance of the beta estimation in asset pricing makes this research economically relevant (Black, 1993). In addition, to the best of my knowledge, no other research exists that examines the difference in the performance of the BAB factor when using different beta estimations. In addition, when significant results are obtained when controlling for market volatility, it would question the statements made in Novy-Marx and Velikov (2018) research. Thus, the results of this research contribute to a better understanding and composition of a trading strategy.

First, this research provides evidence that the novel procedure is a better predictor of the actual beta, since it has a lower mean squared error than the standard method to estimate beta. The actual daily beta is based on the regression coefficient of the market excess returns of that month on the market returns. The average of those beta estimations per day in one month is taken as the actual beta of that month. It is relevant to look at this actual beta, since researchers care about the realized covariance of their assets and not the statistics of the past. After that, this study constructed a betting against beta (BAB) factor to find evidence for the low-beta anomaly. When using the novel procedure betas and actual betas, the BAB factor becomes statistically significant, indicated by a statistically significant CAPM alpha, three-factor alpha, four-factor alpha, and five-factor alpha. Contrary, no evidence is found for a statistical significant alpha when using the standard beta estimation technique. It seems that investors indeed seem to care more about the actual betas since the BAB factor has higher and statistical significant alphas. Results seem to solve a current debate among researchers, concerning the beta estimation in the BAB-strategy. Finally, this study performs two robustness checks: quintile portfolios and controlling for market volatility. Both robustness checks indicate that the factor is somewhat robust, but all factor models' alphas in all panels increase. Consequently, this research proves that the low-beta anomaly is economically and statistically significant when controlling for market volatility.

The remainder of this research is structured as follows. Section 2 discusses relevant existing literature concerning the low-beta anomaly and clarifies the study's expectations. Section 3 describes the data along with its descriptive statistics. The next section describes the methodology, including the empirical models used in this research. Section 5 provides the estimated results and an interpretation of the tested hypothesis. At last, Section 6 reviews the research and discusses the conclusion.

SECTION 2: LITERATURE REVIEW

This section discusses the existing literature that is relevant for this study. The first paragraph reviews the relationship between risk and return, historical asset pricing models and their main limitations. After that, the relevant existing literature concerning the low-beta anomaly and the corresponding betting against beta factor. The last section discusses the estimation of beta shortly.

2.1 ASSET PRICING MODELS

In general, the CAPM is based on Markowitz's Modern Portfolio Theory (MPT) (Markowitz, 1952). This theory states a positive relationship between risk and return, where higher risk is compensated with a higher return. According to this theory, adding more assets to the portfolio will automatically result in lower volatility, since idiosyncratic volatility is decreasing. A rational investor will maximize its return and minimize its variance. The CAPM, developed by Sharpe (1964) and Lintner (1965), distinguishes systematic risk and idiosyncratic risk. Idiosyncratic risk is the risk directly related to the firm and can be diversified away by holding the market portfolio. Systematic risk is the risk of the overall market and cannot be diversified away. Consequently, systematic risk is the most relevant risk measure. Since an investor can diversify its idiosyncratic risk away, only systematic risk is considered and denoted as beta. According to the above theory, the return of a stock is based on the stock's exposure to market volatility. The model builds on the idea that risk and return have a positive relationship, meaning that the expected return of a stock increases when its systematic risk also increases. The following equation describes the relationship:

$$E(R_i) = R_f + \beta[E(R_m) - R_f]$$

Where $E(R_i)$ is the expected return of stock *i*, $E(R_m)$ represents the market's expected return, and R_f denotes the risk-free rate of return. β_i denotes the systematic risk of stock i. A graphical representation of the CAPM results in de Security Market Line (SML), a linear relationship between systematic risk and return, representing the best possible performance given the level of risk (Boguth and Simutin, 2016). However, empirical evidence shows that the expected return of stocks does not equal the actual return of that stock (Bodie, Kane and Marcus, 2018). The systematic difference is denoted with alpha (α), which is the difference between the SML and the actual return. A positive alpha is captured when the actual return on an asset is higher than the expected return. When alpha is equal to zero, the market is considered efficient (Bodie et al., 2018). The unsystematic difference between the SML and actual return is denoted with an error term (ε). This results in the following equation:

$$E(R_i) = \alpha + R_f + \beta[E(R_m) - R_f] + \varepsilon$$

According to the above equation, a portfolio is sufficiently diversified when the error term equals zero. Moreover, many professional investors use α as a measure to evaluate the performance of the portfolio.

The CAPM model is developed as a theoretical model and depends on restrictive assumptions, such as no information asymmetry, rational investors, and no borrowing constraints. These assumptions do not seem to reflect the actual financial market. In addition, the above model does not incorporate taxes or any transaction costs. Many kinds of research have tested the CAPM, and violations were found of this fundamental relationship between risk and return. Moreover, many of them attempted to improve the model, trying to relax the strict assumptions of CAPM. For example, Black (1972) found that the SML is flatter than predicted by the CAPM, indicating that high beta assets earn less reward given the level of risk, while low beta assets earn relatively more for a given level of risk. Black's (1972) evidence is the basis of the paper of Frazinni and Pedersen (2014); *Betting Against Beta*. These newer models aim to explain asset returns better than the original CAPM, for example, multifactor models.

Fama and French (1992) proposed their three-factor model to explain the variation in the cross-section of stock returns. They combine and examine the following factors: market risk, size and book-to-market value. The size risk is conducted through 'small-minus-big' (SMB) and the value risk factor 'high-minus-low' (HML). Fama and French's model adjusts that small-caps and value stocks tend to outperform the market by adding those factors. One of the main findings of this paper is that the relationship between beta and expected returns does not hold for small firms. Moreover, when controlling for size, the relationship between beta and return proposed by the CAPM is flat. Consequently, their research findings indicate that their returns predictions are more accurate than the CAPM estimations. The three-factor model of Fama and French was a building stone for other researchers to find other statistically significant factors to include in this model, trying to achieve the model that predicts returns the most accurately. The formulas of the corresponding model are given in the methodology in Section 3.

Jagadeesh and Titman (1993) built upon this paper and conducted the momentum factor. This factor represents a portfolio that goes long in stocks that have performed well in the previous six months and short stocks that have performed poorly over the previous six months. Carhart (1997) added this momentum factor to the three-factor model of Fama and French (1992) on the performance of mutual funds, resulting in the four-factor model. Empirical evidence shows that

this portfolio generates significant excess returns. Again, corresponding formulas are given in the methodology in Section 3.
Another well-known multifactor model is the five-factor model of Fama and French (2015). They were inspired by the amount of evidence that three factors were not enough. The five-factor model of Fama and French build on their three-factor model by testing two new factors: investments (stocks of companies with high total asset growth have below-average returns) and profitability (stocks of companies with high operating profitability perform better). These two new factors are also known as quality factors (Robeco, 2018). Quality factors refer to the tendency of high-quality stocks, typically have more stable earning and higher margins. It forms a part of an investment strategy that selects high-quality companies.

All these factor models are designed to explain asset returns better than the original CAPM. Therefore, a better investment strategy can be applied. However, some anomalies are observed in the market, which cannot entirely be explained by these factor models.

2.2 BETTING AGAINST BETA

One of those anomalies is the low-beta anomaly. The low-beta anomaly is observed in capital markets but cannot be explained by the risk factors of standard asset pricing models. The theory says that higher risk should be compensated with higher return (Baker et al., 2011), reflected in the security market line. However, in reality, the opposite is observed. For example, Baker et al. (2011) show that a large sample of U.S. stocks of low volatility portfolios produces higher returns. They used U.S. stock data from 1968 till 2008. They rank the sample stocks according to ex-ante beta and assign them equally into quintile portfolios. Beta is estimated using the previous 60-month returns, the portfolios are monthly rebalanced, and transaction costs are ignored. Their results show that the low-beta portfolio outperforms the high-beta portfolio. However, when only the top 1000 stocks according to market capitalization are considered, the difference between performances of high- and low-beta portfolios is lower.

Frazzini and Pedersen (2014) extend this approach by constructing a *Betting against Beta* (BAB) factor. The BAB strategy goes long in low-beta assets and short in high-beta assets. The way the BAB factor is constructed is similar to previous factors like *small minus big* (SMB) or *high minus low* (HML). By leveraging the low-beta portfolio and deleveraging the high-beta portfolio, they achieve a market neutral position. In addition, they suggest an offsetting position in the risk-free rate to make their portfolio self-financing. As a result, investors should now capture the positive alpha in low-beta assets and negative alpha in high beta assets. They differ from the research of Baker et al. (2011) in their methodology. Frazzini and Pedersen (2014) use decile portfolios and do not weigh each stock equally, overweighing lower-ranked stocks and under-weighing higher ranked stocks, and calculate ex-ante beta using a rolling window of at least six months of daily returns for estimating volatility and a rolling window of three years of daily returns for the estimation of correlation. Their U.S. stock sample covers 1926 till 2012, while the other developed internal equity market sample covers data from 1989 till 2012. Notably, the U.S. stock market reveals a nearly monotonically decline in alpha across beta ranked portfolios, supporting the theory behind the low-beta anomaly. Also, the portfolios are rebalanced monthly, and they do not include any transaction costs. One of the main findings of this paper is that they find a positive alpha for the high-beta decile and a negative alpha for the low-beta decile using the CAPM, three-factor regression of Fama and French, and four-factor regression of Carhart. Frazzini and Pedersen (2014) examine this effect in the equity market and among different asset classes, where they find a significant positive excess return. Their results suggest that the low-beta anomaly is not only captured in the U.S. equity market but is present internationally and among different asset classes (Treasury Bonds, corporate bonds and futures).

After Frazzini and Pedersen's (2014) research, more researches examined the low-beta anomaly. The low-beta anomaly is found in the Indian market, indicating overweighting of riskier assets by leveraged constrained investors (Agarwalla, Jacob, Varma, & Vasudevan, 2014). Also, when using the Black-Sholes-Merton model to derive expressions for the model pricing errors under the standard CAPM, it results in a negative pricing error that becomes economically large as firm leverage increases (Buchner & Wagner, 2016), which is consistent with the theory of Frazzini and Pedersen (2014). Furthermore, the BAB strategy can be seen as combining three investable component portfolios, where BAB stems for the time-series component and has the modest portfolio turnover. The betting against correlation (BAC) only provides hedging benefits in market turndowns. Consequently, the time-series component remains the more robust source for profits of BA-type strategies (Han, 2019).

On the other hand, the research of Frazinni and Pedersen (2014) has its shortcomings. First, their research shows that the conditional beta for the high-minus-low beta portfolio negatively correlates with equity premium and positive with market volatility. Modelling the conditional market risk for beta-sorted portfolios using instrumental variables would lead to the conditional CAPM resolving the low-beta anomaly (Cederburg & O'Doherty, 2016). After that, Frazzini and Pedersen (2014) do not take into account any transaction costs. In addition, the investment universe contains an extensive range of stocks, while for investors, this may not be achievable. Novy-Marx and Velikov (2014) examined the BAB strategy, including transaction costs. When accounting for transaction costs, they find the BAB's profitability reduces by almost 60%, while it still has positive and significant returns. Novy-Marx and Velikov (2018) argue that this positive and significant result is earned by tilting toward profitability and investment. However, Novy-Marx and Velikov (2018) do not only criticize Frazinni and Pedersen (2014) for the exclusion of transaction cost. They also criticize them because of the non-standard procedures, specifically the use of novel beta estimation technique. They estimate beta by combining market correlations estimated using five years of overlapping three-day returns with volatilities estimated using one year of daily data, instead of just estimating beta as the slope of the CAPM regressions. According to Novy-Marx and Velikov (2018), FP betas are not yielding market betas, and the FP betas are highly predictable by market volatility and are therefore biased. Besides, Novy-Marx and Velicov believe that any variable correlated with market volatility will predict beta compression when betas are estimated using a non-standard method. However, Novy-Marx and Velikov do not examine whether this non-standard method may be a better predictor of beta than the standard way. After that, Novy-Marx and Velikov believe that the betas are biased because these are highly predictable by market volatility. However, they do not show results when controlling for market volatility.

2.3 THE ESTIMATION OF BETA

The beta of an asset measures how much risk investment will add to the portfolio that looks like the market. Thus, it indicates how volatile the price of an asset is compared to the overall stock market. When beta is greater than one, the asset is riskier compared to the market, and vice versa. In order to construct a BAB-strategy, assets need to be sorted into low-beta assets and high-beta assets according to their estimated ex-ante beta. When the ex-ante beta is estimated, it can be assigned to one of the beta-ranked sub-portfolios. Usually, the ex-ante beta is calculated by dividing covariance between an asset and market by the market variance. Researchers have different opinions on how to calculate those variables. Bodie et al. (2018) compute ex-ante beta suggesting 60 months of return data. Frazzini and Pedersen (2014) use daily return data if possible. If there is no daily return data available, they supplement their data with monthly data. When the sample size increases, the accuracy of the covariance estimations also increases. In addition, Frazzini and Pedersen (2014) calculate variance and correlation separately for two simple reasons: I.) They use one-year rolling returns to calculate volatility and at least three years of rolling returns to calculate covariance since volatility is changing faster than covariances and II.) They use one-day log returns for calculating volatility and three-days overlapping for calculating covariances. Moreover, Frazzini and Pedersen (2014) calculate beta concerning the market portfolio specific to an asset class. When this is not available, a global market portfolio is used.

After that, Frazzini and Pedersen (2014) use shrinkage factors to reduce outliers in the data. They follow Vasicek (1973) and Elton, Gruber, Brown and Goetzmann (2003), who shrink the time series estimate of beta toward the cross-sectional mean. According to Frazzini and Pedersen (2014), these shrinkage factors would not affect how securities are sorted since typical shrinkage does not change the ranks of the security's beta. However, it does influence the construction of BAB portfolios. To account for this effect, they focus on realized abnormal returns.

The way Frazzini and Pedersen (2014) calculate the ex-ante beta is a non-standard procedure (Novy-Marx & Velikov, 2018). The cross-section betas combine market betas with stock volatilities. Moreover, according to Novy-Marx and Velikov (2018), the betas are biased in highly predictable ways by market volatility in the time series, where market volatility explains 47% of its time-series variation. Consequently, the biased beta estimates drive the results of Frazzini and Pedersen (2014) on beta compression. Novy-Marx and Velikov (2018) state that every variable correlated with market volatility results in beta compression when beta is estimated using the novel procedure. However, the research of Novy-Marx and Velikov (2018) does not show results of the performance of BAB when controlling for market volatility. They only show some results that using a novel method has little impact on the performance of the BAB factor. At the same time, they do not substantiate their opinion that the correlation with market volatility impacts the results. Consequently, it can make one questioning their findings and judgements.

2.4 HYPOTHESIS DEVELOPMENT

As discussed in the introduction, this study will contribute to the existing literature in the following ways. First, this study uses more recent data to verify the existence of the low-beta anomaly in the U.S. stock market. Second, the low-beta anomaly is examined under different beta estimation techniques to give more insights into which beta estimation is a better predictor and will provide the best results. Finally, this study uses several robustness checks to determine whether the results change and indicate validity and reliability. The contribution to the existing literature is based on the lack of research on the low-beta anomaly concerning beta estimation. The beta estimation is a crucial subject since it is the basis of the betting against beta factor; it creates a need for investigation.

To provide an answer to the research question, the following hypotheses are formulated:

I) The novel procedure for estimating beta is a better predictor of beta than the standard method since the novel procedure has a lower MSE than the standard method.

II) The low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, using the novel procedure for estimating beta.

III) The low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, when controlling for market volatility.

SECTION 3: METHODOLOGY

This section provides the methodology used in this study. The first paragraph describes how the ex-ante betas are estimated, explaining two methods: the novel procedure (Frazzini & Pedersen, 2014) and the standard method (Novy-Marx & Velikov, 2018). Then, in order to compare whether the FP betas or NM betas do a better job, another beta is introduced called the 'actual beta'. After that, section 4.2 provides the construction of the BAB portfolio. Finally, the last section describes the risk models used to examine the performance of the BAB factor.

3.1 BETA ESTIMATION

In order to construct the betting against beta strategy, or in other words, the BAB factor, assets need to be sorted into low-beta assets and high-beta assets according to their estimated ex-ante beta. After that, the betas are assigned to one of the beta-ranked sub-portfolios. This section is divided into two paragraphs. The first section elaborates the novel procedure Frazzini and Pedersen (2014), used to calculate their ex-ante betas. After that, the second paragraph provides more information about the standard method to calculate betas, according to Novy-Marx and Velikov (2018), using the slope of CAPM. Furthermore, the third paragraph elaborates the actual beta, which is used to as the realized beta to compare the previous two beta estimation techniques.

3.1.1 FRAZZINI AND PEDERSEN (2014)

The first method is the novel procedure that Frazzini and Pedersen (2014) use in their research to estimate beta. In the literature review (Section 2), this novel procedure is briefly explained. This research follows the estimation technique of their paper. Therefore, this research estimate betas from rolling regressions of excess returns on market excess return (Frazzini and Pedersen, 2014), using daily data when possible. Moreover, this research calculates FP beta concerning the market portfolio specific to an asset class. When this is not available, a global market portfolio is used. Furthermore, this research also uses a one-year rolling standard deviation for estimating volatilities and a five-year horizon for correlation. This results in the following equation:

$$\beta_{i,FP}^{TS} = \rho_{i,m} \frac{\sigma_i}{\sigma_m}$$

Where σ_i and σ_m represent the volatilities of the asset, and the market and ρ represent their correlation. After that, this study follows Vasicek (1973) and Elton, Gruber, Brown and Goetzmann (2003) to reduce outliers, where they shrink the time series estimate of beta toward the cross-sectional mean, using shrinkage factors. Consequently, these shrinkage factors are used to make the beta estimates more realistic. This leads to the following equation for beta:

$$\beta_{i,FP} = w_i\, \beta_{i,FP}^{TS} + (1 - w_i)\beta^{XS}$$

Instead of having asset-specific and time-varying shrinkage factors as in Vasicek (1973), this study sets (as in Frazzini and Pedersen (2014)) w = 0.6 and $\beta^{XS} = 1$ for all periods and across all assets. This method of reducing outliers does not affect how securities are sorted since common shrinkage does not change the ranks of the security's beta. However, it does influence the construction of BAB portfolios. To account for this effect, this study focuses on realized abnormal returns. The final equation for the novel procedure beta is:

$$\beta_{i,FP} = 0.6\, \beta_{i,FP}^{TS} + 0.4$$

The FP beta of the previous month (t-1) is used to predict the FP beta of next month (t), using a rolling linear regression. Finally, the predicted FP beta will be compared to the actual beta. The procedure of the FP beta is also performed on the Novy-Marx beta, followed by an analysis of which beta estimation technique has the lowest *Mean Squared Error (MSE)* to conclude the better predictor.

3.1.2 NOVY-MARX AND VELIKOV (2018)

The second method used to calculate the beta is the standard method Novy-Marx and Velikov (2018) suggest in their research. This beta is estimated as the slope of the CAPM. In order to be able to compare the two methods which each other, this study uses the same method to make the betas more realistic. Therefore, using the shrinkage factors of Vasicek (1973) and Elton, Gruber, Brown and Goetzmann (2003). This leads to the following equation:

$$\beta_{i,NM} = 0.6\, \beta_{i,NM}^{TS} + 0.4$$

The NM beta of the previous month (t-1) predicts the NM beta of the following month (t), using a rolling linear regression, which is the same method used to predict the FP beta. The predicted NM beta will be compared to the actual beta using MSE.

3.1.3 ACTUAL BETA

This third beta estimation technique is used to mimic the realized covariance of the assets in the portfolio. This actual beta is calculated as the regression coefficient of the market excess returns of that month onto the Fama and French market returns (MKT), retrieved from Kenneth and French Library (2021). This study uses 252 days rolling window of stocks returns and market returns to estimate daily betas. Then the average of those beta estimations per day in one month is taken as the actual beta of that month.

3.2 CONSTRUCTION OF BAB FACTOR

The BAB portfolio can be constructed in several ways. The main idea behind this portfolio is a strategy that shorts a selection of high-beta assets and longs in a selection of low-beta assets. Frazzini and Pedersen (2014) rank the assets according to the estimated beta. Their strategy is to short all stocks above the estimated median beta and go long in the stocks below this estimated median beta. The weights assigned to each stock depend on the estimated beta. Consequently, beta-sorted stocks with a higher beta have a higher weight in the high-beta sub-portfolio and vice versa, meaning they are not equally weighted. Once the weights are assigned to each asset, the BAB factor can be established by estimating the beta of each portfolio. This results in the following equation:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r_f\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r_f\right)$$

Where β_t^L and β_t^H are the beta of the low-beta and high-beta portfolios, respectively, at *t=0*, r_{t+1}^L and r_{t+1}^H represent the log return of the low-beta and high-beta portfolio respectively, and r_f represents the risk-free rate. So, in each portfolio the securities are weighted by the ranked betas. The weights are given by:

$$w_H = k(z - \bar{z})^+$$
$$w_L = k(z - \bar{z})^-$$

Where k is a normalizing constant $k = 2/1_n|z - \bar{z}|$ and x^+ and x^- are the positive and negative elements for vector x. In this study $1_n w_H = 1$ and $1_n w_L = 1$, following Frazzini and Pedersen (2014).

The composition and weights of the portfolios are calculated per month, whereas the returns are calculated per day. The resulting return is the excess return of the BAB portfolio, denoted with r_{t+1}^{BAB}. The alpha of the BAB portfolio will be tested against several risk models explained in the next section. Furthermore, this study uses the same procedure to construct the BAB portfolio for each beta estimation technique.

3.3 RISK MODELS

This research compares the returns of both the beta estimations, to examine whether calculating beta differently results in differences in performance and magnitude. To test whether the BAB portfolios experience excess returns, it is expedient to run time-series regressions of excess returns on some explanatory variables. This research uses Jensen's alpha as a measure for performance. According to Bodie et al. (2018), alpha is the return attributable to factors that are not part of the factor model. After that, the research of Frazzini and Pedersen (2014) provide alphas estimated by regressing excess return of risk factors of CAPM, three-factor model, four-factor model and the five-factor model. As discussed in the literature review, empirical evidence is found on multiple sources of risk not captured by the portfolio's exposure to market risk (of beta). In order to discuss the results of the performance of the strategy taking into account multiple sources of risk. The main idea behind adding more risk factors is that it should diminish the absolute value of alpha, as more risk loadings are considered in estimating excess returns. In this study, the alpha of the BAB portfolio is estimated concerning CAPM, Fama and French (1993) three-factor model, Carhart (1997) four-factor model, and Fama and French (2015) five-factor model. The first-factor model, concerning the CAPM:

$$r_{t+1}^{BAB} = \alpha_{CAPM} + \beta_m\left(R_m - R_f\right) + \varepsilon_i$$

The CAPM alpha is the portfolio's excess return over the market index (or respective stock exchange). The second factor model, concerning Fama and French's three-factor model:

$$r_{t+1}^{BAB} = \alpha_{three-factor} + \beta_m\left(R_m - R_f\right) + \beta_{SMB}SMB + \beta_{HML}HML + \varepsilon_i$$

Where β_{SMB} and β_{HML} are the factor exposures of the BAB portfolio to the size risk factor (SMB) and value risk factor (HML). Data regarding risk factors are obtained from the website of Kenneth French. The construction of those factors is explained in the Appendix. The third factor model, concerning Carhart's four-factor model:

$$r_{t+1}^{BAB} = \alpha_{four-factor} + \beta_m\left(R_m - R_f\right) + \beta_{SMB}SMB + \beta_{HML}HML + \beta_{UMD}UMD + \varepsilon_i$$

Where β_{UMD} is the factor exposure of the BAB portfolio to the momentum risk factor (UMD). The last factor model, concerning Fama and French's five-factor model:

$$r_{t+1}^{BAB} = \alpha_{five-factor} + \beta_m\left(R_m - R_f\right) + \beta_{SMB}SMB + \beta_{HML}HML + \beta_{RMW}RMW + \beta_{CMA}CMA + \varepsilon_i$$

Where β_{RMW} and β_{CMA} are the factor exposures to the BAB portfolio to the profitability risk factor (RMW) and the investment risk factor (CMA).

3.4 ROBUSTNESS CHECKS

This section provides the robustness checks used in this study. They function to strengthen the robustness of the results, making them more reliable. Each robustness check applies to all the samples with different beta estimations; FP betas, NM betas and actual betas.

3.4.1 USING QUANTILE PORTFOLIOS

Frazzini and Pedersen (2014) use two portfolios in their strategy, the low-beta portfolio (containing stocks that have a beta below the median beta) and the high-beta portfolio (containing stocks that have a beta above the median beta). Other researchers that examine factor anomalies use decile (Fama and French, 1992 – 1993) or quintile portfolios (Ang et al., 2009) for their strategy, which is more common. This study follows the methodology of Frazzini and Pedersen (2014), therefore using only two portfolios to go long in (low-beta portfolio) and short (high-beta portfolio). This study performs a quintile portfolio analysis to verify its effects on the results and test the robustness of the BAB factor. Every beta above the 80% is assigned to the high-beta portfolio and every beta below the 20% is assigned to the low-beta portfolio.

3.4.2 CONTROLLING FOR MARKET VOLATILITY

One of the main criticisms of Frazzini and Pedersen (2014) research is that their estimated beta (FP beta) is correlated with market volatility, leading to biased estimates and results. Therefore, the results are controlled for market volatility to eliminate the problem of correlated market volatility and verify its effects on results. This also builds on the answer to the central question of this research. Also, NM beta and actual beta are controlled for market volatility. This study used the daily standard deviation of the Fama and French market returns (MKT) as market volatility. This daily standard deviation is taken into the factor model regressions as an extra factor to control for market volatility. This variable is taken into account as an explanatory variable of the BAB returns.

SECTION 4: DATA AND DESCRIPTIVE

This section presents the data used in this study and its descriptive statistics. The first section describes the data retrieved from the Center for Research in Security Prices (CRSP) and the adjustments made to the data. The next section discusses the dataset retrieved from Kenneth-French database. The last section provides the descriptive statistics of both datasets.

4.1 CRSP DATASET

The first dataset constructed is the CRSP dataset. The daily U.S. stock return data is retrieved from the CRSP database, containing only U.S. common stocks, with share code equal to 10 or 11, traded on NASDAQ, AMEX or NYSE covering 1963 till 2020. This study made some adjustments to the original dataset in order to work with reliable data. In the original dataset, negative prices were included in the case of bid-ask average is used instead of the closing price (CRSP, 2021). Hence, negative prices are adjusted with their absolute value. After that, this study removed observations from the sample that have a particular return code. To indicate that return data is missing or invalid, CRSP uses four different return codes, including -55, -66, -88, -99. Another adjustment made is that delisting returns are incorporated via a newly created variable: *return_adjusted*. This to encounter the threat of survivorship bias in the dataset.

4.2 KENNETH-FRENCH DATASET

The second dataset used in this study is retrieved from the Kenneth-French dataset (Kenneth French Data Library, 2021). This dataset is used in this study for the primary regression, containing the daily Fama and French (1992, 1993) and Carhart (1997) factors. These factors are obtained from the Kenneth and French data library (Kenneth French Data Library, 2021) with no further adjustment. The main regressions use the following factors: MKTRF, SMB, HML, MOM, RMW, and CMA. MKTRF represents the return on the market in excess of the risk-free rate. SMB represents the size factor and is the excess return spread of small minus big stocks. Moreover, HLM represents the growth/value factor and is the excess return spread on stocks of companies with a high market-to-book ratio minus companies with a low market-to-book ratio. MOM represents the momentum factor and is the excess return spread of stocks that experienced a positive return the past six months minus stocks that experienced a negative return. RMW represents the profitability factor and is the excess return spread of the most profitable firms minus the least profitable firms. Finally, CMA represents the investing factor and is the excess return spread of firms that invest conservatively minus aggressively.

The last step to complete the whole dataset is merging the CRSP dataset and Kenneth-French dataset by date. Furthermore, variables for adjusted excess return and excess return are created.

4.3 DESCRIPTIVE STATISTICS

Figure 1 provides the average daily number of stocks per exchange per day over the entire sample. NASDAQ starts before 1973, more specifically originated in 1971. NASDAQ exhibits a large increase in the volume of stocks around 1980 and 1996, with its highest point at 5089 stocks. After 1996, the number of stocks decreased. The figure shows that the AMEX is overall constant with a slight decrease. Furthermore, the NYSE is also overall constant, with one increase in the '00s. After the increase, it decreased back to the original level.

Figure 1: Average daily number of securities by exchange

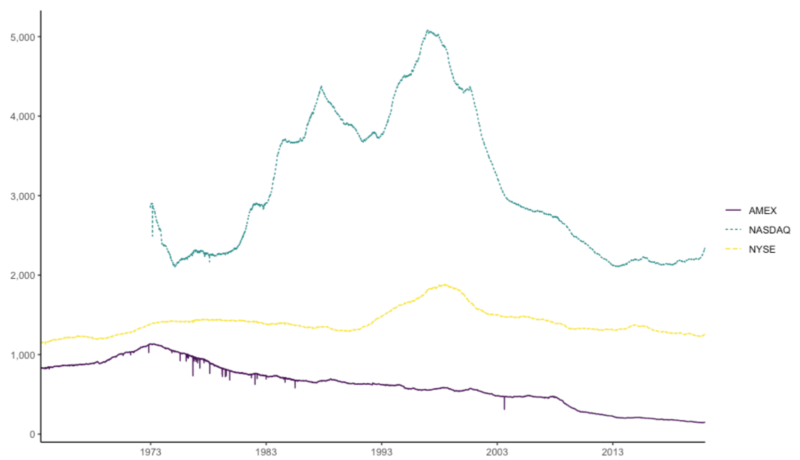

Figure 1 provides the average daily number of securities by exchange. The solid line represents the AMEX, the striped line represents the NYSE, and the dotted line represents the NASDAQ. The x-axis represents the year, from 1971 until 2020, whereas the y-axis represents the number of stocks.

Figure 2 provides the market value distribution in billions of dollars per exchange per day over the entire sample. NYSE has the highest total market value per day, while AMEX has the lowest total market value. The NASDAQ exhibited a higher decrease during the Dot.com bubble, might be caused by the index's more extensive amount of technology stocks. The figure shows that the NASDAQ and NYSE overall experience an increase of average daily total market value.

Figure 2: Average daily total market value by exchange of sample

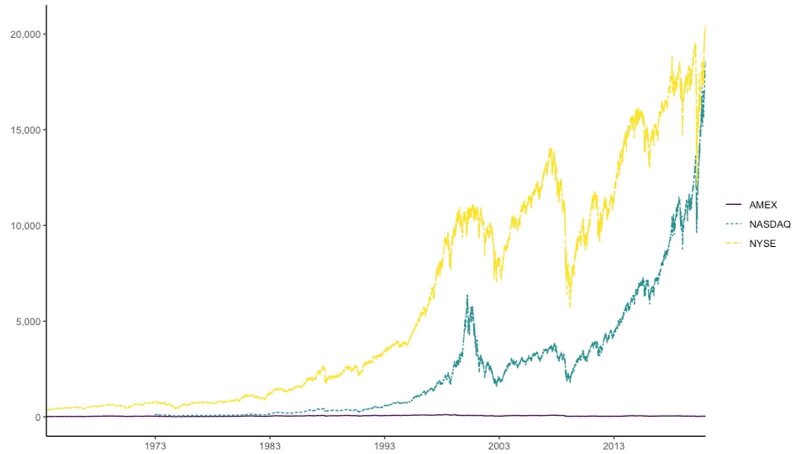

Figure 2 shows the average daily total market value by exchange. The solid line represents the AMEX, the striped line represents the NYSE, and the dotted line represents the NASDAQ. The x-axis represents the year, from 1971 until 2020, whereas the y-axis represents the market value.

Table 1 provides the average number of stocks per exchange per day and the average total market value in billions of dollars per exchange per day. The table shows that the NYSE is the largest exchange among them, with an average of roughly 2891 stocks and an average total market value of $5940 billion per day.

Table 1: Exchange distribution of sample

Exchange	*Average number of stocks*	*Market value*
NYSE	1403	5940.36
	[27%]	[69%]
NASDAQ	3119	2607.54
	[61%]	[30%]
AMEX	619	50.91
	[12%]	[1%]

Table 1: shows the exchange distribution, including the average number of stocks per exchange per day and the average total market value in billions of dollars per exchange per day. The percentages below the average number of stocks and market value are calculated by dividing the average number of stocks (market value) of the exchange by the sum of the average number of stocks of all exchanges.

Moreover, the NYSE is the exchange with the most extensive stocks on average. It has an average total market value in billions of dollars of 4.23, while it is 0.84 for NASDAQ and 0.08 for AMEX.

Table 2: Summary statistics main variables

		Predictions				*Portfolios*	
	Return	$\boldsymbol{\beta_{i,FP}}$	$\boldsymbol{\beta_{i,NM}}$	$\boldsymbol{\beta_{actual}}$	**BAB FP**	**BAB NM**	**BAB actual**
Mean	0.057	0.887	0.837	0.782	0.463	-0.186	-0.341
Median	-0.024	0.839	0.826	0.735	0.578	-0.339	-0.253
1st quintile	-1.277	0.630	0.698	0.345	-1.949	-2.746	-3.333
3rd quintile	1.138	1.087	0.966	1.155	2.950	2.269	2.515
Standard deviation	4.645	0.336	0.283	0.608	4.545	5.030	5.620

Table 2: Summary statistics of the main variables used in this study. The variables are the excess return of the dataset, the predicted values of the FP betas, the predicted values of the NM betas, the values for the actual betas, the excess return of the three portfolios based on the three different beta estimations: BAB FP, BAB NM, BAB actual. All values are denoted in percentages (except the standard deviation, which represents the volatility).

Table 2 shows the summary statistics of the main variables used in this study. The table shows that the FP beta has the highest average monthly excess return (0.887%), and the actual beta has the lowest (0.782%) but are overall close to each other. After that, the table shows that the actual beta is more volatile than the FP beta and NM beta, 0.608, 0.336 and 0.283, respectively. The average returns of the three BAB portfolios reveal a noticeable difference. Where the sample of FP BAB has an average monthly excess return of 0.463%, the average excess return for the NM BAB and actual BAB portfolios are much lower, even negative, -0.186% and -0.341%, respectively.

SECTION 5: EMPIRICAL RESULTS

This section discusses the results from the methodology described in Section 3. The first paragraph discusses the comparison of the different beta estimation techniques. They form the results for the first hypothesis. The second paragraph discusses the three different BAB portfolios. Here the BAB portfolios are constructed as described in the methodology. The alpha of the BAB portfolio is estimated concerning CAPM, three-factor model, four-factor model, and the five-factor model. They form the results for the second hypothesis. The last section discusses the results concerning the robustness checks and form results for the third hypothesis.

5.1 BETA REGRESSIONS

To provide an answer to the main question, several hypotheses are formulated, as discussed in Section 2. The first hypothesis is:

I) The novel procedure for estimating beta is a better predictor of beta than the standard method since the novel procedure has a lower MSE than the standard method.

Table 3 provides the results of the beta regressions. As the table shows, both coefficients are highly statistically significant, with a value of 1.323 for the FP beta and 1.121 for the NM beta. After that, the coefficient of the NM beta is closer to one than the FP beta. The standard deviation of the NM beta is more than twice as big as the standard deviation of the FP beta. The MSE of the beta regressions is 0.195 for the FP beta and 0.274 for the NM beta. However, FP beta has the lowest MSE. Table 4 shows the correlation matrix between the three betas. The FP beta has a higher correlation with the actual beta compared to the NM beta. Consequently, this study fails to reject the first hypothesis. There is evidence to believe that the novel procedure for estimating beta is a better predictor of beta than the standard method, based on the lowest MSE.

Table 3: Beta regressions (Newey west)

β_{actual}	$\beta_{i,FP}$	$\beta_{i,NM}$
Coefficient	1.323****	1.121****
	[180.25]	[74.01]
Standard deviation	0.007	0.015
MSE	0.195	0.274

*Table 3 provides the beta regressions based on Newey West standard errors of the three different beta estimations. **** denotes a significance of <0.001, *** denotes a significance of <0.01, ** denotes a significance of <0.05 and * denotes a significance of <0.1. The T-statistics are given in parentheses.*

Table 4: Correlation matrix

	β_{actual}	$\beta_{i,FP}$	$\beta_{i,NM}$
β_{actual}	1	0.731	0.521
$\beta_{i,FP}$	0.731	1	0.368
$\beta_{i,NM}$	0.521	0.368	1

Table 4 provides the correlation matrix between the three different beta estimations.

5.2 PORTFOLIO REGRESSIONS

As described in the methodology, each stock is assigned to a portfolio, low or high, based on their estimated betas. Returns reflect a one-month holding period. After that, this study regresses the returns on CAPM, three-factor, four-factor, and the five-factor model. The second hypothesis that is formulated is in order to answer the main question is:

II) The low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, using the novel procedure for estimating beta.

This section is divided into four parts. The first part discusses the overall common features of the results of all three portfolios. The second part discusses the portfolio based on FP betas. After that, the portfolio based on NM and the last part provides the results concerning the portfolio based on the actual betas.

5.2.1 COMMON FEATURES OF THE PORTFOLIO REGRESSIONS

Table 5 provides the total return performances of the sub-portfolios and the BAB portfolios based on the three beta estimation techniques applying monthly rebalancing. It provides the factor loadings and Jensen's Alphas along with its Newey-West t-statistics. In tendency, high ex-ante beta portfolios should yield higher returns, whereas the average excess return is not increasing monotonically. Contradictory the CAPM, table 5 shows that the average excess return is not proportionally higher with a higher beta for all portfolios. This supports the finding that the security market line is indeed flatter than theory suggests and even negative.

Thereafter, table 5 shows that the average beta is increasing, where the low portfolio has the lowest average beta, and the high portfolio has the highest average beta. This is in line with our strategy since the strategy contains sorting on beta. Consequently, the table shows that the portfolios with the lowest betas have the highest excess returns on average. Furthermore, all the low portfolios have the most extensive abnormal returns compared to the high portfolios when estimated using the different factor models.

5.2.2 PORTFOLIO REGRESSION BASED ON FRAZZINI AND PEDERSEN (2014) BETA

This section discusses the results concerning the portfolio constructed with FP beta. Therefore, the variables named in this section only relate to this portfolio. Table 5 shows that the standard deviation shows a slight downward trend, except P4, P9 and the high portfolio, but is nearly constant. The average excess returns are also in a downward trend, as discussed above. However, the average excess return decrease more compared to the standard deviation, with a declining trend in Sharpe ratios as a consequence. Thus, investors that seek high risk-adjusted returns benefit from choosing the low-beta portfolio. After that, the market capitalization until P8 is increasing. Afterwards, P9 and the high portfolio shows a slight decrease compared to P8.

When estimated relative to the factor models, the portfolio alphas decline almost monotonically from low-beta to high-beta portfolios. For all factor models, the alphas are, until P5, all significant. Then, for P6, only the CAPM alpha is significant. For P7, the CAPM, three-factor and five-factor alpha are significant. For P8, the three-factor, four-factor and five-factor alpha, and for P9, only the five-factor alpha. Lastly, for the high portfolio, none of the alphas is significant.

The right-most column in table 5 provides the returns of the BAB factor. This is the portfolio that shorts the high-beta stocks and longs the low-beta stocks. The BAB factor experiences a high average excess return and a high alpha. First, the BAB factor has Fama and French (1993) abnormal returns of 0.837% per month (t-statistic is 5.69). Second, the BAB factor yields abnormal returns of 0.626% per month (t-statistic is 4.03) when adjusting returns by adding the Carhart (1997) momentum factor. Lastly, by adding an investment factor and profitability factor to the three-factor model, the BAB factor earns abnormal returns of 0.662% per month (t-statistic is 4.66). Consequently, the BAB factor earns significant abnormal returns when estimated with the CAPM, three-factor, four-factor, and five-factor models. In addition, adding those comprehensive sources of risk factors do lower the alpha, indicating that these factors used to adjust for risk explains at least part of the excess returns found. These results are in line with the results from Frazzini and Pedersen (2014).

5.2.3 PORTFOLIO REGRESSION BASED ON NOVY-MARX (2018) BETA

This section discusses the results concerning the portfolio constructed with NM beta. Therefore, the variables named in this section only relate to this portfolio. Compared to the FM beta portfolio, the standard deviation of the NM beta portfolio is not decreasing throughout the portfolios. However, while decreasing until P5, it shows an increase in P6, P9 and the high portfolio. Consequently, the Sharpe ratios do not decrease monotonically throughout the portfolios but fluctuate between 0.050 and 0.067. Thereafter, the market capitalization increases until P7, whereas it decreases from P8 until the high portfolio.

The portfolio alphas do not monotonically decrease they decrease until P5 and afterwards show an increase (except the CAPM alphas). Here the alpha of the low portfolio has the highest alpha estimated with CAPM, three-factor, four-factor, and the five-factor model. The CAPM alphas are all significant, except P9. The three-factor model and five-factor model have seven portfolios with significant alphas, whereas the four-factor model has eight portfolios with a significant alpha. The two lowest portfolios and the highest portfolio have all significant alphas estimated with the different factor models.

The column on the right in table 5 provides the returns of the BAB factor. The BAB factor experiences a negative average excess return and lower alphas than the BAB portfolio constructed with FP betas. First, the BAB factor has Fama and French (1993) abnormal returns of 0.271% per month (t-statistic is 1.70). Second, the BAB factor yields abnormal returns of 0.209% per month (t-statistic is 1.28) when adjusting returns by adding the Carhart (1997) momentum factor. Lastly, by adding an investment factor and profitability factor to the three-factor model, the BAB factor earns abnormal returns of 0.157% per month (t-statistic is 0.99). Consequently, the BAB factor only earns significant abnormal returns when estimated with the three-factor model. What is noteworthy is the negative excess return of the BAB portfolio. One possible reason for this is that the risk-free rate is greater than the portfolio's return or that the portfolio's return is even negative. This also explains the negative Sharpe ratio of the BAB portfolio.

5.2.4 PORTFOLIO REGRESSION BASED ON ACTUAL BETA

This section discusses the results concerning the portfolio constructed with actual beta. Therefore, the variables named in this section only relate to this portfolio. The standard deviation of the low and high portfolios are outliers. For the remaining portfolios, the standard deviation stays on a constant level, around 19. Thus, the Sharpe ratio first shows an overall decrease. Thereafter, the market capitalization increases until P7, whereas it decreases from P8 until the high portfolio.

The portfolio alphas do not monotonically decrease but show a decrease overall. After that, for every risk model used, the first five portfolios have significant alphas. In addition, P6 has only a significant CAPM alpha. Furthermore, the alphas of P7 till the high portfolio are all insignificant.

The column on the right in table 5 provides the returns of the BAB factor. The BAB factor experiences a positive average excess return and even higher alphas than the BAB portfolio constructed with FP and NM betas. First, the BAB factor has Fama and French (1993) abnormal returns of 0.968 per month (t-statistic is 7.20). Second, the BAB factor yields abnormal returns of 0.844% per month (t-statistic is 5.49) when adjusting returns by adding the Carhart (1997) momentum factor. Lastly, by adding an investment factor and profitability factor to the three-factor model, the BAB factor earns abnormal returns of 0.800 per month (t-statistic is 5.50). Consequently, all of these abnormal returns estimated with factor models are significant. This BAB portfolio also has the highest Sharpe ratio (0.116) compared to the two other BAB portfolios (FP; 0.102, NM; -0.037).

Concluding the results of the portfolio regressions indicate that the alphas estimated with CAPM, three-factor, four-factor and five-factor models are statistically significant when the portfolio is based on Frazzini and Pedersen (2014) betas and when based on actual betas. Whereas the BAB portfolio based on Novy-Marx (2018) betas only have a statistically significant alpha when estimated with the three-factor model of Fama and French (1993). Consequently, this study fails to reject the second hypothesis. There is evidence to believe that the low-beta anomaly is economically and statistically significant using the novel procedure for estimating betas. Additionally, there is evidence to believe that the low-beta anomaly is statistically significant when using actual betas. The empirical evidence shows that portfolios of high-beta assets have lower alphas and Sharpe ratios than portfolios of low-beta assets.

Table 5: Performances of sub-portfolios, monthly rebalanced

Panel A: Strategy approach based on Frazzini and Pedersen (2014) estimated betas

Variable	**Low**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**High**	**BAB**
Excess return	2.119	1.586	1.465	1.516	1.222	0.987	1.031	0.928	0.928	0.557	0.463
Std. dev	23.461	20.268	20.244	27.432	19.934	20.740	20.388	20.657	22.108	28.887	4.545
Sharpe ratio	0.090	0.078	0.072	0.055	0.061	0.048	0.051	0.045	0.042	0.019	0.102
Av. Beta	0.431	0.535	0.630	0.713	0.796	0.844	0.979	1.088	1.233	1.568	
Market cap	0.34%	1.83%	3.60%	7.71%	10.50%	14.26%	19.13%	19.31%	14.54%	8.79%	
CAPM alpha	1.355****	0.935****	0.804****	0.746****	0.509***	0.353*	0.375**	0.258	0.092	-0.323	0.865****
	[5.65]	[4.91]	[4.25]	[3.79]	[2.59]	[1.85]	[2.04]	[1.52]	[0.46]	[-1.18]	[5.51]
3-factor alpha	1.265****	0.835****	0.706****	0.646****	0.433**	0.295	0.322*	0.232*	0.083	-0.364	0.837****
	[5.77]	[4.62]	[3.81]	[3.65]	[2.11]	[1.48]	[1.81]	[1.68]	[0.46]	[-0.54]	[5.69]
4-factor alpha	1.266****	0.877****	0.767****	0.710***	0.493*	0.398	0.475	0.451**	0.359	0.051	0.626****
	[5.16]	[4.49]	[3.48]	[3.21]	[1.91]	[1.38]	[1.63]	[2.27]	[1.55]	[0.18]	[4.03]
5-factor alpha	1.286****	0.815****	0.707****	0.659***	0.429*	0.346	0.459**	0.439***	0.326*	0.020	0.662****
	[5.92]	[4.68]	[4.01]	[3.42]	[1.76]	[1.56]	[2.58]	[2.75]	[1.66]	[0.08]	[4.66]

Panel B: Strategy approach based on Novy-Marx (2018) estimated betas

Variable	**Low**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**High**	**BAB**
Excess return	2.540	1.024	1.023	0.989	0.939	0.911	1.049	1.106	1.147	1.610	-0.186
Std. dev	38.549	21.794	18.231	17.704	16.133	18.188	16.662	16.546	19.440	31.450	5.030
Sharpe ratio	0.066	0.047	0.056	0.056	0.058	0.050	0.063	0.067	0.059	0.051	-0.037
Av. Beta	0.386	0.637	0.698	0.751	0.801	0.851	0.904	0.967	1.056	1.323	
Market Cap	0.10%	2.43%	4.61%	8.69%	13.16%	17.15%	18.55%	16.86%	11.93%	5.61%	
CAPM alpha	1.190***	0.497***	0.411**	0.340**	0.345**	0.343**	0.371**	0.333**	0.272	0.484*	0.239
	[3.30]	[2.81]	[2.32]	[2.03]	[2.11]	[2.01]	[2.23]	[2.13]	[1.51]	[1.69]	[1.51]
3-factor alpha	1.086***	0.396***	0.320**	0.297	0.240	0.253*	0.302**	0.270**	0.226	0.427*	0.271*
	[3.00]	[2.89]	[2.01]	[1.34]	[1.52]	[1.87]	[2.23]	[2.52]	[1.62]	[1.77]	[1.70]
4-factor alpha	1.318****	0.463***	0.388**	0.373	0.302	0.313**	0.375**	0.385***	0.392**	0.729**	0.209
	[3.62]	[3.28]	[2.21]	[1.26]	[1.36]	[2.04]	[2.21]	[3.18]	[2.20]	[2.29]	[1.28]
5-factor alpha	1.208***	0.371***	0.250	0.222	0.176	0.217*	0.314**	0.315***	0.374***	0.685**	0.157
	[3.06]	[2.67]	[1.64]	[0.95]	[1.13]	[1.73]	[2.44]	[2.87]	[2.59]	[2.45]	[0.99]

Table 5 cont.

Panel C: Strategy approach based on actual betas

Variable	**Low**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**High**	**BAB**
Excess return	2.627	1.361	1.317	1.307	1.173	0.965	0.831	0.826	0.799	1.133	0.558
Std. dev	36.392	19.197	17.917	20.259	18.723	19.271	17.971	18.872	20.301	29.638	4.809
Sharpe ratio	0.072	0.071	0.074	0.065	0.063	0.050	0.046	0.044	0.039	0.038	0.116
Av. Beta	-0.148	0.178	0.345	0.502	0.658	0.813	0.975	1.157	1.397	1.941	
Market cap	0.85%	1.68%	3.71%	7.54%	10.88%	16.01%	18.71%	18.59%	14.01%	8.01%	
CAPM alpha	1.448****	0.805****	0.638***	0.624***	0.528***	0.350*	0.245	0.120	0.002	0.011	1.017****
	[4.54]	[4.05]	[3.06]	[3.16]	[2.71]	[1.83]	[1.21]	[1.04]	[0.01]	[0.04]	[6.64]
3-factor alpha	1.343****	0.689****	0.522**	0.532***	0.446**	0.287	0.182	0.181	-0.001	0.014	0.968****
	[3.73]	[3.56]	[2.56]	[2.76]	[2.24]	[1.41]	[0.62]	[0.86]	[-0.05]	[0.06]	[7.20]
4-factor alpha	1.440****	0.764****	0.592**	0.612***	0.508**	0.327	0.251	0.266	0.161	0.326	0.844****
	[3.77]	[3.43]	[2.41]	[2.69]	[2.05]	[1.30]	[0.54]	[0.80]	[0.75]	[1.15]	[5.49]
5-factor alpha	1.402****	0.670****	0.495**	0.434**	0.398*	0.282	0.230	0.325	0.192	0.366	0.800****
	[3.76]	[3.60]	[2.39]	[2.43]	[1.92]	[1.19]	[0.24]	[1.12]	[1.34]	[1.41]	[5.50]

*Table 5 shows the beta-sorted portfolio returns. At the beginning of the month, stocks are ranked in ascending order based on their estimated beta. The ranked stocks are assigned to one of ten decile portfolios. The portfolios are monthly rebalanced. The right-most column represents the returns of the betting against beta (BAB) factor. To construct the BAB factor, all stocks are assigned to the low or high beta portfolio. Stocks are weighted by their ranked betas, whereas the lower beta security has a larger weight in the low-beta portfolio and vice versa. The BAB factor goes long in the low-beta portfolio and shorts the high beta portfolio. This table includes all common stocks on the CRSP database (NYSE, AMEX and NASDAQ) covering 1963 till 2020. Alpha is the intercept in a regression of monthly excess returns. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Fama and French (1995) investment and profitability factor. Returns and alphas are in monthly percent. Returns are adjusted for delisting, winsorized at a 1% level and in excess of the risk-free rate. To indicate whether the estimates are significant, the corresponding Newey-west t-statistic is given in parentheses where *=10%, **5%, ***1%, ****<0.1%.*

5.3 ROBUSTNESS CHECKS

This study examines whether results are robust for changes in measurement. This section discusses the excess return and the Jensen's alpha of the long-short BAB portfolios, which is the main interest of this study. The sub-portfolios concerning the quintile analysis and the sub-portfolios for the analysis of controlling for market volatility can be found in the Appendix. Table 6 shows the results concerning the performed robustness checks.

Table 6: Robustness results

Panel A: Strategy approach based on Frazzini and Pedersen (2014) estimated betas		
	Quintile portfolio	**Controlling for market volatility**
Excess return BAB	0.843	0.463
CAPM alpha	1.436****	2.048****
	[5.78]	[8.36]
3-factor alpha	1.406****	1.999****
	[6.25]	[9.74]
4-factor alpha	1.082****	1.487****
	[4.59]	[6.34]
5-factor alpha	1.145****	1.893****
	[5.12]	[8.95]
Panel B: Strategy approach based on Novy-Marx (2018) estimated betas		
	Quintile portfolio	**Controlling for market volatility**
Excess return BAB	-0.169	-0.186
CAPM alpha	0.320*	1.253****
	[1.69]	[5.07]
3-factor alpha	0.353*	1.407****
	[1.86]	[5.41]
4-factor alpha	0.288	1.301****
	[1.47]	[4.87]
5-factor alpha	0.215	1.353****
	[1.13]	[5.29]
Panel C: Strategy approach based on actual betas		
	Quintile portfolio	**Controlling for market volatility**
Excess return BAB	1.020	0.558
CAPM alpha	1.186****	1.822****
	[6.82]	[5.79]
3-factor alpha	1.608****	1.685****
	[7.33]	[6.12]
4-factor alpha	1.407****	1.385****
	[6.38]	[4.43]
5-factor alpha	1.360****	1.612****
	[5.67]	[5.91]

*Table 6 shows the results of the robustness checks performed in this study. It reports the 1-month holding period return in the BAB portfolio. The second column represents the results of the quintile portfolio, where the right-most column represents the results when controlled for market volatility. Alpha is the intercept in a regression of monthly excess returns. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Fama and French (1995) investment and profitability factor. Returns and alphas are in monthly percent. Returns are adjusted for delisting, winsorized at a 1% level and in excess of the risk-free rate. To indicate whether the estimates are significant, the corresponding Newey-west t-statistic is given in parentheses where *=10%, **5%, ***1%, ****<0.1%.*

5.3.1 QUINTILE PORTFOLIOS

The second column of table 6 provides a quintile portfolio analysis instead of the two portfolio analysis Frazzini and Pedersen (2014) perform.

For Panel A the results seem to be robust over the sample. The average excess return changes to 0.843%, which is slightly higher than before (0.463%). The alphas stay highly statistically significant, and the magnitude of the alphas increased for all factor models.

For Panel B the results maintain reasonably the same for the sample. The average excess return changes to -0.169%, which is slightly higher than before (-0.186%). Using quintile portfolios results in alphas of two factor models being statistically significant: CAPM and three-factor model. The alphas of the four-factor and five-factor models remain statistically insignificant. Likewise, as in Panel A, the alphas are slightly higher when using quintile portfolios.

For Panel C the results seem to be robust over the sample. The average excess return changes from 0.558% to 1.020%, which indicates an increase. The alphas stay highly statistically significant, and the magnitude of the alphas increased for all factor models.

Concluding the results of the quintile portfolios comes down to the fact that two things change. First, when looking at Panel B, the alpha of the CAPM becomes statistically significant, and all other alphas that were already significant using the two portfolio strategy (based on median) remain statistically significant when using quintile portfolios. The second thing that changes is the magnitude of all alphas. When using quintile portfolios, all alphas of all factor models in all Panels increase.

5.3.2 CONTROLLING FOR MARKET VOLATILITY

One of the main criticisms of Frazzini and Pedersen (2014) research is that their estimated beta (FP beta) is correlated with market volatility, leading to biased estimates and results. Therefore, the results are controlled for market volatility to eliminate the problem of correlated market volatility and verify its effects on results. Controlling for market volatility shows how robust the BAB factor is, and helps to answer the central question of this research. As described in the methodology, each stock is assigned to a portfolio, low-beta or high-beta, based on their estimated betas. The same strategy is performed as discussed in the methodology; only in this section the results are controlled for market volatility. The third hypothesis that is formulated is in order to answer the main question is:

III) The low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, when controlling for market volatility.

The right-most column of table 6 provides the performance of the BAB factor when controlling for market volatility.

For Panel A the results seem to be robust over the sample. Although the magnitude has changed, all alphas are still statistically significant at the highest level. Consequently, when controlling for market volatility, the alphas increase.

For Panel B the results do not seem to be robust over the sample. When not controlling for market volatility, only the three-factor model alpha is significant. This changes when the results are controlled for market volatility; namely, all factor model alphas become statistically significant. The alphas also increased when controlling for market volatility.

For Panel C the results seem to be robust over the sample. Although the magnitude has changed, all alphas are still statistically significant at the highest level. Consequently, when controlling for market volatility, the alphas increase.

Concluding the results when controlling for market volatility indicates that the results remain statistically significant for Panel A and Panel C, whereas in Panel B, the alphas become statistically significant. After that, since all alphas estimated with the concerning factor models are significant, this study fails to reject the third hypothesis. Consequently, there is evidence to believe that the low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, when controlling for market volatility.

SECTION 6: DISCUSSION AND CONCLUSION

In tendency, high ex-ante beta portfolios should yield higher returns. Contradictory to CAPM, empirical findings show that the security market line is flatter than theory suggests and even negative. This study provides empirical evidence of the relationship between low-beta stocks and expected stock returns based on different estimation techniques for beta. The research of Frazzini and Pedersen (2014) is the basis of this study, where the findings in this study are in line with the findings of Frazzini and Pedersen (2014). One of the main criticisms of this paper is the non-standard method for estimating the beta (Novy-Marx and Velikov, 2018). This research examined whether the novel method to calculate beta is a better predictor than the standard method, whether it matters in performance when calculating beta differently, and what happened when controlling for market volatility when using the novel procedure. This study tries to answer whether Novy-Marx and Velikov (2018) criticism is justified. This resulted in the following research question:

"Which beta estimation technique is the best beta predictor concerning the betting against beta factor?"

First, this research provides evidence that the novel procedure is a better predictor of the actual beta, since it has a lower mean squared error than the standard method to estimate beta (using only CAPM regressions). Where the actual daily beta is based on regression coefficient of the market excess returns of that month onto the market returns, using a 252 days rolling window of stock and market returns. The average of those beta estimations per day in one month is taken as the actual beta of that month. It is relevant to look at this actual beta, since researchers care about the realized covariance of their assets and not the statistics of the past.

After that, this study constructed a betting against beta (BAB) factor to find evidence for the low-beta anomaly. When using the novel procedure betas and actual betas, the BAB factor becomes statistically significant, indicated by a statistically significant CAPM alpha, three-factor alpha, four-factor alpha, and five-factor alpha. Contrary, no evidence is found for a statistically significant alpha when using the standard beta estimation technique. Only the three-factor model alpha was slightly statistically significant. It seems that investors indeed seem to care more about the actual betas since the BAB factor has higher and statistical significant alphas. Thus, since FP betas are more correlated with the actual beta, it explains why the BAB factor with FP betas work better than the BAB factor with NM betas.

This study performed two robustness checks. Frazzini and Pedersen (2014), which methodology this study follows, use two portfolios in their strategy, the low-beta portfolio (containing stocks that have a beta below the median beta) and the high-beta portfolio (containing stocks that have a beta above the median beta). This study also performed a quintile portfolio analysis to verify its effects on the results and test the robustness of the BAB factor. Concluding the results of the quintile portfolios indicates two things change. First, the alpha of the CAPM using the standard beta estimation method becomes statistically significant, and all other alphas that were already significant using the first BAB portfolio strategy (based on median) remain statistically significant when using quintile portfolios. Secondly, when using quintile portfolios, all alphas of all factor models in all panels increase. The second robustness check performed is that the results are controlled for market volatility, since one of the main criticisms of the research of Frazzini and Pedersen (2014) is that their estimated beta (FP beta) is correlated with market volatility leading to biased estimates and results. All portfolios with different beta estimation techniques are controlled for market volatility. Concluding the results when controlling for market volatility indicates that the results remain statistically significant for Panel A and Panel C, whereas in Panel B, the alphas become statistically significant. Consequently, there is evidence to believe that the low-beta anomaly is economically and statistically significant for the U.S. stock market between 1968 till 2020, when controlling for market volatility.

Although the BAB factor based on FP betas is the better predictor of actual betas, it needs to be analyzed with caution. This is because of the differences in sub-portfolios based on FP betas and actual betas in magnitude. After that, the BAB factor is also constructed under certain assumptions; the assumption that investors can borrow at a risk-free rate or no transaction costs are unrealistic. Concerning future academic research, a reliable BAB factor for markets could be constructed, with a beta estimation technique that can predict beta even better than the Frazzini and Pedersen (2014) beta. Thus, it could be used as an enhancement to existing widely acknowledged factor models. This research only compares the Novy-Marx (2018) beta with the Frazzini and Pedersen (2014) beta, but does not rule out any other beta estimation technique that can be a better predictor. Hence, these results can be helpful for investors in making their investment choice and trading strategy since the BAB factor can be used as a control variable in future research. The return of the BAB factor rivals those of the standard asset pricing factors in terms of economic magnitude and statistical significance.

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SECTION 8: APPENDIX

8.1 TABLE 7: PERFORMANCES OF SUB-PORTFOLIOS, QUINTILE PORTFOLIOS

Table 7: Performances of sub-portfolios using quintile portfolios, monthly rebalanced

Panel A: Strategy approach based on Frazzini and Pedersen (2014) estimated betas

	Low	**2**	**3**	**4**	**High**	**BAB**
Excess return	1.852	1.490	1.104	0.980	0.742	0.843
Std. dev	21.924	24.108	20.341	20.523	25.723	6.817
Sharpe ratio						
Av beta	0.488	0.672	0.840	1.034	1.400	
Market cap	2.16%	11.31%	24.76%	38.44%	23.33%	
CAPM alpha	1.144****	0.774****	0.432**	0.321*	-0.108	1.436****
	[5.60]	[4.14]	[2.25]	[1.89]	[-0.48]	[5.78]
3-factor alpha	1.047****	0.674****	0.362*	0.279*	-0.130	1.406****
	[5.14]	[3.63]	[1.75]	[1.88]	[-0.69]	[6.25]
4-factor alpha	1.078****	0.737***	0.440	0.462**	0.213	1.082****
	[4.94]	[3.25]	[1.57]	[1.97]	[0.83]	[4.59]
5-factor alpha	1.043****	0.682****	0.381	0.449***	0.178	1.145****
	[5.46]	[3.80]	[1.54]	[2.92]	[0.83]	[5.12]

Panel B: Strategy approach based on Novy-Marx (2018) estimated betas

	Low	**2**	**3**	**4**	**High**	**BAB**
Excess return	1.782	1.006	0.925	1.078	1.379	-0.169
Std. dev	31.322	17.969	17.191	16.604	26.145	5.879
Sharpe ratio						
Av beta	0.511	0.724	0.826	0.935	1.190	
Market cap	3.43%	13.31%	30.91%	35.41%	17.54%	
CAPM alpha	0.885****	0.407**	0.346**	0.352**	0.399*	0.320*
	[3.65]	[2.16]	[2.16]	[2.19]	[1.73]	[1.69]
3-factor alpha	0.782****	0.309*	0.248*	0.283**	0.354**	0.353*
	[3.55]	[1.66]	[1.78]	[2.57]	[2.00]	[1.86]
4-factor alpha	0.904****	0.380*	0.309*	0.376***	0.572**	0.288
	[4.14]	[1.66]	[1.87]	[2.82]	[2.35]	[1.47]
5-factor alpha	0.820****	0.237	0.196	0.311***	0.574***	0.215
	[3.54]	[1.30]	[1.48]	[2.96]	[2.83]	[1.13]

Panel C: Strategy approach based on actual betas

	Low	**2**	**3**	**4**	**High**	**BAB**
Excess return	1.994	1.312	1.069	0.828	0.966	1.020
Std. dev	29.101	19.124	18.999	18.427	25.403	7.191
Sharpe ratio						
Av beta	0.015	0.436	0.735	1.067	1.669	
Market cap	2.53%	11.25%	26.89%	37.31%	22.02%	
CAPM alpha	0.196****	0.633***	0.440**	0.223	-0.025	1.186****
	[4.95]	[3.20]	[2.31]	[1.18]	[-0.11]	[6.82]
3-factor alpha	1.086****	0.528***	0.364*	0.180	-0.019	1.608****
	[4.26]	[2.67]	[1.74]	[0.67]	[0.10]	[7.33]
4-factor alpha	1.171****	0.603**	0.414	0.257	0.221	1.407****
	[3.98]	[2.54]	[1.59]	[0.48]	[0.84]	[6.38]
5-factor alpha	1.106****	0.488**	0.333	0.270	0.284	1.360****
	[4.27]	[2.44]	[1.45]	[0.48]	[1.40]	[5.67]

*Table 7: Table 5 shows the beta-sorted portfolio returns based on quintile portfolios. At the beginning of the month, stocks are ranked in ascending order based on their estimated beta. The ranked stocks are assigned to one of five quintile portfolios. The portfolios are monthly rebalanced. The right-most column represents the returns of the betting against beta (BAB) factor. To construct the BAB factor, all stocks are assigned to the low or high beta portfolio. Stocks are weighted by their ranked betas, whereas the lower beta security has larger weight in the low-beta portfolio and vice versa. The BAB factor goes long in the low-beta portfolio and shorts the high beta portfolio. This table includes all common stocks on the CRSP database (NYSE, AMEX and NASDAQ) covering 1963 till 2020. Alpha is the intercept in a regression of monthly excess returns. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Fama and French (1995) investment and profitability factor. Returns and alphas are in monthly percent. Returns are adjusted for delisting, winsorized at a 1% level and in excess of the risk-free rate. To indicate whether the estimates are significant, the corresponding Newey-west t-statistic is given in parentheses where *=10%, **5%, ***1%, ****<0.1%.*

8.2 TABLE 8: PERFORMANCES OF SUB-PORTFOLIOS WHEN CONTROLLED FOR MARKET VOLATILITY

Table 8: Performances of sub-portfolios when controlled for market volatility, monthly rebalanced

Panel A: Strategy approach based on Frazzini and Pedersen (2014) estimated betas

Variable	**Low**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**High**	**BAB**
CAPM alpha	1.848****	0.963**	0.671	0.415	0.323	0.068	-0.352	-0.603	-1.091***	-2.418****	2.048****
	[3.65]	[2.45]	[1.63]	[0.92]	[1.00]	[0.18]	[-1.02]	[-1.60]	[-2.83]	[-4.07]	[8.36]
3-factor alpha	1.543****	0.603	0.311	0.041	0.039	-0.158	-0.578*	-0.739**	-1.180****	-2.658****	1.999****
	[3.41]	[1.26]	[0.65]	[0.08]	[0.11]	[-0.40]	[-1.85]	[-2.23]	[-3.62]	[-4.21]	[9.74]
4-factor alpha	1.561***	0.706	0.456	0.183	0.180	0.100	-0.212	-0.196	-0.497	-1.653**	1.487****
	[3.13]	[1.38]	[0.99]	[0.31]	[0.37]	[0.19]	[-0.51]	[-0.59]	[-1.18]	[-2.19]	[6.34]
5-factor alpha	1.545****	0.585	0.301	0.036	0.037	-0.130	-0.492	-0.626*	-1.042***	-2.424****	1.893****
	[3.36]	[1.24]	[0.62]	[0.07]	[0.09]	[-0.27]	[-1.24]	[-1.93]	[-3.00]	[-3.50]	[8.95]

Panel B: Strategy approach based on Novy-Marx (2018) estimated betas

Variable	**Low**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**High**	**BAB**
CAPM alpha	-0.284	0.495	0.317	0.131	0.070	0.061	-0.022	-0.253	-0.730**	-1.364**	1.253****
	[-0.28]	[1.41]	[0.84]	[0.40]	[0.26]	[0.21]	[-0.07]	[-0.87]	[-2.24]	[-1.98]	[5.07]
3-factor alpha	-0.728	0.130	-0.015	-0.250	-0.318	-0.277	-0.292	-0.508**	-0.940***	-1.651**	1.407****
	[-0.76]	[0.41]	[-0.03]	[-0.52]	[-0.94]	[-0.87]	[-0.87]	[-2.35]	[-3.07]	[-2.26]	[5.41]
4-factor alpha	-0.184	0.298	0.152	-0.072	-0.180	-0.141	-0.124	-0.239	-0.553*	-0.942	1.301****
	[-0.22]	[0.81]	[0.29]	[-0.11]	[-0.39]	[-0.40]	[-0.30]	[-1.11]	[-1.70]	[-1.06]	[4.87]
5-factor alpha	-0.648	0.113	-0.057	-0.292	-0.364	-0.306	-0.956	-0.494**	-0.856***	-1.442*	1.353****
	[-0.69]	[0.37]	[-0.13]	[-0.62]	[-1.09]	[-0.99]	[-0.83]	[-2.19]	[-2.64]	[-1.87]	[5.29]

Table 8 cont.

Panel C: Strategy approach based on actual betas

Variable	Low	2	3	4	5	6	7	8	9	High	BAB
CAPM alpha	0.158	0.861***	0.757**	0.281	0.195	0.027	-0.062	-0.378	-0.748*	-1.447**	1.822****
	[0.16]	[2.66]	[1.97]	[0.74]	[0.62]	[0.09]	[-0.19]	[-1.03]	[-1.77]	[-2.57]	[5.79]
3-factor alpha	-0.268	0.445	0.342	-0.066	-0.118	-0.215	-0.306	-0.475	-0.828**	-1.506***	1.685****
	[-0.26]	[1.29]	[0.84]	[-0.17]	[-0.32]	[-0.75]	[-0.70]	[-1.31]	[-2.56]	[-2.77]	[6.12]
4-factor alpha	-0.092	0.635*	0.523	0.121	0.021	-0.135	-0.144	-0.279	-0.410	-0.740	1.385****
	[-0.09]	[1.77]	[1.24]	[0.27]	[0.05]	[-0.37]	[-0.20]	[-0.53]	[-1.03]	[-1.26]	[4.43]
5-factor alpha	-0.229	0.432	0.319	-0.087	-0.124	-0.211	-0.287	-0.389	-0.724**	-1.311**	1.612****
	[-0.23]	[1.18]	[0.76]	[-0.23]	[-0.32]	[-0.69]	[-0.24]	[-0.84]	[-2.53]	[-2.46]	[5.91]

*Table 8 shows the beta-sorted portfolio returns controlled for market volatility. At the beginning of the month, stocks are ranked in ascending order based on their estimated beta. The ranked stocks are assigned to one of ten decile portfolios. The portfolios are monthly rebalanced. The right-most column represents the returns of the betting against beta (BAB) factor. To construct the BAB factor, all stocks are assigned to the low or high beta portfolio. Stocks are weighted by their ranked betas, whereas the lower beta security has larger weight in the low-beta portfolio and vice versa. The BAB factor goes long in the low-beta portfolio and shorts the high beta portfolio. This table includes all common stocks on the CRSP database (NYSE, AMEX and NASDAQ) covering 1963 till 2020. Alpha is the intercept in a regression of monthly excess returns. The explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios, Carhart (1997) momentum factor and Fama and French (1995) investment and profitability factor. Returns and alphas are in monthly percent. Returns are adjusted for delisting, winsorized at a 1% level and in excess of the risk-free rate. To indicate whether the estimates are significant, the corresponding Newey-west t-statistic is given in parentheses where *=10%, **5%, ***1%, ****<0.1%.*

8.3 DESCRIPTION OF FAMA/FRENCH FACTORS

Data returns: July 1, 1926 – April 20, 2021

Construction: The Fama/French factors are constructed using six value-weight portfolios formed on size and book-to-market.

SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios:

SMB = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth).

HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios:

HML = 1/2 (Small Value + Big Value) - 1/2 (Small Growth + Big Growth).

RMW (Robust Minus Weak) is the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios:

RMW = 1/2 (Small Robust + Big Robust) – 1/2 (Small Weak + Big Weak).

CMA (Conservative Minus Aggressive) is the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios.

CMA = 1/2 (Small Conservative + Big Conservative) – 1/2 (Small Aggressive + Big Aggressive).

MOM (in this research denoted with UMD) is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios.

MOM = 1/2 (Small High + Big High) – 1/2 (Small Low + Big Low)

Rm-Rf, the excess return on the market, value-weight return of all CRSP firms incorporated in the U.S. and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares and price data at the beginning of t, and good return data for t minus the one-month Treasury bill rate (from Ibbotson Associates). See Fama and French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," Journal of Financial Economics, for a complete description of the factor returns.

Stocks: Rm-Rf includes all NYSE, AMEX, and NASDAQ firms. SMB and HML for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for December of t-1 and June of t, and (positive) book equity data for t-1 (Kenneth & French, 2021).